Spin decomposition in QCD

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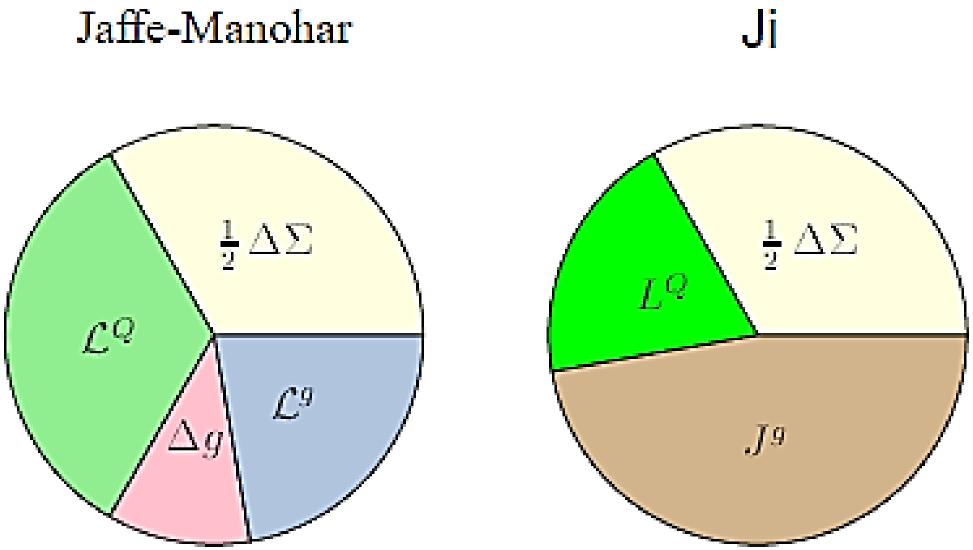
Outline

- Old problem and new idea
- `Physical’ and `pure gauge’ parts of the gauge field
- Orbital angular momentum

Dilemma in spin decomposition



Enormous experimental and theoretical efforts to determine ΔG ← nonexistent in Ji scheme…

Jaffe-Manohar scheme not gauge invariant

The commutation relation $\vec{L}\times\vec{L}=i\vec{L}$ often abandoned.

A new proposal

Chen, Lu, Sun, Wang, Goldman (2008, 2009)

Identify the ``physical” and ``pure gauge” parts

$$A^\mu = A^\mu_{phys} + A^\mu_{pure}$$

$$F^{\mu\nu}_{pure} = 0$$

$$A^\mu_{phys} \to U^\dagger A^\mu_{phys} U \,,$$

$$A^\mu_{pure} \to U^\dagger A^\mu_{pure} U - \frac{i}{g} U^\dagger \partial^\mu U$$

A complete, gauge invariant decomposition

Covariant form

Wakamatsu (2010)

$$M^{\mu\nu\lambda}_{\text{quark-spin}} = -\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_5\gamma_\sigma\psi \,,$$

$$M^{\mu\nu\lambda}_{\text{quark-orbit}} = \bar{\psi}\gamma^\mu(\underline{x^\nu iD^\lambda_{\text{pure}} - x^\lambda iD^\nu_{\text{pure}}})\psi \,,$$

$$M^{\mu\nu\lambda}_{\text{gluon-spin}} = F^{\mu\lambda}_a A^{\nu a}_{\text{phys}} - F^{\mu\nu}_a A^{\lambda a}_{\text{phys}} \,,$$

$$M^{\mu\nu\lambda}_{\text{gluon-orbit}} = F^{\mu\alpha}_a\Big(x^\nu(D^\lambda_{\text{pure}}A^{\text{phys}}_\alpha)_a - x^\lambda(D^\nu_{\text{pure}}A^{\text{phys}}_\alpha)_a\Big)$$

$\vec{L} = \vec{x} \times i\vec{D}_{pure}$ satisfies the commutation relation

Goal

Revamp the approach by Chen et al.

Obtain a practical, gauge invariant scheme in which the gluon helicity is given by ΔG.

