Orbital Angular Momentum in QCD,
Institute for Nuclear Theory
Feb. 7, 2012

Large-x structure functions and OAM

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Outline

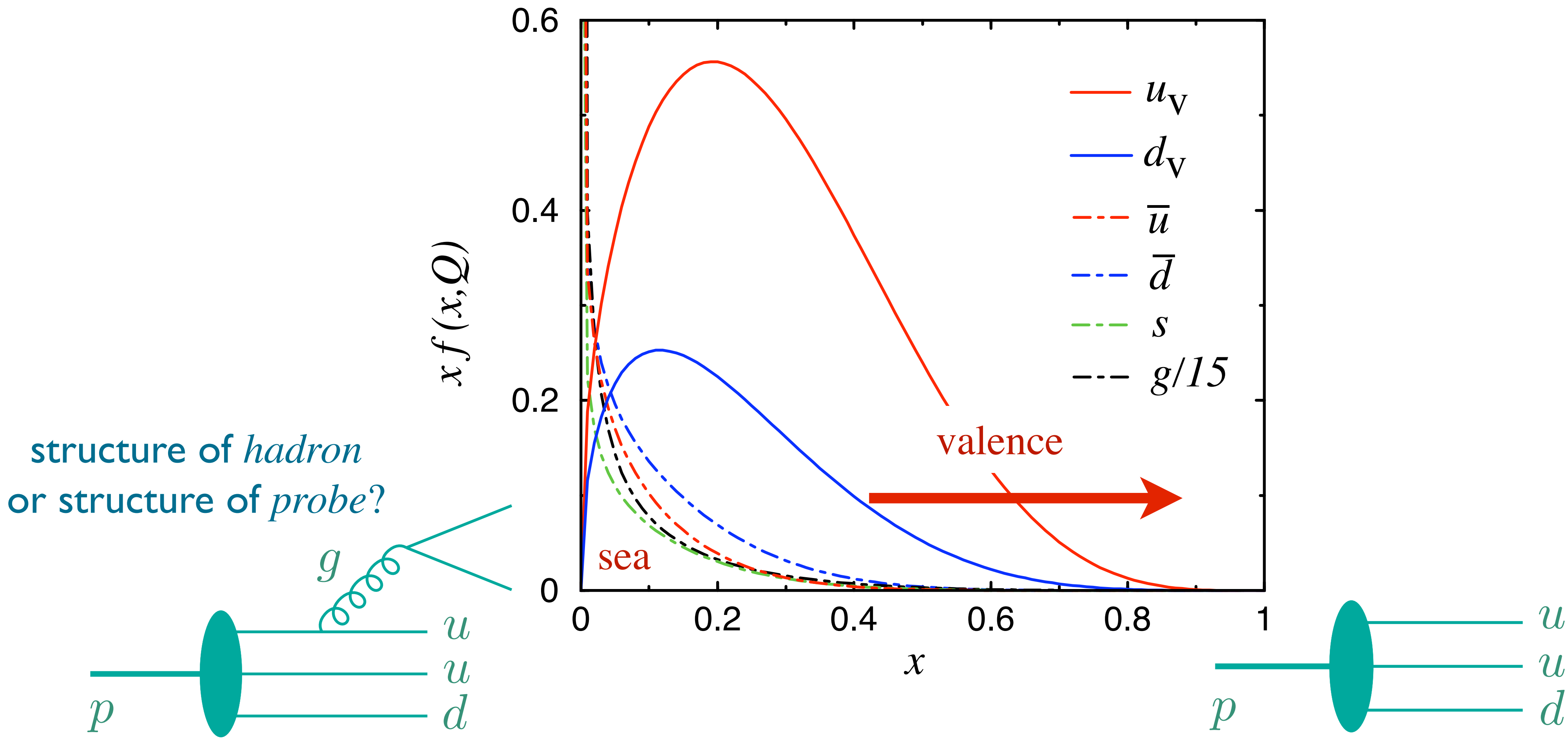
- Why large-x quarks are important
  - $\longrightarrow$ valence quarks, relation with high-t form factors
- $x \to 1$ behavior from perturbative QCD
  - $\longrightarrow$ $L_z = 0$ analysis; suppression of helicity-flip
- Role of OAM
  - $\longrightarrow$ log enhancement of helicity-flip amplitudes
- Phenomenological implications
  - $\longrightarrow$ CJ (CTEQ-JLab) large-x global analysis
  - $\longrightarrow$ challenges for empirical $x \to 1$ analysis

Why large x?

Large-x PDFs

- Most direct connection between quark distributions and models of nucleon structure (*e.g.* leading Fock state of wfn) is via *valence* quarks

  → most cleanly revealed at $x > 0.4$



Large-x PDFs

- Ideal testing ground for nonperturbative & perturbative models of the nucleon

  → *e.g.* ratio of d to u PDFs sensitive to spin-flavor dynamics

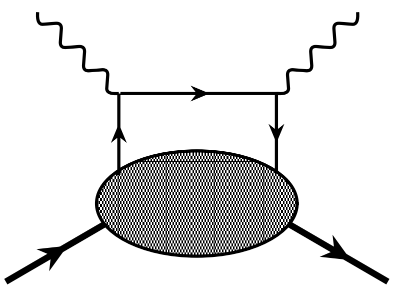
SU(6) proton wave function

$$p^{\uparrow} = -\frac{1}{3}d^{\uparrow}(uu)_1 - \frac{\sqrt{2}}{3}d^{\downarrow}(uu)_1 + \frac{\sqrt{2}}{6}u^{\uparrow}(ud)_1 - \frac{1}{3}u^{\downarrow}(ud)_1 + \frac{1}{\sqrt{2}}u^{\uparrow}(ud)_0$$

interacting quark

spectator "diquark"

diquark spin



Large-x PDFs

- Ideal testing ground for nonperturbative & perturbative models of the nucleon

→ *e.g.* ratio of d to u PDFs sensitive to spin-flavor dynamics

- $d/u \to 1/2$ SU(6) symmetry
- $d/u \to 0$ $S=0$ qq dominance
- $d/u \to 1/5$ $S_z=0$ qq dominance
- $d/u \to \dfrac{4\,\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2}$ local quark-hadron duality* ($\mu_{p,n}$ magnetic moments)

