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Orbital Angular Momentum in QCD

Interpretation & models of QCD energy-momentum tensor

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based on works with:
Manuel Mai (UConn, Heidelberg, Yale), Matt Neubelt (UConn graduating senior) forthcoming (2012)
K. Goeke, M. V. Polyakov, A. Silva et al PRD75 (2007) 094021, PRC75 (2007) 055207, NPA794 (2007) 87
PS, Boffi, Colli, Radici PRD66 (2002) 114004, PRD 67 (2003) 114022

Overview:

- introduction: what do EMT and d_1 tell us about the nucleon?
- rely on insights from models (consistency of models!)
- chiral quark soliton, Skyrme, bag model
- stability and sign of D-term d_1
- lesson from Q-balls
- conclusions

1. Introduction How to learn about nucleon?

$|N\rangle$ = **strong** interaction particle.

em:	$\partial_\mu J^\mu_{\mathbf{em}} = 0$	$\langle N'\|J^\mu_{\mathbf{em}}\|N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \ldots$
weak:	PCAC	$\langle N'\|J^\mu_{\mathbf{weak}}\|N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\mathbf{grav}}\|N\rangle$	$\longrightarrow$	$M,\ J,\ d_1,\ \ldots$

1st global properties:

$$
\begin{aligned}
Q_{\text{prot}} &= 1.602176487(40)\times 10^{-19}\text{C}\\
\mu_{\text{prot}} &= 2.792847356(23)\mu_N\\
g_A &= 1.2694(28)\\
g_p &= 8\text{-}12\\
M &= 938.272013(23)\,\text{MeV}\\
J &= \tfrac{1}{2}\\
d_1 &= \textbf{???}
\end{aligned}
$$

2nd partonic structure: $\ldots\ \ldots\ \ldots\ \ldots$

$\ldots\ \ldots\ \ldots$

$\ldots\ \ldots$

$\ldots$

can access from **GPDs** in hard exclusive reactions

Müller et al, Ji, Radyushkin, Collins et al 1990s

$\longrightarrow$ only way for **EMT form factors**

How do EMT form factors look like?

- insights from models, lattice QCD, χPT
  (see below)

Highlight number 1:

- spin decomposition!
  Xiang-Dong Ji, PRL 78 (1997) 610
  discussions in literature, at this workshop, . . .

What is d_1?

- *last unknown* global nucleon characteristics
  (certainly not last. But interesting!)

- what's the physical meaning?
  M.V.Polyakov, PLB 555, 57 (2003)

- which value does it have?
  (models, lattice, χPT)

Recall definition of 'unpolarized GPDs'

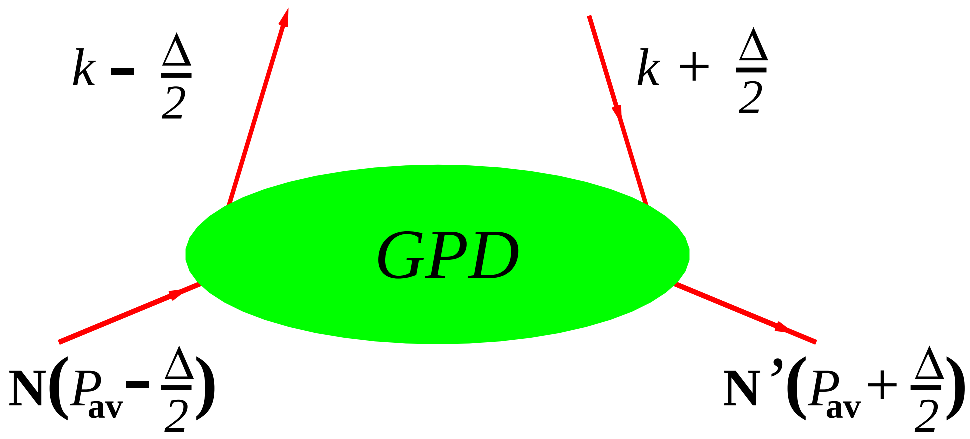
(everything starts & ends with GPDs)

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle N(P')|\overline{\psi}_q(-\tfrac{\lambda n}{2})\, n_\mu \gamma^\mu \, [-\tfrac{\lambda n}{2}, \tfrac{\lambda n}{2}] \psi_q(\tfrac{\lambda n}{2})\, |N(P)\rangle$$

$$= \overline{u}(p')\, n_\mu \gamma^\mu \, u(p)\; H^q(x,\xi,t) + \overline{u}(p')\, \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M_N}\, u(p)\; E^q(x,\xi,t)$$

$P_{\text{av}} = (P' + P)/2$
$\Delta = P' - P$
$t = \Delta^2$
$\xi = (n \cdot \Delta)/(n \cdot P_{\text{av}})$
$n =$ light-like vector
$k = x P_{\text{av}}$
analog gluon GPDs

(scale dependence not indicated)



Polynomiality $\longleftarrow$ Lorentz invariance, time-reversal, hermiticity in QCD

Mellin moments of $H^q(x,\xi,t)$, $E^q(x,\xi,t)$ = polynomials in ξ^2

even N:

$$\int dx\, x^{N-1}\, H^q(x,\xi,t) = h_0^{q(N)}(t) + h_2^{q(N)}(t)\,\xi^2 + \ldots + h_N^{q(N)}(t)\,\xi^N$$

$$\int dx\, x^{N-1}\, E^q(x,\xi,t) = e_0^{q(N)}(t) + e_2^{q(N)}(t)\,\xi^2 + \ldots + e_N^{q(N)}(t)\,\xi^N$$

with $h_N^{q(N)}(t) = -\,e_N^{q(N)}(t)$ for spin $\frac{1}{2}$ particle

odd N: highest power is ξ^{N-1}

minimal requirement no. 1 for a consistent model

x-dependence of $H^q(x,\xi,t)$, $E^q(x,\xi,t)$

$\Leftrightarrow$

infinitely many form factors:

Each form factor:

- related to a local matrix element
- contains different information

$N=1$: $h_0^{f(1)}(t)$, $e_0^{f(1)}(t)$

$N=2$: $h_0^{f(2)}(t)$, $h_2^{f(2)}(t)$, $e_0^{f(2)}(t)$

$N=3$: $h_0^{f(3)}(t)$, $h_2^{f(3)}(t)$, $e_2^{f(3)}(t)$, $e_0^{f(3)}(t)$

$N=4$: $h_0^{f(4)}(t)$, $h_2^{f(4)}(t)$, $h_4^{f(4)}(t)$, $e_2^{f(4)}(t)$, $e_0^{f(4)}(t)$

$N=5$: $h_0^{f(5)}(t)$, $h_2^{f(5)}(t)$, $h_4^{f(5)}(t)$, $e_4^{f(5)}(t)$, $e_2^{f(5)}(t)$, $e_0^{f(5)}(t)$

$N=6$: $h_0^{f(6)}(t)$, $h_2^{f(6)}(t)$, $h_4^{f(6)}(t)$, $h_6^{f(6)}(t)$, $e_4^{f(6)}(t)$, $e_2^{f(6)}(t)$, $e_0^{f(6)}(t)$

$N=7$: $\cdots$ $\cdots$ $\cdots$ $\cdots$ $\cdots$ $\cdots$ $\cdots$ $\cdots$

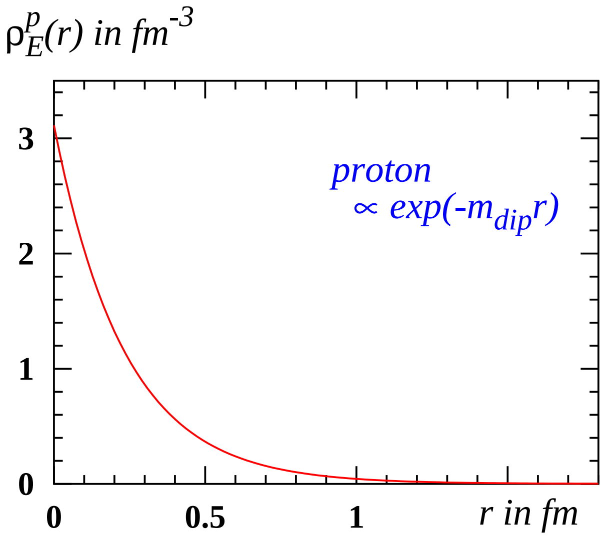
$h_N^{f(N)}(t)$ $(N=2,4,6,\ldots)$ Mellin moments of D-term (Polyakov, Weiss 1999)

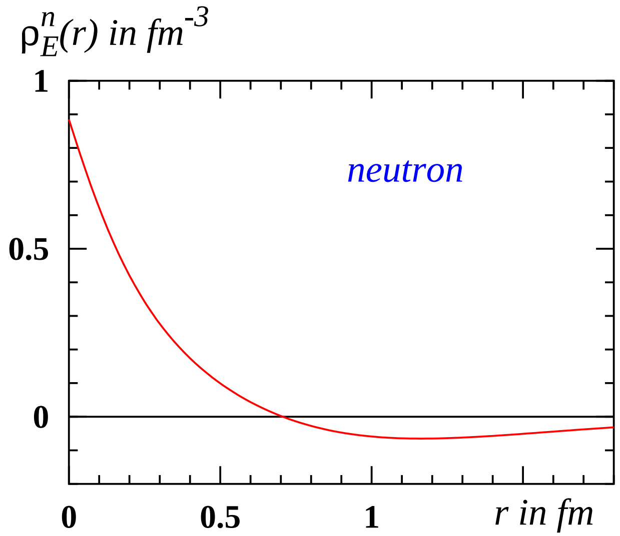
$N = 1$: electromagnetic form factors   Hofstadter et al, 1950s ...

