

High t Form Factors and OAM

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Connection between elastic form factors
and OAM through models

Model wave functions, compute form
factors

OAM content of Models

History -Definitions

$$\langle N, \lambda' p' | J^\mu | N, \lambda p \rangle = \bar{u}_{\lambda'}(p') [F_1(Q^2) \gamma^\mu + F_2(Q^2) \sigma^{\mu\nu} \frac{(p' - p)_\nu}{2M_p}] u_\lambda(p)$$

$$G_E = F_1 - \frac{Q^2}{4M_N^2} F_2, \quad G_M = F_1 + F_2$$

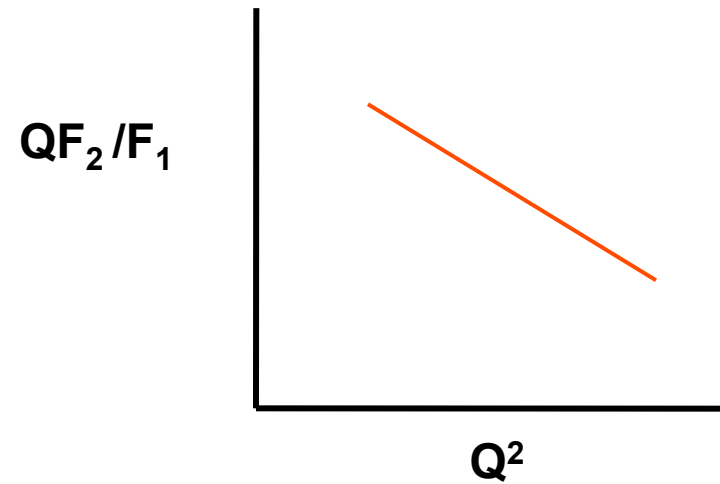
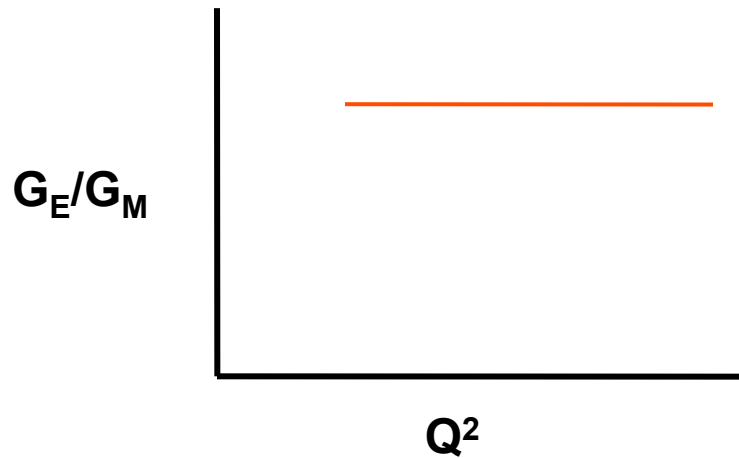
$$F_1(Q^2) = \langle N, \uparrow | J^+ | N, \uparrow \rangle, \quad QF_2(Q^2) = -2M_p \langle N, \uparrow | J^+ | N, \downarrow \rangle$$

old pQCD:

$$\frac{QF_2(Q^2)}{2M_N F_1} \sim \frac{m_{\text{quark}}}{Q} \rightarrow \frac{G_E}{G_M} = \text{const}$$

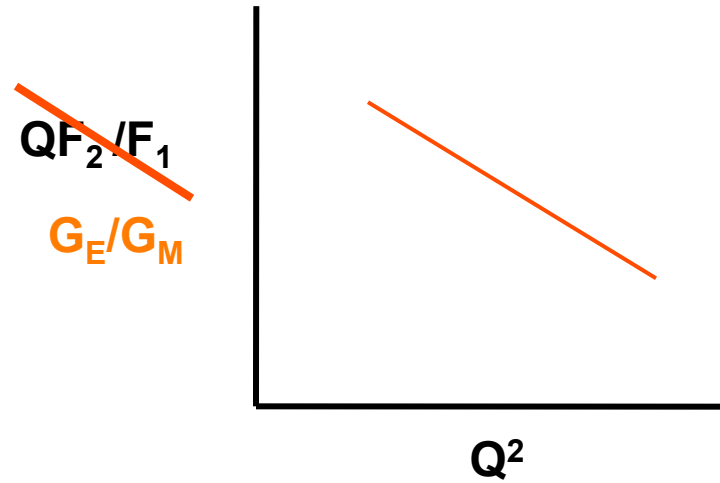
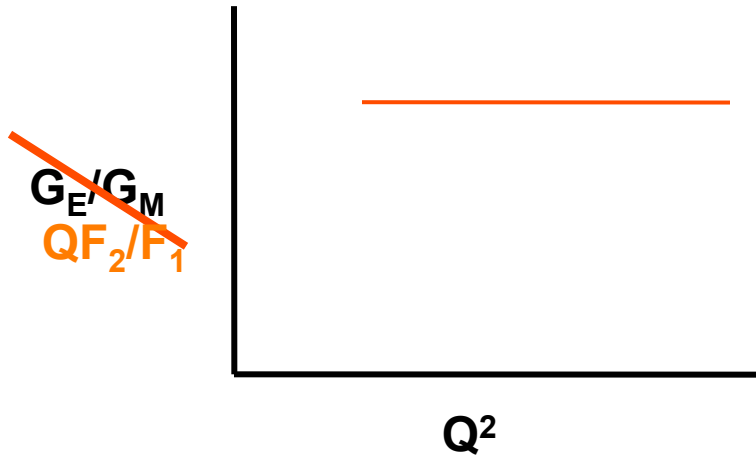
Same as non-relativistic

