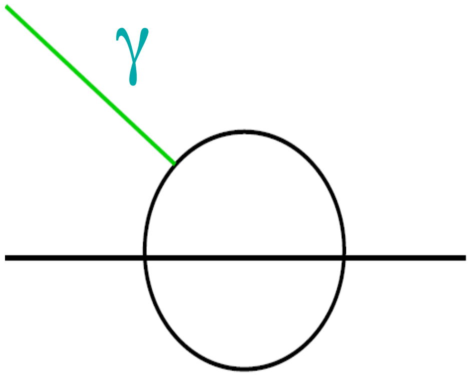
T even TMDs and Nucleon Deformation

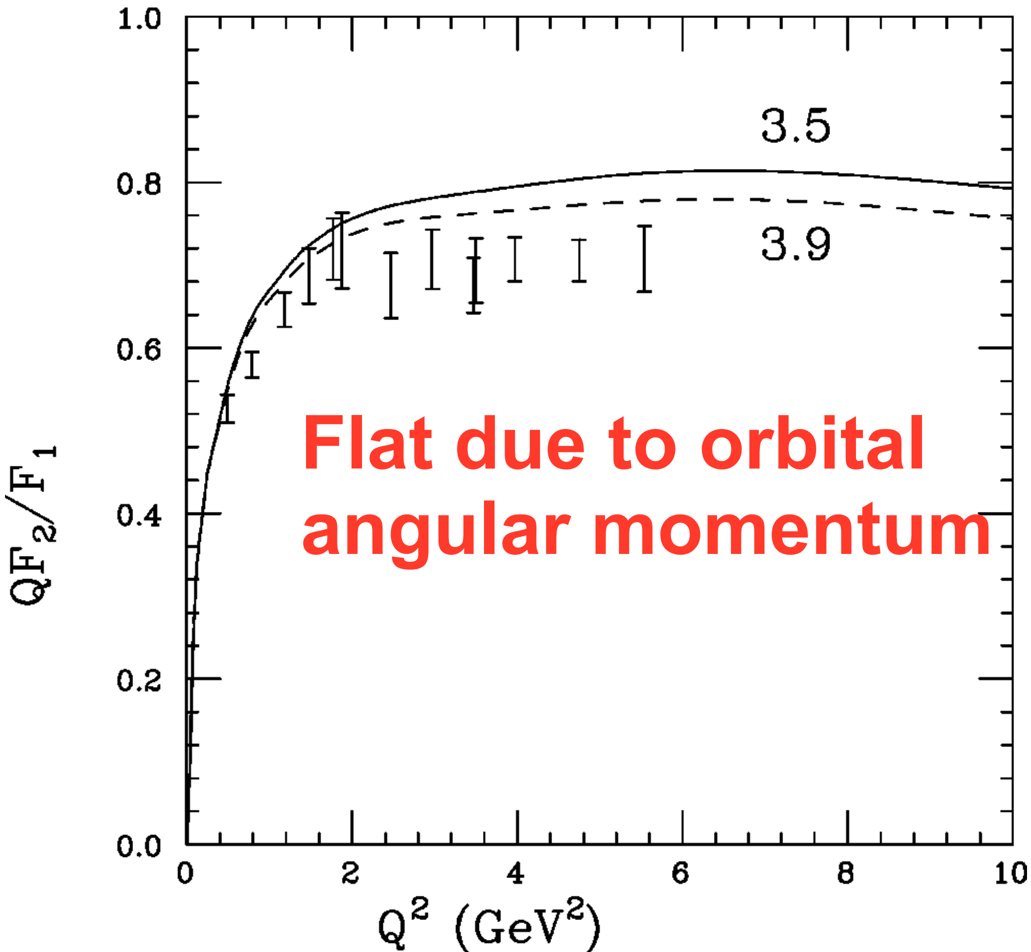
Gerald A. Miller

- Proton form factor, model calculation-
  proton not round via spin dependent density
- Model independent neutron charge density
- Measure shape of proton on lattice (impact parameter dependent GPD) coordinate-space probability, and in experiment (TMD): TMD is momentum-space probability
- GAM “Transverse Charge Densities” arXive: 1002.0355, Ann.Rev.Nucl.Part.Sci. 60 (2010) 1-25