*structure function at $x \to 1$ given by elastic form factor at $Q^2 \to \infty$

see e.g. WM, Ent, Keppel Phys. Rep. ***406****, 127 (2005)*

Large-x PDFs

- Ideal testing ground for nonperturbative & perturbative models of the nucleon

$\longrightarrow$ *e.g.* ratio $\Delta q/q$ even more sensitive

- $\Delta u/u \to 2/3$, $\Delta d/d \to -1/3$ — SU(6) symmetry

- $\Delta u/u \to 1$, $\Delta d/d \to -1/3$ — $S=0$ qq dominance

- $\Delta u/u \to 1$, $\Delta d/d \to 1$ — $S_z=0$ qq dominance <u>*or*</u> local duality

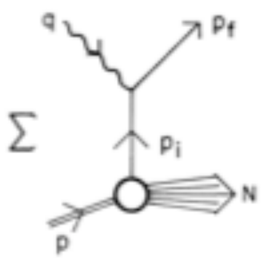
Inclusive-exclusive connection

Drell-Yan-West relation

$$G_M(Q^2) \sim \left(\frac{1}{Q^2}\right)^n \quad \Longleftrightarrow \quad F_2(x) \sim (1-x)^{2n-1}$$

- Drell & Yan: field-theoretical model of strongly interacting $N, \overline{N}$ & π "partons" in infinite momentum frame
  PRL **24**, 181 (1970)

- West: covariant model with single *scalar* quark, assuming amplitude for proton → quark + spectator behaves as
  PRL **24**, 1206 (1970)



$$f(p_i^2, p_{\rm spec}^2) \sim \left(\frac{1}{p_i^2}\right)^n g(p_{\rm spec}^2), \quad p_i^2 \to \infty$$

→ for several flavors, in general $\sum_i e_i^2 \neq \left(\sum_i e_i\right)^2$

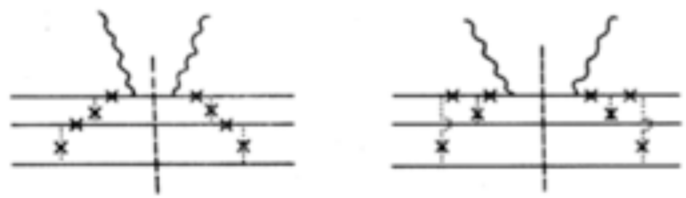
→ how does duality arise?

Close, Isgur, PLB **509**, 81 (2001)

Perturbative QCD

Perturbative QCD

- In QCD, "exceptional" $x \to 1$ configurations of proton wave function generated from "typical" wave function (for which $x_i \sim 1/3$) by exchange of ≥ 2 hard gluons, with mass $k^2 \sim -\langle k_\perp^2 \rangle/(1-x)$



Farrar, Jackson, PRL ***35****, 1416 (1975)*

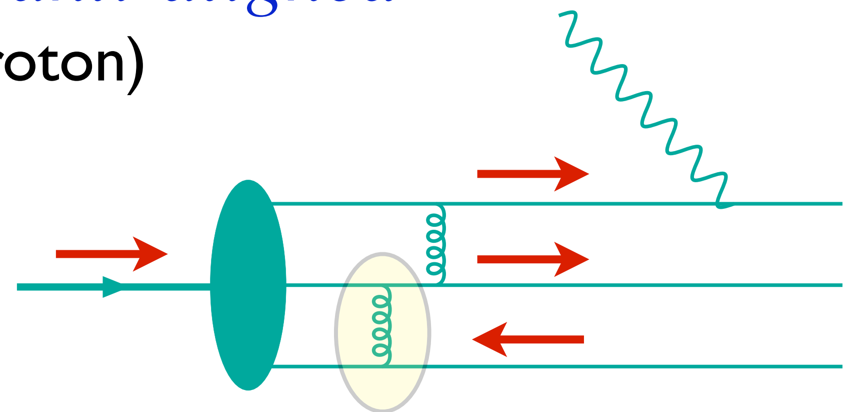
- Since $|k^2|$ is large, coupling at q-g vertex is small

  ⟶ use lowest-order perturbation theory!

- Assume wave function vanishes sufficiently fast as $|k^2| \to \infty$ <u>and</u> *unperturbed* wave function dominated by 3-quark Fock component with $\mathrm{SU}(2) \times \mathrm{SU}(3)$ symmetry

Perturbative QCD

- If spectator "diquark" spins are *anti-aligned*
  (helicity of struck quark = helicity of proton)
  $\longrightarrow$ can exchange *transverse or longitudinal* gluon



- If spectator "diquark" spins are *aligned*
  (helicity of struck quark ≠ helicity of proton)
  $\longrightarrow$ can exchange *only longitudinal* gluon

- Coupling of (large-k^2) longitudinal gluon to (small-p^2) quark is suppressed by $(p^2/k^2)^{1/2} \sim (1-x)^{1/2}$ w.r.t. transverse

$$\longrightarrow \quad q^{\downarrow} \sim (1-x)^2 \, q^{\uparrow} \sim (1-x)^5$$

Perturbative QCD

- Phenomenological consequences of $S_z = 0$ qq dominance*

→ assuming unperturbed $\mathrm{SU}(6)$ wave function,

$$F_2^n/F_2^p \rightarrow 3/7$$

→ dominance of helicity-$1/2$ photoproduction cross section

$$\sigma_{1/2} \gg \sigma_{3/2}$$

→ for all quark flavors q,

$$\Delta q/q \rightarrow 1$$

and therefore all polarization asymmetries $A_1 \rightarrow 1$

→ for pion, expect

$$F_2^\pi \sim (1-x)^2$$

* valid in Abelian & non-Abelian theories

Role of orbital angular momentum

- Above results assume quarks in lowest Fock state are in relative s-wave

  → higher Fock states and nonzero quark OAM will in general introduce additional suppression in $(1-x)$

- BUT nonzero OAM can provide logarithmic enhancement of *helicity-flip* amplitudes!

