Angular Momentum Sum Rule in the Deuteron

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Our framework: investigate the issue of partonic interpretations of AM SR's

1. Episode 1: a SR is suggested identifying the elements of the Energy Momentum Tensor (EMT).
   (Jaffe&Manohar (JM))

2. Episode 2: to have a "simple" partonic picture, work directly in $A^+=0$ – a SR follows where the quark and gluon spin components are identified with the n=1 moments of spin dependent s.f.s' from DIS, $\Delta\Sigma$ and ΔG. The OAM ones have not yet been found to be associated to moments in QCD
   (however… see X.Ji's talk INT2012)

3. Episode 3: New processes (DVCS …) are thought of, whose structure functions – the GPDs - admit n=1 moments that coincide only with the (spin+OAM) quark and gluon components of the SR (X.Ji)

4. New Episode: OAM might be associated with twist-3 GPDs
   (X. Ji, "The man who kicked the hornet's nest")

QCD's prediction of a breakdown into quark and gluon terms is not unique. At present there are several possible interpretations:

- Jaffe Manohar (NPB, 1990)
- X.Ji (PRL, 1997)
- Chen, Lu, Sun, Wang, Goldman (arXiv, 2009)
- Wakamatsu (PRD, 2010)
- Leader (PRD83, 2011)

A possible approach

"We cannot prefer one mechanism vs. the other but all methods are good so long as a connection with experiment, for each case, can be established."

Unfortunately, so far

"None of the mechanisms above connect directly with experiment"

Perhaps suggest a way of relating approaches like was done for scheme dependence

- Other spin systems, Spin=1, and Spin=0, might help shed light/add probes, possibilities, new insights

- In DVCS: spin 1 system, due to the presence of additional L components (D-waves) provides a crucial test the working of the angular momentum sum rules

Interest in Spin 1 targets: deuterium, ^{6}Li, …

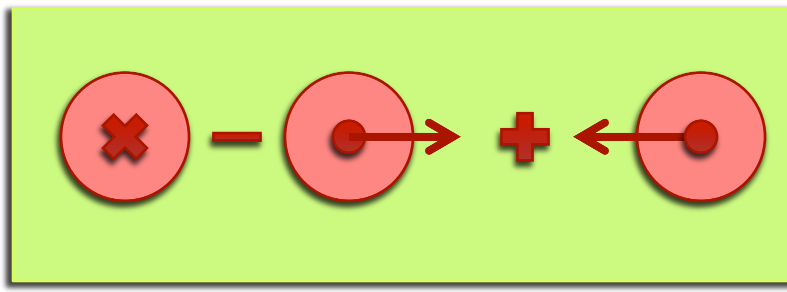
✓ In DIS: Unique possibility to study how the deep inelastic structure of nuclei differs from a system of free nucleons.

➔ One more distribution w.r.t. spin 1/2

Tensor Structure Function

Hoobhoy, Jaffe, Manohar (1989)

$$\mathbf{b_1(x)=\frac{1}{2}\left[2q_{\uparrow}^{0}-\left(q_{\uparrow}^{1}+q_{-}^{1}\right)\right]}$$



$b_1(x)\rightarrow 0$ for free nucleons

$b_1(x)\neq 0$ in bound systems

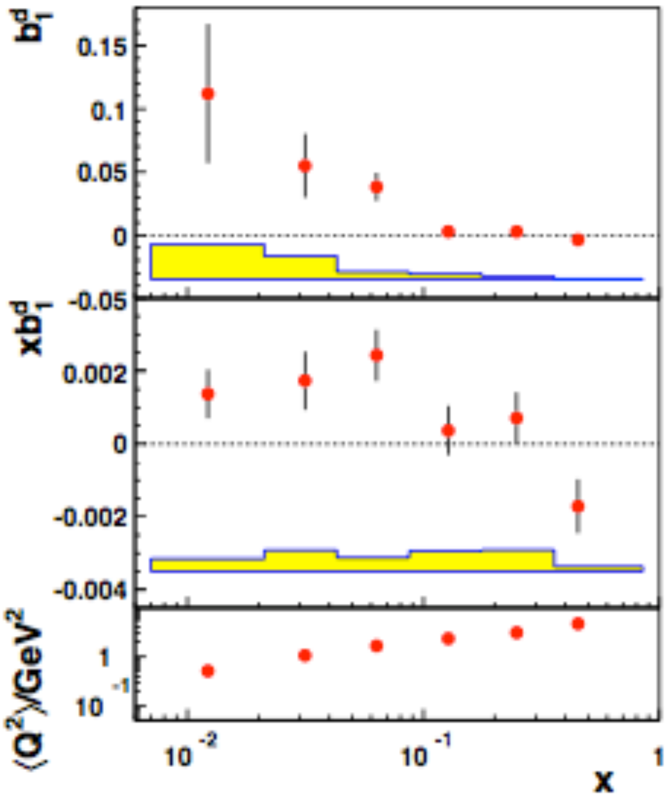
➔ Role of D wave!

