Angular Momentum in Light-Front Dynamics

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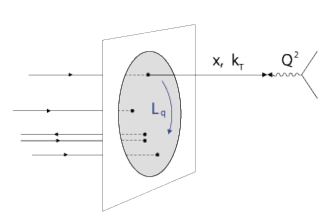
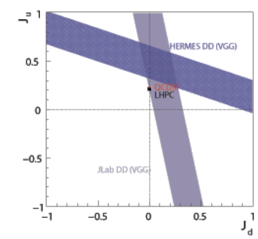
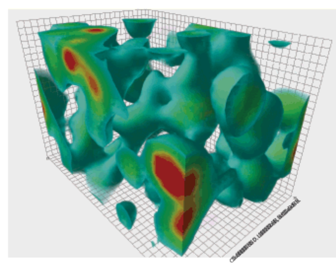
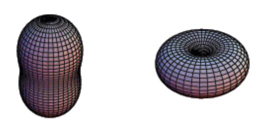








INT Workshop INT-12-49W
Orbital Angular Momentum in QCD

Seattle, WA, Feb. 7, 2012

Poincaré Algebra

$$[p^\mu, p^\nu] = 0 \qquad [p^\mu, J^{\rho\sigma}] = i(g^{\mu\rho} p^\sigma - g^{\mu\sigma} p^\rho)$$

$$[J^{\mu\nu}, J^{\rho\sigma}] = i(g^{\mu\sigma} J^{\nu\rho} + g^{\nu\rho} J^{\mu\sigma} - g^{\mu\rho} J^{\nu\sigma} - g^{\nu\sigma} J^{\mu\rho})$$

$$p^\nu = \begin{bmatrix} p^0 \\ p^1 \\ p^2 \\ p^3 \end{bmatrix} = \int d^3x\, T^{0\nu} \qquad J^{\mu\nu} = \begin{bmatrix} 0 & K^1 & K^2 & K^3 \\ -K^1 & 0 & J^3 & -J^2 \\ -K^2 & -J^3 & 0 & J^1 \\ -K^3 & J^2 & -J^1 & 0 \end{bmatrix} = \int d^3x\, M^{0\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = 0 \quad ; \quad T^{\mu\nu} = \sum_k \frac{\partial L}{\partial(\partial_\mu \phi_k)} \partial^\nu \phi_k - g^{\mu\nu} L$$

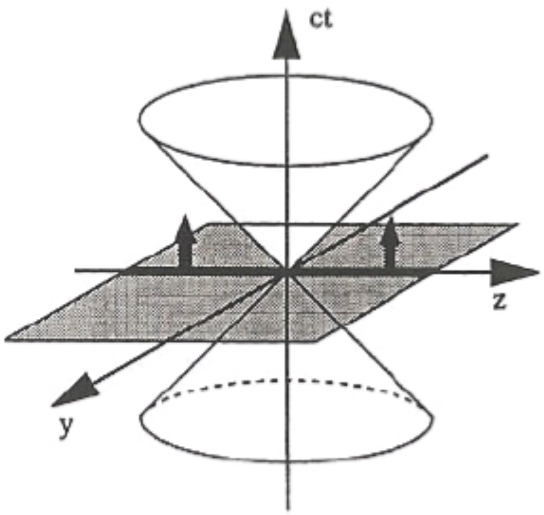
$$M^{\mu\nu\lambda} = M^{\mu\nu\lambda}_{"O"} + M^{\mu\nu\lambda}_{"S"}$$

$$M^{\mu\nu\lambda}_{"O"} = T^{\mu\nu} x^\lambda - T^{\mu\lambda} x^\nu$$

$$\partial_\mu M^{\mu\nu\lambda}_{"O"} = 0 = T^{\lambda\nu} - T^{\nu\lambda} \quad ; \quad \partial_\mu M^{\mu\nu\lambda}_{"S"} = 0$$

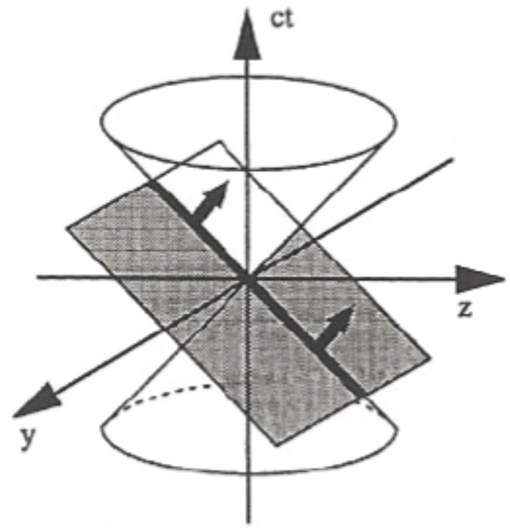
F. J. Belinfante, Physica 6, 887 (1939);
L. Rosenfeld, Mem. Acad. Roy. Belg. 18, 6 (1940).

Dirac's Proposition

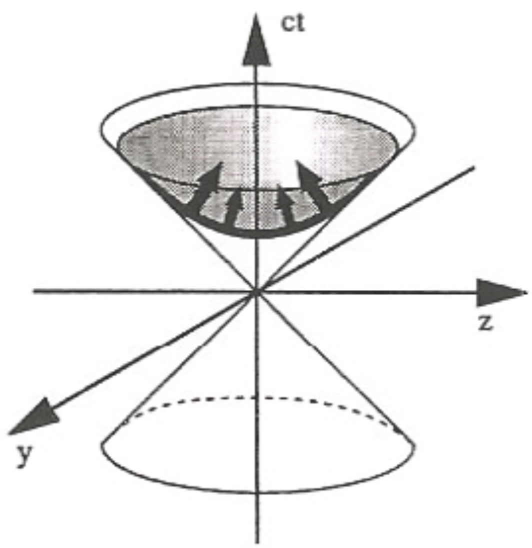


The instant form


1949



The front form



The point form

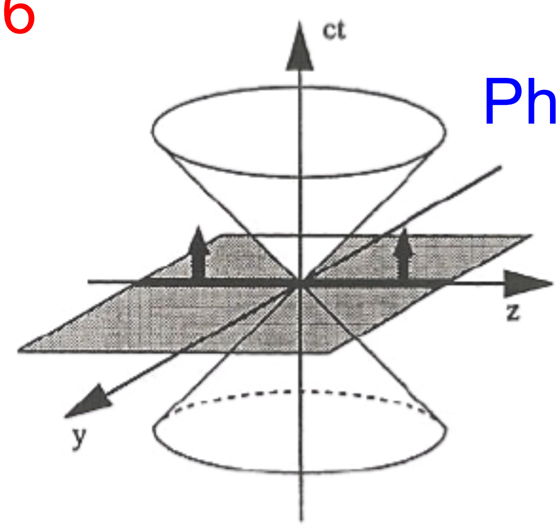
Stability Group

Just Formal?

or

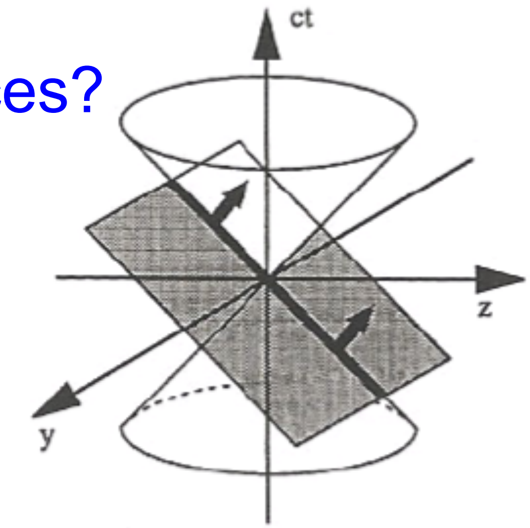
Physical Consequences?