Ratio of Pauli to Dirac Form Factors 1995 Frank,Jennings, Miller theory, data 2000

Impulse approximation





Model proton wave function $\Psi(\mathbf{k}_\perp, \mathbf{K}_\perp, \xi, \eta)$

Poincare invariant

Light front variables for boost: $\mathbf{K} \to \mathbf{K} + \eta \mathbf{q}_\perp$

Dirac spinors carry orbital angular momentum

Model exists

- lower components of Dirac spinor
- orbital angular momentum
- shape of proton?? Wigner Eckart
  no quadrupole moment
- spin dependent densities SDD

  non-relativistic example

I: Non-Rel. $p_{1/2}$ proton outside 0^+ core

$$\langle \mathbf{r}_p | \psi_{1,1/2s} \rangle = R(r_p) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}}_p | s \rangle$$

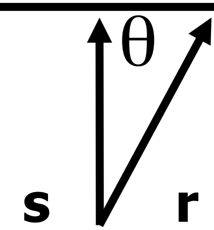
Binding pot'l rotationally invariant

$$\rho(r) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) | \psi_{1,1/2s} \rangle = R^2(r)$$

probability proton at $\mathbf{r}$ & spin direction $\mathbf{n}$:

$$\rho(\mathbf{r}, \mathbf{n}) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) \frac{(1 + \boldsymbol{\sigma} \cdot \mathbf{n})}{2} | \psi_{1,1/2s} \rangle$$

$$= \frac{R^2(r)}{2} \langle s | \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} (1 + \boldsymbol{\sigma} \cdot \mathbf{n}) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} | s \rangle$$



$\mathbf{n} \parallel \hat{\mathbf{s}}$: $\quad \rho(\mathbf{r}, \mathbf{n} = \hat{\mathbf{s}}) = R^2(r) \cos^2 \theta$

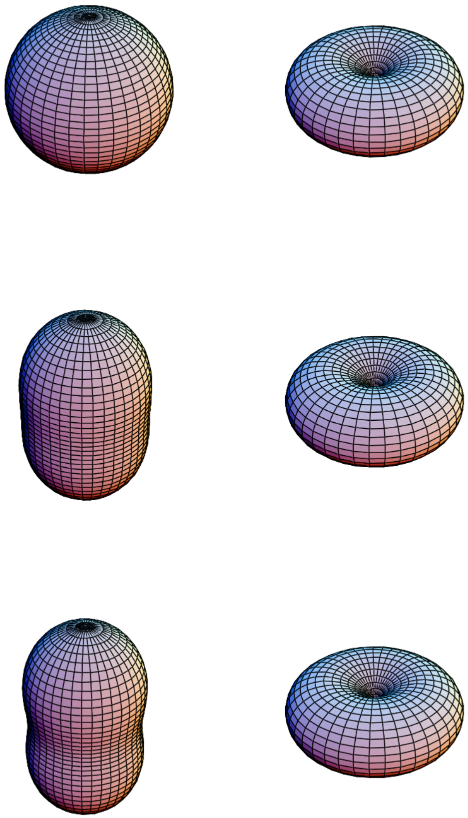
$\mathbf{n} \parallel -\hat{\mathbf{s}}$: $\quad \rho(\mathbf{r}, \mathbf{n} = -\hat{\mathbf{s}}) = R^2(r) \sin^2 \theta$

non-spherical shape depends on spin direction

Shapes of the proton

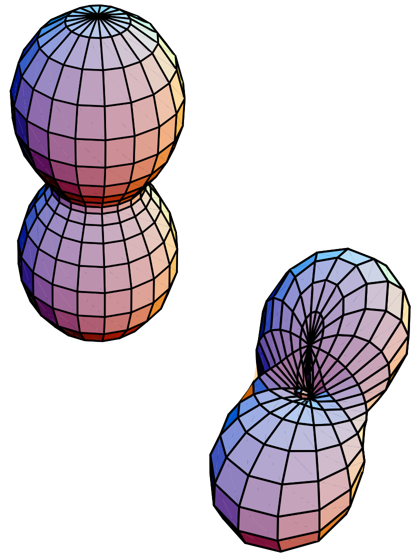
Phys.Rev. C68 (2003) 022201

Momentum space



vectors n, K, S

Coordinate space



Pretzelocity

How to measure?-Lattice and/or experiment

Relation between coordinate and momentum space densities? Model independent technique needed.

