

A short review of axion and axino parameters

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What can be there beyond SM?
New CP? Axions? SUSY? String?

1. Strong CP and axions

2. Axino mass

3. Pseudoscalar boson at 125 GeV ?



Dark matter \longrightarrow observed in the Universe
Axion \longrightarrow natural sol. of strong CP
Supersymmetry \longrightarrow natural sol. of the
Higgs mass problem

All three are related to this talk.



1. Strong CP and axions

Axion is a Goldstone boson arising when the PQ global symmetry is spontaneously broken.

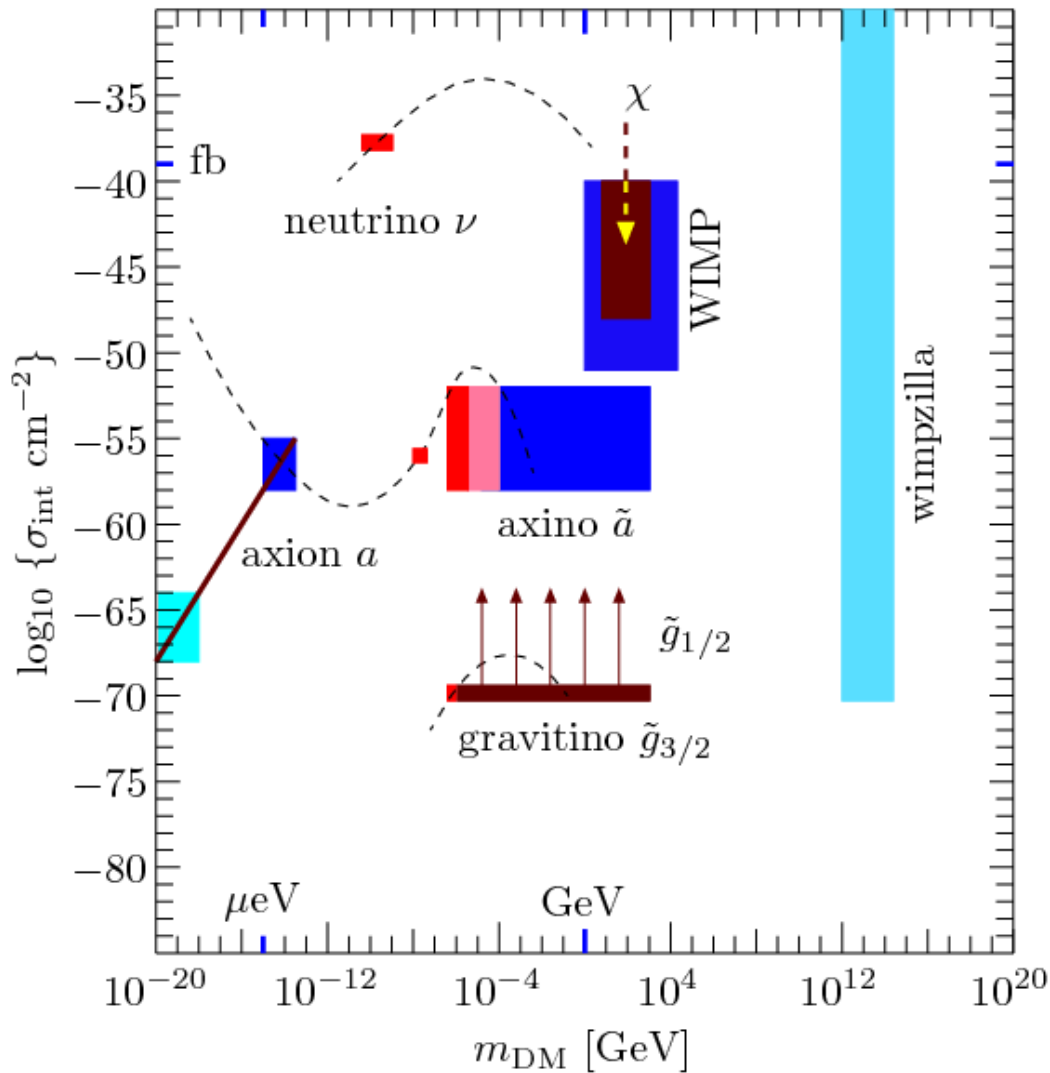
PQ symmetry: Wigner-Weyl realization

Axion: Nambu-Goldstone realization

The axion models have the spontaneous symmetry breaking scale V and the axion decay constant f_a which are related by $V = N_{DW} f_a$.

Here, I present the general idea on axions and its SUSY partner axino.





A rough sketch of masses and cross sections. Bosonic DM with collective motion is always CDM.

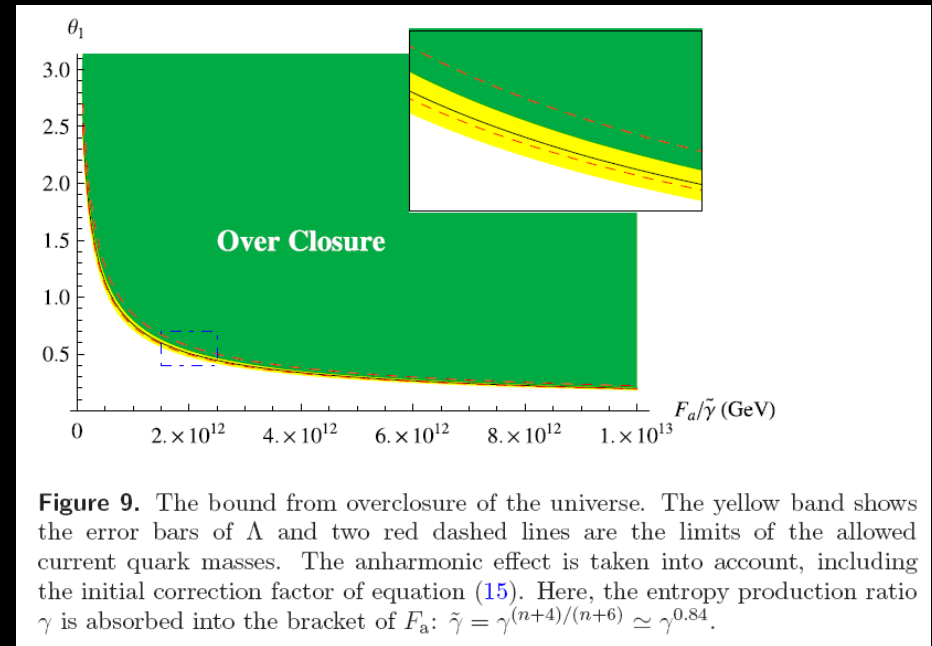
[Kim-Carosi , 2008]

