OAM in collinear factorization

- Does A_N come from parton orbital motion?

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Orbital Angular Momentum in QCD
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Seattle, WA, February 10, 2012

Kang, Qiu, Vogelsang, Yuan, arXiv: 1103.1591, PRD 83, 2011
Kang, Prokudin, arXiv: 1201.5427

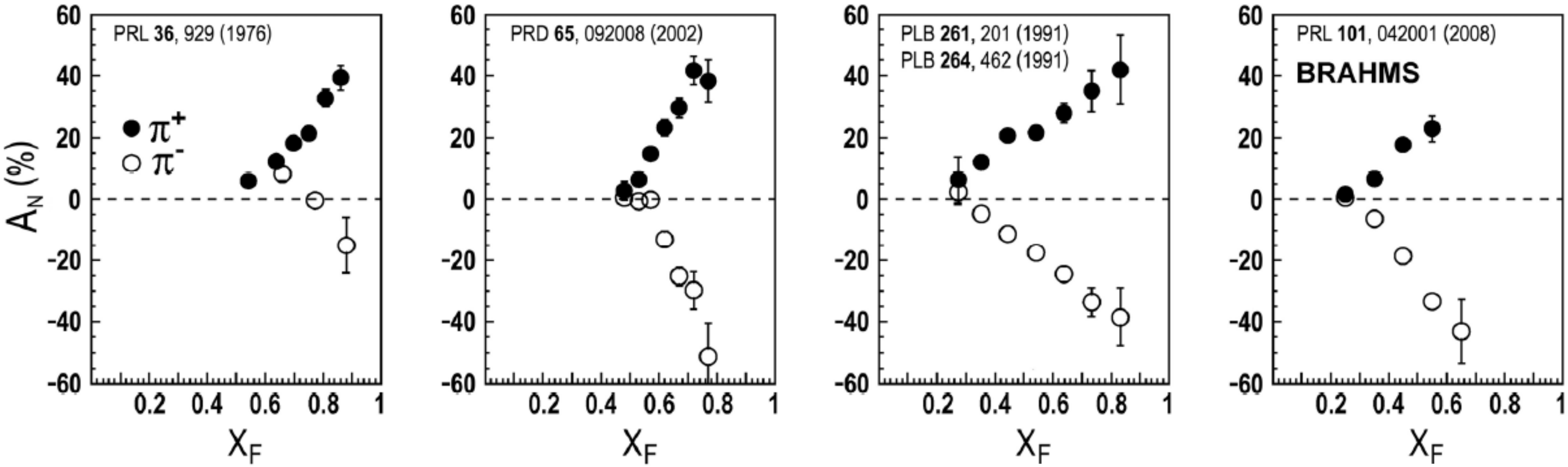
Outline

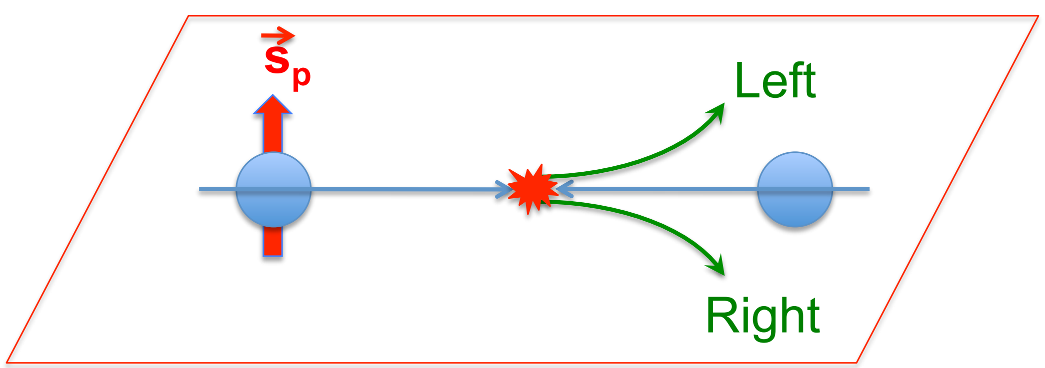
- Single transverse spin asymmetry
  - SIDIS and PP: Sivers vs ETQS
  - Process dependence: sign change from SIDIS to DY
- Unified picture
  - Global fitting of SIDIS: Sivers function
  - Global fitting of PP: ETQS function
  - Connection: sign mismatch
- Global fitting of SIDIS and PP: an attempt
  - Node in kt
  - Node in x
  - discussions
- Conclusion

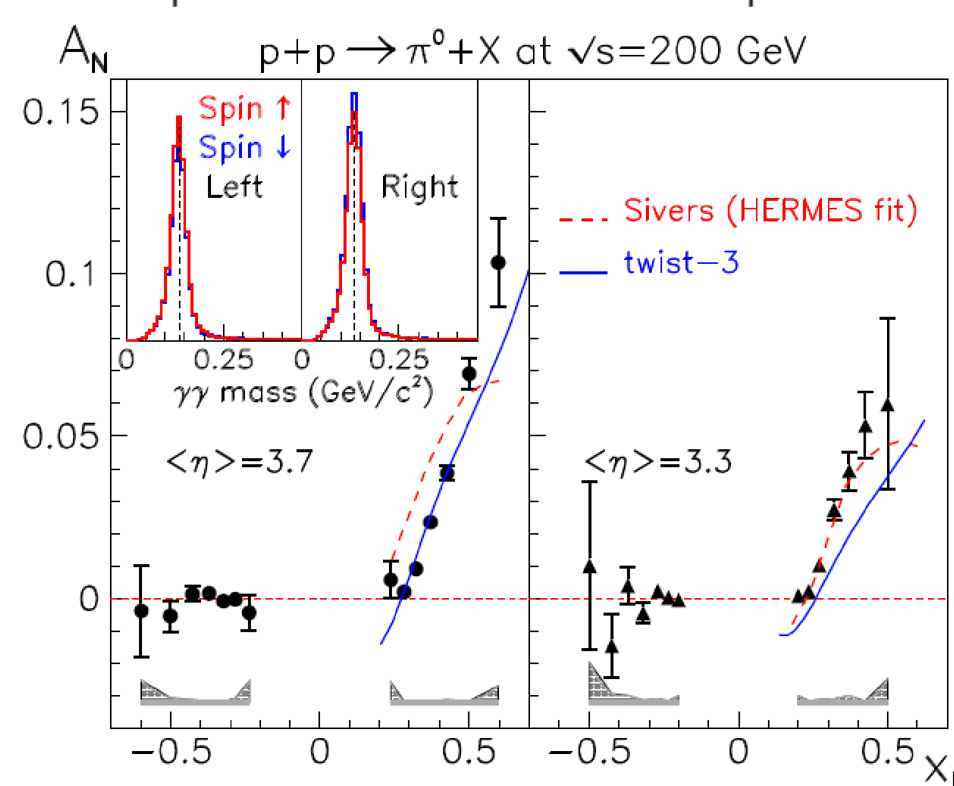
Single transverse-spin asymmetry (SSA)

- A_N for single inclusive hadron production in pp collisions: $p^{\uparrow} + p \to h + X$

ANL $\sqrt{s}$=4.9 GeV | BNL $\sqrt{s}$=6.6 GeV | FNAL $\sqrt{s}$=19.4 GeV | RHIC $\sqrt{s}$=62.4 GeV





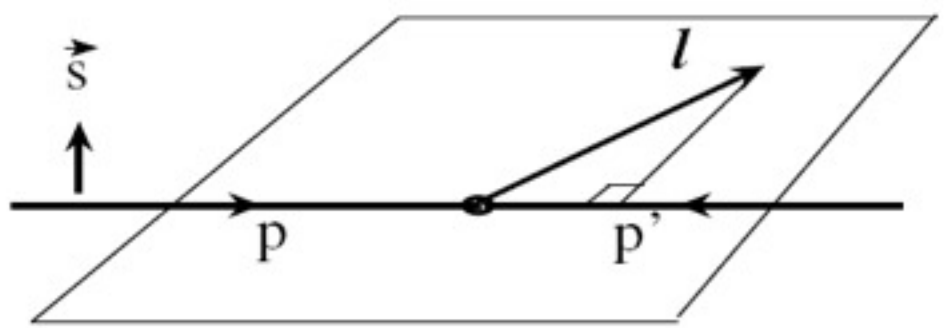


$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

SSA corresponds to a T-odd triplet product

- SSA measures the correlation between the hadron spin and the production plane, which corresponds to $\vec{s}_p \cdot (\vec{p} \times \vec{\ell})$

$$\begin{aligned} \Delta\sigma(\ell, \vec{s}) &\propto \sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s}) \\ &\propto \vec{s}_p \cdot \left(\vec{p} \times \vec{\ell}\right) \\ &\sim A_N \sin(\phi_s - \phi_h) \end{aligned}$$



$$p^{\uparrow} p \to \pi(\ell) X$$

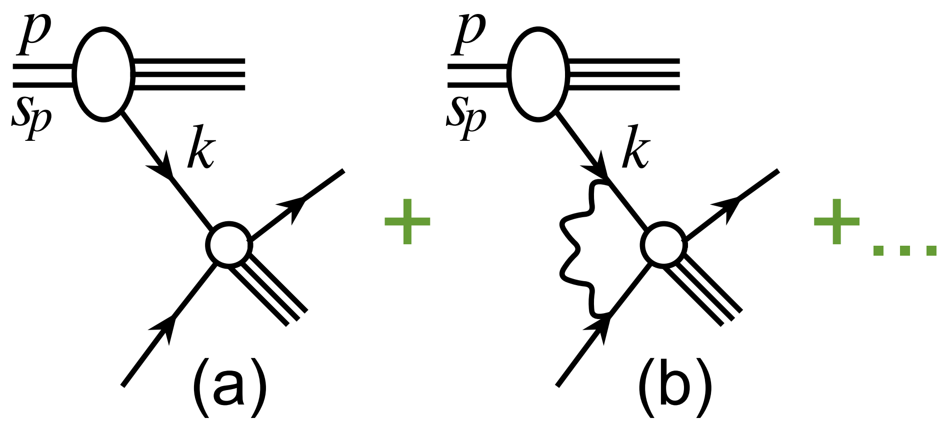
- Such a product is (naive) odd under time reversal (T-odd), thus they can arise in a time-reversal invariant theory (eg, QCD) only when there is a phase between different spin amplitudes

Nonvanishing A_N requires
a phase
a helicity flip
enough vectors to fix a scattering plane

SSA vanishes at leading twist in collinear factorization

- At leading twist formalism: partons are collinear

Kane, Pumplin, Repko, 1978

$\sigma(s_T) \sim \Big|$  $\Big|^2$ ➡ $\Delta\sigma(s_T) \sim$ Re[(a)]·Im[(b)]

  - generate phase from loop diagrams, proportional to α_s
  - helicity is conserved for massless partons, helicity-flip is proportional to current quark mass m_q

Therefore we have

$$A_N \sim \alpha_s \frac{m_q}{P_T} \to 0$$

- $A_N \neq 0$: result of parton's transverse motion or correlations!

Two ways to contain transverse momentum

- One could immediately think of two ways to include parton's transverse momentum into the formalism
  - Generalize the collinear distribution $f(x)$ to $f(x, k_\perp)$
  - Taylor expansion: $f(x, k_\perp) = f(x) + k_\perp f'(x) + \cdots$, where $f'(x) = df(x, k_\perp)/dk_\perp$ at $k_\perp = 0$, then ∫d^{2}kt kt f'(x) = a higher-twist correlation
- The first approach is called TMD approach (transverse-momentum-dependent distribution)
  - Sivers function (in PDFs) Sivers 90
  - Collins function (in FFs) Collins 93
- The second approach is called collinear twist-3 approach
  - Twist-3 three-parton correlation function

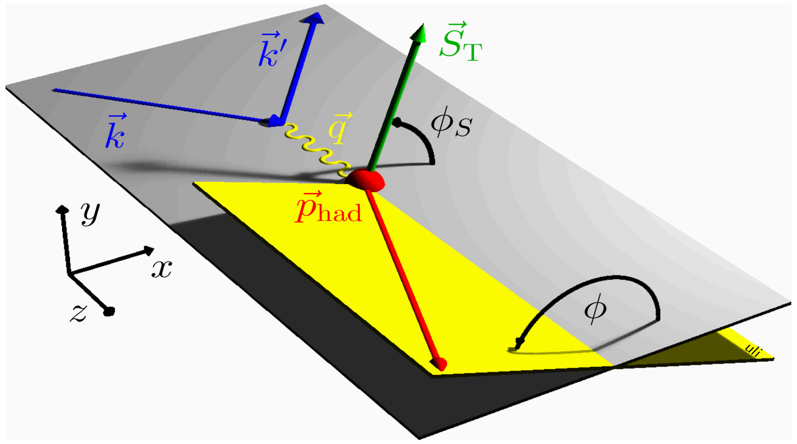
    Efremov-Teryaev 82, 84, Qiu-Sterman 91, 98, …
  - Twist-3 three-parton fragmentation function

    Koike, 02, Kang-Yuan-Zhou 2010, …

They apply in different kinematic domain

- TMD approach: need TMD factorization, applies for the process with two observed momentum scales
  - SIDIS: e+p→e+h+X
  - DY: p+p→e^+e^-(Q, pt)+X

$Q \gg p_\perp$

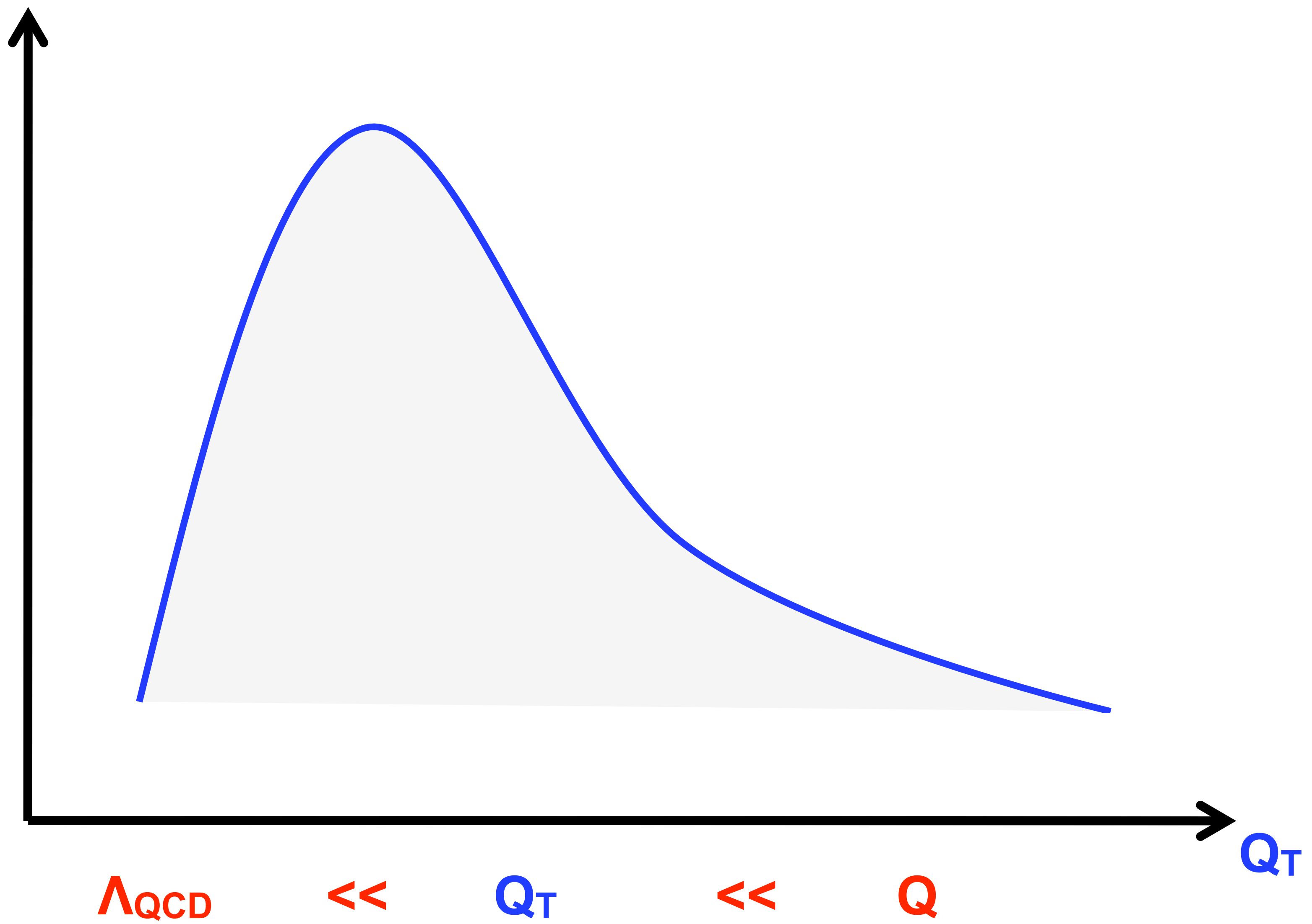


- Collinear approach: applies for the process with one-single hard scale
  - Single inclusive hadron production: p+p→h(pt)+X
  - SIDIS and DY when pt ~ Q >> Λ_{QCD}
- They give the same result in the overlap region where both apply
  - Twist-3 three-parton correlation in distribution ⟷ Sivers function

    Ji-Qiu-Vogelsang-Yuan, 2006, Koike-Tanaka 2010, …
  - Twist-3 three-parton correlation in fragmentation ⟷ Collins function

    Koike 2002, Zhou-Yuan, 2009, Kang-Yuan-Zhou, 2010, …

A unified picture for Drell-Yan (leading Q_T/Q)