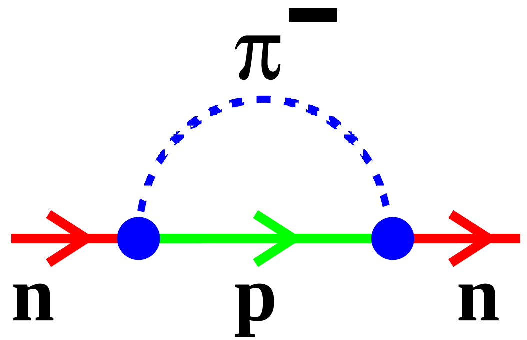
$\sum_q e^q \int dx\, H^q(x,\xi,t) = F_1(t)$ $\sum_q e^q \int dx\, E^q(x,\xi,t) = F_2(t)$ what did we learn?

e.g. $G_E(t) = F_1(t) + \frac{t}{4M_N^2} F_2(t) = \int d^3r\, \rho_E(r)\, e^{i\vec{q}\vec{r}}$ ($t = -\vec{q}^{\,2}$, "textbook interpretation")

$G_E^p(t) \simeq 1/(1 - t/m_{\text{dip}}^2)^2$, $m_{\text{dip}} = 0.84$ GeV, $G_E^n(0) = 0$







pion cloud:
spontaneous breaking of chiral symmetry

continued interest: JLab Hall A $G_E^n(t)$, Hall C Q-weak

e.g. precise data at low t constrain physics at TeV scale

(Young, Carlini, Thomas, Roche, PRL99 (2007) 122003)

Problem: **3D-charge distributions** (Miller, PRC 80 (2009) 045210)

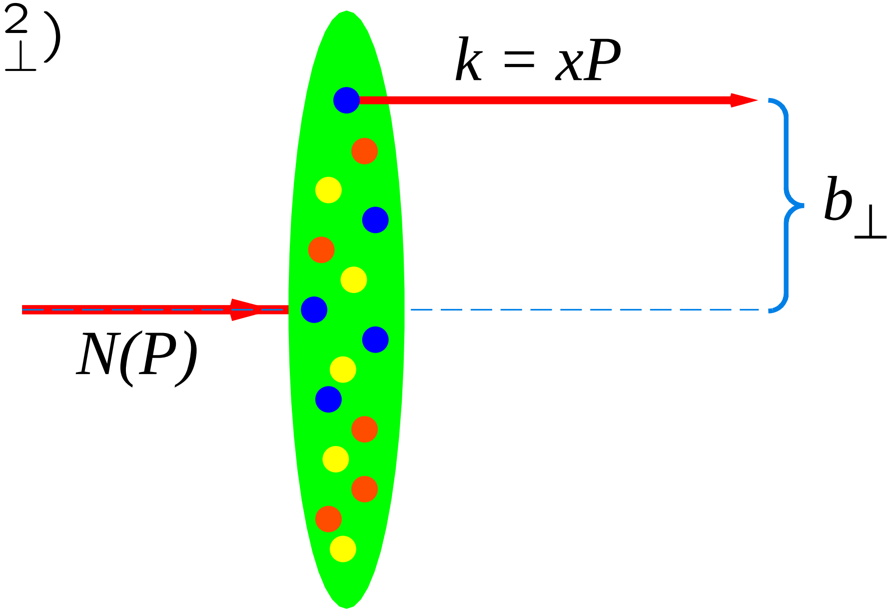
"text book" but rel. corrections

Solution: **2D-impact parameter space** GPDs at $\xi = 0$

exact!

momentum transfer $\perp$ light-cone: $t = \Delta^2 = -\vec{\Delta}_\perp^2$

$$H^q(x, b_\perp) = \int \mathrm{d}^2\Delta_\perp \, e^{i\vec{\Delta}_\perp \vec{b}_\perp} \, H^q(x, 0, -\vec{\Delta}_\perp^2)$$



$b_\perp =$ impact parameter

Matthias Burkardt, 2000

exact interpretation:

probability

$$\longrightarrow \quad \int \mathrm{d}\boldsymbol{x} \, \mathrm{GPD}(\boldsymbol{x}, \boldsymbol{b}_\perp) = \boldsymbol{\rho_E}(\boldsymbol{b}_\perp)$$

$$\boldsymbol{G_E}(\boldsymbol{t}) = \int \mathrm{d}^2\boldsymbol{b}_\perp \, \boldsymbol{\rho_E}(\boldsymbol{b}_\perp) \, \boldsymbol{e}^{i\vec{\boldsymbol{\Delta}}_\perp \vec{\boldsymbol{b}}_\perp}$$

(Miller, Weiss, ... and later this week)

$N=2$: energy-momentum tensor form factors

$$\sum_q \int \mathrm{d}x\, x\, H^q(x,\xi,t) = \qquad +M_2^Q(t) + \tfrac{4}{5}\, d_1^Q(t)\, \xi^2$$

$$\sum_q \int \mathrm{d}x\, x\, E^q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \tfrac{4}{5}\, d_1^Q(t)\, \xi^2$$ gluons analog

we know:

$M_2^Q(0) \approx 0.5$ at few GeV^2: quarks carry half of nucleon momentum

we would like to know ("highlight"):

how do quarks + gluons share nucleon spin? We need:

$$\sum_q \int \mathrm{d}x\, x\, (H^q + E^q)(x,\xi,t) = 2J^Q(t) \quad \text{and} \quad \lim_{t\to 0} J^Q(t) = J^Q(0)$$ Ji, 1997

other open questions:

What's the t-dependence of $M_2(t)$, $J(t)$? What is $d_1(t)$ good for?

$N = 3, 4, 5, \ldots$ further form factors

of what? Of operators $\bar{\psi}\gamma_{\mu_1} D_{\mu_2} D_{\mu_2} \ldots D_{\mu_N} \psi$

scale dependent quantities

each contains information on a different aspect of nucleon

$N = 1, 2$ special

related to conserved (in QCD) currents

global properties $\boldsymbol{Q}$, $\boldsymbol{\mu}$, $\boldsymbol{M_N}$, $\boldsymbol{J}$, $\boldsymbol{d_1}$

↖ the "last" unknown

2. Energy momentum tensor $T^{\mu\nu}$

$T^{\mu\nu}$ fundamental object in field theory. In QCD $T^{Q,G}_{\mu\nu}$ both gauge invariant

$$
\begin{aligned}
\langle P'|T^{Q,G}_{\mu\nu}|P\rangle \;=\; \bar{u}(P')\Big[\, & M_2^{Q,G}(t)\,\frac{P_\mu P_\nu}{M_N} \\
& + J^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} \\
& + d_1^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}\Big]u(p)
\end{aligned}
$$

$$
\begin{aligned}
& A^{Q,G}(t)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} \\
& + B^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M_N} \\
& + C^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{M_N}
\end{aligned}
$$

using Gordon identity
equiv. decomposition:

$M_2(t) = A(t)$

$2\,J(t) = B(t) + A(t)$

$d_1(t) = 5\,C(t)$

$\bar{c}(t)$ because $\hat{T}^{Q,G}_{\mu\nu}$ not conserved separately, drops out from quark+gluon sum.

Properties of form factors

total $T_{\mu\nu} = T^{Q}_{\mu\nu} + T^{G}_{\mu\nu}$ conserved

$$
\begin{aligned}
M_2(t) &= M_2^Q(t) + M_2^G(t), \\
J(t) &= J^Q(t) + J^G(t), \\
d_1(t) &= d_1^Q(t) + d_1^G(t) \quad \text{scale independent (like } F_1(t),\ F_2(t)\text{)}
\end{aligned}
$$

with

$$
\begin{aligned}
M_2^Q(0) + M_2^G(0) &= 1 \quad \text{quarks + gluons carry 100 \% of momentum} \\
J^Q(0) + J^G(0) &= \tfrac{1}{2} \quad \text{quarks + gluons make up the nucleon spin} \\
d_1^Q(0) + d_1^G(0) &\equiv d_1 \quad \text{what is that?}
\end{aligned}
$$

what ever it is (see soon):

$d_1(t)$ dictates asymptotics of unpolarized GPDs

(Goeke, Polyakov & Vanderhaeghen, Prog.Part.Nucl.Phys.47 (2001) 401)

What we (would like to) know

$M_2^Q(0) = \int \mathrm{d}x \sum_q x H^q(x,0,0) \equiv \int \mathrm{d}x \sum_q x f_1^q(x) \sim 0.5$ at $\mu^2 \sim$ few GeV^2

half of momentum of (fast moving) nucleon carried by quarks, rest by gluons.

asymptotically $\mu \to \infty$: $\quad M_2^Q(0) = \dfrac{3n_f}{16+3n_f}$ and $M_2^G(0) = \dfrac{16}{16+3n_f}$

(Gross, Wilczek, 1974)

$2J^Q(0) = \int \mathrm{d}x \sum_q x(H^q + E^q)(x,0,0) =$ **?**
i.e. percentage of nucleon spin due to quarks?

asymptotically $\mu \to \infty$: $\quad 2J^Q(0) = \dfrac{3n_f}{16+3n_f}$ and $2J^G(0) = \dfrac{16}{16+3n_f}$

like M_2 (Ji, 1997)

Meaning of $d_1(t)$ (textbook interpretation ... Formulae nevertheless correct!!!)

M.V.Polyakov, PLB 555, 57 (2003)

Go to Breit frame where $\vec{P}' = -\vec{P}$, i.e. $E' = E$ and $\Delta^\mu = (0, \vec{\Delta})$.

Define static EMT: $T^Q_{\mu\nu}(\vec{r}, \vec{s}) = \frac{1}{2E}\int\frac{d^3\vec{\Delta}}{(2\pi)^3}\, e^{i\vec{\Delta}\,\vec{r}}\, \langle P'|\hat{T}^Q_{\mu\nu}|P\rangle$

with $\vec{s} =$ spin vector of respective nucleon at rest.

