# *OAM in T-odd TMDs and FSIs Bessel Weighted Asymmetries*



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Boer,LG,Musch,Prokudin JHEP 2011 LG, Schlegel PLB 2010, in prep



- **Review transverse spin Effects TSSAs**
	- Transverse Spin Effects-twist 3 & TMD twist 2
- **• Color Gauge Inv. & Gauge links "T-odd" TMDs**
- **• Role of Gauge Links (hard processes)-**

 **"process dependence", Soft Factor (in SIDIS)**

- **•** On the merit of Bessel Weighted asymmetries "S/T" pic of SIDIS
- Fourier Transformed SIDIS cross section & "FT" TMDs
- Cancellation of the Soft Factor from WA
- **• T-odd PDFs & moments via ISI/FSIs ...Lensing QCD-Phases**
- Some pheno results

### Comments Importance of TMDs



- Single inclusive hadron production in hadronic collisions largest/ oldest observed TSSAs
- From theory view notoriously challenging from partonic picture twist-3 power suppressed in hard scale (vs. w/ SIDIS, DY, e<sup>+</sup>e<sup>-</sup>)
- Connection w/ twist 2 "TMD" approach
	- Operator level ETQS fnct 1<sup>st</sup> moment of Sivers

$$
gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x,k_T^2) + \text{``UV''} ...
$$
  
= 
$$
-2M f_{1T}^{\perp(1)}(x)
$$
Z.Kang & A.Prokudin

$$
\tilde{f}_{1T}^{\perp(1)}(x,|\bm{b}_T|) = \int d^2p_T \frac{|p_T|}{|\bm{b}_T|M^2} J_1(|\bm{b}_T||p_T|) f_{1T}^{\perp}(x,p_T^2)
$$

**Boer, LG, Musch, Prokudin JHEP-2011--arXiv:1107.529** 

### Comments Importance of TMDs

**Belitsky, Ji , Yuan (2004 PRD) [Meißner, Metz, Schlegel (2009 JHEP)]**



### Connection of twist 3 and twist 2 approach "overlap regime"





• Explore role parton model processes in twist-2&3 approaches LG & Z. Kang PLB 2011, D'Alesio, LG, Z. Kang, C.Pisano PLB 2011 "exploring impact of Gauge Inv"

# Two methods to account for SSA in QCD

• Depends on momentum of probe  $q^2 = -Q^2$  and momentum of produced hadron  $P_{h\perp}$  relative to hadronic scale  $k_T^2 (\equiv k_\perp^2) \sim \Lambda_{\rm QCD}^2$  $\bm{P}_{h\perp}$  $q^2 = -Q^2$ 



•  $k_{\perp}^2 \sim P_{h\perp}^2 \ll Q^2$  two scales-TMDs •  $k_{\perp}^2 \ll P_{h\perp}^2 \sim Q^2$  twist 3 factorization-ETQSs  $k_{\perp}^2 \ll P_{h\perp}^2 \sim Q^2$  $\Delta\sigma(P_h,S) \sim \Delta f_{a/A}^\perp(x,p_\perp)\otimes D_{h/c}(z,K_\perp)\otimes \hat{\sigma}_{\rm parton}$  $\Delta\sigma(P_h,S) \sim \frac{1}{Q}\, f_{a/A}^\perp(x)\otimes f_{b/B}(x)\otimes D_{h/c}(z)\otimes \hat{\sigma}_{\text{parton}}$ 

#### −<br>− P P ST L Ingredients transverse SPIN-Orbit observables

### kinematics



• Single Spin Asymmetry

<sup>−</sup> <sup>P</sup><sup>⊥</sup>

**Parity Conserving interactions: SSAs Transverse Scatt**<br> $\Lambda \sigma \approx \dot{\delta} S_{\overline{\sigma} \sigma} \left( \overline{\mathbf{P}} \times \overline{P}^{\pi} \right)$ Parity Conserving interactions: SSAs Transverse Scattering plane

- $\Delta \sigma \sim \imath \mathcal{S}_T \cdot (\mathbf{P} \times P_{\perp}^n)$  $\Delta \sigma \sim i S_T \cdot ({\bf P} \times P_\bot^\pi)$
- $\Rightarrow$  Left-Right Asymmetry <sup>=</sup><sup>⇒</sup> <sup>∆</sup><sup>σ</sup> <sup>∼</sup> <sup>i</sup>S<sup>T</sup> · (<sup>P</sup> <sup>×</sup> <sup>P</sup> <sup>π</sup> • Rotational invariance  $\sigma^{\downarrow}(x_F, p_{\perp}) = \sigma^{\uparrow}(x_F, -p_{\perp})$

$$
A_N=\tfrac{\sigma^\uparrow(x_F,\pmb{p}_\perp)-\sigma^\uparrow(x_F,-\pmb{p}_\perp)}{\sigma^\uparrow(x_F,\pmb{p}_\perp)+\sigma^\uparrow(x_F,-\pmb{p}_\perp)}\equiv\Delta\sigma
$$



Reaction Mechanism w/ Partonic Description

 $\Delta\sigma^{pp^+\to\pi X}\thicksim f_a\otimes f_b\otimes\Delta\hat\sigma\otimes D^{q\to\pi}$ Collinear factorized QCD parton dynamics



**Interference of helicity flip and non-flip amps 1) requires breaking of chiral symmetry** *mq /E* **2) relative phases require higher order corrections**

### Factorization Theorem at Partonic level



**•Born amps are real -- need "loops"----> phases •QCD interactions conserve helicity up to corrections** 



Twist three and trivial in chiral limit

 $A_N \propto$  $m_q^{}$ 

 $\frac{u}{E} \alpha_s$  at the partonic level Kane & Repko, PRL: 1978

artonic Picture" *Q* ∼ *P<sup>T</sup> >>* Λqcd One scale Collinear fact Twist 3 Phases in soft poles of prop hard processes Efremov & Teryaev PLB 1982  **Twist 3 ETQS approach-"Partonic Picture"**



Phases from interference two parton three parton scattering amplitudes

Factorization and Pheno: Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..., Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ... Kouvaris Ji, Qiu,Vogelsang! 2006, Vogelsang and Yuan PRD 2007



# Factorization parton model  $P_T$  of hadron small sensitive to intrinsic transv. momentum of partons

$$
W^{\mu\nu}(q, P, S, P_h) = \int \frac{d^2 \mathbf{p}_T}{(2\pi)^2} \int \frac{d^2 \mathbf{k}_T}{(2\pi)^2} \delta^2(\mathbf{p}_T - \frac{\mathbf{p}_{h\perp}}{z_h} - \mathbf{k}_T) \text{Tr} \left[ \Phi(x, \mathbf{p}_T) \gamma^{\mu} \Delta(z, \mathbf{k}_T) \gamma^{\nu} \right]
$$
  
\n
$$
\Phi(x, \mathbf{p}_T) = \int dp^{-1} \Phi(p, P, S)|_{p^+ = x_B P^+}, \quad \Delta(z, \mathbf{k}_T) = \int dk^{-1} \Delta(k, P_h)|_{k^- = \frac{p-1}{z_h}}
$$
  
\n*Small transverse momentum III*  
\n*final transverse momentum III*  
\n*real*  
\n**Minimal requirement satisfy color gauge invariance**  
\n*(r, e)*  
\n*(p, x)*  
\n*(p, x)*  
\n*(p, x)*

#### $u$  Co *Minimal Requirement* Color Gauge Inv. Reaction Mechanism

Sivers function are process-dependent



Gauge Link determined by Gauge link determined remming leading gluon interactions btwn soft and hard Process Dependence bre  $\mathbb{R}^2$  and  $\mathbb{R}^2$  in the interaction in  $\sigma \sim$ " ! + + +... May 2007 + + +... PDFs with SIDIS gauge link  $P \psi \mathcal{P} e^{i \theta}$  by  $Q$  with  $Q$ *P e*  $e^{i\theta}$ .<br>پ *d*λ*·A*(λ)  $\mathcal{P}$   $\mathcal{F}$  $ig \int_{\partial \Omega} -\infty$  $\frac{d}{d\alpha} \int d\alpha A(\lambda)$ γ∗ *q q*¯  $q$   $\gamma$  γ  $\sim$ *q* and the manufacture of the signal of the signal of the signal of the signal state of the signal state of the signal of the signal state of the  $q^q$  or  $\gamma^*$   $q^q$  or  $\gamma^*$  in the interaction in the interaction of  $\gamma$  $\begin{picture}(180,10) \put(0,0){\line(1,0){10}} \put(10,0){\line(1,0){10}} \put(10,0){\line($ + + +...  $\mathbb{R}^n \sim 1$ + + +...  $\mathbf{P}$  $\ddot{p}$   $\mu$   $e^{i\theta}$   $\delta y$   $\theta$  $P$  $e^{i\theta}$  $g$  $\bigcirc \bigcirc \partial d \lambda$ *y d*λ*·A*(λ)  $P$   $e$ <sup> $\cdot$ </sup>  $\int_{\mathcal{U}}\int_{\mathcal{U}}^{\mathcal{U}}d\alpha$  $P_{\mathbf{p}}^{ig}$  is  $\int_{y}^{d\alpha} d\mathbf{x}^{i} A(\lambda)$ γ∗ *q*  $q_1$  $\sim$ <sup>*q* γ ν</sup> *q* active particular particular particular continued to n interaction of the state interaction of  $\gamma$ " !  $\frac{1}{2}q$  +  $\sigma \sim$ " m m/18 + + +... PDFs with SIDIS gauge link **PDFs with DY gauge link of the Strategy of th**  $P_{g}$ ig PDFS<br>. *y d*λ*·A*(λ) *P e*  $\iota\!\!\!\!i\, {\cal D}\, \rho^{\dot{\imath}} g$  .  $P$ <sup>*ep*</sup> *ey*  $P$  *e*  $\int_y^{\infty} d\lambda \cdot A$ γ∗ *q*  $\bar{\pmb{q}}$  $q \sim \sqrt[3]{x}$ *q*  $\overline{11}$   $\overline{12}$   $\overline{13}$   $\overline{21}$ Sivers filoofiare parameter are processes are processed in the process umming leading gluon interactions btwn soft and hard Dreesee Dependence breek dewn of Universality  $\frac{1}{\sqrt{2}}$   $\frac{1}{\sqrt{2}}$   $\frac{1}{\sqrt{2}}$   $\frac{1}{\sqrt{2}}$  $\alpha$  in  $\alpha^*$  $\mathbf{r}$  $\sigma$  - $\begin{picture}(180,10) \put(0,0){\line(1,0){10}} \put(10,0){\line(1,0){10}} \put(10,0){\line($ + + +...  $\overline{\mathbf{0}}$ " !  $\sim$   $\frac{1}{2}$  +  $\sim$   $\frac{1}{2}$   $\sim$   $\frac{1}{2}$   $\sim$   $\frac{1}{2}$   $\sim$   $\frac{1}{2}$   $\sim$ PDFs with SIDIS gauge link PDFs with SIDIS gauge link *P e*  $kgf$  6 *y*  $\partial$ *β*<sup>*A*(*λ*)</sup><sub>*M*</sub>  $\mathbf{y} \int_{\alpha}^{\hat{i}} \mathcal{D}$  $\frac{d}{dy}$   $\mathcal{P}$   $e^{ig\int y}$  $\gamma^*$ *q*  $\bar{q}$  $q \searrow q^*$ *q* ra Donondonco broak down of Universality  $\mathcal{H}_{\alpha,*}$  and  $\mathcal{H}_{\alpha}$  in  $\mathcal{H}_{\alpha}$  in  $\mathcal{H}_{\alpha}$  $\mathbb{R}^n$  drell-Yan: interaction in terms in the interaction in terms in the interaction interaction in the interaction in the interaction of  $\mathbb{R}^n$  $\bigcup$ The Company of the Co<br>The Company of the C + + +...  $\overline{\phantom{a}}$ " PRETEN + + +...  $P_{c}^{\prime}e$ *ig* 9 | ∞ *y d*λ*·A*(λ) *P e*  $P e^{ig \int_y}$ *y d*λ*·A*(λ)  ${\widehat{\eta}^\ast}^\ast$ *q*  $\bar{q}$  $q$  γ γ γ γ γ γ *q*  $\mathbb{I}$   $\mathbb{$ Siders function are processed are provided in the process of the contract of the process of the process of the  $\frac{1}{2}$  is a sure of the Sixteen the interaction of the interaction on  $\frac{1}{2}$ active production and the remaining participant of the remaining of the remaining of the hadron (process-dependent) Dupundunud bildin  $\frac{q}{r}$  $\frac{1}{1}$ + + +...  $\frac{1}{\sqrt{2}}$ " 18 50 + + +... PDFs with SIDIS gauge link PDFs with DY gauge link  $P_{\rm e}$  $\dot{e}$  $ig$   $\left( \bigotimes_{i=1}^{\infty} \right)$  $\boldsymbol{\mathcal{\widetilde{Y}}}$ *d*λ*·A*(λ)  $ig \int_{v} = \infty$ *y*  $d\lambda$ *·* $A(\lambda)$ γ∗ *q*  $\overline{q}$  $q$   $q^*$ *q Gauge Link determined by Gauge link determined resumming leading gluon interactions btwn soft and hard Process Dependence break down of Universality*

