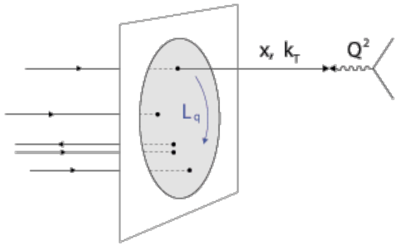
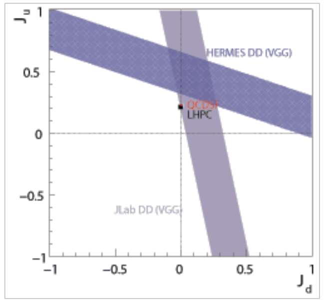
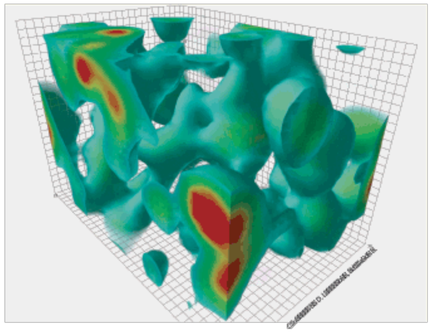
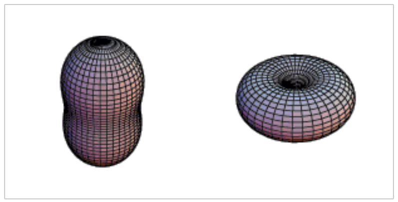








Dispersion Analysis of DVCS

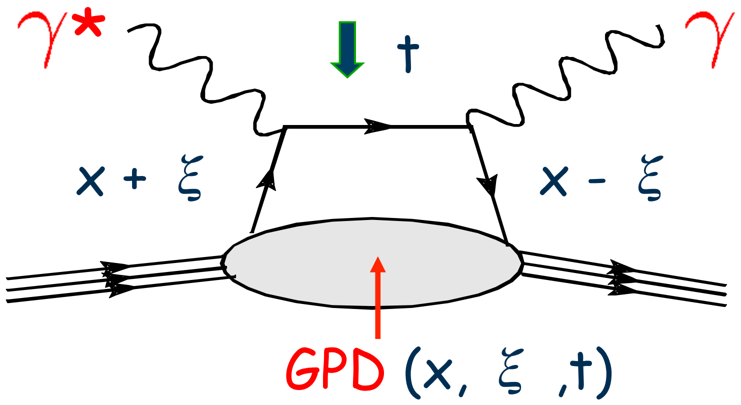
Marc Vanderhaeghen
Johannes Gutenberg Universität, Mainz

INT Workshop "Orbital Angular Momentum in QCD"

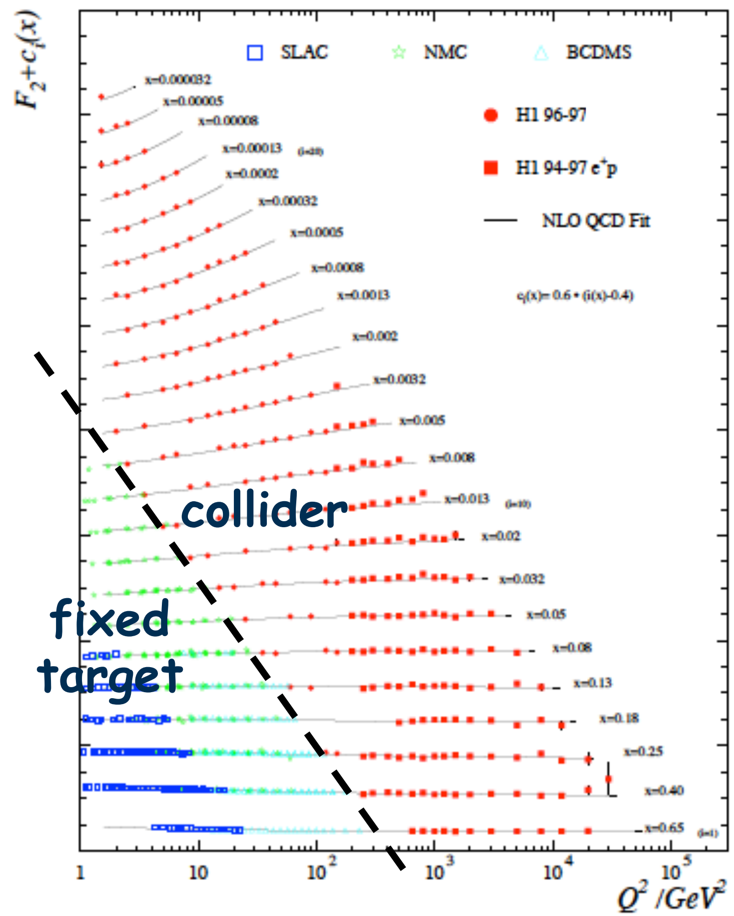
Seattle, February 6 - 17, 2012

QCD factorization : tool to access GPDs

Q^2 >> 1 GeV^2



world data on proton **F2**



➡ at large Q^2 : **QCD factorization theorem** :
hard exclusive process described by GPDs
model independent !

Müller et al. (1994),
Ji (1995), Radyushkin (1995),
Collins, Frankfurt, Strikman (1996)

➡ **KEY** Q^2 leverage required to test
QCD scaling

Outline

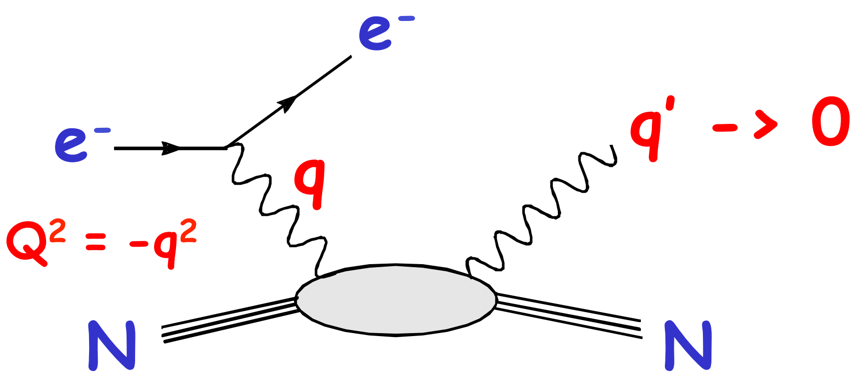
- Dispersion analysis for Virtual Compton Scattering (VCS) at low-energy
- Dispersion formalism for twist-2 DVCS amplitudes in terms of GPDs
- DVCS observables

Introduction :

Dispersion Analysis
for
VCS at low-energy

VCS in low-energy region

VCS :



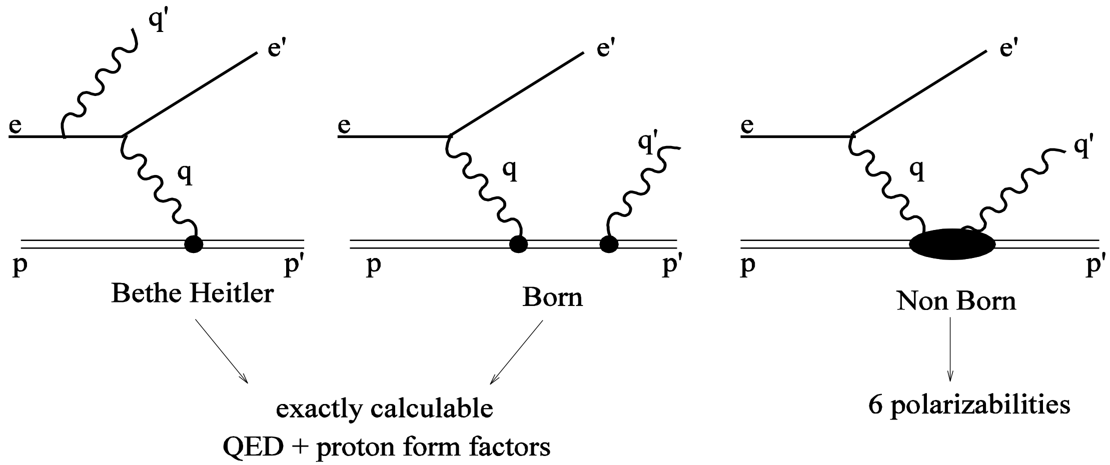
low energy outgoing photon plays
role of applied e.m. dipole field

nucleon response :
Generalized Polarizabilities (GP)
$\alpha(Q^2)$, $\beta(Q^2)$, and 4 spin GPs

Guichon et al. (1995) Drechsel et al. (1998)