Then (gluon analog): $J^Q(t) + \frac{2t}{3} J^{Q\prime}(t) = \int d^3r\, e^{i\,\vec{r}\,\vec{\Delta}}\, \varepsilon^{ijk}\, s_i\, r_j\, T^Q_{0k}(\vec{r}, \vec{s})$

$$d_1^Q(t) + \frac{4t}{3}\, d_1^{Q\prime}(t) + \frac{4t^2}{15}\, d_1^{Q\prime\prime}(t) = -\frac{M_N}{2} \int d^3r\, e^{i\,\vec{r}\,\vec{\Delta}} \left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right) T^Q_{ij}(\vec{r})$$

$$M_2(t) - \frac{t}{4M_N^2}\left(M_2(t) - 2J(t) + \frac{4}{5}\, d_1(t)\right) = \frac{1}{M_N} \int d^3r\, e^{i\,\vec{r}\,\vec{\Delta}}\, T_{00}(\vec{r}, \vec{s})$$

in last equation (which is quark+gluon) take $t \to 0$: $M_2(0) = \frac{1}{M_N} \int d^3r\, T_{00}(\vec{r}, \vec{s}) = 1$

with

$$\int \mathrm{d}^3 r \, T_{00}(\vec{r}) = M_N \qquad \text{known}$$

$$J(0) = \int \mathrm{d}^3 r \, \varepsilon^{ijk} \, s_i \, r_j \, T_{0k}(\vec{r}) = \frac{1}{2} \qquad \text{known}$$

$$d_1(0) = -\frac{M_N}{2} \int \mathrm{d}^3 r \, \left(r^i r^j - \frac{r^2}{3} \delta^{ij} \right) T_{ij}(\vec{r}) \equiv \mathbf{d_1} \qquad \textbf{new!}$$

for spin 0 or $\frac{1}{2}$: $$\boldsymbol{T_{ij}(\vec{r}) = s(r) \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \, \delta_{ij}}$$

$\boldsymbol{p(r)}$ distribution of *pressure* inside hadron
$\boldsymbol{s(r)}$ related to distribution of *shear forces*
$\longrightarrow$ **"mechanical properties"**

conservation of EMT

$$\partial^\mu T_{\mu\nu} = 0 \quad \Leftrightarrow \quad \nabla^i T_{ij}(\vec{r}) = 0 \quad \text{implies:} \quad \frac{2}{3}\, s'(r) + \frac{2}{r}\, s(r) + p'(r) = 0$$

$\longrightarrow$ "stability condition" $$\boxed{\int_0^\infty \mathrm{d}r\; r^2\, p(r) = 0}$$

minimal requirement no. 2 for consistent model: stability condition

finally $$\boldsymbol{d_1(t)} = \frac{15\, M_N}{2\, t} \int \mathrm{d}^3 r\; r^2 j_0(r\sqrt{-t})\, \boldsymbol{p(r)}$$

$\longrightarrow$ **d_1 vs. pressure:** $$\boxed{d_1 = 5\pi M_N \int_0^\infty \mathrm{d}r\; r^4\, p(r)}$$

or $\longrightarrow$ $$\boldsymbol{d_1} = -\frac{1}{3}\, M_N \int \mathrm{d}^3 r\; r^2 \boldsymbol{s(r)}$$

What do we know about d_1?

- pion: $\frac{4}{5} d_1 = -M_2$ soft-pion theorems (M.V.Polyakov and C.Weiss, 1999)

- nucleus liquid drop model $d_1 < 0 \leftrightarrow$ "surface tension" (M.V.Polyakov, 2003)
  (explicit calculations V.Guzey, M.Siddikov, 2006)

- nucleon
  large N_c limit: $|d_1^u + d_1^d| = \mathcal{O}(N_c^2) \gg |d_1^u - d_1^d| = \mathcal{O}(N_c)$
  chiral quark soliton model: $d_1^Q \approx -4$ (Petrov et al 1999, Kivel et al, high scale)
  χPT (Chen, Ji 2001, Diehl Manashov, Schäfer 2005, Ando, Chen, Kao 2006, ...)
  lattice QCD at 2 GeV: $d_1^Q < 0$ (LHPC 2003, QCDSF 2004)
  chiral quark soliton model: $d_1 = -4.8$ (Goeke et al 2007, Wakamatsu)
  Skyrme model: $d_1 = -6.6$ (chiral limit, less with m_π Cebulla et al 2007)
  bag model: $d_1 = -1.4$ (Matt Neubelt, PS, New England APS Fall Meeting 2011)
  Q-balls: $d_1 < 0$ always (Manuel Mai, masters thesis 2009; Mai & PS forthcoming)

Observation: d_1 seems always negative

Can we understand why?

Yes! (see models below)

When using models:

- **minimal requirement no. 1: polynomiality (kinematics, Lorentz inv.)**

$$\int \mathrm{d}x\, x^{N-1}\, \mathrm{GPD}(x,\xi,t) = a(t)\,\xi^0 + b(t)\,\xi^2 + c(t)\,\xi^4 \ldots + d(t)\,\xi^N \qquad \text{(even } N\text{)}$$

- **minimal requirement no. 2: stability condition (dynamics!)**

$$\int_0^\infty \mathrm{d}r\; r^2\, p(r) = 0$$

3. Intuition on pressure, shear forces, d_1

"Liquid drop"

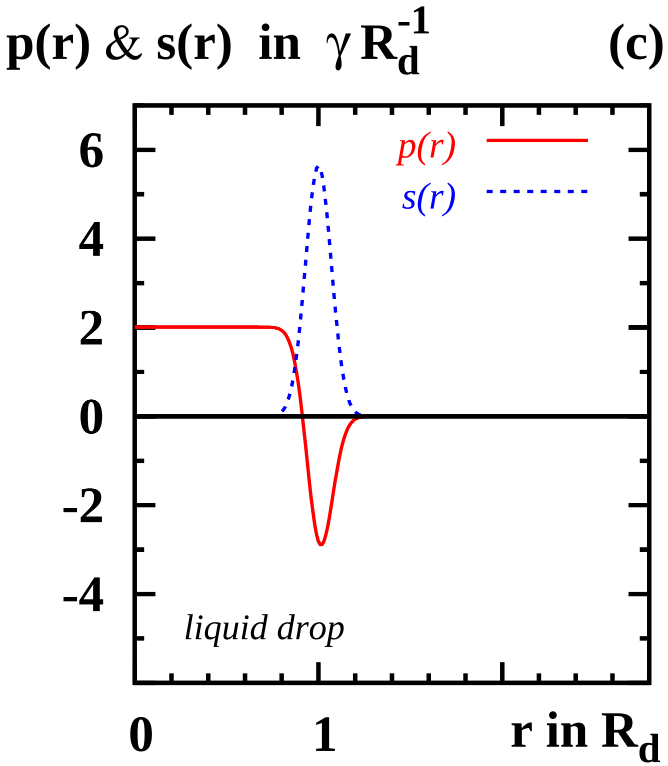
$$p(r) = p_0\,\theta(R_d - r) - \frac{1}{3}\,p_0 R_d\,\delta(R_d - r)$$

$$s(r) = \gamma\,\delta(R_d - r)$$

$$\gamma = \frac{1}{2}\,p_0 R_d$$

$$= \text{surface tension (Kelvin, 1858)}$$

$$d_1 = -\frac{4}{3}\pi\,\gamma\,R_d^4$$



- $d_1^{\text{nucleus}} \propto A^{4/3}$ (since $R \propto A^{1/3}$, M.V.Polyakov, 2003)
- nuclei can be approximated as "liquid drops" (Guzey, Siddikov, 2006)
- nucleon is more diffuse, no edge. How does it look like?

3. Chiral quark-soliton model

Chiral action $S_{\text{eff}} = \int d^4x\, \overline{\Psi}(i\not{\partial} - M\,U^{\gamma_5} - m)\Psi$ with $U^{\gamma_5} = \exp(i\gamma_5\tau^a\pi^a/f_\pi)$

- derived from instanton model of QCD vacuum Diakonov, Petrov 1984, ...
  ("instanton packing fraction" $\rho_{\text{av}}/R_{\text{av}} \sim \frac{1}{3}$ small,
  ρ_{av} = instanton size, R_{av} = instanton separation)

- $M = 350\,\text{MeV}$, $\Lambda_{\text{cut}} \sim \rho_{\text{av}}^{-1} \sim 600\,\text{MeV}$, no adjustable parameters!

- integrate out quarks: $\mathcal{L}_{\text{eff}} = \frac{1}{4} f_\pi^2 \partial^\mu U \partial_\mu U^\dagger$ + Gasser-Leutwyler-terms + ...
  with all coefficients known.

