Quark and Glue Momenta and Angular Momenta in the Nucleon – Lattice Results

- Update of lattice calculations of $\langle x \rangle_{u-d}$, $\langle x \rangle_s / \langle x \rangle_u$ (D.I.) ratio — connected sea partons.
- Status of experimental and lattice results on quark spin and glue spin
- Quark (CI and DI) and glue momenta and angular momenta in a lattice calculation

χ QCD Collaboration:

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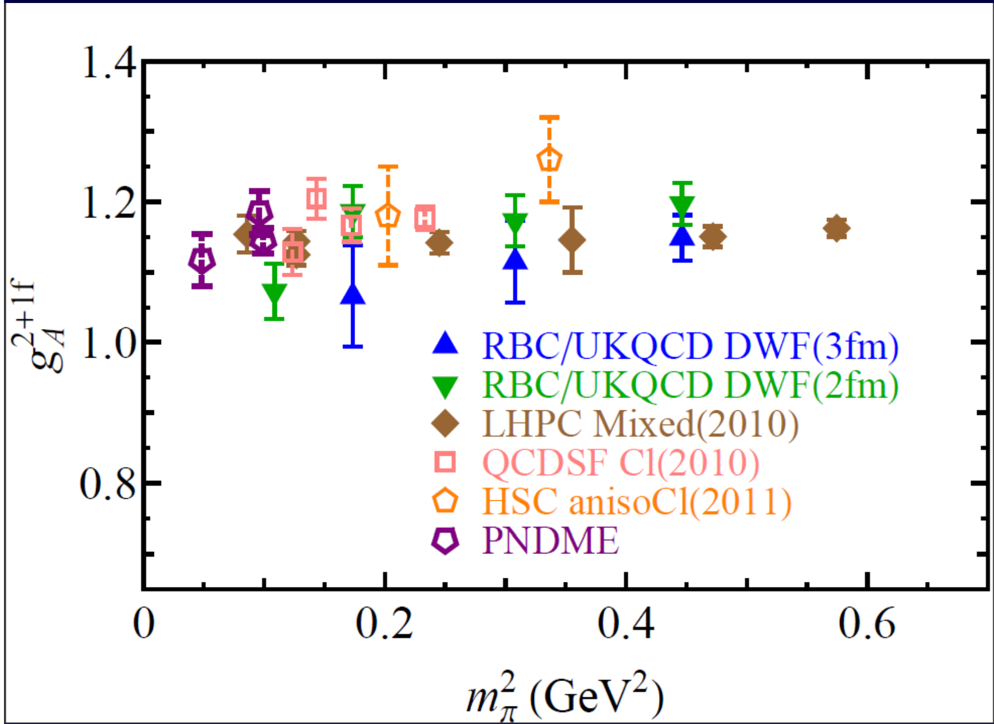
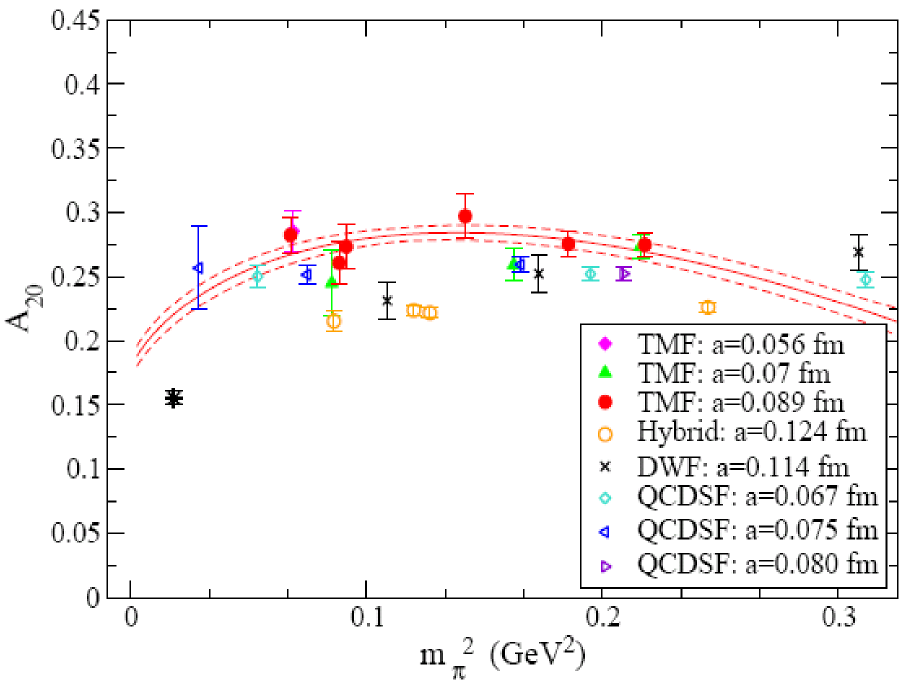
INT 2012

Lattice Calculation of g_A^3 and $<x>_{u-d}$

Isovector g_A

$<x>_{u-d}$

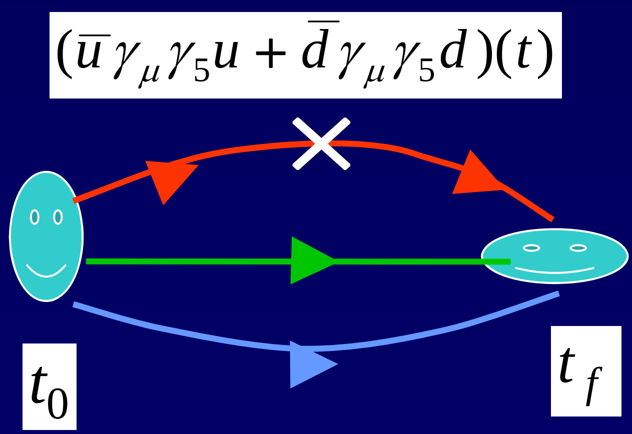


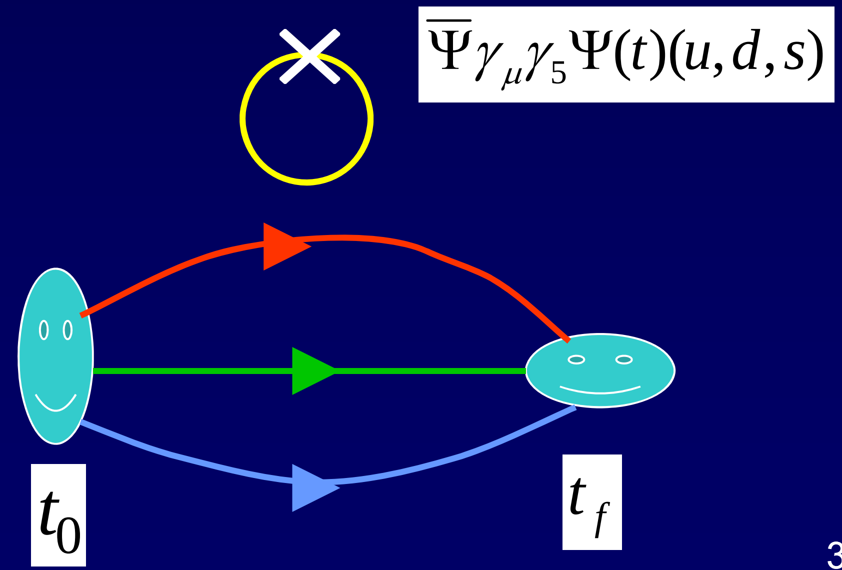


These involve connected insertion calculations.

Hadron Structure with Disconnected Insertion Calculation

- Pion-Nucleon Sigma Term, Strangeness Content in N
- Quark Spin and Orbital Angular Momentum in Nucleon
- Sea Quark Contributions in <x>, and <x²>
- Strangeness Electric and Magnetic Form Factors
- Muon Anomalous M. M. (g-2) (light-by-light)
- Neutron Electric Dipole Moment





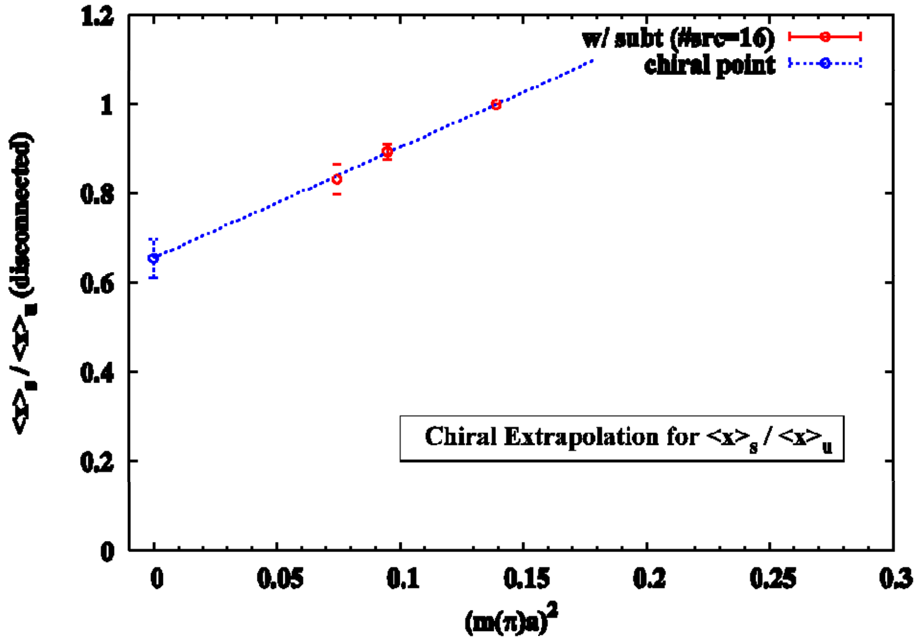
Strange Parton Moments

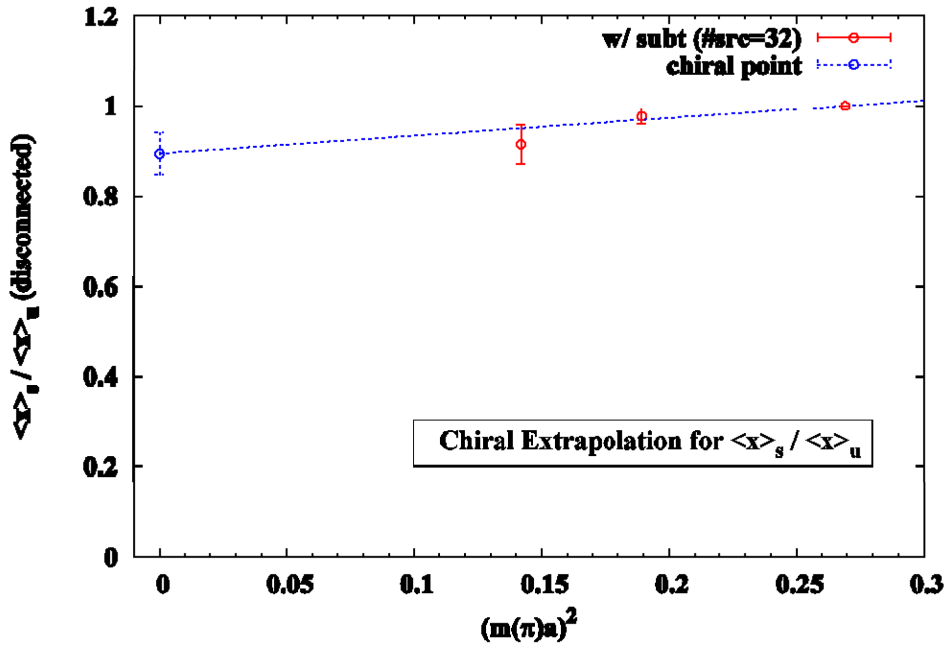
- Strange parton distribution is not well known – H.L. Lai et al. (CTEQ), JHEP 0704:089 (2007)
- $0.018 < \langle x \rangle_s < 0.040$
- NuTeV measurement of $\sin^2\theta_W$ is 3 σ above the standard model ⟶ strangeness asymmetry, i.e. $s(x) \neq \bar{s}(x)$?
- CTEQ: the sign of $\int dx\, x[s(x) - \bar{s}(x)]$ is uncertain.
- Lattice can calculate $\langle x^2 \rangle_s = \int dx\, x^2[s(x) - \bar{s}(x)]$

$$\frac{\langle x \rangle_{s+\bar{s}}}{\langle x \rangle_{u+\bar{u}}\,(\mathrm{DI})}$$

