

With Gary Goldstein, J.Osvaldo Gonzalez Hernandez

### Outline

 How reliably can GPDs be measured? Towards a global fit: <u>models, parameters, theoretical errors, resolution</u>? (GGL, PRD 2011)

2) Exclusive  $\pi^{\circ}$  electroproduction  $\rightarrow$  chiral odd sector (Ahmad et al. PRD 2009, Goldstein et al., hep-ph/1201.6088)



# Extraction of GPDs from experimental data

→ <u>Define</u> "what type of information"

→ <u>Define</u> "the way to access it"

#### Wigner Dist'ns and OAM from Experiment????

b

k<sub>T</sub>

$$F(X,b) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{-i\vec{\Delta}_T \cdot \vec{b}} F(X,0,t \equiv -\Delta_T^2)$$

O. Gonzalez Hernandez, 2011

0.12

0.08

0.06

0.04 0.02

0

0.1







#### "Slices" of Wigner Distn's (S.L., S.Taneja, PRD'04)

Simonetta Liuti: Study of Parton Interactions in Nuclei using Wigner Distributions

EIC Working Group, Editors: K. Hafidi et al.



2

Off forward Parton Distributions (GPDs) are embedded in soft matrix elements for deeply virtual exclusive experiments



) 
$$\frac{1}{(p+q)^2 - m^2 + i\varepsilon} = PV \frac{1}{(p+q)^2 - m^2} - i\pi \delta((p+q)^2 - m^2)$$
 Both Re and Im parts are present

(2) Quarks momenta and spins on LHS can be different from the RHS

(1)



1

H, E, ...

# Quark-Proton Helicity Amplitudes



# Diquark Model

$$A_{++,++} = \int d^{2}k_{\perp}\phi_{++}^{*}(k',P')\phi_{++}(k,P)$$

$$A_{++,++} = \int d^{2}k_{\perp}\phi_{++}^{*}(k',P')\phi_{++}(k,P)$$

$$A_{++,-+} = \int d^{2}k_{\perp}\phi_{++}^{*}(k',P')\phi_{-+}(k,P).$$

$$\phi_{\Lambda,\lambda}(k,P) = \Gamma(k) rac{ar{u}(k,\lambda)U(P,\Lambda)}{k^2 - m^2}$$

$$\phi^*_{\Lambda'\lambda'}(k',P')=\Gamma(k')rac{\overline{U}(P',\Lambda')u(k',\lambda')}{k'^{\,2}-m^2},$$

$$H = \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_{\perp} \frac{\left[ (m + MX) \left( m + M \frac{X - \zeta}{1 - \zeta} \right) + \mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp} \right]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2} + \frac{\zeta^2}{4(1 - \zeta)} E,$$

$$E ~=~ \mathcal{N} rac{1-\zeta/2}{1-X} \int d^2k_\perp rac{-2M(1-\zeta)\left[(m+MX)rac{ ilde{k}\cdot\Delta}{\Delta_\perp^2} - \left(m+Mrac{X-\zeta}{1-\zeta}
ight)rac{k_\perp\cdot\Delta}{\Delta_\perp^2}
ight]}{(k^2-M_\Lambda^2)^2(k'^2-M_\Lambda^2)^2}$$

$$\widetilde{H} \;=\; \mathcal{N} \frac{1-\zeta/2}{1-X} \int d^2 k_\perp \frac{\left[ (m+MX) \left( m+M \frac{X-\zeta}{1-\zeta} \right) - \mathbf{k}_\perp \cdot \tilde{\mathbf{k}}_\perp \right]}{(k^2-M_\Lambda^2)^2 (k'^2-M_\Lambda^2)^2} + \frac{\zeta^2}{4(1-\zeta)} \widetilde{E}$$

$$\widetilde{E} \ = \ \mathcal{N} \frac{1-\zeta/2}{1-X} \int d^2 k_{\perp} \frac{-\frac{4M(1-\zeta)}{\zeta} \left[ (m+MX) \frac{\widetilde{k} \cdot \Delta}{\Delta_{\perp}^2} + \left(m+M\frac{X-\zeta}{1-\zeta}\right) \frac{k_{\perp} \cdot \Delta}{\Delta_{\perp}^2} \right]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2}$$



# Crossing Symmetries



Parametric Form

$$F(X,\zeta,t) = \mathcal{N}G_{M_X,m}^{M_\Lambda}(X,\zeta,t) R_p^{\alpha,\alpha'}(X,\zeta,t)$$
  
