Comments on angular momentum definitions

Dieter Müller
BNL/Ruhr-University Bochum

Overview of proposed nucleon spin s_z=1/2 decompositions

Reminder: observables in Classical and Quantum Mechanics

What to use in an interacting Field Theory such as QCD?

QCD sum rules for momentum and angular momentum

Conclusions

Overview of proposed decompositions

1987/88 EMC measurements (partonic vs. effective degrees of freedom)

⇨ $\Delta q/2 = s_Q \neq 1/2\,, \quad s_Q = \sum_{q=u,d,s,\cdots} s_q$ spin crises of the quark model

1990 Manohar & Jaffe $\Delta q/2 + L_Q^{\rm can} + \Delta g + L_G^{\rm JM} = 1/2$ ⟺ "Coloumb gauge"

there are also gluons and orbital angular momentum
(derived from free field theories – however, finally correct)

1996 Ji $J_Q^{\rm Ji} + J_G^{\rm Ji} = 1/2\,, \quad J_Q^{\rm Ji} = \Delta q/2 + L_Q\,, \quad J_G^{\rm Ji} = J_G$ ⟺ $J^{\rm Ji}_{\cdots} = J^{\rm W}_{\cdots}$

separation of quark and gluon degrees of freedom in gauge invariant quantities

1999 Bashinsky/Jaffe
another derivation of Manohar/Jaffe decomposition and reinterpretation

2007 Chen, Lu, Sun, Wang & Goldman $\Delta q/2 + L_Q^{\rm CLSWG} + s_G^{\rm CLSWG} + L_G^{\rm CLSWG} = 1/2$

2010 Cho, Ge & Zhang $\Delta q/2 + L_Q^{\rm CGZ} + s_G^{\rm CGZ} + L_G^{\rm CGZ} = 1/2$

fixing gauge and defining new momentum and angular momentum expressions

2010 Wakamatsu $\Delta q/2 + L_Q + J_G = 1/2\,, \quad J_G = s_G^{\rm CLSWG} + L_G^{\rm CLSWG} + L_Q^{\rm can} - L_Q$

another decomposition of angular momentum

2011 Leader $\Delta q/2 + L_Q^{\rm can} + \cdots = 1/2$

likes to take canonical instead kinetic operators for definition

The attitudes (what confuses me):

Ji

"...The total quark (and hence the quark orbital) contribution is shown to be ***measurable*** *through virtual Compton scattering in a special kinematic region where single quark scattering dominates. This deeply-virtual Compton scattering (DVCS) has much potential to unravel the quark and gluon structure of the nucleon."*

Chen et al.

"... Explicitly gauge invariant spin and orbital angular momentum operators of quarks and gluons are obtained. This was previously thought to be an impossible task, and opens a ***more promising avenue*** *towards the understanding of the nucleon spin structure."*

"We examine the conventional picture that gluons carry about half of the nucleon momentum in the asymptotic limit. We show that this large fraction is due to an ***unsuitable definition of the gluon momentum*** *in an interacting theory.* ***If defined in a gauge-invariant and consistent way****, the asymptotic gluon momentum fraction is computed to be* ***only about one fifth****. This result suggests that the asymptotic limit of the nucleon spin structure should also be reexamined. A possible* ***experimental test*** *of our finding is discussed in terms of novel PDFs."*

- partonic parts are not observables, might be called quasi-observables, which in the best cases are only accessible or **"measurable"**
- all decompositions are correct, since their sums provide the QCD result, based on

$$\langle\widetilde{T}_{\alpha\mu}\rangle = 2p_\alpha p_\mu\,, \quad \langle\widetilde{M}_{\alpha\mu\nu}\rangle = \mathrm{FT}\left(\langle x_\mu T_{\alpha\nu}\rangle - \{\mu\leftrightarrow\nu\}\right)$$

$$= 2i\partial_\nu p_\alpha p_\mu + \tfrac{1}{2}\epsilon_{\mu\nu pS}\,p_\alpha - \tfrac{1}{2}\epsilon_{\alpha\mu pS}\,p_\nu - \{\mu\leftrightarrow\nu\}$$

- What is the preferred, i.e., phenomenological and theoretical usable, decomposition?

