

INT 12-49W Workshop on "Orbital Angular Momentum in QCD"
February 6th-17th, 2012 - INT

What semi-inclusive DIS has taught us about TMDs

... they exist!

Spin-Momentum Structure of the Nucleon

$$\frac{1}{2} \text{Tr} \left[(\gamma^+ + \lambda \gamma^+ \gamma_5) \Phi \right] = \frac{1}{2} \left[f_1 + S^i \epsilon^{ij} k^j \frac{1}{m} f_{1T}^\perp + \lambda \Lambda g_1 + \lambda S^i k^i \frac{1}{m} g_{1T} \right]$$

$$\frac{1}{2} \text{Tr} \left[(\gamma^+ - s^j i \sigma^{+j} \gamma_5) \Phi \right] = \frac{1}{2} \left[f_1 + S^i \epsilon^{ij} k^j \frac{1}{m} f_{1T}^\perp + s^i \epsilon^{ij} k^j \frac{1}{m} h_1^\perp + s^i S^i h_1 \right. \\ \left. + s^i (2k^i k^j - \mathbf{k}^2 \delta^{ij}) S^j \frac{1}{2m^2} h_{1T}^\perp + \Lambda s^i k^i \frac{1}{m} h_{1L}^\perp \right]$$

quark pol.

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

nucleon pol.

- each TMD describes a particular spin-momentum correlation
- functions in black survive integration over transverse momentum
- functions in green box are chirally odd
- functions in red are naive T-odd

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helicity

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Sivers

worm-gear

transversity

pretzelosity

Boer-Mulders

TMD "discovery" status

quark pol.

nucleon pol.

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TMD "discovery" status

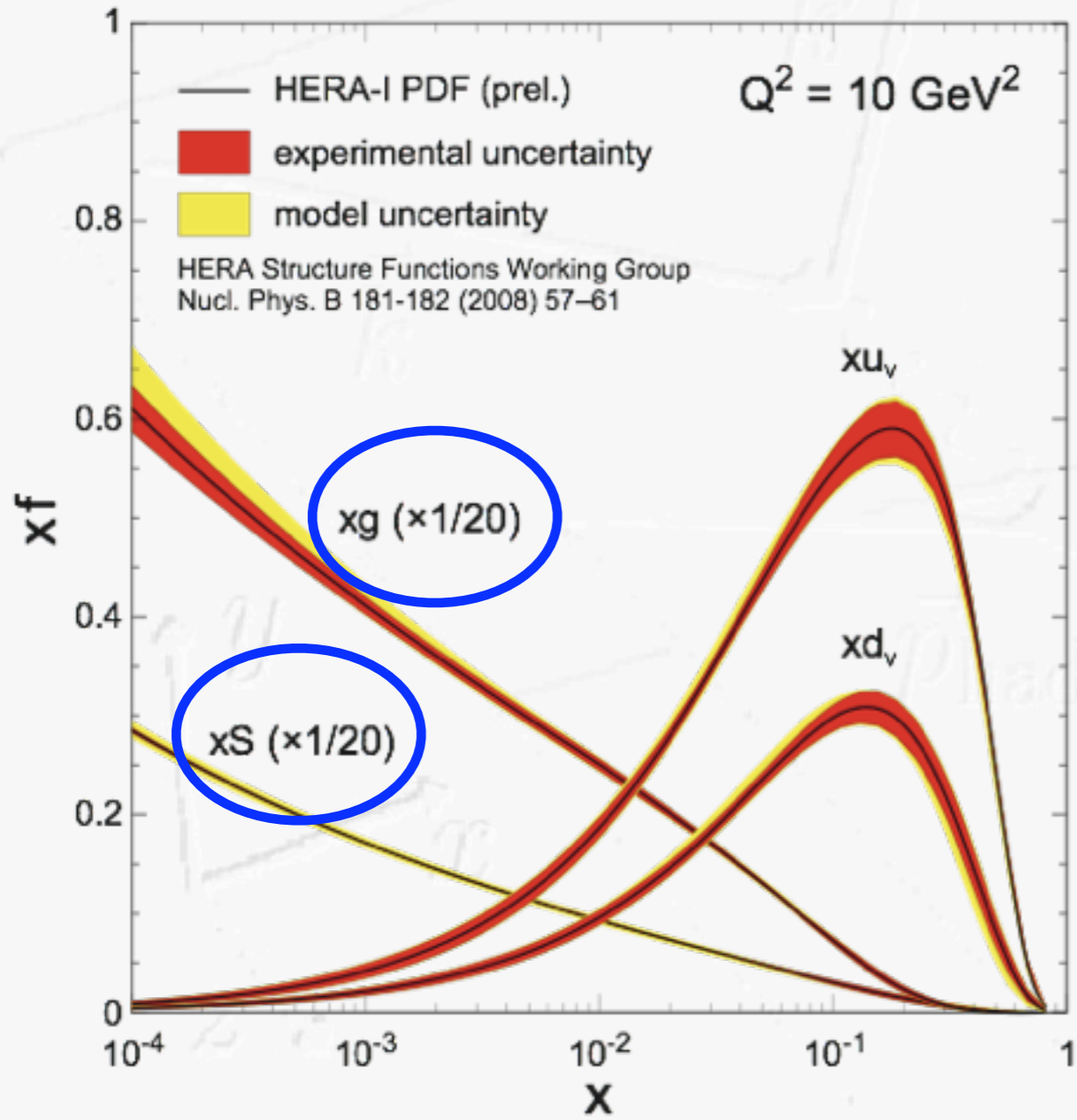
quark pol.

nucleon pol.

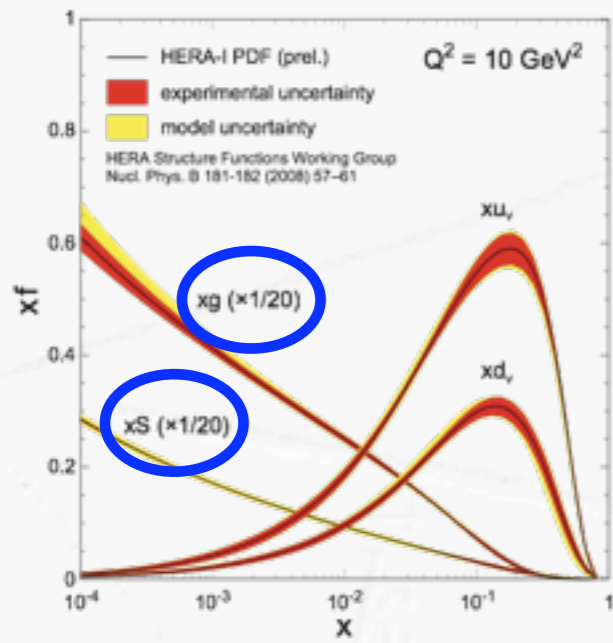
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- flavor dependence (partially) explored
- transverse-momentum dependence (largely) unknown

snapshots



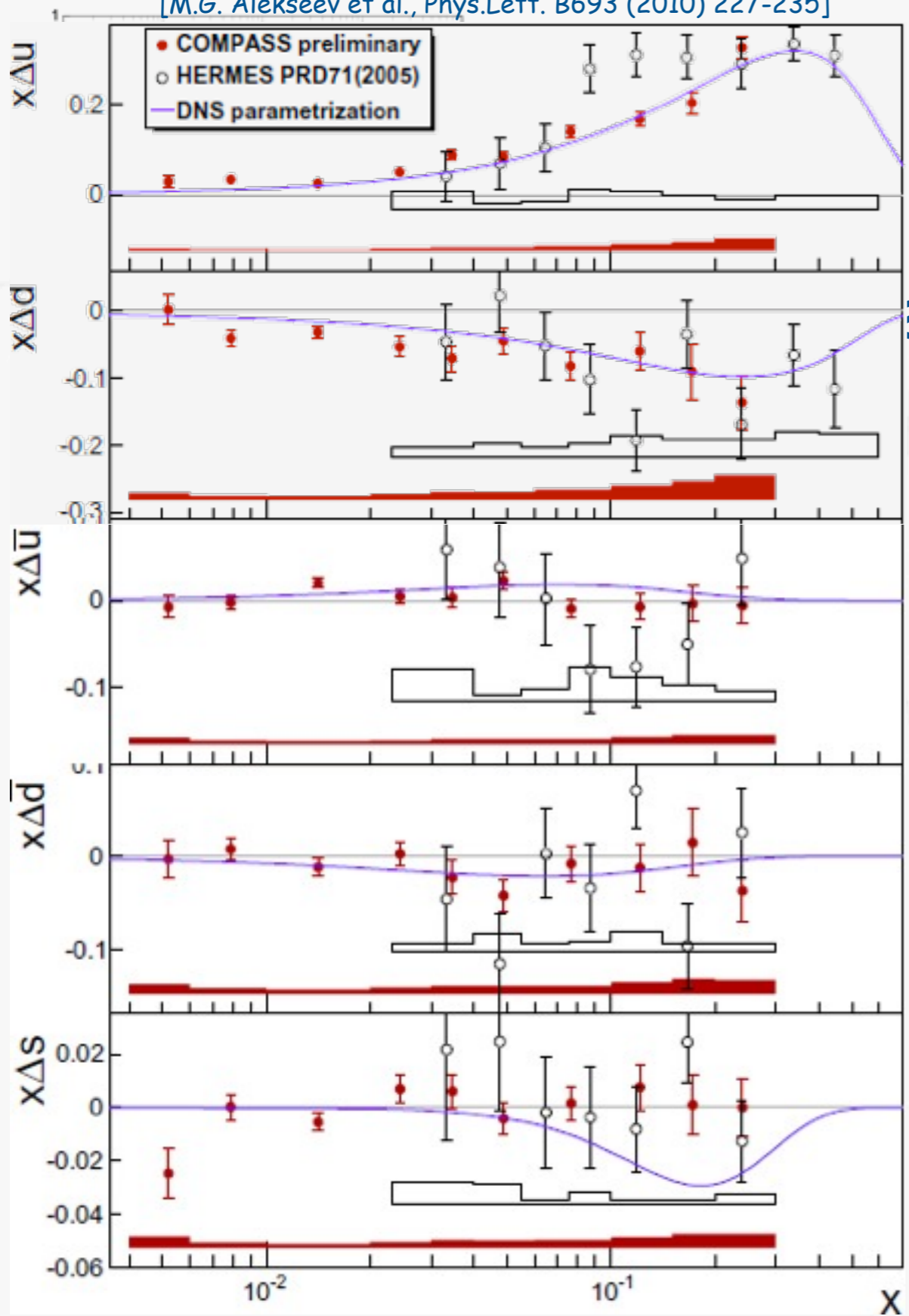
snapshots



need p_T dependence for f_1 📱 6mins*)

*) numbers in $A=0$ (= no Audience) gauge

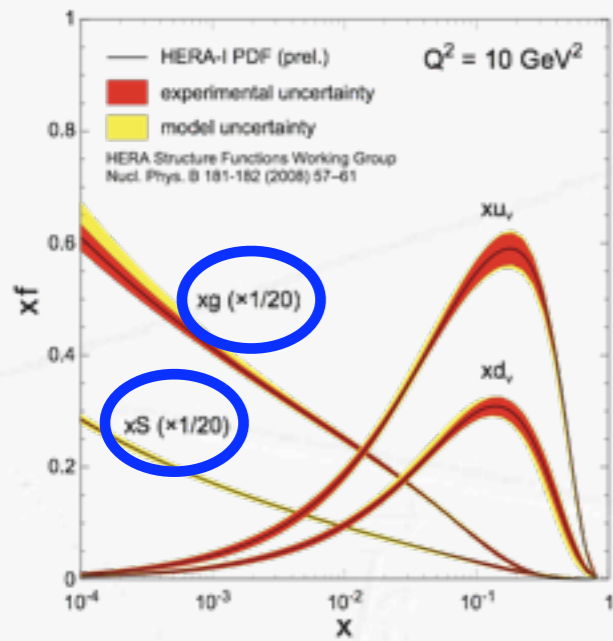
snapshots



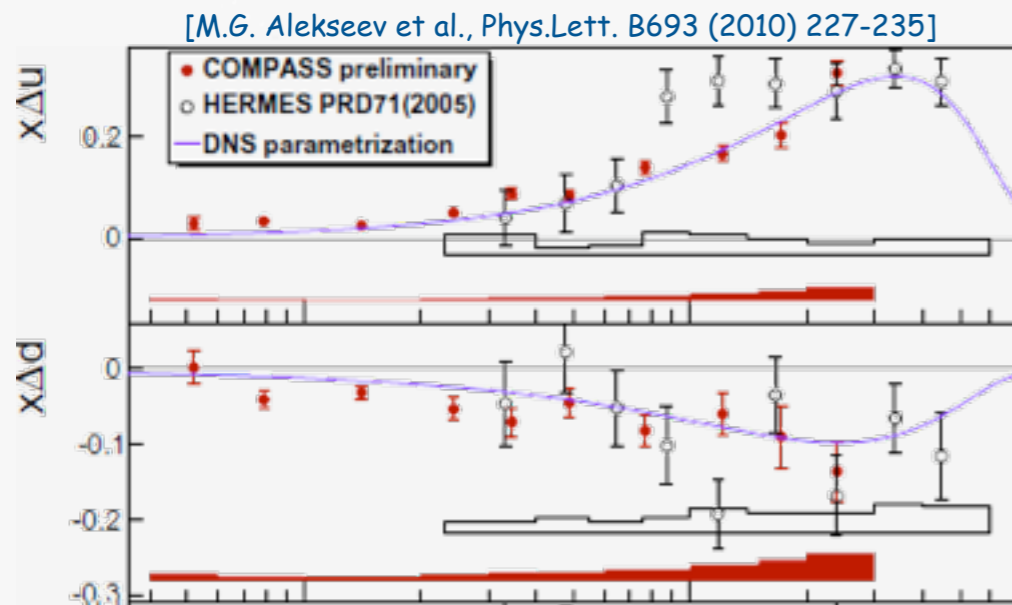
dependence for f_1 (pointing to 6mins*)

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snapshots



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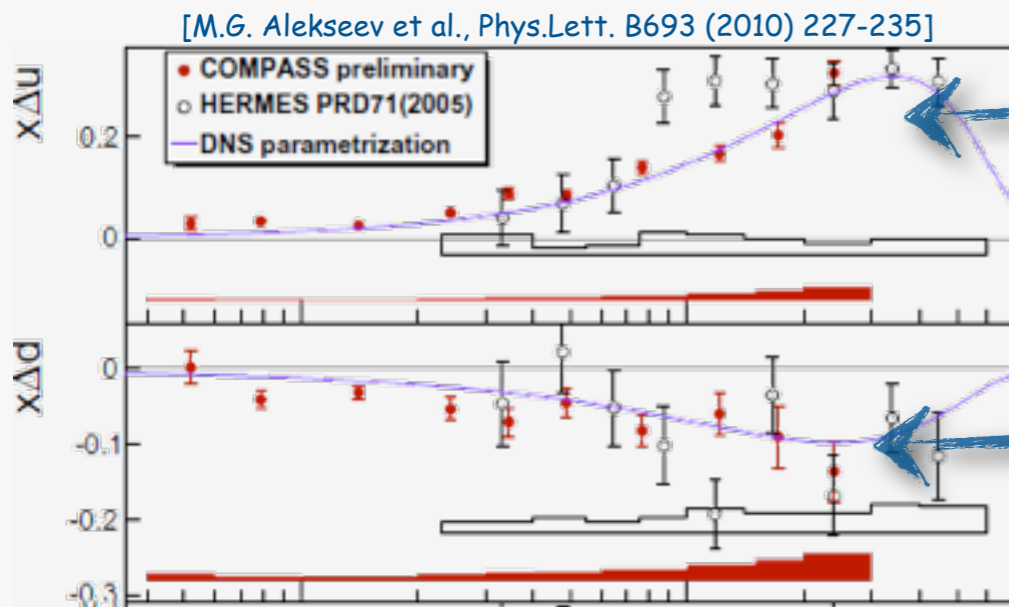
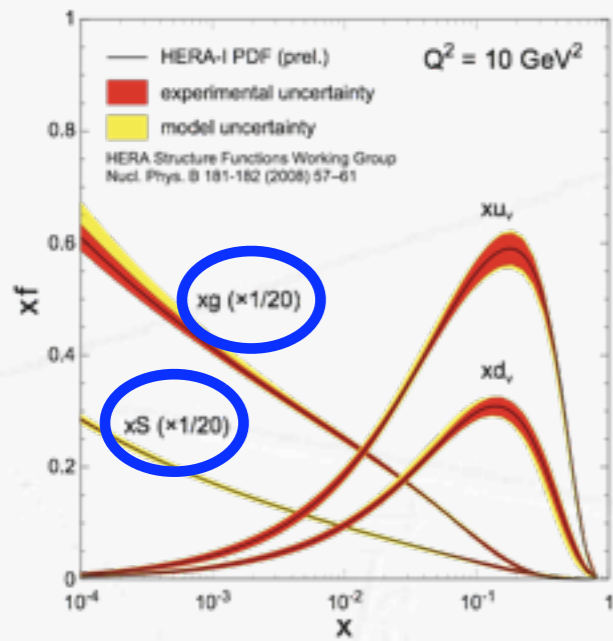


p_T dependence for g_1 📱 10mins*)

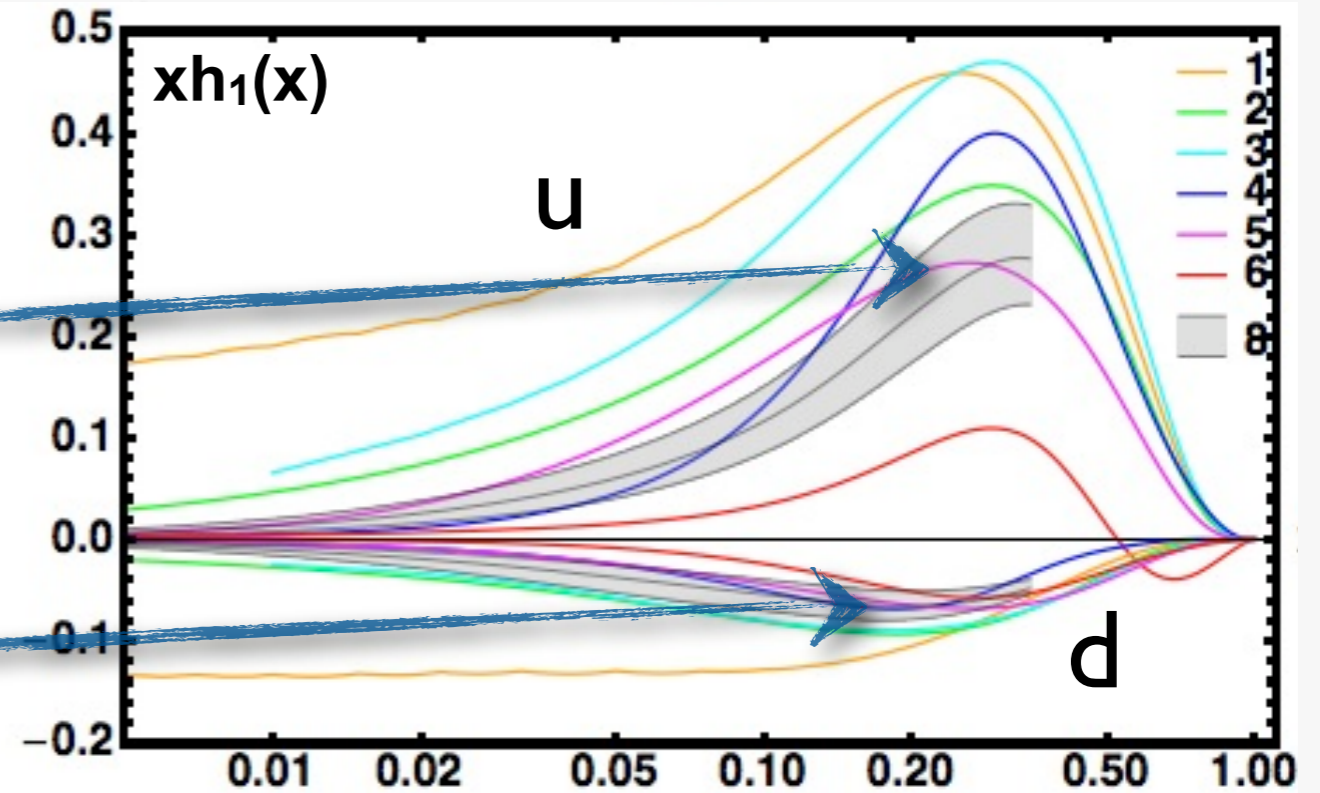
*) numbers in $A=0$ (= no Audience) gauge

snapshots

need p_T dependence for f_1 (6mins*)



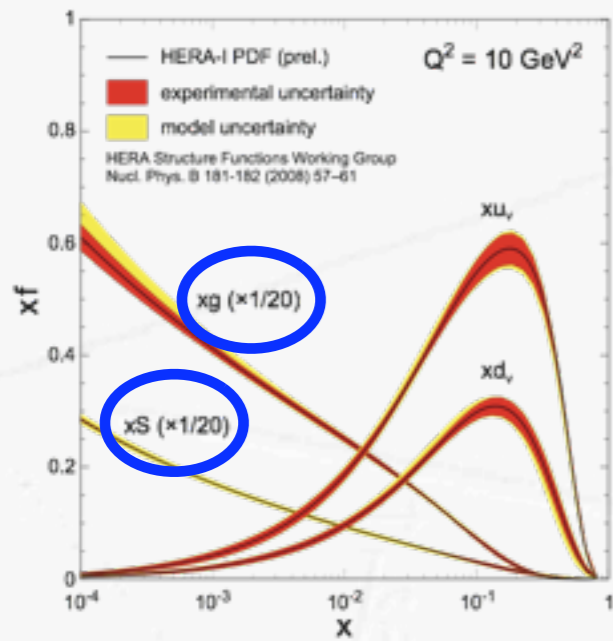
p_T de



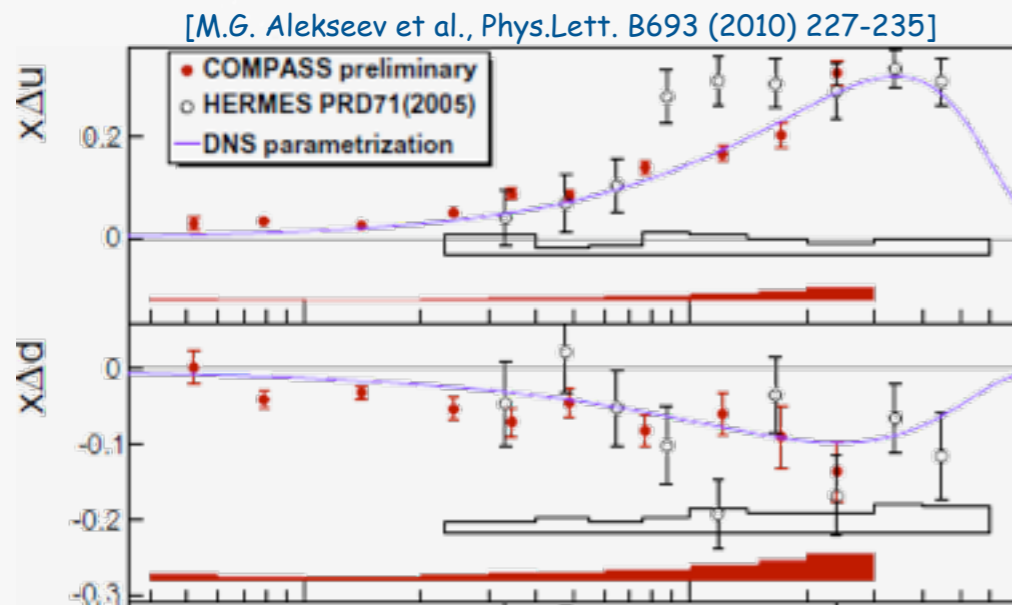
1-6 ... various model curves

*) numbers in $A=0$ (= no Audience) gauge

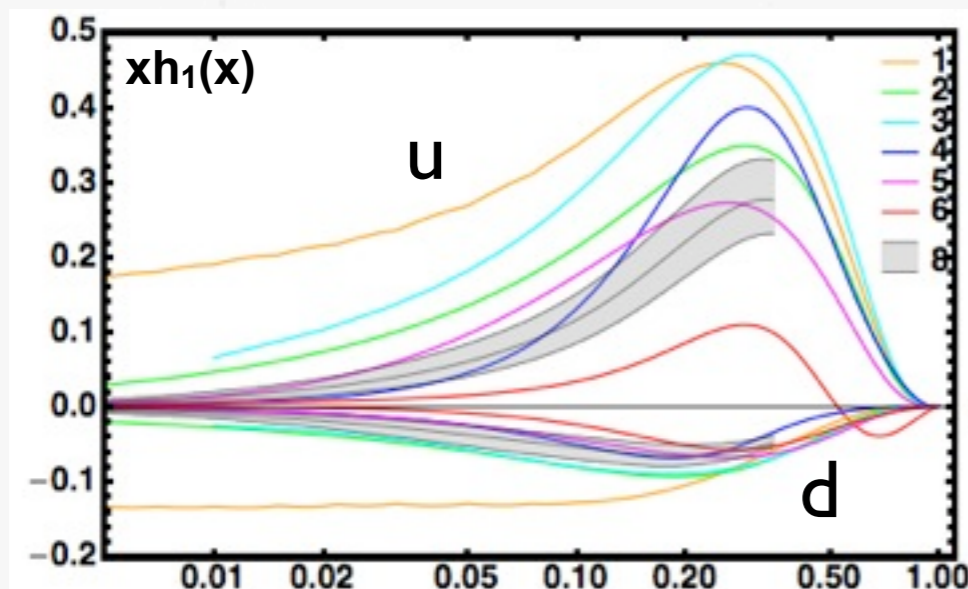
snapshots



need p_T dependence for f_1 📱 6mins*)



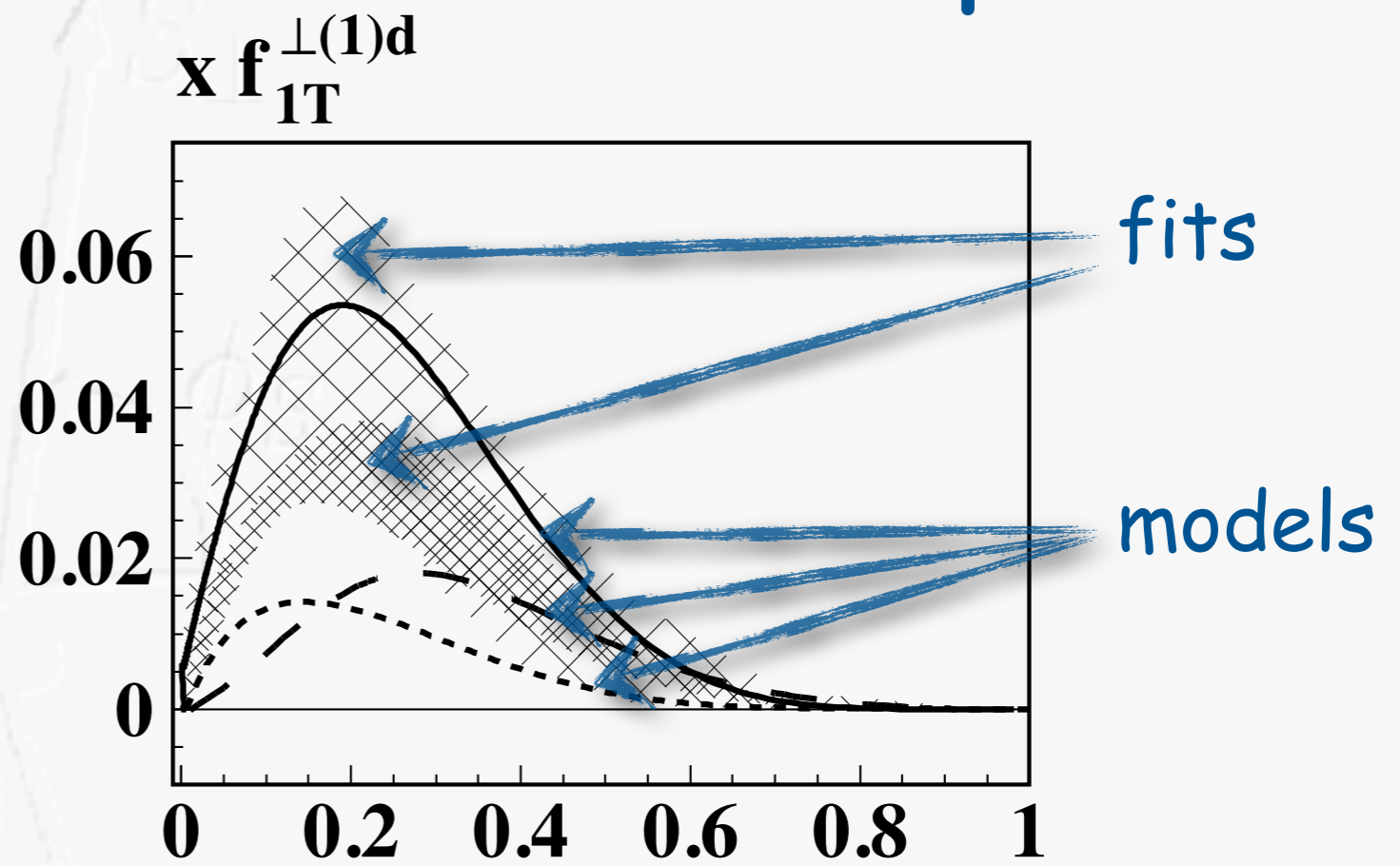
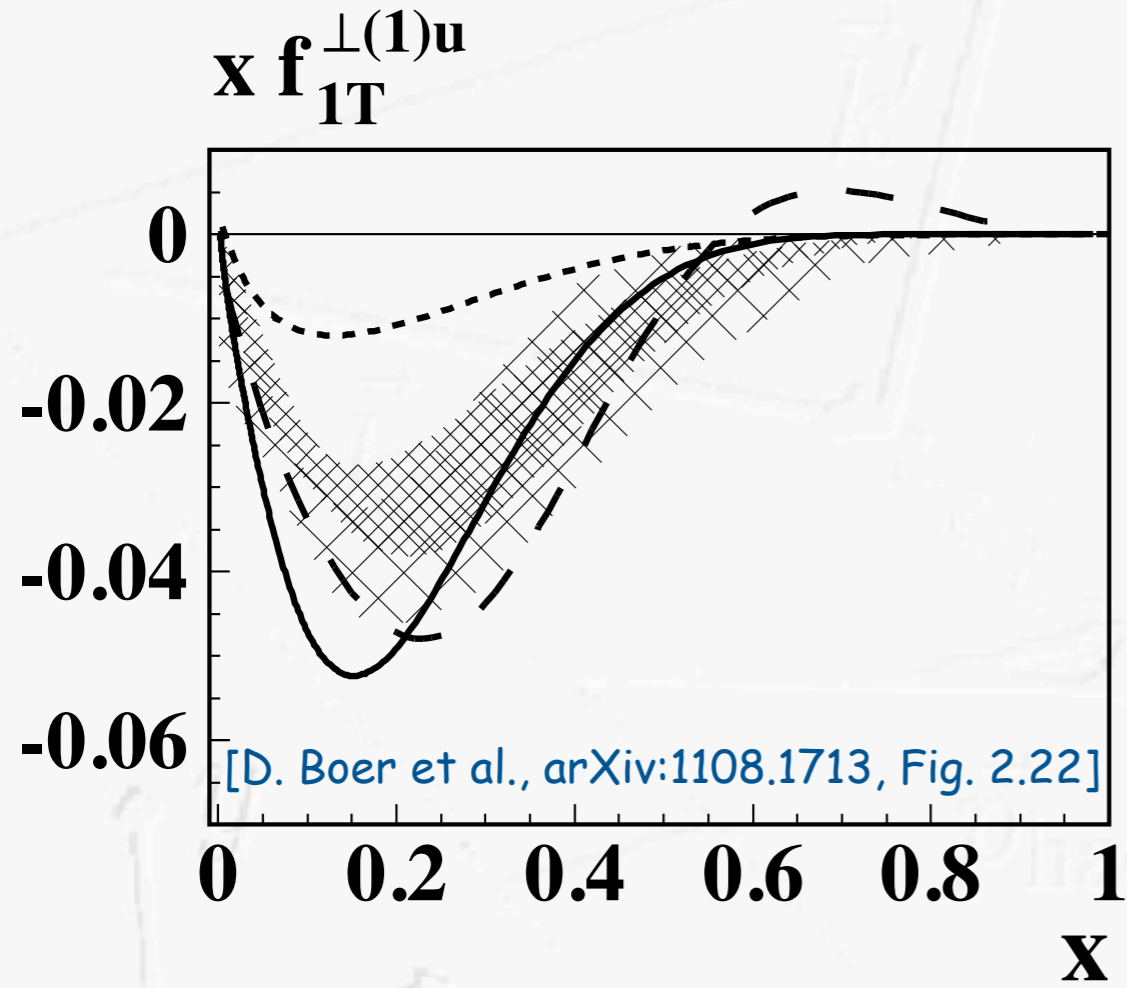
p_T dependence for g_1 📱 10mins*)



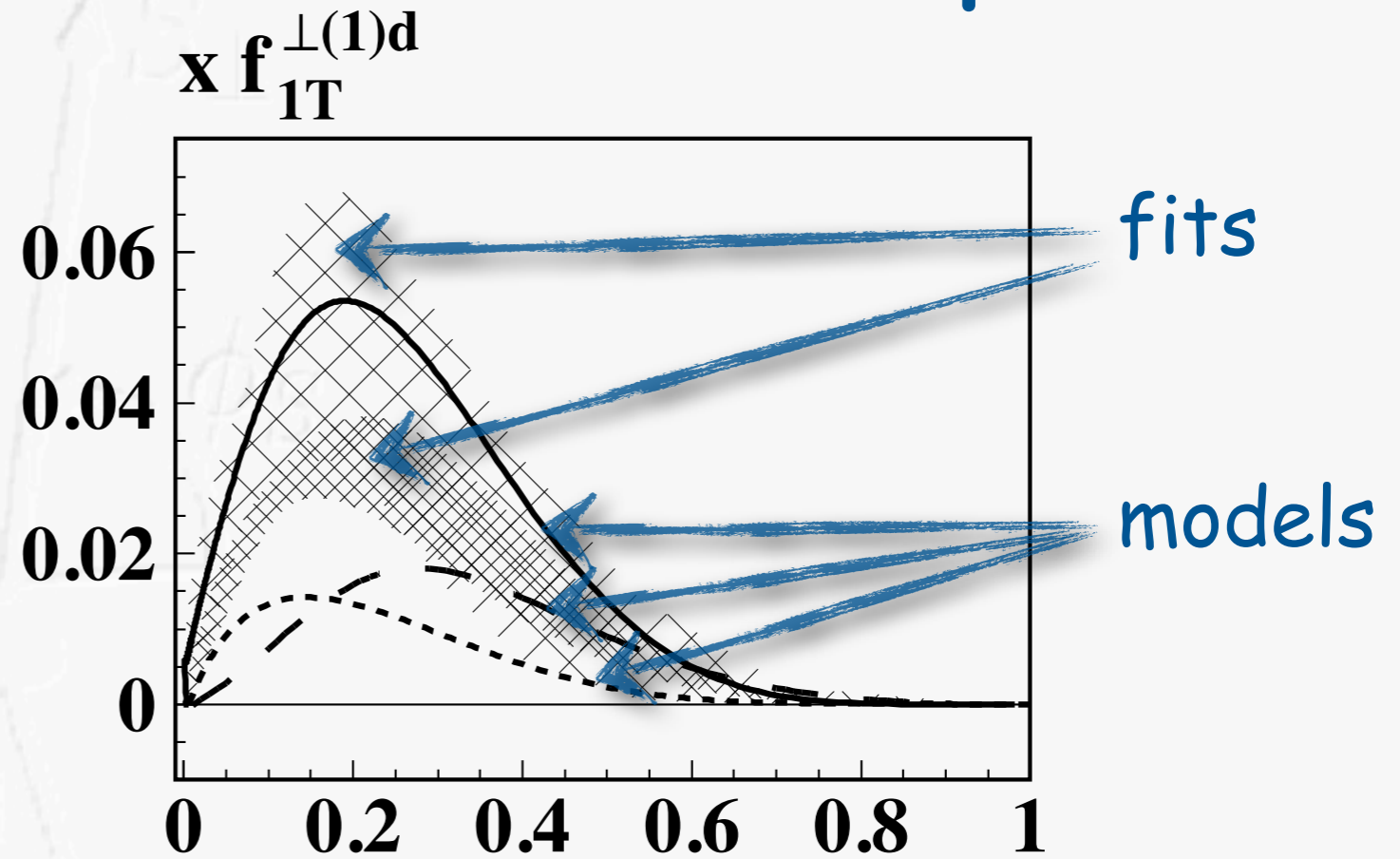
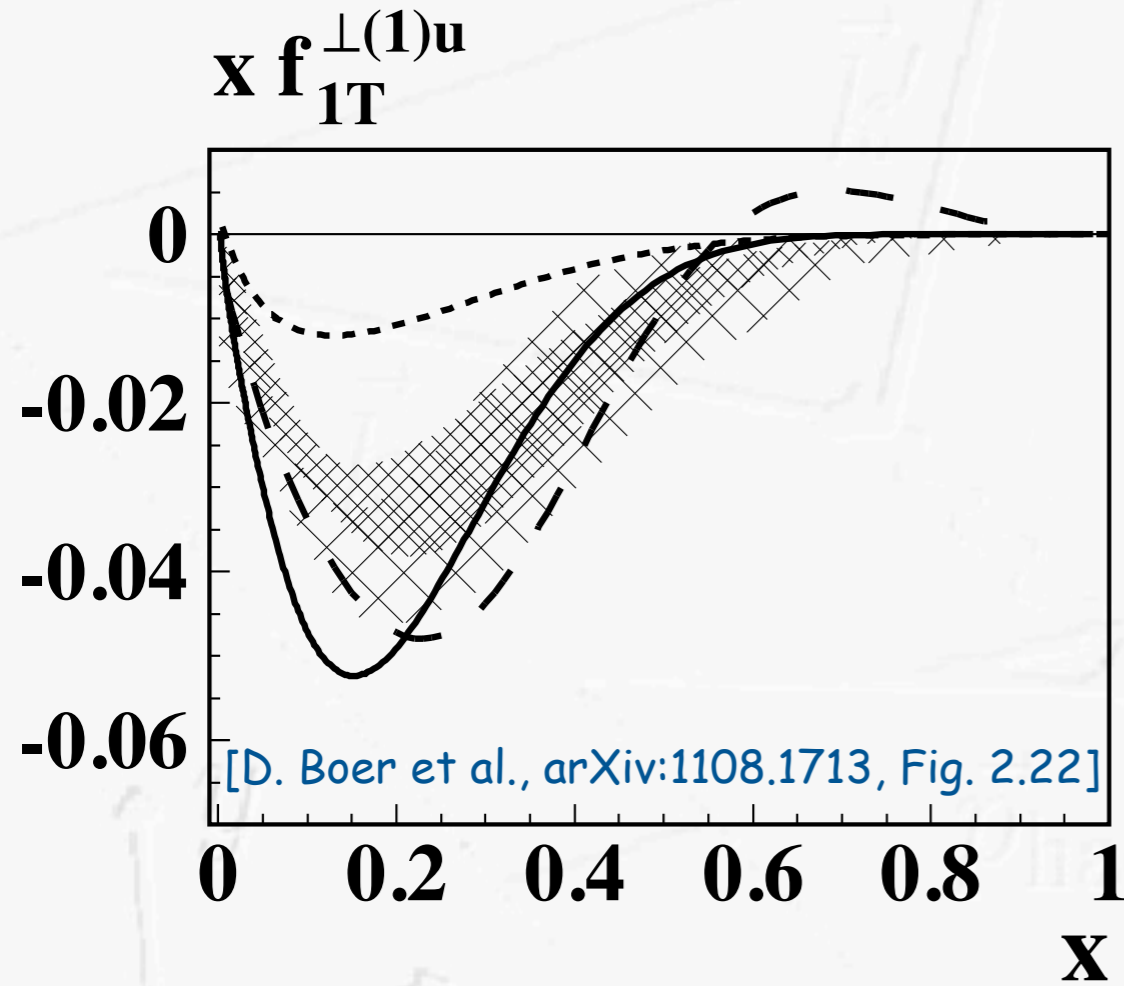
the data for h_1 📱 15mins*)

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snapshots

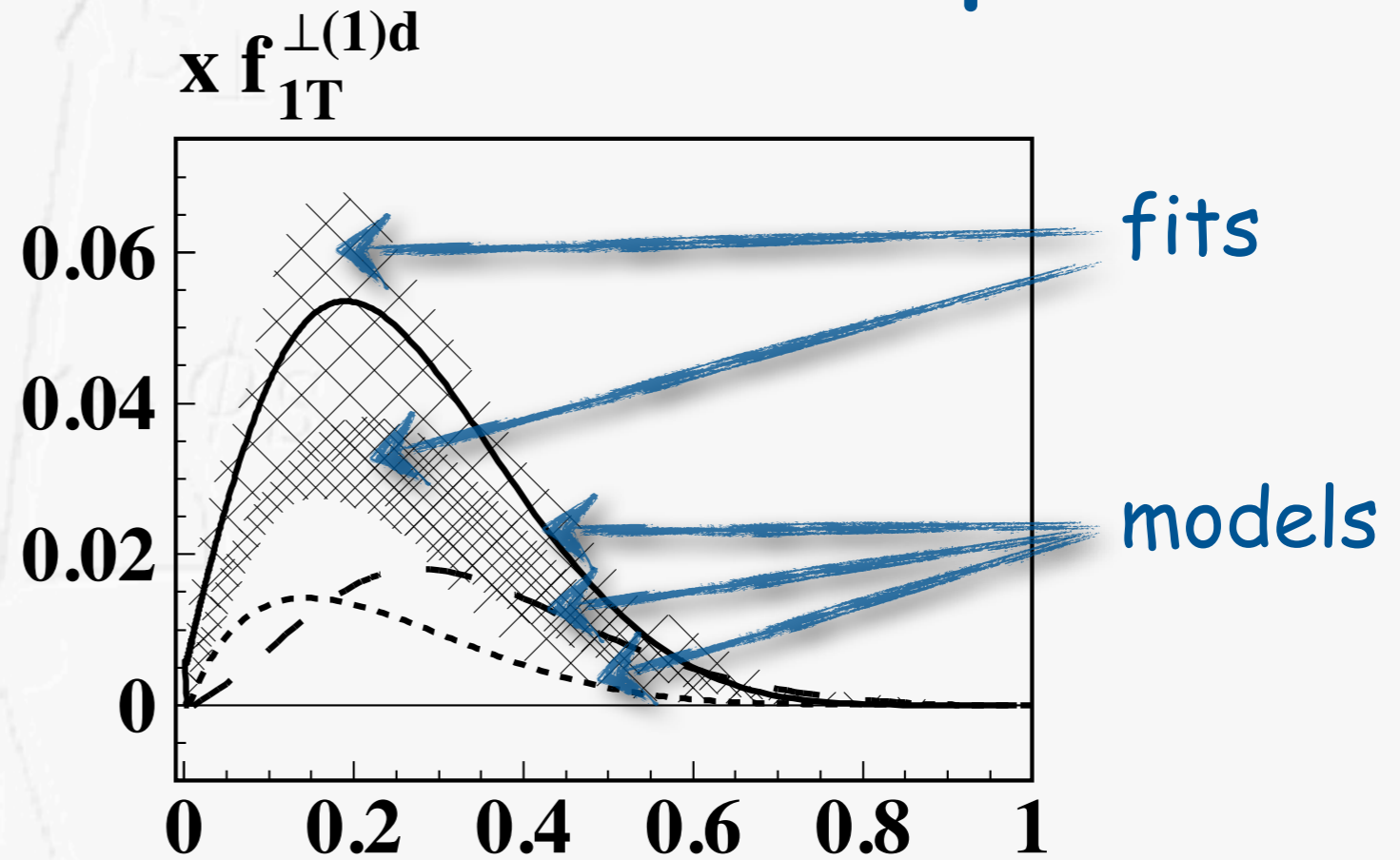
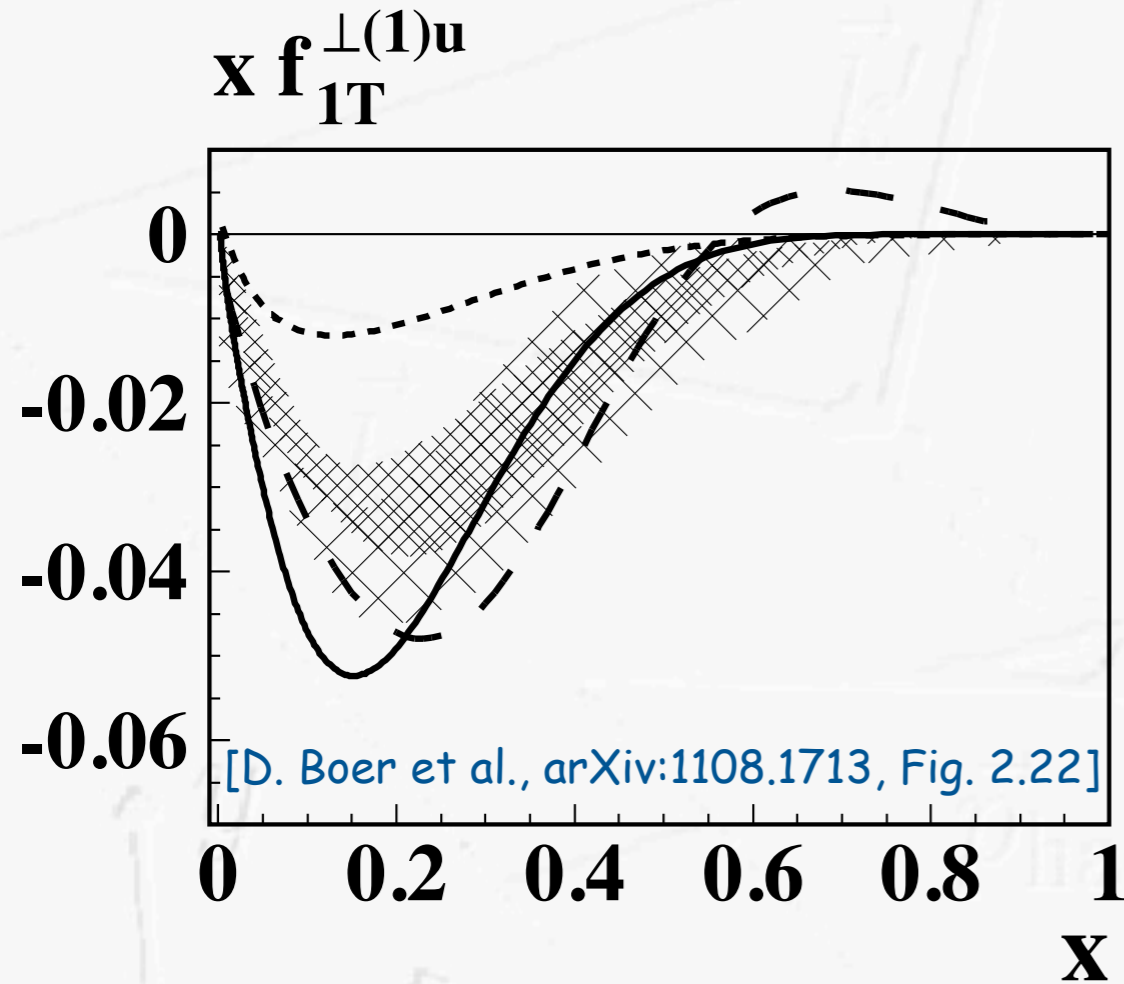


snapshots



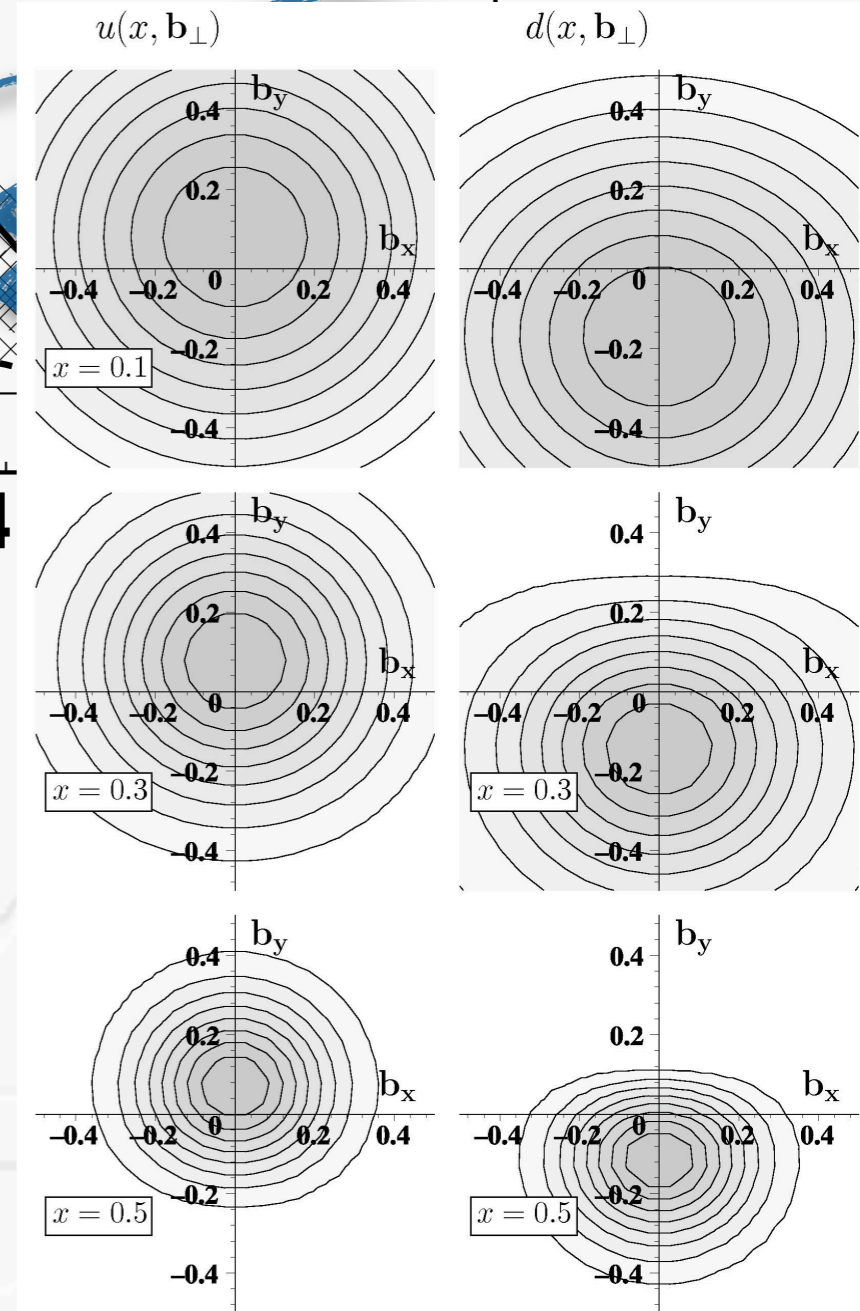
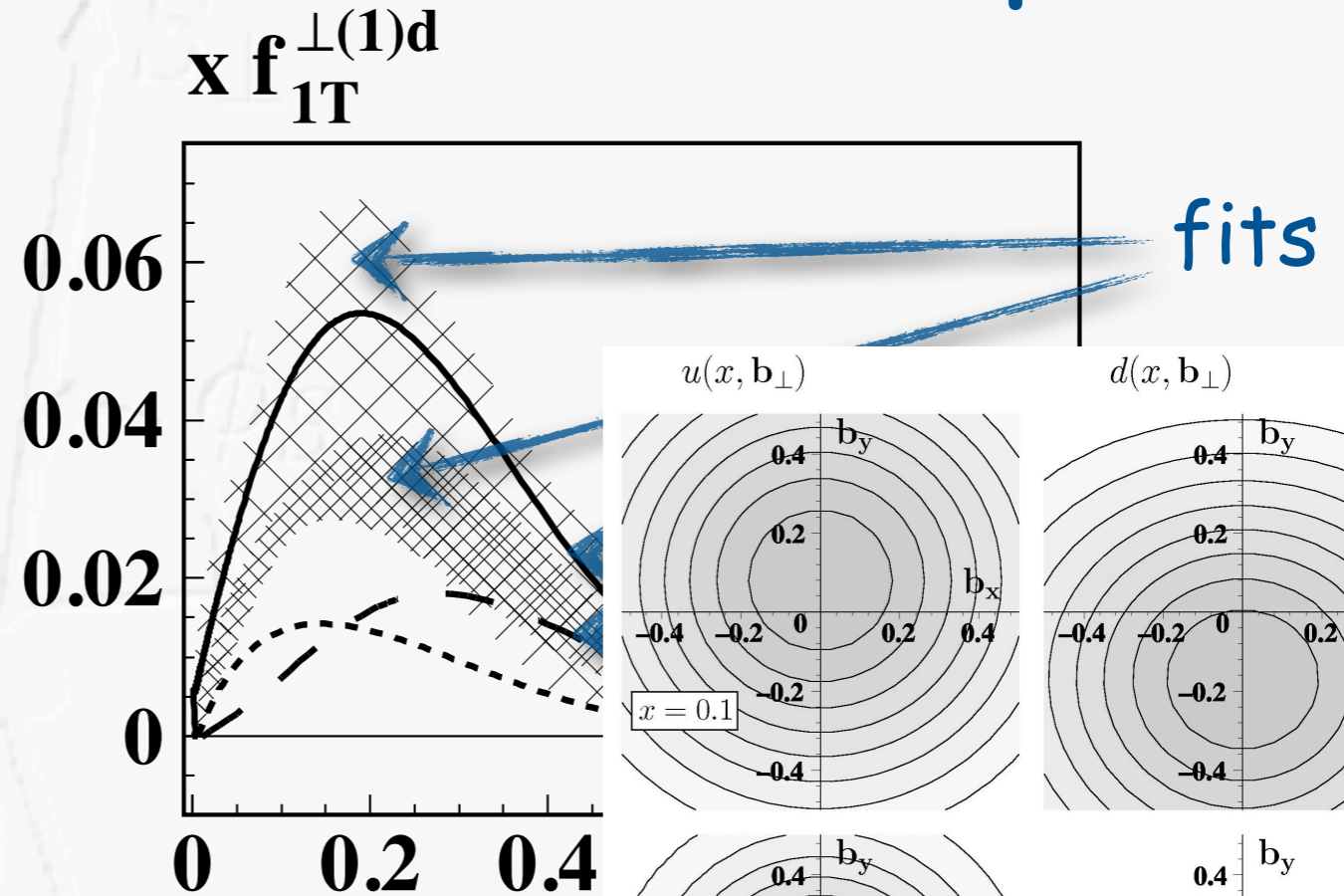
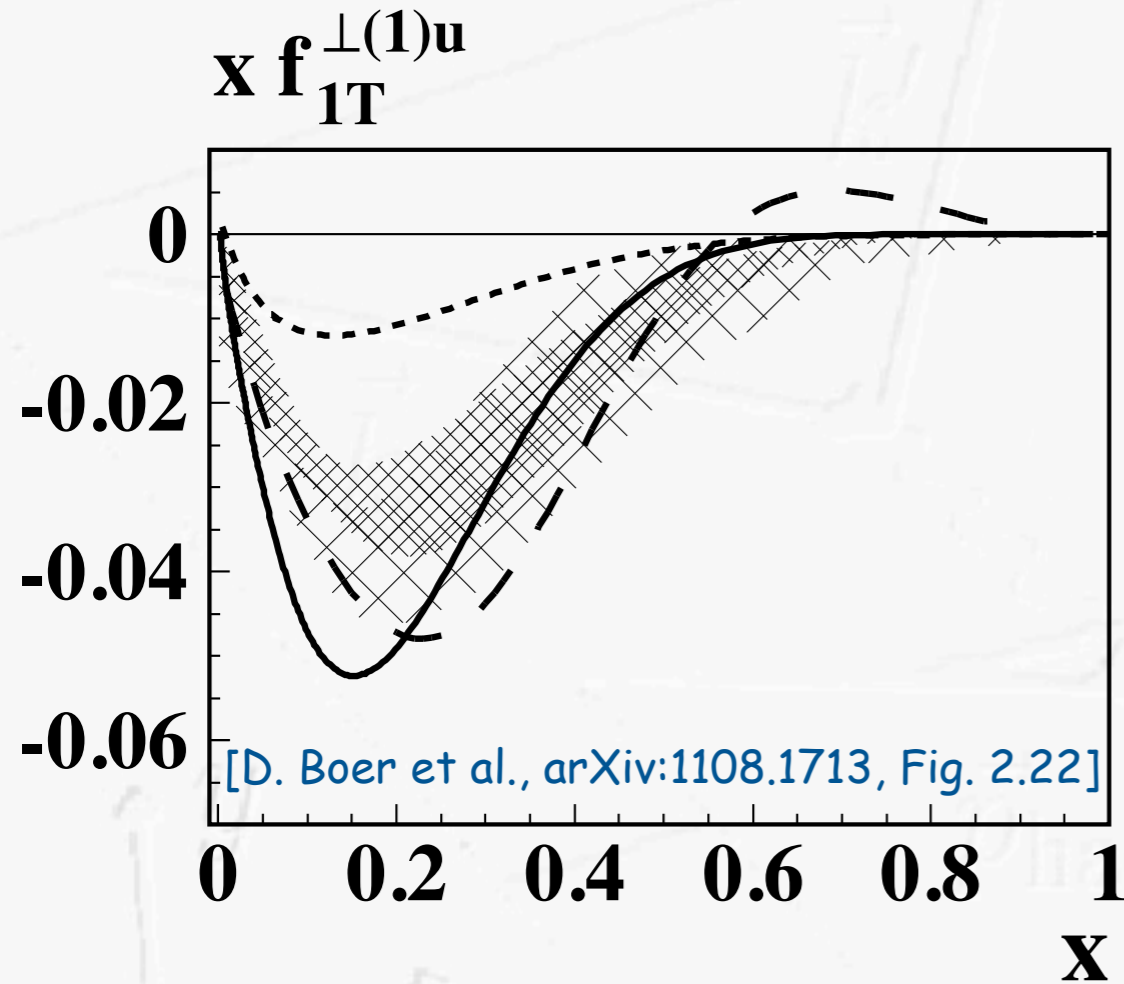
- down-Sivers opposite in sign to up-Sivers and large

snapshots



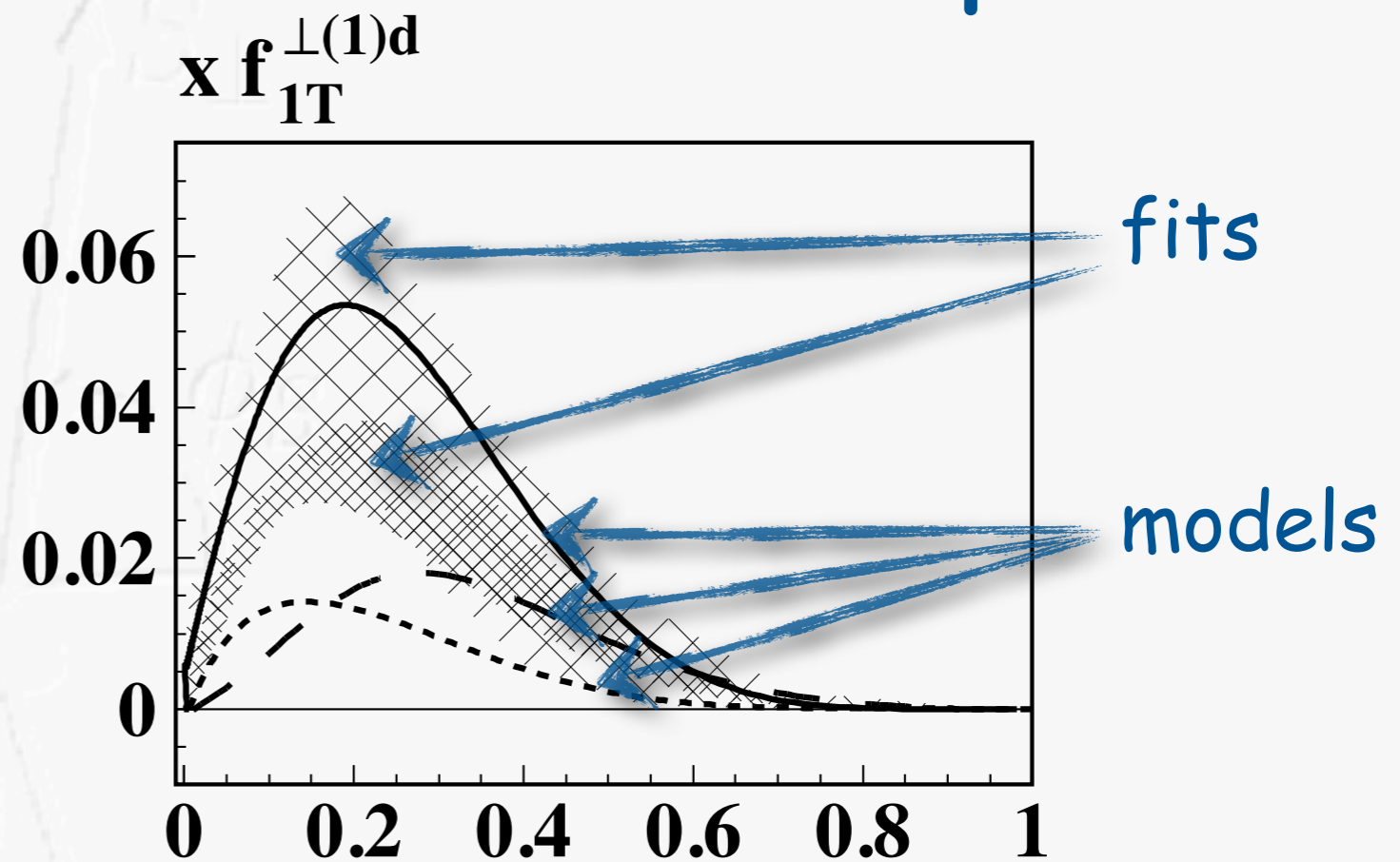
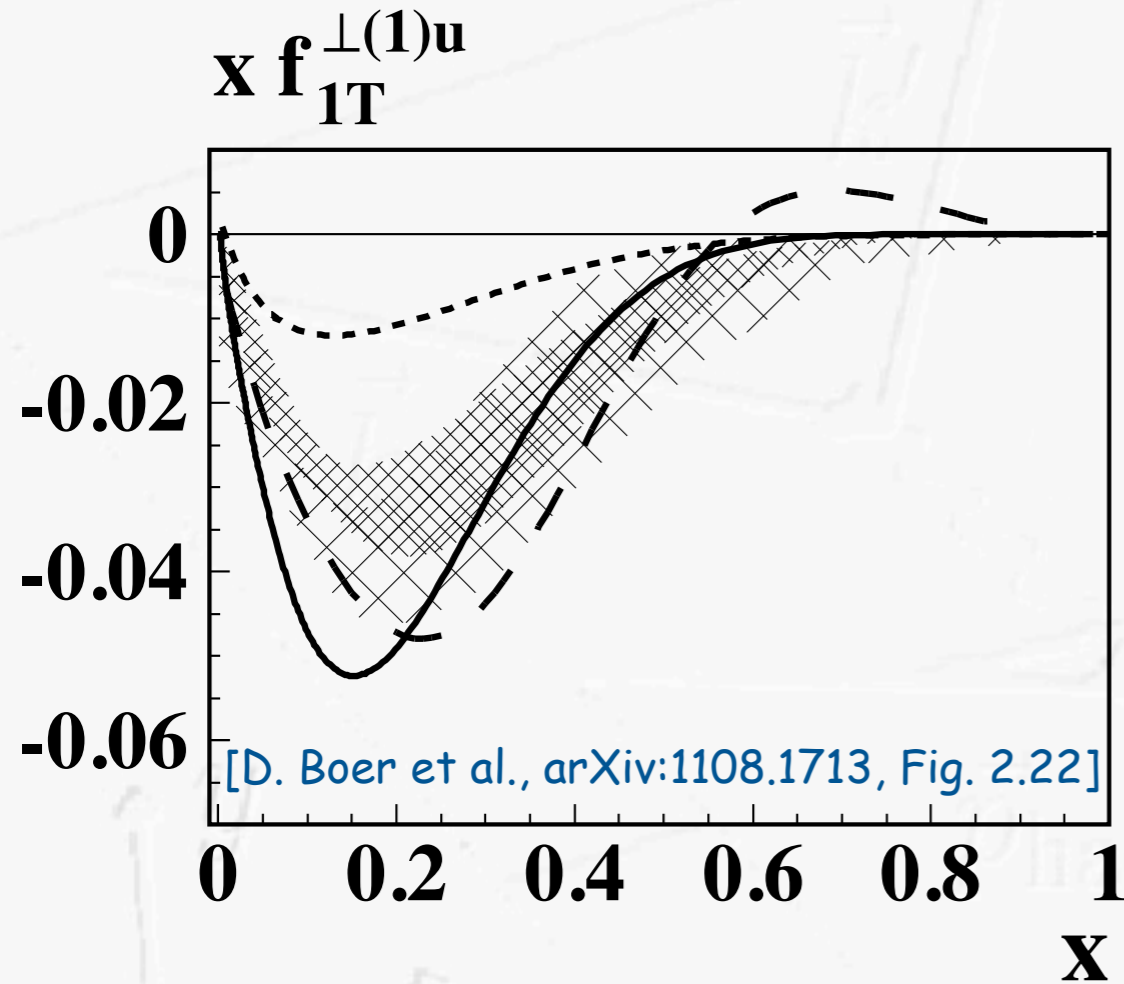
- down-Sivers opposite in sign to up-Sivers and large
- consistent to picture of flavor dipole in impact parameter space
- 👉 M. Burkardt's talk