6

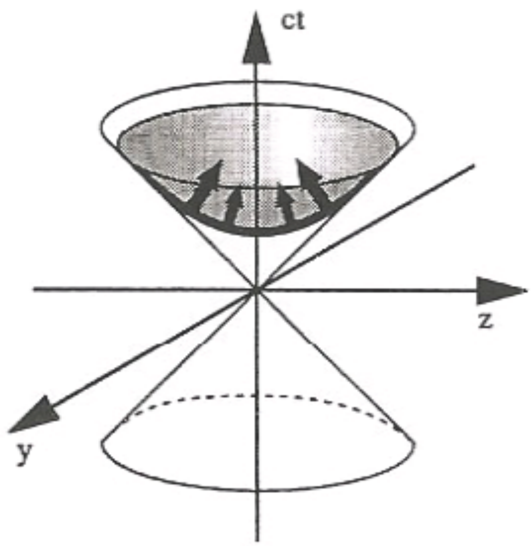


The instant form

7



The front form



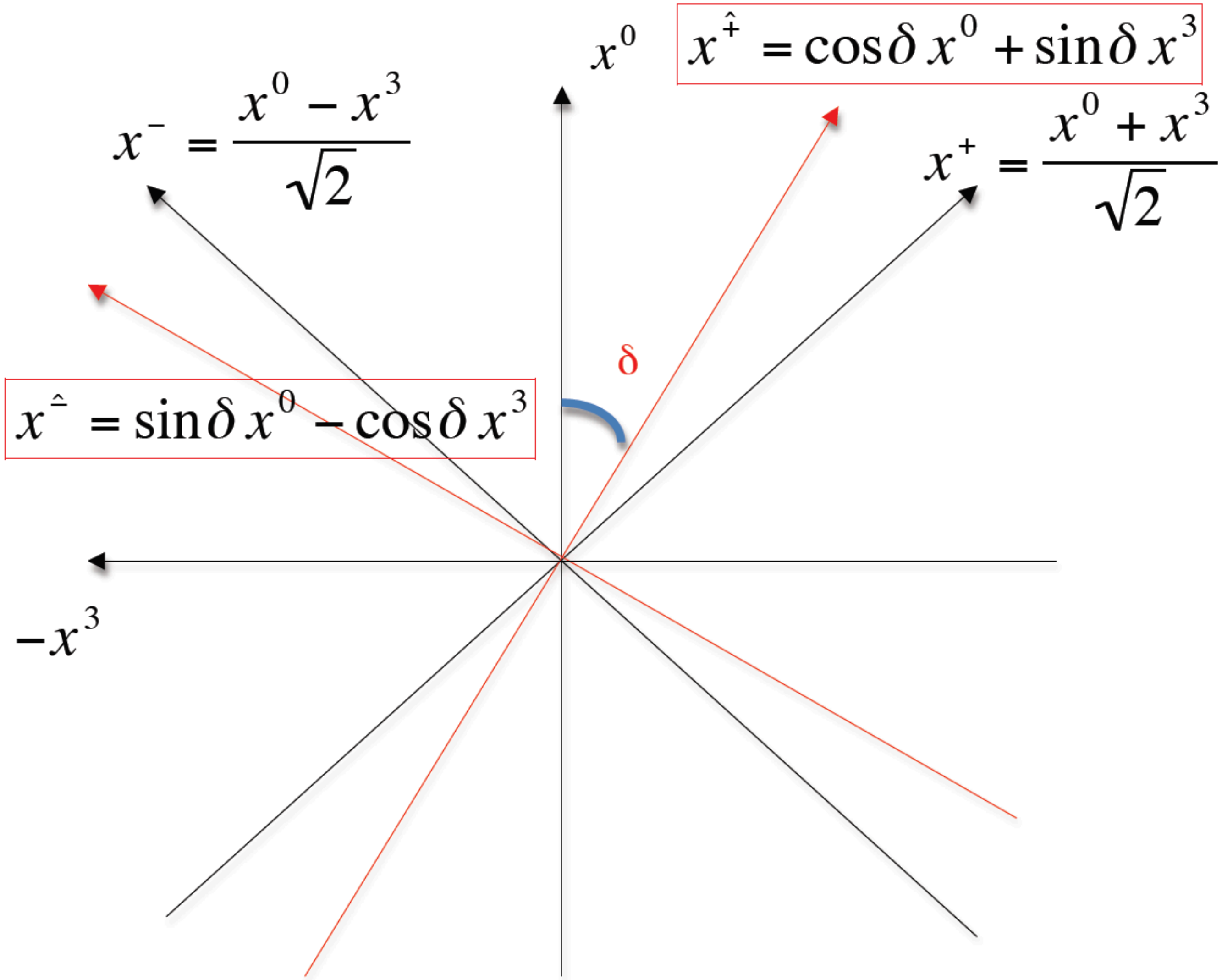
The point form

6

Outline

- Why LFD?
  - Interpolation between Instant and Front Forms
  - Distinguished Features in LFD
- Angular Momentum in LFD
  - Light-Front Helicities
  - Swap of Helicity Amplitudes
- Model Independent General Angular Condition
  - N-Δ Transition Process
  - Deuteron Form Factors
- Conclusion

Interpolation between Instant and Front Forms



K. Hornbostel, PRD45, 3781 (1992);
C.Ji and C. Mitchell, PRD64,085013 (2001).

$$g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \longrightarrow g^{\hat{\mu}\hat{\nu}} = \begin{bmatrix} \cos 2\delta & 0 & 0 & \sin 2\delta \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin 2\delta & 0 & 0 & -\cos 2\delta \end{bmatrix}$$

$$J^{\mu\nu} = \begin{bmatrix} 0 & K^1 & K^2 & K^3 \\ -K^1 & 0 & J^3 & -J^2 \\ -K^2 & -J^3 & 0 & J^1 \\ -K^3 & J^2 & -J^1 & 0 \end{bmatrix} \longrightarrow J^{\hat{\mu}\hat{\nu}} = \begin{bmatrix} 0 & \hat{E}^1 & \hat{E}^2 & -K^3 \\ -\hat{E}^1 & 0 & J^3 & -\hat{F}^1 \\ -\hat{E}^2 & -J^3 & 0 & -\hat{F}^2 \\ K^3 & \hat{F}^1 & \hat{F}^2 & 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} \hat{E}^1 &= J^2 \sin\delta + K^1 \cos\delta \\ \hat{E}^2 &= K^2 \cos\delta - J^1 \sin\delta \\ \hat{F}^1 &= K^1 \sin\delta - J^2 \cos\delta \\ \hat{F}^2 &= J^1 \cos\delta + K^2 \sin\delta \end{aligned}}$$

$$\delta = 0$$
$$p_0 = p^0$$
$$-p_3 = p^3$$

$\longleftarrow$

$$\boxed{\begin{array}{c} 0 < \delta < \pi/4 \\ p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta \\ p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta \end{array}}$$

$\longrightarrow$

$$\delta = \pi/4$$
$$p_+ = p^-$$
$$p_- = p^+$$

Transverse Boost : $$[\hat{E}^i, p_{\hat{-}}] = 0$$

$$(i = 1, 2)$$

Transverse Rotation : $$[\hat{F}^i, p_{\hat{+}}] = 0$$

Longitudinal Boost : K^3

Longitudinal Rotation : J^3

$\left.\begin{array}{c} K^3 \\ J^3 \end{array}\right\}$ Immune to Interpolation

Kinematic Operators

(Members of Stability Group)

$$Exp\left(-i\omega\hat{\aleph}^{i}\right)|x^{\hat{+}}> \propto |x^{\hat{+}}>$$

$$[\hat{\aleph}^{i}, p^{\hat{+}}] = 0$$

$$\hat{\aleph}^{i} = \hat{F}^{i}\cos 2\delta - \hat{E}^{i}\sin 2\delta$$

$\delta = 0$

$-J^2$

J^1

$\longleftarrow$

$$\boxed{\begin{aligned}\hat{\aleph}^{1} &= -J^2\cos\delta - K^1\sin\delta \\ \hat{\aleph}^{2} &= J^1\cos\delta - K^2\sin\delta\end{aligned}}$$

$\longrightarrow$

$\delta = \pi/4$

$-E^1 = -(J^2 + K^1)/\sqrt{2}$

$E^2 = (J^1 - K^2)/\sqrt{2}$

$$(J^3, p^1, p^2, p_{\hat{-}})$$

Corresponding Dynamic Operators

$$\hat{\wp}^i = -\hat{F}^i \sin 2\delta - \hat{E}^i \cos 2\delta$$

$$J^{\hat{\mu}\hat{\nu}} = \begin{bmatrix} 0 & \hat{E}^1 & \hat{E}^2 & -K^3 \\ -\hat{E}^1 & 0 & J^3 & -\hat{F}^1 \\ -\hat{E}^2 & -J^3 & 0 & -\hat{F}^2 \\ K^3 & \hat{F}^1 & \hat{F}^2 & 0 \end{bmatrix} \longrightarrow J_{\hat{\mu}\hat{\nu}} = \begin{bmatrix} 0 & \hat{\wp}^1 & \hat{\wp}^2 & K^3 \\ -\hat{\wp}^1 & 0 & J^3 & -\hat{\aleph}^1 \\ -\hat{\wp}^2 & -J^3 & 0 & -\hat{\aleph}^2 \\ -K^3 & \hat{\aleph}^1 & \hat{\aleph}^2 & 0 \end{bmatrix}$$

$\delta = 0$

$-K^1$

$-K^2$

$$\boxed{\begin{aligned} \hat{\wp}^1 &= -K^1 \cos\delta + J^2 \sin\delta \\ \hat{\wp}^2 &= -K^2 \cos\delta - J^1 \sin\delta \end{aligned}}$$

$\delta = \pi/4$

$-F^1 = -(K^1 - J^2)/\sqrt{2}$

$-F^2 = -(J^1 + K^2)/\sqrt{2}$

$$(K^3 \,, p_{\hat{+}})$$

Longitudinal Boost K^3

$$[K^3, p^{\hat{+}}] = -ip_{\hat{-}} = i(p^{\hat{-}} \cos 2\delta - p^{\hat{+}} \sin 2\delta)$$

$$\Big\downarrow \quad \delta \to \pi/4$$

$$[K^3, p^{+}] = -ip^{+}$$

$$Exp\left(-i\omega K^3\right) | x^{+} > \propto | x^{+} >$$

One more kinematic generator appears only in the front form.