Model independent transverse charge density

$$J^+(x^-, \mathbf{b}) = \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b)$$

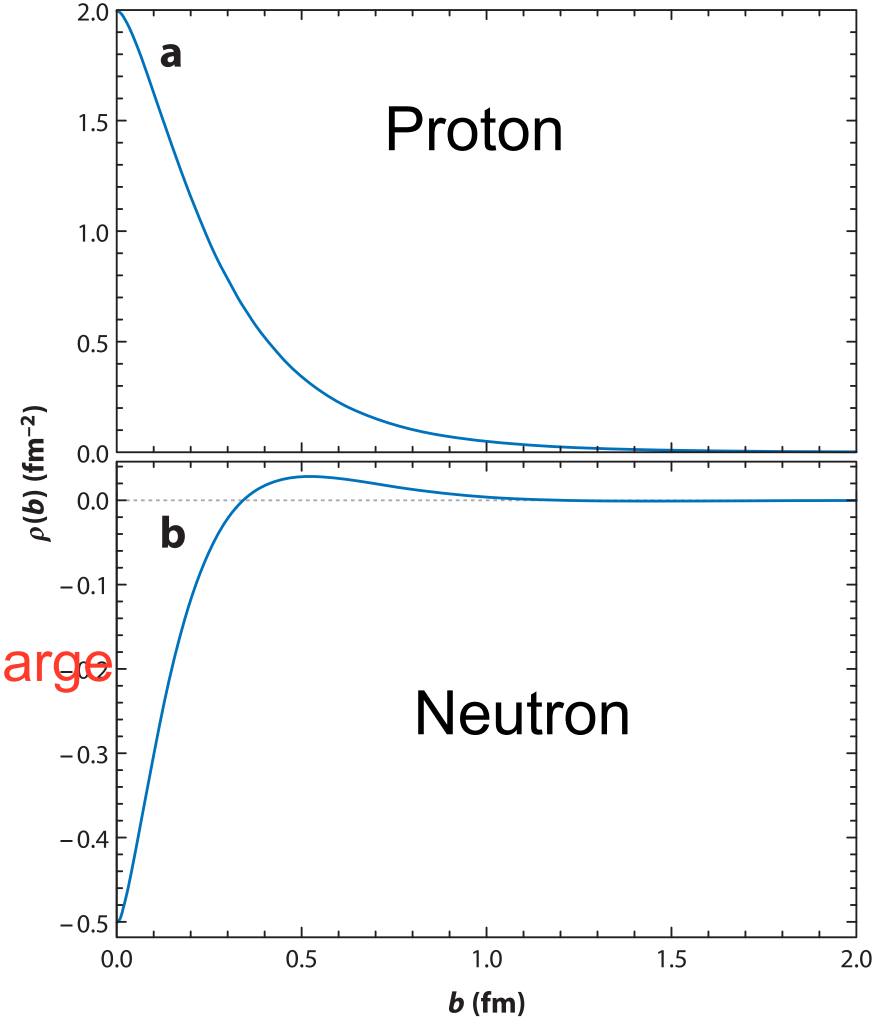
Charge Density operator IMF

$$\rho_\infty(x^-, \mathbf{b}) = \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle$$

$$F_1 = \langle p^+, \mathbf{p}', \lambda | J^+(0) | p^+, \mathbf{p}, \lambda \rangle$$

$$\rho(b) \equiv \int dx^- \rho_\infty(x^-, \mathbf{b}) = \int \frac{QdQ}{2\pi} F_1(Q^2) J_0(Qb)$$

Transverse charge densities from parameterizations (Alberico)