Quenched

2+1 Flavor Full QCD





Implication on Fitting of PDF

$$\frac{\langle x \rangle_{s+\bar{s}}}{\langle x \rangle_{u+\bar{u}}(\mathrm{DI})} = \begin{cases} 0.655(43)\ [\text{quenched}] \\ 0.894\ (47)\ [\text{full QCD}] \end{cases}$$

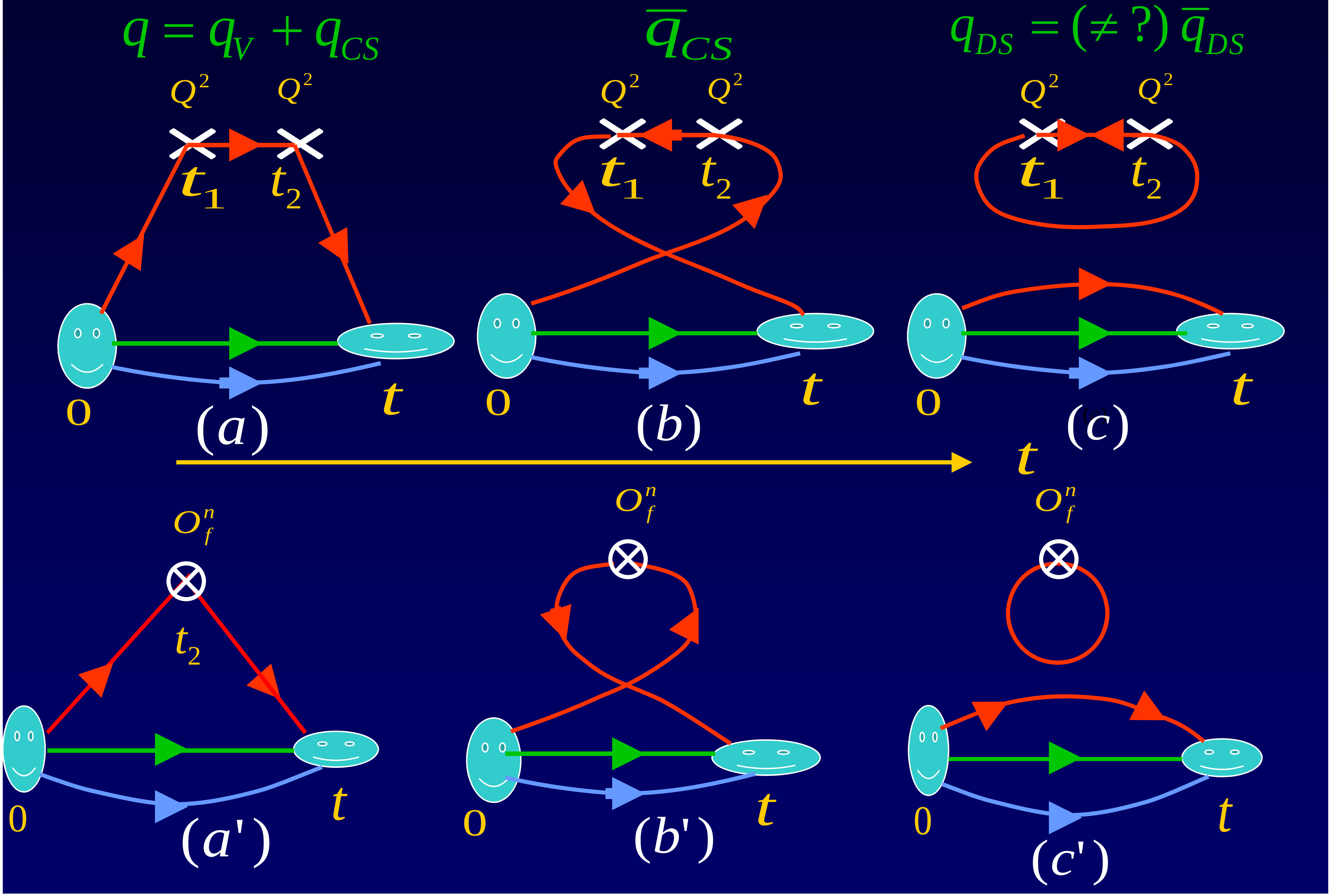
$$\frac{\langle x \rangle_{\bar{s}}}{\langle x \rangle_{(\bar{u}+\bar{d})/2}} \sim 0.4$$

Global PDF Fitting à la CTEQ

Is there a discrepancy?

Path Integral Formulation of DIS

K.F. Liu, PRD (2000)



- $W_{\mu\nu}(p,q) = -W_1(q^2,\nu)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + W_2(q^2,\nu)(p_\mu - \frac{p\cdot q}{q^2}q_\mu)(p_\nu - \frac{p\cdot q}{q^2}q_\nu)$

- Large momentum frame

$$\nu W_2(q^2,\nu) \xrightarrow[|\vec{p}|>>|\vec{q}|]{} F_2(x,Q^2) = x\sum_i e_i^2 (q_i(x,Q^2) + \bar{q}_i(x,Q^2)); \quad x = \frac{Q^2}{2M_N \nu}$$

- Parton degrees of freedom: valence, connected sea and disconnected sea

u	d	s
$u_V(x) + u_{CS}(x)$	$d_V(x) + d_{CS}(x)$	
$\bar{u}_{CS}(x)$	$\bar{d}_{CS}(x)$	
$u_{DS}(x) + \bar{u}_{DS}(x)$	$d_{DS}(x) + \bar{d}_{DS}(x)$	$s_{DS}(x) + \bar{s}_{DS}(x)$

Physical Consequences

- Is it necessary to separate out the CS from the DS?

1) Small x behavior

- $q_V \, , q_{CS} \, , \overline{q}_{CS} \sim_{x \to 0} \; x^{-\alpha_R} (x^{-1/2})$

  (Reggeon exchange, no pomeron exchange)

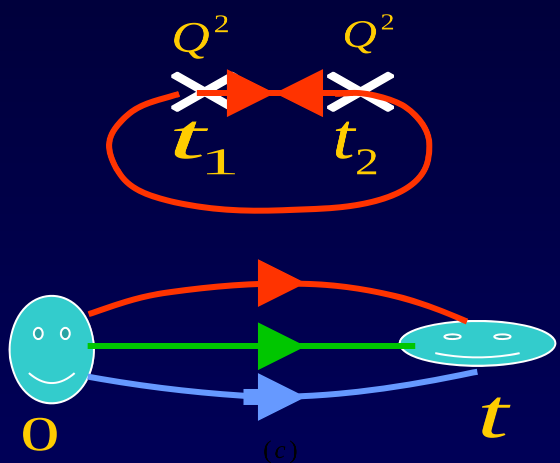
- $q_{DS} \, , \overline{q}_{DS} \sim_{x \to 0} \; x^{-1}$

  (Pomeron exchange)

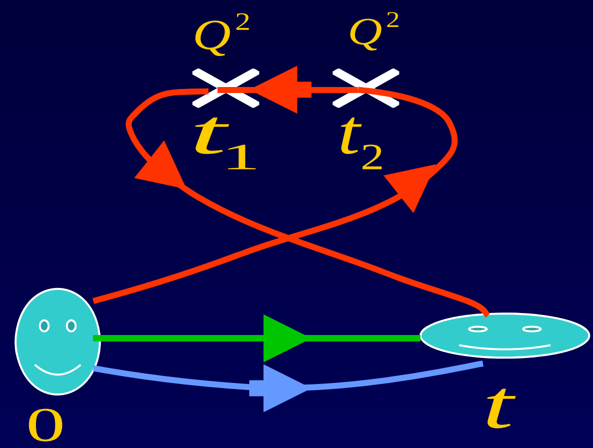
2) Gottfried Sum Rule Violation

$$S_G(0,1;Q^2) = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\,(\bar{u}_p(x) - \bar{d}_p(x)); \quad S_G(0,1;Q^2) = \frac{1}{3}\,\text{(Gottfried Sum Rule)}$$

NMC: $S_G(0,1;4\,\text{GeV}^2) = 0.240 \pm 0.016\ (5\sigma \text{ from GSR})$



two flavor traces ($\bar{u}_{DS} = \bar{d}_{DS}$)