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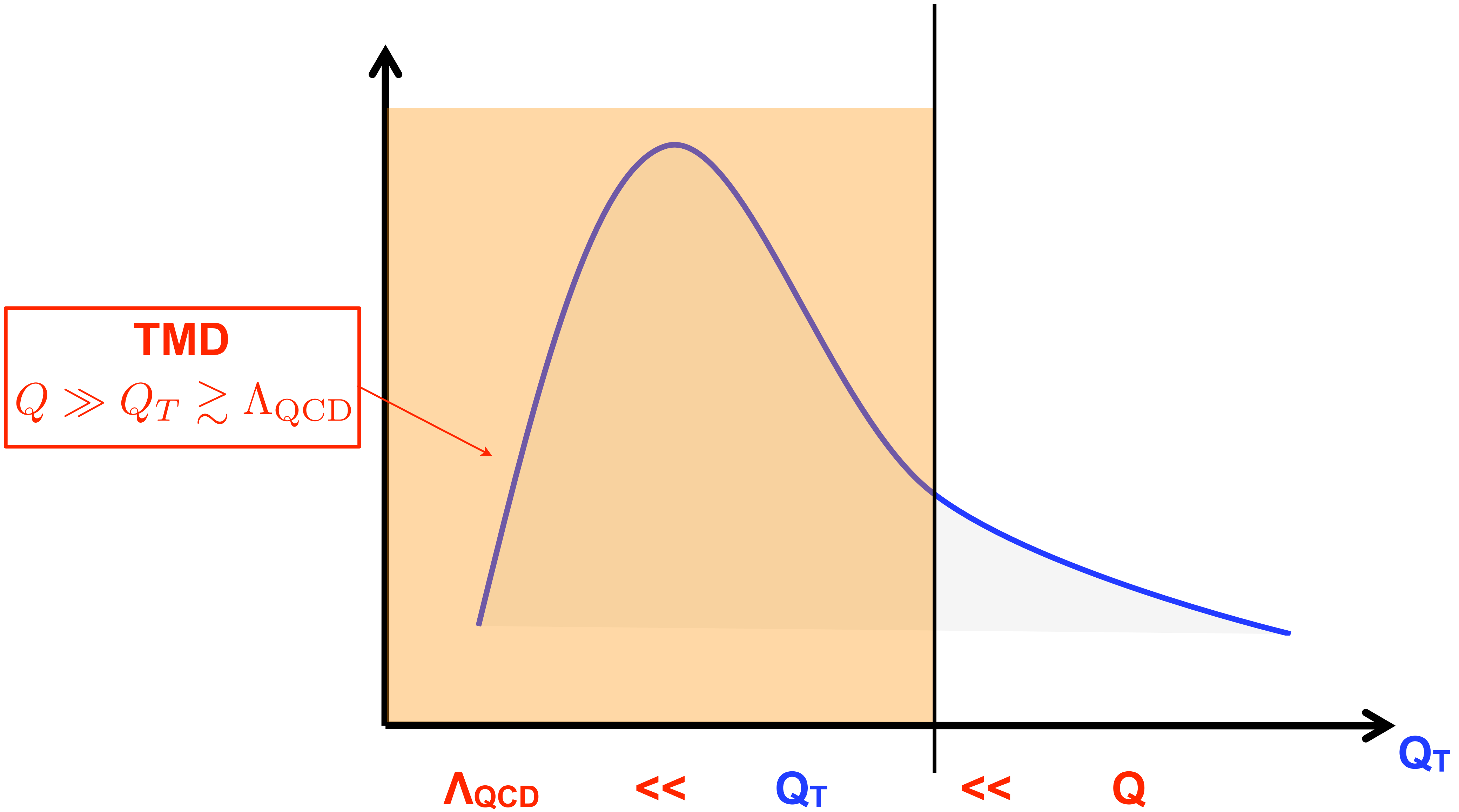
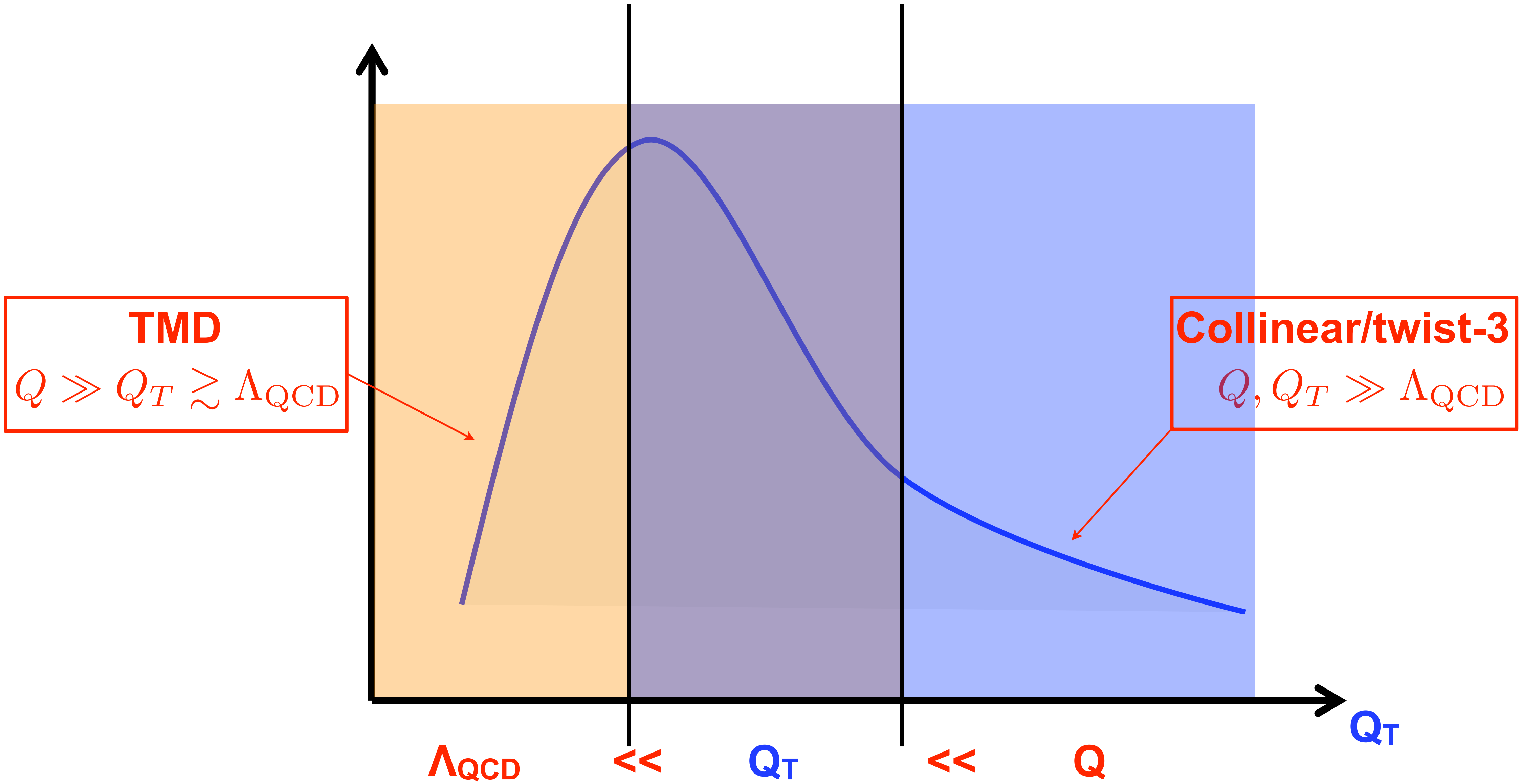
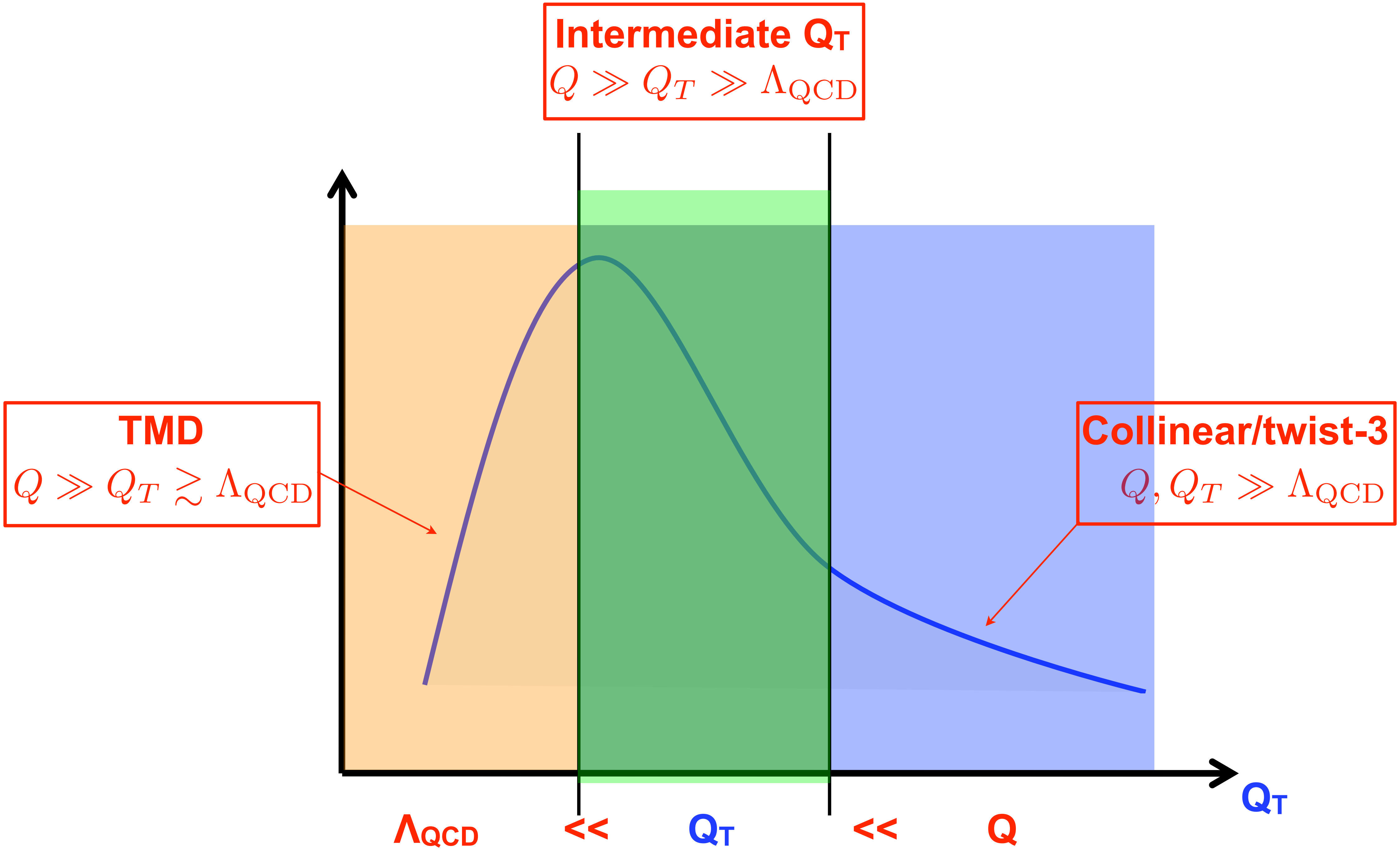


A unified picture for Drell-Yan (leading Q_T/Q)



A unified picture for Drell-Yan (leading Q_T/Q)



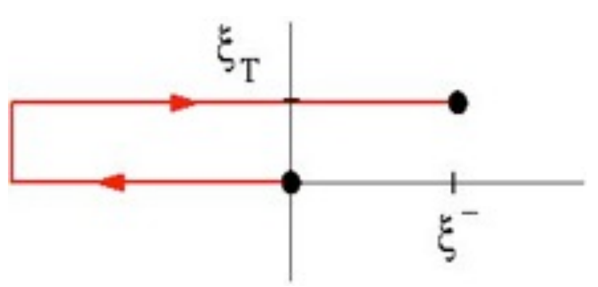
History of Sivers function (1)

- 1990: Sivers function PRD41, 83 (1990); PRD43, 261 (1991)
  - introduce kt dependence of PDFs, generate the SSA through a correlation between the hadron spin and the parton kt
- 1993: Collins function NPB396, 161 (1993)
  - introduce kt in TMD fragmentation function, generate the SSA through a correlation with the quark spin and the parton kt
  - show Sivers function vanishes due to time-reversal invariance
- 2002: S. J. Brodsky, D. S. Hwang, I. Schmidt PLB530, 99 (2002)
  - Explicit model calculation show the existence of the Sivers function in SIDIS
- 2002: J. Collins PLB536, 43 (2002)
  - Original proof missed the gauge link (needed to properly define gauge-invariant distribution)
  - Add gauge link: Sivers function in SIDIS = (-1) * Sivers function in DY
- 2002: S. J. Brodsky, D. S. Hwang, I. Schmidt NPB642, 344 (2002)
  - Verified the sign change through model calculation in DY

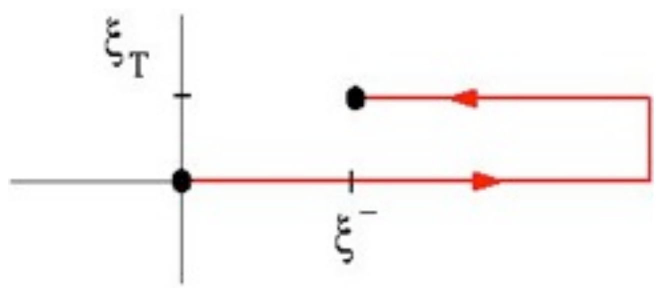
History of Sivers function (2)

- 2002: X. Ji, F. Yuan, A. V. Belitsky PLB543, 66 (2002); NPB656, 165 (2003)
  - the results by S. Brodsky, et.al is equivalent to introduce a transverse gauge link in the TMD distribution to make it fully gauge invariant
- 2003: Boer, Mulders, Pijlman NPB667, 201 (2003)
  - Use Feynman diagram approach to derive the gauge links
    - Resum collinear gluons => gauge links along the light-cone
    - Resum transverse gluons => transverse gauge links

DY



SIDIS



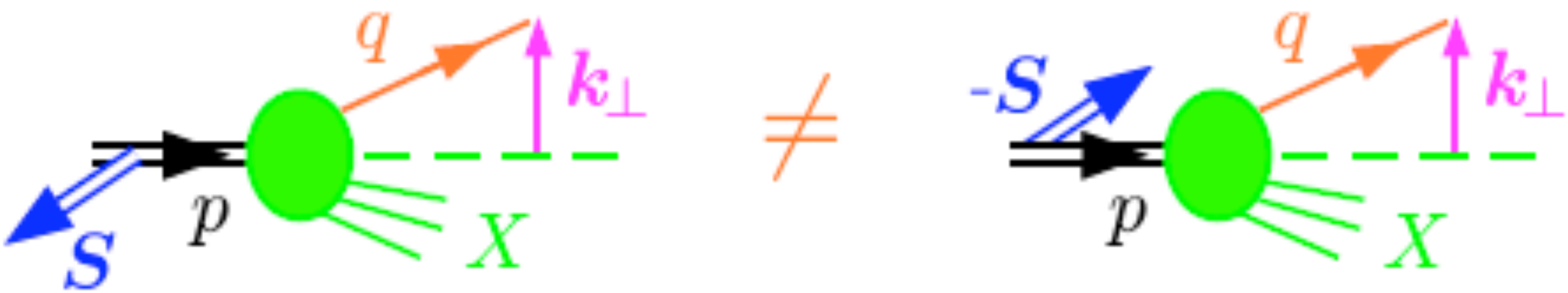
Transverse momentum dependent distribution (TMD)

- Sivers function: an asymmetric parton distribution in a polarized hadron
  - kt correlated with the spin of the hadron
  - Sivers function will vanish if no parton orbital motion

Spin-dependent

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

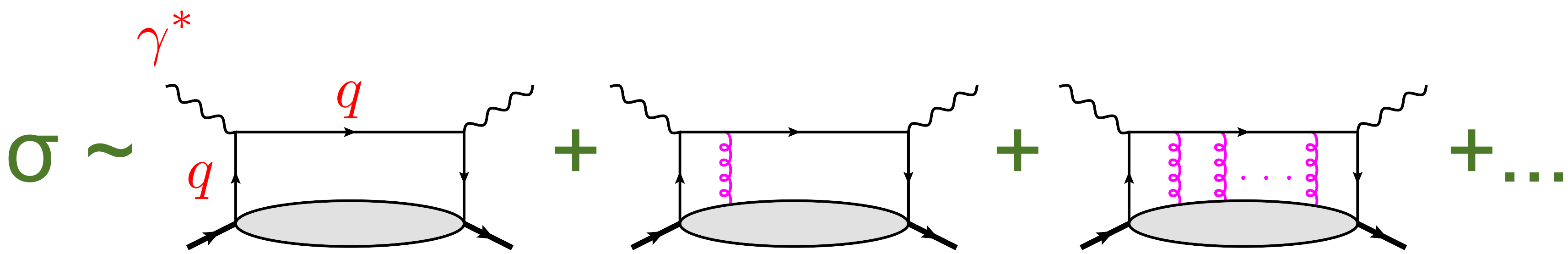
Spin-independent

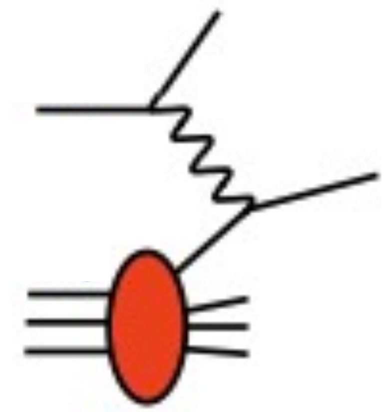


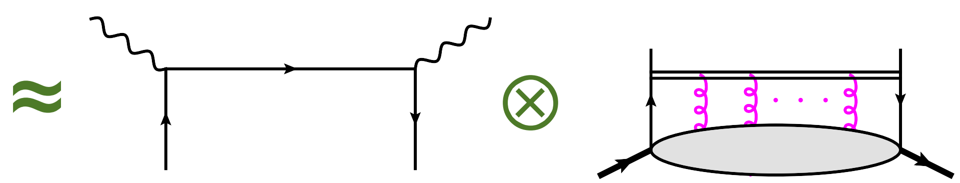
- Where does the phase come from?