The cosmic axion density is,

$$10^9 \text{ GeV} < F_a < \{10^{12} \text{ GeV ?}\}$$

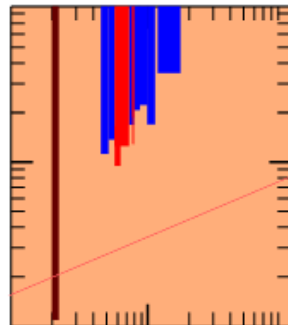
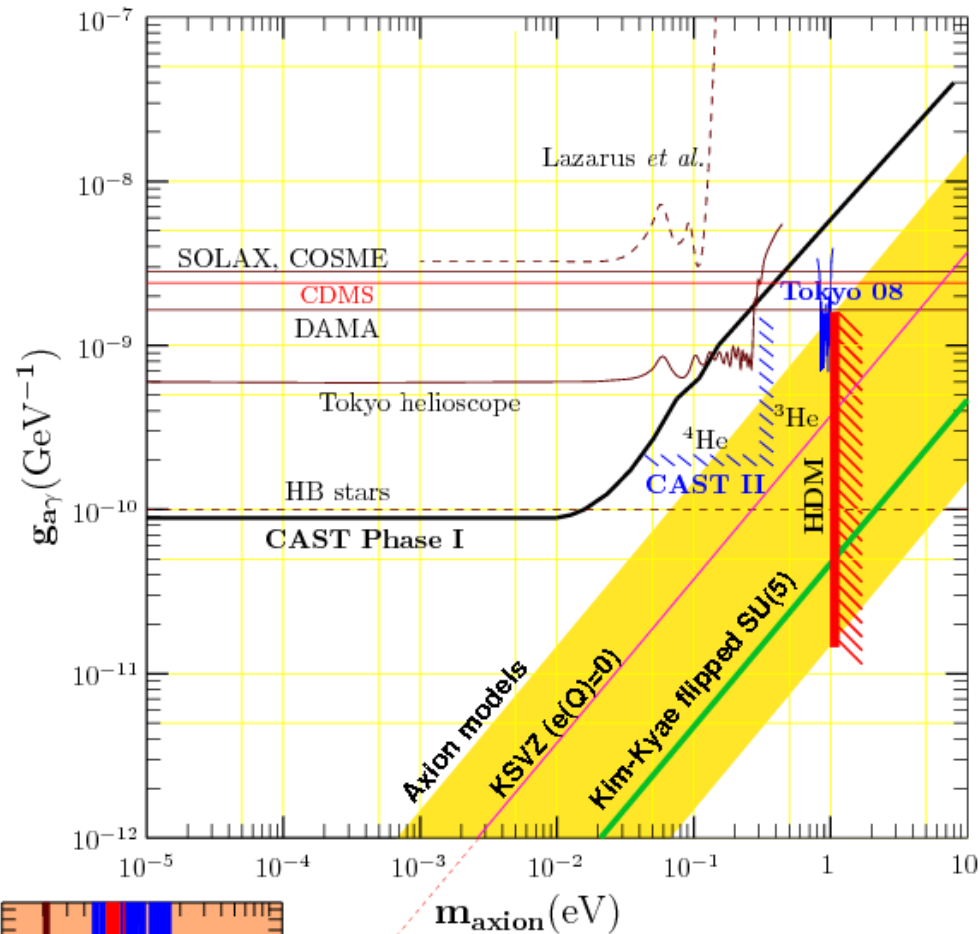
Turner (86), Grin et al (07),
Giudice-Kolb-Riotto (08),
Bae-Huh-K (JCAP 08,
[arXiv:0806.0497]):
recalculated
including the anharmonic
term carefully with the new data
on light quark masses.

It is the basis of using the anthropic
argument for a large f_a .



Many lab. searches were made, and we hope the axion be discovered.

The status was (Kim-Carosi, RMP, 2010)



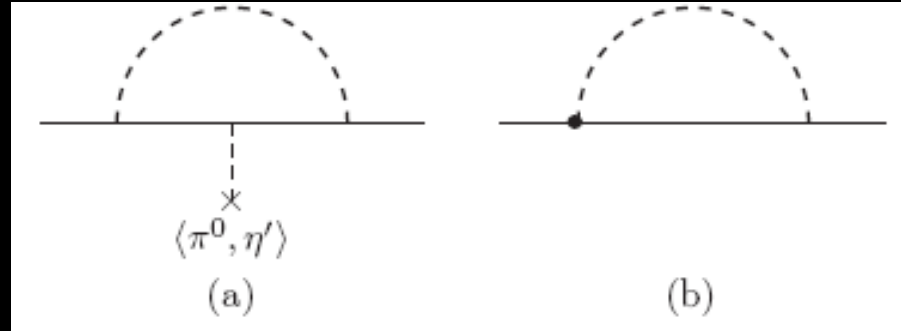
Strong CP problem

The instanton solution introduces the so-called θ term, and the resulting θ bound from NEDM.

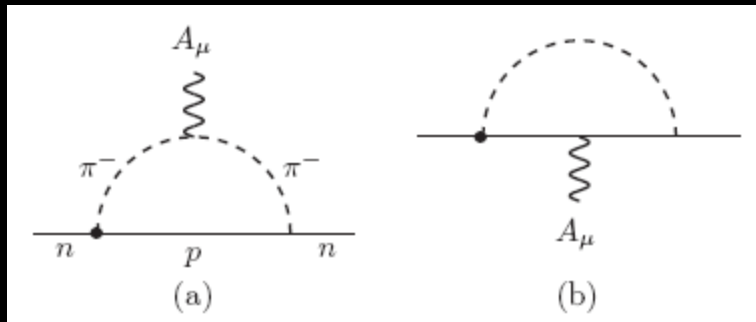


Look for the neutron mass term by CPV meson VEVs

$$g_{\pi NN} \frac{i}{-m_n} \langle m_n e^{-i\pi/f_\pi} \rangle$$



The NMDM and NEDM terms



The mass term and the NMDM term have the same chiral transformation property. So, (b)s are simultaneously removed.

(a) So, $d(\text{proton}) = -d(\text{neutron})$. is the NEDM contribution.

In our study, so the VEV of pi-zero determine the size of NEDM.

$$\overline{g_{\pi NN}} = -\bar{\theta} \frac{Z}{(1+Z)} \simeq -\frac{\bar{\theta}}{3}$$

$$d_n = \frac{g_{\pi NN} \overline{g_{\pi NN}}}{4\pi^2 m_N} \ln \left(\frac{m_N}{m_\pi} \right) e_{cm}$$

$$|\bar{\theta}| < 0.7 \times 10^{-11}$$

It is an order of magnitude stronger than Crewther et al bound.

The strong CP problem is solved by

1. Calculable θ , 2. Massless up quark (X)
3. Axion

1. Calculable θ

The Nelson-Barr CP violation

Here, the weak CP violation must be spontaneous so that θ_0 must be 0. It can be achieved at high energy scale.



2. Massless up quark

Suppose that we chiral-transform a quark,

$$q \rightarrow e^{i\gamma_5\alpha} q: \int d^4x \left(-m\bar{q}q + \frac{\theta}{32\pi^2} F\tilde{F} \right)$$
$$\rightarrow \int d^4x \left(-m\bar{q}qe^{2i\gamma_5\alpha} + \frac{\theta - 2\alpha}{32\pi^2} F\tilde{F} \right)$$

If $m=0$, it is equivalent to changing $\theta \rightarrow \theta - 2\alpha$. Thus, there exists a shift symmetry $\theta \rightarrow \theta - 2\alpha$. Here, α is not physical, and there is no strong CP problem. The problem is, “Is massless up quark phenomenologically viable?”

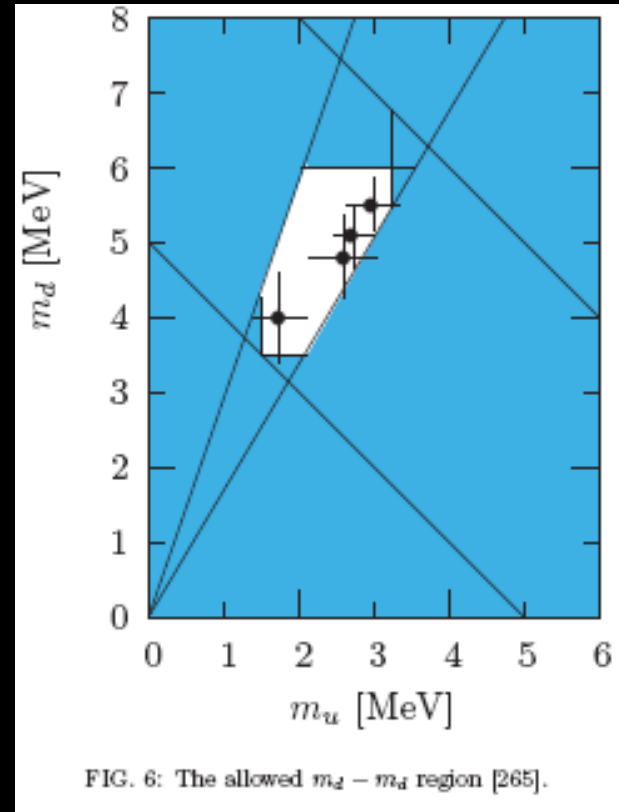
$$\frac{m_u}{m_d} = 0.5,$$

$$m_u = 2.5 \mp 1 \text{ MeV},$$

$$m_d = 5.1 \pm 1.5 \text{ MeV}$$

(Manohar-Sachrajda)

Excluding the lattice cal., this is convincing that $m_u=0$ is not a solution now.



Particle Data (2010)

3. Axions

Historically, Peccei-Quinn tried to mimick the symmetry $U(1)_{PQ}$, $-2\pm$, by the full electroweak theory. They found such a symmetry if H_u is coupled to up-type quarks and H_d couples to down-type quarks,

$$L = \bar{q}_L u_R H_u + \bar{q}_L d_R H_d - V(H_u, H_d) + \dots$$

$$q \rightarrow e^{i\gamma_5 \alpha} q: \{H_u, H_d\} \rightarrow e^{i\beta} \{H_u, H_d\}:$$

$$\rightarrow \int d^4 x \left(-H_u e^{i\beta} \bar{u} e^{i\gamma_5 \alpha} u - H_d e^{i\beta} \bar{d} e^{i\gamma_5 \alpha} d + \frac{\theta - 2\alpha}{32\pi^2} F\tilde{F} \right)$$

Eq. $\beta=\alpha$ achieves the same thing as the $m=0$ case.