Find a way to access matrix elements involving D^{μ}_{pure}

The physical part A^{μ}_{phys}

Our proposal (working in the infinite momentum frame)

$$A^{\mu}_{phys}(x) = -\int dy^{-}\mathcal{K}(y^{-}-x^{-})P\exp\left(-ig\int_{y^{-}}^{x^{-}} A^{+}(z^{-})dz'^{-}\right)F^{+\mu}(y^{-})$$

where $\mathcal{K}(y^{-}) = \frac{1}{2}\epsilon(y^{-})$, or $\theta(y^{-})$, or $-\theta(-y^{-})$

Transformation rule OK $A^{\mu}_{\text{phys}} \to U^{\dagger} A^{\mu}_{\text{phys}} U$

The gluon helicity $M^{\mu\nu\lambda}_{\text{gluon-spin}} = F^{\mu\lambda}_{a} A^{\nu a}_{\text{phys}} - F^{\mu\nu}_{a} A^{\lambda a}_{\text{phys}}$ coincides with ΔG

Need to check if $A^{\mu}_{pure} = A^{\mu} - A^{\mu}_{phys}$ is pure gauge

Hint: $A^{+}_{phys} = 0$

The pure gauge part A^{μ}_{pure}

$$A^{+}_{phys} = 0$$

→ $A^{+}_{pure} = A^{+} = -\frac{i}{g} V \partial^{+} V^{\dagger}$ $\qquad V(x) = P \exp\left(-ig \int_{\pm\infty}^{x^{-}} A^{+}(x'^{-}, \vec{x}) dx'^{-} \right)$

→ $A^{+}_{pure} = -\frac{i}{g} V W \partial^{+} (V W)^{\dagger}$

$W(\pm\infty)$: independent of x^{-}

→ $A^{\mu}_{pure} = -\frac{i}{g} V W \partial^{\mu} (V W)^{\dagger}$

VW is the rotation matrix to the LC gauge $A^{+} = 0$

W represents the residual gauge freedom

$A^{\mu}_{pure} = 0$ in the LC gauge

Proof of $A^\mu = A^\mu_{phys} + A^\mu_{pure}$

With our choice of A^μ_{pure} , the defining relation for A^μ_{phys} is actually an identify

$$A^\mu_{phys}(x) = -\frac{1}{2}\int dy^- \epsilon(y^- - x^-)\underbrace{P\exp\left(-ig\int_{y^-}^{x^-} \overset{A^+_{pure}}{\swarrow} A^+(z^-)dz^-\right)}_{VW(x^-)\underbrace{(VW)^\dagger(y^-)}_{F^{+\mu}_{LC} = \frac{\partial}{\partial y^-}A^\mu_{LC}}} F^{+\mu}(y^-)$$

$$= A^\mu_{phys}(x) - \frac{1}{2}VW(x)\underbrace{\left(A^\mu_{LC}(\infty) + A^\mu_{LC}(-\infty)\right)}$$

Boundary terms at $x^- = \pm\infty$

Boundary conditions in the LC gauge

$$A^{\mu}_{phys}(x) = -\frac{1}{2}\int dy^{-}\epsilon(y^{-}-x^{-})P\exp\left(-ig\int_{y^{-}}^{x^{-}} A^{+}(z^{-})dz^{-}\right)F^{+\mu}(y^{-})$$

$$\Longrightarrow \qquad A^{\mu}_{LC}(\infty) + A^{\mu}_{LC}(-\infty) = 0$$

$\mathcal{K}(y^{-}) = \frac{1}{2}\epsilon(y^{-}),$	Antisymmetric	$A^{\mu}_{LC}(\infty) = -A^{\mu}_{LC}(-\infty)$
$\theta(y^{-})$	Advanced	$A^{\mu}_{LC}(\infty) = 0$
$-\theta(-y^{-})$	Retarded	$A^{\mu}_{LC}(-\infty) = 0$

Orbital angular momentum

“Canonical” OAM

Chen et al. (2008)

Ji (1997)

$$\begin{aligned}\bar{\psi}\gamma^{\mu}(x^{\nu}iD^{\lambda}-x^{\lambda}iD^{\nu})\psi &= \bar{\psi}\gamma^{\mu}(x^{\nu}iD^{\lambda}_{pure}-x^{\lambda}iD^{\nu}_{pure})\psi \\ &+ \bar{\psi}\gamma^{\mu}(x^{\lambda}A^{\nu}_{phys}-x^{\nu}A^{\lambda}_{phys})\psi\end{aligned}$$