Sivers function are process-dependent

- Existence of the Sivers function relies on the interaction between the active parton and the remnant of the hadron (process-dependent)
  - SIDIS: final-state interaction



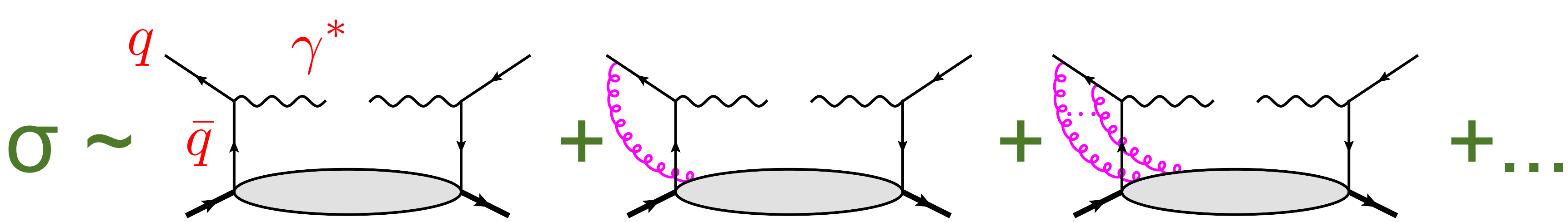


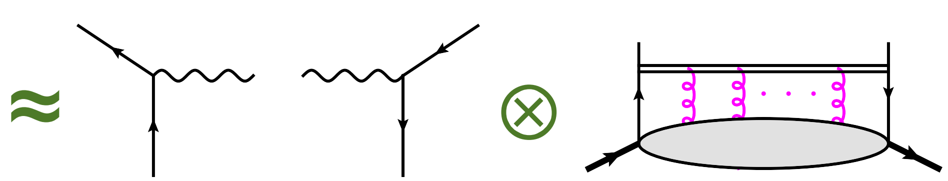


PDFs with SIDIS gauge link

$$\mathcal{P}\, e^{ig\int_y^{\infty} d\lambda\cdot A(\lambda)}$$

  - Drell-Yan: initial-state interaction





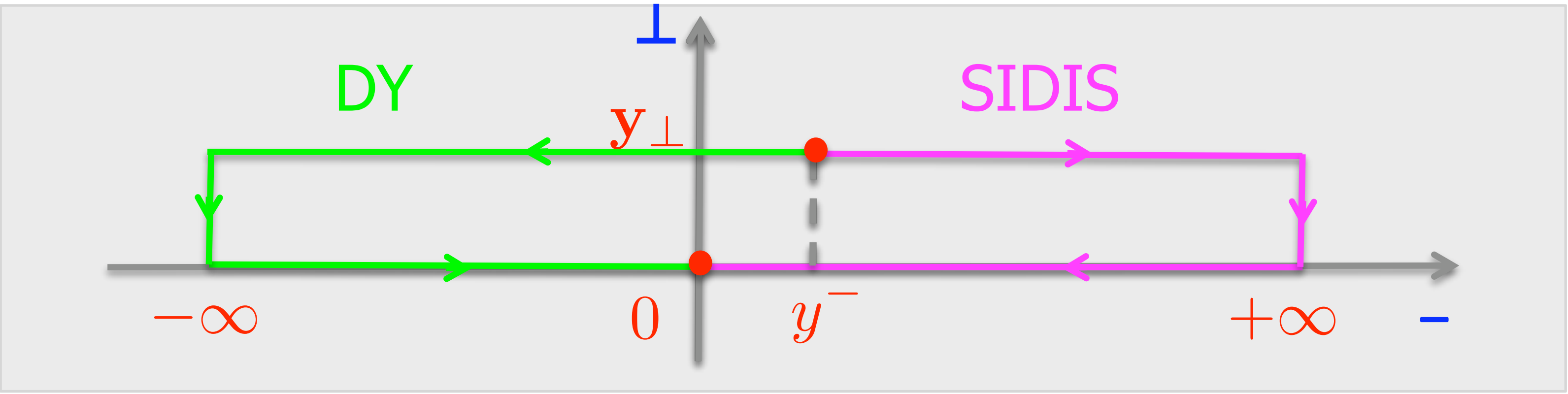
PDFs with DY gauge link

$$\mathcal{P}\, e^{ig\int_y^{-\infty} d\lambda\cdot A(\lambda)}$$

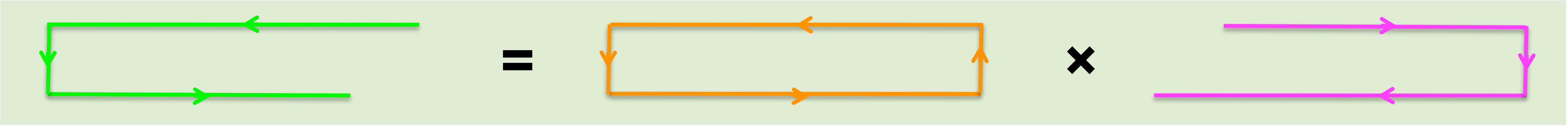
Time-reversal modified universality of the Sivers function

- Different gauge link for gauge-invariant TMD distribution in SIDIS and DY

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\boxed{\text{Gauge link}}\frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$



Wilson Loop ~ $\exp\left[-ig\int_\Sigma d\sigma^{\mu\nu}F_{\mu\nu}\right]$ Area is NOT zero



- Parity and time-reversal invariance:

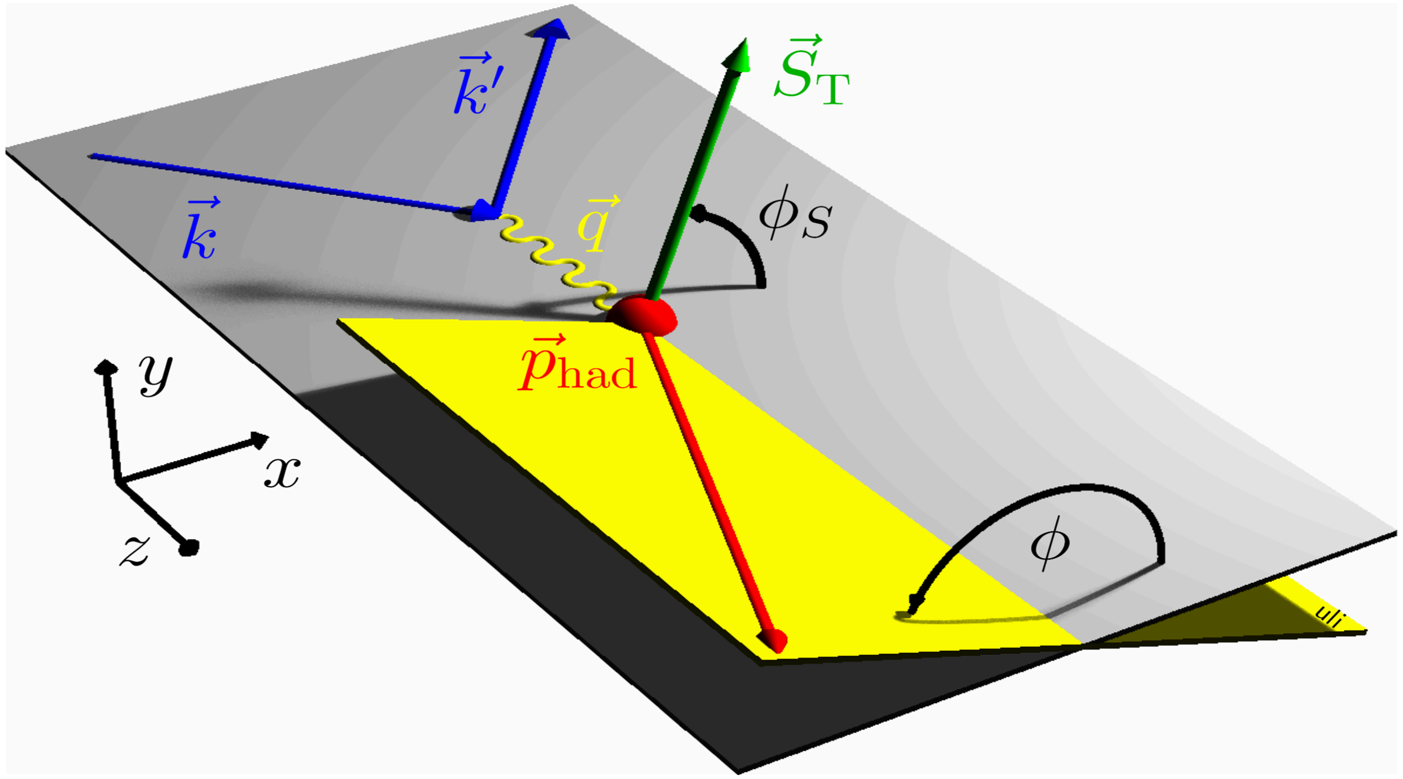
$$\Delta^N f_{q/h^\uparrow}^{\text{SIDIS}}(x,k_\perp) = - \Delta^N f_{q/h^\uparrow}^{\text{DY}}(x,k_\perp)$$

Most critical test for TMD approach to SSA

Current Sivers function from SIDIS

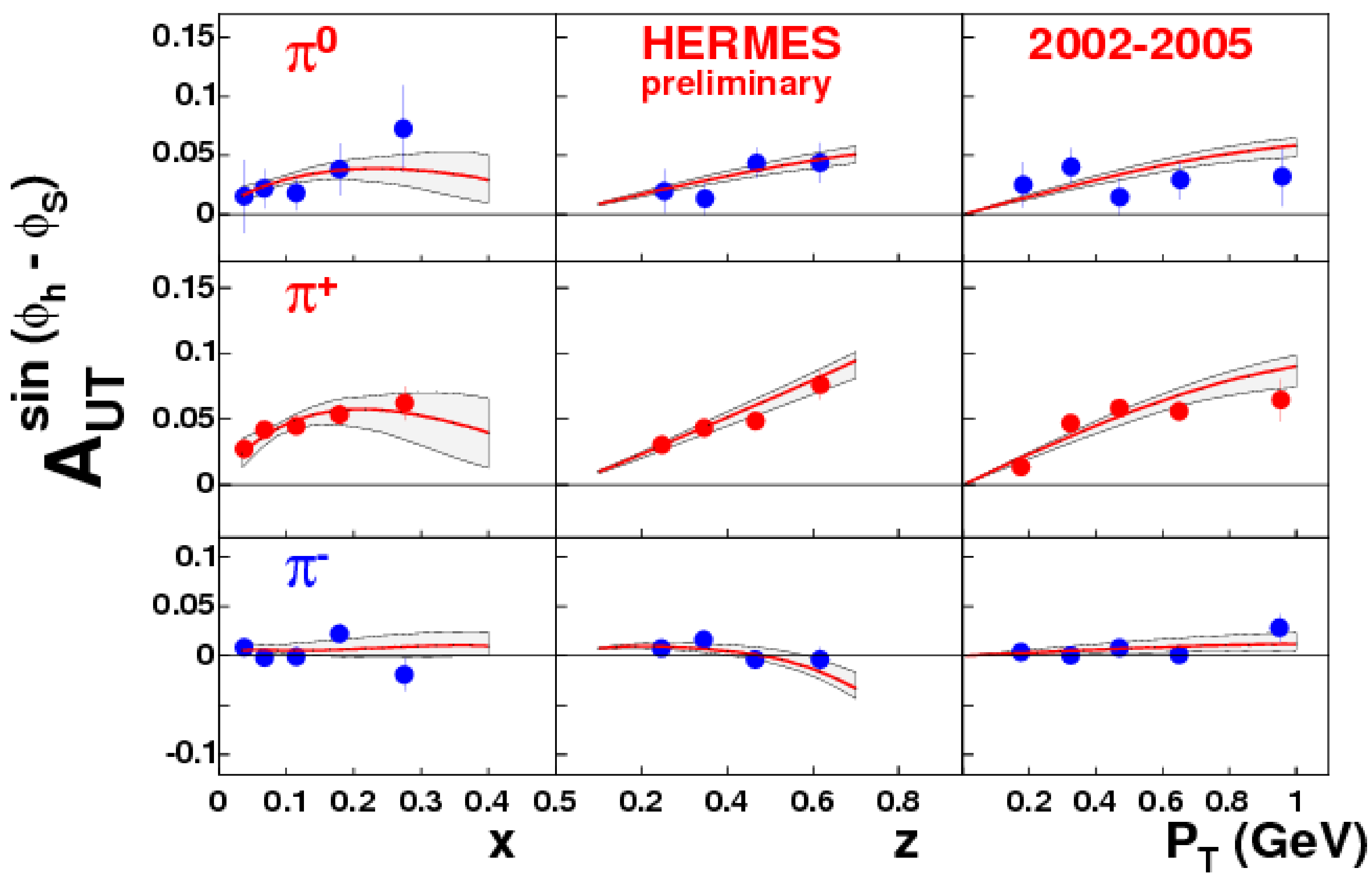
- Sivers and Collins can be separately extracted from SIDIS

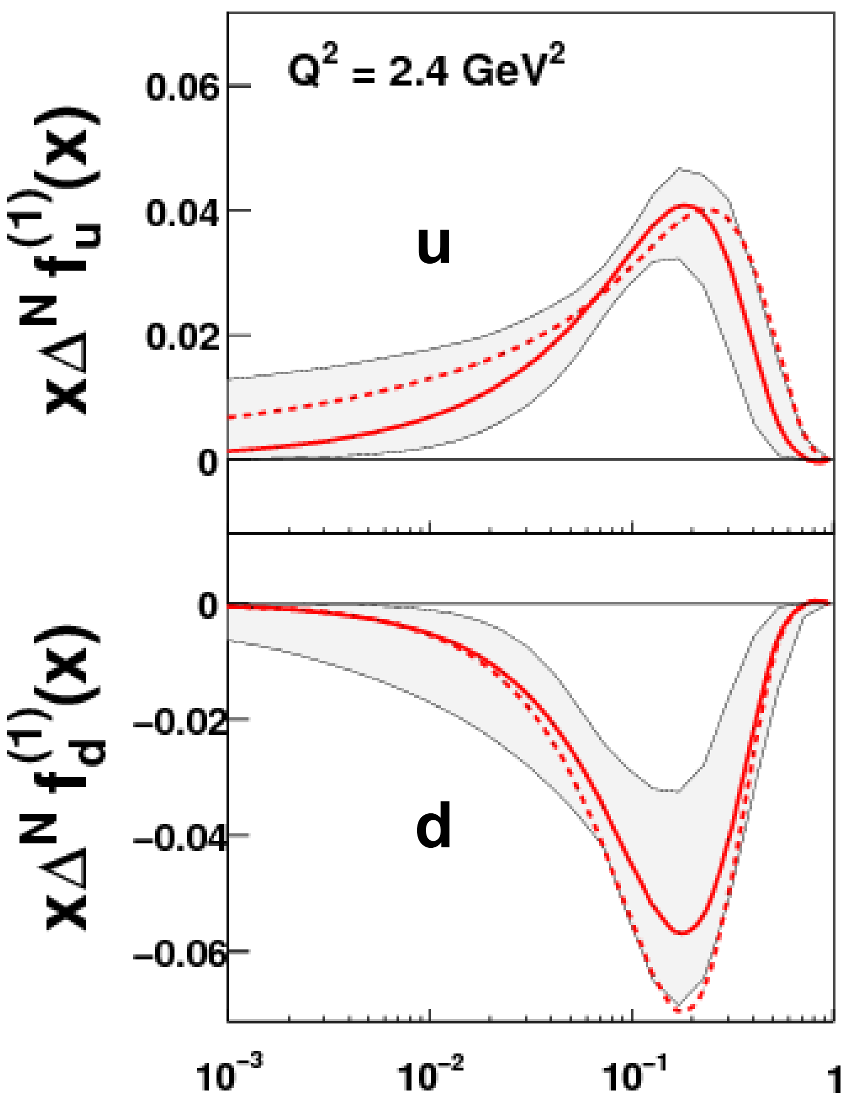
$$\Delta\sigma \propto A_{UT}^{Collins}\sin(\phi+\phi_S) + A_{UT}^{Sivers}\sin(\phi-\phi_S)$$



Sivers function from SIDIS $\ell + p^\uparrow \to \ell' + \pi(p_T) + X: \quad p_T \ll Q$

