Partonic pictures of the nucleon spin

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Outline

- The simple parton picture of the nucleon spin exists only for transverse polarization
- Transverse polarization: sum rule or relation?
- Helicity sum rule, the gauge invariant version, lattice calculation, twist-3 GPDs
- Helicity sum rule, the light-cone version, twist-3 GPDs, Wigner distribution

Why Parton pictures for the nucleon spin?

- What is the spin structure of the nucleon?
- Theorists: any frame, any gauge..
- Experiments:
  - Infinite momentum frame: how the nucleon spin is made of parton constituents
  - Gauge invariance: either textbook type of gauge symmetry, or GIE through light-cone gauge $A^+ = 0$.

Isolating the proton spin

- The proton has spin, but can also have orbital motion itself. When we considering the spin structure, we cannot mix in the orbital motion. Therefore, it is critical that what type of proton states that one chooses.
- The safest approach is to choose a plane wave proton state, just like what experimenters prepare in lab.

Matrix element of AM density

- Matrix element in the plane wave state

$$\langle PS| \int d^4\xi M^{\mu\alpha\beta}(\xi)|PS\rangle = J\frac{2S_\rho P_\sigma}{M^2}(2\pi)^3\delta^3(0)$$
$$\times\left(\epsilon^{\alpha\beta\rho\sigma}P^\mu + \epsilon^{[\alpha\mu\rho\sigma}P^{\beta]} - (\text{trace})\right) + \cdots\ , \qquad ($$

- The leading light-cone component is + + ⊥, thus,

$$\langle PS| \int d^4\xi M^{++\perp}|PS\rangle = J\left[\frac{2(P^+)^2 S^{\perp'}}{M^2}\right](2\pi)^3\delta^3(0)$$

Transverse spin has a leading light-cone interpretation!

Light-cone picture of $S_\perp$

- Burkardt (2005)
- Important point:

$$J_q = \frac{1}{2}\sum_i \int dx x \left[q_i(x) + E_i(x,0,0)\right]$$

works for transverse spin!

- Need a wave packet?
- Works only for the sum? (relation)

if a parton picture works, one has to show that a parton of momentum x will carry angular momentum x(q(x)+E(x)).

A plane-wave derivation of transverse-spin parton sum rule

- Consider parton momentum density

$$\rho^+(x,\xi,S^\perp) = x\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle PS^\perp|\overline{\psi}(-\frac{\lambda n}{2},\xi)\gamma^+\psi(\frac{\lambda n}{2},\xi)|PS^\perp\rangle$$

It has a "distribution" term depending on coordinate ξ,

$$\rho^+(x,\xi,S^\perp) \propto x\left(q(x)+\frac{1}{2}(q(x)+E(x))\partial_\xi^{\perp\prime}\right)$$

- Calculating its contribution to the transverse spin density

$$J(x) = 2\int d^2\xi \xi^{\perp\prime}\rho^+(x,\xi,S^\perp) \quad = \quad (x/2)(q(x)+E(x))$$

comments

- It is a leading-twist parton picture
- It is a true density, and therefore,

$$\int J(x)dx = \frac{1}{2}$$

Is a parton sum rule.

- It cannot be separate into quark spin and orbital contribution because the energy-momentum tensor involved is T^{++} component.
- One can try to develop a parton picture involving quark spin from $T^{+\perp}$, one gets a twist-three picture. which will involve g_2 structure function

Helicity sum rule

- $$\langle PS|\int d^4\xi M^{\mu\alpha\beta}(\xi)|PS\rangle = J\frac{2S_\rho P_\sigma}{M^2}(2\pi)^3\delta^3(0)$$
  $$\times\left(\epsilon^{\alpha\beta\rho\sigma}P^\mu + \epsilon^{[\alpha\mu\rho\sigma}P^{\beta]} - (\text{trace})\right) + \cdots \ , \qquad ($$
- The next most important component is + ⊥⊥

$$\langle PS|\int d^4\xi M^{+12}|PS\rangle = J(2P^+)(2\pi)^3\delta^3(0)$$

One has the J_3 helicity sum rule, subleading in LC, compared to transverse case, one cannot escape from the twist-3 !

Parton transverse momentum and transverse coordinates

- Why twist-three? Because the AM operator involves parton transverse momentum!

$$J^3 = \int d^3\vec{\xi} M^{+12}(\xi)$$
$$= \int d^3\vec{\xi} \left[\overline{\psi}\gamma^+ (\frac{\Sigma^3}{2})\psi + \overline{\psi}\gamma^+ \left(\xi^1(iD^2) - \xi^2(iD^1)\right)\psi \right] .$$

- It has involves parton transverse coordinates!
- To develop a parton picture, we need a wigner distribution in the transverse direction.