**"Generalized Universality" Fund. Prediction of QCD Factorization** 



#### Thus 8 "LI" TMDs: Correlation Matrix Dirac space ,, TMDs: Correlation  $M$ 1atrix Dii MDs: Correlation Matrix Dirac space Thus 8 "LT" TMDs: Correlation Matrix Dirac space

$$
\Phi^{[\gamma^+]}(x, \mathbf{p}_T) \equiv f_1(x, \mathbf{p}_T^2) + \frac{\epsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{1T}^{\perp}(x, \mathbf{p}_T^2)
$$
\n
$$
\Phi^{[\gamma^+ \gamma_5]}(x, \mathbf{p}_T) \equiv \lambda g_{1L}(x, \mathbf{p}_T^2) + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{p}_T^2)
$$
\n
$$
\Phi^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{p}_T) \equiv S_T^i h_{1T}(x, \mathbf{p}_T^2) + \frac{p_T^i}{M} \left( \lambda h_{1L}^{\perp}(x, \mathbf{p}_T^2) + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} h_{1T}^{\perp}(x, \mathbf{p}_T^2) \right)
$$
\n
$$
\text{quark}
$$

 $\epsilon_{\vec{\textit{\i}} }^{i}$ 

 $M \rightarrow \infty$  , where  $\mathcal{A}^{2}$  and  $\mathcal{A}^{2}$  and  $\mathcal{A}^{2}$  and  $\mathcal{A}^{2}$ 

 $\frac{1}{r}$ 

 $M$ 

 $T = \frac{1}{M} \int_0^L (u, \nu) d\nu$ 

M h<br>M h⊥ h<br>M h⊥ h

 $\frac{1}{2}$   $h_1^{\perp}(x, p_2^2)$ 

 $\frac{\epsilon_T^{ij} p_T^j}{M} \; h_1^\perp(x,\bm p_T^2)$ 

 $\mathcal{F}_{\mathcal{A}}^{(n)}$  (x, p2)  $\mathcal{F}_{\mathcal{A}}^{(n)}$ 

 $i j \frac{\partial}{\partial x}$ 

|<br>|<br>|

 $\overline{1}$ 

 $+$ 

 $\mathcal{O}(\mathcal{O}(\log n))$  "  $\mathcal{O}(\log n)$  "  $\mathcal{$ 



### CS EXPRESSED INOUEL INOPEN. CHI'U SU'UCLUI'E IUNCU SIDIS- CS expressed model indpen. thru structure functions

$$
\frac{d\sigma}{dx_B dy dy dz_h d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} \right.
$$
\n
$$
+ \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{UU}^{\sin \phi_h}
$$
\nKotzinian NPB 95,  
\n**Muders Tangerman NPB 96,**\n
$$
+ S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right]
$$
\n
$$
+ S_{\parallel}\lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h F_{UL}^{\cos \phi_h} \right]
$$
\n
$$
+ |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right]
$$
\n
$$
+ \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \right]
$$
\n
$$
+ \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \right]
$$
\n
$$
+ \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S}
$$
\n
$$
+ \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^
$$

Structure functions projecte *, z<sup>h</sup>* <sup>=</sup> *<sup>P</sup> ·P<sup>h</sup>* For our purposes, we may assume *x* ≈ *xB*, *z* ≈ *z<sup>h</sup>* and γ ≈ 0. Individual structure functions can be projected from Structure functions projected from cross section  $\bullet$  ected from cross section

$$
A_{XY}^{\mathcal{F}} \equiv 2 \frac{\int d\phi_h \, d\phi_S \, \mathcal{F}(\phi_h, \phi_S) \, (d\sigma^{\uparrow} - d\sigma^{\downarrow})}{\int d\phi_h d\phi_S \, (d\sigma^{\uparrow} + d\sigma^{\downarrow})} \,, \qquad \mathbf{C}.\mathbf{g}. \, \mathcal{F}(\phi_h, \phi_S) = \sin(\phi_h - \phi_S).
$$

#### $\mathsf{P}_2$ tonic picture of nucleon SEs a  $D$ Fsin <sup>φ</sup><sup>h</sup> LU = Q C ' <sup>−</sup>h<sup>ˆ</sup> ·k<sup>T</sup>  $r_{\mathbf{A}}$ xe H<sup>⊥</sup> <sup>1</sup> + <sup>M</sup> <sup>f</sup><sup>1</sup> z  $\mathbf{r}$ ic pict 2M  $\overline{C}$ of n <u>"</u> cleon SFs  $\overline{\phantom{a}}$   $\overline{\phantom{a}}$ Partonic picture of nucleon SFs

**Partonic picture of nucleon SFs**  
\n
$$
\mathcal{C}[wfD] = x \sum_{c} e_{a}^{2} \int d^{2}p_{T} d^{2}k_{T} \delta^{(2)}(p_{T} - k_{T} - P_{h\perp}/z) w(p_{T}, k_{T}) f^{a}(x, p_{T}^{2}) D^{a}(z, k_{T}^{2})
$$
\n
$$
F_{UU,T} = \mathcal{C}[f_{1}D_{1}], \qquad F_{LL} = \mathcal{C}[g_{1L}D_{1}],
$$
\n
$$
F_{UT,T}^{\sin(\phi_{h} - \phi_{S})} = \mathcal{C}\left[-\frac{\hat{h} \cdot p_{T}}{M} f_{1T}^{\perp} D_{1}\right], \qquad F_{UT}^{\sin(\phi_{h} + \phi_{S})} = \mathcal{C}\left[-\frac{\hat{h} \cdot k_{T}}{M_{h}} h_{1} H_{1}^{\perp}\right],
$$
\n
$$
F_{UL}^{\sin(2\phi_{h}} = \mathcal{C}\left[-\frac{2(\hat{h} \cdot k_{T})(\hat{h} \cdot p_{T}) - k_{T} \cdot p_{T}}{M M_{h}} h_{1} H_{1}^{\perp}\right], \qquad F_{UU}^{\cos 2\phi_{h}} = \mathcal{C}\left[-\frac{2(\hat{h} \cdot k_{T})(\hat{h} \cdot p_{T}) - k_{T} \cdot p_{T}}{M M_{h}} h_{1}^{\perp} H_{1}^{\perp}\right],
$$
\n
$$
F_{UT}^{\sin(3\phi_{h} - \phi_{S})} = \mathcal{C}\left[\frac{2(\hat{h} \cdot p_{T})(p_{T} \cdot k_{T}) + p_{T}^{2}(\hat{h} \cdot k_{T}) - 4(\hat{h} \cdot p_{T})^{2}(\hat{h} \cdot k_{T})}{2M^{2}M_{h}} h_{1T}^{\perp} H_{1}^{\perp}\right]
$$

#### Sensitivity to process-dend to process-dend to process-dend to process-dend to process-dend to pay the term of  $\sim$ **TSSAs thru "T-odd" non-pertb. spin-orbit correlations....**<br>Sivers function are process-dependent *f sensitivity to*  $p_T \sim k_T \ll \sqrt{Q^2}$ Sivers function are process-dependent

• Sivers PRD: <sup>1990</sup> TSSA is associated w/ correlation transverse spin and • Sivers PRD: <sup>1990</sup> TSSA is associated w/ correlation transverse spin and momenta in initial state hadron momenta in initial state hadron  $\blacksquare$ SSA is associated w/ correlation *transverse* spin and



#### Weighted asymmetries Model independent deconvolution of cross section in terms of moments of in a model independent way the cross sections and asymmetries in terms of the transverse in terms of the transvers in a model independent way the cross sections and asymmetries in terms of the transverse (momentum) moments of TMD PDFs. A comprehensive list of  $\mathcal{M}$  and we ights was derived some time ago in refs. [6, 7]. Using the technique of weighting enables one to disentangle

Kotzinian, Mulders PLB 97, Boer, Mulders PRD 98 weighted Sivers asymmetry, obtained from the differential cross section does not according to the differential

$$
A_{UT,T}^{w_1 \sin(\phi_h - \phi_S)} = 2 \frac{\int d|\mathbf{P}_{h\perp}||\mathbf{P}_{h\perp}| d\phi_h d\phi_S w_1(|\mathbf{P}_{h\perp}|) \sin(\phi_h - \phi_S) \left\{ d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi) \right\}}{\int d|\mathbf{P}_{h\perp}| d\phi_h |\mathbf{P}_{h\perp}| d\phi_S w_0(|\mathbf{P}_{h\perp}|) \left\{ d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi) \right\}},
$$

(momentum) moments of TMD PDFs. A comprehensive list of  $\mathcal{M}$  and  $\mathcal{M}$  and  $\mathcal{M}$  was derived was derive

$$
\textbf{e.g.} \qquad \mathcal{W}_{\text{Sivers}} = \frac{|\boldsymbol{P}_{h\perp}|}{zM} \sin(\phi_h - \phi_S)
$$

$$
A_{UT}^{\frac{|P_{h\perp}|}{z_hM}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2 f_{1T}^{\perp(1)}(x) D_1^{a(0)}(z)}{\sum_a e_a^2 f_1^{a(0)}(x) D_1^{a(0)}(z)}
$$
  
Undefined who regularization  
to subtract infinite contribution at  
large transverse momentum  
Backetta et al. JHEP 08

Comments

- Propose generalize Bessel Weights-"BW"
- BW procedure has advantages
	- ★ Structure functions become simple product *P*[ ] rather than convolution  $C$ <sup>[</sup>]
	- ★ CS has simplier s/t interpretation as a *b<sup>T</sup>* [GeV−<sup>1</sup>]multipole expansion in terms of  $P_{h\perp}$ conjugate to
	- ★ Use Fourier Bessel tranforms-
	- ★ The usefulness of Fourier-Bessel transforms in studying the factorization as well as the scale dependence of transverse momentum dependent cross section has been known for quite sometime CS(82), Ellis,Fleishon,Stirling (81), Ji,Ma,Yuan (05), Collins, Foundations of Perturbative QCD, Cambridge University Press, Cambridge(11)

