Physical Degrees of Freedom for Gauge Fields and the Question of Spin

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A BASIC PRINCIPLE ?

Operators must satisfy
the commutation relations
of the physical quantities
that they are meant to represent
in order for their matrix elements
to be identified
with those physical quantities.

Outline

I. Spin, boosts and angular momentum
II. Lorentz irreps and angular momentum
III. Aside: Gauge non-Invariance of Hydrogen Eigenenergies
IV. Gauge invariance and canonical angular momentum in QED
V. Gauge Invariance and canonical commutation relation in QCD (for nucleon spin operators)
VI. Conclusion
VII. Additional Material (Expt+Thy)

I. Spin, boosts and angular momentum

Binding moves spin from static non-relativistic view to include orbital contributions

e.g. Electron in hydrogenic atom: (where $\gamma = \sqrt{1 - Z^2\alpha^2}$)

$$\psi \propto \begin{bmatrix} 1 \\ 0 \\ -\imath\frac{(1-\gamma)}{Z\alpha}\cos\theta \\ \imath\frac{(1-\gamma)}{Z\alpha}\sin\theta e^{\imath\phi} \end{bmatrix}$$

but norm

$$\psi^\dagger\psi \propto \{1 + \left[\frac{(1-\gamma)}{Z\alpha}\right]^2 [(\cos\theta)^2 + (\sin\theta)^2]\}$$

{1st correction at $\mathcal{O}(Z^4\alpha^4)$ }

and so "spin"

$$\psi^\dagger\Sigma_3\psi \propto \frac{1}{1 + \left[\frac{(1-\gamma)}{Z\alpha}\right]^2}\{1 + \left[\frac{(1-\gamma)}{Z\alpha}\right]^2 [(\cos\theta)^2 - (\sin\theta)^2]\}$$

which integrates to

$$\frac{1}{1 + \left[\frac{(1-\gamma)}{Z\alpha}\right]^2} < 1$$

Difference must be made up by orbital

Axial current or generator of rotations?

$$\frac{1}{2}\bar{\psi}\gamma^3\gamma^5\psi = \frac{1}{2}\psi^\dagger\Sigma_3\psi \qquad \Sigma_3 = \begin{bmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{bmatrix}$$

from generator of 3-axis rotations:

$$\frac{1}{2}\bar{\psi}\sigma_{12}\psi = \frac{1}{2}\psi^\dagger \begin{bmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{bmatrix} \psi$$

Same effect for bound state wavefunctions,

but -- $\int(c^2 - s^2) = \int(s^2 - c^2) = 0$

Or recall Melosh: $\vec{S}_\perp$ and $\vec{S}_\parallel$ boost differently + Wigner rotation

Basic Boosts:

Accelerating a polarized fermion from rest distributes angular momentum from spin to spin plus orbital angular momentum

Rest Frame solution of Dirac equation for spin up fermion:

$$\psi(x,t) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} e^{-\imath(mt = p^{\mu} x_{\mu})}$$

Boost FRAME - along spin direction

$$\Psi(p^\mu x_\mu) \rightarrow e^{-i\sigma_{03}\omega/2}\,\Psi(p^\mu x_\mu)$$

$$= e^{\left\{-\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}\omega/2\right\}}\,\Psi(p^\mu x_\mu)$$

$$= \left\{\cosh(\omega/2)\mathbf{1} - \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}\sinh(\omega/2)\right\}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-i(Et-pz)}$$

$$= \begin{pmatrix} \cosh(\omega/2) \\ 0 \\ \sinh(\omega/2) \\ 0 \end{pmatrix} e^{-i(Et-px)}$$

$$\cosh(\omega) = E/m$$

$$\sinh(\omega) = p/m$$

$$\cosh(\omega/2) = \sqrt{\{[1+\cosh(\omega)]/2\}}$$

$$\sinh(\omega/2) = \sqrt{\{[\cosh(\omega)-1]/2\}}$$

or in terms of energy and momentum

$$\Psi_L(p^\mu x_\mu) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix} e^{-i(Et-px)}$$

Cf. σ•p/(E+m) → Spin-flip + Orbital L=1

¡ *lim* p → E → ∞ ! c.f. p = 0

$$\psi^\dagger \Sigma_3 \psi = \frac{(E+m)^2 + p^2}{2m(E+m)} = \frac{E}{m}$$

$$\bar{\psi} \sigma_{12} \psi = \frac{(E+m)^2 - p^2}{2m(E+m)} = 1$$

Boost FRAME- transverse to spin direction

$$\Psi(p^\mu x_\mu) \rightarrow e^{-i\sigma_{01}\omega/2}\,\Psi(p^\mu x_\mu)$$

$$= e^{-\left\{\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}\omega/2\right\}}\,\Psi(p^\mu x_\mu)$$

$$= \left\{ \cosh(\omega/2)\mathbf{1} - \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}\sinh(\omega/2)\right\}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-i(Et-pz)}$$

$$= \begin{pmatrix} \cosh(\omega/2) \\ 0 \\ 0 \\ -\sinh(\omega/2) \end{pmatrix} e^{-i(Et-px)}$$

$$\cosh(\omega) = E/m$$

$$\sinh(\omega) = p/m$$

$$\cosh(\omega/2) = \sqrt{\{[1+\cosh(\omega)]/2\}}$$

$$\sinh(\omega/2) = \sqrt{\{[\cosh(\omega)-1]/2\}}$$

or in terms of energy and momentum

$$\Psi_T(p^\mu x_\mu) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -p/(E+m) \end{pmatrix} e^{-i(Et-px)}$$

$$\psi^\dagger \Sigma_3 \psi = \frac{(E+m)^2 - p^2}{2m(E+m)} = 1$$

$$\bar{\psi}\sigma_{12}\psi = \frac{(E+m)^2 + p^2}{2m(E+m)} = \frac{E}{m}$$

II. Lorentz irreps and angular momentum

Rest Frame Spin-j "Weyl" Spinors

$$\phi_j^R(0^\mu) = \frac{m^j}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \phi_{j-1}^R(0^\mu) = \frac{m^j}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

$$\cdots \qquad \phi_{-j}^R(0^\mu) = \frac{m^j}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

Weinberg-Soper front-form formalism

$$0^\mu \equiv (p^+ = m, p^1 = p^2 = 0, p^- = m)$$

Column Index = $\dot{\zeta}$

Wigner Conjugation Operators

$$\Theta_{[1/2]} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

transforms R = (1/2, 0) Weyl spinor into L = (0,1/2) spinor

$$\psi^{R}_{\frac{1}{2}\dot{\zeta}} \rightarrow \psi^{L}_{\frac{1}{2}\zeta}$$

$$\Theta_{[1]} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

transforms R = (1,0) Weyl-like spinor into L = (0,1) spinor

$$\phi^{R}_{1\dot{\zeta}} \rightarrow \phi^{L}_{1\zeta}$$

Two independent R's, one transformed to L make (1/2, 0) ⊕ (0,1/2) Dirac bispinor

One R ⊕ same transformed to L makes self (or anti-self)-conjugate Majorana bispinor

Similarly for spin-1: self-conjugate bi"spinor" has no charge

After boosting along the 3-axis (quantization axis):

$$\phi_1^R \to \frac{m}{\sqrt{2}} \begin{bmatrix} \frac{p^+}{m} \\ 1 \\ \frac{m}{p^+} \end{bmatrix}$$

In limit p+ → ∞, bispinor → $\phi_1^\xi \propto \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ Wigner-Weyl representation

$\xi \sim \{\zeta\dot{\zeta}\}$

ignoring the plane wave factor.

Note that only helicity = ±1 survive in the massless limit -- and only +1 or -1, violating parity, for each part {(1,0) or (0,1)} separately.

2nd state $\phi_1 \sim \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Structure of spin-1 "Weyl"-spinor, ϕ_1^ξ interaction with spin-½ Weyl-spinor field, χ_ζ :

$$\chi_\zeta^\dagger \bar{\Gamma}_\xi^{\zeta\zeta'} \phi_1^\xi \chi_{\zeta'} \quad \text{using} \quad (\frac{1}{2},0) \otimes (\frac{1}{2},0) = (1,0) \oplus (0,0)$$

Is there a Lorentz group Clebsch-Gordan coefficient $\tilde{\Gamma}_\xi^\mu$

to form a relation to the conventional photon field:

$$\tilde{\Gamma}_\xi^\mu \phi_1^\xi \leftrightarrow \text{ parts of } A^\mu$$

RHS in $(\frac{1}{2}, \frac{1}{2})$ irrep

(J = 0 + 1)

i.e., in Dirac form:

$$\bar{\Psi} \Gamma_\xi \phi_1^\xi \Psi \quad : \quad \Gamma_\xi \leftrightarrow \gamma_\mu \tilde{\Gamma}_\xi^\mu$$

No?

This leaves some of the possible A_μ components without values.