“Potential” OAM

Burkardt (2009), Wakamatsu (2010)

Because $A^{\mu}_{pure}=0$ in the LC gauge in our scheme,

$$L_{Chen}=L_{Jaffe-Manohar}$$

Potential angular momentum (I)

$$\epsilon^{ij} L_{\rm pot} = \frac{1}{2P^+}\langle PS| x^i \bar{\psi}(x)\gamma^+(-g)(x^i A^j_{\rm phys} - x^j A^i_{\rm phys})\psi(x)|PS\rangle$$

$$= \frac{1}{2P^+}\langle PS|\Big\{x^i \bar{\psi}(x)\gamma^+ \int dy^- \mathcal{K}(y^- - x^-)\mathcal{W}^-_{xy}\, gF^{+j}(y^-,\vec{x})\mathcal{W}^-_{yx}\psi(x)$$

$$-x^j \bar{\psi}(x)\gamma^+ \int dy^- \mathcal{K}(y^- - x^-)\mathcal{W}^-_{xy}\, gF^{+i}(y^-,\vec{x})\mathcal{W}^-_{yx}\psi(x)\Big\}|PS\rangle\,.$$

Related to a nonforward matrix element

$$\langle P'S'|\bar{\psi}(0)\gamma^+ \int dy^- \mathcal{K}(y^-)\,\mathcal{W}^-_{0y}\, gF^{+i}(y^-)\mathcal{W}^-_{y0}\psi(0)|PS\rangle = i\epsilon^{ij}\Delta_j \bar{S}^+ \underline{f(\xi)} + \cdots\,,$$

$$\xi = -\frac{\Delta^+}{2P^+}$$

$$L_{\rm pot} = f(0)\frac{S^+}{P^+}$$

Potential angular momentum (II)

Nonforward generalization of the Qiu-Sterman matrix element

$$
\begin{aligned}
T^{\mu\nu}(x_1,x_2,\xi) = &\int \frac{dy^- dz^-}{(2\pi)^2} e^{\frac{i}{2}(x_1+x_2)\bar{P}^+ z^- + i(x_2-x_1)\bar{P}^+ y^-} \\
&\times \langle P'S'|\bar{\psi}(-z^-/2)\gamma^+ W^-_{\frac{-z}{2}y}\, gF^{\mu\nu}(y^-) W^-_{y\frac{z}{2}}\psi(z^-/2)|PS\rangle \\
&= \frac{1}{\bar{P}^+}\epsilon^{\mu\nu\rho\sigma}\bar{S}_\rho \underbrace{\bar{P}_\sigma \Psi(x_1,x_2,\xi)}_{\text{SSA}} + \frac{1}{\bar{P}^+}\epsilon^{\mu\nu\rho\sigma}\bar{S}_\rho \Delta_\sigma \underbrace{\Phi(x_1,x_2,\xi)}_{\text{OAM}} + \cdots .
\end{aligned}
$$

$$
L_{\text{pot}} = \int dX\, dx\, \mathcal{K}(x)\Phi(X,x,0)
$$

$X = \frac{x_1+x_2}{2},\ x = x_1 - x_2.$

$$
\mathcal{K}(x) = p.v.\frac{1}{x}, \quad \frac{1}{x \pm i\epsilon}
$$

Canonical angular momentum

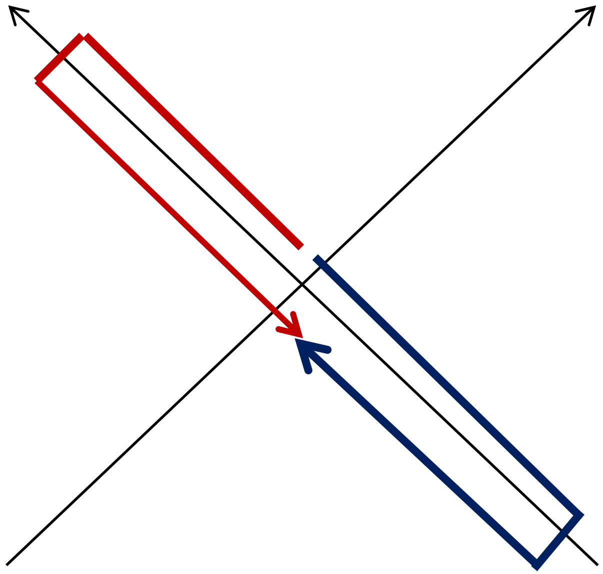
Inspired by the relation between the Sivers function approach and the twist-three approach to SSA