- Extract Sivers function from SIDIS (HERMES&COMPASS): a fit





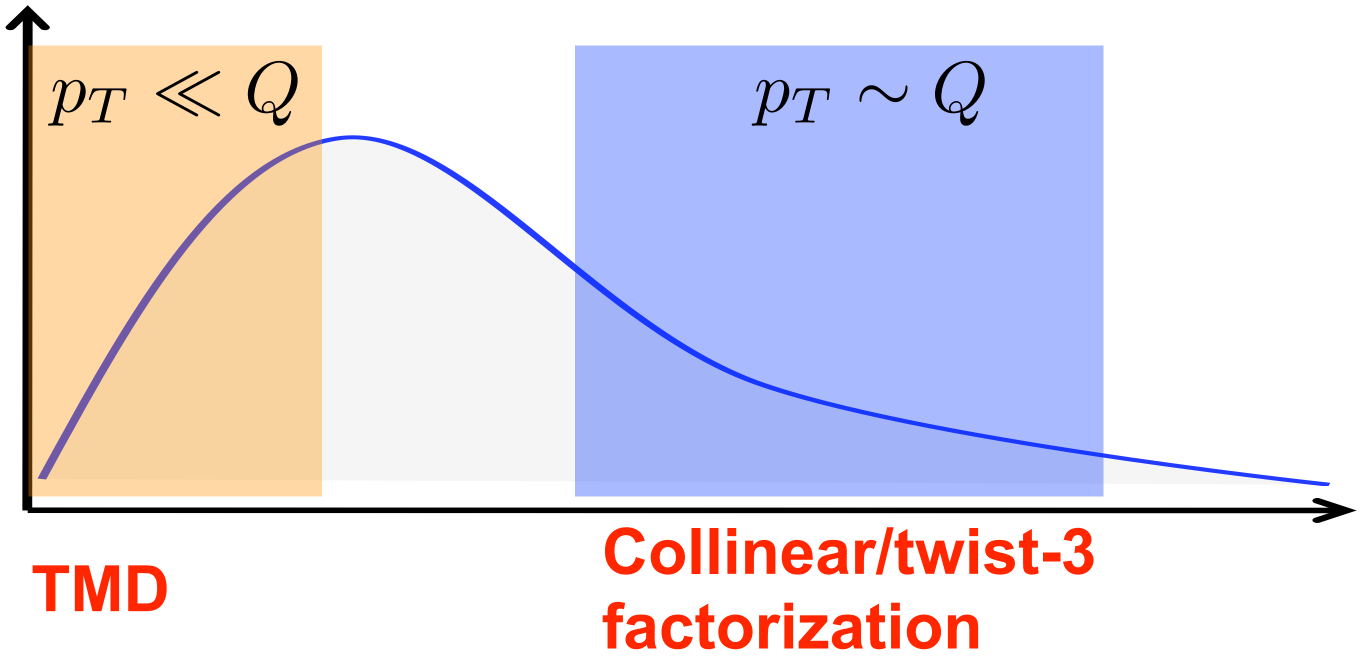
Anselmino, et.al., 2009

  - u and d almost equal size, different sign
  - d-Sivers is slightly larger

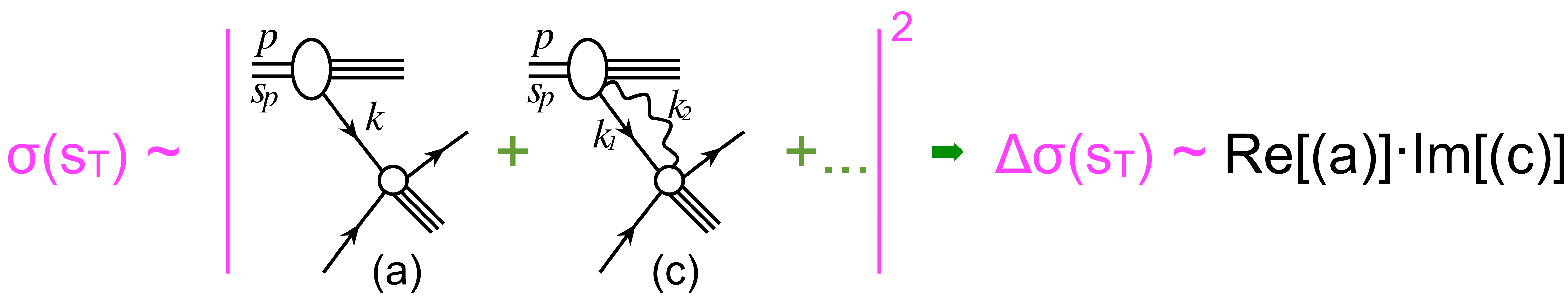
- Still needs DY results to verify the sign change, thus fully understand the mechanism of the SSAs

TMD factorization to collinear factorization

- Transition from low p_T to high p_T

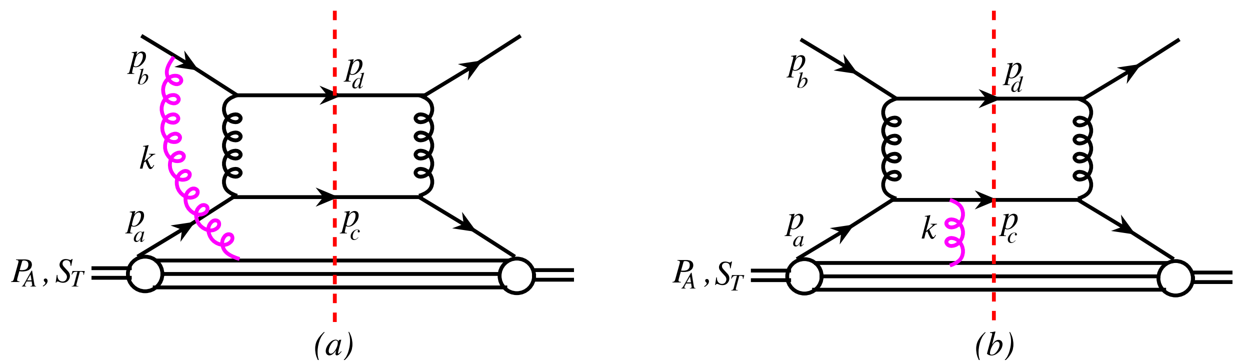


- Collinear twist-3 factorization approach: Efremov-Teryaev 82, 84, Qiu-Sterman 91, 98



Both initial- and final-state interactions

- For the process $pp^{\uparrow} \to \pi + X$, one of the partonic channel: $qq' \to qq'$



phase: from hard scattering amplitudes (unpinched pole)

$$E_h \frac{d\Delta\sigma}{d^3 P_h} \propto \epsilon^{P_{hT} S_A n \bar{n}} \sum_{a,b,c} D_{h/c}(z_c) \otimes f_{b/B}(x_b) \otimes T_{a,F}(x,x) \otimes H^{\mathrm{Siv}}_{ab\to c}$$

Efremov-Teryaev-Qiu-Sterman (ETQS) function

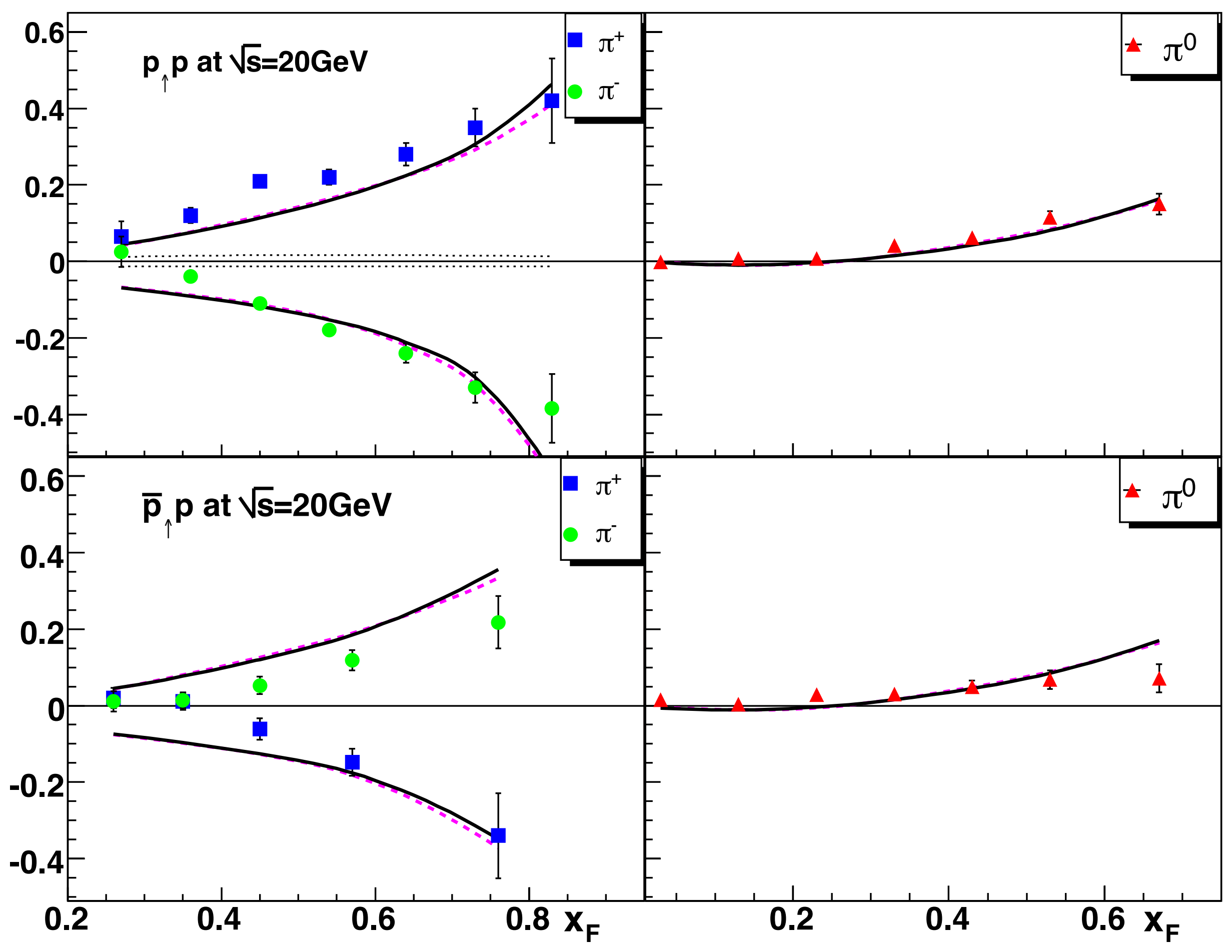
- The effects of initial- and final-state interaction are absorbed to $H^{\mathrm{Siv}}_{ab\to c}$
- ETQS function $T_{q,F}(x,x)$ is universal

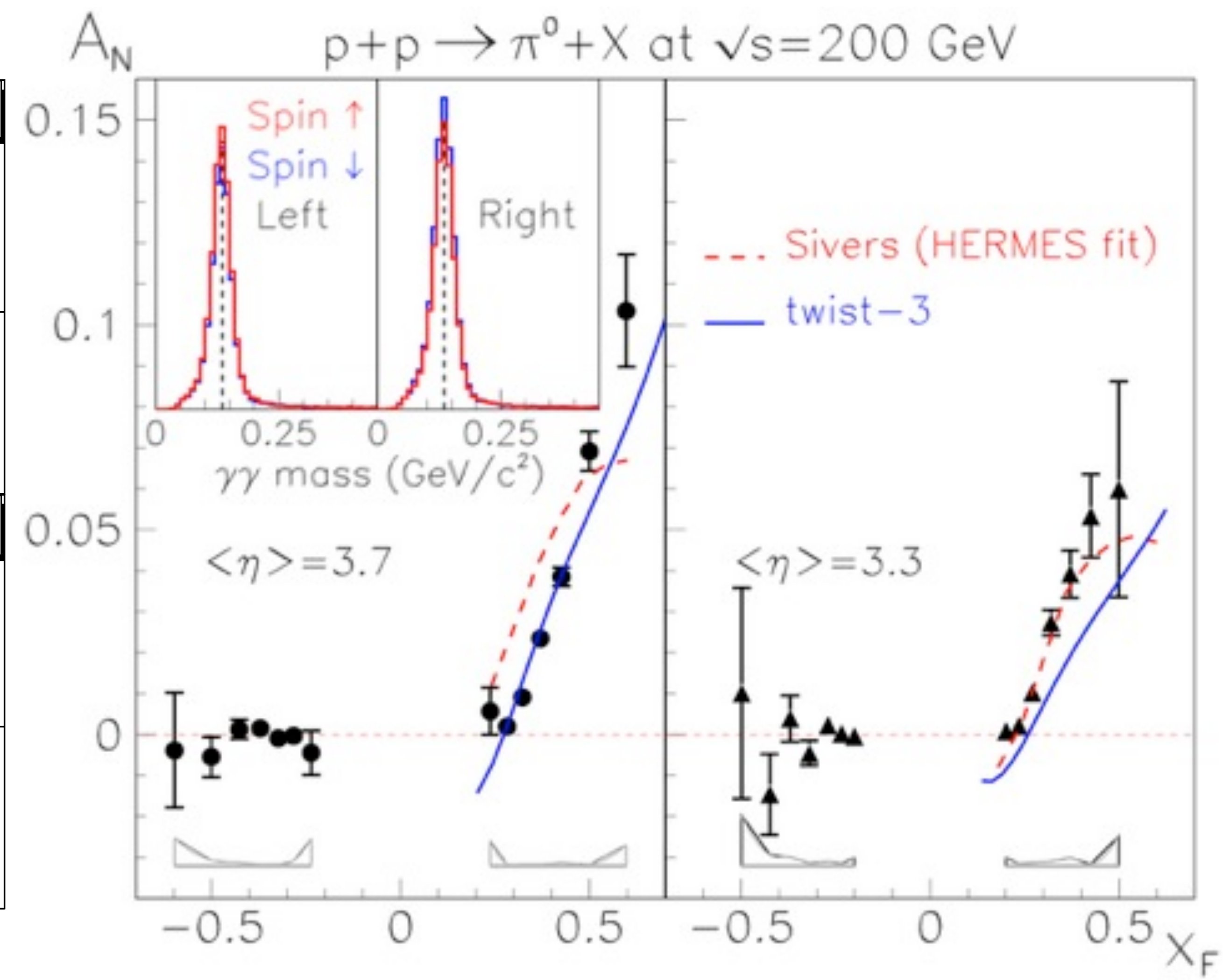
Initial success of twist-3 approach

- Describe both fixed-target and RHIC well: a fit

$$T_{q,F}(x,x) = N_q x^{\alpha_q}(1-x)^{\beta_q}\phi_q(x)$$

Kouvaris-Qiu-Vogelsang-Yuan, 2006





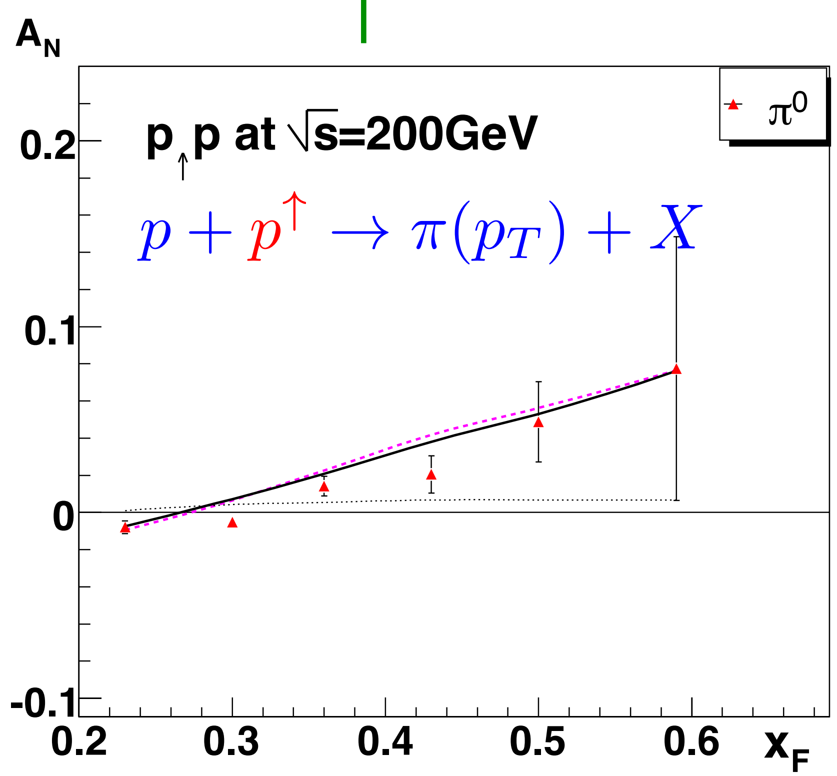
- See also the fit by Koike and Tanaka 2011

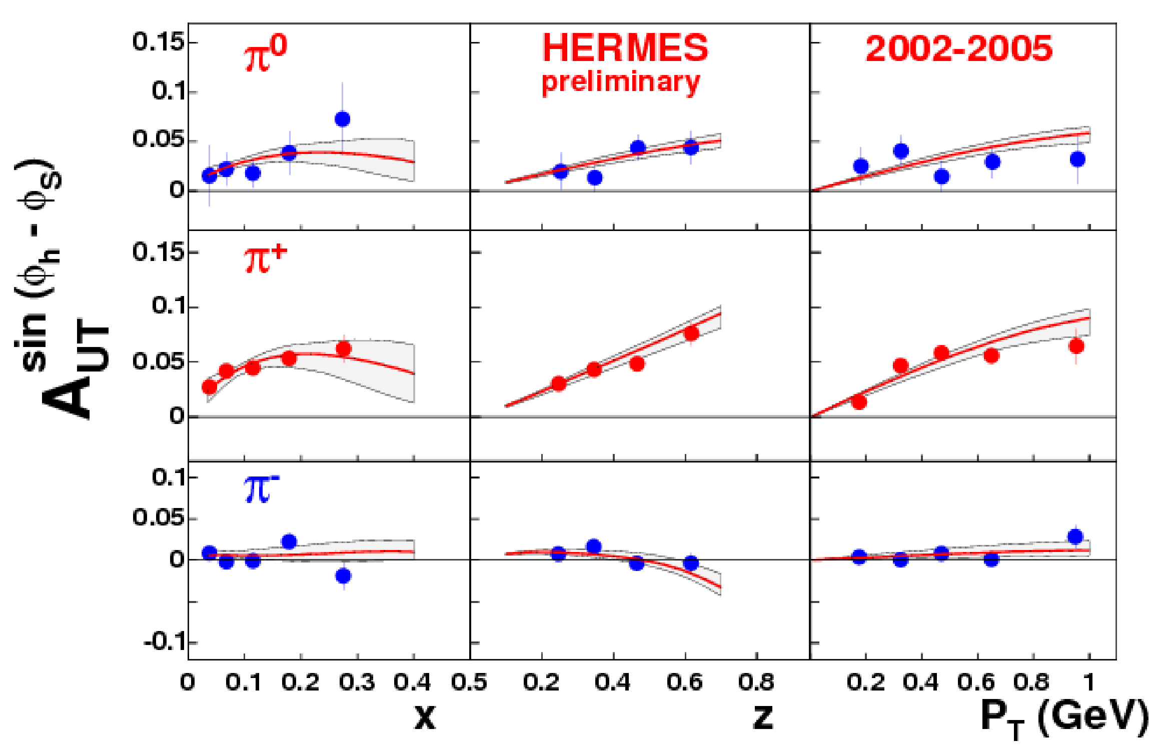
What about the connection?