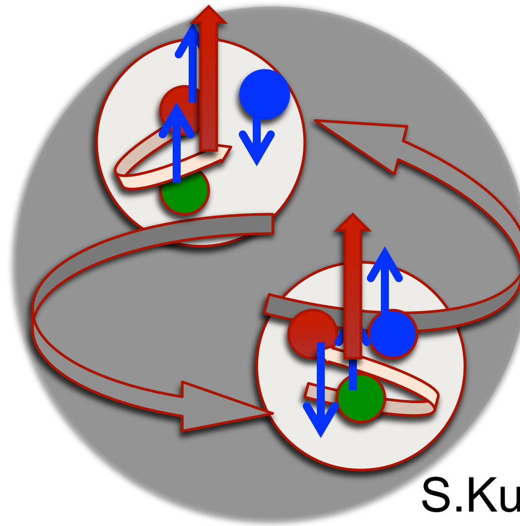
Sum Rule

$$\int \mathbf{b_1(x)\,dx} = \frac{1}{9}\int\left\{4\left[\bar{u}_{\uparrow}^{0}-\left(\bar{u}_{\uparrow}^{1}+\bar{u}_{\downarrow}^{1}\right)\right]+\left[\bar{d}_{\uparrow}^{0}-\left(\bar{d}_{\uparrow}^{1}+\bar{d}_{\downarrow}^{1}\right)\right]+\left[\bar{s}_{\uparrow}^{0}-\left(\bar{s}_{\uparrow}^{1}+\bar{s}_{\downarrow}^{1}\right)\right]\right\}dx$$

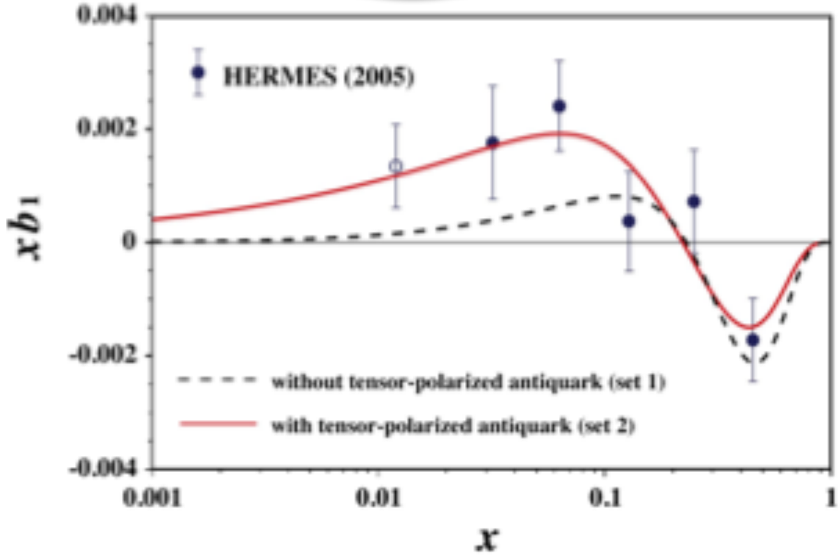
Close and Kumano (1990)

