p_T-dependent semi-inclusive scattering in QCD

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In some sense, all processes used to determine the partonic structure of hadrons are "semi inclusive" and involve high-p_T final states....

Today: discuss a few pQCD aspects relevant to processes "sensitive to OAM"

Outline:

- Introduction
- Single-scale processes
- Two-scale processes

Introduction

Reactions with measured p_T play crucial role in QCD:

- Probes of nucleon structure
- Involved in most of today's Hadron Collider physics ("New Physics")
- Test our understanding of QCD at high energies, and our ability to do "first-principles" computations

Cornerstones: factorization & asymptotic freedom

Distinguish:

Processes with single measured hard scale p_T	Two-scale problems: small measured q_T *and* hard scale Q
• Examples: $pp \to \pi X \quad \gamma p \to \pi X$	• Examples: "TMD-SIDIS", Higgs-q_T
• Collinear factorization	• For simplest observables: TMD factorization
• Typically, fixed-order hard scattering (NLO, ...)	• Perturbation theory: *double*-logs $\alpha_s^k \log^{2k}(q_T/Q)$
• "DGLAP" evolution = resummation of logs $\alpha_s^k \log^k(p_T/Q_0)$ to all orders	• TMD evolution = resummation of these logs

Connections between the two:

- q_T - integrated (weighted) cross sections revert to single-scale problem

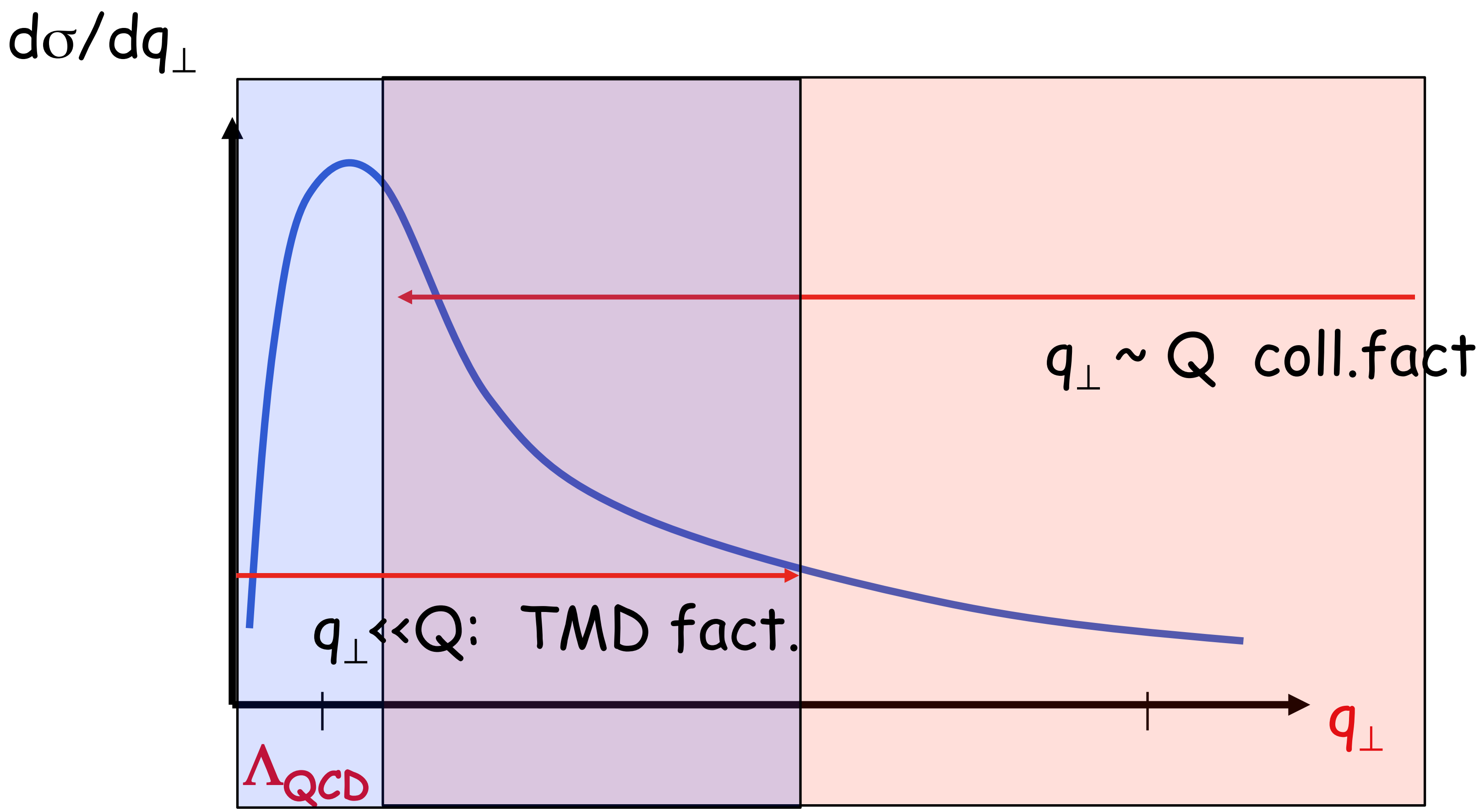
- this typically involves formal relations such as

$$T_F(x,x) = -\int \frac{d^2\vec{k}_\perp}{2\pi} \frac{\vec{k}_\perp^2}{M^2} \left(f_{1T}^\perp(x,k_\perp)\right)_{\rm DIS}$$

("sign puzzle" Kang,Qiu,WV,Yuan - see Kang's talk)

- likewise:

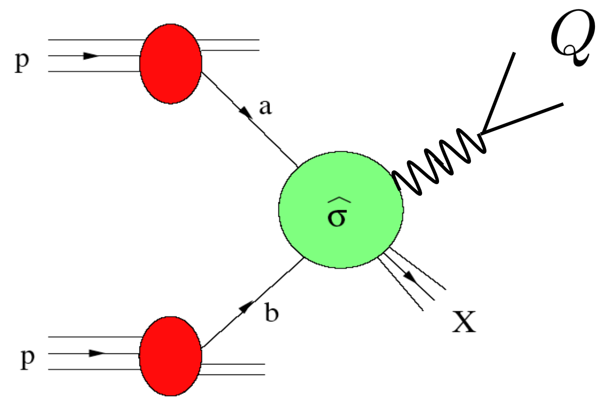
Ji, Qiu, WV, Yuan;
Koike, WV, Yuan; Zhou, Yuan, Liang;
Bacchetta, Boer, Diehl, Mulders



$\Lambda_{QCD} \ll q_\perp \ll Q$ same physics

Single-scale processes

Factorized cross section:
e.g. Drell-Yan



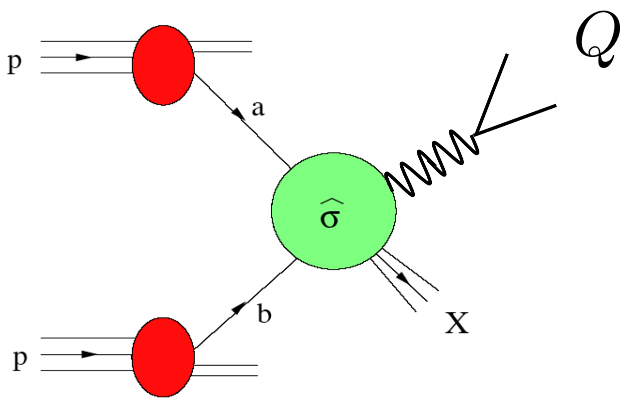
$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b\, f_a(x_a,\mu)\, f_b(x_b,\mu)\, \omega_{ab}\left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu}\right) + \ldots$$

- $f_{a,b}$ parton distributions: non-pert., but universal
- ω_{ab} partonic cross sections: process-dep., but pQCD

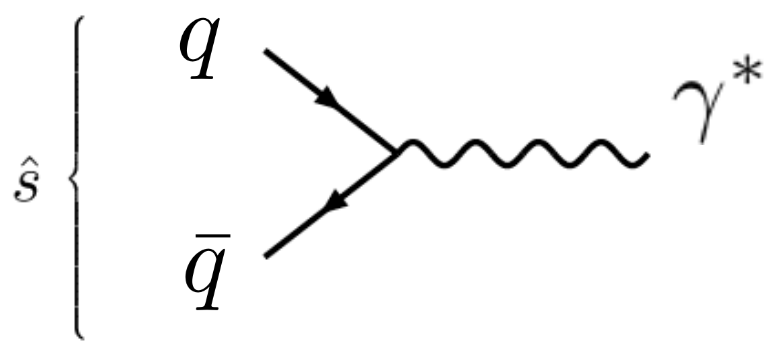
$$\omega_{ab} = \omega_{ab}^{(\mathrm{LO})} + \frac{\alpha_s}{2\pi}\omega_{ab}^{(\mathrm{NLO})} + \ldots$$

- $\mu \sim Q$ factorization / renormalization scale

Numerous applications: $f(x)\,,\ \Delta f(x)\,,\ T_F(x,x')\,,\ldots$



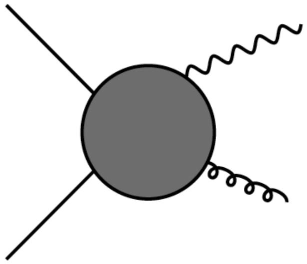
LO :



$$z = \frac{Q^2}{\hat{s}}$$

$$\omega_{q\bar{q}}^{(\mathrm{LO})} \propto \delta(1-z)$$

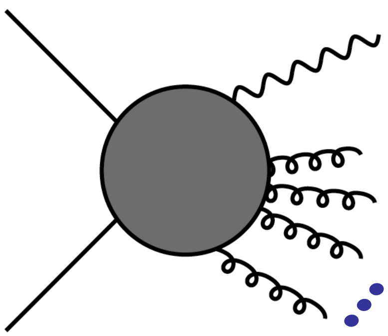
- NLO correction:



$$z \rightarrow 1:$$

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \propto \alpha_s \left(\frac{\log(1-z)}{1-z} \right)_+ + \ldots$$

- higher orders:



$$\omega_{q\bar{q}}^{(\mathrm{N^kLO})} \propto \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z} \right)_+ + \ldots$$

"threshold logarithms"

- for z->1 real radiation inhibited
- (so, not really a "single-scale" problem)

- logs emphasized by parton distributions :

$$d\sigma \sim \int_\tau^1 \frac{dz}{z} \mathcal{L}_{q\bar{q}} \left(\frac{\tau}{z}\right) \omega_{q\bar{q}}(z) \qquad \tau = \frac{Q^2}{S}$$

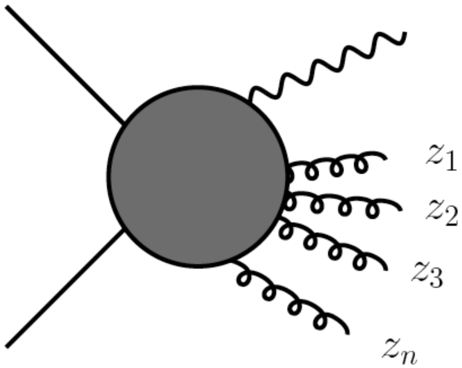
z = 1 relevant,
in particular as $\tau \to 1$

- logs more relevant at lower hadronic energies

Large logs can be resummed to all orders

Catani, Trentadue; Sterman; …

- factorization of matrix elements in soft limit
- and of phase space when integral transform is taken:



$$\delta\left(1-z-\sum_{i=1}^{n} z_i\right) = \frac{1}{2\pi i}\int_C dN\, \mathrm{e}^{N\left(1-z-\sum_{i=1}^{n} z_i\right)}$$

$$z_i = \frac{2E_i}{\sqrt{\hat{s}}}$$

$\overline{\text{MS}}$ scheme

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \propto \exp\left[2\int_0^1 dy\, \frac{y^N-1}{1-y}\int_{\mu^2}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) + \ldots\right]$$

$$A_q(\alpha_s) = C_F\left\{\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{C_A}{2}\left(\frac{67}{18}-\zeta(2)\right)-\frac{5}{9}T_R n_f\right]+\ldots\right\}$$

- logs enhance cross section !