Further Comments

- Introduces a free parameter  $\mathcal{B}_T$  [GeV<sup>-1</sup>] that is Fourier conjugate to *P <sup>h</sup>*<sup>⊥</sup>
- Provides a regularization of infinite contributions at lg. transverse momentum when  $\mathcal{B}_T^2$  is non-zero for moments
- Addtnl. bonus soft factor eliminated from weighted asymmetries
- Possible to compare observables at different scales.... could be useful for an EIC

#### Advantages of Bessel Weighting we take advantage of the rotational invariance of  $\mathcal{L}$  the rotational invariance of  $\mathcal{L}$ First we use the representation of the δ-function Advantages of Bessel Weighting δ = 200 minutes = 200 minut First we use the representation of the function of the  $\sim$  $\mathbf{F} = \mathbf{F} \cdot \mathbf{F} = \mathbf$ =  $a$ i ita $\frac{1}{6}$ αι θα παραπομένουμε από το παραπομένουμε από το παραπομένουμε από το παραπομένο το παραπομένο το παραπομένο το<br>P, S = παραπομένουμε από το παραπομένο το παραπομένο το παραπομένο το παραπομένο το παραπομένο το παραπομένο **C** "  $\Lambda$ dvantages of  $\Gamma$  $\mathbf{F}$  is representation of the  $\mathbf{F}$

, (2.8)

 $\frac{1}{2}$ du<br>du 1. Deconvolution olution"-of CS--struct f  $\mathcal{L}$  control simple product  $\mathcal{L}$ olution' of CS--struct finct simple product " $\mathcal{P}^{\epsilon}$ JHEP10(2011)021  $\frac{1}{2}$ 

**Advantages of Bessel Weighting**  
1."**Deconvolution\*-of CS-struct fact simple product "
$$
\mathcal{P}^{\epsilon}
$$
  

$$
W^{\mu\nu}(P_{h\perp}) = \int \frac{d^2b_T}{(2\pi)^2} e^{-ib_T \cdot P_{h\perp}} \tilde{W}^{\mu\nu}(b_T),
$$

$$
\tilde{\Phi}_{ij}(x, zb_T) \equiv \int d^2p_T e^{izb_T \cdot p_T} \Phi_{ij}(x, p_T)
$$

$$
\tilde{\Delta}_{ij}(z, b_T) \equiv \int d^2K_T e^{ib_T \cdot K_T} \Delta_{ij}(z, K_T)
$$**

$$
\frac{d\sigma}{dx_B dy dy dz_h d\phi_h |\mathbf{P}_{h\perp}|d|\mathbf{P}_{h\perp}|} = \int \frac{d^2 \mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{b}_T \cdot \mathbf{P}_{h\perp}} \left\{ \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x_B} \right) L_{\mu\nu} \tilde{W}^{\mu\nu} \right\}.
$$

$$
2M\tilde{W}^{\mu\nu} = \sum_a e_a^2 \operatorname{Tr}\left(\tilde{\Phi}(x,z\boldsymbol{b}_T)\gamma^{\mu}\tilde{\Delta}(z,\boldsymbol{b}_T)\gamma^{\nu}\right).
$$

 $1.6D$ hadronic tensor and the equation of the equation of the equation constraints in the calculation of the calcu 1. Deco  $\overline{\Omega}$ VII V voluti  $\overline{\mathsf{C}}$ n"-Sivers rs su  $\overline{\mathbf{D}}$ t fnct si  $\overline{\mathbf{r}}$ ıple pr  $\overline{d}$ duct  $\overline{\phantom{a}}$  $\overline{\mathbf{c}}$ 1. "Deconvolution"-Sivers struct fnct simple product "P"

$$
F_{UT,T}^{\sin(\phi_h - \phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],
$$
  

$$
\mathcal{C}\left[wfD\right] = x \sum_{\alpha} e_a^2 \int d^2 \boldsymbol{p}_T d^2 \boldsymbol{k}_T \,\delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z) \, w(\boldsymbol{p}_T, \boldsymbol{k}_T) \, f^a(x, p_T^2) \, D^a(z, k_T^2)
$$

$$
F_{UT,T}^{\sin(\phi_h - \phi_S)} = -x_B \sum_a e_a^2 \int \frac{d|\bm{b}_T|}{(2\pi)} |\bm{b}_T|^2 J_1(|\bm{b}_T| |P_{h\perp}|) Mz \ \tilde{f}_{1T}^{\perp a(1)}(x, z^2 \bm{b}_T^2) \tilde{D}_1^a(z, \bm{b}_T^2).
$$

 $\tilde{f}_1, \, \tilde{f}_{1T}^{\perp (1)},$  and  $\tilde{D}_1$  are Fourier Transf. of TMDs/FFs and finite  $1\hspace{-.08in},\hspace{.08in} J_1\overset{\rightharpoonup}{T}$   $\hspace{-.08in},\hspace{.08in}$  all  $d D$  $_1$  are  $L^{(1)}_{T}, \, \text{and} \, \tilde{D}_1$  are Fourier Transf. of TMDs/FFs and fil 2MM<sup>h</sup>  $1\,\tilde{D}_1$  are F  $\overline{\text{m}}$ urie<br>'  $\overline{T}$ ้ว − .<br>:f MD: TMD:  $\mathsf{L}$ s/F 1T \_<br>DTC : and finite ● Peter's and Barbara's (Jerry's as well ..Bagel)-Pretzelosity = <del>paroure</del>  $\bullet$  Peter's and Barbara's (lerry's as well Bagel)-Pretzelosity hadronic tensor and using the equation-of-motion constraints just discussed, one can calcu-

$$
F_{UT}^{\sin(3\phi_h-\phi_S)}\!=\mathcal{C}\!\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\!\cdot\!\boldsymbol{k}_T\right)+\boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)-4\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)}{2M^2M_h}\,h_{1T}^{\perp}H_1^{\perp}\right]
$$

$$
\mathcal{C}[wfD] = x \sum e_a^2 \int d^2p_T d^2k_T \delta^{(2)}(p_T - k_T - P_{h\perp}/z) w(p_T, k_T) f^a(x, p_T^2) D^a(z, k_T^2)
$$
  

$$
\sum_{\substack{\text{sin}(3\phi_h - \phi_S) \\ \text{UT}}} \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^4 J_3(|b_T||P_{h\perp}|) \frac{M^2 M_h z^3}{4} \tilde{h}_{1T}^{\perp a(2)}(x, z^2 b_T^2) \tilde{H}_1^{\perp a(1)}(z, b_T^2).
$$
  
Simple product "  $\mathcal{P}$  "

$$
F_{UT}^{\sin(3\phi_h-\phi_S)}=x_{\!B}\sum_a e_a^2\int\!\frac{d|\bm{b}_T|}{(2\pi)}|\bm{b}_T|^4\,J_3(|\bm{b}_T|\,|\bm{P}_{h\perp}|)\frac{M^2M_hz^3}{4}\,\tilde{h}_{1T}^{\perp a(2)}(x,z^2\bm{b}_T^2)\,\tilde{H}_1^{\perp a(1)}(z,\bm{b}_T^2)\,.
$$

 $\blacksquare$  E Cancellation of the soft factor in the Sivers asymmetry asymmetry asymmetry asymmetry asymmetry asymmetry  $\bullet$ Simpl  $\blacksquare$ Juupic product / Simple product "  $p$  "

a) F.T. SIDIS cross section w/ following defintions

$$
\tilde{f}(x, \mathbf{b}_T^2) \equiv \int d^2 \mathbf{p}_T e^{i\mathbf{b}_T \cdot \mathbf{p}_T} f(x, \mathbf{p}_T^2)
$$
\n
$$
= 2\pi \int d|\mathbf{p}_T||\mathbf{p}_T| J_0(|\mathbf{b}_T||\mathbf{p}_T|) f^a(x, \mathbf{p}_T^2) ,
$$
\n
$$
\tilde{f}^{(n)}(x, \mathbf{b}_T^2) \equiv n! \left( -\frac{2}{M^2} \partial_{\mathbf{b}_T^2} \right)^n \tilde{f}(x, \mathbf{b}_T^2)
$$
\n
$$
= \frac{2\pi n!}{(M^2)^n} \int d|\mathbf{p}_T||\mathbf{p}_T| \left( \frac{|\mathbf{p}_T|}{|\mathbf{b}_T|} \right)^n J_n(|\mathbf{b}_T||\mathbf{p}_T|) f(x, \mathbf{p}_T^2) ,
$$

b) n.b. connection to  $\,\bm{p}_{T}\,$  moments

$$
\tilde{f}^{(n)}(x,0) = \int d^2 \mathbf{p}_T \left( \frac{\mathbf{p}_T^2}{2M^2} \right)^n f(x, \mathbf{p}_T^2) \equiv f^{(n)}(x)
$$

#### ★ CS has simpler S/T interpretation as a multipole  $\star$  CS has simpler S/L interpretation as a r  $\frac{1}{2}$  and  $\frac{1}{2}$  in the side  $\frac{1}{2}$  in the value of ref.  $\frac{1}{2}$  is the  $\frac{1}{2}$  content of  $\frac{1}{2}$ expansion in terms of  $b_T$  GeV  $^{-1}$  conjuga called Trento conventions [22]), as  $e^{\gamma}$  as a comugate to  $\boldsymbol{P}_{h+1}$  $b_T\,[\text{GeV}^{-1}]$  conjugate to  $\boldsymbol{P}_{h\perp}$

**expansion in terms of** 
$$
b_T
$$
 [GeV<sup>-1</sup>] **conjugate to**  $P_{h\perp}$   
\n
$$
\frac{d\sigma}{dx_g dy \, d\phi_s \, dz_h \, d\phi_h |P_{h\perp}|d|P_{h\perp}|} =
$$
\n
$$
\frac{\alpha^2}{x_n y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_n}\right) \int \frac{d|b_T|}{(2\pi)} |b_T| \left\{ J_0(|b_T||P_{h\perp}|) \mathcal{F}_{UU,T} + \varepsilon J_0(|b_T||P_{h\perp}|) \mathcal{F}_{UU,L} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h J_1(|b_T||P_{h\perp}|) \mathcal{F}_{UU}^{\cos \theta_h} + \varepsilon \cos(2\phi_h) J_2(|b_T||P_{h\perp}|) \mathcal{F}_{UU}^{\cos(2\phi_h)}
$$
\n
$$
+ \lambda_k \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h J_1(|b_T||P_{h\perp}|) \mathcal{F}_{LU}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) J_2(|b_T||P_{h\perp}|) \mathcal{F}_{UL}^{\sin 2\phi_h} \right]
$$
\n
$$
+ S_{\parallel} \lambda_{\rm e} \left[ \sqrt{1-\varepsilon^2} J_0(|b_T||P_{h\perp}|) \mathcal{F}_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h J_1(|b_T||P_{h\perp}|) \mathcal{F}_{LL}^{\cos \phi_h} \right]
$$
\n
$$
+ |S_{\perp}| \left[ \sin(\phi_h - \phi_S) J_1(|b_T||P_{h\perp}|) \mathcal{F}_{UTT}^{\sin(\phi_h - \phi_S)} + \varepsilon \mathcal{F}_{UTL}^{\sin(\phi_h - \phi_S)} \right]
$$
\n
$$
+ \varepsilon \sin(3\phi_h - \phi_S) J_3(|b_T||P_{h\perp}|) \mathcal{F}_{UTT}^{\sin(3\phi_h - \phi_S)} + \varepsilon \mathcal{F}_{UTL}^{\sin(\phi_h - \phi_S)} \right]
$$
\n
$$
+ \sqrt{2\varepsilon(1+\varepsilon)} \
$$