  → quark OAM modifies asymptotic behavior of nucleon's Pauli form factor

$$F_2(Q^2) \;\sim\; \log^2(Q^2/\Lambda^2)\frac{1}{Q^6}$$

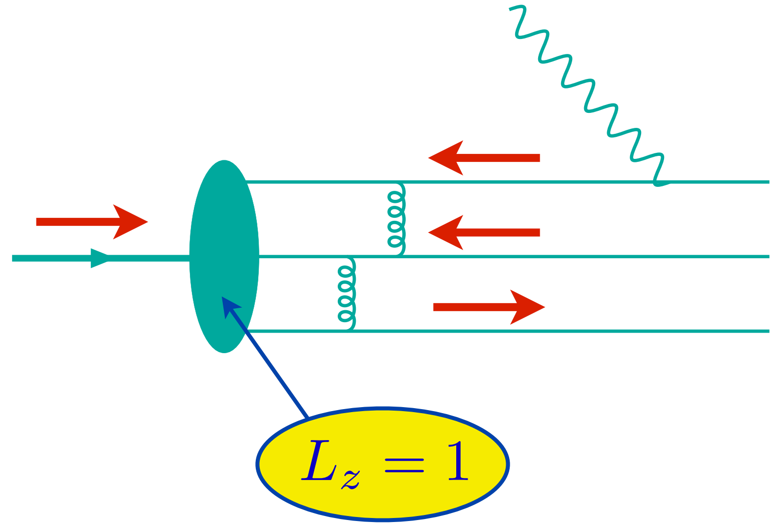
Belitsky, Ji, Yuan
PRL ***91***, *092003 (2003)*

  → consistent with surprising Q^2 dependence of proton's G_E/G_M form factor ratio

Role of orbital angular momentum

- For $L_z = 1$ Fock state, expand hard scattering amplitude in powers of $k_\perp$ ("collinear expansion")

→ logarithmic singularities arise when integrating over longitudinal momentum fractions x_i of soft quarks



→ leads to additional $\log^2(1-x)$ enhancement of $q^\downarrow$

$$q^\downarrow \sim (1-x)^5 \log^2(1-x)$$

Avakian, Brodsky, Deur, Yuan, PRL ***99****, 082001 (2007)*

(similar contributions to positive helicity $q^\uparrow$ are power-suppressed)

Role of orbital angular momentum

- $k_\perp$-odd transverse momentum dependent (TMD) distributions (vanish after $k_\perp$ integration)

  → arise from *interference* between $L_z = 0$ and $L_z = 1$ states

- T-even TMDs

  → g_{1T} (*longitudinally* polarized q in a *transversely* polarized N)

  h_{1L} (*transversely* polarized q in a *longitudinally* polarized N)

- T-odd TMDs

  → $f_{1T}^{\perp}$ (*unpolarized* q in a *transversely* polarized N – "Sivers")

  $h_1^{\perp}$ (*transversely* polarized q in an *unpolarized* N – "Boer-Mulders")

- Each behaves in $x \to 1$ limit as

$$\mathrm{TMD} \ \sim \ (1-x)^4$$

Brodsky, Yuan
PRD ***74****, 094018 (2006)*

Phenomenological implications

Phenomenological implications

- Power counting rule constraints used in exploratory fit to limited set of inclusive DIS spin structure function data

$$q^{\uparrow} = x^{\alpha}\left[A(1-x)^3 + B(1-x)^4\right]$$

$$q^{\downarrow} = x^{\alpha}\left[C(1-x)^5 + D(1-x)^6\right]$$

Brodsky, Burkardt, Schmidt
NPB ***441****, 197 (1995)*

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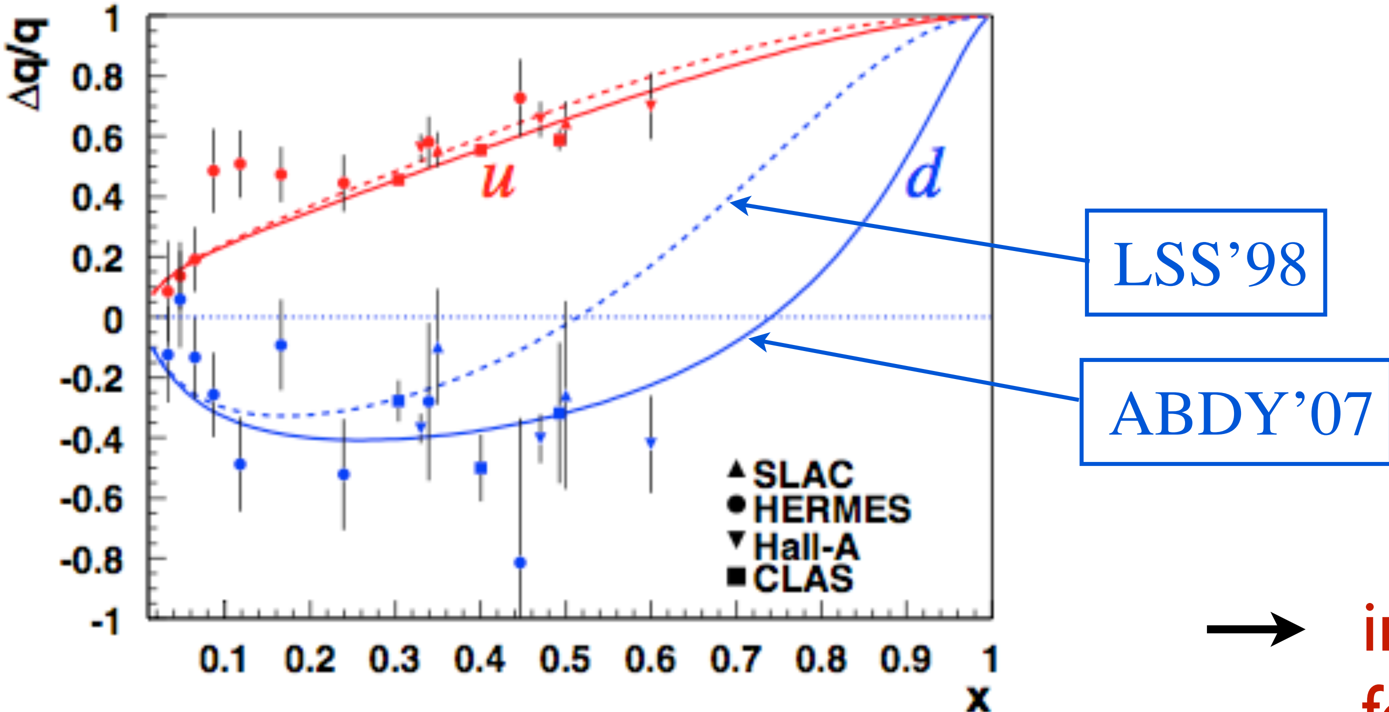
$$q^{\uparrow} = x^{\alpha}\left[A(1-x)^3 + B(1-x)^4\right]$$