VCS observables in low-energy region

$$T^{ee'\gamma} = T^{BH} + T^{VCS}_{Born} + T^{VCS}_{non-Born}$$



➡ Low-energy expansion (in q') : only valid below pion threshold

$$d\sigma \sim \frac{C_{-2}}{q'^2} + \frac{C_{-1}}{q'} + C_0 + \mathcal{O}(q')$$

C_{-2} and C_{-1} : completely fixed by BH + Born

C_0 : depends linearly on 6 low-energy constants -> generalized polarizabilities

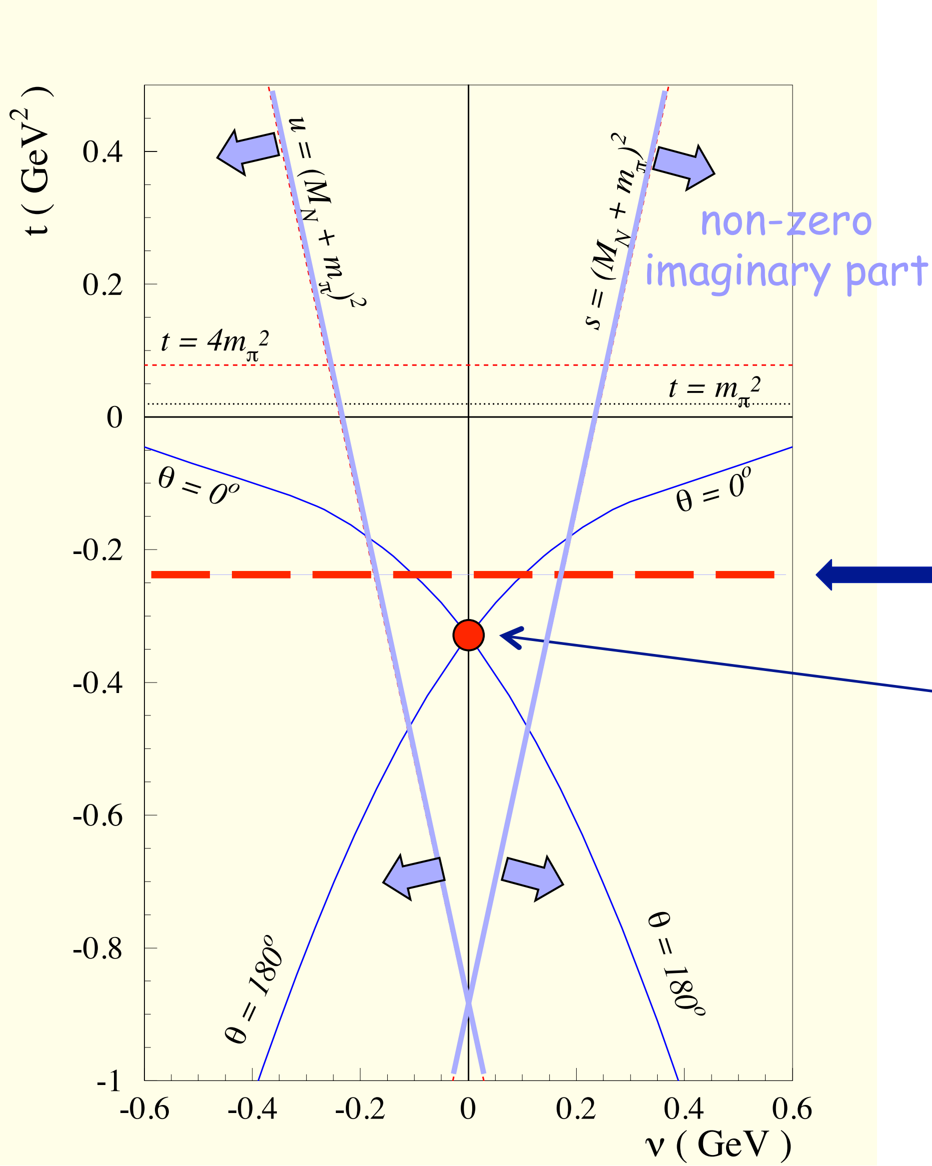
Guichon, Liu, Thomas (1995)

➡ Dispersion relations (in q' or s) : valid both below and above pion threshold

describe full energy dependence : low-energy constants are extracted as subtraction constants from data

Pasquini, Drechsel, Gorchtein, Metz, Vdh (2001)

Mandelstam plane for VCS



crossing symmetry variable

$$\nu = \frac{s-u}{4M_N}$$

Dispersion Relation at

fixed t (for a **fixed Q²**)

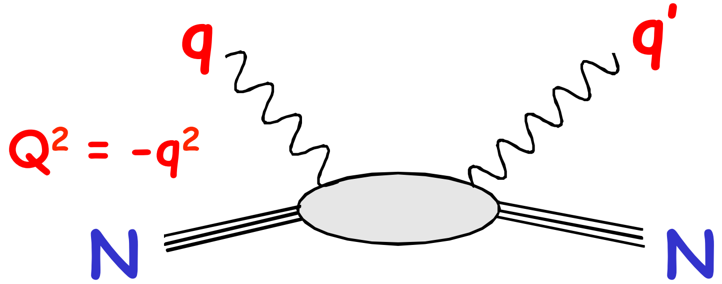
Point where subtraction constants are specified

(in case of polarizabilities :

$\nu = 0$, $t = -Q^2$)

Drechsel, Pasquini, Vdh
Phys. Rept. 378 (2003)

Dispersion formalism for VCS : general



VCS : 12 independent helicity amplitudes

$$H^{\mu\nu} = -i \int d^4x e^{-iq\cdot x} \langle p', \lambda'_N | T\left[J^\mu(x), J^\nu(0)\right] | p, \lambda_N \rangle = \sum_{i=1}^{12} F_i(Q^2, \nu, t)\, h_i^{\mu\nu}$$

invariant amplitudes

independent tensors

$$F_i(Q^2, \nu, t) = F_i^{pole} + F_i^{inel}$$

for 10 amplitudes : **unsubtracted DR**

Unitarity input : γ* N -> X

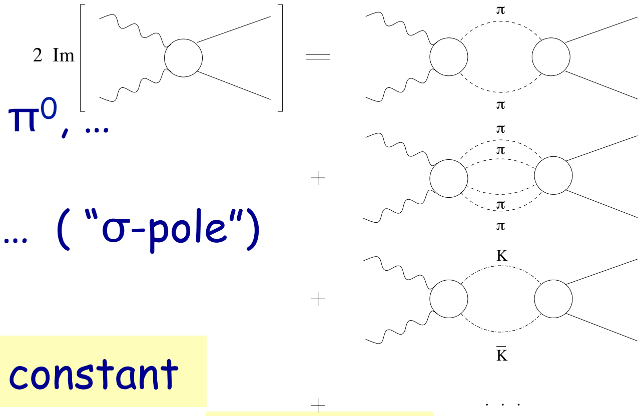
$$\mathrm{Re} F_i^{inel}(Q^2, \nu, t) = \frac{2}{\pi} \mathcal{P} \int_{\nu_0}^{+\infty} d\nu' \frac{\nu' \, \mathrm{Im}_s F_i(Q^2, \nu', t)}{\nu'^2 - \nu^2}$$

Dispersion formalism for VCS : general

➡ 2 amplitudes : **subtracted DR**

Subtraction function

- $$\mathrm{Re}F_i^{inel}(Q^2,\nu,t) = F_i^{inel}(Q^2,0,t) + \frac{2}{\pi}\,\nu^2\,\mathcal{P}\int_{\nu_0}^{+\infty} d\nu'\,\frac{\mathrm{Im}_s F_i(Q^2,\nu',t)}{\nu'(\nu'^2-\nu^2)}$$



- **physics of subtraction functions :**

fixed t-channel poles → Pseudo-scalars : π⁰, …

fixed t-channel poles → J = 0 state : ππ, … ("σ-pole")

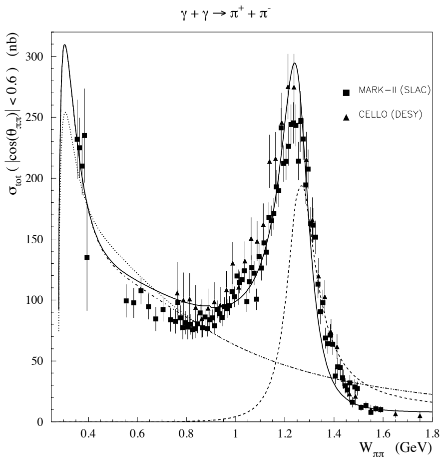
- **DR in t for subtraction function :**

$$F_i^{inel}(Q^2,0,t) = F_i^{inel}(Q^2,0,0)$$ ➡ Subtraction constant

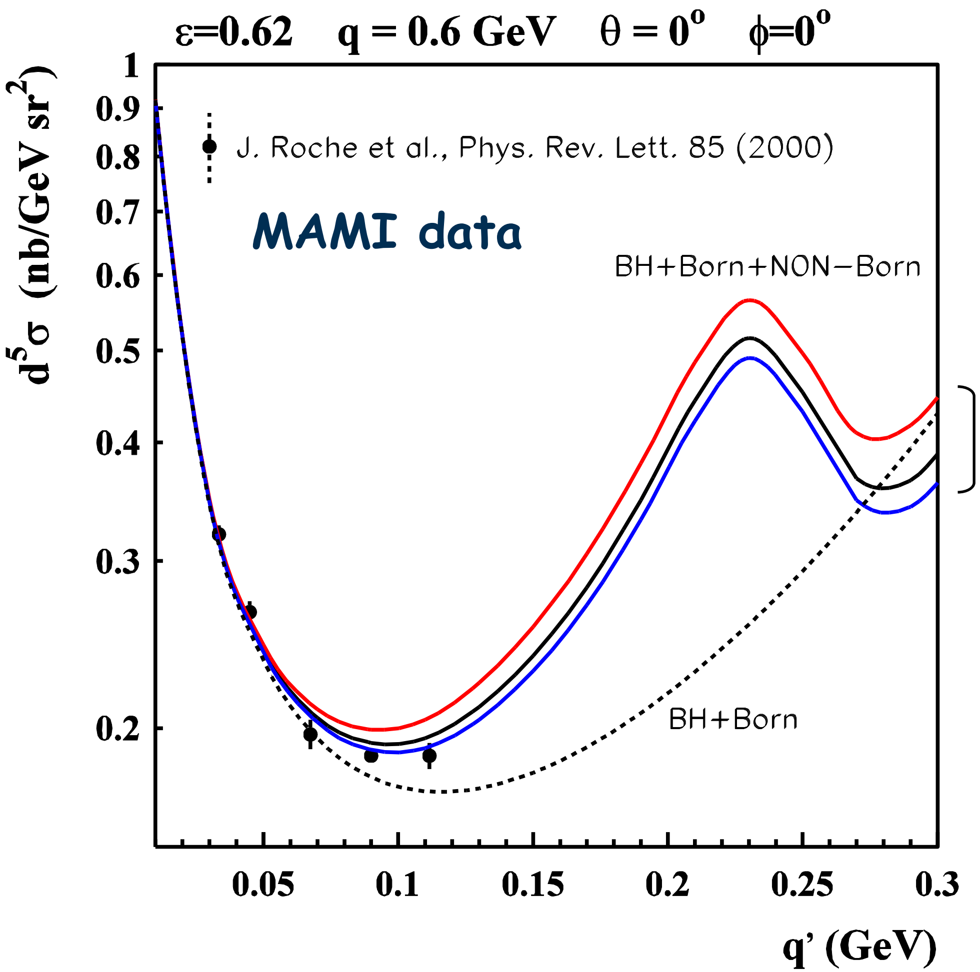
$$+ \left[F_i^{t-pole}(Q^2,0,t) - F_i^{t-pole}(Q^2,0,0)\right]$$ ➡ π⁰ pole

$$+ \frac{t}{\pi}\int_{(2m_\pi)^2}^{+\infty} dt'\,\frac{\mathrm{Im}_t F_i(Q^2,0,t')}{t'\,(t'-t)}$$ ➡ ππ,… (J=0) t-channel state

$$- \frac{t}{\pi}\int_{-\infty}^{t_0} dt'\,\frac{\mathrm{Im}_t F_i(Q^2,0,t')}{t'\,(t'-t)}$$ ➡ negative t-cut : reflection of s-channel cut