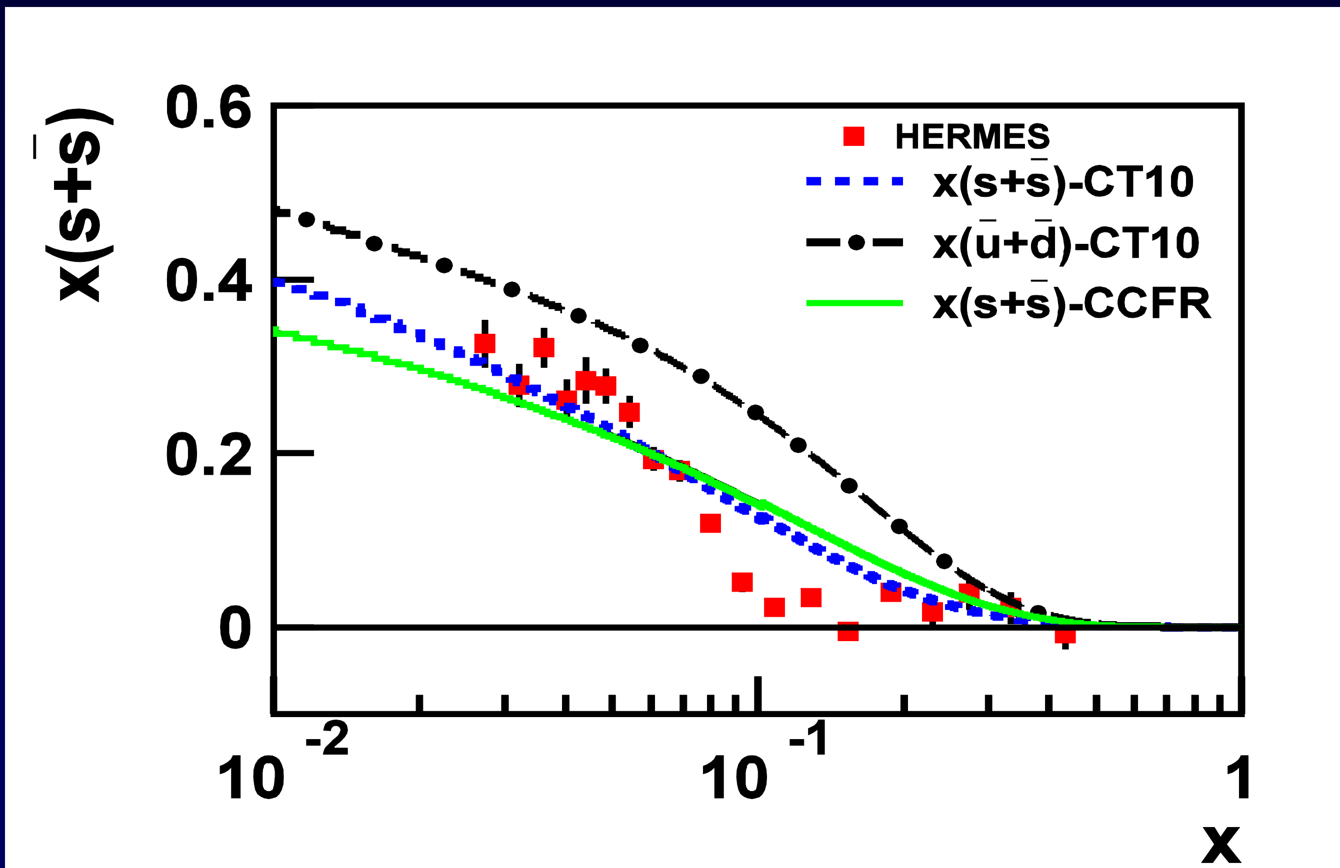
one flavor trace ($\bar{u}_{CS} \neq \bar{d}_{CS}$)

K.F. Liu and S.J. Dong, PRL 72, 1790 (1994)

$$Sum = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\,(\bar{u}_{CS}(x) - \bar{d}_{CS}(x)),$$

$$= \frac{1}{3} + \frac{2}{3}\left[n_{\bar{u}_{CS}} - n_{\bar{d}_{CS}}\right](1 + O(\alpha_s))$$

HERMES – Kaon production in DIS, PL B666, 446 (2008)



3) Fitting of experimental data

K.F. Liu, PRD (2000)

$$\bar{u} - \bar{d} \xrightarrow[x \to 0]{} x^{-1/2} \qquad \text{O.K.}$$

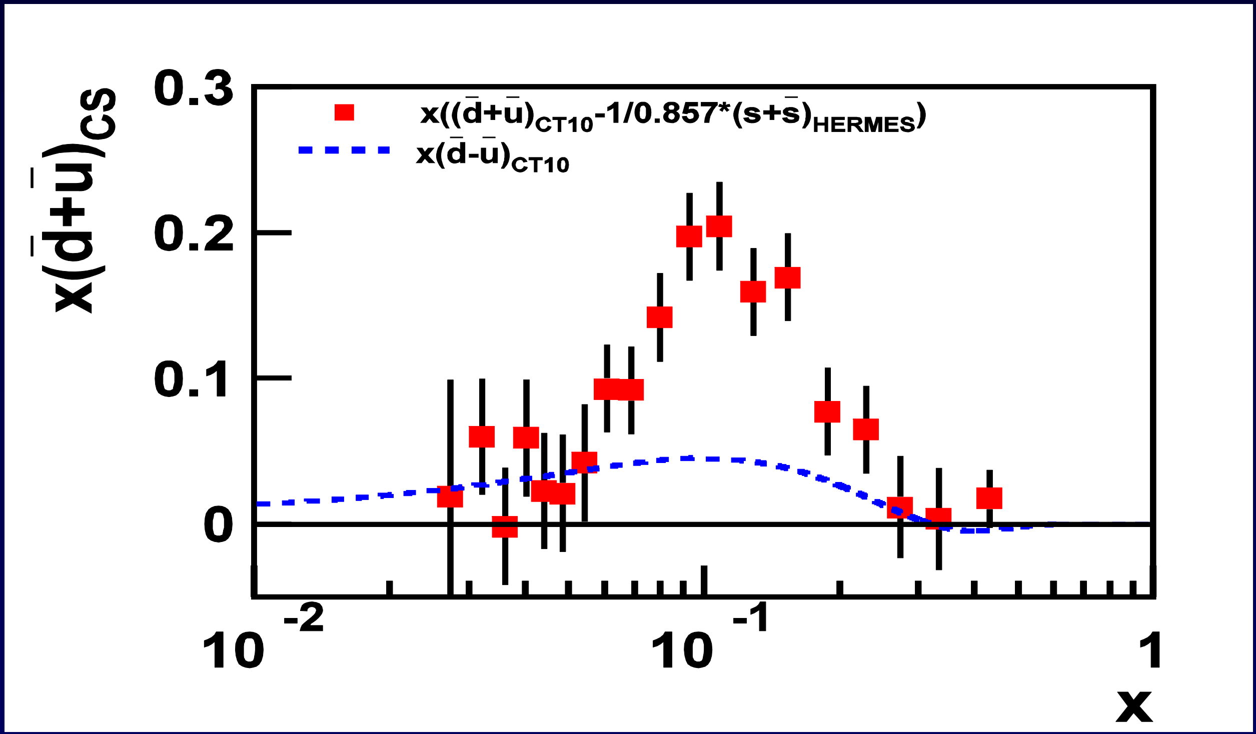
But $\bar{u} + \bar{d} \propto \bar{s}$ is not adequate.

A better fit $$\frac{\bar{u}(x) + \bar{d}(x)}{2} = f\ \bar{s}(x) + c(x), \quad f \approx 1$$

where $c(x) \xrightarrow[x \to 0]{} x^{-1/2}$ like in $\bar{u}(x) - \bar{d}(x)$

4) Unlike DS, CS evolves the same way as the valence.

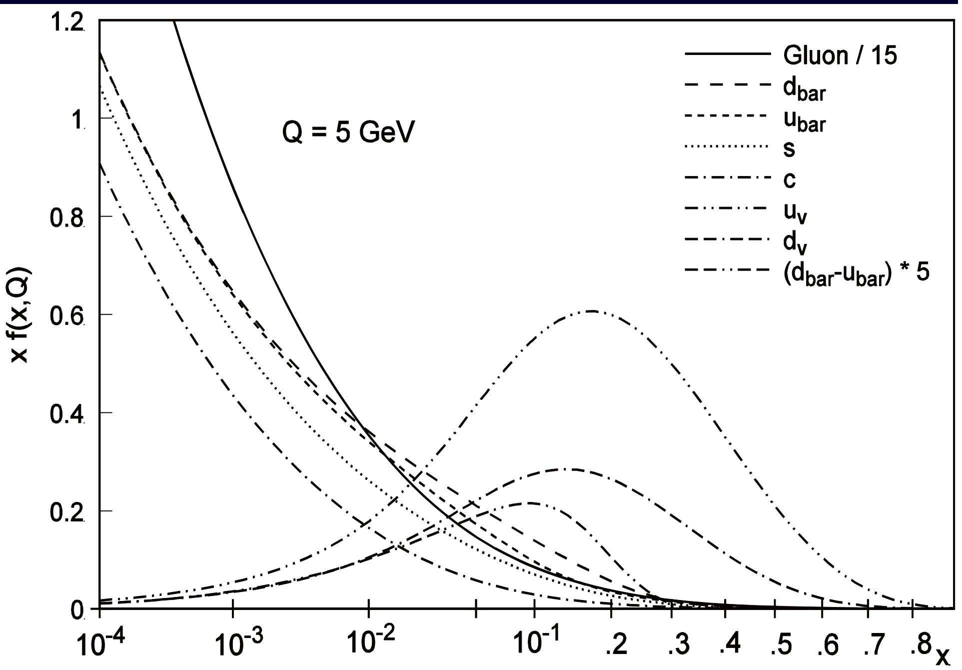
Connected Sea Partons

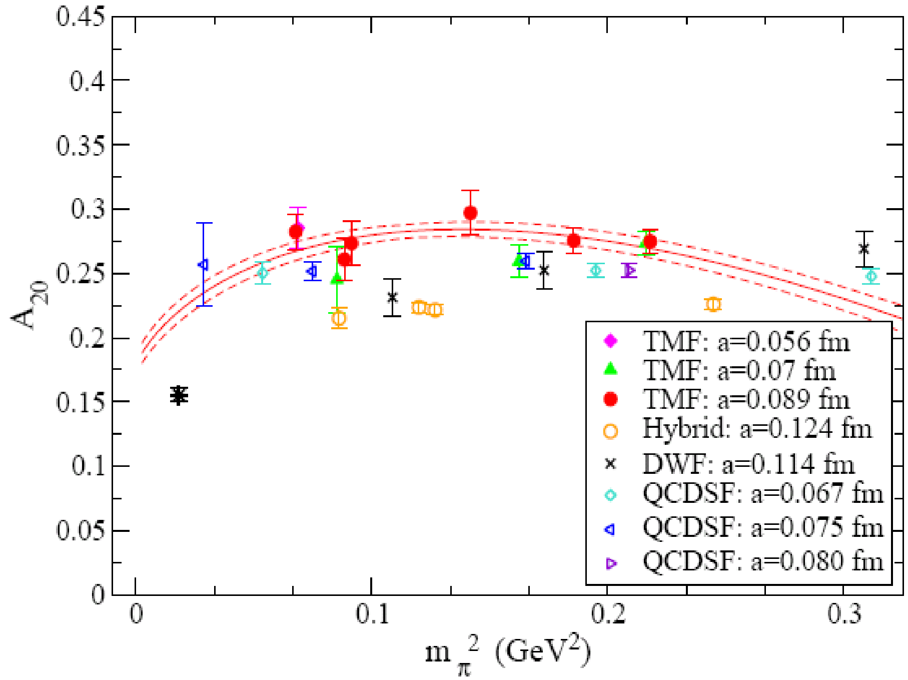


$$x(\bar{d}+\bar{u})_{CS}(x) = x(\bar{d}+\bar{u})(x) - \frac{1}{R}x(s+\bar{s})(x);$$

CT10 lattice expt

$$R = \frac{\langle x \rangle_s}{\langle x \rangle_u (DI)} (\text{lattice}) \sim 0.857$$