The Lagrangian is invariant under changing $\theta \rightarrow \theta - 2\pi$. Thus, it seems that θ is not physical, since it is a phase of the PQ transformation. But, θ is physical. At the Lagrangian level, there seems to be no strong CP problem. But $\langle H_u \rangle$ and $\langle H_d \rangle$ breaks the PQ global symmetry and there results a Goldstone boson, axion a [Weinberg, Wilczek]. Since θ is a phase field, the original $\cos \theta$ dependence becomes the potential of the axion a .

If its potential is of the $\cos \theta$ form, always $\theta = a/Fa$ can be chosen at 0 [Instanton physics, PQ, Vafa-Witten]. So the PQ solution of the strong CP problem is that the vacuum chooses

$$\theta = 0$$

Historically: The Peccei-Quinn-Weinberg-Wilczek axion is short lived , and the so-called invisible axions [KSVZ , DFSZ] are searched for.

KSVZ:
$$L = \bar{Q}_L Q_R S - V(S, H_u, H_d) + \dots$$

Now the couplings of S determines the axion interaction. Because it is a Goldstone boson, the couplings are of the derivative form except the anomaly term.

Let the PQ symmetry operator be: Γ

When it is realized by the Wigner-Weyl manner, the PQ charged fields ϕ_i have PQ quantum numbers, Γ_i ,

$$\Gamma \phi_i = \Gamma_i \phi_i$$

But for axion, we describe in the Nambu-Goldstone manner where Γ is not unitarily implementable. Still the PQ quantum numbers Γ_i of the fields ϕ_i are useful since they are just numbers.

And we use the Nambu-Goldstone fields a_i . Let the axion field be in the phase of ϕ which was a linear combination of axion components a_i which were in the phase:

$$\phi = \sum_i c_i \phi_i$$

The PQ transformation here is the shift of the Nambu-Goldstone fields,

$$\delta a = f_a \delta \theta$$

The PQ transformation here is the shift of the Nambu-Goldstone fields,

$$\delta a = f_a \delta \theta$$

$$\delta a_i = \Gamma_i f_a \delta \theta$$

We obtain the relation, considering the $\delta \theta$ coefficient, to encode the shift symmetry in the Nambu-Goldstone phase,

$$e^{ia/f_a} \propto \sum c_i \Gamma_i (\text{number}) e^{i\Gamma_i a/f_a}$$

Where the number is VEV of some field operator. Let the field be

$$\varphi(x)$$

So, axion a is defined as with $\langle \varphi \rangle = N_{DW} f_a$:

$$\Phi = \varphi e^{ia/f_a} [NG] \leftrightarrow \sum_i c_i \varphi_i e^{i\Gamma_i \theta} [WW]$$

We use only one a because we need one Γ

PQ: WW, axion:NG

Two real fields are needed to describe axion with

$\varphi, a,$

and

$$\langle \varphi \rangle = N_{DW} f_a, \quad \langle a \rangle = 0.$$

If the VEV of a were not zero, it breaks the CP symmetry.

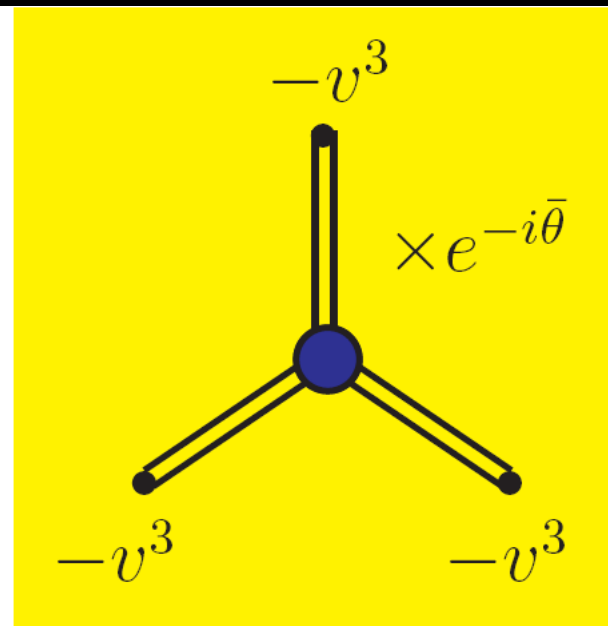
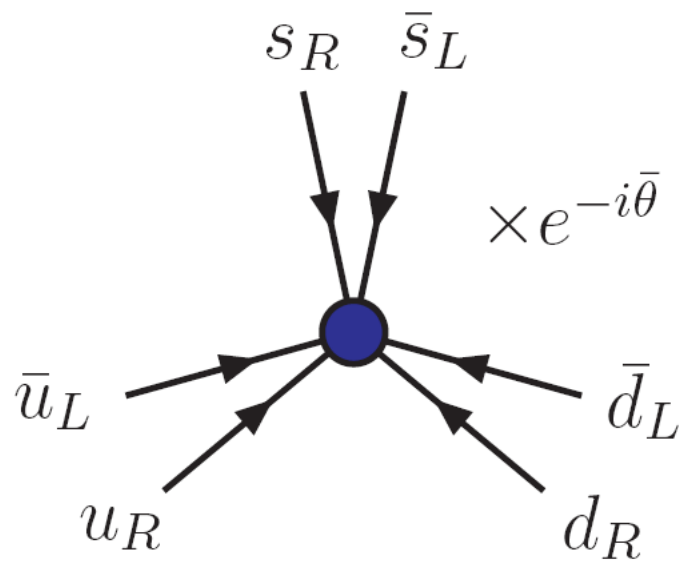
Heavy
Q's are
Integrated
out

$$\begin{aligned}
 \mathcal{L}_\theta = & \frac{1}{2} f_S^2 \partial^\mu \theta \partial_\mu \theta - \frac{1}{4g_c^2} G_{\mu\nu}^a G^{a\mu\nu} + (\bar{q}_L i \not{D}_{q_L} + \bar{q}_R i \not{D}_{q_R}) \\
 & + c_1 (\partial_\mu \theta) \bar{q} \gamma^\mu \gamma_5 q - (\bar{q}_L m q_R e^{ic_2 \theta} + \text{H.c.}) \\
 & + c_3 \frac{\theta}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} (\text{or } \mathcal{L}_{\text{det}}) + c_{\theta\gamma\gamma} \frac{\theta}{32\pi^2} F_{\text{em}}^i{}_{\mu\nu} \tilde{F}_{\text{em}}^{i\mu\nu} \\
 & + \mathcal{L}_{\text{leptons}, \theta}, \tag{19}
 \end{aligned}$$