VCS in low-energy region



Dispersive integrals :
evaluated using $\gamma^* N \to \pi N$ amplitudes as input

sensitivity to low-energy constants

2 fit parameter (GPs) formalism

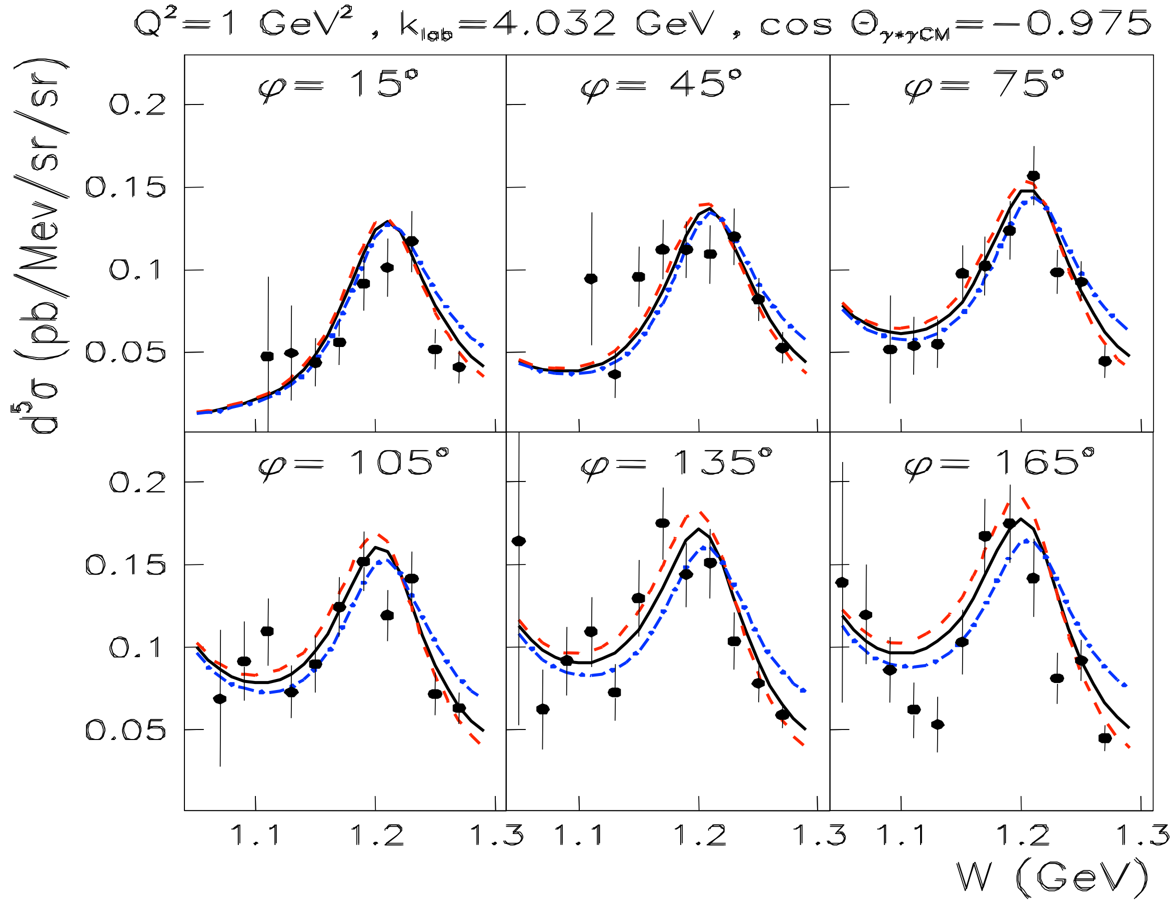
Electric : $\alpha(Q^2)$

Magnetic : $\beta(Q^2)$

DR calculation
- $(\Lambda_\alpha\ \ \Lambda_\beta) = (1.0,\ 0.4)$ GeV
- $(\Lambda_\alpha\ \ \Lambda_\beta) = (1.0,\ 0.7)$ GeV
- $(\Lambda_\alpha\ \ \Lambda_\beta) = (1.0,\ 0.4)$ GeV

Pasquini, Drechsel, Gorchtein, Metz, Vdh (2001)

VCS in low-energy region (W < 1.3 GeV)



JLab data

Laveissiere et al. (2003)

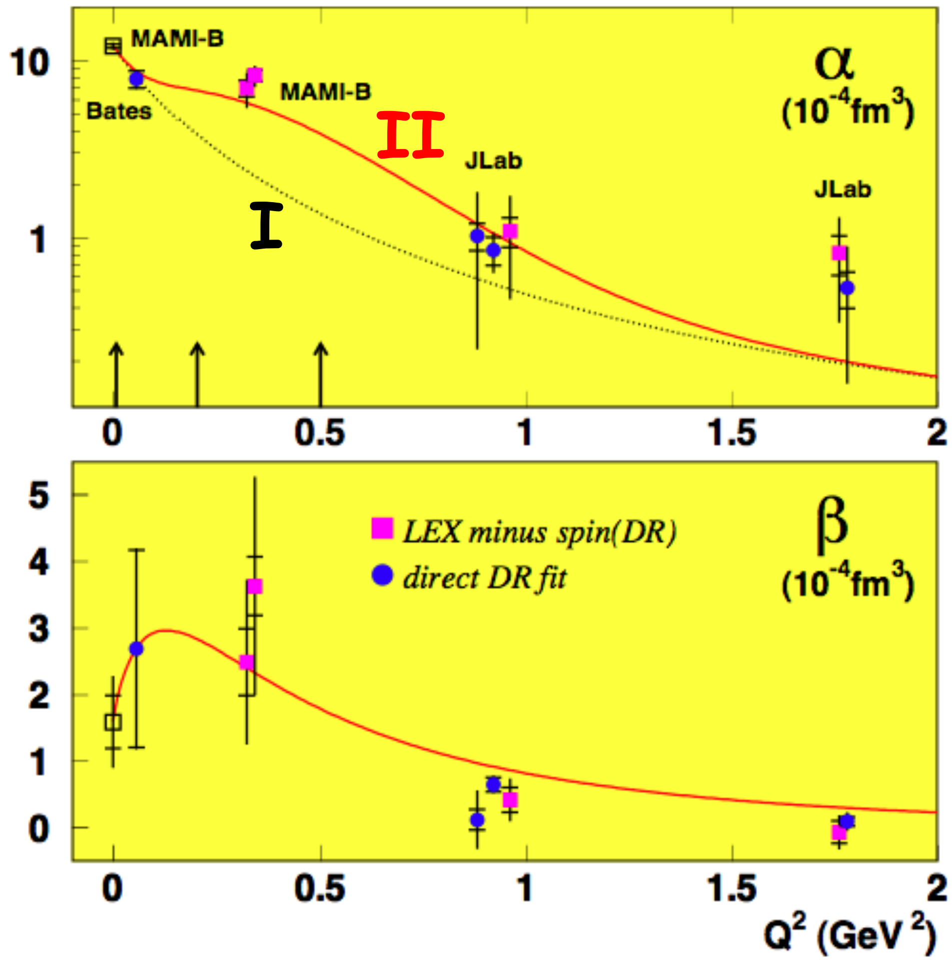
DR calculation

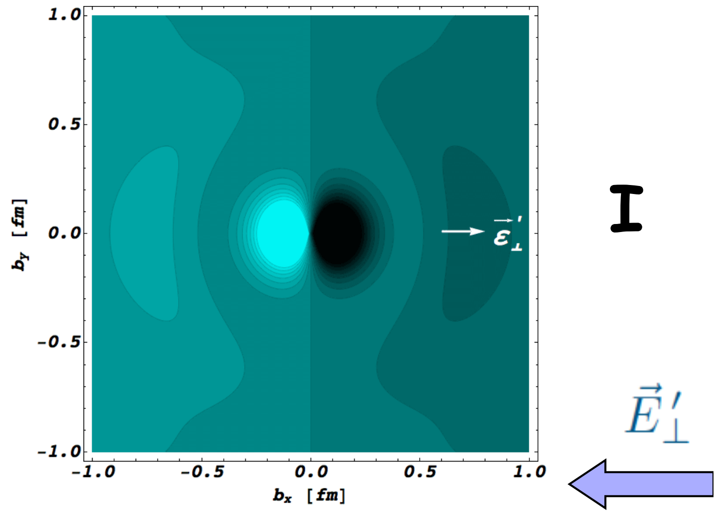
- $(\Lambda_\alpha \;\; \Lambda_\beta) = (0.7, \; 0.6)$ GeV
- $(\Lambda_\alpha \;\; \Lambda_\beta) = (0.9, \; 0.8)$ GeV
- $(\Lambda_\alpha \;\; \Lambda_\beta) = (0.5, \; 0.4)$ GeV