Expectations- Pre Jlab



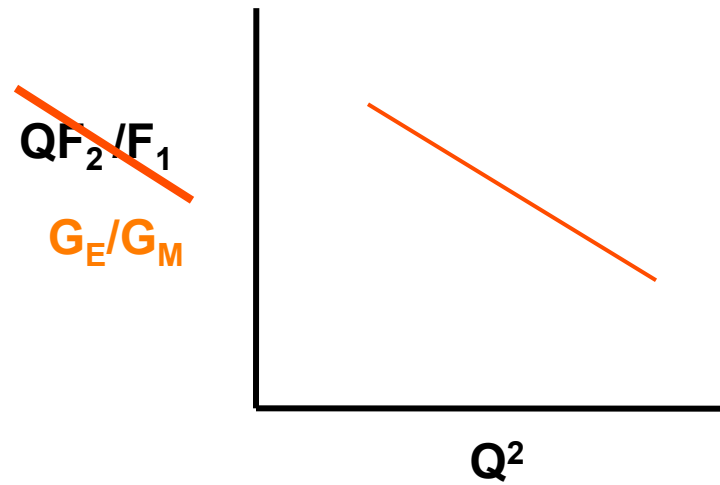
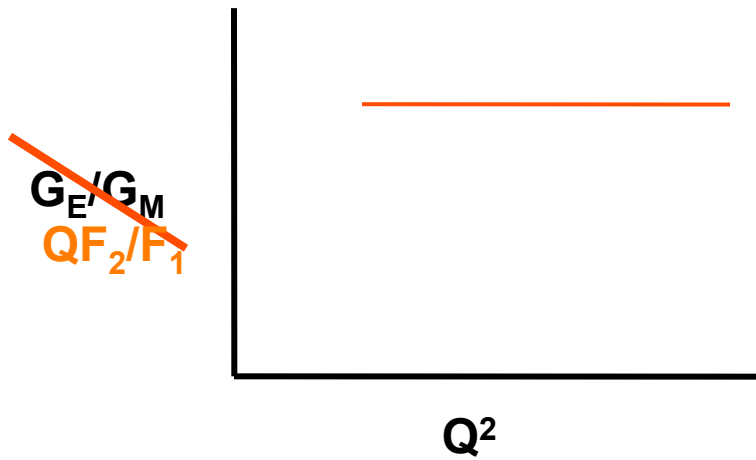
Frank, Jennings, Miller PR C54, 920 (1996)
Relativistic model for color transparency

Expectations- Pre Jlab



Frank, Jennings, Miller PR C54, 920 (1996)
Relativistic model for color transparency

Jlab



Frank, Jennings, Miller PR C54, 920 (1996)
Relativistic model for color transparency

Relativistic Wave function

- 3 quark anti-symmetric
- relative variables, frame independent **Light front variables**
- eigenstate of spin operator- rotational invariant
- reduces to non-relativistic if $m \rightarrow \infty$

$$\Psi = \Phi(M_0^2) u(p_1) u(p_2) u(p_3 = K) \psi(s_i, t_i) \text{ Terentev, Coester}$$