Gottfried Sum Rule Violation

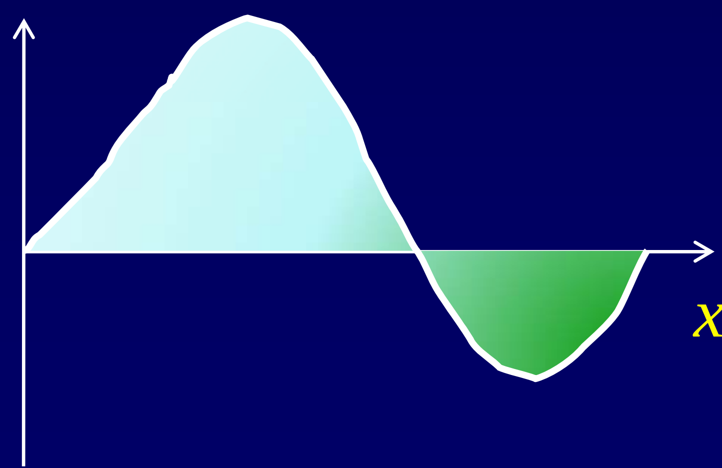
$$\Rightarrow \int_0^1 dx\ (\bar{d}_{CS}(x) - \bar{u}_{CS}(x)) > 0$$

$$\langle x \rangle_{u-d}\,(\text{lattice}) = \langle x \rangle_{u_{val}-d_{val}} + 2\langle x \rangle_{\bar{u}_{CS}-\bar{d}_{CS}}$$

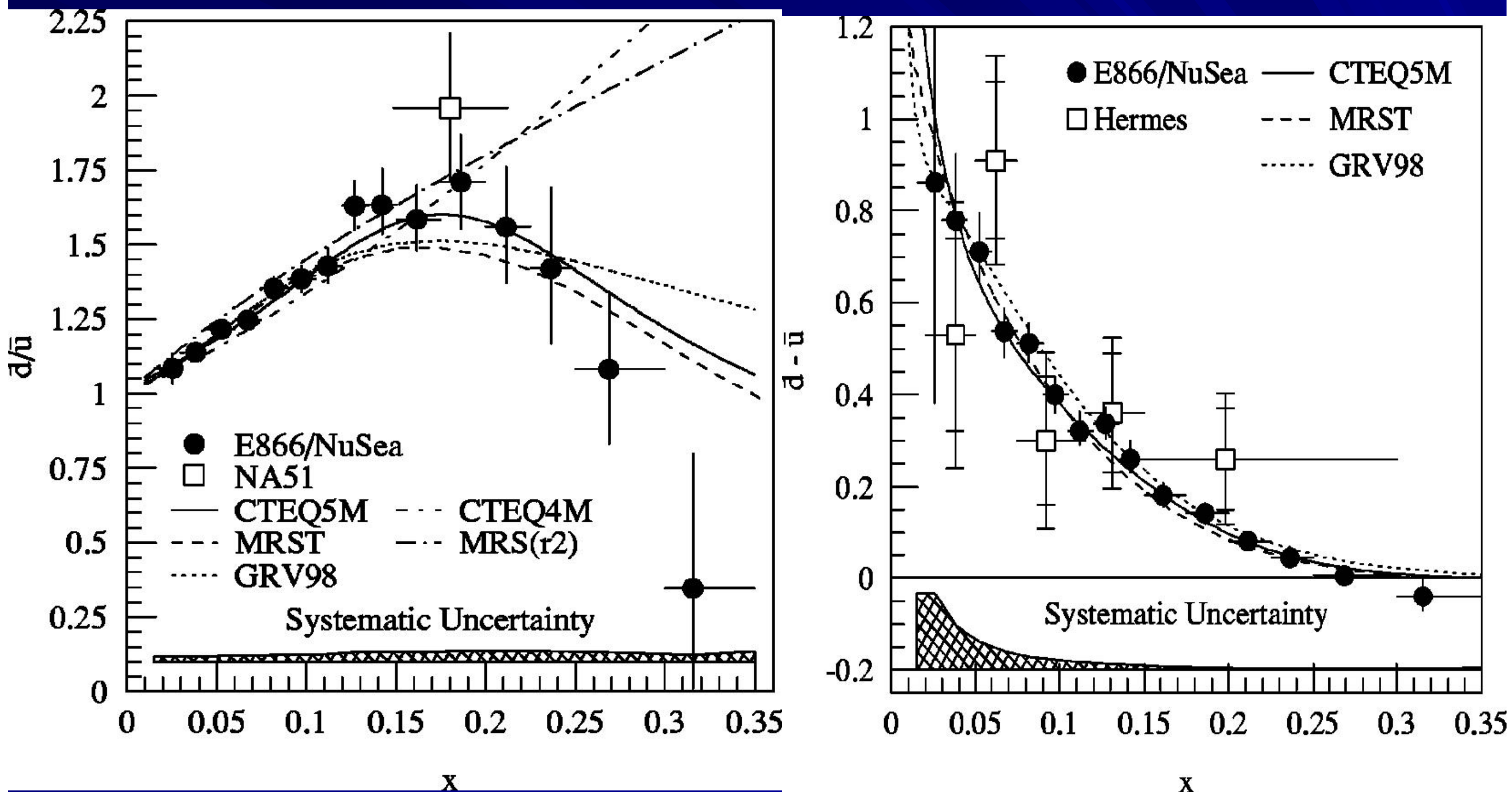
$$> \langle x \rangle_{u_{val}-d_{val}}\,(\text{expt})$$

$$\Rightarrow\ \langle x \rangle_{\bar{d}_{CS}-\bar{u}_{CS}} < 0$$

$\bar{d}_{CS} - \bar{u}_{CS}$



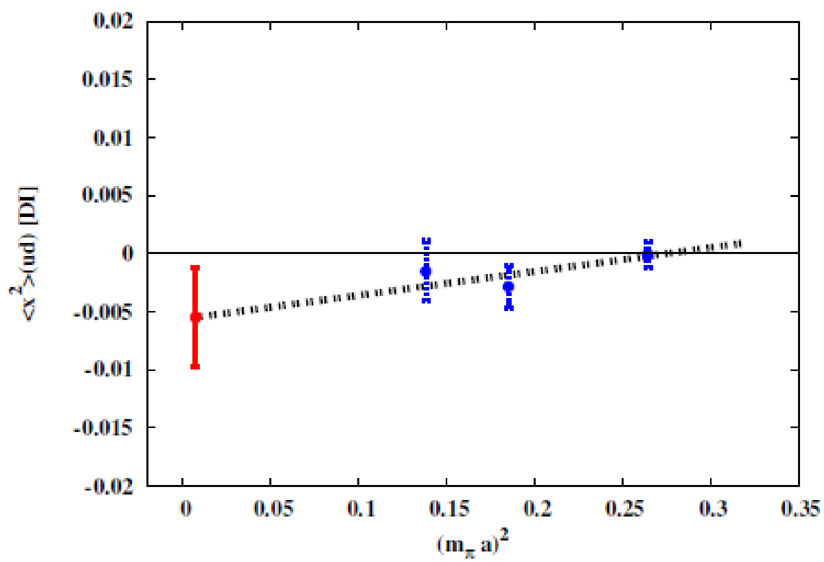
Results from Drell-Yan Experiment – E866/NuSea



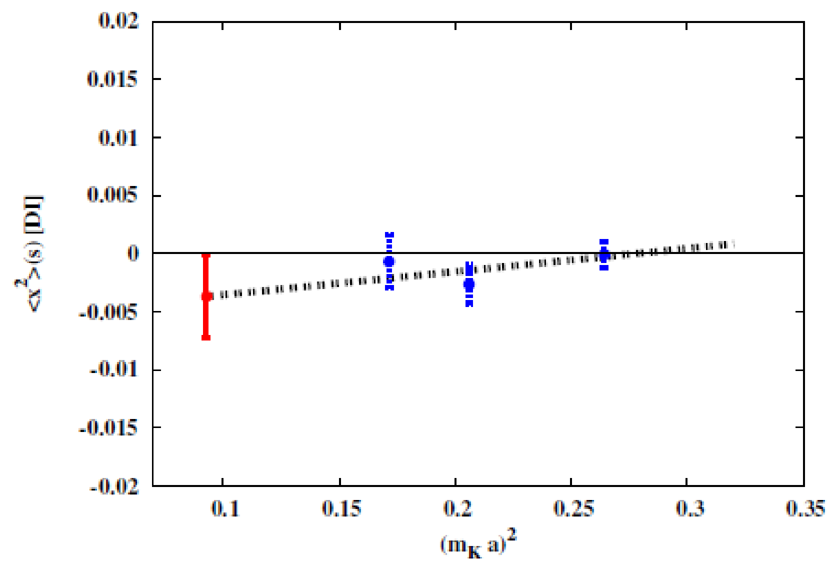
Results for <x²>

Nf=2+1

Preliminary



<x²>(ud) [DI]



<x²>(s)

c.f. $\langle x \rangle_{s-\bar{s}} = 0.0038 \rightarrow$ No NuTeV anomaly

$Z_{\text{pert}}(\mu, a) \simeq 1.1$

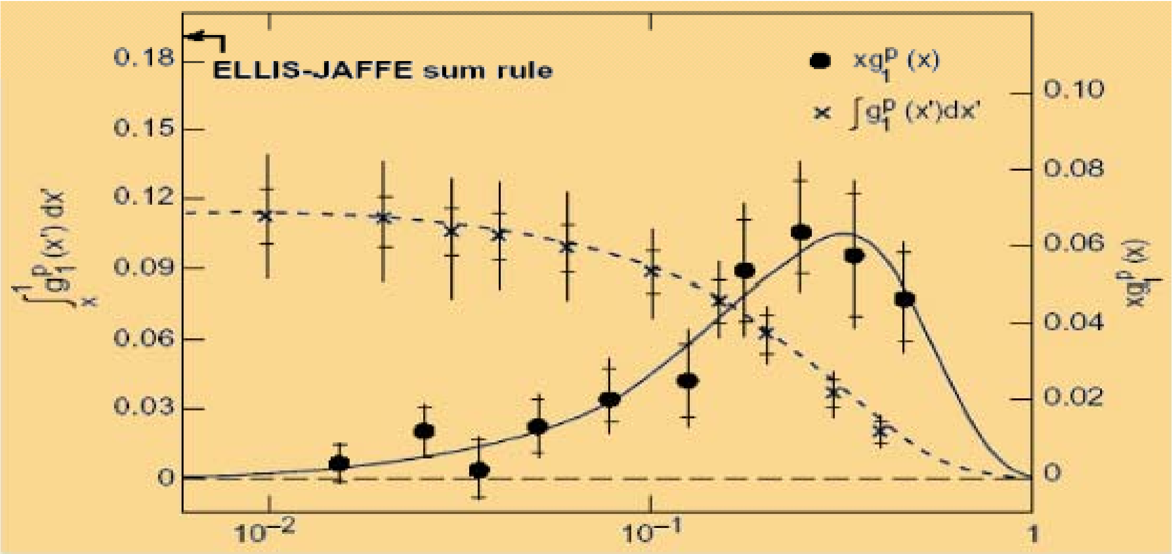
μ=2GeV

Quark and Glue Momenta and Angular Momenta

- Quark and glue spin – experiments
- Glue spin operator and gauge invariance
- Momentum and angular momentum sum rules
- Lattice calculation