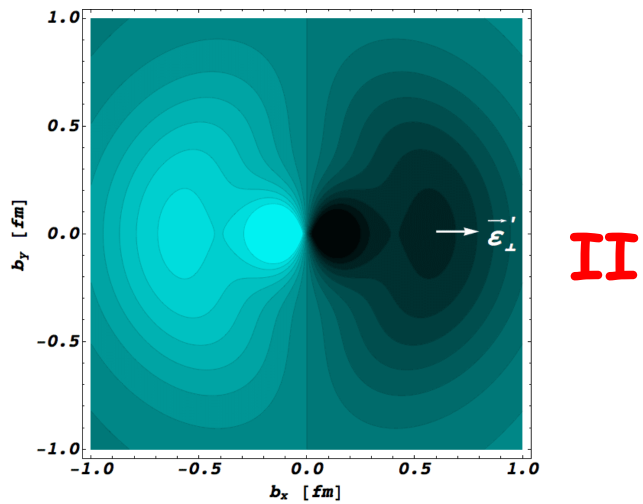
Pasquini, Drechsel, Gorchtein, Metz, Vdh (2001)

Induced polarization in proton

$$\vec{P}_0(\vec{b}) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{b}}\,\vec{P}_0(\vec{q}_\perp)$$
$$= \hat{b}\int_0^\infty \frac{dQ}{(2\pi)}\,Q\,J_1(b\,Q)\,A(Q^2)$$







data compilation : Fonvieille

Gorchtein, Lorcé, Pasquini, Vdh (2009)

Dispersion Formalism for DVCS (twist-2)

helicity averaged twist-2 DVCS amplitude

- twist-2 DVCS amplitude for GPD H : convolution integral

$$A(\xi, t) \equiv - \int_0^1 dx \, H^{(+)}(x, \xi, t) \left[\frac{1}{x - \xi + i\varepsilon} + \frac{1}{x + \xi - i\varepsilon} \right]$$

involves singlet GPD $\quad H^{(+)}(x, \xi, t) \equiv H(x, \xi, t) - H(-x, \xi, t)$

- **SSA** measures **Im part** : $\quad Im A(\xi, t) = \pi \, H^{(+)}(\xi, \xi, t)$

- **Re part** involves **convolution integral** :

$$Re A(\xi, t) \equiv -PV \int_0^1 dx \, H^{(+)}(x, \xi, t) \left[\frac{1}{x - \xi} + \frac{1}{x + \xi} \right]$$

Dispersion relation for helicity- averaged twist-2 DVCS

- energy variables $\nu = \dfrac{Q^2}{4M_N\xi}, \qquad \nu' = \dfrac{Q^2}{4M_N x}$
- helicity averaged ampl : even in ν $\qquad \bar{A}(\nu, t) = \bar{A}(-\nu, t)$
- once subtracted fixed-t dispersion relation (analyticity, crossing)

$$\boxed{Re\bar{A}(\nu, t) = \bar{A}(0, t) + \frac{\nu^2}{\pi}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{Im\bar{A}(\nu', t)}{\nu'^2 - \nu^2}}$$

subtraction at ν = 0

$$\nu = \nu_0 = \frac{Q^2}{4M_N} \longrightarrow \xi = 1$$
$$\nu = 0 \longrightarrow \xi \to \infty$$
$$\nu \to \infty \longrightarrow \xi = 0$$

DR for DVCS (cont'd)

➡ once subtracted fixed-t dispersion relation in variable x

$$Re A(\xi, t) = \Delta(t) + \frac{2}{\pi} PV \int_0^1 \frac{dx}{x} \frac{Im A(x,t)}{(\xi^2/x^2 - 1)}$$

Subtraction function

accessible through spin asymmetries

➡ link with twist-2 GPD : $Im A(x,t) = \pi\, H^{(+)}(x,x,t)$

$$\boxed{Re A(\xi, t) = \Delta(t) - PV \int_0^1 dx\, H^{(+)}(x,x,t) \left[\frac{1}{x-\xi} + \frac{1}{x+\xi} \right]}$$

DR for DVCS amplitudes (in terms of GPDs)

Anikin, Teryaev (2007) Diehl, Ivanov (2007) Polyakov, Vdh (2008)

Kumericki-Passek, Mueller, Passek (2008) Goldstein, Liuti (2009)

…

Subtraction function for DVCS

- difference between convolution and DR integrals :

$$\Delta(t) = PV \int_0^1 dx \left[H^{(+)}(x,\xi,t) - H^{(+)}(x,x,t)\right] \left[\frac{1}{\xi - x} - \frac{1}{\xi + x}\right]$$

- Subtraction function is independent of ξ ➡ formally put ξ = 0

$$\Delta(t) = -PV \int_{-1}^1 dx \, \frac{1}{x} \left[H^{(+)}(x,0,t) - H^{(+)}(x,x,t)\right]$$

- time reversal : GPD even in ξ (2nd argument)

$$H(x,-x,t) = H(x,x,t)$$

$$\boxed{\Delta(t) = -2 \; PV \int_{-1}^1 dx \, \frac{1}{x} \left[H(x,0,t) - H(x,x,t)\right]}$$

Subtraction function for DVCS : relation with D-term (I)

- Lorentz invariance ➡ polynomiality of Mellin moments of GPDs

$$\int_{-1}^{1} dx\, x\, H(x,\xi,t) = A(t) + C(t)\,\xi^2$$

$$\int_{-1}^{1} dx\, x^n\, H(x,\xi,t) = h_0^{(n)}(t) + h_2^{(n)}(t)\,\xi^2 + ... + h_{n+1}^{(n)}(t)\,\xi^{n+1}$$

- highest moment generated by

  Polyakov-Weiss D-term contribution to GPD

$$h_{n+1}^{(n)}(t) = \frac{1}{N_f}\int_{-1}^{1} dz\, z^n\, D(z,t)$$

Subtraction function for DVCS : relation with D-term (II)

$$
\begin{aligned}
& \int_{-1}^{1} \frac{dx}{x} \left[H(x, \xi + x, t) - H(x, \xi, t) \right] \\
= \; & \int_{-1}^{1} \frac{dx}{x} \sum_{n=1}^{\infty} \frac{x^n}{n!} \frac{\partial^n}{\partial \xi^n} H(x, \xi, t) \\
= \; & \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \int_{-1}^{1} dx x^n \frac{\partial^{n+1}}{\partial \xi^{n+1}} H(x, \xi, t) \\
= \; & \sum_{n = odd}^{\infty} h_{n+1}^{(n)}(t) \\
= \; & \frac{1}{N_f} \int_{-1}^{1} dz \, \frac{D(z, t)}{1 - z} \equiv \frac{2}{N_f} \, D(t)
\end{aligned}
$$

Subtraction function for DVCS : relation with D-term (III)

➡ $$\Delta(t) = -2\ PV \int_{-1}^{1} dx\, \frac{1}{x}\, [H(x,0,t) - H(x,x,t)]$$

$$\boxed{\Delta(t) = \frac{4}{N_f}\, D(t) \quad \text{with} \quad D(t) \equiv \frac{1}{2}\int_{-1}^{1} dz\, \frac{D(z,t)}{1-z}}$$

➡ Gegenbauer expansion of D-term

$$D(z,t) = (1-z^2) \sum_{\substack{n=1 \\ n\ odd}}^{\infty} d_n(t)\, C_n^{(3/2)}(z)$$

Gegenbauer polynomials

$$C_1^{(3/2)}(z) = 3z$$

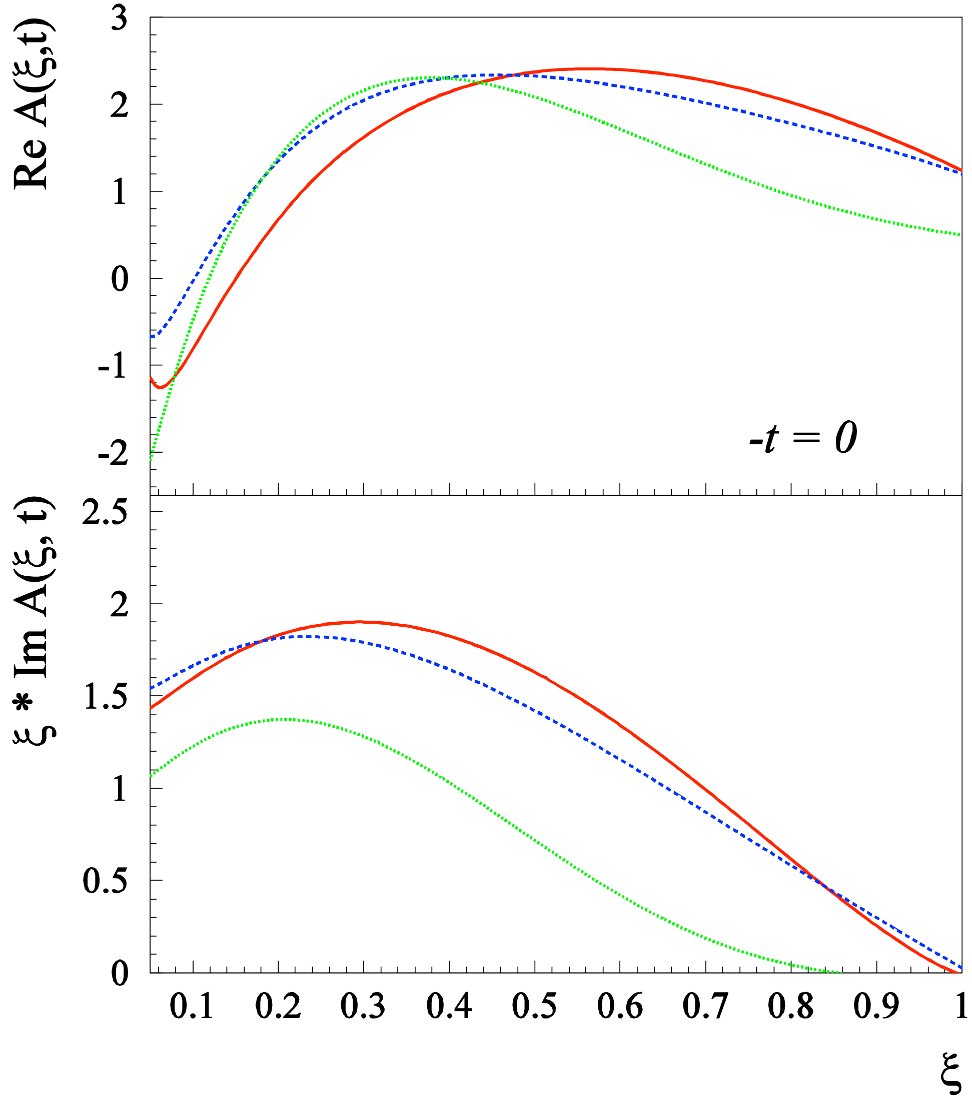
$$\boxed{D(t) = \sum_{n=1}^{\infty} d_n(t)}$$

in χQSM at t = 0 : d_1 = -4.0, d_3 = -1.2, d_5 = -0.4

Goeke, Polyakov, Vdh (2001)

also calculable in lattice QCD

DR evaluation for DVCS : GPD H



Double Distribution model

- - - - $b_v = b_s = 1$

.... $b_v = b_s = 20$

Dual model

——— based on same forward distr.