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the data 👉 25mins*)

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The details

Chiral-odd distribution

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
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chiral-odd distributions involve quark helicity flip

$$f_1^q = \text{[red circle with white center]}$$

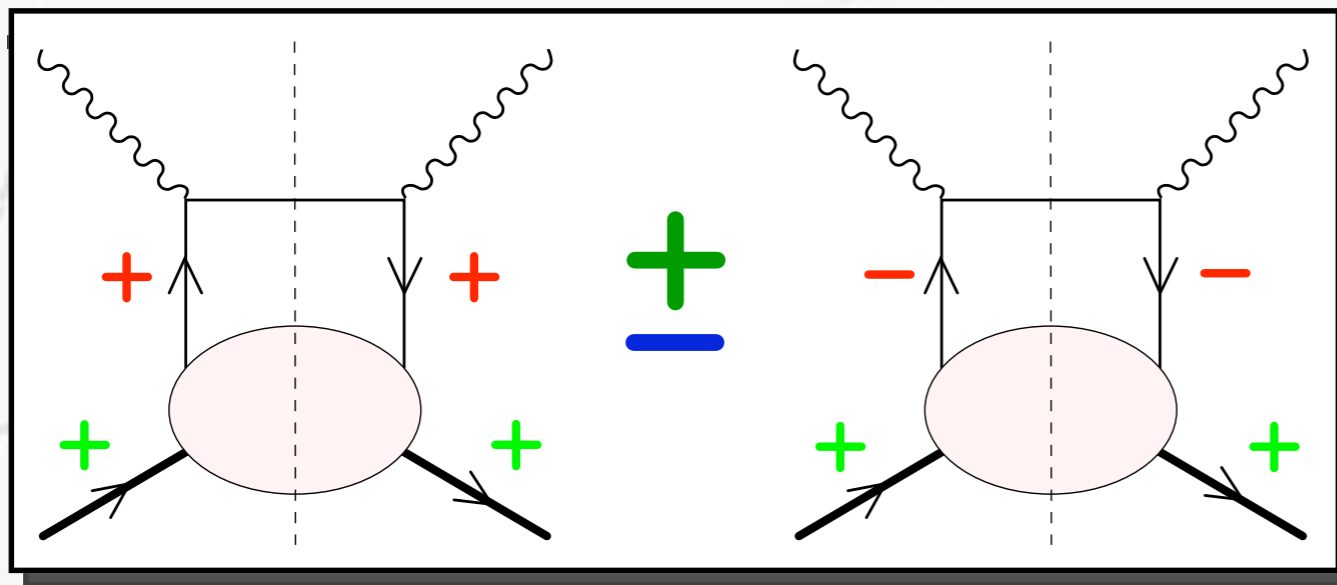
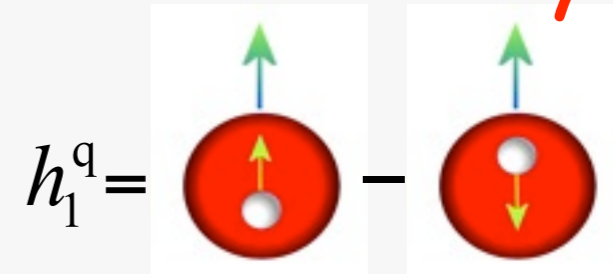
$$g_1^q = \text{[red circle with white center, right-pointing yellow arrow]} - \text{[red circle with white center, left-pointing yellow arrow]}$$

$$h_1^q = \text{[red circle with white center, up-pointing blue arrow]} - \text{[red circle with white center, down-pointing blue arrow]}$$

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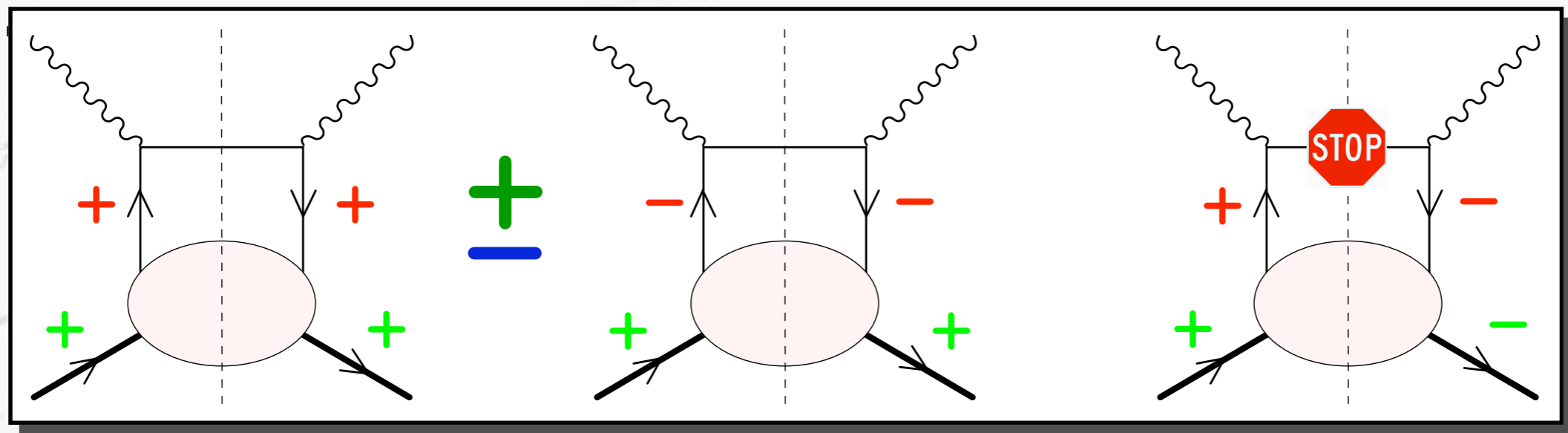
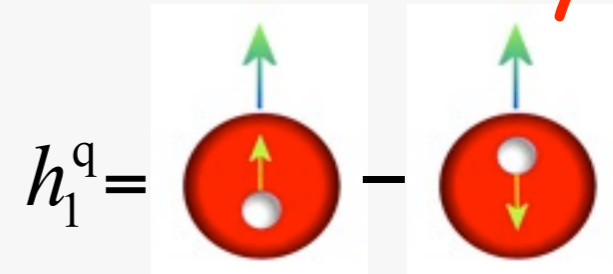
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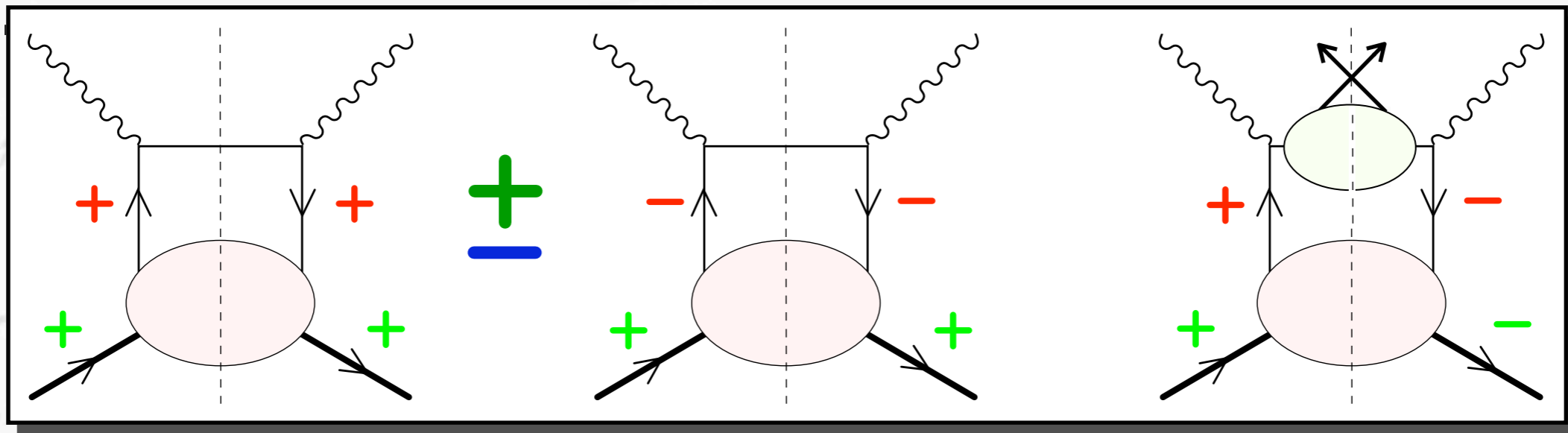
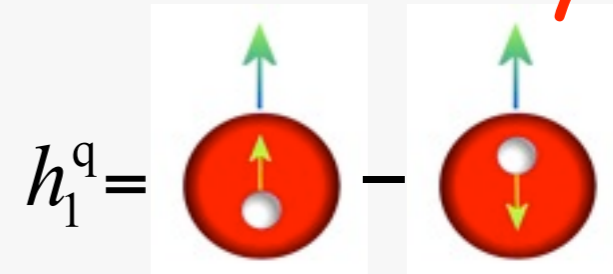
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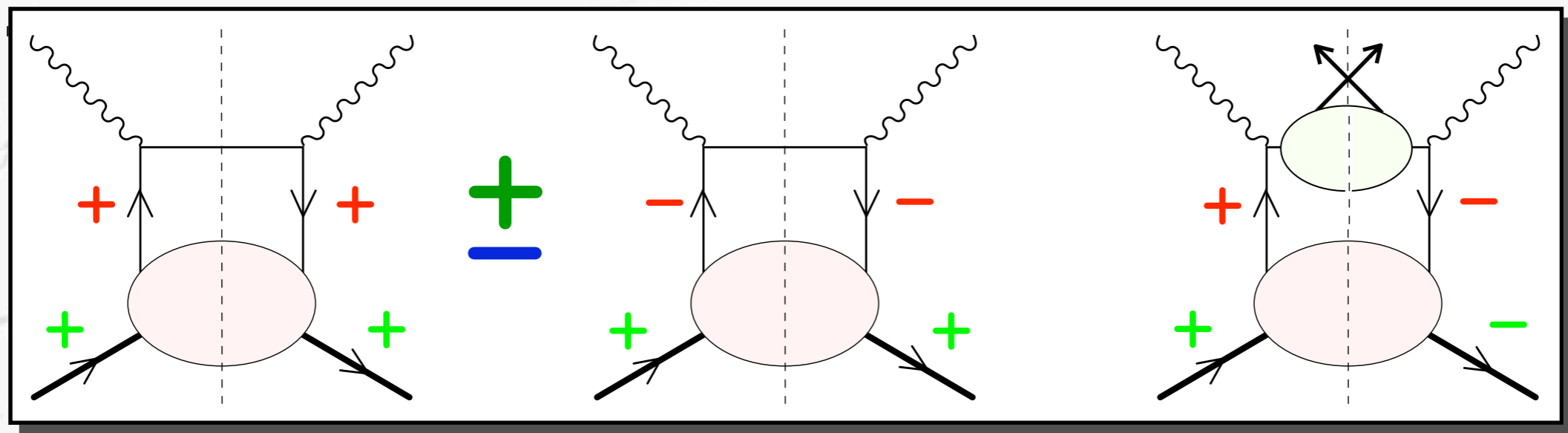
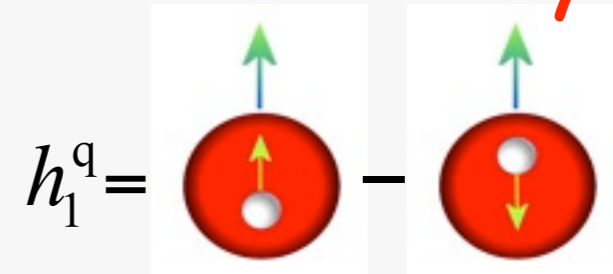


need to couple to chiral-odd fragmentation function:

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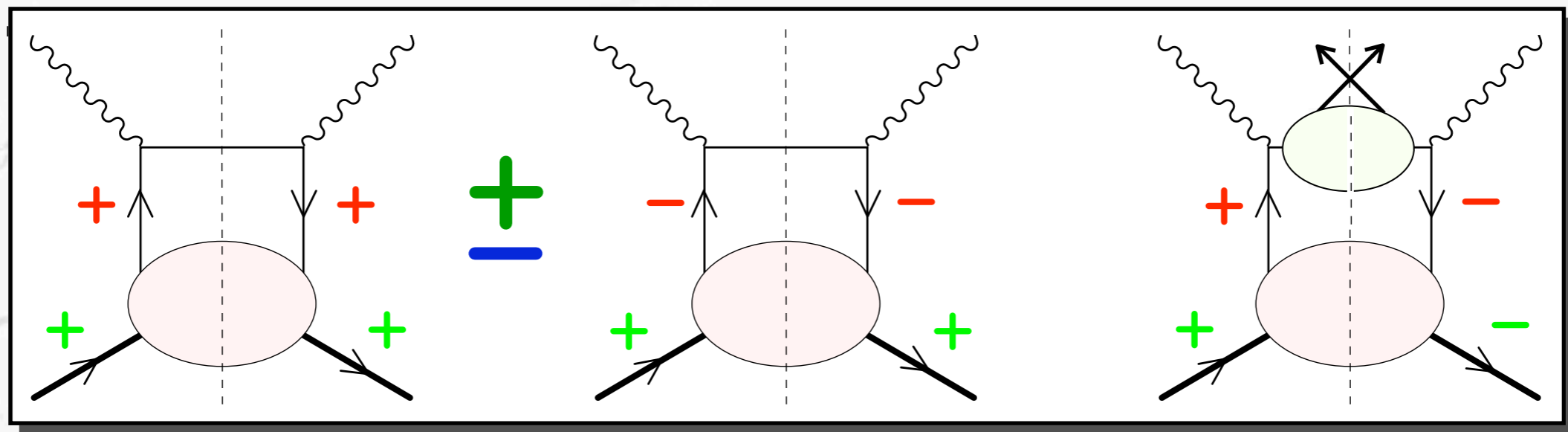
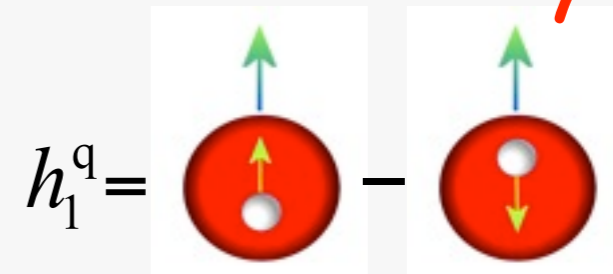
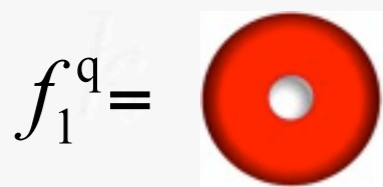
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□ transverse spin transfer (polarized final-state hadron)

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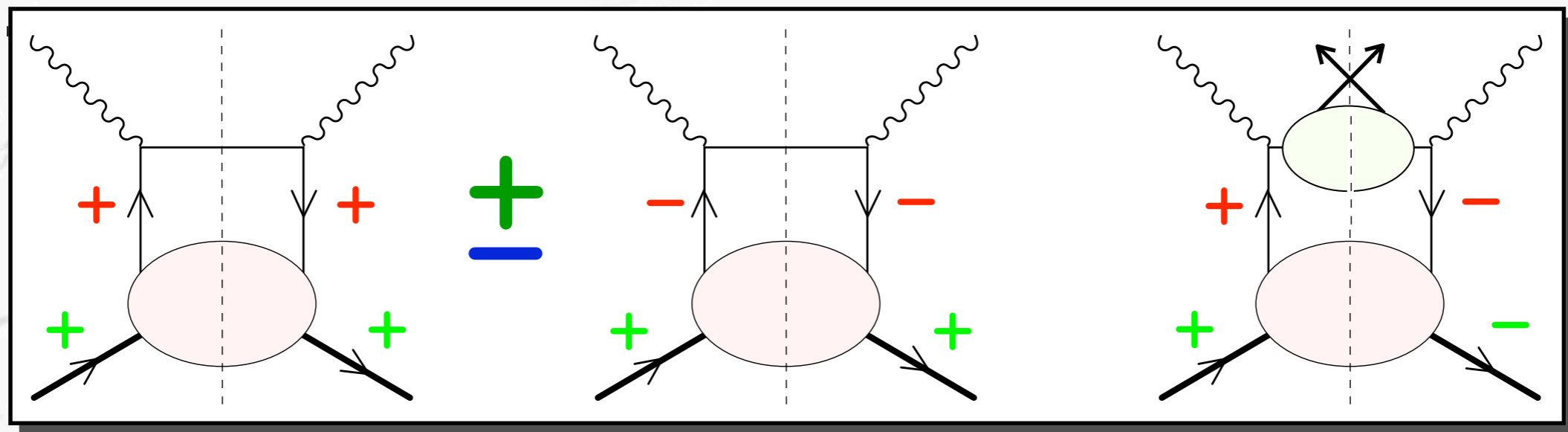
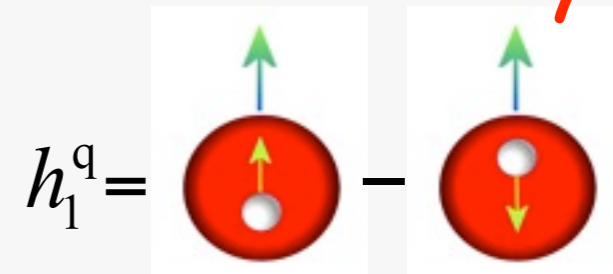
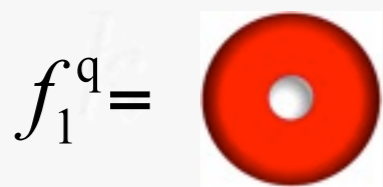
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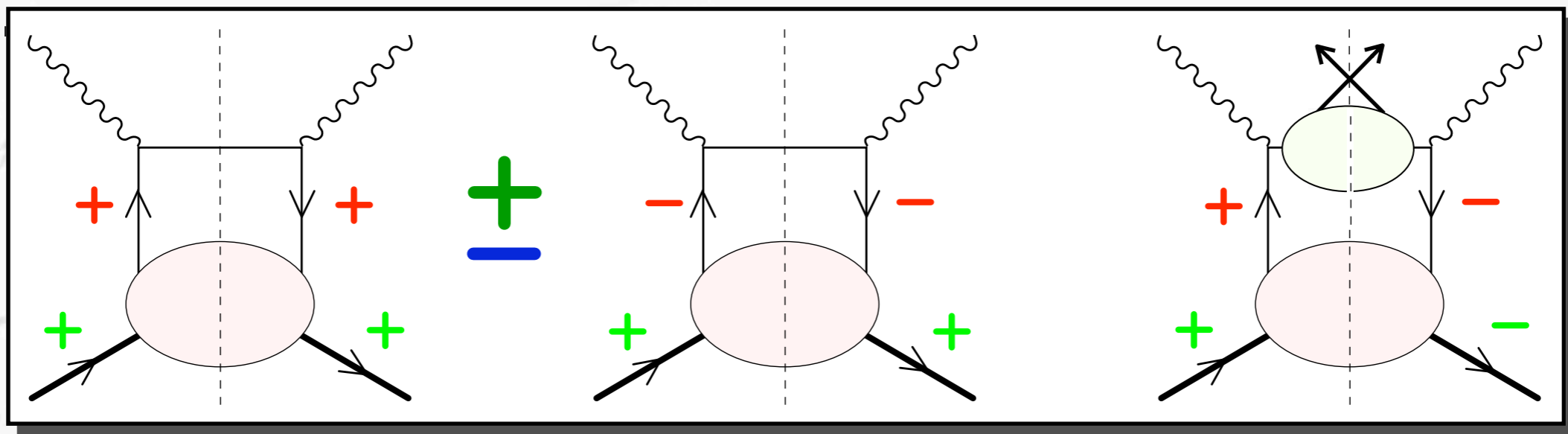
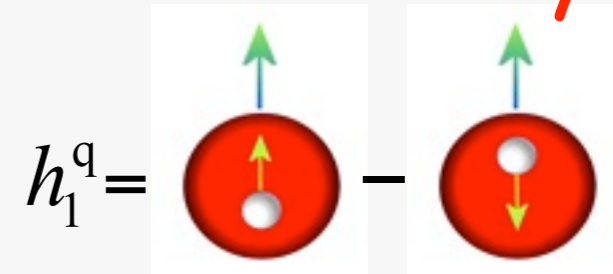
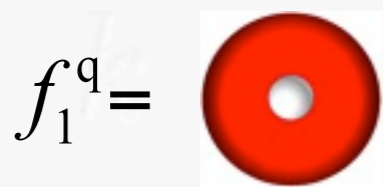
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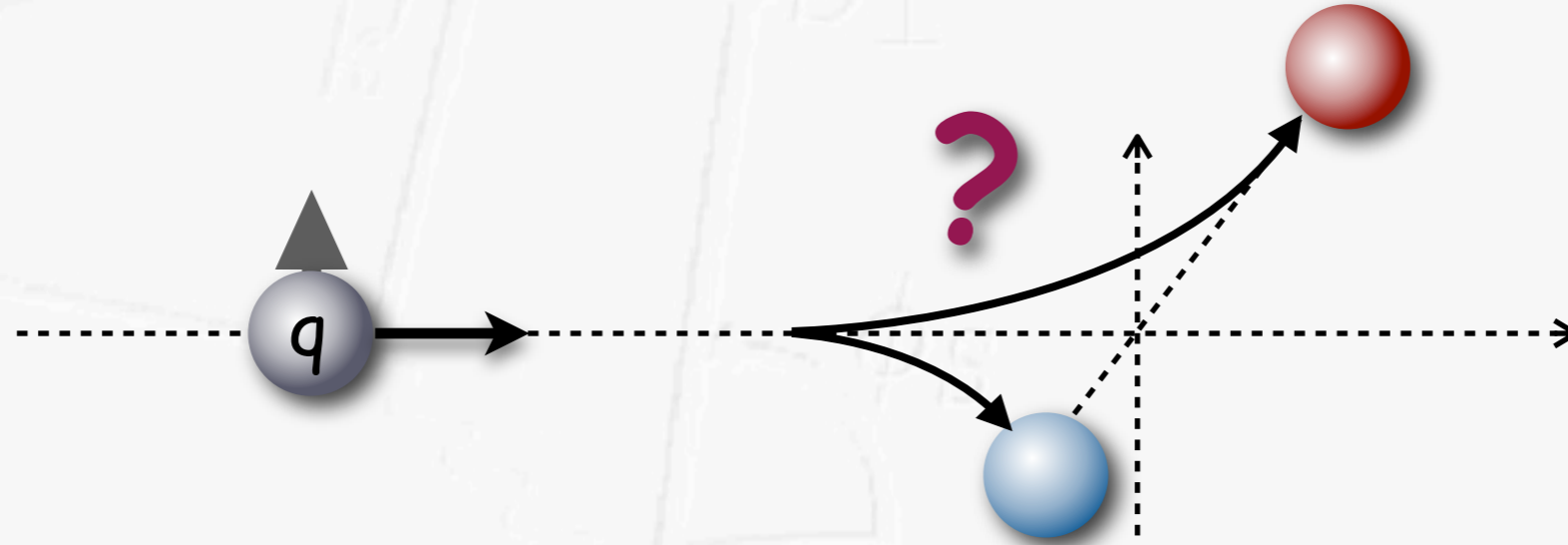


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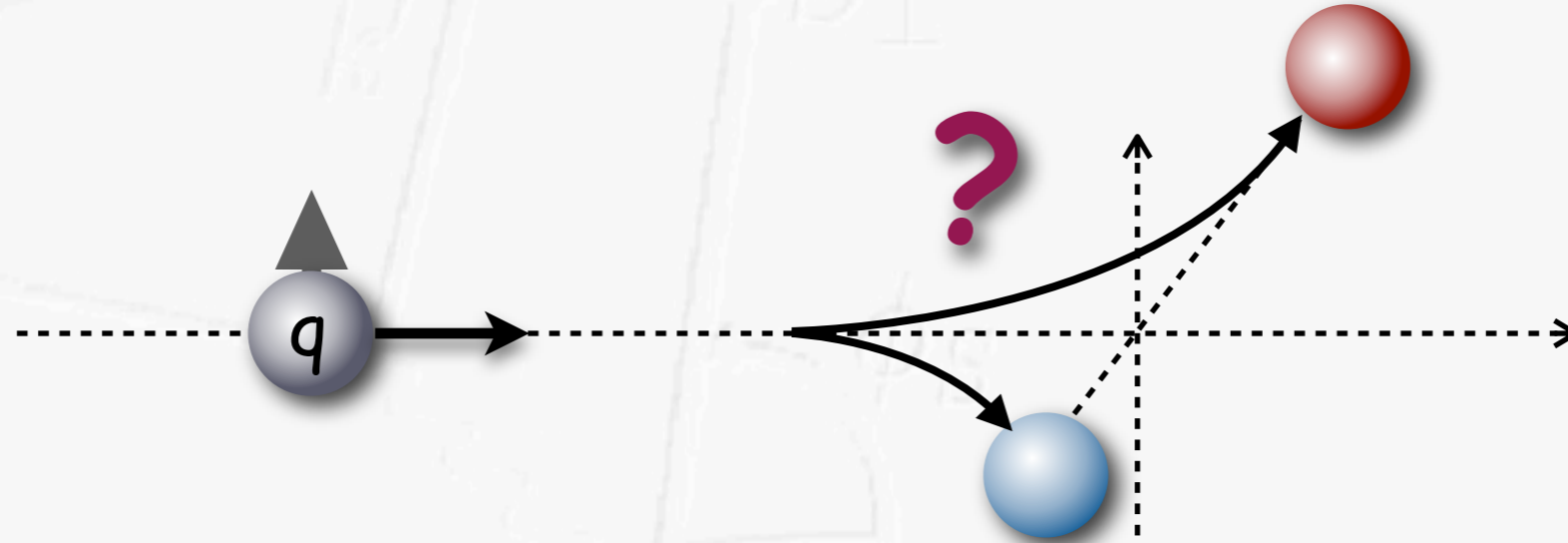
data / details R. Seidl

Collins fragmentation



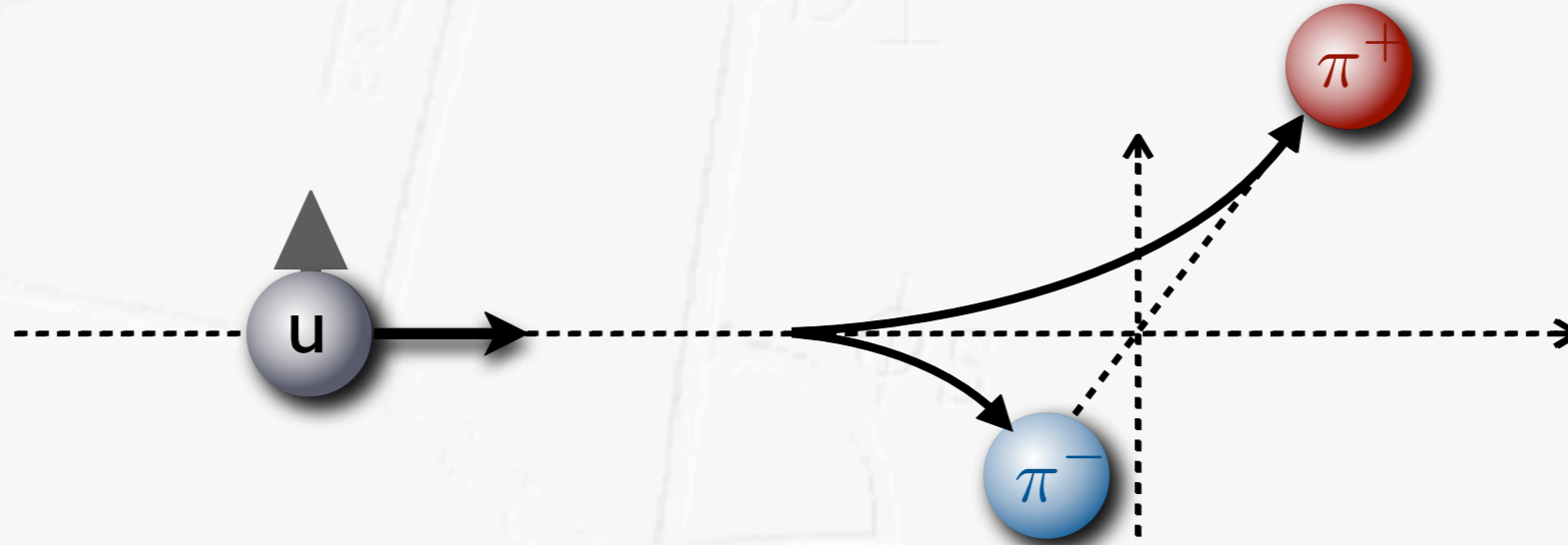
- spin-dependence in fragmentation
- left-right asymmetry in hadron direction transverse to both quark spin and momentum

Collins fragmentation



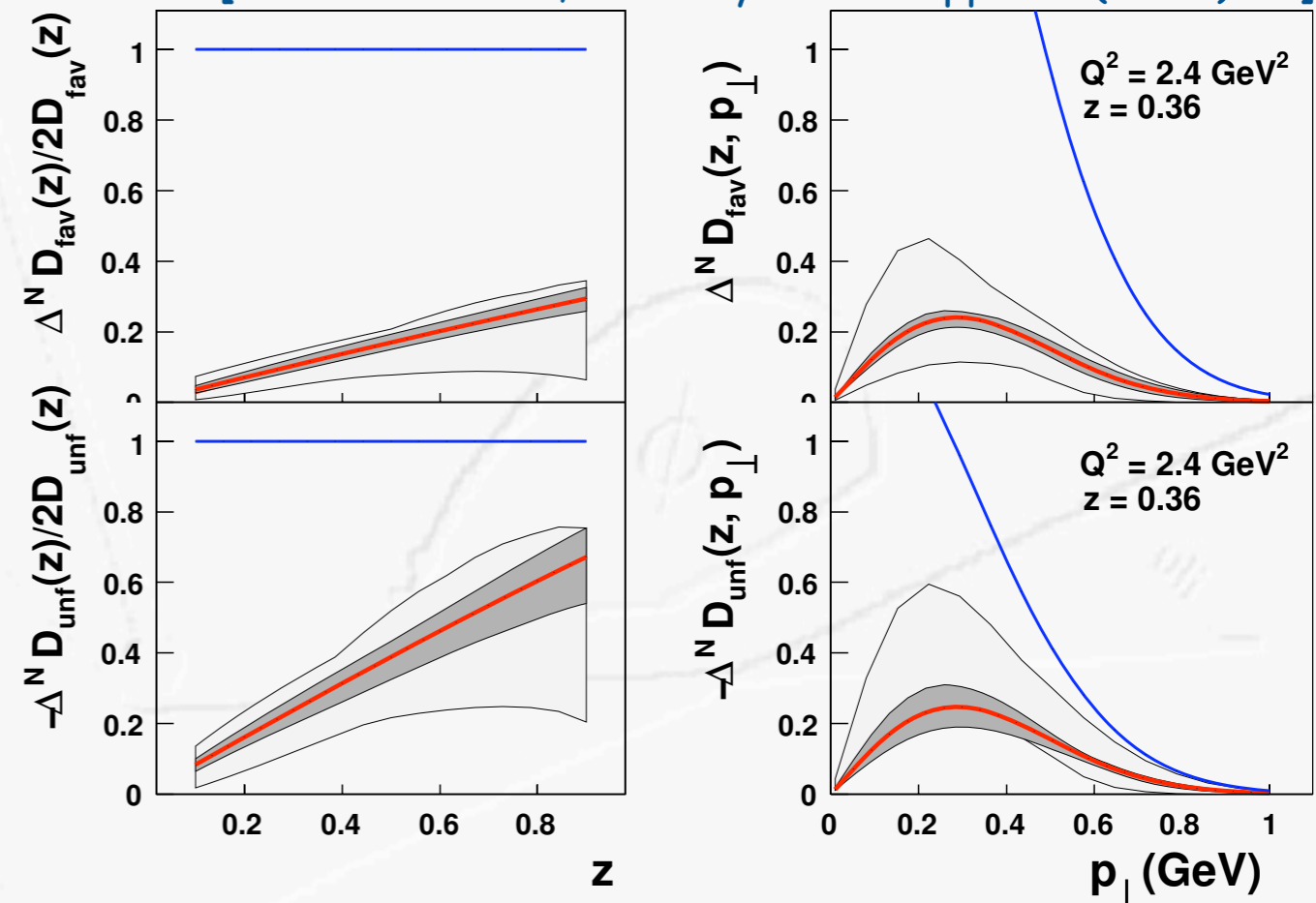
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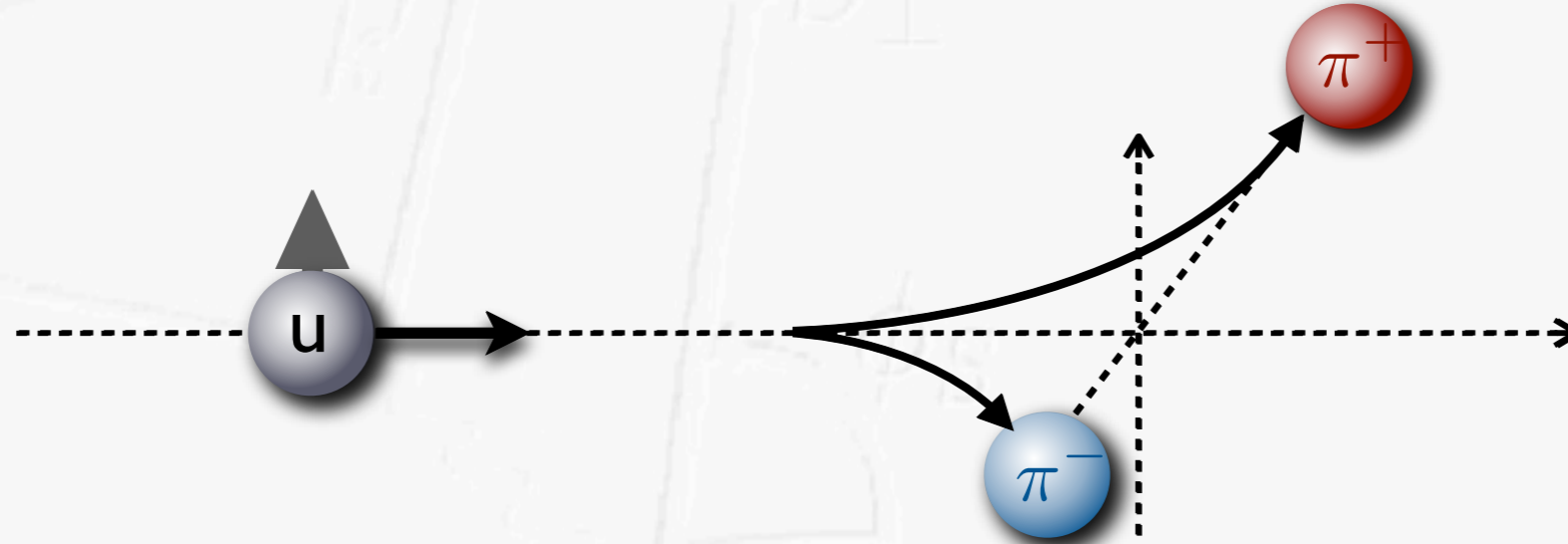


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[Anselmino et al., Nucl.Phys.Proc.Suppl.191 (2009) 98]

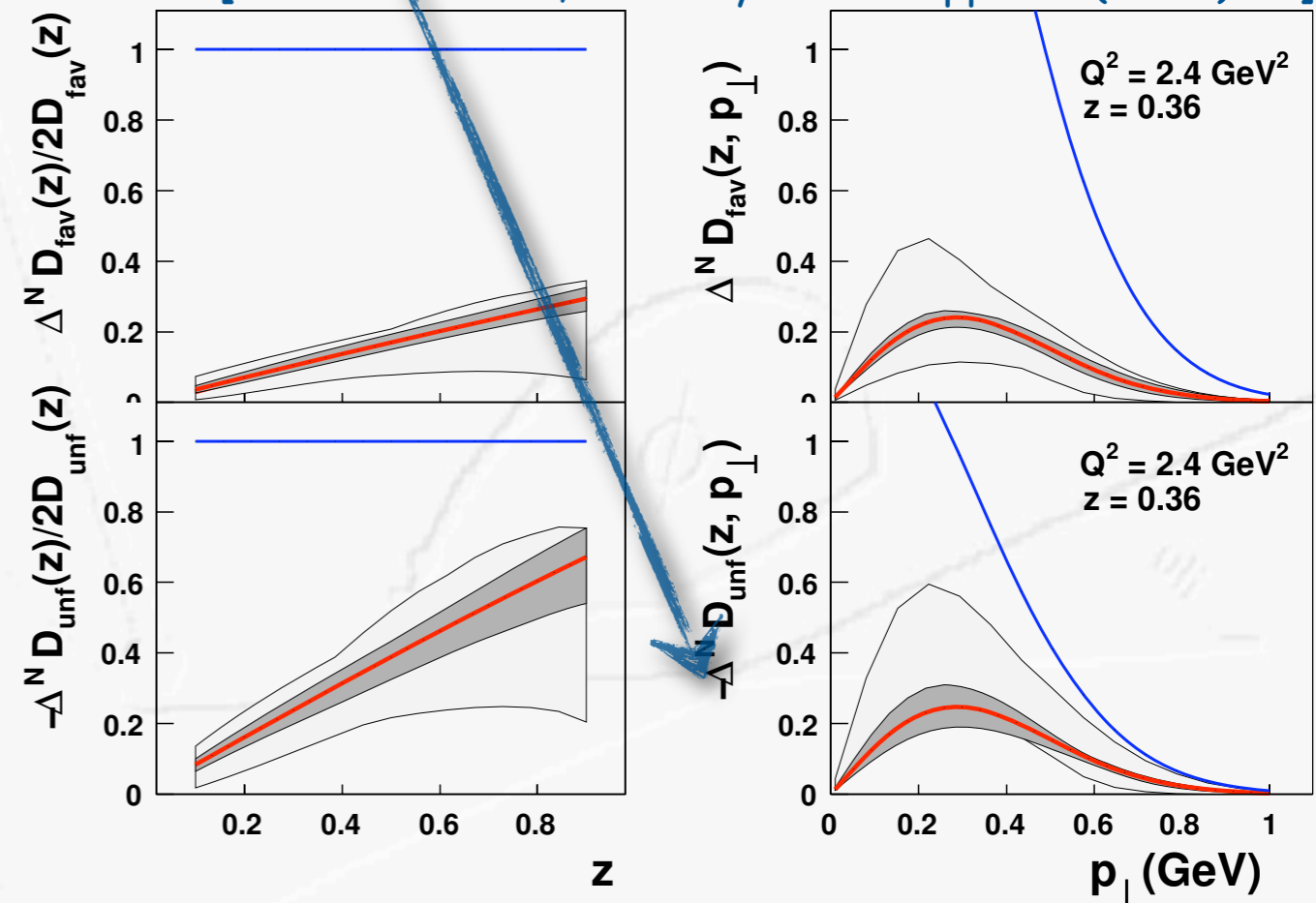


Collins fragmentation

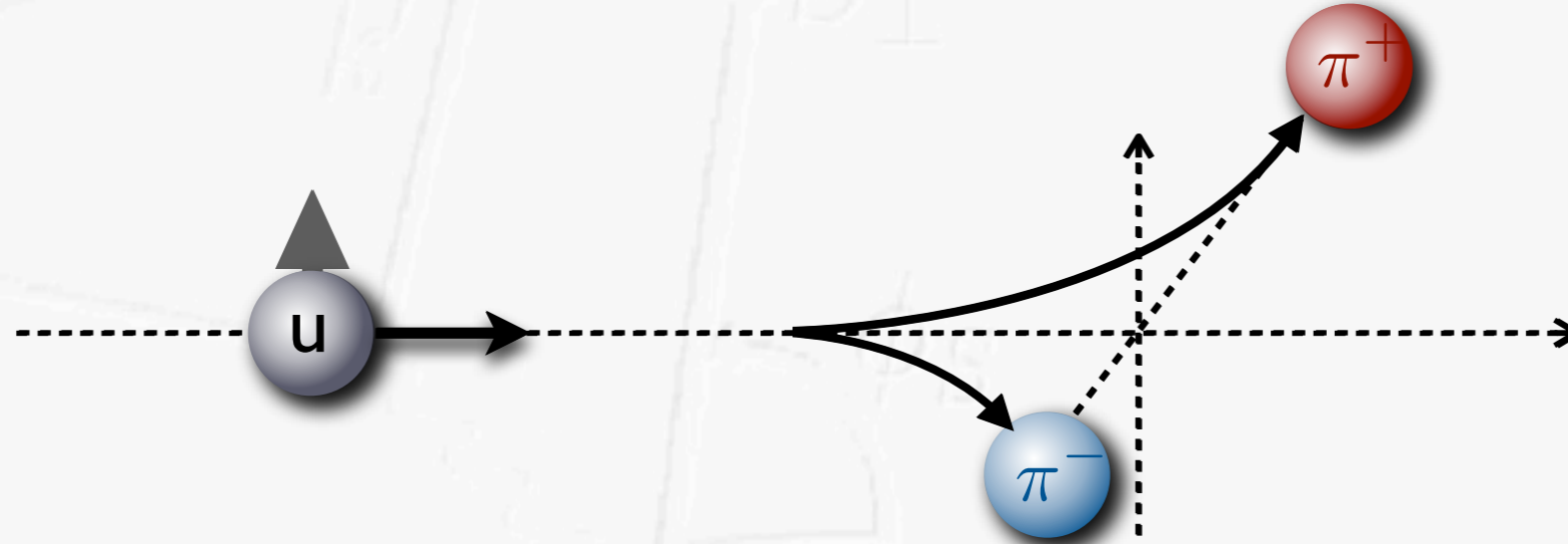


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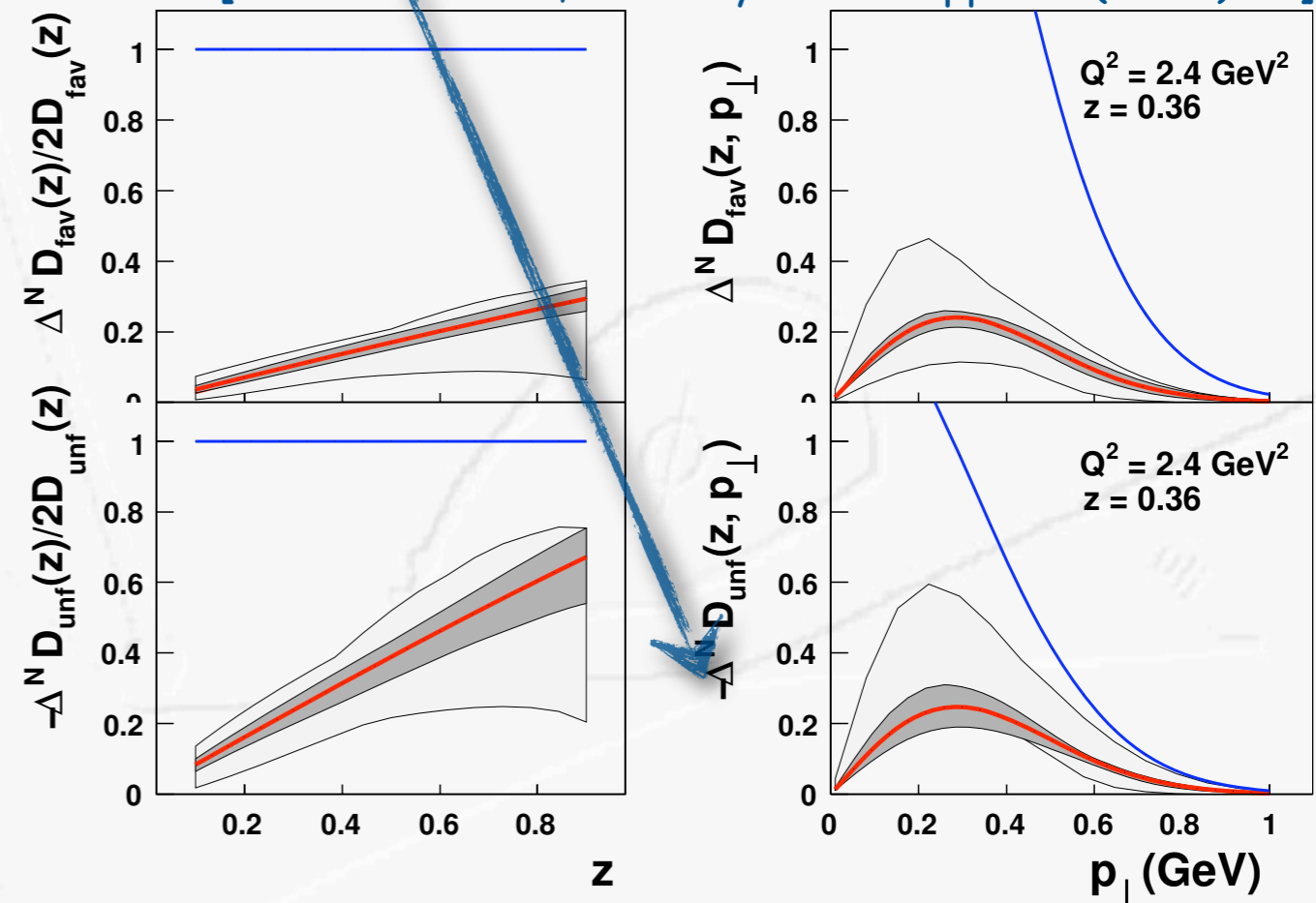


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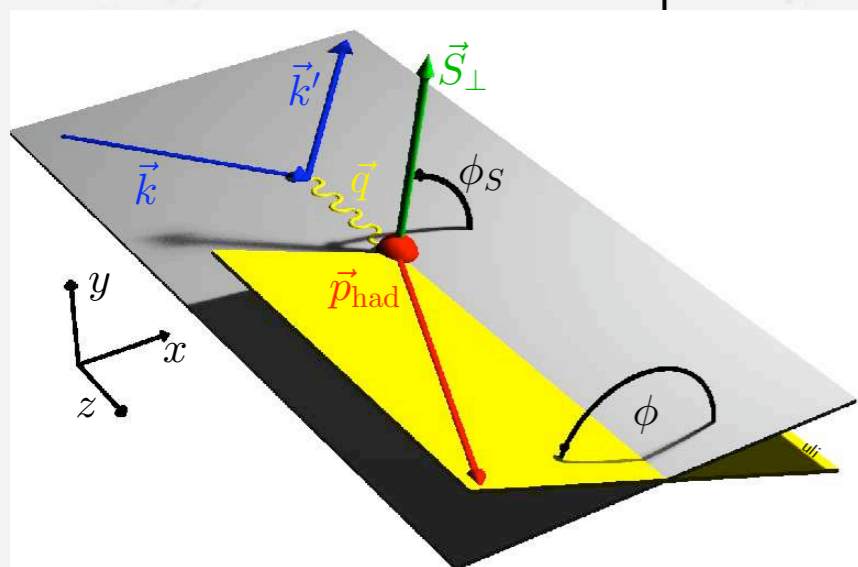
[Anselmino et al., Nucl.Phys.Proc.Suppl.191 (2009) 98]



1-Hadron Production ($ep \rightarrow ehX$)

$$\begin{aligned}
 d\sigma = & d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3 \\
 & + S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\} \\
 & + S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \frac{1}{Q} \right. \\
 & \quad \left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right. \\
 & \quad \left. + \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}
 \end{aligned}$$

σ_{XY}
 ↙ ↘
Beam **Target**
Polarization



Mulders and Tangermann, Nucl. Phys. B 461 (1996) 197

Boer and Mulders, Phys. Rev. D 57 (1998) 5780

Bacchetta et al., Phys. Lett. B 595 (2004) 309

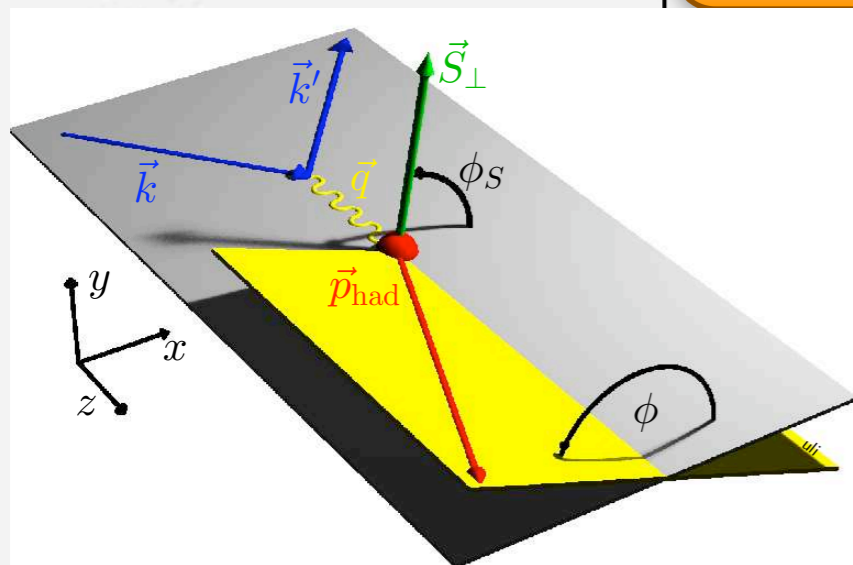
Bacchetta et al., JHEP 0702 (2007) 093

“Trento Conventions”, Phys. Rev. D 70 (2004) 117504

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 & + S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\} \\
 & + S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \frac{1}{Q} \right. \\
 & \quad \left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right. \\
 & \quad \left. + \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}
 \end{aligned}$$