LL :

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \;\propto\; \exp\left[+ \frac{2C_F}{\pi}\,\alpha_s \ln^2 N + \ldots \right]$$

to NLL :

Catani,Mangano,Nason,Trentadue

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \;\propto\; \exp\left\{ 2\ln\bar{N}\; h^{(1)}(\lambda) + 2h^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right) \right\}$$

LL NLL

$$\lambda = \alpha_s(\mu^2)\, b_0\, \log(N e^{\gamma_E})$$

$$h^{(1)}(\lambda) \;=\; \frac{A_q^{(1)}}{2\pi b_0 \lambda}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right] \qquad h^{(2)} \;=\; \ldots$$

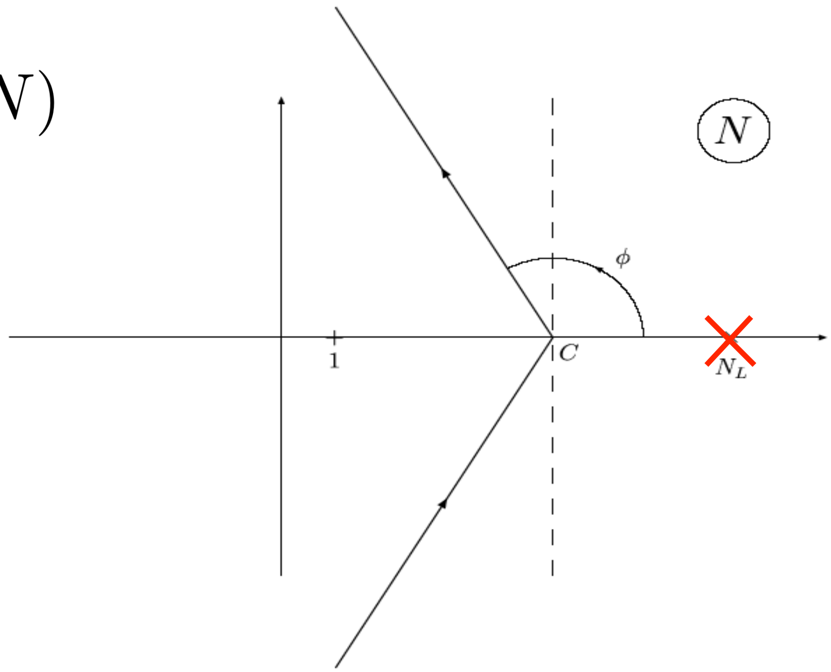
Note,

$$\tau = \frac{Q^2}{S}$$

$$\int_0^1 d\tau\, \tau^{N-1} \frac{d\sigma}{dQ^2} \propto \sum_{ab} \left(\int_0^1 dx_a\, x_a^N\, f_a \right) \left(\int_0^1 dx_b\, x_b^N\, f_b \right) \tilde{\omega}_{ab}(N)$$

Inverse transform:

$$\sigma^{\rm res} = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} dN\, \tau^{-N}\ \tilde{\sigma}^{\rm res}(N)$$



"Minimal prescription"

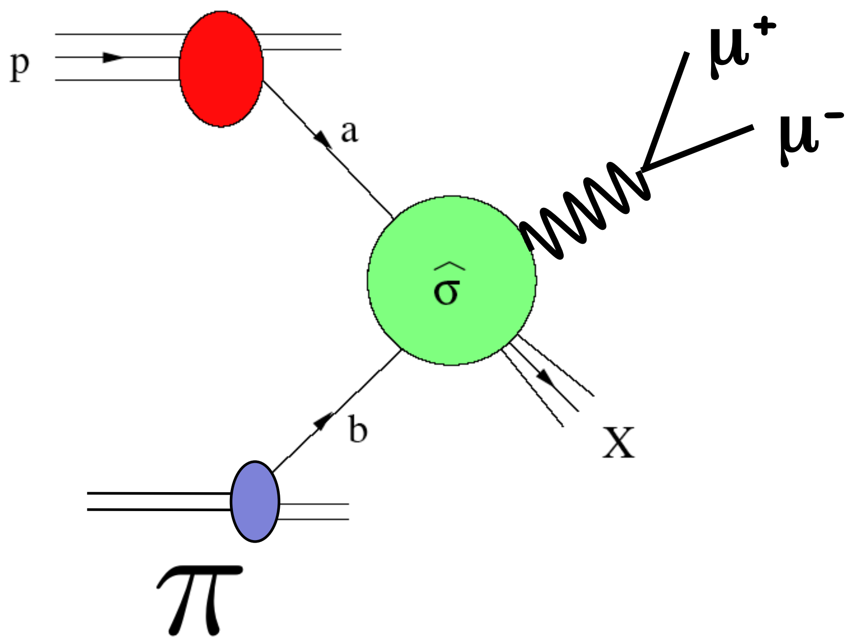
Catani,Mangano,Nason,Trentadue

Drell-Yan process in πN scattering

M. Aicher, A.Schäfer, WV

• Drell-Yan process has been main source of information on pion structure:

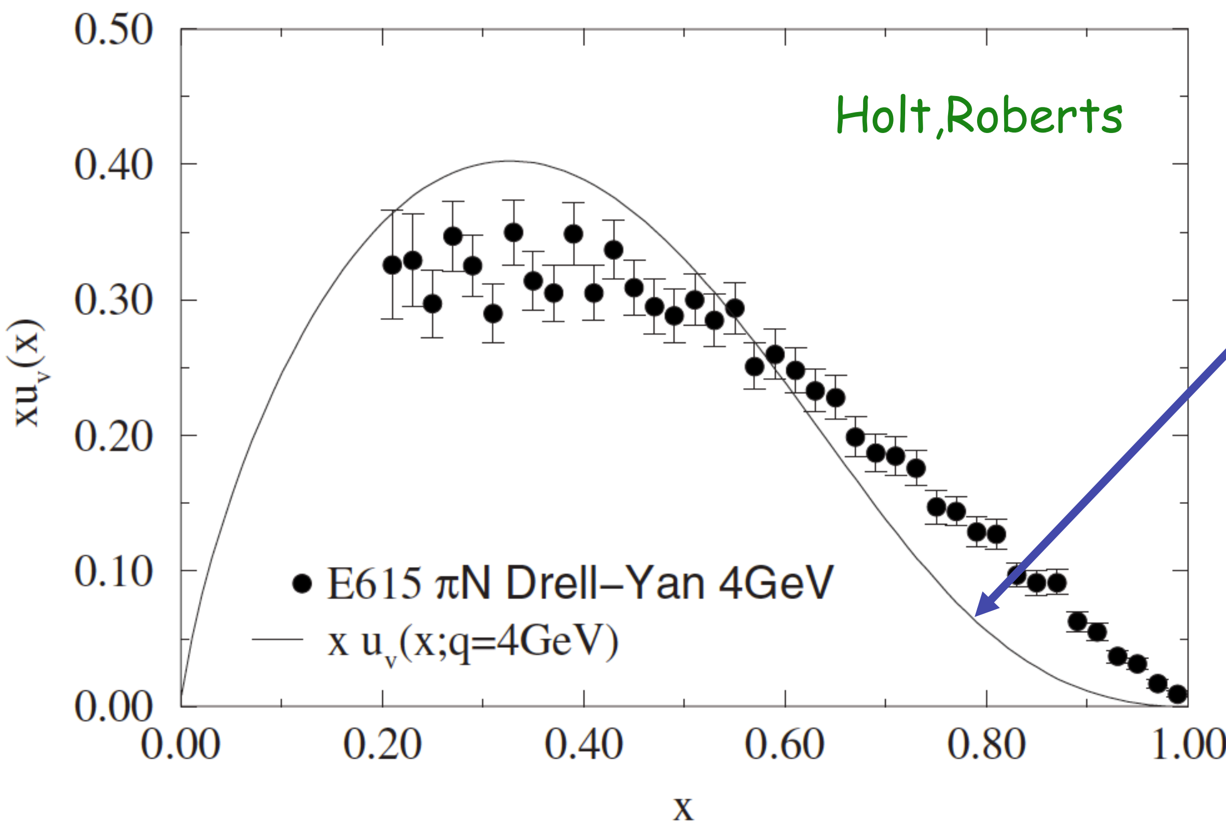
E615, NA10



$$d\sigma = \sum_{ab} \int dx_a \int dx_b \, f_a^\pi(x_a, \mu) f_b(x_b, \mu) \, d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu)$$

• Kinematics such that data mostly probe valence region: ~200 GeV pion beam on fixed target

- LO extraction of u_v from E615 data: $\sqrt{S} = 21.75\,\mathrm{GeV}$



$\sim (1-x)^2$

QCD counting rules

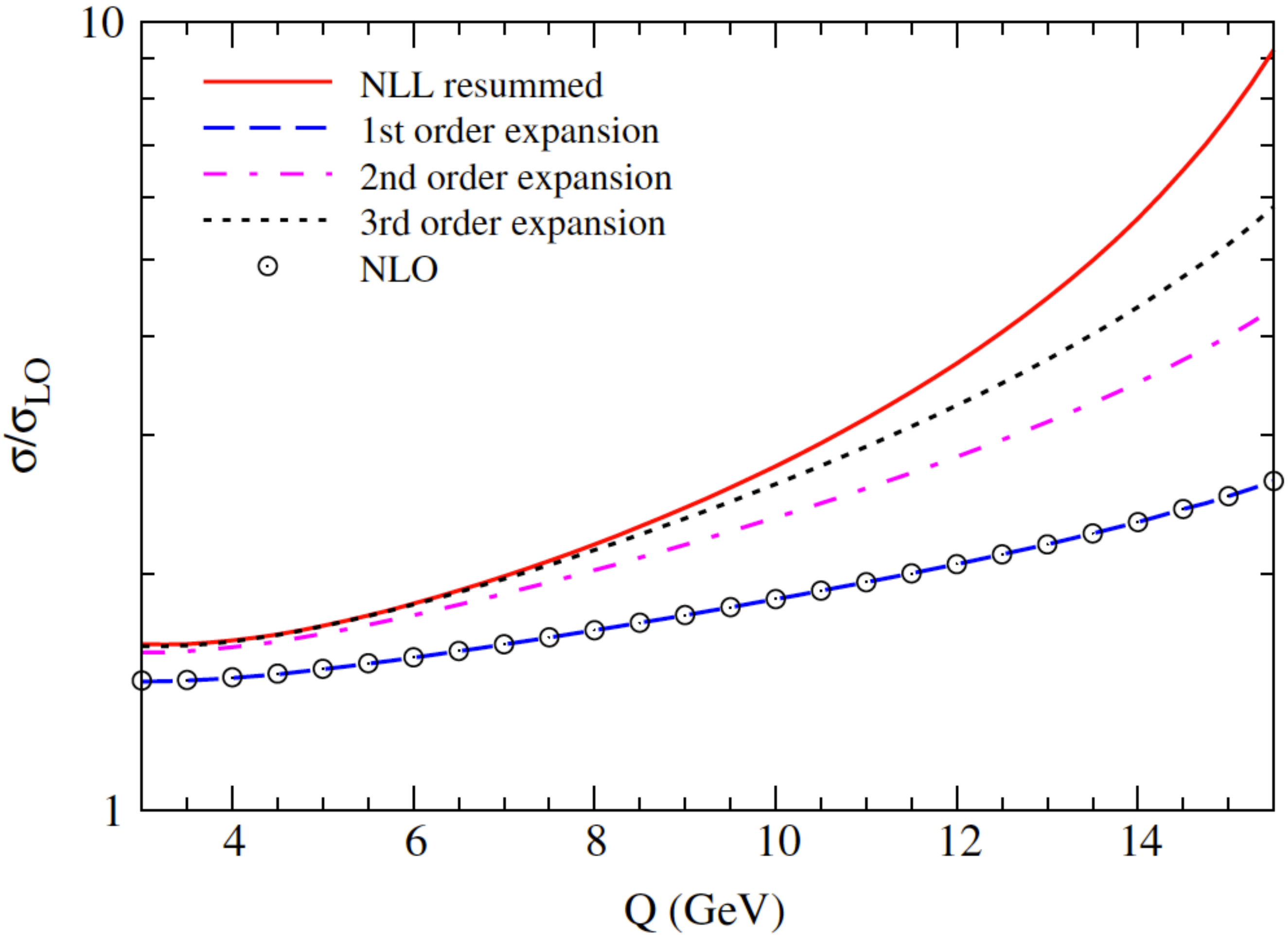
Farrar,Jackson;
Berger, Brodsky; Yuan
Blankenbecler,Gunion,
Nason

Dyson-Schwinger

Hecht et al.