III. Gauge non-Invariance of Pauli Hamiltonian Hydrogen Eigenenergies

$$H_D\psi = \imath\frac{\partial}{\partial t}\psi$$
$$= E\psi$$

$$A_0 = \frac{Z}{r}$$

$$H_D = \vec{\gamma}\cdot(\vec{p} - e\vec{A}) + m - eA_0$$

$$\vec{J} = \vec{L} + \vec{S}$$
$$= \vec{r}\times\vec{p} + \frac{1}{2}\vec{\Sigma}$$

$$\vec{\Sigma} = \begin{bmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{bmatrix}$$

Bjorken & Drell, Vol.1 (1964) p.52

T. Goldman, *Phys. Rev.* D15 (1977) 1063.

See also: Wei-min Sun: Time Evolution Op $\neq \int T_{00}$

$$i\frac{\partial\psi'}{\partial t} = \left(UHU^{-1} - iU\frac{\partial U^{-1}}{\partial t}\right)\psi' \equiv H'\psi'$$

$$U = \exp[-iHf(t)]$$

$$H_P = UH_D U^{-1} - iU\frac{\partial}{\partial t}U^{-1}$$

$$\int d^3x\, \psi'^\dagger H'\psi' = (1+\dot{f})\sum_n |c_n|^2 E_n$$

$$\neq \sum_n |c_n|^2 E_n = \int d^3x\, \psi^\dagger H\psi$$

Pauli and Dirac Hamiltonians are not unitarily equivalent

$$U = \exp[-iHf(t)] \rightarrow U = e^{[\beta\vec{\alpha}\cdot(\vec{p}-e\vec{A})/2m]}$$

Foldy-Wouthuysen

$$H_P = UH_D U^{-1} - iU\frac{\partial}{\partial t}U^{-1}$$

$$\vec{E} = -\vec{\nabla}A_0 - \frac{\partial\vec{A}}{\partial t}$$

No problem only if:

$$\vec{\nabla}\cdot\vec{A} = 0$$
$$\vec{\sigma}\cdot\frac{\partial\vec{A}}{\partial t}\times\vec{p} = 0$$
$$\vec{\sigma}\cdot\vec{\nabla}\times\frac{\partial\vec{A}}{\partial t} = 0$$

$$\simeq \beta\left[m + \frac{(\vec{p}-e\vec{A})^2}{2m}\right] - eA_0 - \frac{e}{2m}\beta\vec{\sigma}\cdot\vec{B}$$

$$-\frac{ie}{8m^2}\vec{\sigma}\cdot\vec{\nabla}\times\vec{E} - \frac{e}{4m^2}\vec{\sigma}\cdot\vec{E}\times\vec{p} - \frac{e}{2m^2}\vec{\nabla}\cdot\vec{E}$$

i.e., OK only in Coulomb gauge

Gauge-invariant hydrogen-atom Hamiltonian

Wei-Min Sun, Xiang-Song Chen, Xiao-Fu Lü and Fan Wang

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Time Evolution Op ≠ ∫T_{00} : Generator of time translation is not identical to energy operator

Identifying $A^\mu = (A^0, \vec{A}) = (\frac{e}{4\pi r} - \frac{\partial f}{\partial t}, \vec{\nabla} f)$ as the em field produced by the proton in $M_p=\infty$ limit, and solving for H with gauge $\vec{\nabla}\cdot\vec{A} = \nabla^2 f(\vec{x},t)$

The standard Lagrangian gives the energy of the electron in the field of the proton as: $-e^2/4\pi r$ and hence the Hydrogen Hamiltonian

$$H = \vec{\alpha}\cdot(\vec{p} + e\vec{\nabla} f) + \beta m - \frac{e^2}{4\pi r} = \vec{\alpha}\cdot(\vec{p} + e\vec{A}) + \beta m - eA^0_{phys}$$

Whereas the Dirac equation

$$\imath\frac{\partial}{\partial t}\psi_e = \left(-\imath\vec{\alpha}\cdot\vec{\nabla} + e\vec{\alpha}\cdot\vec{\nabla} f + \beta m - \frac{e^2}{4\pi r} + e\frac{\partial f}{\partial t}\right)\psi_e = H_D\psi_e$$

gives $\quad H_D = \vec{\alpha}\cdot(\vec{p} + e\vec{A}) + \beta m - eA^0$

which generates time translations but is not equal to the energy (except in Coulomb gauge)

$$e^{-\imath E_n t}\psi_n(\vec{x}) \rightarrow e^{-\imath e f} e^{-\imath E_n t}\psi_n(\vec{x})$$

Some details

$$\partial^2 A^0 - \frac{\partial}{\partial t}\{\frac{\partial A^0}{\partial t} + \vec{\nabla}\cdot\vec{A}\} = -\nabla^2 A^0 - \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = e\delta^3(x) + \rho_e$$

$$\partial^2 \vec{A} + \vec{\nabla}\{\frac{\partial A^0}{\partial t} + \vec{\nabla}\cdot\vec{A}\} = \vec{j}_e$$

$$A^0(\vec{x},t) = \frac{e}{4\pi r} - \frac{\partial}{\partial t} f(\vec{x},t) + \frac{1}{4\pi}\int d^3y \frac{\rho_e(\vec{y},t)}{|\vec{x}-\vec{y}|}$$

$$\vec{A}(\vec{x},t) = \vec{\nabla} f(\vec{x},t) + (\partial^2)^{-1}\vec{j}_e(\vec{x},t) + \frac{1}{4\pi}\int d^3y(\partial^2)^{-1}\left(\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^3}\frac{\partial}{\partial t}\rho_e(\vec{y},t)\right)$$

The difference of Canonical and Belinfante energy-momentum tensor is a surface term which makes the Belinfante tensor symmetric and gauge invariant. But this does not solve the problem that the average value of the Hamiltonian is gauge dependent.

(EM) multipole radiation has the same problem

$$A^0_{phys} = -\frac{1}{\nabla^2}(\rho_p + \rho_e)$$ does not have a unique solution.

IV. Gauge invariance and canonical angular momentum

Straightforward angular momentum decomposition not gauge invariant:

$$
\begin{aligned}
\vec{J}_{QED} &= \vec{S}_e + \vec{L}_e + \vec{S}_\gamma + \vec{L}_\gamma \\
\vec{S}_e &= \int d^3x \, \psi^\dagger \frac{\vec{\Sigma}}{2} \psi \\
\vec{L}_e &= \int d^3x \, \psi^\dagger \vec{x} \times \frac{1}{i} \vec{\nabla} \psi \\
\vec{S}_\gamma &= \int d^3x \, \vec{E} \times \vec{A} \\
\vec{L}_\gamma &= \int d^3x \, \vec{x} \times E^i \vec{\nabla} A^i
\end{aligned}
$$

BUT gauge invariant form does not obey canonical commutation relations:

$$\vec{J}_{QED} = \vec{S}_e + \vec{L}'_e + \vec{J}'_\gamma$$

$$\vec{S}_e = \int d^3x\, \psi^\dagger \frac{\vec{\Sigma}}{2} \psi$$

$$\vec{L}'_e = \int d^3x\, \psi^\dagger \vec{x} \times \frac{1}{\imath} \vec{D} \psi$$

$$\vec{J}'_\gamma = \int d^3x\, \vec{x} \times (\vec{E} \times \vec{B})$$

Therefore, despite the labels, $\vec{L}'_e$ and $\vec{J}'_\gamma$ are NOT angular momenta!

QM example: $[(\vec{x} \times \frac{1}{i}\vec{\nabla})_j, (\vec{x} \times \frac{1}{i}\vec{\nabla})_k] = \imath\epsilon_{jkl}[\vec{x} \times \frac{1}{i}\vec{\nabla}]_l$

Using the gauge invariant "mechanical" momentum generates an extra term

$$[(\vec{x} \times \frac{1}{i}(\vec{\nabla} - ie\vec{A}))_j, (\vec{x} \times \frac{1}{i}(\vec{\nabla} - ie\vec{A}))_k]$$
$$= \imath\epsilon_{jkl}\{[\vec{x} \times \frac{1}{i}(\vec{\nabla} - ie\vec{A})]_l + ex_l\vec{x} \cdot (\vec{\nabla} \times \vec{A})\}$$

But OK if we define a part of the vector field as $\vec{A} = \vec{A}_{pur}$

such that $\vec{\nabla} \times \vec{A}_{pur} = 0$

See, e.g.: D. Singleton and V. Dzhunushaliev, *Found. Phys.* **30** (2000) 1093.