- apply to the description of nucleon $\rightarrow$ chiral quark soliton model
  Diakonov, Petrov, Pobylitsa 1988

- nucleon = soliton of chiral field U, in limit number of colors $N_c \rightarrow$ large
  Witten 1979

- interpolates: QCD $(q, \bar{q}, g) \longleftrightarrow$ model $(q, \bar{q}, \pi) \longleftrightarrow$ χPT $(N,\ \pi)$

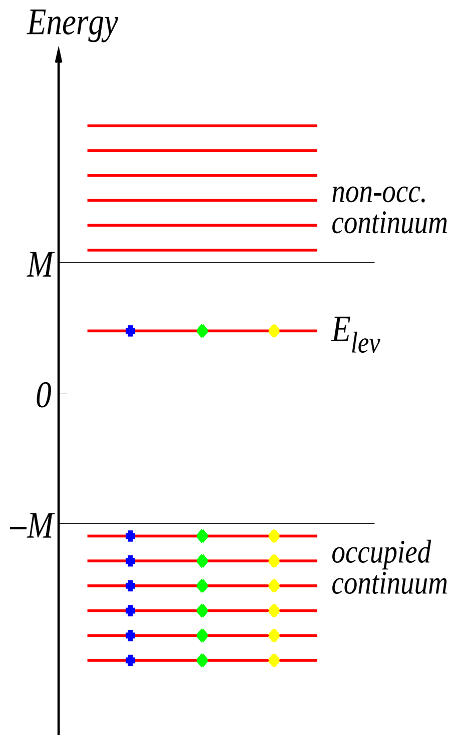
Description of nucleon

Euclidean correlation function

$\int \mathcal{D}\Psi^\dagger \int \mathcal{D}\Psi \int \mathcal{D}U \, J_N(T/2) J_N^\dagger(-T/2) \, e^{-S_{\rm eff}} \sim e^{-M_N T}$ as $T \to \infty$

Solve in limit $N_c \to$ **large** $\Rightarrow$ minimize action/soliton energy $\delta M_N = 0$
yields 'self consistent' field $U_s = \exp(i\tau^a e_r^a P_s(r))$

$$\longrightarrow \quad M_N = N_c \sum_{\rm occupied} \Big(E_n - E_{n,0}\Big)_{\rm reg}$$



$H\phi_n = E_n \, \phi_n$, $H = -i\gamma^0\gamma^k\partial_k + \gamma^0 M U^{\gamma_5} + \gamma^0 m$
$E_{n,0}$ eigenvalues of H_0 (H with $U^{\gamma_5} \to 1$)

Features:

- Field theory!
- theoretically consistent
- phenomenologically successful
- applications $\langle N'|\overline{\Psi}(0)\Gamma\Psi(z)|N\rangle = A \sum_n \overline{\phi}_n(0)\Gamma\phi_n(z) + \ldots$

Applications of the model

- 'static properties' (baryon mass splittings, . . .)$\surd$

- form factors (em, axial) up to $|t| \sim \mathcal{O}(1\,\text{GeV}^2)$$\surd$

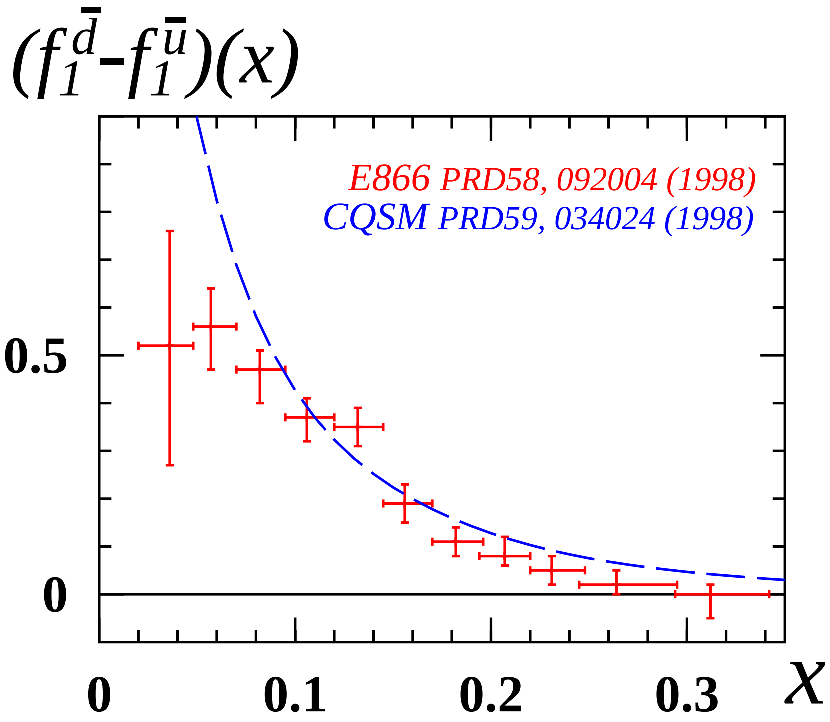
- $f_1^a(x)$, $g_1^a(x)$, $h_1^a(x)$ at $\mu \sim \rho_{\text{av}}^{-1} \sim 0.6\,\text{GeV}$
  satisfy sum rules, positivity, inequalities!$\surd$

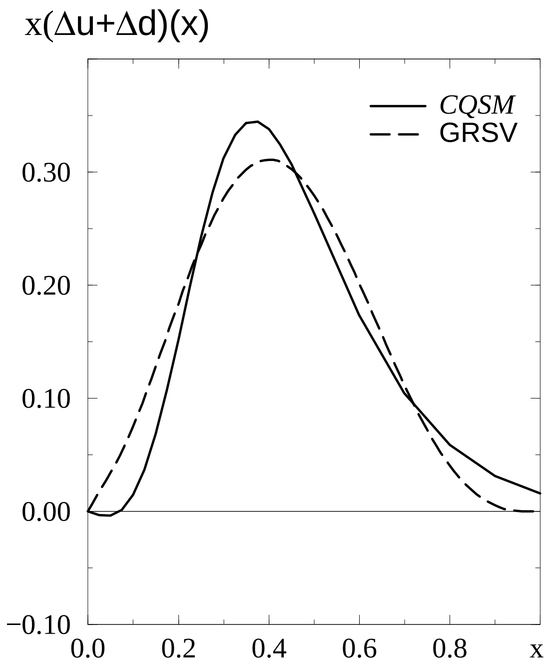
- GPDs ('discovery of D-term')
  satisfy all requirements$\surd$
  including polynomiality!!!$\surd$

- EMT form factors
  same from $T_{\mu\nu}$ and GPDs!$\surd$

accuracy (10-30)% (higher orders in $1/N_c$, instanton vacuum)
catches properties of nucleon due to chiral physics

Example:





$f_1^{\bar q}(x)$ at $Q = 7.35\,\text{GeV}$

$g_1^q(x)$ at low scale vs. GRSV

disconnected diagrams included!

(DPPPW, Wakamatsu et al) satisfactory! ✓

Many more examples.

Form factors of $T_{\mu\nu}$ in model

in model $T^{Q}_{\mu\nu}$ is total $T_{\mu\nu}$ (gluons suppressed in instanton vacuum)

given by $\hat{T}_{\mu\nu} = \frac{1}{4}\,\bar{\psi}(x)\left(i\gamma^{\mu}\overrightarrow{\partial}^{\nu} + i\gamma^{\nu}\overrightarrow{\partial}^{\mu} - i\gamma^{\mu}\overleftarrow{\partial}^{\nu} - i\gamma^{\nu}\overleftarrow{\partial}^{\mu}\right)\psi(x)$

$$\langle p'|\hat{T}_{\mu\nu}(0)|p\rangle = \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\; e^{i\mathbf{p}'\mathbf{y}-i\mathbf{p}\mathbf{x}} \int \mathcal{D}\psi\,\mathcal{D}\bar{\psi}\,\mathcal{D}U\; J_{N'}(-\tfrac{T}{2},\mathbf{y})\; T^{\mathrm{eff}}_{\mu\nu}(0) J^{\dagger}_{N}(\tfrac{T}{2},\mathbf{x})\; e^{iS_{\mathrm{eff}}}$$

results refer to $|t| = \mathcal{O}(N_c^0) < M_N^2 = \mathcal{O}(N_c^2)$

$$M_2(t) - \frac{t}{5M_N^2} d_1(t) = \frac{1}{M_N} \int \mathrm{d}^3\mathbf{r}\; \rho_E(r)\; j_0(r\sqrt{-t})$$

$$d_1(t) = \frac{15M_N}{2} \int \mathrm{d}^3\mathbf{r}\;\; p(r)\;\; \frac{j_0(r\sqrt{-t})}{t}$$

$$J(t) = 3 \int \mathrm{d}^3\mathbf{r}\; \rho_J(r)\; \frac{j_1(r\sqrt{-t})}{r\sqrt{-t}}$$

$$\bar{c}(t) = 0$$ quark $T_{\mu\nu}$ conserved by itself in model √

with the "densities" ...

with the "densities" defined as:

$$\rho_E(r) = N_c \sum_{n,\text{occ}} E_n\, \phi_n^\dagger(\vec{r})\phi_n(\vec{r})|_{\text{reg}} \equiv T_{00}(r) \quad \text{energy density}$$

$$p(r) = \frac{N_c}{3} \sum_{n,\text{occ}} \phi_n^\dagger(\vec{r})\,(\gamma^0 \vec{\gamma}\,\widehat{\vec{p}}\,)\,\phi_n(\vec{r})|_{\text{reg}} \equiv \quad \text{pressure}$$

$$\rho_J(r) = -\frac{N_c}{24I} \sum_{\substack{n,\text{occ} \\ j,\text{non}}} \epsilon^{abc} r^a \phi_j^\dagger(\vec{r})(2\widehat{p}^b + (E_n + E_j)\gamma^0\gamma^b)\phi_n(\vec{r})\,\frac{\langle n|\tau^c|j\rangle}{E_j - E_n}\Big|_{\text{reg}}$$

"angular momentum density"

Now: test the consistency

important (technical) remark:
analytical manipulations in terms of evaluated quark wave-functions,
i.e. no operator identities

but equations of motion $\Leftrightarrow$ $\delta M_N = 0$ collective many-body phenomenon

Consistency

I $\quad M_N = \int d^3r\, T_{00}(r) \;\;\Leftrightarrow\;\; M_2(0) = 1 \;\surd$

II $\quad J(0) = \int d^3\mathbf{r}\, \rho_J(r) = \;\dots\; = \frac{1}{2}\surd$ (decomposition, evolution → Thomas, Wakamatsu)

III $\quad \int_0^\infty dr\, r^2 p(r) \propto \sum_{n,\mathrm{occ}} \langle n|\,(\gamma^0 \vec{\gamma}\,\widehat{\vec{p}}\,)\,|n\rangle \overset{!}{=} 0 \surd$ if evaluated at true minimum U_s!