Maximum number (7) of members in the stability group.

TABLE I. Kinematic and dynamic generators on an arbitrary interpolation front.

	Kinematic	Dynamic
$\delta = 0$	$\mathcal{K}_1 = -J_2$, $\mathcal{K}_2 = J_1$, J_3, P^1, P^2, P^3	$\mathcal{D}_1 = -K_1$, $\mathcal{D}_2 = -K_2$, K_3, P^0
$0 \leq \delta < \frac{\pi}{4}$	$\mathcal{K}_1$, $\mathcal{K}_2$, J_3, P^1, P^2, P_-	$\mathcal{D}_1$, $\mathcal{D}_2$, K_3, P_+
$\delta = \frac{\pi}{4}$	$\mathcal{K}_1 = -E_1$, $\mathcal{K}_2 = -E_2$, J_3, K_3, P^1, P^2,P^+	$\mathcal{D}_1 = -F_1$, $\mathcal{D}_2 = -F_2$, P^-

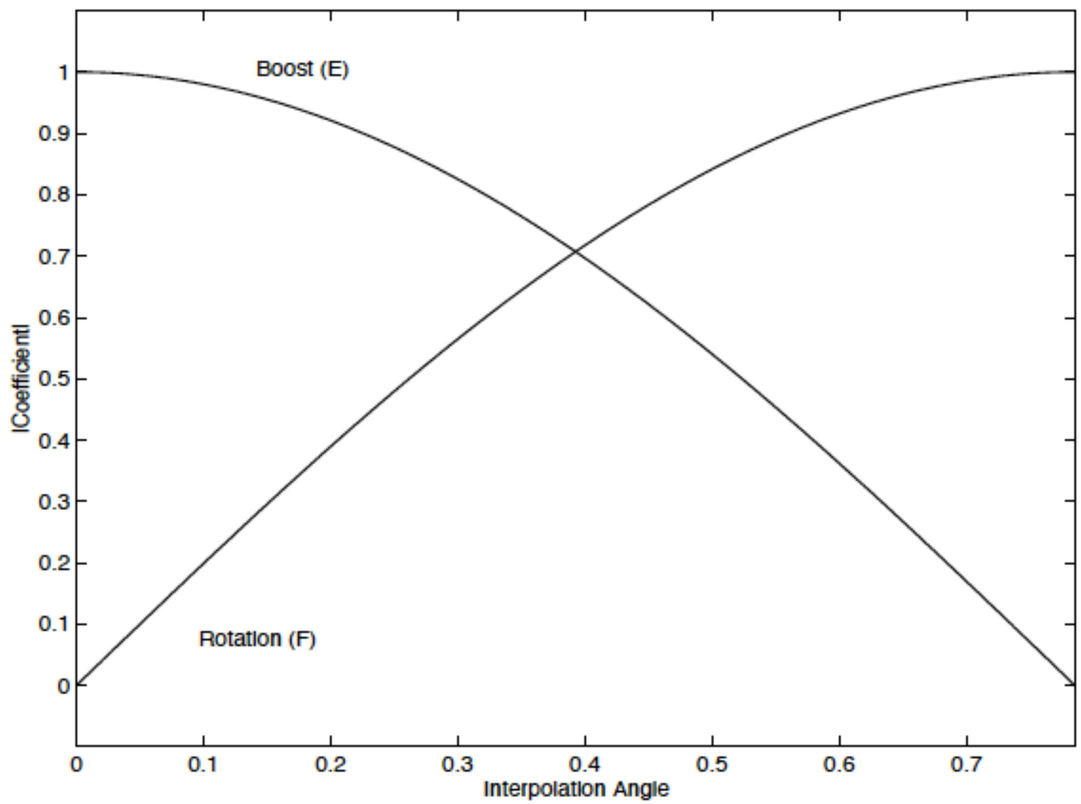


FIG. 1. The smooth exchange of the roles of rotation and boost is displayed. The coefficients of rotation F_i and boost E_i in the dynamic generator $\mathcal{D}_i$ are plotted versus interpolation angle δ.

Kinematic Transformation

$$p^1 = 0, p^2 = 0, p_{\hat{-}} = M \sin\delta$$

$$(p^0 = M, p^3 = 0)$$

$$\Big\downarrow \quad T = Exp\left\{-i(\beta_1 \hat{\aleph}^1 + \beta_2 \hat{\aleph}^2)\right\}$$

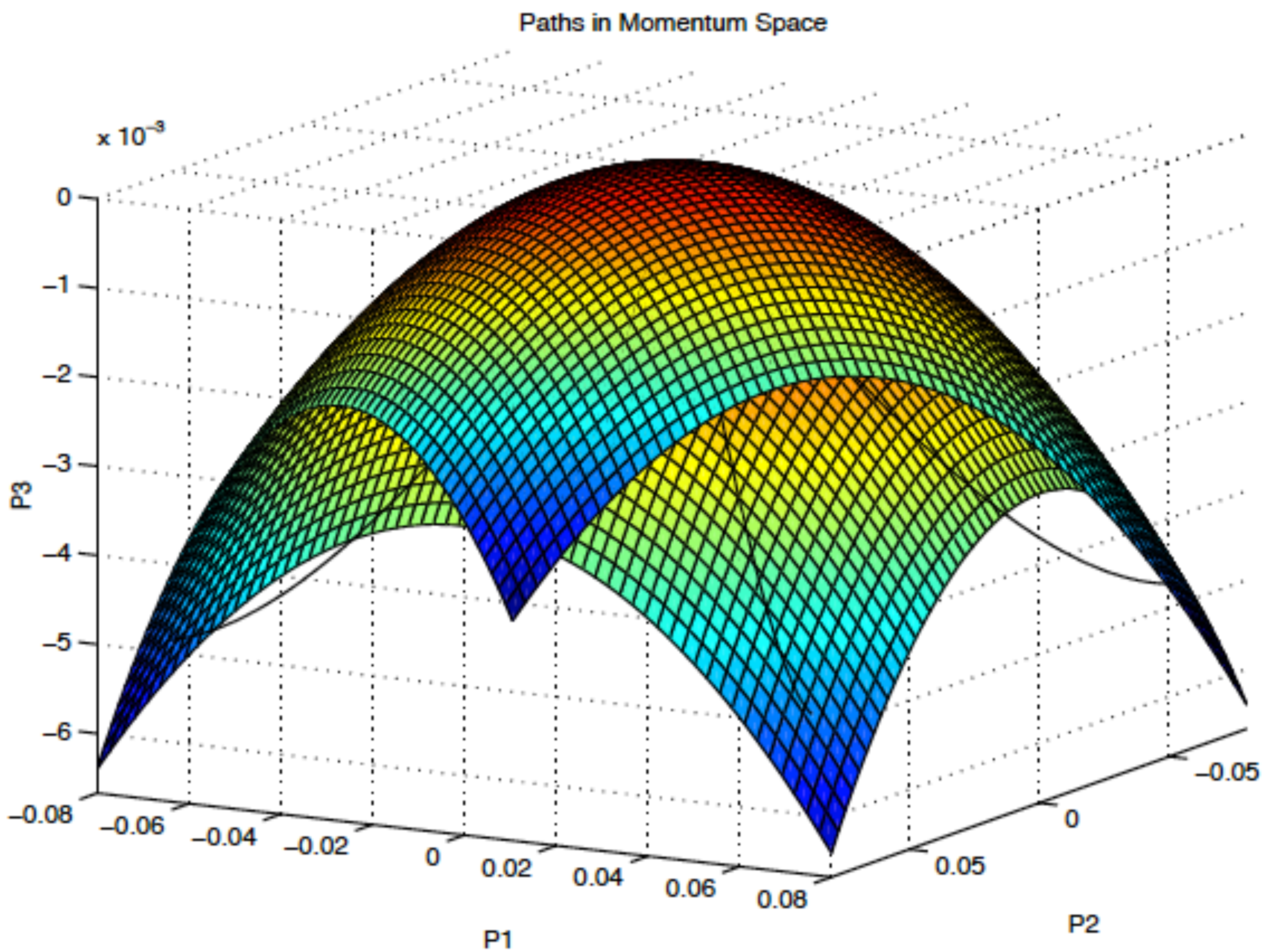
$$p^1 = -\beta_1 M \sin\delta \sin\alpha / \alpha, p^2 = -\beta_2 M \sin\delta \sin\alpha / \alpha,$$

$$p_{\hat{-}} = M \sin\delta \cos\alpha \,;\, \alpha = \sqrt{(\beta_1^2 + \beta_2^2)\cos 2\delta}$$