Both requirements can be satisfied by identifying physical and pure gauge parts of the gauge field:

$$\vec{A} \equiv \vec{A}_{fys} + \vec{A}_{pur} \, , \; \vec{D}_{pur} \equiv \vec{\nabla} - \imath e \vec{A}_{pur}$$

$$\vec{\nabla} \cdot \vec{A}_{fys} = 0 \, , \; \vec{\nabla} \times \vec{A}_{pur} = 0$$

$$\begin{aligned}
\vec{J}_{QED} &= \vec{S}_e + \vec{L}''_e + \vec{S}''_\gamma + \vec{L}''_\gamma \\
\vec{S}_e &= \int d^3x \, \psi^\dagger \frac{\vec{\Sigma}}{2} \psi \\
\vec{L}''_e &= \int d^3x \, \psi^\dagger \vec{x} \times \frac{1}{\imath} \vec{D}_{pur} \psi \\
\vec{S}''_\gamma &= \int d^3x \, \vec{E} \times \vec{A}_{fys} \\
\vec{L}''_\gamma &= \int d^3x \, \vec{x} \times E^i \vec{\nabla} A^i_{fys}
\end{aligned}$$

NOT Coulomb gauge: $\vec{\nabla}\cdot\vec{A} \neq 0$

$$\vec{A} \equiv \vec{A}_{fys} + \vec{A}_{pur} \quad \text{only} \quad \vec{\nabla}\cdot\vec{A}_{fys} = 0$$

This defines $\vec{A}_{pur}$ piece

$$\begin{aligned} -\vec{E}_{pur} &= F^{i0}_{pur} \\ &= \partial^i A^0_{pur} - \partial^0 A^i_{pur} \\ &= 0 \end{aligned}$$

Full constraint: $F^{\mu\nu}_{pur} = 0$

$$-(\vec{\nabla})^2 A^0_{pur} - \partial_t \vec{\nabla}\cdot\vec{A}_{pur} = 0$$

So $\vec{A}_{pur}$ does not contribute to charge either: $\vec{\nabla}\cdot\vec{E}_{pur} = 0$

Cf.: Momentum operator in quantum mechanics

$$\vec{p} = m\dot{\vec{r}} + q\vec{A} = m\dot{\vec{r}} + q\vec{A}_{\perp} + q\vec{A}_{||}$$

$$\vec{p} - q\vec{A}_{||} = m\dot{\vec{r}} + q\vec{A}_{\perp}$$

$$\vec{\nabla} \cdot \vec{A}_{\perp} = 0 \quad \vec{\nabla} \times \vec{A}_{||} = 0$$

Generalized momentum for a charged particle moving in EM field:

- ➥1st form is not gauge invariant, but satisfies the canonical momentum commutation relation.
- ➥2nd form is both gauge invariant and the canonical momentum commutation relation is satisfied.

We recognize

$$\vec{D}_{pur} = \vec{p} - q\vec{A}_{||} = \frac{1}{\imath}\vec{\nabla} - q\vec{A}_{||}$$

as the physical momentum.

It is neither the canonical momentum:

$$\vec{p} = m\dot{\vec{r}} + q\vec{A} = \frac{1}{\imath}\vec{\nabla}$$

nor the mechanical momentum:

$$\vec{p} - q\vec{A} = m\dot{\vec{r}} = \frac{1}{\imath}\vec{D}$$

Gauge transformation

$$\psi' = e^{\imath q \omega(x)} \psi \,, \qquad A'_\mu = A_\mu + \partial_\mu \omega(x),$$

only affects the longitudinal part of the vector potential:

$$\vec{A'}_{||} = \vec{A}_{||} + \vec{\nabla}\omega(x),$$

and the time component:

$$\phi' = \phi - \partial_t \omega(x).$$

It does not affect the transverse part:

$$\vec{A'}_{\perp} = \vec{A}_{\perp},$$

so $A_\perp$ is physical.

Hamiltonian of hydrogen atom

Coulomb gauge:

$$\vec{A}^{c}_{//} = 0, \qquad \vec{A}^{c}_{\perp} \neq 0, \qquad A^{c}_{0} = \varphi^{c} \neq 0.$$

Hamiltonian of a nonrelativistic particle:

$$H_{c} = \frac{(\vec{p} - q\vec{A}^{c}_{\perp})^{2}}{2m} + q\varphi^{c}.$$

Gauge transformed becomes:

$$\vec{A}_{//} = \vec{A}^{c}_{//} + \vec{\nabla}\omega(x) = \vec{\nabla}\omega(x), \vec{A}_{\perp} = \vec{A}^{c}_{\perp}, \varphi = \varphi^{c} - \partial_{t}\omega(x)$$

$$H = \frac{(\vec{p} - q\vec{A})^{2}}{2m} + q\varphi = \frac{(\vec{p} - q\vec{\nabla}\omega - q\vec{A}^{c}_{\perp})^{2}}{2m} + q\varphi^{c} - q\partial_{t}\omega.$$

Following this recipe, we introduce a new Hamiltonian:

$$H_{fys} = H + q\partial_t \omega(x) = \frac{(\vec{p} - q\vec{\nabla}\omega - q\vec{A}_\perp^c)^2}{2m} + q\varphi^c$$

The matrix elements are gauge invariant, i.e.,

$$\langle \psi \mid H_{fys} \mid \psi \rangle = \langle \psi^c \mid H_c \mid \psi^c \rangle$$

i.e., the hydrogen energy states calculated in Coulomb gauge are both gauge invariant and physical.

See also Wei-min Sun.

Coulomb gauge Lorentz invariant: $\partial_k [A^k, J^{ab}] = 0$

-- E. B. Manoukian, *J. Phys. G: Nucl.Phys.* **13** (1987) 1013.

QED:

$$\vec{A}_{pur} = \vec{A} - \vec{A}_{fys}$$

$$F^{\mu\nu}_{pur} = 0 \ ; \ F^{\mu\nu}_{fys} = F^{\mu\nu}$$

$$\vec{\nabla} \times \vec{A}_{fys} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \cdot \vec{A}_{fys} = 0 \ ; \ \vec{A}_{fys}(|x| \rightarrow \infty\ ) = 0$$

$$\vec{A}_{fys}(x) = \vec{\nabla} \times \frac{1}{4\pi} \int d^3y \frac{\vec{\nabla}_{(y)} \times \vec{A}(y)}{|\vec{x} - \vec{y}|}$$

$$\vec{A}'_{fys} = \vec{A}_{fys} \ ; \ \vec{A}'_{pur} = \vec{A}'_{pur} - \vec{\nabla}\omega$$

$$A^0_{fys}(x) = \int_{-\infty}^{x} dx^i (\partial_i A^0 + \partial_t A^i - \partial_t A^i_{fys})$$

$$\phi(x) = -\frac{1}{4\pi} \int d^3y \frac{\vec{\nabla}_{(y)} \cdot \vec{A}(y)}{|\vec{x} - \vec{y}|} + \phi_0(x)$$

$$\vec{A}_{pur} = -\vec{\nabla}\phi(x) \ ; \ A^0_{pur} = \partial_t \phi(x) \ ; \ \nabla^2 \phi_0(x) = 0$$

Multipole Radiation

Multipole radiation analysis is based on the decomposition of EM vector potential in Coulomb gauge. The results are physical and gauge invariant, i.e., gauge transformed to other gauges one obtains the same results.

$2P_{3/2} \rightarrow 2P_{1/2} \leftrightarrow$ spin-flip

$2P_{1/2} \rightarrow 1S_{1/2} \leftrightarrow \Delta L$ of 1

Similarly in Dalitz plot analysis to determine particle spin.

V. Gauge Invariance and canonical commutation relation in QCD (for nucleon spin operators)

From the QCD Lagrangian, one can obtain the total angular momentum by a Noether theorem:

$$
\begin{aligned}
\vec{J} &= \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g \\
\vec{S}_q &= \int d^3x\, \psi^\dagger \frac{1}{2}\vec{\Sigma}\, \psi \\
\vec{L}_q &= \int d^3x\, \psi^\dagger\, (\vec{x} \times \frac{1}{i}\vec{\nabla})\psi \\
\vec{S}_g &= 2\int d^3x\, \mathrm{Tr}\,\{\vec{E} \times \vec{A}\} \\
\vec{L}_g &= 2\int d^3x\, \mathrm{Tr}\,\{\vec{x} \times E^i\, \vec{\nabla} A^i\}
\end{aligned}
$$

- Each term in this decomposition satisfies the canonical angular momentum algebra, so they may properly be called, respectively, quark spin, quark orbital angular momentum, gluon spin and gluon orbital angular momentum operators.
- However they are not individually gauge invariant, except for the quark spin.

→ physical meaning obscure

A Gauge Invariant Decomposition:

$$\vec{J} = \vec{S}_q + \vec{L'}_q + \vec{J'}_G$$

$$\vec{S}_q = \frac{1}{2}\int d^3x \, \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L'}_q = \int d^3x \, \psi^\dagger \vec{x} \times \frac{\vec{D}}{i} \psi$$

$$\vec{J'}_G = 2\int d^3x \, \{\vec{x} \times (\vec{E}^a \times \vec{B}^a)\}$$

- These terms do not separately satisfy the canonical angular momentum algebra (except the quark spin). In this sense the second and third terms are not quark orbital and gluon angular momentum operators.
- The physical meaning of these operators is obscure also.
- Gluon spin and orbital angular momentum operators are not separately gauge invariant; only the total angular momentum of the gluon is gauge invariant.