σ_{XY}
 ↙ ↘
Beam Target
Polarization



Mulders and Tangermann, Nucl. Phys. B 461 (1996) 197

Boer and Mulders, Phys. Rev. D 57 (1998) 5780

Bacchetta et al., Phys. Lett. B 595 (2004) 309

Bacchetta et al., JHEP 0702 (2007) 093

“Trento Conventions”, Phys. Rev. D 70 (2004) 117504

Cross section without polarization

$$F_{XY,Z} = F_{XY,Z}(x, y, z, P_{h\perp})$$

target polarization \downarrow
 beam polarization \uparrow virtual-photon polarization \uparrow

$$\frac{d^5\sigma}{dx dy dz d\phi_h dP_{h\perp}^2} \propto \left(1 + \frac{\gamma^2}{2x}\right) \{F_{UU,T} + \epsilon F_{UU,L}\}$$

$$\gamma = \frac{2Mx}{Q}$$

$$\epsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

[see, e.g., Bacchetta et al., JHEP 0702 (2007) 093

INT 12-49W, February 10th, 2012

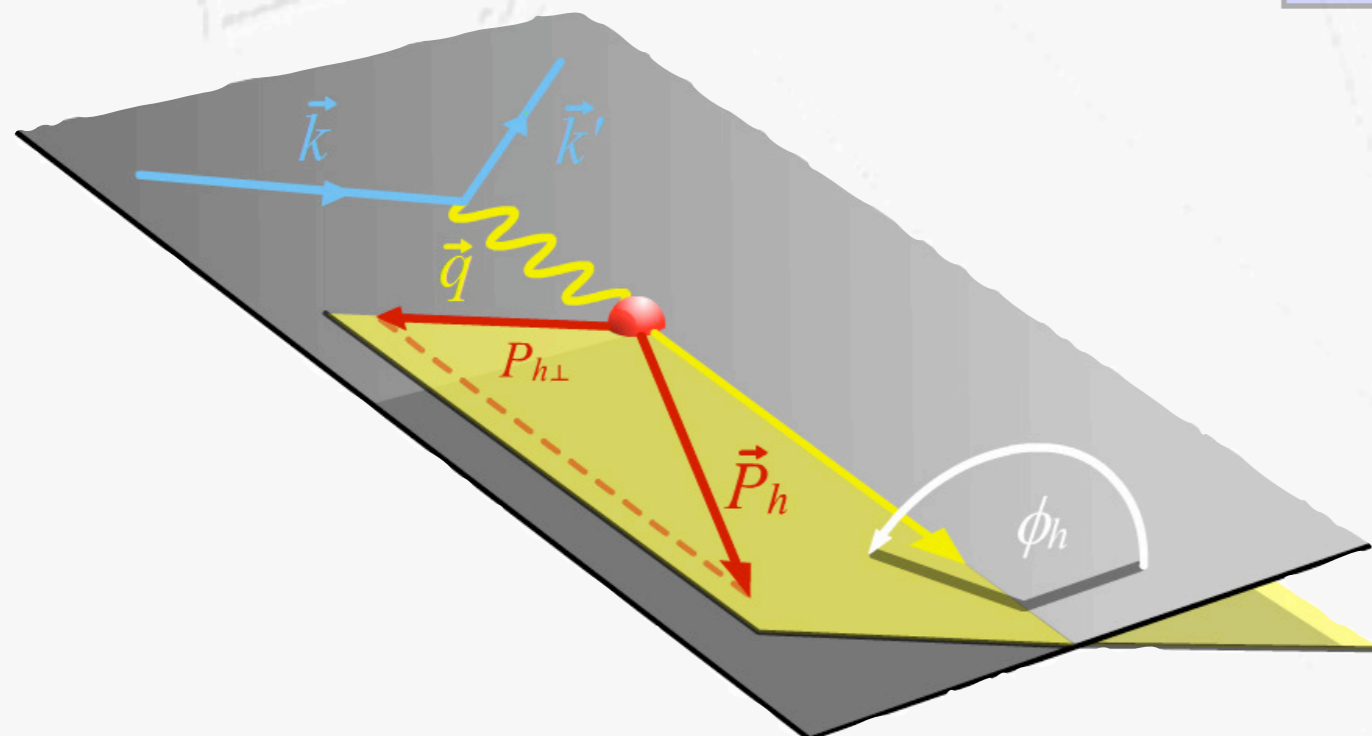
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$$\left. + \sqrt{2\epsilon(1-\epsilon)} F_{UU}^{\cos\phi_h} \cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h \right\}$$



$$\gamma = \frac{2Mx}{Q}$$

$$\epsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

[see, e.g., Bacchetta et al., JHEP 0702 (2007) 093
INT 12-49W, February 10th, 2012

... possible measurements

$$\frac{d^5\sigma}{dx dy dz d\phi_h dP_{h\perp}^2} \propto \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \epsilon F_{UU,L} \right. \\ \left. + \sqrt{2\epsilon(1-\epsilon)} F_{UU}^{\cos\phi_h} \cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h \right\}$$

... possible measurements

hadron multiplicity:
normalize to inclusive DIS
cross section

$$\frac{d^5\sigma}{dx dy dz d\phi_h dP_{h\perp}^2} \propto \left(1 + \frac{\gamma^2}{2x}\right) \{F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1-\epsilon)} F_{UU}^{\cos\phi_h} \cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h\}$$

... possible measurements

hadron multiplicity:
normalize to inclusive DIS
cross section

$$\frac{d^2 \sigma^{\text{incl. DIS}}}{dx dy} \propto F_T + \epsilon F_L$$

$$\frac{d^4 \mathcal{M}^h(x, y, z, P_{h\perp}^2)}{dx dy dz dP_{h\perp}^2} \propto \left(1 + \frac{\gamma^2}{2x}\right) \frac{F_{UU,T} + \epsilon F_{UU,L}}{F_T + \epsilon F_L}$$

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$$\approx \frac{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^{q \rightarrow h}(z, K_T^2)}{\sum_q e_q^2 f_1^q(x)}$$

$$\frac{d^5 \sigma}{dx dy dz d\phi_h dP_{h\perp}^2} \propto \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1-\epsilon)} F_{UU}^{\cos \phi_h} \cos \phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h \right\}$$

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moments:
normalize to azimuth-
independent cross-section

... possible measurements

hadron multiplicity:
normalize to inclusive DIS
cross section

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$$2 \langle \cos 2\phi \rangle_{UU} \equiv 2 \frac{\int d\phi_h \cos 2\phi d\sigma}{\int d\phi_h d\sigma} = \frac{\epsilon F_{UU}^{\cos 2\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

moments:
normalize to azimuth-
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... possible measurements

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moments:
normalize to azimuth-
independent cross-section

$$\approx \epsilon \frac{\sum_q e_q^2 h_1^{\perp,q}(x, p_T^2) \otimes_{\text{BM}} H_1^{\perp,q \rightarrow h}(z, K_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^{q \rightarrow h}(z, K_T^2)}$$

... possible measurements

hadron multiplicity:
normalize to inclusive DIS
cross section

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$$\approx \frac{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^{q \rightarrow h}(z, K_T^2)}{\sum_q e_q^2 f_1^q(x)}$$

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$$+ \sqrt{2\epsilon(1-\epsilon)} F_{UU}^{\cos \phi_h} \cos \phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h\}$$

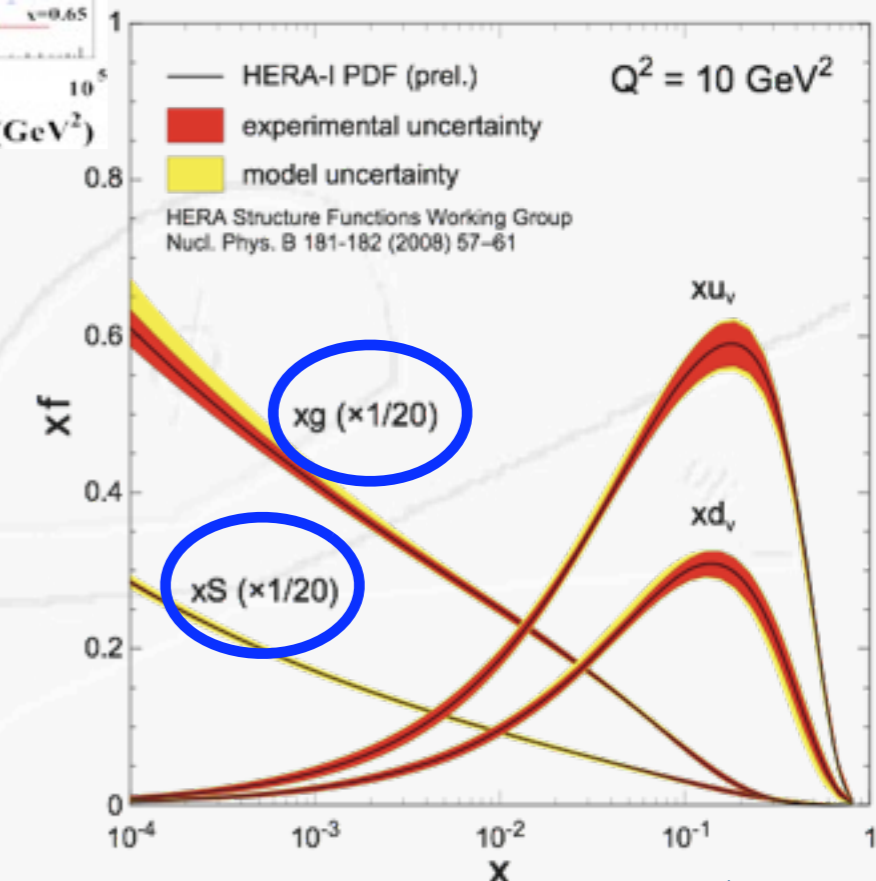
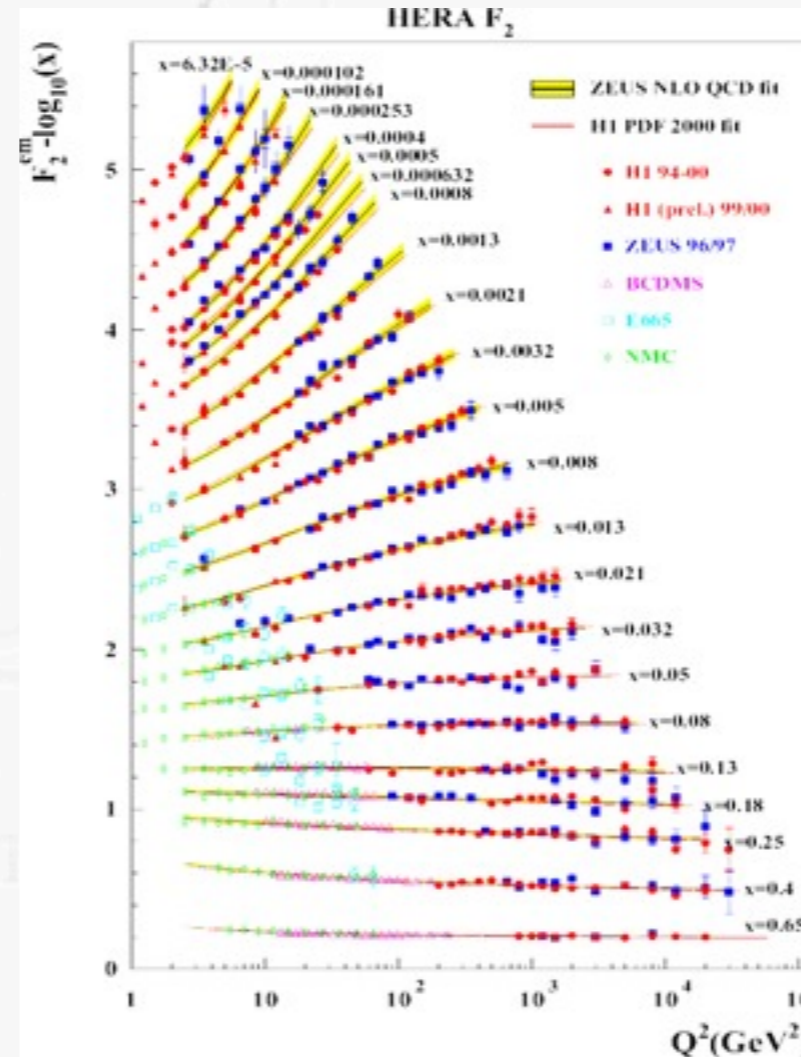
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moments:
normalize to azimuth-
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Momentum density

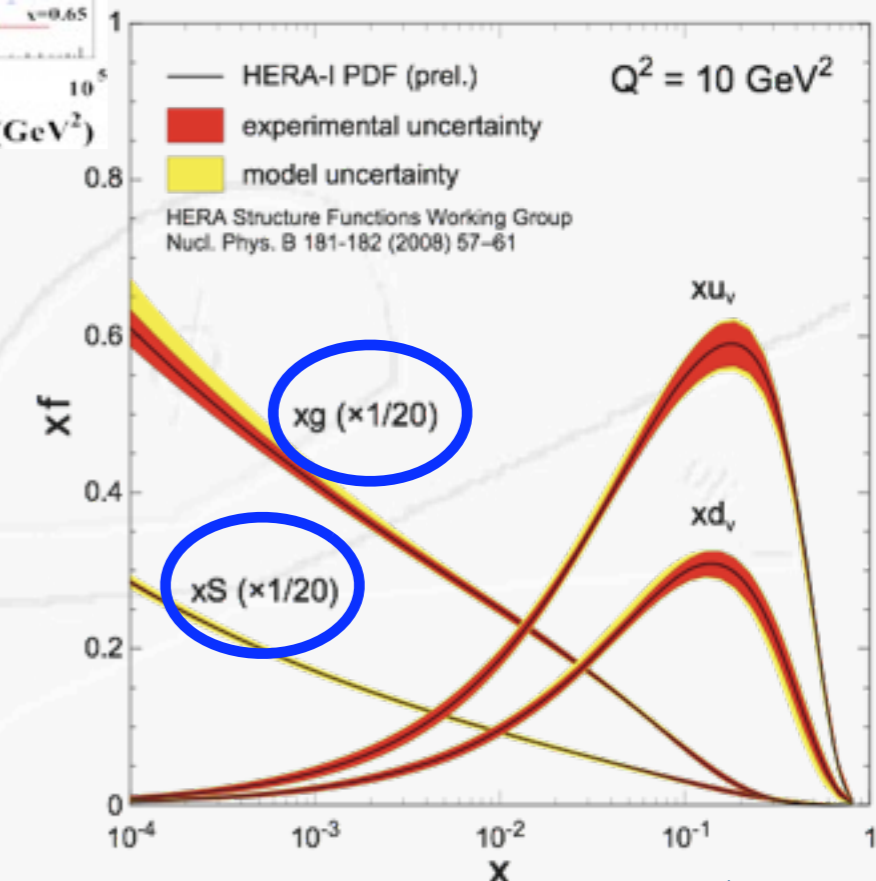
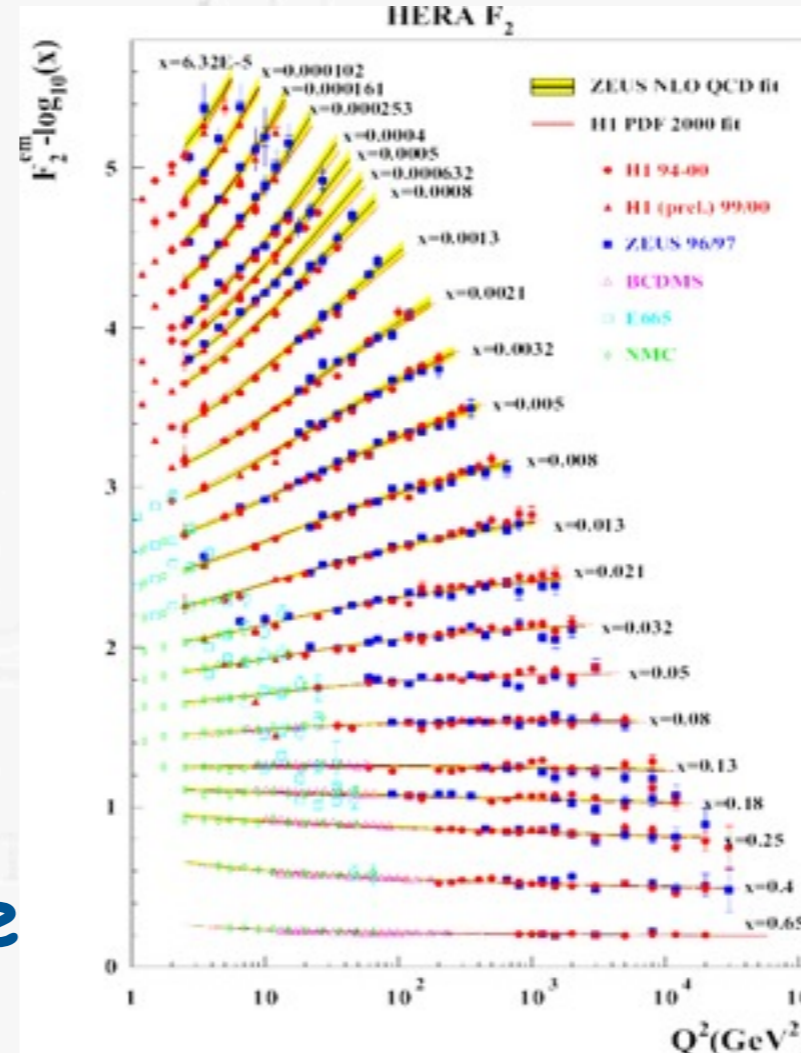
	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



- plenty of data available
- but only for integrated version of f_1

Momentum density

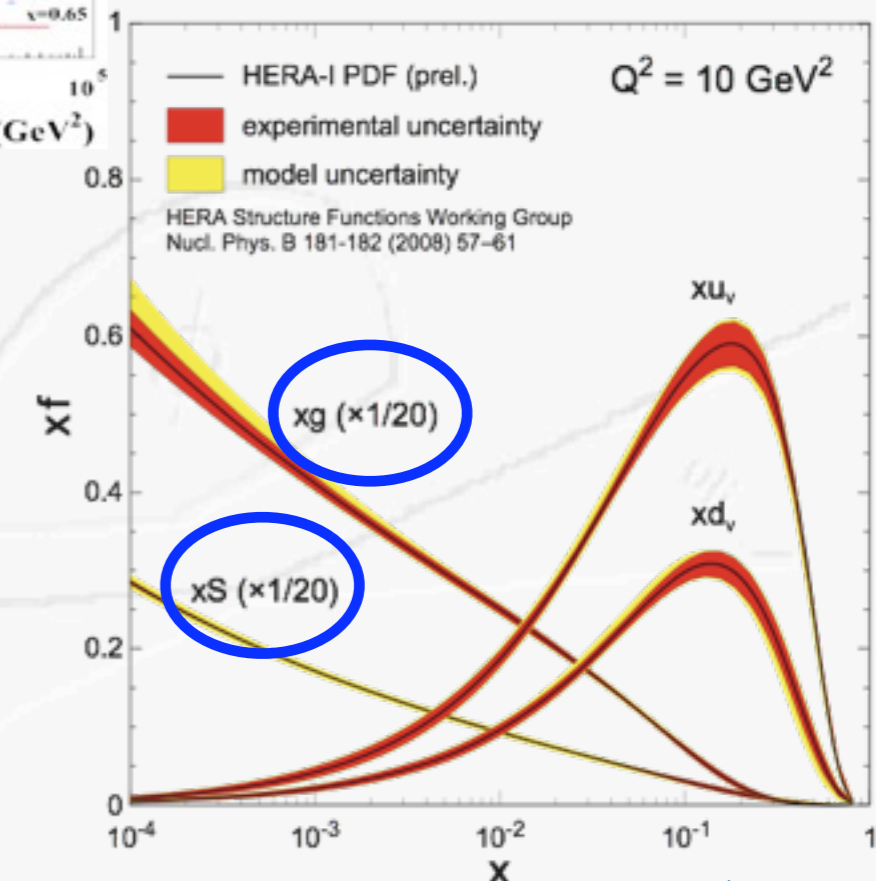
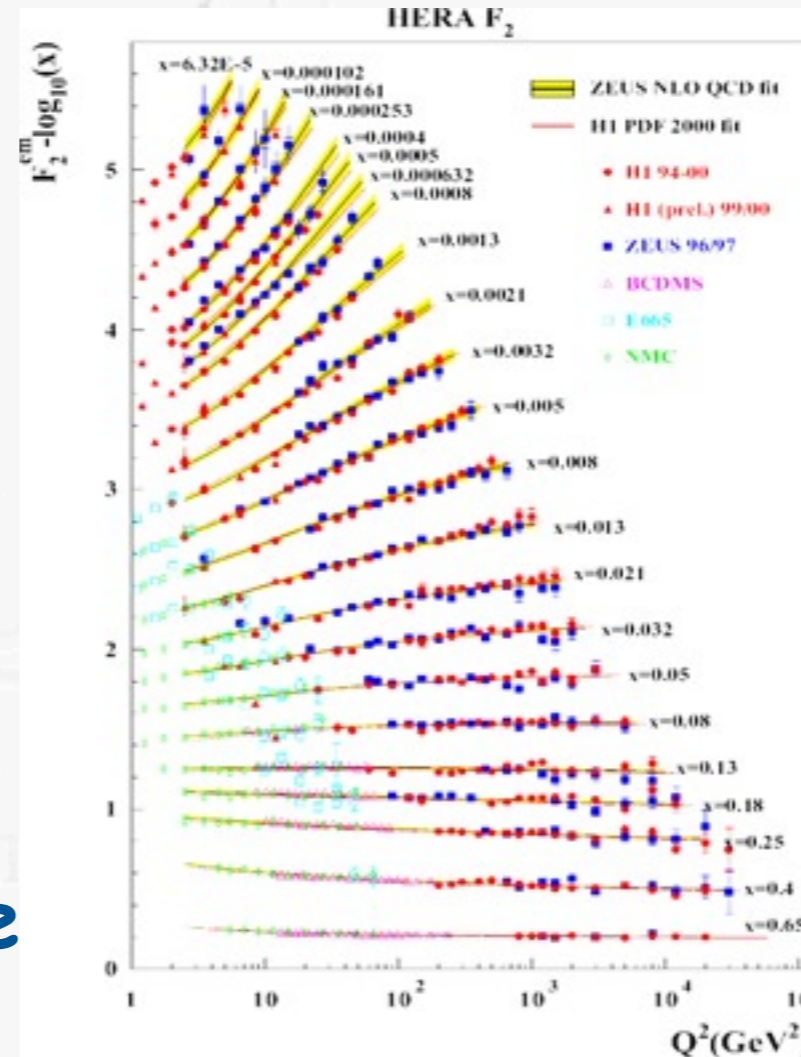
	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



- plenty of data available
- but only for integrated version of f_1
- spin asymmetries involve unintegrated f_1 in denominator!

Momentum density

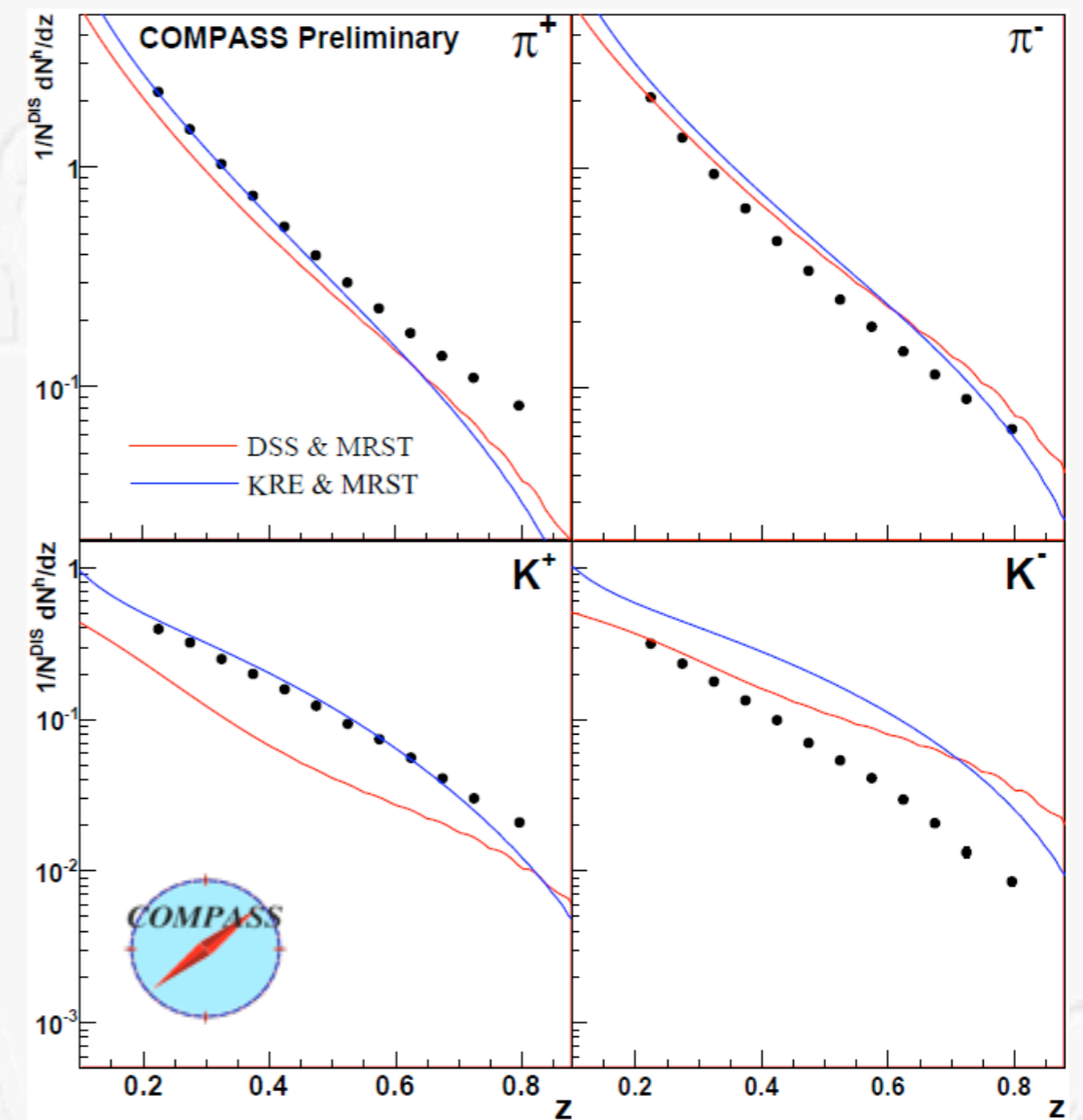
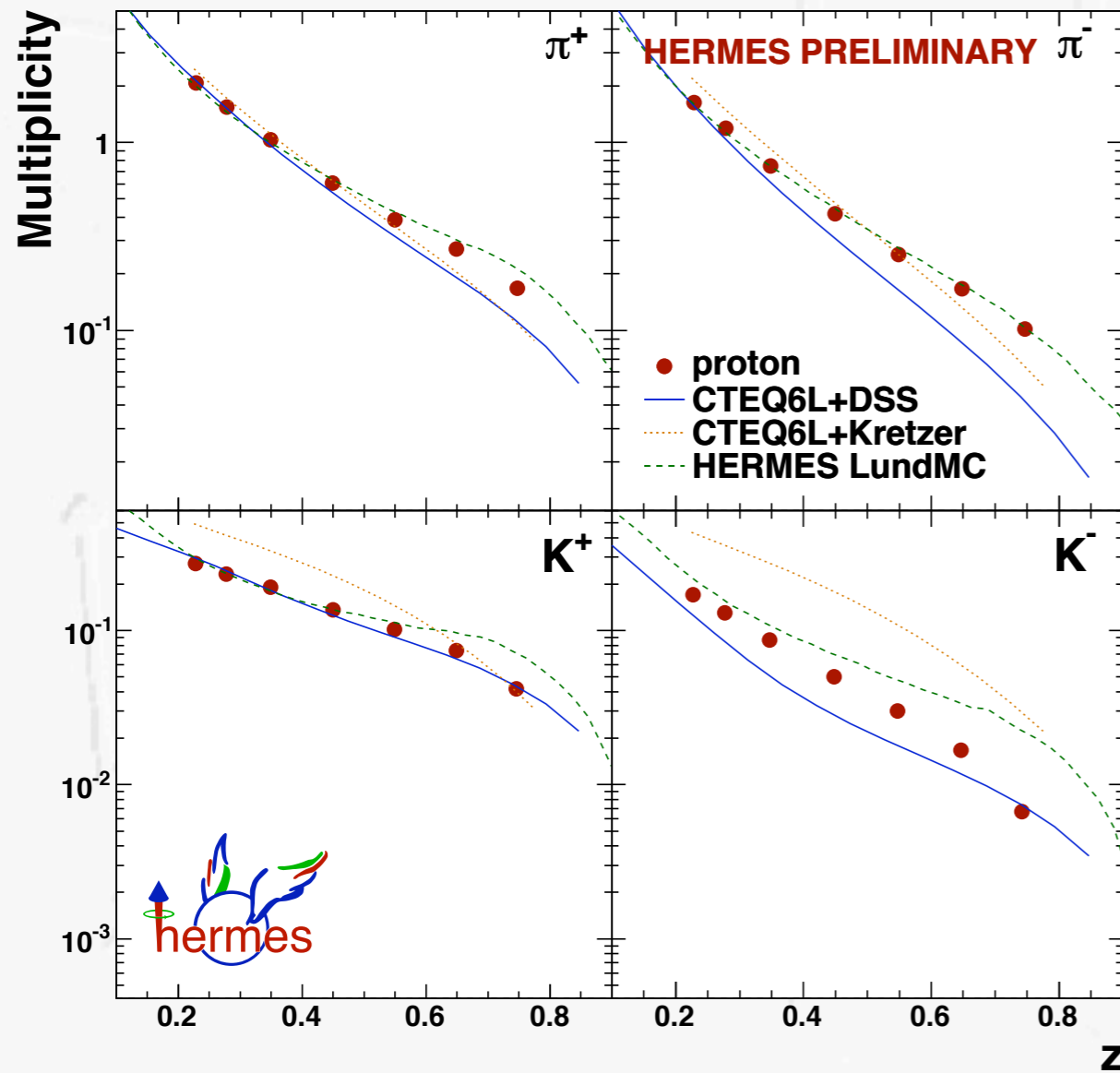
	U	L	T
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L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



- plenty of data available
- but only for integrated version of f_1
- spin asymmetries involve unintegrated f_1 in denominator!
- need multiplicities and fragmentation functions not only binned in z but also in $P_{h\perp}$

Hadron multiplicities

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

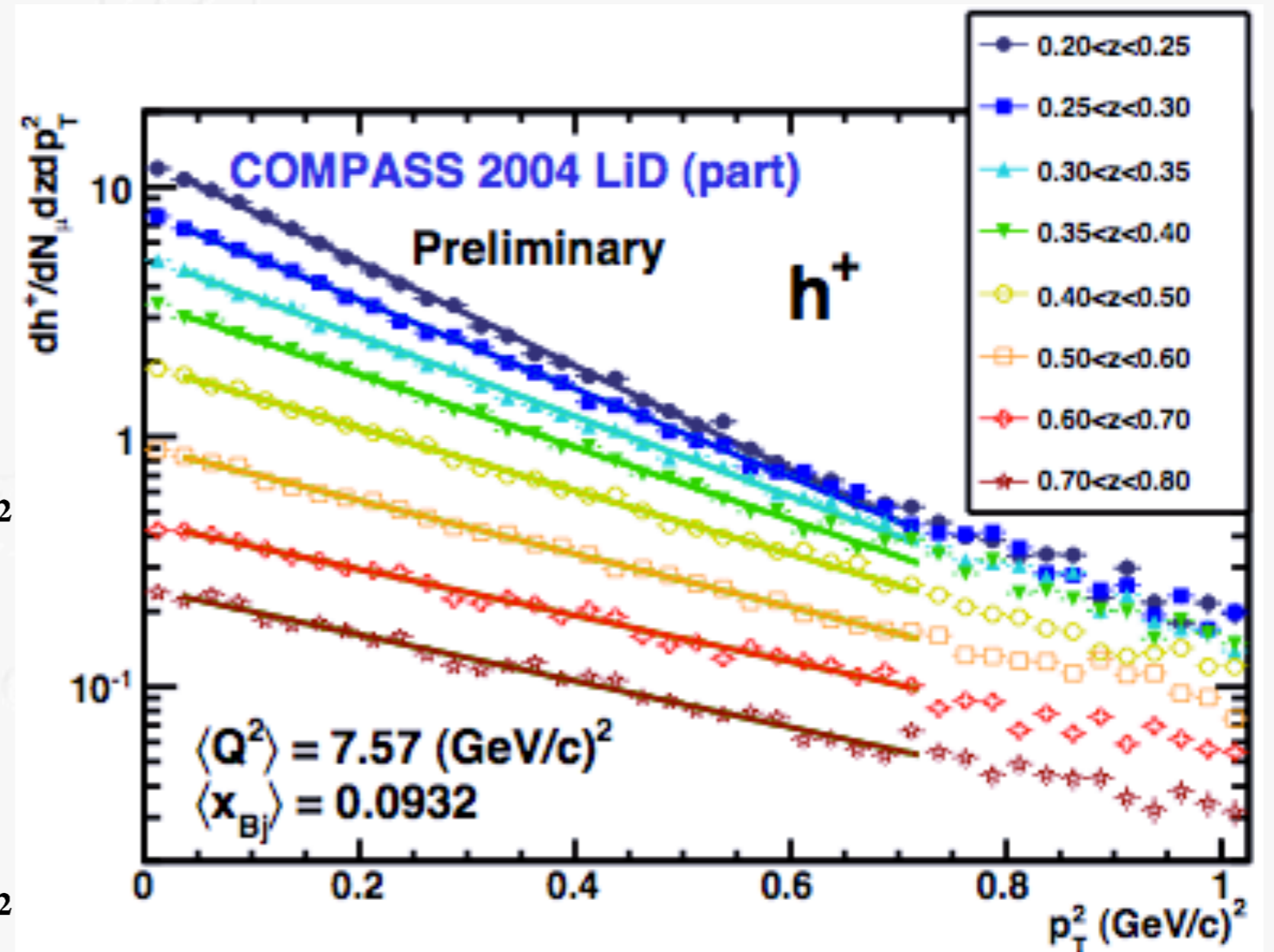
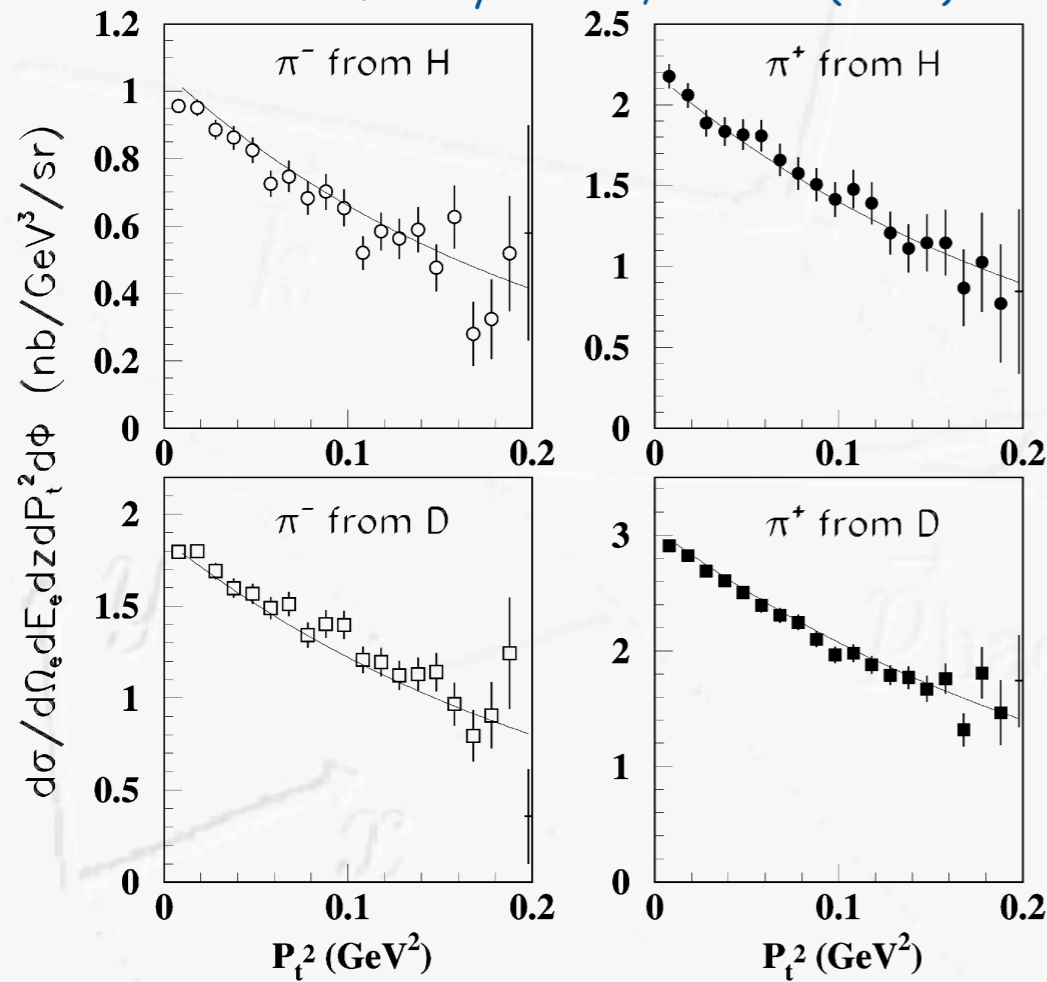


- P_{h_\perp} -integrated multiplicities ideal input for FF fits and tests
- kaons difficult to describe

Disentangle z and $P_{h\perp}$ -dependence

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

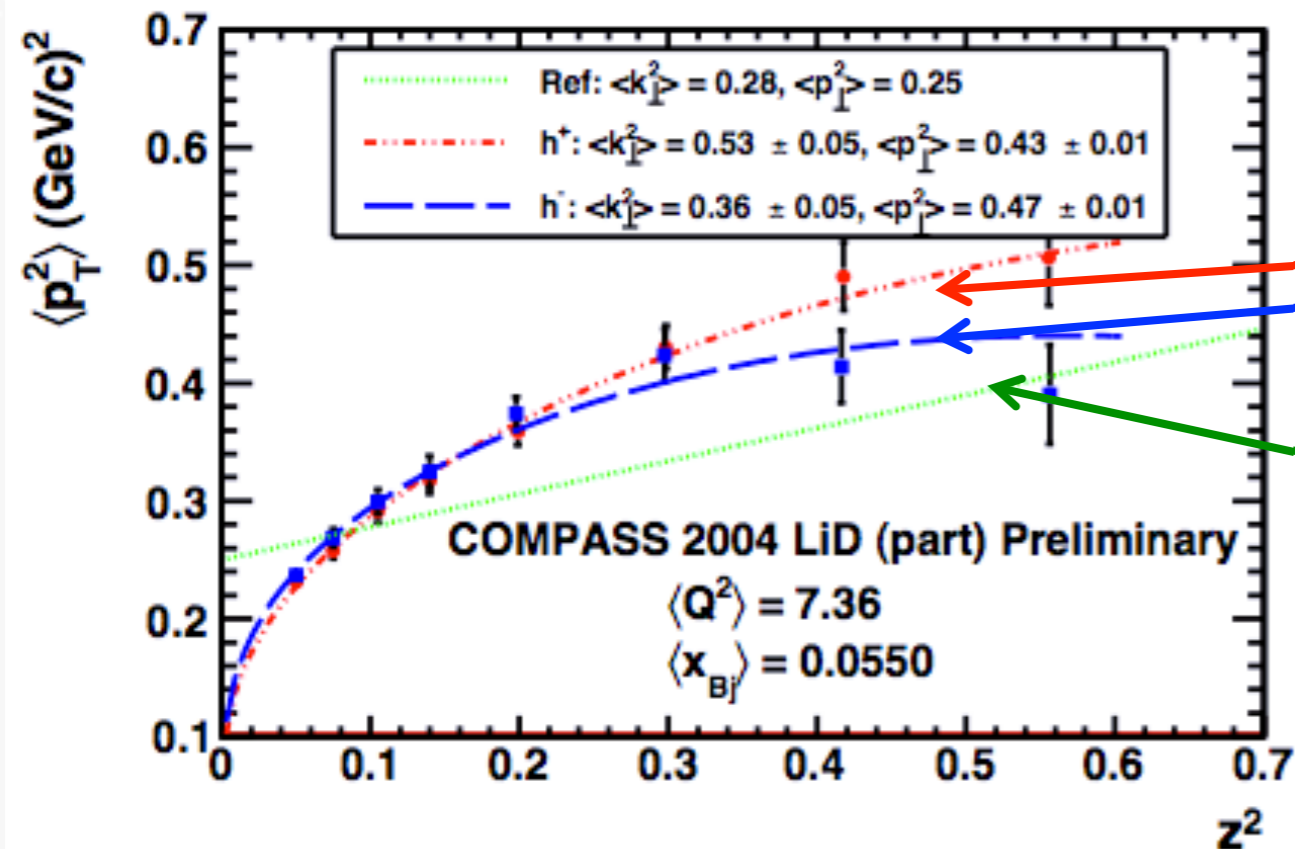
Mkrtchyan et al., PLB 665 (2008) 20



- study $P_{h\perp}$ -dependence → access to TMDs

Disentangle z and $P_{h\perp}$ -dependence

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



$$\langle p_T^2 \rangle = z^\alpha (1 - z)^\beta \langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle$$

$$\alpha = 0.5; \beta = 1.5$$

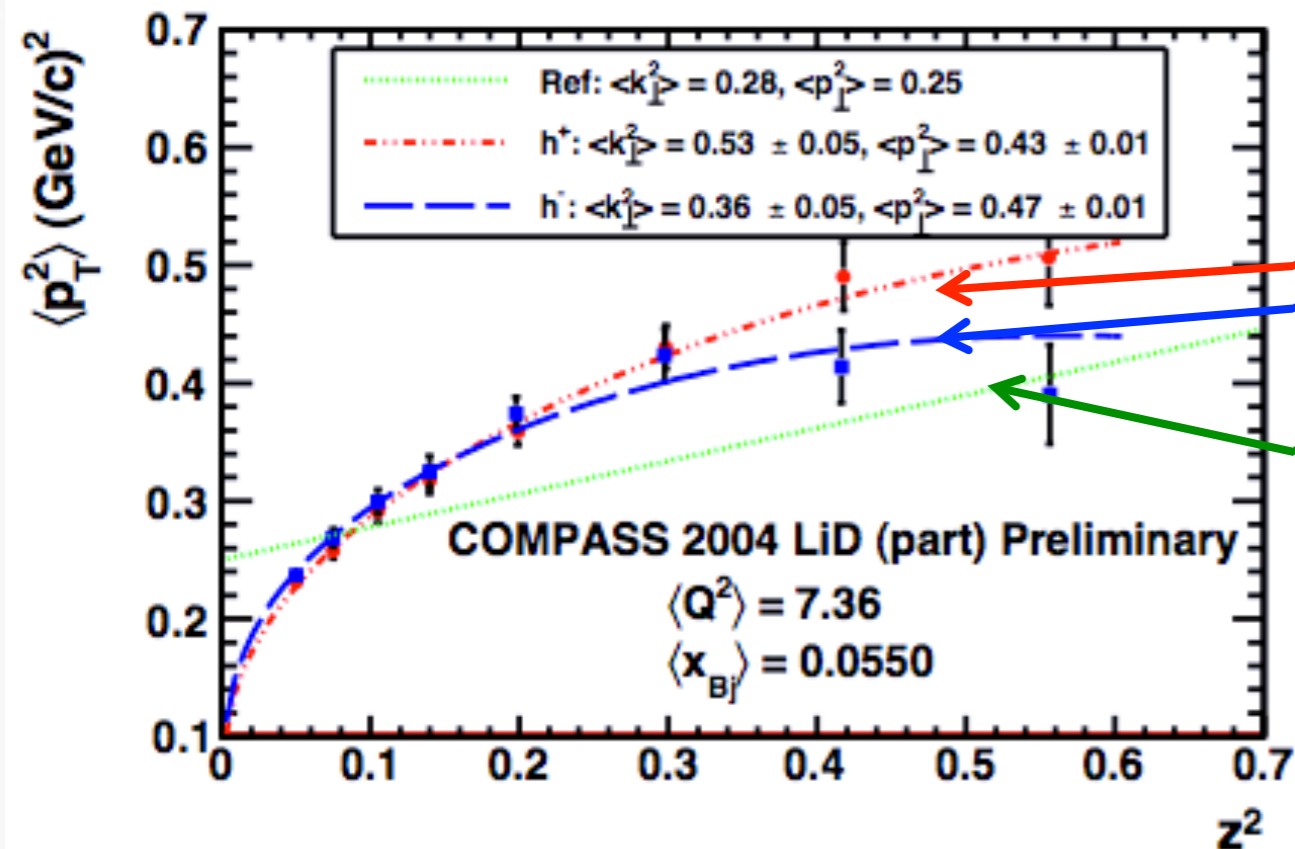
$$\langle p_T^2 \rangle = \langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle$$

Claude Marchand - DIS 2011

- study $P_{h\perp}$ -dependence \rightarrow access to TMDs
- constant average (fragmentation) p_\perp excluded

Disentangle z and $P_{h\perp}$ -dependence

	U	L	T
U	f_1		h_1^\perp
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$$\langle p_T^2 \rangle = z^\alpha (1 - z)^\beta \langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle$$

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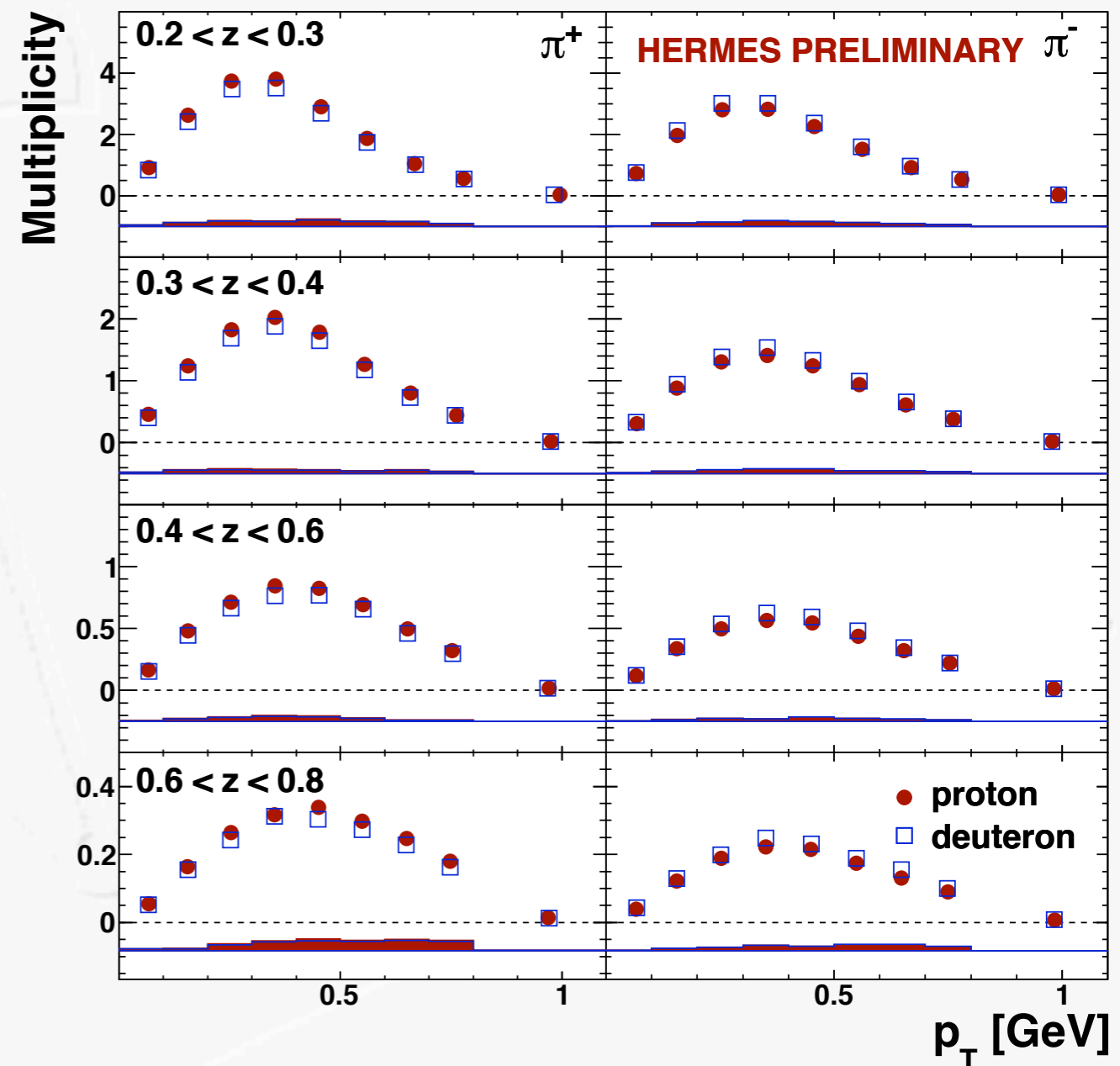
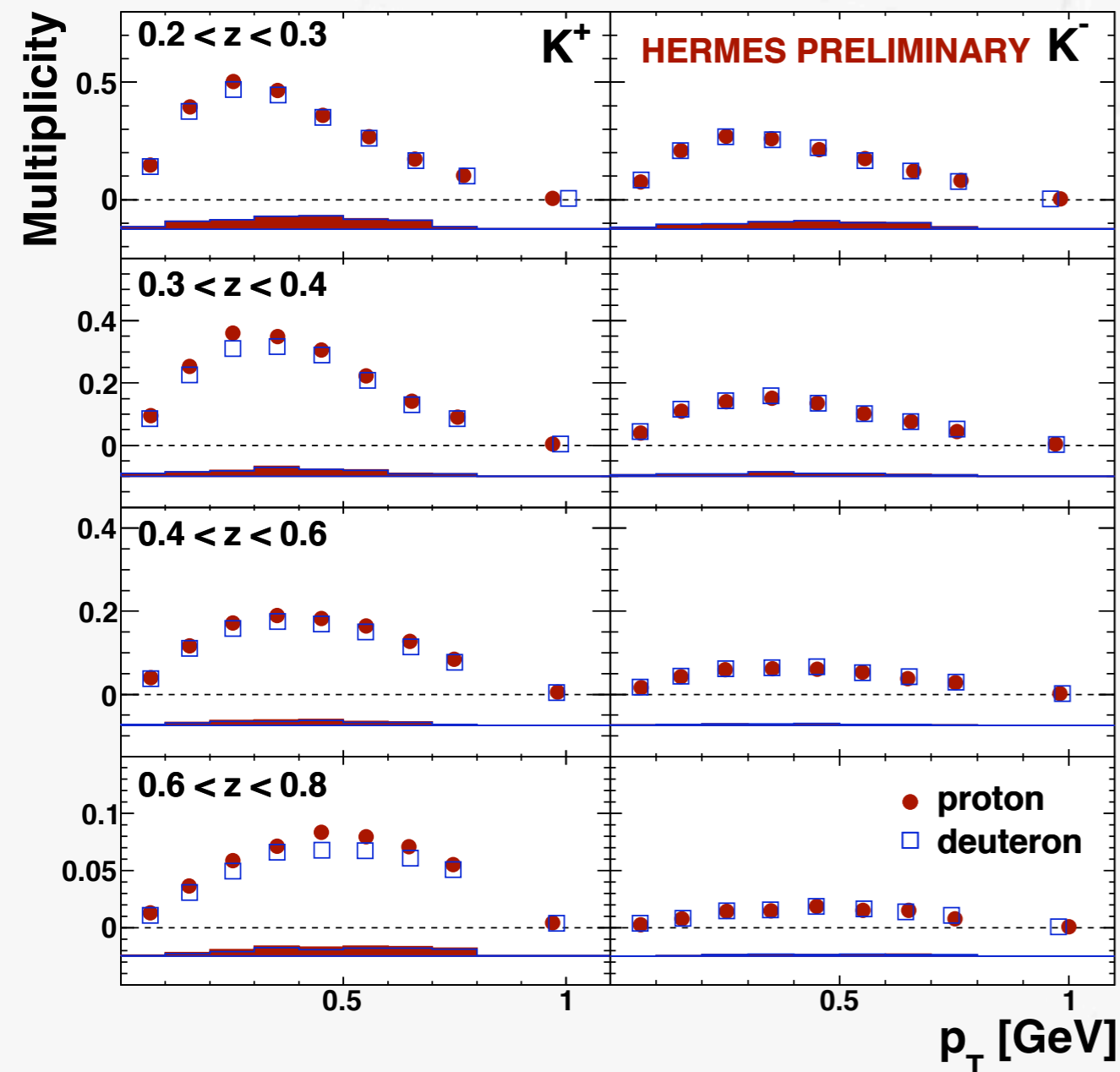
Claude Marchand - DIS 2011

- study $P_{h\perp}$ -dependence \rightarrow access to TMDs
- constant average (fragmentation) p_\perp excluded
- difference in h^+ and h^- behavior \rightarrow flavor dependence

Disentangle z and $P_{h\perp}$ -dependence

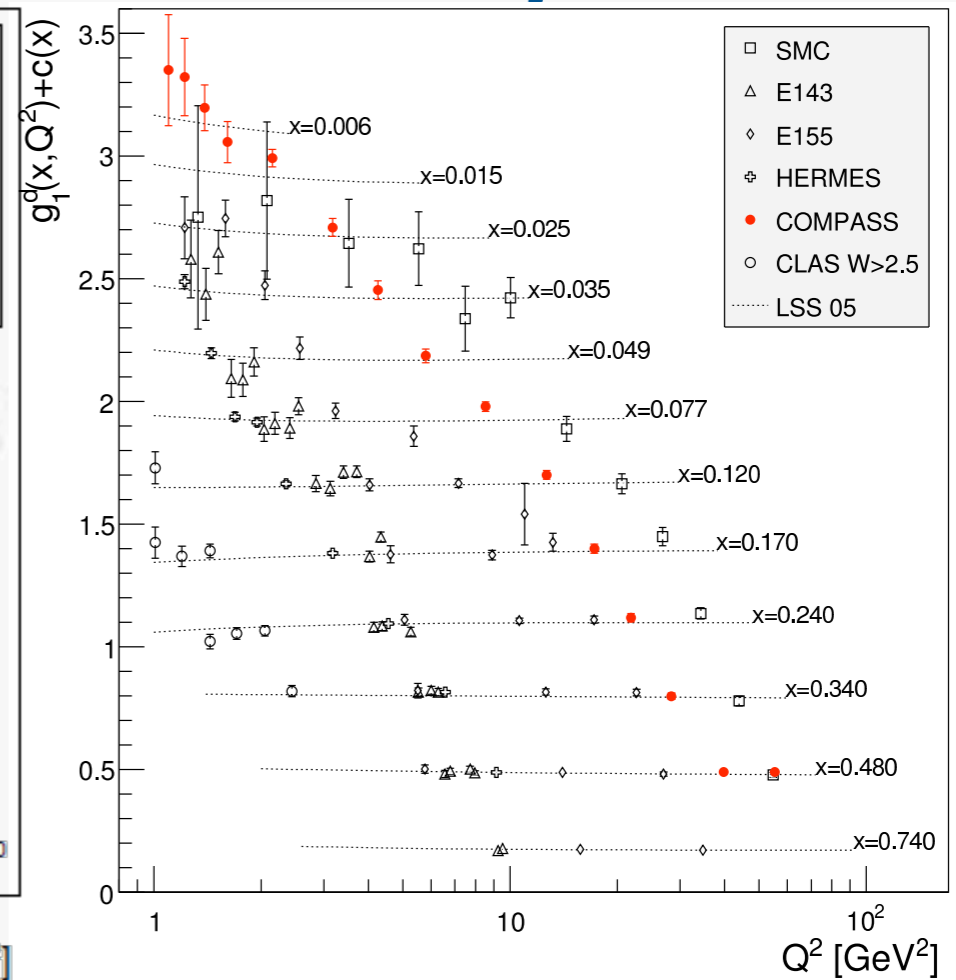
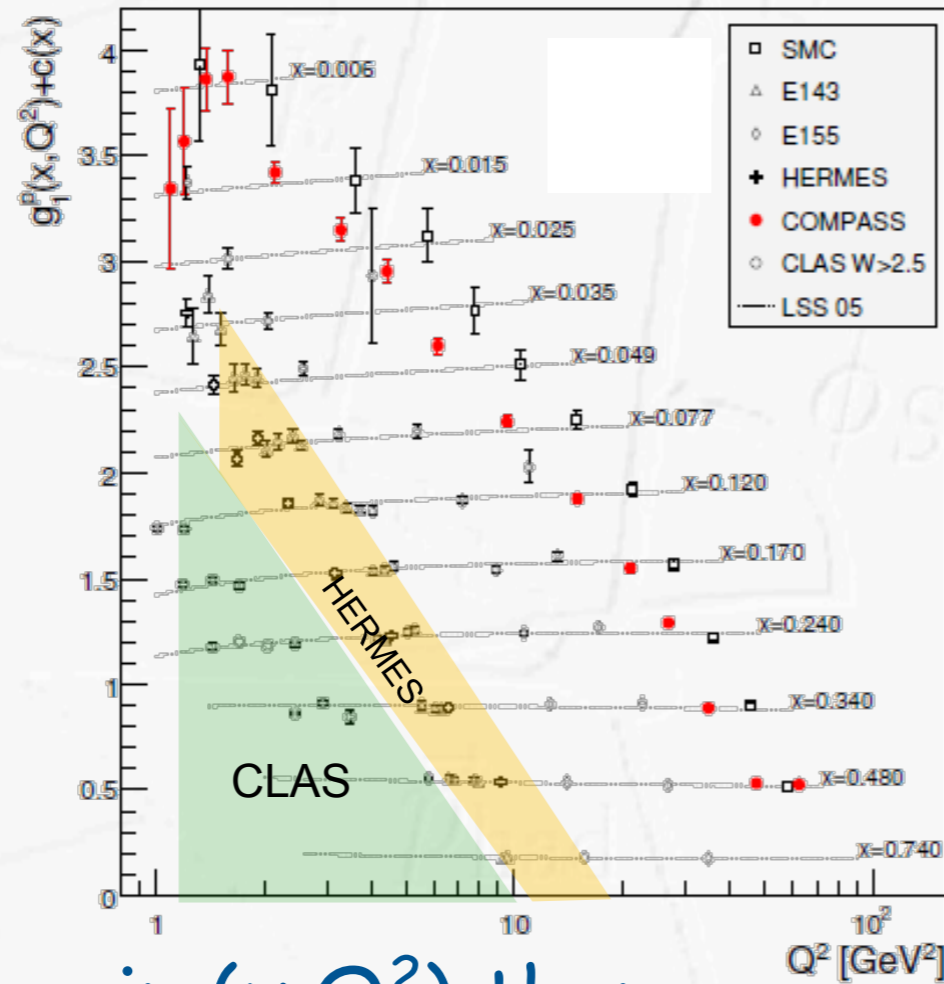
	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