(Compass kinematics)

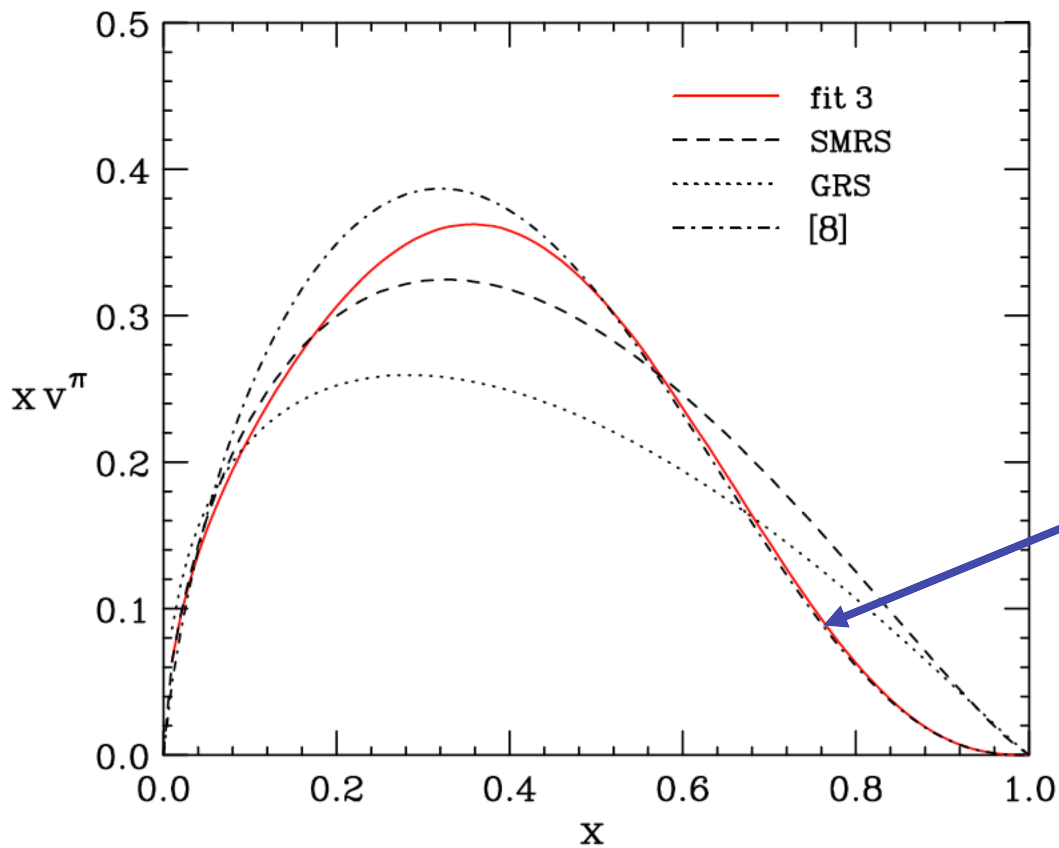
$\sqrt{S} = 19\,\mathrm{GeV}$



Aicher,Schäfer, WV
(earlier studies: Shimizu,Sterman,WV,Yokoya)

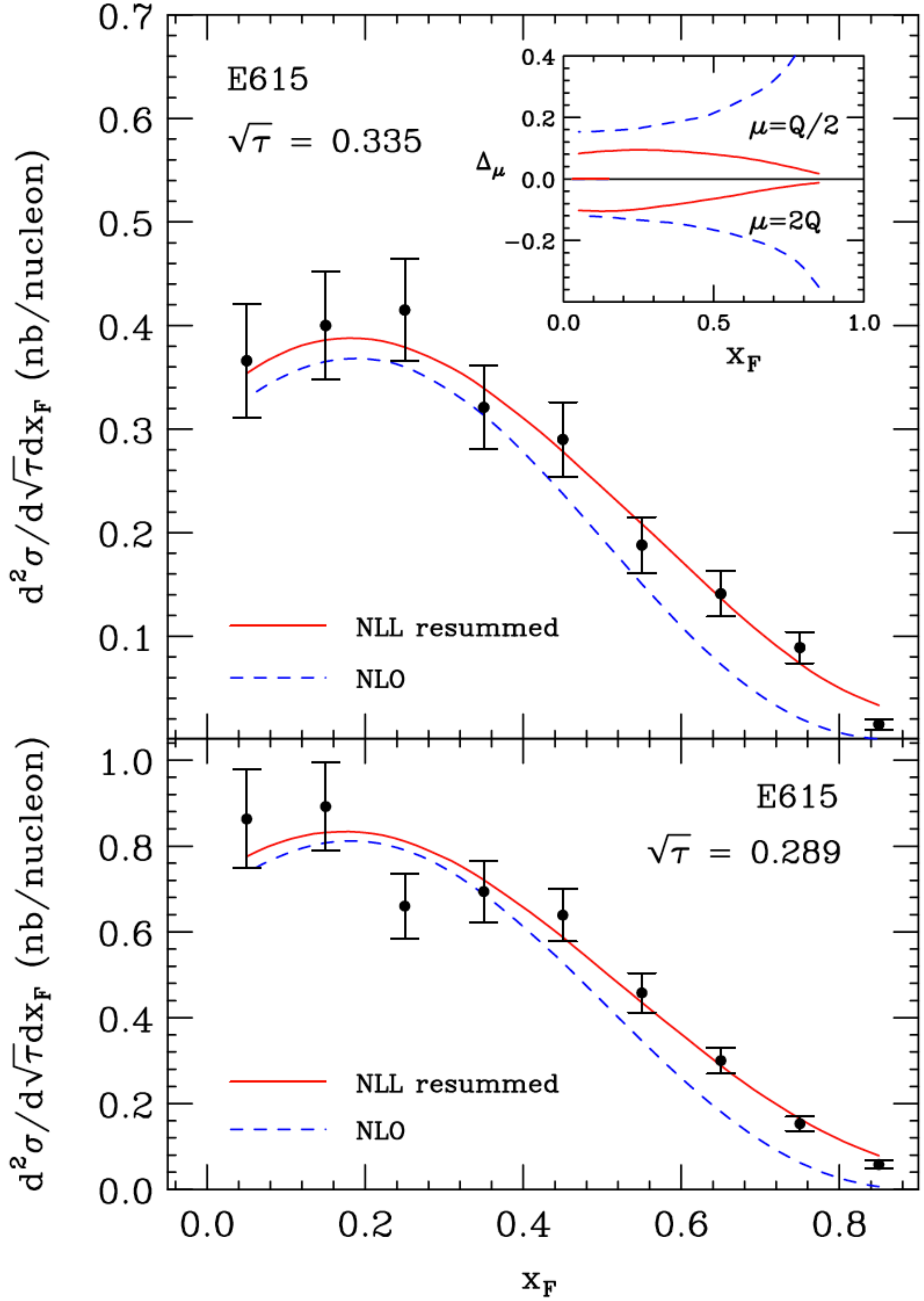
$$xv^\pi(x, Q_0^2) = N_v x^\alpha (1 - x)^\beta (1 + \gamma x^\delta)$$

Fit	$2\langle xv^\pi \rangle$	α	β	γ	K	χ^2 (no. of points)
1	0.55	0.15 ± 0.04	1.75 ± 0.04	89.4	0.999 ± 0.011	82.8 (70)
2	0.60	0.44 ± 0.07	1.93 ± 0.03	25.5	0.968 ± 0.011	80.9 (70)
3	0.65	0.70 ± 0.07	2.03 ± 0.06	13.8	0.919 ± 0.009	80.1 (70)
4	0.7	1.06 ± 0.05	2.12 ± 0.06	6.7	0.868 ± 0.009	81.0 (70)

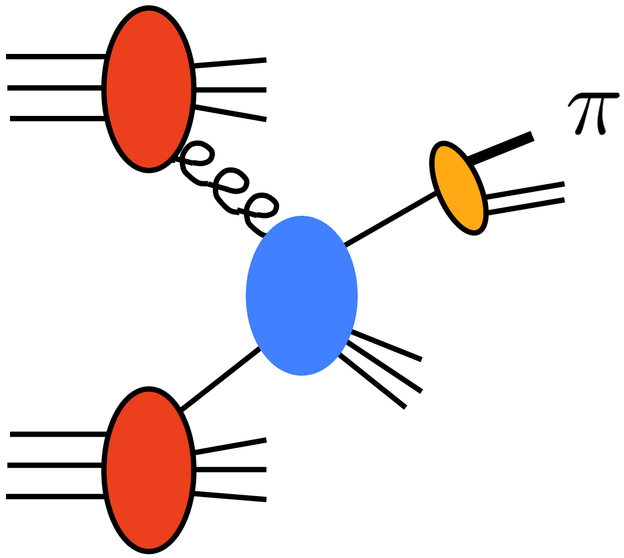


$Q = 4$ GeV

$\sim (1-x)^{2.34}$


E615
$\sqrt{\tau}$ = 0.335
NLL resummed
NLO
Δ_μ
μ=Q/2
μ=2Q
x_F
E615
$\sqrt{\tau}$ = 0.289
NLL resummed
NLO
$d^2\sigma/d\sqrt{\tau}dx_F$ (nb/nucleon)
x_F

Hadron production



$$\Big\}\; p_T\,,\eta \qquad x_T = \frac{2p_T}{\sqrt{S}}$$

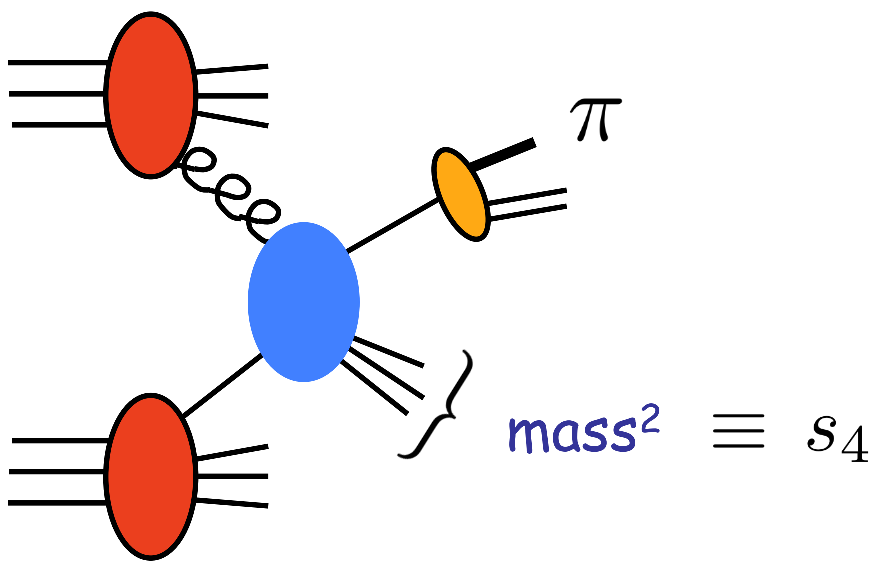
$$\frac{d\sigma}{dp_T d\eta} = \int_{x_a^0}^{1} dx_a \int_{x_b^0}^{1} dx_b \int_{z_c^0}^{1} dz_c \; f_a(x_a)\, f_b(x_b)\, \frac{d\hat{\sigma}_{ab\to c}}{dp_T d\eta}\, D_c(z_c)$$

$$x_a^0 = \frac{x_T \mathrm{e}^{\eta}}{2 - x_T \mathrm{e}^{-\eta}} \qquad x_b^0 = \frac{x_a x_T \mathrm{e}^{-\eta}}{2x_a - x_T \mathrm{e}^{\eta}}$$

$$z_c^0 = \frac{2p_T}{\sqrt{\hat{s}}}\cosh\left(\eta - \frac{1}{2}\ln\frac{x_a}{x_b}\right)$$