$$q^{\downarrow} = x^{\alpha}\left[C(1-x)^5 + D(1-x)^6 + C'(1-x)^5 \log^2(1-x)\right]$$

additional $L_z = 1$ term



Avakian, Brodsky, Deur, Yuan
PRL ***99****, 082001 (2007)*

→ improved fit for $\Delta d/d$

Phenomenological implications

- Determining $x \to 1$ behavior experimentally is problematic

  → simple $x^\alpha (1-x)^\beta$ parametrizations inadequate for describing *high-precision* data, and global fits typically require more complicated x dependence, $e.g.$

$$q \;\sim\; x^\alpha (1-x)^\beta \,(1 + \gamma\sqrt{x} + \eta\, x)$$

  → recent global fits of spin-dependent PDFs find (at $Q^2 \sim 5\ \mathrm{GeV}^2$)

$$\beta \;\approx\; 3.3\ (\Delta u_V),\ 3.9\ (\Delta d_V)$$

de Florian et al.
*PRD **80**, 034030 (2009)*

  but with $\gamma, \eta \sim \mathcal{O}\,(10\text{–}100)$

- Challenge to perform constrained *global* fit to all DIS, SIDIS & $\vec{p}\,\vec{p}$ scattering data

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→ recent global fits of spin-dependent PDFs find (at $Q^2 \sim 5$ GeV^2)

$$\beta \approx 3.3\ (\Delta u_V),\ 4.1\ (\Delta d_V)$$

Leader, Sidorov, Stamenov PRD ***82****, 114018 (2010)*

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  → recent global fits of spin-dependent PDFs find (at $Q^2 \sim 5\ \mathrm{GeV}^2$)

$$\beta \approx 3.0\ (\Delta u_V),\ 4.1\ (\Delta d_V)$$

Bluemlein, Boettcher NPB ***841***, *205 (2010)*

  but with $\gamma, \eta \sim \mathcal{O}(10\text{–}100)$

- Challenge to perform constrained *global* fit to all DIS, SIDIS & $\vec{p}\vec{p}$ scattering data

Phenomenological implications

- Challenges for large-x PDF analysis

  → at fixed Q^2, increasing x corresponds to decreasing W
  – eventually run into nucleon *resonance* region as $x \to 1$
  – impose cuts (usual solution) or utilize quark-hadron duality (theoretical bias)

  → subleading $1/Q^2$ corrections (target mass, higher twists)

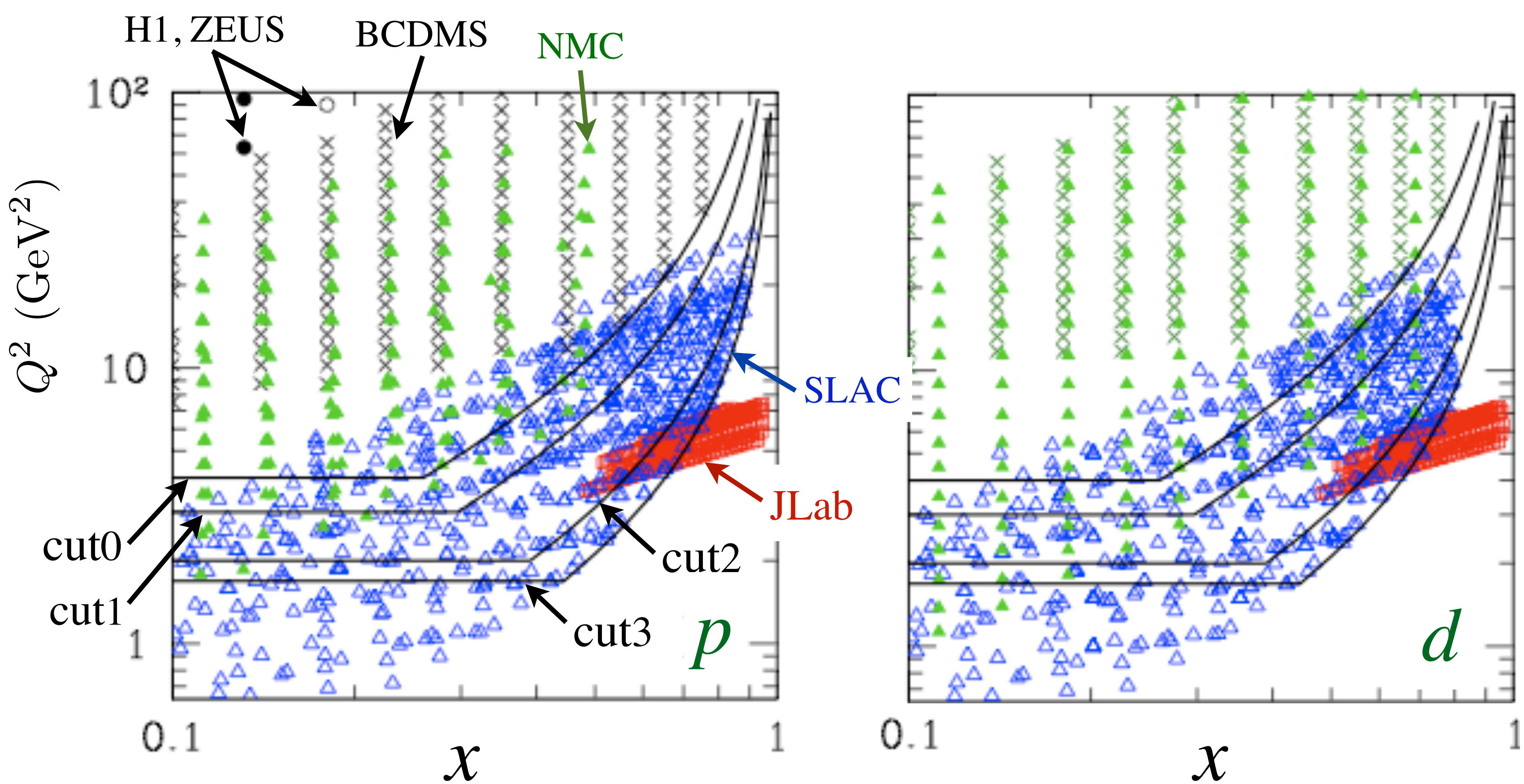
  → nuclear corrections in extraction of *neutron* information from nuclear (deuterium, ^{3}He) data