spatial dist **DIRAC SPINORS** spin-ispin color amp

Schlumpf Mom space wf $\Phi(M_0) = N / (M_0^2 + \beta^2)^\gamma$

$$\beta = 0.607 \text{ GeV} \quad \gamma = 3.5 \quad m = 0.267 \text{ GeV}$$

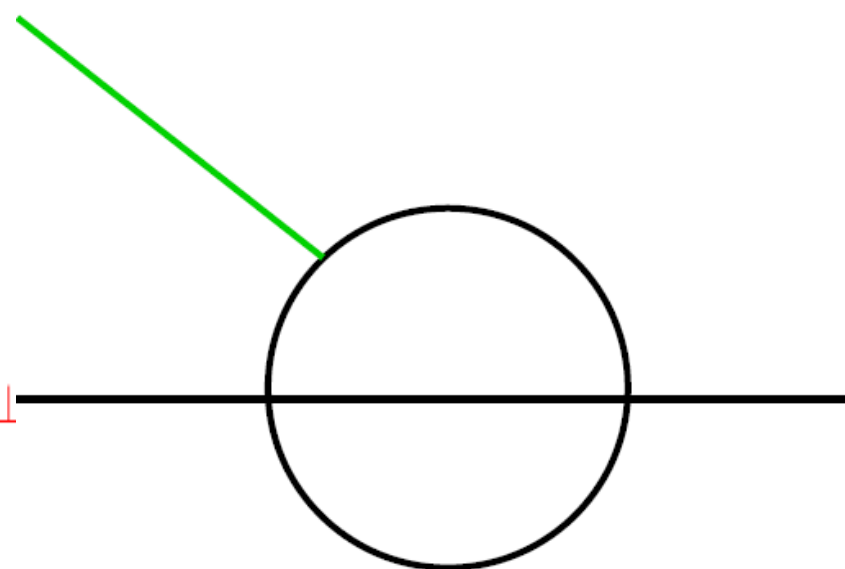
Impulse Approximation

Model proton wave function $\Psi(\mathbf{k}_\perp, \mathbf{K}_\perp, \xi, \eta)$

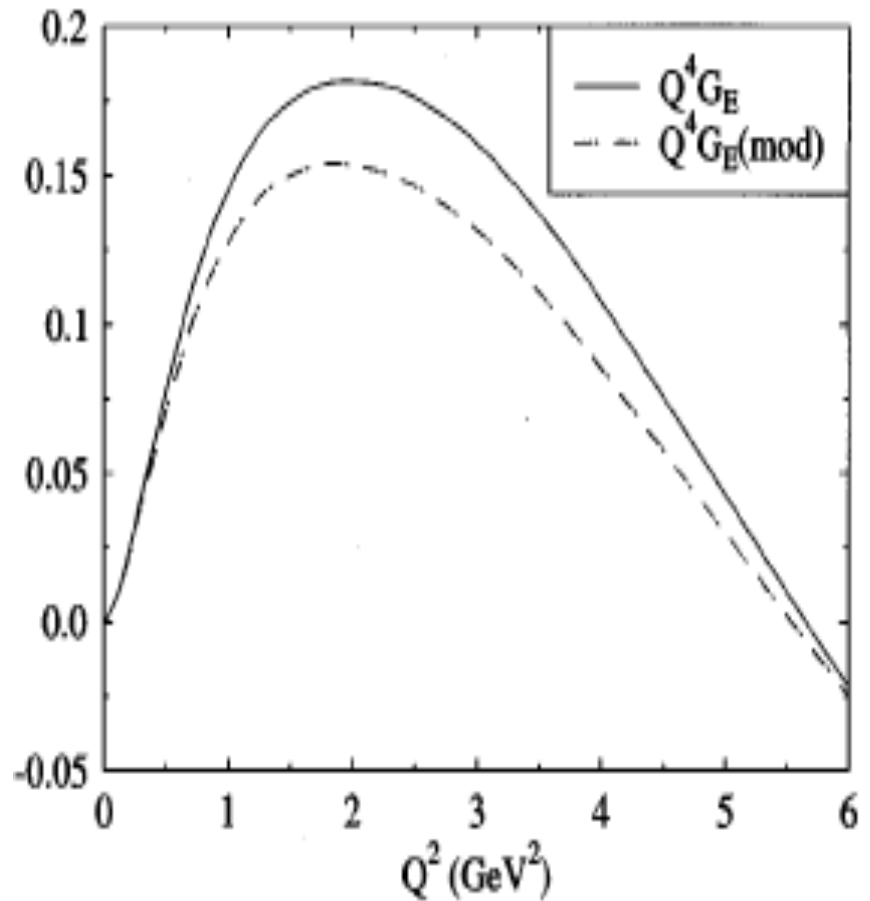
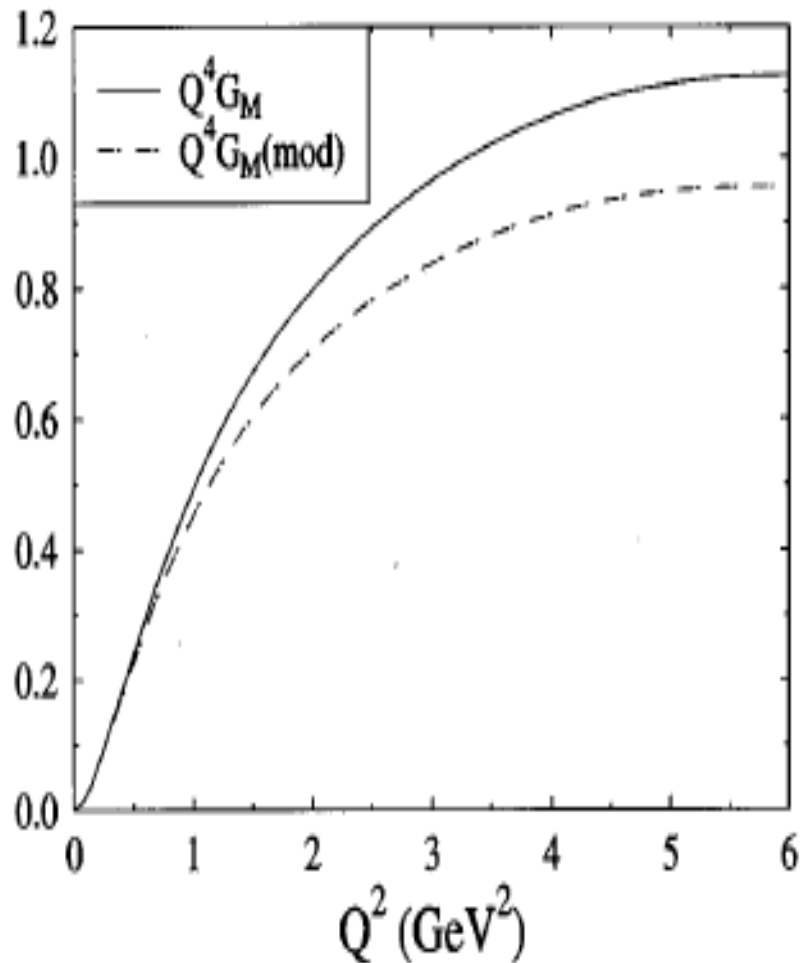
Poincare invariant

Light front variables for boost: $\mathbf{K} \rightarrow \mathbf{K} + \eta \mathbf{q}_\perp$