Partonic variables: $\hat{x}_T = \frac{2p_T}{z_c\sqrt{\hat{s}}}$ $\hat{\eta} = \eta - \frac{1}{2}\ln\frac{x_a}{x_b}$

$$\frac{d\sigma}{dp_T d\eta} = \int_{x_a^0}^{1} dx_a \int_{x_b^0}^{1} dx_b \int_{z_c^0}^{1} dz_c\, f_a(x_a)\, f_b(x_b)\, \frac{d\hat{\sigma}_{ab\to c}(\hat{x}_T, \hat{\eta})}{dp_T d\eta} D_c(z_c)$$



$$s_4 = \hat{s}\,(\,1 - \hat{x}_T\,\cosh\hat{\eta}\,)$$

LO :

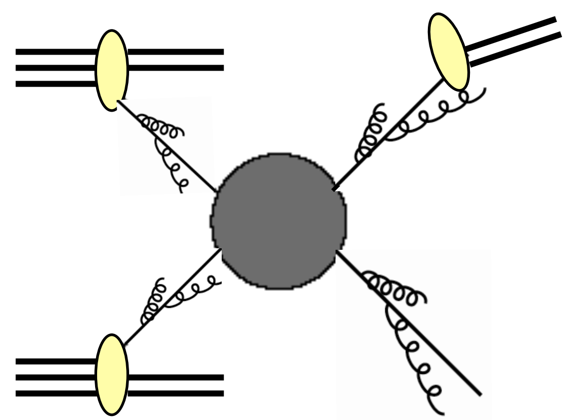
$$d\hat{\sigma}^{(\mathrm{LO})}_{ab\to c} \propto \delta\left(\frac{s_4}{\hat{s}}\right)$$

NLO :

$$d\hat{\sigma}^{(\mathrm{NLO})}_{ab\to c} \propto \alpha_s \left(\frac{\log(s_4/\hat{s})}{s_4/\hat{s}}\right)_+ + \ldots$$

yet higher orders:

$$d\hat{\sigma}^{(\mathrm{N^kLO})}_{ab\to c} \propto \alpha_s^k \left(\frac{\log^{2k-1}(s_4/\hat{s})}{s_4/\hat{s}}\right)_+ + \ldots$$



Resummation is more complicated now:

- color structure of hard scattering

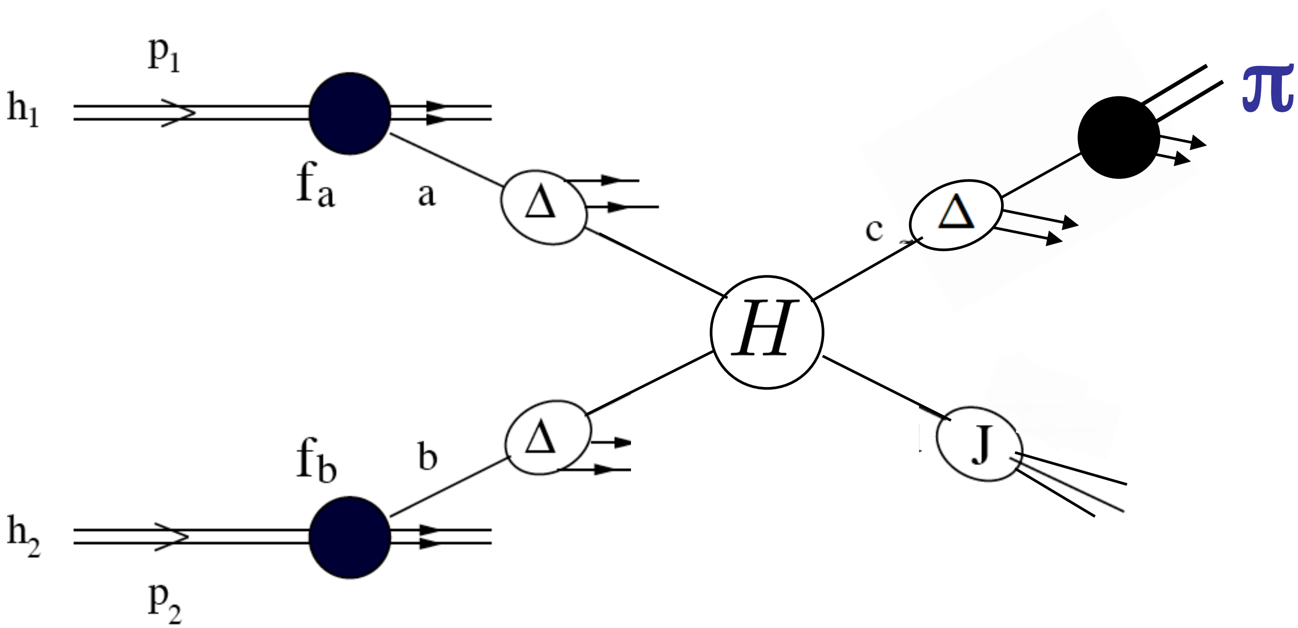
- cross section does not simplify under Mellin-moments

$$\int_0^1 \frac{ds_4}{\hat{s}} \left(1 - \frac{s_4}{\hat{s}}\right)^N \frac{d\hat{\sigma}^{\text{resum}}_{ab\to c}(\hat{x}_T, \hat{\eta})}{dp_T d\eta} = \Delta_a^N \, \Delta_b^N \, \Delta_c^N \; J^N_{\text{recoil}}$$

like DY

$$\times \sum_{IK} [H^{ab\to cd}_{IK} \, S^{ab\to cd}_{KI}] (\hat{\eta}, N)$$

Kidonakis,Oderda,Sterman
Bonciani,Catani,Mangano,Nason
Banfi,Salam,Zanderighi
Dokshitzer,Marchesini

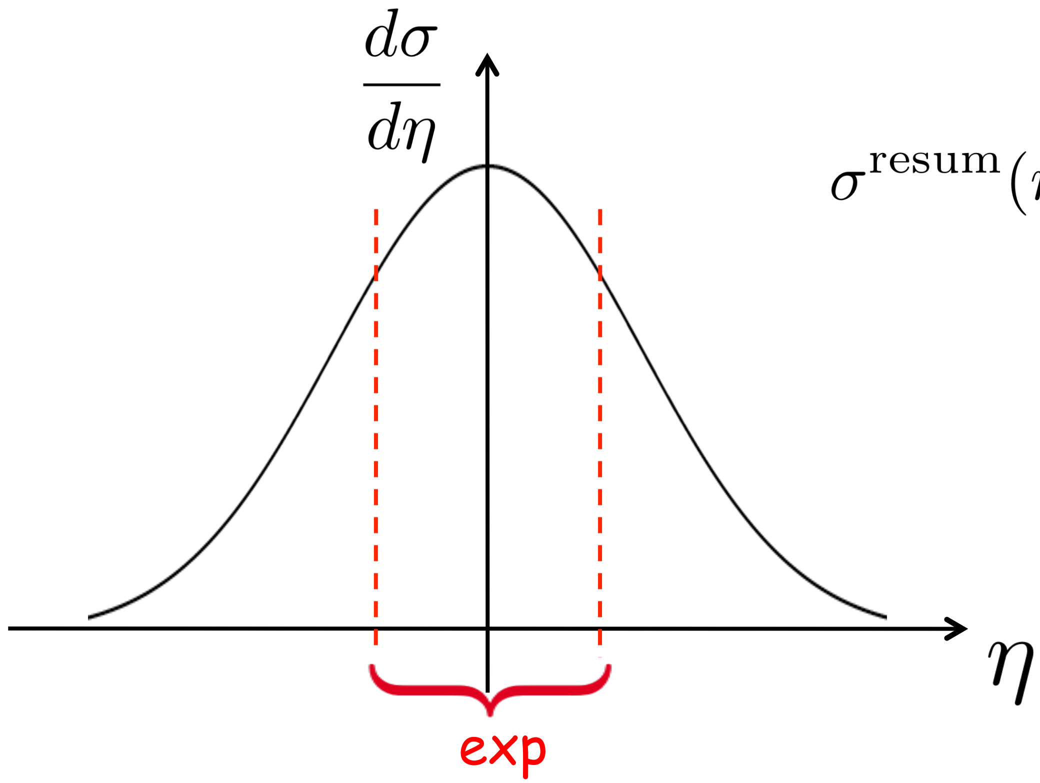


$$\int_{\text{full}} d\eta \;\; \frac{d\sigma}{dp_T d\eta}$$

$$= \int_{x_T^2}^{1} dx_a \int_{x_T^2/x_a}^{1} dx_b \int_{x_T/\sqrt{x_a x_b}}^{1} dz_c \; f_a(x_a)\, f_b(x_b)\, \frac{d\hat{\sigma}_{ab\to c}\left(\hat{x}_T = \frac{x_T}{z_c\sqrt{x_a x_b}}\right)}{dp_T}\, D_c(z_c)$$

→ used in studies until recently

de Florian, WV, ...

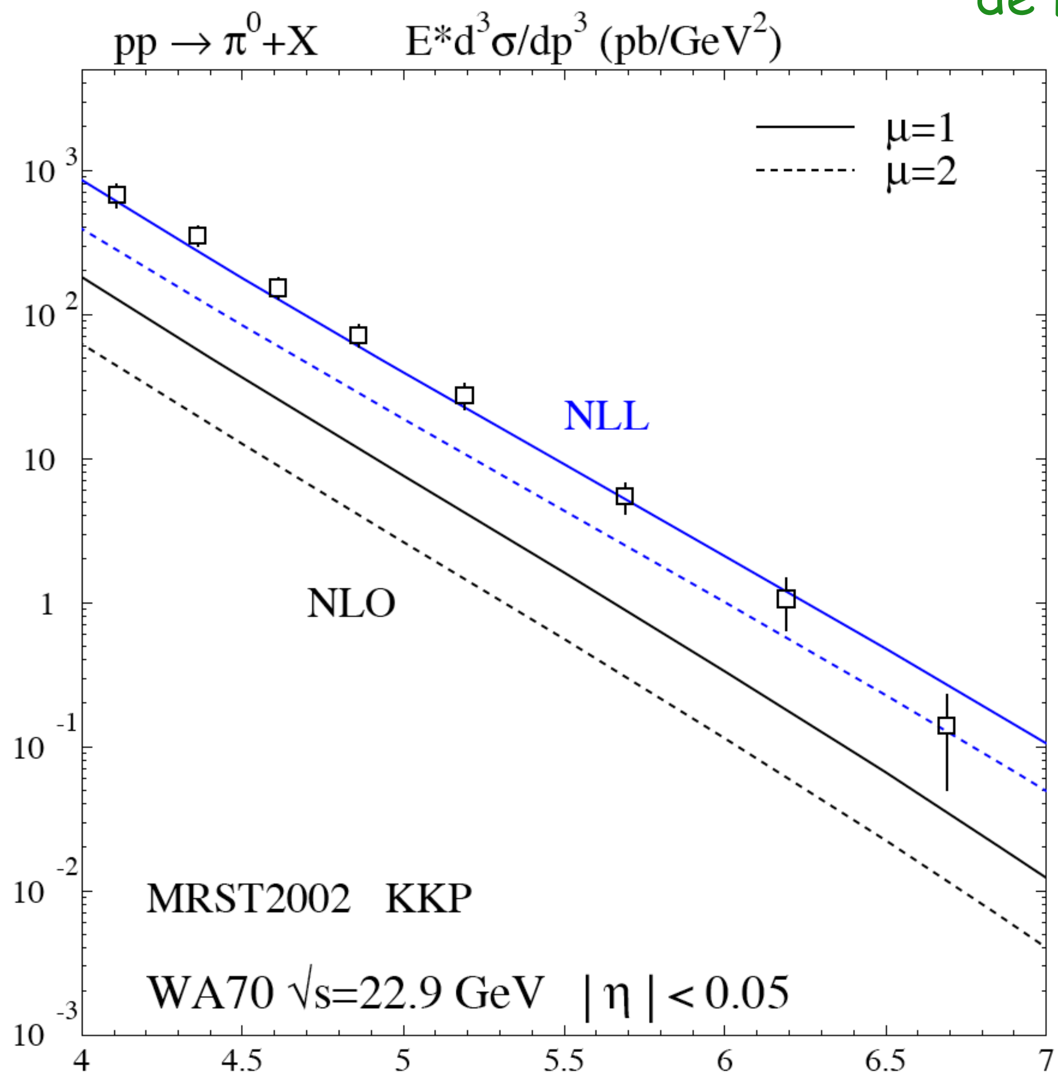


$$\sigma^{\text{resum}}(\eta \in \text{exp}) \;\approx\; \sigma^{\text{resum}}(\text{all}\ \eta)$$