  → dependence on choice of PDF parametrization

- New CTEQ-JLab ("CJ") global PDF analysis* (unpolarized) dedicated to describing large-x region

* CJ collaboration: A. Accardi, J. Owens, WM (theory) + E. Christy, C. Keppel, P. Monaghan, L. Zhu (expt.)

CJ global analysis



cut0: $Q^2 > 4$ GeV^2, $W^2 > 12.25$ GeV^2

cut1: $Q^2 > 3$ GeV^2, $W^2 > 8$ GeV^2

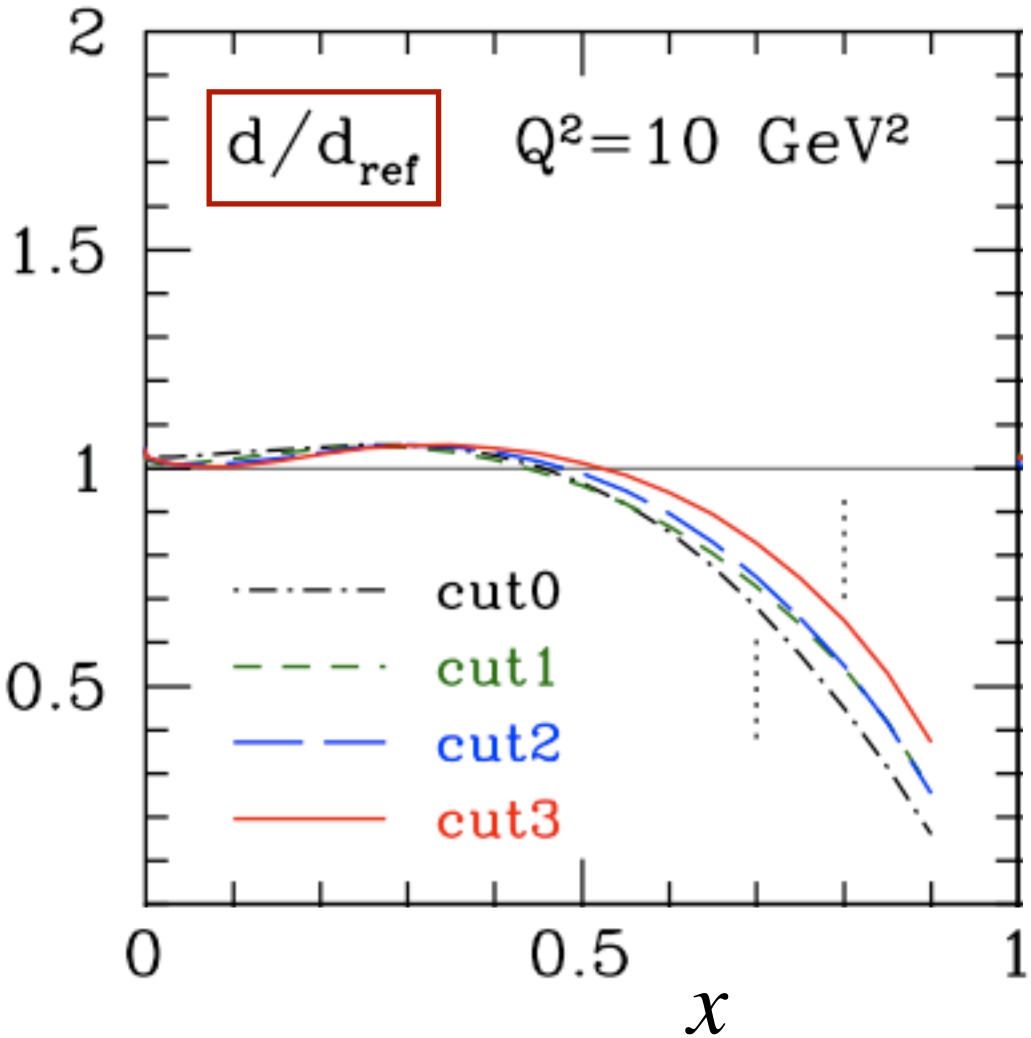
cut2: $Q^2 > 2$ GeV^2, $W^2 > 4$ GeV^2

cut3: $Q^2 > m_c^2$, $W^2 > 3$ GeV^2

factor 2 increase in DIS data from cut0 $\longrightarrow$ cut3

CJ global analysis

- Systematically reduce Q^2 & W cuts
- Fit includes TMCs, HT term, nuclear corrections