Hermes 2005





S.Kumano PRD82,2010



Spin One Sum Rule Derivation

OAM Sum Rule: Operators

$$M^{\mu\nu\lambda} = x^{\mu} T^{\nu\lambda} - x^{\lambda} T^{\mu\nu}$$

$$J^{i}_{q,g} = \frac{1}{2} \varepsilon^{ijk} \int d^3x \left(x^{j} T^{0k}_{q,g} - x^{k} T^{0j}_{q,g} \right)^{\mu}$$

$$T^{\mu\nu}_{q} = \frac{1}{2} \left[\bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + \bar{\psi} \gamma^{(\mu} i \overleftarrow{D}^{\nu)} \psi \right]$$

$$T^{\mu\nu}_{g} = \frac{1}{4} g^{\mu\nu} F - F^{\mu\rho} F^{\nu}_{\rho}$$

EMT Matrix Element

$$
\begin{aligned}
\langle p'|T^{\mu\nu}|p\rangle &= -\frac{1}{2}P^\mu P^\nu(\epsilon'^*\epsilon)\mathcal{G}_1(t) \\
&- \frac{1}{4}P^\mu P^\nu \frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_2(t) - \frac{1}{2}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right](\epsilon'^*\epsilon) \\
&\times \mathcal{G}_3(t) - \frac{1}{4}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_4(t) \\
&+ \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) + \epsilon^\mu(\epsilon'^* P)\right)P^\nu + \mu \leftrightarrow \nu\right]\mathcal{G}_5(t) \\
&+ \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) - \epsilon^\mu(\epsilon'^* P)\right)\Delta^\nu + \mu \leftrightarrow \nu \right. \\
&+ \left. 2g_{\mu\nu}(\epsilon P)(\epsilon'^* P) - (\epsilon'^{*\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu)\Delta^2\right]\mathcal{G}_6(t) \\
&+ \frac{1}{2}\left[\epsilon^{*\prime\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu - \frac{1}{2}g^{\mu\nu}(\epsilon'^*\epsilon)\right]M^2\mathcal{G}_7(t) \\
&+ g^{\mu\nu}(\epsilon'^*\epsilon)M^2\mathcal{G}_8(t) \qquad (4)
\end{aligned}
$$

7 (conserved f.f.'s➔ $\mathcal{G}_1 - \mathcal{G}_7$) + 1 (non-conserved ➔ $\mathcal{G}_8$)

Compare to spin 1/2

$$\langle p', s|T_{\mu\nu}(0)|p, s\rangle = A_0(k^2)P_\mu P_\nu + iA_1(k^2)(\epsilon_{\mu\alpha\beta\sigma}P_\nu + \epsilon_{\nu\alpha\beta\sigma}P_\mu)k^\alpha P^\beta s^\sigma + O(k^2)\,, \quad (6.9)$$

JM (1990)

where $P^\mu = \frac{1}{2}(p+p')^\mu$. Both of these terms are conserved ($k^\mu P_\mu = 0$), symmetric, and have the correct parity. An example of a form factor which is ignored is $(k^2 g_{\mu\nu} - k_\mu k_\nu)A_2(k^2)$. It makes a vanishingly small contribution near $k^2 = 0$ since

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')\left[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M\right]U(P)\,,$$

Ji (1997)

Compare to spin 0

$$\left\langle p' \left| T^{\mu\nu}_{q,g} \right| p \right\rangle = A_{q,g}(t)\bar{P}^{\mu}\bar{P}^{\nu} + C_{q,g}(t)(\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^{2}) / M$$

Only 1 chiral even GPD

General rule to count form factors: t-channel J^{PC} q. numbers

Haegler, PLB(2004)
Z.Chen&Ji, PRD(2005)

n	$J^{PC}(S;L)$				
0	0^{+-}	$1^{--}(1;0,2)$			
1	$0^{++}(1;1)$	1^{-+}	$2^{++}(1;1,3)$		
2	0^{+-}	$1^{--}(1;0,2)$	2^{+-}	$3^{--}(1;2,4)$	
3	$0^{++}(1;1)$	1^{-+}	$2^{++}(1;1,3)$	3^{-+}	$4^{++}(1;3,5)$
...		...			

TABLE III: J^{PC} of the vector operators with $(S;L,L')$ for the corresponding $N\bar{N}$ state. Where there are no $(S;L,L')$ values there are no matching quantum numbers for the $N\bar{N}$ system.

Nucleon

	$L=0$	1	2	3	4	...
$S=0$	J^{PC} 0^{-+}	1^{+-}	2^{-+}	3^{+-}	4^{-+}	
$S=1$	1^{--}	0^{++}	1^{--}	2^{++}	3^{--}	
		1^{++}	2^{--}	3^{++}	4^{--}	
		2^{++}	3^{--}	4^{++}	5^{--}	

TABLE I: J^{PC} of the $N\bar{N}$ states.