The gauge invariant approach

- Define the Wigner operator

$$\hat{W}(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\xi/2)\gamma^{+}\Psi(\vec{r}+\xi/2)e^{ik\cdot\xi}d^{4}\xi \ ,$$

where Ψ is a gauge-invariant quark field which gauge links going to infinity along the radial direction,

$$\Psi_{FS}(\xi) = P\left[\exp\left(-ig\int_{0}^{\infty} d\lambda\xi\cdot A(\lambda\xi)\right)\right]\psi(\xi) \ ,$$

The link vanishes in the Fock-Schwinger gauge: $\xi \circ A=0$

Wigner and OAM distributions

- Distribution in x, b, k

$$W(k^+ = xP^+, \vec{b}_\perp, \vec{k}_\perp)$$
$$= \frac{1}{2P^+} \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{dk^-}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \left\langle \frac{\vec{q}_\perp}{2} \left| \hat{\mathcal{W}}(0,k) \right| -\frac{\vec{q}_\perp}{2} \right\rangle$$

No known experimental measurement, but can be calculated in lattice QCD!

- Parton OAM,

$$L(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$

which is experimentally measureable!

Moments of L(x)

- $$\int x^{n-1} L_{FS}(x)dx = \langle PS| \int d^3\vec{r} \sum_{i=0}^{n-1} \frac{1}{n} \overline{\psi}(\vec{r})(in\cdot D)^i$$
$$\times(\vec{r}_\perp \times i\vec{D}_\perp)(in\cdot D)^{n-1-i}\psi(\vec{r})|PS\rangle \ . \qquad (16$$

Which can be extracted from the twist-three GPDs. In particular, the first moment reduces to the gauge invariant version of the OAM,

$$\int L(x)dx = \langle PS| \int d^3\vec{r}\, \overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{D}_\perp)\psi(\vec{r})|PS\rangle$$

A simpler parton picture?

- Presence of $A_\perp$ in the covariant derivative spoils simple parton picture! Let's get rid of it.
- One can work in the fixed gauge A+=0

$$J^3 = \int d^3\vec{\xi}\left[\overline{\psi}\gamma^+(\vec{\xi}\times i\vec{\partial})^3\psi + \frac{1}{2}\overline{\psi}\gamma^+\Sigma^3\psi + E^i(\vec{\xi}\times\vec{\partial})^3 A^i + (\vec{E}\times\vec{A})^3\right] ,$$

All operator are bilinear in fields, which leads to a spin decomposition in light-cone gauge

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \tilde{L}_q + \Delta G + \tilde{L}_g \ .$$

GIE

- A gauge-dependent quantity is usually not measurable in experiment
- However, if one is lucky enough, a GIE is measurable experimentally.
- Example is the gluon spin operator in the light-cone gauge.
  - It is not known that the gluon spin operator in any other gauge has a GIE that is measurable.
- It is possible to measure the matrix element of the GIE of the OAM in LC? Yes.

Partial derivative made gauge invariant

- Partial derivative in LC gauge can be gauge-invariantized as follows

$$i\partial_\xi^\perp = iD_\xi^\perp + \int^{\xi^-} d\eta^- L_{[\xi^-,\eta^-]} gF^{+\perp}(\eta^-,\xi_\perp) L_{[\eta^-,\xi^-]} \,.$$

- The second term is a non-local operator along the lightcone.
- Question is can one measure something like this?

OAM density

- OAM density

$$\tilde{L}^q(x) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} d^2\xi \langle PS|\overline{\psi}(-\frac{\lambda n}{2}, \xi)\gamma^+ \times(\xi^1 i\partial^2 - \xi^2 i\partial^1)\psi(\frac{\lambda n}{2}, \xi)|PS\rangle$$

Bashingsky, Jaffe, Hagler, Schaefer…

- which also has GIE in the light-gauge.

Relation to twist-three GPDs

- All GIE of the OAM in LC can be related to the matrix elements of twist-three GPDs.
- The gauge-variant AMO density in LC is measurable in experiment!

- X. Ji, X. Xiong, F. Yuan, To be published.

Wigner distribution

- Define the gauge-invariant quark field through LC gauge link

$$\Psi_{LC}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda n\cdot A(\lambda n+\xi)\right)\right]\psi(\xi)\ .$$

- Define a Wigner distribution

$$\hat{\mathcal{W}}(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\xi/2)\gamma^+\Psi(\vec{r}+\xi/2)e^{ik\cdot\xi}d^4\xi\ ,$$

$$W(k^+ = xP^+, \vec{b}_\perp, \vec{k}_\perp)$$
$$= \frac{1}{2P^+}\int\frac{d^2\vec{q}_\perp}{(2\pi)^2}\int\frac{dk^-}{(2\pi)^2}e^{-i\vec{q}_\perp\cdot\vec{b}_\perp}\left\langle\frac{\vec{q}_\perp}{2}\left|\hat{\mathcal{W}}(0,k)\right|-\frac{\vec{q}_\perp}{2}\right\rangle$$

AO density

- Angular momentum density

$$\tilde{L}_q(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$

C. Lorce and B. Pasquini, Phys. Rev. D **84**, 014015 (2011); C. Lorce', B. Pasquini, X. Xiong and F. Yuan, arXiv:1111.4827 [hep-ph].
Y. Hatta, arXiv:1111.3547 [hep-ph].

- Unfortunately, it cannot be calculated on lattice (with finite number of moments)

Measuring Wigner distributions

- W_{LC} (x,b,k) might be measurable in exp. directly.
  to be published, Ji, Yuan and others,
- Interplay with TMDs! One can check the consistency or use the constraint.
- In some sense, GPD is already a Wigner distribution (Belitsky, Ji, Yuan)