$$T_{\alpha\mu} = T^Q_{\alpha\mu} + T^G_{\alpha\mu}\,, \quad M_{\alpha\mu\nu} = M^Q_{\alpha\mu\nu} + M^G_{\alpha\mu\nu}$$

Reminder: observables in Classical and Quantum Mechanics

Hamiltonian Mechanics

consider a particle with mass *m* and charge *q* in an electromagnetic field

$$\text{action:} \quad S = \int_{t_0}^{t_1} dt\, \mathcal{L}\,, \quad \mathcal{L} = T - V = \tfrac{1}{2} m \dot{\mathbf{x}}^2 + q\, \dot{\boldsymbol{x}} \cdot \boldsymbol{A}(t, \boldsymbol{x}) - q\, U(t, \boldsymbol{x})$$

$$\text{canonical momenta:} \quad \mathbf{p} = m\, \dot{\boldsymbol{x}} + q\, \mathbf{A}(t, \mathbf{x})$$

$$\text{dynamics:} \ \dot{\boldsymbol{x}} = \frac{\partial H}{\partial \mathbf{p}} \quad \dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \mathbf{x}} \quad H(t, \mathbf{x}) = \frac{1}{2m} \left(\mathbf{p} - q\, \mathbf{A}(\mathbf{t}, \mathbf{x})\right)^2 + q\, U(t, \mathbf{x})$$

$$\text{or:} \ m \frac{d^2 \boldsymbol{x}}{dt^2} = \mathbf{F}_{\text{Lorentz}}\,, \quad \mathbf{F}_{\text{Lorentz}} = q\, \mathbf{E} + q\, \dot{\mathbf{x}} \times \mathbf{B} \quad \mathbf{E} = -\frac{\partial U}{\partial \mathbf{x}} - \frac{\partial \mathbf{A}}{\partial t} \quad \mathbf{B} = \frac{\partial}{\partial \mathbf{x}} \times \mathbf{A}$$

observed is the trajectory ***x(t)*** or derived truly observables

$$\text{kinetic momenta:} \ \boldsymbol{\pi} = m\, \dot{\mathbf{x}} = \mathbf{p} - q\, \mathbf{A}(t, \mathbf{x})$$

$$\text{kinetic angular momenta:} \ \mathbf{L} = \mathbf{x} \times m\, \dot{\mathbf{x}} = \mathbf{x} \times \left(\mathbf{p} - q\, \mathbf{A}(t, \mathbf{x})\right)$$

lessons from Classical Physics

- observables are independent on the choice of gauge
- kinetic rather canonical quantities are associated with observables
- ***gauge field is an auxiliary tool, providing beauty and simplicity***

➢ cumbersome live: eliminate the unphysical degrees in ***A***, *i.e.*, ***p*** *is measurable, too*

quantization (postulate within simplicity)

- consider position and canonical momentum as operators
  (***x***,***p***) ⇨ (***x***, ***p***=-$i\partial_x$), common commutator ***[x,x] = [p,p] = 0***, ***[x,p] = i 1***

- construct Hamiltonian (take care on operator order, guided by gauge invariance)

$$H(t,\mathbf{x}) = \frac{1}{2m}\left(\boldsymbol{p} - q\,\boldsymbol{A}(t,\boldsymbol{x})\right)^2 + q\,U(t,\boldsymbol{x})\,, \quad \boldsymbol{p} = -i\boldsymbol{\partial_x}$$

- solve Schrödinger equation and calculate observables

$$i\frac{\partial}{\partial t}\psi(t,\boldsymbol{x}) = H\psi(t,\boldsymbol{x}) \quad \text{NOTE: } \psi \text{ depends on } \boldsymbol{A} \text{ (on the gauge choise)}$$

$$\Rightarrow \quad \langle\boldsymbol{x}\rangle(t)\,,\langle\boldsymbol{\pi}\rangle(t) \ = \langle\boldsymbol{p} - q\,\boldsymbol{A}\rangle\,,\langle\text{Ł}\rangle(t) = \langle\boldsymbol{x}\times(\boldsymbol{p} - q\,\boldsymbol{A})\rangle$$

Is there a problem with commutator relations?

$$[\pi_i, \pi_j] = iq\,\epsilon_{ijk}B_k$$
$$[L_i, L_j] = i\epsilon_{ijk}\left(L_k + x_k\,q\,\boldsymbol{x}\cdot\boldsymbol{B}\right)$$

No, it is just Quantum Mechanics as it is.
(uncertainty relation, Aharonov-Bohm effect, Zeeman effect)

also Quantum Mechanics is based on gauge invariance
(same applies for Quantum Field Theory – not discussed**)**

Orbital angular momentum of light

angular momentum of light:

$$\boldsymbol{J} = \int d^3\boldsymbol{x}\,\boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})$$ *

contains spin and orbital angular momentum
plan wave *radial wave*

suppose a (spread out) light beam interacts with a particle and one might ask How much spin and orbital angular momentum is transferred to the particle?