(Similarly for the photon, but we do have polarized photon beams!)

Our Solution - A different decomposition: Gauge invariance and angular momentum algebra both satisfied for individual terms. Key point is to separate out the transverse and longitudinal parts of the gauge field.

Essential task: to separate properly

the pure gauge field: $\vec{A}_{pur}$

from the physical one: $\vec{A}_{fys}$

$$\vec{A} = \vec{A}_{pur} + \vec{A}_{fys} \qquad \vec{A}_{\Box} = T^a \vec{A}^a_{\Box}$$

Fundamental: $\vec{D}_{pur} = \vec{\nabla} - ig\vec{A}_{pur}$

$$\vec{D}_{pur} \times \vec{A}_{pur} = \vec{\nabla} \times \vec{A}_{pur} - ig\vec{A}_{pur} \times \vec{A}_{pur} = 0$$

Adjoint: $\vec{\mathcal{D}}_{pur} = \vec{\nabla} - ig[\vec{A}_{pur}, \ ]$

$$\vec{\mathcal{D}}_{pur} \cdot \vec{A}_{fys} = \vec{\nabla} \cdot \vec{A}_{fys} - ig[A^i_{pur}, A^i_{fys}] = 0$$

QCD: $$\vec{\nabla} \cdot \vec{A}_{fys} = \imath g[A^i - A^i_{fys}, A^i_{fys}] = \imath g[A^i, A^i_{fys}]$$

$$\vec{\nabla} \times \vec{A}_{fys} = \vec{\nabla} \times \vec{A} = \imath g(A^i - A^i_{fys}) \times (A^i - A^i_{fys})$$

$$\partial_t A^0_{fys} = \partial_i A^0 + \partial_t(A^i - A^i_{fys}) - \imath g[A^i - A^i_{fys}, A^0 - A^0_{fys}]$$

Solve perturbatively: $$\vec{\nabla} \times \vec{A}_{pur} = ig\vec{A}_{pur} \times \vec{A}_{pur}$$

$$\vec{\nabla} \cdot \vec{A}_{pur} = \vec{\nabla} \cdot \vec{A} - ig[A^i_{pur}, A^i]$$

$$\partial_i A^0_{pur} = -\partial_t A^i_{pur} + ig[A^i_{pur}, A^0_{pur}]$$

Gauge transformation: $$\boxed{\vec{A}'_{fys} = U\vec{A}_{fys}U^\dagger}$$

$$\vec{A}'_{pur} = U\vec{A}_{pur}U^\dagger - \frac{i}{g}U\vec{\nabla}U^\dagger$$

New decomposition

$$\vec{J}_{QCD} = \vec{S}_q + \vec{L}''_q + \vec{S}''_g + \vec{L}''_g$$

$$\vec{S}_q = \int d^3x\, \psi^\dagger \frac{\vec{\Sigma}}{2} \psi$$

$$\vec{L}''_q = \int d^3x\, \psi^\dagger\, \vec{x} \times \frac{1}{\imath} \vec{D}_{pur} \psi$$

$$\vec{S}''_g = \int d^3x\, \vec{E} \times \vec{A}_{fys}$$

$$\vec{L}''_g = \int d^3x\, \vec{x} \times E^i\, \vec{\mathcal{D}}_{pur} A^i_{fys}$$

We have chosen a separation between physical and gauge pieces of the gauge vector potential and consistently separated the gauge boson and fermion degrees of freedom in the interacting case.

190 *M. Burkardt*

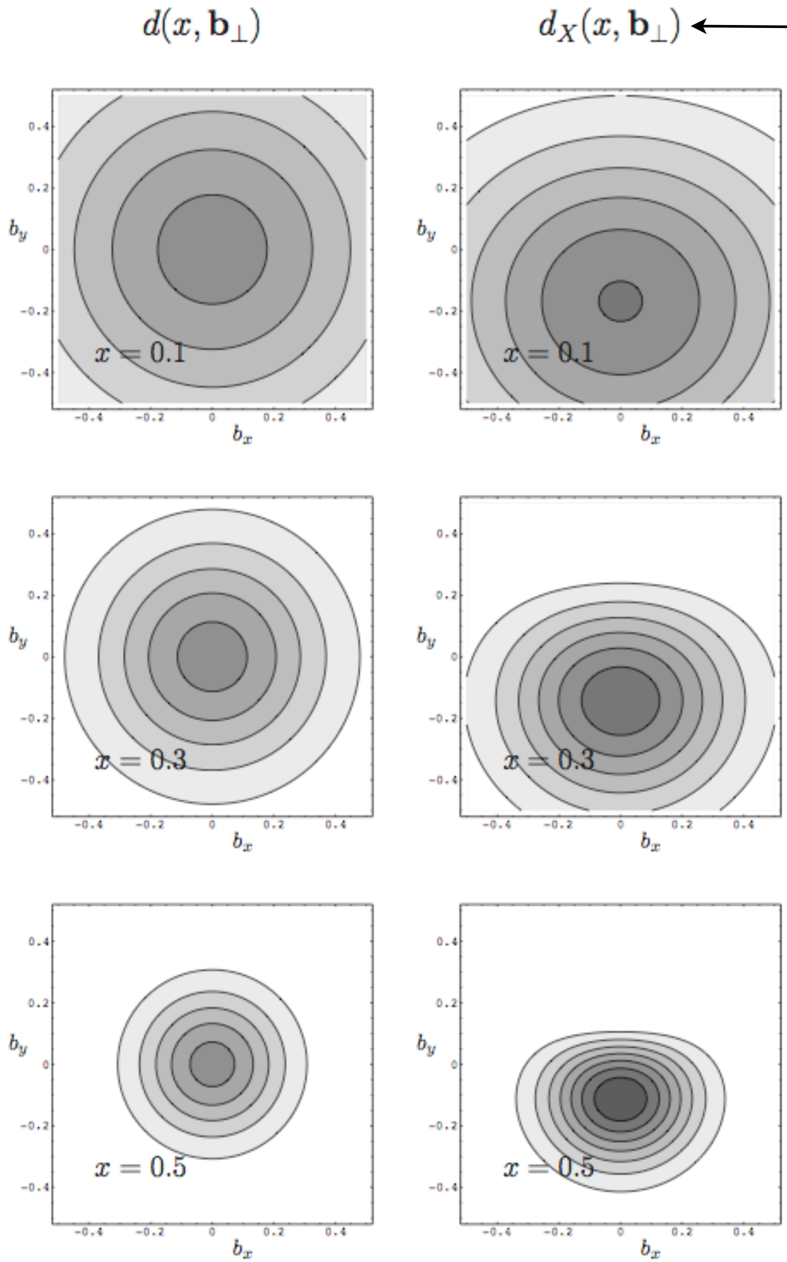


Fig. 4. Same as Fig. 3, but for d quarks.