#### $\Gamma$ transformed  $\Gamma$ . The chiese section are simple products of  $\Gamma$ Structure Functions become

$$
\mathcal{F}_{UU,T} = \mathcal{P}[\tilde{f}_1^{(0)} \tilde{D}_1^{(0)}],
$$
  
\n
$$
\mathcal{F}_{UT,T}^{\sin(\phi_h - \phi_S)} = -\mathcal{P}[\tilde{f}_{1T}^{\perp (1)} \tilde{D}_1^{(0)}],
$$
  
\n
$$
\mathcal{F}_{LL} = \mathcal{P}[\tilde{g}_{1L}^{(0)} \tilde{D}_1^{(0)}],
$$
  
\n
$$
\mathcal{F}_{LT}^{\cos(\phi_h - \phi_s)} = \mathcal{P}[\tilde{g}_{1T}^{(1)} \tilde{D}_1^{(0)}],
$$
  
\n
$$
\mathcal{F}_{UT}^{\sin(\phi_h + \phi_S)} = \mathcal{P}[\tilde{h}_1^{(0)} \tilde{H}_1^{\perp (1)}],
$$
  
\n
$$
\mathcal{F}_{UU}^{\cos(2\phi_h)} = \mathcal{P}[\tilde{h}_1^{\perp (1)} \tilde{H}_1^{\perp (1)}],
$$
  
\n
$$
\mathcal{F}_{UL}^{\sin(2\phi_h)} = \mathcal{P}[\tilde{h}_{1L}^{\perp (1)} \tilde{H}_1^{\perp (1)}],
$$
  
\n
$$
\mathcal{F}_{UT}^{\sin(3\phi_h - \phi_S)} = \frac{1}{4} \mathcal{P}[\tilde{h}_{1T}^{\perp (2)} \tilde{H}_1^{\perp (1)}].
$$

 $\mathcal{P}[\tilde{f}^{(n)}\tilde{D}^{(m)}] \equiv x_P \sum e_a^2 (zM|\bm{b}_T|)^n (zM_h|\bm{b}_T|)^m \tilde{f}^{a(n)}(x, z^2\bm{b}_T^2) \tilde{D}^{a(m)}(z, \bm{b}_T^2)$ . typically derived in bt $\sim$  space and are thus obtained in terms of the same (derivatives of) are the same (derivatives of) and are the same (derivatives of) and are thus obtained in terms of the same (derivatives of) and  $\mathcal{P}[\tilde{f}^{(n)}\tilde{D}^{(m)}]\equiv x_{B}$  $\sum e_{a}^{2}\,(zM|{\bm b}_{T}|)^{n}\,(zM_{h}|{\bm b}_{T}|)^{m}\,\tilde{f}^{a(n)}(x,z^{2}{\bm b}_{T}^{2})\,\tilde{D}^{a(m)}(z,{\bm b}_{T}^{2})\;,$ 

# Correlator w/ explicit spin orbit correlations

 $\tilde{\Phi}^{[\gamma^+]}(x,\boldsymbol{b}_T) = \tilde{f}_1(x,\boldsymbol{b}_T^2) - i\, \epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M \tilde{f}_{1T}^{\perp (1)}(x,\boldsymbol{b}_T^2)\,,$  $\tilde{\Phi}^{[\gamma^+\gamma^5]}(x,\bm{b}_T) = S_L \, \tilde{g}_{1L}(x,\bm{b}_T^2) + i \, \bm{b}_T \cdot \bm{S}_T M \, \tilde{g}^{(1)}_{1T}(x,\bm{b}_T^2) \, ,$  $\tilde{\Phi}^{[i\sigma^{\alpha+}\gamma^5]}(x,\bm{b}_T) \,=\, S_T^{\alpha}\,\tilde{h}_1(x,\bm{b}_T^2) + i\,S_L\,b_T^{\alpha}M\,\tilde{h}_{1L}^{\perp(1)}(x,\bm{b}_T^2)$  $+$ 1 2  $\sqrt{ }$  $b_T^{\alpha}b_T^{\rho}$  + 1 2  $\bm{b}_T^2\, g_T^{\alpha\rho}$ "  $M^2$   $S_{T\rho} \tilde{h}_{1T}^{\perp (2)} (x,\bm{b}_T^2)$  $-i \, \epsilon_T^{\alpha \rho} b_{T \rho} M \tilde{h}_1^{\perp (1)} (x, \boldsymbol{b}_T^2)$  $\left(\frac{Z}{T}\right),$ 

#### **Further Beyond "tree level" factorization**



CS NPB 81,CSS NPB 1985 Collins, Hautman PLB 00, Adilbi, Ji, Ma, Yuan PRD 05, Cherednikov, Karanikas, Stefanis NPB 10, Collins Oxford Press 2011, Abyat, Rogers PRD 2011, Abyat, Collins, Qiu, Rogers arXiv 2011 ...

Soft

- •Extra divergences at one loop and higher
- •Various strategies to address them
- •Extra variables needed to regulate divergences
- •Modifies convolution integral by introduction soft factor
- •Will show cancels in Bessel weighted asymmetries

### **Comments on Soft factor**

- Collective effect soft gluons not associated with distribution frag function-factorizes into a matrix of Wilson lines in QCD vacuum
- Subtracts rapidity (LC) divergences from TMD pdf and FF
- Considered to be universal in hard processes (Collins & Metz PRL 04, Ji, Ma, Yuan PRD 05)



- At tree level (zeroth order  $\alpha_s$  ) unity-parton model
- Absent tree level pheno analyses of experimental data (e.g. Anselmino et al PRD 05 & 07, Efremov et al PRD 07)
- Potentially, results of analyses can be difficult to compare at different energies **issue for EIC**
- Correct description of energy scale dependence of cross section and asymmetries in TMD picture, soft factor must be included ( Ji, Ma, Yuan 2005, Collins Oxford Press 2011, Abyat, Collins, Rogers PRD 2011)
- However, possible to consider observables where it cancels e.g. weighted asymmetries Boer, LG, Musch, Prokudin JHEP 2011

#### Met we know about correlator off light cone **1 . The evolution is not increduced to First summarize what** First summarize what we know about correlator off light cone

gauge links 15 million and 15 million



 $\frac{1}{2}$  funning along a direction given  $i$ *e* $i$  by the for *z d*<sub>2</sub>*d*<sub>7</sub>*T*  $\cot v$ <sup>*v*</sup> Wilson lines starting at infinity running along a direction given by the four-vector  $\boldsymbol{v}$ to an endpoint a are denoted  $\mathcal{L}_n(\infty; a)$ do an endpoint *a* are  $\alpha$ *,* ρ) is formed from vacuum expectation values of Wilson lines involving both framework [18, 22] for definiteness. For any four vector *w*, we introduce light cone coordinates *w* = (*w*−*, w*<sup>+</sup> framework [18, 22] for definiteness. For any four vector *w*, we introduce light cone coordinates *w* = (*w*−*, w*<sup>+</sup> *, w<sup>T</sup>* ) wilson lines starting at infinity rullining along a direction green by the running a direction given by the four-vector  $\mathcal{L}_v(\infty; a)$  and  $\mathcal{L}_v(\infty; a)$ 

*S*  $\frac{24}{\sqrt{24}}$   $\frac{2}{\sqrt{24}}$  represents a nucleon with  $\frac{2}{\sqrt{24}}$  and spin points a spin point  $\frac{2}{\sqrt{24}}$  and spin points a nucleon with  $\frac{2}{\sqrt{24}}$  and spin points a nucleon with  $\frac{2}{\sqrt{24}}$  and spin points  $\rho = \sqrt{v-\tilde{v}^+}/v^+\tilde{v}$ specified in a Lorentz-invariant way by the parameter <sup>ζ</sup>, defined by <sup>ζ</sup> = (2*<sup>P</sup> ·v*)<sup>2</sup>*/v*<sup>2</sup> [check], which represents a rapidity Direction defined in LI way  $\zeta^2 = (2P \cdot v)^2/v^2$  scales from Direction defined in LI way  $\hat{\zeta}^2 = (2P_h \cdot \tilde{v})^2 / \tilde{v}^2$  regulating LC div<br> $\zeta^2 = (2P_h \cdot \tilde{v})^2 / \tilde{v}^2$  gluon rap. cutoff angle between v and  $\tilde{v}$   $\rho = \sqrt{v^+ v^+} v^ \overline{D}$ *Pirection* define  $\overline{\phantom{a}}$  $\hat{\mathcal{L}}$  and  $\hat{\mathcal{L}}$  and  $\hat{\mathcal{L}}$  $\frac{1}{\sqrt{2}}$  $\overline{p}$   $\overline{$ *e*−*i*<sub>*n*</sub> *e*−*i*<sub>*n*</sub> *defined in*<sup>1</sup> Direction defined in LI way <sup>2</sup> #*P, S<sup>|</sup> <sup>q</sup>*¯(*b*)*L†* angle between *v* and  $\tilde{v}$   $\rho = \sqrt{v^2 - \tilde{v}^2 + v^2}$  $\frac{1}{\sqrt{1}}$  $\overline{\phantom{a}}$  $+\tilde{v}$  - **giuon** r Direction defined in LI way  $\hat{\zeta}^2 = (2P_h \cdot \tilde{v})^2 / \tilde{v}^2$  $\zeta^2 = (2P \cdot v)^2/v^2$ angle between *v* and  $\tilde{v}$   $\rho = \sqrt{v-\tilde{v}^+/v^+\tilde{v}^-}$ 

<sup>1</sup>*<sup>T</sup>* (*x*) evolves logarithmically with *Q*<sup>2</sup> just like *f*1(*x*), only falling off faster at a given *x* value as *Q*<sup>2</sup> increases. For

 $\mathsf{c}$  and  $\mathsf{c}$  and  $\mathsf{c}$  and  $\mathsf{c}$ ा।<br>.. *<sup>q</sup>* (*b, P, S*; *v, µ*<sup>2</sup>)  $\mathcal{L}$  0.  $\mathcal{L}$  0.  $\mathcal{L}$  the light case  $\mathcal{L}$  $\sum_{n=1}^{\infty}$  *e*−*i*<sub>*n*</sub> + *i*<sub>x</sub><sup>*p*</sup> + *i*<sup>x</sup> **IS**  $\overline{a}$  $P$   $\sim$  P  $\sim$   $P$ *<sup>v</sup>*(∞; *b*) Γ *Lv*(∞; 0)*q*(0) *|P, S*\$ gluon rap. cutoff scales from

#### **Note that Crucial property of Soft Factor-SIDIS** direction ¯*n*. The soft factor *S*˜<sup>+</sup>(*b<sup>T</sup> , µ*<sup>2</sup> *,* ρ) is formed from vacuum expectation values of Wilson lines involving both *Crucial property of Soft Factor-SIDIS , <u>Crucial</u>* property of soit fact Crucial property of Soft Factor-SIDIS

indicates a choice of link directions appropriate for SIDIS, i.e. *v* ≈ *n*, or, more precisely, *v·P >* 0. As mentioned, Soft factor formed from vacuum expt. value of Wilson lines involving both v and  $\tilde{v}$  thus depends on relative orientation of directions  $\rho = \sqrt{\tilde{v}^2 + \tilde{v}^2 + \tilde{v}^2}$ indicate of  $v$  and  $v$ *<sup>q</sup>* can be decomposed

directions *<sup>v</sup>* and ˜*v*, and thus depends on the relative orientation of these directions, specified by <sup>ρ</sup> <sup>≡</sup> )*v*−*v*˜<sup>+</sup>*/v*<sup>+</sup>*v*˜−.