Deuteron

	$L=0$	1	2	3	4	...
$S=0$	J^{PC} 0^{++}	1^{--}	2^{++}	3^{--}	4^{++}	
$S=1$	1^{+-}	0^{-+}	1^{+-}	2^{-+}	3^{+-}	
		1^{-+}	2^{+-}	3^{-+}	4^{+-}	
		2^{-+}	3^{+-}	4^{-+}	5^{+-}	
$S=2$	2^{++}	1^{--}	0^{++}	1^{--}	2^{++}	
		2^{--}	1^{++}	2^{--}	3^{++}	
		3^{--}	2^{++}	3^{--}	4^{++}	
			3^{++}	4^{--}	5^{++}	
			4^{++}	5^{--}	6^{++}	

TABLE II: J^{PC} of the $d\bar{d}$ states.

Both S and L states considered (related to X.Ji's talk)

Sum Rules in Deuteron

Momentum

$$\left\langle p' \right| \int d^3x \, T^{0i}_{q,g} \left| p \right\rangle = p^i \left\langle p' \middle| p \right\rangle = \mathcal{G}_1^{q,g} p^i \int d^3x \, 2p^0$$

$$\Rightarrow \boxed{\mathcal{G}_1^{q} + \mathcal{G}_1^{q} = 1}$$

OAM

$$\left\langle p' \right| \int d^3x \left(x_1 T^{02}_{q,g} - x_2 T^{01}_{q,g} \right) \left| p \right\rangle = \mathcal{G}_5^{q,g} \int d^3x \, p^0$$

$$\Rightarrow \boxed{\frac{1}{2} \mathcal{G}_5^{q,g} = J_z^{q,g}}$$

$$\left\langle p' \middle| p \right\rangle = 2p^0 \delta^3(p' - p)$$

Use Ji's framework, attention must be payed to singularities (Bakker Leader Trueman), ...

Compare to spin 1/2

Momentum

$$\mathrm{A}^q + \mathrm{A}^g = 1$$

OAM

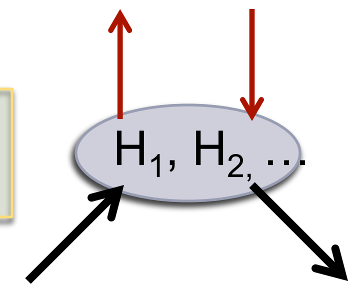
$$\frac{1}{2}(\mathrm{A}^{q,g} + B^{q,g}) = J_z^{q,g}$$

Quark-Hadron Helicity Amplitudes

To proceed with the identification of EMT components with n=1 moments of GPDs, start from quark-hadron helicity amplitudes:

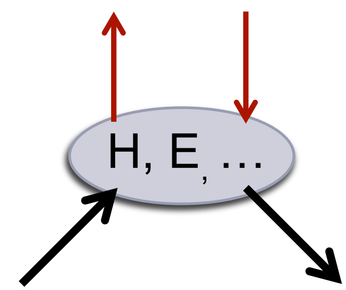
Deuteron

$$V_{\Lambda'\Lambda} = \sum_i \left[\varepsilon^{*\mu}(p',\Lambda') V^i_{\mu\nu} \varepsilon^{\nu}(p,\Lambda) \right] H_i(x,\xi,t)$$



Nucleon

$$F^S_{\Lambda'\Lambda} = \sum_i \left[\bar{U}_\alpha(p',\Lambda') O^i_{\alpha\beta} U_\beta(p,\Lambda) \right] H_i(x,\xi,t)$$



$$H_1 = H, \quad H_2 = E, \quad O^1 = \gamma^+, \; O^2 = \frac{-i\sigma^{+\mu}\Delta_\mu}{2M}$$