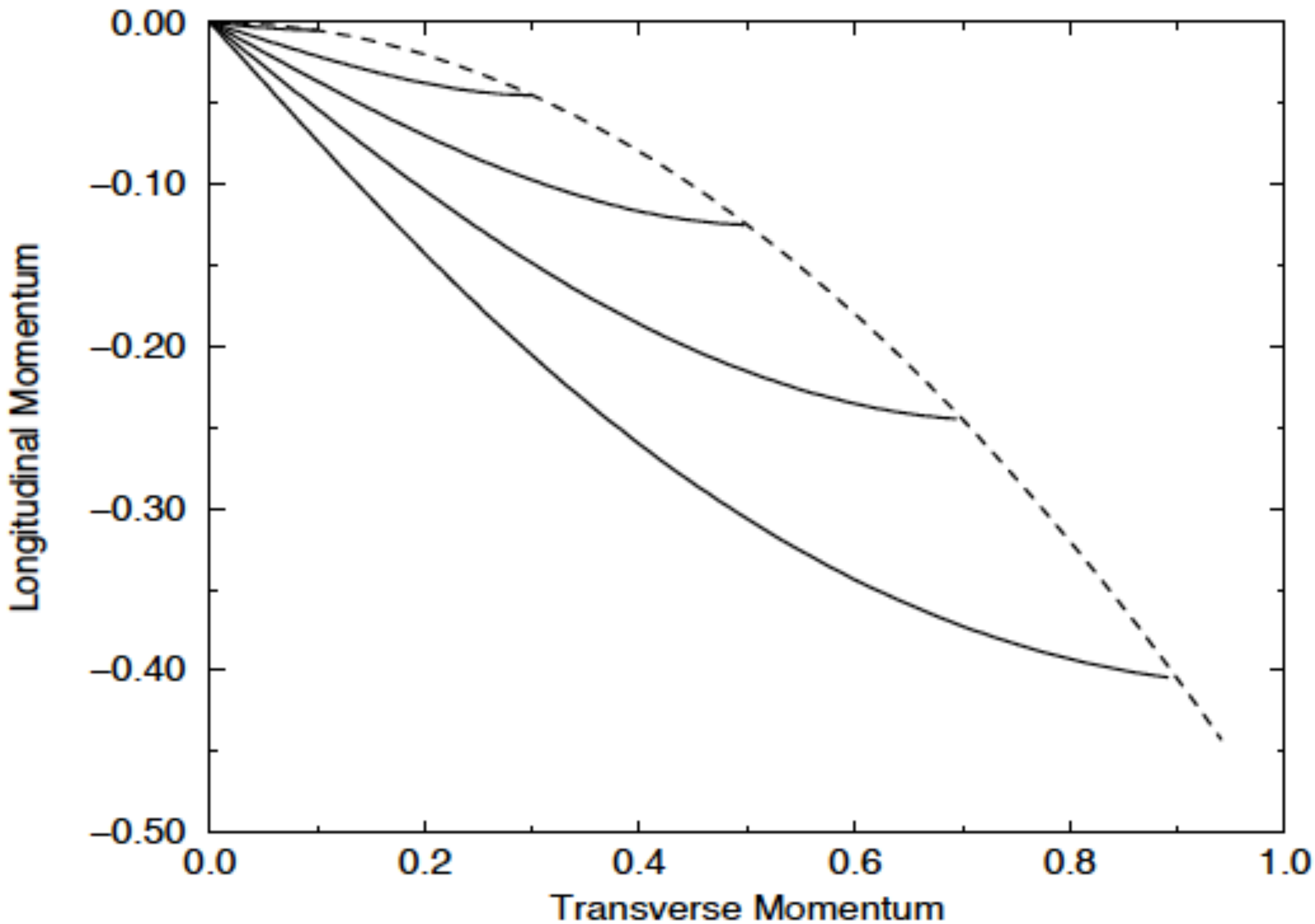
$$\Big\downarrow \quad \delta \to \pi/4$$

$$\vec{p}_\perp = -\vec{\beta}_\perp M / \sqrt{2}\ , p^+ = M / \sqrt{2}$$

$$(p^- = (\vec{p}_\perp^{\,2} + M^2)/\sqrt{2}M\ , p^3 = -\frac{\vec{p}_\perp^{\,2}}{2M})$$



Paths in Momentum Space
x 10^-3
0
−1
−2
−3
−4
−5
−6
P3
−0.08
−0.06
−0.04
−0.02
0
0.02
0.04
0.06
0.08
P1
−0.05
0
0.05
P2



0.00
−0.10
−0.20
−0.30
−0.40
−0.50
0.0
0.2
0.4
0.6
0.8
1.0
Longitudinal Momentum
Transverse Momentum

Angular Momentum

$$\boxed{[J^i, J^j] = i\varepsilon_{ijk} J^k \;, [J^i, M] = 0}$$

$$\Big\downarrow \quad T = Exp\left\{-i(\beta_1 \hat{\aleph}^1 + \beta_2 \hat{\aleph}^2)\right\}$$

$$\boxed{[\hat{\Im}^i, \hat{\Im}^j] = i\varepsilon_{ijk} \hat{\Im}^k \;, [\hat{\Im}^i, M] = 0}$$

$$T \mid n >= \mid p, n >$$

$$\hat{\Im}^i \mid p, n >= T J^i \mid n >$$

$$\hat{\Im}^i = T J^i T^+$$

$$\boxed{\hat{\mathfrak{I}}^3 = \left\{ J^3 p_{\hat{-}} + \hat{z} \cdot (\vec{p}_\perp \times \vec{\hat{\aleph}}_\perp) \right\} / M \sin\delta}$$

$$\vec{\hat{\mathfrak{I}}}_\perp = \vec{J}_\perp + \left\{ \vec{p}_\perp \cos\delta \left(J^3 + \frac{\hat{z} \cdot (\vec{p}_\perp \times \vec{\hat{\aleph}}_\perp)}{p_{\hat{-}} + M \sin\delta} \right) - (\hat{z} \times \vec{p}_\perp) \sin\delta \left(K^3 + \frac{\vec{p}_\perp \cdot \vec{\hat{E}}_\perp}{p_{\hat{-}} + M \sin\delta} \right) \right\}$$

$$\Big\downarrow \quad \delta \to \pi/4$$

$$\boxed{\mathfrak{I}^3 = J^3 + \hat{z} \cdot (\vec{E}_\perp \times \vec{p}_\perp) / p^+}$$

$$\vec{\mathfrak{I}}_\perp = \left\{ \hat{z} \times (p^- \vec{E}_\perp - p^+ \vec{F}_\perp + \vec{p}_\perp K^3) - \frac{\vec{p}_\perp}{p^+} \left(p^+ J^3 + \hat{z} \cdot \vec{E}_\perp \times \vec{p}_\perp \right) \right\} / M$$

$$\boxed{\mathfrak{I}^3 = \frac{W^+}{p^+}} \qquad W^\mu = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} p_\nu J_{\alpha\beta}$$

$$[\mathfrak{I}^3, \textit{Stability Group Members}] = 0$$

Front Form Current Matrix Element
(Light-Front Helicity Amplitude)

$$G^{\mu=+}_{L\lambda'\lambda} = {}_L\langle j'm'; p', \lambda' | J^{\mu=+}(0) | jm; p, \lambda \rangle_L$$

$$\boxed{\Im^3 | jm; p, \lambda >_L = \lambda | jm; p, \lambda >_L}$$

The same in all frames related by front-form boosts for each fixed set of angular momentum indices. The value of this matrix element is independent of all reference frames related by front-form boosts.

e.g. Spin-1 Form Factors

$$G^{\mu}_{h'h} = \varepsilon^{*\alpha}(p', h') J^{\mu}_{\alpha\beta} \varepsilon^{\beta}(p, h)$$