Negative central charge density

Negative central density - GAM PRL '07

Generalized Coordinate Space Densities

$$\rho^{\Gamma}(\mathbf{b}) = \sum_q e_q \int dx^- q_+^{\dagger}(x^-, \mathbf{b}) \gamma^+ \Gamma q_+(x^-, \mathbf{b})$$

$$\Gamma = \frac{1}{2}(1 + \mathbf{n} \cdot \boldsymbol{\gamma}) \text{ gives spin} - \text{dependent density}$$

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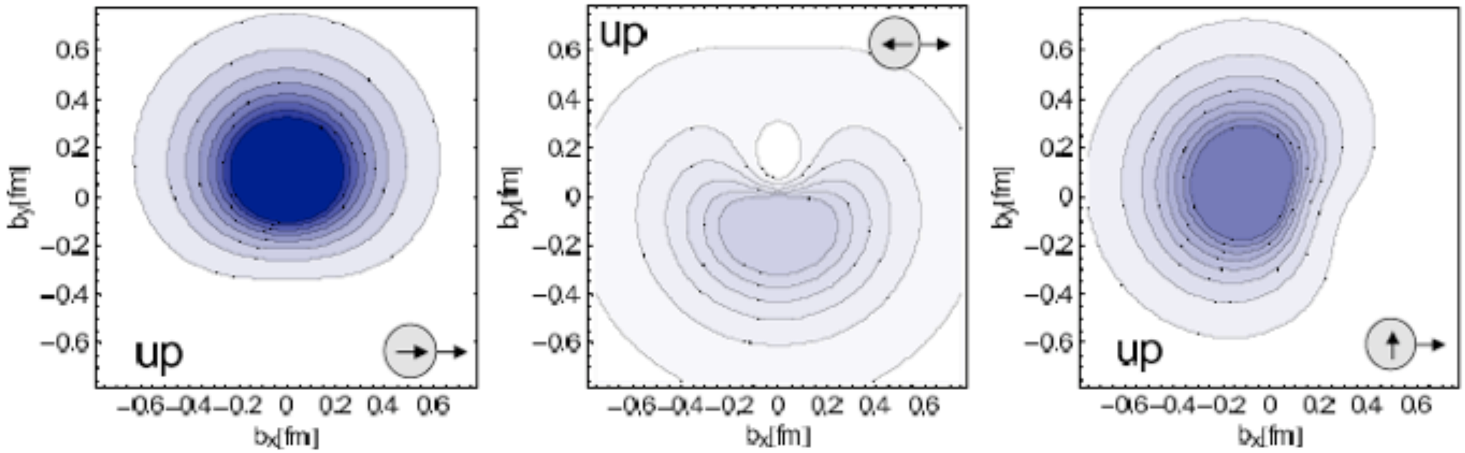
Transverse Spin Structure of the Nucleon from Lattice-QCD Simulations

M. Göckeler,[1] Ph. Hägler,[2,*] R. Horsley,[3] Y. Nakamura,[4] D. Pleiter,[4] P. E. L. Rakow,[5] A. Schäfer,[1] G. Schierholz,[6,4]
H. Stüben,[7] and J. M. Zanotti[3]