LC Correlation Function for Deuteron

$$\int \frac{d\kappa}{2\pi} e^{ix\kappa P.n} \langle p', \lambda' | \bar{\psi}(-\kappa n)\, \gamma.n\, \psi(\kappa n)\, | p, \lambda \rangle$$
$$= -(\epsilon'^{*}.\epsilon) H_1 + \frac{(\epsilon.n)(\epsilon'^{*}.P) + (\epsilon'^{*}.n)(\epsilon.P)}{P.n} H_2$$
$$- \frac{(\epsilon.P)(\epsilon'^{*}.P)}{2M^2} H_3 + \frac{(\epsilon.n)(\epsilon'^{*}.P) - (\epsilon'^{*}.n)(\epsilon.P)}{P.n} H_4$$
$$+ \left\{ 4M^2 \frac{(\epsilon.n)(\epsilon'^{*}.n)}{(P.n)^2} + \frac{1}{3}(\epsilon'^{*}.\epsilon) \right\} H_5 \qquad (7)$$

Berger, Cano, Diehl, Pire, PRL(2001)

Compare to spin 1/2

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \overline{\psi}(-\lambda n/2) \gamma^{\mu} \psi(\lambda n/2) | P \rangle = H(x, \Delta^2, \xi) \overline{U}(P') \gamma^{\mu} U(P) + E(x, \Delta^2, \xi) \overline{U}(P') \frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2M} U(P)$$

Physical Interpretation of the various deuteron GPDs

Form Factors

$$\int H_1(x,\xi,t)\,dx = G_1(t)$$
$$\int H_2(x,\xi,t)\,dx = G_2(t)$$
$$\int H_3(x,\xi,t)\,dx = G_3(t)$$
$$\int H_4(x,\xi,t)\,dx = 0$$
$$\int H_5(x,\xi,t)\,dx = 0$$

$$G_C(t) = G_1(t) + \frac{2}{3}\eta\, G_Q(t)$$
$$G_M(t) = G_2(t)$$
$$G_Q(t) = G_1(t) - G_2(t) + (1+\eta)\, G_3(t)$$

$$G_C(0) = 1$$
$$G_M(0) = \frac{M_D}{M_N}\mu_D = 1.714$$
$$G_Q(0) = M_D^2 Q_D = 25.83$$

$$\eta = \frac{t}{2M_D^2}$$

Connection with PDFs

$$H_1(x,0,0) = \frac{1}{3}\left(q^1(x) + q^{-1}(x) + q^0(x)\right) = f_1(x)$$

$$H_5(x,0,0) = \left(q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2}\right) = b_1(x)$$

The momentum and OAM SRs are obtained by connecting GPDs Mellin Moment n=1 with the EMT components (X.Ji, J.Phys G, 1998)

Tower of vector operators: $O_V^n = \bar{\psi}(0)\gamma^{\{\mu} i\overleftrightarrow{D}^{\mu_1} \ldots i\overleftrightarrow{D}^{\mu_n\}}\psi(0)$

Moments

$$H_{n+1}(\xi,t) \equiv \int_{-1}^{1} dx\, x^n H(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (-2\xi)^i A_{n+1,i}(\Delta^2) + (-2\xi)^{n+1} C_{n+1,0}(\Delta^2)|_{n\ \text{odd}},$$

$$E_{n+1}(\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (-2\xi)^i B_{n+1,i}(\Delta^2) - (-2\xi)^{n+1} C_{n+1,0}(\Delta^2)|_{n\ \text{odd}}.$$

"Parametrization" of spin ½ matrix elements

$$\begin{aligned}
&\langle P'|\bar{\psi}(0)\gamma^{\{\mu} iD^{\mu_1}\cdots iD^{\mu_n\}}\psi(0)|P\rangle \\
&= \bar{U}(P')\Bigg[\sum_{\substack{i=0\\ \text{even}}}^{n}\Big\{\gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\cdots\bar{P}^{\mu_n\}} A_{n+1,i}(\Delta^2) \\
&\quad - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\cdots\bar{P}^{\mu_n\}} B_{n+1,i}(\Delta^2)\Big\} + \frac{\Delta^{\mu}\cdots\Delta^{\mu_n}}{m} C_{n+1,0}(\Delta^2)\Big|_{n\ \text{odd}}\Bigg]U(P).
\end{aligned}$$

n=1, spin 1/2

"Parametrization" of spin ½ matrix elements

$$\left\langle p' \left| \bar{\psi}(0)\gamma^{\mu} iD^{\nu}\psi(0) \right| p \right\rangle = \bar{U}(p',\Lambda')\gamma^{\mu}U(p,\Lambda)\bar{P}^{\nu} A_{20}(t) -$$

$$\bar{U}(p',\Lambda')\frac{i\sigma^{\alpha\mu}\Delta_{\mu}}{2M}U(p,\Lambda)\bar{P}^{\nu} B_{20}(t) + \frac{\Delta_{\mu}\Delta_{\nu}}{M}\bar{U}(p',\Lambda')U(p,\Lambda)C_{20}(t)$$