Recursive fit, STEP 1  

$$H^q(X,0,0,Q^2) = f_1^q(X,Q^2) \equiv q_v(X)$$
  

$$\widetilde{H}^q(X,0,0,Q^2) = g_1^q(X,Q^2) \equiv \Delta q_v(X)$$

Constraints from PDFs

Recursive fit, STEP 2

$$egin{aligned} &\int_0^1 H^q(X,\zeta,t) &= F_1^q(t) \ &\int_0^1 E^q(X,\zeta,t) &= F_2^q(t) \ &\int_0^1 \widetilde{H}^q(X,\zeta,t) &= G_A^q(t) \ &\int_0^1 \widetilde{E}^q(X,\zeta,t) &= G_P^q(t) \end{aligned}$$

#### Constraints from FFs

We asked the question: "What is the minimal number of parameters necessary to fit X and t?" Can be addressed with Recursive Fit

Parameters	Н	E	$\widetilde{H}$	$\widetilde{E}$
$m_u$ (GeV)	0.420	0.420	2.624	2.624
$M^u_X$ (GeV)	0.604	0.604	0.474	0.474
$M^u_{\Lambda}~({ m GeV})$	1.018	1.018	0.971	0.971
$\alpha_u$	0.210	0.210	0.219	0.219
$\alpha'_u$	$2.448 \pm 0.0885$	$2.811\pm0.765$	$1.543\pm0.296$	$5.130\pm0.101$
$p_u$	$0.620 \pm 0.0725$	$0.863\pm0.482$	$0.346\pm0.248$	$3.507\pm0.054$
$\mathcal{N}_u$	2.043	1.803	0.0504	1.074
$\chi^2$	0.773	0.664	0.116	1.98
$m_d~({ m GeV})$	0.275	0.275	2.603	2.603
$M^d_X$ (GeV)	0.913	0.913	0.704	0.704
$M^d_{\Lambda}~({ m GeV})$	0.860	0.860	0.878	0.878
$\alpha_d$	0.0317	0.0317	0.0348	0.0348
$lpha_d'$	$2.209 \pm 0.156$	$1.362\pm0.585$	$1.298\pm0.245$	$3.385 \pm 0.145$
$p_d$	$0.658 \pm 0.257$	$1.115 \pm 1.150$	$0.974\pm0.358$	$2.326\pm0.137$
$\mathcal{N}_d$	1.570	-2.800	-0.0262	-0.966
$\chi^2$	0.822	0.688	0.110	1.00

#### Flexible Model ('04-'07-'09'-'11-...) S.Ahmad, H. Honkanen, S. Taneja -G.Goldstein, O.Gonzalez Hernandez





Goldstein et al. arXiv:1012.3776

ξ=0, t=0

#### Compton Form factors vs. $\zeta$



 $Q^2=2 GeV^2$ 

#### Polynomiality!

#### Goldstein et al. arXiv:1012.3776



#### Comparison with lattice



# Implementing DVCS data...

$$R = X^{-[\alpha + \alpha'(X)t + \beta(\zeta)t]},$$

#### extra term





#### Hall A



#### Having fitted Jlab data, we predict Hermes





 $\pi^{o}$  and  $\eta$  production probing the GPD chiral-odd sector Goldstein et al., arXiv:hep-ph/1201.6088

Issue in a nutshell:

"Collinear factorization approach" for chiral-even process

$$g_{0,+;0,+} \approx \frac{1}{Q} \int d\tau \frac{\phi_{\pi}(\tau)}{\tau} C^{+} \Rightarrow \frac{d\sigma_{L}^{even}}{dt} \propto \frac{1}{Q^{6}}$$
$$g_{1,+;0,+} \approx \frac{1}{Q^{2}} \int d\tau \frac{\phi_{\pi}(\tau)}{\tau} C^{+} \Rightarrow \frac{d\sigma_{T}^{even}}{dt} \propto \frac{1}{Q^{8}}.$$

"Collinear factorization approach" for chiral-odd process

$$g_{0+,0-} pprox rac{d\sigma_L^{odd}}{dt} \propto rac{1}{Q^{10}}, \quad g_{1+,0-} pprox rac{d\sigma_T^{odd}}{dt} \propto rac{1}{Q^8}.$$

