Vector Meson Production in QCD

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GPD representations

DVMP in the collinear factorization framework

Other frameworks

NLO corrections

DVCS+DVMP small x_B fits at LO

in collaboration with

M. Meškauskas, K. Passek-Kumerićki, T. Lautenschlager, A. Schäfer K. Kumerićki

M. Meśkauskas and D.M., 1112.2597 [hep-ph]
flexible GPD model fits@LO for small *x* and fits to vector mesons/DVCS H1/ZEUS data
C. Bechler and D.M., 0906.2571 [hep-ph]
GPD description of *π*+ electroproduction from HERMES/JLAB
T. Lautenschlager, DM, K.Passek-Kumerićki, A. Schäfer, to be polished
NLO corrections to DVMP in conformal space

Overview: GPD representations

``light-ray spectral functions"

diagrammatic α-representation DM, Robaschik, Geyer, Dittes, Hořejši (88 (92) 94)

called *double distributions* A. Radyushkin (96)

$\begin{array}{c} \underset{k+p_{1}}{\overset{\underset{k+p_{2}}{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\atopk+p_{2}}{\overset{\underset{k+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{\atopk+p_{2}}}{\overset{t$

light cone wave function overlap

(Hamiltonian approach in light-cone quantization)

SL(2,R) (conformal) expansion

(series of local operators)

one version is called Shuvaev transformation, used in `dual' (*t*-channel) GPD parameterization

Diehl, Feldmann, Jakob, Kroll (98,00) Diehl, Brodsky, Hwang (00)

Radyushkin (97); Belitsky, Geyer, DM, Schäfer (97); DM, Schäfer (05);

Shuvaev (99,02); Noritzsch (00) Polyakov (02,07)

each representation has its own *advantages*, 2 however, they are *equivalent* (clearly spelled out in [Hwang, DM 07])

Summing up conformal PWs

• GPD support is a consequence of Poincaré invariance (polynomiality)

$$H_j(\eta, t, \mu^2) = \int_{-1}^{1} dx \, c_j(x, \eta) H(x, \eta, t, \mu^2) \,, \qquad c_j(x, \eta) = \eta^j C_j^{3/2}(x/\eta)$$

• conformal moments evolve autonomous (to LO and beyond in a special scheme)

$$\mu \frac{d}{d\mu} H_j(\eta, t, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \gamma_j^{(0)} H_j(\eta, t, \mu^2)$$

• inverse relation is given as series of mathematical distributions:

$$H(x,\eta,t) = \sum_{j=0}^{\infty} (-1)^{j} p_{j}(x,\eta) H_{j}(\eta,t) , \ p_{j}(x,\eta) \propto \theta(|x| \le \eta) \frac{\eta^{2} - x^{2}}{\eta^{j+3}} C_{j}^{3/2}(-x/\eta)$$

various ways of resummation were proposed:

smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]

- mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
- dual parameterization [M. Polyakov, A. Shuvaev (02)]
- based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
- *Mellin-Barnes integral* [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

Sommerfeld-Watson transform

 \checkmark rewrite sum as an integral around the real axis:

$$F(x,\eta,\Delta^2) = \frac{1}{2i} \oint_{(0)}^{(\infty)} dj \, \frac{1}{\sin(\pi j)} p_j(x,\eta) \, F_j(\eta,\Delta^2)$$

 find appropriate analytic continuation of p_j and F_j (Carlson's theorem)

$$p_{j}(x,\eta) = \theta(\eta - |x|)\eta^{-j-1}\mathcal{P}_{j}\left(\frac{x}{\eta}\right) + \theta(x-\eta)\eta^{-j-1}\mathcal{Q}_{j}\left(\frac{x}{\eta}\right)$$
$$\mathcal{P}_{j}(x) = \frac{2^{j+1}\Gamma(5/2+j)}{\Gamma(1/2)\Gamma(1+j)}(1+x) {}_{2}F_{1}\left(\frac{-j-1,j+2}{2}\Big|\frac{1+x}{2}\right)$$
$$\mathcal{Q}_{j}(x) = -\frac{\sin(\pi j)}{\pi} {}_{x}r^{-j-1} {}_{2}F_{1}\left(\frac{(j+1)/2,(j+2)/2}{5/2+j}\Big|\frac{1}{x^{2}}\right)$$

change integration path so that singularities remain on the l.h.s.