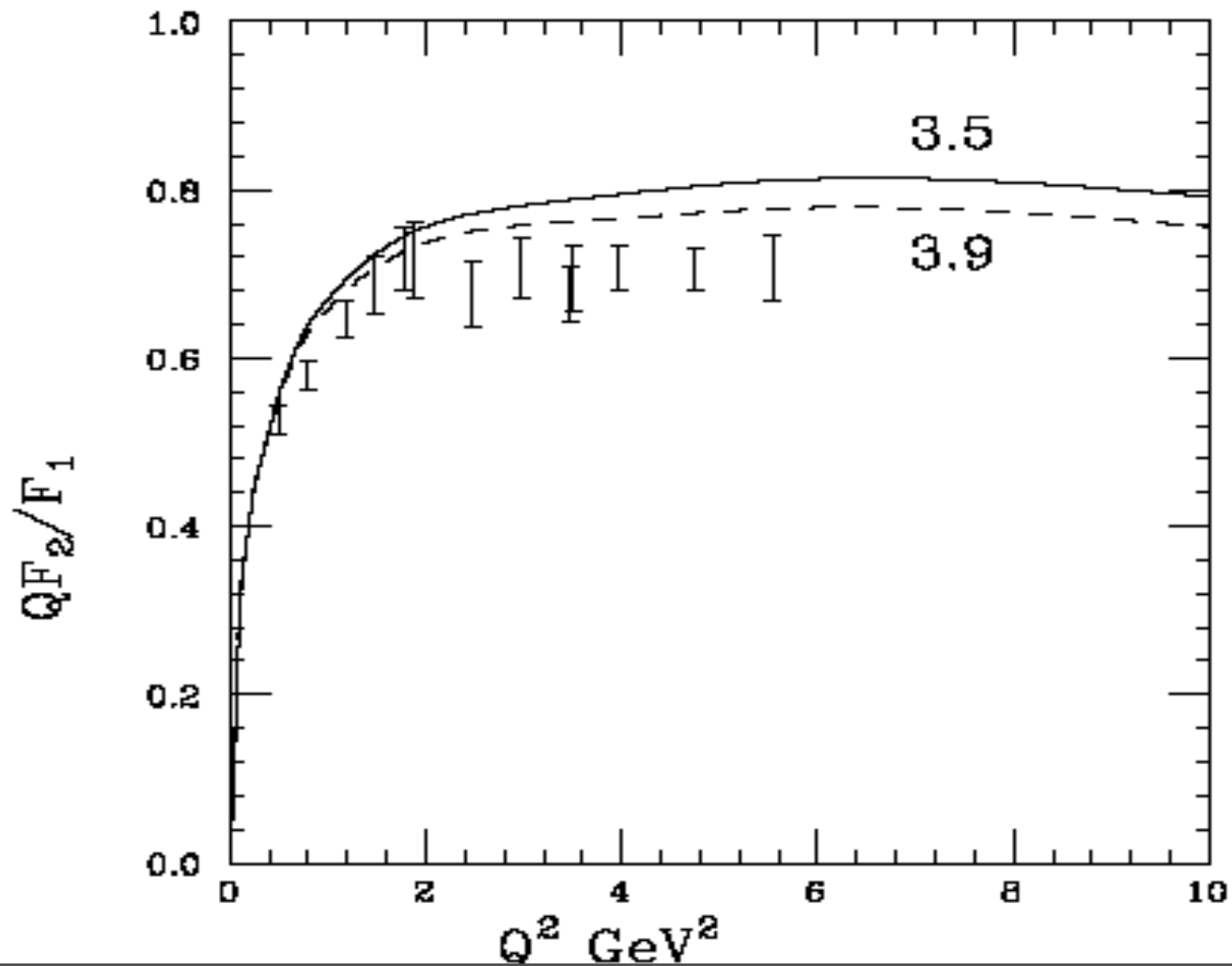
Dirac spinors



1995 Frank, Jennings, Miller



Ratio of Pauli to Dirac Form Factors 1995



Relativistic Explanation

J^+ acts on third quark, other two have 0 spin

$$u(K, s) = \begin{pmatrix} (E(K) + m)|s\rangle \\ \boldsymbol{\sigma} \cdot \mathbf{K}|s\rangle \end{pmatrix}$$

$\sigma_y|s\rangle$: quark spin \neq proton ang mom

lower components $\equiv L_z \neq 0$

$$\bar{u}(K', s')\gamma^+u(K, s) \sim \langle s'|K^+ + i\sigma_y Q|s\rangle \text{ Large } Q$$

spin non-flip $F_1(Q^2) = \int \dots Q\Phi\Phi$, flip $QF_2 = \int \dots Q\Phi\Phi$

$$\frac{QF_2}{F_1} \sim \text{Constant}$$

Miller, Frank **Phys.Rev. C65 (2002) 065205**

Spin content - OAM

$$s_{\mu} \Delta q = \langle N, s | \bar{q} \gamma_{\mu} \gamma_5 q | N, s \rangle$$

$$\Sigma = \Delta u + \Delta d + \Delta s$$

75 % of proton angular momentum carried by quark spin

Textbook reduction of axial vector coupling constant

Neutron: Need π cloud effect at low Q^2

Cloudy Bag Model 1980

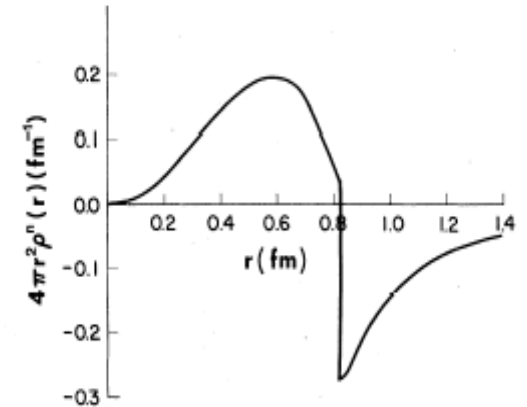
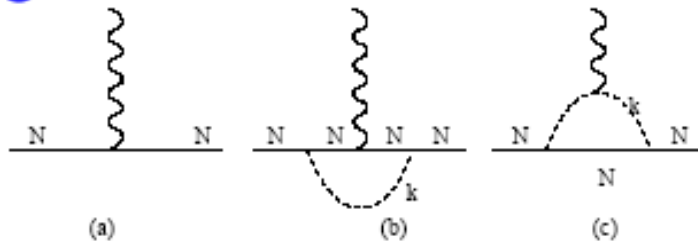


FIG. 11. Neutron charge density.