Twenty years since the “spin crisis”

- EMC experiment in 1988/1989 – “the plot”:



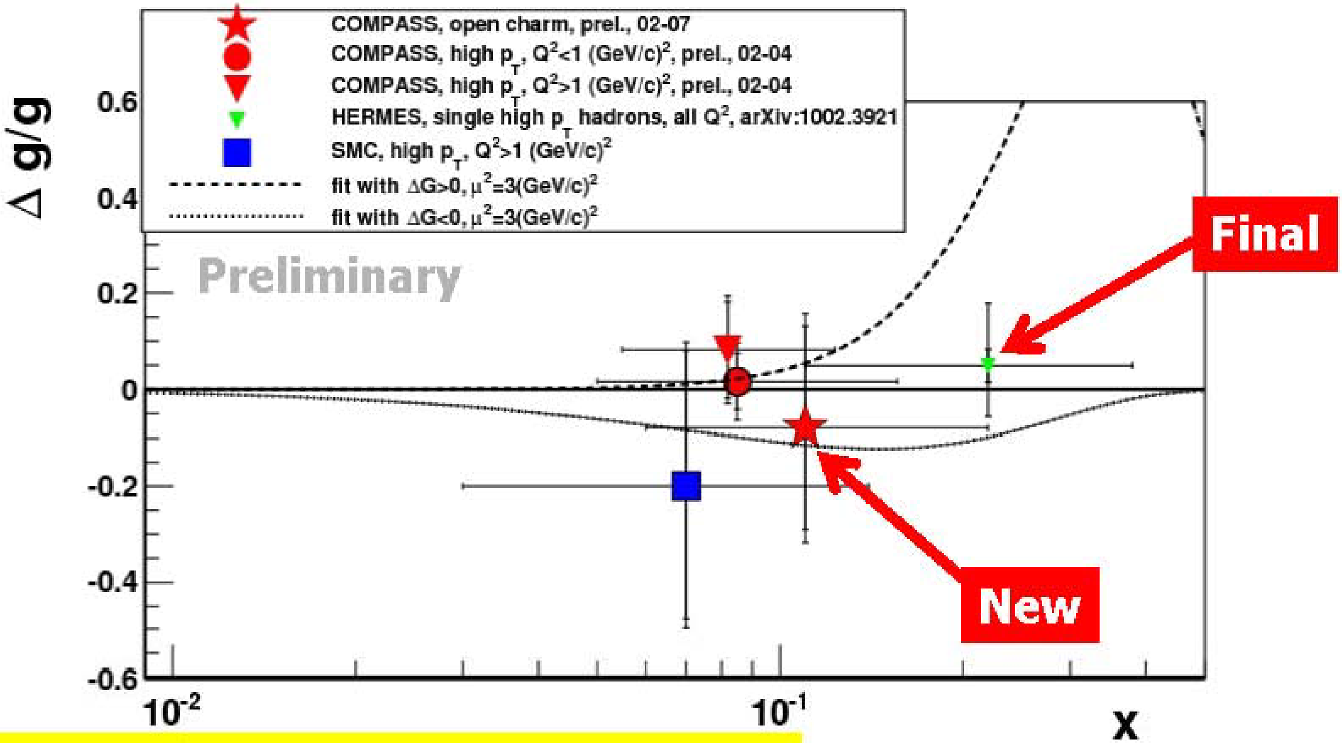
$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{||}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{||}\rangle$$

- “Spin crisis” or puzzle: $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} = 0.245 \pm 0.05$

Summary Gluon Polarization

Presently all Analysis in LO only



COMPASS Open Charm:

ΔG/G=-0.08+- 0.21(stat) +- 0.11(sys.)
(Systematic error still under investigations)

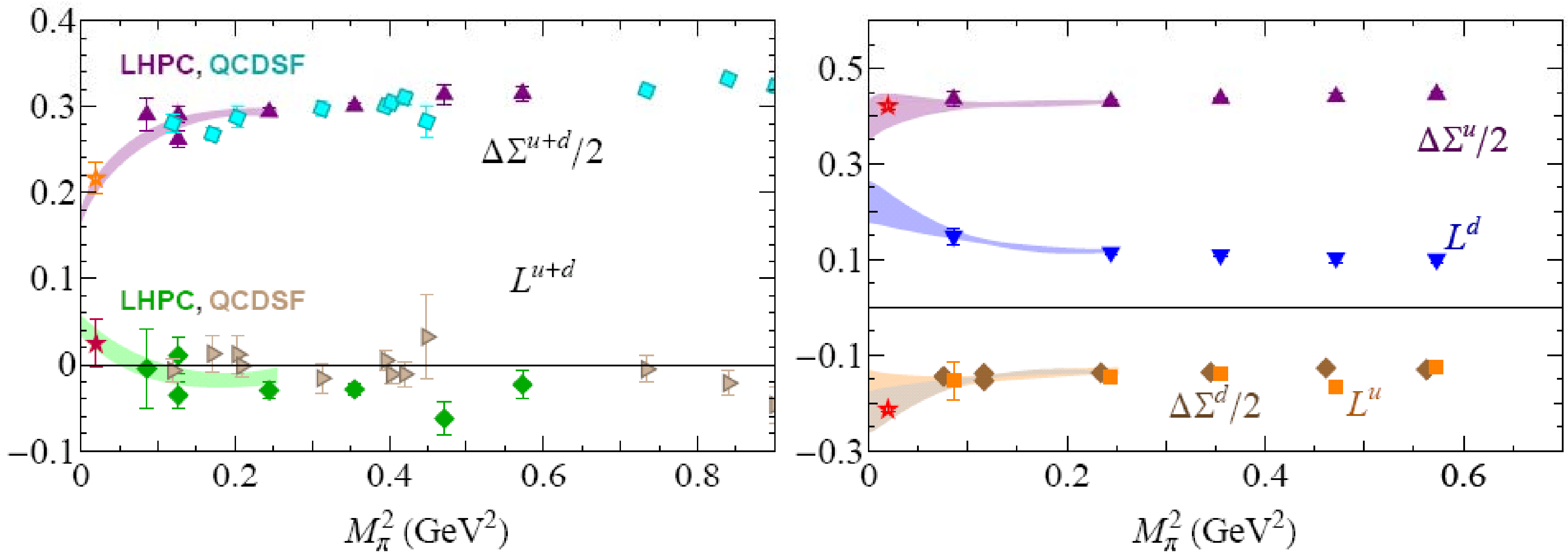
(Value supersedes previous publication)

C.Franco

See Talk 1193 by F. Kunne

Horst Fischer DIS2010

Quark Orbital Angular Momentum (connected insertion)



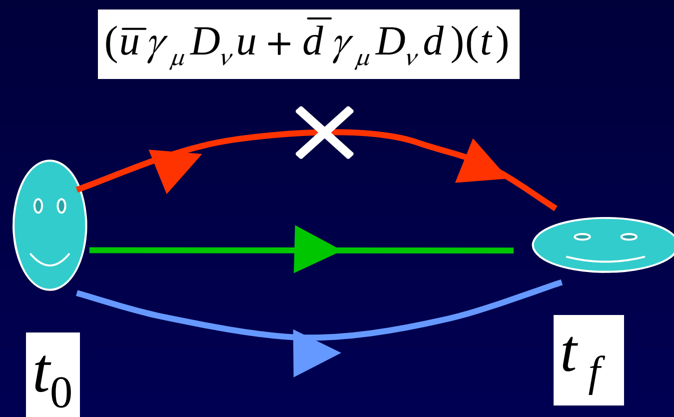
Nucleon Spin

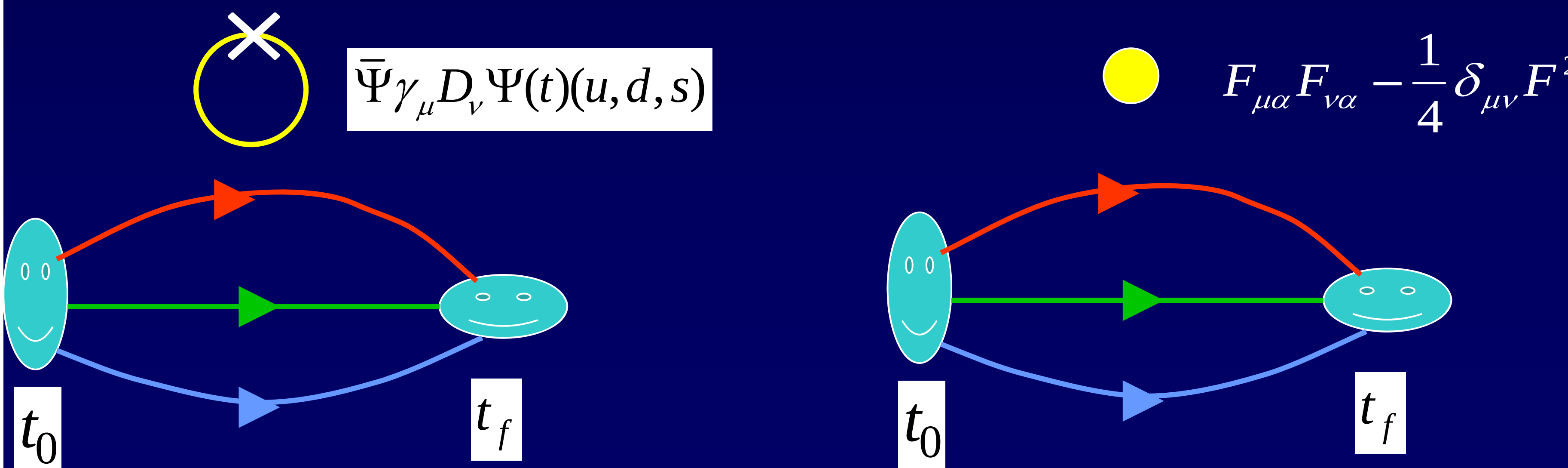
- Quark spin $\Delta\Sigma \sim 20 - 30\%$ of proton spin (DIS, Lattice)
- Quark orbital angular momentum? (lattice calculation (LHPC,QCDSF)→ ~ 0)
- Glue spin $\Delta G/G$ small (COMPASS, STAR) ?
- Glue orbital angular momentum is small (Brodsky and Gardner) ?

→ Dark Spin ?