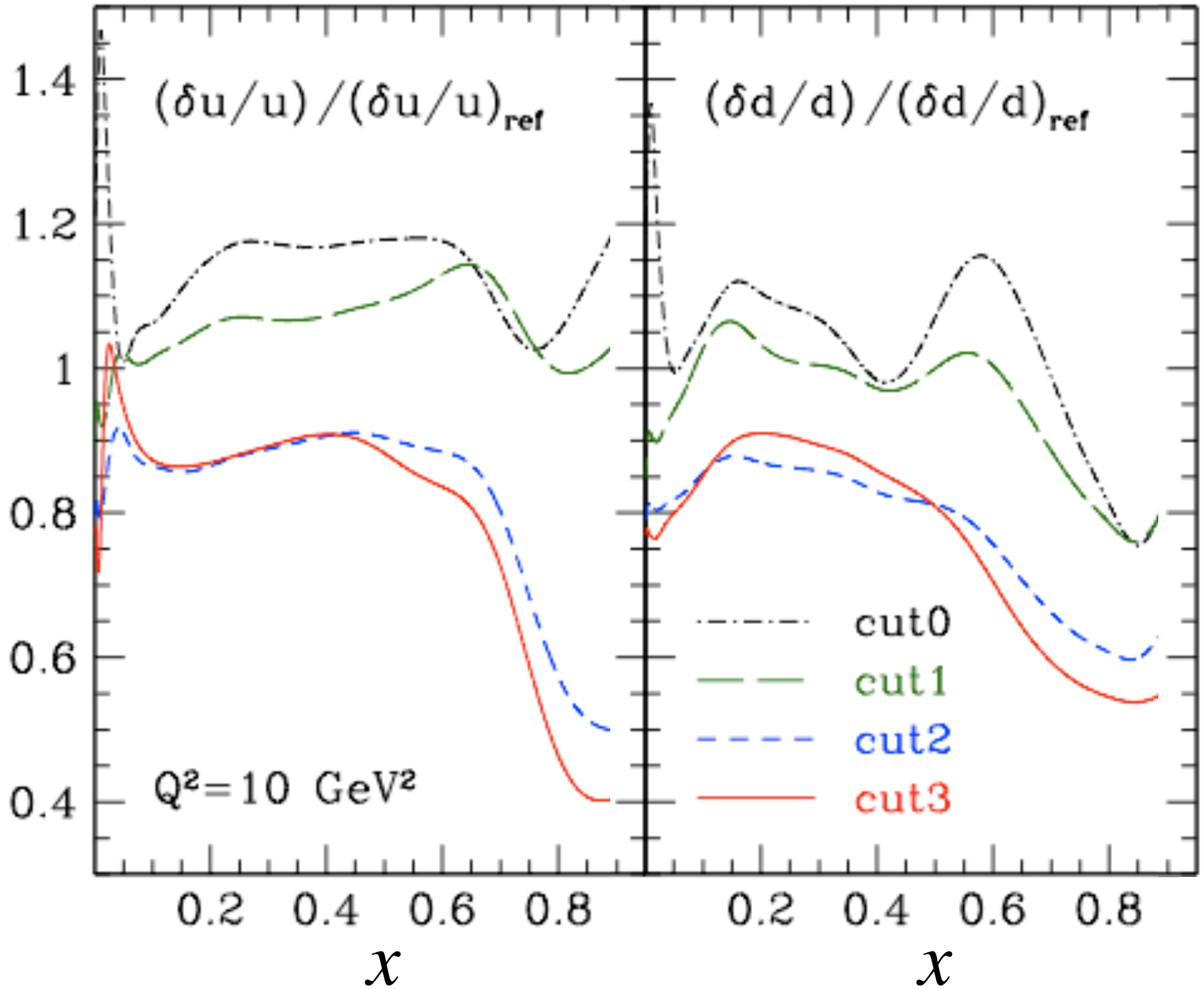


→ *stable* with respect to cut reduction

→ d quark suppressed by ~ 50% for $x > 0.5$ (driven by nuclear corrections)

Accardi et al., PRD ***81****, 034016 (2010)*

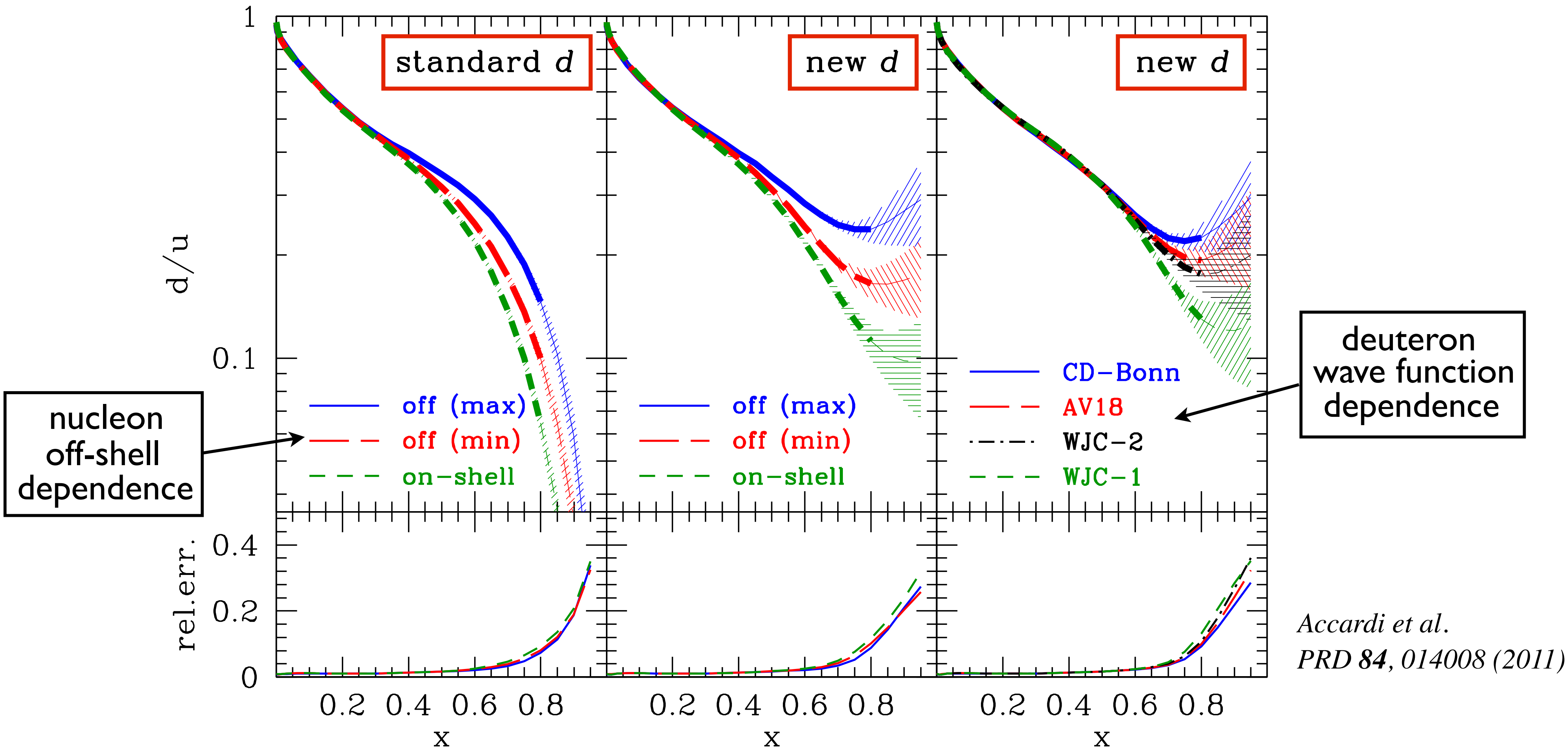
CJ global analysis



Accardi et al.
PRD ***81****, 034016 (2010)*

→ larger database with weaker cuts leads to significantly *reduced errors*, esp. at large x