Which components of ***E*** and ***B*** provide orbital angular momentum transfer?

I. to answer experimentalists use * **[Allen, Beijersbergen, Spreeuw, Woerdman 1992]**

II. reshuffle terms in ***J***, use Helmholtz decomposition and Field equations

$$\boldsymbol{A}(t,\boldsymbol{x}) = \boldsymbol{A}_\perp(t,\boldsymbol{x}) + \boldsymbol{A}_{||}(t,\boldsymbol{x}) \quad \text{with} \quad \boldsymbol{\partial}_{\boldsymbol{x}}\cdot\boldsymbol{A}_\perp(t,\boldsymbol{x}) = 0\,, \quad \boldsymbol{\partial}_{\boldsymbol{x}}\times\boldsymbol{A}_{||}(t,\boldsymbol{x}) = 0$$

to find *one answer* (monochromatic light **[Humblet, 1943]**) *out of three* **[Stewart, A.M. 10]**

$$\boldsymbol{J} = \boldsymbol{S} + \boldsymbol{L}\,, \quad \boldsymbol{S} = \int d^3\boldsymbol{x}\,\boldsymbol{E} \times \boldsymbol{A}_\perp\,, \quad \boldsymbol{L} = \int d^3\boldsymbol{x} \sum_{i=1}^{3} \boldsymbol{E}_i\,(\boldsymbol{x} \times \boldsymbol{\partial}_{\boldsymbol{x}} \boldsymbol{A}_{\perp i})$$

where $\boldsymbol{A}_\perp(t,\boldsymbol{x}) = \boldsymbol{\partial}_{\boldsymbol{x}} \times \int d^3y\, \frac{\boldsymbol{B}(t,\boldsymbol{y})}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}$ is gauge invariant

Does *S* (*L*) *component only transfer spin (orbital angular momenta)*?
Assuming displayed one holds, is an analogous QCD experiment doable?
(gluon with J_G scatters on quark with s_q and L_q and then "measure" *s* and *L* transfer)
Certainly, not in DIS nor in DVCS.

What to use in an interacting Field Theory?

- Noether theorem provides canonical energy and angular momentum tensor
- one might use equation-of-motions (EOM) and drop surface terms for beauty
  (not necessary and not relevant for physics, however, alters partonic interpretation)
- partonic decomposition of physical quantities yields quasi-observables

➢ interacting terms might be ambiguously reshuffled by using EOM

➢ even quasi-observables should be gauge invariantly defined

➢ have in mind which quasi-observables can be "measured"

$T^{\alpha\mu}(x) = T_Q^{\alpha\mu}(x) + T_G^{\alpha\mu}(x)$ decomposition as done for momentum sum rule

$$T_Q^{\alpha\mu}(x) \;=\; \frac{1}{4}\overline{\psi}(x)\left[\gamma^\alpha i\overleftrightarrow{D}^\mu + \gamma^\mu i\overleftrightarrow{D}^\alpha\right]\psi(x) + \text{counterterms} + \delta^{\text{BRST}}\cdots\,,$$

$$T_G^{\alpha\mu}(x) \;=\; -F^\alpha{}_\kappa(x)F^{\mu\kappa}(x) + \frac{1}{4}g^{\alpha\mu}F^{\kappa\lambda}(x)F_{\kappa\lambda}(x) - \text{counterterms} + \delta^{\text{BRST}}\cdots\,,$$

one might adopt this decomposition for the orbital angular momentum, too

$M_{\cdots}^{\alpha\mu\nu} = x^\mu T_{\cdots}^{\alpha\nu} - x^\nu T_{\cdots}^{\alpha\mu}$ **NOTE:** $\partial_\alpha M_{\mathrm{Q}}^{\alpha\mu\nu} = -\partial_\alpha M_{\mathrm{G}}^{\alpha\mu\nu} + \mathrm{EOM} + \delta^{BRST}\cdots$