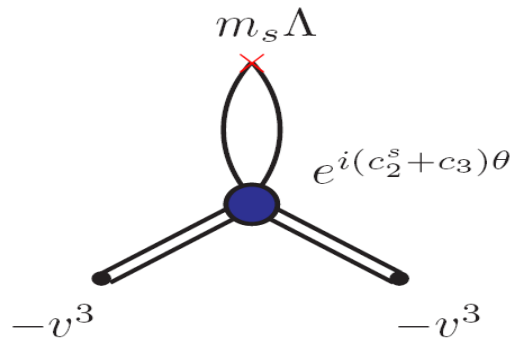
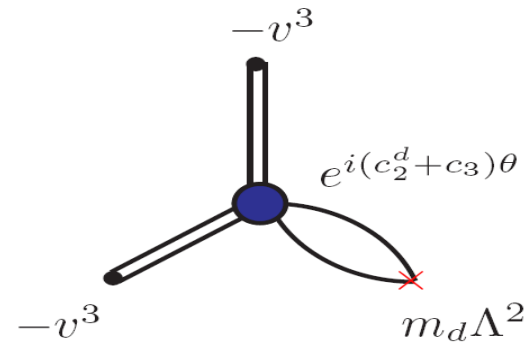
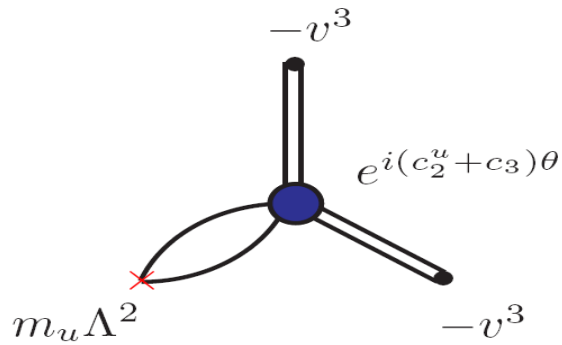
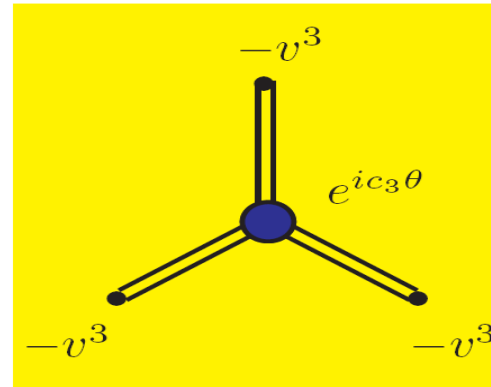
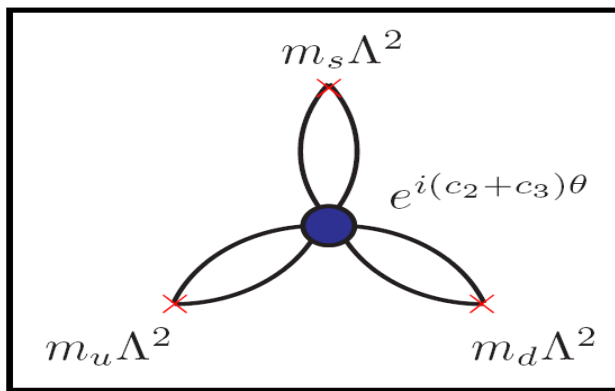
$$\mathcal{L}_{\text{det}} = -2^{-1} ic_3 \theta (-1)^{N_f} \frac{e^{-ic_3 \theta}}{K^{3N_f - 4}} \text{Det}(q_R \bar{q}_L) + \text{H.c.},$$

$$\begin{aligned}
 & \Gamma_{1PI}[a(x), A_\mu^a(x); c_1, c_2, c_3, m, \Lambda_{\text{QCD}}] \\
 & = \Gamma_{1PI}[a(x), A_\mu^a(x); c_1 - \alpha, c_2 - 2\alpha, c_3 \\
 & \quad + 2\alpha, m, \Lambda_{\text{QCD}}].
 \end{aligned}$$

The axion mass depends only on the combination of $(c_2 + c_3)$. The 'hadronic axion' usually means $c_1 = 0$, $c_2 = 0$, $c_3 \neq 0$.



't Hooft determinantal interaction and the solution of the U(1) problem. If the story ends here, the axion is exactly massless. But,....



$$+ \mathcal{O}(m^2 \Lambda^4 v^3)$$

$$\mathcal{L} = -m_u \langle \bar{u}_L u_R \rangle e^{i[(\theta_\pi + \theta_{\eta'}) + c_2^u \theta]} - m_d \langle \bar{d}_L d_R \rangle e^{i[(-\theta_\pi + \theta_{\eta'}) + c_2^d \theta]} + \text{h.c.} + \mathcal{L}_{\text{det}}$$

$$\begin{aligned} -V = & m_u v^3 \cos(\theta_\pi + \theta_{\eta'}) + m_d v^3 \cos(-\theta_\pi + \theta_{\eta'}) + \frac{v^9}{K^5} \cos(2\theta_{\eta'} - (c_2^u + c_2^d + c_3)\theta) \\ & + m_u \frac{\Lambda_u^2 v^6}{K^5} \cos(-\theta_\pi + \theta_{\eta'} - (c_2^u + c_2^d + c_3)\theta) + m_d \frac{\Lambda_d^2 v^6}{K^5} \cos(\theta_\pi + \theta_{\eta'} - (c_2^u + c_2^d + c_3)\theta) \end{aligned}$$

$$M_{a, \eta', \pi^0}^2 = \begin{pmatrix} c^2[\Lambda_{\eta'}^4 + 2\mu\Lambda_{\text{inst}}^3]/F^2 & -2c[\Lambda_{\eta'}^4 + \mu\Lambda_{\text{inst}}^3]/f'F & 0 \\ -2c[\Lambda_{\eta'}^4 + \mu\Lambda_{\text{inst}}^3]/f'F & [4\Lambda_{\eta'}^4 + 2\mu\Lambda_{\text{inst}}^3 + m_+ v^3]/f'^2 & -m_- v^3/ff' \\ 0 & -m_- v^3/ff' & (m_+ v^3 + 2\mu\Lambda_{\text{inst}}^3)/f^2 \end{pmatrix}$$

$$m_{\pi^0}^2 \simeq \frac{m_+ v^3 + 2\mu\Lambda_{\text{inst}}^3}{f_\pi^2}$$

$$m_{\eta'}^2 \simeq \frac{4\Lambda_{\eta'}^4 + m_+ v^3 + 2\mu\Lambda_{\text{inst}}^3}{f_{\eta'}^2}$$

$$m_a^2 \simeq \frac{c^2}{F^2} \frac{Z}{(1+Z)^2} f_\pi^2 m_{\pi^0}^2 (1 + \Delta)$$

$$\Delta = \frac{m_-^2}{m_+} \frac{\Lambda_{\text{inst}}^3 (m_+ v^3 + \mu\Lambda_{\text{inst}}^3)}{m_{\pi^0}^4 f_\pi^4}$$

The instanton contribution is included by Δ .

Numerically, we use

$$-m_u \Lambda^3 \cos \frac{a}{F_a} \Rightarrow m_a = \frac{\sqrt{Z}}{1+Z} \frac{f_\pi m_\pi}{F_a} = 0.6[eV] \frac{10^7 GeV}{F_a}$$

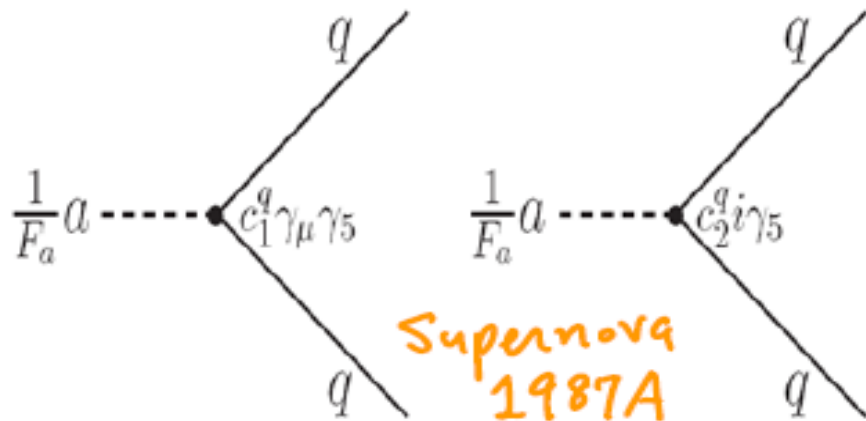
Axion couplings

Above the electroweak scale, we integrate out heavy fields. If colored quarks are integrated out, its effect is appearing as the coefficient of the gluon anomaly. If only bosons are integrated out, there is no anomaly term. Thus, we have

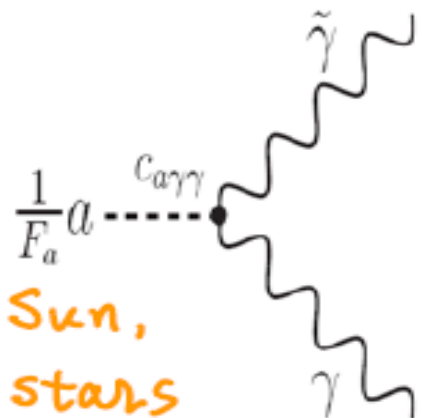
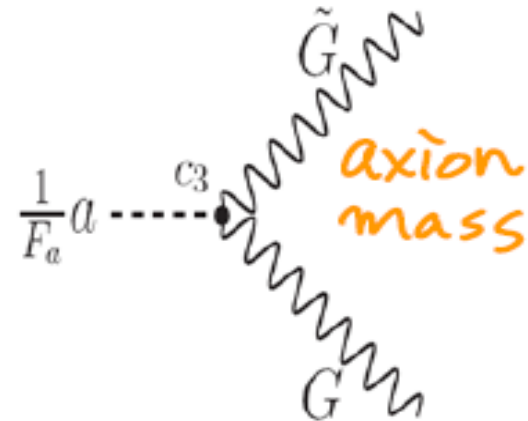
KSVZ: $c_1=0$, $c_2=0$, $c_3=\text{nonzero}$: hadronic

DFSZ: $c_1=0$, $c_2=\text{nonzero}$, $c_3=0$





Supernova
1987A



Hadronic coupling is important for the study of supernovae:
The chiral symmetry breaking is properly taken into account,
using the reparametrization invariance so that $c_3'=0$.