"Parametrization" of EMT

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\,[A_{q,g}(\Delta^2)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(\Delta^2)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_{\alpha}/2M + C_{q,g}(\Delta^2)(\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^2)/M$$
$$+ \bar{C}_{q,g}(\Delta^2)g^{\mu\nu}M]U(P),$$

$$\frac{1}{2}\int dx\, x\Big(H(x,0,0) + E(x,0,0)\Big) = J_z^{q,g}$$

n=1, spin 1

Spin 1/2

$$\left\langle p' \middle| n_\mu n_\nu \bar{\psi}(0)\gamma^\mu i D^\nu \psi(0) \middle| p \right\rangle = \sum_i \left[\bar{U}_\alpha(p',\Lambda') O^i_{\alpha\beta} U_\beta(p,\Lambda) \right] \int dx\, x\, H_i(x,\xi,t)$$

Spin 1

$$\left\langle p' \middle| n_\mu n_\nu \bar{\psi}(0)\gamma^\mu i D^\nu \psi(0) \middle| p \right\rangle = \sum_i \left[\varepsilon^{*\mu}(p',\Lambda') V^i_{\mu\nu} \varepsilon^\nu(p,\Lambda) \right] \int dx\, x\, H_i(x,\xi,t)$$

n=1, spin 1

"Parametrization" of EMT

$$
\begin{aligned}
\langle p'|T^{\mu\nu}|p\rangle &= -\frac{1}{2}P^\mu P^\nu(\epsilon'^*\epsilon)\mathcal{G}_1(t) \\
-\ & \frac{1}{4}P^\mu P^\nu\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_2(t) - \frac{1}{2}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right](\epsilon'^*\epsilon) \\
\times\ & \mathcal{G}_3(t) - \frac{1}{4}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_4(t) \\
+\ & \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) + \epsilon^\mu(\epsilon'^* P)\right)P^\nu + \mu\leftrightarrow\nu\right]\mathcal{G}_5(t) \\
+\ & \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) - \epsilon^\mu(\epsilon'^* P)\right)\Delta^\nu + \mu\leftrightarrow\nu\right. \\
+\ & \left. 2g_{\mu\nu}(\epsilon P)(\epsilon'^* P) - (\epsilon'^{*\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu)\Delta^2\right]\mathcal{G}_6(t) \\
+\ & \frac{1}{2}\left[\epsilon^{*\prime\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu - \frac{1}{2}g^{\mu\nu}(\epsilon'^*\epsilon)\right]M^2\mathcal{G}_7(t) \\
+\ & g^{\mu\nu}(\epsilon'^*\epsilon)M^2\mathcal{G}_8(t) \qquad (4)
\end{aligned}
$$

Angular Momentum Sum Rule

$$\frac{1}{2}\mathcal{G}_5^{q,g} = \frac{1}{2}\int dx\; xH_2(x,0,0) = J_z^{q,g}$$

Other relations

$$\int dx x[H_1(x,\xi,t) - \frac{1}{3}H_5(x,\xi,t)] = \mathcal{G}_1(t) + \xi^2\mathcal{G}_3(t) \quad (7)$$ Momentum