Hadron Structure with Quarks and Glue

- Quark and Glue Momentum and Angular Momentum in the Nucleon





Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \;\rightarrow\; \vec{\mathrm{J}}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \qquad \rightarrow\; \vec{\mathrm{J}}_g = \int d^3x \left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\left\langle p,s \,|\, T_{\mu\nu} \,|\, p's'\right\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu \bar{p}_\nu - T_2(q^2)\bar{p}_\mu \sigma_{\nu\alpha} q_\alpha / 2m$$

$$\text{-}i\mathrm{T}_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu} q^2)/m + T_4(q^2)\delta_{\mu\nu} m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g} \;\; [\mathrm{OPE}] \rightarrow \langle x\rangle_{q/g}(\mu,\overline{\mathrm{MS}}), \quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\mathrm{MS}})$$

T_1 (q^2) and T_2 (q^2)

- 3-pt to 2-pt function ratios

$$G^{3pt}_{\mu\nu}(\vec{p},t_2;\vec{q},t_1)=\sum_{\vec{x}_1,\vec{x}_2} e^{-i\vec{p}\cdot\vec{x}_2+i\vec{q}\cdot\vec{x}_1}\left\langle 0\,|\,T\left[\chi_N(\vec{x}_2,t_2)T_{\mu\nu}\bar{\chi}_N(0)\right]\right\rangle;$$

$$\mathrm{Tr}\left[\Gamma_m G^{3pt}_{\mu\nu}(\vec{p}=0,t_2;\vec{q},t_1)\right]=We^{-m(t_2-t_1)}e^{-Et_1}\left[T_1(q^2)+T_2(q^2)\right]$$

- Need both polarized and unpolarized nucleon and different kinematics (p_i, q_j, s) to separate out T_1 (q^2), T_2 (q^2) and T_3 (q^2)

Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q\langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g\langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$Z_q\langle x\rangle_q^L + Z_g\langle x\rangle_g^L = 1,$$

$$Z_q J_q^L + Z_g J_g^L = \frac{1}{2} \Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q(T_1^q + T_2^q)(0) + Z_g(T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

Mixing

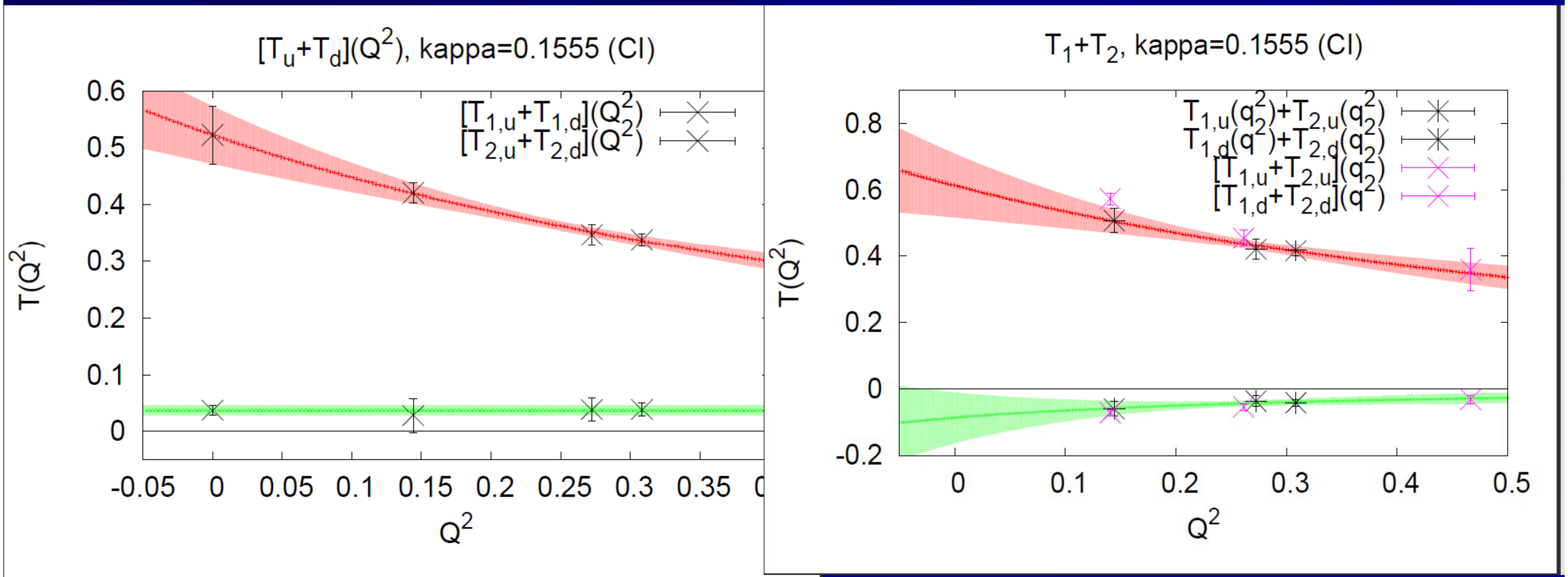
$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

Lattice Parameters

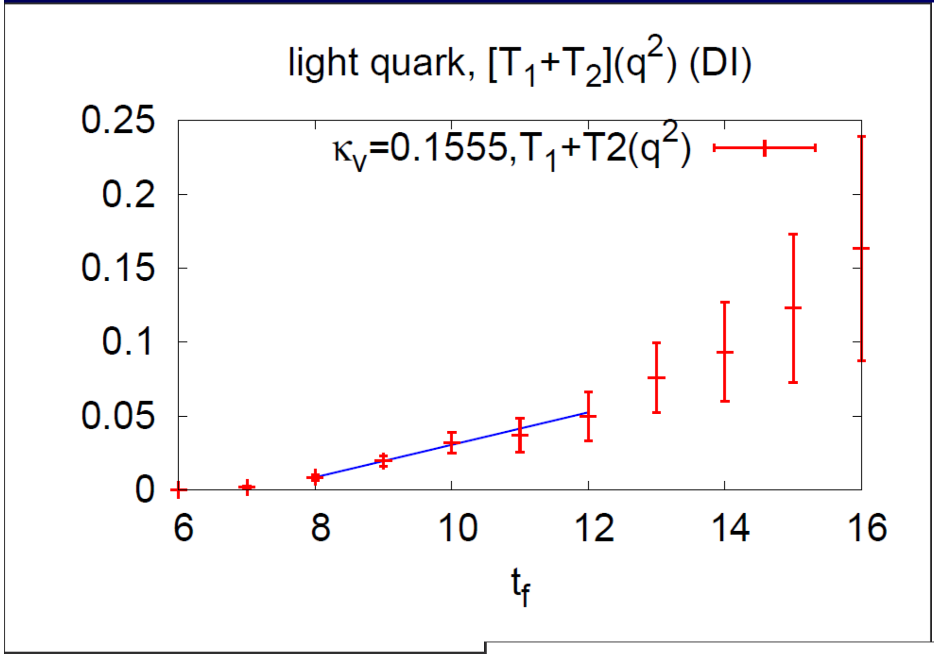
- Quenched 16^3 x 24 lattice with Wilson fermion
- Quark spin and <x> were calculated before for both the C.I. and D.I.
- κ = 0.154, 0.155, 0.1555 (m_π = 650, 538, 478 MeV)
- 500 configurations
- 400 noises (Optimal Z_4 noise with unbiased subtraction) for DI
- 16 nucleon sources

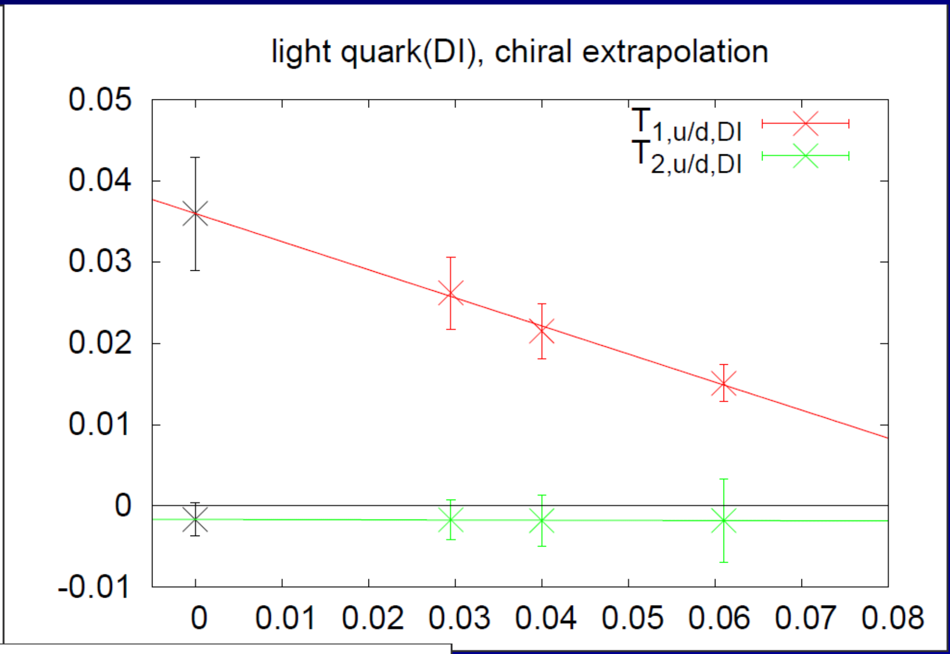
Connected Insertions of T_1 (q^2) and T_2 (q^2) for u/d Quarks

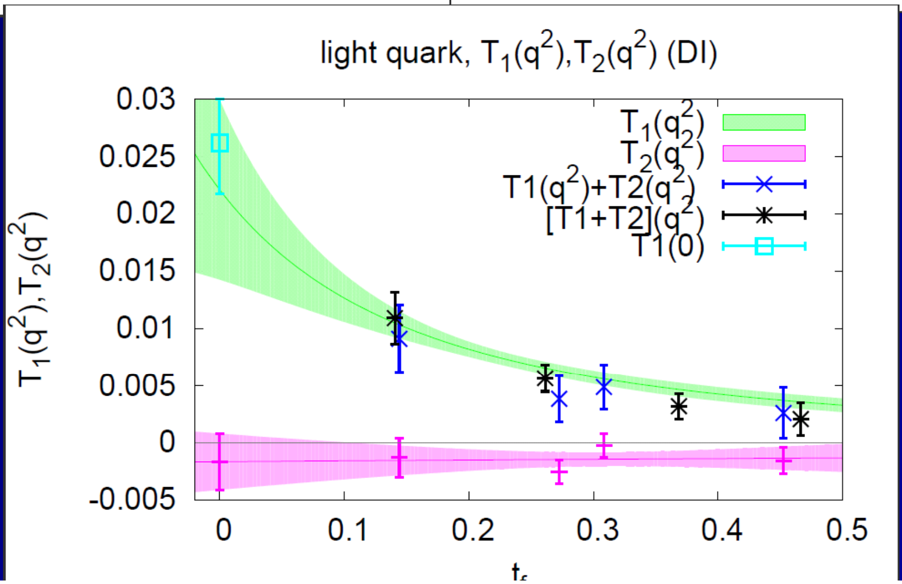
cross check



Disconnected Insertions of T_1 (q^2) and T_2 (q^2) for u/d Quarks







Gauge Operators from the Overlap Dirac Operator

- Overlap operator

$$D_{ov} = 1 + \gamma_5 \varepsilon(H); \quad H = \gamma_5 D_W(m_0)$$

- Index theorem on the lattice (Hasenfratz, Laliena, Niedermayer, Lüscher)

$$\text{index } D_{ov} = -Tr\gamma_5 (1 - \frac{a}{2} D_{ov})$$

- Local version (Kikukawa & Yamada, Adams, Fujikawa, Suzuki)

$$q_L(x) = -tr\gamma_5 (1 - \frac{a}{2} D_{ov}(x,x)) \xrightarrow[a \to 0]{} a^4 q(x) + O(a^6)$$