KSVZ:

$$\bar{c}_1^{u,d} = \frac{1}{2}\bar{c}_2^{u,d}$$
$$\bar{c}_2^u = \frac{1}{1+Z}, \quad \bar{c}_2^d = \frac{Z}{1+Z}$$

DFSZ:

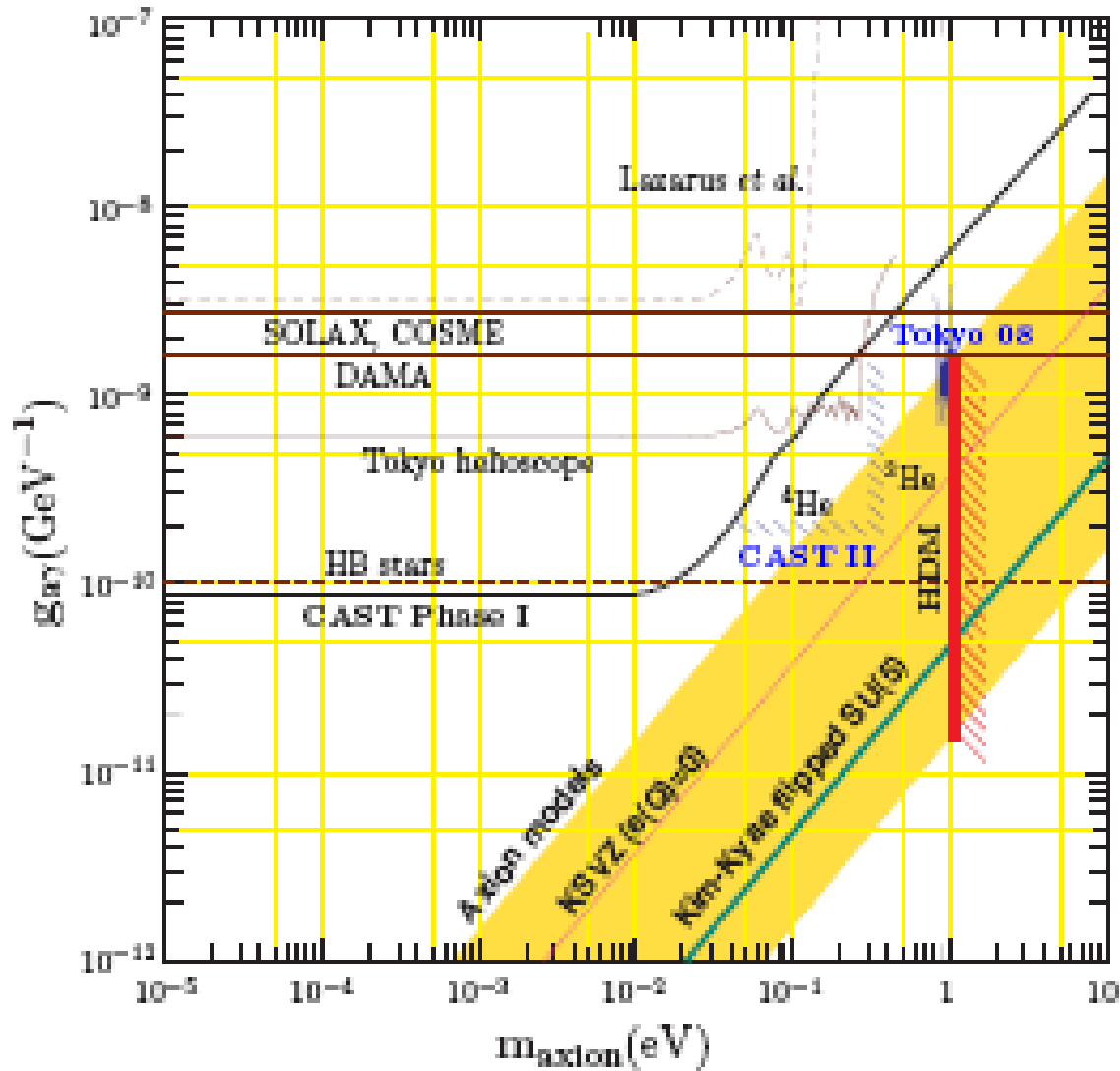
$$\bar{c}_1^u = -\frac{|v_d|^2}{2v_{EW}^2} + \frac{1}{2}\bar{c}_2^u, \quad \bar{c}_1^d = -\frac{|v_u|^2}{2v_{EW}^2} + \frac{1}{2}\bar{c}_2^d,$$
$$\bar{c}_2^u = \frac{1}{1+Z}, \quad \bar{c}_2^d = \frac{Z}{1+Z},$$

The KSVZ axion has been extensively studied. Now the
DFSZ axion can be studied, too.

For axion detection, we need to know the axion-photon-photon coupling strength which is parametrized by $c_{a\gamma\gamma}$

TABLE I. $c_{a\gamma\gamma}$ in several field theoretic models. The left block is for the KSVZ and the right block is for the DFSZ. (m, n) in the KSVZ block denotes m copies of $Q_{em} = \frac{2}{3}$ and n copies of $Q_{em} = -\frac{1}{3}$ heavy quarks with the same PQ charge. In the DFSZ block $x = \tan \beta = v_u/v_d$.

Q_{em}	$c_{a\gamma\gamma}$	x	one Higgs couples to	$c_{a\gamma\gamma}$
0	-1.95	any x	(d^c, e)	0.72
$\pm \frac{1}{3}$	-1.28	any x	(u^c, e)	-1.28
$\pm \frac{2}{3}$	0.72			
± 1	4.05			
(m, m)	-0.28			



String models
give definite
numbers. [I-W
Kim-K]

There exist only
one calculation
in string
compactification,
In a model
explaining all
MSSM
phenomenology.

Axions in the universe

The axion potential is of the form



The vacuum stays there for a long time, and oscillates when the Hubble time($1/H$) is larger than the oscillation period($1/m_a$)

$$3H < m_a$$

This occurs when the temperature is about 0.92 GeV.
[Preskill-Wise-Wilczek; Dine-Fischler, Abbott-Sikivie, 1983]

$$\rho_a(T_\gamma = 2.73\text{K}) = m_a(T_\gamma)n_a(T_\gamma)f_1(\theta_2) = \frac{\sqrt{Z}}{1+Z}m_\pi f_\pi \frac{3 \cdot 1.66g_{*s}(T_\gamma)T_\gamma^3}{2\sqrt{g_*(T_1)}M_{\text{P}}} \frac{F_a}{T_1} \frac{\theta_2^2 f_1(\theta_2)}{\gamma} \left(\frac{T_2}{T_1}\right)^{-3-n/2}$$