some well-known algebra provides for the dimensional regularized theory

$$M_Q^{\alpha\mu\nu}(x) = \frac{1}{2}\overline{\psi}(x)\gamma^\alpha\Sigma^{\mu\nu}\psi(x) + \frac{1}{2}\overline{\psi}(x)\gamma^\alpha\left[x^\mu i\vec{D}^\nu - x^\nu i\vec{D}^\mu\right]\psi(x) + \text{h.c.} + \cdots$$

$$M_G^{\alpha\mu\nu}(x) = x^\nu F^\alpha{}_\kappa(x)F^{\mu\kappa}(x) + \frac{1}{4}x^\mu g^{\alpha\nu}F^{\kappa\lambda}(x)F_{\kappa\lambda}(x) - \{\mu \leftrightarrow \nu\} + \cdots$$

Comparison with Ji`s expressions

$$M_{Q,Ji}^{\alpha\mu\nu} = \lim_{n\to 4} M_Q^{\alpha\mu\nu} + \overline{\psi}\left[\gamma^\alpha\Sigma^{\mu\nu} - i\epsilon^{\alpha\mu\nu\kappa}\gamma_\kappa\gamma^5\right]\psi$$

$$M_{G,Ji}^{\alpha\mu\nu} = \lim_{n\to 4} M_G^{\alpha\mu\nu}$$

Is there a γ^5 problem if we switch in the sum rule to polarized PDF moment?

Comparison with free field expressions (Manohar & Jaffe derivation)

$$M_{Q,JM}^{\alpha\mu\nu} + g\,\overline{\psi}\gamma^\alpha t^a\psi\left(x^\mu A^{a\nu} - x^\nu A^{a\mu}\right) \simeq M_Q^{\alpha\mu\nu}$$

$$M_{G,JM}^{\alpha\mu\nu} + \left[D_\kappa F^{\alpha\kappa}\right]^a\left(x^\mu A^{a\nu} - x^\nu A^{a\mu}\right) \simeq M_G^{\alpha\mu\nu}$$

NOTE: EOM tells that heuristic derivation of Manohar & Jaffe is correct

$$\left[D_\kappa F^{\alpha\kappa}(x)\right]^a + g\,\overline{\psi}(x)\gamma^\alpha t^a\psi(x) = \delta^{\text{BRST}}(\cdots) \simeq 0$$

Chen, Lu, Sun, Wang & Goldman expressions

$$A^{a\mu} = A_\perp^{a\mu} + A_\parallel^{a\mu}\,, \quad f^{abc}\vec{A}_\perp^b\,\vec{E}_\perp^c = 0\,, \quad \left[\vec{D}_\parallel \times \vec{A}_\parallel\right]^a = 0$$

$$M^{\alpha\mu\nu}_{Q,\mathrm{CLSWG}} + g\,\overline{\psi}\gamma^\alpha t^a\psi\,(x^\mu A_\perp^{a\nu} - x^\nu A_\perp^{a\mu}) \simeq M_Q^{\alpha\mu\nu}$$

$$M^{\alpha\mu\nu}_{G,\mathrm{CLSWG}} + [D_\kappa F^{\alpha\kappa}]^a\,(x^\mu A_\perp^{a\nu} - x^\nu A_\perp^{a\mu}) \simeq M_G^{\alpha\mu\nu}$$

Cho, Ge & Zhang another gauge fixing to find a gauge invariant decomposition

Wakamatsu suggestion:

$$M_Q^{\alpha\mu\nu}\,, \quad M^{\alpha\mu\nu}_{G,\mathrm{CLSWG}} - g\,\overline{\psi}\gamma^\alpha t^a\psi\,(x^\mu A_\perp^{a\nu} - x^\nu A_\perp^{a\mu}) \simeq M_G^{\alpha\mu\nu}$$

Pragmatic suggestion (already used):

$$M_Q^{\alpha\mu\nu}\,, \quad M^{\alpha\mu\nu}_{G,\mathrm{JM}} + [D_\kappa F^{\alpha\kappa}]^a\,(x^\mu A^{a\nu} - x^\nu A^{a\mu}) \simeq M_G^{\alpha\mu\nu}$$

yielding $J_G = s_G + L_G$ with s_G given by the lowest moment of $\Delta g(x,Q^2)$

