CORRECTED VERSION INCLUDING SOME AUDIENCE COMMENTS

On the controversy concerning the definition of quark and gluon angular momentum

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For a detailed treatment see: PRD **83**, 096012 (2011)

Important question: how are the momentum and angular momentum of a nucleon built up from the momenta and angular momenta of its constituents?

• Controversy in QCD : how to split the total angular momentum into separate quark and gluon components
• Ji vs Chen, Lu, Sun, Wang and Goldman (Chen etal) vs Wakamatsu and Hatta vs Canonical
• Ji stresses: gauge invariant operators; covariance; local operators

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• Different results for momentum and angular momentum carried by quarks and gluons e.g. as $\mu^2 \to \infty$

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Since problem already arises in QED, will illustrate via QED

There are four versions of $\boldsymbol{J}$
Canonical (can), Belinfante (bel)= Ji, Chen at al (chen), Wakamatsu (wak)

$$
\begin{aligned}
\boldsymbol{J}_{can} &= \int d^3x\,\psi^\dagger \boldsymbol{\gamma}\gamma_5\psi + \int d^3x\,\psi^\dagger[\boldsymbol{x}\times(-i\boldsymbol{\nabla})]\psi \\
&+ \int d^3x\,(\boldsymbol{E}\times\boldsymbol{A}) + \int d^3x\,E^i[\boldsymbol{x}\times\boldsymbol{\nabla}A^i] \\
&= \boldsymbol{S}_{can}(el) + \boldsymbol{L}_{can}(el) + \boldsymbol{S}_{can}(\gamma) + \boldsymbol{L}_{can}(\gamma)
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{J}_{bel} &= \int d^3x\,\psi^\dagger \boldsymbol{\gamma}\gamma_5\psi + \int d^3x\,\psi^\dagger[\boldsymbol{x}\times(-i\boldsymbol{D})]\psi \\
&+ \int d^3x\,\boldsymbol{x}\times(\boldsymbol{E}\times\boldsymbol{B}) \\
&= \boldsymbol{S}_{bel}(el) + \boldsymbol{L}_{bel}(el) + \boldsymbol{J}_{bel}(\gamma)
\end{aligned}
$$

Note: $\boldsymbol{J}_{bel}(\gamma)$ NOT split into spin and orbital parts.

$$
\begin{aligned}
\boldsymbol{J}_{chen} &= \int d^3x\, \psi^\dagger \boldsymbol{\gamma}\gamma_5 \psi + \int d^3x\, \psi^\dagger [\boldsymbol{x} \times (-i\boldsymbol{D}_{pure})]\psi \\
&+ \int d^3x\, (\boldsymbol{E} \times \boldsymbol{A}_{phys}) + \int d^3x\, E^i [\boldsymbol{x} \times \boldsymbol{\nabla} A^i_{phys}] \\
&= \boldsymbol{S}_{ch}(el) + \boldsymbol{L}_{ch}(el) + \boldsymbol{S}_{ch}(\gamma) + \boldsymbol{L}_{ch}(\gamma)
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{J}_{wak} &= \int d^3x\, \psi^\dagger \boldsymbol{\gamma}\gamma_5 \psi + \int d^3x\, \psi^\dagger [\boldsymbol{x} \times (-i\boldsymbol{D})]\psi \\
&+ \int d^3x\, (\boldsymbol{E} \times \boldsymbol{A}_{phys}) \\
&+ \left[\int d^3x\, E^i (\boldsymbol{x} \times \boldsymbol{\nabla} A^i_{phys}) + \int d^3x\, \psi^\dagger (\boldsymbol{x} \times e\boldsymbol{A}_{phys})\psi \right] \\
&= \boldsymbol{S}_{wak}(el) + \boldsymbol{L}_{wak}(el) + \boldsymbol{S}_{wak}(\gamma) + \boldsymbol{L}_{wak}(\gamma)
\end{aligned}
$$

In this version the very last term $\int d^3x\, \psi^\dagger (\boldsymbol{x} \times e\boldsymbol{A}_{phys})\psi$ has been shifted from Chen et al's electron orbital term to the photon's orbital angular momentum.

As usual $D^\mu = \partial^\mu - ieA^\mu$
Chen et al: $\boldsymbol{A} = \boldsymbol{A}_{phys} + \boldsymbol{A}_{pure}$

$$\boldsymbol{\nabla}.\boldsymbol{A}_{phys} = 0 \qquad \boldsymbol{\nabla} \times \boldsymbol{A}_{pure} = 0$$

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You have to choose:
Does this hold in every Lorentz frame? If yes: A^{μ} does *not* transform as a 4-vector
If no, the splitting is different in every Lorentz frame.