Boer, Mulders, Pijlman (2003)

$$f(x,q_T,\Delta) \equiv \int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+z^- - iq_T\cdot z_T}$$
$$\times \langle P'S'|\bar{\psi}(-z^-/2,-z_T/2)\gamma^+ W^-_{\frac{-z}{2},\pm\infty} W^T_{\frac{-z_T}{2},\frac{z_T}{2}} W^-_{\pm\infty,\frac{z}{2}} \psi(z^-/2,z_T/2)|PS\rangle$$

↑

nonforward TMD



``Generalized parton correlation function"

Meissner, Metz, Schlegel (2009)

$$\int dx \int d^2q_T \, q_T^i f(x, q_T, \Delta)$$ ← double moment in x, q_T

$$= \frac{1}{\bar{P}+}\left\{\frac{1}{2}\langle P'S'|\bar{\psi}(0)\gamma^+(i\overrightarrow{D}^i - i\overleftarrow{D}^i)\psi(0)|PS\rangle \right.$$

$$\left. -\langle P'S'|\bar{\psi}(0)\gamma^+ \int dy^- \mathcal{K}(y^-)\mathcal{W}^-_{0y}\, gF^{+i}(y^-)\mathcal{W}^-_{y0}\psi(0)|PS\rangle \right\}$$

$$= \frac{1}{\bar{P}+}\left\{\frac{1}{2}\langle P'S'|\bar{\psi}(0)\gamma^+(i\overrightarrow{D}^i - i\overleftarrow{D}^i)\psi(0)|PS\rangle + \langle P'S'|\bar{\psi}(0)\gamma^+ A^i_{\text{phys}}\psi(0)|PS\rangle\right\}$$

$$= \frac{1}{2\bar{P}+}\langle P'S'|\bar{\psi}(0)\gamma^+(i\overrightarrow{D}^i_{\text{pure}} - i\overleftarrow{D}^i_{\text{pure}})\psi(0)|PS\rangle$$

$$f(x, q_T, \Delta) \sim \frac{i}{\bar{P}+}\epsilon^{+-ij} S^+ q_{Ti} \Delta_j \tilde{f}(x, q_T^2, \xi, \Delta_T \cdot q_T)$$

➡ $$L_{can} = \frac{S^+}{P^+}\frac{1}{2}\int dx d^2q_T \, q_T^2 \tilde{f}(x, q_T^2)$$

No sign flip !

cf. Wigner function approach Lorce, Pasquini (2011)

Gluon OAM

Similarly we define "Generalized gluon correlation function"

$$g(x,q_T,\Delta) \equiv -i\int \frac{dz^-d^2z_T}{(2\pi)^3}e^{ixP^+z^- - iq_T\cdot z_T}$$

$$\times \langle P'S'|F^{+\alpha}(-z^-/2,-z_T/2)\mathcal{W}^-_{\frac{-z}{2},\pm\infty}\mathcal{W}^T_{\frac{-z_T}{2},\frac{z_T}{2}}\mathcal{W}^-_{\pm\infty,\frac{z}{2}}A^{\rm phys}_\alpha(z^-/2,z_T/2)|PS\rangle$$

$$= \frac{i}{P^+}\epsilon^{+-ij}S^+q_{Ti}\Delta_j\tilde{g}(x,q_T^2)+\cdots.$$

$$L_{gluon} = \frac{S^+}{P^+}\frac{1}{2}\int dx d^2q_T\, q_T^2\tilde{g}(x,q_T^2)$$

Summary

With our specific choice of A^{μ}_{phys},

- The gluon helicity coincides with Δg
- Canonical OAM by Chen et al. = gauge invariant generalization of Jaffe-Manohar and Lorce-Pasquini (from Wigner distribution)
- All components are manifestly gauge invariant, could be observable on a lattice

  or in high energy processes.