Transverse component seems to be larger than naively expected

$$f_{\Lambda_{\gamma},\Lambda;0,\Lambda'}(\xi,t) = \sum_{\lambda,\lambda'} \int dx d^2 k_{\perp} g_{\Lambda_{\gamma},\lambda;0,\lambda'}(x,k_{\perp},\xi,t) A_{\Lambda',\lambda';\Lambda,\lambda}(x,k_{\perp},\xi,t)$$

$$q, \Lambda_{\gamma}$$

$$p, \lambda$$

$$P = K f_{\pi} \{\gamma_5 \not \phi_{\pi}(\tau) + \gamma_5 \mu_{\pi} \phi_{\pi}^{(3)}(\tau)\}$$

$$p' = p - \Delta \cdot \lambda' \qquad \gamma_{\mu} \gamma_5 \qquad \gamma_5$$

$$P' = P - \Delta, \Lambda'$$

$$g_{T} = g_{\pi}^{odd}(Q) \left[ \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right] = g_{\pi}^{odd}(Q) C^{+}$$
  
$$g_{L} = g_{\pi}^{odd}(Q) \sqrt{\frac{t_{o} - t}{Q^{2}}} \left[ \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right] = g_{\pi}^{odd}(Q) \sqrt{\frac{t_{o} - t}{Q^{2}}} C^{+},$$

$$egin{aligned} f_1 &= f_{1+,0+} &= g_{1+,0-} \otimes A_{+-,++} \ f_2 &= f_{1+,0-} &= g_{1+,0-} \otimes A_{--,++} \ f_3 &= f_{1-,0+} &= g_{1+,0-} \otimes A_{+-,-+} \ f_4 &= f_{1-,0-} &= g_{1+,0-} \otimes A_{--,-+}, \end{aligned}$$

#### **Cross** Section

$$\begin{split} \frac{d^4\sigma}{d\Omega d\epsilon_2 d\phi dt} &= \Gamma \left\{ \frac{d\sigma_T}{dt} + \epsilon_L \frac{d\sigma_L}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon_L(\epsilon+1)} \cos \phi \frac{d\sigma_{LT}}{dt} \right. \\ &+ h \sqrt{2\epsilon_L(\epsilon-1)} \frac{d\sigma_{L'T}}{dt} \sin \phi \right\}, \end{split}$$

 $_{4}ec{S}_{\perp}$ 

 $\phi_{S}$ 

-Nunnu

φ

 $\vec{k'}$ 

 $ec{k}$ 

$$\begin{split} \frac{d\sigma_T}{dt} &= \mathcal{N} \left( \mid f_1 \mid^2 + \mid f_2 \mid^2 + \mid f_3 \mid^2 + \mid f_4 \mid^2 \right) \\ \frac{d\sigma_L}{dt} &= \mathcal{N} \left( \mid f_5 \mid^2 + \mid f_6 \mid^2 \right), \\ \frac{d\sigma_{TT}}{dt} &= 2\mathcal{N} \, \Re e \left( f_1^* f_4 - f_2^* f_3 \right). \\ \frac{d\sigma_{LT}}{dt} &= 2\mathcal{N} \, \Re e \left[ f_5^* (f_2 + f_3) + f_6^* (f_1 - f_4) \right]. \\ \frac{d\sigma_{LT'}}{dt} &= 2\mathcal{N} \, \Im m \left[ f_5^* (f_2 + f_3) + f_6^* (f_1 - f_4) \right]. \end{split}$$

#### In terms of GPDs

$$\begin{split} \epsilon_T^{\mu} T_{\mu}^{\Lambda\Lambda'} &= e_q \int_{-1}^1 dx \, \frac{g_T}{2\overline{P}^+} \, \overline{U}(P',\Lambda') \left[ i\sigma^{+i} H_T^q(x,\xi,t) + \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2M} E_T^q(x,\xi,t) \right. \\ \left. \frac{\overline{P}^+ \Delta^i - \Delta^+ \overline{P}^i}{M^2} \widetilde{H}_T^q(x,\xi,t) + \frac{\gamma^+ \overline{P}^i - \overline{P}^+ \gamma^i}{2M} \widetilde{E}_T^q(x,\xi,t) \right] U(P,\Lambda), \end{split}$$

# M. Diehl, 2001

 $\mathcal{H}_{_{T}}, \mathcal{E}_{_{T}}, ilde{\mathcal{E}}_{_{T}}, ilde{\mathcal{E}}_{_{T}}$ 