- flavor info via target variation and hadron ID



Helicity density

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

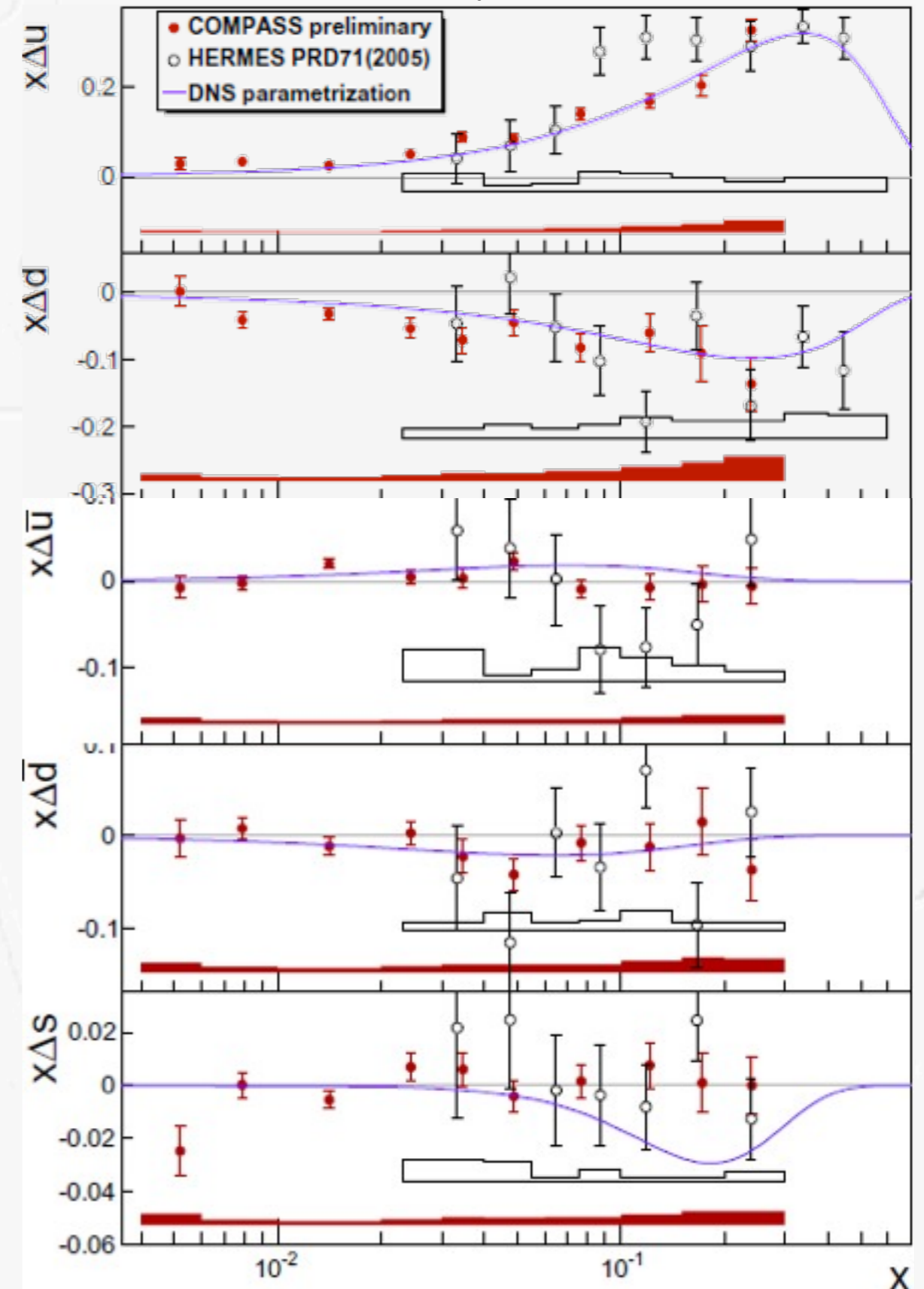


- smaller range in (x, Q^2) than for f_1
- data mainly for integrated version of g_{1L}
- need asymmetries not only binned in x but also in $P_{h\perp}$

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

Helicity density

[M.G. Alekseev et al., Phys.Lett. B693 (2010) 227-235]

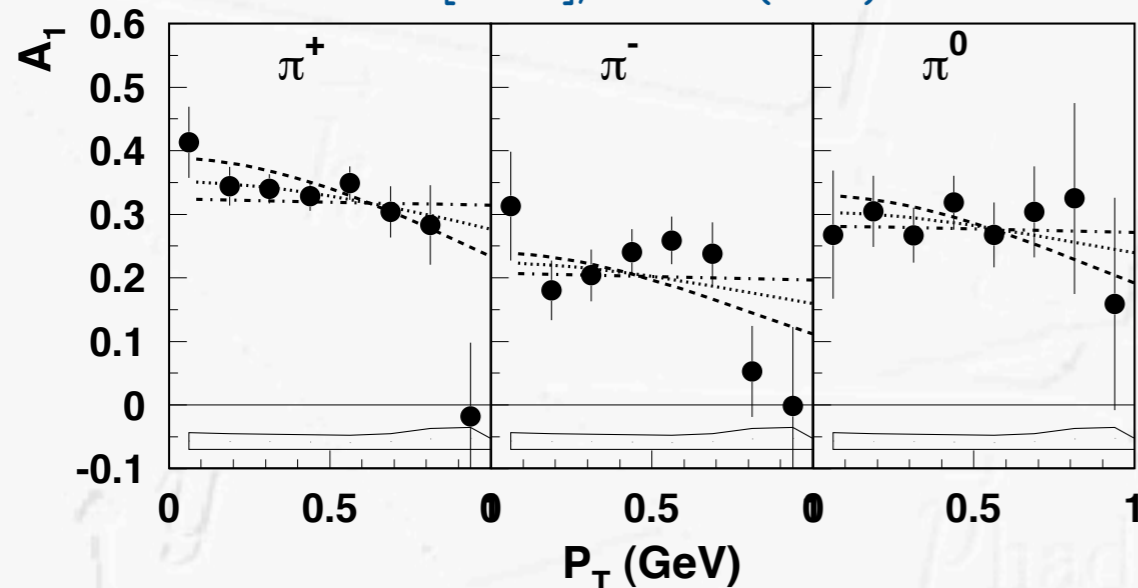


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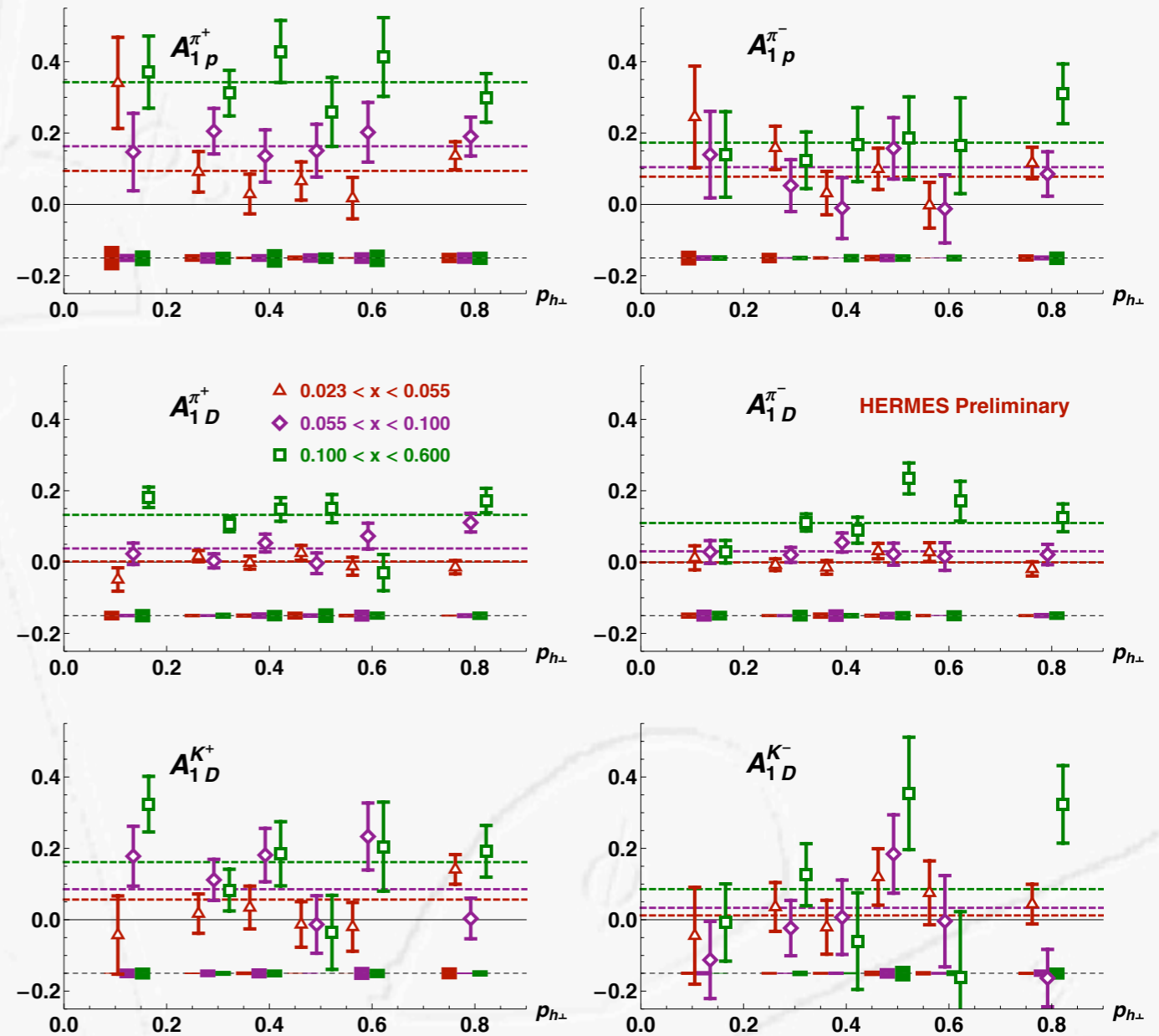
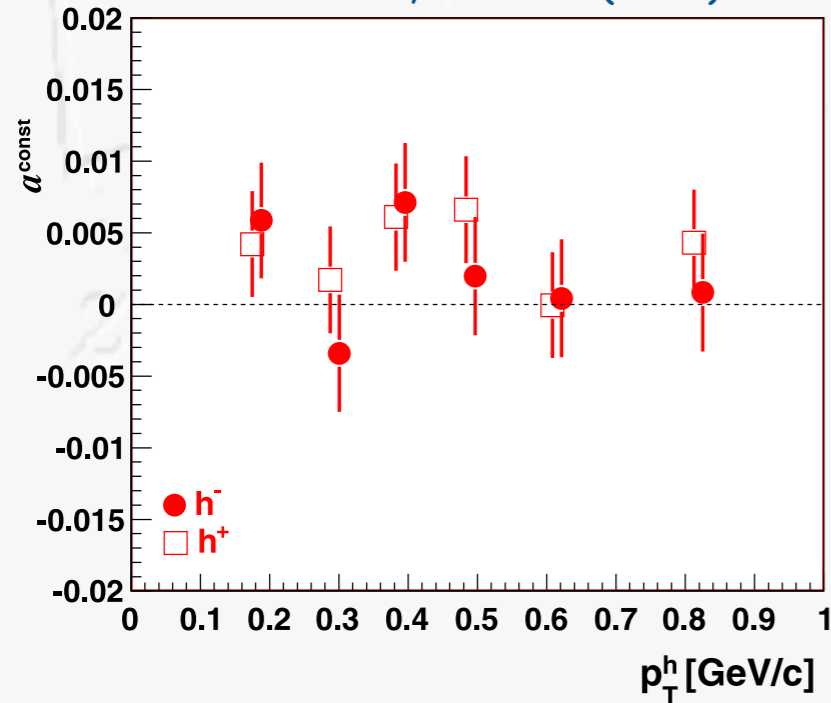
Helicity density (unintegrated)

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

Avakian et al. [CLAS], PRL 105 (2010) 262002

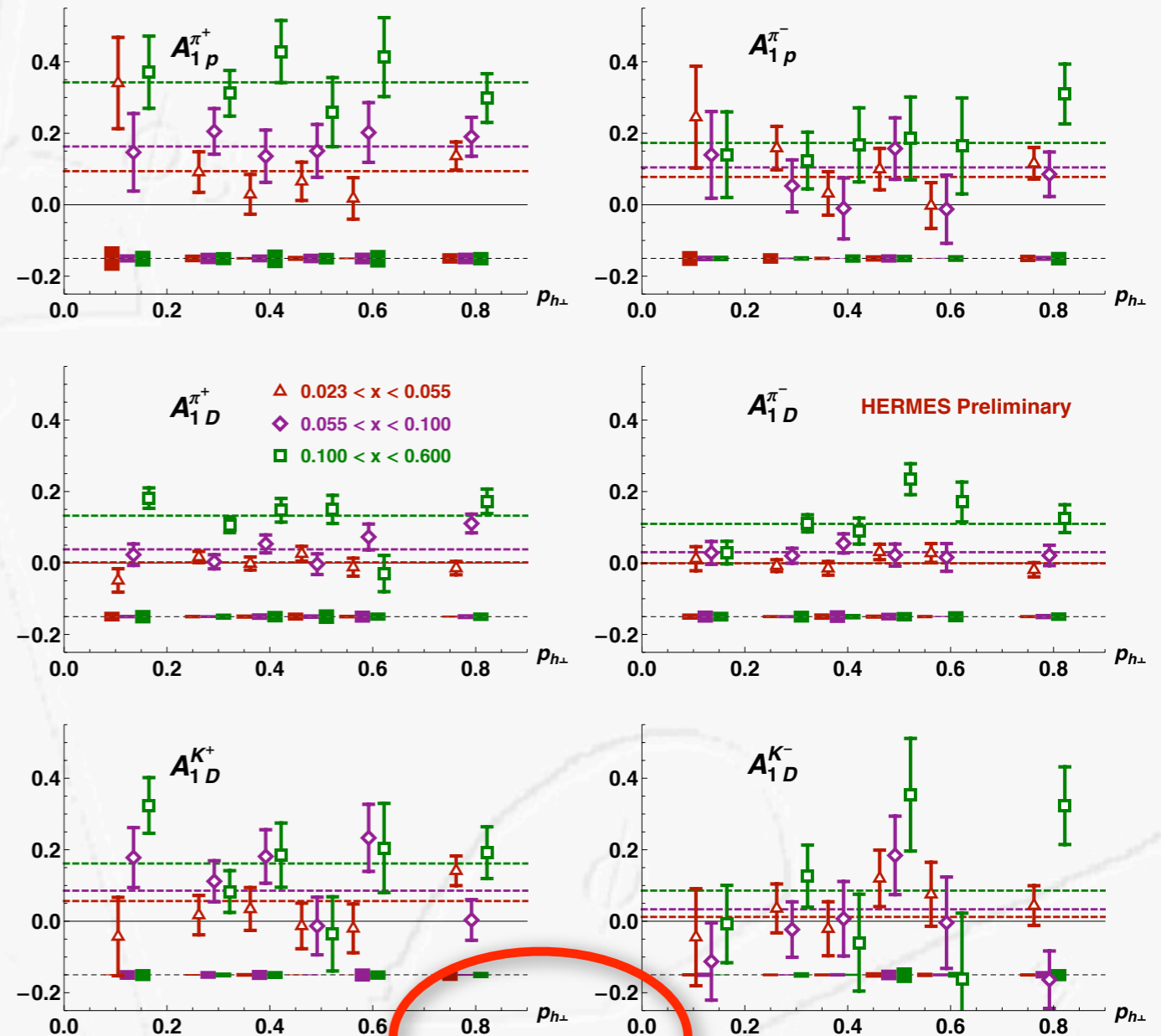
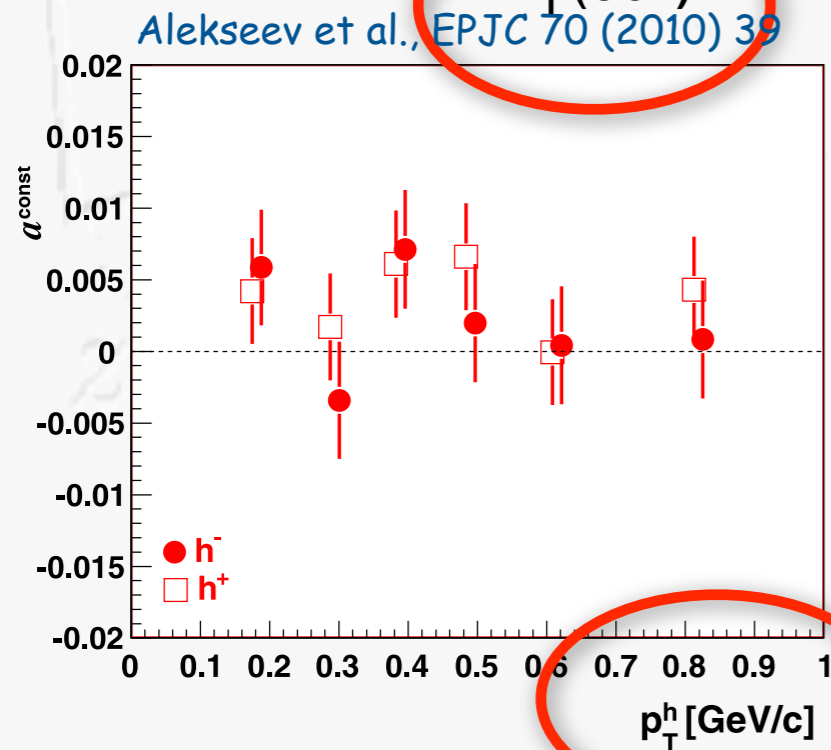
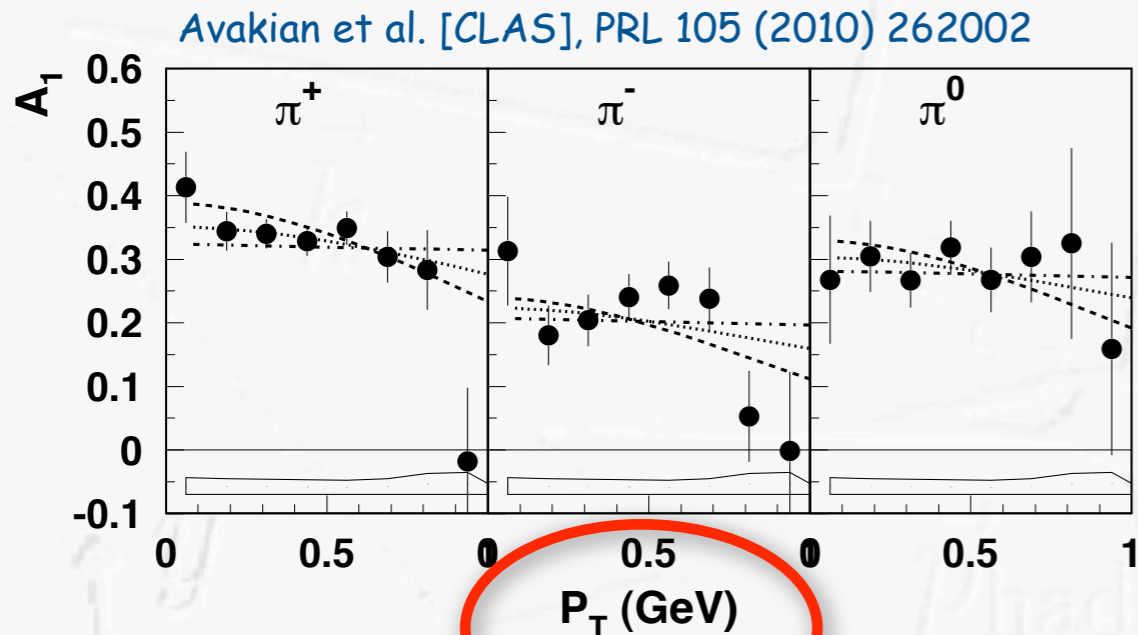


Alekseev et al., EPJC 70 (2010) 39



Helicity density (unintegrated)

	U	L	T
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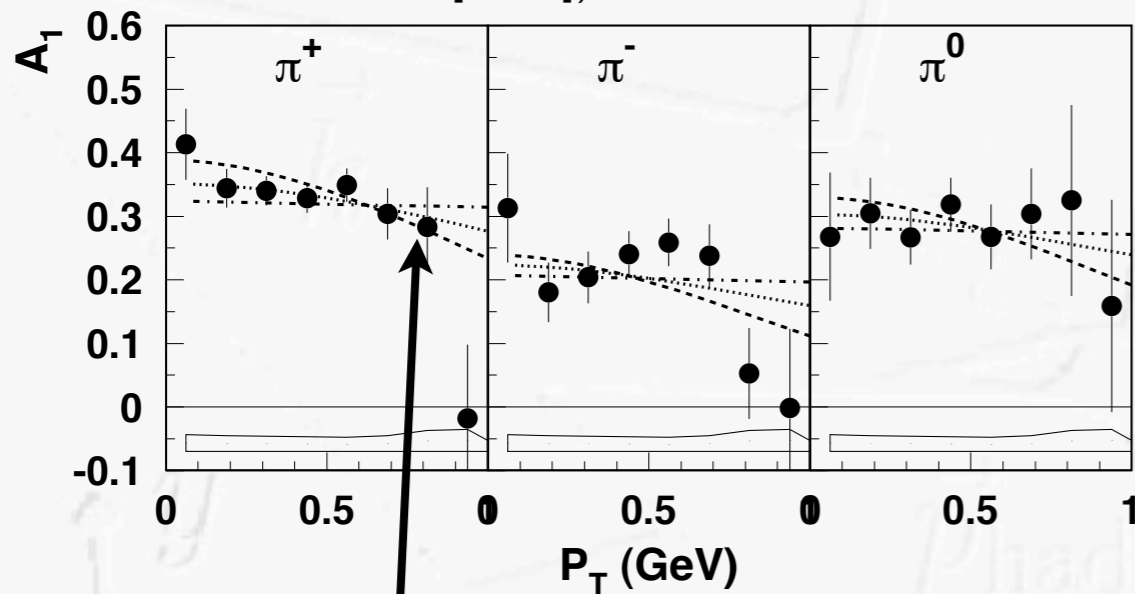


only weak if any dependence on $P_{h\perp}$ seen

Helicity density (unintegrated)

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

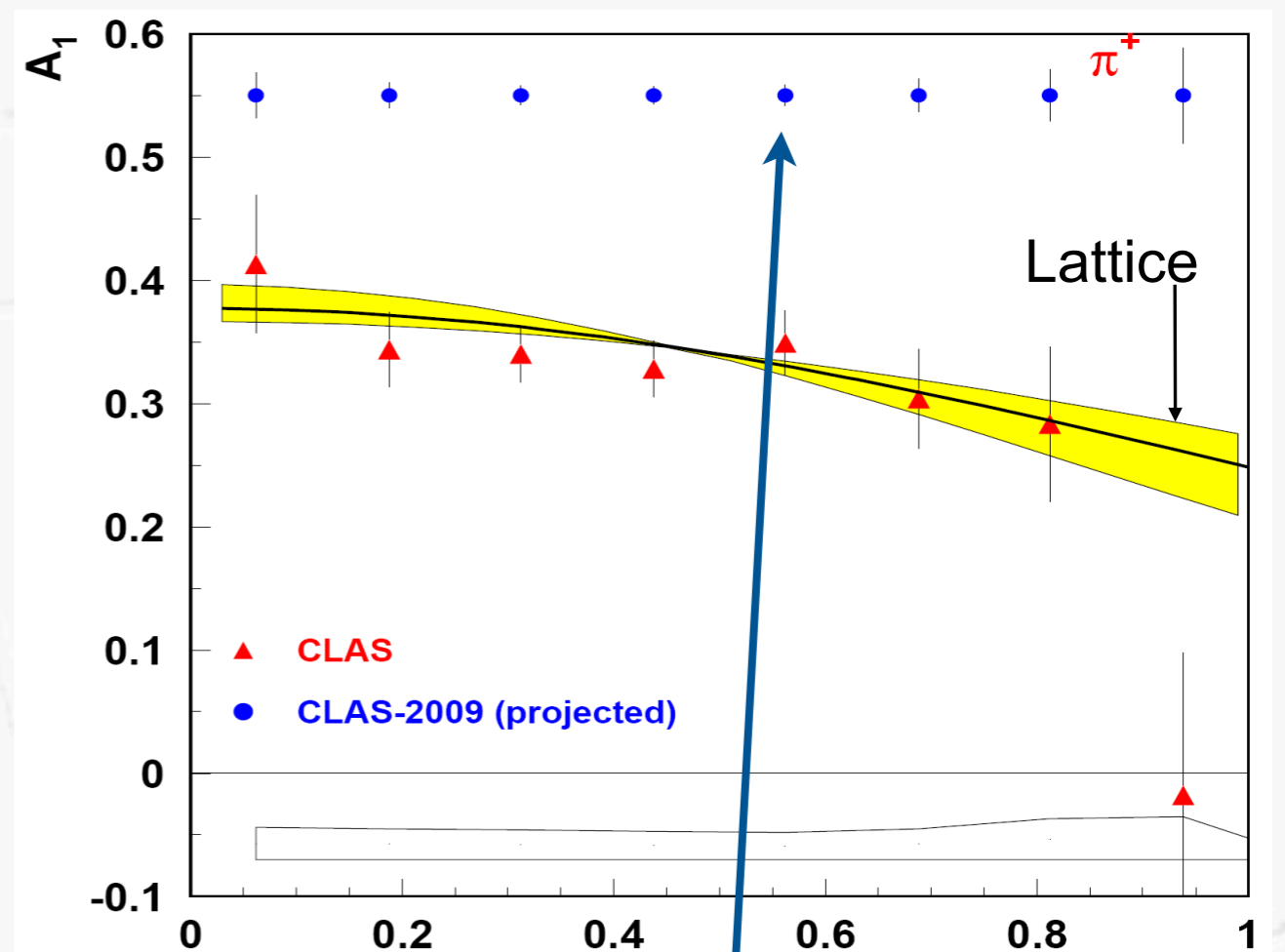
Avakian et al. [CLAS], arXiv:1003.4549



CLAS data hints at width μ_2 of g_1 that is less than the width μ_0 of f_1

$$f_1^q(x, k_T) = f_1(x) \frac{1}{\pi \mu_0^2} \exp\left(-\frac{k_T^2}{\mu_0^2}\right)$$

$$g_1^q(x, k_T) = g_1(x) \frac{1}{\pi \mu_2^2} \exp\left(-\frac{k_T^2}{\mu_2^2}\right)$$

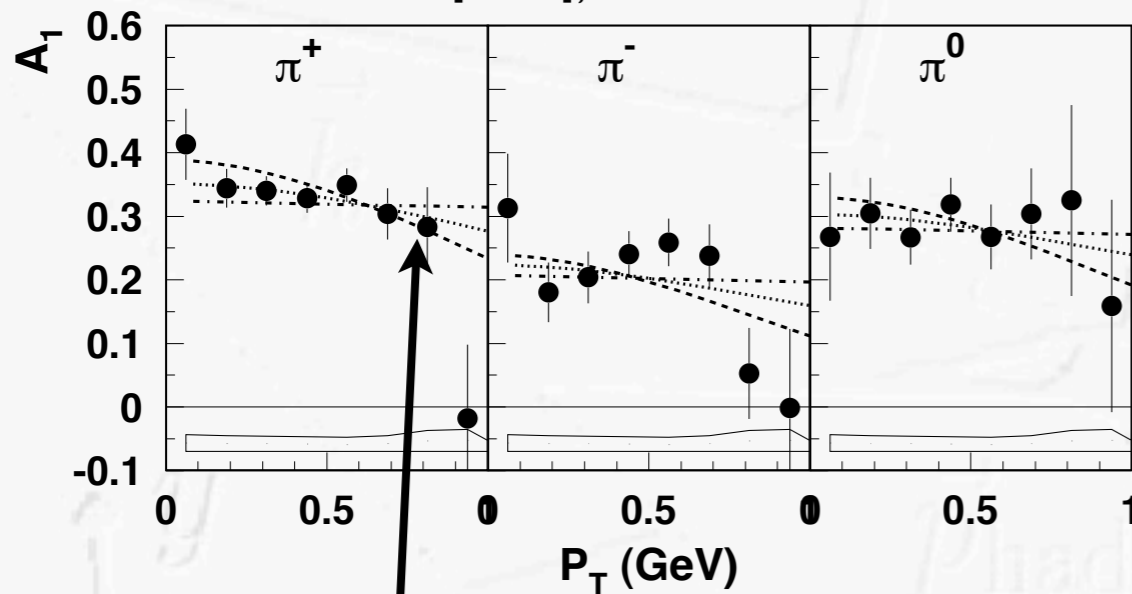


New CLAS data will allow multi-D binning to study $P_{h\perp}$ dependence for fixed x

Helicity density (unintegrated)

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
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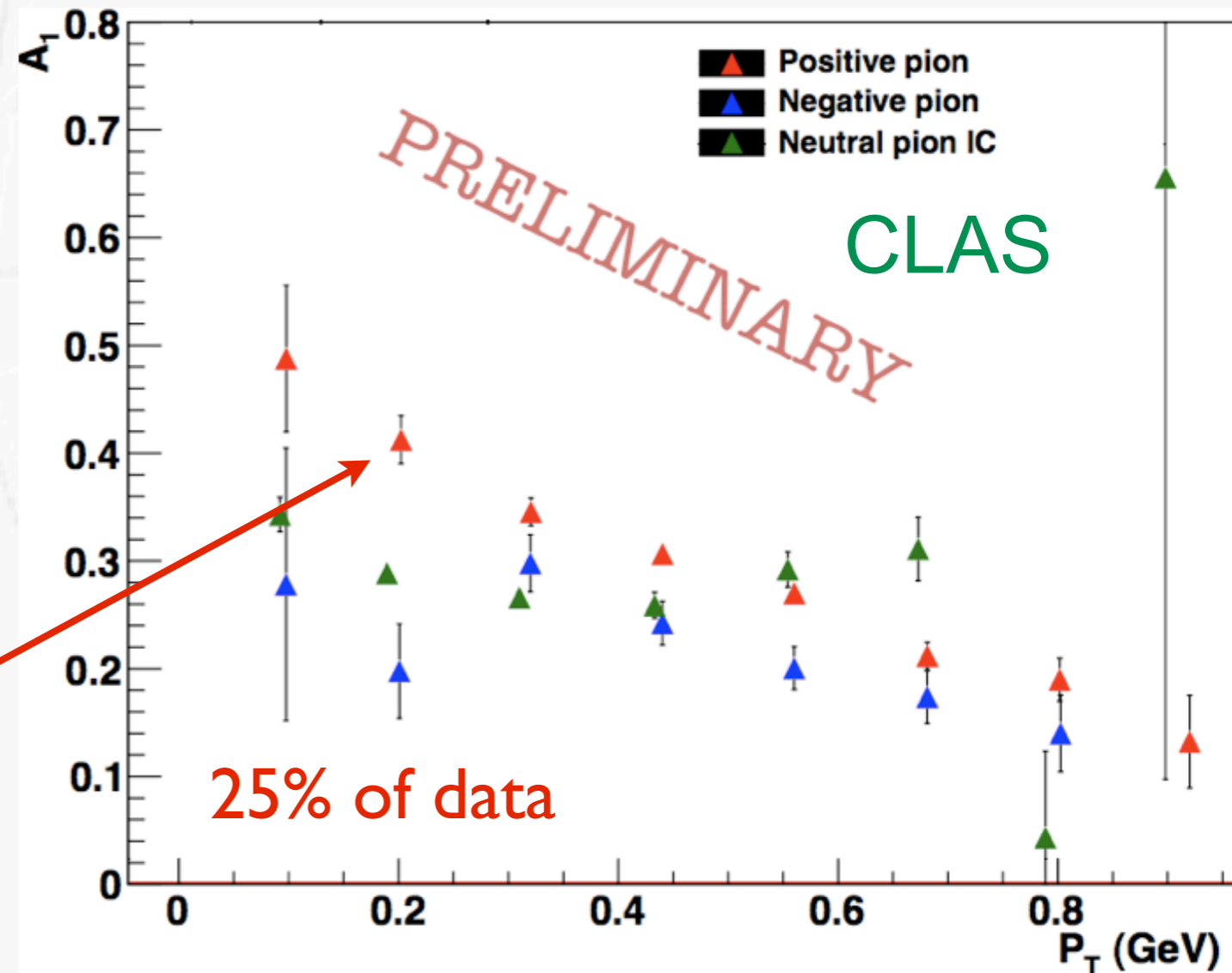
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25% of data

New CLAS data will allow multi-D binning to study $P_{h\perp}$ dependence for fixed x

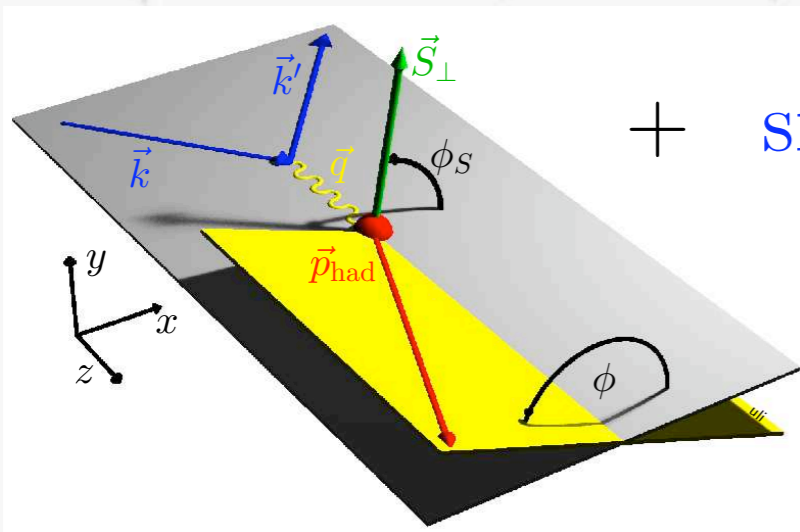
The quest for transversity

Measuring azimuthal spin asymmetries

$$A_{UT}(\phi, \phi_S) = \frac{1}{\langle |S_{\perp}| \rangle} \frac{N_h^{\uparrow}(\phi, \phi_S) - N_h^{\downarrow}(\phi, \phi_S)}{N_h^{\uparrow}(\phi, \phi_S) + N_h^{\downarrow}(\phi, \phi_S)}$$

$$\sim \sin(\phi + \phi_S) \sum_q e_q^2 \mathcal{I} \left[\frac{k_T \hat{P}_{h\perp}}{M_h} h_1^q(x, p_T^2) H_1^{\perp,q}(z, k_T^2) \right]$$

$$+ \sin(\phi - \phi_S) \sum_q e_q^2 \mathcal{I} \left[\frac{p_T \hat{P}_{h\perp}}{M} f_{1T}^{\perp,q}(x, p_T^2) D_1^q(z, k_T^2) \right]$$



+ ... $\mathcal{I}[\dots]$: convolution integral over initial (p_T) and final (k_T) quark transverse momenta

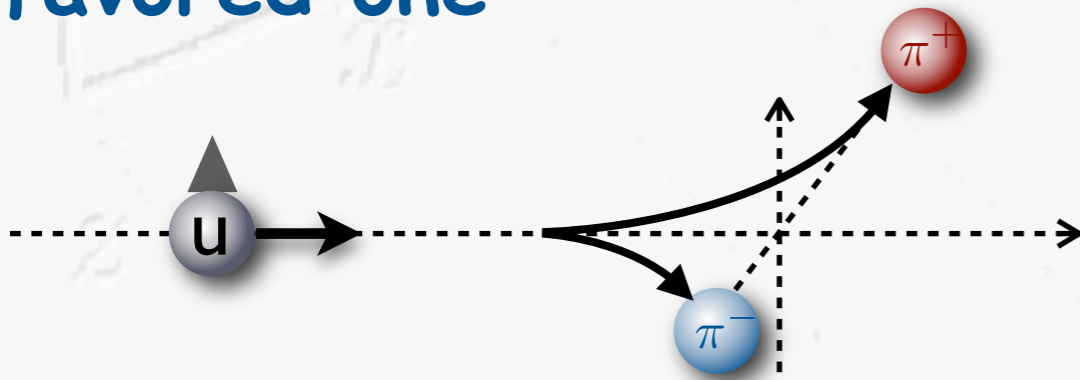
\Rightarrow 2D Max.Likelihood fit of to get Collins and Sivers amplitudes:

$$PDF(2\langle \sin(\phi \pm \phi_S) \rangle_{UT}, \dots, \phi, \phi_S) = \frac{1}{2} \{ 1 + P_T (2\langle \sin(\phi \pm \phi_S) \rangle_{UT} \sin(\phi \pm \phi_S) + \dots) \}$$

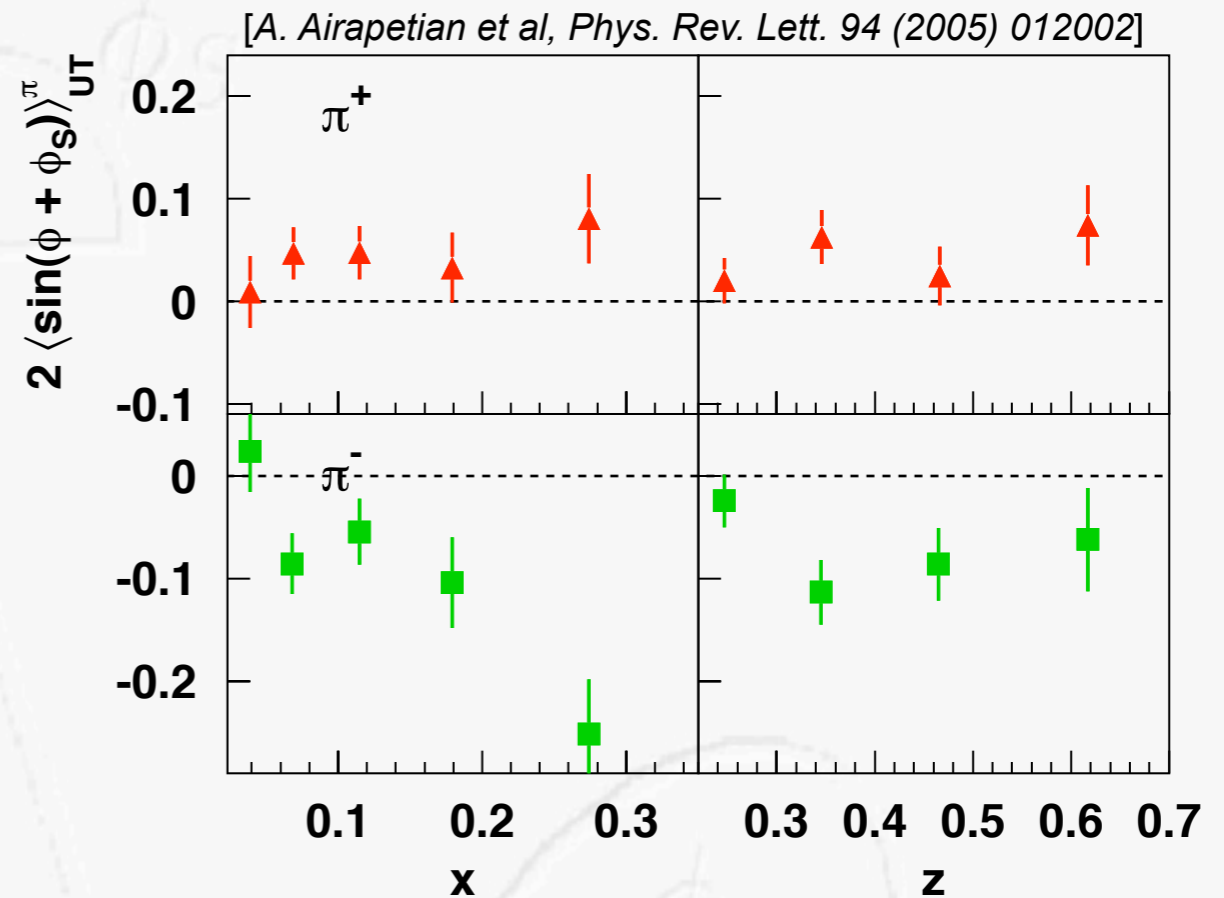
Transversity distribution (Collins fragmentation)

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

- significant in size and opposite in sign for charged pions
- disfavored Collins FF large and opposite in sign to favored one



- leads to various cancellations in SSA observables

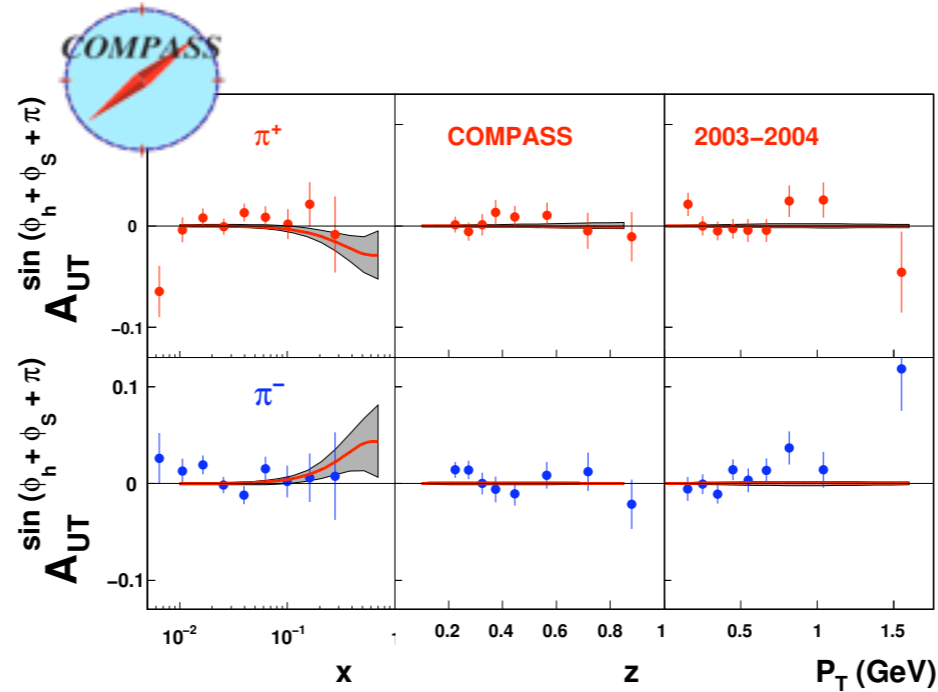
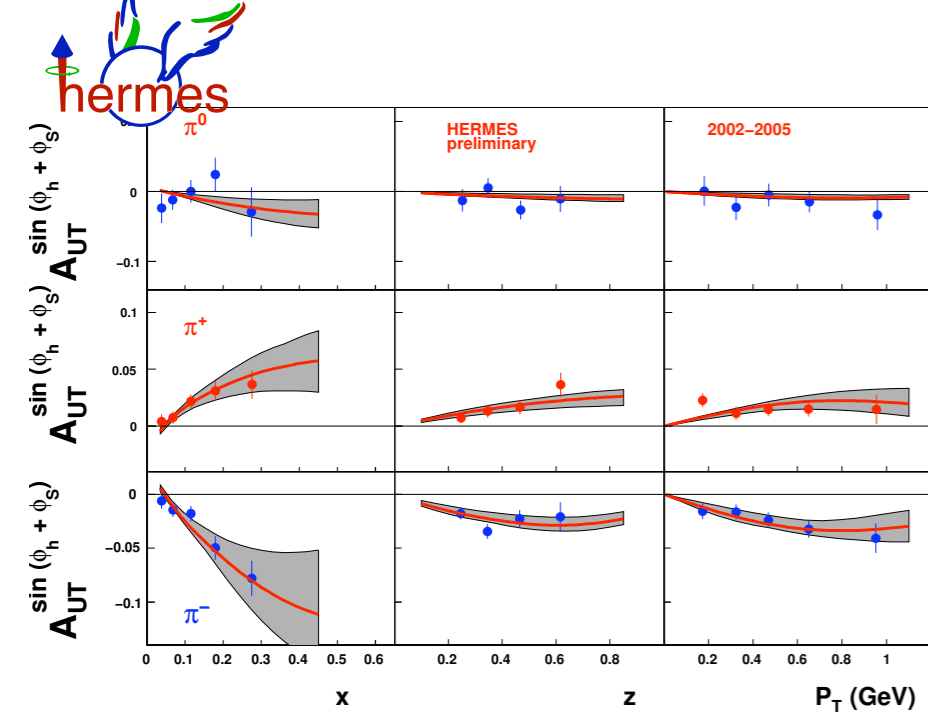


2005: First evidence from HERMES
SIDIS on proton

Non-zero transversity
Non-zero Collins function

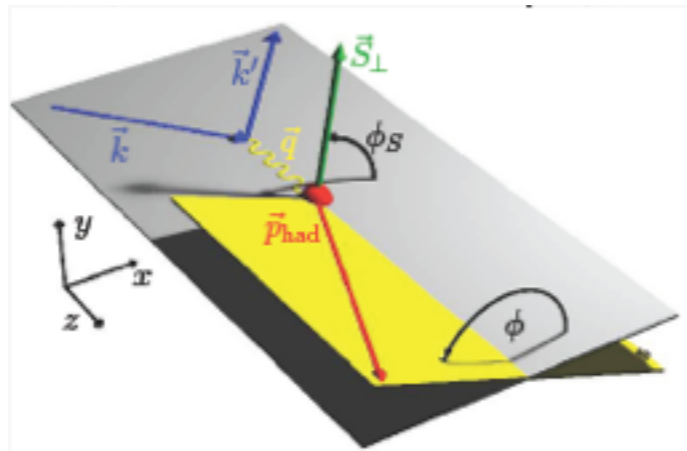
Fit of Collins amplitudes

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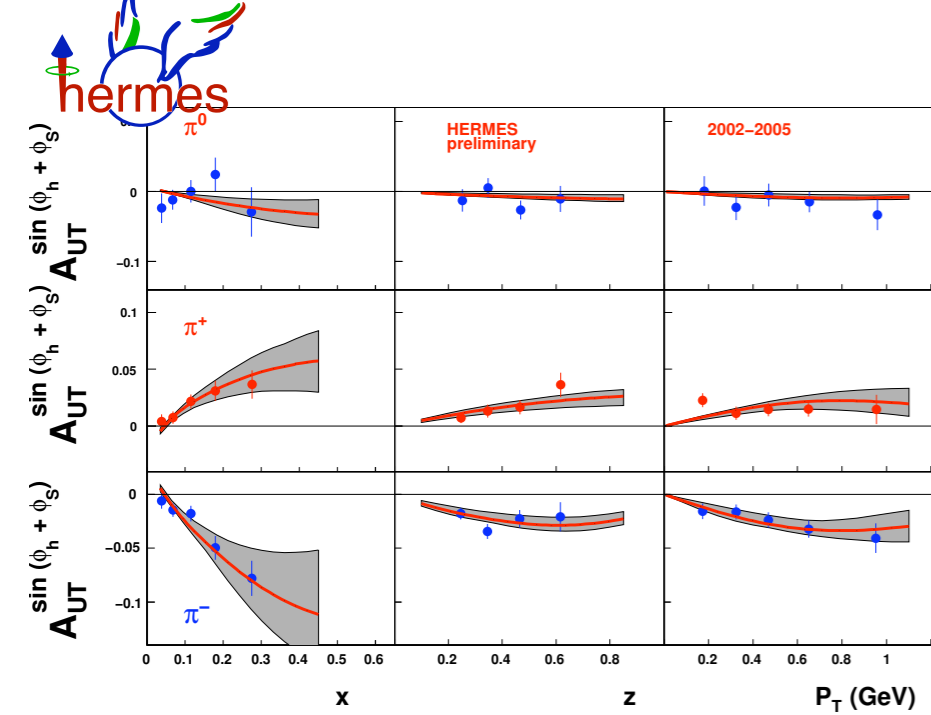
$$e^\pm p^\uparrow \rightarrow e^\pm \pi X$$

$$\mu^\pm d^\uparrow \rightarrow \mu^\pm \pi X$$

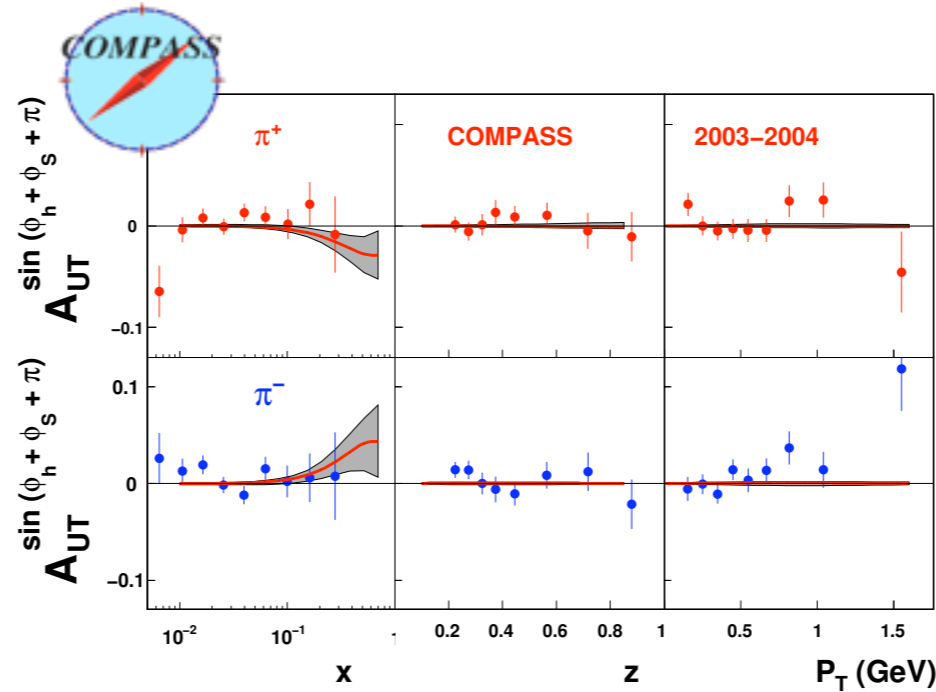


Fit of Collins amplitudes

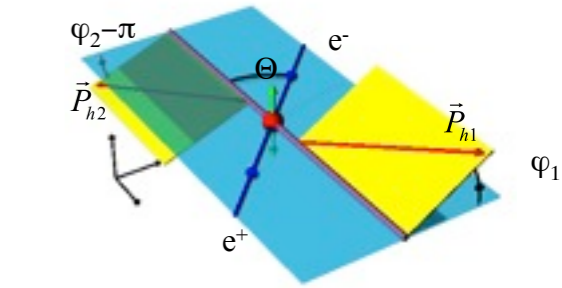
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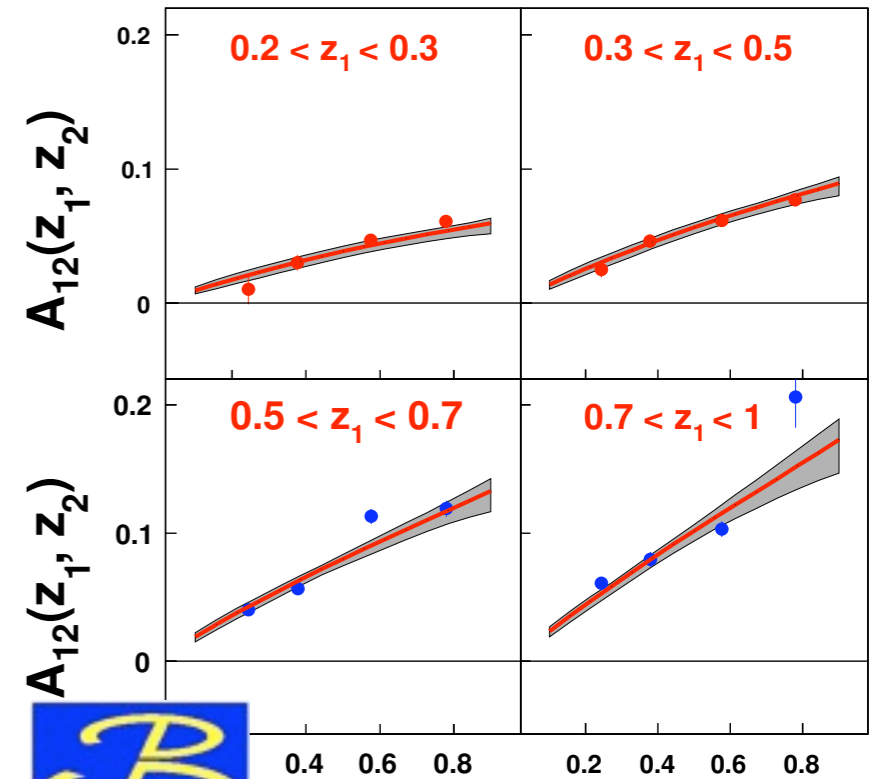
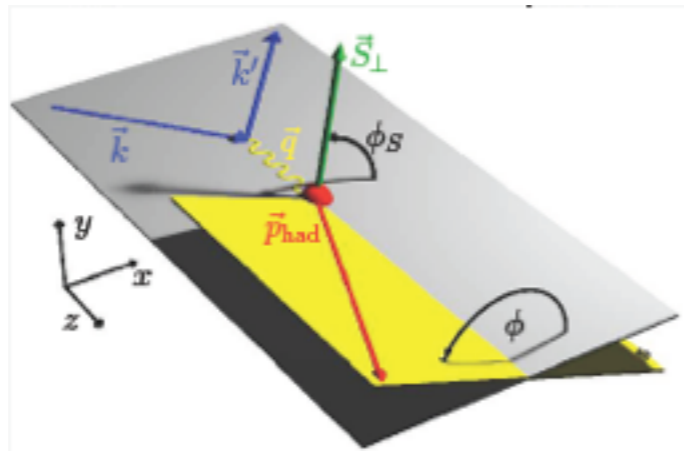
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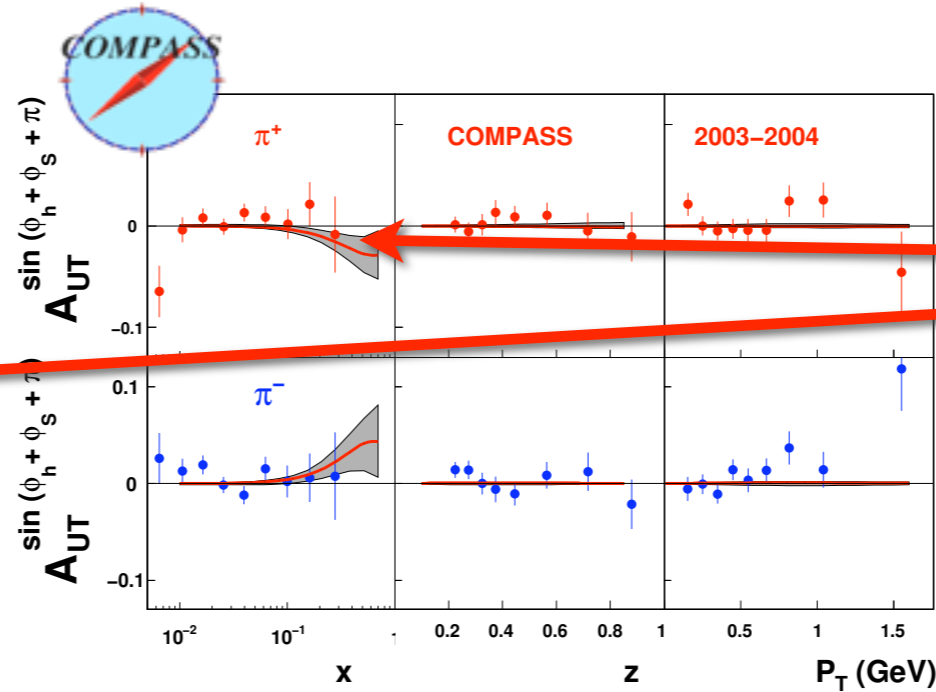
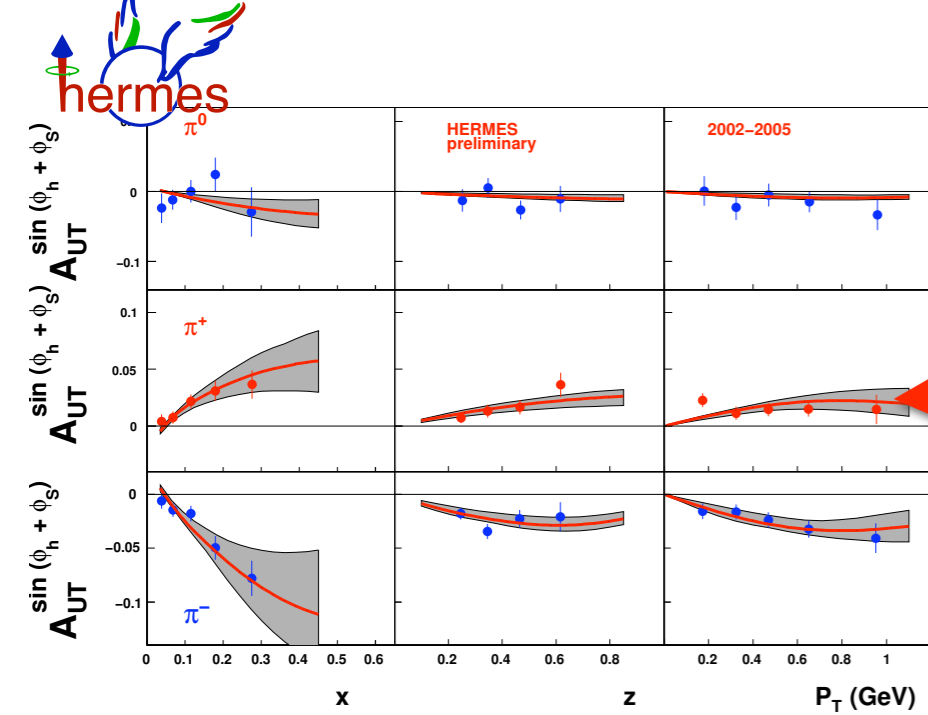


$$e^+ e^- \rightarrow \pi \pi X$$

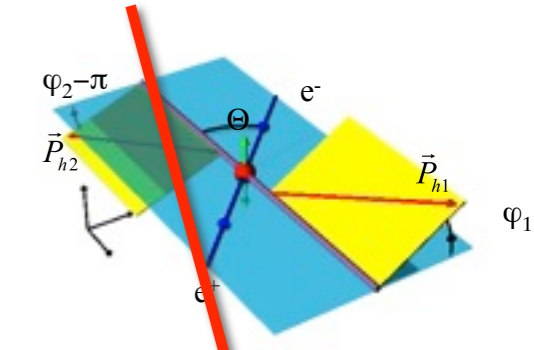


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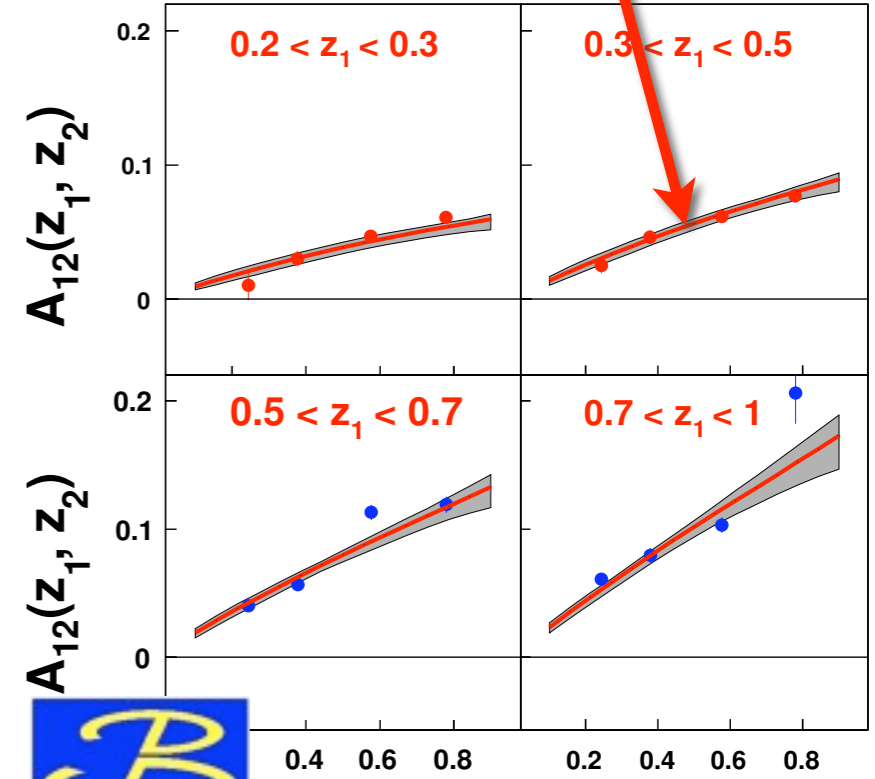
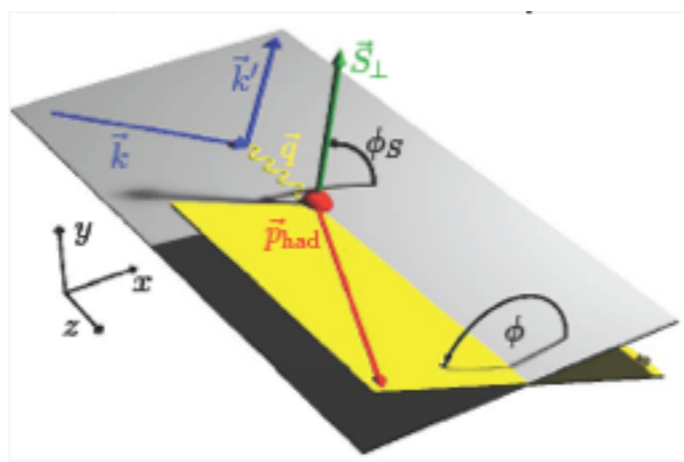
fit to data



$$e^\pm p^\uparrow \rightarrow e^\pm \pi X$$

$$\mu^\pm d^\uparrow \rightarrow \mu^\pm \pi X$$

$$e^+e^- \rightarrow \pi\pi X$$



Transversity: models and fits

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[0] Barone et al. PLB 390 (97)