Pros and cons for decomposition of nucleon spin $s_z = 1/2$

Jaffe & Manohar decomposition

- L_Q, s_G, and L_G are expected to be gauge dependent
- J_Q and J_G are not accessible in phenomenology and are ambiguous
- $s_Q = \Delta q/2$ and in light-cone gauge $s_G = \Delta g$ holds

Ji`s decomposition is preferred

- gauge invariance and no gauge fixing is used in quasi-observables
- J_Q and J_G are (somehow) “measurable” in phenomenology and lattice QCD
- $J_Q = \Delta q/2 + L_Q$ is given as a gauge invariant decomposition
- J_G can be decomposed as one likes, e.g., $J_G = \Delta g + L_G$

Chen, Lu, Sun, Wang & Goldman and Cho, Ge & Zhang decomposition

- their L_Q, s_G, and L_G are defined via gauge fixing (looks cumbersome to me)
- not a unique decomposition – what is the physical motivation?
- quantities are mostly not related to elaborated pQCD framework
- How to “measure” them in phenomenology and lattice QCD?

Leader`s comment

- Is the canonical $\langle L \rangle$ gauge independent? (for me canonical $\langle p \rangle$ is gauge depender

QCD sum rules for momentum and angular momentum

momentum sum rule follows from gauge invariant decomposition of $T^{++}=1$

$$\sum_q \langle P|\mathrm{R}\bar{\psi}_q\gamma^+ D^+\psi_q|_{\mu^2}|P\rangle + \langle P|\mathrm{R}F^{+\mu}F_\mu{}^+|_{\mu^2}|P\rangle = 1\,, \quad P^+ = 1$$

and fits together with the pQCD framework

$$\langle P|\mathrm{R}\bar{\psi}\gamma^+ D^+\psi|_{\mu^2}|P\rangle = \int_0^1 dx\, xq(x,\mu^2)\,, \quad \langle P|\mathrm{R}F^{+\mu}F_\mu{}^+|_{\mu^2}|P\rangle = \int_0^1 dx\, xg(x,\mu^2)$$

where the quark/gluon separation is provided by the factorization scheme, i.e., normalization conditions for R-operation are implicit

$$\begin{aligned} F_2(x_B,Q^2) \;&=\; \sum_q \int_{x_B}^1 \frac{dx}{x}\, C_q\left(\frac{x_B}{x}, \frac{Q^2}{\mu^2}, \alpha_s(\mu)\right) q(x,\mu^2) \\ &\quad + \int_{x_B}^1 \frac{dx}{x}\, C_g\left(\frac{x_B}{x}, \frac{Q^2}{\mu^2}, \alpha_s(\mu)\right) g(x,\mu^2) \end{aligned}$$

scheme used for evaluation of *C*`s determines the value of *q* and *g* PDFs
momentum sum rule is used as a constrain in global fits (not a quantitative test)

the conventional pQCD separation of quarks and gluons provide:

- that at asymptotical large scales *<x q>* $\approx$ *<x g>* $\approx$*1/2*
- this prediction agrees (? accidently) roughly with phenomenology findings
- the phenomenological values are rather robust against radiative corrections

Chen, Lu, Sun, Wang & Goldman and also Cho, Ge & Zhang suggestion:

$$\langle P|\bar{\psi}_q \gamma^+ D^+ \psi_q|P\rangle = \langle P|\bar{\psi}_q \gamma^+ D^+_{\|} \psi_q|_{\mu^2}|P\rangle + \langle P|\bar{\psi}_q \gamma^+ g A^+_{\perp} \psi_q|P\rangle$$

$$\langle P|F^{+\mu} F_{\mu}{}^{+}|P\rangle \overset{\mathrm{EOM}}{=} \langle P|(F^{+i} D^+_{\|} A^i_{\perp})|P\rangle - \sum_q \langle P|\bar{\psi}_q \gamma^+ g A^+_{\perp} \psi_q|P\rangle$$

- factorization theorems are valid for common PDFs not for the suggested ones
- increases the number of PDFs
- Are their new PDFs (matrix elements) closed under renormalization?
- claim that in the asymptotic limit their *<x g>* $\approx$ *<x q> /4* $\approx$*1/5*
  (at least LO calculation of spin-2 anomalous dimensions should be given)
- Can this be understood as an additive **infinite** scheme transformation?
  (Is this consistent with a NLO calculation of DIS structure functions?)