result for real part shown for $\Delta = 0$

Polyakov, Vdh (2008)

DR for DVCS amplitudes involving GPD H-tilde

- Under crossing : amplitude odd in ν $\quad \bar{A}(\nu, t) = -\bar{A}(-\nu, t)$

- assume unsubtracted dispersion relation (cfr. Bjorken sum rule, GDH sum rule,... in forward case)

$$Re\bar{A}(\nu, t) = \frac{2\nu}{\pi} PV \int_{\nu_0}^{\infty} d\nu' \, \frac{Im\bar{A}(\nu', t)}{\nu'^2 - \nu^2}$$

$$\boxed{Re\tilde{A}(\xi, t) = -\frac{1}{\pi} PV \int_0^1 dx \, Im\tilde{A}(x, t) \left[\frac{1}{x - \xi} - \frac{1}{x + \xi} \right]}$$

- link with GPD

$$\begin{aligned} Im\tilde{A}(x, t) &= \pi \, \tilde{H}^{(+)}(x, x, t) \\ \tilde{H}^{(+)}(x, \xi, t) &\equiv \tilde{H}(x, \xi, t) + \tilde{H}(-x, \xi, t) \end{aligned}$$

DR for DVCS amplitudes involving GPDs E and E-tilde

➡ **GPD E : once-subtracted DR**

GPD (H + E) is unsubtracted

subtraction constant for GPD E is ➡ **- $\Delta(t)$**

(cfr. - D-term)

➡ **GPD Etilde : once-subtracted DR**

subtraction constant for GPD Etilde ➡ **sum of pseudoscalar meson poles**

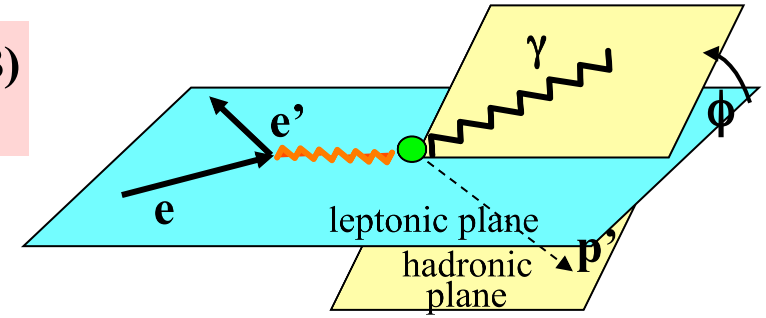
(π^0, η , …)

DVCS Observables

Accessing GPDs from DVCS observables

$$A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} = \frac{\Delta\sigma}{2\sigma}$$

$\xi = xB/(2-xB)$
$k = -t/4M^2$



Polarized beam, unpolarized proton target:

$\Delta\sigma LU \sim \sin\phi\ \mathbf{Im}\{F1H + \xi(F1+F2)\tilde{H} + kF2E\}d\phi$ → **Hp**, $\tilde{H}$p, Ep

Kinematically suppressed

Unpolarized beam, longitudinal proton target:

$\Delta\sigma UL \sim \sin\phi\mathbf{Im}\{F1\tilde{H}+\xi(F1+F2)(H + \ldots\}d\phi$ → Hp, **$\tilde{H}$p**

Unpolarized beam, transverse proton target:

$\Delta\sigma UT \sim \sin\phi\mathbf{Im}\{k(F2H - F1E) + \ldots..\}d\phi$ → **Hp, Ep**

Polarized beam, unpolarized neutron target:

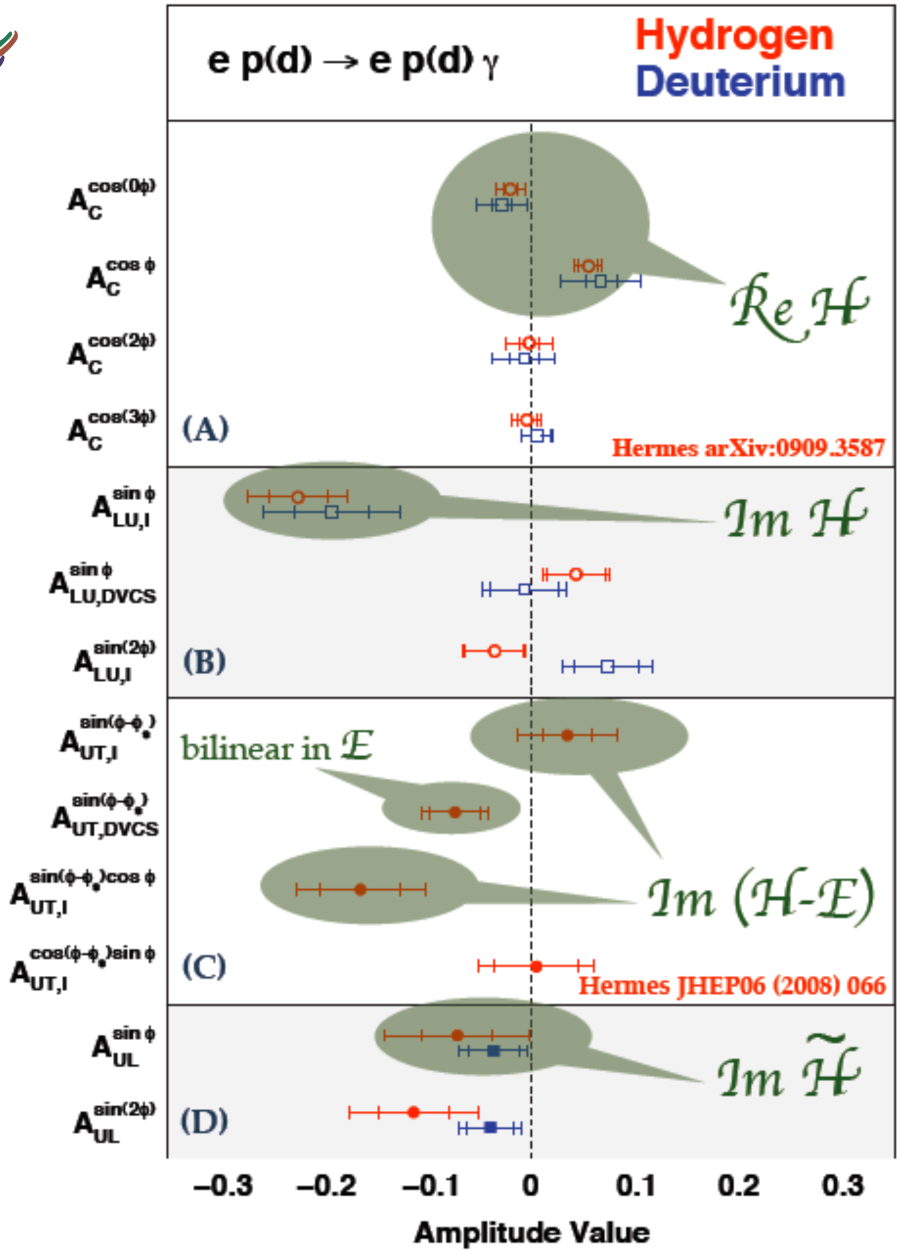
$\Delta\sigma LU \sim \sin\phi\ \mathbf{Im}\{F1H + \xi(F1+F2)\tilde{H} - kF2E\}d\phi$ → Hn, $\tilde{H}$n, **En**

Suppressed because F1(t) is small

Suppressed because of cancellation between PPD's of u and d quarks

Hp(x, ξ, t) = 4/9 Hu(x, ξ, t) + 1/9 Hd(x, ξ, t)
Hn(x, ξ, t) = 1/9 Hu(x, ξ, t) + 4/9 Hd(x, ξ, t)