- Study of topological structure of the vacuum
  - Sub-dimensional long range order of coherent charges (Horvàth et al; Thacker talk in Lattice 2006)
  - Negativity of the local topological charge correlator (Horvàth et al)

- We obtain the following result

$$\mathbf{tr}_s \sigma_{\mu\nu} a D_{ov}(x,x) = c^T a^2 F_{\mu\nu} + O(a^3),$$

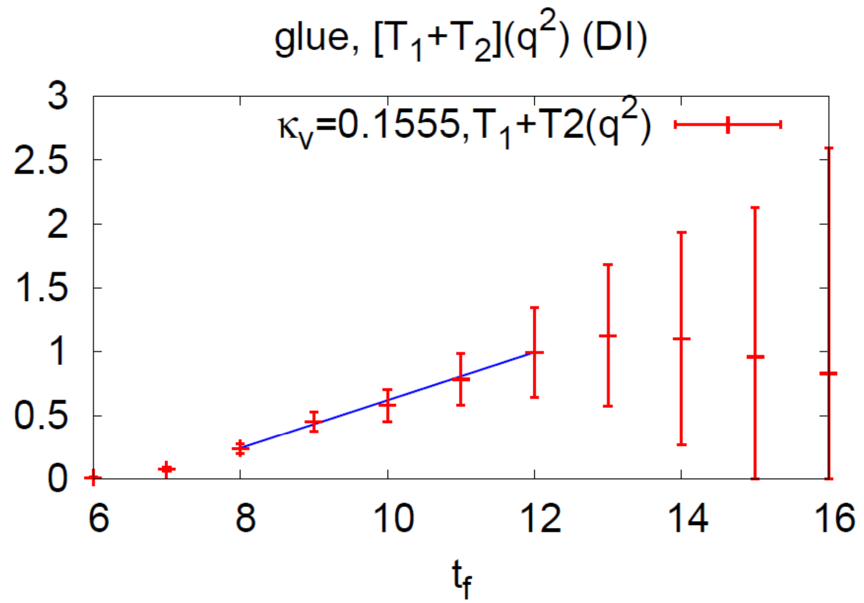
$$c^T = \rho \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{2\left[(\rho + r\sum_{\lambda}(c_\lambda - 1))c_\mu c_\nu + 2rc_\mu s_\nu^2 \right]}{(\sum_\mu s_\mu^2 + [\rho + \sum_\nu (c_\nu - 1)]^2)^{3/2}}$$

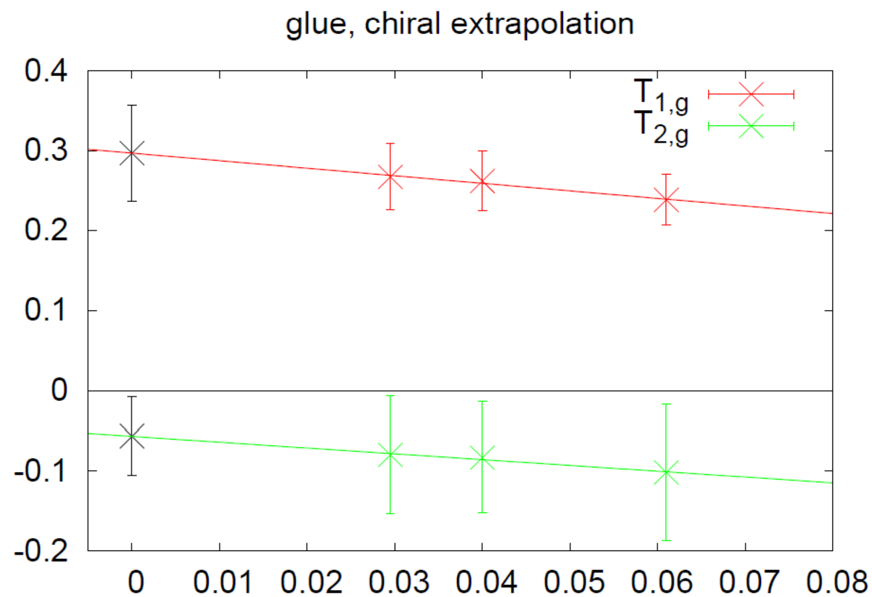
where, $r = 1, \quad \rho = 1.368, \quad c^T = 0.11157$

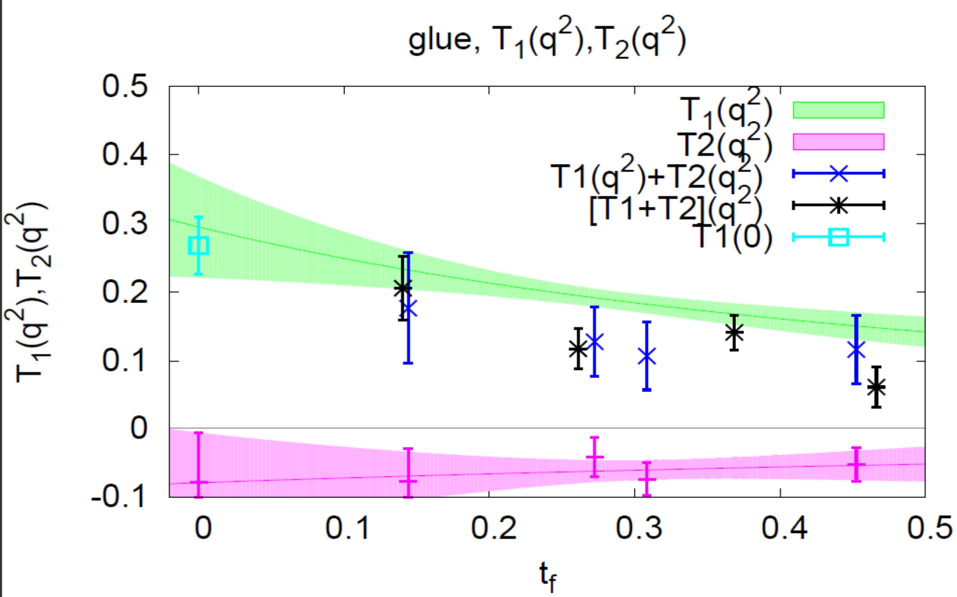
Liu, Alexandru, Horvath – PLB 659, 773 (2007)

- Noise estimation $D_{ov}(x,x) \to \left\langle \eta_x^\dagger (D_{ov}\eta)_x \right\rangle$
with Z_4 noise with color-spin dilution and some dilution in space-time as well.

Glue $T_1(q^2)$ and $T_2(q^2)$







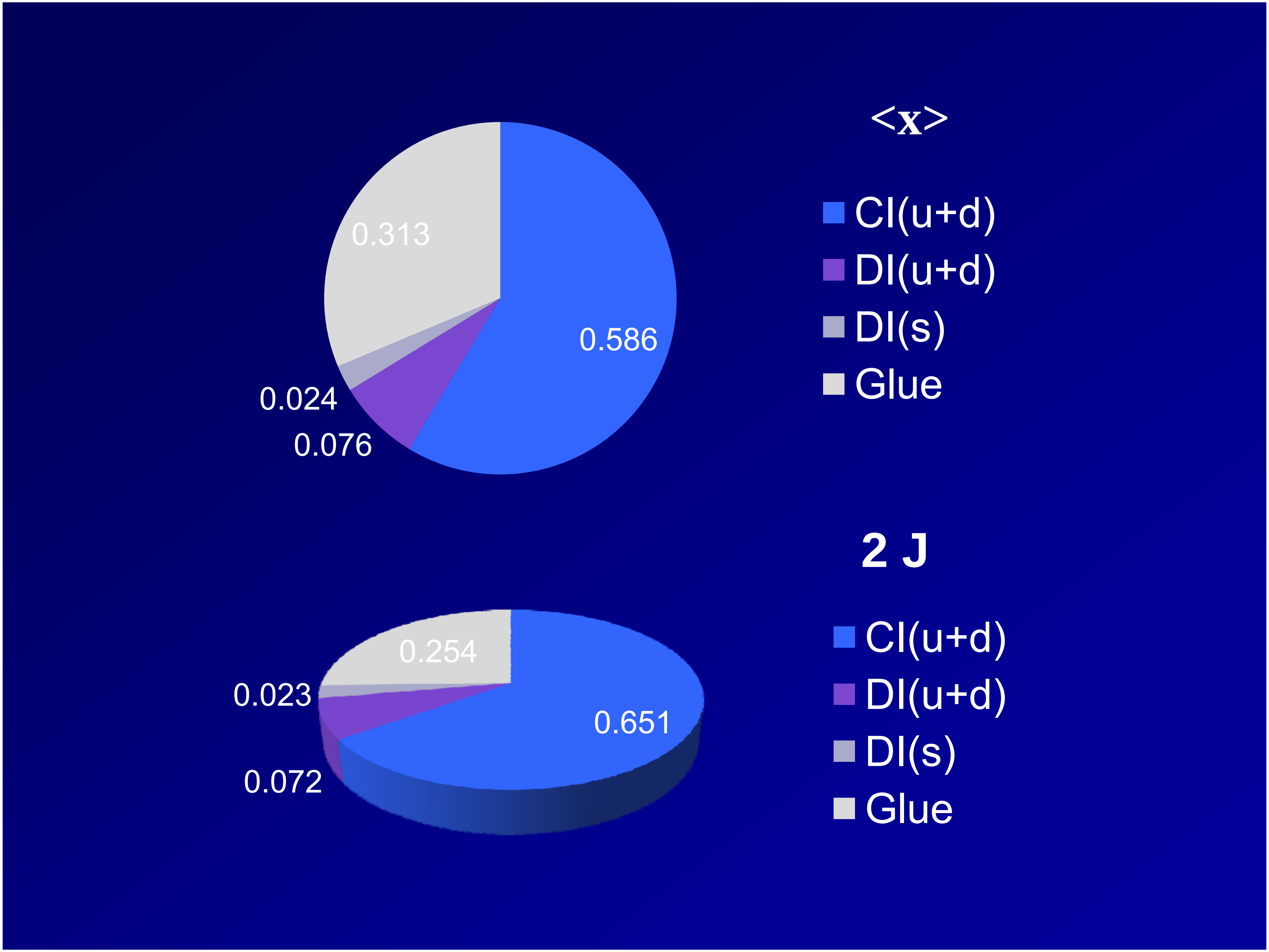
Renormalized results: $Z_q = 1.05$, $Z_g = 1.05$