There is an overshoot factor of 1.8. So we use θ_2 , rather than θ_1 . If F_a is large ($> 10^{12}$ GeV), then the axion energy density dominates. Since the energy density is proportional to the number density, it behaves like a CDM, but

$$10^9 \text{ GeV} < F_a < 10^{12} \text{ GeV},$$

The axion field evolution eq. and time-varying Lagrangian

$$\ddot{\theta} + 3H\dot{\theta} + \frac{1}{F_a^2}V'(\theta) = 0$$

$$L = R^3(F_a^2\dot{\theta}^2 - V(\theta))$$

$$V = m_a^2 F_a^2 (1 - \cos \theta)$$

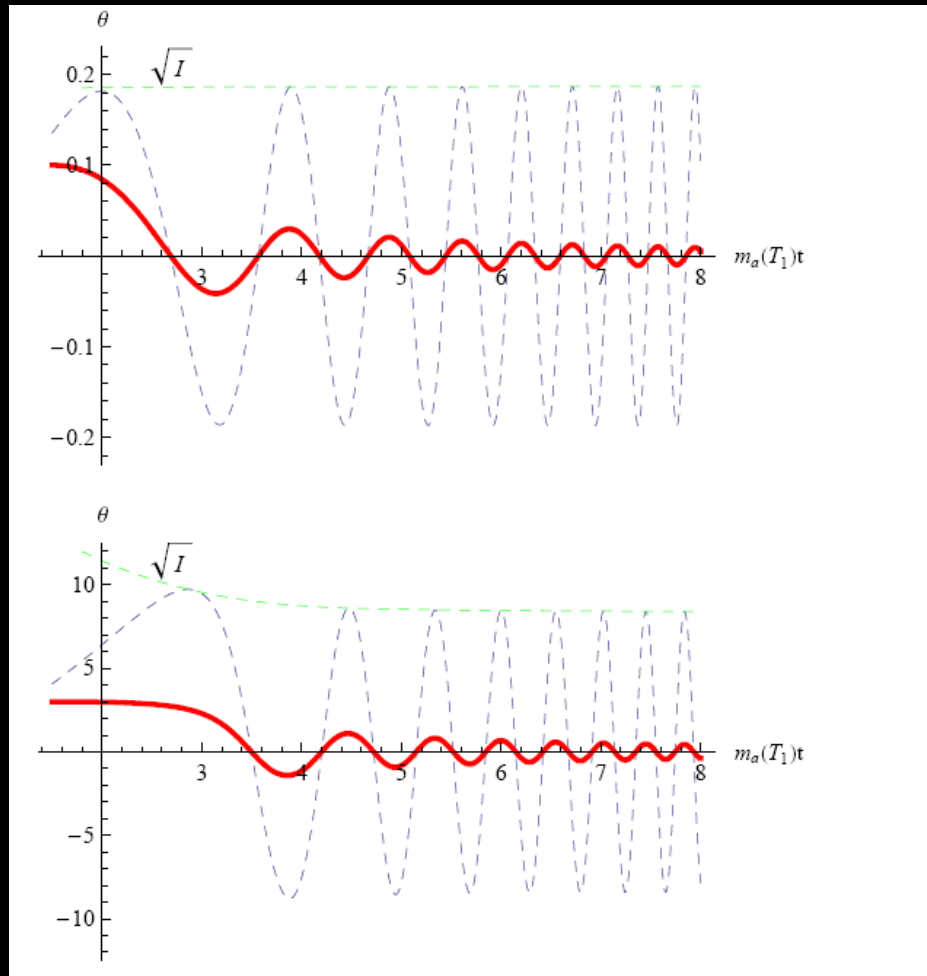
The adiabatic condition:

$$H, \dot{m}_a \ll m_a$$

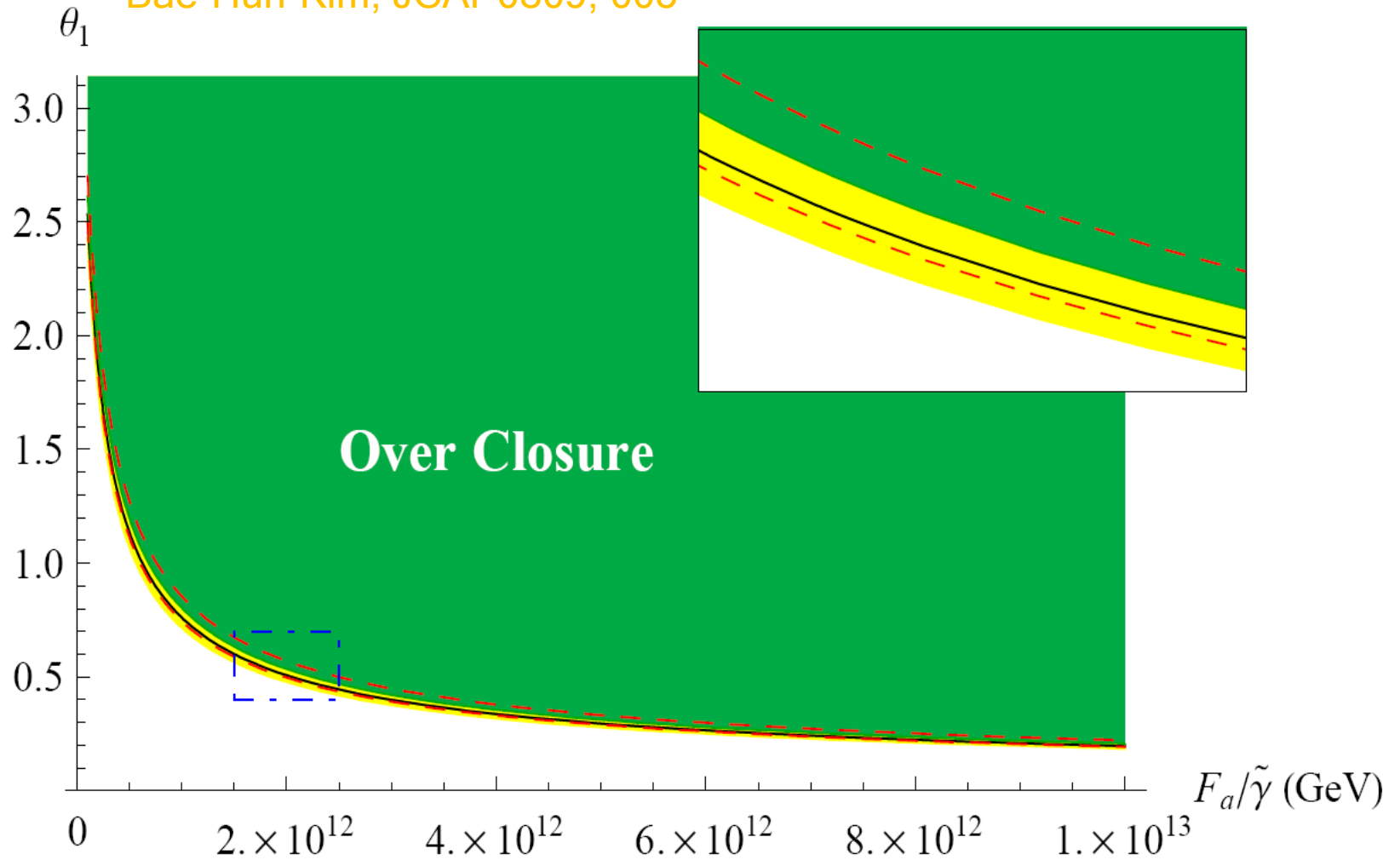
The adiabatic invariant quantity:

$$R^3 m_a \bar{\theta}^2 f_1(\bar{\theta})$$

$$f_1(\bar{\theta}) = \frac{2\sqrt{2}}{\pi\bar{\theta}} \int_{-\bar{\theta}}^{\bar{\theta}} d\theta' \sqrt{\cos \theta' - \cos \bar{\theta}}$$



Bae-Huh-Kim, JCAP0809, 005



2. Axino mass



Each real field is made SUSY multiplets with

$$\varphi \rightarrow \text{chiral } \varphi, \quad a \rightarrow \text{chiral } A,$$

The axion a was defined as with $\langle \varphi \rangle = f_a$:

$$\Gamma_A \varphi_A e^{A/f_a} = \sum_i \frac{v_i}{V_a} \Gamma_i \varphi_i e^{A/f_a}, \quad \varphi_i = v_i + \rho_{i\perp}$$

$$\langle \varphi_A \rangle = V_a, \quad \langle A \rangle = 0.$$

The axion is defined in SUSY as

$$\frac{1}{N_{DW}} \varphi_A e^{A/f_a} \equiv \sum_i \frac{v_i}{V_a} \Gamma_i \varphi_i e^{A/f_a}$$

$$\begin{aligned} \langle \varphi_i \rangle &= v_i, \\ \langle \varphi_A \rangle &= V_a, \quad \langle A \rangle = 0 \end{aligned}$$

To preserve the shift symmetry of A , SUSY couplings in the Kaehler potential and W are

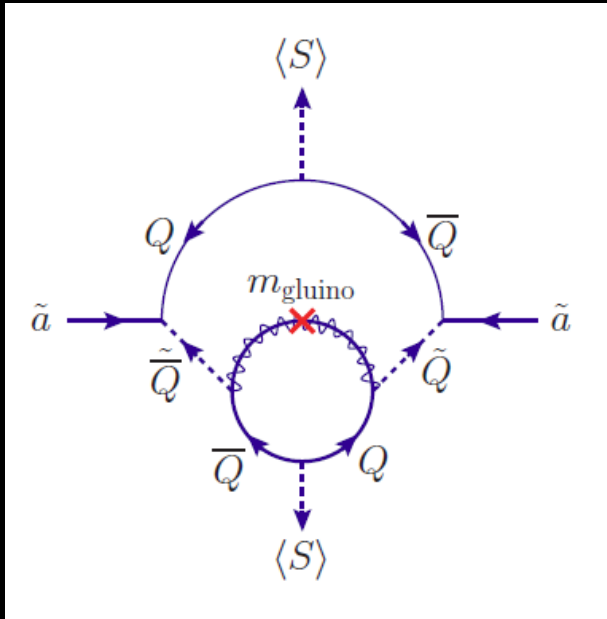
$K = \text{function of } (A+A^*), \quad W(A)=0.$

So, the axino mass can be calculated when the Kaehler geometry is completely known. But, we can guess its form,

$$m_{\tilde{a}} = \left(\xi_{\text{goldstino}} + \sum_{\text{terms in } W_{\text{int}}} \xi_I^{\text{anomalous}} \right) m_{3/2} + \xi_{\text{gaugino } a} m_{\text{gaugino } a}$$

If the gravitino mass is not dominating, the axino mass can be calculated in the GMSB scenario.