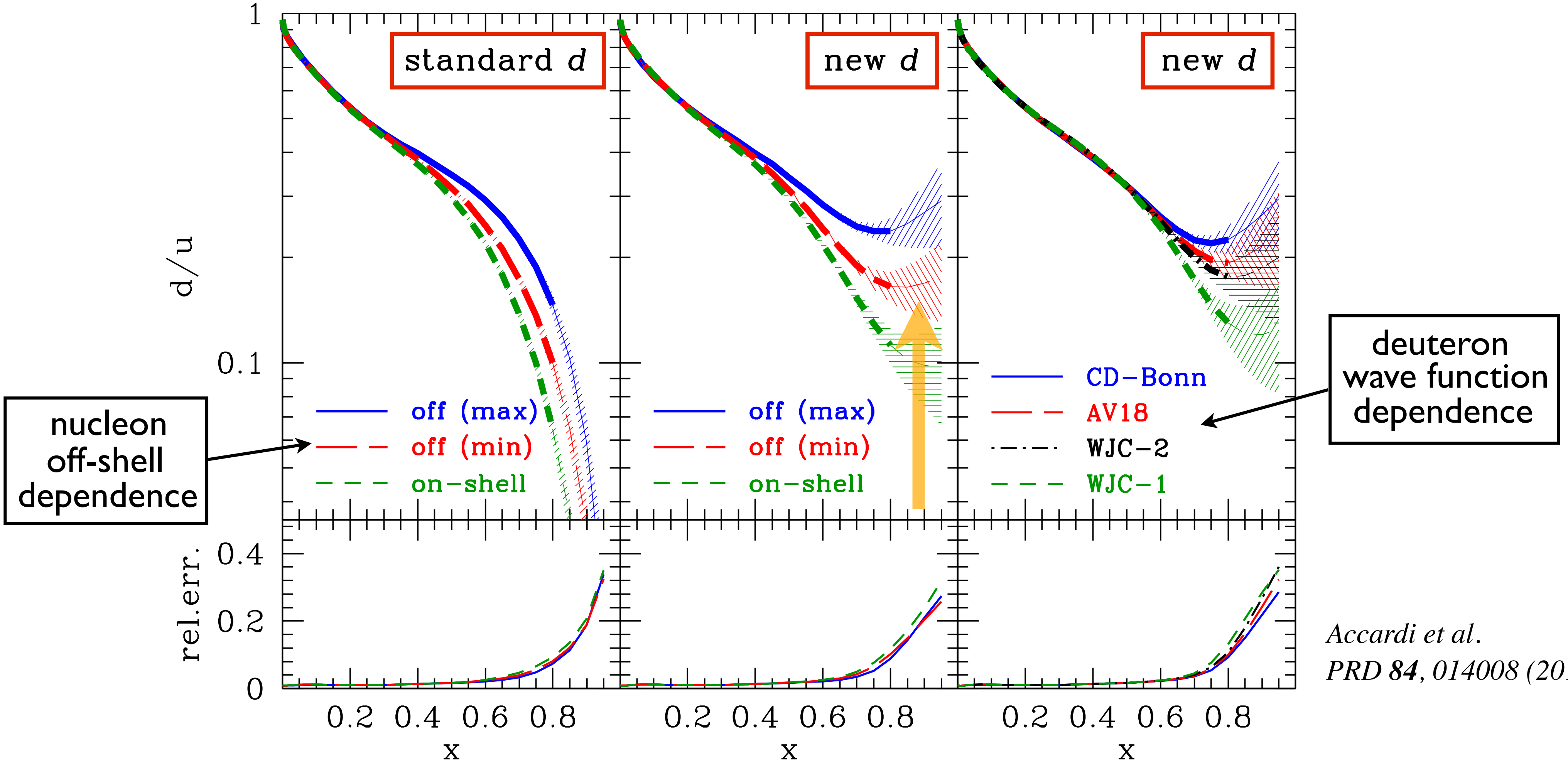
CJ global analysis



→ large nuclear correction uncertainties at $x > 0.5$

→ $x \to 1$ limiting value depends on deuteron model

CJ global analysis



Accardi et al. *PRD* ***84***, *014008 (2011)*

→ dramatic increase in d PDF in $x \to 1$ limit with more flexible parametrization $d \to d + a\,x^b\,u$

(allows for finite, nonzero d/u in $x = 1$ limit)

Outlook

- Nuclear correction uncertainties expected to be resolved with new experiments at JLab-12 GeV uniquely sensitive to d quarks (up to $x \sim 0.85$)

  → "spectator" protons tagged in SIDIS from deuterium
  $e\ d \rightarrow e\ p_{\rm spec}\ X$ ("BoNuS")

  → DIS from ^{3}He-tritium mirror nuclei
  $e\ {}^3{\rm He}({}^3{\rm H}) \rightarrow e\ X$ ("MARATHON")

  → PVDIS from protons
  $\vec{e}_L(\vec{e}_R)\ p \rightarrow e\ X$ ("SOLID")

- Constraints from W production in pp collisions at high (lepton & W boson) rapidities

