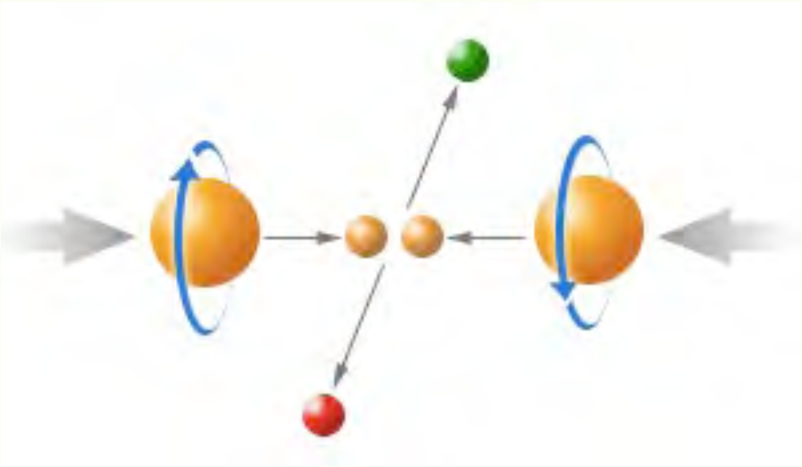
OAM in T-odd TMDs and FSIs
Bessel Weighted Asymmetries



10 February 2012

INT Univ. of Washington

Leonard Gamberg Penn State University-Berks

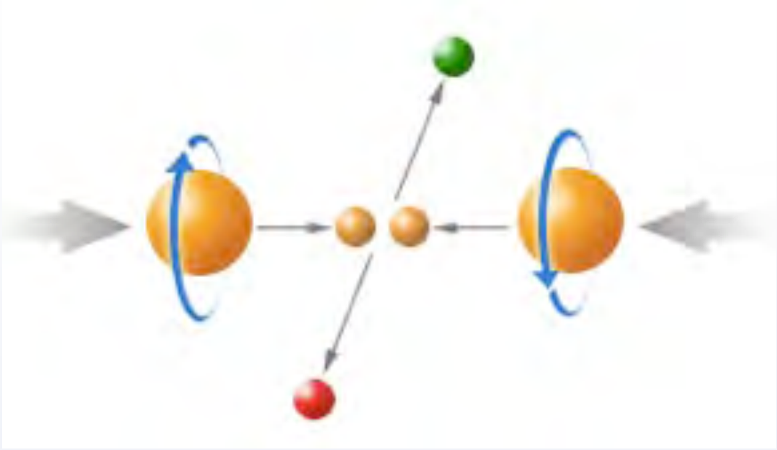
Boer,LG,Musch,Prokudin JHEP 2011
LG, Schlegel PLB 2010, in prep

Outline

- **Review transverse spin Effects - TSSAs**
  - Transverse Spin Effects-twist 3 & TMD twist 2
- **Color Gauge Inv. & Gauge links - “T-odd” TMDs**
- **Role of Gauge Links (hard processes)-**

  “process dependence”, Soft Factor (in SIDIS)
- On the merit of Bessel Weighted asymmetries “S/T” pic of SIDIS
- Fourier Transformed SIDIS cross section & “FT” TMDs
- Cancellation of the Soft Factor from WA
- **T-odd PDFs & moments via ISI/FSIs ...Lensing QCD-Phases**
- Some pheno results

Comments Importance of TMDs



- Single inclusive hadron production in hadronic collisions largest/ oldest observed TSSAs

- From theory view notoriously challenging from partonic picture twist-3 power suppressed in hard scale (vs. w/ SIDIS, DY, e^+e^-)

- Connection w/ twist 2 "TMD" approach

  - Operator level ETQS fnct 1st moment of Sivers

$$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x,k_T^2) + \text{"UV"} \ldots$$

$$= -2M f_{1T}^{\perp(1)}(x)$$

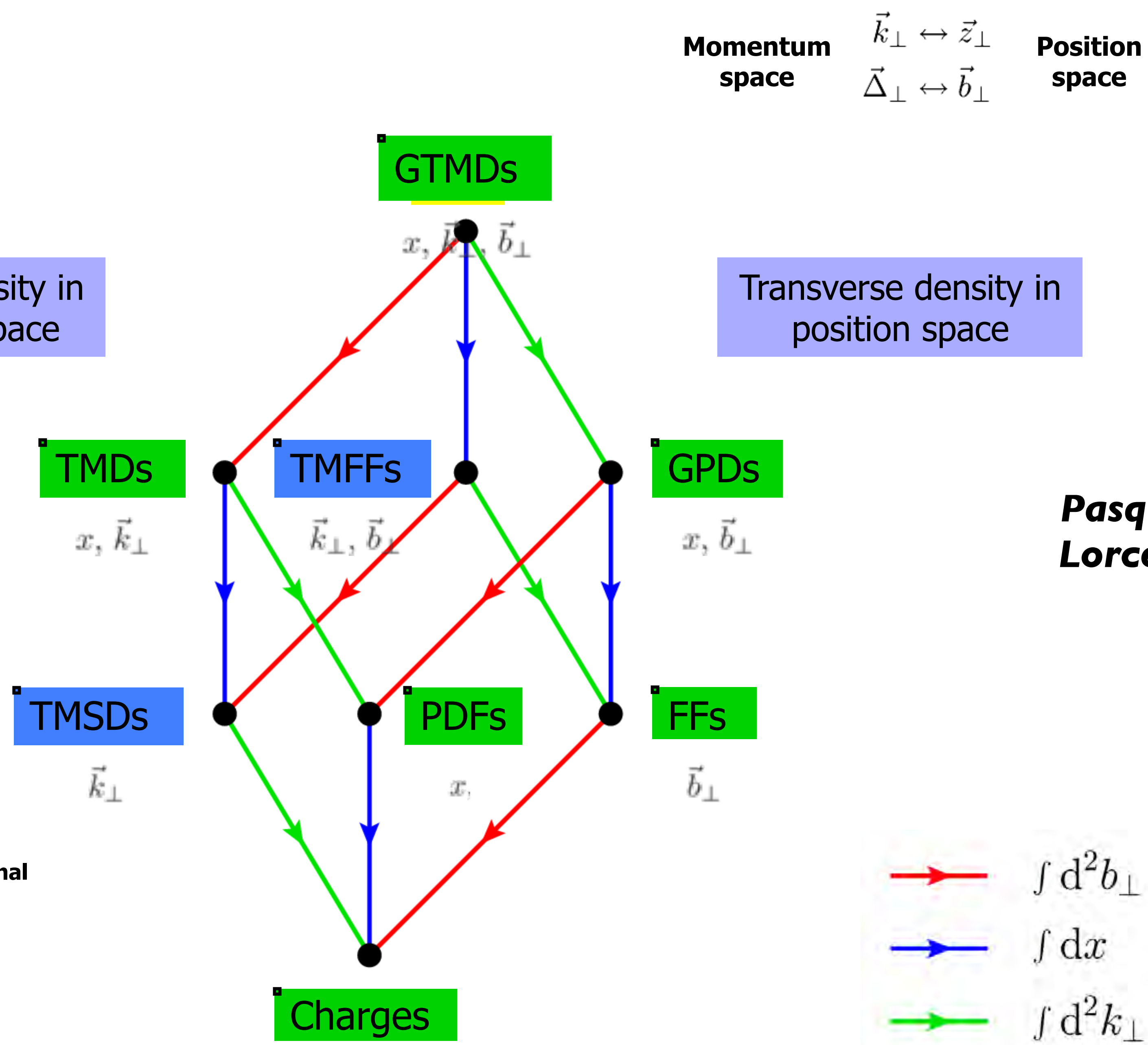
Z.Kang & A.Prokudin

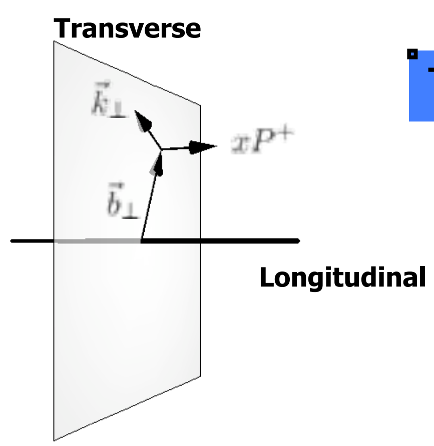
$$\tilde{f}_{1T}^{\perp(1)}(x,|\boldsymbol{b}_T|) = \int d^2p_T \frac{|p_T|}{|\boldsymbol{b}_T|M^2} J_1(|\boldsymbol{b}_T||p_T|) f_{1T}^{\perp}(x,p_T^2)$$

Boer, LG, Musch, Prokudin JHEP-2011--arXiv:1107.529

Comments Importance of TMDs

Belitsky, Ji , Yuan (2004 PRD)
[Meißner, Metz, Schlegel (2009 JHEP)]

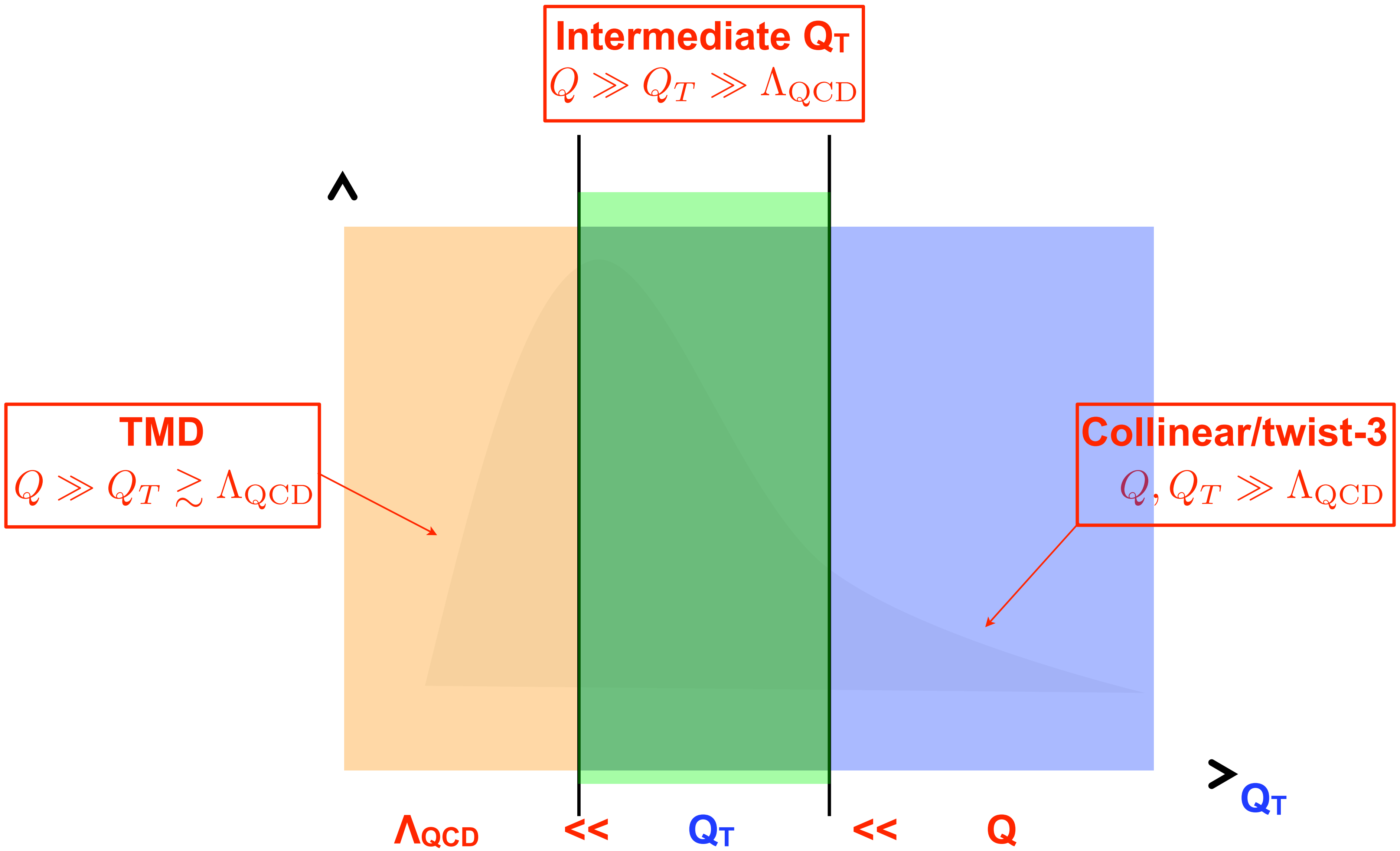




Pasquini & Lorce Slide

Connection of twist 3 and twist 2 approach “overlap regime”

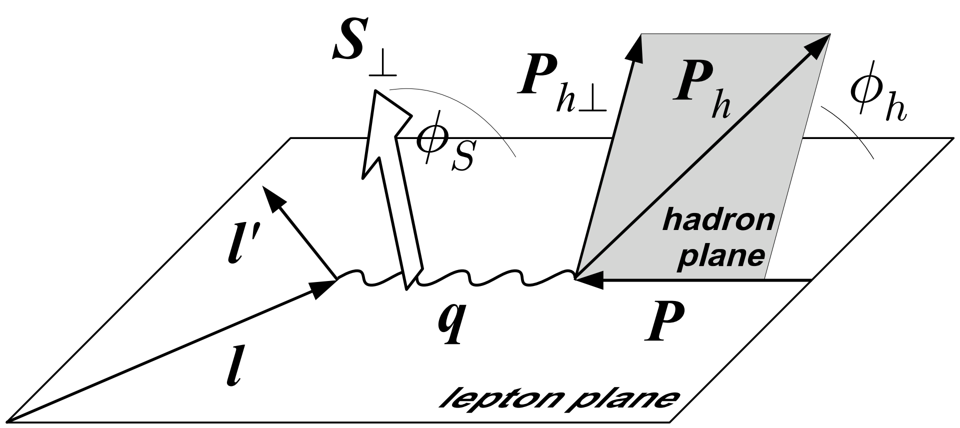
Ji, Qiu, Vogelsang, Yuan PRL 2006 ...
Bacchetta, Boer, Diehl, Mulders JHEP 2008

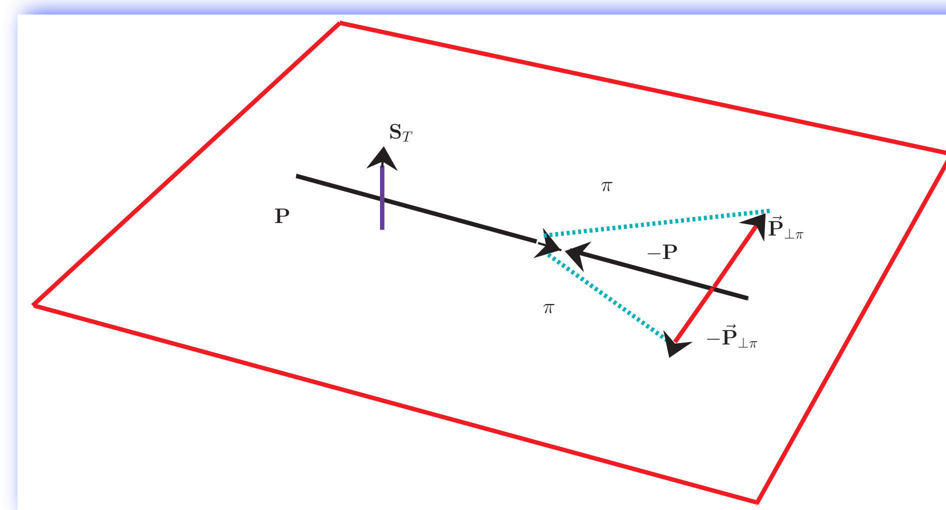


- Explore role parton model processes in twist-2&3 approaches
  LG & Z. Kang PLB 2011, D’Alesio, LG, Z. Kang, C.Pisano PLB 2011
  “exploring impact of Gauge Inv”