- Both seem to describe the data well (in their own kinematic region), but what about their connections?
  - At the operator level, ETQS function is related to the first kt-moment of the Sivers function

Boer, Mulders, Pijlman, 2003
Ji, Qiu, Vogelsang, Yuan, 2006

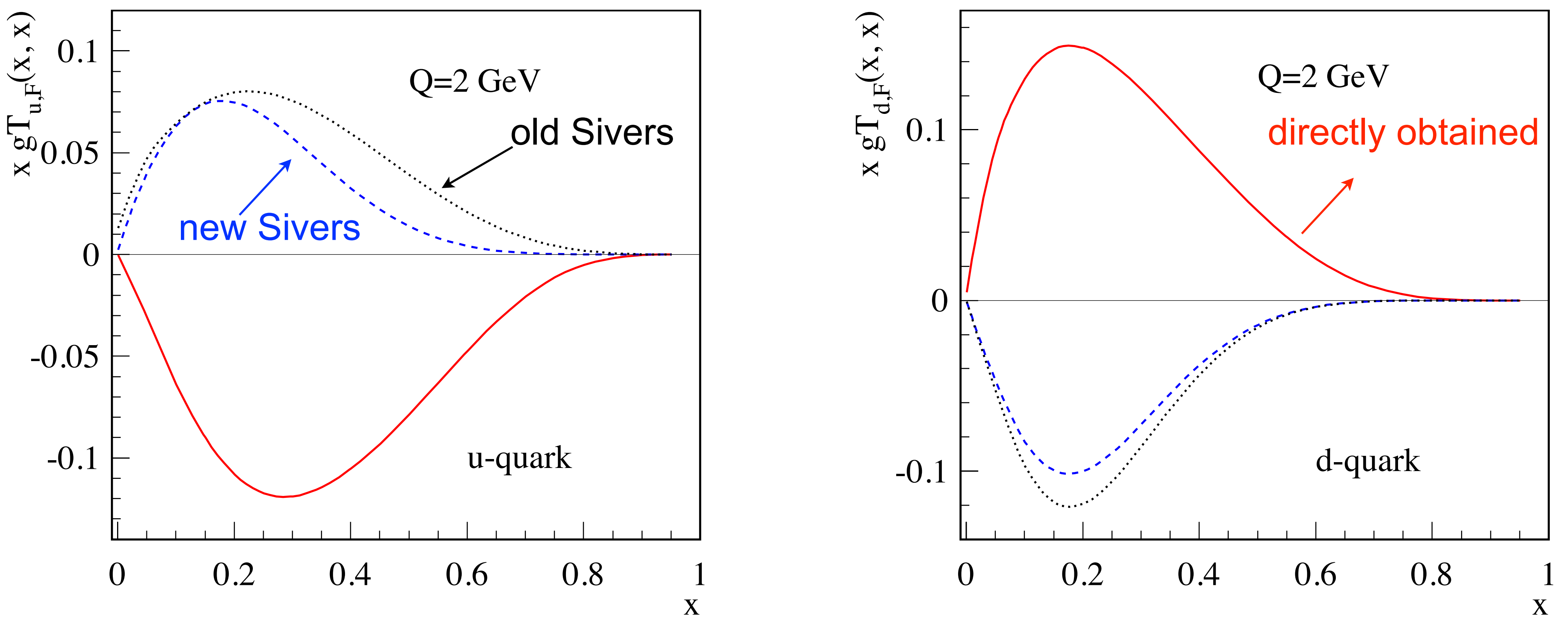
$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\text{SIDIS}}$$





Directly obtained ETQS function

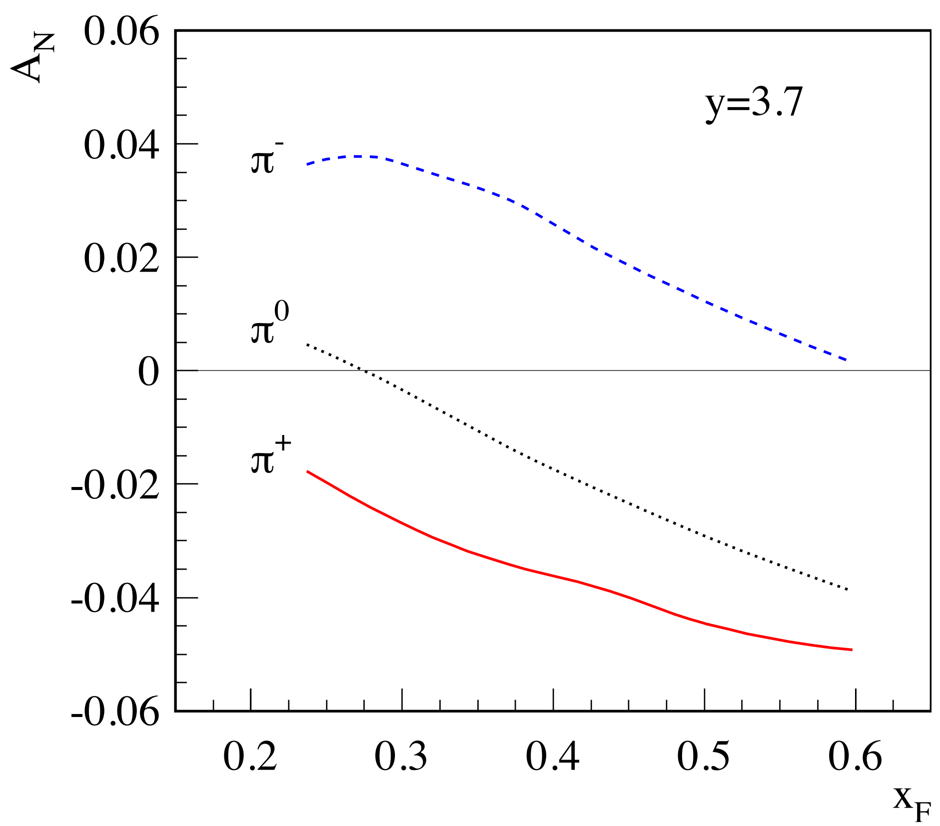
- ETQS function could be directly obtained from the global fitting of inclusive hadron production in hadronic collisions



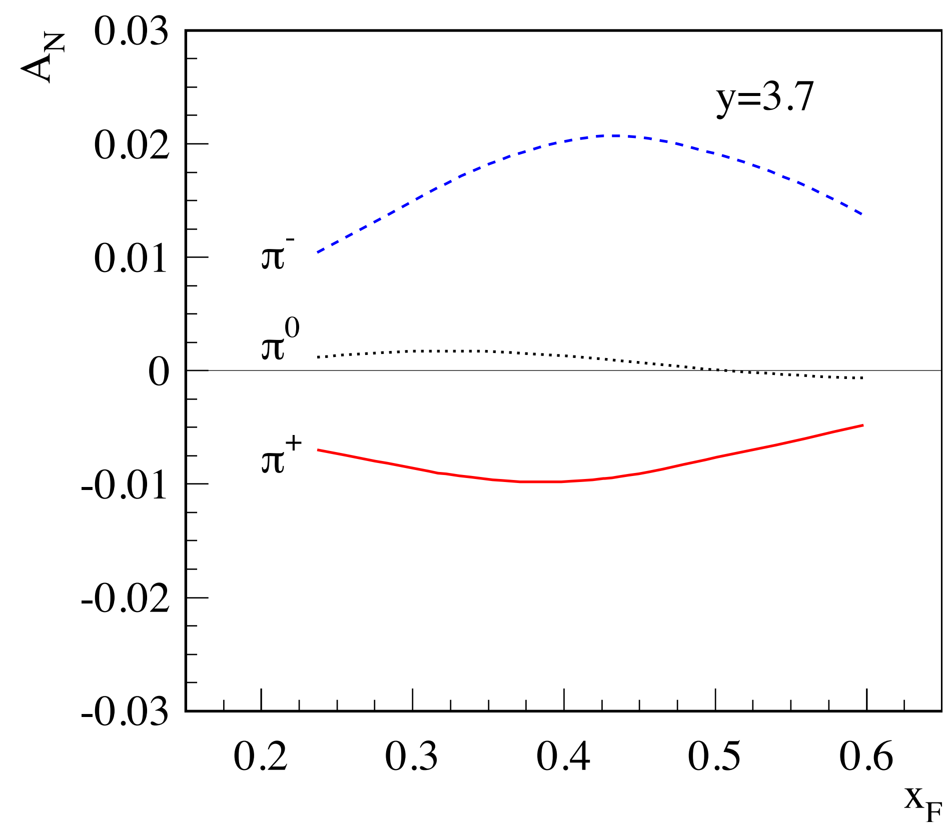
- directly obtained ETQS functions for both u and d quarks are opposite in sign to those indirectly obtained from the kt-moment of the quark Sivers function - "a sign mismatach"

Extrapolation from SIDIS to PP

- Use the ETQS function derived from the old Sivers and new Sivers functions, one could make predictions for the single inclusive hadron production. We find they are opposite to the experimental observations.



old Sivers

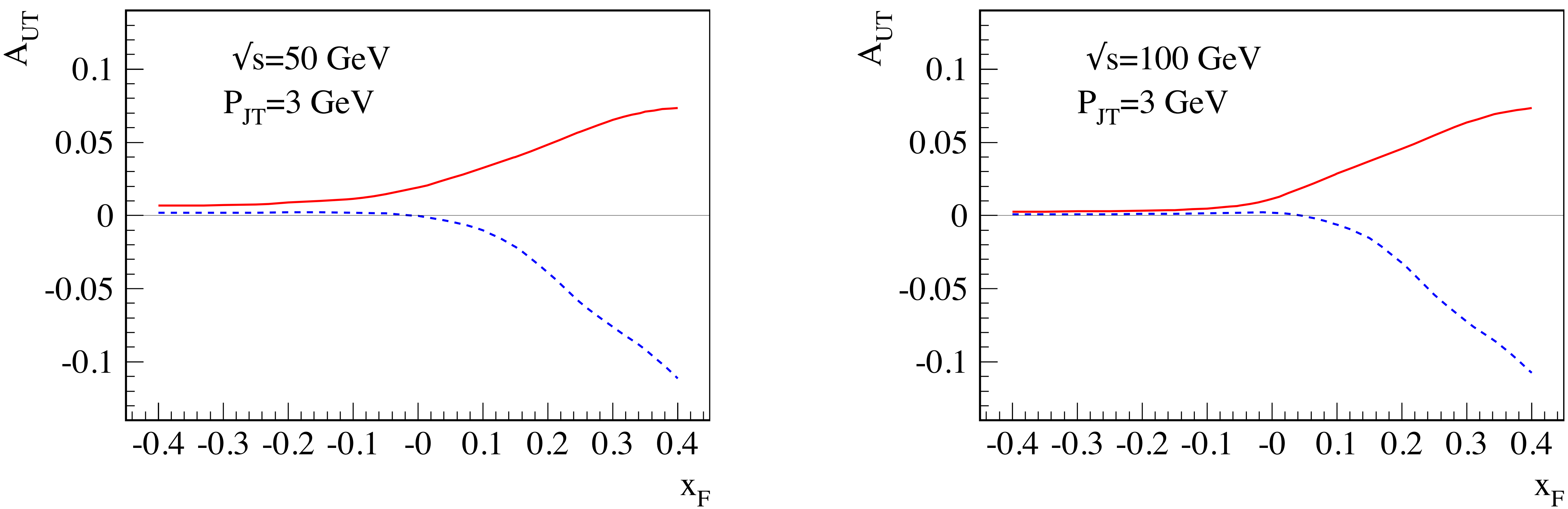


new Sivers

$$p^{\uparrow}p \to \pi + X$$

A plot for HERMES: e+p→h+X

- Solid line is $T_F(x,x)$ calculated from SIDIS itself, dashed line is $T_F(x,x)$ from the inclusive hadron data at RHIC: here is for jet



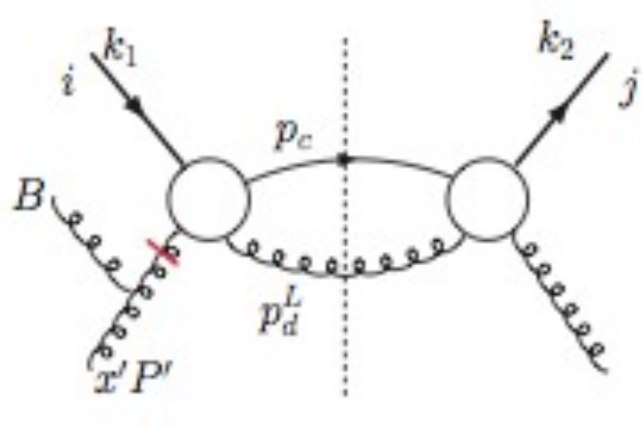
Kang-Metz-Qiu-Zhou, arXiv: 1106.3514, PRD84, 2011

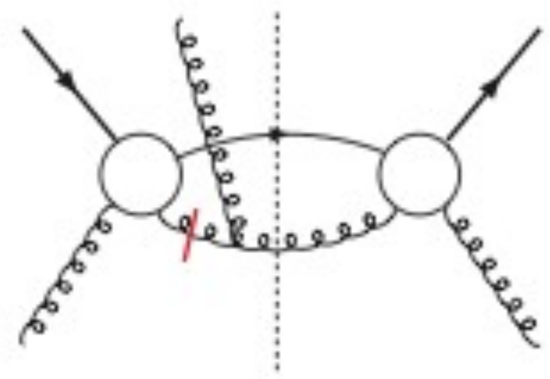
- The solid line is almost the same as Anselmino, et.al. in a TMD formalism

Anselmino, et.al., arXiv: 0911.1744, PRD81, 2010

Initial- and final-state interaction in pp collisions

- The dominant channel is qg → qg





$$H^U_{qg\to qg} = \frac{N_c^2-1}{2N_c^2}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]\left[1-\frac{2N_c^2}{N_c^2-1}\frac{\hat{s}\hat{u}}{\hat{t}^2}\right] \xrightarrow{|\hat{t}|\ll\hat{s}\sim|\hat{u}|} \left[\frac{2\hat{s}^2}{\hat{t}^2}\right]$$

$$H^I_{qg\to qg} = \frac{1}{2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]\left[1-N_c^2\frac{\hat{u}^2}{\hat{t}^2}\right] \xrightarrow{|\hat{t}|\ll\hat{s}\sim|\hat{u}|} \left[-\frac{N_c^2}{2(N_c^2-1)}\right]\left[\frac{2\hat{s}^2}{\hat{t}^2}\right]$$

$$H^F_{qg\to qg} = \frac{1}{2N_c^2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]\left[1+2N_c^2\frac{\hat{s}\hat{u}}{\hat{t}^2}\right] \xrightarrow{|\hat{t}|\ll\hat{s}\sim|\hat{u}|} \left[-\frac{1}{N_c^2-1}\right]\left[\frac{2\hat{s}^2}{\hat{t}^2}\right]$$

- Sivers effect in single hadron production is more similar to DY

Question

Does this apparent sign "mismatch" indicate an inconsistency in our current QCD formalism for describing the SSAs?

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An attempt: try to reconcile the SIDIS and PP data

find a solution to sign mismatch

Scenario I

- Let us assume the directly obtained ETQS function from inclusive hadron production reflects the true sign of these functions.

- In such case, to make everything consistent, we need to explain how the sign of the kt-moment of the Sivers function is different from the sign of the Sivers function.