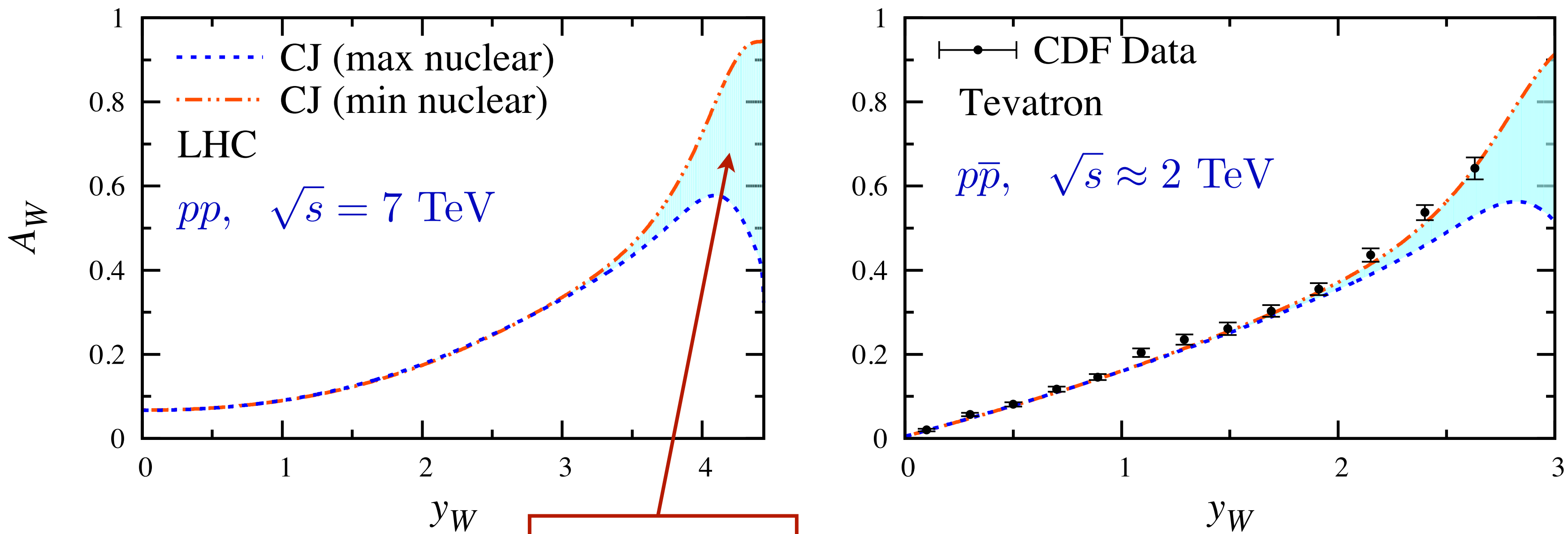
  → CDF & D0 at Fermilab, LHCb at CERN

W boson asymmetries

- Large-x PDF uncertainties affect observables at large *rapidity* y, with

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) \quad \rightarrow \quad \boxed{x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y}}$$

e.g. $W^{\pm}$ asymmetry



sensitive to d at high x

Brady, Accardi, WM, Owens
arXiv:1110:5398 [hep-ph]

Outlook

- New JLab–$12\ \mathrm{GeV}$ precisions measurements of A_1^n & A_1^p hope to constrain $\Delta d/d$ up to $x \sim 0.8$

  → new (non-inclusive DIS) experiments to reduce nuclear dependence

- Parametrization dependence of $x \to 1$ limit may be eliminated through *e.g.* "neural network" PDFs

  → thus far applied mainly to unpolarized PDFs

- New global analysis of *spin-dependent* PDFs dedicated to large-x, moderate-Q^2 region

  → JLab Angular Momentum ("JAM") collaboration*

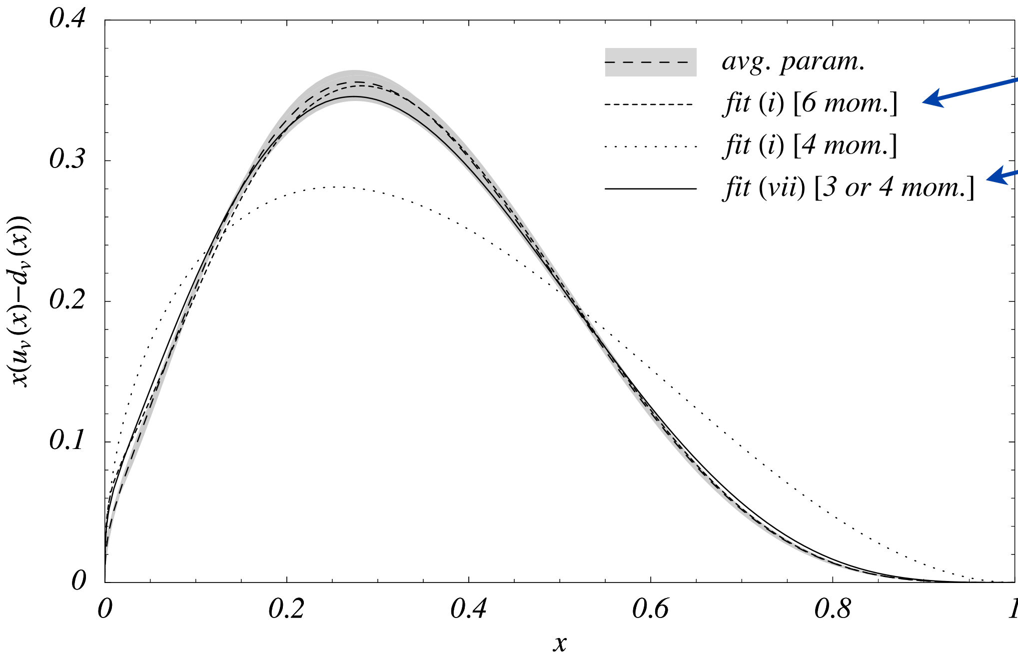
  → initial focus on helicity PDFs; later expand scope to TMDs

* JAM collaboration: P. Jimenez-Delgado, A. Accardi, WM (theory) + JLab Halls A, B, C (expt.)

Outlook

- Large-x PDFs from lattice?

  → need many moments to reconstruct x dependence



fit (i): $\alpha, \beta, \gamma, \eta$ unconstrained

fit (vii): γ, η constrained

assume functional form

$$q(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

Detmold, WM, Thomas
MPLA ***18****, 2681 (2003)*

- Need new ideas

  → *e.g.* compute Compton scattering tensor
  directly by coupling to fictitious heavy quark
  (remove all-to-all propagators, and operator mixing)

Detmold, Lin
PRD ***73****, 014501 (2006)*

The End