IV same form factors from GPDs $\surd$

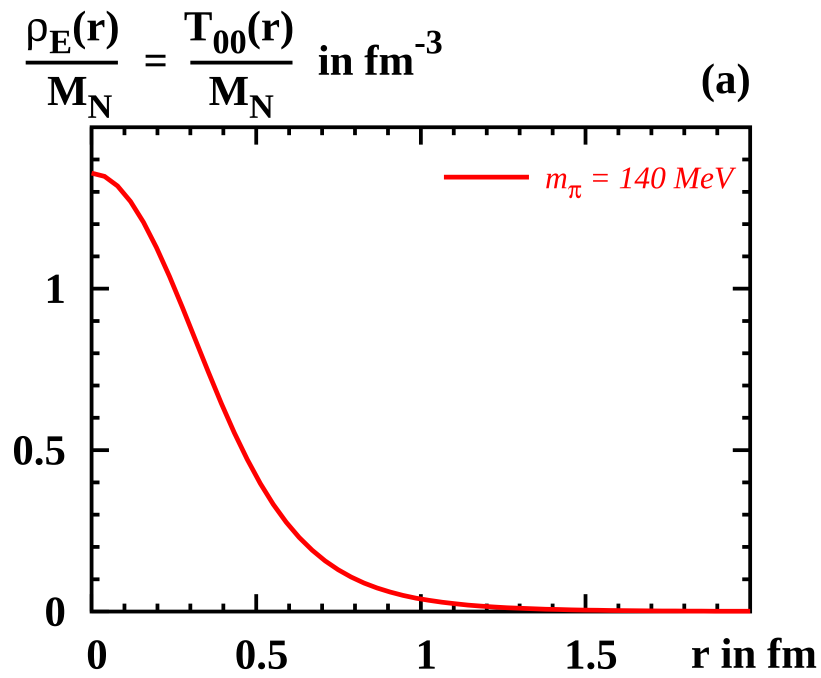
V $\quad \int dx \sum_q x H^q(x,\xi,t) = M_2(t) + \frac{4}{5}\, d_1(t)\, \xi^2 \;\surd$

VI $\quad \int dx \sum_q x(H^q + E^q)(x,\xi,t) = 2J(t) \;\surd$

Ossmann et al, PRD71, 034011 (2005)

Model is consistent.
Let us see what it predicts.

Results: energy distribution



- $\rho_E(0) = 1.7$ GeV fm^{-3} $= 3.0 \times 10^{15}$g/cm^3
  $\sim 13\times$nuclear matter equilibrium density

- distributed 'similar' to electric charge

- chiral limit: $\rho_E(r) \sim 3\left(\frac{3g_A}{8\pi f_\pi}\right)^2 \frac{1}{r^6}$ at large r

- $\langle r_E^2 \rangle \equiv \dfrac{\int \mathrm{d}^3\mathbf{r}\; r^2 \rho_E(r)}{\int \mathrm{d}^3\mathbf{r}\; \rho_E(r)} = 0.7$ fm^2 similar to proton electric charge radius

- leading non-analytic term $\langle r_E^2 \rangle = \langle \overset{\circ}{r}{}_E^2 \rangle - \dfrac{81\, g_A^2}{64\pi f_\pi^2 M_N}\, m_\pi +$ higher orders

- for $m_\pi \to 0$ nucleon 'grows' (range of pion cloud increases)

- reasonable & consistent picture

Compare: energy vs. baryon number density

in Skyrme model   Cebulla et al

- baryon number density

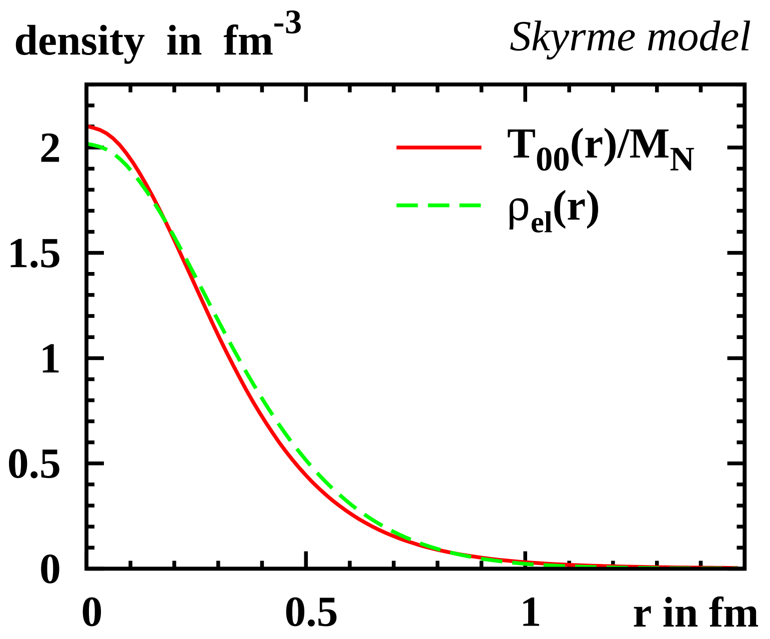
  $=$ isoscalar charge density

- distributions similar at intermediate r

- significant differences at large r

  $\rho_E(r) \propto \frac{1}{r^6}$ vs. $\rho_{el}(r) \propto \frac{1}{r^9}$

  different chiral physics!



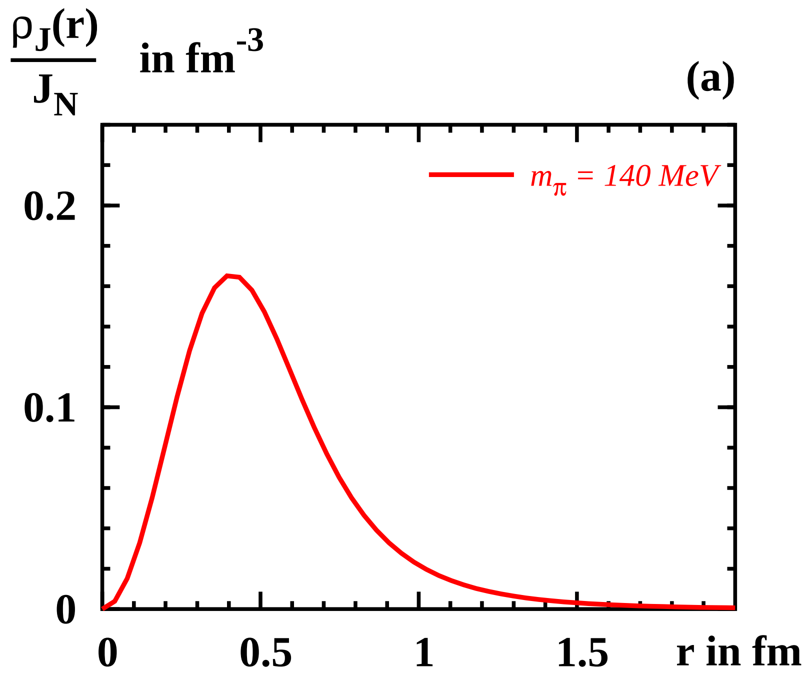
Picture in model.
And, in nature?

Angular momentum distribution

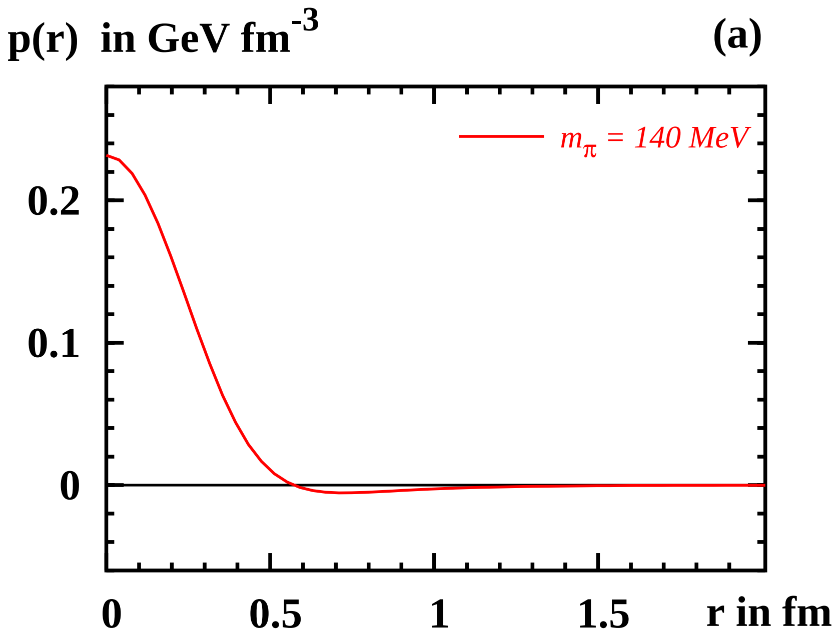
- $\rho_J(r) \propto r^2$ at small r
- $\langle r_J^2 \rangle = 1.3\ \text{fm}^2$

  2 times larger than $\langle r_E^2 \rangle$ or $\langle r_{em}^2 \rangle$
- in chiral limit: $\rho_J(r) \sim \frac{1}{r^4}$ at large r

  such that $\langle r_J^2 \rangle$ diverges



Pressure



- $p(0) = 0.23\,\text{GeV/fm}^3 = 4\cdot 10^{34}\,\text{N/m}^2$

  $\sim$ (10–20) $\times$ (pressure in neutron star)

- $p(0) \times$ (typical hadronic area $1\,\text{fm}^2$)