DVCS : asymmetries

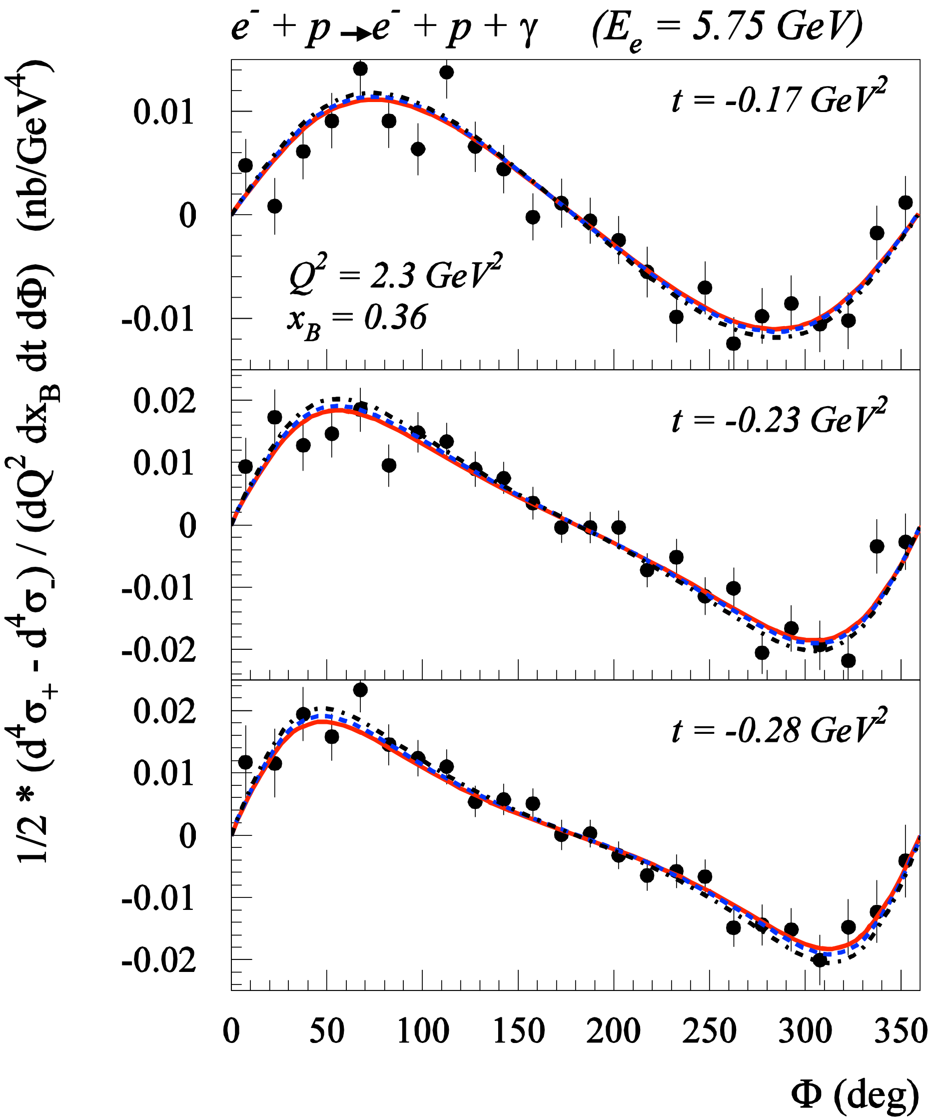


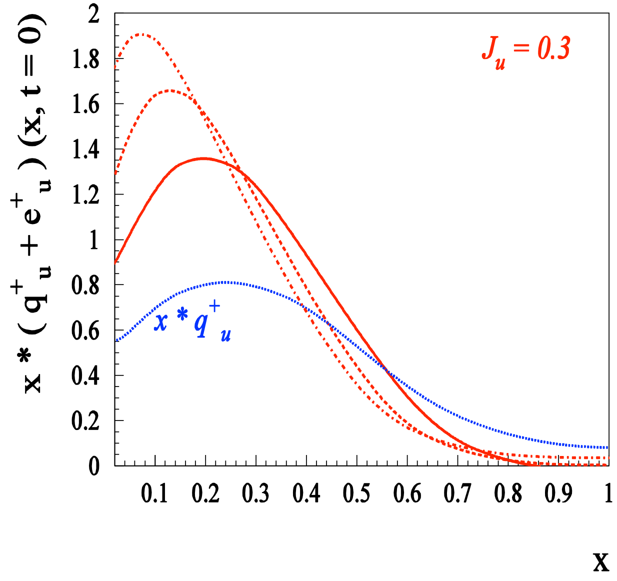
beam *charge* asymmetry

beam *spin* asymmetry

T target spin asymmetry

L target spin asymmetry

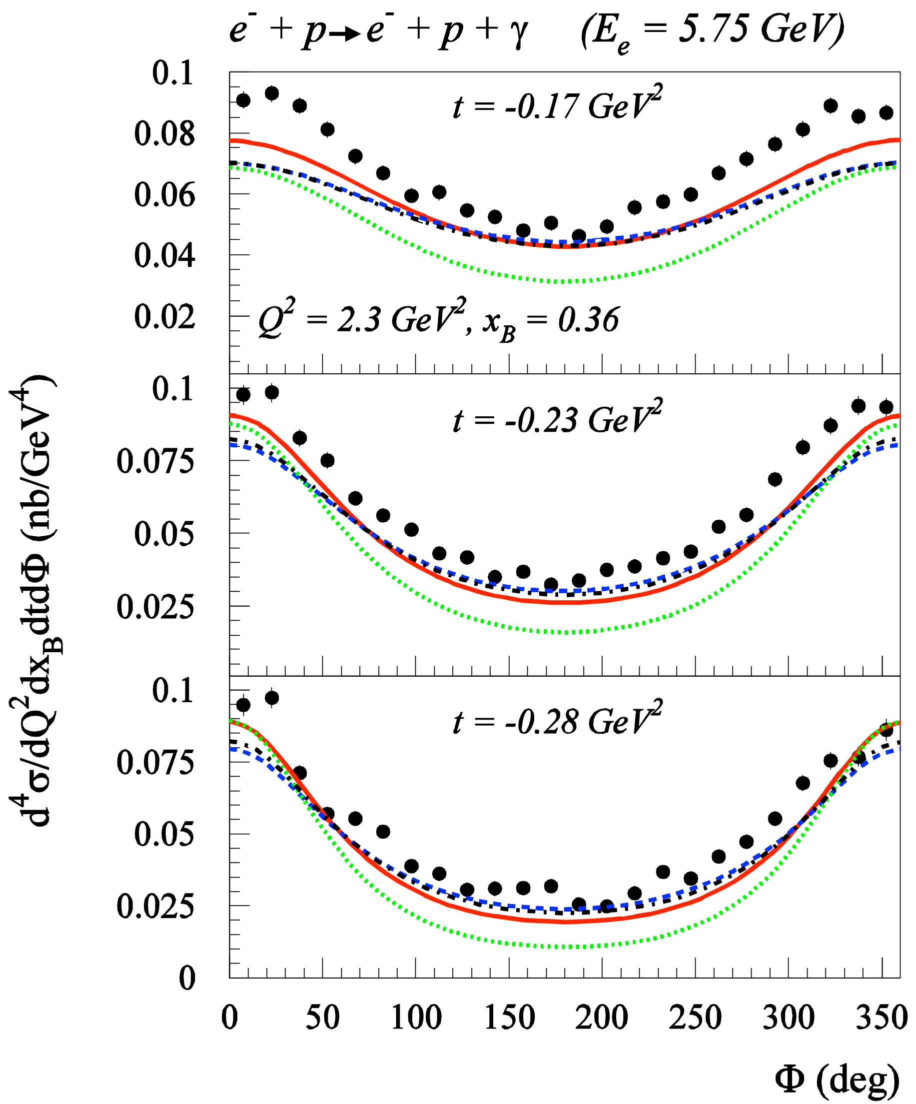




DR evaluation (dual model for GPDs)

Polyakov, Vdh (2008)

data : JLab/Hall A Munoz Camacho et al. (2006)



Bethe-Heitler

DR evaluation

(dual model for GPDs)

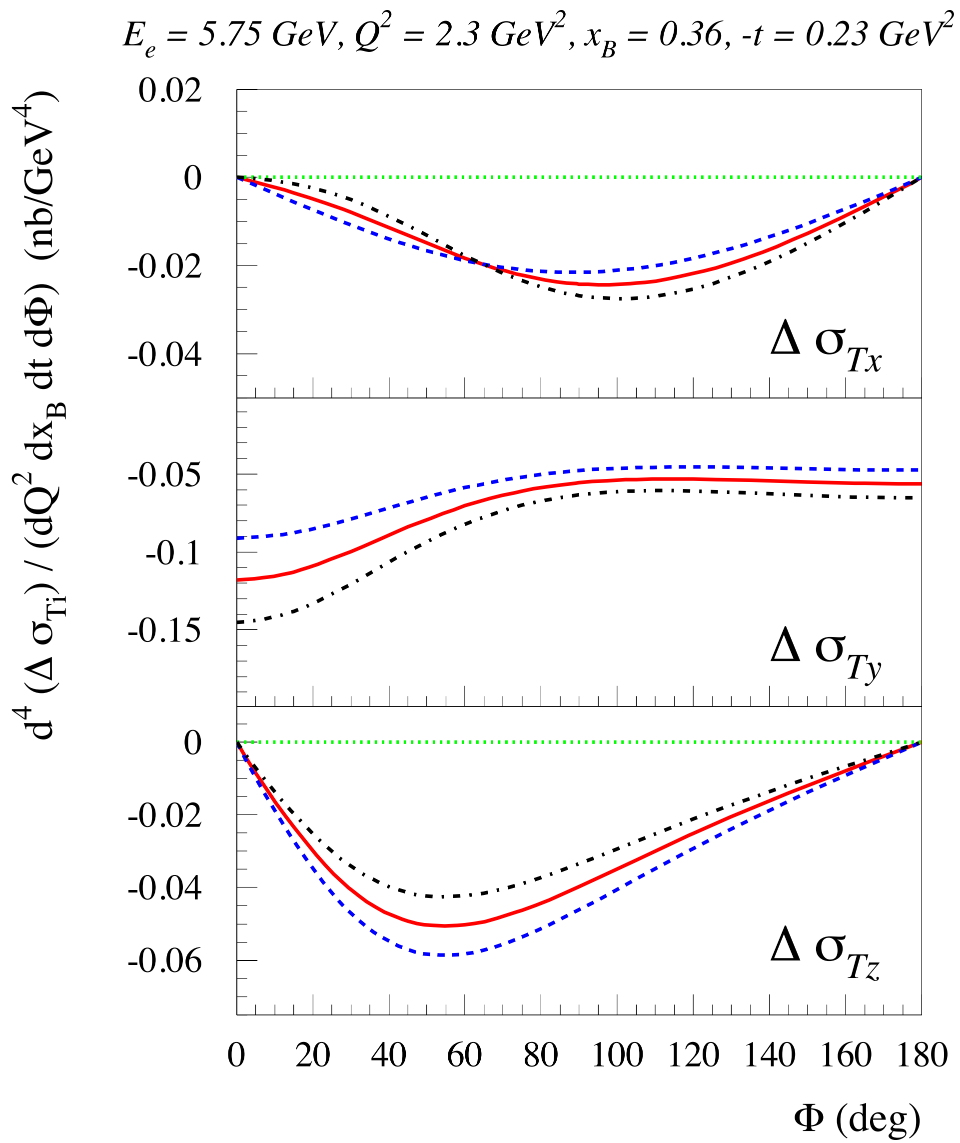
Polyakov, Vdh (2008)

data :