$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\text{SIDIS}}$$

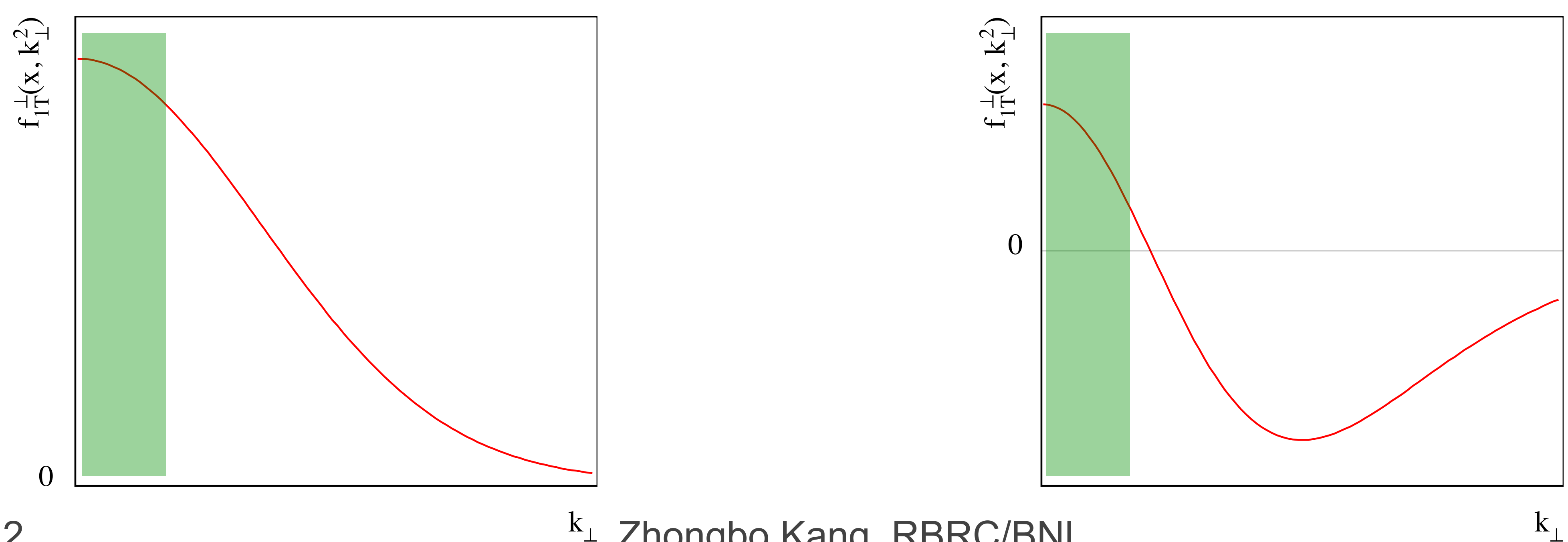
What could go wrong - Scenario I

- To obtain ETQS function, one needs the full kt-dependence of the quark Sivers function

$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\text{SIDIS}}$$

- However, the Sivers functions are extracted mainly from HERMES data at rather low $Q^2 \sim 2.4\ \text{GeV}^2$, and TMD formalism is only valid for the kinematic region kt << Q.
  - HERMES data only constrain the behavior (or the sign) of the Sivers function at very low kt ~ Λ_{QCD}.

$$\Delta^N f_{q/h^\uparrow}(x,k_\perp)\, \vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp = f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) - f_{q/h^\uparrow}(x,\mathbf{k}_\perp,-\vec{S})$$



Sivers function: the requirements for node in kt (1)

- change kt-dependence: difference between two gaussian

$$f_{1T}^{\perp q}(x,k_\perp^2) = -\mathcal{N}_q(x)h(k_\perp)f_{q/A}(x,k_\perp^2)$$

NOW

$$h(k_\perp) = \sqrt{2e}\,M\left[\frac{e^{-\mathbf{k}_\perp^2/M_1^2}}{M_1} - \frac{e^{-\mathbf{k}_\perp^2/M_2^2}}{M_2}\right]$$

BEFORE

$$h(k_\perp) = \sqrt{2e}M\frac{e^{-k_\perp^2/M_1^2}}{M_1}$$

  - In order to have the same sign at low kt like before, one requires

$$M_2 > M_1$$

  - Now we hope the high-kt part weighs over the low-kt part, thus it gives the correct sign of $T_F(x,x)$

$$T_{q,F}(x,x) = \sqrt{2e}\langle k_\perp^2\rangle\left[\frac{M_1^3}{(\langle k_\perp^2\rangle + M_1^2)^2} - \frac{M_2^3}{(\langle k_\perp^2\rangle + M_2^2)^2}\right]\mathcal{N}_q(x)f_{q/A}(x)$$

$$\Rightarrow \frac{M_2^3}{(\langle k_\perp^2\rangle + M_2^2)^2} > \frac{M_1^3}{(\langle k_\perp^2\rangle + M_1^2)^2}$$

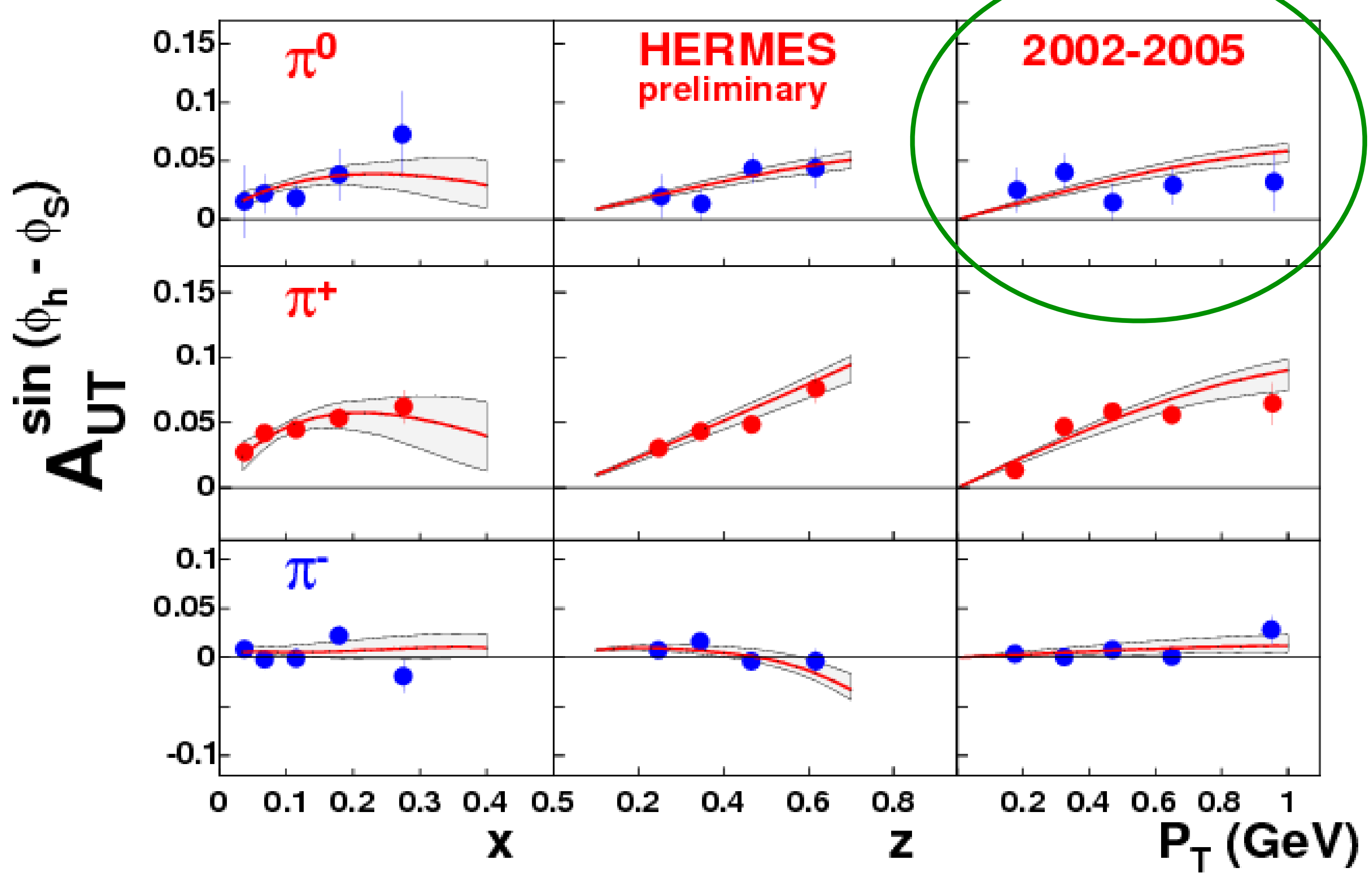
Sivers function: the requirements for node in kt (2)

- At the same time, take into account that the asymmetry follows the same sign up to pt ~ 1 GeV

$$\gamma(z_h)^{-1} = 1 + z_h^2 \langle k_\perp^2 \rangle / \langle p_T^2 \rangle$$

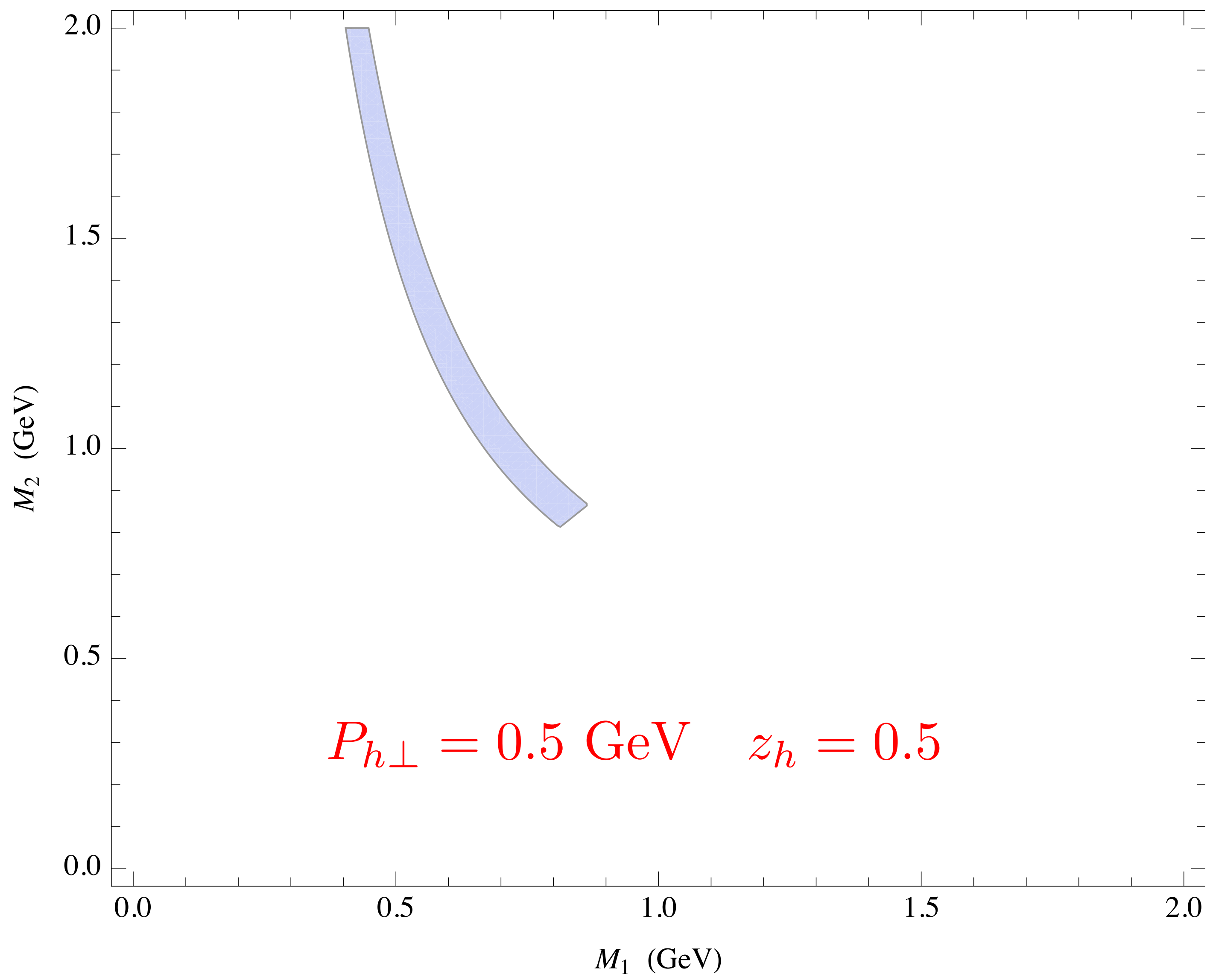
$$F_{UT,T}^{\sin(\phi_h-\phi_s)} \propto P_{h\perp} \left[\frac{M_1^3}{(\langle k_\perp^2 \rangle + M_1^2)^2} \frac{1}{\langle P_{h\perp 1}^2 \rangle^2} e^{-\frac{P_{h\perp}^2}{\langle P_{h\perp 1}^2 \rangle}} - \frac{M_2^3}{(\langle k_\perp^2 \rangle + M_2^2)^2} \frac{1}{\langle P_{h\perp 2}^2 \rangle^2} e^{-\frac{P_{h\perp}^2}{\langle P_{h\perp 2}^2 \rangle}} \right]$$

$$\Rightarrow \frac{M_2^3}{(M_2^2 + \gamma(z)\langle k_\perp^2 \rangle)^2} < \frac{M_1^3}{(M_1^2 + \gamma(z)\langle k_\perp^2 \rangle)^2}$$



The allowed parameter space for M_1 and M_2 is very small

- Parameter space becomes even smaller if one increase pt and/or decrease z_h, thus node in kt may be not a natural solution for the sign mismatch

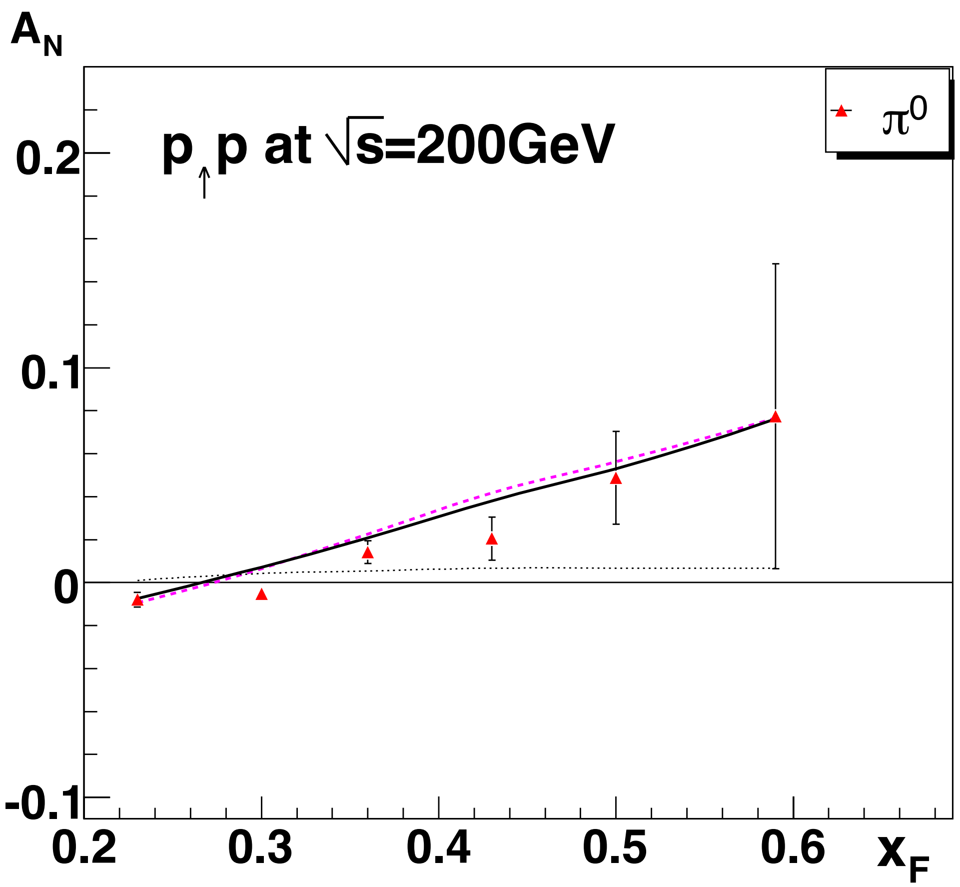


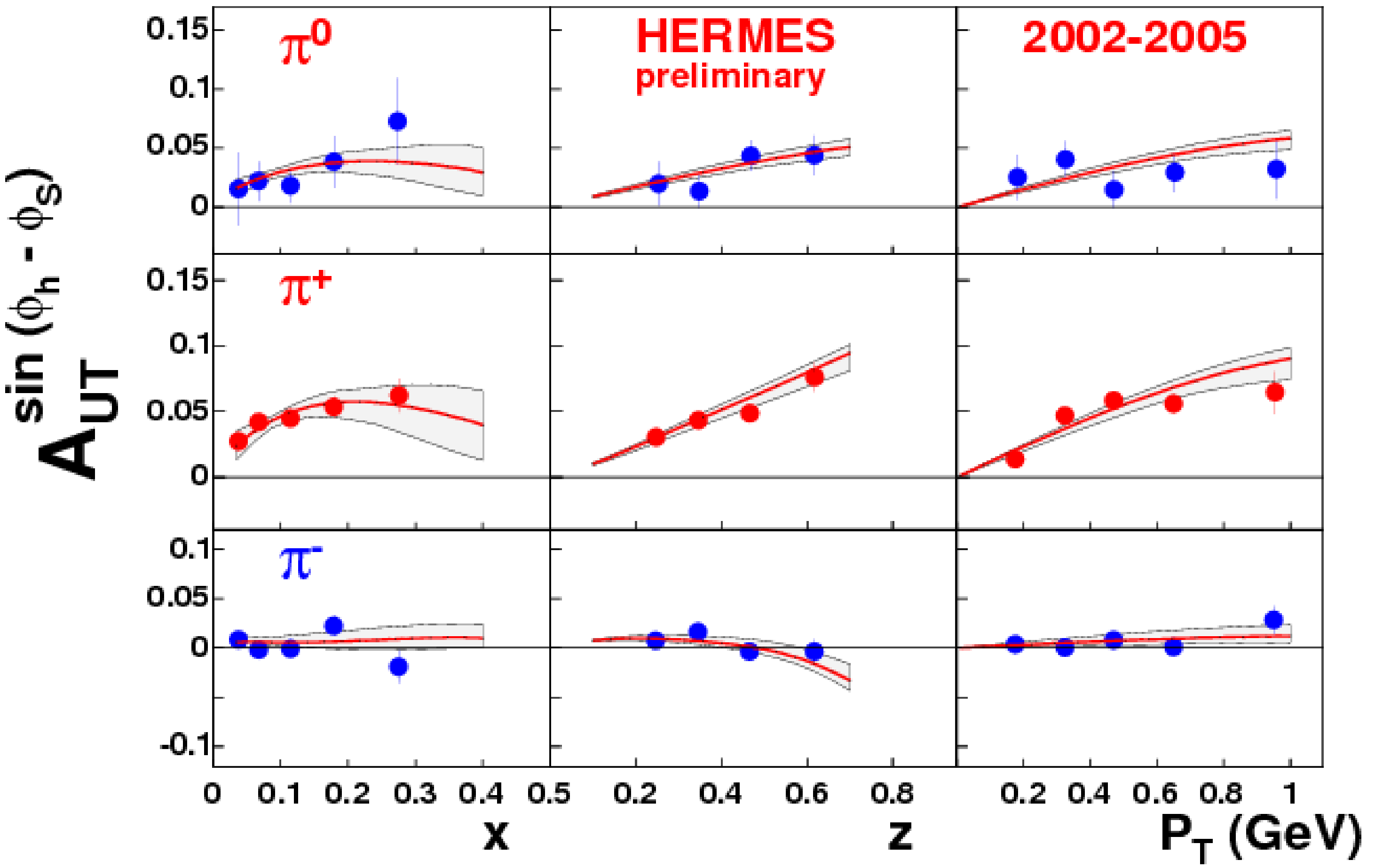
- Other kt-dependence leads to similar conclusion (suggested by Boer)

$$h(k_\perp) = \sqrt{2e}\frac{M}{M_1}e^{-\mathbf{k}_\perp^2/M_1^2}(1-\eta\,\mathbf{k}_\perp^2)$$