  $\sim\ 0.2\,\text{GeV/fm} \sim \dfrac{1}{5} \times \{\text{string tension}\}$

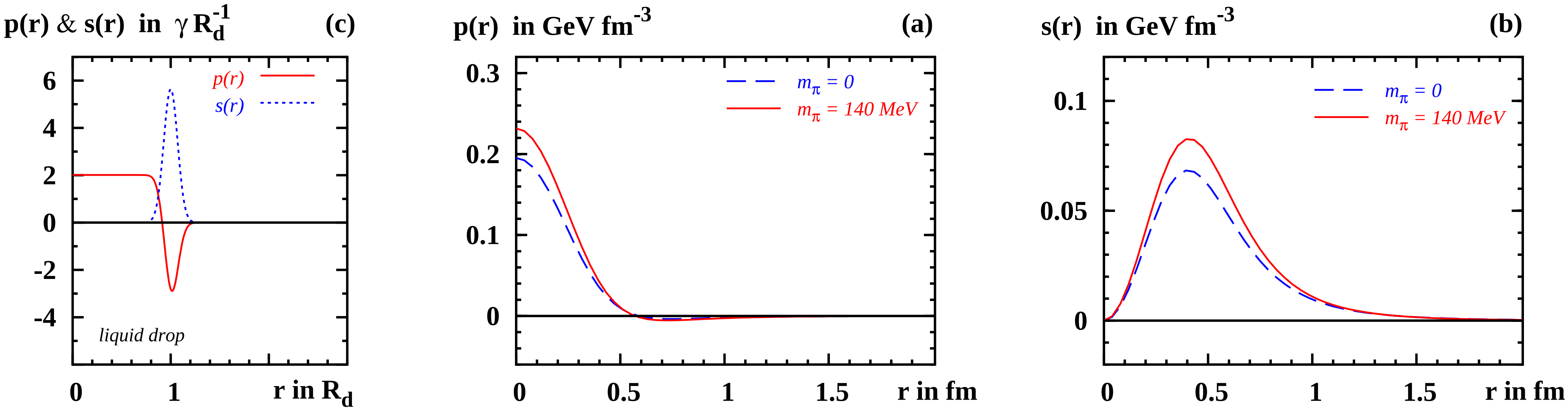
- chiral limit: $p(r) \sim\ -\left(\frac{3g_A}{8\pi f_\pi}\right)^2 \frac{1}{r^6}$ at large r

- consequence: derivative $d_1'(0) = -\dfrac{3\,g_A^2\,M_N}{32\,\pi f_\pi^2\,m_\pi} + \dots$ diverges in chiral limit

- $r < 0.57\,\text{fm}$: $\boldsymbol{p(r) > 0}$ $\Rightarrow$ **repulsion** $\leftrightarrow$ quark core, Pauli principle

- $r > 0.57\,\text{fm}$: $\boldsymbol{p(r) < 0}$ $\Rightarrow$ **attraction** $\leftrightarrow$ pion cloud, binding forces

Compare to liquid drop



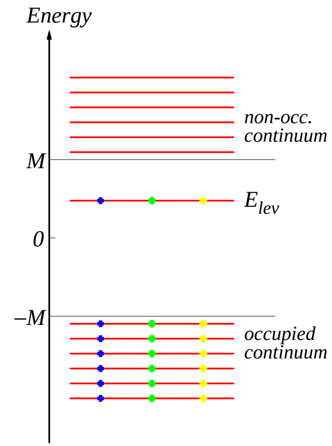
nucleon does not resemble much a liquid drop

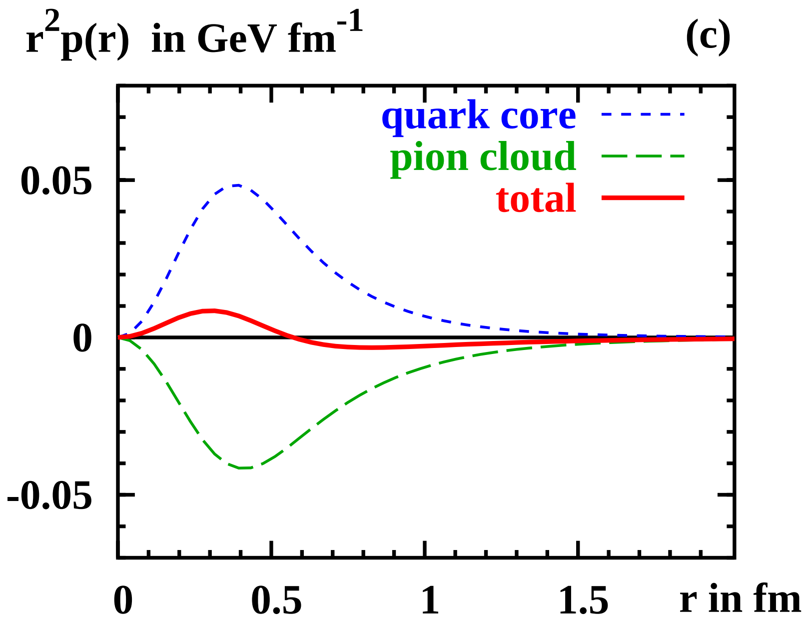
the “edge” is very diffuse (of course)

concept more useful for nuclei

Insights into stability of soliton

$M_N = \min_U E_{\text{sol}}[U], \quad E_{\text{sol}}[U] = N_c(E_{\text{lev}} + E_{\text{cont}})$ Recall:





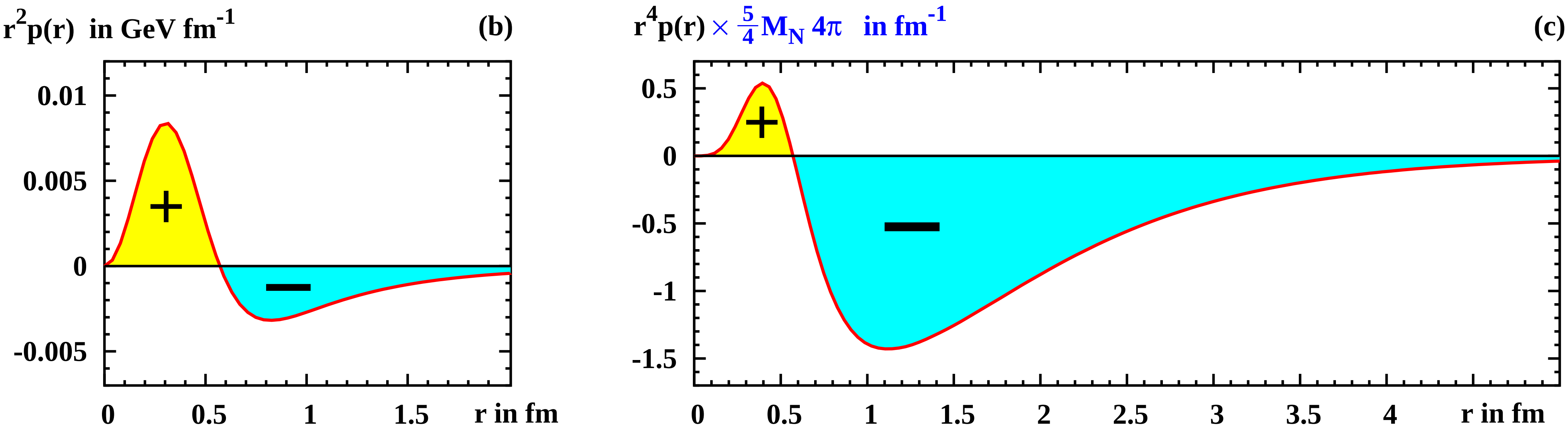
level part: **"quark core"** → **repulsion**

continuum: **"pion cloud"** → **attraction**

we learn:

- strong forces in nucleon really strong (even without glue)
- what we always knew (but now can quantify): **nucleon stability** due to subtle balance between **repulsive quark core** and **attractive pion cloud**!

Stability & sign of D-term



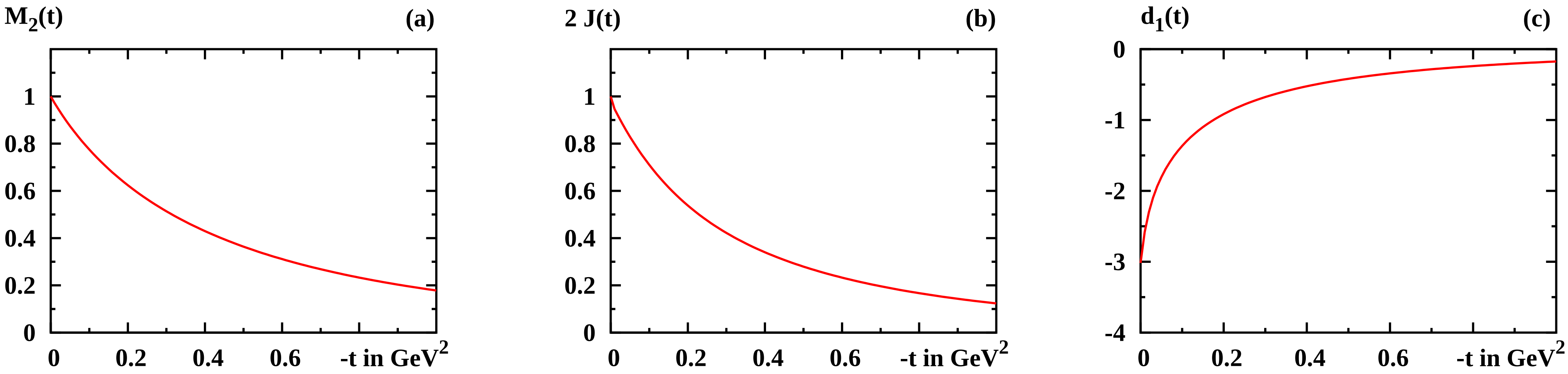
- $\int_0^\infty \mathrm{d}r\ r^2 p(r) = 0$ ✓

- $\int_0^\infty \mathrm{d}r\ r^4 p(r) < 0,$ **of course!**

- $d_1 = \frac{5}{4} M_N \int \mathrm{d}^3\mathbf{r}\ r^2 p(r) < 0$ **natural consequence of stability!**

d_1 negative, could be a theorem! Remains to be proven in general.