Two methods to account for SSA in QCD

- Depends on momentum of probe $q^2 = -Q^2$ and momentum of produced hadron $\boldsymbol{P}_{h\perp}$ relative to hadronic scale $k_T^2 (\equiv k_\perp^2) \sim \Lambda_{\rm QCD}^2$





- $k_\perp^2 \sim P_{h\perp}^2 \ll Q^2$ two scales-TMDs

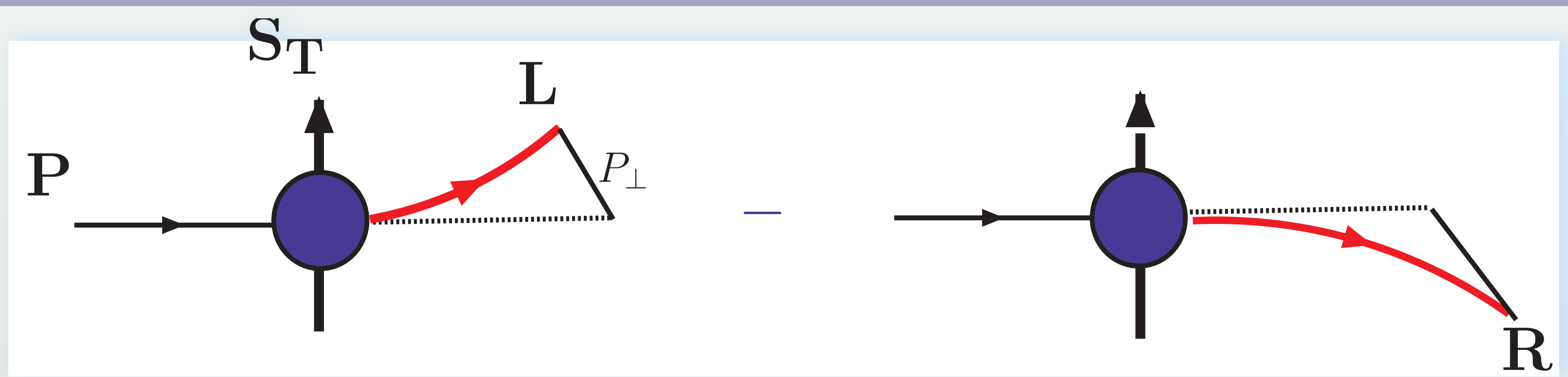
$$\Delta\sigma(P_h, S) \sim \Delta f_{a/A}^{\perp}(x, p_\perp) \otimes D_{h/c}(z, K_\perp) \otimes \hat{\sigma}_{\rm parton}$$

- $k_\perp^2 \ll P_{h\perp}^2 \sim Q^2$ twist 3 factorization-ETQSs

$$\Delta\sigma(P_h, S) \sim \frac{1}{Q}\, f_{a/A}^{\perp}(x) \otimes f_{b/B}(x) \otimes D_{h/c}(z) \otimes \hat{\sigma}_{\rm parton}$$

Ingredients transverse SPIN-Orbit observables kinematics

- **Single Spin Asymmetry**



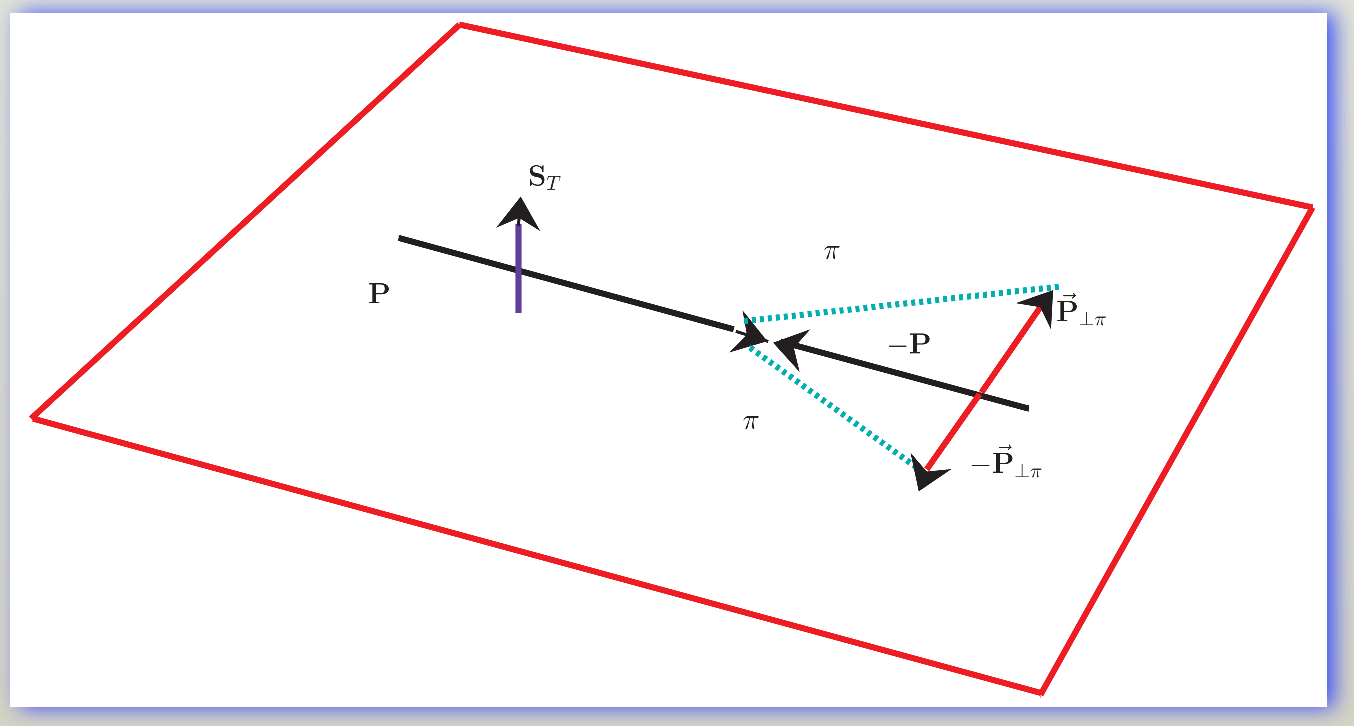
Parity Conserving interactions: SSAs Transverse Scattering plane

$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_\perp^\pi)$$

- **Rotational invariance** $\sigma^\downarrow(x_F, \boldsymbol{p}_\perp) = \sigma^\uparrow(x_F, -\boldsymbol{p}_\perp)$

  $\Rightarrow$ ***Left-Right Asymmetry***

$$\boldsymbol{A}_N = \frac{\sigma^\uparrow(x_F,\boldsymbol{p}_\perp) - \sigma^\uparrow(x_F,-\boldsymbol{p}_\perp)}{\sigma^\uparrow(x_F,\boldsymbol{p}_\perp) + \sigma^\uparrow(x_F,-\boldsymbol{p}_\perp)} \equiv \Delta\sigma$$



Reaction Mechanism w/ Partonic Description

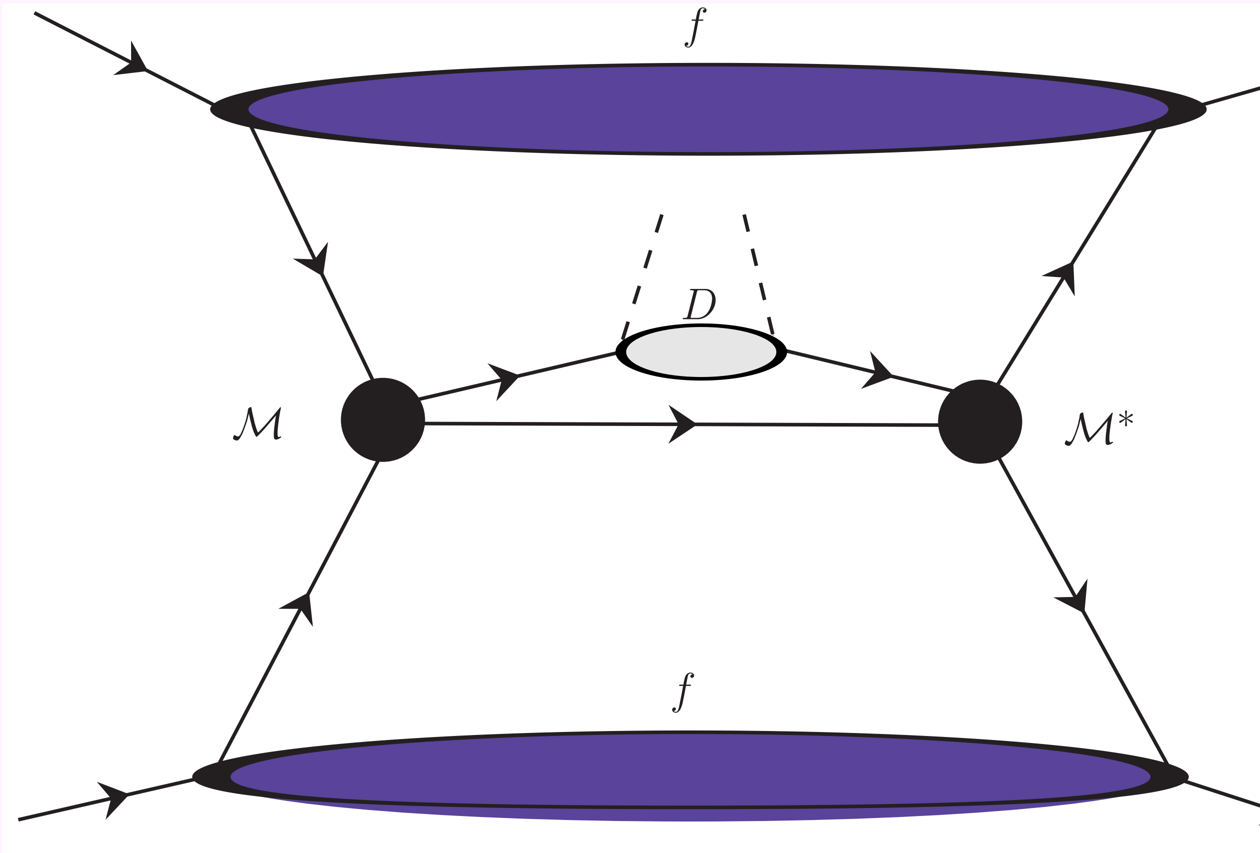
Collinear factorized QCD parton dynamics

$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$

$$\Delta\hat{\sigma} \equiv \hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}$$

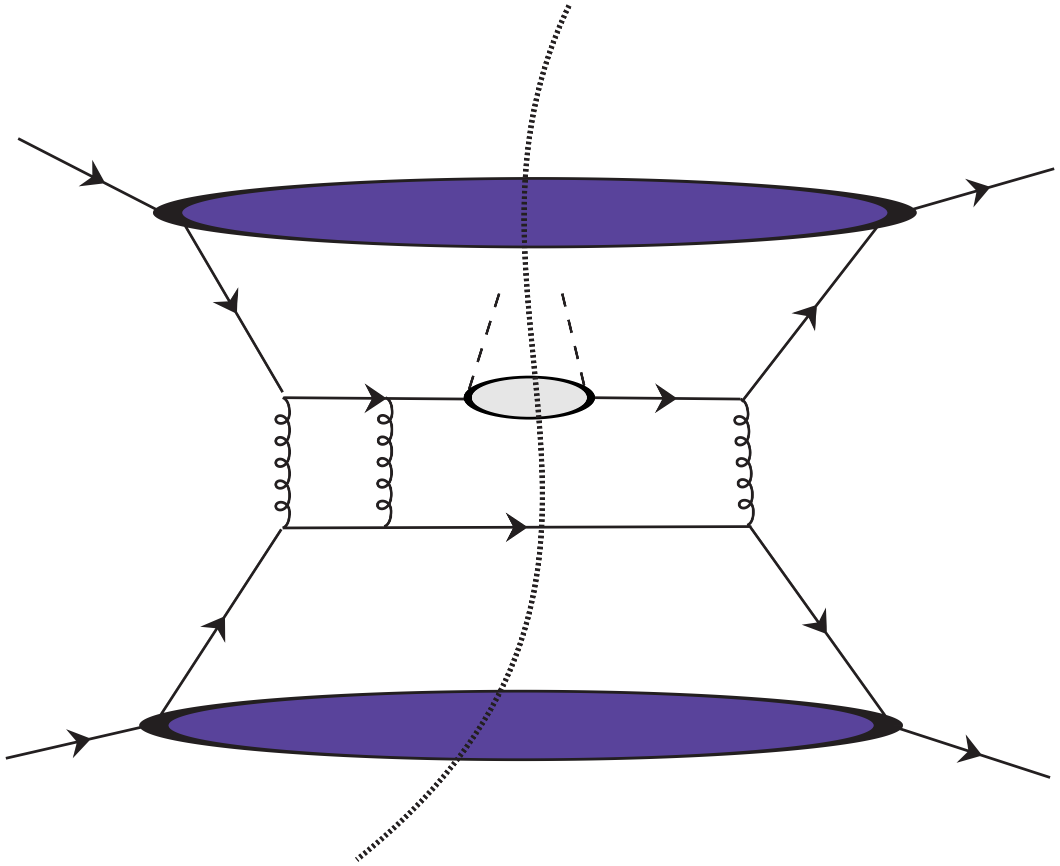
$$|\uparrow / \downarrow\rangle = (|+\rangle \pm i|-\rangle)$$

$$\hat{a}_N = \frac{\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}}{\hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow}} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^{-}\right)}{|\mathcal{M}^{+}|^2 + |\mathcal{M}^{-}|^2}$$

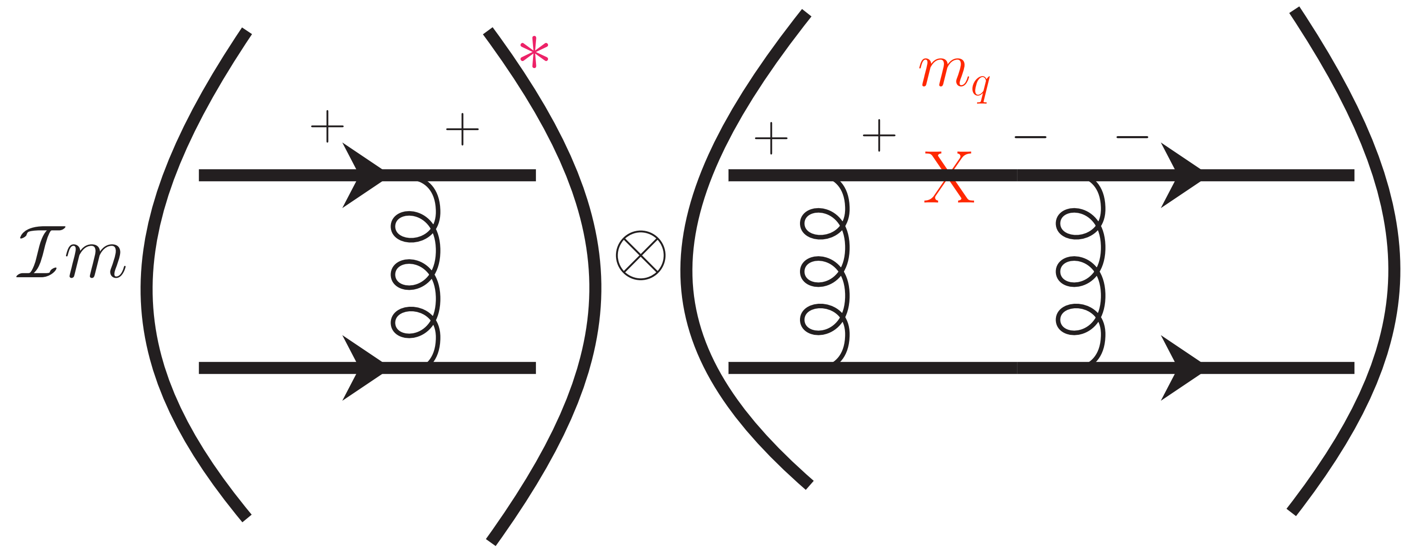


Interference of helicity flip and non-flip amps
1) requires breaking of chiral symmetry m_q/E
2) relative phases require higher order corrections