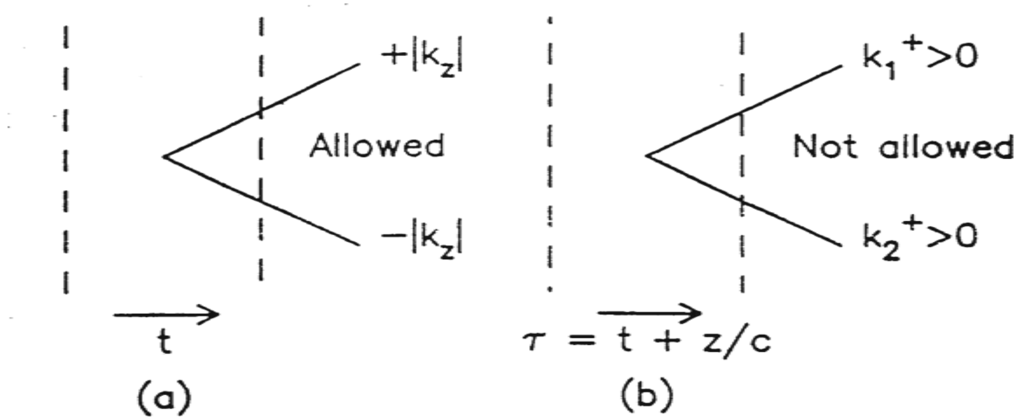
$$J^{\mu}_{\alpha\beta} = \left\{ -(p+p')^{\mu} g_{\alpha\beta} F_1(q^2) + (g^{\mu}_{\alpha} q_{\beta} - g^{\mu}_{\beta} q_{\alpha}) F_2(q^2) + \frac{q_{\alpha} q_{\beta}}{2M_W^2} (p+p')^{\mu} F_3(q^2) \right\}$$

Distinguished Features in LFD

$$\begin{array}{ccc} \textit{Equal } t & & \textit{Equal } \tau \\ p^0 & \leftrightarrow & p^- = p^0 - p^3 \\ (p^1, p^2) & \leftrightarrow & \vec{p}_\perp \\ p^3 & \leftrightarrow & p^+ = p^0 + p^3 \end{array}$$

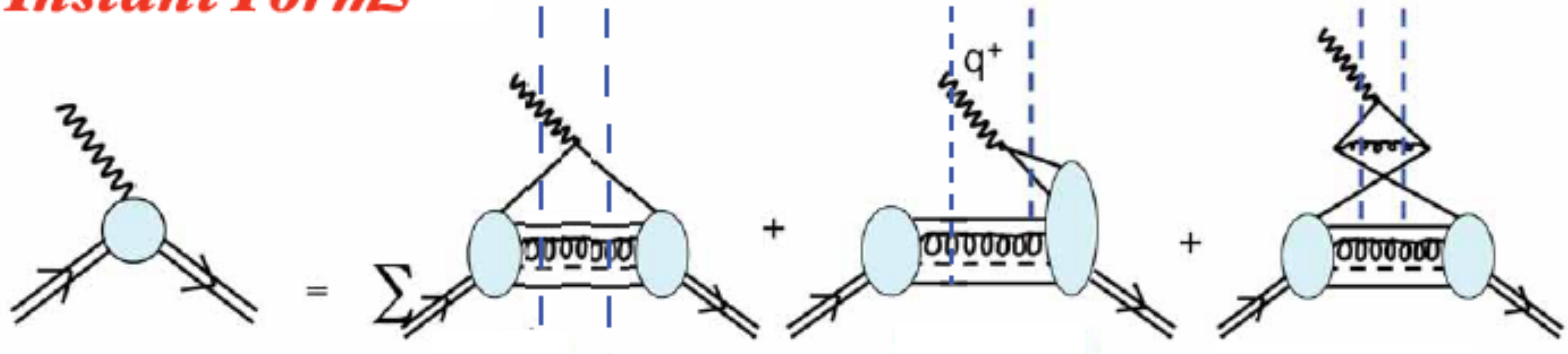
Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad\qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



Calculation of Form Factors in Equal-Time Theory

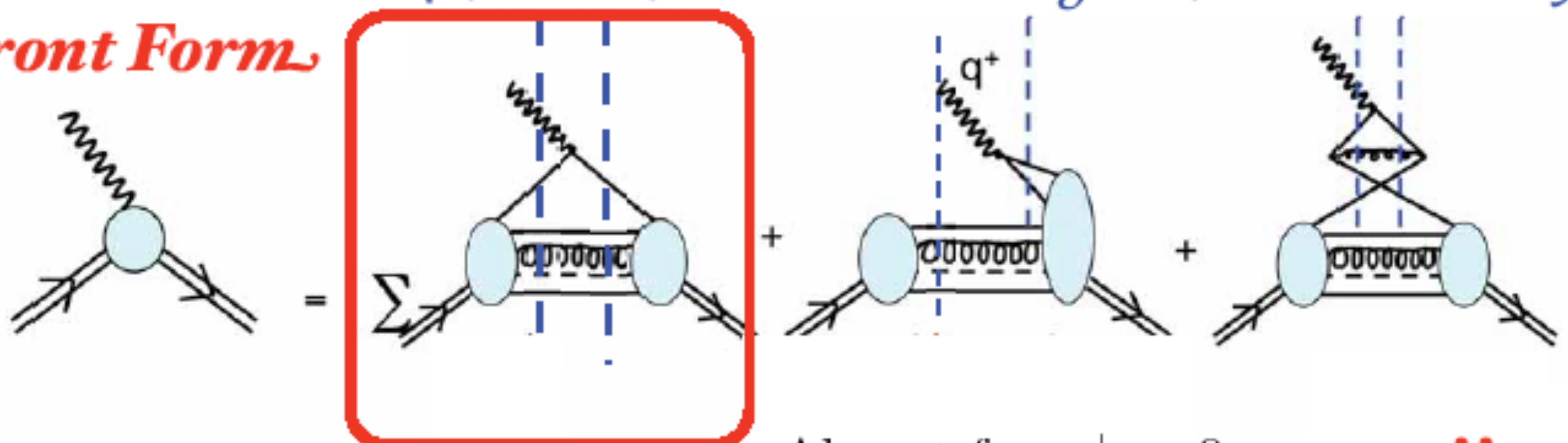
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

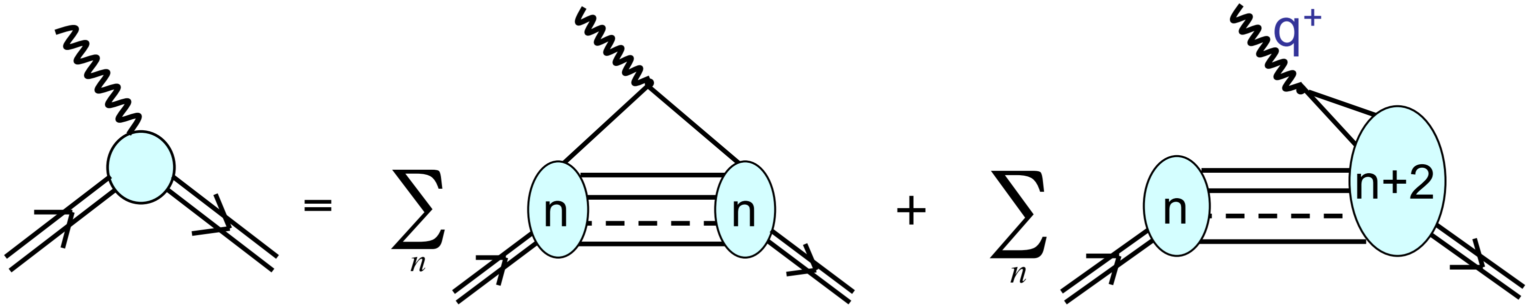
Front Form



Absent for $q^+ = 0$ **zero !!**

Zero-Mode Issue in LFD

- Valence and Nonvalence Contributions

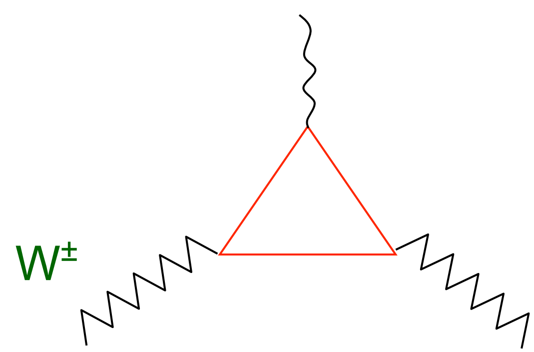


- Even if $q^+ \to 0$, the off-diagonal elements do not go away in some cases.