← for transversely polarized proton

Lattice

M. Burkhardt, IJMPA 18 (2003) 173

184 *M. Burkardt*

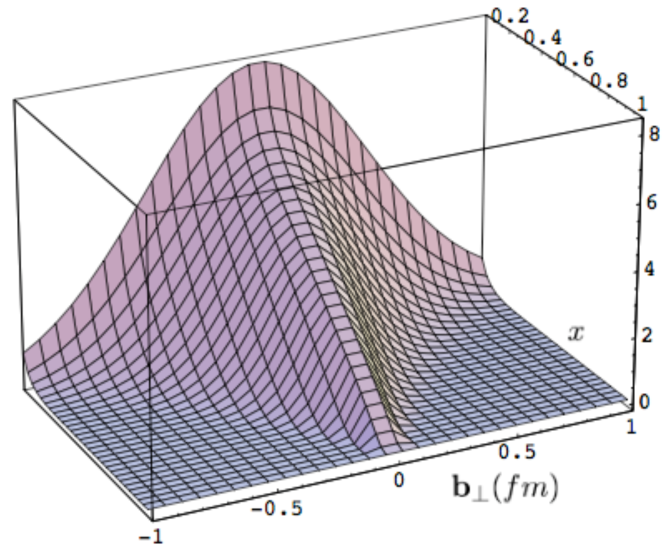
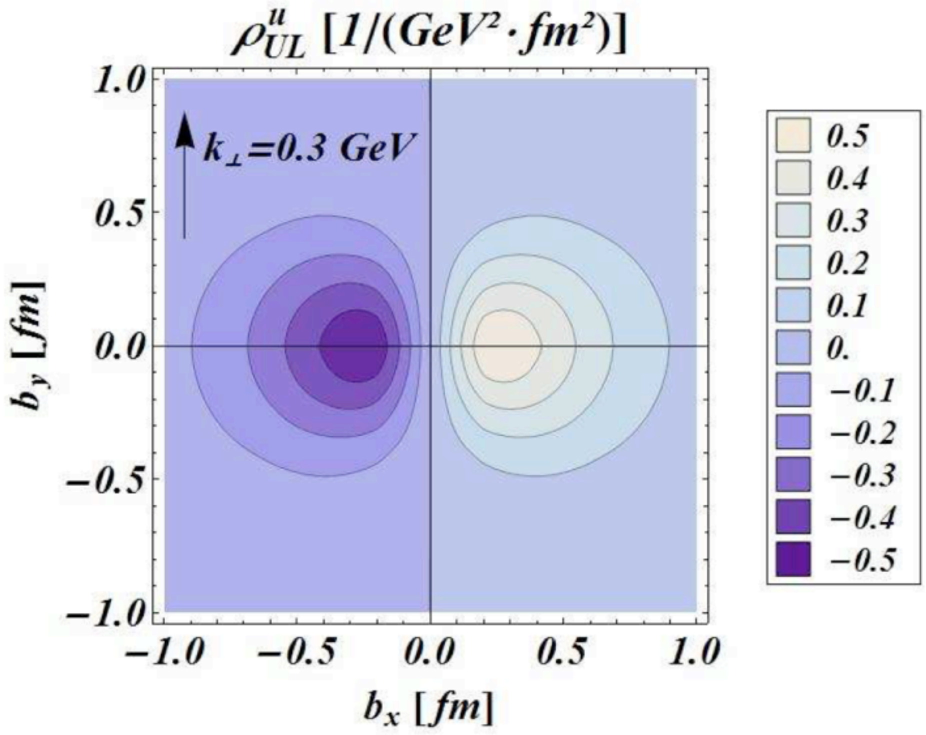


Fig. 1. Impact parameter dependent parton distribution $u(x, \mathbf{b}_\perp)$ for the simple model (31).

Lorce, Pasquini arXiv:1106.0139



see also: G.A. Miller, arXiv:0802.3731v1

Hagler et al., arXiv:0908.1283

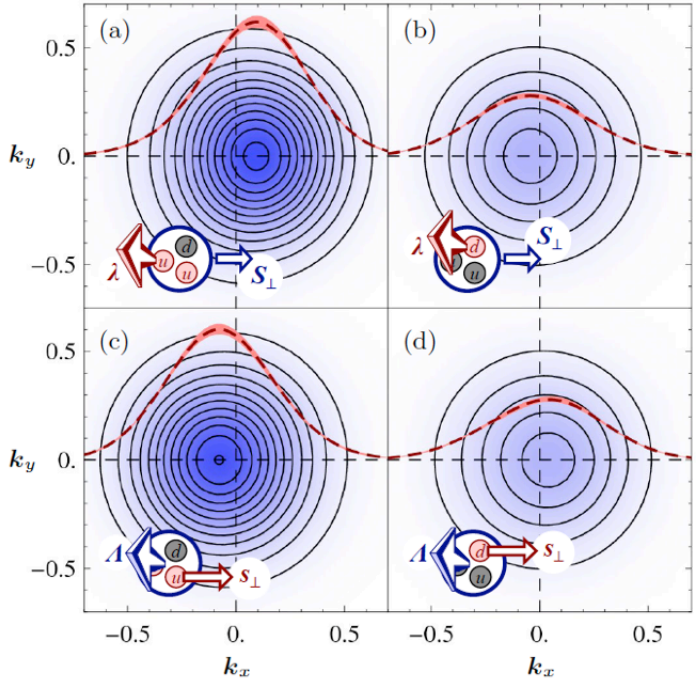


FIG. 3: Quark densities in the $\boldsymbol{k}_\perp$-plane, for $m_\pi \approx 500$ MeV. (a) ρ_L for u-quarks and $\lambda = 1$, $\boldsymbol{S}_\perp = (1,0)$, (b) the same for d-quarks, (c) ρ_T for u-quarks and $\Lambda = 1$, $\boldsymbol{s}_\perp = (1,0)$, (d) the same for d-quarks. The error bands show the density profile at $\boldsymbol{k}_y = 0$ as a function of $\boldsymbol{k}_x$ (scale not shown).

from Bass review: arXiv:hep-ph/0411005v2 10 Jun 2005

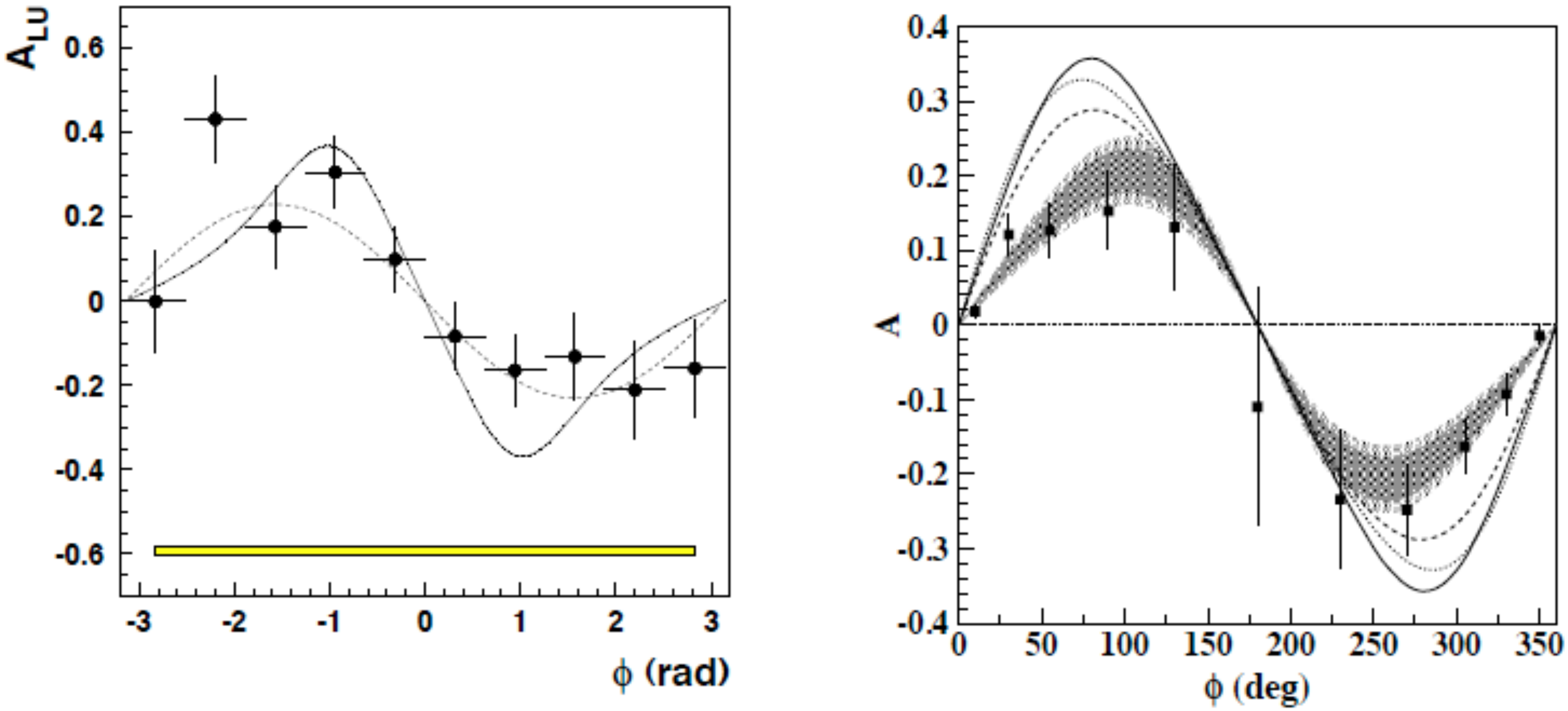
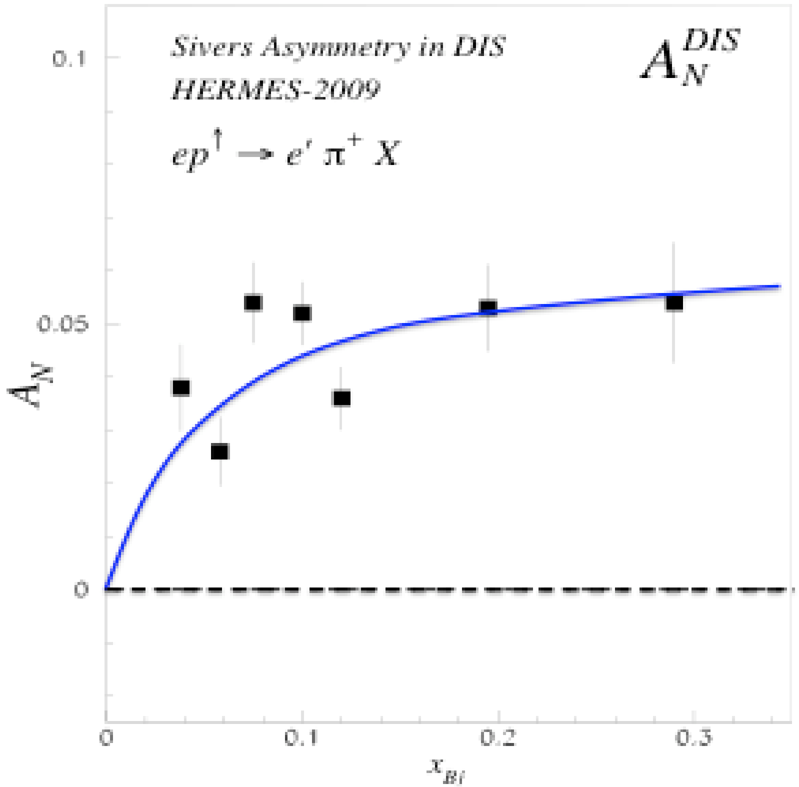