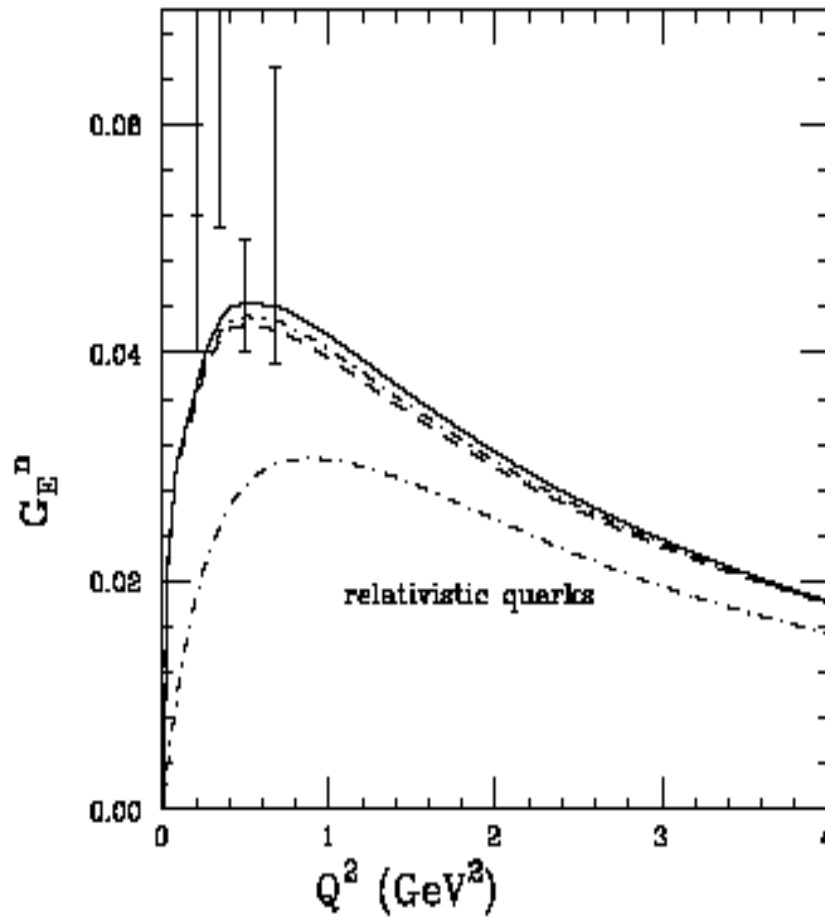
Relativistic treatment needed Feynman graphs, $\int dk^-$

Light front cloudy bag model LFCBM 2002

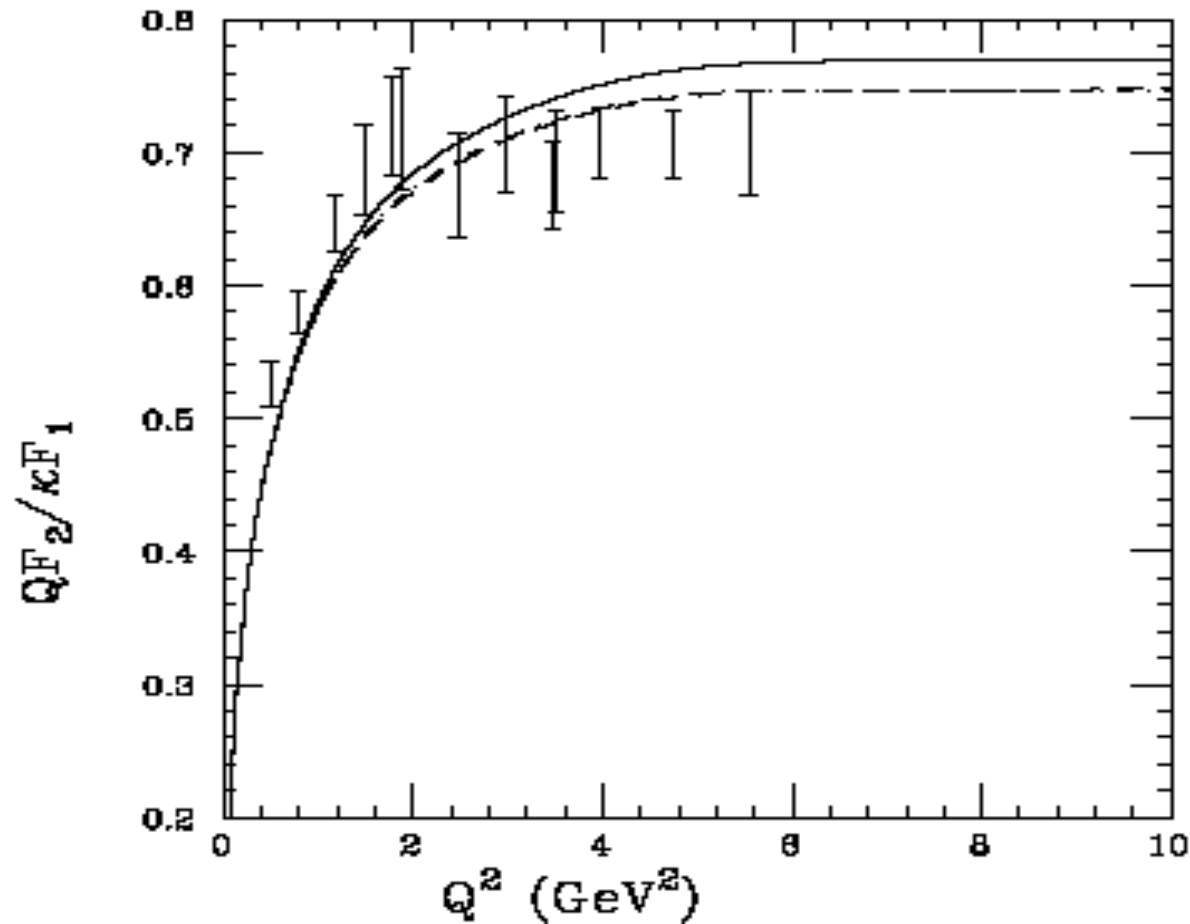
Miller Phys.Rev. C66 (2002) 032201

- γN form factors from model (our model)
- rel. πN form factor $\Lambda_{\pi N}$
- Model parameters: $m, \beta, \gamma, \Lambda_{\pi N}$