Results for form factors



$M_{\text{dip}}(M_2) = 0.91\,\text{GeV}$ $\qquad$ $M_{\text{dip}}(J) = 0.75\,\text{GeV}$ $\qquad$ $M_{\text{dip}}(d_1) = 0.65\,\text{GeV}$

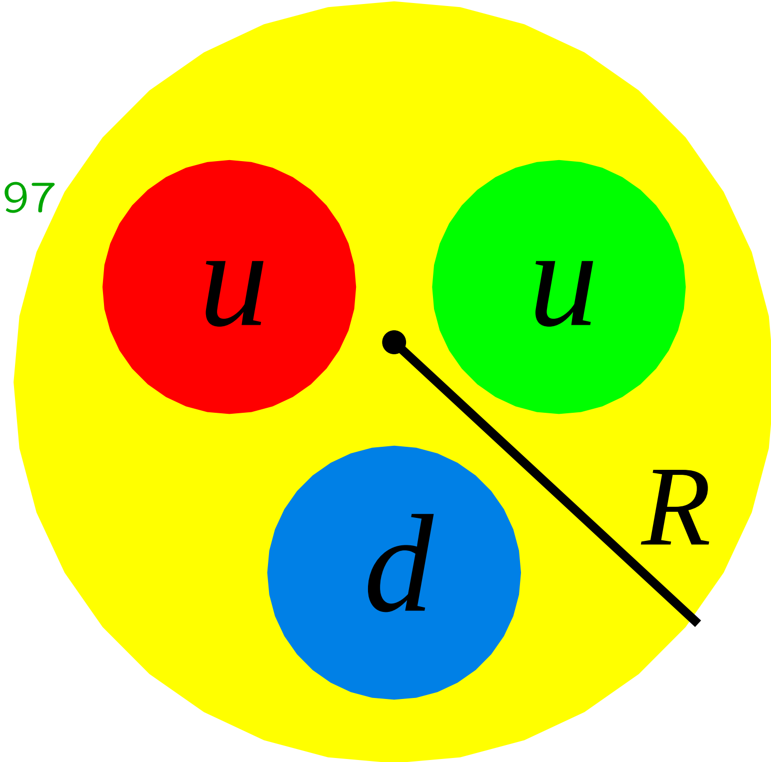
for $|t| \lesssim 1\,\text{GeV}^2$ reasonable approximation $\boldsymbol{F(t) \approx \dfrac{F(0)}{(1 - t/M_{\text{dip}}^2)^2}}$ with

vs. electromagnetic form factors, for example, $G_E^p(t)$ with $M_{\text{dip}} \approx 0.91\,\text{GeV}$

$\Rightarrow$ $\boldsymbol{M_2(t)}$ similar to em form factors, $\boldsymbol{J(t)}$ and $\boldsymbol{d_1(t)}$ different
Instructive: need to extrapolate from data at $t < 0$ to get $J^Q(0)$!

4. MIT bag model

GPS & $A(t)$, $B(t)$, $C(t)$ Ji, Melnitchouk, Song 1997



But how do the densities look like?

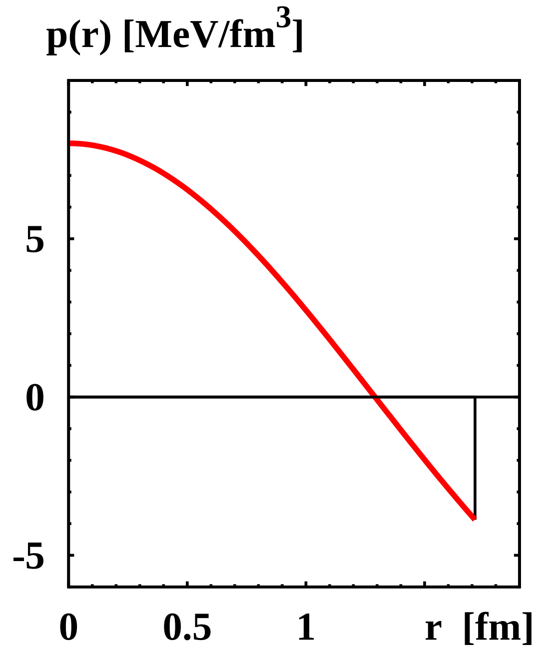
Matt Neubelt, UConn graduating senior

$$M_N = \int \mathrm{d}^3x\, T_{00}(r) = \min_R \Big(\frac{3\omega}{R} + \frac{4\pi}{3} R^3 B \Big)$$

$$\frac{\partial M_N}{\partial R} = 0 \;\Rightarrow\; M_N = \frac{4\omega}{R} \;\Rightarrow\; \int_0^R \mathrm{d}r\, r^2 p(r) = 0$$

simple but fully consistent model!

$d_1 = -1.4 < 0$ negative!



vs. chiral models: magnitude smaller

How does it look like in cloudy bag model?

6. Q-balls S. Coleman, NPB262 (1985) 263, Lee, Wick, Friedberg, Sirlin 1970s

- Look for static solution ("solitons") in: $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$

  static: $H = \int d^D x \left(\frac{1}{2}\nabla^i\phi\nabla^i\phi + V(\phi)\right)$

  dilations $x \to \lambda x$: $M(\lambda) = \frac{1}{2} E_{\text{surf}}\lambda^{2-D} + U_{\text{pot}}\lambda^{-D}$

  $M'(\lambda) = 0$ possible only in $D = 1$ space dimension

  $\to$ no static soliton solutions in $D \geq 2$ dimensions (Derrick's theorem)

- Coleman: consider $\mathcal{L} = \frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi + \frac{1}{2}\partial_\mu^*\phi\partial^\mu\phi - V(|\phi|)$
  with glogal continous symmetry $\phi(x) \to e^{i\omega t}\phi(r)$

  conserved Noether charge $Q = \omega \int d^D x\, \phi(r)^2 = \omega\, I$

  $M = \int d^D x \left(\frac{1}{2}\omega^2\phi^2 + \frac{1}{2}\nabla^i\phi\nabla^i\phi + V(\phi)\right)$

now dilations $x \to \lambda x$: $\quad M(\lambda) = \lambda^3 \frac{Q^2}{2I} + \frac{1}{2}\lambda^{2-D} E_{\text{surf}} + \lambda^{-D} U_{\text{pot}}$

$$M'(\lambda) = \frac{3Q^2}{2I}\,\lambda^2 + E_{\text{surf}}\,\frac{(2-D)}{2\lambda^{D-1}} - U_{\text{pot}}\,\frac{D}{\lambda^{D+1}} = 0 \quad \text{possible in } D > 2$$

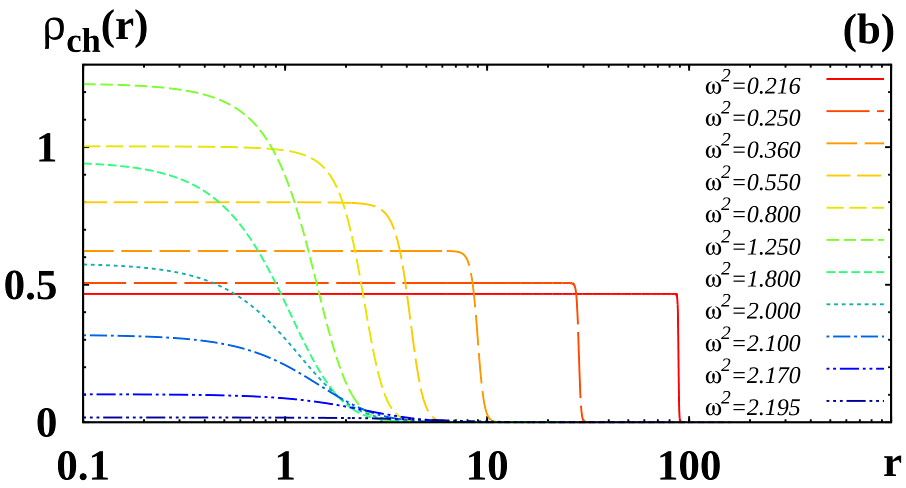
more precisely, if $\omega^2_{\text{min}} < \omega^2 < \omega^2_{\text{max}}$, solitons may exist! $\quad \omega^2_{\text{min}} = \min_\phi V(\phi)/\phi^2$, $\omega^2_{\text{max}} = V''(\phi)|_{\phi=0}$

Coleman: solitons do exist in limit $\omega \to \omega_{\text{min}}$,
extended objects with constant density inside,
and sharp edge: Q-balls

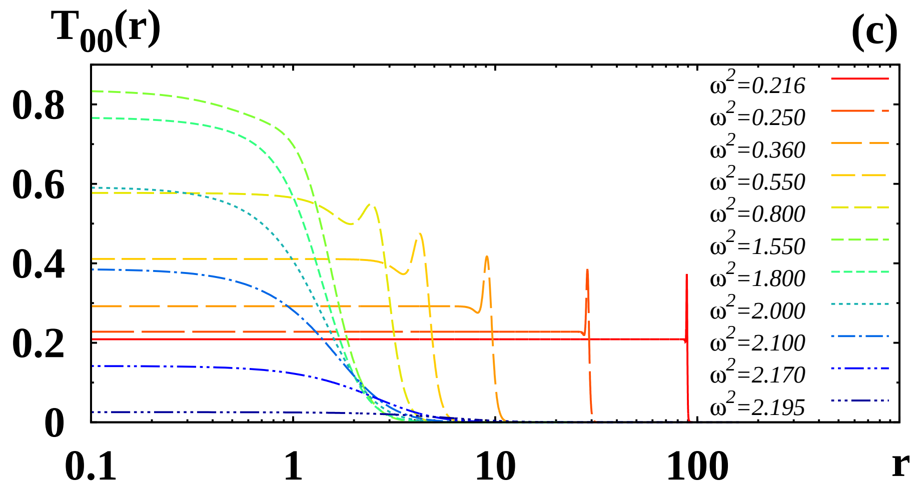
- what are Q-balls good for?
  can form in early universe (degrees of freedom, thermodynamics) (Coleman)
  exist in SUSY extensions of standard model (Kusenko & Shaposhnikov 1998)
  dark matter? baryon asymmetry in univers? neutron stars? (...)