$$\lim_{q^+ \to 0} \int_{p^+}^{p^+ + q^+} dk^+ (....) \neq 0$$

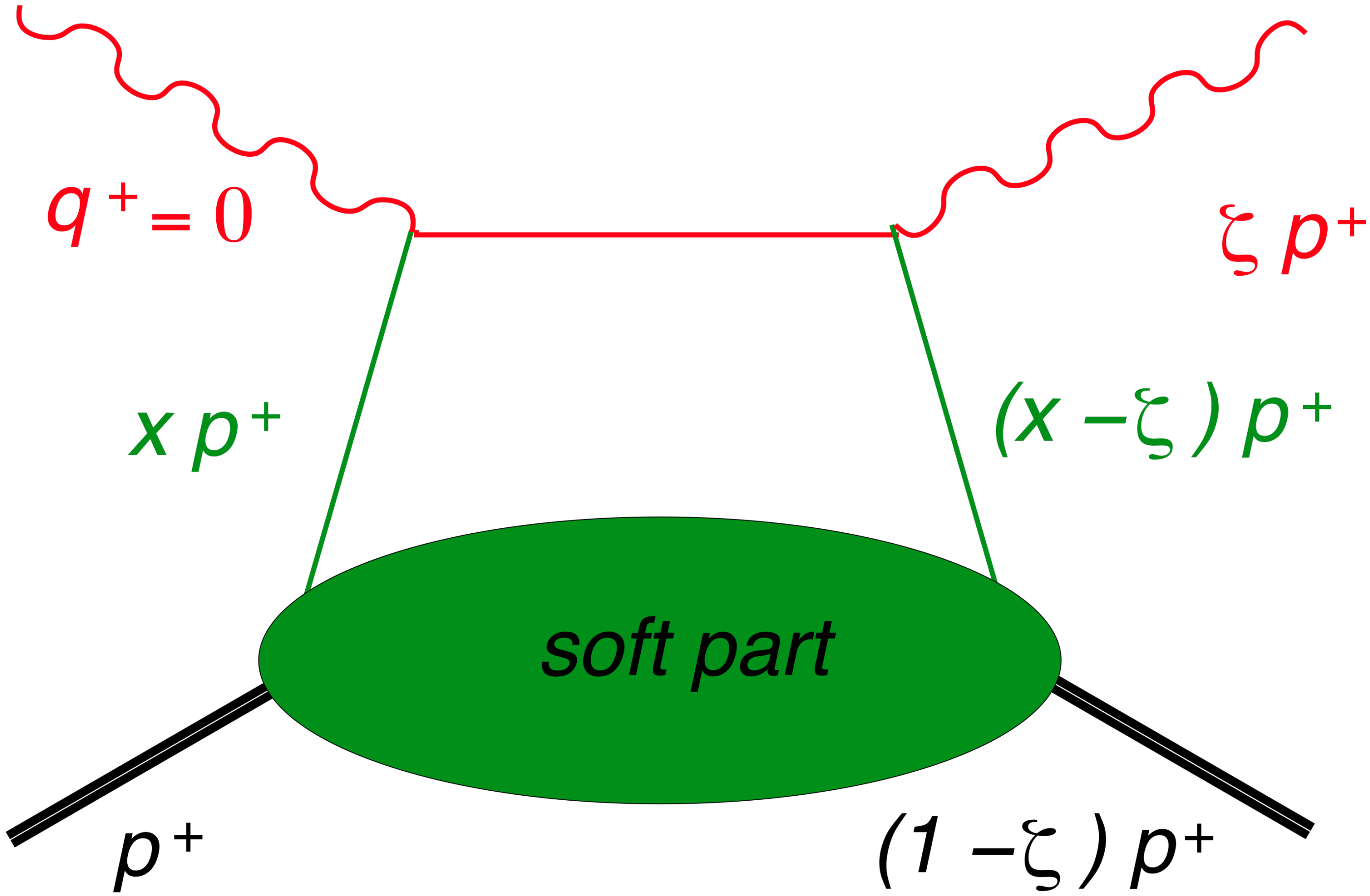
For example, G_{00}^+ has the zero-mode contribution in the Standard Model W$^\pm$ form factors.

$$\left(G_{00}^+\right)_{Z.M.} = \frac{g^2 Q_f p^+}{2\pi^3 M_W^2} \int_0^1 dx \int d^2 k_\perp \frac{k_\perp^2 + m_1^2 - x(1-x)Q^2}{k_\perp^2 + m_1^2 + x(1-x)Q^2} \neq 0$$



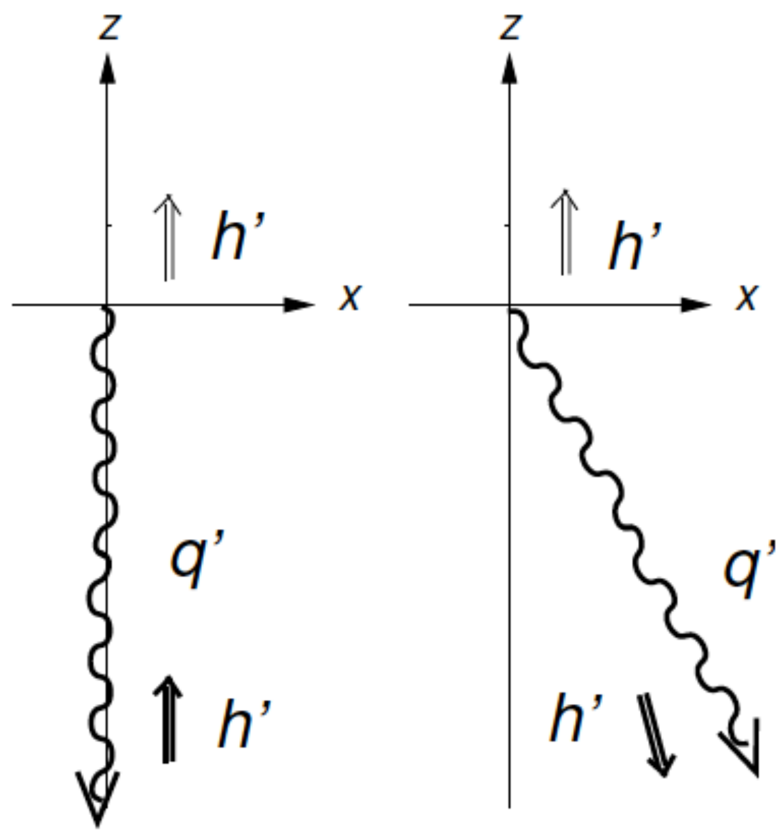
B.Bakker and C.Ji, PRD71,053005(2005)

GPDs rely on the handbag dominance in DVCS; i.e.
Q^2 >> any soft mass scale



$q^2 = q^+ q^- - q_\perp^2 = -q_\perp^2 = -Q^2 < 0$, e.g.
S.J.Brodsky,M.Diehl,D.S.Hwang, NPB596,99 (2001)

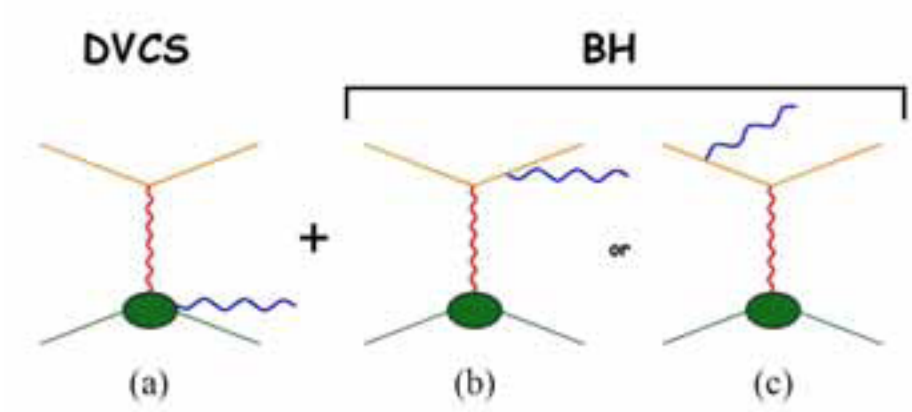
Swap

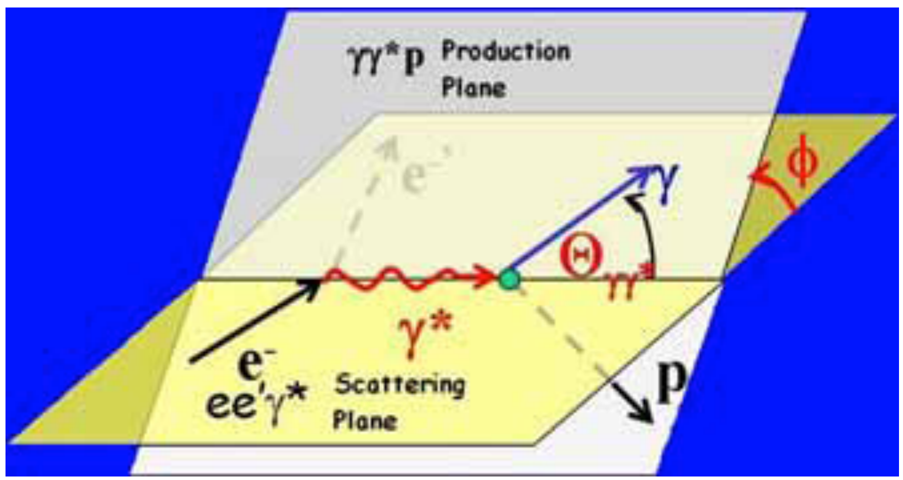


Definition of Light-Front Helicity

B.Bakker and C.Ji, Phys.Rev.D83,091502(R) (2011)