Factorization Theorem at Partonic level



$$\Delta\hat{\sigma} \sim \mathcal{I}m[M^*{}_+ M_-]$$



- Born amps are real -- need “loops”----> phases
- QCD interactions conserve helicity up to corrections $\mathcal{O}\left(\frac{m_q}{E_q}\right)$

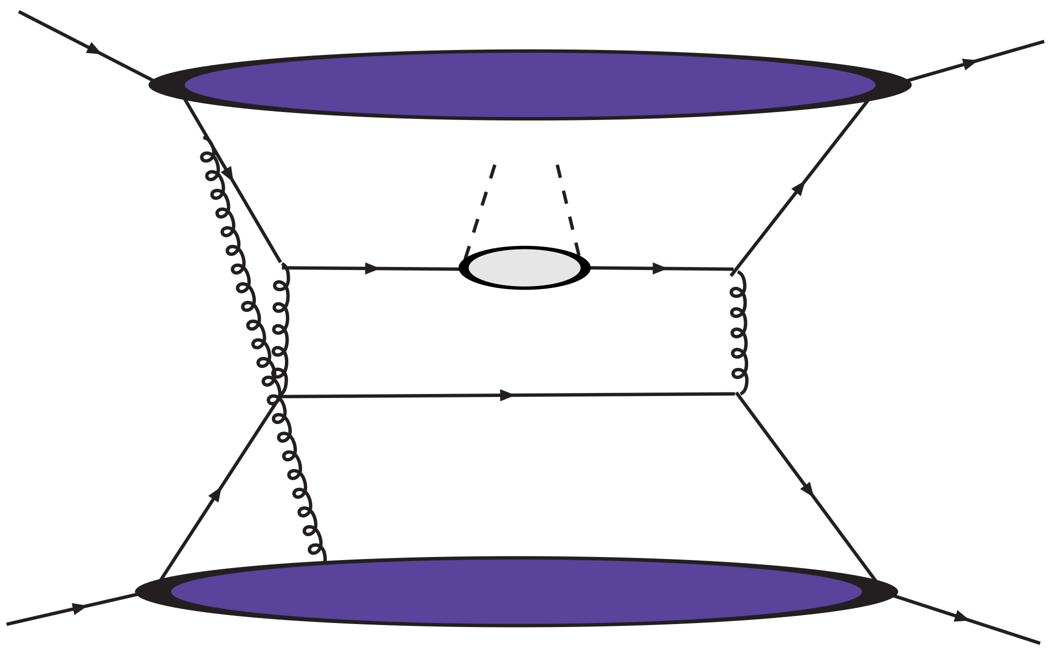
Twist three and trivial in chiral limit

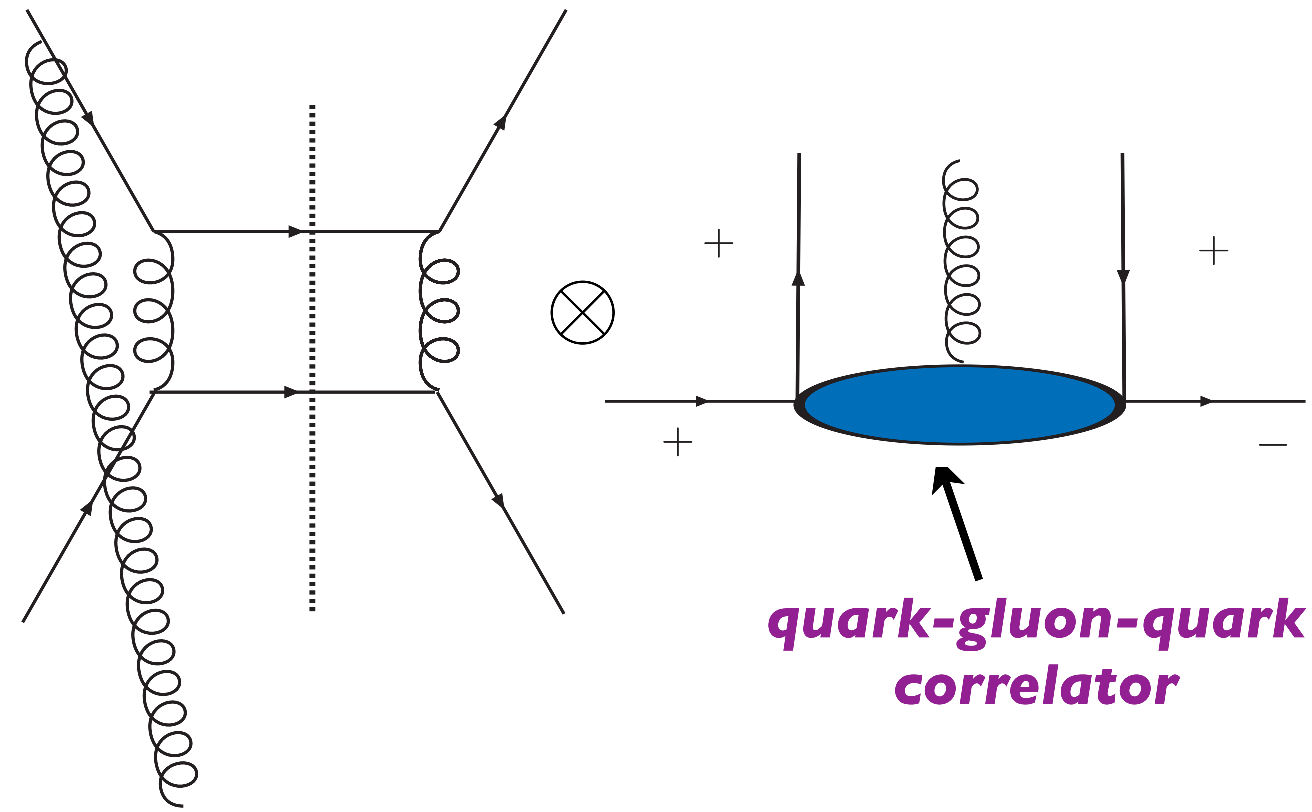
$$A_N \propto \frac{m_q}{E}\alpha_s$$ at the partonic level Kane & Repko, PRL: 1978

Twist 3 ETQS approach-"Partonic Picture"

$Q \sim P_T >> \Lambda_{\rm qcd}$ One scale Collinear fact Twist 3

Phases in soft poles of prop hard processes Efremov & Teryaev PLB 1982





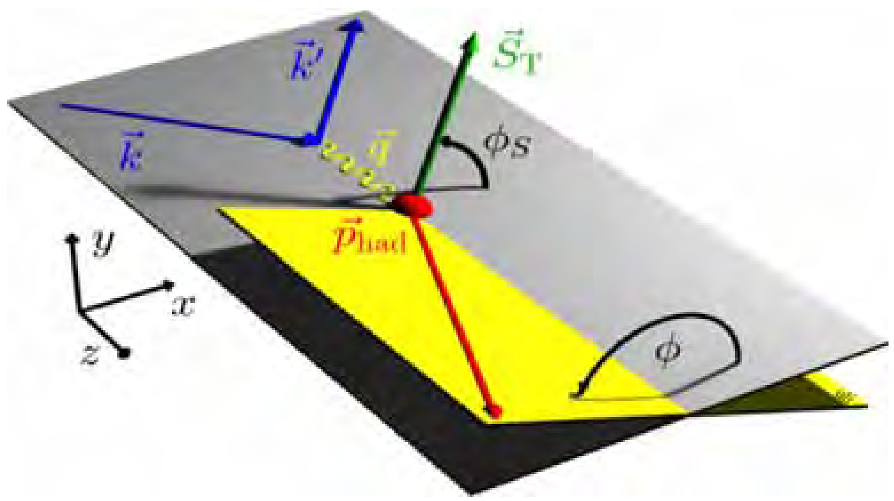
$$\Delta\sigma \sim f_a \otimes T_F \otimes H_{ETQS} \otimes D^{q\to h}$$

$$\frac{1}{xs + i\epsilon} = \mathcal{P}\left(\frac{1}{xs}\right) \pm i\pi\delta(xs)$$

- Phases from interference two parton three parton scattering amplitudes

- Factorization and Pheno: Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..., Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ... Kouvaris Ji, Qiu, Vogelsang! 2006, Vogelsang and Yuan PRD 2007

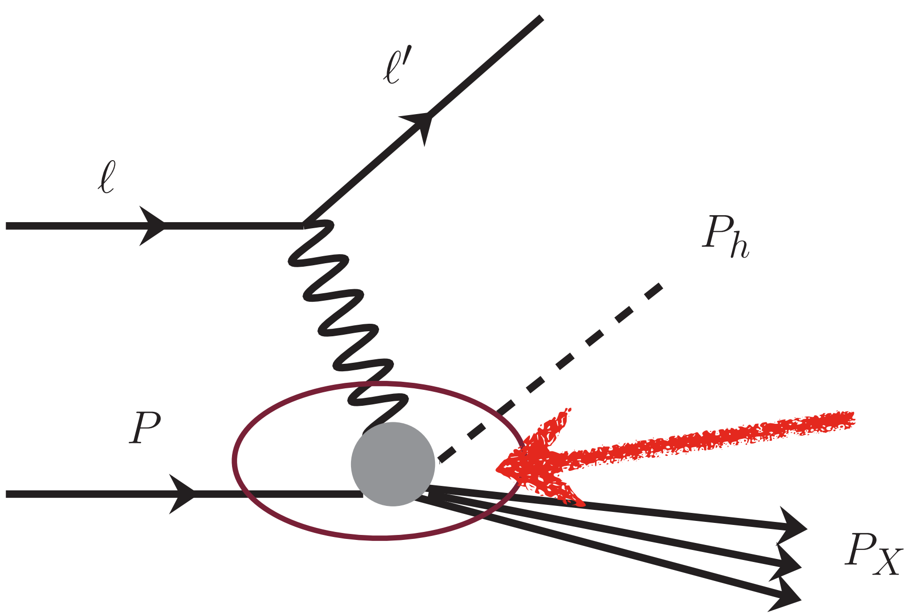
Factorization in Parton Model



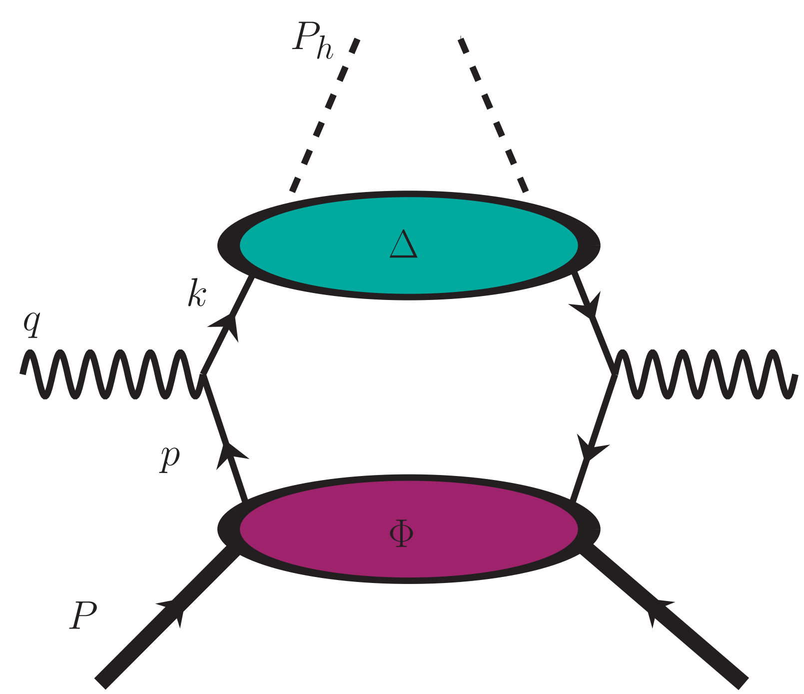
$$x_B = \frac{Q^2}{2P\cdot q}$$

$$z_h = \frac{P\cdot P_h}{P\cdot q} \approx \frac{P_h^-}{q^-}$$

Parton model & DIS kinematics



Factorize



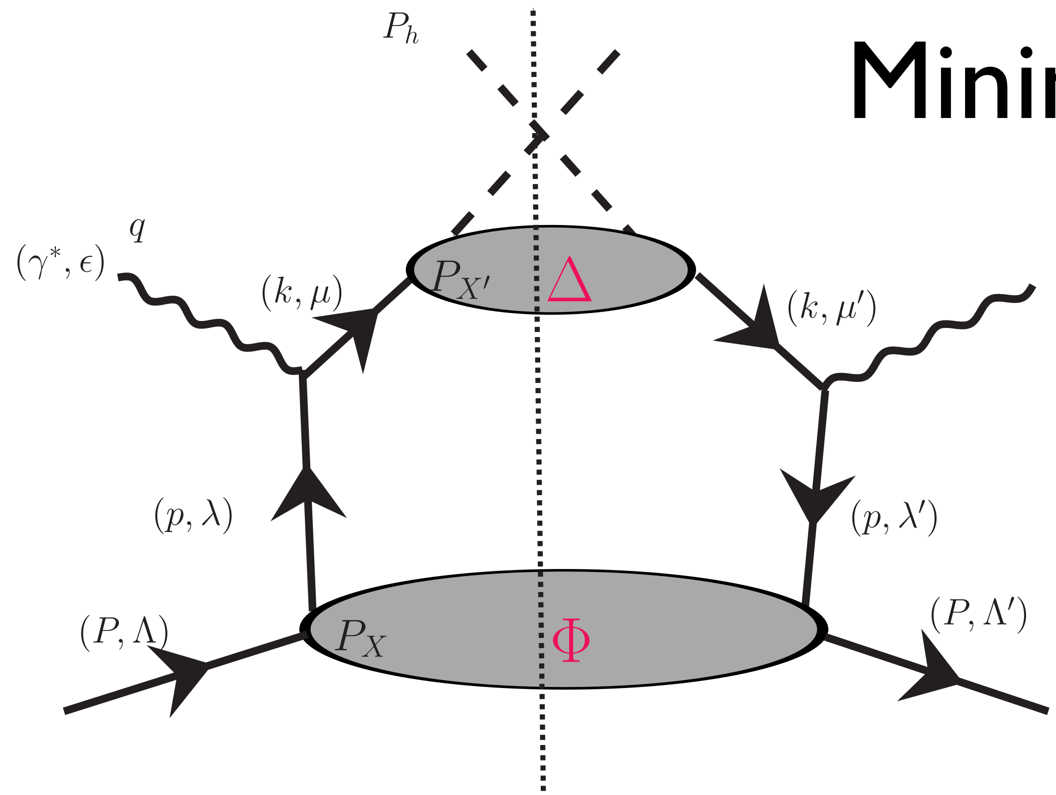
$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}|\, d|\boldsymbol{P}_{h\perp}|} = \frac{\alpha^2}{x_B y Q^2}\, \frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right) L_{\mu\nu}W^{\mu\nu},$$

Factorization parton model P_T of hadron small sensitive to intrinsic transv. momentum of partons

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^2} \int \frac{d^2\mathbf{k}_T}{(2\pi)^2} \delta^2\left(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z_h} - \mathbf{k}_T\right) \mathrm{Tr}\left[\Phi(x,\mathbf{p}_T)\gamma^\mu \Delta(z,\mathbf{k}_T)\gamma^\nu\right]$$

$$\Phi(x,\mathbf{p}_T) = \int dp^- \Phi(p,P,S)|_{p^+=x_B P^+}\,, \qquad \Delta(z,\mathbf{k}_T) = \int dk^- \Delta(k,P_h)|_{k^-=\frac{P^-}{z_h}}$$

Small transverse momentum !!!