In the KSVZ model, a two loop diagram can be drawn



The axino mass is estimated as

$$m_{\tilde{a}} \sim \frac{f_Q^2 g_c^2}{(4\pi^2)^2} m_{\text{gluino}} \simeq \frac{\alpha f_Q \alpha_c}{\pi^2} m_{\text{gluino}}.$$

We can take it as 1-100 GeV mass.

In the DFSZ model, a two loop diagram with the top mass gives a much smaller axino mass

$$m_{\tilde{a},DFSZ} \approx \frac{m_t}{m_Q} m_{\tilde{a},KSVZ}$$

We can take it as about 10 eV mass.

3. Higgs boson at 125 GeV ?



It seems that there is something new
particle is present around 125 GeV.
Most probably, it looks like a Higgs boson.
The SM Higgs boson or the lightest or the
next lightest scalar Higgs boson, or
something else?

It has to be determined. In SUSY models,
many models with a singlet X is introduced.



We point out that a pseudoscalar boson with a singlet X is of general phenomenon. The pseudoscalar possibility of the 125 GeV two photon mode is discussed.

In addition, the strong CP problem must be solved. The MSSM cannot do it because $\mu H_u H_d$ breaks the PQ symmetry.

Strong CP solution in NMSSM \longrightarrow pseudoscalar at EW

JEK+Nilles+Seo, arXiv:1201.6547.



	H_u	H_d	S_1	S_2	Z_1	Z_2	X	X'	\overline{X}
Q_{PQ}	+1	+1	-1	+1	0	0	-2	-2	+2
R	+1	+1	0	0	2	2	0	0	2

This is a typical example, leading to the EW scale singlet, starting with the PQ symmetry. After the PQ symmetry is broken at the intermediate scale of $10^{10} - 10^{12}$ GeV, there exist the invisible axion.

The invisible axion not explicitly written,

$$W = -H_u H_d X + m X \bar{X} - \eta \bar{X} S_1^2 - \xi H_u H_d X' + m' X' \bar{X} + Z_1 (S_1 S_2 - F_1^2) + Z_2 (S_1 S_2 - F_2^2)$$



$$W_{EW} = -\mu H_u H_d + \tilde{\mu} X_{ew}^2 - f_h H_u H_d X_{ew} + \dots$$

Survives down to the EW scale

In our new language, X_{ew} is the φ type field. It must survive down to the EW scale.

The surviving fields are just

A and the φ type field.

$$W_{\text{EW}} = -\mu H_u H_d + \tilde{\mu} \varphi^2 - f_h \varphi H_u H_d + \dots$$

We showed in a specific model that $\tilde{\mu}=0$.

But, the μ term is present and a TeV scale by the Kim-Nilles[PLB 138, 150].

Remember that SUSY models have no A dependence in the superpotential.
Any global symmetry realized at low energy in NG manner includes A . What is the other?
In supersymmetrizing its nonSUSY part, we double the degrees: supersymmetrize the ϕ type field and also a field.

$$W = R(S_1 S_2 - f_a^2)$$

S_1 : 2 components S_2 : 2 components



Doubling S1 and S2 by

$$\varphi_1, \varphi_2, A_1, A_2$$

Each carries two components. To have the NG global symmetry, we need $A = A_1 - A_2$ in the exponent should not appear. But $A' = A_1 + A_2$ in the exponent can appear. It is superheavy and removed at low energy. The same remark applies to φ_1 and φ_2 , and only X_{ew} survives to low energy. This is the counting of degrees in the NG realization of the four components.

JEK+Nilles+Seo, arXiv:1201.6547: A does not appear in W , but X_{ew} appears as $X_{ew} H_u H_d$. And μ -term. If X_{ew}^3 appears, I will be very much surprised.



Now, the EW scale Lagrangian has an additional symmetry,

$$W_{\text{EW}} = -\mu H_u H_d - f_h H_u H_d X_{\text{ew}}$$

Survives down to the EW scale

Treating μ 's as a perturbation, there exists a symmetry

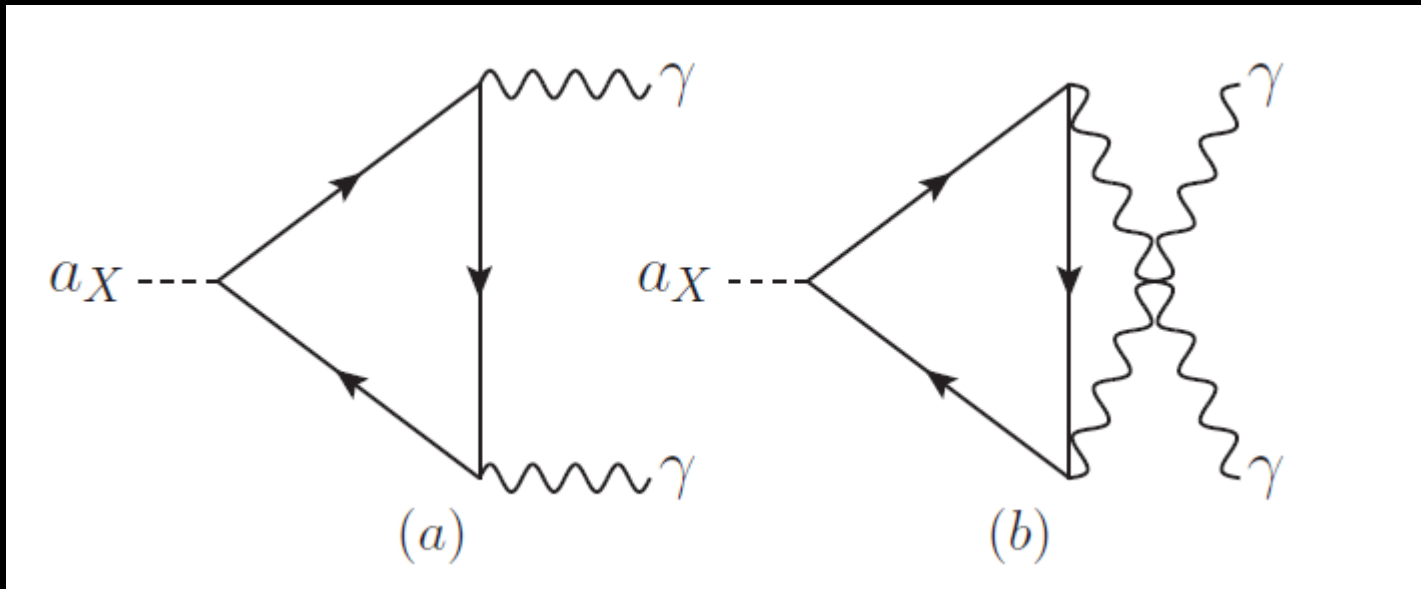
$$H_u \rightarrow e^{i\delta} H_u, \quad H_d \rightarrow e^{i\delta} H_d$$
$$X_{\text{ew}} \rightarrow e^{-2i\delta} X_{\text{ew}}$$