[1] Soffer et al. PRD 65 (02)

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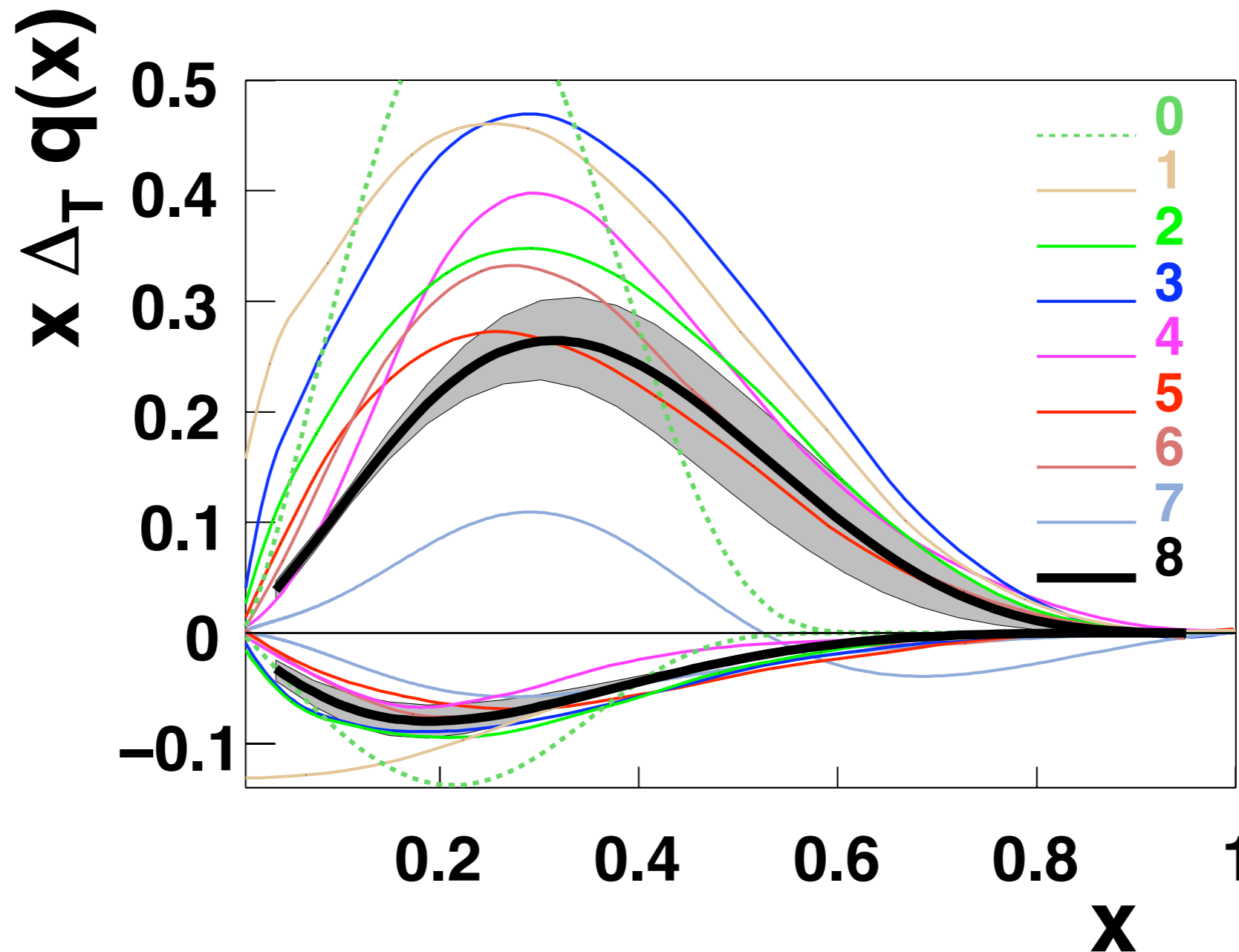
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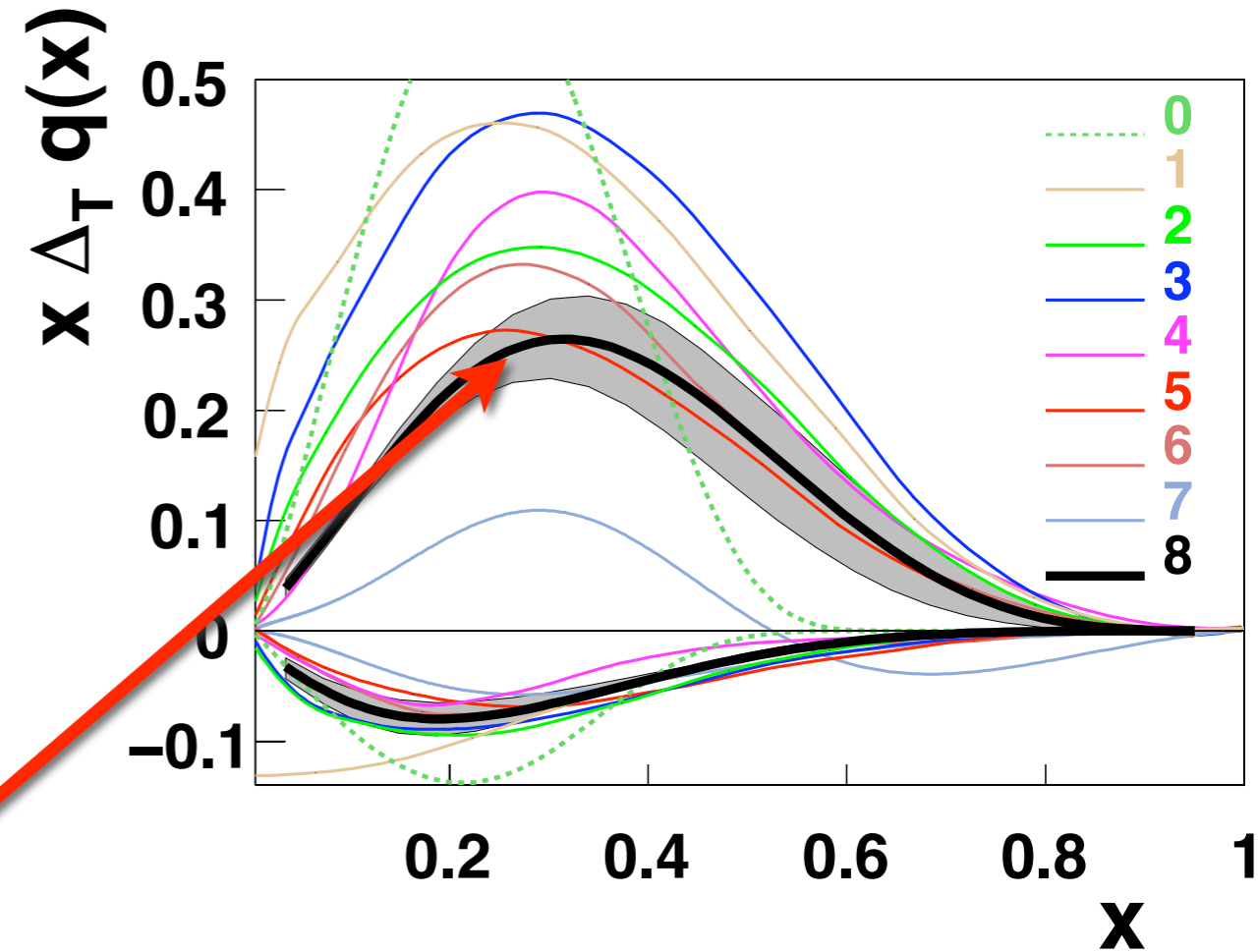
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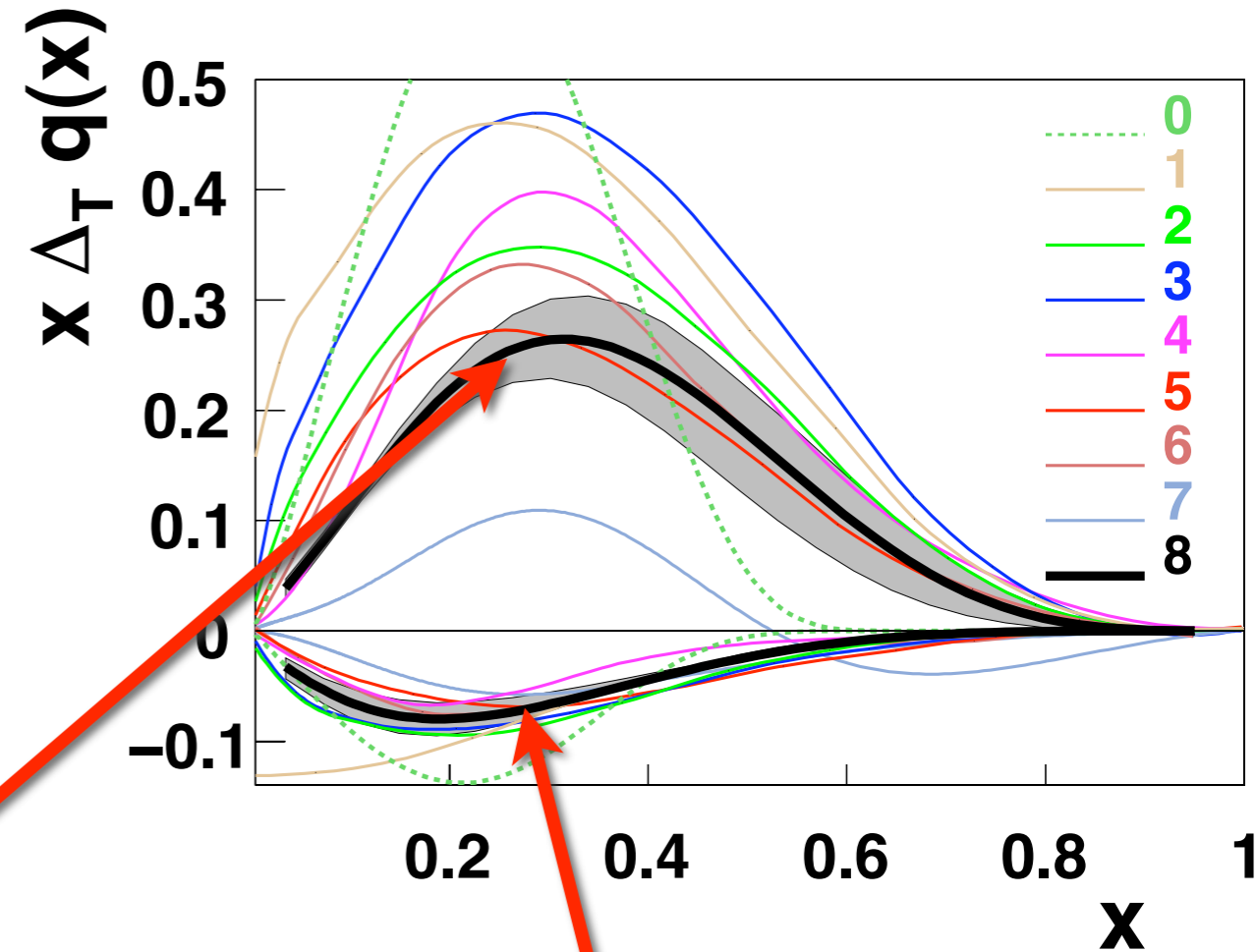
☑ u quark transversity along nucleon spin

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u quark transversity along nucleon spin

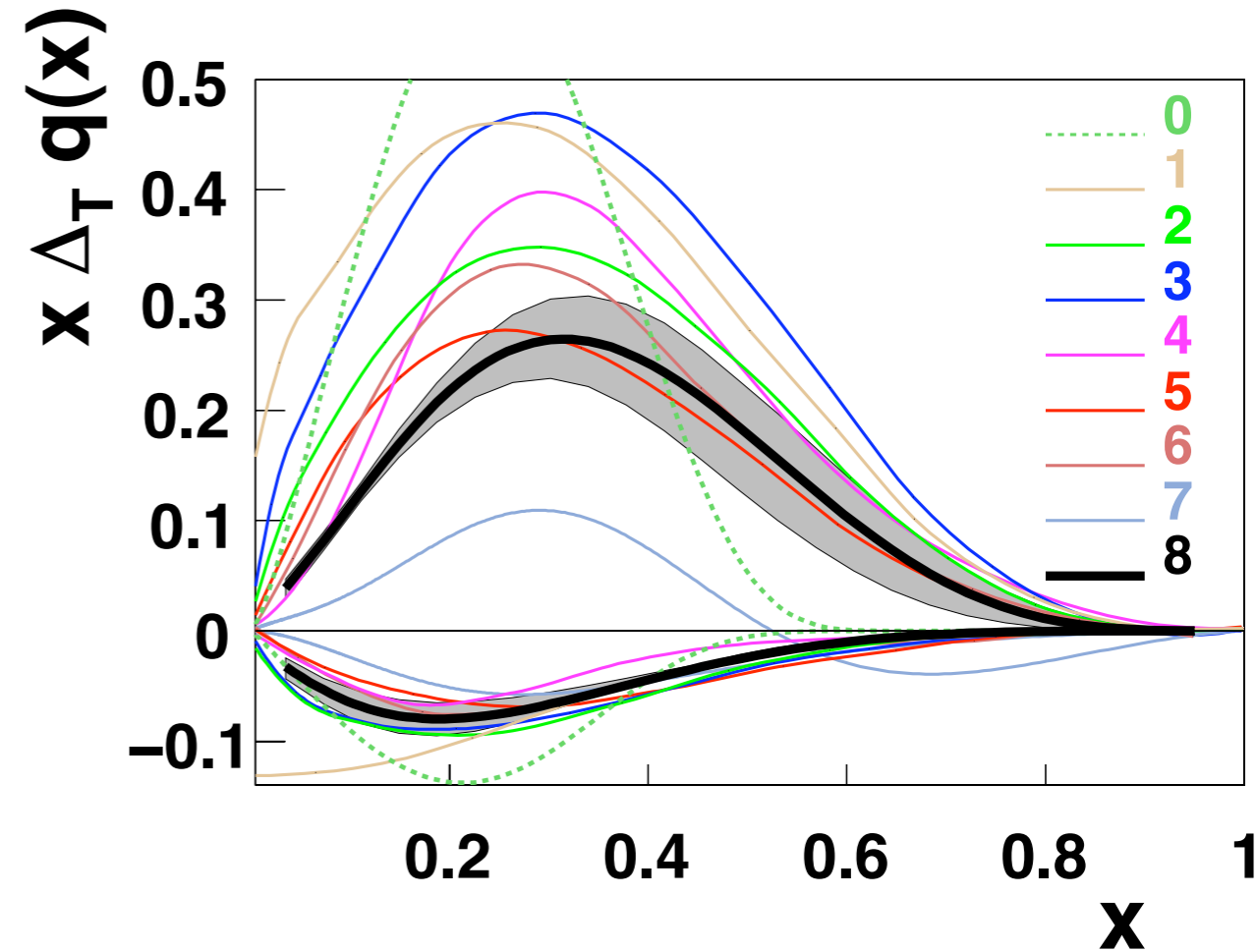
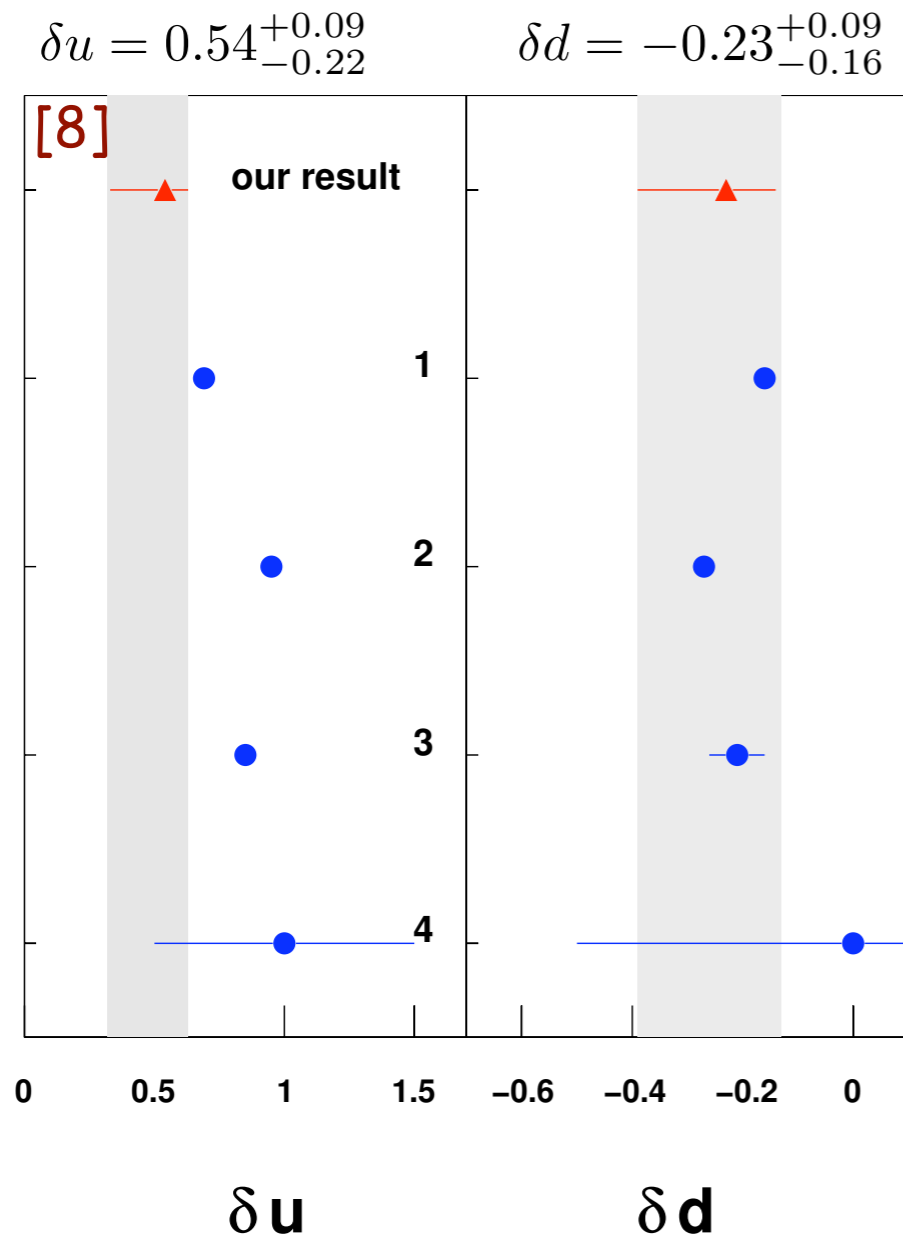
d quark transversity anti-parallel to nucleon spin

Transversity: models and fits

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tensor charge:

$$\delta q \equiv \int_0^1 dx [h_1^q(x) - h_1^{\bar{q}}(x)]$$

Collins amplitudes

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
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- **wealth of new results available and/or analyses ongoing**

- **JLab**

- **COMPASS**

- **HERMES**

- **BELLE**

- **BaBar**

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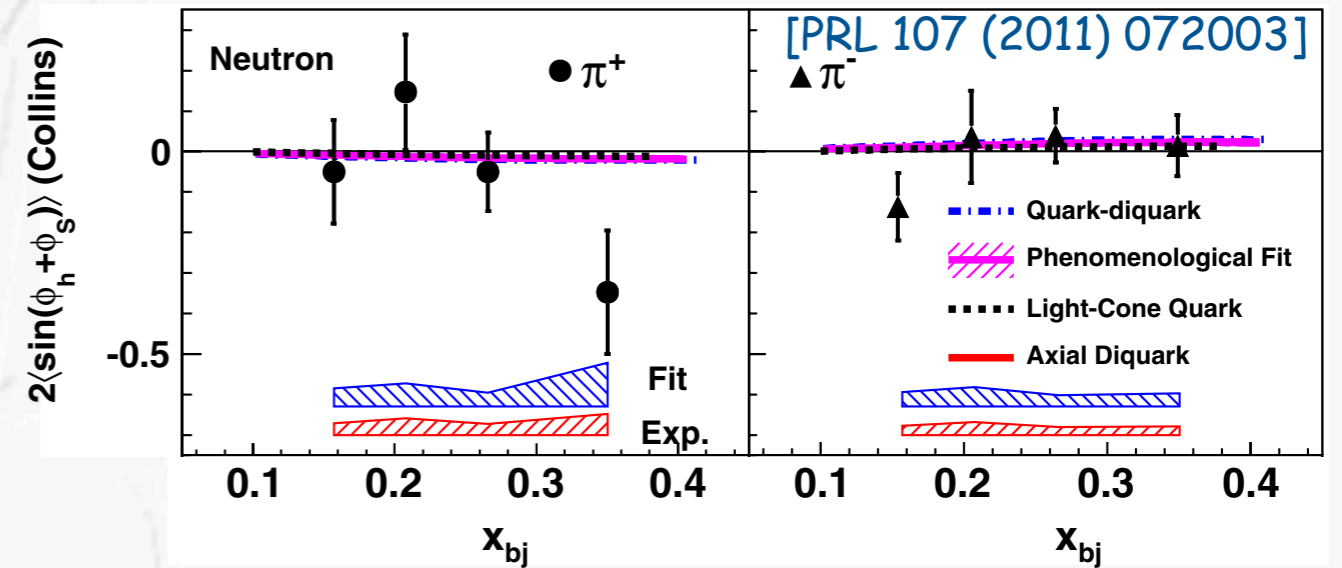
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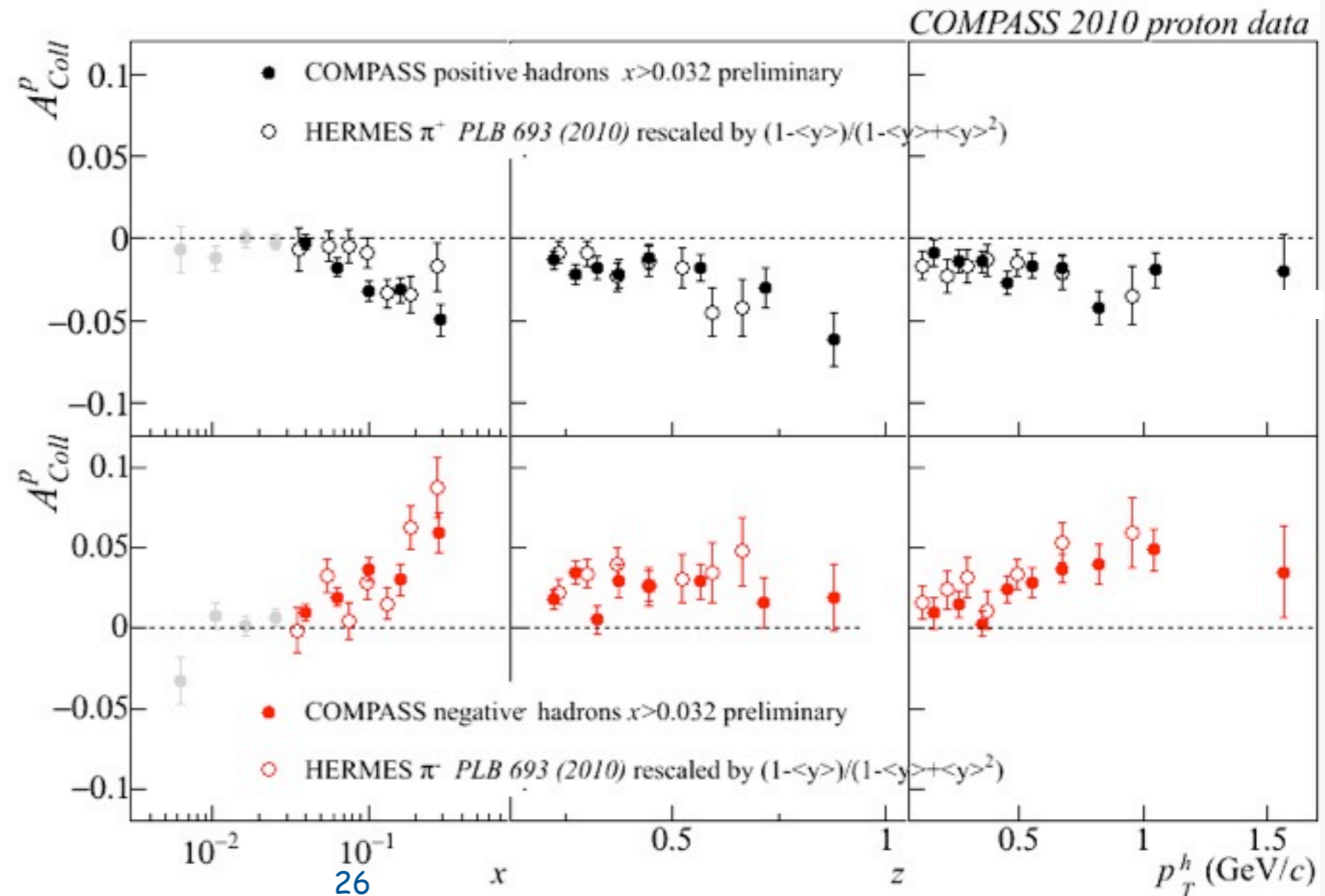
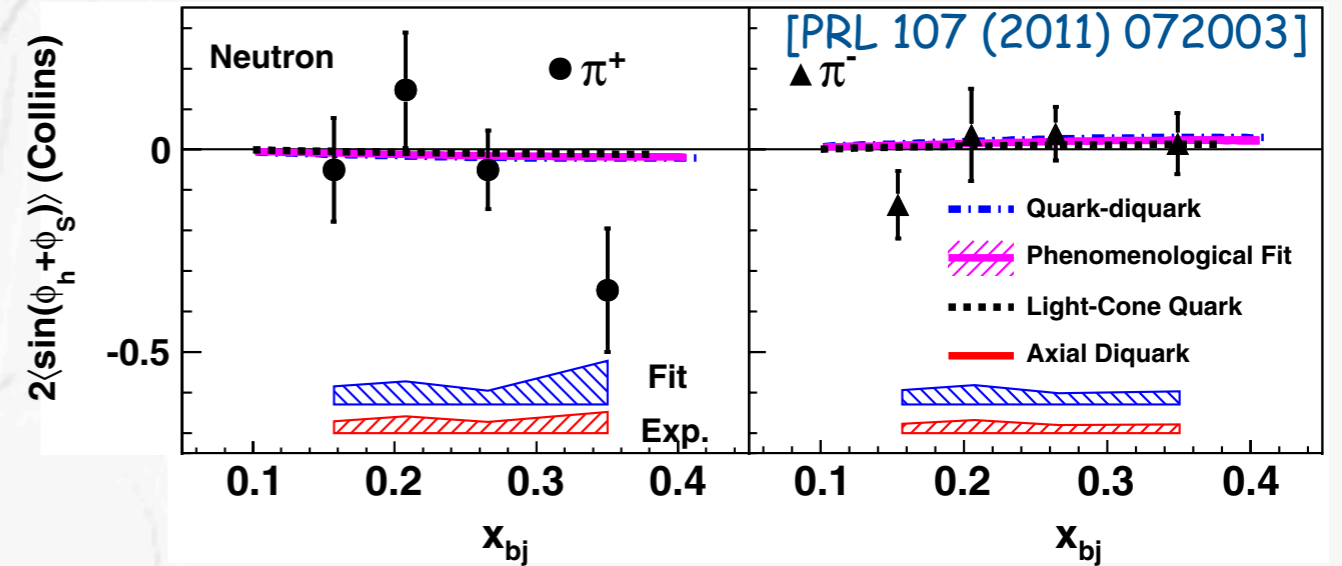
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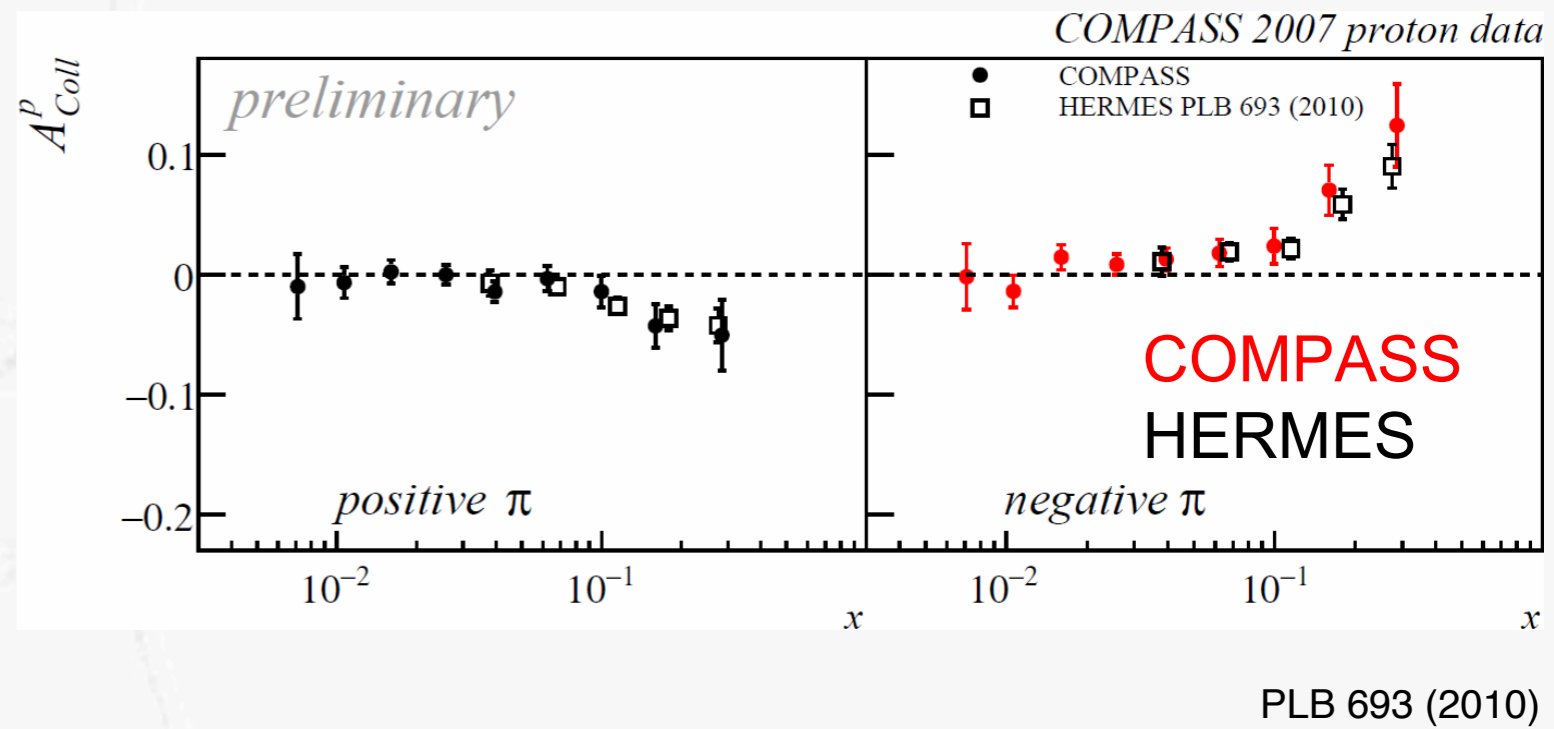
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Collins amplitudes

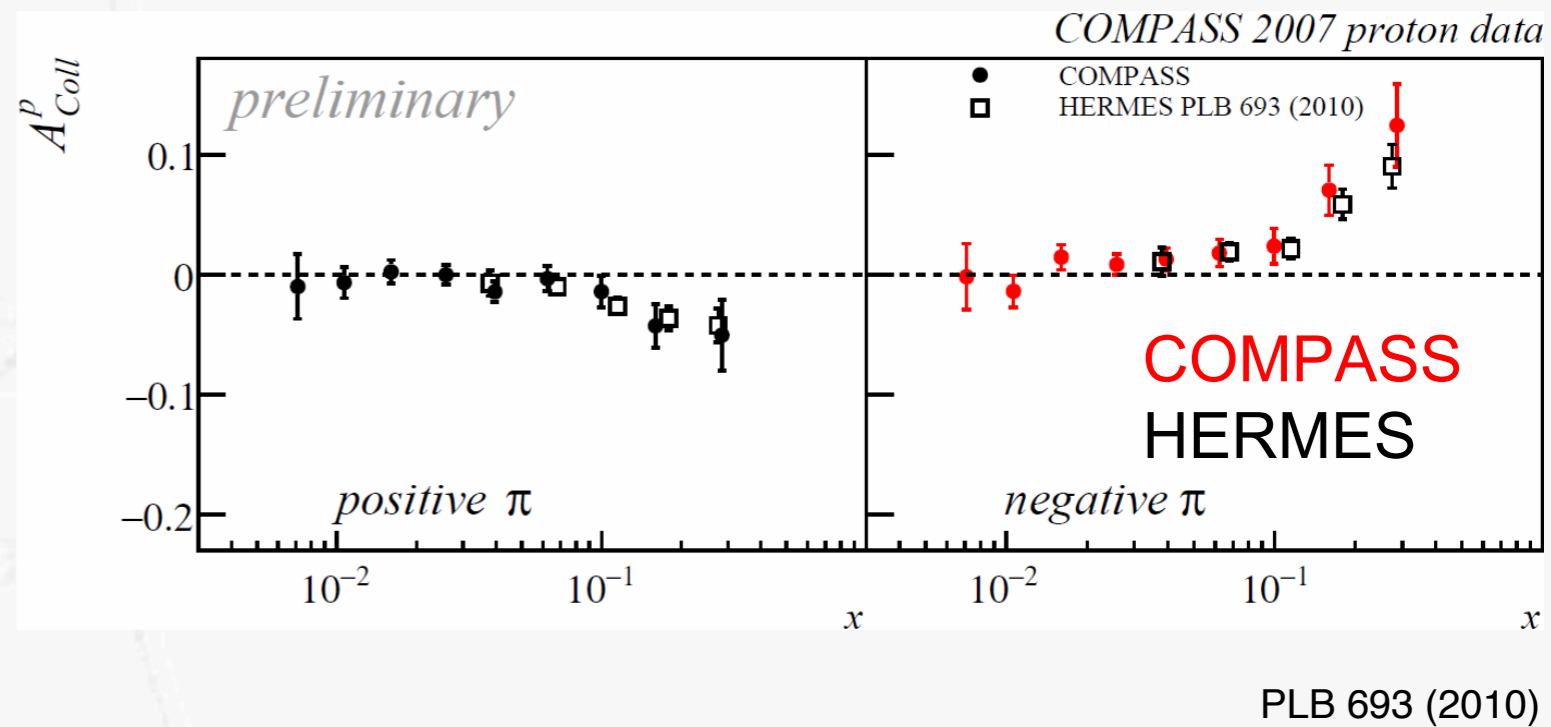
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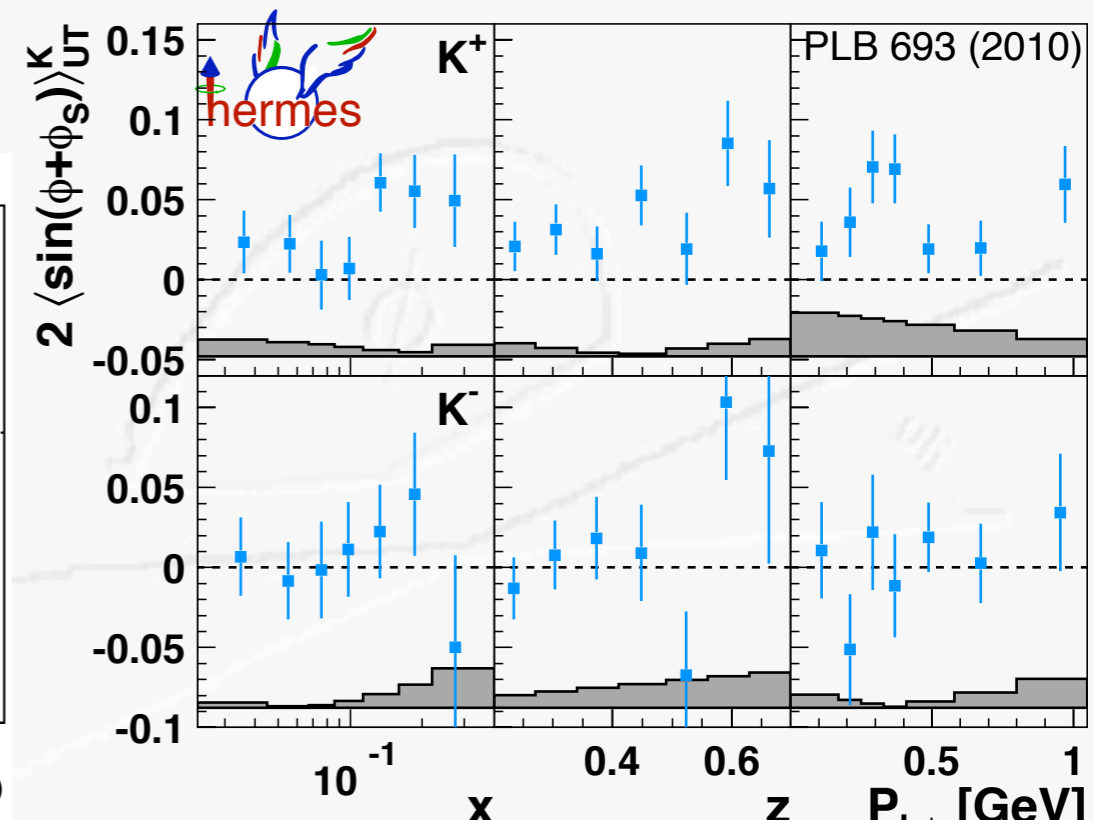
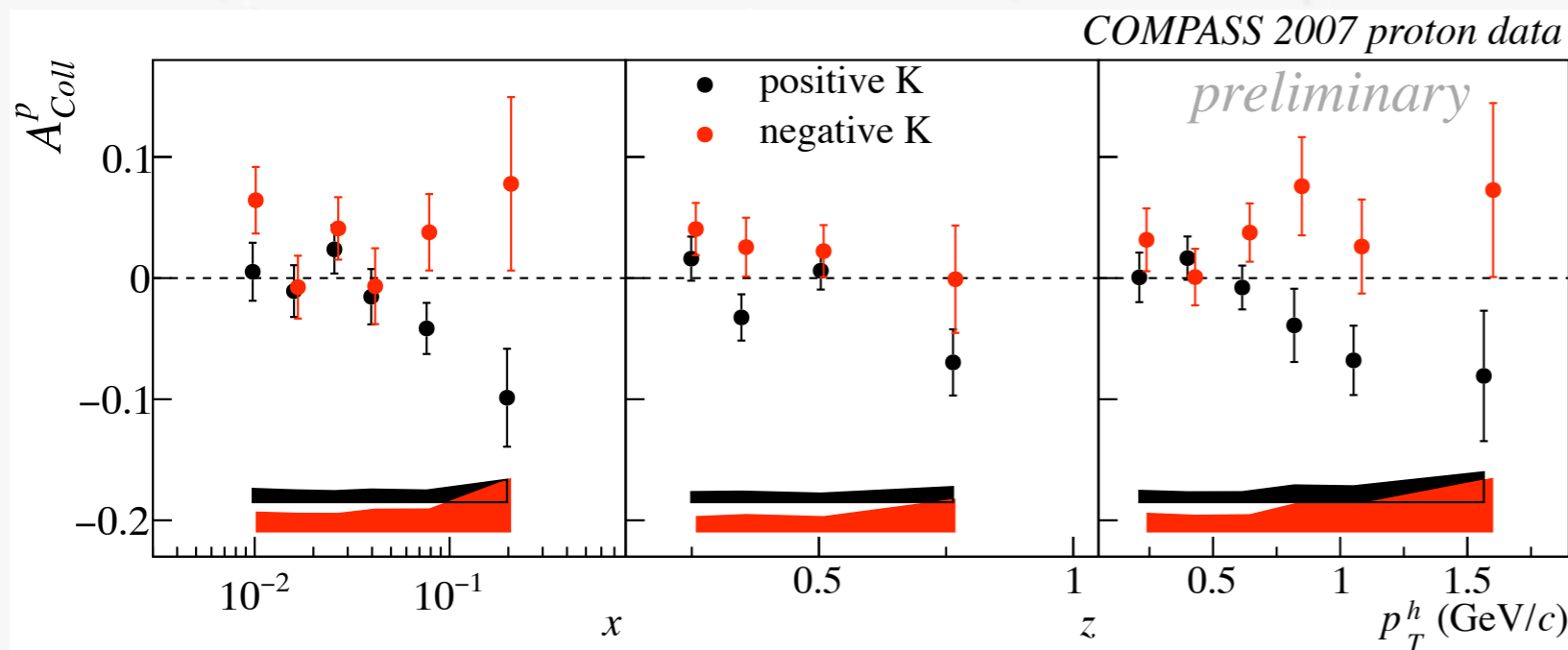
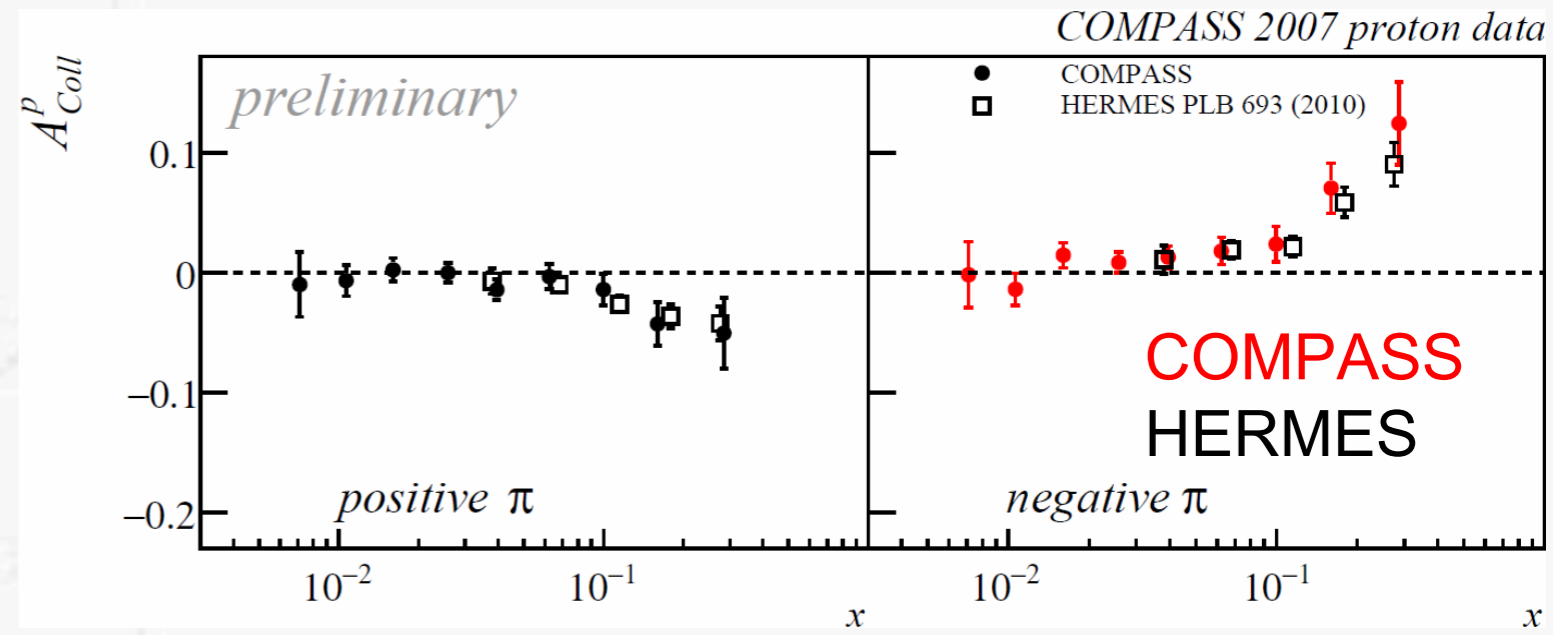
- similar behavior for pions



Collins amplitudes

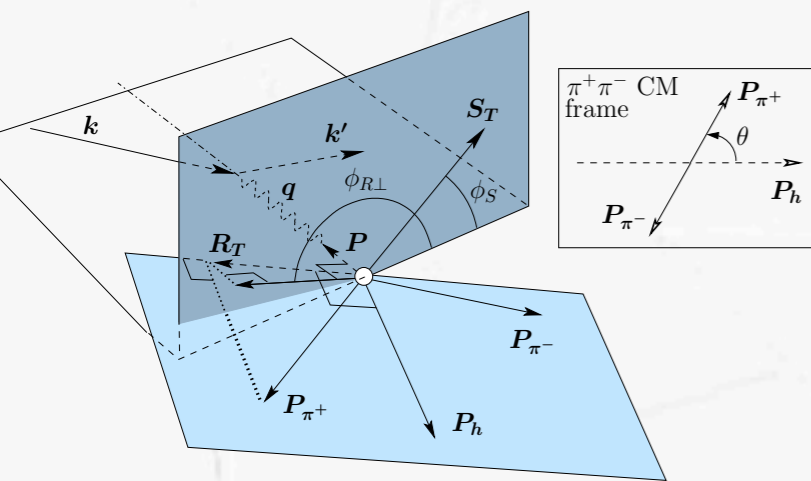
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- similar behavior for pions
- similar behavior for K^+
- different trend for K^-
- (opposite sign conventions!)

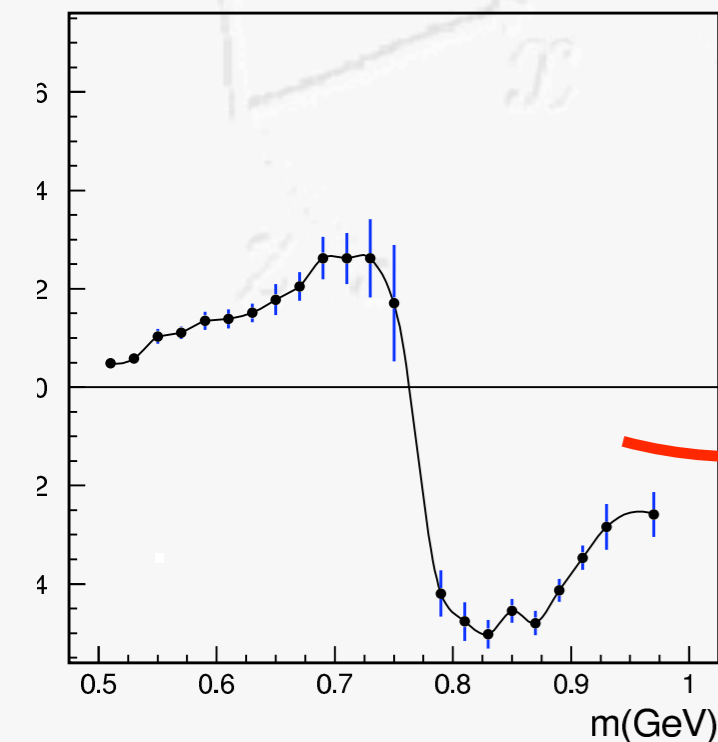


Transversity distribution (2-hadron fragmentation)

	U	L	T
U	f_1		h_1^\perp
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T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



$$A_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sin\theta h_1 H_1^\Delta$$



Jaffe et al. [hep-ph/9709322]:

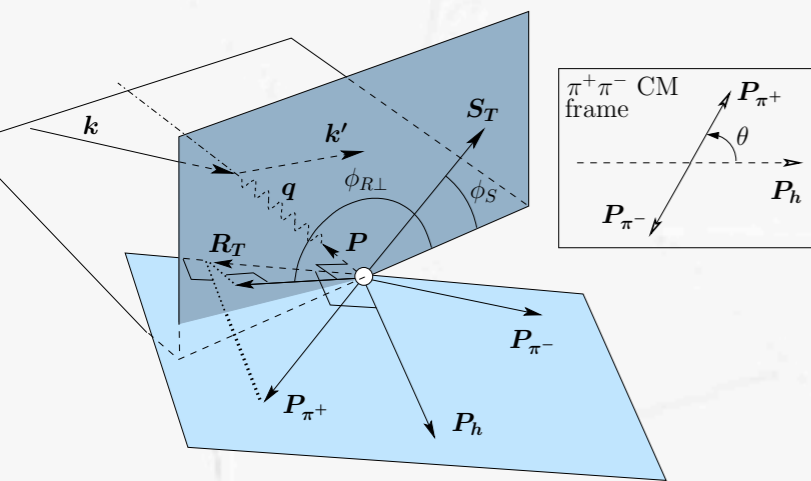
$$H_1^{\Delta, sp}(z, M_{\pi\pi}^2) = \frac{\sin\delta_0 \sin\delta_1 \sin(\delta_0 - \delta_1) H_1^{\Delta, sp'}(z)}{\delta_0 (\delta_1) \rightarrow \text{S(P)-wave phase shifts}}$$

$$= \mathcal{P}(M_{\pi\pi}^2) H_1^{\Delta, sp'}(z)$$

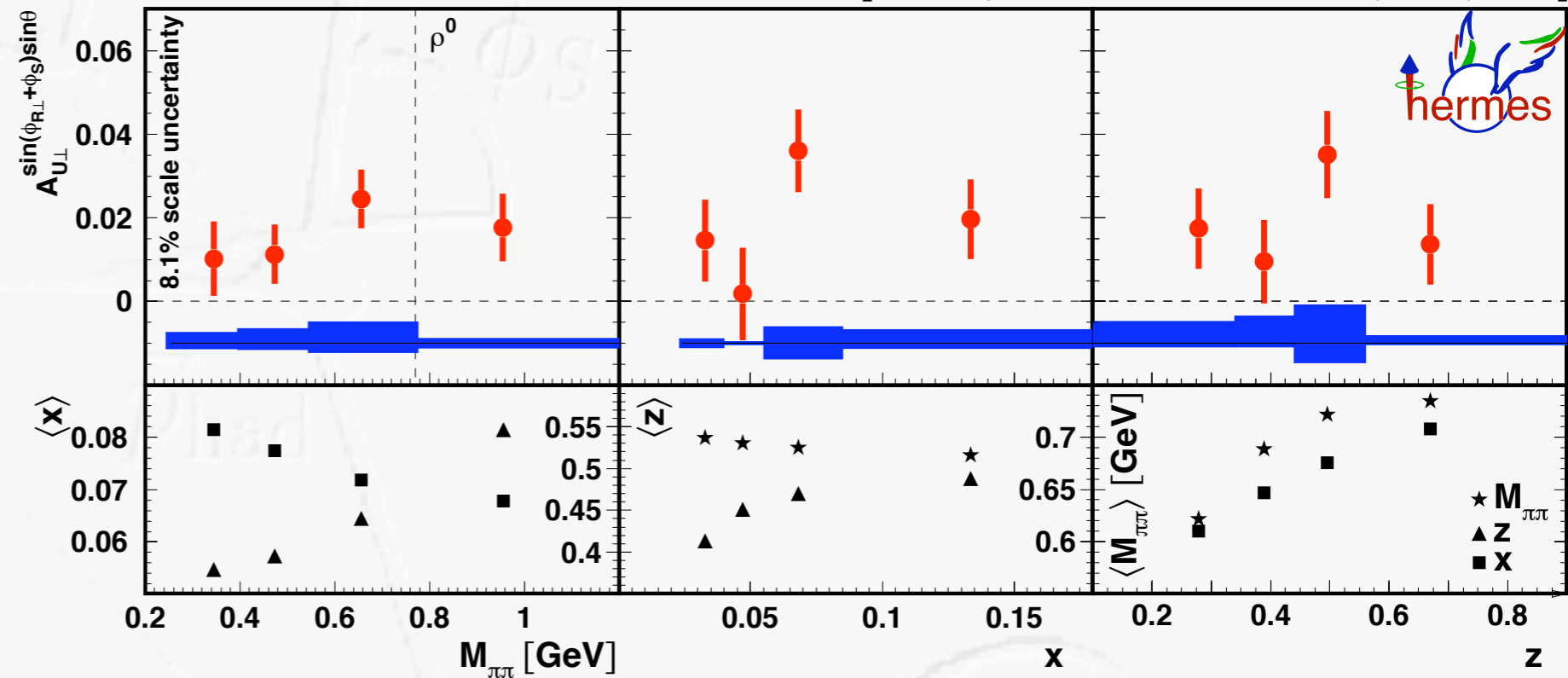
$\Rightarrow A_{UT}$ might depend strongly on $M_{\pi\pi}$

Transversity distribution (2-hadron fragmentation)

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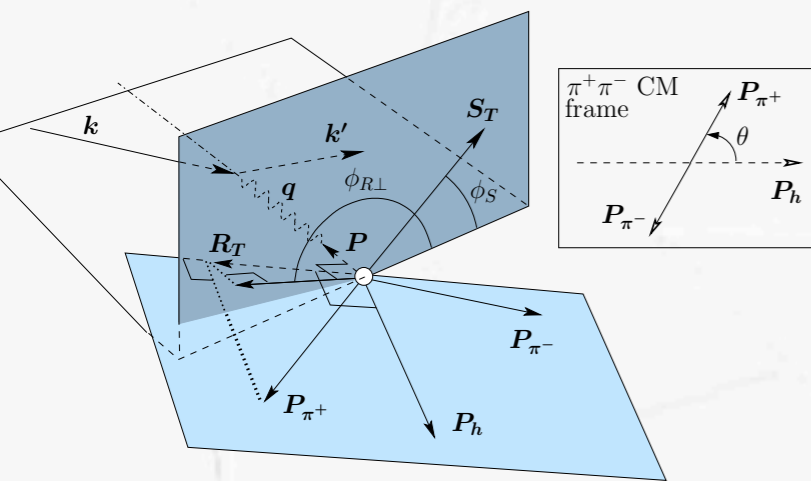
[A. Airapetian et al., JHEP 06 (2008) 017]



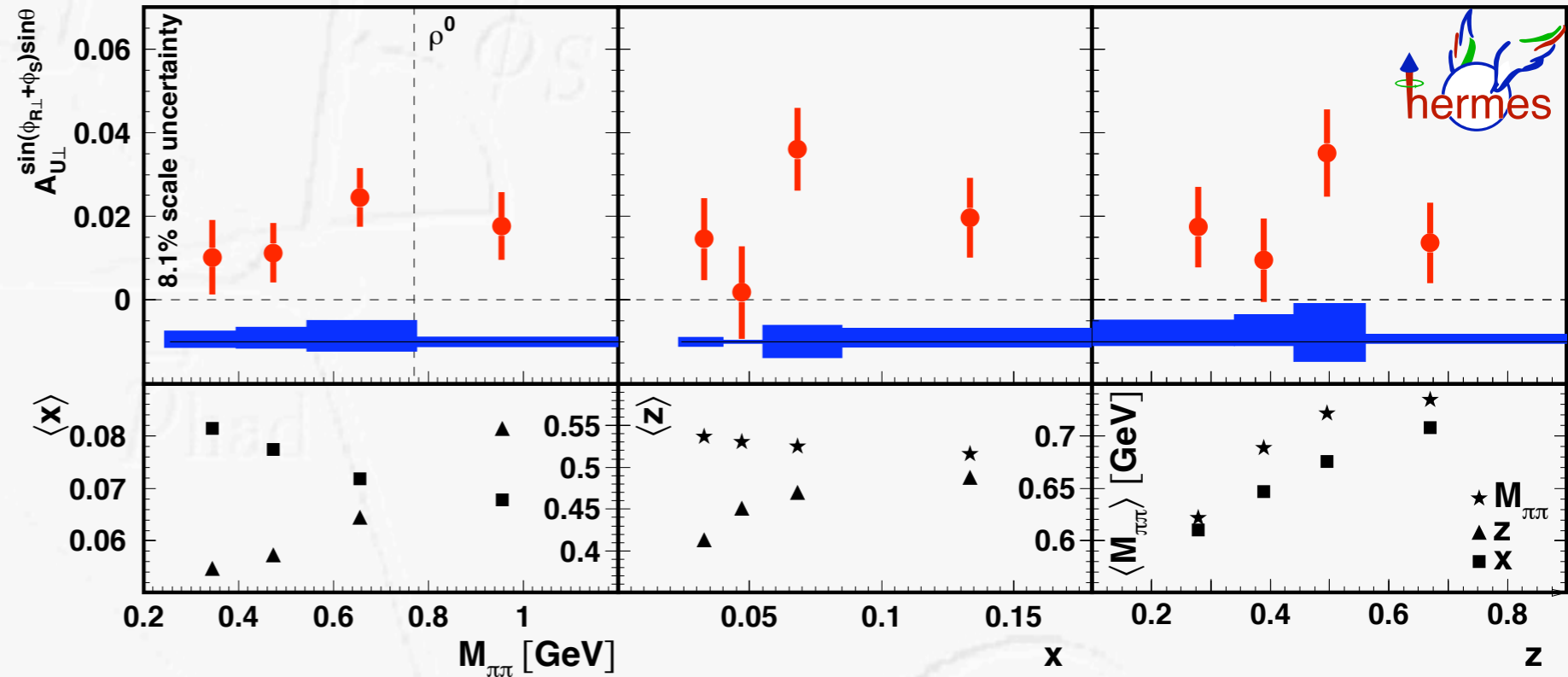
✓ first evidence for T-odd 2-hadron fragmentation function in semi-inclusive DIS!