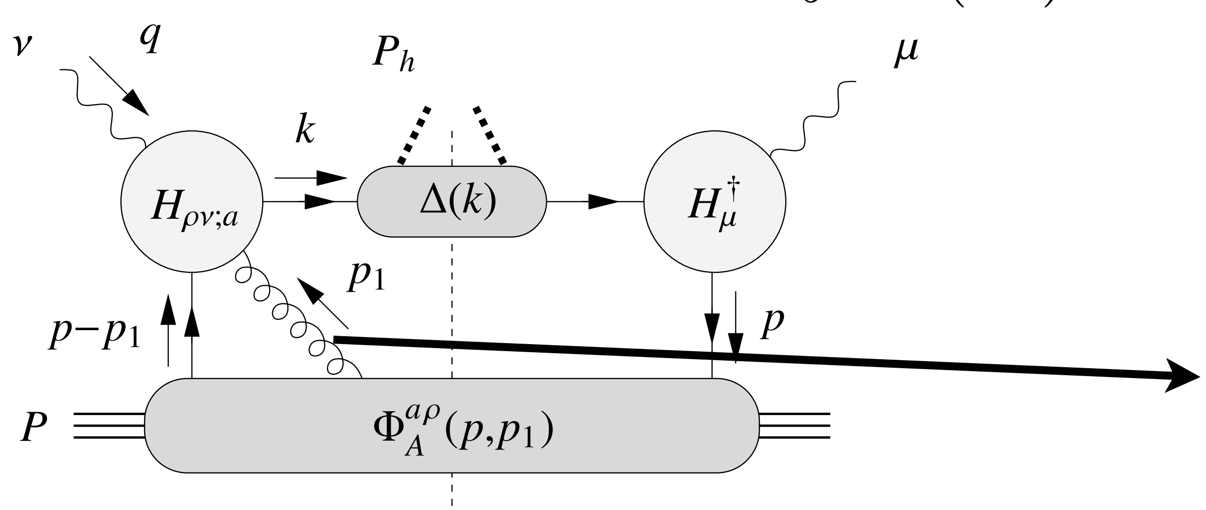
Minimal requirement satisfy color gauge invariance

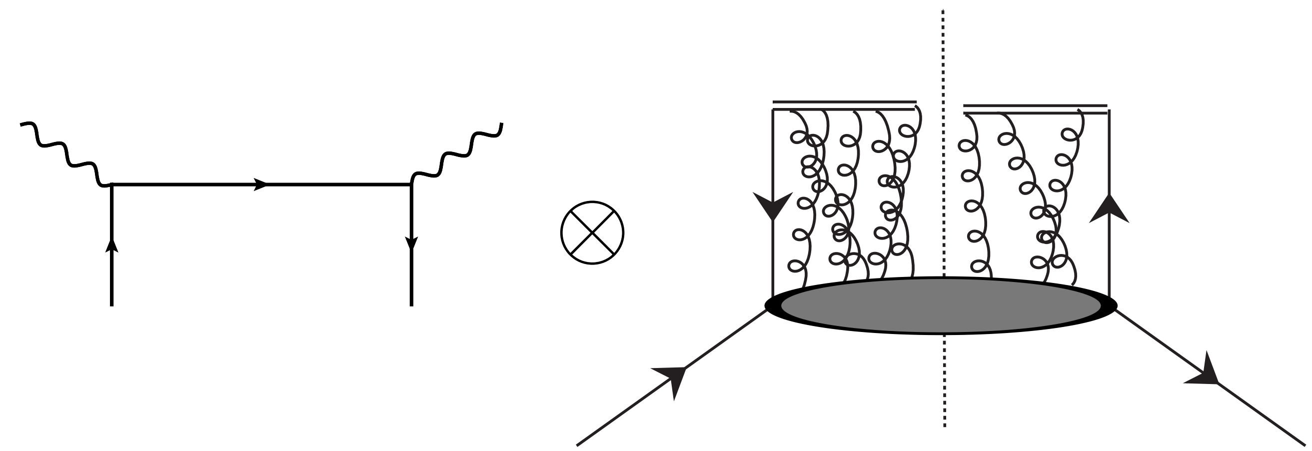
Minimal Requirement Color Gauge Inv. Reaction Mechanism

Gauge link determined re-summing leading gluon interactions btwn soft and hard

Efremov,Radyushkin Theor. Math. Phys. 1981,Belitsky, Ji, Yuan NPB 2003,
Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

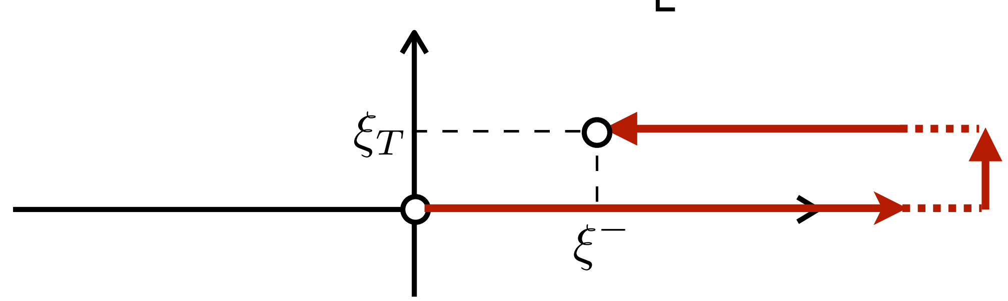
$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-, \xi_T)|P\rangle|_{\xi^+=0}$$





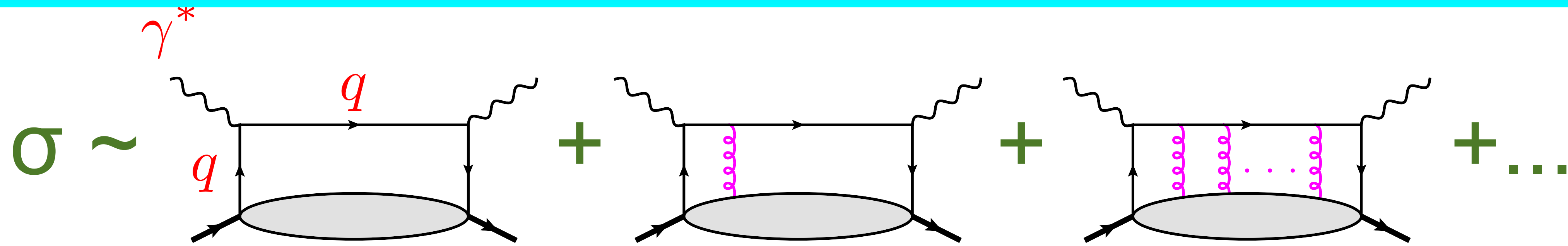
- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

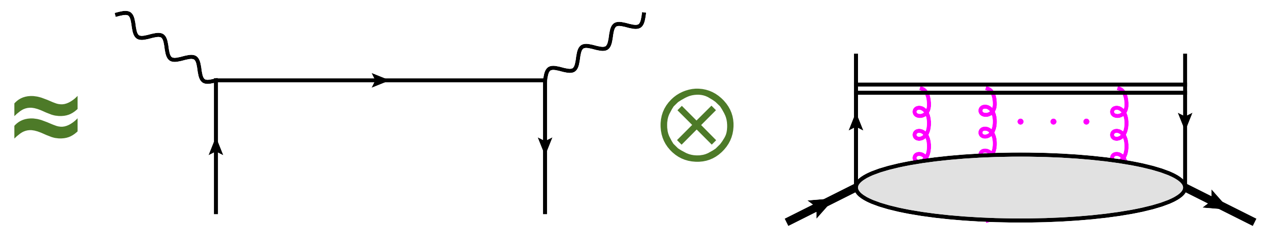
$$W_{\mu\nu}(q, P, S, P_h) = \int d^4p\, d^4k\, \delta^4(p + q - k) \mathrm{Tr}\left[\Phi^{\mathcal{U}^{[C]}_{[\infty;\xi]}}(p) H^\dagger_\mu(p, k) \Delta(k) H_\nu(p, k)\right]$$



$U_{[+]}$

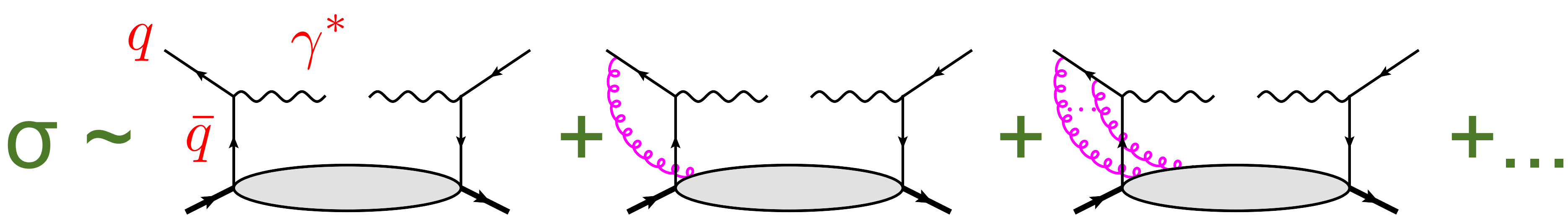
Gauge Link determined by Gauge link determined re-summing leading gluon interactions btwn soft and hard Process Dependence break down of Universality

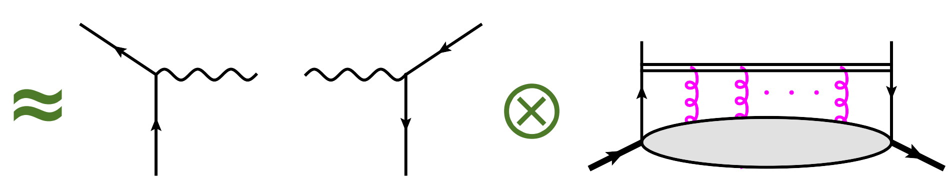




PDFs with SIDIS gauge link

$$\mathcal{P}\, e^{ig\int_y^{\infty} d\lambda\cdot A(\lambda)}$$





PDFs with DY gauge link

$$\mathcal{P}\, e^{ig\int_y^{-\infty} d\lambda\cdot A(\lambda)}$$

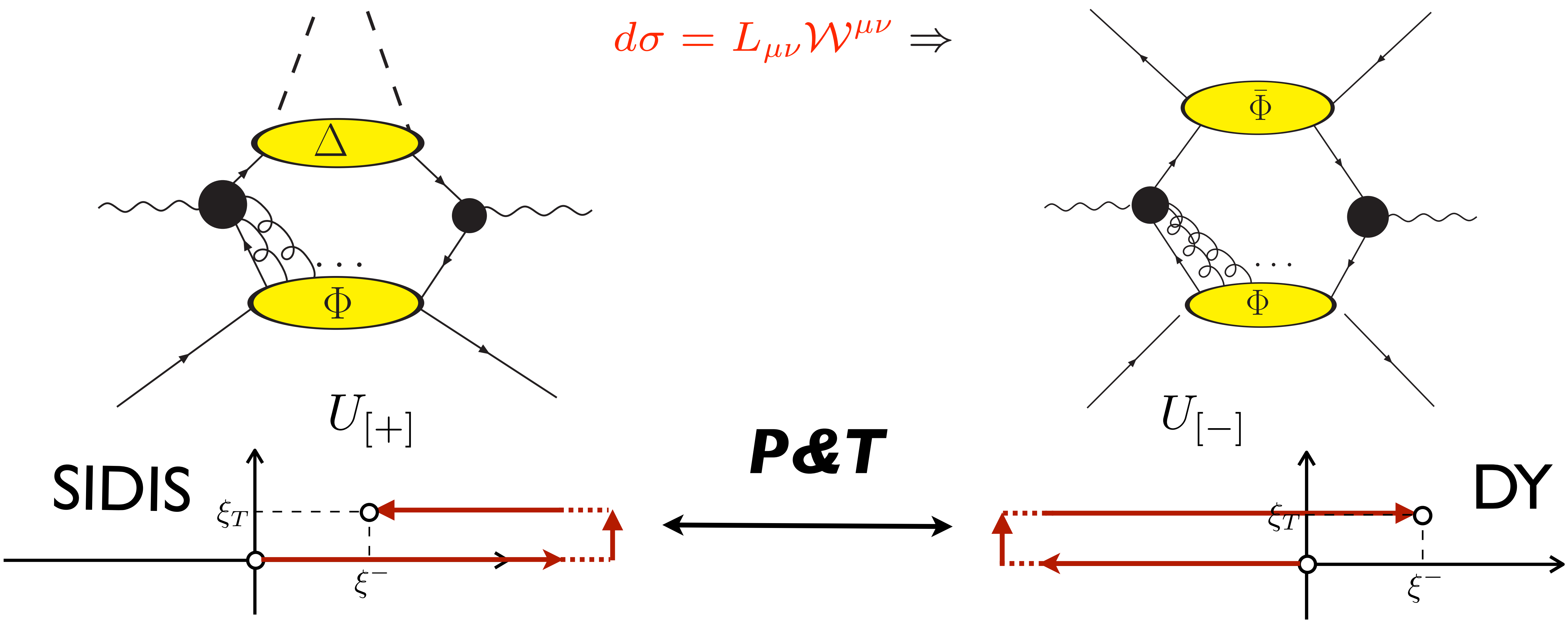
"Generalized Universality" Fund. Prediction of QCD Factorization

$$f_{1T\,sidis}^{\perp}(x,k_T) = -f_{1T\,DY}^{\perp}(x,k_T) \qquad p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

EIC conjunction with DY exp. E906-Fermi, RHIC II, Compass, JPARC

Process Dependence, Collins PLB 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...