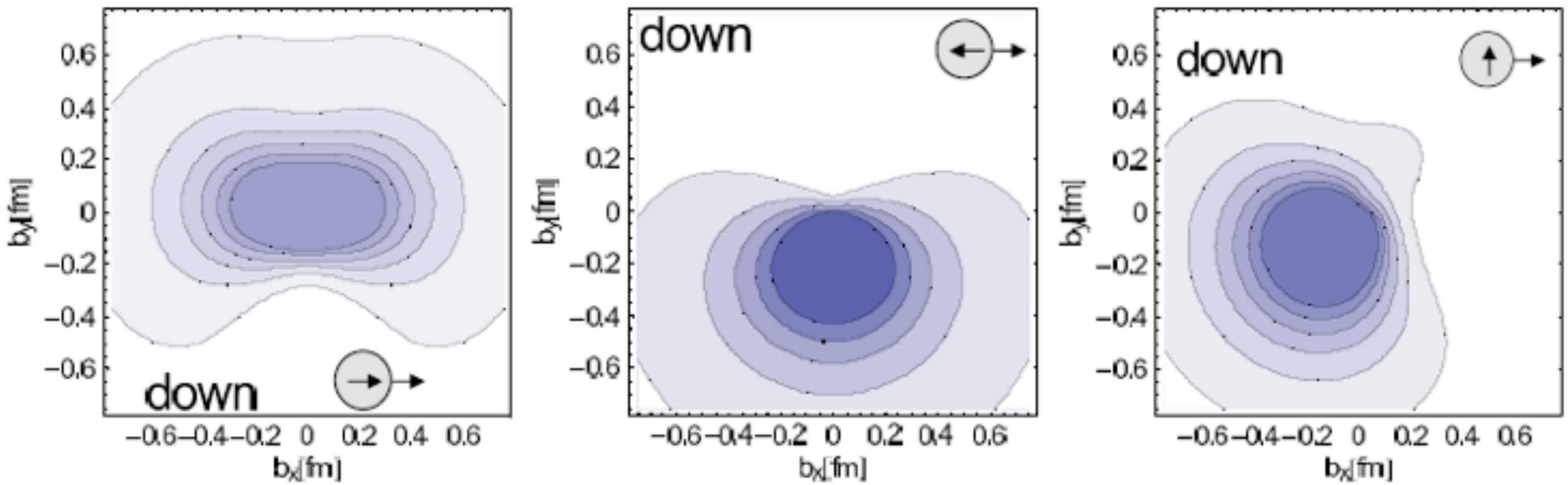
$$\begin{aligned}\rho^n &= \int_{-1}^{1} dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) \\ &= \frac{1}{2}\bigg\{A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \bigg(A_{Tn0}(b_\perp^2) - \frac{\Delta_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2)}{4m^2}\bigg) \\ &\quad + \frac{b_\perp^j \epsilon^{ji}}{m}[S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2)] \\ &\quad + s_\perp^i (2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2)\bigg\}, \qquad (1)\end{aligned}$$

spin-dependent density
-depends on direction
of **b**: proton is not round

Spin dependent densities-transverse-Lattice QCDSF, Zanotti, Schierholz...

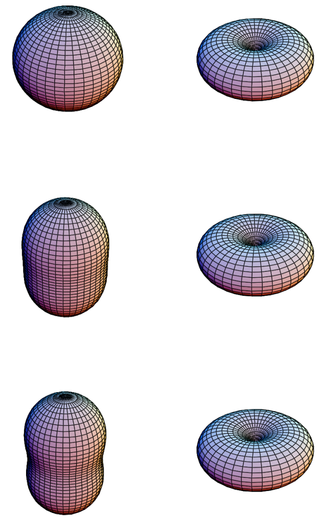


This is not zero! proton is not round



Shapes of the proton

- Relate spin dependent density to experiment
- **Phys.Rev.C76:065209,2007**



Field-theoretic spin dependent momentum density is related to the transverse momentum distribution $h_{1T}^{\perp}$

$$\Phi^{[\Gamma]}(x, \mathbf{K}_T) \;=\; \int \frac{d\xi^- d^2\xi_T}{2\,(2\pi)^3}\, e^{iK\cdot\xi}\, \langle P, S|\overline{\psi}(0)\,\Gamma\,\mathcal{L}(0,\xi;n_-)\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=0}$$

Mulders Tangerman'96

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{K}_T) = S_T^i\, h_1(x, K_T^2) + \frac{\left(K_T^i K_T^j - \frac{1}{2}K_T^2\delta_{ij}\right) S_T^j}{M^2}\, h_{1T}^{\perp}(x, K_T^2)$$

$$\sigma^{i+}\gamma^5 \sim \gamma^0\gamma^+\sigma^i,$$

then relate equal time to $\xi^+ = 0$ by integration over x

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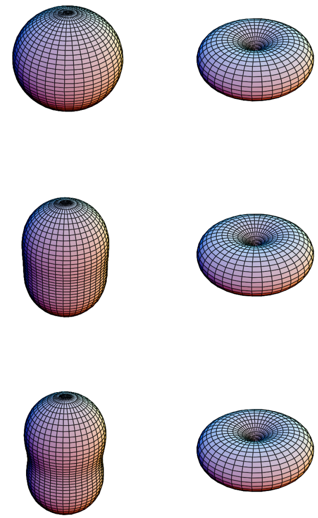