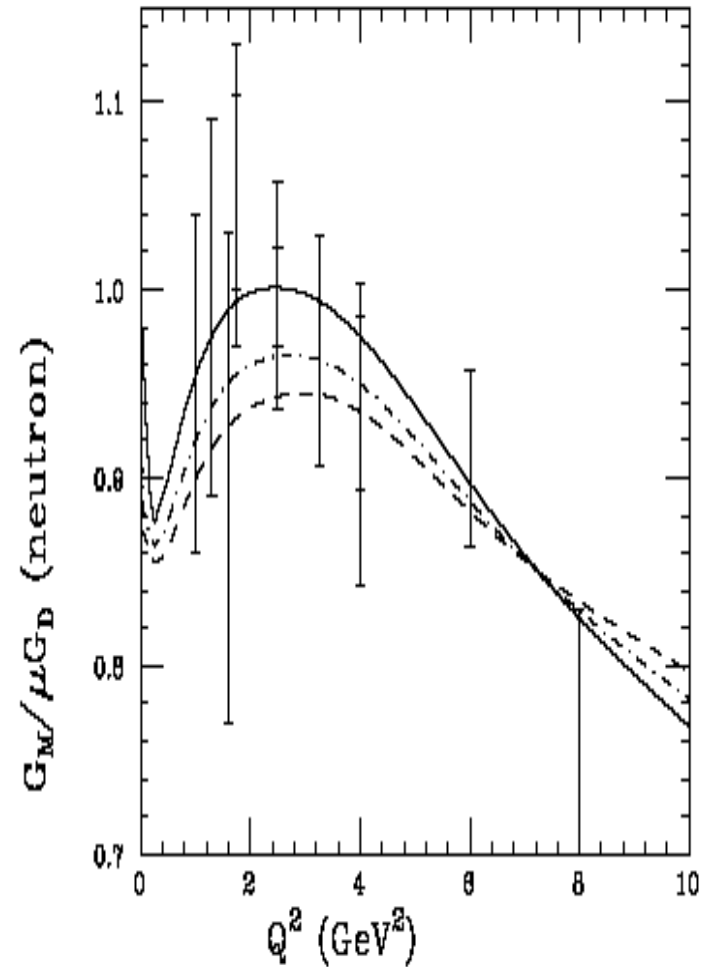
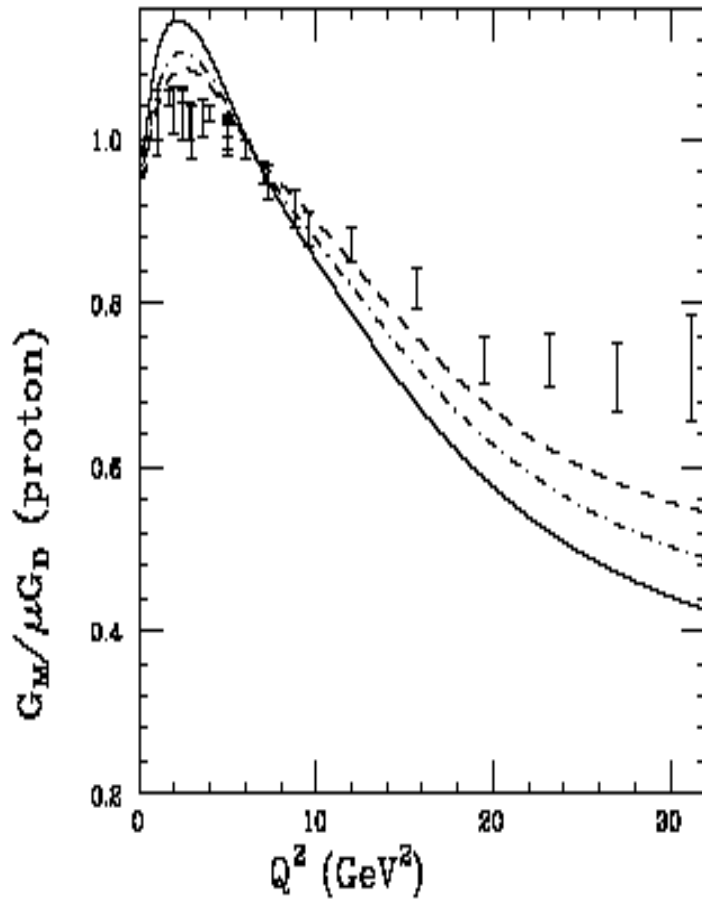
Neutron Electric Form Factor



Ratio of Pauli to Dirac Form



Two More Form Factors Needed



OAM content of light front cloudy bag model

$$\Sigma \rightarrow \left(Z - \frac{1}{3} P_{N\pi} + \frac{5}{3} P_{\Delta\pi} \right) \Sigma$$

Schreiber, Thomas PLB215, 141(88)

$$*LFCBM* : P_{N\pi} \approx .25, P_{\Delta\pi} = 0$$

$$\Sigma \rightarrow \frac{2}{3} \Sigma \sim \frac{2}{3} \frac{3}{4} = \frac{1}{2}$$

Can include 

Alberg, Miller [arXiv:1201.4184](https://arxiv.org/abs/1201.4184)

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2011 Update model

- In LFCBM G_E/G_M falls too fast with Q^2
- New data -slower fall, flavor decomposition not good, get smaller quark spin?
- Many invariant forms of nucleon wave function
- Cloet & Miller quark di-quark model:
- uses other invariant wave functions

(Brodsky, Hiller, Karmanov, Hwag)

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Cloet Miller 2011-12

Scalar diquark

$$\Phi_{\lambda_q \lambda_D}^{\lambda_N}(k, p) = \bar{u}(k, \lambda_q) \left[\varphi_1^s + \frac{M}{p^+} \gamma^+ \varphi_2^s \right] u_N(p, \lambda_N) \\ + \bar{u}(k, \lambda_q) \varepsilon_\nu^*(q, \lambda_D) \gamma^\nu \gamma_5 \left[\varphi_1^a + \frac{M}{p^+} \gamma^+ \varphi_2^a \right] u_N(p, \lambda_N)$$

Axial vector diquark

$$|p\rangle = \frac{1}{\sqrt{2}} |u S_0\rangle + \frac{1}{\sqrt{6}} |u T_0\rangle - \frac{1}{\sqrt{3}} |d T_1\rangle,$$

Plus pion cloud- 9 parameters

χ^2	M	M_s	M_a	c_s	β_s	γ_s	c_a	β_a	γ_a	Λ	μ_p	μ_n
0.078516	0.191	0.414	0.167	1.509	1.226	5.719	0.008	1.104	8.586	1.035	2794	-1.849