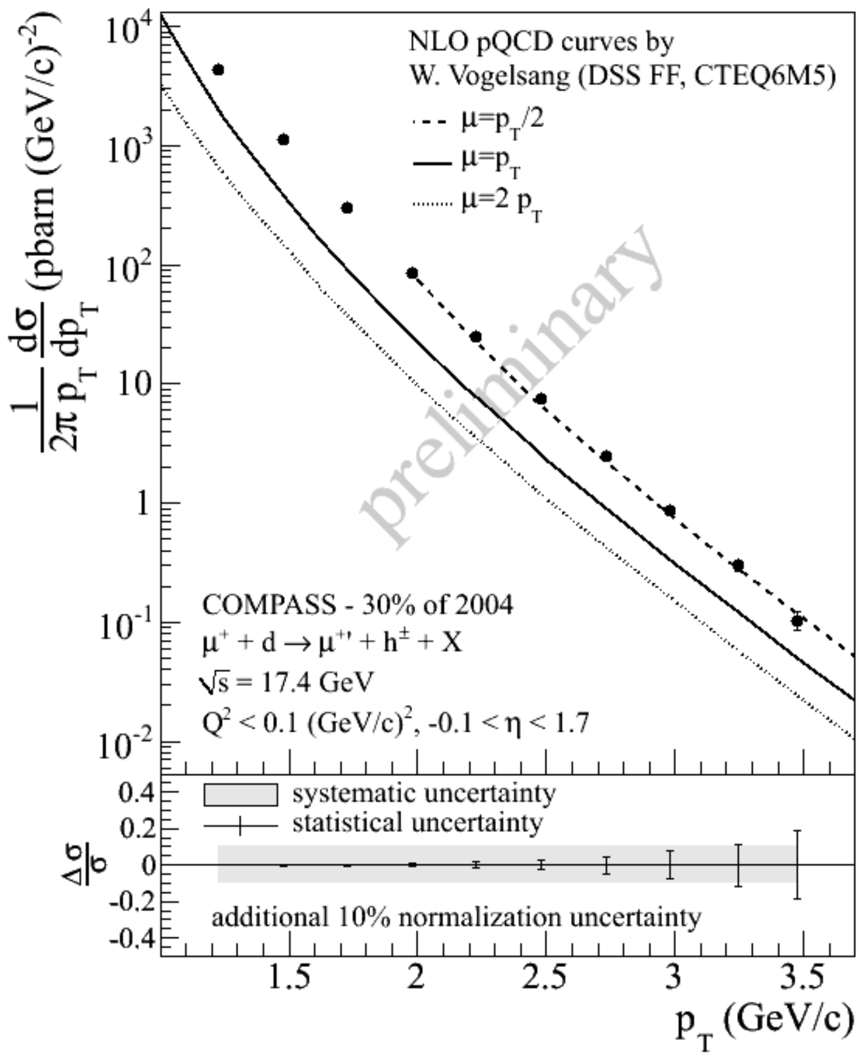
$$\times\; \frac{\sigma^{\text{NLO}}(\eta \in \text{exp})}{\sigma^{\text{NLO}}(\text{all}\ \eta)}$$

WA70

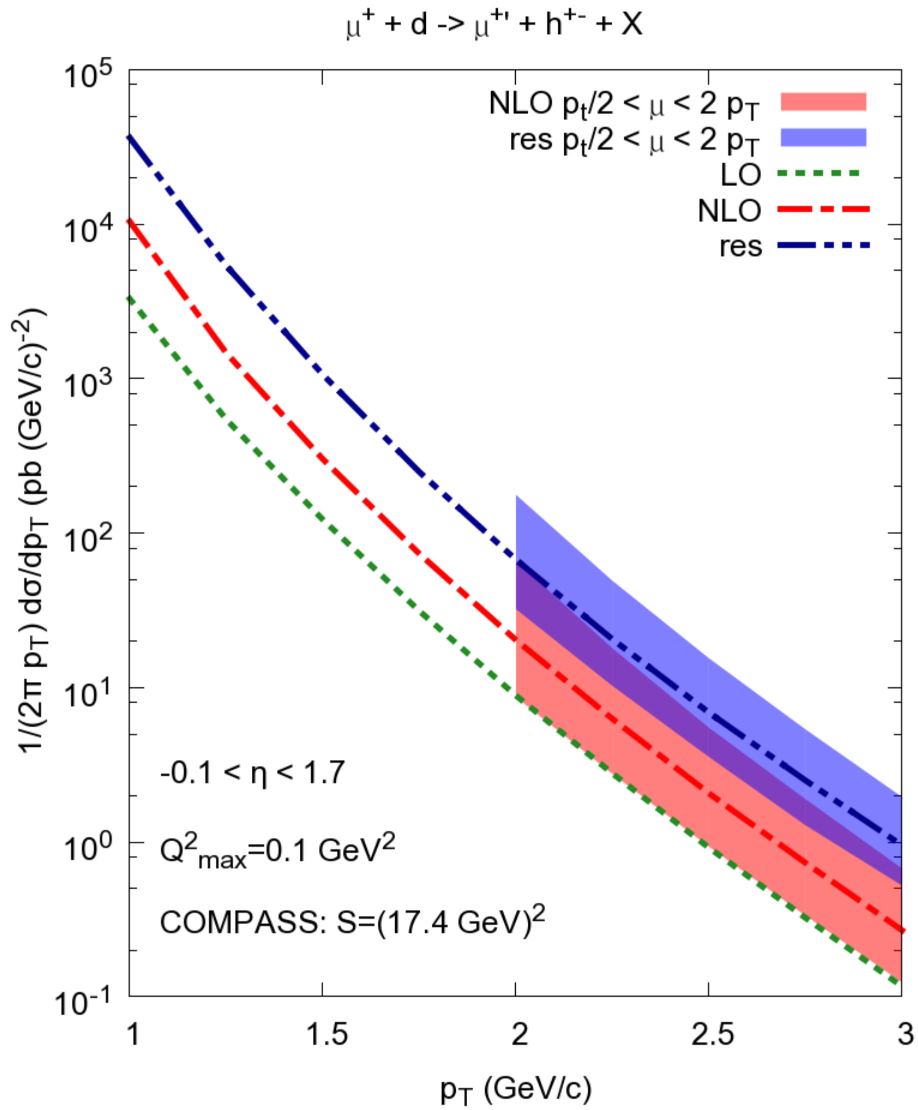
de Florian, WV



(effects at RHIC more modest)



COMPASS



de Florian, Pfeuffer, Schäfer, WV (prel.)

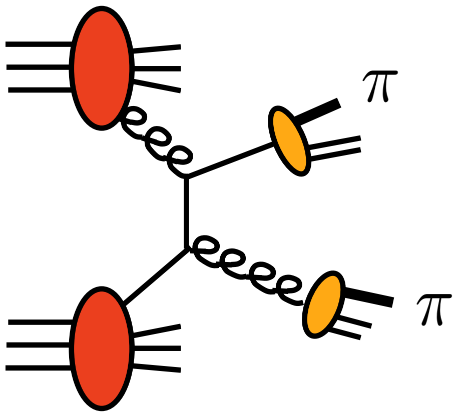
$$\frac{d\sigma}{dp_T d\eta} = \int_{x_a^0}^1 dx_a\, f_a(x_a) \int_{x_b^0}^1 dx_b\, f_b(x_b) \boxed{\int_{z_c^0}^1 dz_c\, \frac{d\hat{\sigma}_{ab\to c}(\hat{x}_T, \hat{\eta})}{dp_T d\eta} D_c(z_c)}$$

$$\left(\frac{z_c^0}{z_c}, \hat{\eta}\right)$$

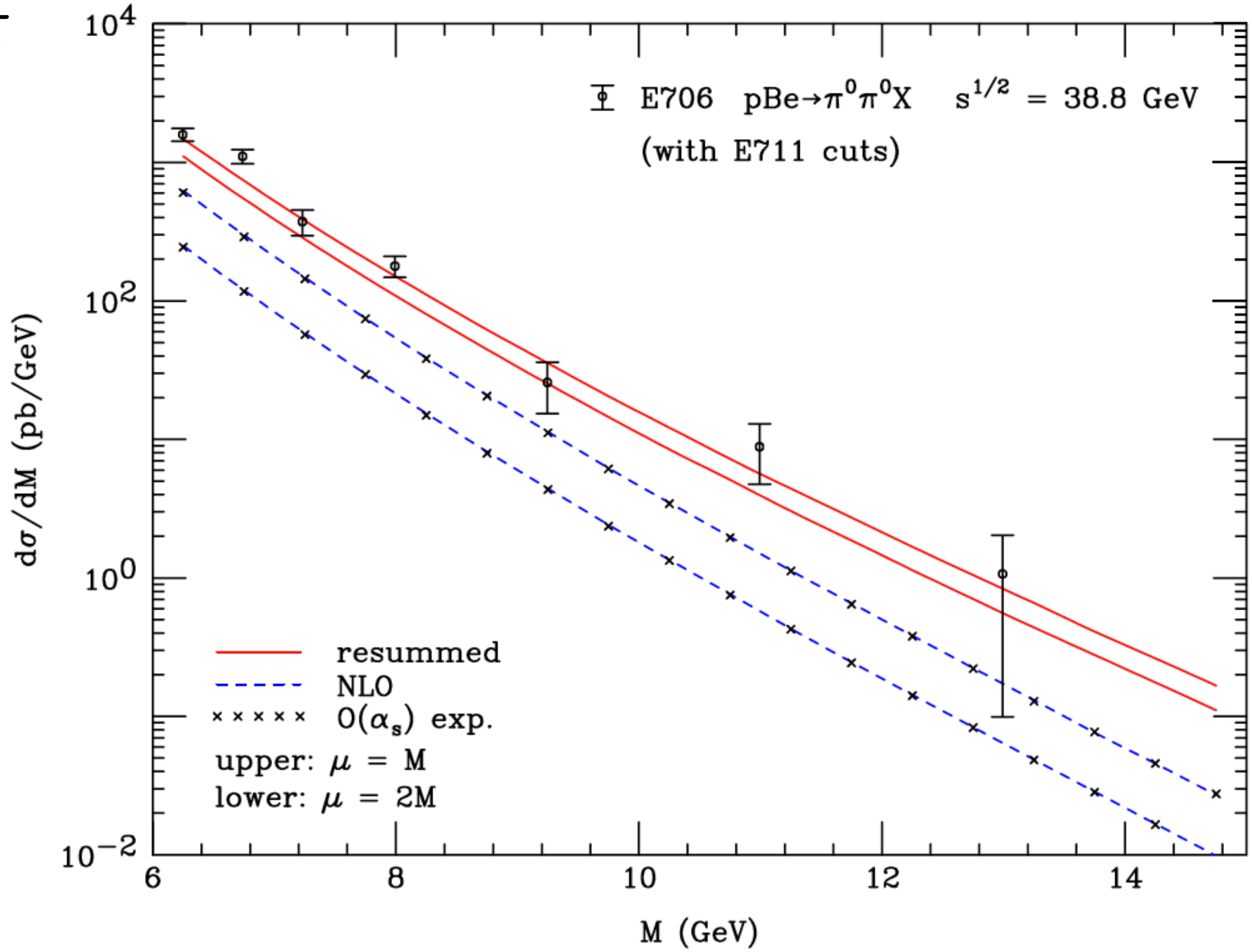
$$z_c^0 = \frac{2p_T}{\sqrt{\hat{s}}} \cosh(\hat{\eta}) = z_c\, \hat{x}_T \cosh(\hat{\eta})$$

- [box] factorizes under Mellin-moments !