JLab/Hall A

Munoz Camacho et al. (2006)

proton target spin asymmetry in DVCS :



sensitivity to J_u

$J_u = 0.1$

$J_u = 0.3$ $\qquad J_d = 0$

$J_u = 0.5$

Bethe-Heitler

DR evaluation

(dual model for GPDs)

In hard exclusive process @ leading twist : one accesses 8 GPD quantities

Observables : Compton Form Factors

REAL parts of CFF

$$P\int_0^1 dx\,[H(x,\xi,t)-H(-x,\xi,t)]\,C^+(x,\xi), \qquad (1)$$

$$P\int_0^1 dx\,[E(x,\xi,t)-E(-x,\xi,t)]\,C^+(x,\xi), \qquad (2)$$

$$P\int_0^1 dx\left[\tilde{H}(x,\xi,t)+\tilde{H}(-x,\xi,t)\right]C^-(x,\xi), \qquad (3)$$

$$P\int_0^1 dx\left[\tilde{E}(x,\xi,t)+\tilde{E}(-x,\xi,t)\right]C^-(x,\xi), \qquad (4)$$

IMAG parts of CFF

$$H(\xi,\xi,t)-H(-\xi,\xi,t), \qquad (5)$$

$$E(\xi,\xi,t)-E(-\xi,\xi,t), \qquad (6)$$

$$\tilde{H}(\xi,\xi,t)+\tilde{H}(-\xi,\xi,t), \qquad (7)$$

$$\tilde{E}(\xi,\xi,t)+\tilde{E}(-\xi,\xi,t) \qquad (8)$$

with

$$C^\pm(x,\xi)=\frac{1}{x-\xi}\pm\frac{1}{x+\xi} \qquad (9)$$

which we can call, respectively, in a symbolic notation, $Re(H)$, $Re(E)$, $Re(\tilde{H})$, $Re(\tilde{E})$, $Im(H)$, $Im(E)$, $Im(\tilde{H})$ and $Im(\tilde{E})$.

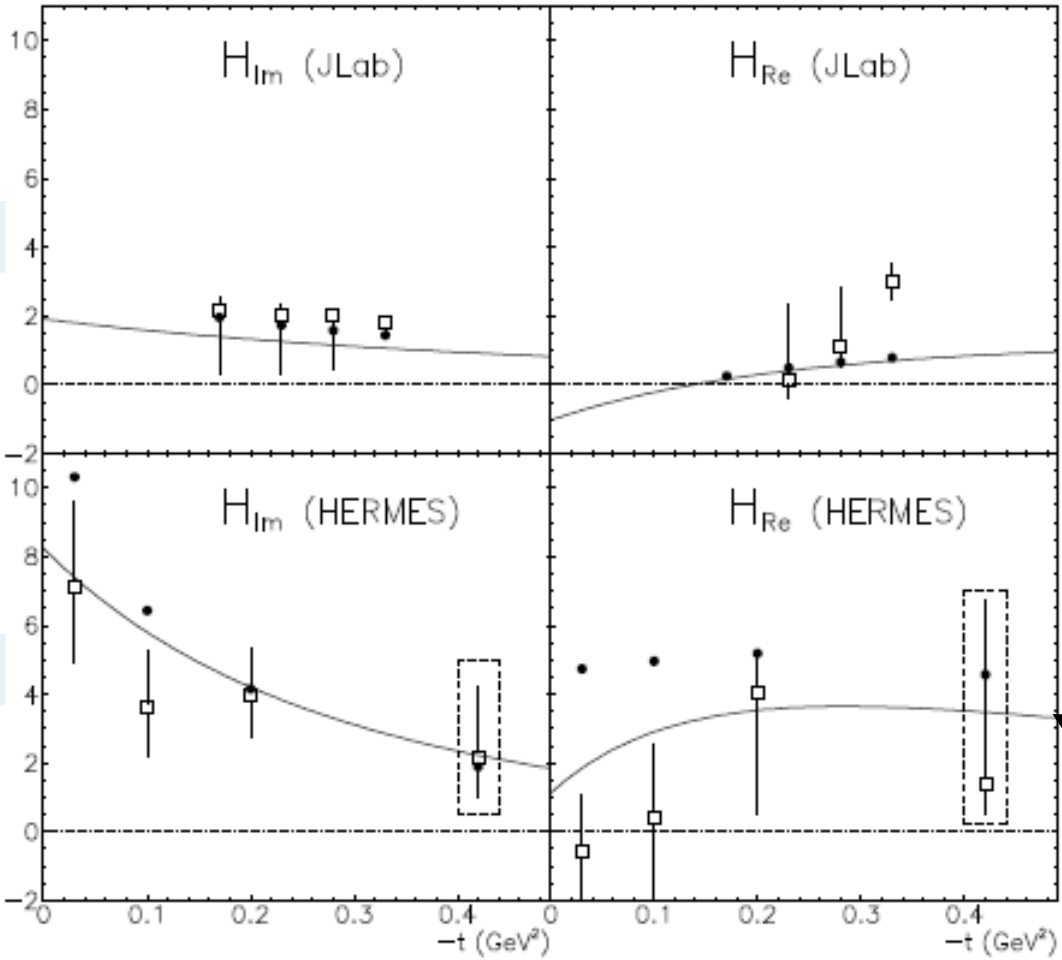
CFF from DVCS :
model independent fit extractions (I)

JLab

x_B=0.36,Q^2=2.3

HERMES

x_B=0.09,Q^2=2.5



as energy increases:

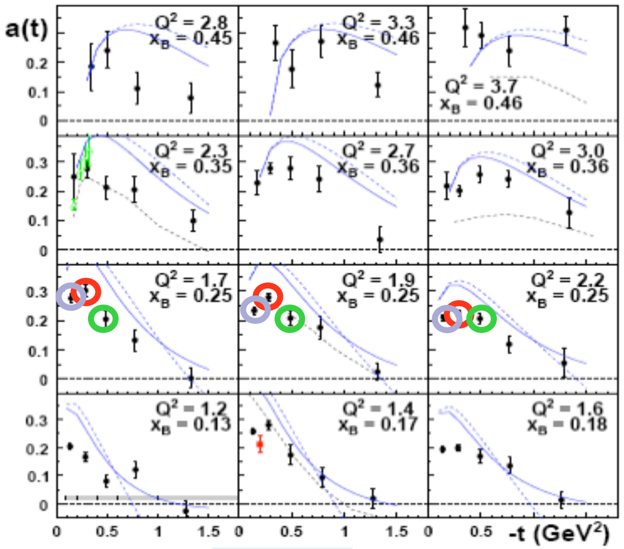
- « Shrinkage » of Im(H)
- Im(H) > Re(H)
- different t-behavior for Im(H) & Re(H)

solid circles : VGG(1998)

model dependent fit of D. Muller, K.Kumericki(2009)

Guidal, Moutarde (2009)

CFF from DVCS : fits (II)

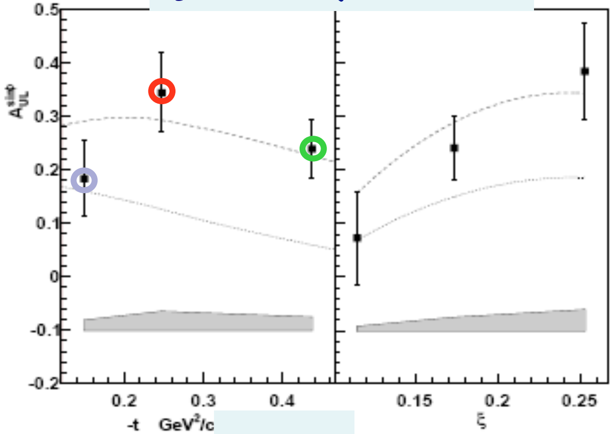


Jlab/CLAS

Girod et al. (2006)

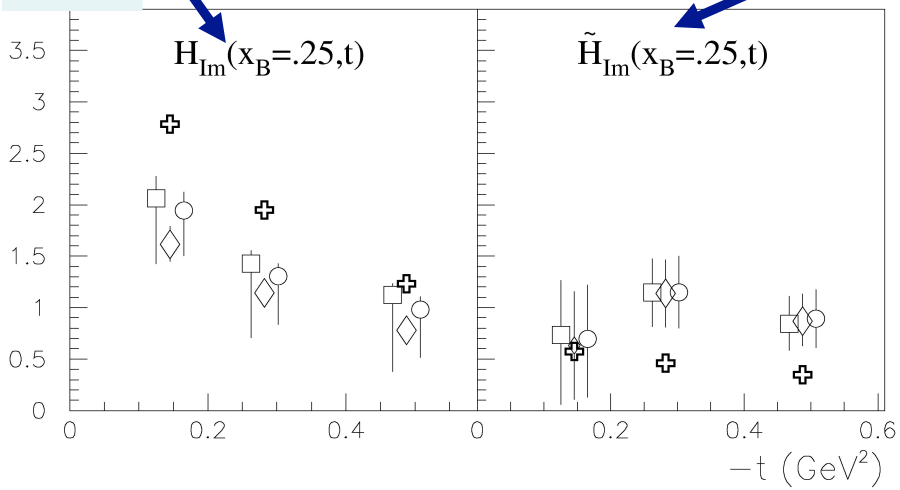
Chen et al. (2008)