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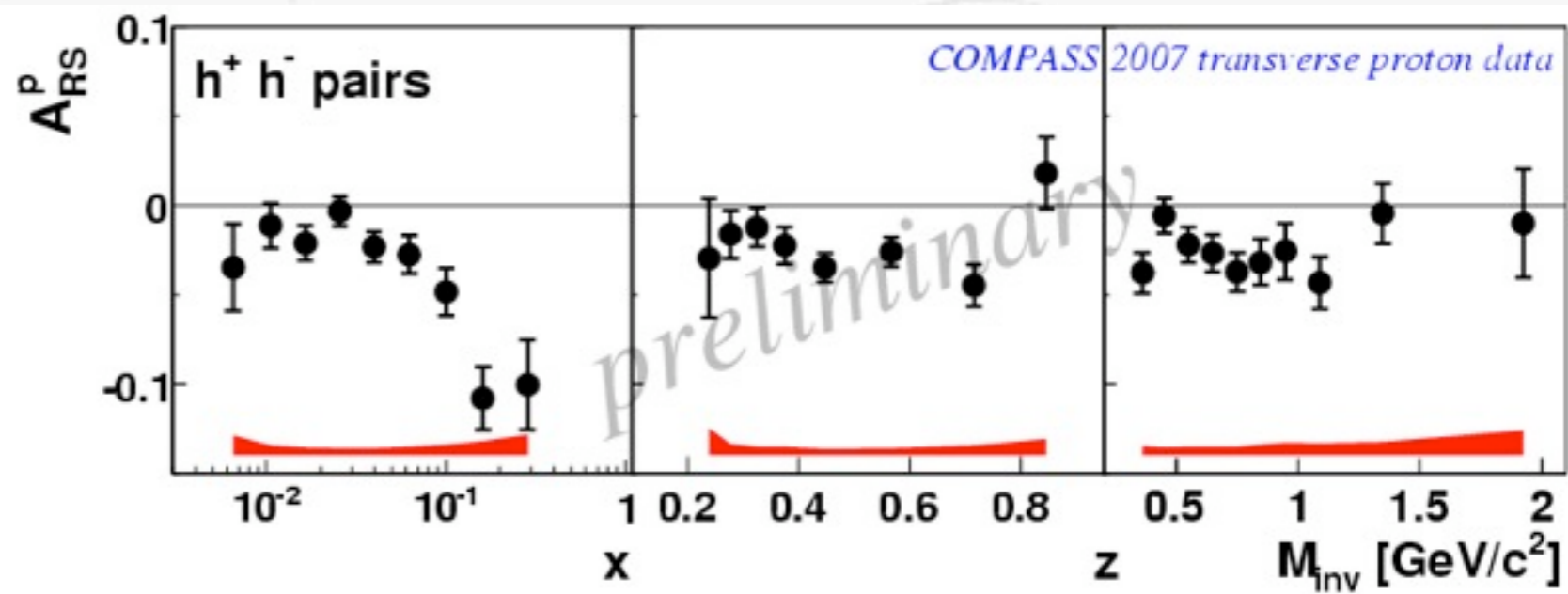
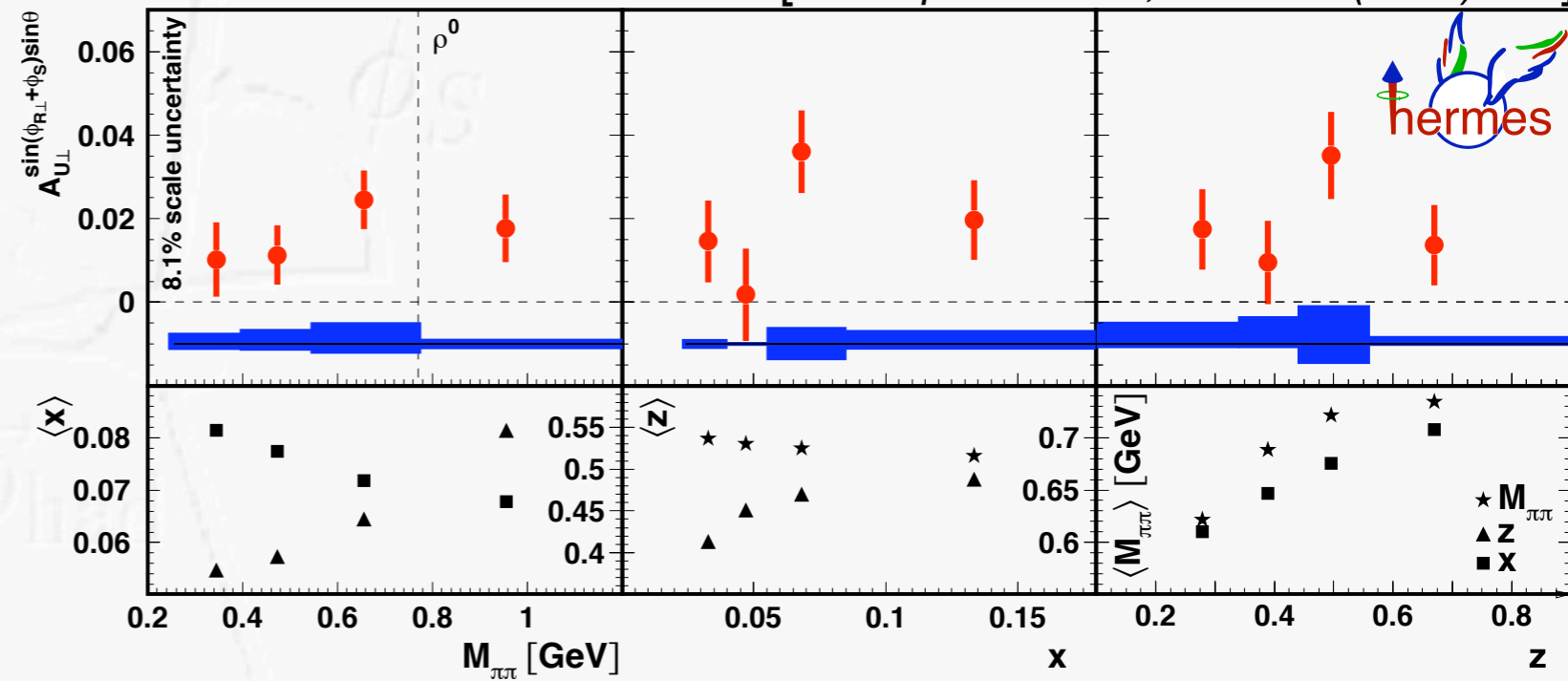


- ✓ first evidence for T-odd 2-hadron fragmentation function in semi-inclusive DIS!
- ✓ invariant-mass dependence rules out Jaffe model predicting a sign change to rho mass

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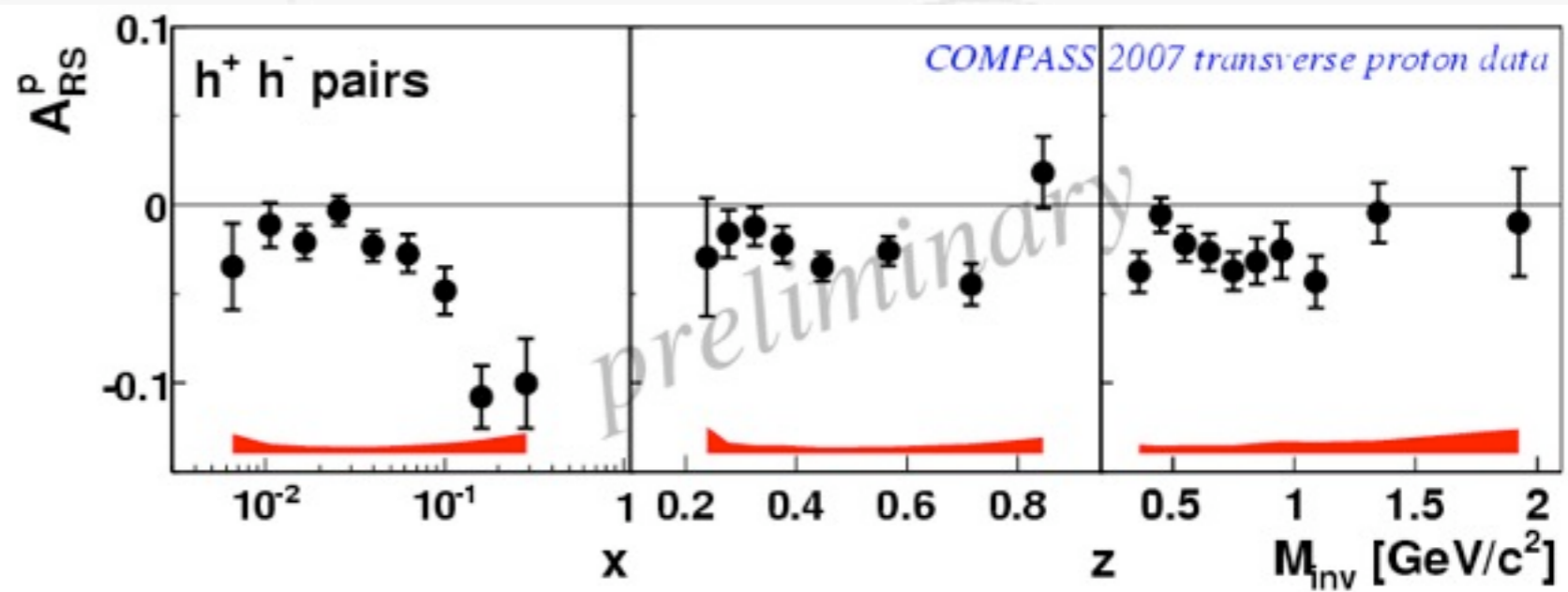
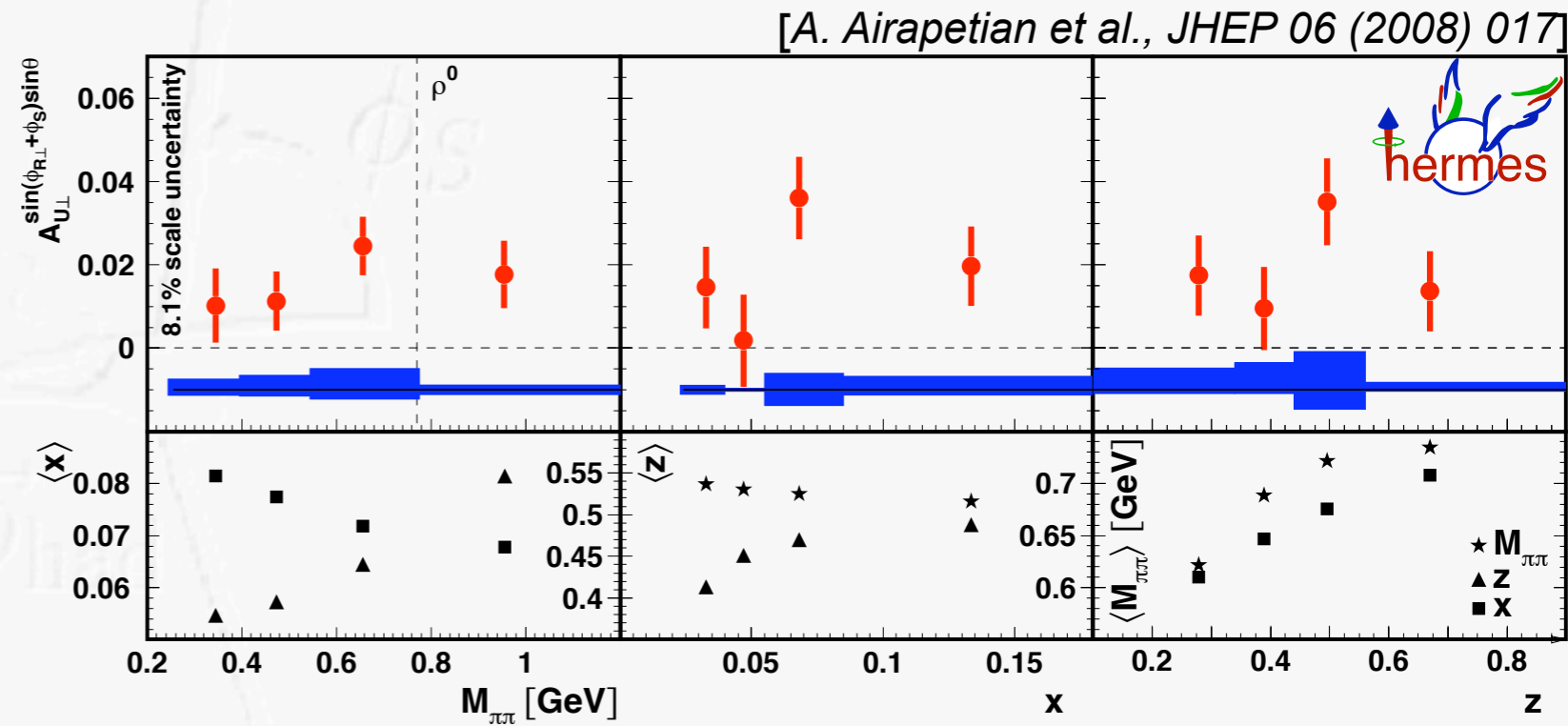
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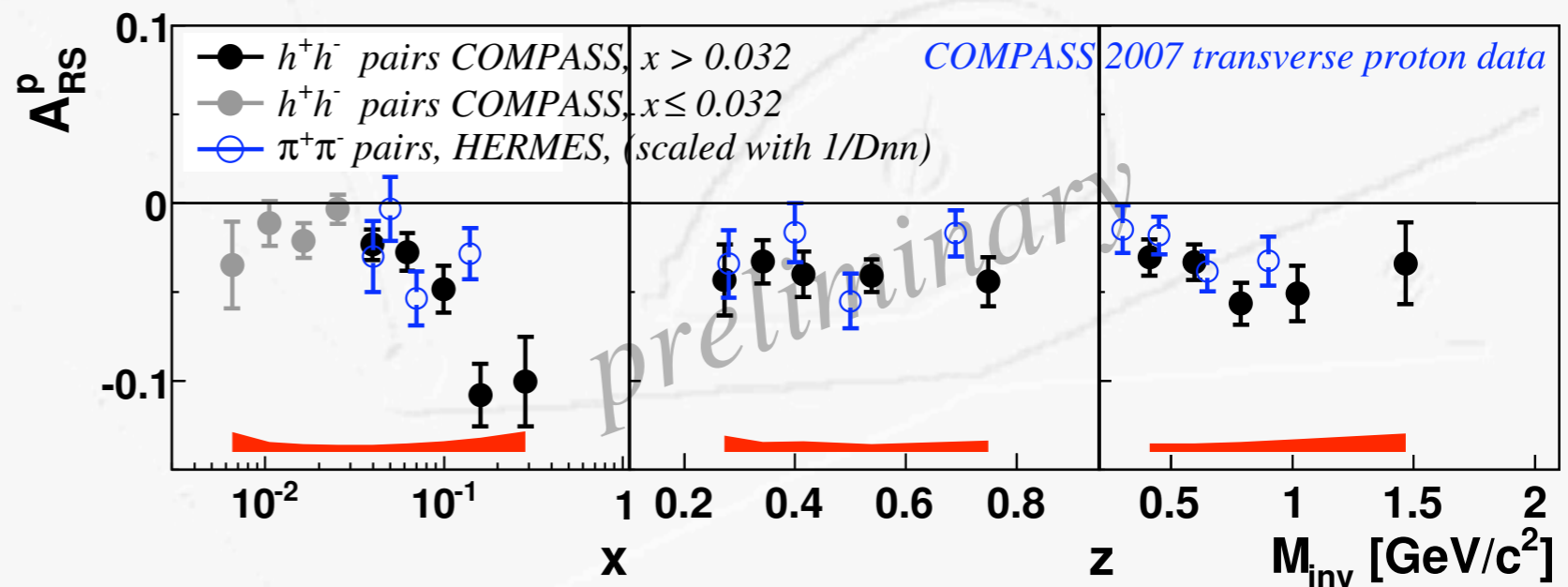
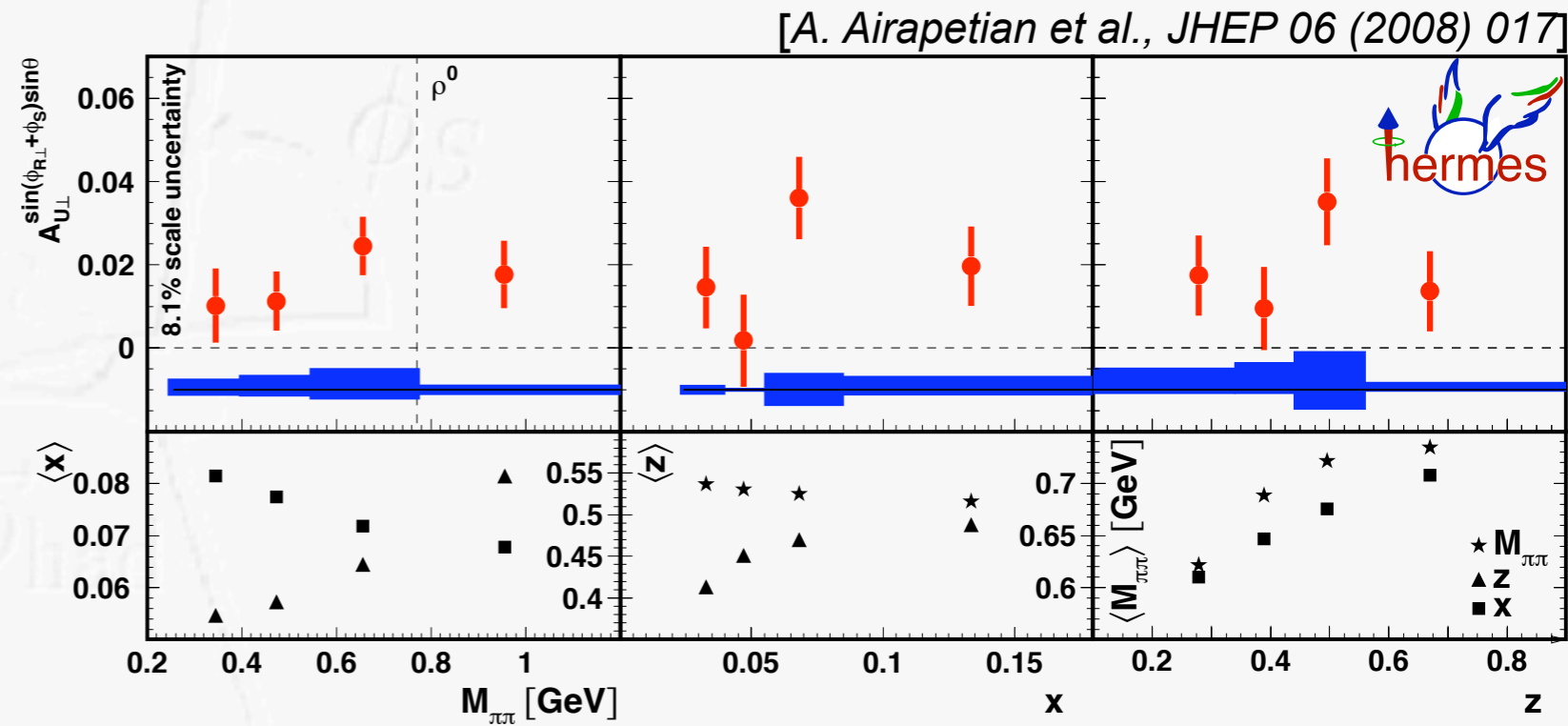
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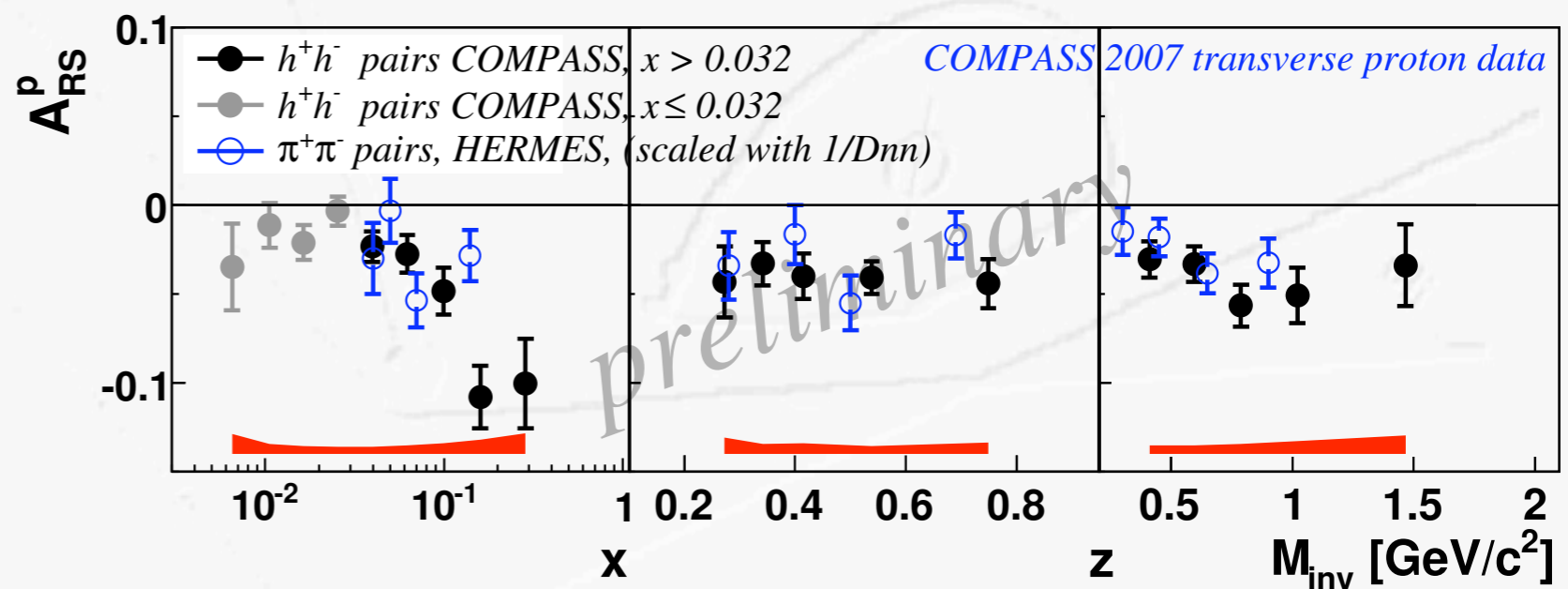
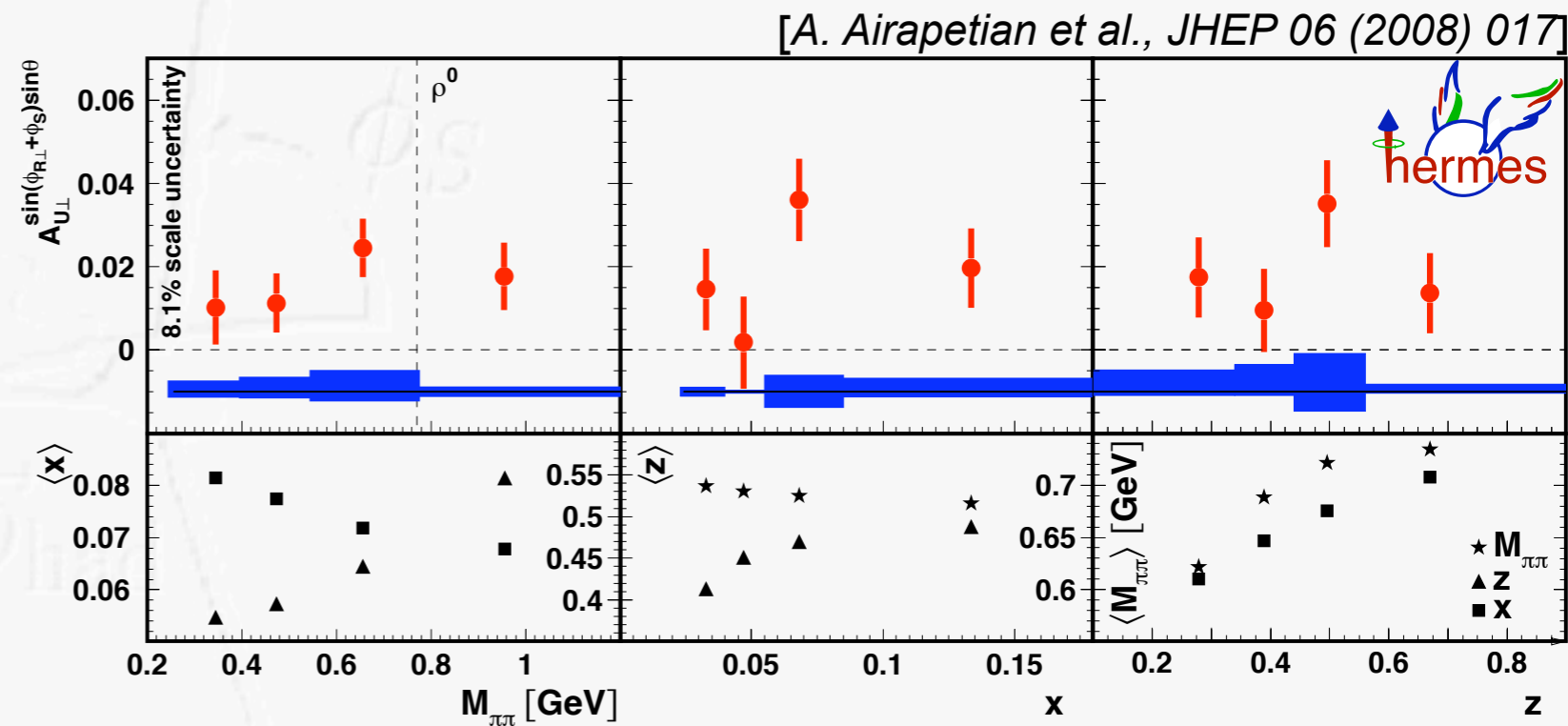
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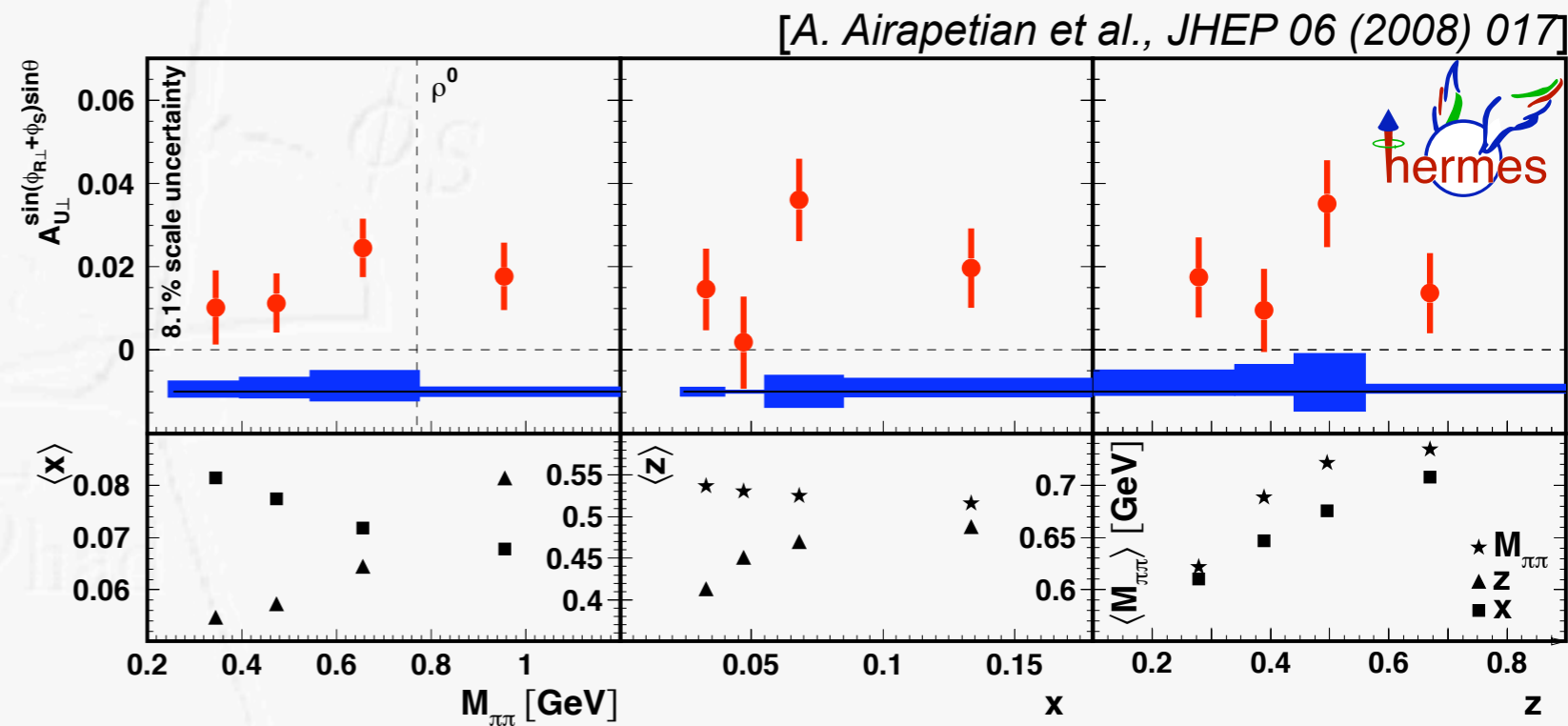
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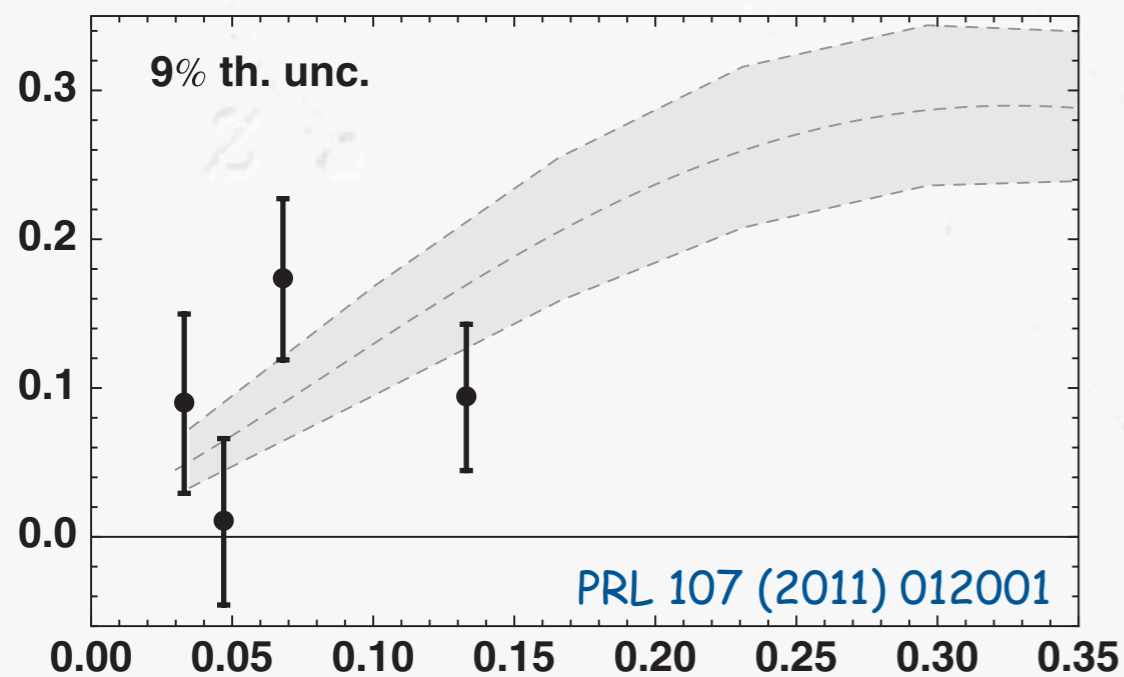
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$$x h_1^{u_v}(x) - x h_1^{d_v}(x)/4$$



- first (collinear) extraction of transversity (compared to Anselmino et al.)

Transversity's friends

Pretzelosity

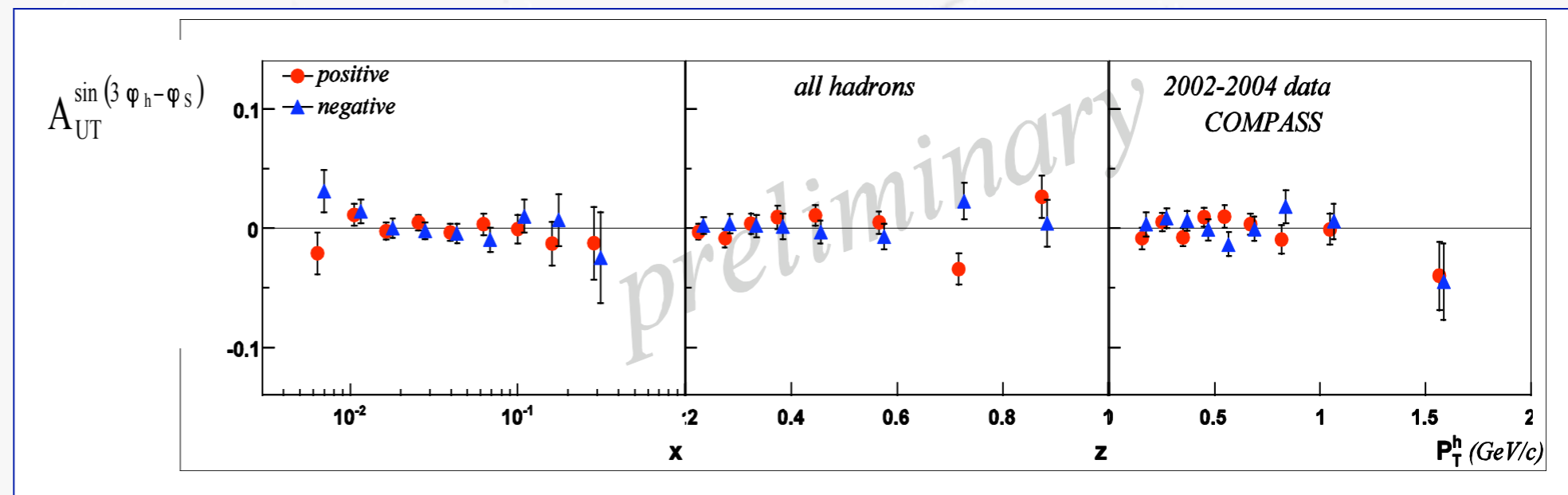
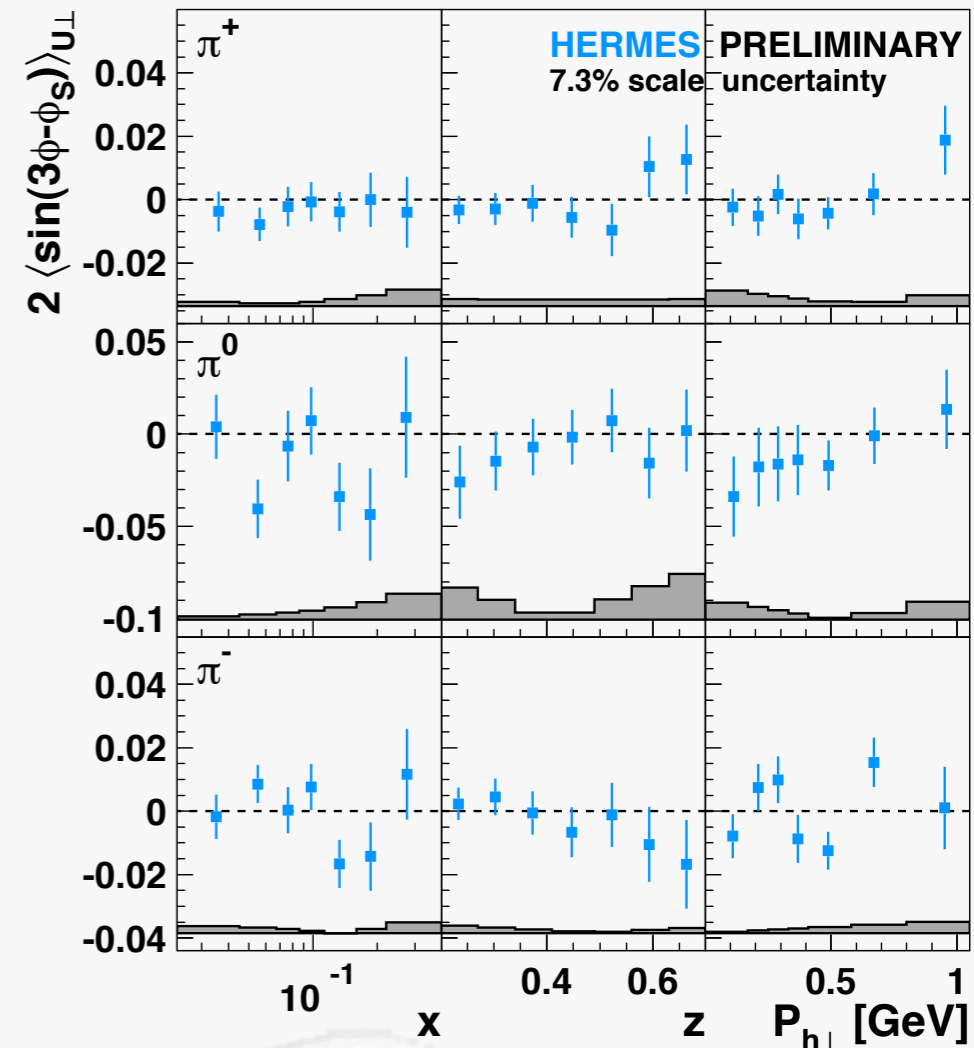
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- leads to $\sin(3\phi - \phi_s)$ modulation in A_{UT}
- proton and deuteron data consistent with zero
- cancelations? pretzelosity=zero?
or just the additional suppression by two powers of $P_{h\perp}$

Pretzelosity

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- proton and deuteron data consistent with zero
- cancelations? pretzelosity=zero?
or just the additional suppression by two powers of $P_{h\perp}$

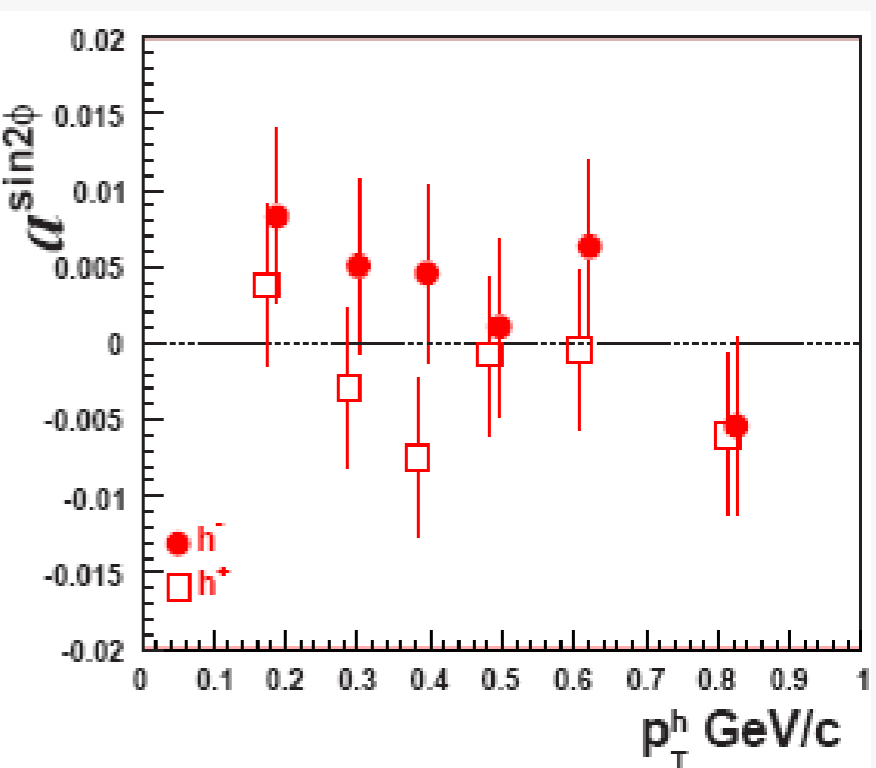
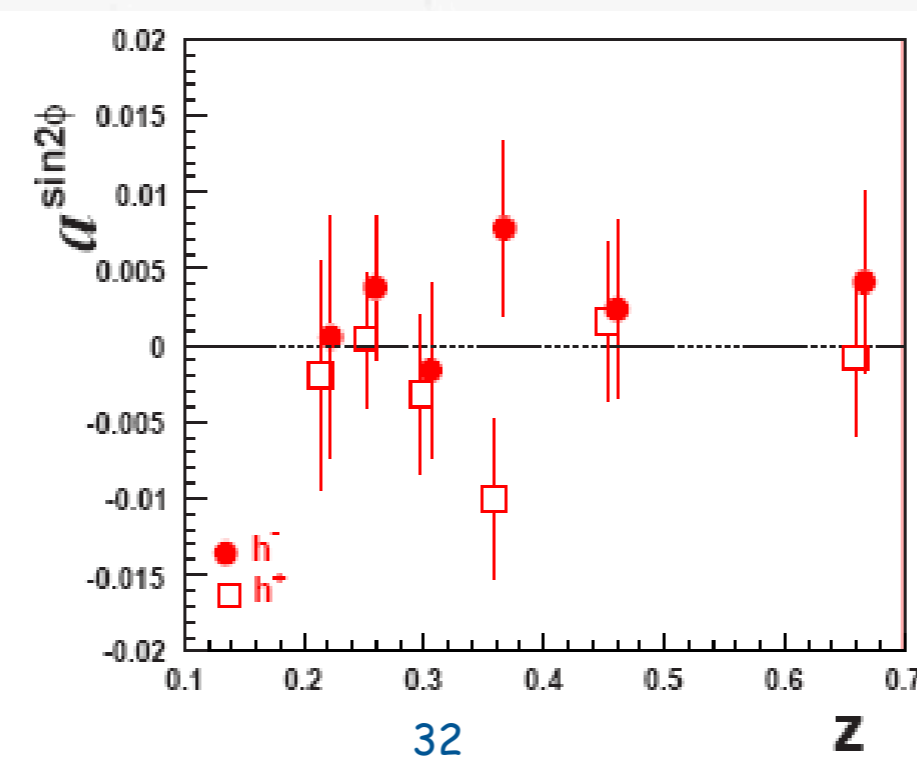
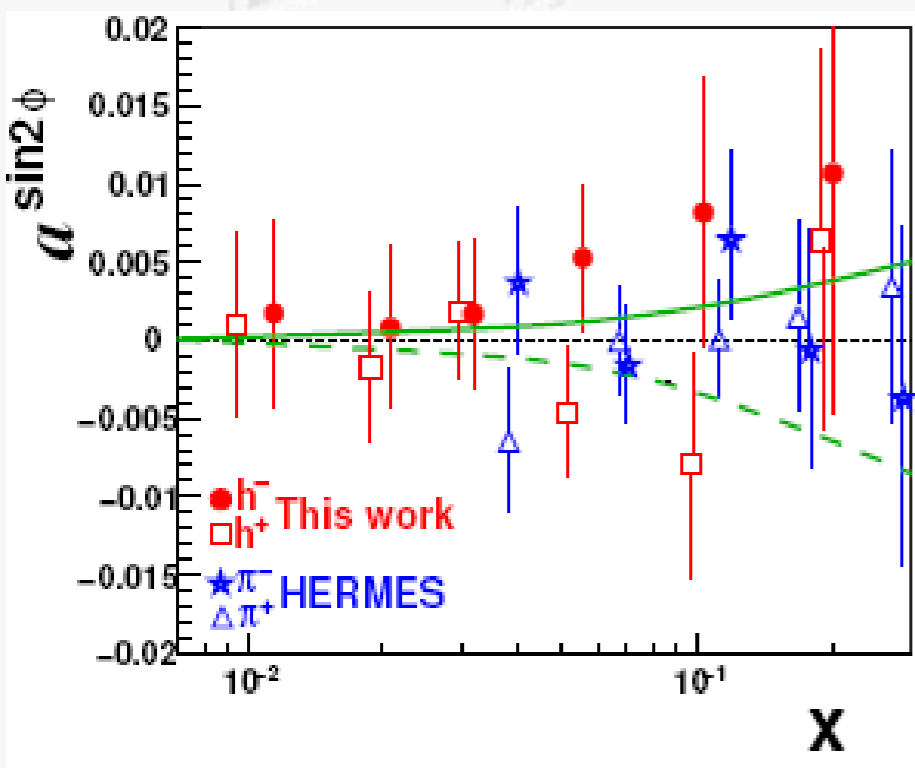
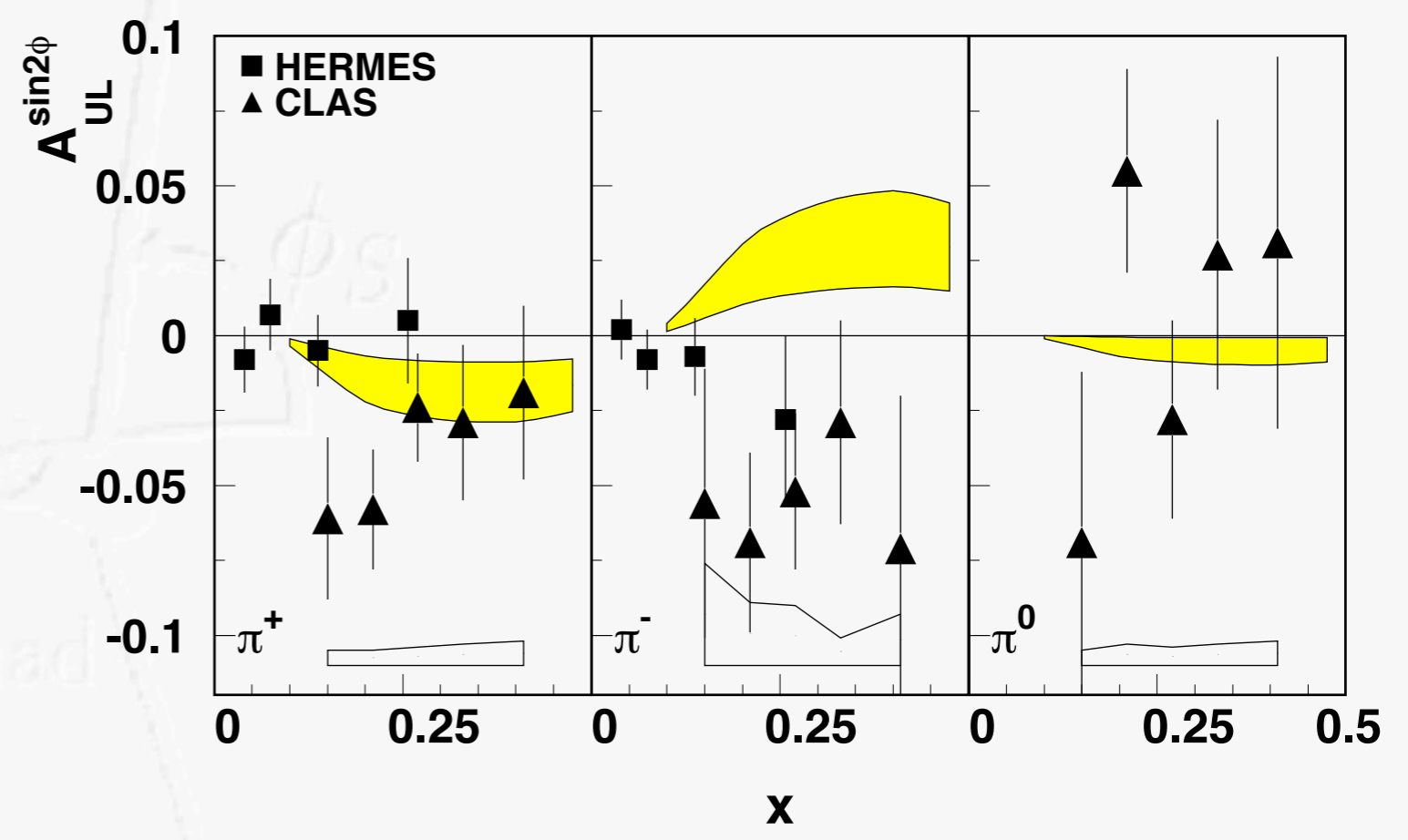


Worm-Gear I

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



- again chiral-odd
- evidence from CLAS (violating isospin symmetry?)
- consistent with zero at COMPASS and HERMES

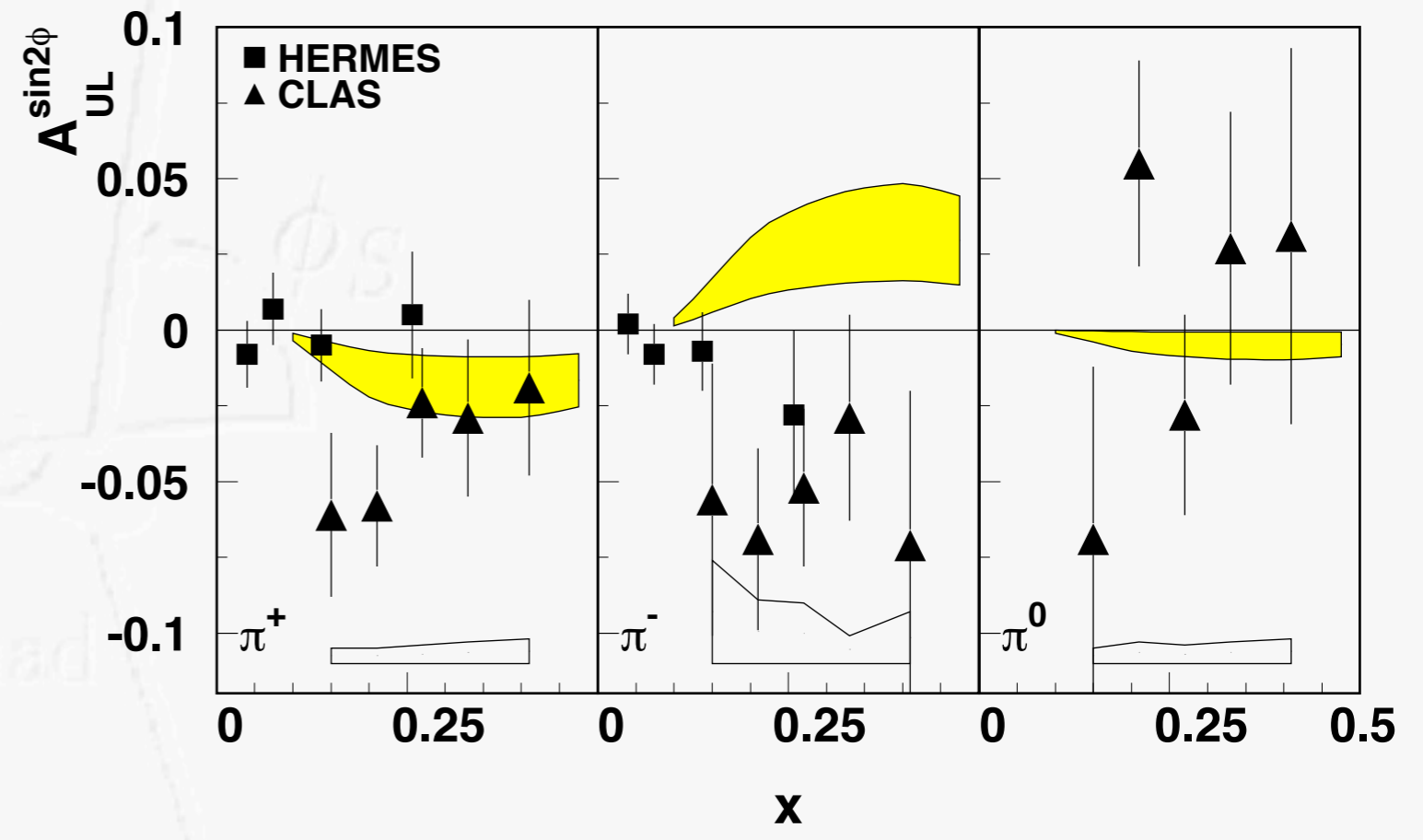


Worm-Gear I

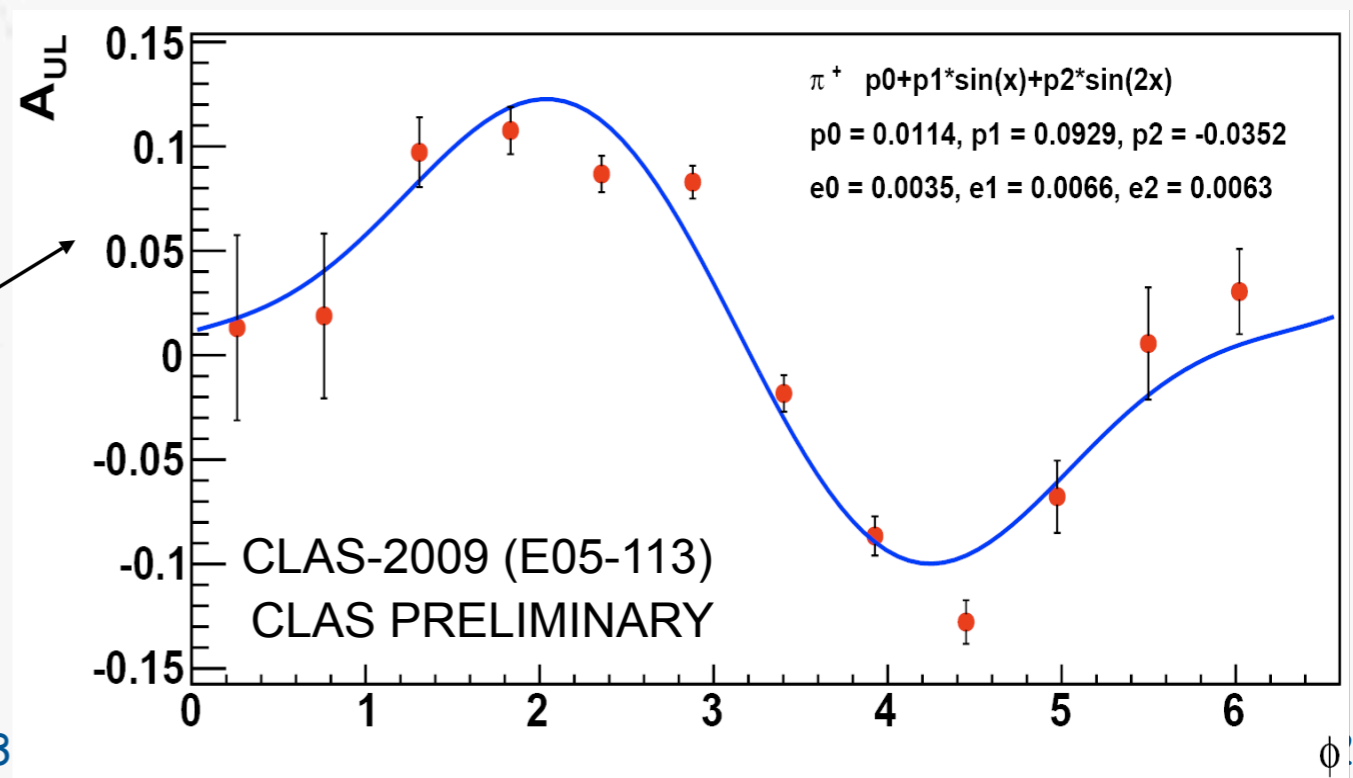
	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