$$d\sigma = L_{\mu\nu}\mathcal{W}^{\mu\nu} \Rightarrow$$



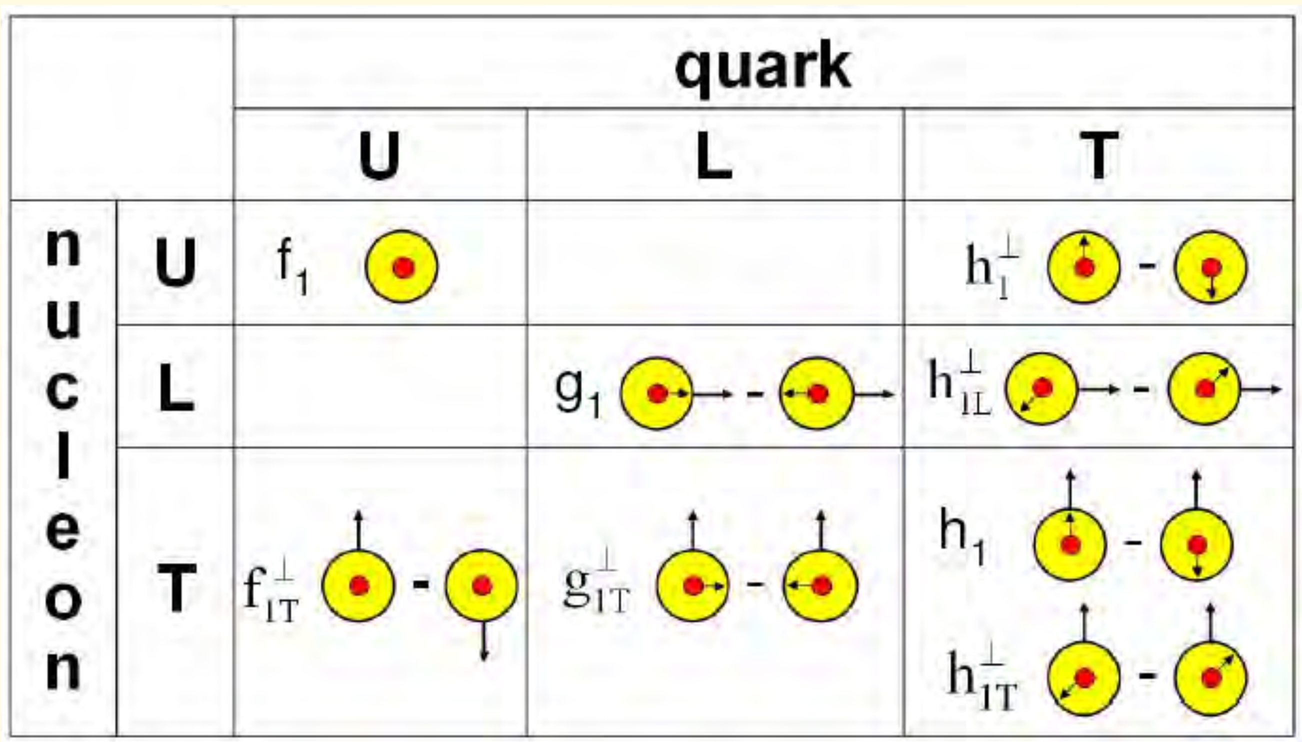
$$\Phi^{[+]*}(x,p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x,p_T)i\gamma^1\gamma^3$$

Thus 8 “LT” TMDs: Correlation Matrix Dirac space

$$\Phi^{[\gamma^+]}(x,\boldsymbol{p}_T) \equiv f_1(x,\boldsymbol{p}_T^2) + \frac{\epsilon_T^{ij} p_{Ti} S_{Tj}}{M}\, f_{1T}^{\perp}(x,\boldsymbol{p}_T^2)$$

$$\Phi^{[\gamma^+\gamma_5]}(x,\boldsymbol{p}_T) \equiv \lambda\, g_{1L}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T\cdot \boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{p}_T^2)$$

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x,\boldsymbol{p}_T) \equiv S_T^i\, h_{1T}(x,\boldsymbol{p}_T^2) + \frac{p_T^i}{M}\left(\lambda\, h_{1L}^{\perp}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T\cdot \boldsymbol{S}_T}{M}\, h_{1T}^{\perp}(x,\boldsymbol{p}_T^2)\right)$$



$$+ \frac{\epsilon_T^{ij} p_T^j}{M}\, h_1^{\perp}(x,\boldsymbol{p}_T^2)$$

SIDIS- CS expressed model indpen. thru structure functions

$$\begin{aligned}
\frac{d\sigma}{dx_B\,dy\,d\psi\,dz_h\,d\phi_h\,dP_{h\perp}^2} &= \frac{\alpha^2}{x_B y Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&+ \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&+ S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+ S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+ |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&+ \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+ |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}$$

Kotzinian NPB 95,
Mulders Tangermann NPB 96,
Bacchetta et al JHEP 08

Structure functions projected from cross section

$$A_{XY}^{\mathcal{F}} \equiv 2\,\frac{\int d\phi_h\,d\phi_S\,\mathcal{F}(\phi_h,\phi_S)\,\left(d\sigma^\uparrow-d\sigma^\downarrow\right)}{\int d\phi_h d\phi_S\,(d\sigma^\uparrow+d\sigma^\downarrow)}\,,$$

X Y-polarization

e.g. $\mathcal{F}(\phi_h,\phi_S)=\sin(\phi_h-\phi_S).$

Partonic picture of nucleon SFs

$$\mathcal{C}\big[w\,f\,D\big] = x\sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big], \qquad F_{LL} = \mathcal{C}\big[g_{1L} D_1\big],$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right], \qquad F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

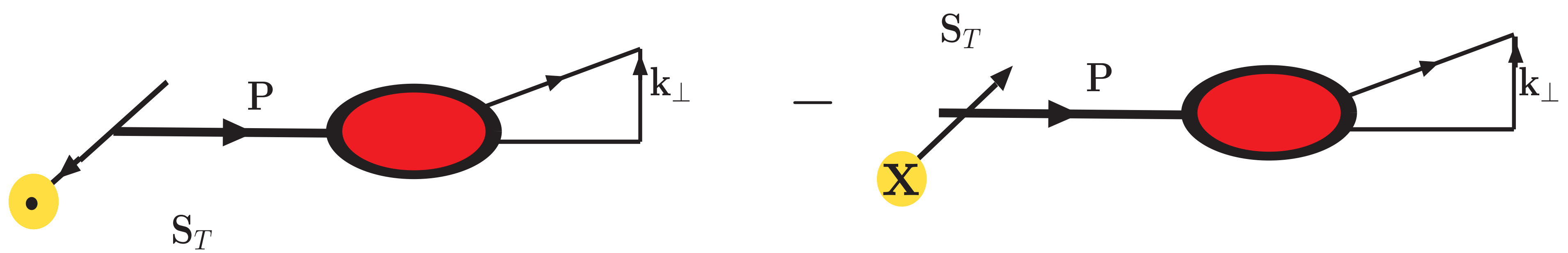
$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right], \qquad F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^{\perp} H_1^{\perp}\right],$$

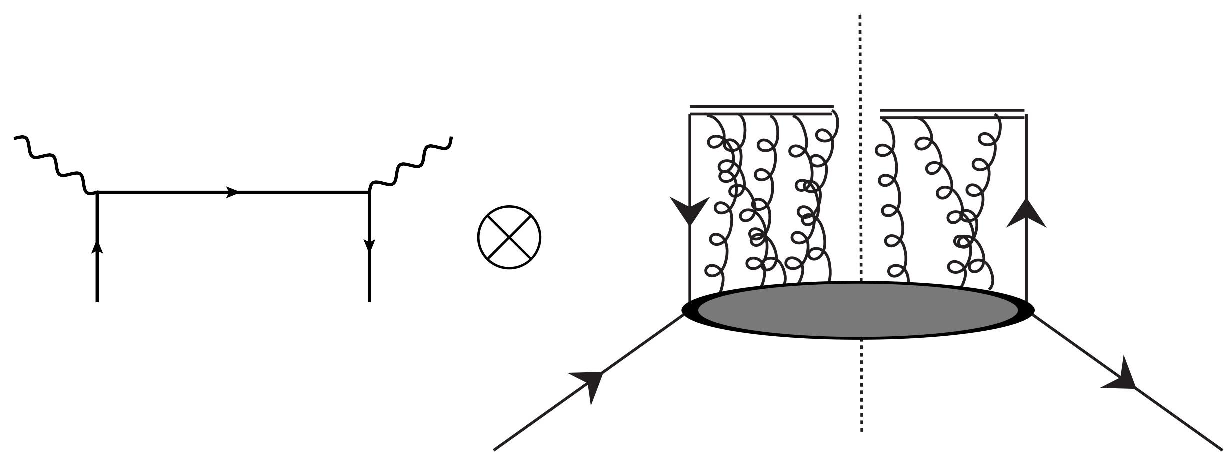
$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,(\boldsymbol{p}_T\cdot\boldsymbol{k}_T) + \boldsymbol{p}_T^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - 4\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{2M^2 M_h}\, h_{1T}^{\perp} H_1^{\perp}\right]$$

TSSAs thru "T-odd" non-pertb. spin-orbit correlations....

Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

- Sivers PRD: 1990 TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron





$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim D \otimes f \otimes \Delta f^{\perp} \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta f^{\perp}(x, k_{\perp}) = iS_T \cdot (P \times k_{\perp})}\, f_{1T}^{\perp}(x, \boldsymbol{k}_{\perp})$$

Weighted asymmetries Model independent deconvolution of cross section in terms of moments of TMDs

Kotzinian, Mulders PLB 97, Boer, Mulders PRD 98

$$A_{UT,T}^{w_1 \sin(\phi_h-\phi_S)} = 2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}| d\phi_h\, d\phi_S\, w_1(|\boldsymbol{P}_{h\perp}|)\ \sin(\phi_h-\phi_S)\ \{d\sigma(\phi_h,\phi_S)-d\sigma(\phi_h,\phi_S+\pi)\}}{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d\phi_S\, w_0(|\boldsymbol{P}_{h\perp}|)\ \{d\sigma(\phi_h,\phi_S)+d\sigma(\phi_h,\phi_S+\pi)\}},$$

e.g. $$\mathcal{W}_{\text{Sivers}} = \frac{|\boldsymbol{P}_{h\perp}|}{zM}\sin(\phi_h-\phi_S)$$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\,\frac{\sum_a e_a^2\ f_{1T}^{\perp(1)}(x)\ D_1^{a(0)}(z)}{\sum_a e_a^2\ f_1^{a(0)}(x)\ D_1^{a(0)}(z)}$$

Undefined w/o regularization to subtract infinite contribution at large transverse momentum

Bacchetta et al. JHEP 08

Comments

- Propose generalize Bessel Weights-"BW"
- BW procedure has advantages
  - ★ Structure functions become simple product $\mathcal{P}[\quad]$ rather than convolution $\mathcal{C}[\quad]$
  - ★ CS has simplier s/t interpretation as a $b_T\ [\mathrm{GeV}^{-1}]$ multipole expansion in terms of conjugate to $\boldsymbol{P}_{h\perp}$
  - ★ Use Fourier Bessel tranforms-
  - ★ The usefulness of Fourier-Bessel transforms in studying the factorization as well as the scale dependence of transverse momentum dependent cross section has been known for quite sometime
    CS(82), Ellis,Fleishon,Stirling (81), Ji,Ma,Yuan (05), Collins, Foundations of Perturbative QCD, Cambridge University Press, Cambridge(11)

Further Comments

- Introduces a free parameter $\mathcal{B}_T\,[\mathrm{GeV}^{-1}]$ that is Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero for moments
- Addtnl. bonus soft factor eliminated from weighted asymmetries
- Possible to compare observables at different scales.... could be useful for an EIC

Advantages of Bessel Weighting

1.“Deconvolution”-of CS--struct fnct simple product “$\mathcal{P}$“

$$W^{\mu\nu}(\boldsymbol{P}_{h\perp}) \equiv \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{-i\boldsymbol{b}_T\cdot\boldsymbol{P}_{h\perp}}\, \tilde{W}^{\mu\nu}(\boldsymbol{b}_T)\,,$$

$$\tilde{\Phi}_{ij}(x, z\boldsymbol{b}_T) \equiv \int d^2\boldsymbol{p}_T\, e^{iz\boldsymbol{b}_T\cdot\boldsymbol{p}_T}\; \Phi_{ij}(x,\boldsymbol{p}_T)$$

$$\tilde{\Delta}_{ij}(z, \boldsymbol{b}_T) \equiv \int d^2\boldsymbol{K}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{K}_T}\; \Delta_{ij}(z,\boldsymbol{K}_T)$$

$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot\boldsymbol{P}_{h\perp}} \left\{ \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_B}\right) L_{\mu\nu}\tilde{W}^{\mu\nu} \right\}.$$

$$2M\tilde{W}^{\mu\nu} = \sum_a e_a^2\, \mathrm{Tr}\left(\tilde{\Phi}(x, z\boldsymbol{b}_T)\gamma^\mu \tilde{\Delta}(z,\boldsymbol{b}_T)\gamma^\nu\right)\,.$$

1.“Deconvolution”-Sivers struct fnct simple product “$\mathcal{P}$“

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$\mathcal{C}\big[w\, f\, D\big] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

★ $$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -x_B \sum_a e_a^2 \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|^2\, J_1(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|) M z\ \tilde{f}_{1T}^{\perp a(1)}(x, z^2\boldsymbol{b}_T^2)\, \tilde{D}_1^a(z, \boldsymbol{b}_T^2).$$

$\tilde{f}_1,\ \tilde{f}_{1T}^{\perp(1)}$, and $\tilde{D}_1$ are Fourier Transf. of TMDs/FFs and finite

- Peter's and Barbara's (Jerry's as well ..Bagel)-Pretzelosity

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)+\boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)-4\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)}{2M^2M_h}\,h_{1T}^{\perp}H_1^{\perp}\right]$$

$$\mathcal{C}\left[w\,f\,D\right] = x\sum_a e_a^2\int d^2\boldsymbol{p}_T\,d^2\boldsymbol{k}_T\,\delta^{(2)}\left(\boldsymbol{p}_T-\boldsymbol{k}_T-\boldsymbol{P}_{h\perp}/z\right)w(\boldsymbol{p}_T,\boldsymbol{k}_T)\,f^a(x,p_T^2)\,D^a(z,k_T^2)$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = x_B\sum_a e_a^2\int\frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|^4\,J_3(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|)\frac{M^2M_hz^3}{4}\,\tilde{h}_{1T}^{\perp a(2)}(x,z^2\boldsymbol{b}_T^2)\,\tilde{H}_1^{\perp a(1)}(z,\boldsymbol{b}_T^2)\,.$$

Simple product " $\mathcal{P}$ "

1. "Deconvolution"-SIDIS structure functions simple products

a) F.T. SIDIS cross section w/ following defintions

$$\tilde{f}(x,\boldsymbol{b}_T^2) \equiv \int d^2\boldsymbol{p}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T}\; f(x,\boldsymbol{p}_T^2)$$

$$= 2\pi \int d|\boldsymbol{p}_T||\boldsymbol{p}_T|\; J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|)\; f^a(x,\boldsymbol{p}_T^2)\ ,$$

$$\tilde{f}^{(n)}(x,\boldsymbol{b}_T^2) \equiv n!\left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \tilde{f}(x,\boldsymbol{b}_T^2)$$

$$= \frac{2\pi\ n!}{(M^2)^n}\int d|\boldsymbol{p}_T||\boldsymbol{p}_T|\left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}_T|}\right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{p}_T|)\; f(x,\boldsymbol{p}_T^2)\ ,$$

b) n.b. connection to $\boldsymbol{p}_T$ moments

$$\tilde{f}^{(n)}(x,0) = \int d^2\boldsymbol{p}_T \left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n f(x,\boldsymbol{p}_T^2) \equiv f^{(n)}(x)$$