$$F(x,\eta,\Delta^2) = \frac{i}{2} \int_{c-i\infty}^{c+i\infty} dj \, \frac{1}{\sin(\pi j)} p_j(x,\eta) \, F_j(\eta,\Delta^2)$$

Model based on SL(2,R) and SO(3) PWE

• SL(2,R) GPD moments: $F_j(\eta, t) = \sum_{J=J^{\min}} f_j^J(t) \eta^{j+1-J} \hat{d}_J(\eta)$

partial wave amplitudes reduced Wigner depending on j and J rotation matrices

 taking 2 better 3 SO(3) PWs: (two parameters s₂ and s₄)

$$f_j^{j-1}(t) = s_2 f_j^{j+1}(t),$$

$$f_j^{j-3}(t) = s_4 f_j^{j+1}(t),$$

• resulting CFF easy to handle:

 $\mathcal{F} = \frac{1}{2i} \sum_{k=0}^{4} \int_{c-i\infty}^{c+i\infty} dj \,\xi^{-j-1} \frac{2^{j+1+k}\Gamma(5/2+j+k)}{\Gamma(3/2)\Gamma(3+j+k)} \left(i - \frac{\cos(\pi j) \mp 1}{\sin(\pi j)}\right) \times s_k E_{j+k}(\mathcal{Q}^2) f_j^{j+1}(t) \hat{d}_j(\xi), \quad s_0 = 1$ • zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha't} \left(1 - \frac{t}{M_j^2}\right)^{-p}$ • zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha't} \left(1 - \frac{t}{M_j^2}\right)^{-p}$ • zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha't} \left(1 - \frac{t}{M_j^2}\right)^{-p}$ • zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha't} \left(1 - \frac{t}{M_j^2}\right)^{-p}$ • zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha't} \left(1 - \frac{t}{M_j^2}\right)^{-p}$ • zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha't} \left(1 - \frac{t}{M_j^2}\right)^{-p}$ • zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha't} \left(1 - \frac{t}{M_j^2}\right)^{-p}$ • zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha't} \left(1 - \frac{t}{M_j^2}\right)^{-p}$ • zero-skewness GPD: $h_j^{j+1} = q_j \frac{j+1-\alpha(0)}{j+1-\alpha(0)-\alpha't} \left(1 - \frac{t}{M_j^2}\right)^{-p}$

Collinear factorization framework

at large Q² **longitudinal** DVMP amplitude factorizes at twist-two level into a hard scattering part, GPD, and a meson distribution amplitude (DA) [Collins, Frankfurt, Strikman 98], e.g.,



collinear factorization = integrating out k_{τ}

perturbative + non-perturbative corrections

$$\mathcal{F}(x_{\mathrm{B}}, t, \mathcal{Q}^{2}) = \frac{C_{F} f_{\mathrm{M}}}{N_{c} \mathcal{Q}} F(x, \xi, t, \mu_{\mathrm{F}}^{2}) \overset{x}{\otimes} T\left(\frac{\xi - x}{2\xi}, \bar{v} \middle| \cdots\right) \overset{v}{\otimes} \varphi(v, \mu_{\varphi}^{2}) \Big|_{\substack{\xi = \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}}}{\overset{1}{\bar{u}} = 1 - u}, \quad \bar{v} = 1 - v$$

at LO in α_s twist-two amplitudes "factorize"

$$\mathcal{H}^{pV}\left(x_{\mathrm{B}}, t, \mathcal{Q}^{2}\right) \stackrel{\mathrm{LO}}{=} \frac{4\alpha_{s}(\mu_{\mathrm{R}})}{9} \frac{f_{V}}{\mathcal{Q}} 3\mathcal{I}^{V}\left(\mu^{2}\right) \mathcal{H}^{pV}\left(x_{\mathrm{B}}, t, \mu^{2}\right)$$