- technique allows to do resummation at fixed rapidity

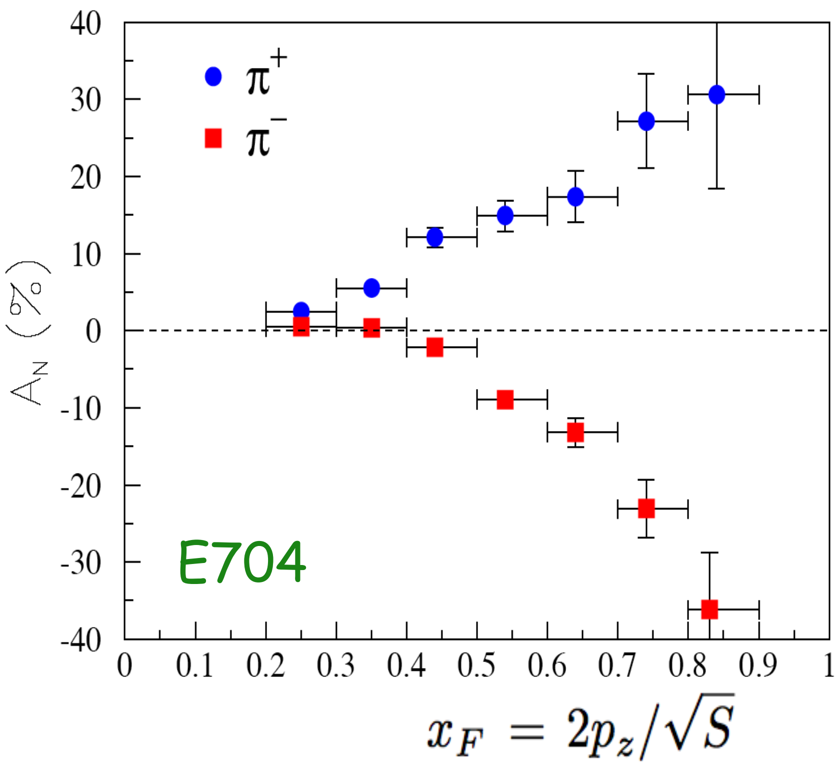


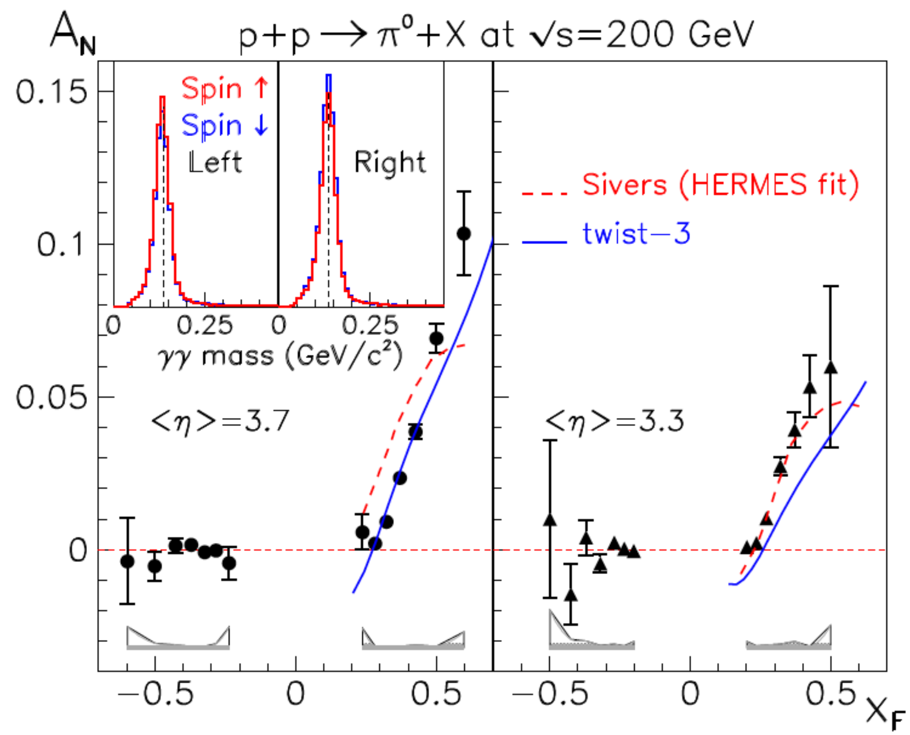
Almeida,Sterman,WV



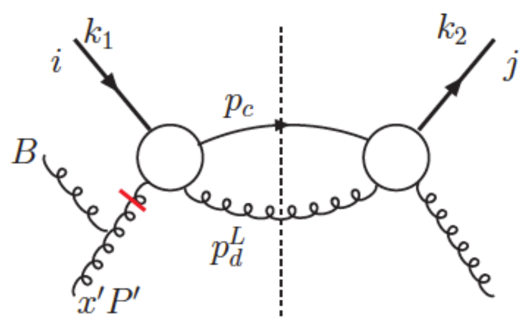
- should be very relevant for single-spin asymmetries in pp → πX

STAR





Used to extract TF:



Qiu,Sterman
Kouvaris et al.
Kanazawa,Koike
Kang,Prokudin

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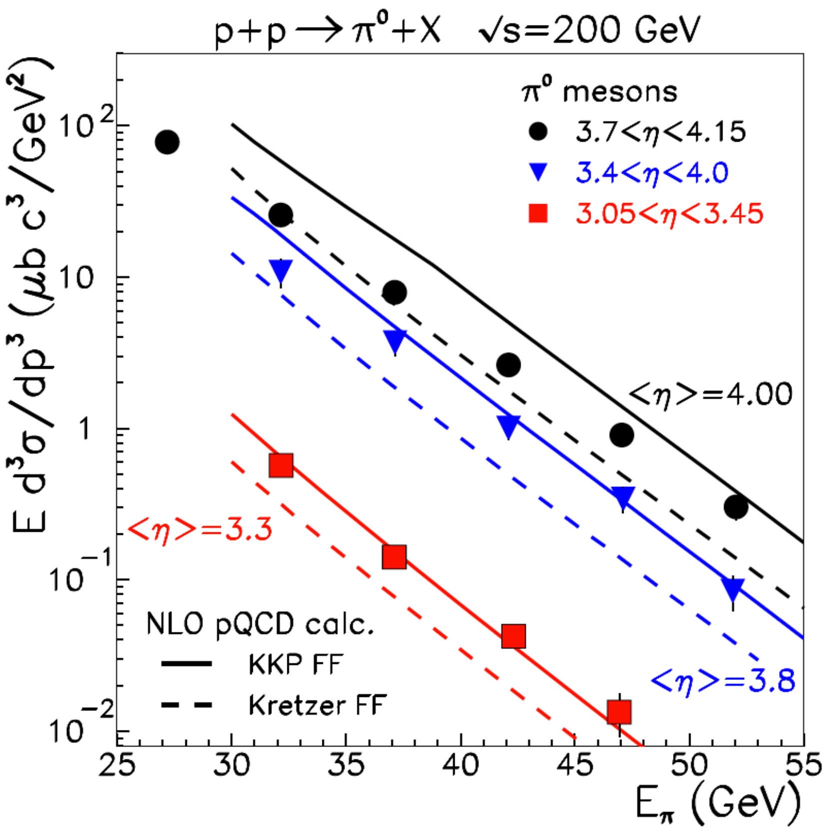
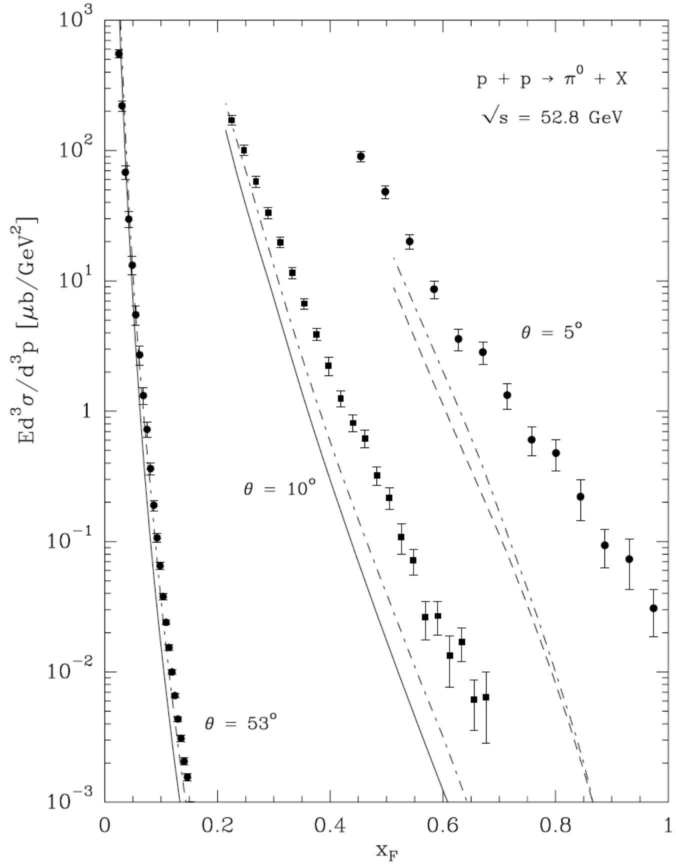




Bourrely,Soffer

- expect large corrections at high-x_F
- cf. NLO calculation of Drell-Yan single-spin asymm.