is the Fourier transform (21) of the same soft factor as the one in the convolution integral Eq. (8). Moreover, the  $\tilde{S}^+(b_T, \rho, \mu)$  is invariant under rotations of the *b*<sub>*T*</sub>-vector (provided *b* · *v* = 0). for TMDs we always consider the case *b*<sup>+</sup> = 0, we have *b*<sup>2</sup> *T* = −−*b2, so that we can write the soft factor as a function*  $\alpha$  *factor as a function*  $\alpha$  $t_{\rm eff}$  factor is considered to be universal in hard processes  $\frac{1}{\sqrt{2}}$  $S^{\top}(\bm{b}_{T}, \rho)$ u) is invariant under rotations of the  $\mathbf{b}_T$ -vector (provided  $\hat{\mathbf{b}} \cdot \mathbf{v} = 0$  $\int$ is the Fourier transform (21) of the same soft factor as the one in the convolution integral Eq. (8). Moreover, the *x*<sup>*γ*</sup>, *piz***<sub>***x***</sub></sup>,** *p***<sup>***x***</sup>,** *n***<sup>2</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***i***</sup>** *s***<sup>***x***</sup>,** *p***<sup>***i***</sup>** *s***<sup>***x***</sup>,** *p***<sup>***i***</sup>** *s***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x***</sup>,** *p***<sup>***x*</sup> <sup>1</sup>*<sup>T</sup>* (*x, p<sup>T</sup>* ; *µ*<sup>2</sup>*, x*ζ*,* ρ) (31) *,* <sup>ρ</sup>*, µ*). In the following section, we will consider the case <sup>Γ</sup> <sup>=</sup> <sup>γ</sup><sup>+</sup>. The correlator <sup>Φ</sup>+[γ+]

 $Since$  $\frac{1}{\sqrt{2}}$  is invariant under rotations of the *s*<sup> $\frac{1}{\sqrt{2}}$ </sup>  $\frac{1}{\sqrt{2}}$   $\frac{1}{\sqrt{2}}$ for TMDs we always consider the case  $b^+ = 0$ , we have  $b_T^2 = -b^2$ , *g q* in terms of *aluming*  $\alpha$  *aluming* consider the case  $h^+ - 0$ , we have  $h^2 - 1$ .  $\frac{1}{\sqrt{2}}$  for  $\frac{1}{\sqrt{2}}$  we drivay contract the ease  $\sigma = 0$ , we have  $\sigma$ <sup>1</sup>,  $\sigma$  $\longrightarrow S^+(b^2,\rho,\mu)$ only a direction, the amplitudes must remain invariant under rescaling of *v*, i.e., under the substitution *v* → η*v*, for always consider the case  $b^+=0$ , we have  $b^2_T = -b^2$ .  $\frac{b}{\alpha}$  **e**−i**p**  $\frac{c}{\alpha}$ *<sup>q</sup>* in terms of real-valued Lorentz-invariant amplitudes *A*1, *. . .*, *A*12, see, e.g., Ref. [1]. It has been  $\rightarrow$   $S^+(h^2 \cap \mathbb{R})$  $\rightarrow$   $S^+(b^2, \rho, \mu)$  $\tilde S^+(b^2)$  $, \rho, \mu)$ 

Decompose TMDs taking into account  $\upsilon$  dependence Goeke, Metz, Schlegel PLB 05 **the "4" Goeke, Metz, Schlegel PLB 05** only a direction, the amplitudes must remain invariant under rescaling of *v*, i.e., under the substitution *v* → η*v*, for Ds taking into account *1*, dependence the "+" indicates the sign of *v·P* and where *v·p/*(*v·P*) ≈ *x* for *v* ≈ *n*. into account  $\eta$  dependence

 $\widetilde{\Phi}_q^{[\Gamma]}$  $q^{[1]}(b, P, S, v) \in A_i^+(b^2, b \cdot P, b \cdot v/(v \cdot P), \zeta, \mu)$  and  $B_i^+(b^2, b \cdot P, b \cdot v/(v \cdot P), \zeta, \mu)$  $\alpha$ -dependent matrix  $\alpha$ -dependent matrix  $\alpha$  $(\widetilde{B}, v) \in \widetilde{A}_i^+(b^2, b \cdot P, b \cdot v/(v \cdot P), \zeta, \mu)$  and  $\widetilde{B}_i^+(b^2, b \cdot P, b \cdot v/(v \cdot P), \zeta, \mu)$ conventional parametrization in terms of the *A*<sup>+</sup> *<sup>i</sup>* and *B*<sup>+</sup> *<sup>i</sup>* by making the replacement *<sup>k</sup>* <sup>→</sup> *im*<sup>2</sup> 2  $h \cdot P h \cdot \frac{1}{2}$  $P, b \cdot v/(v \cdot P), \zeta, \mu)$ 

# Momentum space convolution

Adilbi, Ji, Ma, Yuan PRD 05 ....

Find

\n
$$
\mathcal{C}[H; w f S D] = x_B H(Q^2, \mu^2, \rho) \sum_{a} e_a^2 \int d^2p_T d^2K_T d^2\ell_T \delta^{(2)}(z p_T + K_T + \ell_T - P_{h\perp}) w \left( p_T, -\frac{K_T}{z} \right)
$$
\n
$$
\times f^a(x, p_T^2, \mu^2, x\zeta, \rho) \frac{S(\ell_T^2, \mu^2, \rho)}{D^a(z, K_T^2, \mu^2, \zeta/z, \rho)}
$$



#### Products in terms of  $"b_T$  moments"  $f(x) = f(x)$ frouncts in terms c

 $\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)}=H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho) \,\,\tilde{S}^{(+)}(\bm{b}_T^2,\mu^2,\rho) \,\, \mathcal{P}[\tilde{f}_{1T}^{(1)}\tilde{D}_1^{(0)}]+\tilde{Y}_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\bm{b}_T^2)\,\,.$ 

The first term in the following referred to as the "TMD expression", dominates in the  $\mathcal{P}[\tilde{f}^{(n)}\tilde{D}^{(m)}] \equiv x_B \sum e_a^2 (zM|\bm{b}_T|)^n (zM_h|\bm{b}_T|)^m \tilde{f}^{a(n)}(x,z^2\bm{b}_T^2,\mu^2,\zeta,\rho) \tilde{D}^{a(m)}(z,\bm{b}_T^2,\mu^2,\zeta,\rho)$  $\sum e_a^2 (zM|\bm{b}_T|)^n (zM_h|\bm{b}_T|)^m \tilde{f}^{a(n)}(x,z^2\bm{b}_T^2,\mu^2,\zeta,\rho) \tilde{D}^{a(m)}(z,\bm{b}_T^2,\mu^2,\hat{\zeta},\rho)$ 

(3.1)

### 2. Bessel Weighting & cancellation of soft factor

Bessel weighting-projecting out Sivers using orthogonality of Bessel Fncts.

$$
\frac{\mathcal{J}_{1}^{\mathcal{B}_{T}}(|\mathbf{P}_{hT}|)}{zM} = \frac{2 J_{1}(|\mathbf{P}_{hT}|\mathcal{B}_{T})}{zM\mathcal{B}_{T}}
$$
\n
$$
A_{UT}^{\frac{\mathcal{J}_{1}^{\mathcal{B}_{T}}(|\mathbf{P}_{hT}|)}{zM}} \sin(\phi_{h} - \phi_{S}) (\mathcal{B}_{T}) =
$$
\n
$$
2 \frac{\int d|\mathbf{P}_{h\perp}| |\mathbf{P}_{h\perp}| d\phi_{h} d\phi_{S} \frac{\mathcal{J}_{1}^{\mathcal{B}_{T}}(|\mathbf{P}_{hT}|)}{zM} \sin(\phi_{h} - \phi_{S}) (d\sigma^{\uparrow} - d\sigma^{\downarrow})}{\int d|\mathbf{P}_{h\perp}| |\mathbf{P}_{h\perp}| d\phi_{h} d\phi_{S} \mathcal{J}_{0}^{\mathcal{B}_{T}}(|\mathbf{P}_{hT}|) (d\sigma^{\uparrow} + d\sigma^{\downarrow})}
$$
\n
$$
A_{UT}^{\frac{\mathcal{J}_{1}^{\mathcal{B}_{T}}(|\mathbf{P}_{hT}|)}{zM} \sin(\phi_{h} - \phi_{s})}(\mathcal{B}_{T}) =
$$

$$
-2\frac{\tilde{S}(\mathcal{B}_{T}^{2}) H_{UT,T}^{\sin(\phi_{h}-\phi_{S})}(Q^{2}) \sum_{a}e_{a}^{2}\tilde{f}_{1T}^{\perp(1)a}(x,z^{2}\mathcal{B}_{T}^{2}) \tilde{D}_{1}^{a}(z,\mathcal{B}_{T}^{2})}{\tilde{S}(\mathcal{B}_{T}^{2}) H_{UU,T}(Q^{2}) \sum_{a}e_{a}^{2}\tilde{f}_{1}^{a}(x,z^{2}\mathcal{B}_{T}^{2}) \tilde{D}_{1}^{a}(z,\mathcal{B}_{T}^{2})}
$$

### Sivers asymmetry with full dependences

A  $\mathcal{J}_1^{\mathcal{B}T}\left(|\bm{P}_{hT}|\right)$  $\frac{\partial T}{\partial T} \frac{(1 + hT)^2}{\partial M} \sin(\phi_h - \phi_s) (\mathcal{B}_T) =$ 

 $-2$  $\tilde{S}(\mathcal{B}_{\mathcal{I}}^2, \mu^2, \rho^2) H_{UT, T}^{\sin(\phi_h - \phi_S)}(Q^2, \mu^2, \rho) \, \sum_a e_a^2 \, \tilde{f}_{1T}^{\perp (1)a}(x, z^2 \mathcal{B}_T^2; \mu^2, \zeta, \rho) \, \tilde{D}_1^a(z, \mathcal{B}_T^2; \mu^2, \hat{\zeta}, \rho)$  $\tilde{S}(\mathcal{B}^2_T, \mu^2, \rho^2) H_{UU,T}(Q^2, \mu^2, \rho) \sum_a e_a^2 \ \tilde{f}_1^a(x, z^2 \mathcal{B}^2_T; \mu^2, \zeta, \rho) \, \tilde{D}^a_1(z, \mathcal{B}^2_T; \mu^2, \hat{\zeta}, \rho)$ 

### 3. Circumvents the problem of ill-defined  $p_T$  moments

$$
A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|P_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) =
$$

$$
-2\frac{\tilde{S}(\mathcal{B}_{T}^{2},\mu^{2},\rho^{2})H_{UT,T}^{\sin(\phi_{h}-\phi_{S})}(Q^{2},\mu^{2},\rho)\sum_{a}e_{a}^{2}\tilde{f}_{1T}^{\perp(1)a}(x,z^{2}\mathcal{B}_{T}^{2};\mu^{2},\zeta,\rho)\tilde{D}_{1}^{a}(z,\mathcal{B}_{T}^{2};\mu^{2},\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_{T}^{2},\mu^{2},\rho^{2})H_{UU,T}(Q^{2},\mu^{2},\rho)\sum_{a}e_{a}^{2}\tilde{f}_{1}^{a}(x,z^{2}\mathcal{B}_{T}^{2};\mu^{2},\zeta,\rho)\tilde{D}_{1}^{a}(z,\mathcal{B}_{T}^{2};\mu^{2},\hat{\zeta},\rho)}
$$