Two important points:

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• $\boldsymbol{A}_{phys}$ is not a local field:

$$\boldsymbol{A}_{phys} = \boldsymbol{A} - \frac{1}{\nabla^2}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{A})$$

Which is “correct”?

What is the criterion for deciding?

Similar differences in definitions of linear momentum. Asymptotically what fraction of total momentum is carried by gluons?

Ji: $\frac{16}{16+3n_f} \simeq 1/2$ for $n_f = 5$

Chen et al: $\frac{8}{8+6n_f} \simeq 1/5$ for $n_f = 5$!

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Beware quantization conditions! Belinfante, as used, does not correspond to covariant quantization.

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• "If E and F are interacting particles, definition of e.g. $\boldsymbol{J}(E)$ should satisfy $[J^i(E), J^j(E)] = i\,\epsilon^{ijk} J^k(E)$"
Impossible. Cannot be checked!

Will only have time to discuss some aspects of these problems

Many of the problems involved also apply to **linear momentum**.

Also many apply in **QED**

Much simpler, therefore illustrate them using linear momentum in QED.

The momentum operator in gauge-invariant theories

Theory invariant under translations; Noether construction, from classical Lagrangian; canonical e-m density $t^{\mu\nu}_{can}(x)$. A conserved density, generally not symmetric under $\mu \leftrightarrow \nu$.

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Canonical total linear momentum operator P^j_{can}

$$P^j_{can} = \int d^3x \; t^{0j}_{can}(x)$$

independent of time.

Canonical momentum operator as generator of translations

Classically : P^j_{can} generates spatial translations.
Quantum theory: check correct commutation relations with fields i.e. for any field $\phi(x)$

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Interacting theory: cannot calculate arbitrary commutation relation between the fields.
But Equal Time Commutators (ETC) fixed in quantizing theory. Thus can check because P^j_{can} independent of time. Take time variable of fields in P^j_{can} to coincide with time variable in $\phi(x) \equiv \phi(t, \boldsymbol{x})$.

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Will be crucial when discussing division of total momentum into contributions from different fields .

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Differs from $t^{\mu\nu}_{can}(x)$ by a divergence term:

$$t^{\mu\,\nu}_{bel}(x) = t^{\mu\,\nu}_{can}(x) + \frac{1}{2}\partial_\rho[H^{\rho\mu\nu} - H^{\mu\rho\nu} - H^{\nu\rho\mu}]$$

where $H^{\rho\mu\nu} = -H^{\rho\nu\mu}$ and is a local operator.(See my pedagogical lecture)

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For a classical *c-number* field it is meaningful to argue that the field vanishes at infinity. Much less obvious what this means for a quantum operator.

Is it safe to throw away integral of this divergence ??

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It had better be, otherwise a catastrophe

Would find that P^j does not commute with itself !

Non-gauge invariance of the QED momentum operator

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where $\Lambda(x)$ is a c-number field satisfying $\Box\Lambda(x) = 0$ and vanishing at infinity.

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Let F be the generator of gauge transformations, so that

$$i[F, A^\mu(x)] = \partial^\mu \Lambda(x)$$

Consider the Jacobi identity which holds for any three operators:

$$[F,\,[P^\mu,A^\nu]]+[A^\nu,\,[F,P^\mu]]+[P^\mu,\,[A^\nu,F]]=0$$

Now $[P^\mu,\,[A^\nu,F]]=0$ since $[A^\nu,F]$ is a c-number. Thus

$$[[F,P^\mu],\,A^\nu]=[F,\,[P^\mu,A^\nu]] \qquad (\alpha)$$

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Hence from Eq. (α)

$$[[F, P^\mu],\, A^\nu] \neq 0$$

Therefore

$$[F, P^\mu] \neq 0 \tag{1}$$

so that P^μ is not gauge invariant.
Wang objected on the grounds that Strocchi and Wightman had proved that covariantly quantized QED is NON invariant under c-number gauge transformations. Ironically, I cite SW and I discovered my theorem by trying to find a simple-minded algebraic version of the super-sophisticated axiomatic field theory proof of SW. The point is that they assume, without ever saying so, that their generators are INVARIANT under gauge transformations. Eq. (1) is then a contradiction and leads them to claim that c-number gauge transformations aren't acceptable, contrary to what the authors of the covariant papers state.