FIG. 21 Recent data from HERMES (left) and the CLAS experiment at Jefferson Laboratory (right) in the realm of DVCS Bethe-Heitler interference. The sin ϕ azimuthal dependence of the single spin asymmetry is clearly visible in the data (Airapetian *et al.*, 2001; Stepanyan *et al.*, 2001).

How can there be transverse orbital motion
in the Infinite Momentum Frame? (Large, Finite)



Experimentally, there is!

Sivers Effect

Quantum Mechanics:

P-wave without
classical motion

DVCS

Data

FNAL E906 will test in Drell-Yan

VI. Conclusion

- The physical component of a vector gauge field can be identified in a gauge covariant fashion.
- The gauge covariant derivatives needed to extract orbital angular momentum (and mechanical momentum) of fermions coupled to the gauge field must include only the non-physical, pure gauge part of the vector gauge field so that:
- Both gauge invariance and canonical commutation relations are satisfied in order to allow physical interpretation of the matrix elements of these operators.

Quark and Gluon momentum contributions are also affected by these considerations:

PRL **103,** 062001 (2009) PHYSICAL REVIEW LETTERS week ending 7 AUGUST 2009

Do Gluons Carry Half of the Nucleon Momentum?

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We examine the conventional picture that gluons carry about half of the nucleon momentum in the asymptotic limit. We show that this large fraction is due to an unsuitable definition of the gluon momentum in an interacting theory. If defined in a gauge-invariant and consistent way, the asymptotic gluon momentum fraction is computed to be only about one-fifth. This result suggests that the asymptotic limit of the nucleon spin structure should also be reexamined. A possible experimental test of our finding is discussed in terms of novel parton distribution functions.

$$\mathcal{P}_{q/h}(\xi) = \frac{1}{2}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < \bar{\psi}(0,x^-,0_\perp)\gamma^+ \mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+(0,y^-,0_\perp)}\psi(0) >_h$$

$$\int_{-\infty}^{\infty} d\xi\,\xi\,\mathcal{P}_{q/h}(\xi) = \frac{1}{2(P^+)^2} < \bar{\psi}\gamma^+ \imath D^+ \psi >_h$$

$$\mathcal{P}_{q/h}(\xi) = \frac{1}{2}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < \bar{\psi}(0,x^-,0_\perp)\gamma^+ \mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+_{pur}(0,y^-,0_\perp)}\psi(0) >_h$$

$$\int_{-\infty}^{\infty} d\xi\,\xi\,\mathcal{P}_{q/h}(\xi) = \frac{1}{2(P^+)^2} < \bar{\psi}\gamma^+ \imath D^+_{pur} \psi >_h$$

$$\mathcal{P}_{g/h}(\xi) = \frac{1}{\xi P^+}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < F^{+\nu}(0,x^-,0_\perp)\mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+(0,y^-,0_\perp)}F_\nu{}^+(0) >_h$$

$$\mathcal{P}_{g/h}(\xi) = \frac{1}{\xi P^+}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < F^{+i}(0,x^-,0_\perp)\mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+_{pur}(0,y^-,0_\perp)}A^i_{fys}(0) >_h$$

Polarized glue:

$$\mathcal{P}_{\Delta g/h}(\xi) = \frac{1}{\xi P^+}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < F^{+i}(0,x^-,0_\perp)\mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+_{pur}(0,y^-,0_\perp)}\epsilon_{ij+}A^j_{fys}(0) >_h$$

Conventional gluon momentum defintion:

$$\int d^3x \, \vec{E} \times \vec{B}$$

$$\gamma^{\mathscr{P}} = -\frac{\alpha_s}{4\pi} \begin{pmatrix} -\frac{8}{9}n_g & \frac{4}{3}n_f \\ \frac{8}{9}n_g & -\frac{4}{3}n_f \end{pmatrix}$$

$$\hookrightarrow \quad \vec{\mathscr{P}}_g^R = \frac{2n_g}{2n_g + 3n_f} \vec{P}_{\text{total}}$$

becomes

$$\int d^3x \, E^i \vec{\mathcal{D}}_{pur} A^i_{fys}$$

$$\gamma^P = -\frac{\alpha_s}{4\pi} \begin{pmatrix} -\frac{2}{9}n_g & \frac{4}{3}n_f \\ \frac{2}{9}n_g & -\frac{4}{3}n_f \end{pmatrix}$$

$$\hookrightarrow \quad \vec{P}_g^R = \frac{\frac{1}{2}n_g}{\frac{1}{2}n_g + 3n_f} \vec{P}_{\text{total}}.$$

for $n_f = 5$:
gluon momentum fraction

1/2 ➡ 1/5

There is no proton spin crisis but only quark spin-axial charge confusion

The quark spin contributions measured in DIS are:

$$\Delta u + \Delta d + \Delta s$$

$$= \begin{cases} 0.82(6) - 0.44(6) - 0.10(7) & = & 0.29(19) \\ 0.80(2) - 0.46(2) - 0.12(2) & = & 0.23(6) \\ 0.82(4) - 0.44(4) - 0.11(4) & = & 0.27(12) \end{cases} \quad Q^2 = \begin{cases} 10 \\ 5 \quad GeV^2. \\ 3 \end{cases}$$

while the pure valence q^3 S-wave quark model calculated values are:

$$\Delta u = \frac{4}{3}, \Delta d = -\frac{1}{3}, \Delta s = 0$$

More recent values for sum:

$$\Sigma = 0.330 \pm 0.011(\mathrm{thry}) \pm 0.025(\mathrm{exp}) \pm 0.028(\mathrm{evol}) \quad \mathrm{Hermes}$$

$$\Sigma = 0.33 \pm 0.03(\mathrm{stat}) \pm 0.05(\mathrm{syst}) \qquad \mathrm{COMPASS}.$$

- To clarify, first recognize that the value measured in DIS is the matrix element of the quark axial-vector current operator in a nucleon state:

$$2a_0 S^\mu = < ps| \int d^3x \, \bar{\psi} \, \gamma^\mu \gamma^5 \, \psi \, |ps >$$

Here, $a_0 = \Delta u + \Delta d + \Delta s$ which is not the quark spin contribution calculated in the CQM. The value calculated in the CQM is the matrix element of the Pauli spin part only.

The axial-vector current operator can be expanded as:

$$\int d^3x \bar{\psi}\vec{\gamma}\gamma^5\psi = \sum_{i\lambda\lambda'} \int d^3k \chi^\dagger_\lambda \vec{\sigma}\chi_{\lambda'}(a^\dagger_{i\vec{k}\lambda}a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'}b_{i\vec{k}\lambda})$$

$$-\sum_{i\lambda\lambda'} \int d^3k \chi^\dagger_\lambda \frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m_i)} i\vec{\sigma}\vec{k}\chi_{\lambda'}$$

$$\times(a^\dagger_{i\vec{k}\lambda}a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'}b_{i\vec{k}\lambda})$$

$$+\sum_{i\lambda\lambda'} \int d^3k \chi^\dagger_\lambda \frac{i\vec{\sigma}\times\vec{k}}{k_0}\chi_{\lambda'}a^\dagger_{i\vec{k}\lambda}b^\dagger_{i-\vec{k}\lambda'} + \text{H.c.}$$

Spin is 1/2 of this.

- Only the first term of the axial-vector current operator, which is the Pauli spin part, has been calculated in non-relativistic quark models.
- The second term, the relativistic correction, has not been included in non-relativistic quark model calculations. The relativistic quark model does include this correction and it reduces the quark spin contribution by about 25%.
- The third term, $q\bar{q}$ creation and annihilation, does not contribute in a model with only valence quark configurations and so it has not been calculated in any quark model to our knowledge.