- again chiral-odd
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- new data from CLAS



~10% of E05-113 data

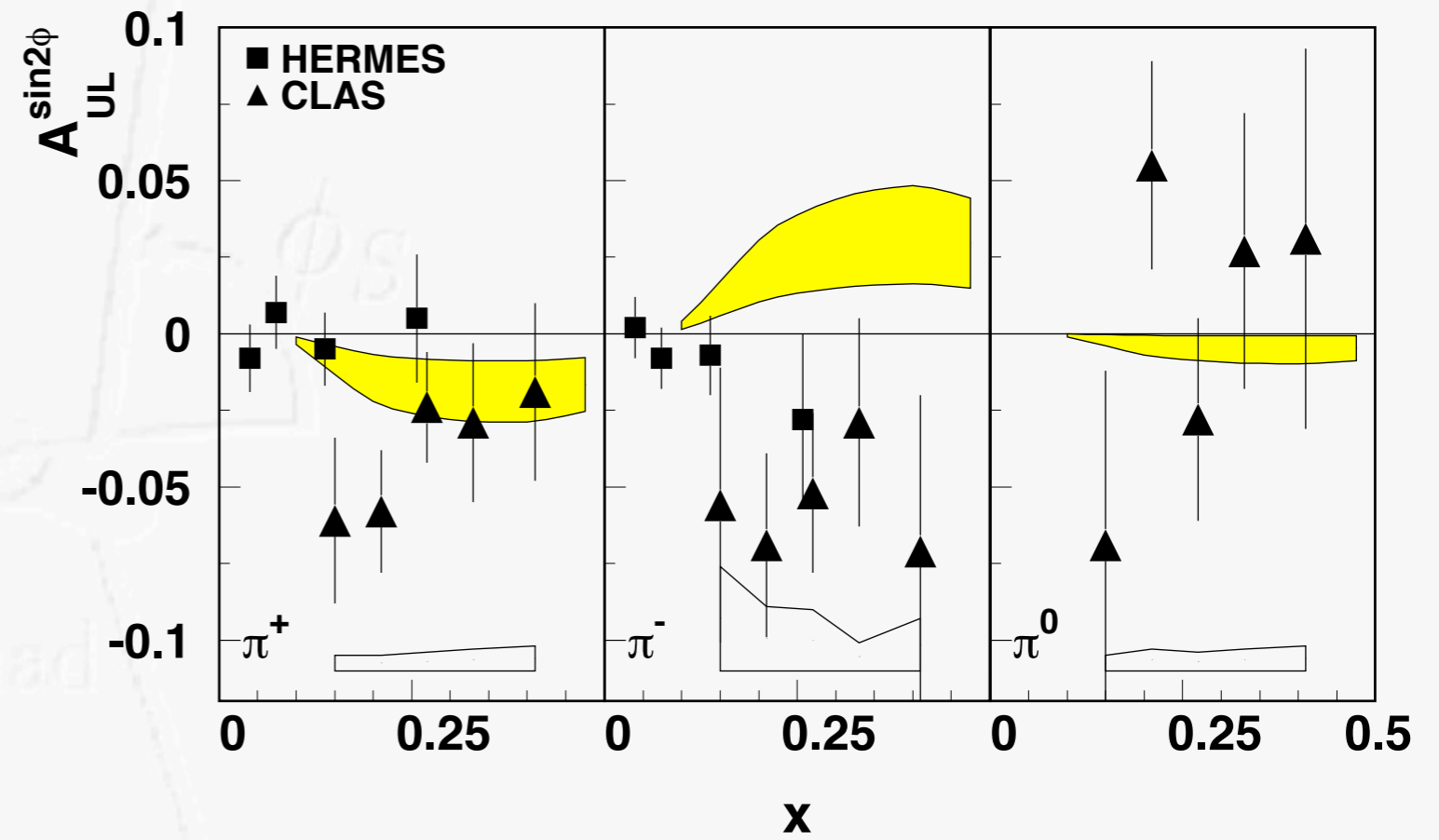


Worm-Gear I

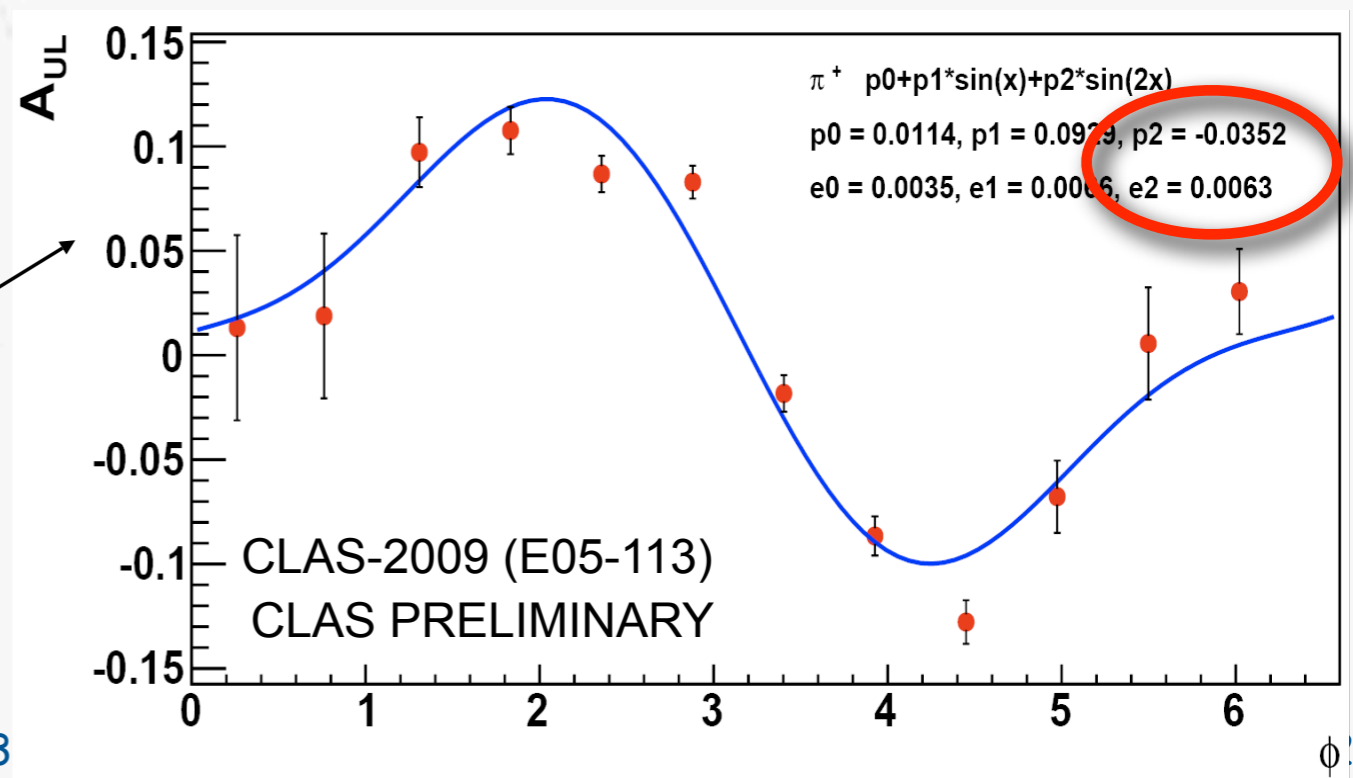
	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



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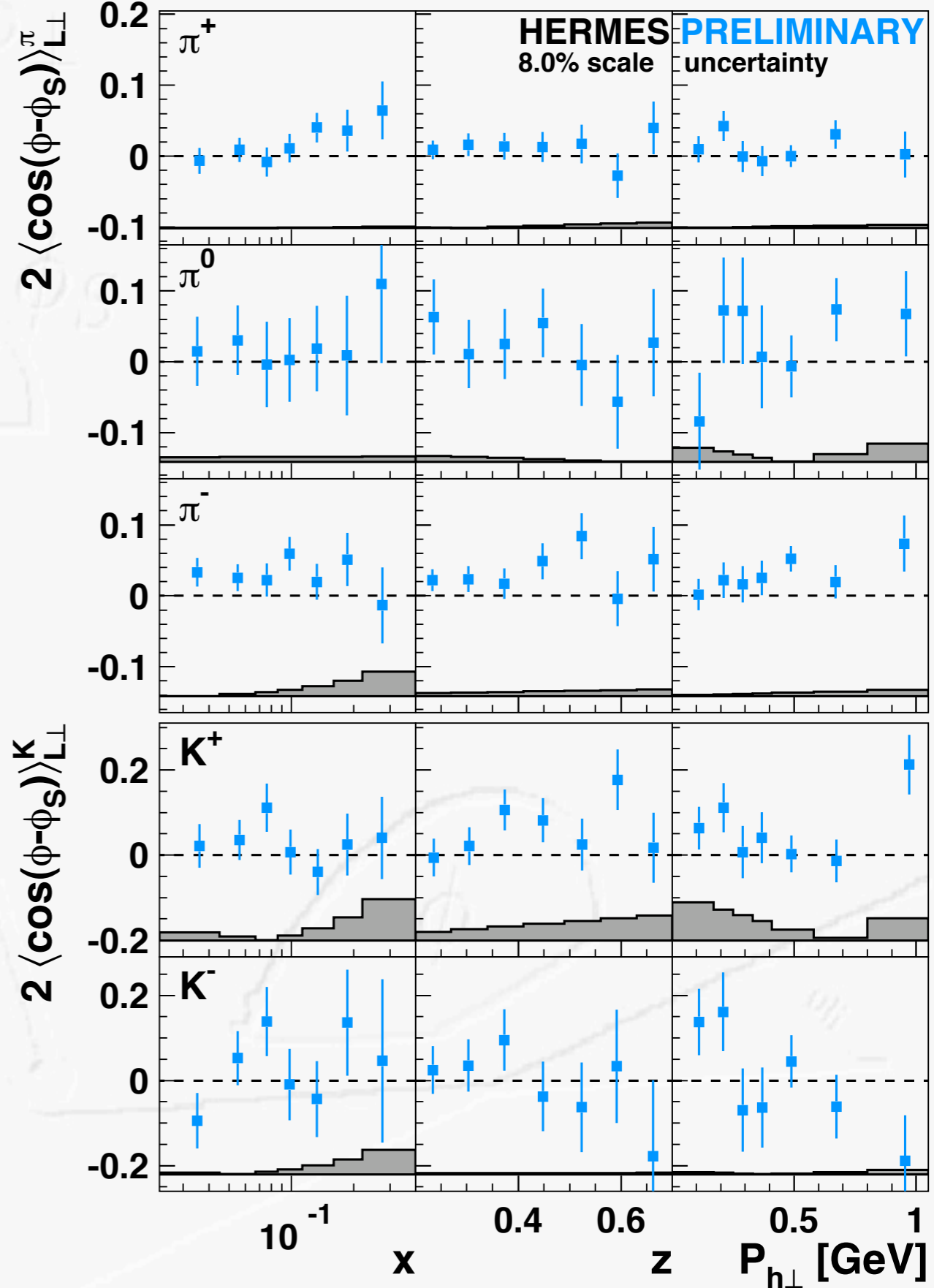


	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

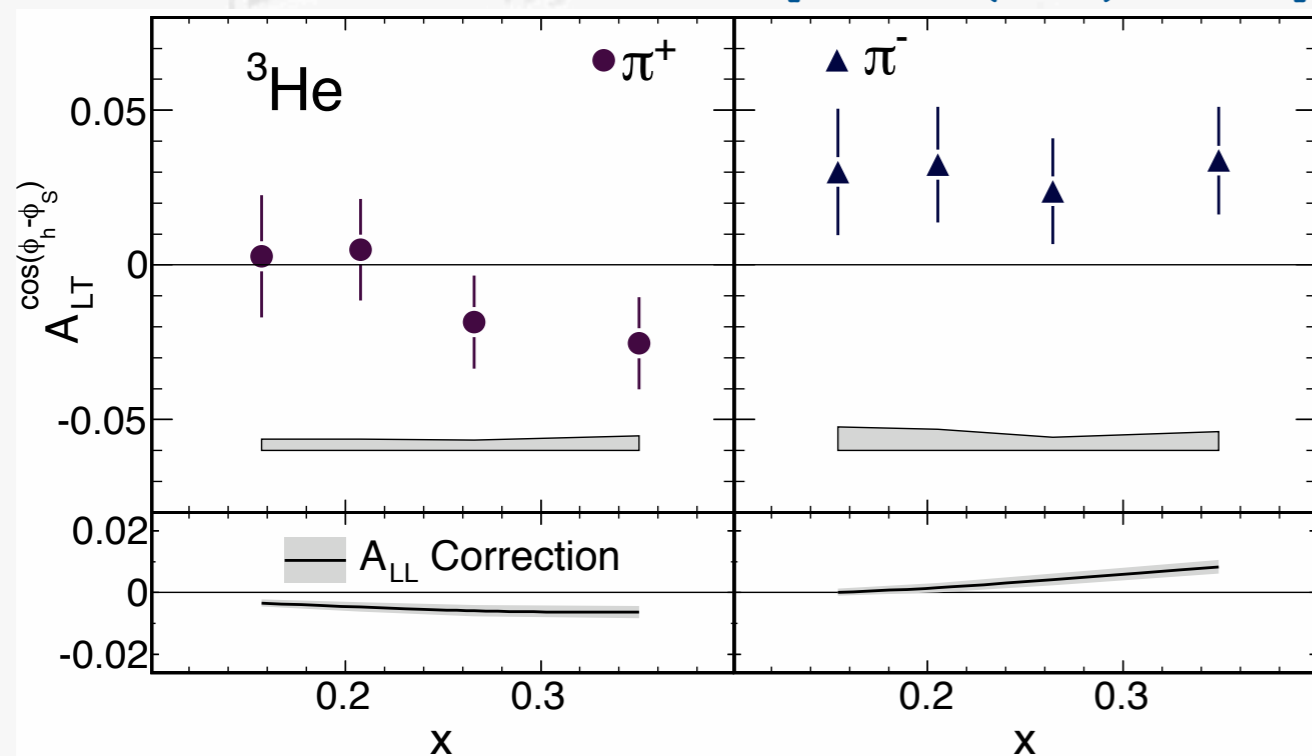


Worm-Gear II

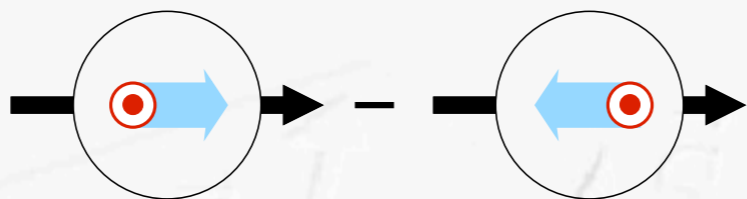
- chiral even
- first direct evidence for worm-gear g_{1T} on
- ^3He target at JLab
- H target at HERMES



[PRL 108 (2012) 052001]

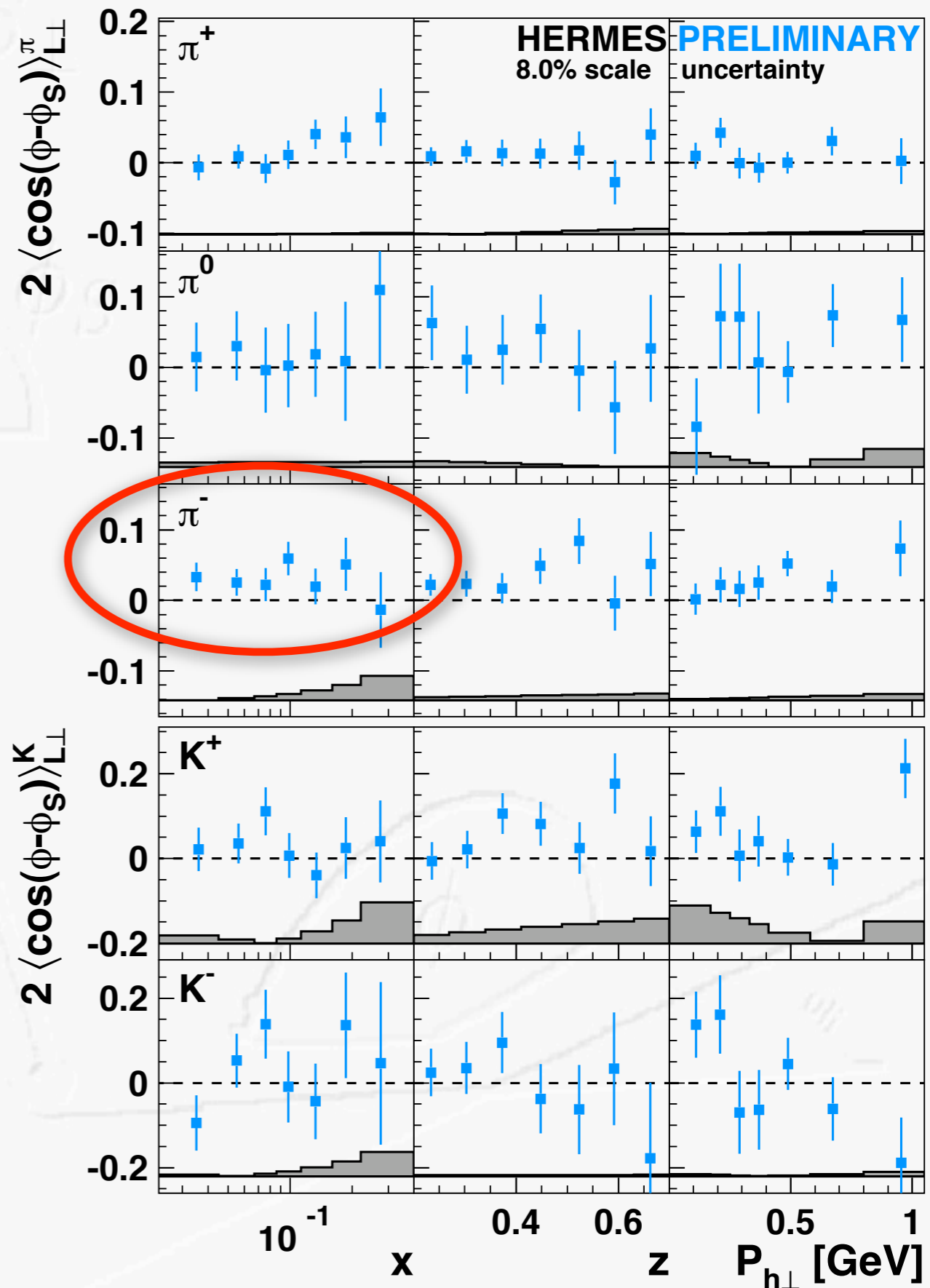


	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

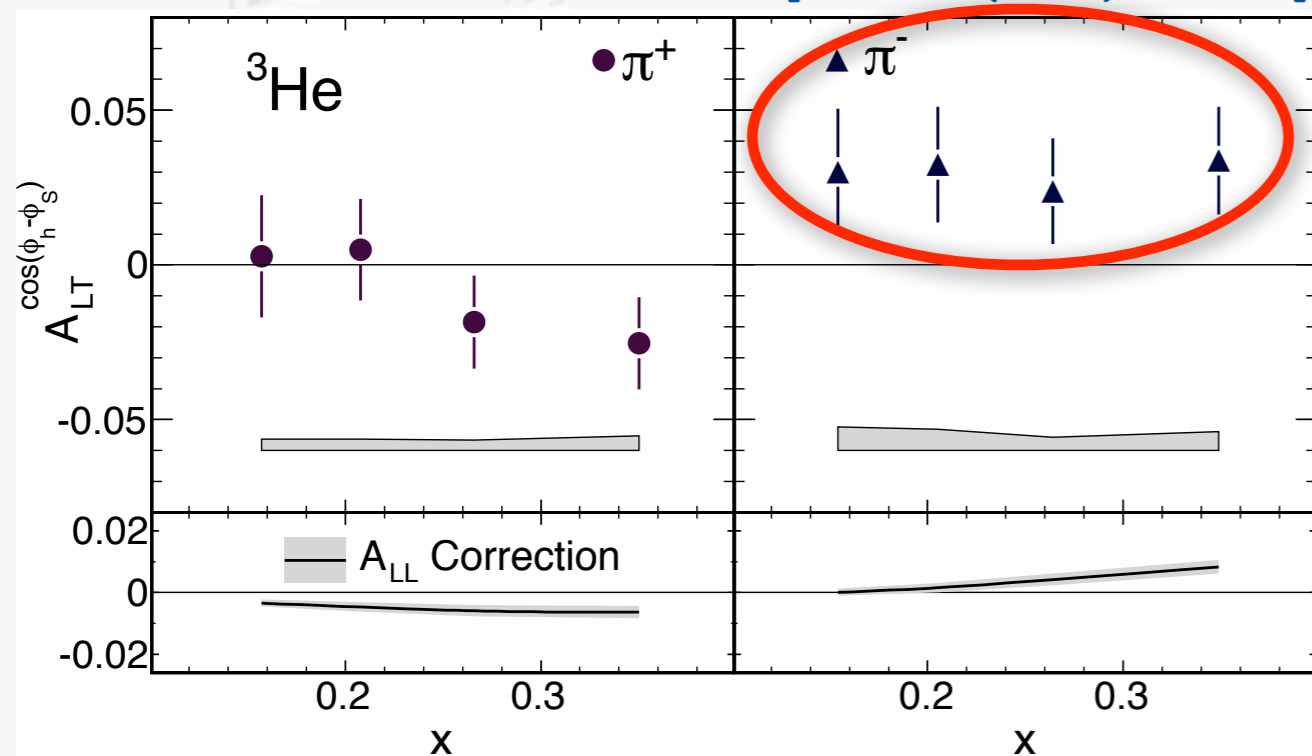


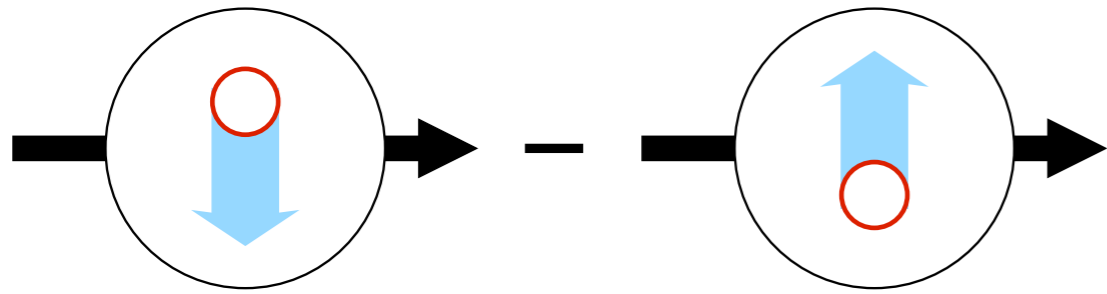
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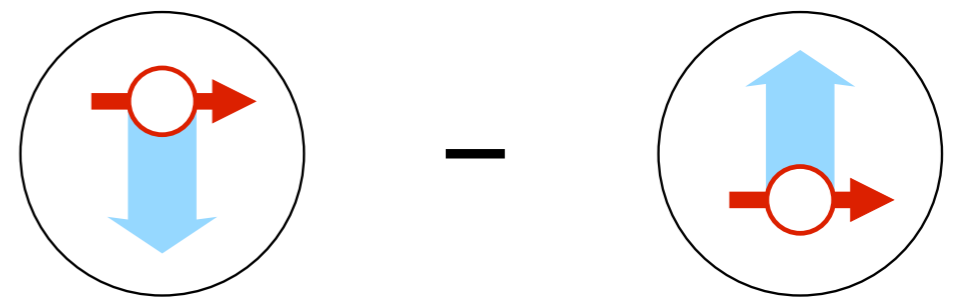


[PRL 108 (2012) 052001]





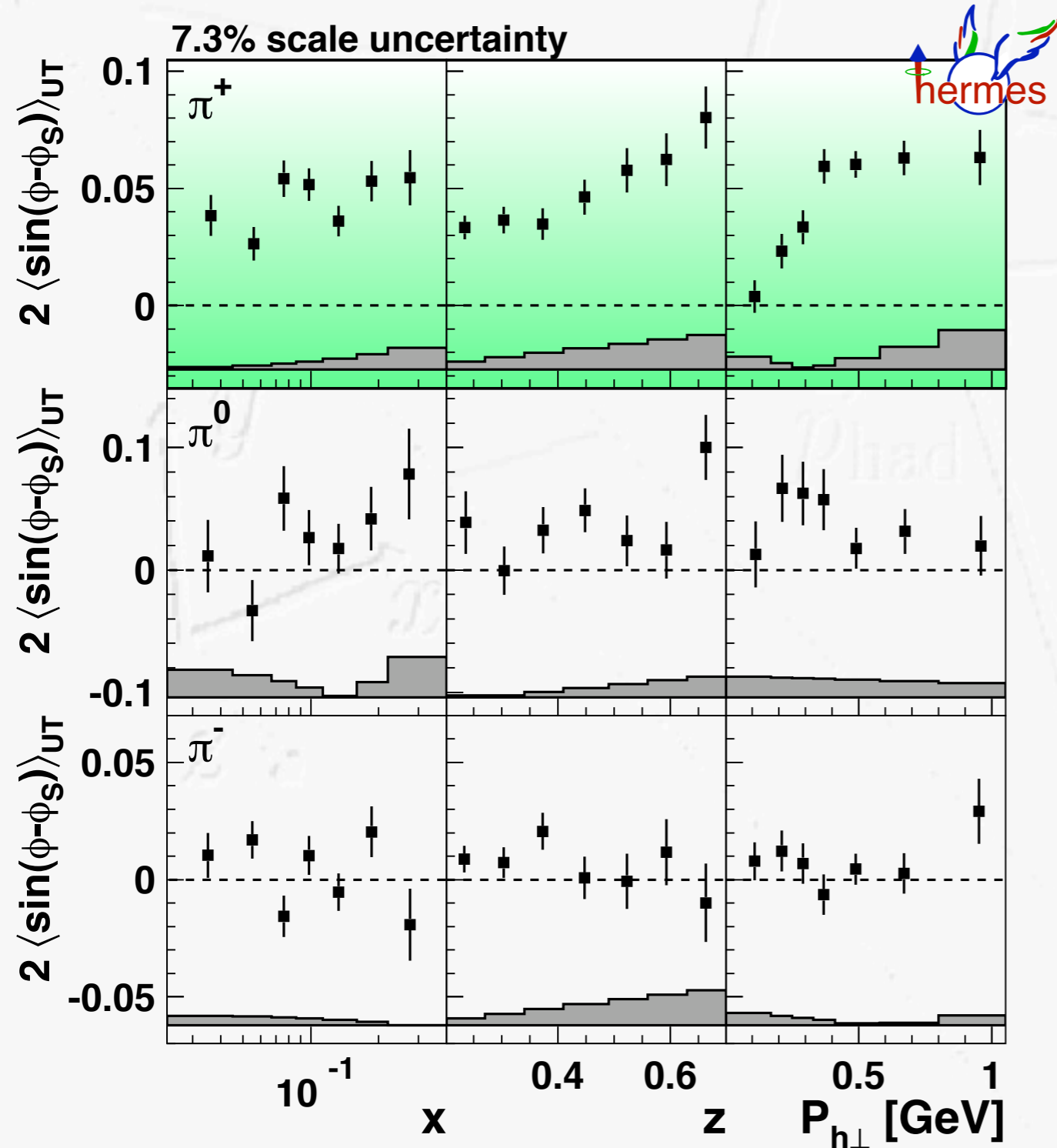
“gauge-link physics”
naively T-odd distributions



Sivers amplitudes for pions

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

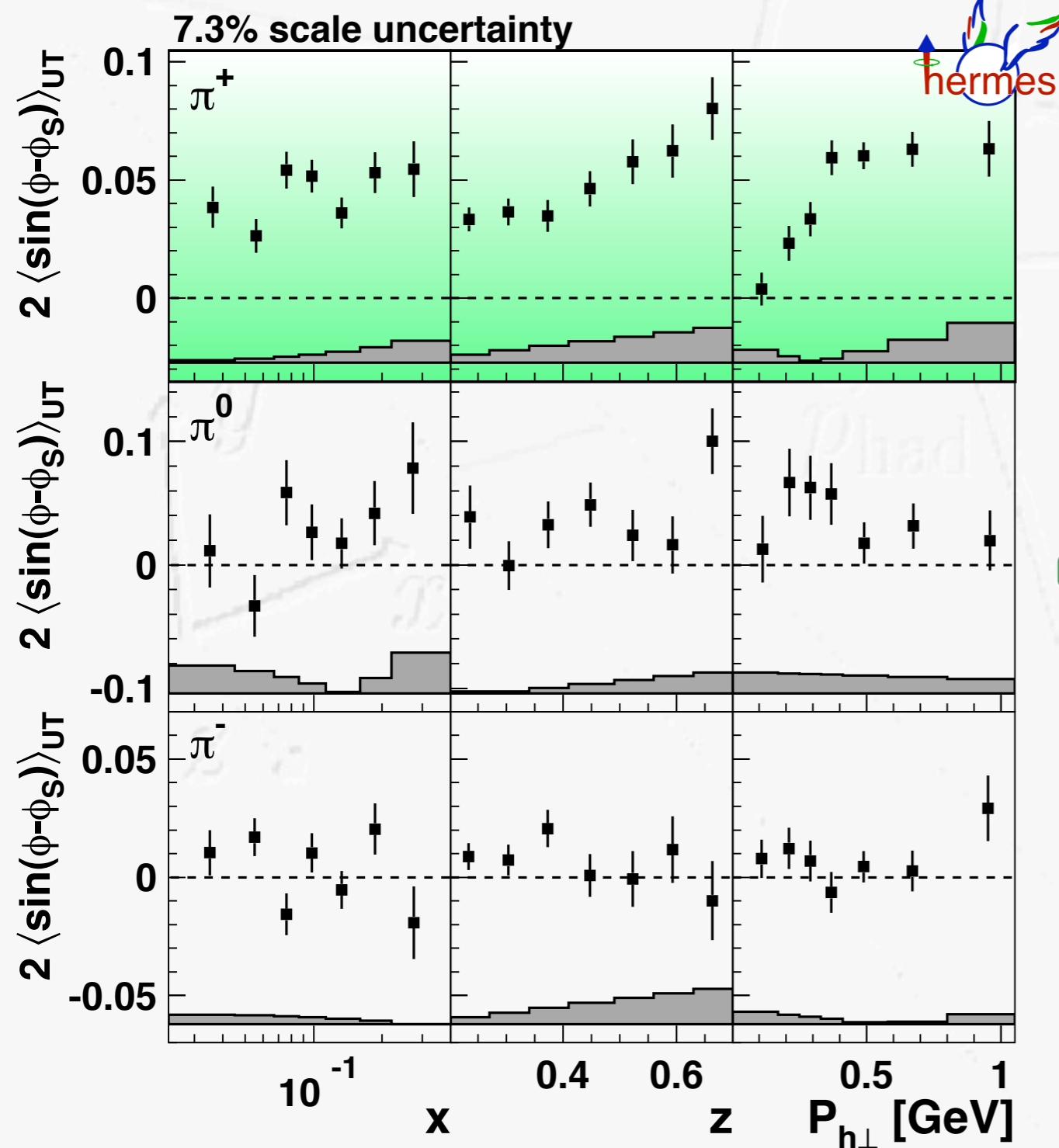
$$2\langle \sin(\phi - \phi_S) \rangle_{UT} = - \frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x, p_T^2) \otimes_{\mathcal{W}} D_1^q(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)}$$



Sivers amplitudes for pions

	U	L	T
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π^+ dominated by u-quark scattering:

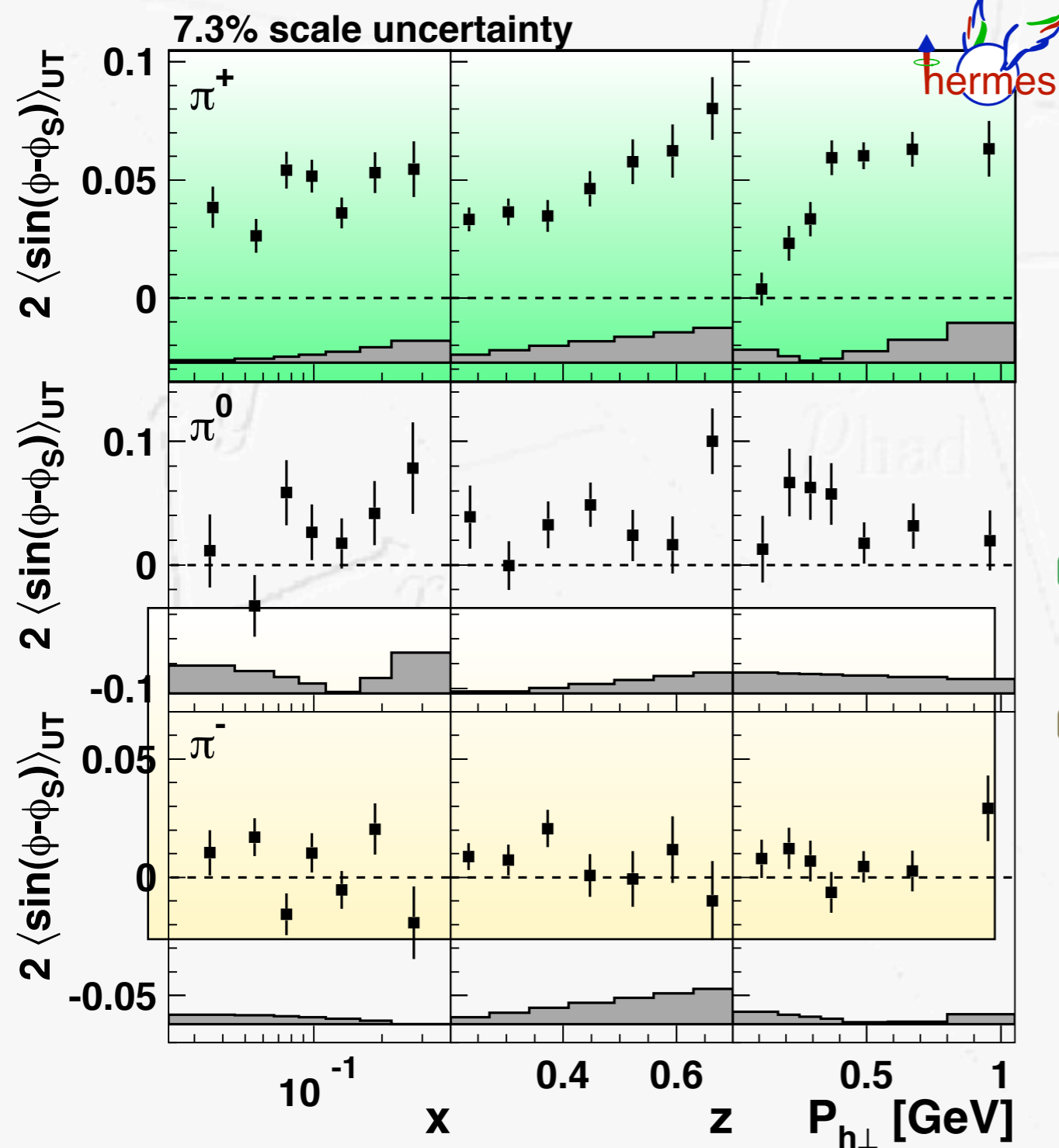
$$\simeq - \frac{f_{1T}^{\perp,u}(x, p_T^2) \otimes_{\mathcal{W}} D_1^{u \rightarrow \pi^+}(z, k_T^2)}{f_1^u(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+}(z, k_T^2)}$$

u-quark Sivers DF < 0

Sivers amplitudes for pions

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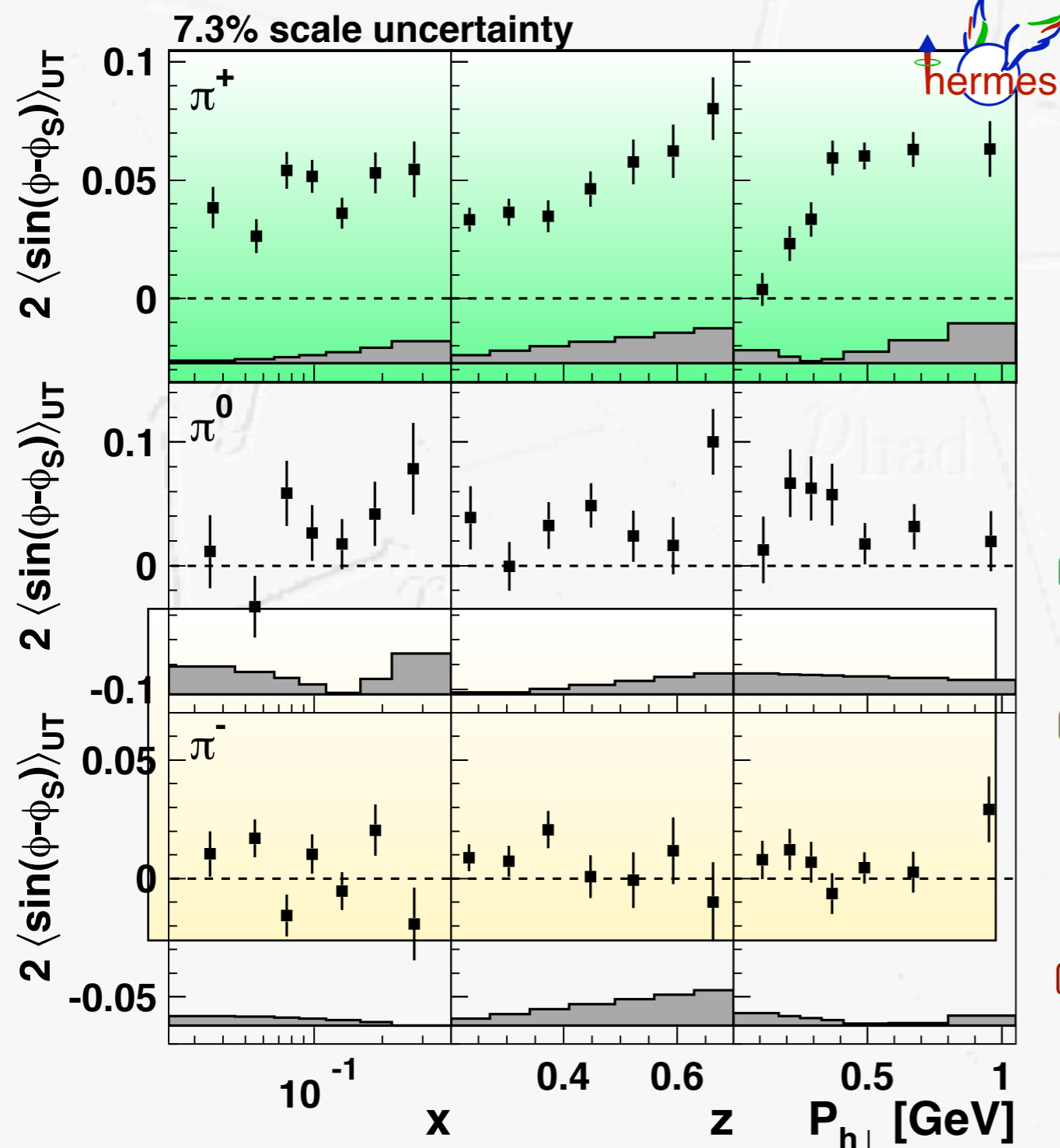
☞ u-quark Sivers DF < 0

☞ d-quark Sivers DF > 0
(cancelation for π^-)

Sivers amplitudes for pions

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

$$2\langle \sin(\phi - \phi_S) \rangle_{UT} = - \frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x, p_T^2) \otimes_{\mathcal{W}} D_1^q(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)}$$



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$$\simeq - \frac{f_{1T}^{\perp,u}(x, p_T^2) \otimes_{\mathcal{W}} D_1^{u \rightarrow \pi^+}(z, k_T^2)}{f_1^u(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+}(z, k_T^2)}$$

☞ u-quark Sivers DF < 0

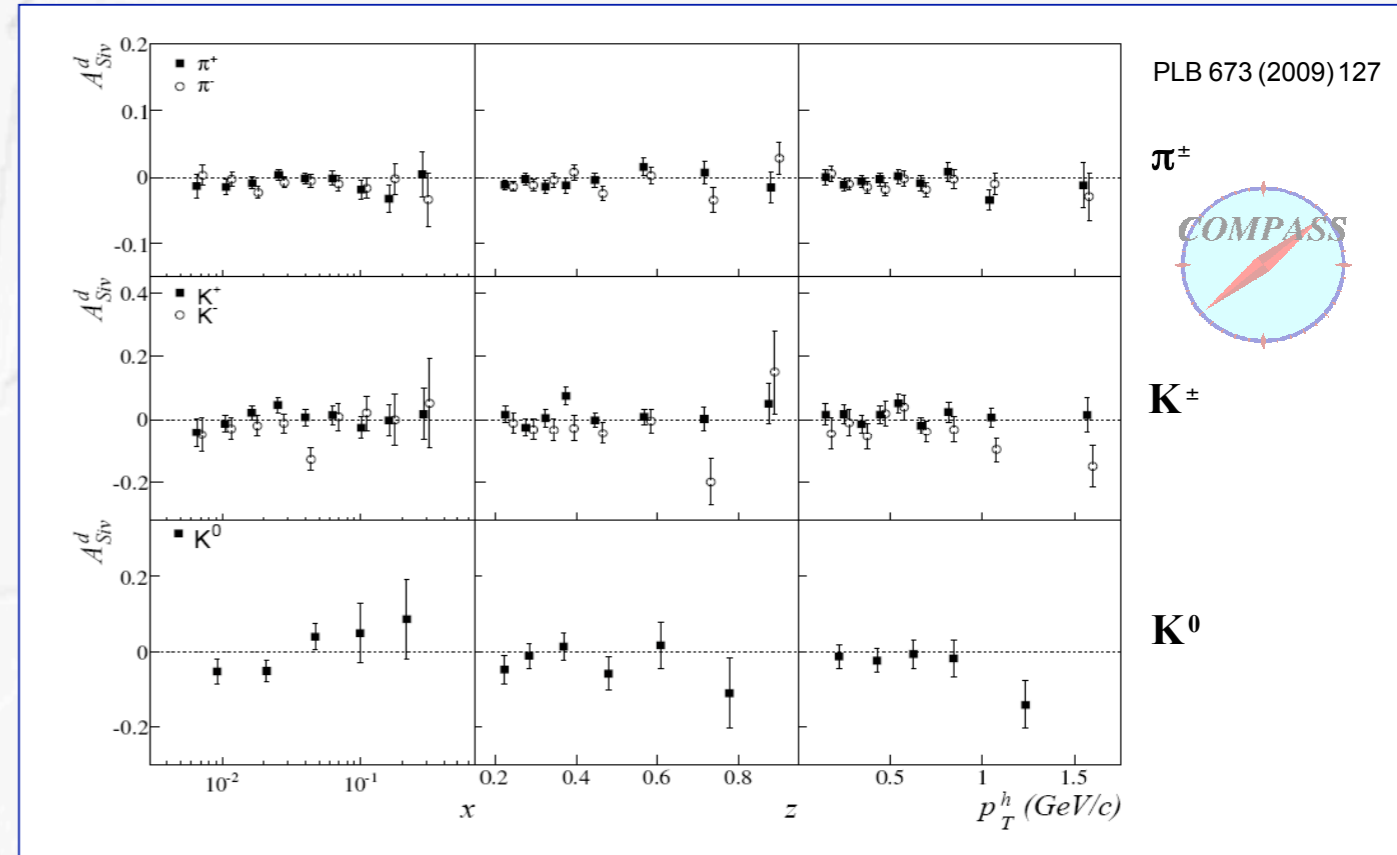
☞ d-quark Sivers DF > 0
(cancellation for π^-)

☞ u-d cancellation supported by COMPASS D data

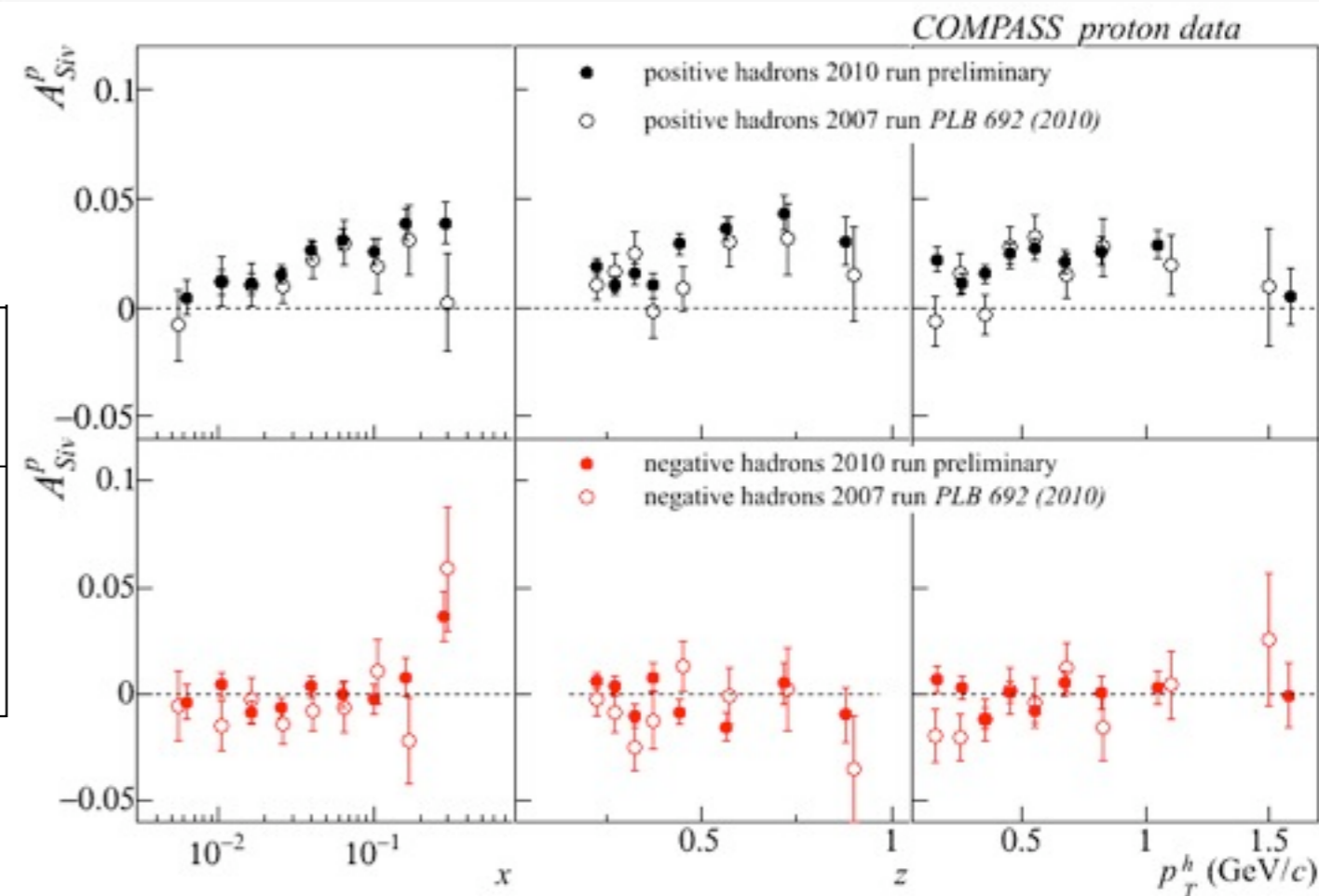
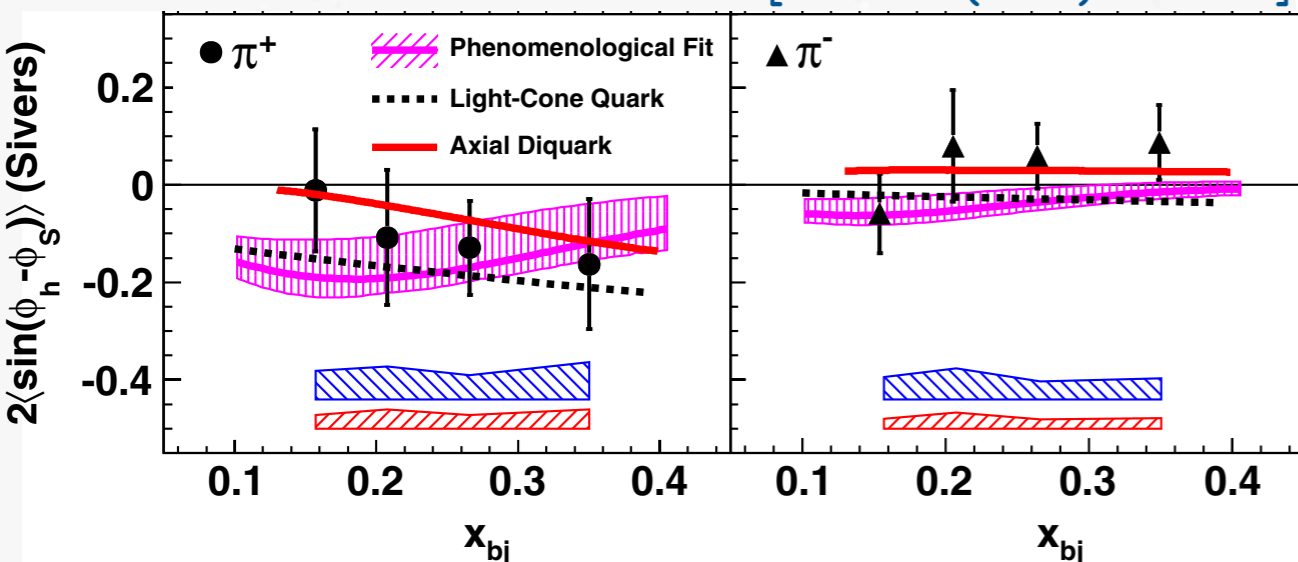
Sivers function

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

- cancelation for D target supports opposite signs of u and d Sivers
- new results from JLab using ^3He target and from COMPASS for proton target

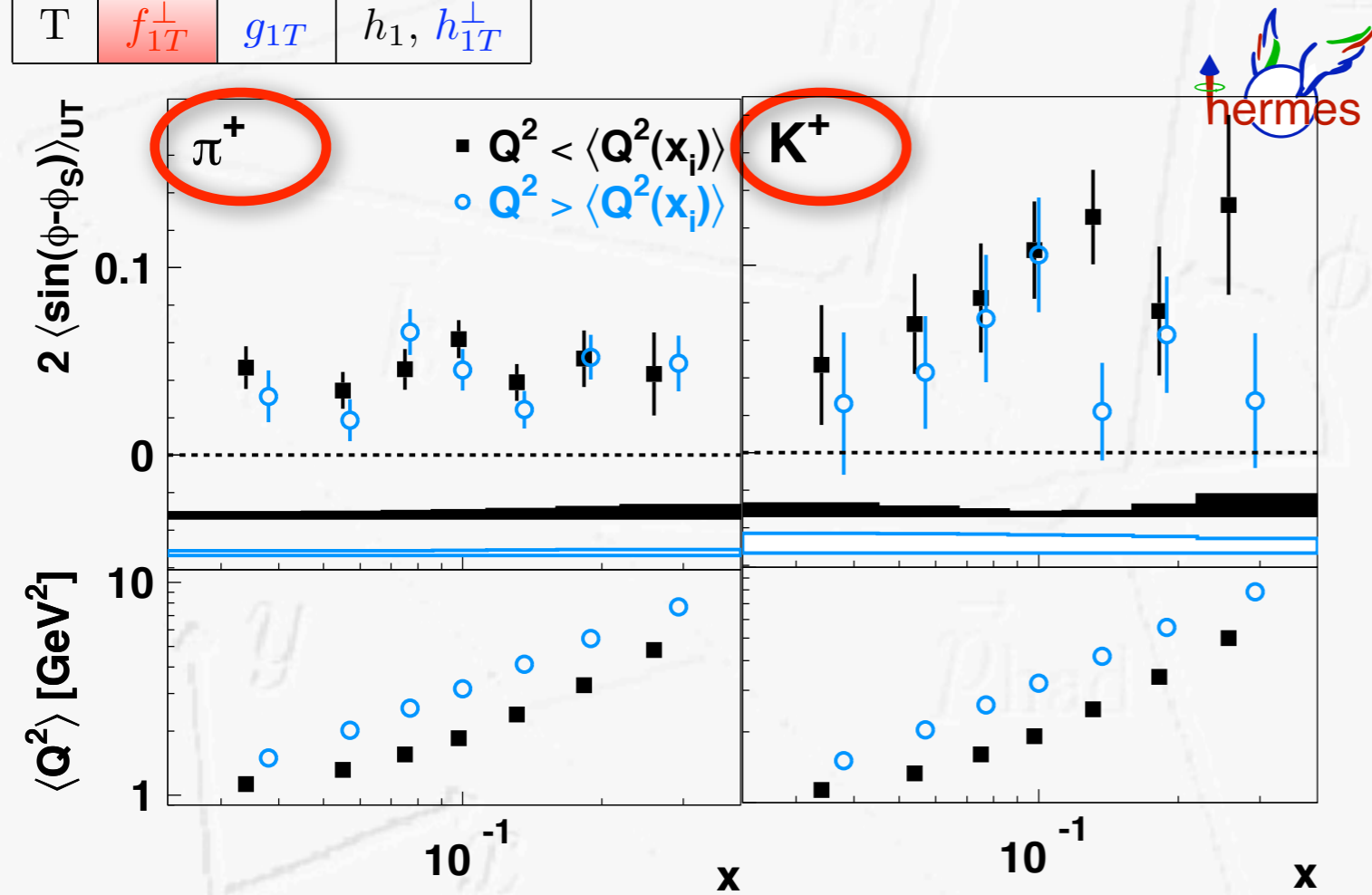


[PRL 107 (2011) 072003]



Sivers amplitudes

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

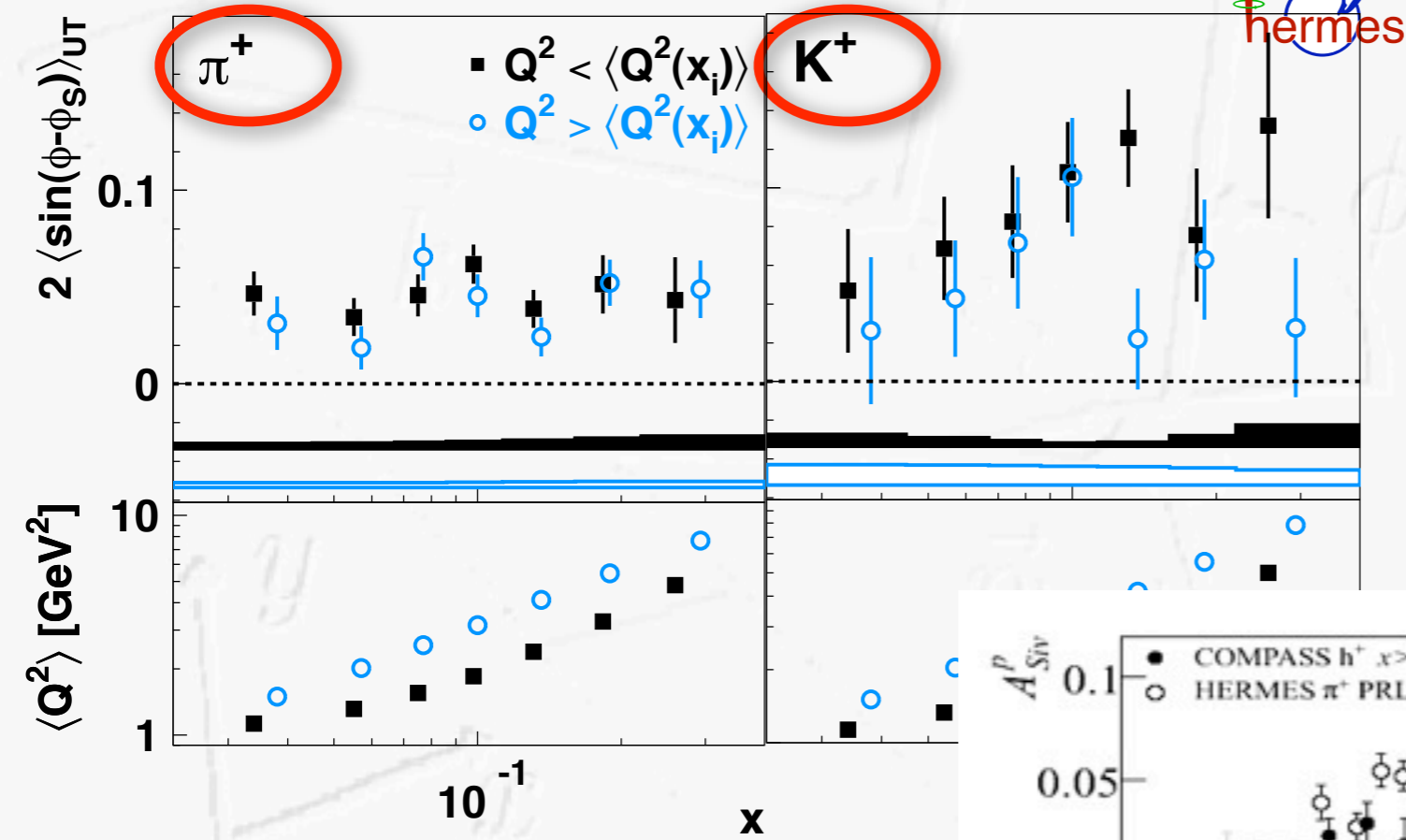


somewhat unexpected if dominated by scattering off u-quarks:

$$\approx - \frac{f_{1T}^{\perp,u}(x, p_T^2) \otimes_{\mathcal{W}} D_1^{u \rightarrow \pi^+/\text{K}^+}(z, k_T^2)}{f_1^u(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+/\text{K}^+}(z, k_T^2)}$$