Traditional weighted asymmetry recovered but UV divergent

$$
\lim_{\mathcal{B}_T\to 0} w_1 = 2J_1(|\mathbf{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |\mathbf{P}_{h\perp}|/zM
$$

$$
A_{UT}^{\frac{|P_h \perp|}{z_h M} \sin(\phi_h - \phi_s)} = -2 \frac{\sum_a e_a^2 f_{1T}^{\perp(1)}(x) D_1^{a(0)}(z)}{\sum_a e_a^2 f_1^{a(0)}(x) D_1^{a(0)}(z)}
$$

*undefined w/o*  **Bacchetta et al. JHEP 08** *regularization* 

# **4.** More sensitive to low  $P_{h\perp}$  region

 $\mathcal{B}_T$  can serve as a lever arm to enhance the low  $P_{h\perp}$ description and possibly dampen lg. momentum tail of cross section. We can use it to scan the cross section



### Cancellation of Soft Factor on level of the Matrix elements *(summarize)*

- So far we get ratios of moments of TMDs and FFs that are free of soft factor
- It was not necessary to specify explicit def. of TMDs and FFs
- We also analyze ratio of moments of TMDs directly on level of matrix elements of TMDs & FFs
- Again we find cancellation of soft factors in ratio
- Impact for Lattice calculation of moments of<br>TMDS. Musch. Ph. Hagler. M. Engelhardt. I.W. Negele, A. Schafer arXiv 2011 Musch, Ph. Hagler, M. Engelhardt, J.W. Negele, A. Schafer arXiv 2011

#### $\mathcal{A}$  . The nucleon quark-Subtracted correlator off light cone <sup>i</sup> that depend on b<sup>2</sup> , b·P, v·b/(v·P) and  $\tilde{P}$  and  $\tilde{$

gauge links 15  $\mu$  m and 15  $\mu$ 

 $\mathcal{A}(\mathcal{A})$ 

<sup>i</sup> and <sup>B</sup>!(+)



#### $\overline{\phantom{a}}$ *<sup>q</sup>* (*x, <sup>p</sup><sup>T</sup> , P, S, µ*<sup>2</sup>*, x*ζ*,* <sup>ρ</sup>) = ! *db*<sup>−</sup> **Again consider JMY framework**  $\mathbf{f} = \mathbf{f} \mathbf{f} + \mathbf{f} \mathbf{f}$  of  $\mathbf{f} = \mathbf{f} \mathbf{f} + \mathbf{f} \mathbf{f}$  and  $\mathbf{f} \mathbf{f} = \mathbf{f} \mathbf{f} + \mathbf{f} \mathbf{f}$  and  $\mathbf{f} \mathbf{f} = \mathbf{f} \mathbf{f} + \mathbf{f} \mathbf{f}$ Again consider JMY framework

 $q$  -correlator defining  $\overline{q}$  -correlator defining TMDs has the form of  $\overline{q}$ 

$$
\Phi^{(+)[\Gamma]}(x, \mathbf{p}_T, P, S, \mu^2, \zeta, \rho) = \int \frac{db^-}{(2\pi)} e^{ixb^-P^+} \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{p}_T \cdot \mathbf{b}_T}
$$

$$
\times \frac{1}{2} \langle P, S | \bar{\psi}(0) \mathcal{U}[\mathcal{C}_b] \Gamma \psi(b) | P, S \rangle \Bigg/ \widetilde{S}^{(+)}(\mathbf{b}_T^2, \mu^2, \rho) \Bigg|_{b^+ = 0},
$$

$$
\widetilde{\Phi}^{[\Gamma]}_{\text{unsub}}(b, P, S; v, \mu^2)
$$

#### **discript in the discriming term is denoted and find the discriming term is denoted and for the discriming term is a set of the discriming of the disc**  $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$   $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$   $\overline{\phantom{a}}$ = *M* Soft Factor cancels Generalized av. quark trans. momentum shift  $\sim$   $\sim$   $\sim$ i Soft Fa



 $\langle p_y \rangle_{TU} := \text{average quark momentum in}$ transverse *y*-direction measured in a proton polarized in transverse *x*-direction.  $\sigma$   $\alpha$   $\alpha$ mon entum in

> "dipole moment", "shift"  $\mathbf{r}$ moment' ment", "shift" .<br>.

attention divergences from high- $p_T$ -tails!  $\mathbf{P} \mathbf{x}$  are seen an long of Eq. (6.1) is the Eq.

$$
\langle p_y(x) \rangle_{TU}^{\mathcal{B}_T} = \left. \frac{\int d|\mathbf{p}_T| |\mathbf{p}_T| \int d\phi_p \frac{2 J_1(|\mathbf{p}_T| \mathcal{B}_T)}{\mathcal{B}_T} \sin(\phi_p - \phi_S) \Phi^{(+)[\gamma^+]}(x, \mathbf{p}_T, P, S, \mu^2, \zeta, \rho)}{\Phi^{(+)[\gamma^+]}(x, \mathbf{p}_T, P, S, \mu^2, \zeta, \rho)} \right|_{|\mathbf{S}_T|=1}
$$

$$
\langle \boldsymbol{p}_y \rangle_{TU}(\mathcal{B}_T) \equiv M \frac{\int dx \tilde{f}_{1T}^{(1)}(x, \mathcal{B}_T^2)}{\int dx \tilde{f}_1^{(0)}(x, \mathcal{B}_T^2)} = \frac{\tilde{S}(\mathcal{B}_T^2, \dots) \tilde{A}_{12B}(\mathcal{B}_T^2, 0, 0, \tilde{\zeta}, \mu)}{\tilde{S}(\mathcal{B}_T^2, \dots) \tilde{A}_{2B}(\mathcal{B}_T^2, 0, 0, \tilde{\zeta}, \mu)}
$$

- **Propose generalize Bessel Weights**
- Theoretical weighting procedure w/ advantages
- Introduces a free parameter  $\mathcal{B}_T$  [GeV<sup>-1</sup>] that is Fourier conjugate to *P <sup>h</sup>*<sup>⊥</sup>
- Provides a regularization of infinite contributions at lg. transverse momentum when  $\mathcal{B}_T^2$  is non-zero
- Addtnl. bonus soft factor eliminated from weighted asymmetries
- Possible to compare observables at different scales.... could be useful for an EIC

Fourier transform of GPD  $F(x, 0, \vec{\Delta}_T)$   $\circled{a}$   $\xi = 0$ 



### *Burkardt PRD 00, 02, 04...*

Localizing partons: impact parameter

 $\blacktriangleright$  states with definite light-cone momentum  $p^+$ and transverse position (impact parameter):

formal: eigenstates of 2 dim. position operator D. Soper '77 Prob. of finding unpol. quark w/ long momentum x at position b<sub>T</sub> in trans. polarized  $S_T$  nucleon: spin independent  $H$  and spin flip part  $E'$ 

$$
\mathcal{F}(x,\vec{b}) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T \cdot \vec{b}} F(x,0,\vec{\Delta}_T) \n= \mathcal{H}(x,\vec{b}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} (\mathcal{E}(x,\vec{b}))' \qquad \vec{b} \leftrightarrow \vec{\Delta}_T
$$

 $\overline{x}$ 

Boer,LG,Musch,Prokudin JHEP (11)

 $\bar{b}$ 

 $b' =$ 

 $\overline{b}$  =

 $=\frac{1}{1-a}$ 

*b*

 $1 - x$ 

$$
\tilde{\Phi}^{[\gamma^+]}(x, \mathbf{b}_T) = \tilde{f}_1(x, \mathbf{b}_T^2) - i \epsilon_T^{\rho \sigma} b_{T\rho} S_{T\sigma} M \tilde{f}_{1T}^{\perp(1)}(x, \mathbf{b}_T^2), \qquad \text{F.T.} \quad \vec{b'} \leftrightarrow \vec{k}_T
$$
\n
$$
\vec{b} \qquad \qquad \text{In Spectator picture}
$$
\nBurkardt, Hwang 2004

\nNeissner Metz Goeke 07 P

 $T_N$ 

1<br>11 L (x, b2)<br>12 L (x, b2)

Burkardt, Hwang 2004 Meissner, Metz, Goeke 07 PRD LG, Schlegel PLB 2010

#### **Non-trivial relations for "T-odd" parton distributions:**

**'#+)+%&)#-(,&-,\*?+3"\*)&'O,1+#5<\*,"#6**

What observable to test this possible connection bth TMD and Impact par. picture? **g**luonic Pole ME

$$
\langle k_T^i \rangle_T(x) \, = \, \int d^2k_T \, k_T^i \, \tfrac{1}{2} \Big[ \text{Tr}[\gamma^+ \Phi(\vec{S}_T)] - \text{Tr}[\gamma^+ \Phi](-\vec{S}_T) \Big]
$$



Phases in soft poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982





# Clarification of Approximate Factorization of Lensing & **Distortion**



- Stay in momentum space in space
- Insert complete sets of momentum states complete sets of momentum  $kT$  $\sqrt{1-\frac{1}{2}}$

$$
-\epsilon_T^{ij}S_T^j f_{1T}^{\perp(1)}(x) = \frac{1}{2M}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P, S_T | \bar{q}(-\frac{z^-}{2}n) \gamma^+[-\frac{z^-}{2}n\, ; \frac{z^-}{2}n] \prod^i (\frac{z^-}{2}n) \Psi(\frac{z^-}{2}n) |P, S_T \rangle.
$$

$$
\sum_{\lambda_{\mathbf{P}}^{\prime}}\sum_{\lambda_{\mathbf{P}}}\ldots \langle \lambda_{\mathbf{P}}^{\prime}|\hat{I}^{i}|\lambda_{\mathbf{P}}\rangle \ldots
$$
 L.G & Schlegel in prep

• Diagonal in momentum eigenstates under assumptions

ft gluon exch<br>ret baskward w/r to target backward and forwards .<br>ว เ 3e, scattered quark and remnant move qu<br>d forwards 1) FSI ... soft gluon exchange, scattered quark and remnant move quasi-collinearly

2) Under these conditions one expects FSIs to be dominated by small transverse momentum of quark and remnant rather than a large momentum. Pole contribution dominates otherwise there large momentum is also transferred 3) under these conditions number of spectators match in intermediate state

#### LG, Schlegel AIP 2011, and in prep



**FIGURE 1.** Left: The matrix element  $W = \langle P - k | [\infty \ n; 0] q(0) | P \rangle$  dressed with the FSIs. The FSIs are described by a nonperturbative scattering amplitude *M* that is calculated in a generalized ladder approximation [20]. Right: The quark-quark correlator with FSIs.