An Extended CQM with Sea Quark Components

- To understand nucleon spin structure quantitatively within the CQM and to clarify the quark spin-axial vector confusion further a CQM was developed with sea quark components:

$$|N >= c_0|q^3 > +\Sigma C_{\alpha\beta}|(q^3)_\alpha(q\bar{q})_\beta >$$

PHYSICAL REVIEW D, VOLUME 58, 114032

Is nucleon spin structure inconsistent with the constituent quark model?

Di Qing, Xiang-Song Chen, and Fan Wang
Department of Physics and Center for Theoretical Physics, Nanjing University, Nanjing 210093, People's Republic of China
(Received 23 February 1998; published 9 November 1998)

TABLE III. The spin contents of the proton.

	q^3	$q^3-q^4\bar{q}$	$q^4\bar{q}-q^4\bar{q}$	sum	exp.	lattice [9]	lattice [9,15]
Δu	0.773	−0.125	0.100	0.75	0.80	0.79(11)	0.638(54)
Δd	−0.193	−0.249	−0.041	−0.48	−0.46	−0.42(11)	−0.347(46)
Δs	0	−0.064	−0.002	−0.07	−0.12	−0.12(1)	−0.109(30)

TABLE I. Proton model wave function.

q^3	$N\eta$	$N\pi$	$\Delta\pi$	$N\eta'$	ΛK	ΣK	$\Sigma^* K$
−0.923	0.044	0.232	−0.252	0.065	0.109	−0.036	−0.106

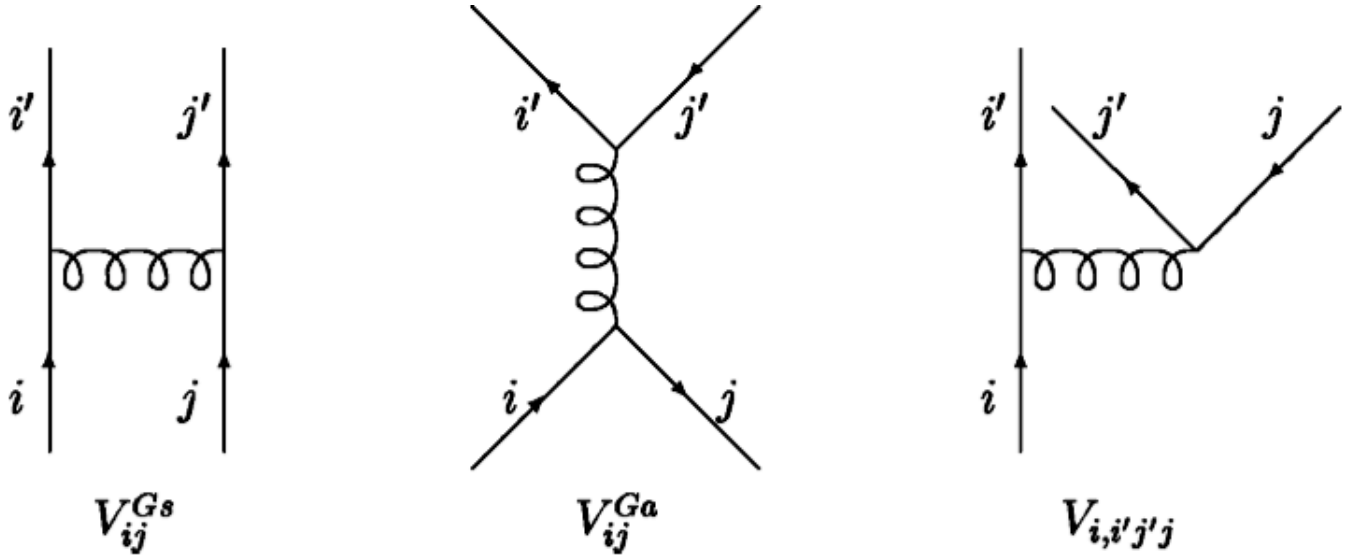


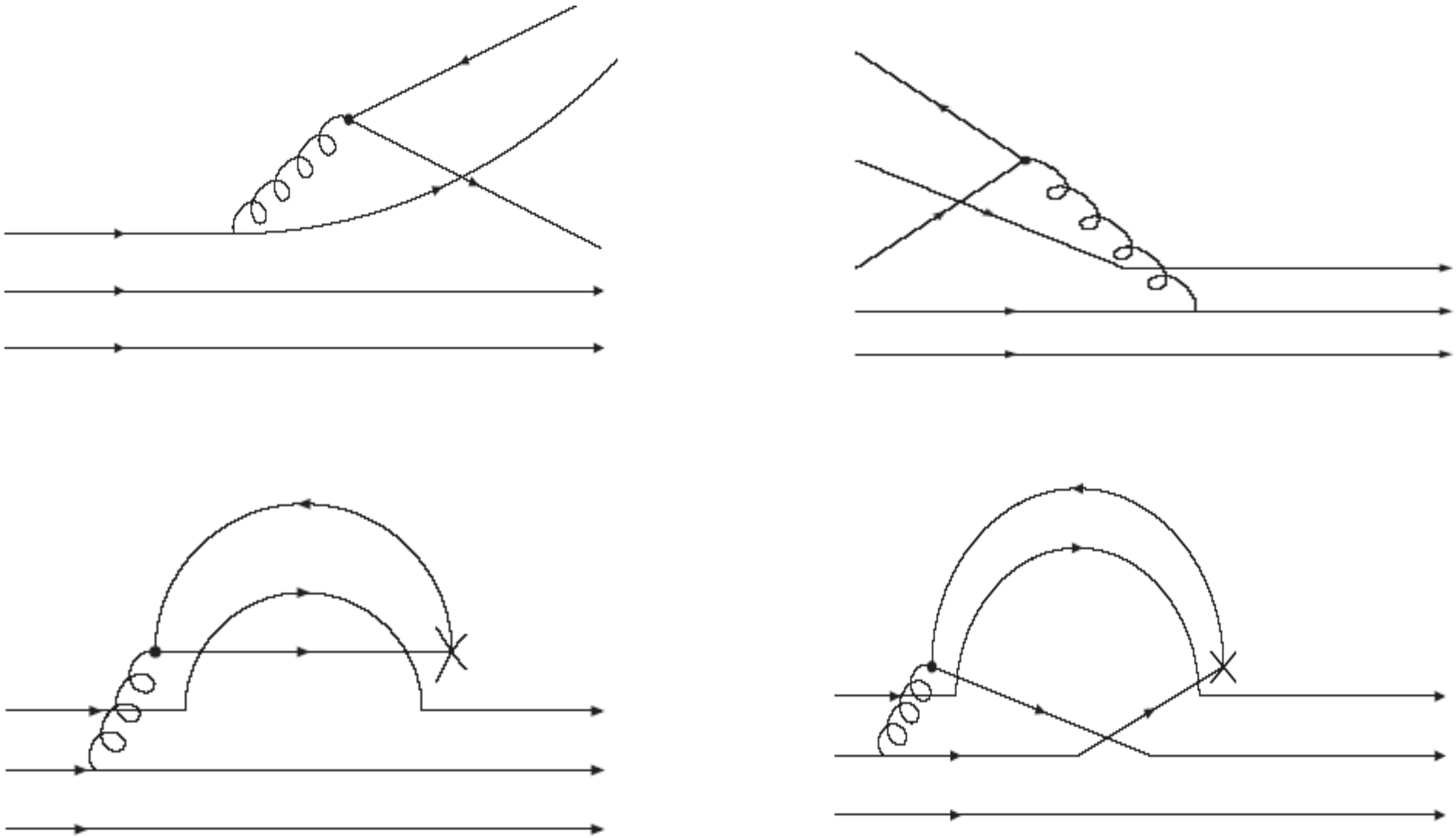
FIG. 2. Quark interaction diagrams.