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
<x>	0.428 (40)	0.156 (20)	0.586 (45)	0.038 (6)	0.024 (6)	0.313 (56)
$T_2(0)$	0.297 (112)	-.218 (80)	0.064 (22)	-0.002 (2)	-.001 (3)	-.059 (52)
2J	0.726 (128)	-.072 (82)	0.651 (51)	0.036 (7)	0.023 (7)	0.254 (76)

$$T_2(0)_{CI} + T_2(0)_{DI} + T_2(0)_g = 0.002(56)$$

S. Brodsky et al. NPB 593, 311(2001) → no anomalous gravitomagnetic moment

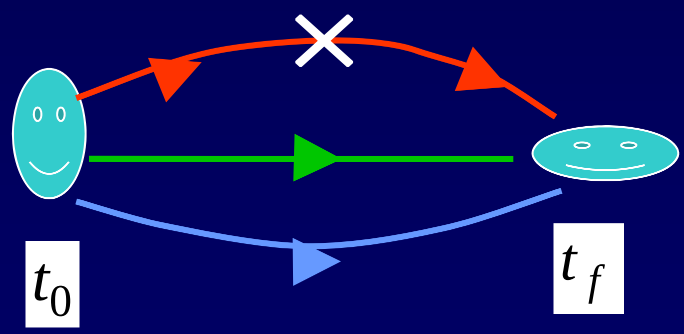
E. Leader, arXiv:1109.1230 → transverse angular momentum


<x>
0.313
0.586
0.024
0.076
CI(u+d)
DI(u+d)
DI(s)
Glue
2 J
0.254
0.023
0.651
0.072
CI(u+d)
DI(u+d)
DI(s)
Glue

Flavor-singlet g_A

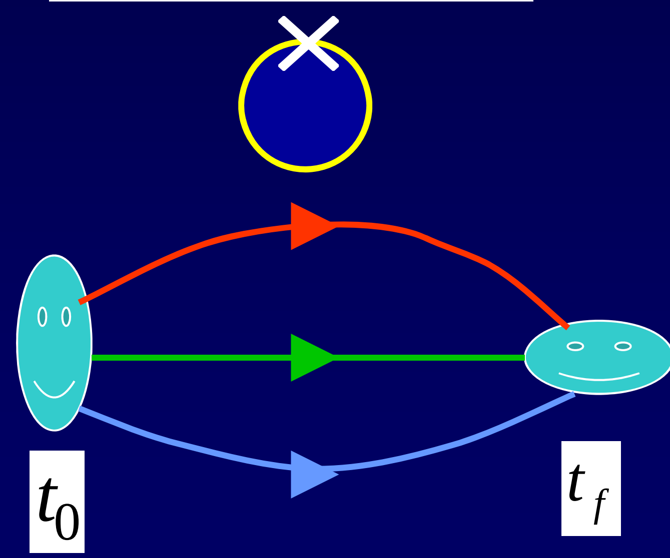
- Quark spin puzzle (dubbed `proton spin crisis')
  - $g_A^0 = \Delta u + \Delta d + \Delta s = \begin{cases} 1 & \text{NRQM} \\ 0.75 & \text{RQM} \end{cases}$
  - Experimentally (EMC, SMC, … $\Delta\Sigma = g_A^0 \sim 0.2 - 0.3$

$(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)(t)$

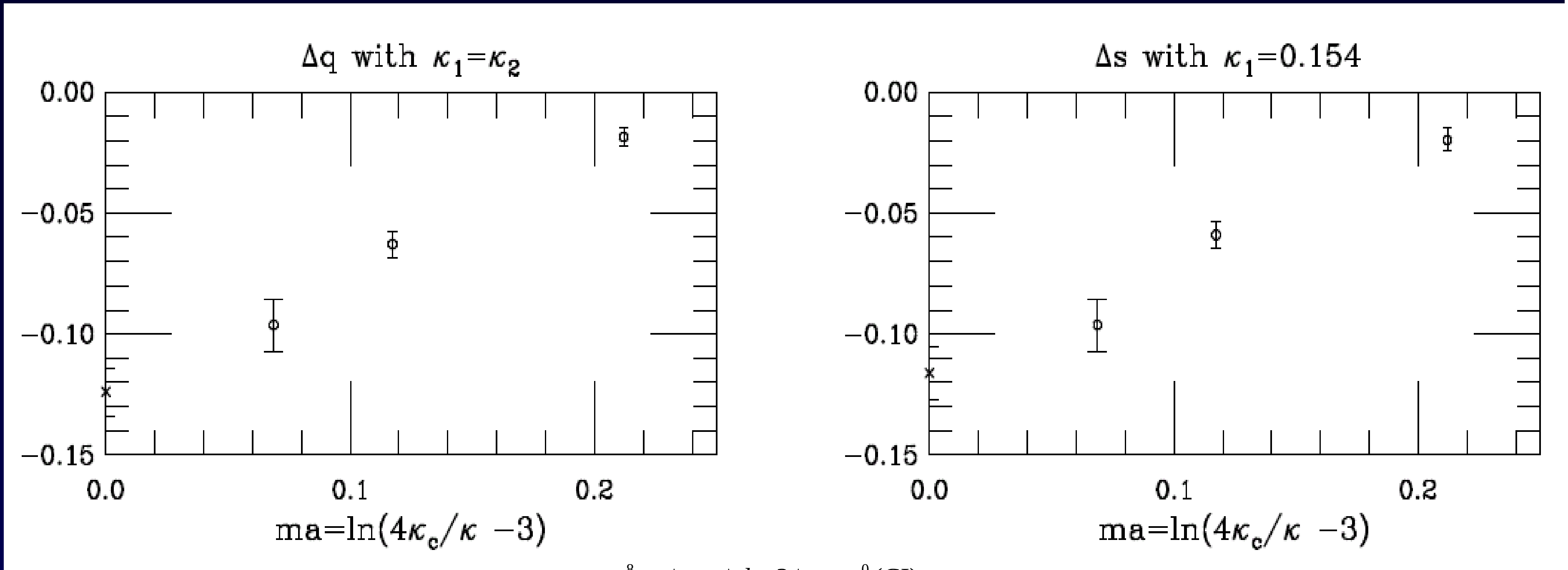


$g^0_{A,con} = (\Delta u + \Delta d)_{con}$

$\overline{\Psi}\gamma_\mu\gamma_5\Psi(t)(u,d,s)$



$g^0_{A,dis} = (\Delta u + \Delta d + \Delta s)_{dis}$



$g_A^8 = \Delta u + \Delta d - 2\Delta s \approx g_A^0(\mathrm{CI})$

S.J. Dong, J.-F. Lagae, and KFL, PRL 75, 2096 (1995)

- DI sea contribution independent of quark mass

$$\Delta u = \Delta d \cong \Delta s$$

- This suggests U(1) anomaly at work.
- $$g_A^8 = \Delta u + \Delta d - 2\Delta s \approx g_A^0(\mathrm{CI})$$

Lattice resolution: U(1) anomaly

$$g_A^0 = (\Delta u + \Delta d)_{con} + (\Delta u + \Delta d + \Delta s)_{dis} = 0.62(9) + 3(-0.12(1)) = 0.25(12)$$

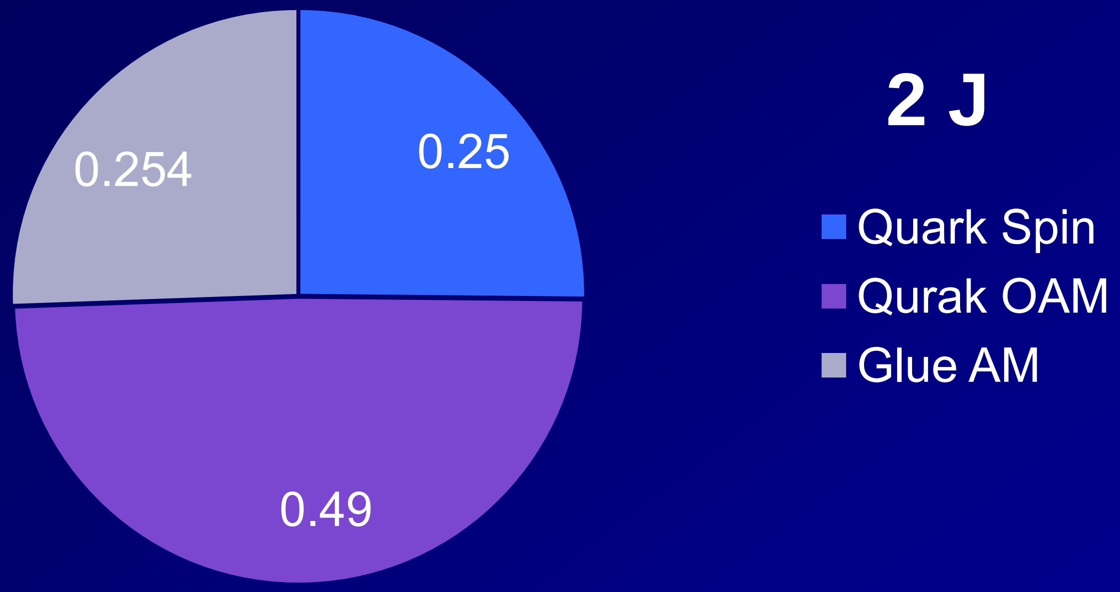
	Lattice	Expt. (SMC)	NRQM	RQM
$g_A^0 = \Delta u + \Delta d + \Delta s$	0.25(12)	0.22(10)	1	0.75
$g_A^3 = \Delta u - \Delta d$	1.20(10)	1.2573(28)	5/3	1.25
$g_A^8 = \Delta u + \Delta d - 2\Delta s$	0.61(13)	0.579(25)	1	0.75
Δu	0.79(11)	0.80(6)	1.33	1
Δd	-.42(11)	-0.46(6)	-0.33	-0.25
Δs	-.12(1)	-0.12(4)	0	0
F_A	0.45(6)	0.459(8)	0.67	0.5
D_A	0.75(11)	0.798(8)	1	0.75
F_A / D_A	0.60(2)	0.575(16)	0.67	0.67

$$F_A = (\Delta u - \Delta s)/2; \quad D_A = (\Delta u - 2\Delta d + \Delta s)/2$$

Renormalized results:

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
2J	0.726 (128)	-.072 (82)	0.651 (51)	0.036 (7)	0.023 (7)	0.254 (76)
g_A	0.95 (11)	-0.32 (12)	0.65 (8)	-0.12 (1)	-0.12 (1)	
2 L	-0.25 (18)	0.26 (14)	0.00 (10)	0.17 (2)	0.15 (2)	

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum



$\Delta q \approx 0.25;$

$2\ L_q \approx 0.49\ (0.0(\mathrm{CI})+0.49(\mathrm{DI}));$

$2\ \mathrm{J}_g \approx 0.25$

Summary

- Path-integral formulation of DIS reveals the CS partons in addition to DS partons.
- The CS partons can be isolated with the help of the combination of experiments and lattice calculations.
- Momentum fraction of quarks (both valence and sea) and glue have been calculated for a quenched lattice:
  - Glue momemtum fraction is ~ 31%.
  - g_A^0 ~ 0.25 in agreement with expt.
  - Glue angular momentum is ~ 25%.
  - Quark orbital angular momentum is large for the sea quarks (~ 50%).

Current project

- Dynamical domain-wall fermion gauge (RBC + UKQCD configurations, lowest pion mass ~ 180 MeV on 4.5 fm box)
  + overlap fermion for the valence.
- Quark loops with low mode averaging and improved nucleon propagator.