 $\tau = (t_0 - t)/2M^2$ 

Physical Interpretation of the various chiral-odd GPDs

Forward limit

$$H_{_T}(x,0,0) = q_{\Uparrow}^{\uparrow}(x) - q_{\Uparrow}^{\downarrow}(x) = h_{_1}(x)$$

Form Factors

$$\int H_T(x,\xi,t) \, dx = \delta_T(t)$$
$$\int \overline{E}_T(x,\xi,t) \, dx = \int \left(2\tilde{H}_T + E_T\right) dx = \kappa_T(t)$$

$$\int \tilde{E}_{T}(x,\boldsymbol{\xi},t)\,dx = 0$$

No direct interpretation of  $E_{\tau}$ 

$$\begin{vmatrix} S = 0 & S = 1 \\ \phi^*_{\Lambda'\lambda'}\phi_{\Lambda\lambda} & \phi^{\mu}_{\Lambda'\lambda'} \left( \sum_{\lambda''} \epsilon^{*\lambda''}_{\mu} \epsilon^{\lambda''}_{\nu} \right) \phi^{\nu}_{\Lambda\lambda} \end{vmatrix}$$

• -/

In order to explain the working of Parity transformations, we write the LHS and RHS of Fig. 1

$$\begin{array}{|c|c|c|c|c|}\hline RHS & LHS \\ \hline S = 0 & \phi^*_{\Lambda'\lambda'} & \phi_{\Lambda\lambda} \\ \hline S = 1 & \phi^\mu_{\Lambda'\lambda'} \epsilon^{*\,\lambda''}_\mu & \epsilon^{\lambda''}_\nu \phi^\nu_{\Lambda\lambda} \end{array}$$

$$\begin{array}{c} \text{Odd} \qquad \begin{array}{l} \text{S=1} \\ \text{Even} \end{array} \\ \begin{array}{c} A_{++,--}^{(0)} = A_{++,++}^{(0)} \\ A_{++,+-}^{(0)} = -A_{++,++}^{(0)} \end{array} \\ A_{++,+-}^{(0)} = -A_{++,++}^{(0)} \end{array} \\ \begin{array}{c} A_{++,+-}^{(1)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X'^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{++,-+}^{(1)} \\ A_{++-,++}^{(1)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X'^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{++,++}^{(1)} \end{array} \\ \begin{array}{c} A_{++-,++}^{(0)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X'^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{++,++}^{(1)} \\ A_{++-,++}^{(0)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{-+,++}^{(1)} \end{array} \\ \end{array} \\ \begin{array}{c} A_{++,-+}^{(0)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{++,-+}^{(1)} \\ A_{++-,++}^{(0)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{-+,++}^{(1)} \end{array} \\ \end{array} \\ \begin{array}{c} A_{++,-+}^{(0)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{-+,++}^{(1)} \\ A_{++-,++}^{(0)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{-+,++}^{(1)} \end{array} \\ \end{array} \\ \begin{array}{c} A_{++,-+}^{(0)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{-+,++}^{(1)} \\ A_{++,-+}^{(1)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{-+,++}^{(1)} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} A_{++,-+}^{(0)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{-+,++}^{(1)} \\ A_{++,-+}^{(0)} = -\sqrt{\frac{\langle \tilde{k}_{\perp}^2 \rangle}{X^2 + \langle \tilde{k}_{\perp}^2 \rangle / P^{+2}}} A_{-+,++}^{(1)} \end{array} \\ \end{array} \\ \end{array} \\ \end{array}$$





X

 $\sum_{\Lambda} \Im m F_{\Lambda+,\Lambda-} \propto h_1^{\perp}(x,k_T^2)$  $A_{\scriptscriptstyle ++,+-} - A_{\scriptscriptstyle +-,++} \propto 2\tilde{H}_{_T} + E_{_T}$  $\sum_{\lambda} \Im m F_{+\lambda,-\lambda} \propto f_{1T}^{\perp}(x,k_T^2)$  $A_{\scriptscriptstyle ++,-+}-A_{\scriptscriptstyle -+,++} \propto E$  $Q^2 = 2 \text{ GeV}^2$   $\zeta = 0.2$  -t = 0.112 1.75 1.5  $(\mathbf{h}_1^{\mathsf{T}}) 2\tilde{H}_T + E_T$ 1.25 1 0.75 0.5 U  $\tilde{E}_{T}$ 0.25 0 0.8 0.6 0.4 0.2 -0 -0.2 -0.4

-0.6 -0.8

0.1

0.2

0.3

0.4

0.5

0.7

0.6

0.8

d

0.9

How well do the parameters fixed with DVCS data reproduce  $\pi^{\circ}$  electroproduction data?