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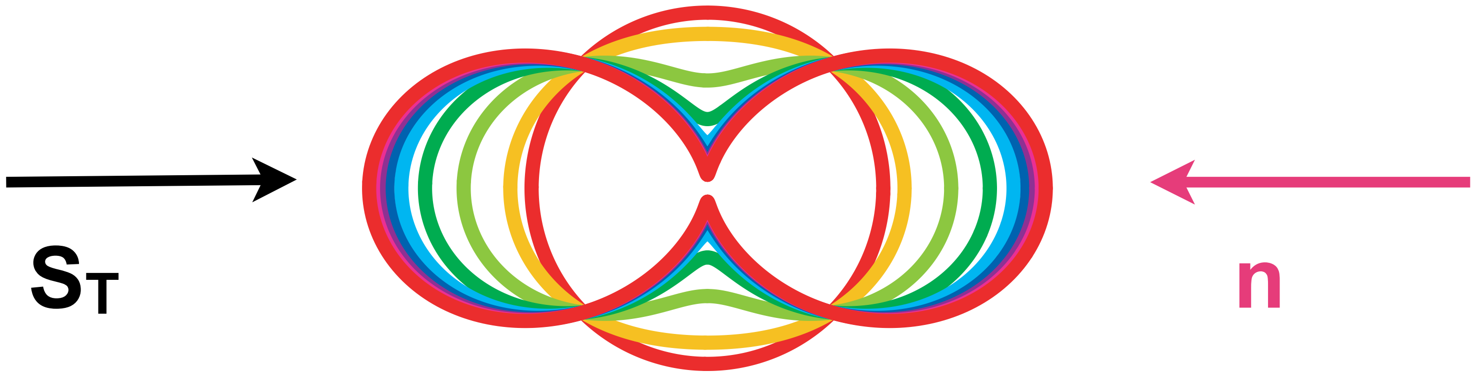
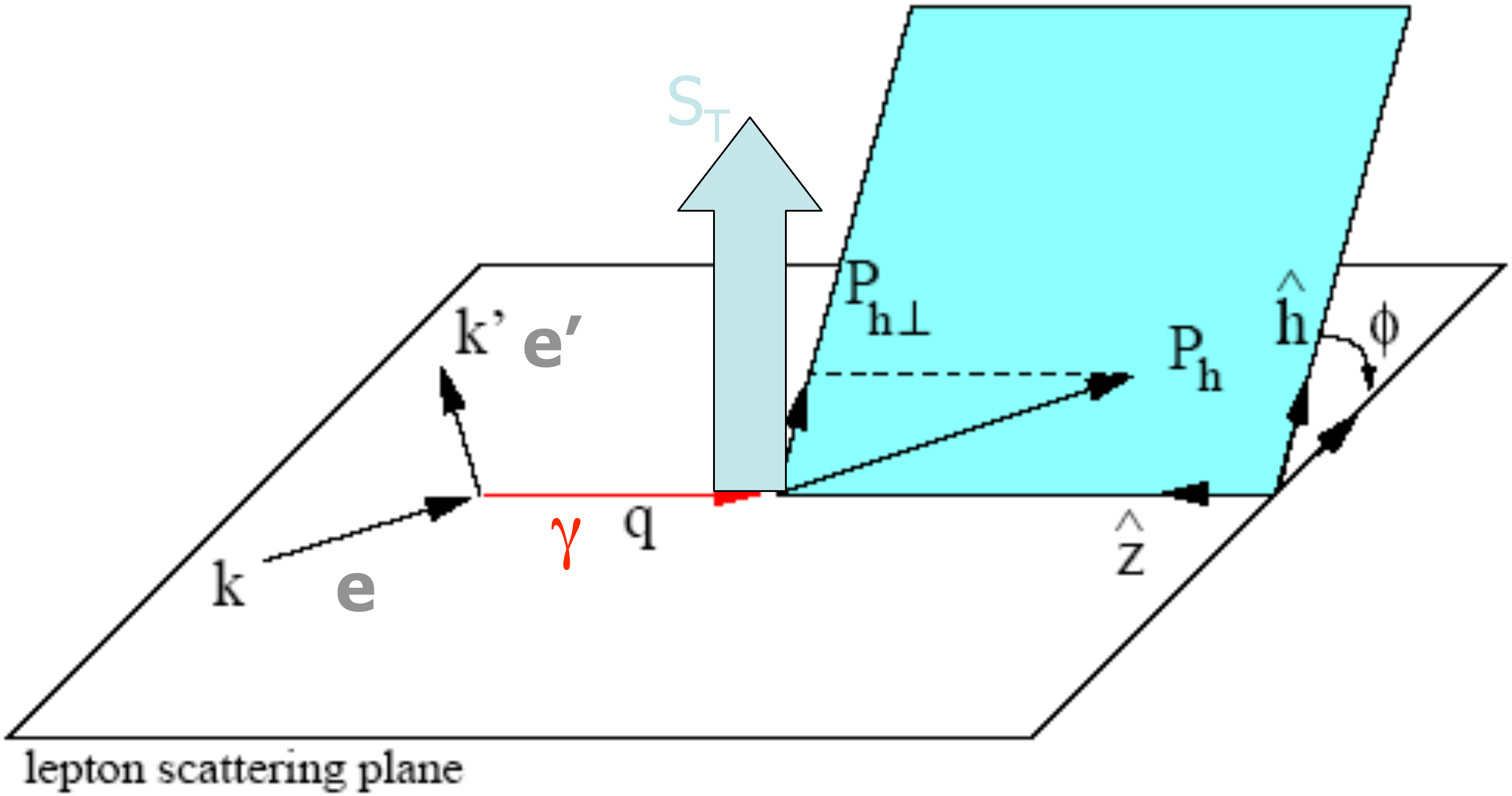
Transverse Shapes of the Proton