But there is no reason to insist that the generators are invariant. So the SW theorem is a corollary of mine: "**IF** the generators are gauge invariant then covariant quantized QED is not invariant under c-number gauge transformations."

However, lack of gauge invariance of no physical significance.

Example, covariantly quantized QED: generator of translations is P_{can}: show that the matrix element of P^j_{can} between any normalizable physical states, unaffected by gauge changes in the operator.

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Lautrup-Nakanishi Lagrangian density: combination of the Classical Lagrangian ($Clas$) and a Gauge Fixing part (Gf) $\quad \mathcal{L} = \mathcal{L}_{Clas} + \mathcal{L}_{Gf}$

$$\mathcal{L}_{Clas} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}[\bar{\psi}(i\,\partial\!\!\!/ - m + e\,A\!\!\!/)\psi + \text{h.c.}]$$

$$\mathcal{L}_{Gf} = B(x)\,\partial_\mu A^\mu(x) + \frac{\text{a}}{2}B^2(x)$$

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$$A_\mu \to A_\mu + \partial_\mu \Lambda(x) \qquad \psi \to \psi + ie\Lambda\psi$$

while $B(x)$ is unaffected by gauge transformations (GTs).

Generator of GTs : $F = \int d^3x \, [(\partial_0 B)\Lambda - B\partial_0\Lambda + \partial_j(F^{0j}\Lambda)]$.

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Generator of GTs : $F = \int d^3x \, [(\partial_0 B)\Lambda - B\partial_0\Lambda + \partial_j(F^{0j}\Lambda)]$ $\qquad (\beta)$

Physical states $|\Psi\rangle$ of the theory defined to satisfy

$$B^{(+)}(x)|\Psi\rangle = 0 \qquad (+) = \text{positive freq part}$$

$$B(x) = B^{(+)}(x) + B^{(-)}(x) \qquad B^{(-)}(x) = [B^{(+)}]^\dagger(x)$$

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$$\langle \Psi' | B(x) | \Psi \rangle = 0 \qquad (\gamma)$$

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Theorem Physical matrix elements of P^j are invariant under gauge transformations.

Proof Consider the general physical matrix element

$$\langle \Psi' | P^j | \Psi \rangle = \int d^3\boldsymbol{p}\, d^3\boldsymbol{p}'\, \phi(\boldsymbol{p})\, \phi'(\boldsymbol{p}')\, \langle \boldsymbol{p}' | P^j | \boldsymbol{p} \rangle$$

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Change induced in $\langle \boldsymbol{p}' | P^j | \boldsymbol{p} \rangle$ is $\langle \boldsymbol{p}' | i[F, P^j] | \boldsymbol{p} \rangle$.

First two terms in F, Eq. (β), give zero because of Eq. (γ) and the fact that Λ is a c-number.

Change induced by the divergence term is

$$\begin{aligned}\int d^3x \langle \boldsymbol{p}' | i[\partial_k (F^{0k}\Lambda), P^j] | \boldsymbol{p} \rangle &= (p' - p)^j [(p^0 - p'^0) \langle \boldsymbol{p}' | A^k(0) | \boldsymbol{p} \rangle \\ &- (p - p')^k \langle \boldsymbol{p}' | A^0(0) | \boldsymbol{p} \rangle] \\ &\times \int d^3x \partial_k [\Lambda(x) \, e^{i(p-p') \cdot x}]\end{aligned}$$

which vanishes after the spatial integration because $\Lambda(x)$ vanishes at infinity.

Hence $\langle \Psi' | P^j | \Psi \rangle$ is indeed invariant under gauge transformations.

The problem of defining separate quark and gluon momenta

Two separate issues:(1) general problem of how to define the separate momenta for a system of interacting particles, (2) more specific to gauge theories and includes the issue of splitting the angular momentum of a gauge particle into a spin and orbital part.

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Note that this expression is totally misleading, and should be written

$$P^j = P^j_E(t) + P^j_F(t)$$

to reflect the fact that the particles exchange momentum as a result of their interaction. [See my Controversy paper for more detail]

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But there is no way we can check this, since $P_E^j(t)$ depends on t and, without solving the entire theory, we are only able to compute equal time commutators .

We suggest, therefore, that *the minimal requirement for identifying an operator* P_E^j *with the momentum carried by* E , is to demand that *at equal times*, for the bare fields

$$i[P_E^j(t)\,,\phi^E(t,\boldsymbol{x})] = \partial^j\phi^E(t,\boldsymbol{x}).$$

We specify "bare" fields since it is not clear whether such a relation will be preserved under renormalizzation.