$\xi=0.16, Q^2=1.82$



AUL

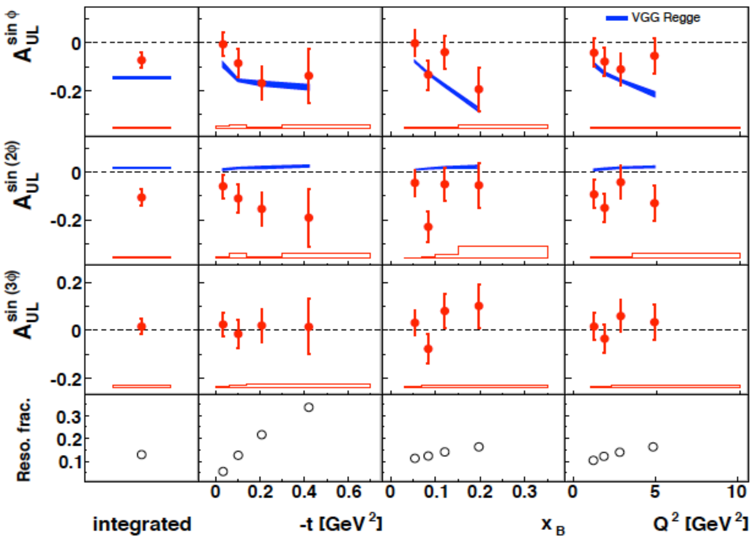
ALU



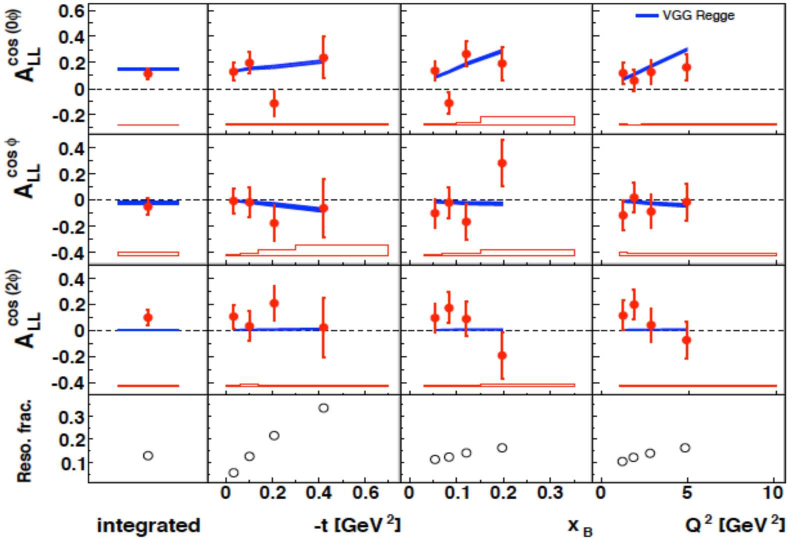
- □ Fit with 7 CFFs (bounds 5xVGG CFFs)
- ○ Fit with 7 CFFs (bounds 3xVGG CFFs)
- ◇ Fit with ONLY $\tilde{H}$ and H CFFs
- ✣ VGG prediction

Guidal (2010)

CFF from DVCS : fits (III)

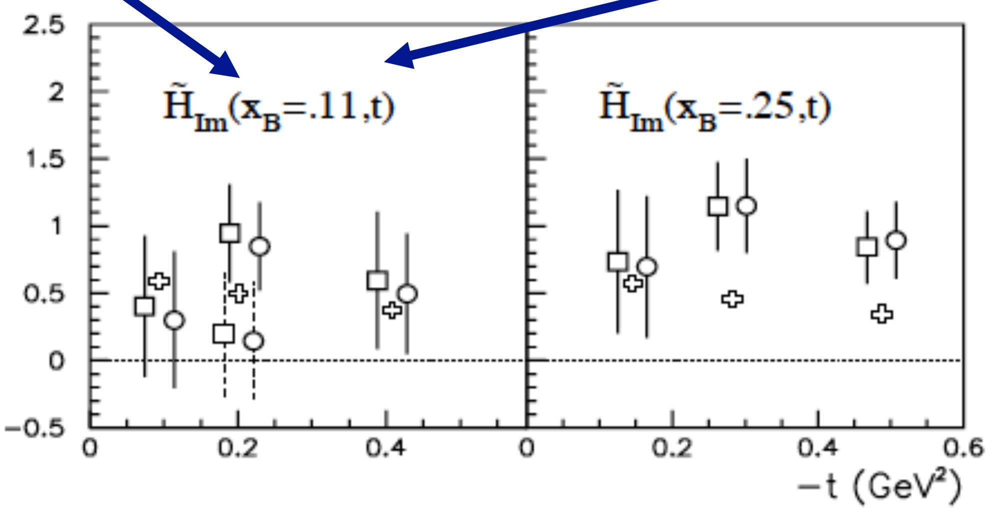


HERMES
(2010)



AUL

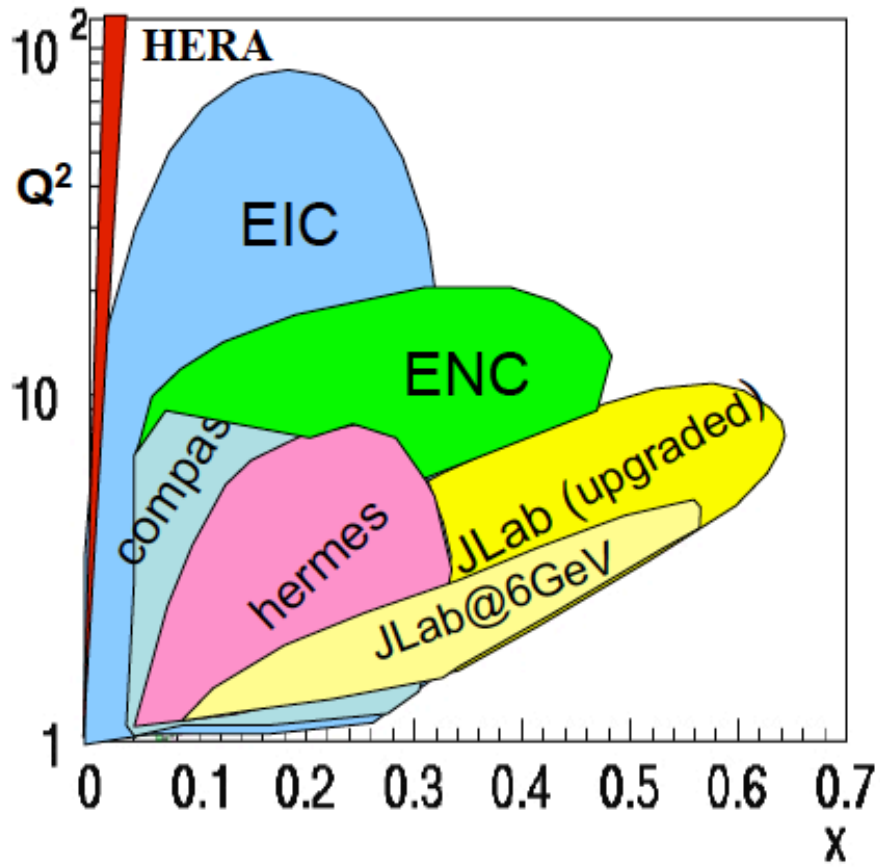
ALL

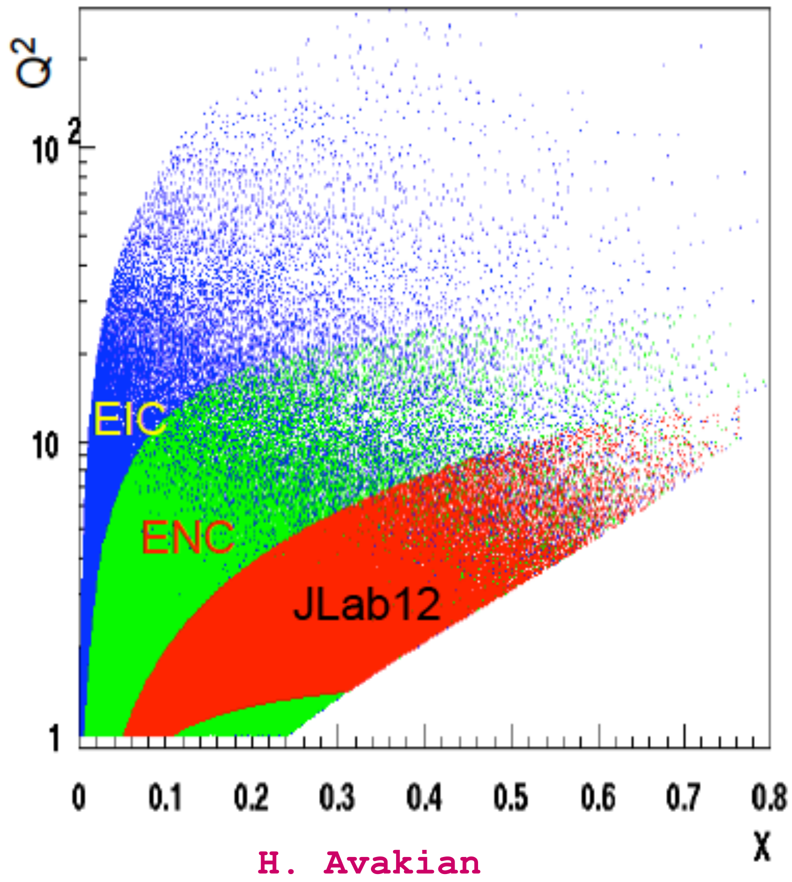


Guidal (2010)

open crosses :
VGG(1998)

The Energy / Luminosity Frontier





H. Avakian

Hard exclusive reactions :

high energy and high luminosity required + polarization

Conclusions

- Dispersion relations for DVCS / DVCS amplitudes

  constraint from analyticity, crossing built in

- Predictive formalism at low energies :

  subtraction constants -> generalized polarizabilities

- Subtraction functions for twist-2 DVCS amplitude :

  for H and E : D-term

  for Htilde : no subtraction (?)

  for Etilde : PS meson poles

- Experimental strategy :

  measurement of **imaginary parts** : single polarization observables over sufficiently broad range in ξ

  real parts : fix the subtraction constants, **cross check** by extracting constants at same t for different values of ξ