★ CS has simpler S/T interpretation as a multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate to $\boldsymbol{P}_{h\perp}$

$$
\begin{aligned}
&\frac{d\sigma}{dx_B\,dy\,d\phi_S\,dz_h\,d\phi_h\,|\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|} = \\
&\quad \frac{\alpha^2}{x_B y Q^2}\,\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\int\frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|\Big\{J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,T}+\varepsilon\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,L} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos\phi_h}+\varepsilon\cos(2\phi_h)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos(2\phi_h)} \\
&\quad + \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LU}^{\sin\phi_h} \\
&\quad + S_{\|}\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin 2\phi_h}\Big] \\
&\quad + S_{\|}\lambda_e\Big[\sqrt{1-\varepsilon^2}\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}^{\cos\phi_h}\Big] \\
&\quad + |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\left(\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,\mathcal{F}_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad + \varepsilon\,\sin(\phi_h+\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&\quad + |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$

$$\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} = \frac{1}{4}\mathcal{P}[\tilde{h}_{1T}^{\perp(2)}\ \tilde{H}_1^{\perp(1)}]$$

Structure Functions become

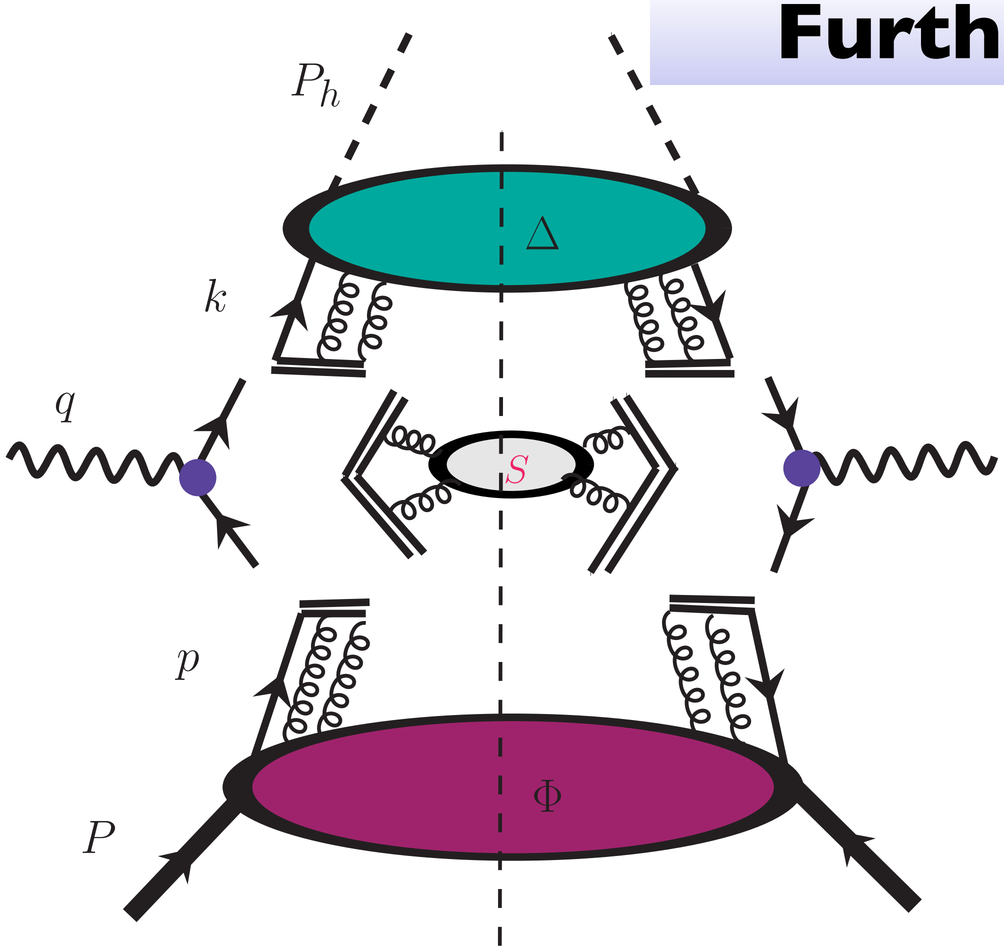
$$\begin{aligned}
\mathcal{F}_{UU,T} &= \mathcal{P}[\tilde{f}_1^{(0)}\ \tilde{D}_1^{(0)}]\,,\\
\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} &= -\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\ \tilde{D}_1^{(0)}]\,,\\
\mathcal{F}_{LL} &= \mathcal{P}[\tilde{g}_{1L}^{(0)}\ \tilde{D}_1^{(0)}]\,,\\
\mathcal{F}_{LT}^{\cos(\phi_h-\phi_s)} &= \mathcal{P}[\tilde{g}_{1T}^{(1)}\ \tilde{D}_1^{(0)}]\,,\\
\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} &= \mathcal{P}[\tilde{h}_1^{(0)}\ \tilde{H}_1^{\perp(1)}]\,,\\
\mathcal{F}_{UU}^{\cos(2\phi_h)} &= \mathcal{P}[\tilde{h}_1^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]\,,\\
\mathcal{F}_{UL}^{\sin(2\phi_h)} &= \mathcal{P}[\tilde{h}_{1L}^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]\,,\\
\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} &= \frac{1}{4}\mathcal{P}[\tilde{h}_{1T}^{\perp(2)}\ \tilde{H}_1^{\perp(1)}].
\end{aligned}$$

$$\mathcal{P}[\tilde{f}^{(n)}\tilde{D}^{(m)}] \equiv x_{_B}\sum e_a^2\,(zM|\boldsymbol{b}_T|)^n\,(zM_h|\boldsymbol{b}_T|)^m\,\tilde{f}^{a(n)}(x,z^2\boldsymbol{b}_T^2)\,\tilde{D}^{a(m)}(z,\boldsymbol{b}_T^2)\ ;$$

Correlator w/ explicit spin orbit correlations

$$\begin{aligned}
\tilde{\Phi}^{[\gamma^+]}(x,\boldsymbol{b}_T) &= \tilde{f}_1(x,\boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M \tilde{f}_{1T}^{\perp(1)}(x,\boldsymbol{b}_T^2)\,,\\
\tilde{\Phi}^{[\gamma^+\gamma^5]}(x,\boldsymbol{b}_T) &= S_L\,\tilde{g}_{1L}(x,\boldsymbol{b}_T^2) + i\,\boldsymbol{b}_T\cdot\boldsymbol{S}_T M\,\tilde{g}_{1T}^{(1)}(x,\boldsymbol{b}_T^2)\,,\\
\tilde{\Phi}^{[i\sigma^{\alpha+}\gamma^5]}(x,\boldsymbol{b}_T) &= S_T^\alpha\,\tilde{h}_1(x,\boldsymbol{b}_T^2) + i\,S_L\, b_T^\alpha M\,\tilde{h}_{1L}^{\perp(1)}(x,\boldsymbol{b}_T^2)\\
&\quad + \frac{1}{2}\left(b_T^\alpha b_T^\rho + \frac{1}{2}\,\boldsymbol{b}_T^2\, g_T^{\alpha\rho}\right) M^2\, S_{T\rho}\tilde{h}_{1T}^{\perp(2)}(x,\boldsymbol{b}_T^2)\\
&\quad - i\,\epsilon_T^{\alpha\rho} b_{T\rho} M \tilde{h}_1^{\perp(1)}(x,\boldsymbol{b}_T^2)\,,
\end{aligned}$$

Further Beyond “tree level” factorization



CS NPB 81,CSS NPB 1985 Collins, Hautman PLB 00,
Adilbi, Ji, Ma, Yuan PRD 05,
Cherednikov, Karanikas, Stefanis NPB 10, Collins
Oxford Press 2011,
Abyat, Rogers PRD 2011,
Abyat, Collins, Qiu, Rogers arXiv 2011 ...

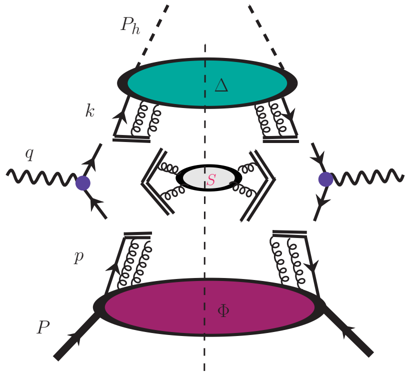
Soft

- Extra divergences at one loop and higher
- Various strategies to address them
- Extra variables needed to regulate divergences
- Modifies convolution integral by introduction soft factor
- Will show cancels in Bessel weighted asymmetries

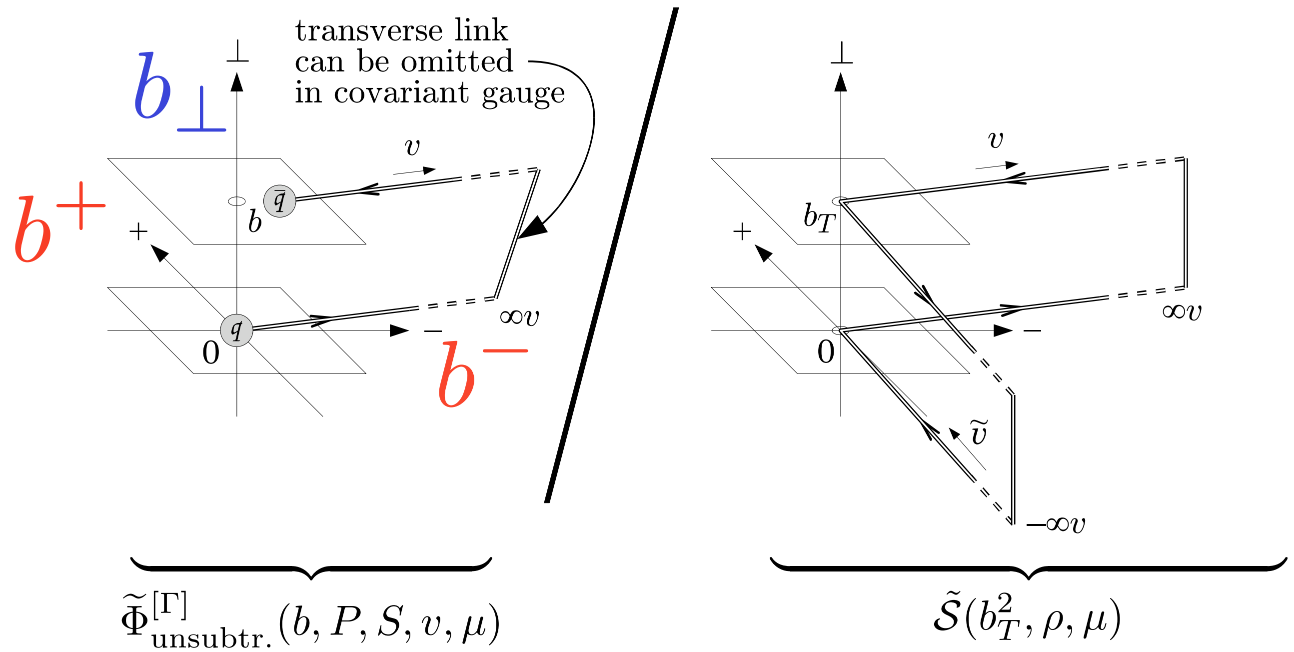
Collins Soper NPB 1981, Collins Metz PRL 2004, Ji, Ma, Yuan PRD 2005, also Bacchetta Boer Diehl Mulders JHEP 2008

Comments on Soft factor

- Collective effect soft gluons not associated with distribution frag function-factorizes into a matrix of Wilson lines in QCD vacuum
- Subtracts rapidity (LC) divergences from TMD pdf and FF
- Considered to be universal in hard processes
(Collins & Metz PRL 04, Ji, Ma, Yuan PRD 05)
- At tree level (zeroth order α_s) unity-parton model
- Absent tree level pheno analyses of experimental data
(e.g. Anselmino et al PRD 05 & 07, Efremov et al PRD 07)
- Potentially, results of analyses can be difficult to compare at different energies **issue for EIC**
- Correct description of energy scale dependence of cross section and asymmetries in TMD picture, soft factor must be included
(Ji, Ma, Yuan 2005, Collins Oxford Press 2011, Abyat, Collins, Rogers PRD 2011)
- However, possible to consider observables where it cancels e.g. weighted asymmetries Boer, LG, Musch, Prokudin JHEP 2011



First summarize what we know about correlator off light cone



$v = (v^-, v^+, 0)$ **w/** lightlike directions $n = (1,0,0)$, $\bar{n} = (0,1,0)$.

is slightly off light-cone direction n & $b \cdot v = 0$

Wilson lines starting at infinity running along a direction given by the four-vector v to an endpoint a are denoted $\mathcal{L}_v(\infty; a)$

Direction defined in LI way $\zeta^2 = (2P \cdot v)^2 / v^2$

Direction defined in LI way $\hat{\zeta}^2 = (2P_h \cdot \tilde{v})^2 / \tilde{v}^2$

angle between v and $\tilde{v}$ $\rho = \sqrt{v^- \tilde{v}^+ / v^+ \tilde{v}^-}$

scales from regulating LC div gluon rap. cutoff

Crucial property of Soft Factor-SIDIS

Soft factor formed from vacuum expt. value of Wilson lines involving both v and $\tilde{v}$ thus depends on relative orientation of directions $\rho = \sqrt{v^- \tilde{v}^+ / v^+ \tilde{v}^-}$

$\tilde{S}^+(\boldsymbol{b}_T, \rho, \mu)$ is invariant under rotations of the $\boldsymbol{b}_T$-vector (provided $b \cdot v = 0$).

Since for TMDs we always consider the case $b^+ = 0$, we have $\boldsymbol{b}_T^2 = -b^2$,

$$\longrightarrow \quad \tilde{S}^+(b^2, \rho, \mu)$$

Decompose TMDs taking into account v dependence

Goeke, Metz, Schlegel PLB 05

$$\widetilde{\Phi}_q^{[\Gamma]}(b, P, S, v) \in \widetilde{A}_i^+(b^2, b{\cdot}P, b{\cdot}v/(v{\cdot}P), \zeta, \mu) \text{ and } \widetilde{B}_i^+(b^2, b{\cdot}P, b{\cdot}v/(v{\cdot}P), \zeta, \mu)$$

Momentum space convolution

Adilbi, Ji, Ma, Yuan PRD 05 ....

Hard

$$\mathcal{C}[H; wfSD] \equiv x_B H(Q^2, \mu^2, \rho) \sum_a e_a^2 \int d^2p_T\, d^2K_T\, d^2\ell_T\, \delta^{(2)}(zp_T + K_T + \ell_T - P_{h\perp})\, w\left(p_T, -\frac{K_T}{z}\right)$$
$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho)\, S(\ell_T^2, \mu^2, \rho)\, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

TMD

Soft

FF

Soft factor in deconvoluted Fourier Bessel rep of CS

$\mathcal{P}$ versus $\mathcal{C}$

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, d|\boldsymbol{P}_{h\perp}|^2} \propto \frac{\alpha^2}{x_B Q^2} \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|\, \tilde{S}(\boldsymbol{b}_T^2) \Big\{ \quad \ldots$$

$$+ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_1\ \tilde{D}_1]$$

Soft factor is

- spin blind
- flavor blind
- factors in $\mathcal{P}$
- Universal

Idilbi,Ji,Ma,Yuan PRD 05

$$+ |\boldsymbol{S}_\perp|\, \sin(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\ \tilde{D}_1]$$

$$+ \varepsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{h}_1^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]$$

$$+ \quad \ldots 15 \quad \text{more} \quad \text{structure} \quad \text{functions}$$

Products in terms of " $\boldsymbol{b}_T$ moments "

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} = H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\ \tilde{S}^{(+)}(\boldsymbol{b}_T^2,\mu^2,\rho)\ \mathcal{P}[\tilde{f}_{1T}^{(1)}\tilde{D}_1^{(0)}] + \tilde{Y}_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\boldsymbol{b}_T^2)\ .$$

$$\mathcal{P}[\tilde{f}^{(n)}\tilde{D}^{(m)}] \equiv x_B \sum e_a^2 (zM|\boldsymbol{b}_T|)^n (zM_h|\boldsymbol{b}_T|)^m \tilde{f}^{a(n)}(x, z^2\boldsymbol{b}_T^2, \mu^2, \zeta, \rho) \tilde{D}^{a(m)}(z, \boldsymbol{b}_T^2, \mu^2, \hat{\zeta}, \rho)$$

2. Bessel Weighting & cancellation of soft factor

Bessel weighting-projecting out Sivers using orthogonality of Bessel Fncts.