The chargino (more generally the Higgsino pair) has the anomaly current ,

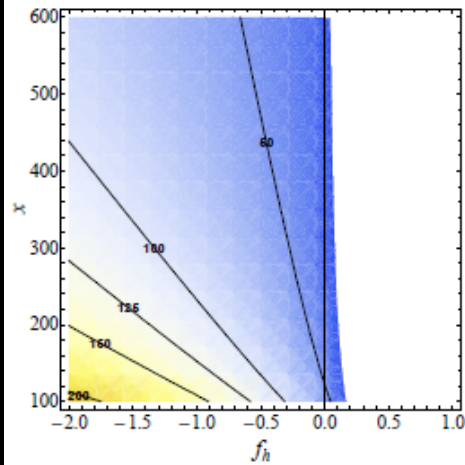
$$J_{\mu}^5 = \bar{\tilde{H}} \gamma_{\mu} \gamma_5 \tilde{H}$$

$$\partial^{\mu} J_{\mu}^5 = \frac{\alpha_{\text{em}}}{2\pi} F_{\text{em} \mu\nu} \tilde{F}_{\text{em}}^{\mu\nu} + 2\mu \bar{\tilde{H}} \gamma_{\mu} \gamma_5 \tilde{H}$$

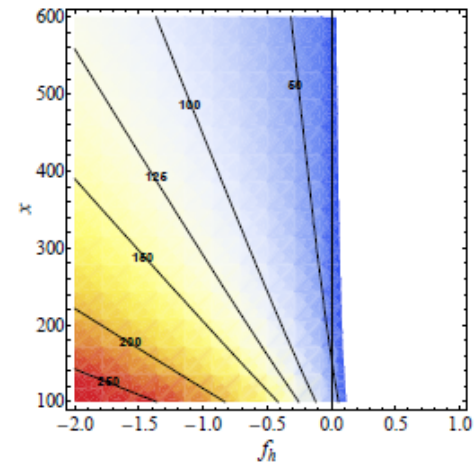
KNS symmetry from the PQ symmetry: An EW scale light pseudoscalar



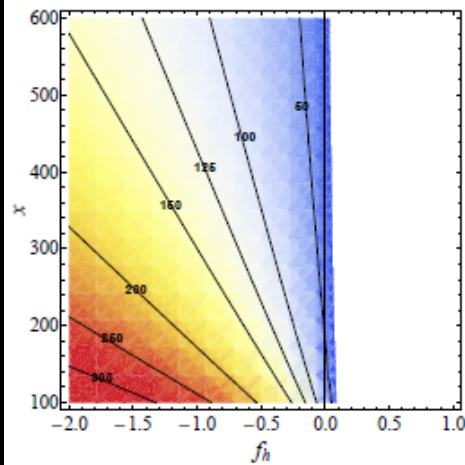
The weak gauge boson fusion gives a correct order of the two photon mode. Still solves the strong CP problem by the invisible axion discussed earlier.



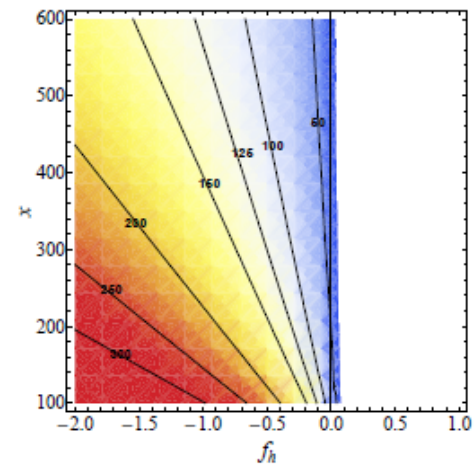
(a)



(b)



(c)



(d)

FIG. 2: a_X mass for $\tan\beta = 10$ and (a) $\mu = 50$ GeV, (b) $\mu = 100$ GeV, (c) $\mu = 150$ GeV, and (d) $\mu = 200$ GeV.

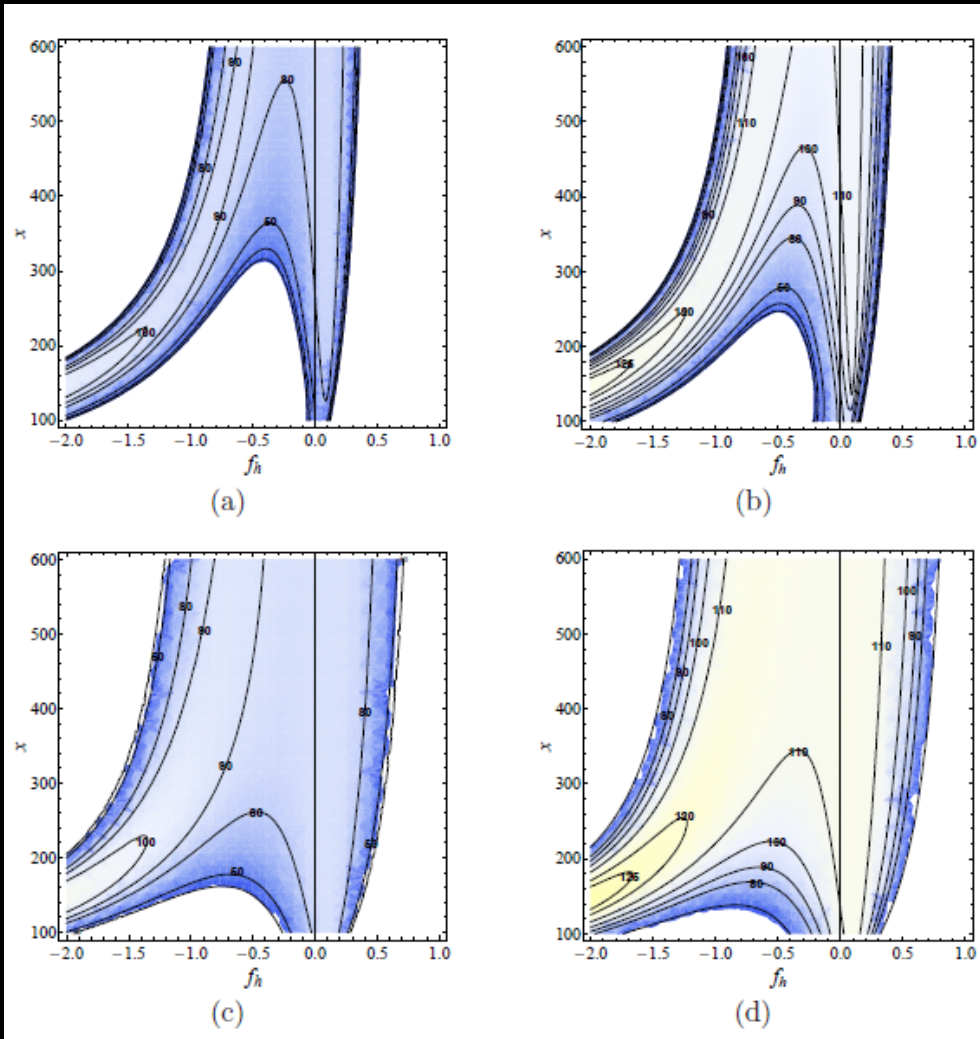
With a singlet X , the scalars take the following mass matrix.

$$M_H^2 = \begin{pmatrix} m_0^2 \cos^2 \beta & \frac{1}{2} \sin 2\beta (f_h^2 v^2 & m_c^2 \sin \beta \\ +M_Z^2 \sin^2 \beta & -m_0^2 - M_Z^2) & -m_c'^2 \cos \beta \\ \frac{1}{2} \sin 2\beta (f_h^2 v^2 & m_0^2 \sin^2 \beta & m_c^2 \cos \beta \\ -m_0^2 - M_Z^2) & +M_Z^2 \cos^2 \beta & -m_c'^2 \sin \beta \\ m_c^2 \sin \beta & m_c^2 \cos \beta & \\ -m_c'^2 \cos \beta & -m_c'^2 \sin \beta & M_E^2 \end{pmatrix}$$

With the off-diagonal term, the lightest Higgs boson mass can be raised:

Without the off-diagonal term, the maximum value of the lightest scalar mass is smaller than that of the MSSM limit M_Z (Weinberg's limit). M. Drees, IJMP A4, 3635 (1989)

With the off-diagonal term, the lightest Higgs boson mass can be raised:



Conclusion

1. Strong CP solution completes QCD's theoretical ends.
2. Invisible axion, working for DM and can be detected.
3. Axino mass can arise from many sources.
4. Some comments on the LHC physics, and set the axion to the recent LHC finding. i.e., **out of the MSSM.**
5. **A new approximate anomalous symmetry at the EW scale: Higgsino symmetry**
An EW scale light pseudoscalar for LHC, not necessarily for 125 GeV. Watch this!!!