#### $T$ ransform to  $h$  space transversely polarized nucleon. Transform to  $\vec{b}$  space *b* Z *d*<sup>2</sup>*p~*<sup>0</sup> h*P*! *; b~ <sup>T</sup>*; *S*j " N '  $2<sup>2</sup>$ **Transform to h s** observation one may hope to find a relation of the type  $\sim$  0.1  $\mu$  the involved GPDs involve impact parameter space. In the case of  $\alpha$ spin asymmetry through the "naive" time reversal odd ( $T_{\rm eff}$  odd) structure,  $\rightarrow$ *ktt* is the target intrinsic transverse momentum and *P* is the momentum of the target  $\frac{1}{2}$  is the single state of the  $\frac{1}{2}$  is the single state of the single state of the single state of the Sivers as  $\frac{1}{2}$   $\blacksquare$  and  $\blacksquare$  and  $\blacksquare$  factorization theorems. Among the most interesting results is interesting results in the most in

difference is the additional factor *Iq;i* and an integration

upon the impact parameter *b*

*<sup>T</sup>*\$ % *b~*<sup>2</sup>

$$
\langle k_T^{q,i}(x) \rangle_{UT} = \int d^2k_T \ k_T^i \ \frac{1}{2} \Big[ \text{Tr}[\gamma^+ \Phi(\vec{S}_T)] - \text{Tr}[\gamma^+ \Phi](-\vec{S}_T) \Big]
$$
  
1) 
$$
\langle k_T^i \rangle(x) = \int \frac{dz^-}{2\pi} e^{ix^p \cdot z^-} \langle P, S_T | \bar{q}(-z^- n/2) \gamma^+ [-z \sqrt{n/2; z^- n/2}] \hat{I}^i(z^- n/2) q(z^- n/2) | P, S_T \rangle,
$$

2) 
$$
\mathcal{F}^{q[\Gamma]}(x, \vec{b}_T; S) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P^+, \vec{0}_T; S | \vec{\psi}(z_1) \Gamma \mathcal{W}(z_1; z_2) \psi(z_2) | P^+, \vec{0}_T; S \rangle, \quad \Gamma \equiv \gamma^+
$$

#### ing expressions difference is additional  $\mathbf{r} \cdot \mathbf{r} = \mathbf{r} \cdot \mathbf{r}$  $\int$ <sup>*T*</sup> and h*P*!  $\int$ *factor,*  $\overline{O}$  and analogous for the higher derivatives of the GPDs X, as as  $\overline{O}$ forms difference is a<br>d integration over Z *dz*% and integration over  $\vec{b}$ Comparing expressions difference is additional factor, *I*  $q^{q,i}$  and integration over  $\vec{b}$ parameter space  $\mathbf{r}$  **because the only if**  $\mathbf{r}$  and  $\mathbf{r}$  on  $\mathbf{r}$  and  $\mathbf{r$ **i**paring expressions difference is additional fac observation one may hope to find a relation of the type relations of first, second, third, and fourth type, depending  $\mathbf{u}$  $parting$  expre *T*" *M A*  $\cdot$ *b b*<sub>2</sub> *b*<sub>2</sub> *d*</sup>  $\cdot$ *d*  $\cdot$ *f*  $\cdot$ *n*  $\cdot$ *f*  $\cdot$  $\overline{\text{ce}}$  $T^{1/2}$  and incegration over .<br>|- $\mathbf{d}$ 1**a** *k~*2 *T* <sup>2</sup>*M*<sup>2</sup> *<sup>h</sup>*?*<sup>q</sup>* Comparing expressions difference is additional factor. Comparing expressions difference is additional factor,<br>*I<sup>Q;i*</sup> and integration over  $\overrightarrow{I}$ *~* **T**  $\overline{a}$  **b** Non-trivial relations for "T-odd" parton distributions: *<u>z* expressions difference is additional factor,</u>

*xP*! 2*!* \* <sup>W</sup>*ab*#*z*1; *<sup>z</sup>*2\$*F*!*<sup>i</sup> <sup>b</sup>* #*z*2\$j*P*! *;* 0*~ <sup>T</sup>*; *S*i*;* (35) \* <sup>W</sup>*ab*#*z*1; *<sup>z</sup>*2\$*F*!*<sup>i</sup> <sup>b</sup>* #*z*2\$j*P*! *Janarat* [*r* 100*i.i* 11y3. 71 M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005] which simply correspond to the trivial relations discussed to the trivial relatio in Sec. III A. Sec. III A.

*x*  $\frac{1}{2}$  (36)  $\frac{1}{2}$  (36)  $\frac{1}{2}$  (36) ate Lens &  $\mathcal{S}(\mathbf{S})$  $(T^{1,3})$ , input GPD chosen, the GPDs *E*~ and *E*~*<sup>T</sup>* do not show up in (39)–(41):  $(\vec{r}_T) \mathcal{F}^q(x, \vec{b}_T; S)$  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  $\mathcal{R}$  $\overline{\phantom{a}}$ chosen, the GPDs *E*~ and *E*~*<sup>T</sup>* do not show up in (39)–(41): the GPD *<sup>E</sup>*<sup>~</sup> is multiplied by the kinematical factor #! " <sup>0</sup> **3)**  $\simeq$  $\int d^2 \vec{b}_T J^{q,i}(x, \vec{b}_T) \mathcal{F}^q(x, \vec{b}_T; S)$  $\cap$  $G \Gamma U$  $\left(1 - a_1\right)$ . It also provides an intervalse understand-sec. It also provides an intervalse understand-sec. It also provides  $a_1$ 3)  $\langle k_T^{q,\nu}(x) \rangle_{UT} \simeq \int d^2 \vec{b}_T I^{q,i}(x, \vec{b}_T) \mathcal{F}^q(x, \vec{b}_T)$  $\int$  does not have the status of a general,  $\int$  $\mathcal{C}$  second type contains  $\mathcal{C}$  $\langle k_T^{q,i}(x) \rangle_{UT} \simeq \int d^2 \vec{b}_T I^{q,i}(x, \vec{b}_T) \mathcal{F}^q(x, \vec{b}_T; S),$ <sup>1</sup>*<sup>T</sup>* \$ \$!E*q=g*" *; h*?*<sup>q</sup>*  $\vec{b}_T$ ; *S*)  $\overline{\phantom{a}}$  $\mathcal{C}$  polarized nucleon:  $\frac{1}{\sqrt{2}}$  ...  $\mathbf{r}$  $\int$   $\alpha$ "  $\overline{T}$  $(x, b)$  (x, b), input GF Calculate Lens & input GPD

### *Conjecture* born out factorization FSI and spatial distortion in eikonal + spectator approximation

$$
\langle k_T^i \rangle (x) = M \epsilon_T^{ij} S_T^i f_{1T}^{\perp (1)} \approx \int d^2 b_T \mathcal{I}^i (x, \vec{b}_T^2) \frac{\vec{b}_T \times \vec{S}_T}{M} \frac{\partial}{\partial b_T^2} \mathcal{E} (x, \vec{b}_T^2)
$$

# $\mathcal{I}^i(x,\vec{b}_{T}^2)$  Lensing Function



### Boer Mulders as well ...

LG, Schlegel PLB 10 LG, Schlegel PLB 10

• Av. transv. momentum of transv. pol. partons in an unpol. hadron:

$$
\langle k_T^i\rangle^j(x) \,=\, \int d^2k_T\, k_T^i\, \tfrac{1}{2}\Big(\Phi^{[i\sigma^{i+}\gamma^5]}(S)+\Phi^{[i\sigma^{i+}\gamma^5]}(-S)\Big)
$$

$$
\sum_{i=1}^n \sqrt{-2M^2 h_1^{\perp,(1)}(x)} \simeq \int d^2b_T \vec{b_T} \cdot \vec{\mathcal{I}}(x, \vec{b_T}) \frac{\partial}{\partial b_T^2} \left( \mathcal{E}_T + 2\tilde{\mathcal{H}}_T \right) (x, \vec{b_T})
$$

ieh<mark>l &</mark> Hagler EJPC .<br>ח ), Burkardt PRD (  $\overline{A}$ Diehl & Hagler EJPC (05), Burkardt PRD (04)

#### **Relativistic Ei elativistic Eikonal models (I dels (II)** • Generalized Ladder approximation:  $\mathbf{B}$  both  $\mathbf{B}$  and  $\mathbf{B}$  and  $\mathbf{B}$  and  $\mathbf{B}$  and  $\mathbf{B}$  and  $\mathbf{B}$ • If T-odd TMDs ! 0: Gauge Link not neglegible, physical effect: "! *!"#\$%&'()\*\$+)&,-./()\$&0123)4& !"#\$%&'()\*\$+)&,-./()\$&0123)4&* Sivers Function in this approach





#### I models: Treat FSI non-perturbatively. Incorporate interactions in the interactions of the contractions of the interactions of the interactio The complete state street street street pos **. Relativistic Eikonal models: Treat FSI non-perturbatively.**

#### eikonal Amerikaanse andere and Coulomb Phase: Eikonal Amplitude andere andere andere andere andere andere ander<br>16 September - Coulomb Phase: Eikonal Amerikaanse andere andere andere andere andere andere andere andere ande Phys.Lett.B685:95-103,2010 *&* in prep for Sivers...AIP 1374 (2011) 309-313 *L.G. & Marc Schlegel*



**?\* \*&+)3%"&'-&)(% 3))+ B),&C/2C%, -D/"-&E\*)44 E5:"&**=>"4.)#\*: ?@"7\*: !)AB: !&: 5-0 CD<: E9FG

 $\frac{1}{2}$  or higher twist T-odd TMDs **r higher twist T-odd TMDs** [Gamberg, Hwang, Metz, MS, PLB 639, 508] q - poles at one loop for higher twist T-odd TMDs [Gamberg, Hwang, Metz, MS, PLB 639, 508] **F ()(\*3/2C- 3/G%&H/3")( 3/(%"I D)#34 "+)/3 ,%3'-/)(A** oop for higher twist 1-odd TMDs [Gamberg, Hwang, Metz, MS, PLB 639, 508]<br>1

Inserting (20) into (20) and a bit of algebra yields the following expressions for the Sivers function,

**?\* \*&+)3%"&'-&)(% 3))+ B),&C/2C%, -D/"-&E\*)44 E5:"&**=>"4.)#\*: ?@"7\*: !)AB: !&: 5-0 CD<: E9FG

 $\frac{1}{\sqrt{2}}$  $\mathbf{F} \cdot \mathbf{G}^{\dagger}$  **:**  $\qquad \qquad \qquad$   $\qquad \qquad$   $\q$ l" picture  $\mathsf{Fix}\ \mathsf{the}\ \mathsf{q}$  +  $\mathsf{pole}$ <br>  $\mathsf{emn}$ mphasizes a "natural" picture of FSI<br>quivalent to Cutkosky cut. assumptions of Step 1 valid in Eikonal models '#"32( ! 2#( #\$&4&) ! **7-%+ J>&<(-%2,'-/)( )0%,&?6 heta integration over your**<br>A **c** through also the control of  $\mathbf{y}$  $\frac{1}{(r)P^+ - a^+ + i\varepsilon} = P$  $\frac{1}{\sqrt{2}}$  $\frac{1}{r \cdot P^+ - a^+} - i \pi \delta ((1-x) P^+ - q^+)$ **%\$+C'"/L%"&'&M('-#,'3M&+/1-#,%&)B&;7<**  $(-x)P^+ - q^+$ "! ! !#" ! ! #! ! \$&'""! ! !#" ! ! #!# **Fix the q+ - pole %?#/0'3%(-&-) N#-G)"GO 1#-I '""#\$+-/)("&)B 7-%+ =&0'3/4 /( P/G)('3 \$)4%3"**  $\overline{(c)P^+ - q^+ + i\varepsilon} = P \overline{(1)}$  $\epsilon$   $(1-x)P^+ - q^+$   $(1-x)$ **P** Step 2: Integration over q<sup>+</sup>: emphasizes a "natural" picture of FSI<br>hequivalent to Cutkosky cut, assumnti  **equivalent to Cutkosky cut, assumptions of Step 1 valid in Eikonal models**  $\bf{l}$  $(1-x)P^+ - q^+ + i\varepsilon$  $= P \frac{1}{(1+P)^{r}}$  $\frac{1}{(1-x)P^{+}-q^{+}}$   $\leftarrow i\pi\delta((1-x)P^{+}-q^{+})$  $\frac{1}{2\pi\delta((1 - r)P^+ - a^+)}$ <u>n v</u>  $\mu$ ral" picture of  $(1-x)P^+ - q^+ + i\varepsilon$  $\alpha$ f Stan 1  $\nu$  $1 +$  b.t., no form factor,  $\mathcal{L} = \mathcal{L} \times \mathcal{L}$ 