in inverse moment of meson distribution amplitude

$$\mathcal{I}^{V}(\mu^{2}) = \frac{1}{3} \int_{0}^{1} du \, \frac{\varphi^{V}(u, \mu^{2})}{u} \,, \qquad \int_{0}^{1} du \, \varphi^{V}(u, \mu^{2}) = 1 \,,$$
and GPD part "CFFs" , e.g.,

$$\mathcal{H}^{q(+)}(x_{\rm B}, t, \mu^2) \stackrel{\rm LO}{=} \int_{-1}^{1} dx \, \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] H^q(x, \xi, t, \mu^2) \Big|_{\xi = x_{\rm B}/(2 - x_{\rm B})}$$

factorization might be broken at twist-3 level due to the initial/final state interaction indicated by endpoint singularities at LO "pragmatic point of view" (introduce a cut-off) [Mankiewicz et. al (98)]



Other frameworks

GPD inspired model of Goloskokov/Kroll (contains quark + gluons)



color dipole models in various fashions [A. Mueller (90), ... ,Kowalski et. al, ...]



applicable at small x, Regge inspired

non-perturbative vector-meson WF

only contains **other** gluons (**universal color dipole amplitude**)

[picture from Kowalski, Motyka, Watt (06)]

Regge inspired models [Laget et al] (also applied at low W)

NLO corrections

- NLO hard scattering coefficients are calculated in momentum fraction space
- i. non-singlet quark-quark channel [Belitsky, DM, 01] (extend form factor hard scattering part
- ii. pure singlet quark-quark channel [Ivanov, Szymanowski, Krasnikov 04, PK et al.]
- iii. gluon-quark channel [Ivanov, Szymanowski, Krasnikov 04]



iv. NLO quark-gluon channel is still missing (needed for η/η')

NLO hard scattering part are to be transformed in conformal Mellin-space (to implement in our flexible GPD model framework)

• NLO evolution kernels in conformal moment space and momentum fraction [Belitsky, DM, 98] and [Belitsky, DM, Freund 00]

NLO example: pure singlet quark-quark channel

momentum fraction representation

$${}^{\Sigma}T^{(1)}\left(\overline{u},\overline{v}\left|\frac{\mathcal{Q}^{2}}{\mu_{\mathrm{F}}^{2}}\right) = C_{\mathrm{F}}{}^{\Sigma}T^{(1,F)}\left(\overline{u},\overline{v}\left|\frac{\mathcal{Q}^{2}}{\mu_{\mathrm{F}}^{2}}\right), \quad \overline{u} = 1-u, \overline{v} = 1-v$$

mathematically trivial terms (factorization in u and v terms except for $\Delta T^{(1,G)}$)



nontrivial term (free of 1/(u-v) singularities) can be nicely written as

$$\Delta^{\Sigma} T^{(1,\mathrm{F})}(u,v) = \frac{d}{dv} \frac{L(u,v)}{u-v} + \frac{\overline{u}-u}{v \overline{v}} \frac{L(u,v)}{u-v}$$
$$L(u,v) = \mathrm{Li}_2(v) - \mathrm{Li}_2(u) + \ln \overline{v} \ln u - \ln \overline{u} \ln u$$

dispersion relation representation – imaginary part

dispersion relation allows to evaluate the real part from the imaginary one

$$\Re \mathfrak{R} \mathfrak{H}^{pV}\left(x_{\mathrm{B}}, t, \mathcal{Q}^{2}\right) = \frac{1}{\pi} \mathrm{PV} \int_{0}^{1} dx \, \frac{2x}{x^{2} - \xi^{2}} \Im \mathfrak{M} \mathcal{H}^{pV}\left(\frac{2x}{1+x}, t, \mathcal{Q}^{2}\right) + \mathcal{C}\left(t, \mathcal{Q}^{2}\right)|_{\xi = \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}}$$