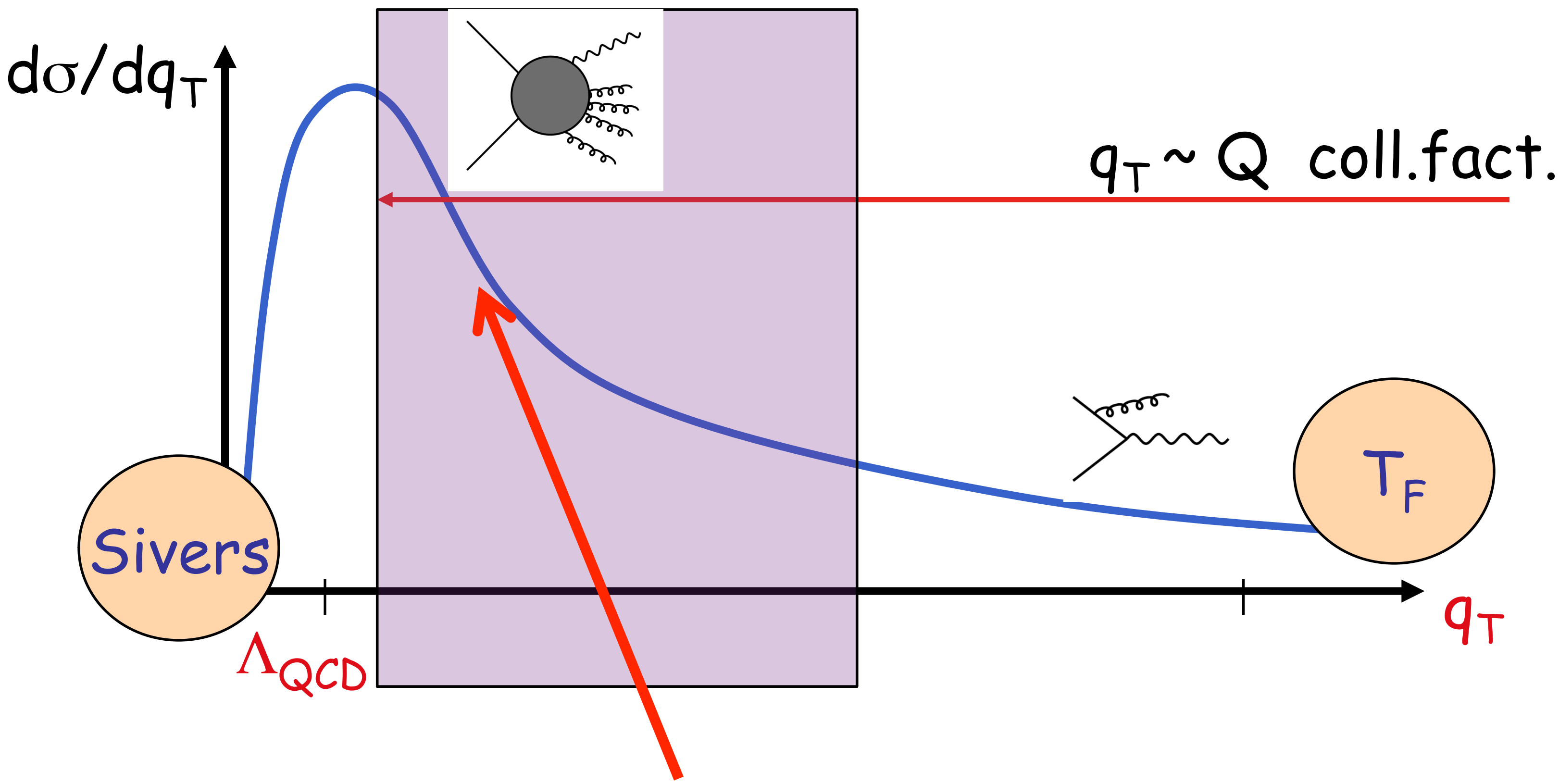
$$\frac{d\langle q_\perp \Delta\sigma(S_\perp)\rangle}{dQ^2}$$

Yuan,WV

$$= \sigma_0 \frac{\alpha_s}{2\pi} \int \frac{dx}{x}\frac{dx'}{x'} T_F(x,x;\mu^2)\bar{q}(x';\mu^2)\left[8C_F\left(\frac{\ln(1-z)}{1-z}\right)_+ + \ldots\right]$$

- affect phenomenological extraction of T_F ?

Two-scale processes



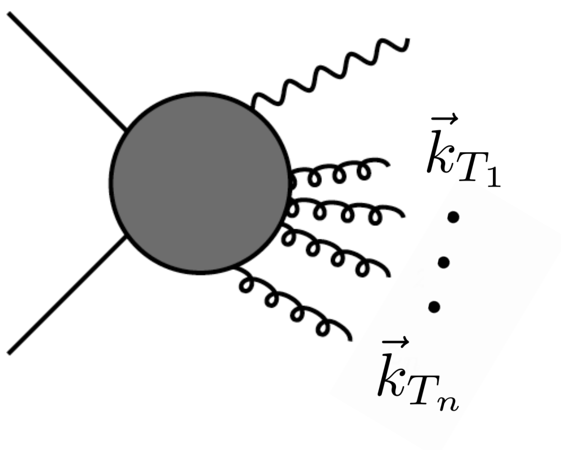
well-known feature: emergence of Sudakov logarithms

$$\alpha_s^k \; \frac{\log^{2k-1}\left(\frac{Q^2}{q_T^2}\right)}{q_T^2} + \ldots$$

- these logs are related (although not identical) to the threshold logs

- all-order resummation for "ordinary" cross section understood for long time

Collins, Soper, Sterman; ...



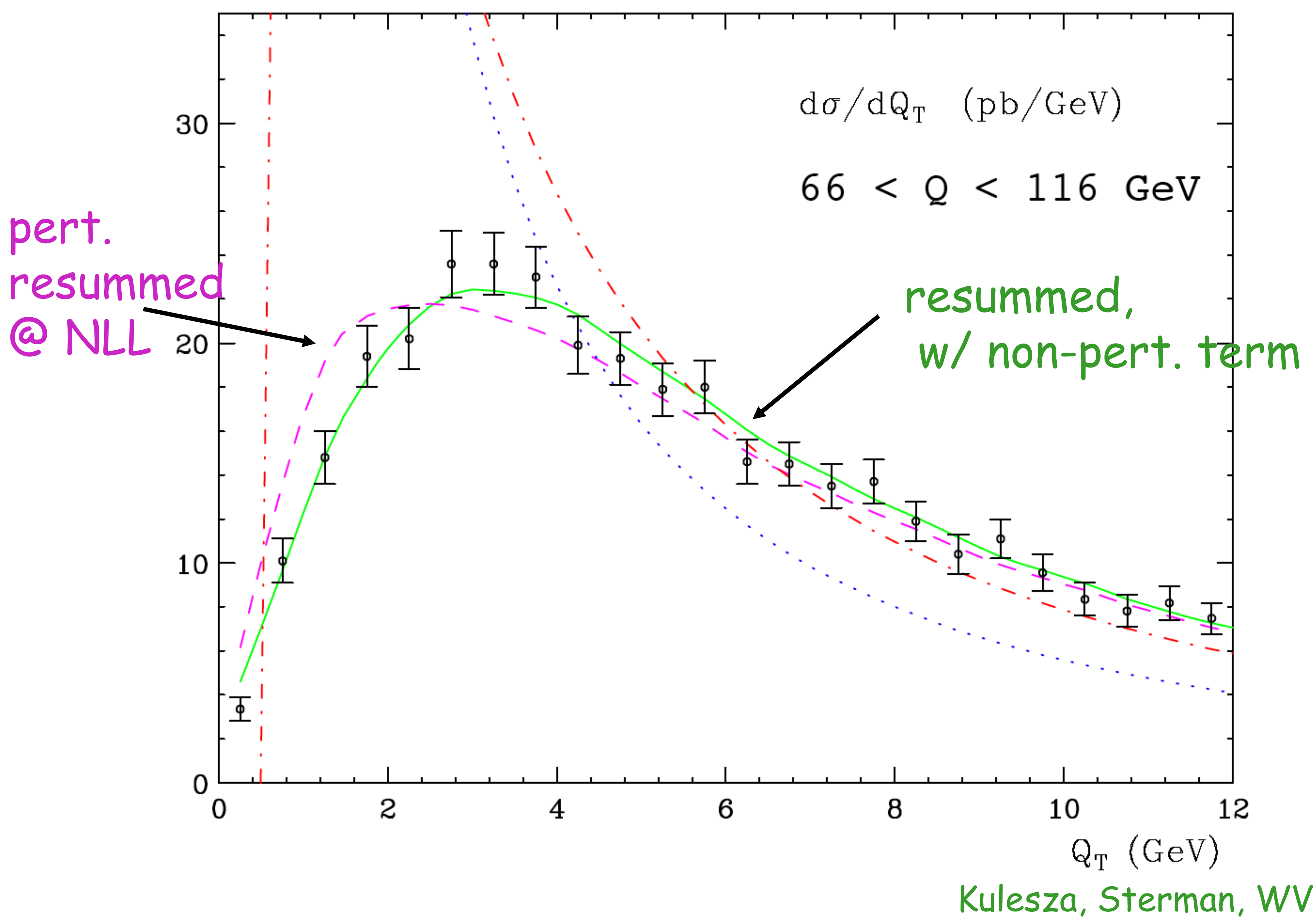
$$\delta^2\left(\vec{q}_T + \sum_i \vec{k}_{T_i}\right) = \frac{1}{(2\pi)^2}\int d^2b\, \mathrm{e}^{-i\vec{b}\cdot\left(\vec{q}_T + \sum_i \vec{k}_{T_i}\right)}$$

$$\frac{\log^{2k-1}\left(\frac{Q^2}{q_T^2}\right)}{q_T^2} \leftrightarrow \log^{2k}(bQ)$$

$$\hat{\sigma}^{(\mathrm{resum})}(b) \propto \exp\left[2\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\left(J_0(bk_\perp)-1\right)\left\{A_q(\alpha_s(k_\perp^2))\log\frac{Q^2}{k_\perp^2}+\ldots\right\}\right]$$

(Sudakov exponent)

- logs suppress cross section !



pert.
resummed
@ NLL
resummed,
w/ non-pert. term
dσ/dQ_T (pb/GeV)
66 < Q < 116 GeV
0
10
20
30
0
2
4
6
8
10
12
Q_T (GeV)
Kulesza, Sterman, WV

- great strides forward recently on resummation formalism for single-spin observables

Kang,Xiao,Yuan
Aybat,Collins,Qiu,Rogers

- Kang,Xiao,Yuan: use full NLO calculation of $d\sigma/dq_T$ (in b-space)

$$\sigma_{UT}(b) \sim \frac{\alpha_s}{2\pi}\left(\frac{-ib_\perp^\alpha}{2}\right)\left\{\left[-\frac{1}{\epsilon}+\ln\frac{4}{b^2\mu^2}e^{-2\gamma_E}\right]\left(\mathcal{P}_{q/q}\otimes\bar{q}(z_2')\right.\right.$$
$$\left.+\mathcal{P}^T_{qg\to qg}\otimes T_F(z_1')\right)+C_F(1-\xi_2)\delta(1-\xi_1)$$
$$+\left(-\frac{1}{2N_c}\right)(1-\xi_1)\delta(1-\xi_2)+\delta(1-\xi_1)\delta(1-\xi_2)$$
$$\left.\times\left[-\ln^2\left(\frac{Q^2b^2}{4}e^{2\gamma_E-\frac{3}{2}}\right)-\frac{23}{4}+\pi^2\right]\right\}$$

- Aybat,Collins,Qiu,Rogers:
  organize in terms of simple parton-model TMD-like formula

$$\frac{d\sigma}{dq_\perp^2} \sim \sigma_0 \int d^2k_{\perp,1} \int d^2k_{\perp,2}\, F(x_1, k_{\perp,1}, Q)\, \bar{F}(x_2, k_{\perp,2}, Q)\, \delta^{(2)}(\vec{k}_{\perp,1} + \vec{k}_{\perp,2} - \vec{q}_\perp)$$

- find (at large Q)

collinear piece

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1\, d\hat{x}_2}{\hat{x}_1\, \hat{x}_2} \tilde{C}^{\rm Sivers}_{f/j}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b))\, T_{F\, j/P}(\hat{x}_1, \hat{x}_2, \mu_b)$$

~ Sudakov

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\}$$

$$\times \exp\left\{ -g^{\rm Sivers}_{j/P}(x, b_T) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}$$

non-perturbative piece

- represents "evolution" of TMDs

Reason for non-perturbative piece:

$$\hat{\sigma}^{(\mathrm{resum})}(b) \;\propto\; \exp\left[\frac{2C_F}{\pi}\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,(J_0(bk_\perp)-1)\,\left\{\alpha_s(k_\perp^2)\,\log\frac{Q^2}{k_\perp^2}+\ldots\right\}\right]$$