JLab Kinematics $t < -|t_{min}| \neq 0$

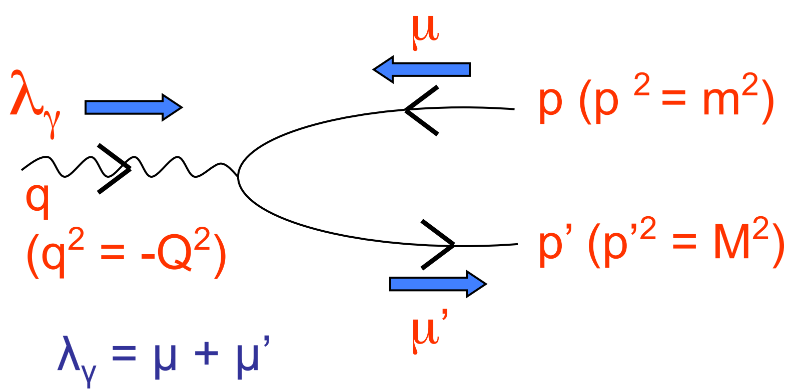




Discretion is advised in applying the $t=0$ formulation of DVCS in terms of GPDs for the data and/or hadron model analysis.

General Angular Condition

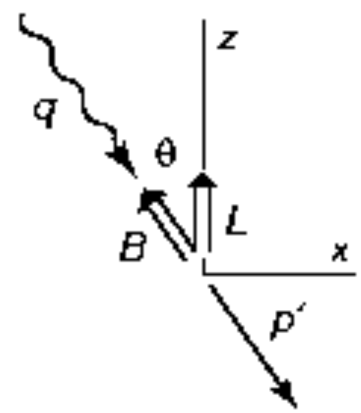
Angular Momentum Conservation in Breit Frame



$$G^{\nu}_{B\mu'\mu} = {}_B\langle p', \mu' \,|\, J^{\nu} \,|\, p, \mu \rangle_B$$
$$= \quad 0 \quad if \quad |\,\mu + \mu'\,| \geq 2$$

Light-front Helicity Amplitude in $q^+ = 0$ Frame

$$G^{\nu}_{L\lambda'\lambda} = {}_L\langle p', \lambda' \,|\, J^{\nu} \,|\, p, \lambda \rangle_L$$
$$= d^{\,j'}_{\mu'\lambda'}(\theta')\; G^{\nu}_{B\mu'\mu}\; d^{\,j}_{\lambda\mu}(-\theta)$$



$$\tan\theta = \frac{2mQ}{Q^2 + M^2 - m^2} \quad , \quad \tan\theta' = -\frac{2MQ}{Q^2 - M^2 + m^2}$$

C.Carlson and C.Ji,
Phys.Rev.D67,116002(03)

N-Δ Transition

$$<\Delta|J^\mu(0)|N>=e\overline{\Psi}_\beta(p')\Gamma^{\beta\mu}\Psi(p)$$

$$\Gamma^{\beta\mu}=G_1(q^\beta\gamma^\mu-\not{q}g^{\beta\mu})\gamma_5$$

$$+G_2(q^\beta\frac{p^\mu+p'^\mu}{2}-\frac{q\cdot(p+p')}{2}g^{\beta\mu})\gamma_5$$

$$+G_3(q^\beta q^\mu-q^2g^{\beta\mu})\gamma_5$$

$$\begin{pmatrix} G_M^* \\ G_E^* \\ G_C^* \end{pmatrix}=\frac{m}{3(M+m)}\begin{pmatrix} (3M+m)(M+m)+Q^2 & M^2-m^2 & -2Q^2 \\ M^2-m^2-Q^2 & M^2-m^2 & -2Q^2 \\ 4M^2 & 3M^2+m^2+Q^2 & 2(M^2-m^2-Q^2) \end{pmatrix}\begin{pmatrix} G_1/M \\ G_2 \\ G_3 \end{pmatrix}$$

$$G^\mu_{h'h}=\bar{u}_\beta(p',h')\Gamma^{\beta\mu}u(p,h)$$

$$G^+_{\frac{3}{2}\frac{1}{2}}=\frac{p^+Q}{\sqrt{2}}\{2G_1+(M-m)G_2\}\;;\;G^+_{-\frac{3}{2}\frac{1}{2}}=\frac{p^+Q^2}{\sqrt{2}}G_2\;;$$

$$G^+_{\frac{1}{2}\frac{1}{2}}=-\frac{p^+Q^2}{\sqrt{6}M}\{2G_1+(2M-m)G_2-2(M-m)G_3\}\;;$$

$$G^+_{-\frac{1}{2}\frac{1}{2}}=\frac{p^+Q^2}{\sqrt{6}M}\{2mG_1+(Q^2+mM-M^2)G_2-2Q^2G_3\}\;;\cdots$$

Helicity Amplitudes in N-Δ Transition

$$G^{+}_{\frac{3}{2}\frac{1}{2}} \qquad G^{+}_{\frac{1}{2}\frac{1}{2}} \qquad G^{+}_{-\frac{1}{2}\frac{1}{2}} \qquad G^{+}_{-\frac{3}{2}\frac{1}{2}}$$

$$G^{+}_{\frac{3}{2}-\frac{1}{2}} \qquad G^{+}_{\frac{1}{2}-\frac{1}{2}} \qquad G^{+}_{-\frac{1}{2}-\frac{1}{2}} \qquad G^{+}_{-\frac{3}{2}-\frac{1}{2}}$$

Helicity Amplitudes in N-Δ Transition

LF Parity: $Y_P = R_y(\pi)P$

$$
\begin{array}{cccc}
 & & G^{+}_{\frac{1}{2}-\frac{1}{2}} & -G^{+}_{\frac{3}{2}-\frac{1}{2}} \\
 & & \parallel & \parallel \\
G^{+}_{\frac{3}{2}\frac{1}{2}} & G^{+}_{\frac{1}{2}\frac{1}{2}} & G^{+}_{-\frac{1}{2}\frac{1}{2}} & G^{+}_{-\frac{3}{2}\frac{1}{2}} \\
G^{+}_{\frac{3}{2}-\frac{1}{2}} & G^{+}_{\frac{1}{2}-\frac{1}{2}} & G^{+}_{-\frac{1}{2}-\frac{1}{2}} & G^{+}_{-\frac{3}{2}-\frac{1}{2}} \\
 & & \parallel & \parallel \\
 & & -G^{+}_{\frac{1}{2}\frac{1}{2}} & G^{+}_{\frac{3}{2}\frac{1}{2}}
\end{array}
$$

$$G^{+}_{-\lambda'-\lambda} = \eta'_P \eta_P (-1)^{j'-j+\lambda'-\lambda} G^{+}_{\lambda'\lambda}$$

C.Carlson and C.Ji,
Phys.Rev.D67,116002(03)

Helicity Amplitudes in N-Δ Transition

$$\mu + \mu' = 2;\ \mu = \frac{1}{2},\ \mu' = \frac{3}{2}$$

$$d^{\frac{3}{2}}_{\lambda' \frac{3}{2}}(-\theta')\, G^{+}_{\lambda'\lambda}\, d^{\frac{1}{2}}_{\frac{1}{2}\lambda}(\theta) = 0$$

$$\begin{array}{cccc}
 & & G^{+}_{\frac{1}{2}-\frac{1}{2}} & -G^{+}_{\frac{3}{2}-\frac{1}{2}} \\
 & & \| & \| \\
G^{+}_{\frac{3}{2}\frac{1}{2}} & G^{+}_{\frac{1}{2}\frac{1}{2}} & G^{+}_{-\frac{1}{2}\frac{1}{2}} & G^{+}_{-\frac{3}{2}\frac{1}{2}} \\
G^{+}_{\frac{3}{2}-\frac{1}{2}} & G^{+}_{\frac{1}{2}-\frac{1}{2}} & G^{+}_{-\frac{1}{2}-\frac{1}{2}} & G^{+}_{-\frac{3}{2}-\frac{1}{2}} \\
 & & \| & \| \\
 & & -G^{+}_{\frac{1}{2}\frac{1}{2}} & G^{+}_{\frac{3}{2}\frac{1}{2}}
\end{array}$$