$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\,J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) =$$

$$2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\,(d\sigma^\uparrow - d\sigma^\downarrow)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\;\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;(d\sigma^\uparrow + d\sigma^\downarrow)}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) =$$

$$-2\,\frac{\tilde{S}(\mathcal{B}_T^2)\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2)\,\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x, z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}{\tilde{S}(\mathcal{B}_T^2)\,H_{UU,T}(Q^2)\sum_a e_a^2\;\tilde{f}_1^a(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}$$

Sivers asymmetry with full dependences

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UU,T}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

3. Circumvents the problem of ill-defined p_T moments

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) =$$

$$-2\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UU,T}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Traditional weighted asymmetry recovered but UV divergent

$$\lim_{\mathcal{B}_T\to 0} w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |\boldsymbol{P}_{h\perp}|/zM$$

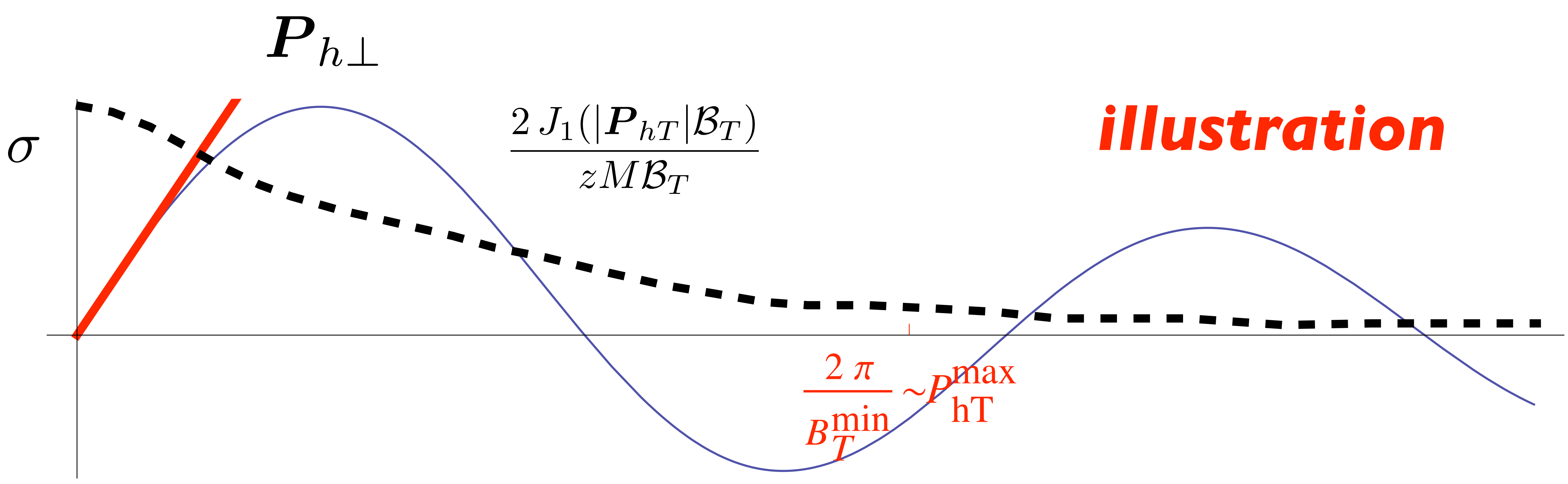
$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\, f_{1T}^{\perp(1)}(x)\;D_1^{a(0)}(z)}{\sum_a e_a^2\, f_1^{a(0)}(x)\;D_1^{a(0)}(z)}$$

undefined w/o regularization

Bacchetta et al. JHEP 08

4. More sensitive to low $\boldsymbol{P}_{h\perp}$ region

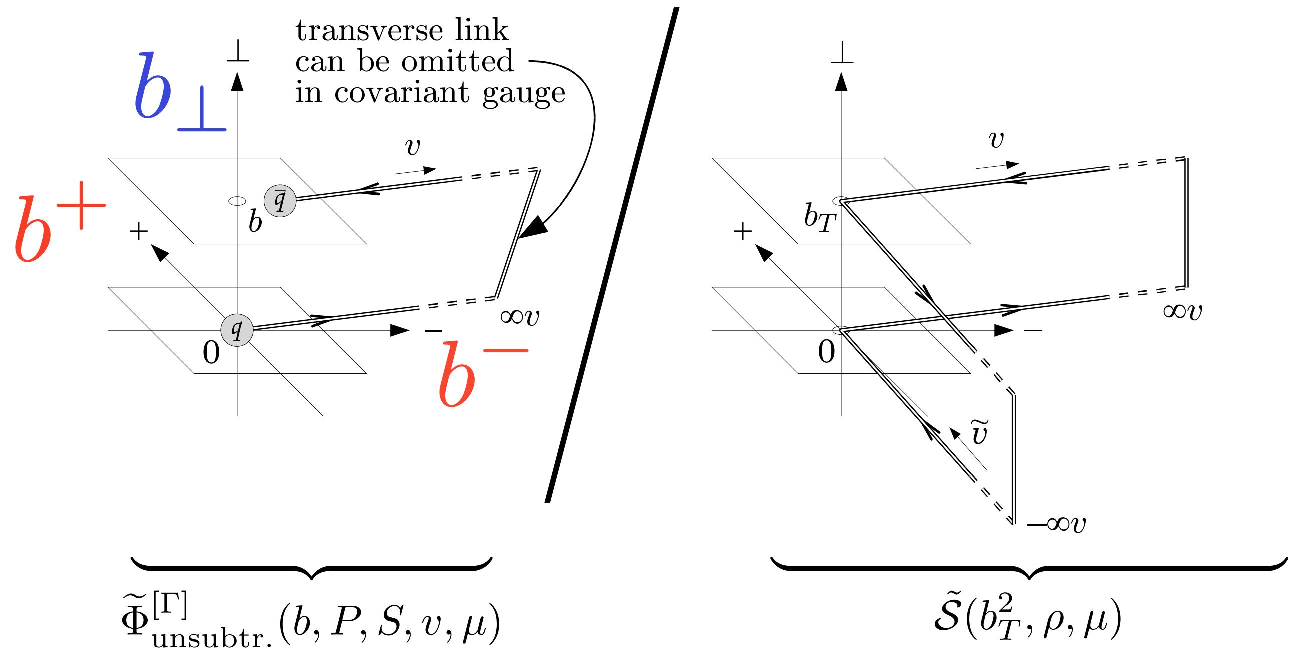
$\mathcal{B}_T$ can serve as a lever arm to enhance the low $\boldsymbol{P}_{h\perp}$ description and possibly dampen lg. momentum tail of cross section. We can use it to scan the cross section



Cancellation of Soft Factor on level of the Matrix elements ***(summarize)***

- So far we get ratios of moments of TMDs and FFs that are free of soft factor
- It was not necessary to specify explicit def. of TMDs and FFs
- We also analyze ratio of moments of TMDs directly on level of matrix elements of TMDs & FFs
- Again we find cancellation of soft factors in ratio
- Impact for Lattice calculation of moments of TMDS, Musch, Ph. Hagler, M. Engelhardt, J.W. Negele, A. Schafer arXiv 2011

Subtracted correlator off light cone



$b \cdot v = 0$

$v = (v^-, v^+, 0)$ w/ lightlike directions $n = (1, 0, 0)$, $\bar{n} = (0, 1, 0)$.

Again consider JMY framework

$$\Phi^{(+)[\Gamma]}(x, \boldsymbol{p}_T, P, S, \mu^2, \zeta, \rho) = \int \frac{db^-}{(2\pi)}\, e^{ixb^- P^+} \int \frac{d^2 \boldsymbol{b}_T}{(2\pi)^2}\, e^{-i\boldsymbol{p}_T \cdot \boldsymbol{b}_T} \times \underbrace{\frac{1}{2} \langle P, S|\ \bar{\psi}(0)\, \mathcal{U}[\mathcal{C}_b]\, \Gamma\, \psi(b)\ |P, S\rangle}_{\widetilde{\Phi}^{[\Gamma]}_{\text{unsub}}(b, P, S; v, \mu^2)} \Big/ \widetilde{S}^{(+)}(\boldsymbol{b}_T^2, \mu^2, \rho) \Big|_{b^+ = 0},$$

Generalized av. quark trans. momentum shift
Soft Factor cancels

$\langle \boldsymbol{p}_y \rangle_{TU}$:= average quark momentum in transverse y-direction measured in a proton polarized in transverse x-direction.

"dipole moment", "shift"

attention divergences from high-$\boldsymbol{p}_T$-tails!

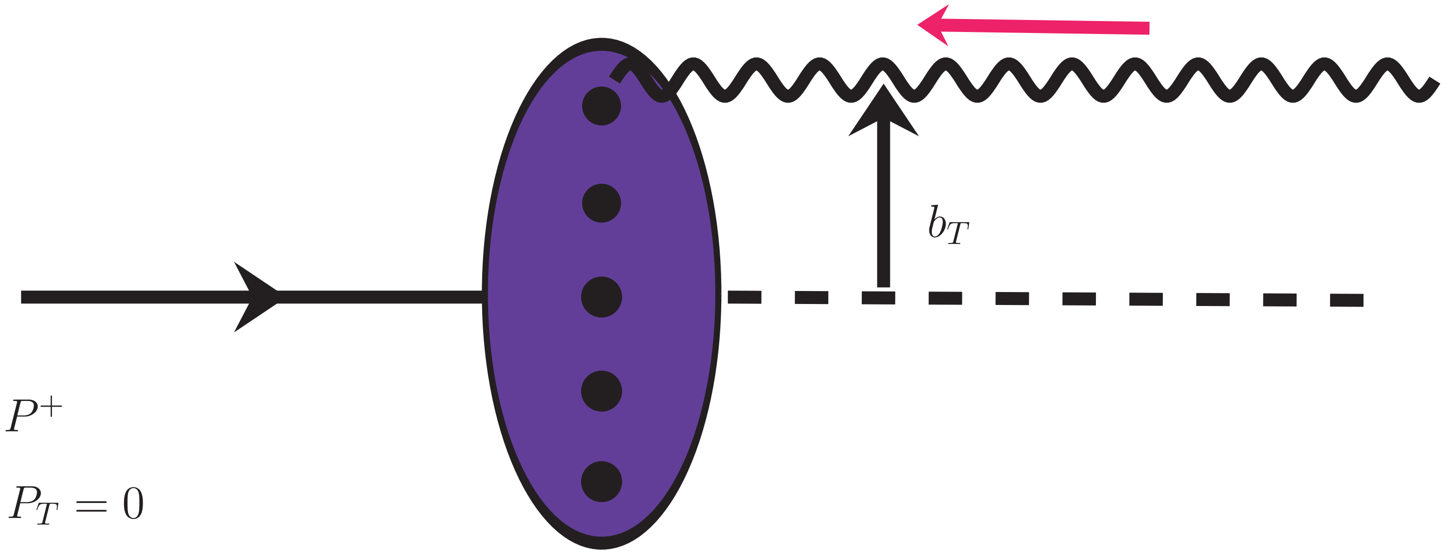
$$\langle p_y(x) \rangle_{TU}^{\mathcal{B}_T} \equiv \left. \frac{\int d|\boldsymbol{p}_T|\, |\boldsymbol{p}_T| \int d\phi_p \frac{2\, J_1(|\boldsymbol{p}_T|\mathcal{B}_T)}{\mathcal{B}_T} \sin(\phi_p - \phi_S)\ \Phi^{(+)[\gamma^+]}(x, \boldsymbol{p}_T, P, S, \mu^2, \zeta, \rho)}{\int d|\boldsymbol{p}_T|\, |\boldsymbol{p}_T| \int d\phi_p J_0(|\boldsymbol{p}_T|\mathcal{B}_T)\ \Phi^{(+)[\gamma^+]}(x, \boldsymbol{p}_T, P, S, \mu^2, \zeta, \rho)} \right|_{|\boldsymbol{S}_T|=1}$$

$$\langle \boldsymbol{p}_y \rangle_{TU}(\mathcal{B}_T) \equiv M \frac{\int dx\, \tilde{f}_{1T}^{\perp(1)}(x, \mathcal{B}_T^2)}{\int dx\, \tilde{f}_1^{(0)}(x, \mathcal{B}_T^2)} = \frac{\tilde{S}(\mathcal{B}_T^2, \ldots)\, \tilde{A}_{12B}(\mathcal{B}_T^2, 0, 0, \tilde{\zeta}, \mu)}{\tilde{S}(\mathcal{B}_T^2, \ldots)\, \tilde{A}_{2B}(\mathcal{B}_T^2, 0, 0, \tilde{\zeta}, \mu)}$$

Conclusions

- Propose generalize Bessel Weights
- Theoretical weighting procedure w/ advantages
- Introduces a free parameter $\mathcal{B}_T\,[\mathrm{GeV}^{-1}]$ that is Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero
- Addtnl. bonus soft factor eliminated from weighted asymmetries
- Possible to compare observables at different scales.... could be useful for an EIC

Fourier transform of GPD $F(x,0,\vec{\Delta}_T)$ @ $\xi = 0$



Burkardt PRD 00, 02, 04...

Localizing partons: impact parameter

- states with definite light-cone momentum p^+ and transverse position (impact parameter):

Soper PRD1977 $|p^+, \boldsymbol{b}\rangle = \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$

Prob. of finding unpol. quark w/ long momentum x at position b_T in trans. polarized S_T nucleon: spin independent $\mathcal{H}$ and spin flip part $\mathcal{E}'$

$$\mathcal{F}(x,\vec{b}) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}}\, F(x,0,\vec{\Delta}_T)$$

$$= \mathcal{H}(x,\vec{b}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M}\left(\mathcal{E}(x,\vec{b})\right)'$$

F.T. $\vec{b} \leftrightarrow \vec{\Delta}_T$

Boer,LG,Musch,Prokudin JHEP (11)

$$\tilde{\Phi}^{[\gamma^+]}(x,\boldsymbol{b}_T) = \tilde{f}_1(x,\boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M\, \tilde{f}_{1T}^{\perp(1)}(x,\boldsymbol{b}_T^2)\,,$$

F.T. $\vec{b}' \leftrightarrow \vec{k}_T$

$$\vec{b}' = \frac{\vec{b}}{1-x}$$

In Spectator picture

Burkardt, Hwang 2004
Meissner, Metz, Goeke 07 PRD
LG, Schlegel PLB 2010

What observable to test this possible connection btnw TMD and Impact par. picture?