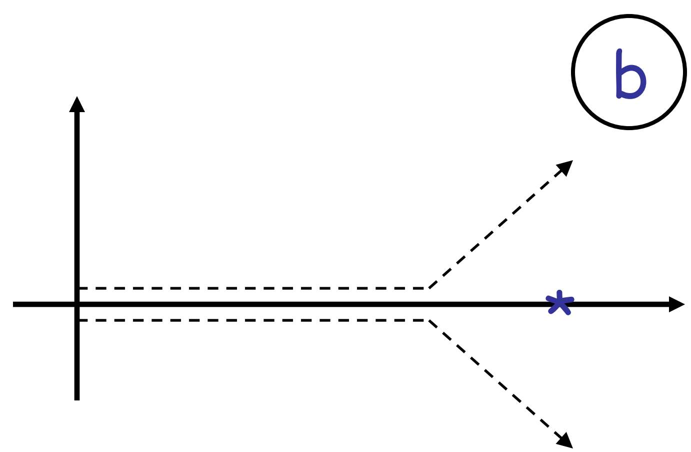
Logarithms are contained in

$$\exp\left[-\frac{2C_F}{\pi}\int_{1/b^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,\left\{\alpha_s(k_\perp^2)\,\log\frac{Q^2}{k_\perp^2}+\ldots\right\}\right]$$

→ needs prescription for dealing with large-b

$$\int d^2b\,\mathrm{e}^{-i\vec{b}\cdot\vec{q}_T}\,[\ldots]$$

e.g. $b^* \equiv \dfrac{b}{\sqrt{1+b^2/b_{\mathrm{max}}^2}}$ or



Contribution from very low $k_\perp$

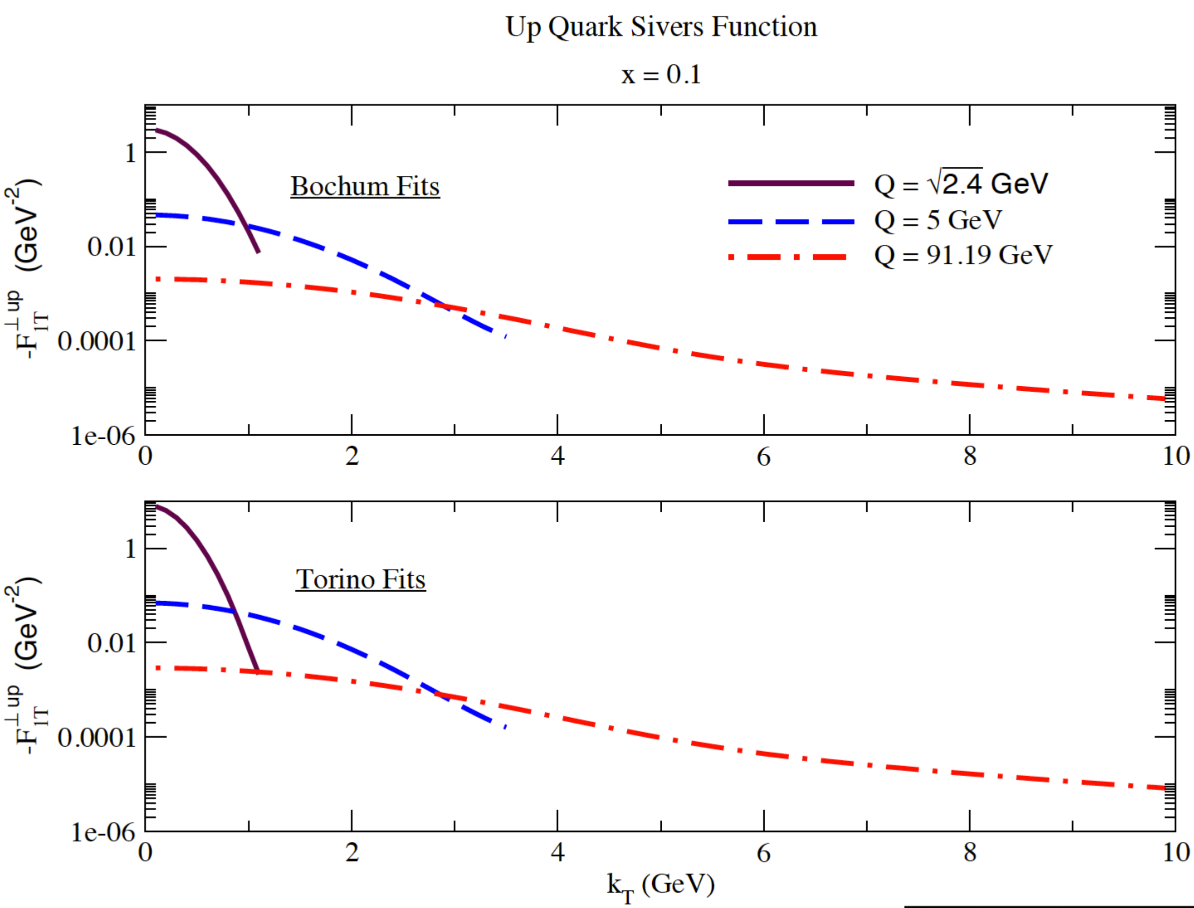
$$\exp\left[-b^2\,\frac{C_F}{\pi}\int dk_\perp^2\,\alpha_s(k_\perp^2)\,\log\left(\frac{Q}{k_\perp}\right)\right]$$

$$\underbrace{\qquad\qquad}_{g_1\;+\;g_2\,\log\left(\frac{Q}{Q_0}\right)}$$

- suggests Gaussian non-pert. contribution with logarithmic Q dependence
- expected to be universal (unpol. <-> Sivers)
- "global" fits

  Davies, Webber, Stirling; Landry et al., Ladinsky, Yuan; Qiu, Zhang; Konychev,Nadolsky

- values of g_1,g_2 depend on treatment of large-b region !

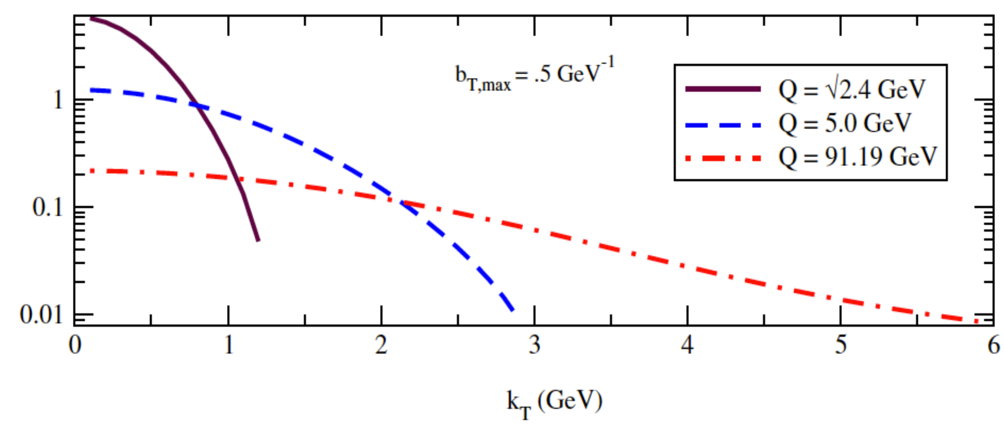


Evolution of Sivers fcts. (Gaussian)

Aybat, Collins, Qiu, Rogers

cf. unpolarized TMD

Aybat, Rogers

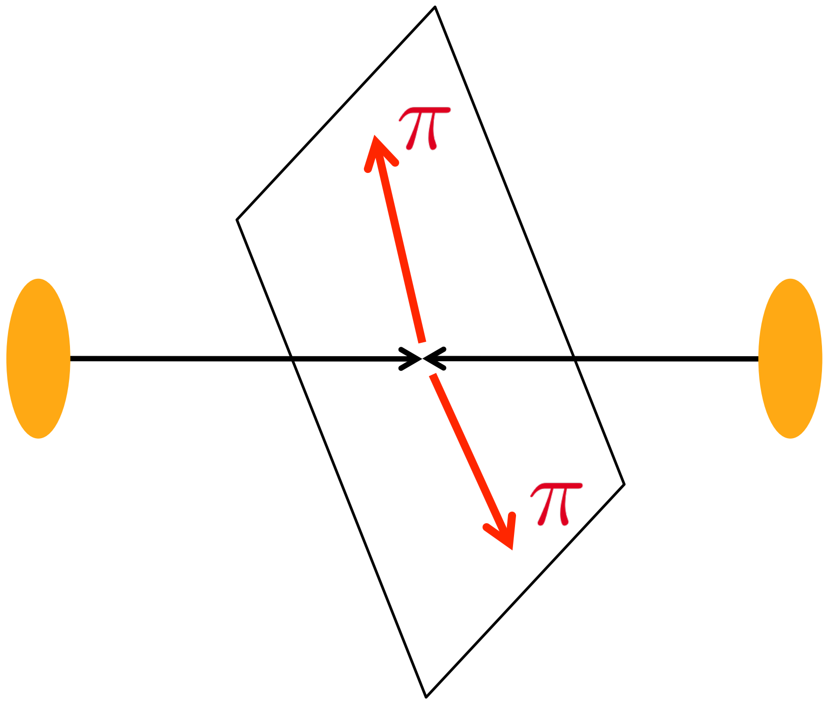


- evolution for Sivers more sensitive to large b

Interesting "follow-up":

- we know there is no TMD factorization for general QCD hard scattering

Bomhof, Mulders, Pijlman;
Collins, Qiu; Mulders,Rogers



- gauge links in parton distributions "know" about full hard process

- what does this imply for perturbative resummation ?

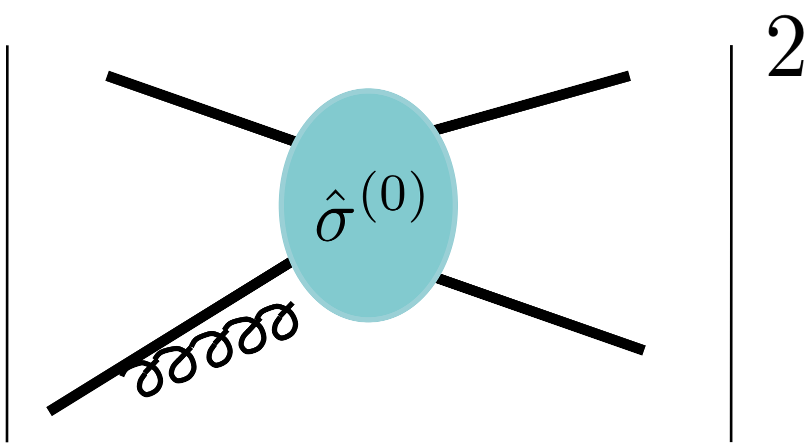
- at some order, breakdown of "standard" formula should occur

- recent study of collinear singularities at high orders

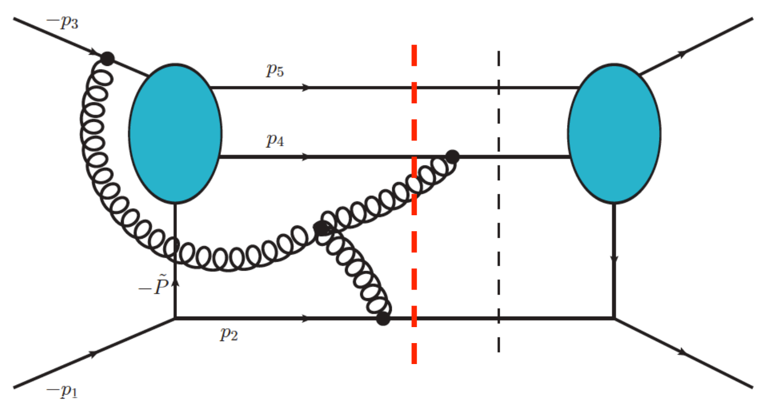
Space-like (vs. time-like) collinear limits in QCD: is factorization violated?

Stefano Catani [(a)], Daniel de Florian [(b)(c)] and Germán Rodrigo [(d)]

usually,



$$\sim \; P_{qq} \otimes \hat{\sigma}^{(0)}$$



Conclusions:

- numerous applications of QCD resummation to hadronic scattering:
  Threshold-resum. / qT-resum. → "joint" resummation?
  Laenen,Sterman,WV
- many are relevant for the processes we use to determine nucleon structure
- great recent progress, in particular in TMD area