Angular Condition

$$G^{+}_{\frac{3}{2}-\frac{1}{2}} = \frac{Q[Q^2 - m(M-m)]G^{+}_{\frac{3}{2}\frac{1}{2}} + \sqrt{3}MQ^2 G^{+}_{\frac{1}{2}\frac{1}{2}} + \sqrt{3}MQ(M-m)G^{+}_{\frac{1}{2}-\frac{1}{2}}}{[(M-m)(M^2-m^2)+mQ^2]}$$

High Q Scaling in QCD (modulo logarithm)

$$G^{+}_{\lambda'\lambda} = \left(\frac{\Lambda}{Q}\right)^{|\lambda'-\lambda_{\min}|+|\lambda-\lambda_{\min}|} G^{+}_{\lambda_{\min}\lambda_{\min}} \propto \left(\frac{\Lambda}{Q}\right)^{2(n-1)+|\lambda'-\lambda_{\min}|+|\lambda-\lambda_{\min}|}$$

n = the number of quarks in the state;

λ_{min}= 0 (bosons) or 1/2 (fermions); Λ = QCD scale;

e.g. X.Ji, J.-P.Ma, and F.Yuan, PRL90, 241601 (2003)

$$G^{+}_{\frac{1}{2}\frac{1}{2}} \sim \frac{1}{Q^4}\,;\; G^{+}_{-\frac{1}{2}\frac{1}{2}} = a\left(\frac{\Lambda}{Q}\right) G^{+}_{\frac{1}{2}\frac{1}{2}}\,;\; G^{+}_{\frac{3}{2}\frac{1}{2}} = b\left(\frac{\Lambda}{Q}\right) G^{+}_{\frac{1}{2}\frac{1}{2}}\,;\; G^{+}_{-\frac{3}{2}\frac{1}{2}} = \left(c\frac{\Lambda}{Q}\right)^2 G^{+}_{\frac{1}{2}\frac{1}{2}}$$

$$\boxed{G^{+}_{\frac{3}{2}-\frac{1}{2}} = \frac{Q[Q^2 - m(M-m)]G^{+}_{\frac{3}{2}\frac{1}{2}} + \sqrt{3}MQ^2 G^{+}_{\frac{1}{2}\frac{1}{2}} + \sqrt{3}MQ(M-m)G^{+}_{\frac{1}{2}-\frac{1}{2}}}{[(M-m)(M^2-m^2)+mQ^2]}}$$

$$\sqrt{3} + \frac{b\Lambda}{M} = 0\,;\; b = -\frac{\sqrt{3}M}{\Lambda} \approx -20$$

Leading PQCD postponed to a larger Q region; 12GeV upgrade anticipated.

Deuteron Form Factors

$$J^{\mu}_{\alpha\beta} = \left\{ -(p+p')^{\mu} g_{\alpha\beta} F_1(q^2) + (g^{\mu}_{\alpha} q_{\beta} - g^{\mu}_{\beta} q_{\alpha}) F_2(q^2) + \frac{q_{\alpha} q_{\beta}}{2M_W^2} (p+p')^{\mu} F_3(q^2) \right\}$$

$$\begin{pmatrix} G_C \\ G_M \\ G_Q \end{pmatrix} = \begin{pmatrix} 1+\frac{2}{3}\eta & \frac{2}{3}\eta & \frac{2}{3}\eta(1+\eta) \\ 0 & -1 & 0 \\ 1 & 1 & 1+\eta \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \qquad \left(\eta = \frac{Q^2}{4M^2} \right)$$

Deuteron Helicity Amplitudes

$$G^{\mu}_{h'h} = \varepsilon^{*\alpha}(p',h') J^{\mu}_{\alpha\beta} \varepsilon^{\beta}(p,h)$$

Deuteron Helicity Amplitudes

$$G^{+}_{++} \quad G^{+}_{+0} \quad G^{+}_{+-}$$

$$G^{+}_{0+} \quad G^{+}_{00} \quad G^{+}_{0-}$$

$$G^{+}_{-+} \quad G^{+}_{-0} \quad G^{+}_{--}$$

Deuteron Helicity Amplitudes

LF Parity: $Y_P = R_y(\pi)P$

$$G^+_{++} \qquad G^+_{+0} \qquad G^+_{+-}$$

$$G^+_{0+} \qquad G^+_{00} \qquad G^+_{0-} = -G^+_{0+}$$

$$G^+_{-+} \qquad G^+_{-0} \qquad G^+_{--} = G^+_{++}$$

$$G^+_{-+} = G^+_{+-} \qquad G^+_{-0} = -G^+_{+0}$$

$$G^+_{-\lambda'-\lambda} = \eta'_P \eta_P (-1)^{j'-j+\lambda'-\lambda} G^+_{\lambda'\lambda}$$

Deuteron Helicity Amplitudes

LF Time-Reversal: $Y_T = R_y(\pi)T$

$$\begin{array}{ccccc}
 & G^+_{++} & G^+_{+0} & G^+_{+-} & \\
-G^+_{+0} = & G^+_{0+} & G^+_{00} & G^+_{0-} & = -G^+_{0+} \\
 & G^+_{-+} & G^+_{-0} & G^+_{--} & = G^+_{++} \\
 & \| & \| & & \\
 & G^+_{+-} & -G^+_{+0} & &
\end{array}$$

$$G^+_{\lambda'\lambda} = (-1)^{\lambda'-\lambda} G^+_{\lambda\lambda'}$$

C.Carlson and C.Ji,
Phys.Rev.D67,116002(03)

$$(2\eta+1)G^{+}_{++} + \sqrt{8\eta}\,G^{+}_{0+} + G^{+}_{+-} - G^{+}_{00} = 0$$

Angular Condition

$$
\begin{array}{ccccc}
 & G^{+}_{++} & G^{+}_{+0} & G^{+}_{+-} & \\
-G^{+}_{+0} = & G^{+}_{0+} & G^{+}_{00} & G^{+}_{0-} & = -G^{+}_{0+} \\
 & G^{+}_{-+} & G^{+}_{-0} & G^{+}_{--} & = G^{+}_{++} \\
 & \parallel & \parallel & & \\
 & G^{+}_{+-} & -G^{+}_{+0} & &
\end{array}
$$

High Q Scaling in QCD

$$G^{+}_{\lambda'\lambda} = \left(\frac{\Lambda}{Q}\right)^{|\lambda'-\lambda_{\min}|+|\lambda-\lambda_{\min}|} \qquad G^{+}_{\lambda_{\min}\lambda_{\min}} \propto \left(\frac{\Lambda}{Q}\right)^{2(n-1)+|\lambda'-\lambda_{\min}|+|\lambda-\lambda_{\min}|}$$

n = the number of quarks in the state;

λ_{min}= 0 (bosons) or 1/2 (fermions);

Λ = QCD scale

$$G^{+}_{00} \sim \frac{1}{Q^4}\ ;\ G^{+}_{+0} = a\left(\frac{\Lambda}{Q}\right)G^{+}_{00}\ ;\ G^{+}_{+-} = \left(b\frac{\Lambda}{Q}\right)^2 G^{+}_{00}\ ;\ G^{+}_{++} = \left(c\frac{\Lambda}{Q}\right)^2 G^{+}_{00}$$

$$\boxed{(2\eta+1)G^{+}_{++} + \sqrt{8\eta}\,G^{+}_{0+} + G^{+}_{+-} - G^{+}_{00} = 0}$$

$$1+\sqrt{2}\,\frac{a\Lambda}{M} - \frac{1}{2}\left(\frac{c\Lambda}{M}\right)^2 = 0\ ;\ a\ or\ c\ must\ be\ \mathrm{O}\left(\frac{M}{\Lambda}\right) \approx 20$$

Leading PQCD postponed to a larger Q region; 12GeV upgrade anticipated.

Conclusion

- Angular momentum in LFD is not just formal but consequential in the analysis of physical observables.

- LF helicity amplitudes are independent of all references frames that are related by LF boosts.

- Model independent constraints can be made using LFD.