GPD in the outer region is needed

"D-term" projection is needed to calculate the subtraction constant

imaginary part can be straightforwardly evaluated $(x > \xi)$

$$\Im \mathfrak{MF}(x_{\mathrm{B}}, t, \mathcal{Q}^{2}) = \frac{C_{F} f_{\mathrm{M}}}{N_{c} \mathcal{Q}} F(x, \xi, t, \mu_{\mathrm{F}}^{2}) \overset{x}{\otimes} T^{\Im \mathrm{m}} \left(\frac{x}{\xi}, \overline{v} \middle| \cdots \right) \overset{v}{\otimes} \varphi(v, \mu_{\varphi}^{2}) \Big|_{\xi = \frac{x_{\mathrm{B}}}{2 - x_{\mathrm{B}}}}$$

$$= \left[\ln \frac{\mathcal{Q}^{2}}{\mu_{\mathrm{F}}^{2}} + \ln \frac{1 - x}{1 + x} + \ln(v\overline{v}) - 1 \right] \frac{1}{x(1 + x)v\overline{v}}$$

$$+ \frac{1 - x}{1 + x} \left[1 - 2\ln \frac{1 + x}{2xv} - \frac{2vx}{1 + x - 2xv} \ln \frac{1 + x}{2xv} \right] \frac{1}{(1 + x - 2vx)\overline{v}}$$

reminder conformal representation of distribution amplitudes

conformal partial wave expansion of meson distribution amplitude

$$\varphi(u,\mu^2) = \sum_{\substack{k=0 \\ \text{even}}}^{\infty} 6u\bar{u} C_k^{3/2}(u-\bar{u}) \varphi_k(\mu^2), \quad \varphi_0 = 1, \quad \bar{u} = 1-u$$

conformal partial wave amplitudes evolve autonomously at LO

$$\int_0^1 du \, T(u)\varphi(u) \quad \Rightarrow \quad \sum_{\substack{k=0 \\ \text{even}}} T_k\varphi_k \qquad T_k = \int_0^1 du \, T(u) \, 6u\bar{u} \, C_k^{3/2}(u-\bar{u})$$

to calculate T_k one might use Rodrigues formula + partial integration

$$T_k = \frac{3(2+k)}{k!} \int_0^1 du \, (u\bar{u})^{k+1} \frac{d^k}{du^k} T(u)$$

simple LO example: $T(u) = \frac{1}{u}$ $T_k = 3(2+k)(-1)^k \int_0^1 du \, u^{k+1} = 3(-1)^k$

all NLO expressions (LO decorated by In and Li₂ functions) are analytically known

fixing $(-1)^k = \sigma = \pm 1$ allows to employ Carlson theorem to find expressions for complex *k*

(analytic continuation of a function with integer argument)

conformal representation of transition form factors

$${}^{\Sigma}T^{(1,\mathrm{F})}(u,v) \quad \Rightarrow \quad {}^{\Sigma}T^{(1,\mathrm{F})}_{jk} \propto \int_{0}^{1} du \int_{0}^{1} dv \, u\bar{u} \, C_{j}^{3/2} (u-\bar{u})^{\Sigma}T^{(1,\mathrm{F})}(u,v) v\bar{v} \, C_{k}^{3/2} (v-\bar{v})$$

NLO hard scattering part posses the form

$$T = \sum_{f,g} f(u) g(v) + \delta T(u,v) \qquad \qquad T_{jk} = \sum_{f,g} f_j g_k + \delta T_{jk}$$
known find appropriate representation

use "double dispersion relation" to get moments of δT_{jk} for complex j (k)

$$\delta^{\Sigma} T^{(1,F)}(u,v) = \int_0^1 dy \int_0^1 dz \, \frac{1}{1-uy} \, \frac{1}{y+z-yz} \left[\frac{y}{z} - 1 - z \, \frac{\vec{d}}{dz} \right] \frac{z}{1-\bar{v}z}$$

conformal moments of 1/(1-uy) are easily calculable and have the needed analytic properties ($v \in \{3/2, 5/2\}$)