Analogously, *for an angular momentum operator* M_E^{ij} *(* $J^i = \epsilon^{ijk} M^{jk}$*) we suggest* that at equal times, for the bare fields,

$$i[M_E^{ij}(t)\,,\phi_r^E(t,\boldsymbol{x})] = (x^i\partial^j - x^j\partial^i)\phi_r^E(t,\boldsymbol{x}) + (\Sigma^{ij})_r{}^s\phi_s^E(t,\boldsymbol{x})$$

where r and s are spinor or Lorentz labels and $(\Sigma^{ij})_r{}^s$ is the relevant spin operator.

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But, if we split P_{can} into $P_{can,E} + P_{can,F}$ and P_{bel} into $P_{bel,E} + P_{bel,F}$, then the integrands of $P_{can,E}$ and $P_{bel,E}$ do *not* differ by a spatial divergence.

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Hence $P_{can,\,E}$ and $P_{bel,\,E}$ do **not** generate the same transformation on $\phi^E(x)$, and similarly for F.

Since, by construction, $P_{can,E}$ and $P_{can,F}$ do generate the correct transformations on $\phi_E(x)$ and $\phi_F(x)$ respectively, we conclude that with the above minimal requirement we are forced to associate the momentum and angular momentum of E and F with the canonical version of the relevant operators.

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This disagrees with Ji, Chen et al, Wakamatsu and Hatta, but agrees with Jaffe and Manohar.

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In fact, no contradiction in the special case of the *longitudinal* components of the momentum and angular momentum.

From the gauge invariant expression for the unpolarized quark number density $q(x)$ (including Wilson line operator) one finds

$$\int_0^1 dx\, x\, [\, q(x) + \bar{q}(x)\,] = \frac{i}{4(P^+)^2} \langle\, P \,|\, \bar{\psi}(0)\, \gamma^+ \overleftrightarrow{D}^+ \psi(0) \,|\, P \,\rangle$$

with

$$\overleftrightarrow{D}^+ = \overrightarrow{\partial}^+ - \overleftarrow{\partial}^+ - 2igA^+(0).$$

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$$\overleftrightarrow{D}^+ = \overrightarrow{\partial}^+ - \overleftarrow{\partial}^+ - 2igA^+(0). \qquad (3)$$

But the quark part of $t^{\mu\nu}_{bel}(qG)$ is given by

$$t^{\mu\nu}_{q,\,bel}(z) = \frac{i}{4}[\bar{\psi}(z)\gamma^{\mu}\overleftrightarrow{D}(z)^{\nu}\psi(z) + (\mu \leftrightarrow \nu)] - g^{\mu\nu}\mathcal{L}_q$$

where $\mathcal{L}_q$ is the quark part of $\mathcal{L}_{qG}$.

Since $g^{++} = 0$

$$t_{q,\,bel}^{++}(0) \,= \frac{i}{2}\{\bar{\psi}(0)\,\gamma^{+}\overleftrightarrow{D}^{+}\,\psi(0)\}$$

so that

$$\int_0^1 dx\,x\,[\,q(x) + \bar{q}(x)\,] = \frac{1}{2(P^{+})^2}\langle\,P\,|\,t_{q,\,bel}^{++}(0)\;|\,P\,\rangle.$$

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Consider the physical interpretation of the LHS in the parton model. The parton model is not synonymous with QCD. It is a picture of QCD in the gauge $A^+ = 0$ and it is in this gauge, and in an infinite momentum frame that x can be interpreted as the momentum fraction carried by a quark in the nucleon.

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Hence the fraction of *longitudinal* momentum carried by the quarks in an infinite momentum frame is given equally well by either the canonical or Belinfante versions of the energy momentum tensor density.

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But this J_z is the Belinfante version! Does it mean that the RHS is not consistent with our canonical our canonical interpretation of the angular momentum?

Need to know connection between matrix elements of $t^{\mu\nu}$ and matrix elements of J. (See my pedagogical lecture).