Sivers amplitudes

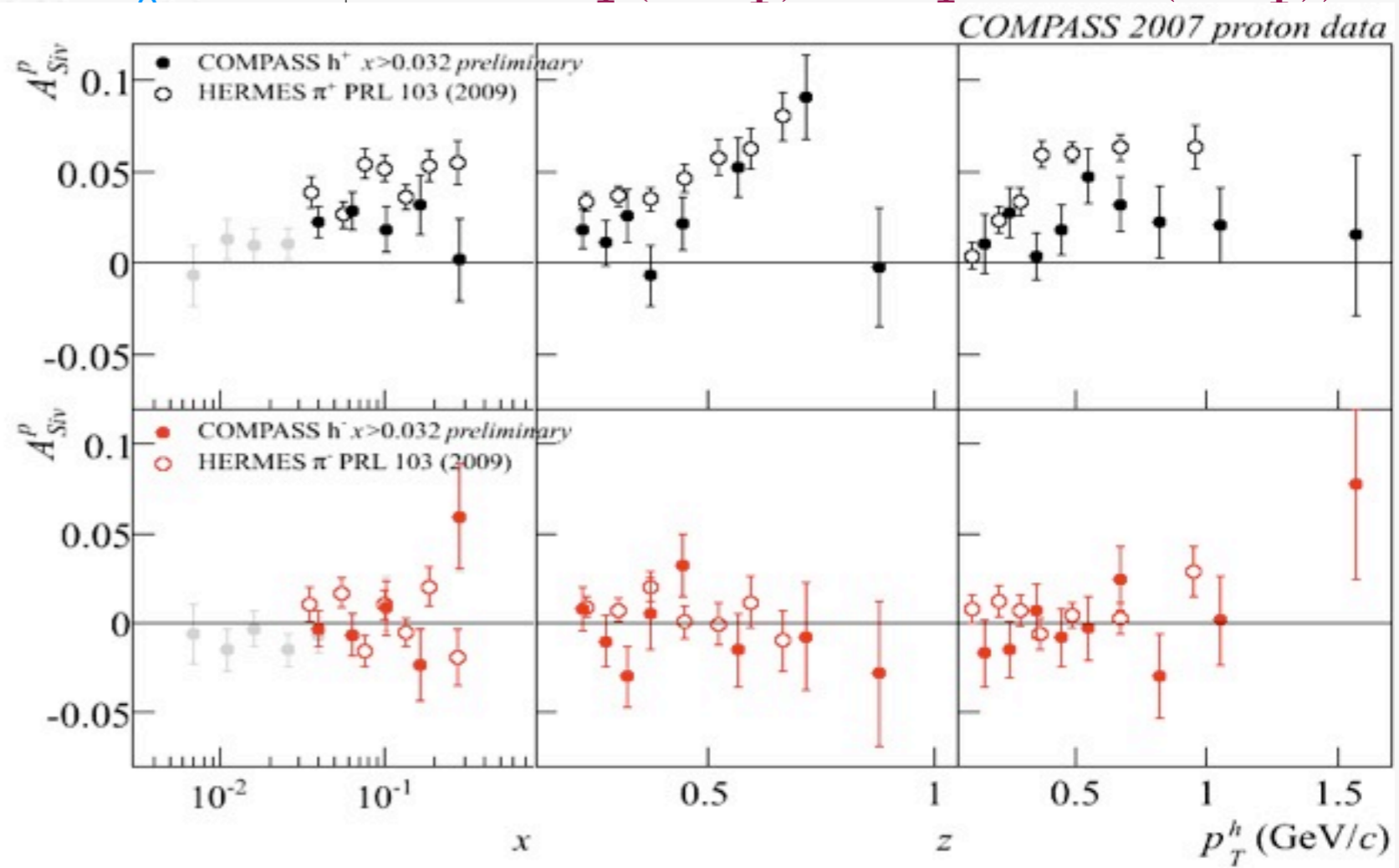
	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp



somewhat unexpected if dominated by scattering off u-quarks:

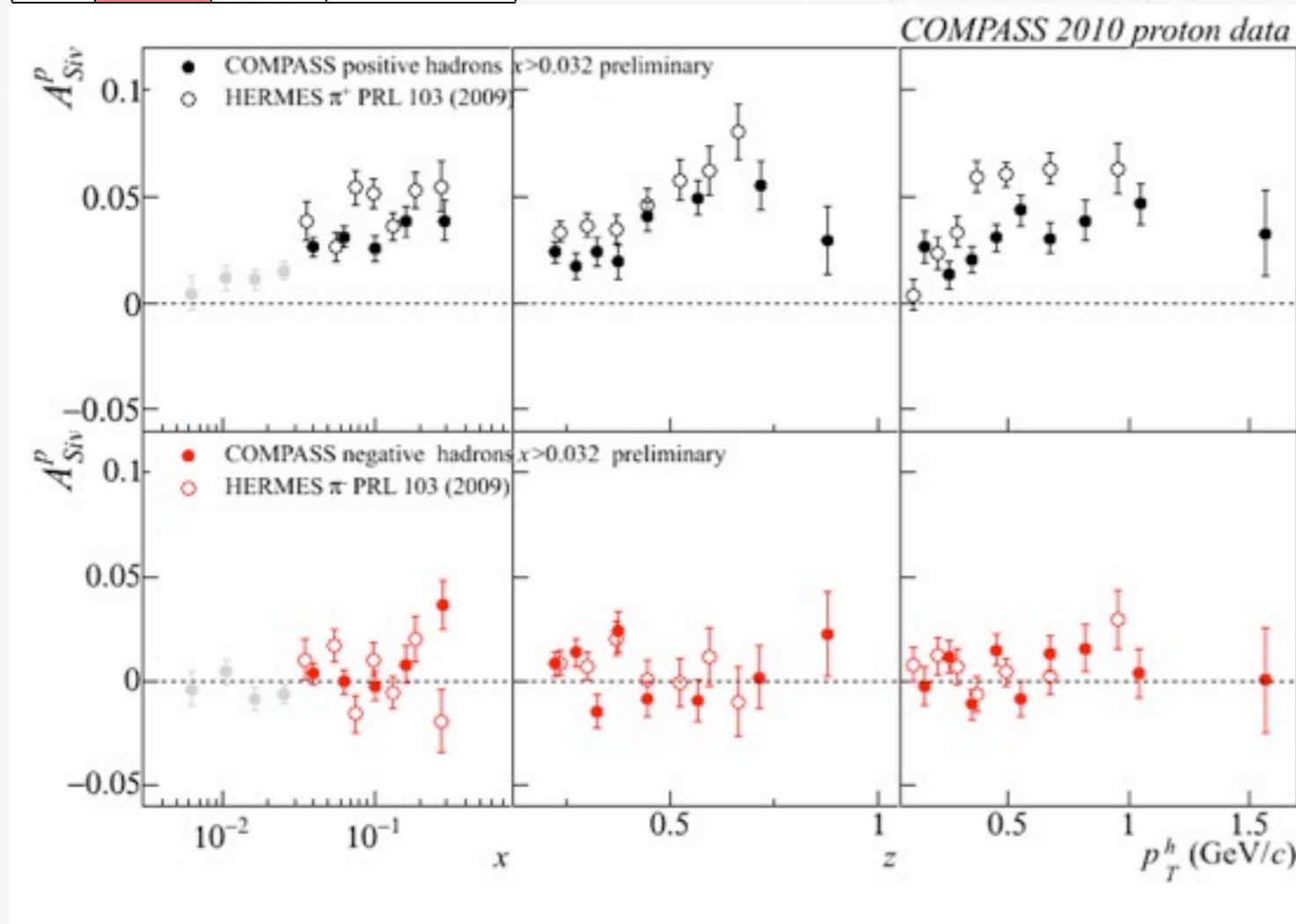
$$\approx - \frac{f_{1T}^{\perp,u}(x, p_T^2) \otimes_{\mathcal{W}} D_1^{u \rightarrow \pi^+/\text{K}^+}(z, k_T^2)}{f_1^u(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+/\text{K}^+}(z, k_T^2)}$$

smaller pion amplitudes seen by COMPASS



Sivers amplitudes

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

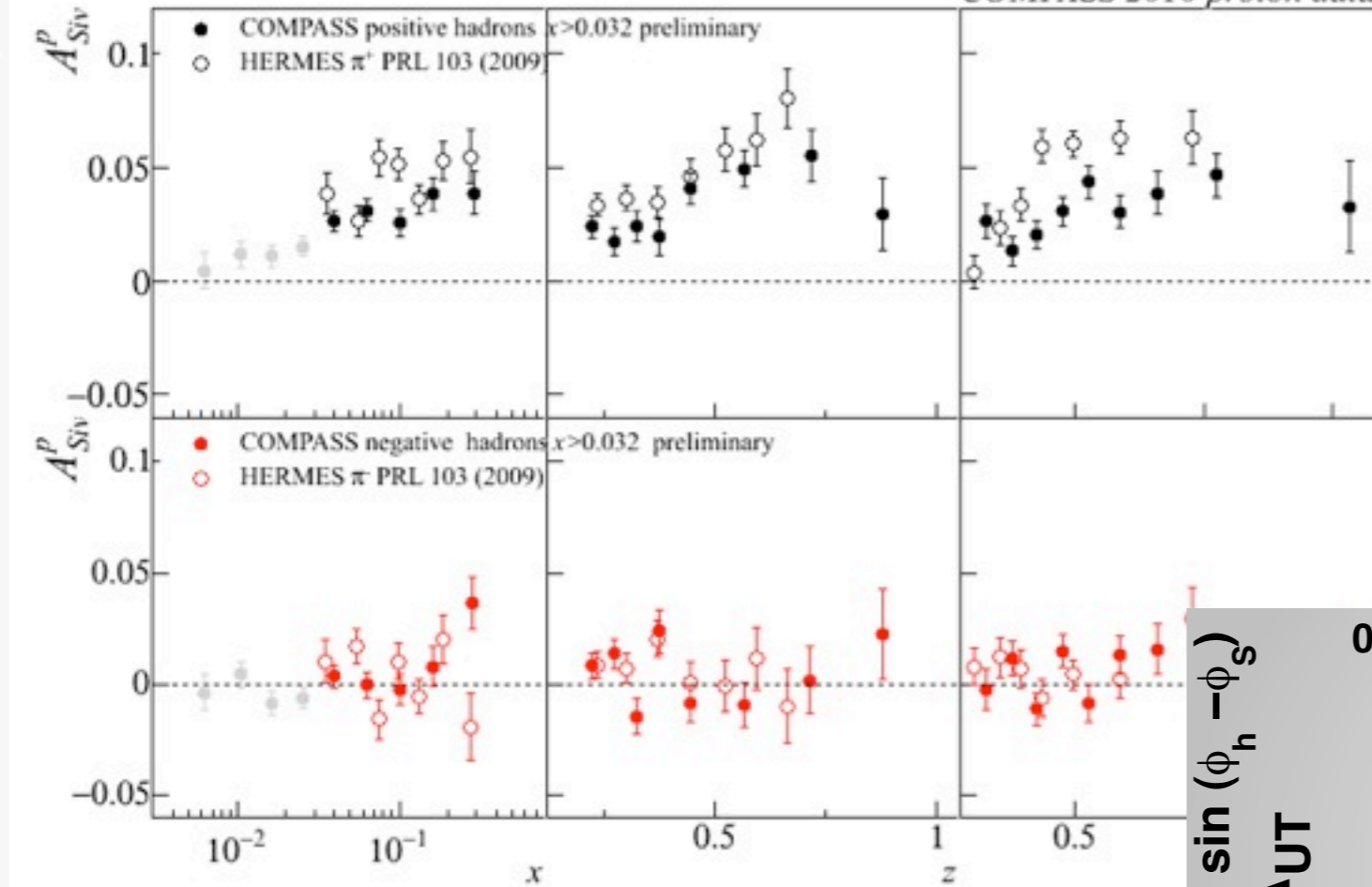


2010 Compass data
closer to HERMES

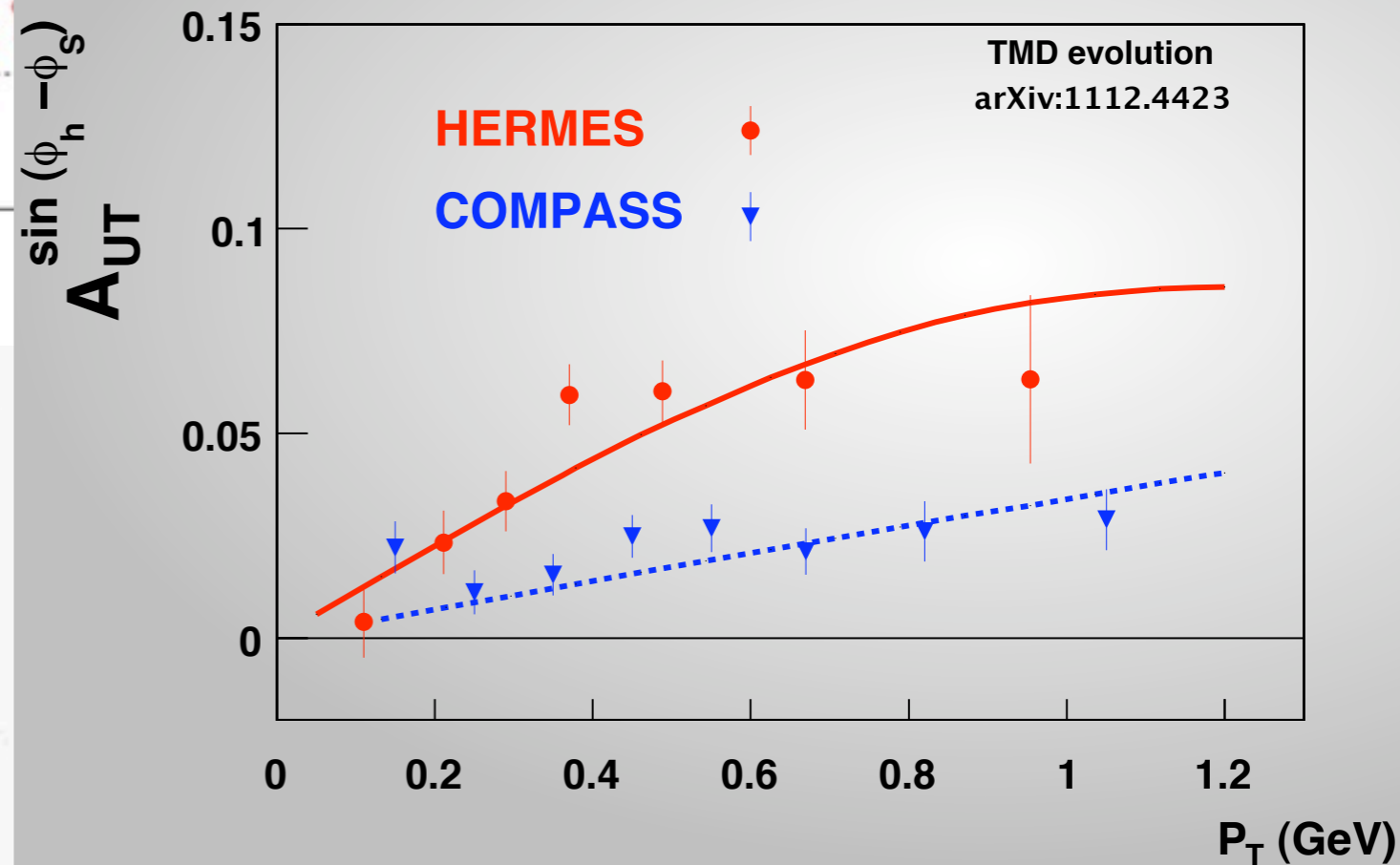
Sivers amplitudes

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

COMPASS 2010 proton data



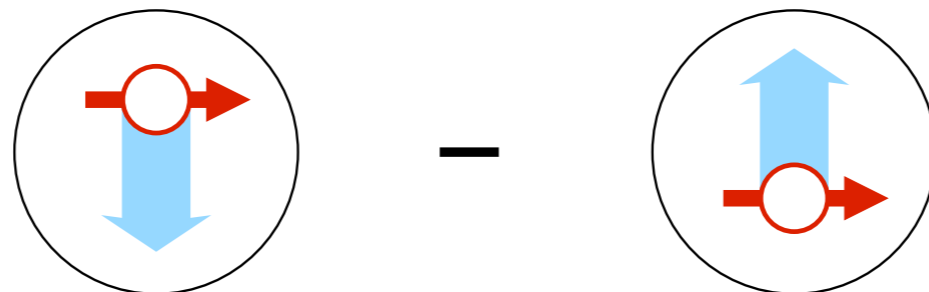
2010 Compass data closer to HERMES



difference from evolution?

Boer-Mulders

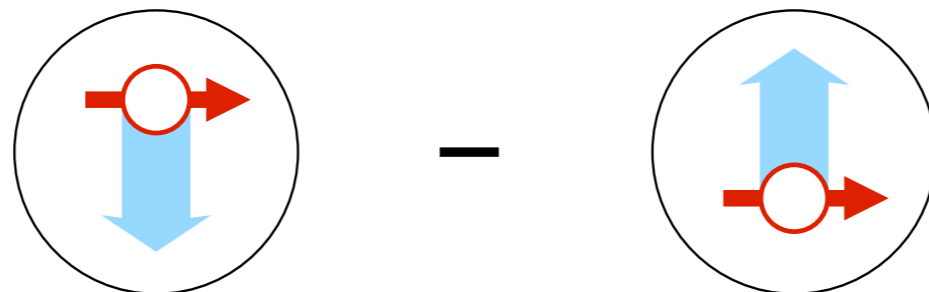
the other naively T-odd distribution



spin effects in unpolarized reactions

Boer-Mulders

the other naively T-odd distribution



... moment measurements

hadron multiplicity:
normalize to inclusive DIS
cross section

$$\frac{d^2 \sigma^{\text{incl. DIS}}}{dx dy} \propto F_T + \epsilon F_L$$

$$\frac{d^4 \mathcal{M}^h(x, y, z, P_{h\perp}^2)}{dx dy dz dP_{h\perp}^2} \propto \left(1 + \frac{\gamma^2}{2x}\right) \frac{F_{UU,T} + \epsilon F_{UU,L}}{F_T + \epsilon F_L}$$

$$\approx \frac{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^{q \rightarrow h}(z, K_T^2)}{\sum_q e_q^2 f_1^q(x)}$$

$$\frac{d^5 \sigma}{dx dy dz d\phi_h dP_{h\perp}^2} \propto \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1-\epsilon)} F_{UU}^{\cos \phi_h} \cos \phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h \right\}$$



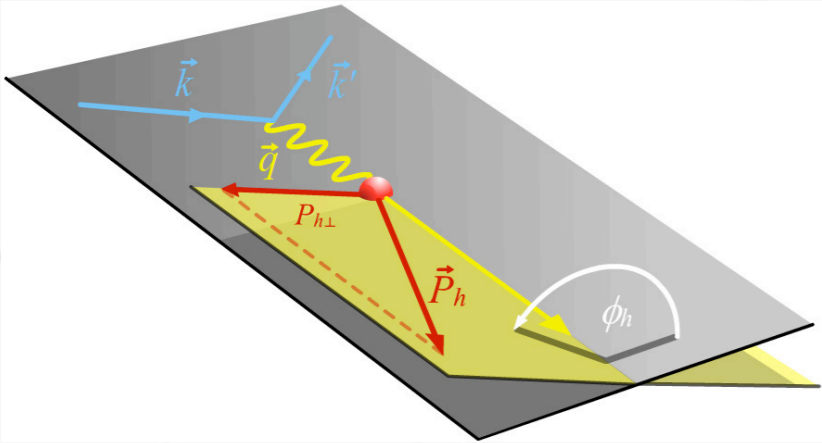
$$2 \langle \cos 2\phi \rangle_{UU} \equiv 2 \frac{\int d\phi_h \cos 2\phi d\sigma}{\int d\phi_h d\sigma} = \frac{\epsilon F_{UU}^{\cos 2\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

moments:
normalize to azimuth-
independent cross-section

$$\approx \epsilon \frac{\sum_q e_q^2 h_1^{\perp,q}(x, p_T^2) \otimes_{\text{BM}} H_1^{\perp,q \rightarrow h}(z, K_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^{q \rightarrow h}(z, K_T^2)}$$

Modulations in spin-independent SIDIS cross section

SIDIS cross section



$$\frac{d^5 \sigma}{dx dy dz d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ A(y) F_{UU,T} + B(y) F_{UU,L} + C(y) \cos \phi_h F_{UU}^{\cos \phi_h} + B(y) \cos 2\phi_h F_{UU}^{\cos 2\phi_h} \right\}$$

leading twist
 $F_{UU}^{\cos 2\phi_h} \propto C \left[\frac{2(\hat{P}_{h\perp} \cdot \vec{k}_T)(\hat{P}_{h\perp} \cdot \vec{p}_T) - \vec{k}_T \cdot \vec{p}_T}{MM_h} h_1^\perp H_1^\perp \right]$ BOER-MULDERS EFFECT

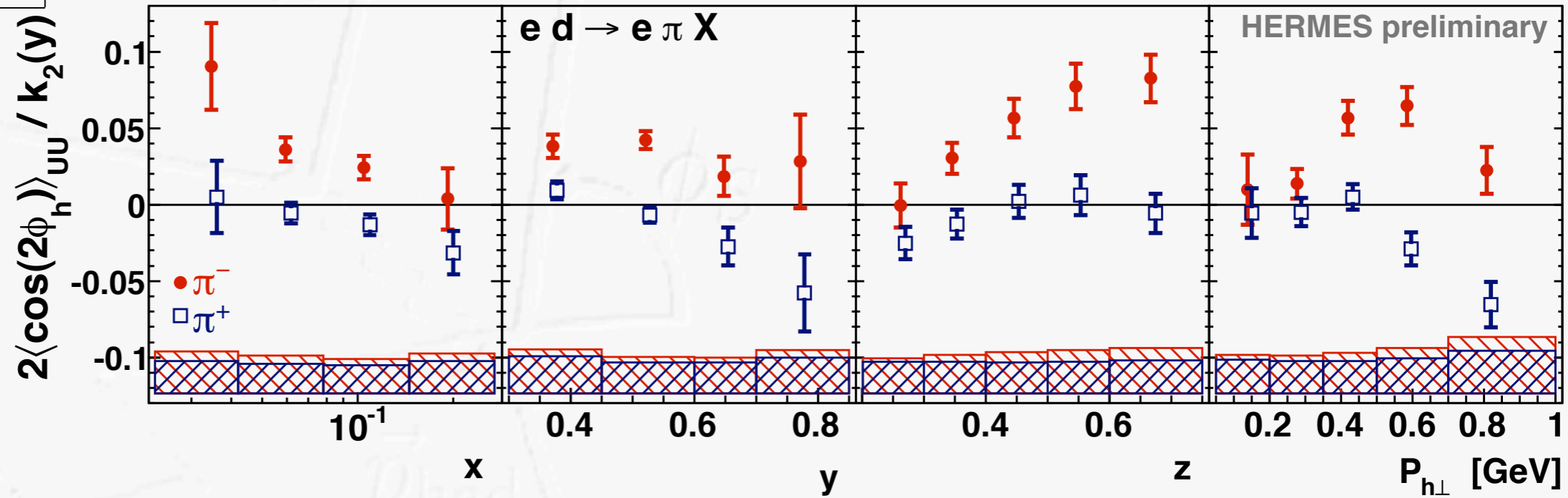
next to leading twist
 $F_{UU}^{\cos \phi_h} \propto \frac{2M}{Q} C \left[\frac{\hat{P}_{h\perp} \cdot \vec{p}_T}{M_h} x h_1^\perp H_1^\perp - \frac{\hat{P}_{h\perp} \cdot \vec{k}_T}{M} x f_1 D_1 + \dots \right]$ CAHN EFFECT

Interaction dependent terms neglected

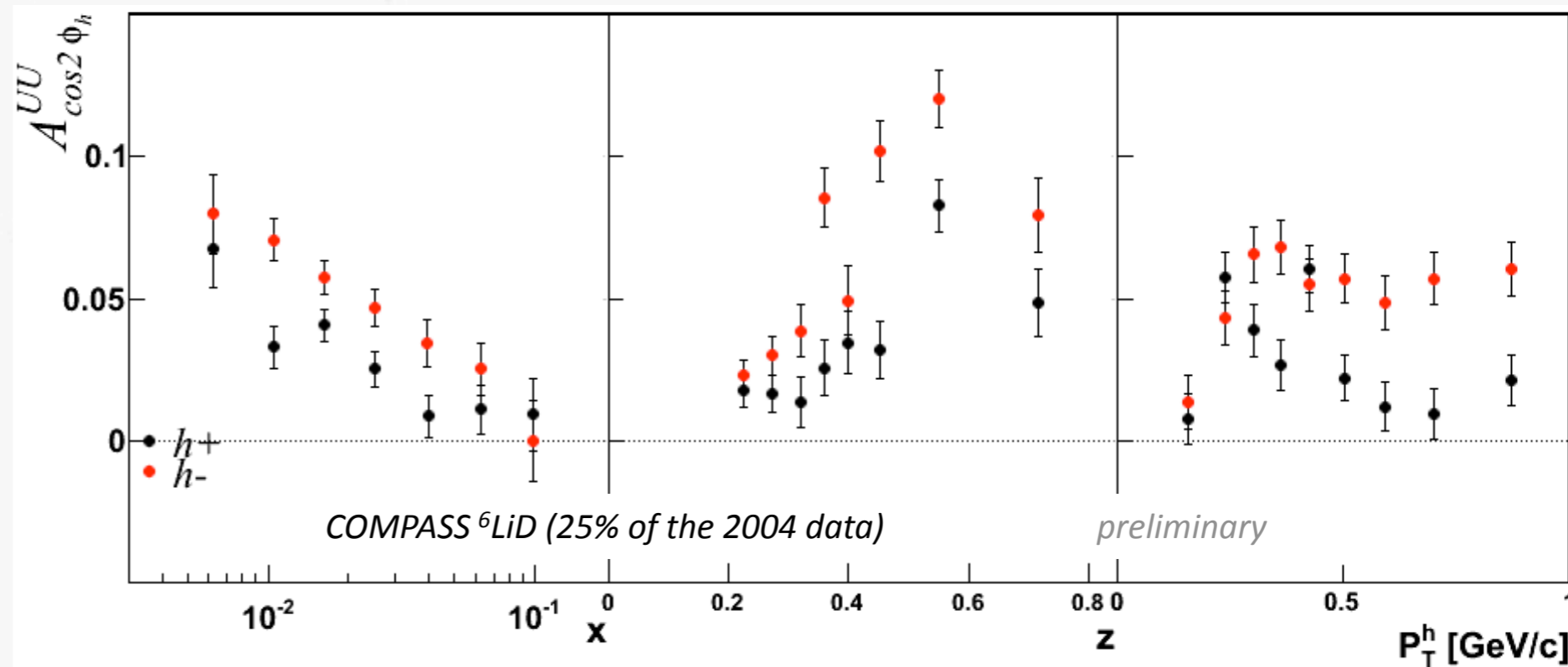
(Implicit sum over quark flavours)

Signs of Boer-Mulders

	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

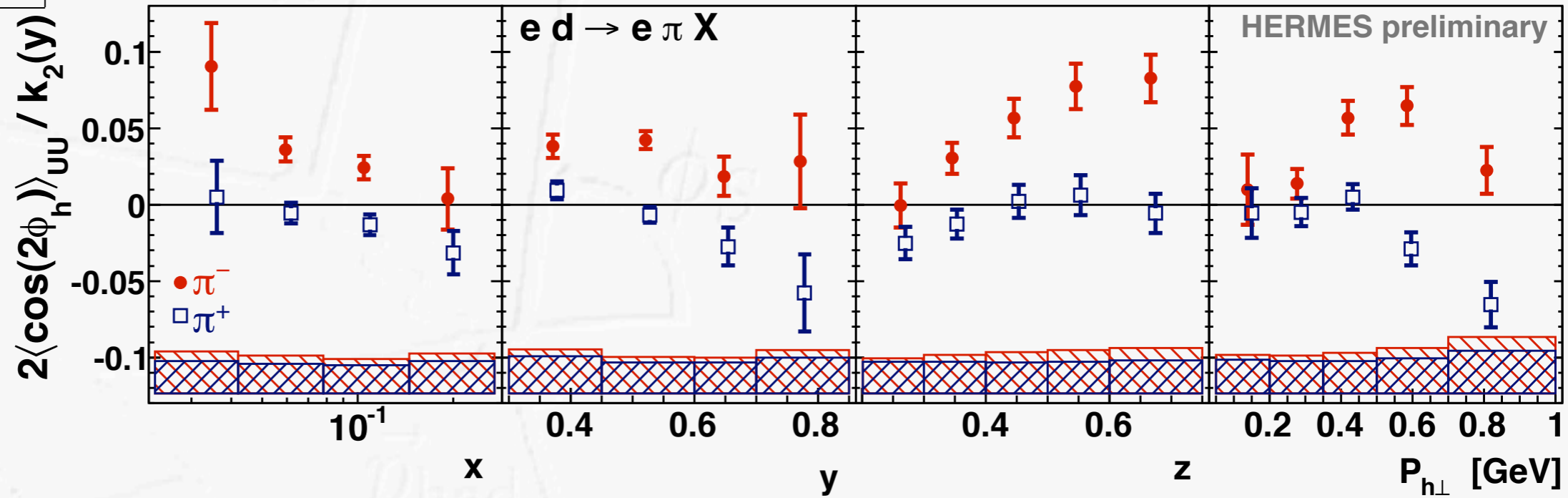


non-zero!

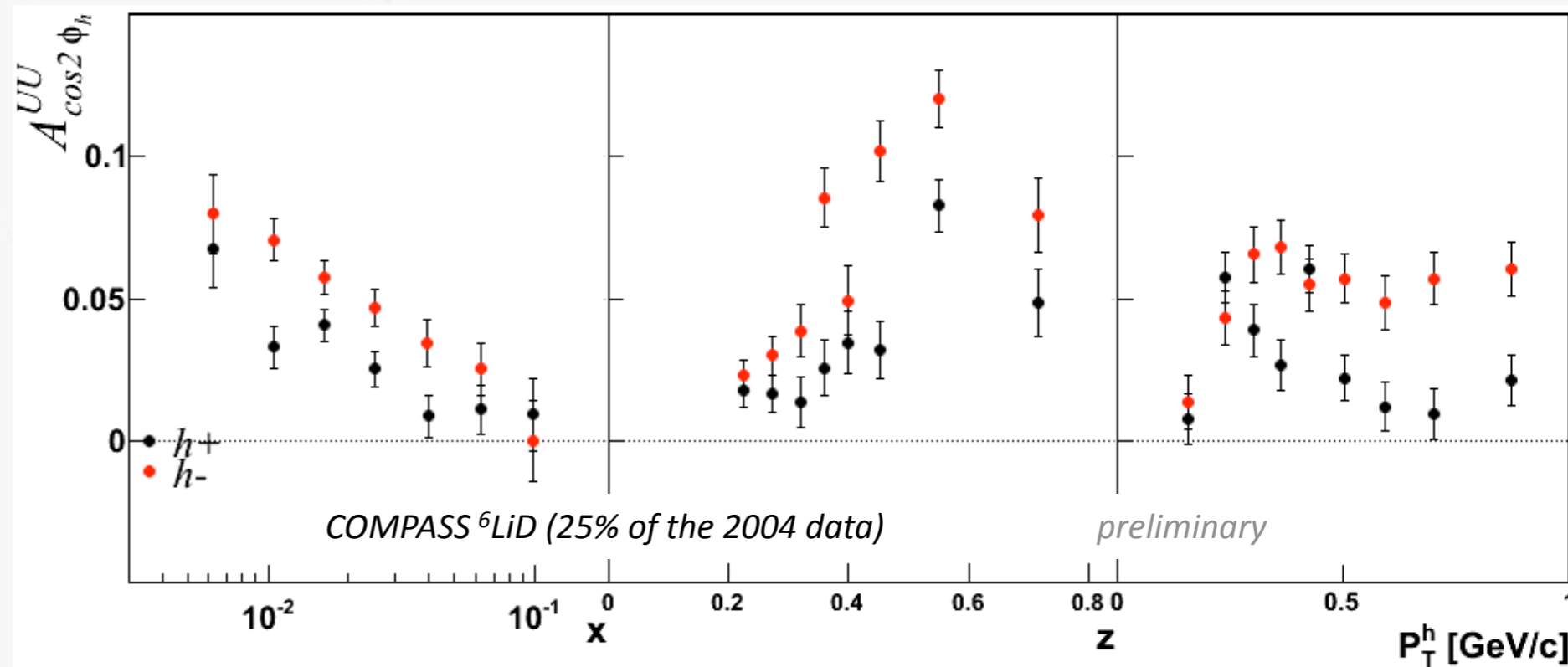


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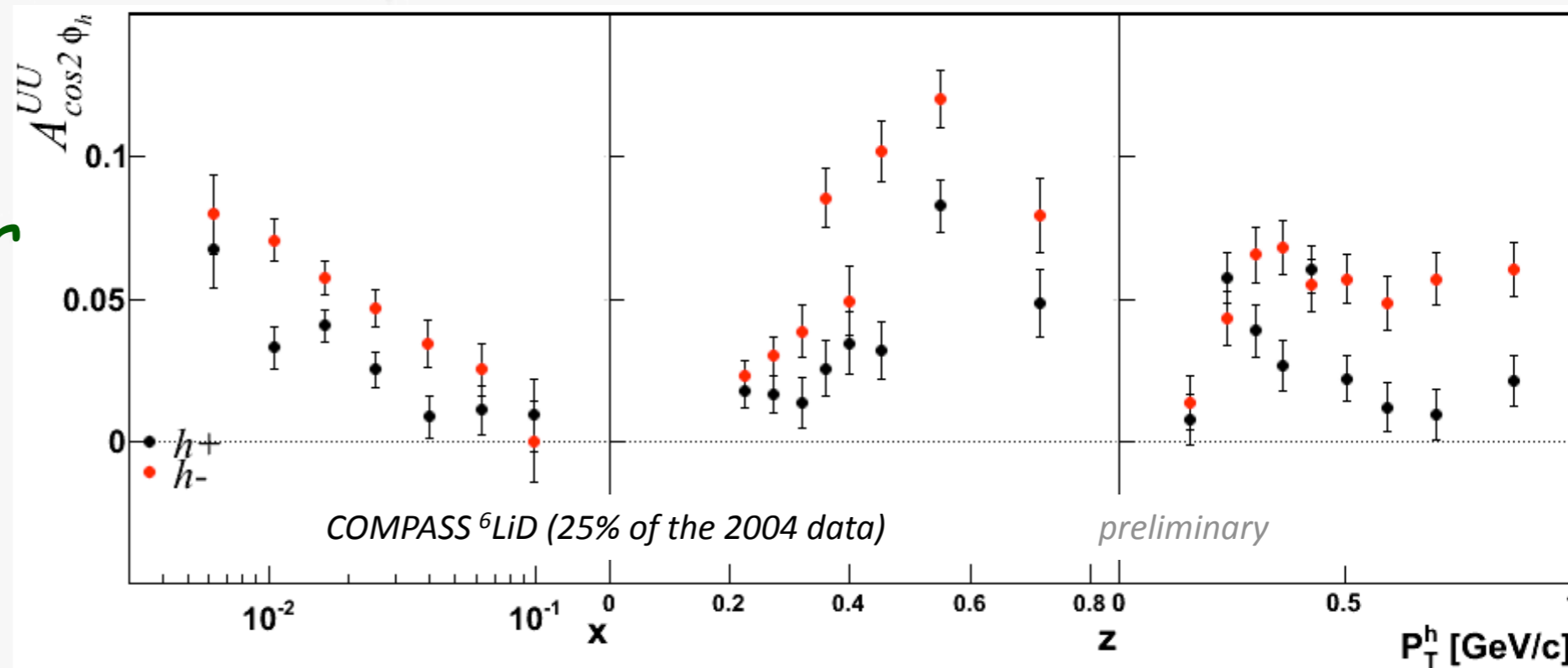
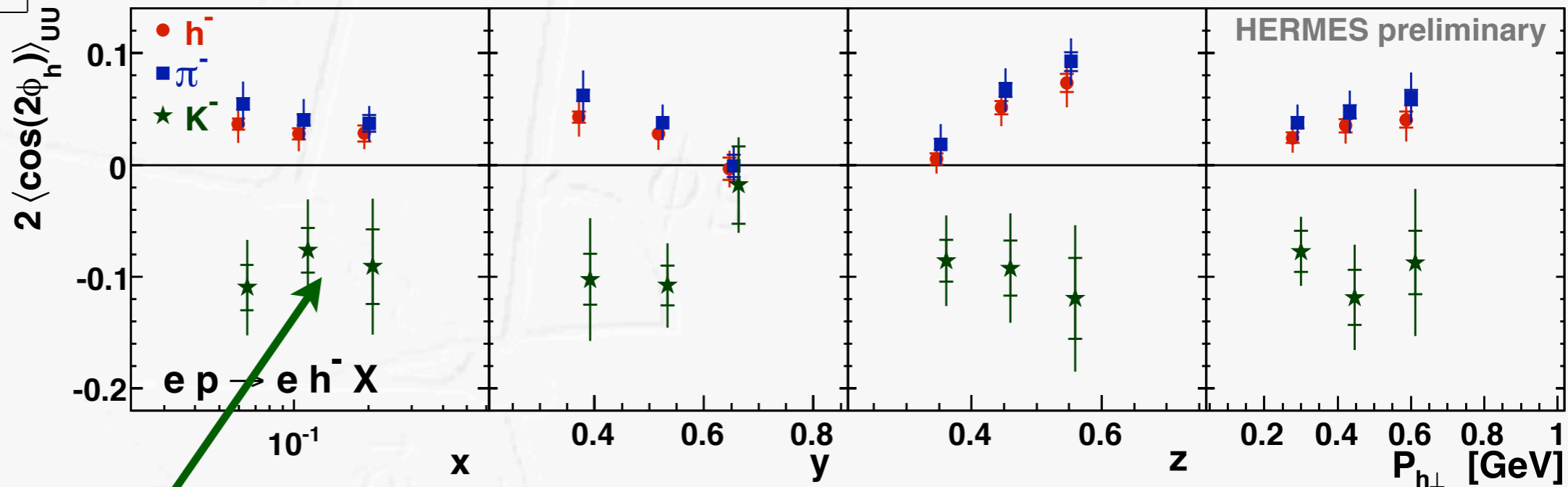


non-zero!
data consistency?



Signs of Boer-Mulders

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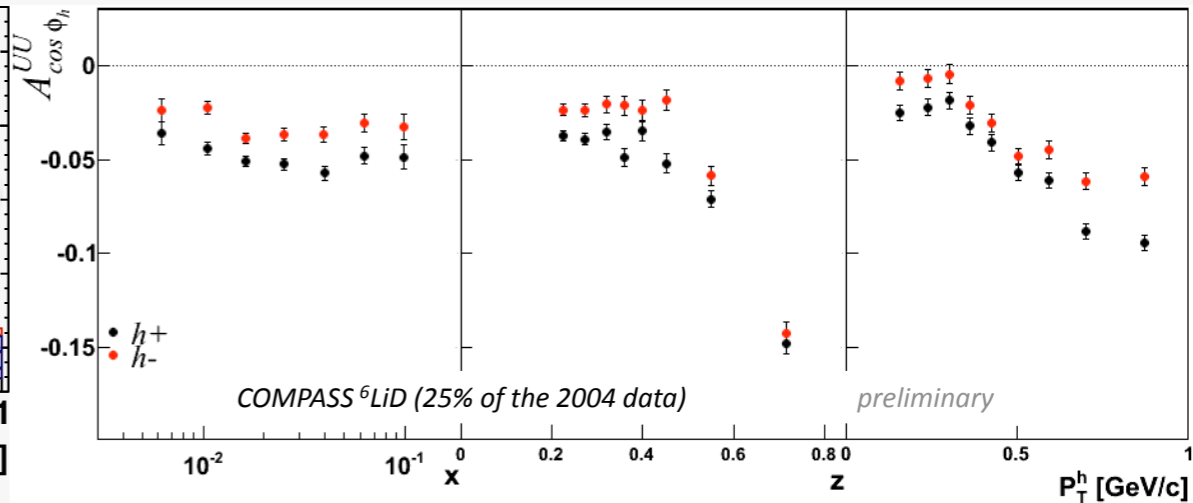
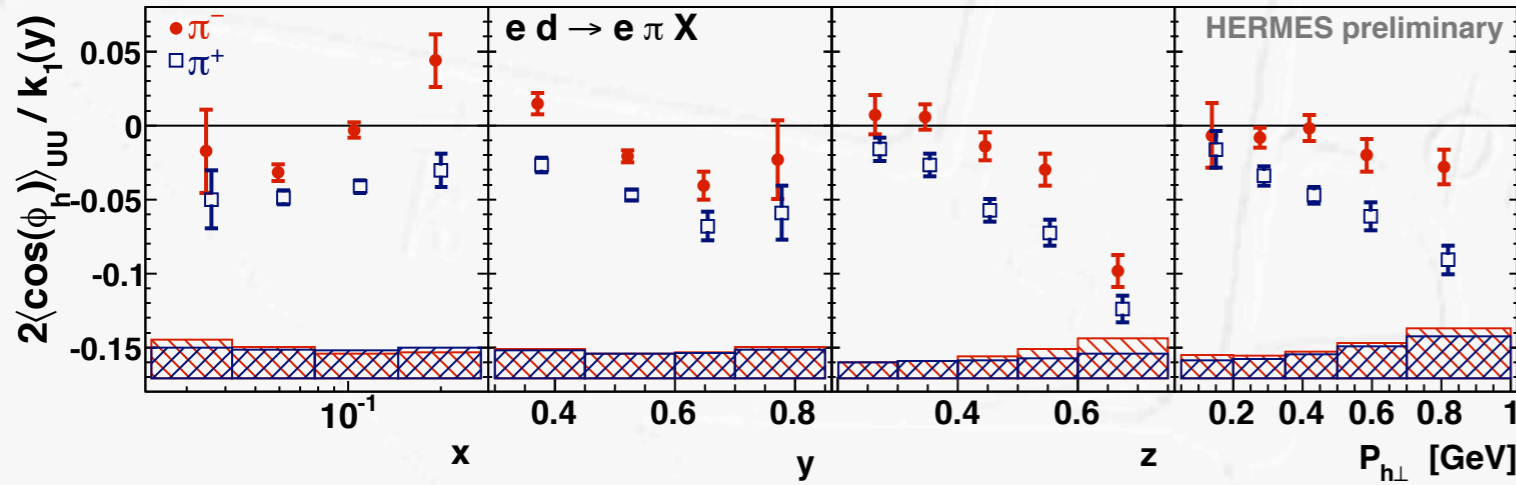
intriguing behavior
for kaons

Cahn effect?

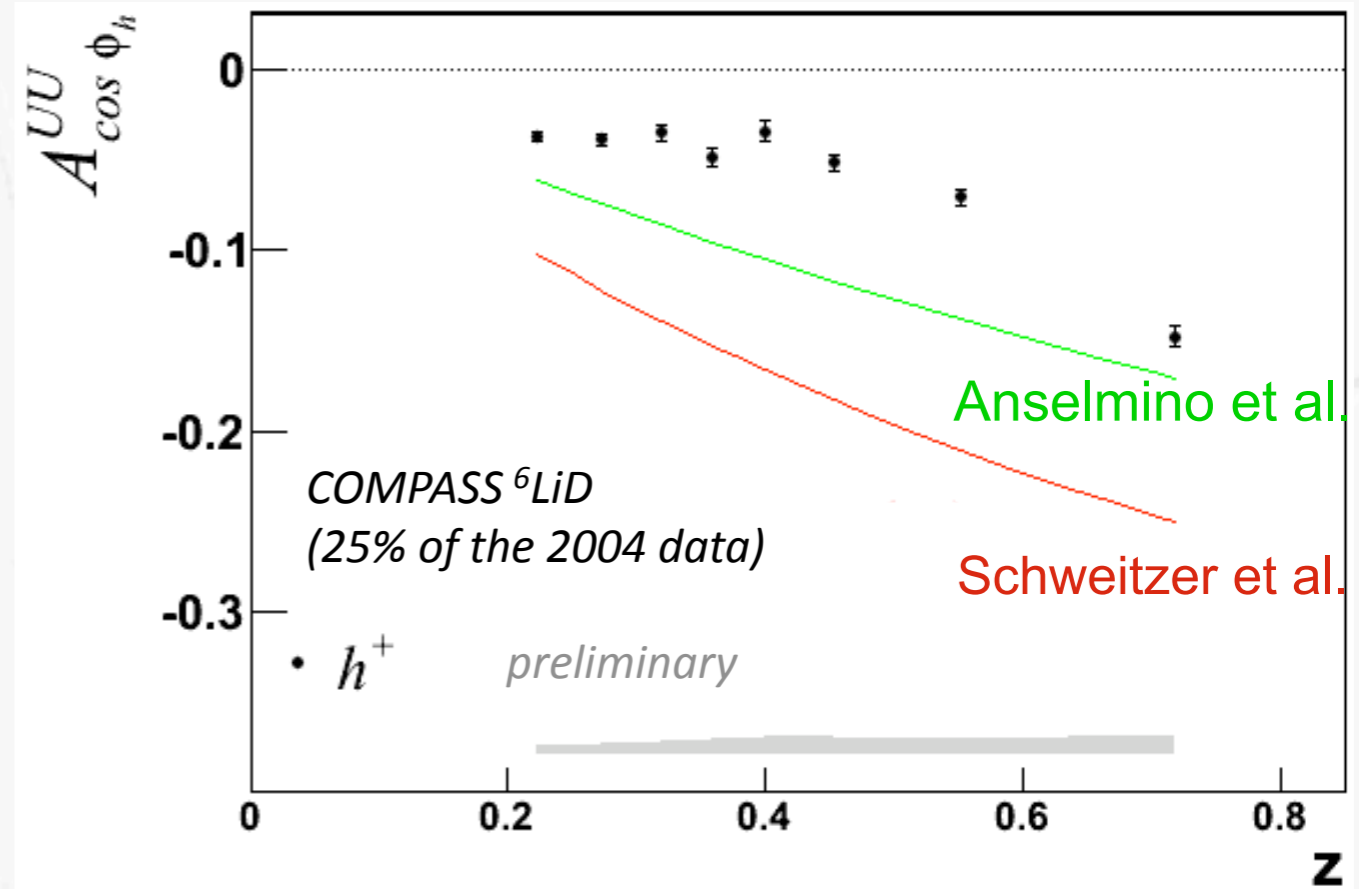
next to leading twist

$$F_{UU}^{\cos\phi_h} \propto \frac{2M}{Q} C \left[\underbrace{-\frac{\hat{P}_{h\perp} \cdot \vec{p}_T}{M_h} x h_1^\perp H_1^\perp}_{\text{BOER-MULDERS EFFECT}} - \underbrace{\frac{\hat{P}_{h\perp} \cdot \vec{k}_T}{M} x f_1 D_1}_{\text{CAHN EFFECT}} + \dots \right]$$

Interaction dependent terms neglected



- no dependence on hadron charge expected
- prediction off from data

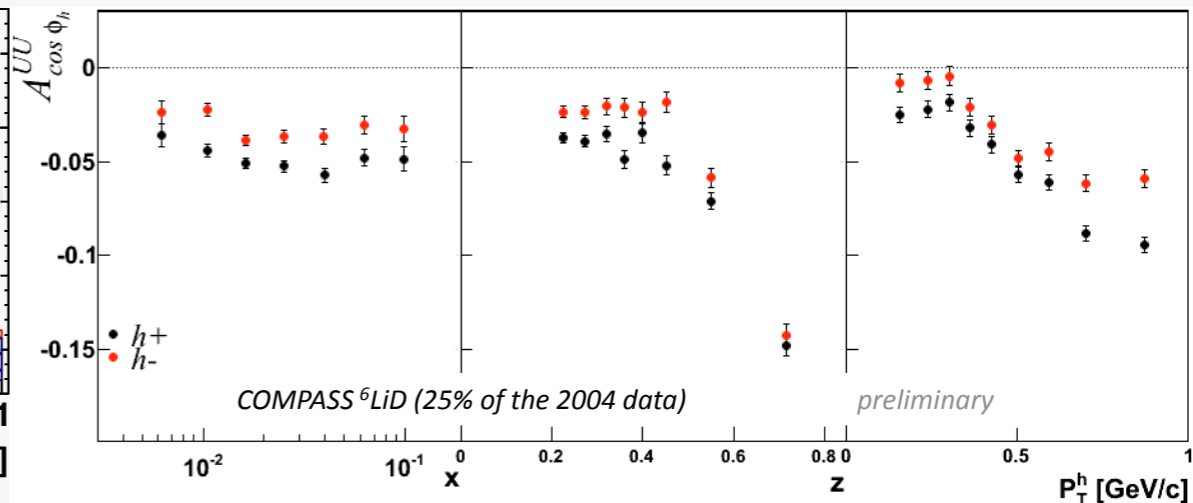
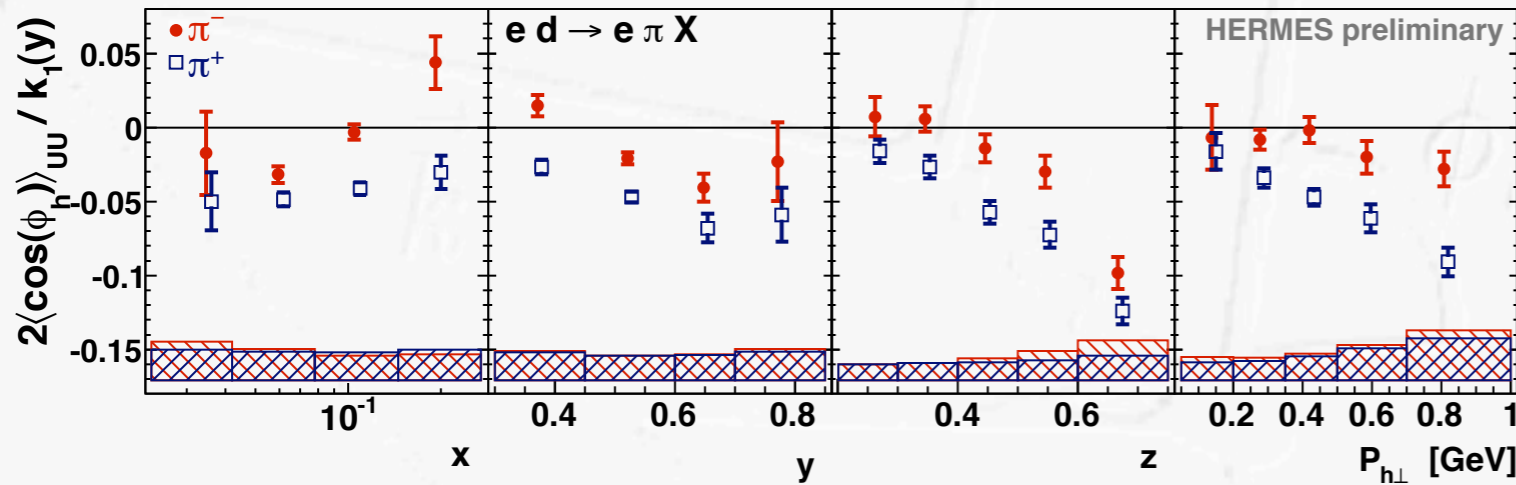


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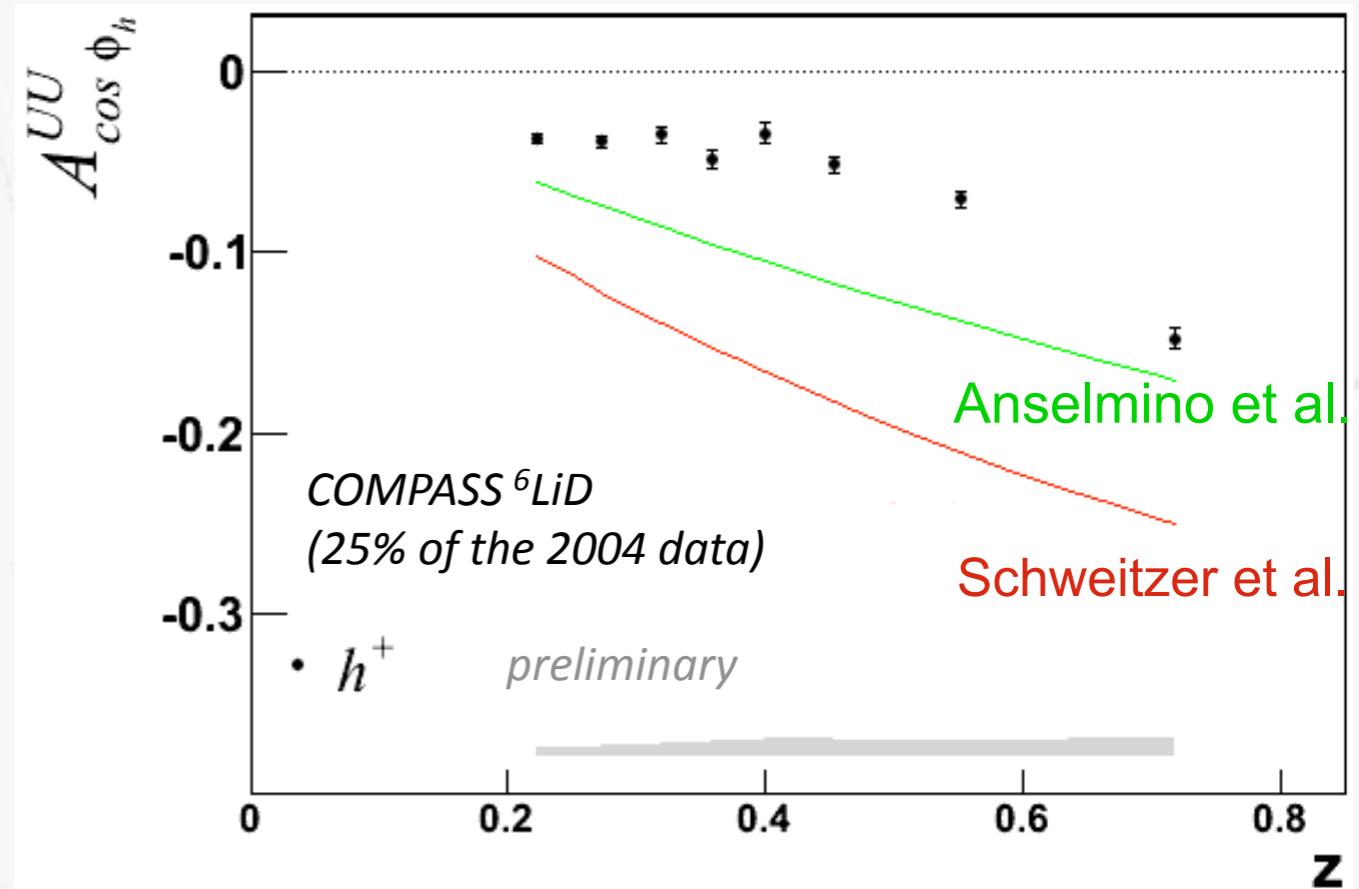
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Interaction dependent terms neglected



- no dependence on hadron charge expected
- prediction off from data
- ➔ sign of Boer-Mulders in $\cos\phi$ modulation or genuin twist-3?




Summary

- first round of SIDIS measurements coming to an end
- transversity is non-zero and quite sizable
 - can be measured, e.g., via Collins effect or s-p interference in 2-hadron fragmentation
- Sivers and Boer-Mulders effects are also non-zero
 - direct probe of "physics of the QCD gauge links"
- so far no sign of a non-zero pretzelosity distribution
- first evidence for non-vanishing worm-gear functions



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 - precision measurements at future SIDIS facilities
 - fundamental QCD tests in Drell-Yan experiments

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 - direct probe of "physics of the QCD gauge links"
- so far no sign of a non-zero pretzelosity distribution
- first evidence for non-vanishing worm-gear functions
- let's prepare for
 - precision measurements at future SIDIS facilities  Harut's talk
 - fundamental QCD tests in Drell-Yan experiments  Les' talk

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