$$\widetilde{p}_k^{(\nu)}(y) \propto \int_0^1 du \, \frac{1}{1 - uy} \, (u\bar{u})^{\nu - 1/2} \, C_k^{\nu}(u - \bar{u}) \, \Rightarrow \, \widetilde{p}_k^{(3/2)}(y) \propto y^k \int_0^1 \, du \, \frac{(u\bar{u})^{k+1}}{(1 - uy)^{k+1}}$$

analytical continuation of δT_{jk} can be numerical performed

$$\delta^{\Sigma} T_{jk}^{(1,F)} = \int_0^1 dy \int_0^1 dz \, \tilde{p}_j^{(3/2)}(y) \left[\frac{y}{z} - 1 - z \, \frac{\vec{d}}{dz}\right] z \tilde{p}_k^{(3/2)}(z) \tag{13}$$

conformal moment representation (pure singlet)

$$T_{j,k}^{\Sigma(1)}(Q^2/\mu_F^2) = T_{j,k}^{(0)} C_{\rm F} {}^{\Sigma} c_{j,k}^{(1,{\rm F})} \left(\frac{Q^2}{\mu_{\rm F}^2}\right)$$

j=0 pole is contained in anomalous dimension

$${}^{\rm G\Sigma}\gamma_j^{(0,{\rm F})} = -2\frac{(j+1)(j+2)+2}{j(j+1)(j+2)}$$

non-factorizable δc_{jk} terms in j,k can be numerically calculated for complex j (k)

- analog structure in NS and gluon channels
- δc_{jk} might be also analytically evaluated (they are harmless at j=0, large j or large k)

Size of NLO corrections

[Belitsky, DM 01] large NLO corrections in the NS sector (proportional to C_{F} , β_0)

known from pion form factor:

- removing β_0 term will push $\alpha_s(\mu_R)$ into non-perturbative region
- C_F proportional corrections indicate Sudakov suppression (different sign)

discussion how to set Brodsky, Lepage, Mackenzie scale to eliminate β_0 proportional hard scattering coefficient (dispersion relation) [Pire et al., Brodsky et al.]

[Ivanov, Szymanowski, Krasnikov 04] big NLO corrections at small x

[Diehl, Kugler 07] model studies without NLO evolution

analytic expressions allows easily to understand nature of NLO corrections

- big at small x (j=0 pole) and large at large x (large j)
- increase with growing k
- still a large scale dependence at NLO

NLO corrections are model dependent

pragmatic point of view:

- not much hope to understand power suppressed corrections
- work hard to evaluate radiative corrections (? resummation)
- explore how the collinear factorization approach works to describe data