To 1st order in $\Delta = P' - P$

$$\langle p+\Delta/2,\, \boldsymbol{s}\,|\, t^{\mu\nu}(0)\,|\, p-\Delta/2,\, \boldsymbol{s}\rangle \;=\; 2\mathbb{D}p^{\mu}p^{\nu} - \frac{i\Delta_{\rho}}{M}\Big\{\mathbb{S}(p^{\mu}\epsilon^{\rho\nu\alpha\beta} + p^{\nu}\epsilon^{\rho\mu\alpha\beta}) + +\mathbb{A}(p^{\mu}\epsilon^{\rho\nu\alpha\beta} - p^{\nu}\epsilon^{\rho\mu\alpha\beta}) + \frac{\mathbb{D}}{M(p_0+M)}p^{\mu}p^{\nu}\epsilon^{0\rho\alpha\beta}\Big\}S_{\alpha}p_{\beta}$$

where

$$u \equiv u(P,S) \qquad u' \equiv u(P',S').$$

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This is generally not taken into account in papers on Angular Momentum Controversy.

Then

$$\langle\, \psi_{\boldsymbol{p},\boldsymbol{s}} \,|\, M_{bel}^{ij} \,|\, \psi_{\boldsymbol{p},\boldsymbol{s}} \,\rangle \;=\; \frac{1}{M}\Big\{\frac{\mathbb{D}_{bel}}{2(p_0+M)}\,(p^j\,\epsilon^{0i\alpha\beta} - p^i\,\epsilon^{0j\alpha\beta}) + \mathbb{S}_{bel}\,\epsilon^{ij\alpha\beta}\Big\}\,S_\alpha\,p_\beta \qquad (4)$$

The $\mathbb{D}_{bel}$ term vanishes in the M_{bel}^{12} if $\boldsymbol{p}$ is along OZ.

Thus, for a longitudinally polarized nucleon moving at high speed in the Z direction $\mathbb{S}_{bel}$ measures the Z-component of $\boldsymbol{J}$.

So, Ji relation becomes

$$\int_{-1}^{1} dx x H(x,0,0) + \int_{-1}^{1} dx x E(x,0,0) = 2\,\mathbb{S}_{q,\,bel}.$$

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Thus $J_{bel,z}$(quarks)=$J_{can,z}$(quarks) and no contradiction with Ji sum rule.

The spin of the photon and the gluon

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It shouldn't: can show projection of the spin terms onto the direction of the photon's (or gluon's) momentum i.e. the photon (and gluon) helicity, is gauge invariant and it is this quantity which can be measured in deep inelastic scattering on atoms or nucleons respectively.

Summary

- There is no need to insist that the operators appearing in expressions for the momentum and angular momentum of the constituents of an interacting system should be gauge invariant, provided that the *physical matrix elements* of these operators are gauge invariant.

- We suggest that *the minimal requirement for identifying an operator P_E^j with the momentum carried by E* , is to demand that *at equal times*, for the bare fields,

$$i[P_E^j(t)\,,\phi^E(t,\boldsymbol{x})] = \partial^j\phi^E(t,\boldsymbol{x}).$$

Analogously, *for an angular momentum operator M_E^{ij} ($J^i = \epsilon^{ijk}M^{jk}$) we suggest* that at equal times

$$i[M_E^{ij}(t)\,,\phi_r^E(t,\boldsymbol{x})] = (x^i\partial^j - x^j\partial^i)\phi_r^E(t,\boldsymbol{x}) + (\Sigma^{ij})_r{}^s\phi_s^E(t,\boldsymbol{x})$$

where r and s are spinor or Lorentz labels and $(\Sigma^{ij})_r{}^s$ is the relevant spin operator.

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- The expressions given by Chen et al and the variants proposed by Wakamatsu, for the momentum and angular momentum operators of quarks and gluons are somewhat arbitrary and do not satisfy the fundamental requirement that the operators should generate the relevant infinitesmal symmetry transformations.

- Demanding that these conditions be satisfied leads to the conclusion that the canonical expressions for the momentum and angular momentum operators are the correct and physically meaningful ones.

- It is then an inescapable fact that the photon and gluon angular momentum operators cannot, in general, be split in a gauge-invariant way into a spin and orbital part. However, the projection of the photon and gluon spin onto their direction of motion i.e. their helicity, is gauge-invariant and is measured in deep inelastic scattering on atoms or nucleons respectively.

- Although Ji's expressions for the quark and gluon angular momenta are the Belinfante versions, it turns out that the expectation value of the Belinfante operator $J_{z,\,bel}$(quark) used by Ji for the *longitudinal* component of the quark angular momentum, which has the nice property that it can be measured in deeply-virtual Compton scattering reactions, coincides, in the gauge $A^+ = 0$, with the Z-component of the canonical angular momentum carried by the quarks in a nucleon moving in the Z direction.