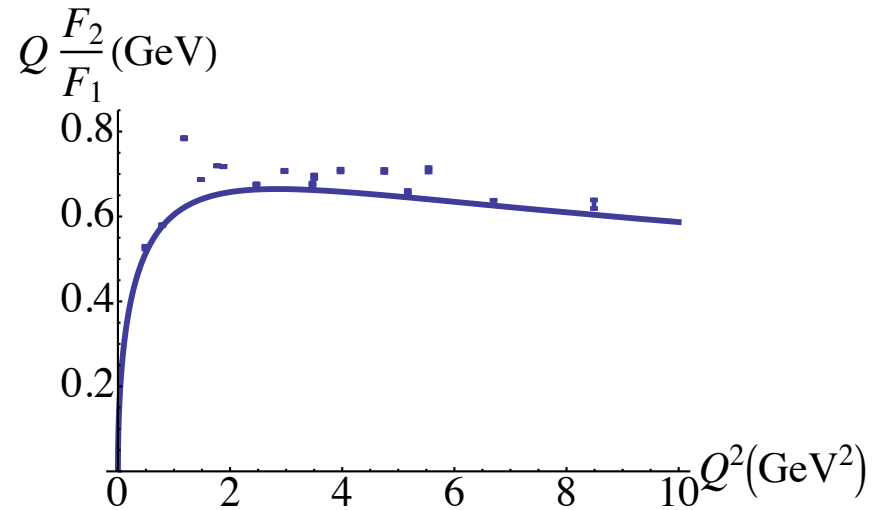
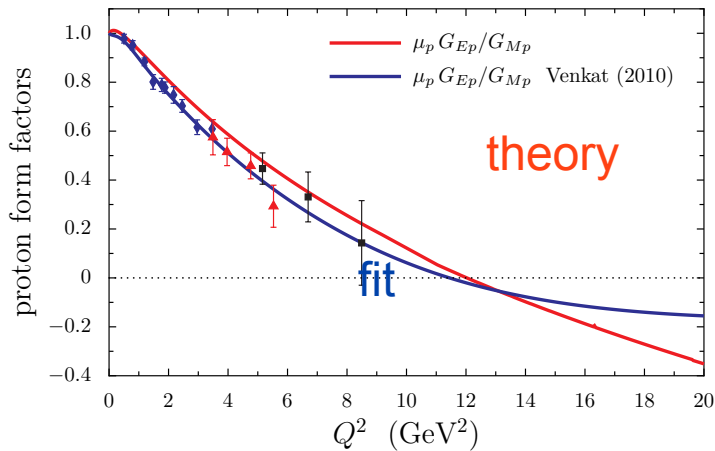
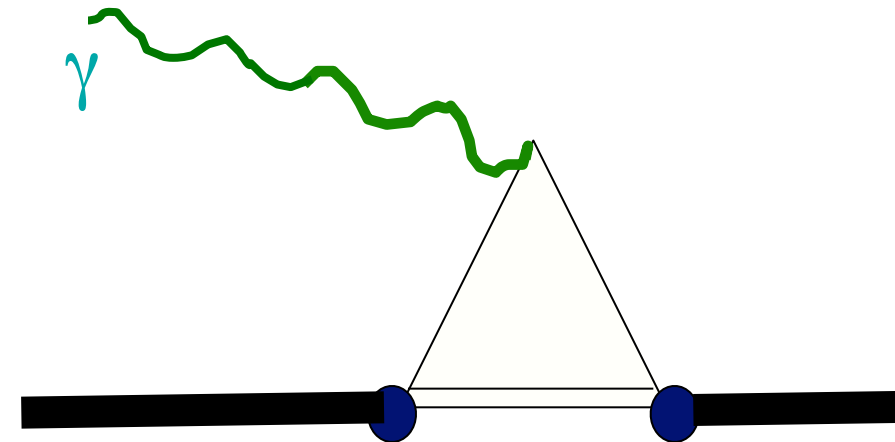
Cloet & Miller '11

Model proton wave function: quark-diquark

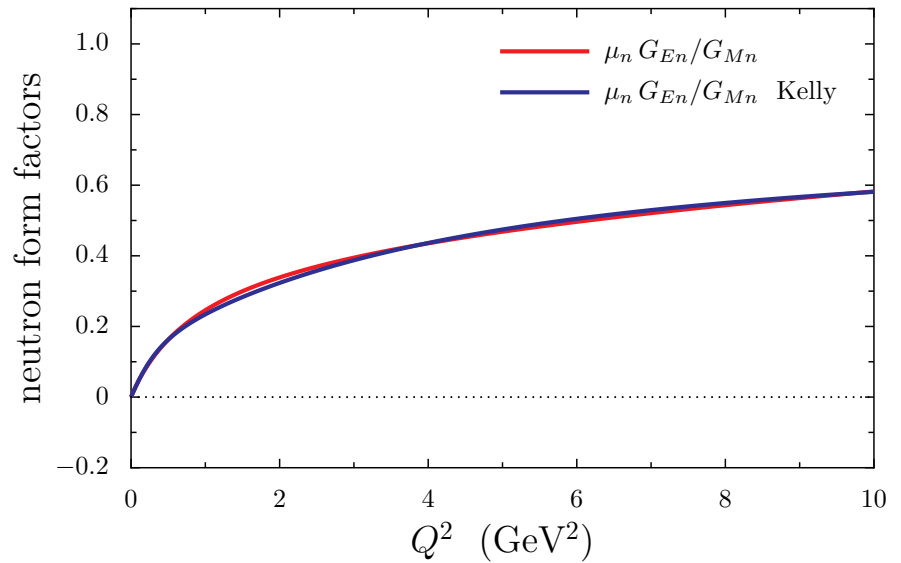
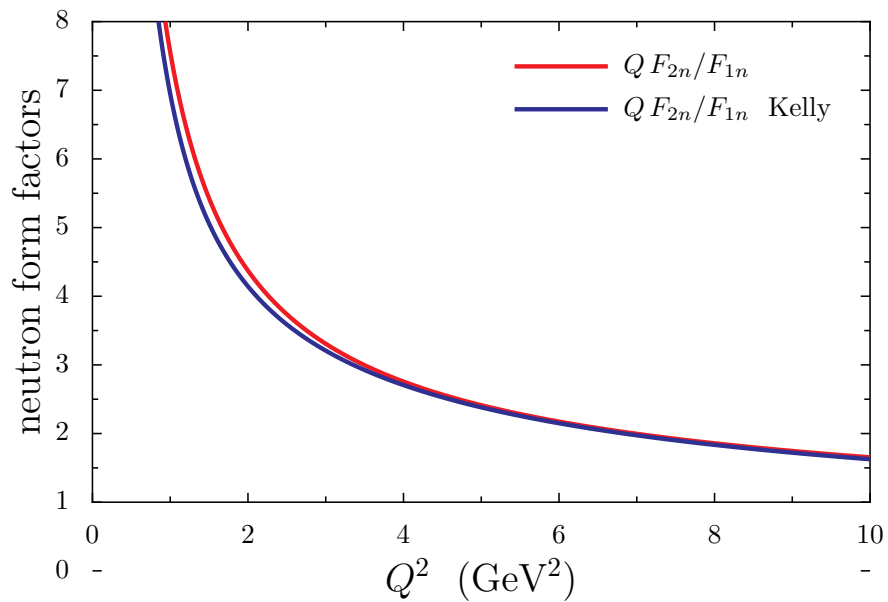
Lorentz and rotationally invariant-different forms!