$$\begin{split} & \frac{d\sigma^{\gamma^{*}p \to \gamma p}}{dt} \stackrel{\mathrm{Tw} - 2}{\approx} \pi \alpha^{2} \frac{x_{\mathrm{B}}^{2}}{Q^{4}} \left| \mathcal{H} \left(x_{\mathrm{B}}, t, Q^{2} \right) \right|^{2} + \cdots \\ \mathcal{H} \stackrel{\mathrm{LO}}{=} \frac{4}{9} \mathcal{H}^{(u)+} + \frac{1}{9} \mathcal{H}^{(d)+} + \frac{1}{9} \mathcal{H}^{(s)+} + \frac{4}{9} \mathcal{H}^{(c)+} \\ & \mathsf{DVMP\ cross\ sections} \qquad \frac{d\sigma^{\gamma^{*}_{L}p \to Vp}}{dt} \stackrel{\mathrm{Tw} - 2}{\approx} 4\pi^{2} \alpha \frac{x_{\mathrm{B}}^{2}}{Q^{4}} \left| \mathcal{H}^{pV} \left(x_{\mathrm{B}}, t, Q^{2} \right) \right|^{2} + \cdots \\ & \mathcal{H}^{pV} \left(x_{\mathrm{B}}, t, Q^{2} \right) \stackrel{\mathrm{LO}}{=} \frac{4\alpha_{s}(\mu_{\mathrm{B}})}{9} \frac{f_{V}}{Q} 3\mathcal{I}^{V} \left(\mu^{2} \right) \mathcal{H}^{pV} \left(x_{\mathrm{B}}, t, \mu^{2} \right) \\ & \mathcal{I}^{V}(\mu^{2}) = \frac{1}{3} \int_{0}^{1} du \frac{\varphi^{V}(u, \mu^{2})}{u} , \qquad \int_{0}^{1} du \varphi^{V}(u, \mu^{2}) = 1 , \\ & \mathcal{H}^{pp^{0}} \stackrel{\mathrm{LO}}{=} \frac{1}{\sqrt{2}} \left(\frac{2}{3} \mathcal{H}^{u(+)} + \frac{1}{3} \mathcal{H}^{d(+)} + \frac{3}{4} \mathcal{H}^{G} \right) \\ & \mathcal{H}^{p\omega} \stackrel{\mathrm{LO}}{=} \frac{1}{\sqrt{2}} \left(\frac{2}{3} \mathcal{H}^{u(+)} - \frac{1}{3} \mathcal{H}^{d(+)} + \frac{1}{4} \mathcal{H}^{G} \right) \\ & \mathcal{H}^{q(+)}(x_{\mathrm{B}}, t, \mu^{2}) \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] \\ & \mathcal{H}^{q}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x_{\mathrm{B}}, t, \mu^{2}) \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} dx \frac{1}{2x} \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} dx \frac{1}{2x} \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} dx \frac{1}{2x} \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})}^{16} \\ & \mathcal{H}^{G}(x, \xi, t, \mu^{2}) \Big|_{\xi = x_{\mathrm$$

Cross sections and R-ratio

most of H1/ZEUS measurements are given for integrated cross section

$$\sigma(W, \mathcal{Q}^2) = \left[\varepsilon(W, \mathcal{Q}^2) + \frac{1}{R^{\exp}(\mathcal{Q}^2)}\right] \int_{|t_{\min}|}^{|t_{\mathrm{cut}}|} dt \, \frac{d\sigma_{\mathrm{L}}(x_{\mathrm{B}}, t, \mathcal{Q}^2)}{dt} \quad |t_{\mathrm{cut}}| < \mathcal{Q}^2$$

R-ratio is extracted via s-channel helicity conservation hypothesis it is assumed to be independent on *W* and *t* (*is not be entirely true*)



DVCS+DVMP fit to H1/ZEUS data

strategies: pure DVCS fit χ^2 /d.o.f. = 130/(126-3) DVCS + H1 DVMP fit χ^2 /d.o.f. = 342/(230-6) DVCS + H1/ZEUS DVMP fit -very soft gluon χ^2 /d.o.f. = 618/(304-7) confronting GK07 model with DVCS (χ^2 /n.o.p. = 226/126)

R and normalization errors are not taken into account, cut $Q^2 > 4$ GeV² for DVMP data

DVCS data dominated by quark GPD gluon GPD is to some extend not pinned down







r^{sea}~1 (small skewness effect at LO) of sea quarks are driven by DVCS data

 $r^{G} < 1$ gluon GPD is suppressed at LO

very soft gluon GPD is disfavored by DIS fit

GK07 model is based on NLO PDFs [(very) good DVCS description at LO] interchange of skewing and evolution provides a very desired GPD behavior remember Freund/McDermott could not reach DVCS description @LO with similar model

Conclusions

• LO fit to DVCS and DVMP works reasonably at small x_B

- contradicts common wisdoms
- exclusive physics in H1/ZEUS kinematics is dominated by gluons
- onset of perturbation regime is at $\sim 15 \text{ GeV}^2$ or so
- GPD interpretation for $Q^2 > 4 \text{ GeV}^2$ states that
- quark exchanges at small x_B are more important as thought
- gluons in off-forward kinematics are suppressed
- NLO corrections are available in conformal moment space
- to be implemented in NLO fitting routines to DVCS and DVMP
- lets see what comes out

another partonic GPD interpretation arises in GK framework