- how can they help us?
  compute d_1, study stability, and learn

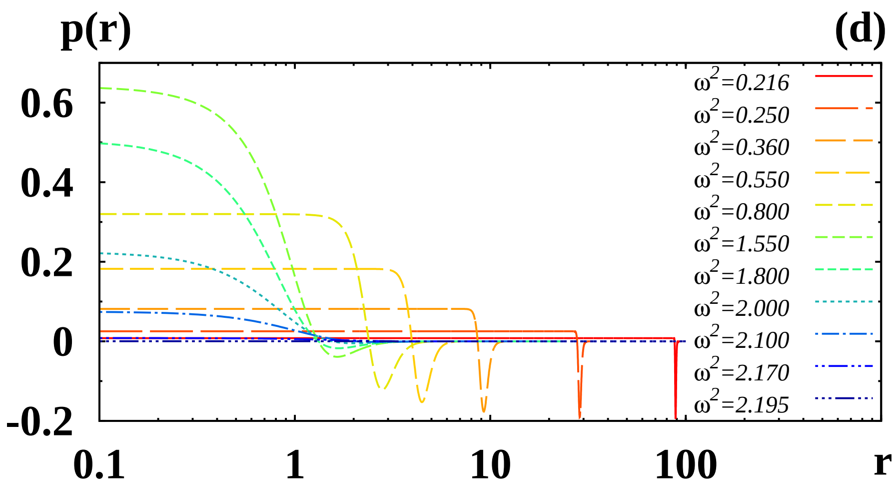
charge distribution



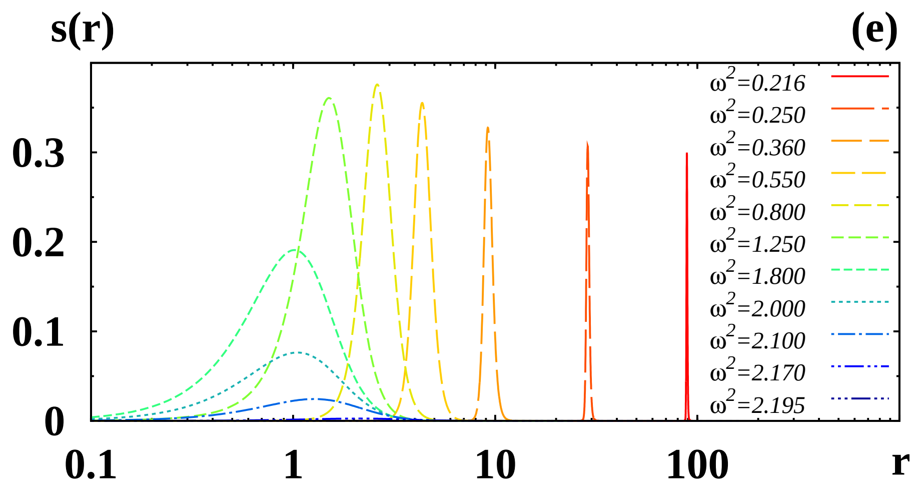
enegergy density



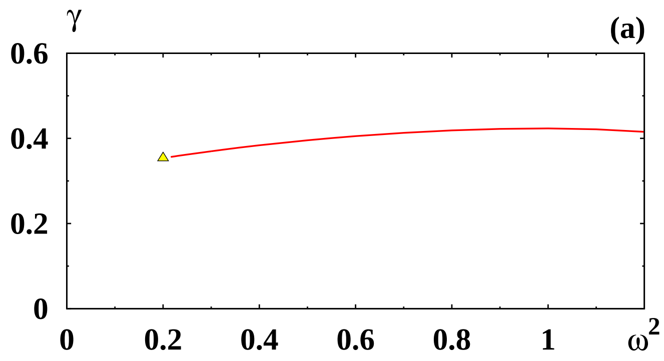
pressure



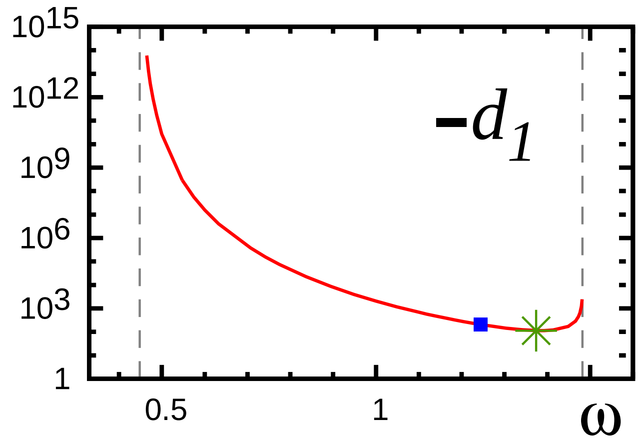
shear forces



- limits $\omega \to \omega_{\text{extreme}}$ under analytical control!
- e.g. surface tension γ



- d_1 varies over 12 orders of magnitude
  feels size of system (and $R \to \infty$)
  $\boldsymbol{d_1 = -\frac{4}{3}\pi\,\gamma\,R^4}$ ✓ Kelvin $\gamma = \frac{1}{2}\,p(0)R$ ✓



- Q-balls **like liquid drops** for $\omega \to \omega_{\text{min}}$
  like nuclei (of course → neutron stars!)
- and **always** $d_1 < 0$
- but Q-balls not always stable: **stability ⇒ d_1 negative**
  ($d_1 <$ 0 necessary [not sufficient] condition for stability) **Can d_1 be positive???**
  (Manuel Mai, masters thesis 2009; Mai & PS forthcoming)

7. Conclusions

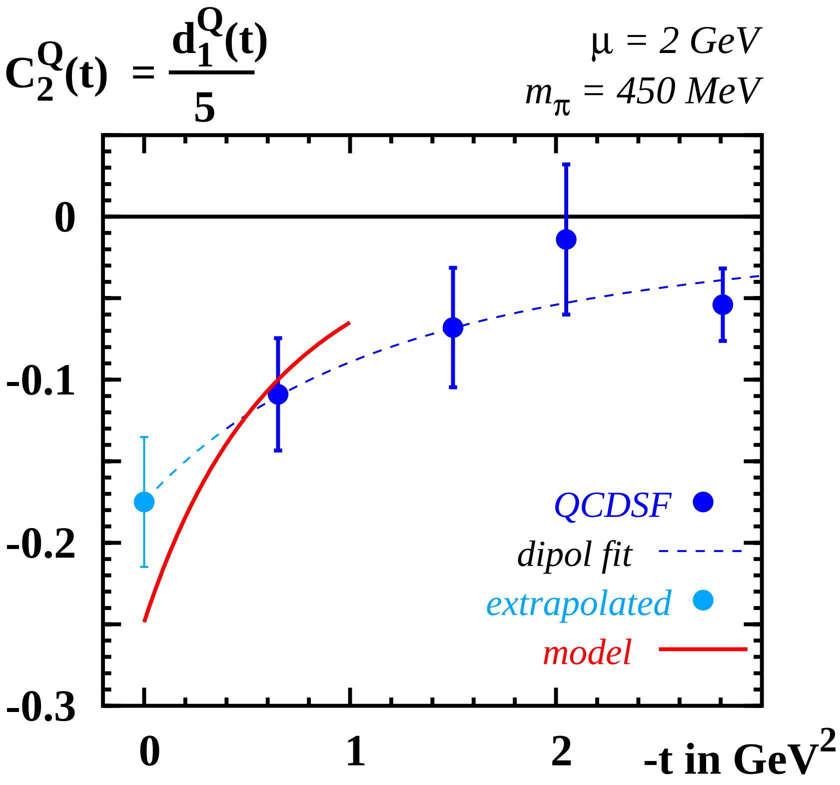
- **EMT form factors** accessible via GPDs, promise fascinating new insights!
- energy density, strong forces (mechanical properties), **rely on models**
  minimal theoretical requirements: polynomiality & stability
- illustration in chiral quark soliton model:
  theoretically consistent (GPDs, polynomiality, stability, etc.)$\surd$
  successful: **chiral dynamics** important for understanding nucleon$\surd$
  (supporting results from Skyrme model)
- interesting lesson: chiral models, bag model, liquid drop: **sign of $d_1 < 0$**
- relation: if **stable** $\Rightarrow$ d_1 negative (not vice versa, lesson from Q-balls)
- unknown: rigorous field theoretical proof that $d_1 < 0$? Can it be positive?
  formulate in terms of 2D distributions? Lattice?? Experiment???

Thank you !!!

Support slide

What about m_π-dependence of EMT form factors?
Focus on $d_1(t)$

QCDSF Göckeler, Horsley, Pleiter, Rakow, Schäfer, Schierholz and Schroers, PRL 92, 042002 (2004); Nucl.Phys.Proc.Suppl. 128, 203 (2004)



K. Goeke et al, PRC75 (2007) 055207

Observations:

- different t-ranges: in model $|t| \ll M_N^2$
- where overlap $\Rightarrow$ **agreement !**
- chiral quark soliton model indicates
  $\Rightarrow$ small t-behaviour different
  $\Rightarrow$ t-extrapolation possibly underestimates $d_1^Q(0)$

$$d_1'(t)|_{t=0} \propto \frac{1}{m_\pi} \quad \textbf{with} \quad m_\pi \to 0$$