 $\sim$ 

T

 $\mathbf{r}$ 

# **Lensing Function**



Assume a non-perturbative scattering amplitude M +

**Separate GPD and FSI via contour integration** 

Contour integration  $\rightarrow$  cut diagram  $\rightarrow$  enforces "natural" picture of FSI

$$
f_{1T}^{\perp,(1)u}(x)=-\tfrac{1}{2(1-x)M^2}\int\tfrac{d^2q_T}{(2\pi)^2}\,q_T^2I^y(x,\vec{q}_T)F^u(x,0,-\tfrac{\vec{q}_T^2}{(1-x)^2})
$$

$$
I^{i}(x, \vec{q}_{T}) = \int \frac{d^{2}p_{T}}{(2\pi)^{2}} (2p_{T} - q_{T})^{i} \Im M_{bc}^{ab}(|\vec{p}_{T}|) \Big( (2\pi)^{2} \delta^{ac} \delta^{(2)}(\vec{p}_{T} - \vec{q}_{T}) + \Re M_{da}^{cd}(|p_{T} - q_{T}|) \Big)
$$

- More or less "realistic" model for  $M \rightarrow$  allows for numerical comparison
- · Sivers function from HERMES/COMPASS data, **GPD E from models or parameterizations**

### $\blacksquare$  Calculation of *M*





 $L.G.$  and L.G. and M. Schlegel Phys. Lett B 10 and in prepr

### Eikonal Color calculation and path ordered gauge link • Generalized Ladder approximation: Color Structure



Abarabanel Itzykson PRL 1970, L.G, Milton PRD 1999 Abarabanel Itzykson PRL 1970, L.G, Milton PRD 1999, Fried et al. 2000  $\frac{1}{2}$   $\frac{1}{2}$  on PR  $1570$ , L.G, Millon FKD 1555, Fried et al. 200

$$
G_{\rm eik}^{ab}(x,y|A) = -i \int_0^\infty ds \, e^{-is(m_q - i0)} \delta^{(4)}(x - y - sv) \left( e^{-ig \int_0^s d\beta \, v \cdot A^\alpha(y + \beta v) \, t^\alpha} \right)_+^{ab}
$$

 $\ddot{\phantom{0}}$ 

#### !\$ \*' ! \$ & Trick to disentangle the A-field and the color matrices t: Functional FT

!!!

**1 TCK to disentangle the A-11 and the color matrices II. Functional F I**  

$$
\left(e^{-ig\int_0^s d\beta v \cdot A^{\alpha}(y+\beta v)t^{\alpha}}\right)_+^{ab} = \mathcal{N}' \int \mathcal{D}\alpha \int \mathcal{D}u \, e^{i\int d\tau \, \alpha^{\beta}(\tau)u^{\beta}(\tau)} e^{ig\int d\tau \, \alpha^{\beta}(\tau) v \cdot A^{\beta}(y+\tau v)} \left(e^{i\int_0^s d\tau \, t^{\beta}u^{\beta}(\tau)}\right)_+^{ab}
$$

#### leads to a well-known eikonal representation [70], which was LUW UNANT IUI GIUURIIUII U **a**<br>*a Mul*  $\frac{1}{2}$ *a*1=1 ... + FLOW CHART for calculation of Boer Mulders ator *I<sup>i</sup>* which represents the FSIs. In various model calculations

 $\ddot{a}$ *A* Mod.Phys.Lett.A24:2 a distortion of the transverse space parton distribution and the *L.G. & Marc Schlegel*   $\frac{1}{6}$   $\frac{1}{6}$   $\frac{1}{6}$   $\frac{1}{6}$ Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

Non-pertb

FSIs in here

$$
2m_{\pi}^{2}h_{1}^{(1)}(x) \simeq \int d^{2}b_{T} \vec{b}_{T} \cdot \vec{I}(x,\vec{b}_{T}) \frac{\partial}{\partial \vec{b}_{T}^{2}} H_{1}^{a}(x,\vec{b}_{T}^{2}),
$$
\n
$$
2m_{\pi}^{2}h_{1}^{(1)}(x) \simeq \int d^{2}b_{T} \vec{b}_{T} \cdot \vec{I}(x,\vec{b}_{T}) \frac{\partial}{\partial \vec{b}_{T}^{2}} H_{1}^{a}(x,\vec{b}_{T}^{2}),
$$
\n
$$
I^{i}(x, \vec{q}_{T}) = \frac{1}{N_{c}} \int \frac{d^{2}p_{T}}{(2\pi)^{2}} (2p_{T} - q_{T})^{i} (3[\bar{M}^{eik}]_{\theta_{\beta\beta}}^{0\sigma}(|\vec{p}_{T}|) \qquad \text{Non-pertb}
$$
\n
$$
((2\pi)^{2}\delta^{\alpha\beta}\delta^{(2)}(\vec{p}_{T} - \vec{q}_{T}) + (2\mathbf{K}[\bar{M}^{eik}]_{\gamma\alpha}^{0\gamma}(|\vec{p}_{T} - \vec{q}_{T}|)). \qquad \text{FSIs in here}
$$
\n
$$
(M^{eik})_{\theta_{\beta\beta}}^{\pi\sigma}(x, |\vec{q}_{T} + \vec{x}_{T}|) = \frac{(1-x)p^{+}}{m_{s}} \int d^{2}z_{T} e^{-iz_{1} \cdot (q_{T} + \vec{x}_{1})} (20)
$$
\n
$$
\times \int d^{N_{c}^{2}-1} \alpha \int \frac{d^{N_{c}^{2}-1}u}{(2\pi)^{N_{c}^{2}-1}} e^{-i\alpha u} (e^{i\chi(|z_{T}|)r\alpha})_{\alpha\delta} (e^{iuu})_{\delta\beta} - \delta_{\alpha\beta}.
$$
\n
$$
f_{\alpha\beta}(x) \equiv \int d^{N_{c}^{2}-1} \alpha \int \frac{d^{N_{c}^{2}-1}u}{(2\pi)^{N_{c}^{2}-1}} e^{-i\alpha u} (e^{i\chi(|z_{T}|)r\alpha})_{\alpha\delta} (e^{iuu})_{\delta\beta} - \delta_{\alpha\beta}
$$
\n
$$
f_{\alpha\beta}(x) = \sum_{n=1}^{\in
$$

*Lensing Function & untangling the COLOR FACTOR*  $\overline{\phantom{a}}$  800  $\overline{\phantom{a}}$  8 INCHON & UNIGHUING THE COLOR FACTOR I analytical result in Eq. (32) for the *SU*(2) color case. The numerical and analytical result agree up to <sup>χ</sup> *tion & untangling the COLOR FACTOR* 0.5 GeV, 0.7 GeV. **Right:** The lensing function <sup>I</sup>*<sup>i</sup>* **ARTICLE IN PRESS**  $\frac{1}{2}$  $\overline{\phantom{a}}$  we use functional methods to incorporate the top  $791$ ! <sup>∞</sup> *d*α <u> |</u> *n* & *ancarymy* 

(*x*, "

17 82

*bT* ) from Eq. (30) for *U*(1), *SU*(2) and *SU*(3) for *x* = 0.2 at a scale ΛQCD = 0.2 GeV. For comparison we also plot the

$$
\mathcal{I}^i(x, \vec{b}_T) = \frac{(1-x)}{2N_c} \frac{b_T^i}{|\vec{b}_T|} \frac{\chi'}{4} C\left[\frac{\chi}{4}\right],
$$





with the parameters  $c = 1.269$ ,  $d = 2.105$ , and  $\delta = -\frac{9}{44}$ . with the parameters  $c = 1.269$ ,  $d = 2.105$ , and  $\delta = -\frac{9}{44}$ . These fits for the running and the gluon propaga-<br>These fits for the gluon propaga-



luon non-p<br>opagator erti<br>E rturbative gluon<br>.  $\begin{array}{c|c}\n\text{Jc V} & \text{I} \\
\text{SaV} & \text{I}\n\end{array}$  and the functions regime  $\frac{1}{\sqrt{1-\frac{1}{2}}}\frac{1}{\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\$  $\frac{1}{1}$  Diopagaton in  $\frac{1}{2}$ •gluon non-perturbative gluon propagator

#### **Lensing Function** To compute the lowest two moments of the density in ndör var andre 10 ðt**b. mars í sem sem sem sem sem sem sem se**

#### **Express Lensing Function in terms of Eikonal Phase:** parameterization with provides and provided with provided with  $p$

$$
\mathcal{I}_{(N=1)}^i(x, \vec{b}_T) = \frac{1}{4} \frac{b_T^i}{|\vec{b}_T|} \chi'(\frac{|\vec{b}_T|}{1-x}) \left[1 + \cos \chi(\frac{|\vec{b}_T|}{1-x})\right]
$$

$$
\mathcal{I}_{(N=3)}^i(x, \vec{b}_T) = \text{numerics}
$$

$$
\mathcal{I}^i_{(N=2)}(x, \vec{b}_T) = \frac{1}{8} \frac{b_T^i}{|\vec{b}_T|} \chi'(\frac{|\vec{b}_T|}{1-x}) \Big[ 3(1 + \cos \frac{\chi}{4}) + \left(\frac{\chi}{4}\right)^2 - \sin \frac{\chi}{4} \left(\frac{\chi}{4} - \sin \frac{\chi}{4}\right) \Big] (\frac{|\vec{b}_T|}{1-x})
$$

*L.G. & Marc Schlegel*  **Phys.Lett.B685:95-103,2010** *&* **Mod.Phys.Lett.A24:2960-2972,2009**.

FSI + distortion

Spin Structure of the Pion



D. Brömmel,<sup>1,2</sup> M. Diehl,<sup>1</sup> M. Göckeler,<sup>2</sup> Ph. Hägler,<sup>3</sup>  $A = \frac{1}{2}$  G. Schierholz,  $\frac{1}{2}$  H. Stuïben,  $\frac{1}{2}$  H. Stuïben,  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  $\mathcal{L}$ , bronned,  $\mathcal{L}$ ,  $\mathcal{L}$  based  $\mathcal{L}$ , with pion masses as  $\mathcal{L}$ 



FSIs are negative and "grow" with Color! error bands in the profile plots show that  $\mu$ <u>ESIs are negat</u> tion. The data lines show the uncertainty from a ChPTT show the uncertainty from a ChPTT show the uncertainty f asymmetry in the spatial distribution of the spatial polarized  $q$ magnitud and analogous assembly as a previously as a previously obtained for  $\alpha$ support the hypothesis that all Boer-Mulders functions are alike. terms of quark and gluon degrees of freedom is trivial. Pion matrix elements of  $\Gamma$ PSI: where, for  $\mathbf{3} \cdot \mathbf{3}$  ,  $\mathbf{4} \cdot \mathbf{3}$ sor generalized for the pion A! are negative  $1 - 5 - 10$ Figure 3: **Left:** The lensing function <sup>I</sup>*<sup>i</sup>*

### Prediction for Boer-Mulders Function of PION

#### *L.G. & Marc Schlegel*

**Phys.Lett.B685:95-103,2010** *&* **Mod.Phys.Lett.A24:2960-2972,2009**.



Relations produce a BM funct. approx equiv. to Sivers from HERMES Expected sign i.e. FSI are negative **Answer will come from pion BM from COMPASS**  $\pi N$  Drell Yan  $\mathbf{a}$ <br> $\vdots$ **b**<br>Tive<br>Tive

, µ2) = *p*<sup>2</sup>

### Study how Sivers function scales with color