Light front variables

Dirac spinors-orbital angular momentum

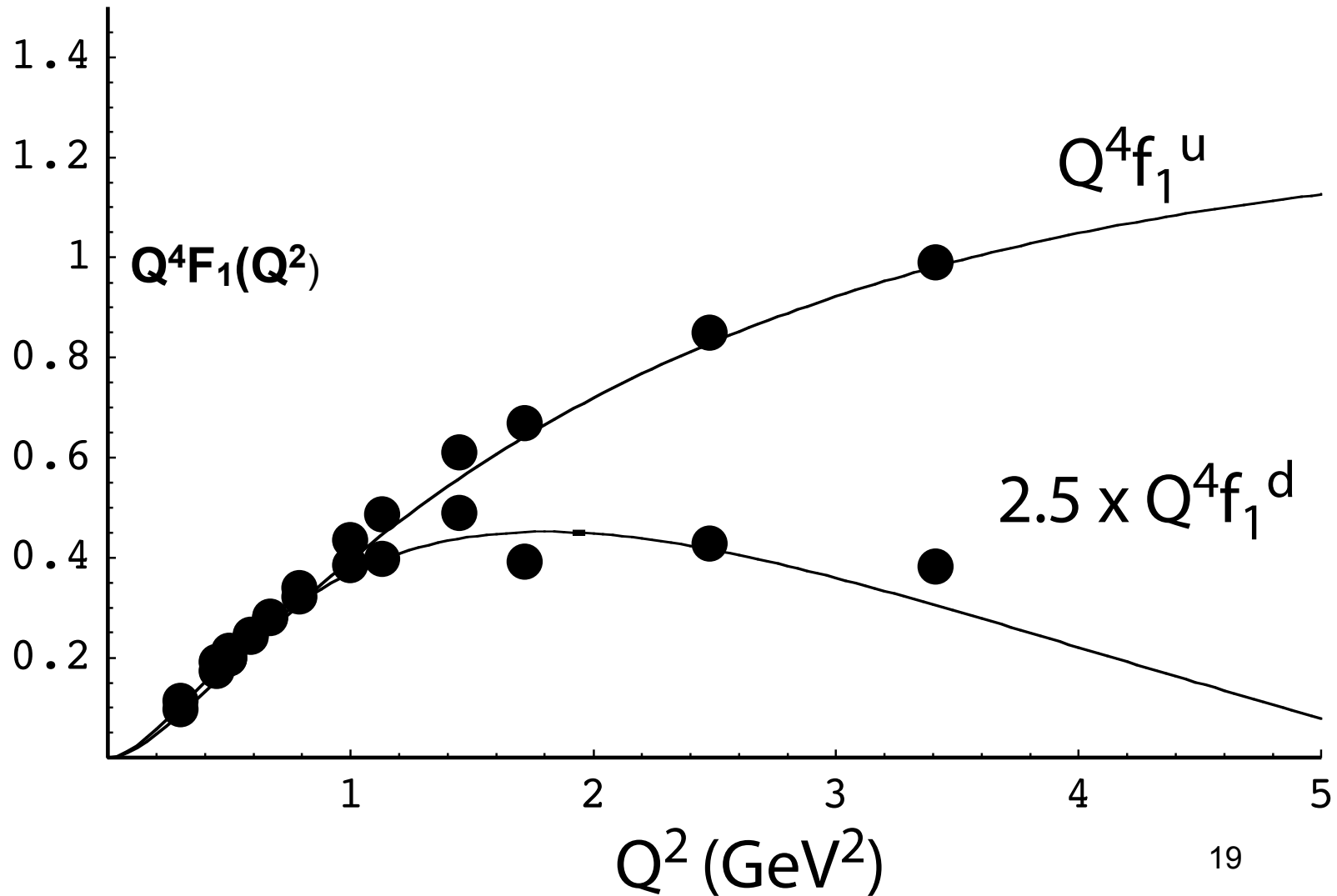


Neutron form factors



Flavor separation: Cates, de Jager, Riordan, Wojtsekhowski

PRL 106,252003



Quark-Diquark model -spin content

- Quark spin fraction:
- $1/2 (.782 \text{ (scalar)} + .167 \text{ (Axial Vector)}) = .474$
- **With pion cloud $0.7 (0.474) = 0.34$**

Summary

- Relativistic light front quark model with pion cloud can reproduce nucleon form factors
- Model quark spin is 35 % of total angular momentum
- Relativistic quark model alive and well