Sivers function: node in x - motivation (1)

- The most accurate (also nonvanishing) SSA data comes from STAR, which typically covers a large x_F region and thus probes relatively large x region of the $T_F(x, x)$
- At the same time, the SIDIS data covers relatively small x region





Sivers function: node in x (2)

- Maybe the Sivers function at small x follow the sign from SIDIS, while at the same time have the opposite sign from PP - a node in x
- The parameterization:

$$f_{1T}^{\perp q}(x, k_\perp^2) = -\mathcal{N}_q(x) h(k_\perp)\, f_{q/A}(x, k_\perp^2)$$

$$\mathcal{N}_q(x) = N_q x^{\alpha_q}(1-x)^{\beta_q} \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}(1-\eta_q x)$$

  - no node in kt space
  - choose Nq to satisfy the positivity bound $N_q^{-1} > \max\{1, |1-\eta_q|\}$
  - if η_q>1, we have a node in x-space
- The fitting procedure: HERMES proton, COMPASS deuteron, STAR pi0, BRAHMS pi+,pi- data
  - use TMD formalism to describe the SIDIS data
  - use collinear twist-3 to describe the PP data: the needed $T_F(x,x)$ function can be obtained from the parameterized Sivers function through the relation

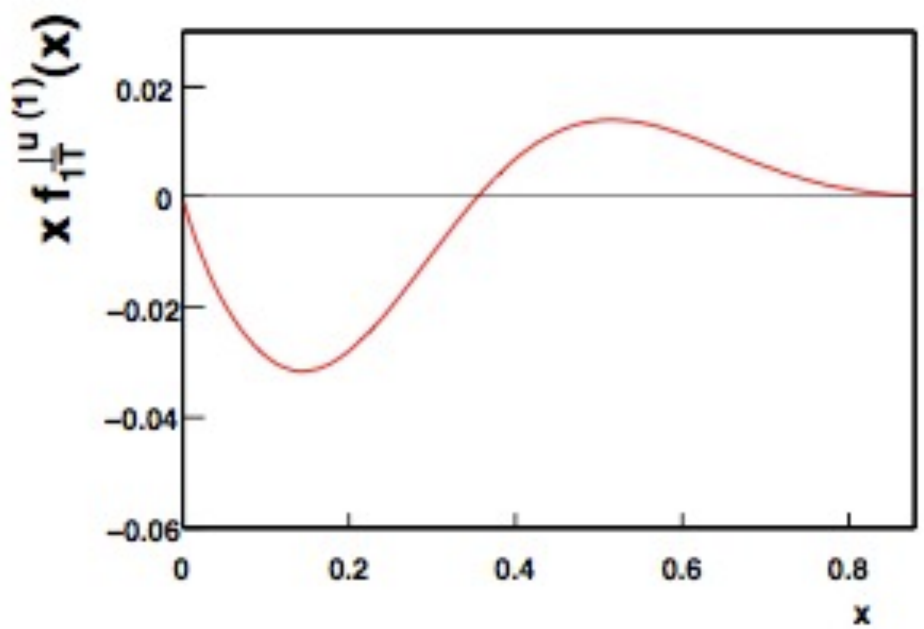
$$T_{q,F}(x,x) = \frac{\sqrt{2e}\langle k_\perp^2\rangle M_1^3}{(\langle k_\perp^2\rangle + M_1^2)^2}\mathcal{N}_q(x) f_{q/A}(x)$$

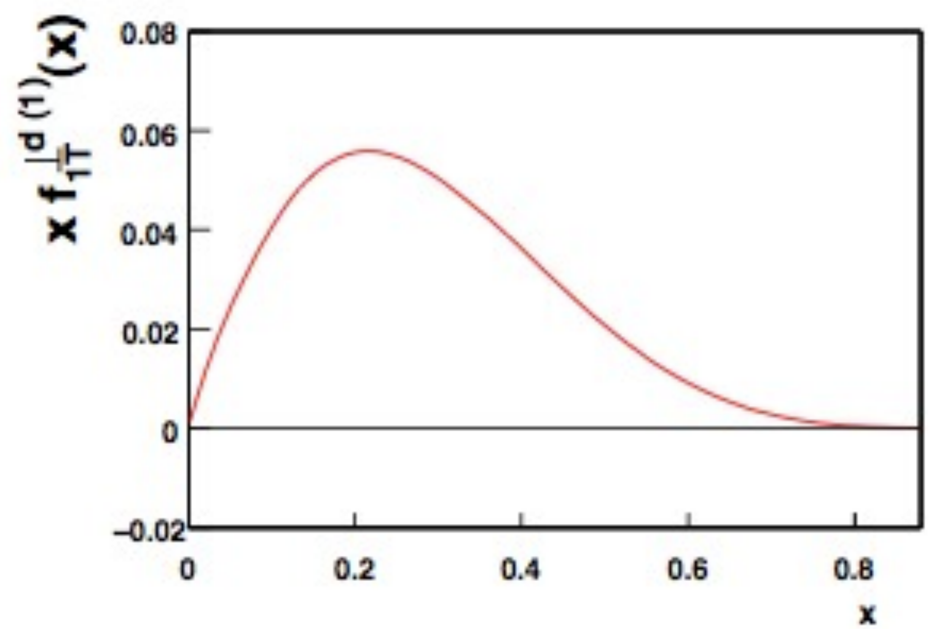
The shape of the Sivers function from the fitting

- 9 parameters

$\chi^2/\text{d.o.f.}$	$= 3.6$		
N_u	$= 1$	N_d	$= -1$
α_u	$= 0.8$	α_d	$= 0.8$
β_u	$= 1.5$	β_d	$= 1$
η_u	$= 2.8$	η_d	$= 0$
M_1^2	$= 0.7\ \text{GeV}^2$		

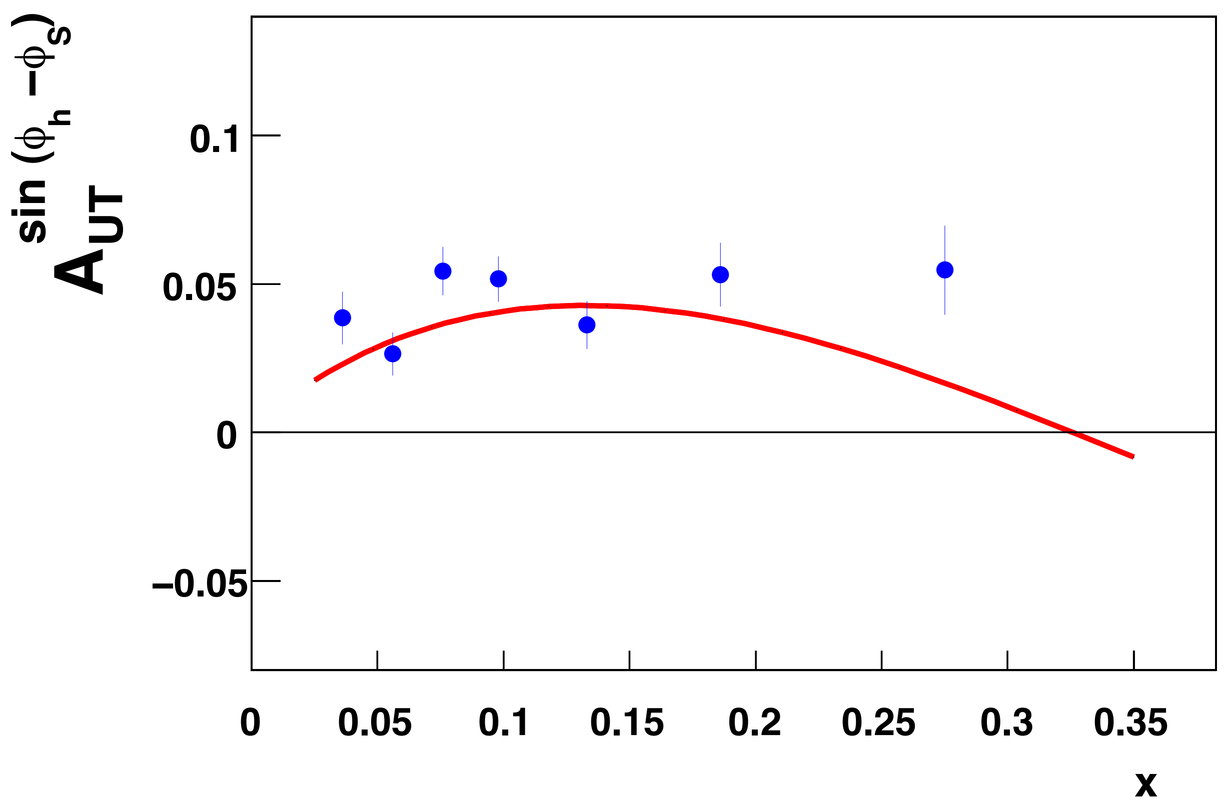
- Sivers function: u-quark has a node at x=0.36, d-quark does not

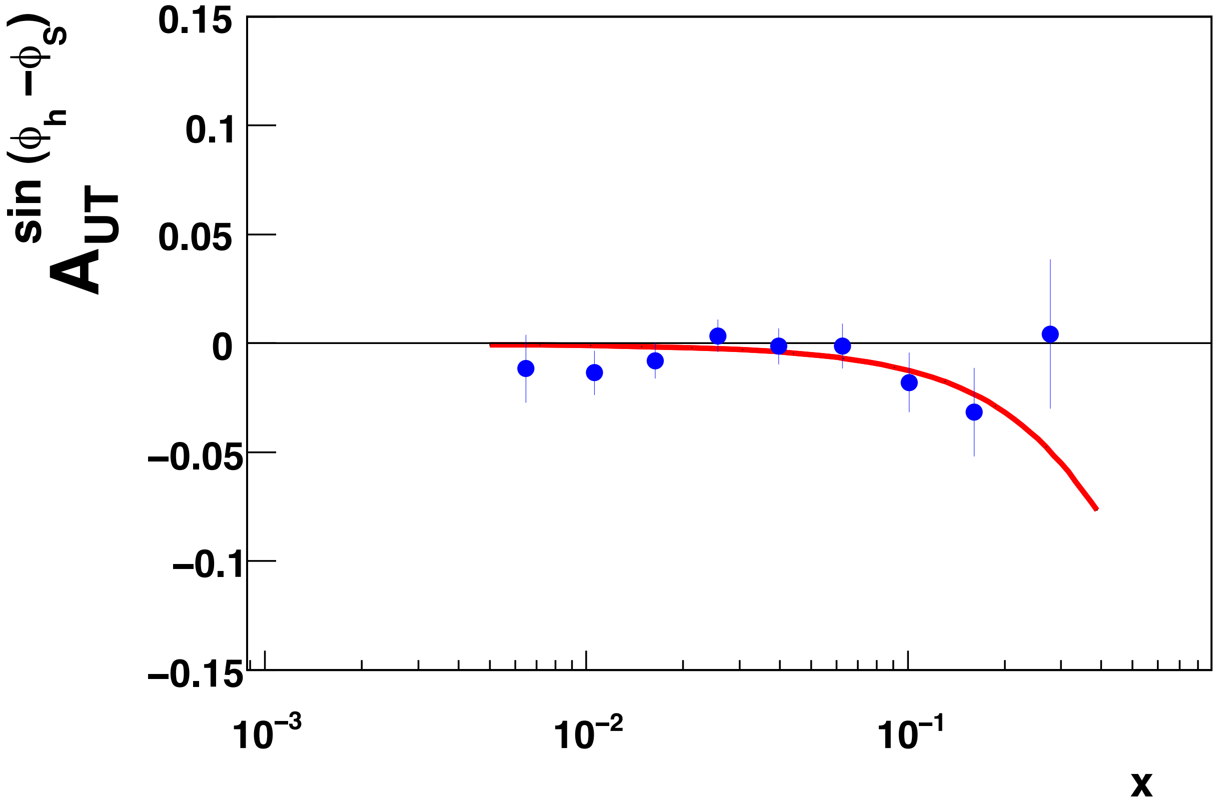




Description of the SIDIS data: satisfactory

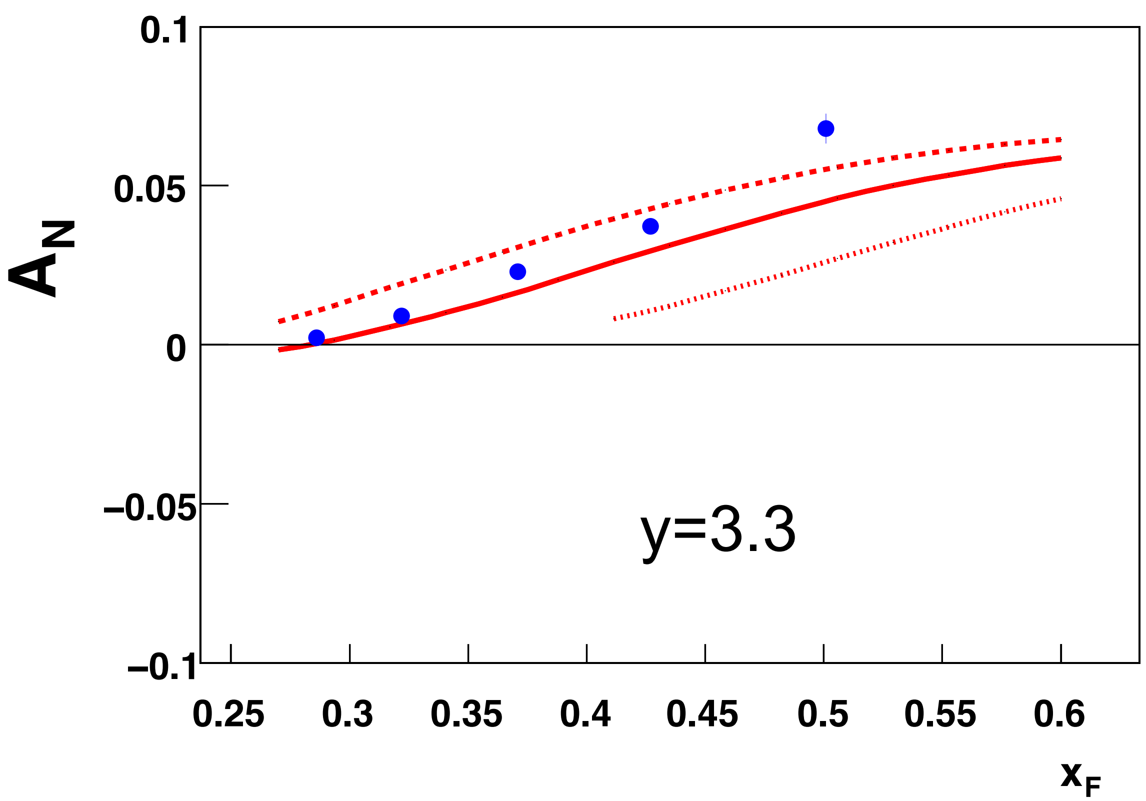
- pi+ asymmetry as a function of xB at HERMES and COMPASS: other dependence are similar, i.e., satisfactory: χ^2/d.o.f ~ 1.5