Uses of angular momentum sum rules

gauge invariant decomposition within no gauge fixing (Ji`s) provides sum rules

$2 J_Q = A_Q + B_Q,\ 2 J_G = A_G + B_G$ *with* $A_Q + A_G = 1$ *and* $B_Q + B_G = 0$

$A_{Q/G}$ and $B_{Q/G}$ are given in terms of the same renormalized twist-two operators

$A_{Q/G}$ and $B_{Q/G}$ are scheme dependent quantities (also on the considered order)
(renormalization is required on Lattice too and has to match the factorization scheme)

the momentum sum rule is implemented in global PDF fits
the gravitomagnetic sum rule can be (is) implemented in global GPD fits

the separation of spin and orbital angular momentum,

$J_Q = s_Q + L_Q$

is based on the use of EOM

requires that the renormalization of J_Q, s_Q, and L_Q matches each other
(note that L_Q might be calculated on Lattice, too)

Does $L_Q = J_Q - s_Q$, $2J_Q = A_Q + B_Q$ holds true in a model calculation?

- equation-of-motion (dynamics) is usually ignored by LCWF model builders take effective two-body LCWF (scalar diquark)

$$\boldsymbol{\psi}^{\rm sca}(X,\mathbf{k}_\perp) = \frac{1}{M}\begin{pmatrix} m+XM \\ -|\mathbf{k}_\perp|e^{i\varphi} \\ |\mathbf{k}_\perp|e^{-i\varphi} \\ m+XM \end{pmatrix} \frac{\overline{\phi}(X,\mathbf{k}_\perp)}{\sqrt{1-X}} \quad \begin{matrix} \Rightarrow\rightarrow & L=0 \\ \Rightarrow\leftarrow & L=+1 \\ \Leftarrow\rightarrow & L=-1 \\ \Leftarrow\leftarrow & L=0 \end{matrix}$$

one might now calculate all uPDFs, uGPDs, ..., FFs and also L from sum rule:

$$\langle L\rangle^{\rm sca} = \left\{\begin{matrix} 0.069 \\ 0.075 \\ -0.03 \end{matrix}\right\}, \quad \langle J\rangle^{\rm sca} = \left\{\begin{matrix} 0.19 \\ 0.20 \\ 0.33 \end{matrix}\right\} \quad \text{for} \quad \left\{\begin{matrix} \text{LCWF}^{\rm pow} \\ \text{LCWF}^{\rm exp} \\ \text{HM07} \end{matrix}\right\}$$

one might directly calculate orbital angular momentum (counting ΔL)

$$\langle\mathcal{L}\rangle^{\rm sca} \stackrel{\rm sca}{=} \int_0^1 dx\,(1-x)\int d^2\mathbf{k}_\perp |\psi_{\stackrel{\Rightarrow}{\leftarrow}}(x,\mathbf{k}_\perp)|^2$$

$$\langle\mathcal{L}\rangle^{\rm sca} = \left\{\begin{matrix} 0.33 \\ 0.33 \\ 0.11 \end{matrix}\right\}, \quad \langle\mathcal{J}\rangle^{\rm sca} = \left\{\begin{matrix} 0.46 \\ 0.46 \\ 0.47 \end{matrix}\right\} \quad \text{for} \quad \left\{\begin{matrix} \text{LCWF}^{\rm pow} \\ \text{LCWF}^{\rm exp} \\ \text{HM07} \end{matrix}\right\}$$

- exceptions: any Field Theory
  illustrated at LO in Yukawa theory and QED by **[Burkardt, BC 08]**

Conclusions

- there are many partonic decompositions of nucleon spin, which can be called correct
- one might even introduce more :-)
- gauge invariant decompositions without constraints are preferred since they fit best the elaborated pQCD formalism
- even then the decomposition of quark angular momentum might be more intricate as thought
- model calculations have a good chance to violate Ji`s sum rule
- spin and orbital angular momentum are just numbers that might be in some future reliable evaluated from Lattice
- these numbers might serve to constrain twist-two parton distributions, extracted from measurements