Measure $h_{1T}^{\perp}$:e,↑ p → e', π X

H. Avakian LOI at Jlab



Cross section has term proportional to cos 3ϕ

Boer Mulders '98 there are other ways to see $h_{1T}^{\perp}$

Boer Mulders, PRD57,5780

TABLE II. Leading order single spin asymmetries for the case of leptoproduction into unpolarized final states.

ABC	W	$\langle W\rangle_{ABC} \cdot \left[4\pi\,\alpha^2\, s/Q^4\right]^{-1}$	T
OLO	$(Q_T^2/4MM_h)\,\sin(2\phi_h^\ell)$	$-\lambda\,(1-y)\sum_{a,\bar{a}} e_a^2\, x_B\, h_{1L}^{\perp(1)a}(x_B) H_1^{\perp(1)a}(z_h)$	eo
OTO	$(Q_T/M_h)\,\sin(\phi_h^\ell + \phi_S^\ell)$	$\lvert \boldsymbol{S}_T\rvert\,(1-y)\sum_{a,\bar{a}} e_a^2\, x_B\, h_1^a(x_B) H_1^{\perp(1)a}(z_h)$	eo
OTO	$(Q_T^3/6M^2M_h)\,\sin(3\phi_h^\ell - \phi_S^\ell)$	$\lvert \boldsymbol{S}_T\rvert\,(1-y)\sum_{a,\bar{a}} e_a^2\, x_B\, h_{1T}^{\perp(2)a}(x_B) H_1^{\perp(1)a}(z_h)$	eo
OTO	$(Q_T/M)\,\sin(\phi_h^\ell - \phi_S^\ell)$	$\lvert \boldsymbol{S}_T\rvert\,\left(1-y+\frac{1}{2}\,y^2\right)\sum_{a,\bar{a}} e_a^2\, x_B\, f_{1T}^{\perp(1)a}(x_B) D_1^a(z_h)$	oe

There have been searches for this term

Summary

- Form factors, GPDs, TMDs, understood from unified light-front formulation, GPD-coordinate space density,TMD momentum space density
- Neutron central transverse density is negative-
- Proton is not round- lattice QCD spin-dependent-density is **not** zero
- Experiment can whether or not proton is round by measuring

$$h_{1T}^{\perp}$$

Summary

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The Proton