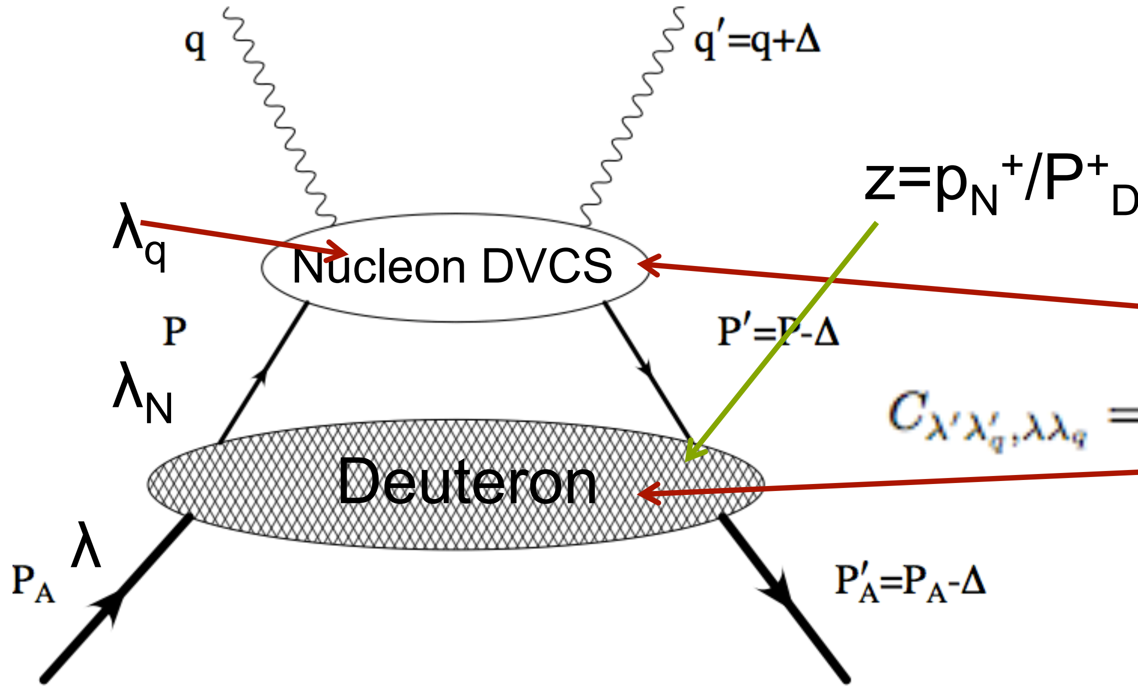
$$\int dx x H_2(x,\xi,t) = \mathcal{G}_5(t) \quad (8)$$ Angular Momentum

$$\int dx x H_3(x,\xi,t) = \mathcal{G}_2(t) + \xi^2\mathcal{G}_4(t) \quad (9)$$

$$\int dx x H_4(x,\xi,t) = \xi\mathcal{G}_6(t) \quad (10)$$

$$\int dx x H_5(x,\xi,t) = \mathcal{G}_7(t) \quad (11)$$ Connected to b_1 SR

What are the quark and gluon angular momenta in the deuteron?



$$C_{\lambda'\lambda'_q,\lambda\lambda_q} = \sum_{\lambda_N,\lambda'_N} B_{\lambda'\lambda'_N,\lambda\lambda_N} \otimes A_{\lambda'_N\lambda'_q,\lambda_N\lambda_q}$$

$$\sqrt{\frac{t_0 - t}{2M^2}} H_2(x,0,0) = 2\left[\left(C_{1+,1+} + C_{1-,1-}\right) - \left(C_{1+,0+} + C_{1-,0-}\right)\right]$$

$$\Rightarrow H_2 \approx \int_x^{M_D/M_N} dz\left[f^{++}(z) H_{ISO}(x/z,0,0) + f^{0+}(z) E_{ISO}(x/z,0,0)\right]$$

$H_{ISO}=H_u+H_d$ $E_{ISO}=E_u+E_d$

Deuteron LC Momentum distribution

$$f^{++}(z) = 2\pi M \int_{p_{min}(z)}^{\infty} dp\, p \sum_{\lambda_N} \chi_+^{*\lambda'_{N_1}\lambda_{N_2}}(z,p)\chi_+^{\lambda_{N_1}\lambda_{N_2}}(z,p) \tag{14}$$

$z = p_N^+/P^+_D$

$$f^{0+}(z) = 4\pi M \int_{p_{min}(z)}^{\infty} dp\, p \sum_{\lambda_N} \chi_0^{*\lambda'_{N_1}\lambda_{N_2}}(z,p)\chi_+^{\lambda_{N_1}\lambda_{N_2}}(z,p). \tag{15}$$