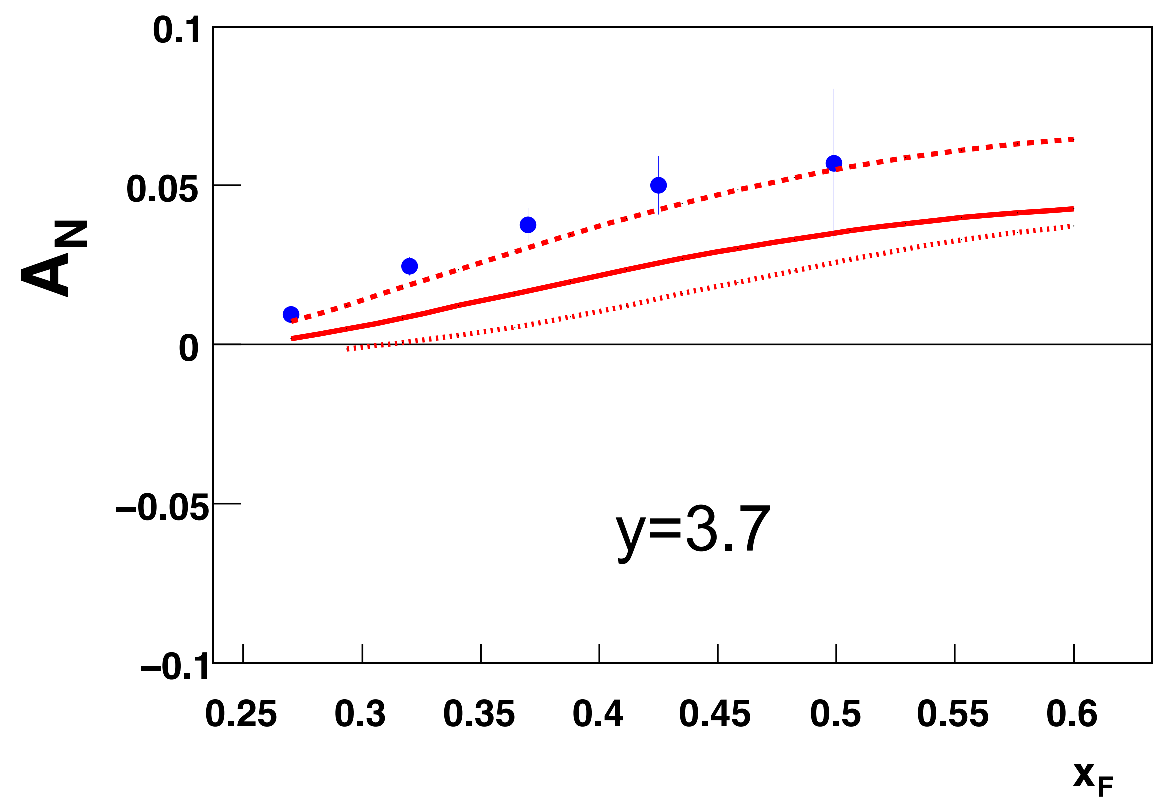




Description of STAR pi0 data

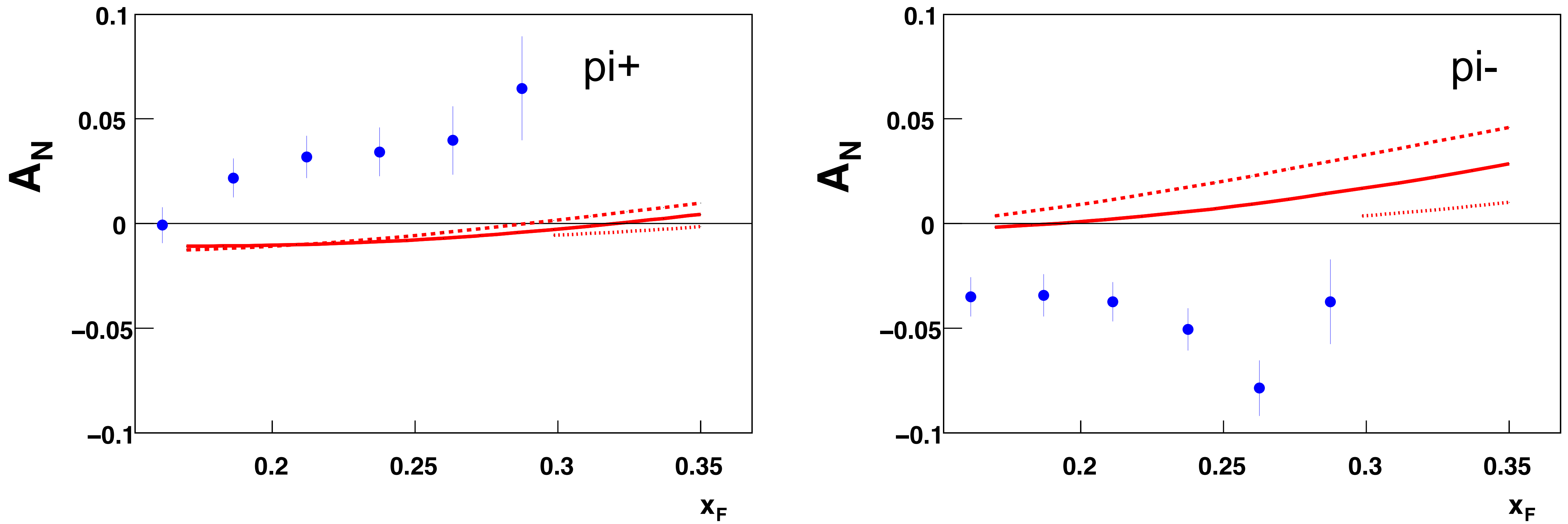
- The description is okay, but worse than SIDIS





Description of BRAHMS pi+, pi- data

- Cannot describe the BRAHMS data, not even the sign



- BRAHMS have the relatively small xF region, which is overlapping with the SIDIS: the opposite sign here is exactly on the heart of the "sign mismatch" paper
  - The sign of BRAHMS is also consistent with the old fixed-target experiments, say, E704

A_N seems not coming from Sivers effect

- Our exercises seem to indicate that the SSA of single inclusive hadron production cannot entirely come from the Sivers effect (partonic orbital motion in the nucleon), if we believe our formalism is consistent
- Also caution
  - relation at the operator level

$$f(x) = \int d^2k_\perp f(x, k_\perp^2)$$

$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x, k_\perp^2)|_{\text{SIDIS}}$$

  - subject to the renormalization

$$gT_{q,F}(x,x,\mu) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x, k_\perp^2)|_{\text{SIDIS}} + \text{UV counter term}$$

Scenario II

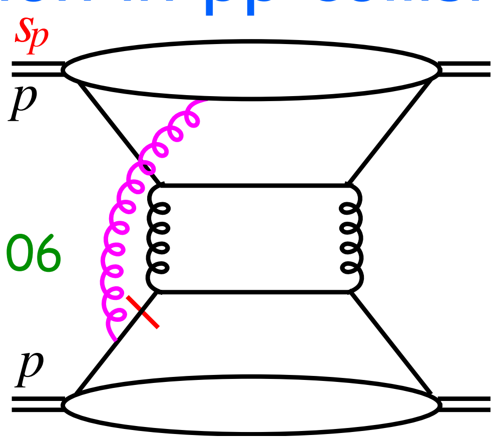
- Let us assume indirectly obtained (from the kt-moment of the Sivers function) ETQS function reflects the true sign of these functions
- In such case, to make everything consistent, we need to explain why we obtain a sign-mismatched ETQS function by analyzing the inclusive hadron data

$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x, k_\perp^2)|_{\text{SIDIS}}$$

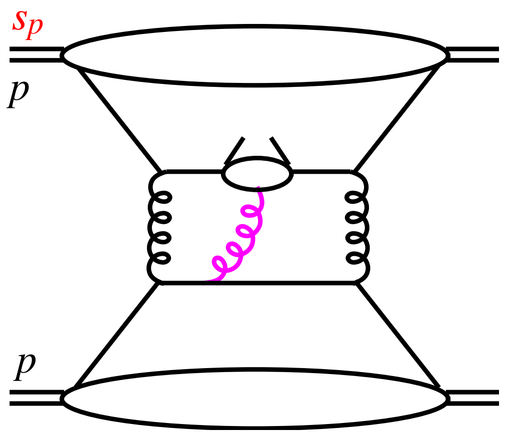
Single inclusive hadron production is complicated

- There are two major contributions to the SSAs of the single inclusive hadron production in pp collisions

Efremov-Teryaev 82, 84,
Qiu-Sterman 91, 98,
Kouvaris-Qiu-Vogelsang-Yuan, 06
Kanazawa-Koike, 11



in PDFs



in FFs

Kang-Yuan-Zhou 2010

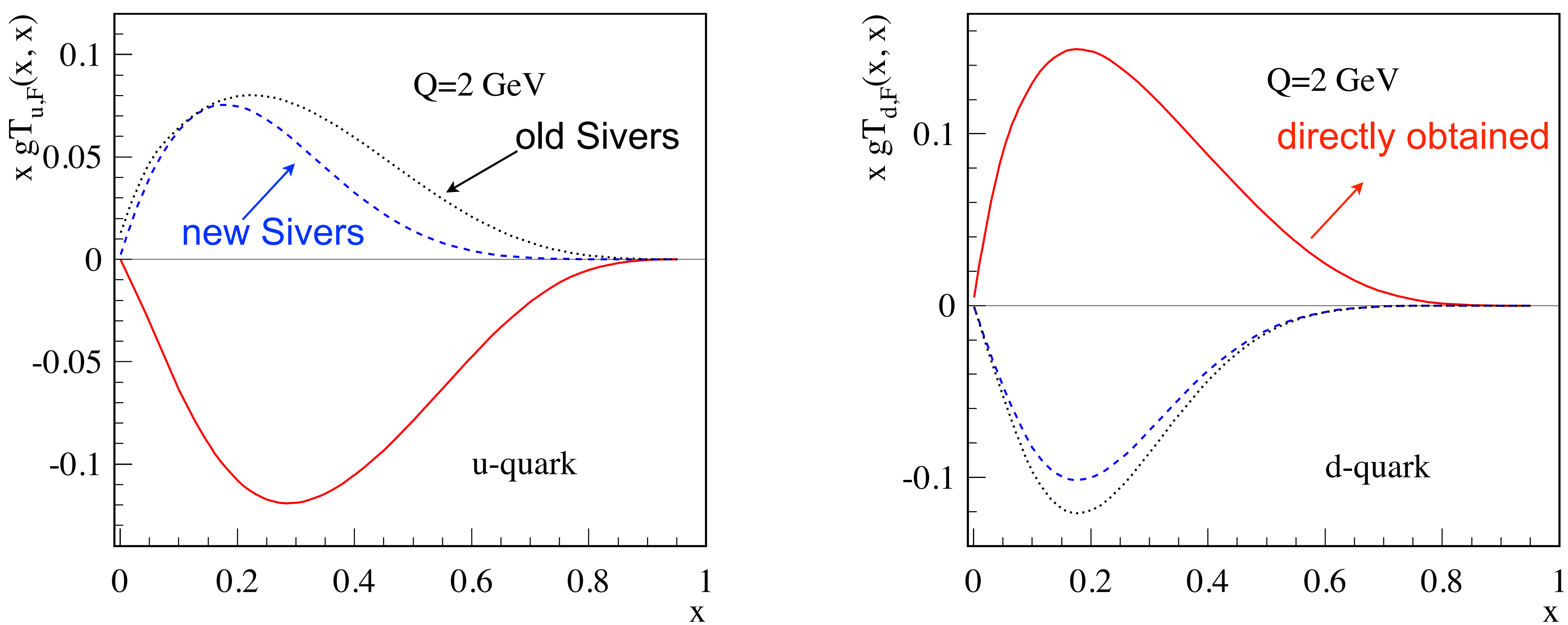
- So far the calculations related to three-parton correlation functions are more complete, while those related to the twist-3 fragmentation functions are available only very recently (not complete)
  - The current available global fittings are based on the assumptions that the SSAs mainly come from the twist-3 correlation functions, which might not be the case
  - If the contribution from the twist-3 fragmentation functions dominates, one might even reverse the sign of the ETQS function?

$$A_N = A_N|^{\rm PDFs} + A_N|^{\rm FFs}$$

If $A_N|^{\rm FFs} > A_N$, sign of $A_N|^{\rm PDFs}$ is opposite to A_N

Distinguish scenario I and II

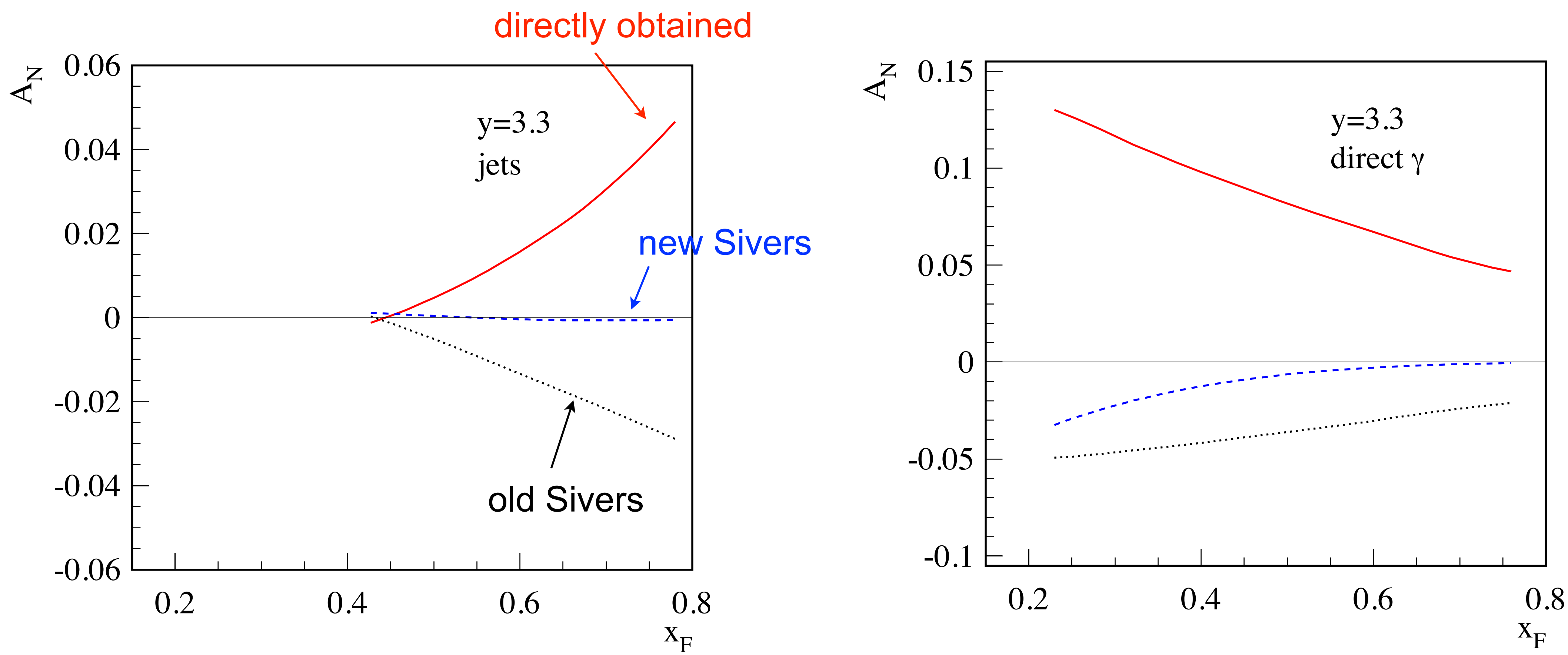
- Scenario I and II are completely different from each other



- To distinguish one from the other, in hadronic machine (like RHIC), one needs to find observables which are sensitive to twist-3 correlation function (not fragmentation function), such as single inclusive jet production, direct photon production

Predictions for jet and direct photon

- at RHIC 200 GeV:



Summary

- The existence of Sivers function relies on the initial and final-state interactions
- Sivers effect is process dependent
  - Test process-dependence is very important to understand the SSAs: sign change between SIDIS and DY
  - Both TMD and collinear twist-3 approaches seem to be successful phenomenologically
- Their connection seems to have a puzzle - sign mismatch
  - The "sign mismatch" is still an open question, we haven't found a solution yet
  - Future experiments could help resolve different scenarios, which will help understand the SSAs and hadron structure better

Summary

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Thank you

Backup

kt-dependence is a Gaussian in current parameterization

- To extract the Sivers function, the following parametrization is used
  - unpolarized PDFs: $f_1^q(x, k_\perp^2) = f_1^q(x) g(k_\perp)$
  - Sivers function: $\Delta^N f_{q/h^\uparrow}(x, k_\perp) = 2\mathcal{N}_q(x) f_1^q(x) h(k_\perp) g(k_\perp)$

$\mathcal{N}_q(x)$ is a fitted function

$$g(k_\perp) = \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

old Sivers: $$h(k_\perp) = \frac{2k_\perp M_0}{k_\perp^2 + M_0^2}$$ Anselmino, et.al, 2005

new Sivers: $$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1} e^{-k_\perp^2/M_1^2}$$ Anselmino, et.al, 2009

- Using $\Delta^N f_{q/A^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp^2)$, one can obtain

$$gT_{q,F}(x,x)|_{\text{old Sivers}} = 0.40 f_1^q(x) \mathcal{N}_q(x)|_{\text{old}}$$

$$gT_{q,F}(x,x)|_{\text{new Sivers}} = 0.33 f_1^q(x) \mathcal{N}_q(x)|_{\text{new}}$$