Hall B data, Kubarovsky& Stoler, PoS ICHEP 2010



#### Vary tensor charge as a parameter to see sensitivity of data



FIG. 9 (color online). (color online) Longitudinal/transverse interference term,  $d\sigma_{\rm LT}/dt$ , Eq. (15), plotted vs -t at  $Q^2 = 2.3 \text{ GeV}^2$ ,  $x_{Bj} = 0.36$ , for different values of the *u* quarks tensor charge,  $\delta u$ , used as a freely varying parameter in the GPD approach. The *d* quark component,  $\delta d$  was taken as  $\delta d = -0.62$ , i.e. equal to the central value extracted in the global fit of Ref. [44].

 $Q^2$  dependence  $\rightarrow$  obviously not predicted by collinear factorization  $\checkmark$  Presence of a large transverse component

"Anomalous" Pion Vertex behavior

#### Explain large T component



CCL or Vivebon nh 1201	6099				
	Chiral Odd GPD	$J^{-C}$		$J^{+C}$	
	$H_T(x,\xi,t) - H_T(-x,\xi,t)$	$ 2^{-+}, 4^{-+},$ (8)	S = 0)	1++, 3++ (	S = 1)
	$E_T(x,\xi,t) - E_T(-x,\xi,t)$	$2^{-+}, 4^{-+}, \dots$ (8)	S = 0)	1++, 3++ (	S = 1)
	$\widetilde{H}_T(x,\xi,t) - \widetilde{H}_T(-x,\xi,t)$			$1^{++}, 3^{++}, \dots$ (	S = 1)
	$\widetilde{E}_T(x,\xi,t) - \widetilde{E}_T(-x,\xi,t)$	$ 2^{-+}, 4^{-+}, \dots$ (8)	S = 0)	$3^{++}, 5^{++} \dots$ (	S = 1)
	$H_T(x,\xi,t) + H_T(-x,\xi,t)$	1, 2, 3	(S = 1)	$1^{+-}, 3^{+-} \dots$	(S=0)
	$E_T(x,\xi,t) + E_T(-x,\xi,t)$	1, 2, 3	(S = 1)	$1^{+-}, 3^{+-} \dots$	(S=0)
	$\widetilde{H}_T(x,\xi,t) + \widetilde{H}_T(-x,\xi,t)$	1, 2, 3	(S = 1)		
	$\widetilde{E}_T(x,\xi,t) + \widetilde{E}_T(-x,\xi,t)$	$2^{}, 3^{}, 4^{} \dots$	(S = 1)	$3^{+-}, 5^{+-} \dots$	(S=0)

All these combinations are possible, therefore...



Now that we have allowed for a large T component, explain the  $Q^2$  dependence....





#### Spin plays a role

LIUTAhmad et al., PHYSICAL REVIEW D 79, 054014 (2009



V=1<sup>--</sup>, 2<sup>--</sup>, 3<sup>--</sup>, ... A=1<sup>+-</sup>, 3<sup>+-</sup>, ...

#### Size of qqbar pair

We obtain a mixture of configurations of different "radii" (and different Q2 dependence)

> ✓ V →  $\pi^{\circ}$  → No change of OAM, ΔL=0 ✓ A →  $\pi^{\circ}$  → One unit change of OAM, ΔL=1

Axial vector transition involves Bessel  $J_1$ 

 $\psi_A^{(1)}(y_1,b) = \int d^2k_T J_1(y_1b)\psi(y_1,k_T), \;\;$  qqbar pair are more separated!

#### Summary of Q<sup>2</sup> dependence

✓ Twist 3 DA has a steeper dependence in the longitudinal variable "x" yields larger contribution

 $\checkmark$  This can compensate for the fall off in Q<sup>2</sup>

✓ Spin plays a role

(A connection is possible with A. Radyushkin's et al. interpretation of Babar data more channels including rho production need to be explored)

# Back up

#### Role of BSA components: H and H-tilde



Goldstein et al. arXiv:1012.3776

#### Preliminary Results (PDFs)

(D. Perry, DIS 2010 and MS Thesis 2010, K. Holcomb, Exclusive Processes Workshop, Jlab 2010)  $Q^2 = 7.5 \text{ GeV}^2$ 