$\lambda_N = \{\lambda_{N1}, \lambda'_{N1} \lambda_{N2}\}$

Deuteron w.f. (momentum space)

$$\chi_\lambda^{\lambda_{N_1},\lambda_{N_2}}(z,p) = \mathcal{N} \sum_{L,m_L,m_S} \begin{pmatrix} j_1 & j_2 & 1 \\ \lambda_{N_1} & \lambda_{N_2} & m_S \end{pmatrix} \begin{pmatrix} L & S & J \\ m_L & m_S & \lambda \end{pmatrix}$$

$$\times\; Y_{L\,m_L}\left(\frac{\mathbf{p}}{p} = \frac{M(1-z)-E}{p}\right) u_L(p),$$

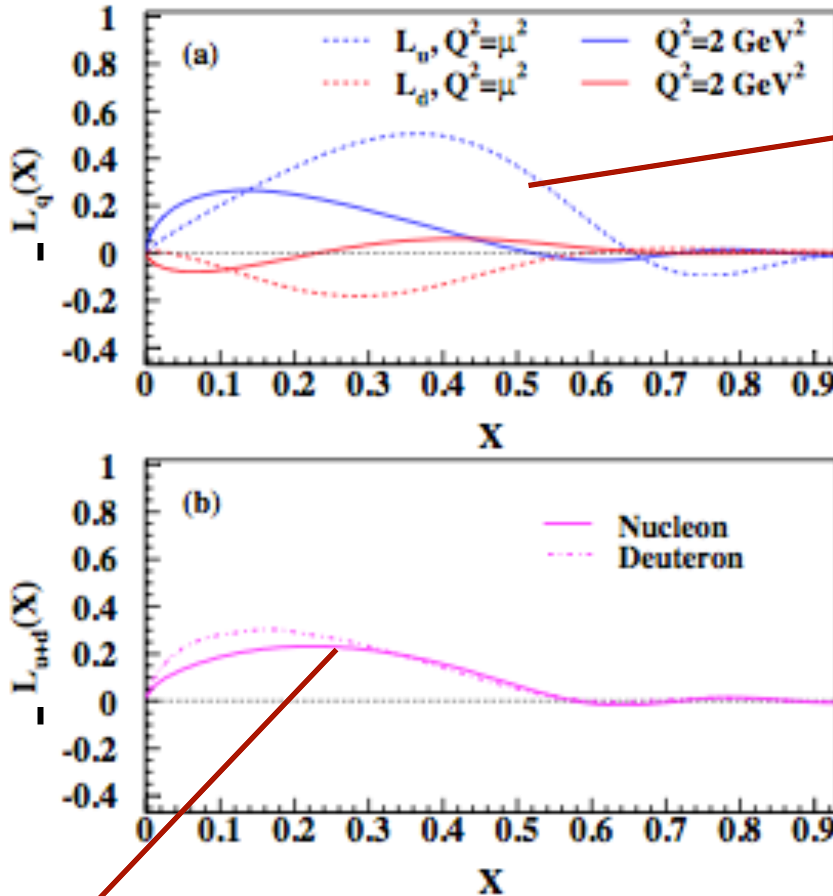
Mixture of S and D components , L=0,2

If $f^{++}(z) = f^{+0}(z)=\delta(1-z)$ then $H_2=H+E$

$$J_q = \frac{1}{2}\int dx\, x\left[H_q(x,0,0)+E_q(x,0,0)\right], \;\longrightarrow\; J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

$F_1+F_2= G_M$ $\quad\quad$ G_M

How does Ji sum rule differ from JM in the deuteron?



Effect of evolution

Model calculations of L with w.f.s' seem to lead to similar conclusions as M.Burkardt, more to explore here… avenue to compare different schemes?

Nuclear effect much larger than in unpolarized scattering

Observable

$$A_{UT} \approx -\frac{4\sqrt{D_0}}{\Sigma} \Im m \left[\mathcal{H}_1^* \mathcal{H}_5 + \left(\mathcal{H}_1^* + \frac{1}{6} \mathcal{H}_5^* \right) (\mathcal{H}_2 - \mathcal{H}_4) \right]$$

subleading

Interest in Spin 0 targets: pion, ^{4}He, …

One less distribution w.r.t. spin ½

Ji's Sum Rule: 0=0

Partonic interpretation of spin and OAM?

L(x) is also 0, no nodes.

Angular momentum sum rule for spin one hadronic systems

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arXiv:1101.0581

Conclusions

✓Spin and OAM with Jaffe-Manohar and Ji approaches: interesting relations are obtained by looking at spin 0, spin ½ and spin 1 hadronic systems

Back up