TABLE II. Masses and magnetic moments of the baryon octet and decuplet. $m=330$ (MeV), $m_s=564$ (MeV), $b=0.61$ (fm), $\alpha_s=1.46$, $a_c=48.2$ (MeV fm^{-2}).

		p	n	Λ	Σ^+	Σ^-	Ξ^0	Ξ^-	Δ	Σ*	Ξ*	Ω
	M(Mev)	939		1116	1193		1346		1232	1370	1523	1659
Theor.	E1(MeV)	2203		2323	2306		2409		2288	2306	2450	2638
	$\mu(\mu_N)$	2.780	−1.818	−0.522	2.652	−1.072	−1.300	−0.412				
	$\sqrt{\langle r^2\rangle}$(fm)	0.802	0.124 ← Improved over Isgur-Karl (all other results almost identical)									
	M(MeV)	939		1116	1189		1315		1232	1385	1530	1672
Exp.	$\mu(\mu_N)$	2.793	−1.913	−0.613	2.458	−1.160	−1.250	−0.651				
	$\sqrt{\langle r^2\rangle}$(fm)	0.836	0.34									

NOTE: $^3S_1$ NOT $^3P_0$ -- Vector Gluons, not 0^+ pairs

Coupling between 3-quark and 5-quark sectors



$$H = \sum_i \left(m_i + \frac{p_i^2}{2m_i} \right) + \sum_{i<j} \left(V_{ij}^c + V_{ij}^G \right)$$

$$+ \sum_{i<j} \left(V_{i,i'j'j} + V_{i,i'j'j}^{\dagger} \right),$$

$$V_{ij}^c = -a_c \vec{\lambda}_i \cdot \vec{\lambda}_j r_{ij}^2,$$

$$V_{ij}^G = V_{ij}^{Gs} + V_{ij}^{Ga},$$

$$V_{ij}^{Gs} = \alpha_s \frac{\vec{\lambda}_i \cdot \vec{\lambda}_j}{4}$$

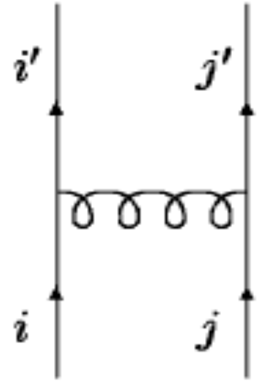
$$\times \left[\frac{1}{r_{ij}} - \frac{\pi}{2} \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4 \vec{\sigma}_i \cdot \vec{\sigma}_j}{3 m_i m_j} \right) \delta(\vec{r}_{ij}) + \cdots \right],$$

$$V_{ij}^{Ga} = \pi \alpha_s \left(\frac{\vec{\lambda}_i + \vec{\lambda}_j}{2} \right)^2 \left(\frac{1}{3} - \frac{\vec{f}_i \cdot \vec{f}_j}{2} \right)$$

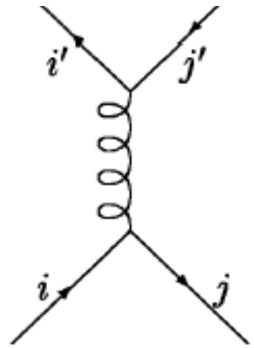
$$\times \left(\frac{\vec{\sigma}_i + \vec{\sigma}_j}{2} \right)^2 \frac{2}{3} \frac{1}{(m_i + m_j)^2} \delta(\vec{r}_{ij}),$$

$$V_{i,i'j'j} = i \alpha_s \frac{\vec{\lambda}_i \cdot \vec{\lambda}_j}{4} \frac{1}{2 r_{ij}}$$

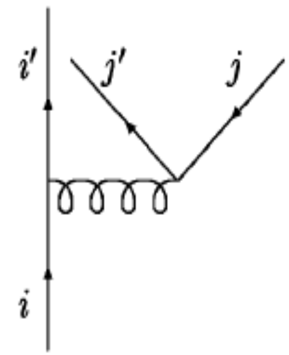
$$\times \left\{ \left[\left(\frac{1}{m_i} + \frac{1}{m_j} \right) \vec{\sigma}_j + \frac{i \vec{\sigma}_j \times \vec{\sigma}_i}{m_i} \right] \cdot \frac{\vec{r}_{ij}}{r_{ij}^2} - \frac{2 \vec{\sigma}_j \cdot \vec{\nabla}_i}{m_i} \right\}$$



V_{ij}^{Gs}



V_{ij}^{Ga}



$V_{i,i'j'j}$

If one allows sea quark Fock component mixing as shown in Eq. (6) used in our model, then the third term of Eq. (11), the quark-antiquark pair creation and annihilation term, will contribute to the matrix element of QAVCO. Table III shows our model results of the quark spin contents Δq of proton, in fact the matrix element of the QAVCO (axial charge). The experimental value and lattice QCD results are listed for comparison. In Table III, the second column is the q^3 valence quark contribution, where

$$\Delta u = \frac{4}{3}(1-0.32)(-0.923)^2,$$

$$\Delta d = -\frac{1}{3}(1-0.32)(-0.923)^2,$$

$$\Delta s = 0, \qquad (12)$$

Motion Fock

the first factors $\frac{4}{3}$, $-\frac{1}{3}$, 0 are the well known proton spin contents of the nonrelativistic quark model. $-0.32 = -1/3m^2b^2$ is the relativistic reduction and -0.923 is the amplitude of the q^3 component of our model. The third column is the contribution of the quark-antiquark pair creation (annihilation) term. It is another important reduction of the quark spin contribution and Δs is mainly due to this term. The fourth column lists the contribution of $q^3 q\bar{q}$ Fock components; due to quark antisymmetrization it cannot be separated into the valence and sea quark part. However, the antiquark contribution is very small (the largest one is $\Delta\bar{d} = 0.004$), and has not been listed in Table III. The fifth column lists the sum. Our model quark spin contents Δu, Δd, and Δs are quite close to the experiment ones in Eq. (3) and column 6, even though we have not made any model parameter adjustments aimed at fitting the proton spin content.

PHYSICAL REVIEW D, VOLUME 58, 114032

Is nucleon spin structure inconsistent with the constituent quark model?

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TABLE I. Proton model wave function.

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−0.923	0.044	0.232	−0.252	0.065	0.109	−0.036	−0.106

- The quark orbital angular momentum operator can be expanded as:

$$\vec{L}_q = \sum_{i\lambda} \int d^3k (a^\dagger_{i\vec{k}\lambda} i\vec{\partial}_k \times \vec{k} a_{i\vec{k}\lambda} + b^\dagger_{i\vec{k}\lambda} i\vec{\partial}_k \times \vec{k} b_{i\vec{k}\lambda})$$

$$+\frac{1}{2}\sum_{\lambda\lambda'} \int d^3k \chi^\dagger_\lambda \frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m)} i\vec{\sigma}\vec{k}\chi_{\lambda'}$$

$$\times(a^\dagger_{i\vec{k}\lambda} a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'} b_{i\vec{k}\lambda})$$

$$-\sum_{i\lambda\lambda'} \int d^3k \chi^\dagger_\lambda \frac{i\vec{\sigma}\times\vec{k}}{2k_0} \chi_{\lambda'} a^\dagger_{i\vec{k}\lambda} b^\dagger_{i-\vec{k}\lambda'} + \text{H.c.}$$

- The first term is the nonrelativistic quark orbital angular momentum operator used in the CQM, which does not contribute to nucleon spin in a pure valence S-wave configuration.
- The second term is a relativistic correction, which undoes the relativistic spin reduction.
- The third term is the $q\bar{q}$ creation and annihilation contribution, which also replaces missing spin.

- The quark orbital angular momentum operator can be expanded as:

$$\vec{L}_q = \sum_{i\lambda} \int d^3k \left(a^\dagger_{i\vec{k}\lambda} i\vec{\partial}_k \times \vec{k} a_{i\vec{k}\lambda} + b^\dagger_{i\vec{k}\lambda} i\vec{\partial}_k \times \vec{k} b_{i\vec{k}\lambda} \right)$$

$$+ \frac{1}{2} \sum_{\lambda\lambda'} \int d^3k \chi^\dagger_\lambda \frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m)} i\vec{\sigma}\vec{k} \chi_{\lambda'}$$

$$\times \left(a^\dagger_{i\vec{k}\lambda} a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'} b_{i\vec{k}\lambda} \right)$$

$$- \sum_{i\lambda\lambda'} \int d^3k \chi^\dagger_\lambda \frac{i\vec{\sigma}\times\vec{k}}{2k_0} \chi_{\lambda'} a^\dagger_{i\vec{k}\lambda} b^\dagger_{i-\vec{k}\lambda'} + \mathrm{H.c.}$$

Add to half of (see next page) cancels 2nd & 3rd terms.

RECALL:: axial-vector current operator can be expanded as:

$$\int d^3x \bar{\psi} \vec{\gamma} \gamma^5 \psi = \sum_{i\lambda\lambda'} \int d^3k \chi_\lambda^\dagger \vec{\sigma} \chi_{\lambda'} (a^\dagger_{i\vec{k}\lambda} a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'} b_{i\vec{k}\lambda})$$

$$- \sum_{i\lambda\lambda'} \int d^3k \chi_\lambda^\dagger \frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m_i)} i\vec{\sigma}\vec{k} \chi_{\lambda'}$$

$$\times (a^\dagger_{i\vec{k}\lambda} a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'} b_{i\vec{k}\lambda})$$

$$+ \sum_{i\lambda\lambda'} \int d^3k \chi_\lambda^\dagger \frac{i\vec{\sigma}\times\vec{k}}{k_0} \chi_{\lambda'} a^\dagger_{i\vec{k}\lambda} b^\dagger_{i-\vec{k}\lambda'} + \mathrm{H.c.}$$

Spin is 1/2 of this.