Gluonic Pole ME

$$\langle k_T^i \rangle_T(x) = \int d^2k_T\, k_T^i\, \tfrac{1}{2}\left[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\right]$$

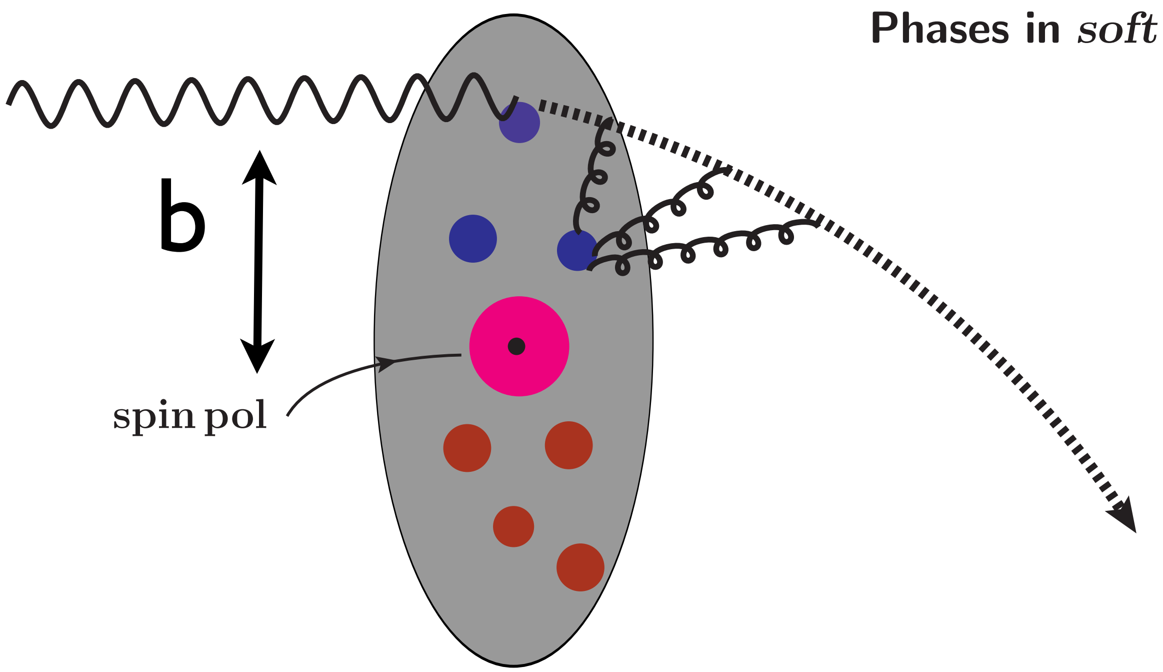
$$\langle k_T^i \rangle(x) = \int d^2b_T \int \frac{dz^-}{2(2\pi)} e^{ixP^+z^-} \langle P^+; \vec{0}_T; S_T | \bar{\psi}(z_1)\gamma^+[z_1; z_2] I^i(z_2) \psi(z_2) | P^+; \vec{0}_T; S_T \rangle$$

$$z_{1/2} = \mp \frac{z^-}{2} n_- + b_T$$

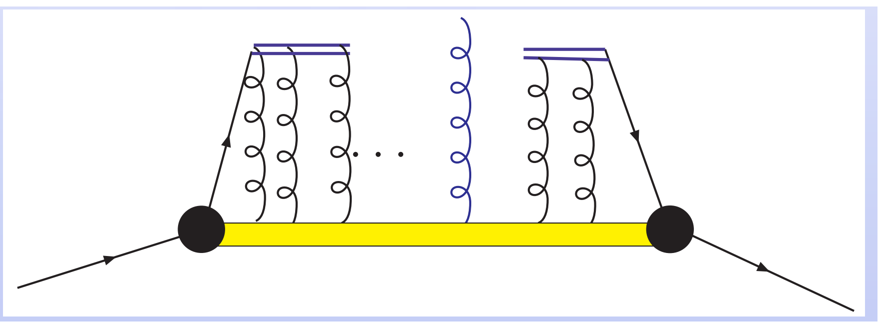
$$I^i(z^-) = \int dy^-[z^-; y^-] gF^{+i}(y^-)[y^-; z^-]$$

Impact parameter rep for GPD E

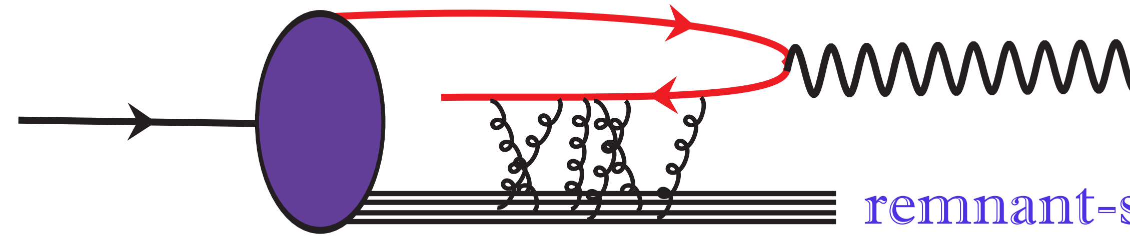
Soft gluonic pole op



Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982



Clarification of Approximate Factorization of Lensing & Distortion



- Stay in momentum space
- Insert complete sets of momentum states

$$-\epsilon_T^{ij} S_T^j f_{1T}^{\perp(1)}(x) = \frac{1}{2M} \int \frac{dz^-}{2\pi} \, \mathrm{e}^{ixP^+z^-} \langle P, S_T | \, \bar{q}(-\tfrac{z^-}{2}n) \gamma^+ [-\tfrac{z^-}{2}n\, ; \, \tfrac{z^-}{2}n] \, I^i(\tfrac{z^-}{2}n) \, q(\tfrac{z^-}{2}n) \, | P, S_T \rangle .$$

$$\sum_{\lambda'_{\mathbf{P}}} \sum_{\lambda_{\mathbf{P}}} \ldots \langle \lambda'_{\mathbf{P}} | \hat{I}^i | \lambda_{\mathbf{P}} \rangle \ldots$$

L.G & Schlegel in prep

- Diagonal in momentum eigenstates under assumptions

1) FSI ... soft gluon exchange, scattered quark and remnant move quasi-collinearly w/r to target backward and forwards

2) Under these conditions one expects FSIs to be dominated by small transverse momentum of quark and remnant rather than a large momentum. Pole contribution dominates otherwise there large momentum is also transferred

3) under these conditions number of spectators match in intermediate state

LG, Schlegel AIP 2011, and in prep

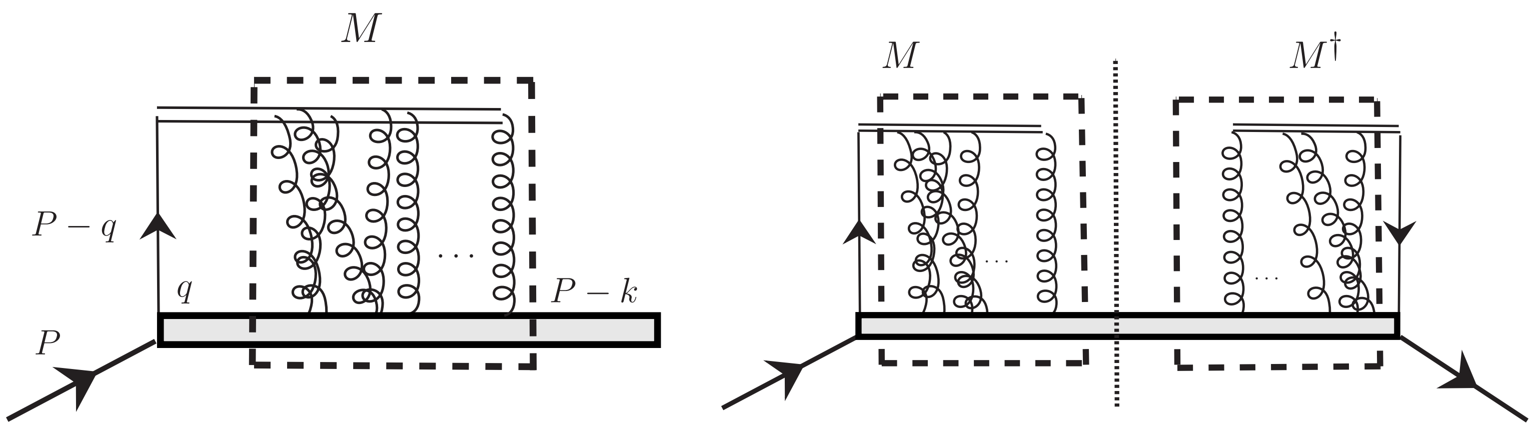


FIGURE 1. Left: The matrix element $W = \langle P-k|[\infty\ n;0]\,q(0)|P\rangle$ dressed with the FSIs. The FSIs are described by a non-perturbative scattering amplitude M that is calculated in a generalized ladder approximation [20]. Right: The quark-quark correlator with FSIs.

Transform to $\vec{b}$ space

$$\langle k_T^{q,i}(x)\rangle_{UT} = \int d^2k_T\, k_T^i\, \tfrac{1}{2}\Big[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\Big]$$

1) $$\langle k_T^i\rangle(x) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-}\langle P,S_T|\bar{q}(-z^- n/2)\gamma^+[-z^- n/2; z^- n/2]\hat{I}^i(z^- n/2)q(z^- n/2)|P,S_T\rangle,$$

2) $$\mathcal{F}^{q[\Gamma]}(x,\vec{b}_T;S) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-}\langle P^+,\vec{0}_T;S|\bar{\psi}(z_1)\Gamma\,\mathcal{W}(z_1;z_2)\psi(z_2)|P^+,\vec{0}_T;S\rangle,$$ $\Gamma \equiv \gamma^+$

Comparing expressions difference is additional factor, $I^{q,i}$ and integration over $\vec{b}$

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]

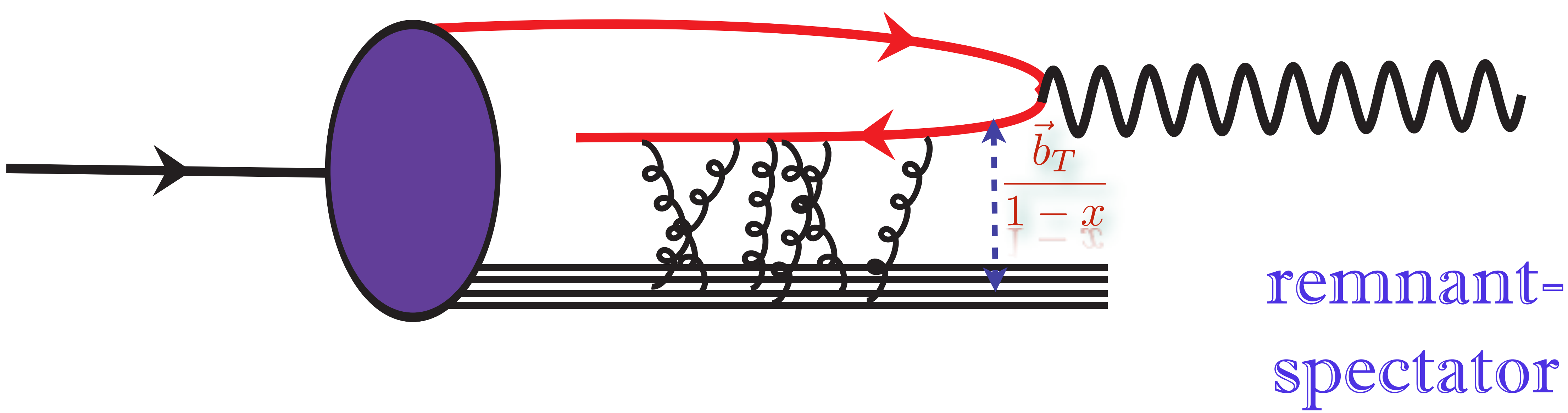
3) $$\langle k_T^{q,i}(x)\rangle_{UT} \simeq \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\mathcal{F}^q(x,\vec{b}_T;S),$$

Calculate Lens & input GPD

Conjecture born out factorization FSI and spatial distortion in eikonal + spectator approximation

$$\langle k_T^i \rangle (x) = M \epsilon_T^{ij} S_T^i f_{1T}^{\perp(1)} \approx \int d^2 b_T \mathcal{I}^i(x, \vec{b}_T^2) \frac{\vec{b}_T \times \vec{S}_T}{M} \frac{\partial}{\partial b_T^2} \mathcal{E}(x, \vec{b}_T^2)$$

$\mathcal{I}^i(x, \vec{b}_T^2)$ Lensing Function



Boer Mulders as well ...

LG, Schlegel PLB 10

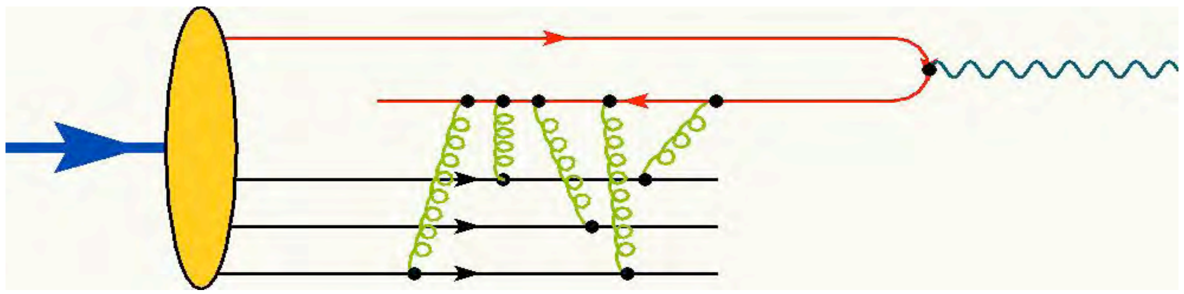
- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

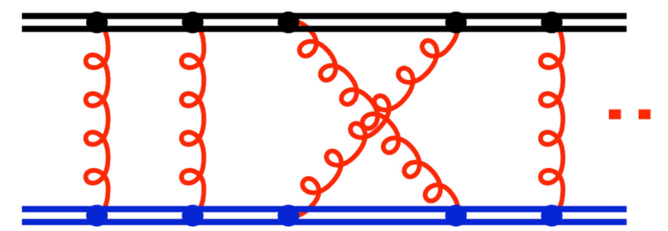
$$\langle k_T^i \rangle^j(x) \;=\; \int d^2k_T\, k_T^i\, \frac{1}{2}\Big(\Phi^{[i\sigma^{i+}\gamma^5]}(S) + \Phi^{[i\sigma^{i+}\gamma^5]}(-S)\Big)$$

$$-2M^2\, h_1^{\perp,(1)}(x) \simeq \int d^2b_T\, \vec{b}_T \cdot \vec{\mathcal{I}}(x,\vec{b}_T)\, \frac{\partial}{\partial b_T^2}\Big(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T\Big)(x,\vec{b}_T^2)$$

Diehl & Hagler EJPC (05), Burkardt PRD (04)

Sivers Function in this approach



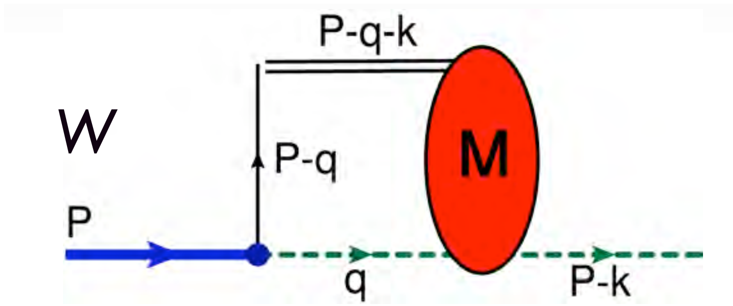


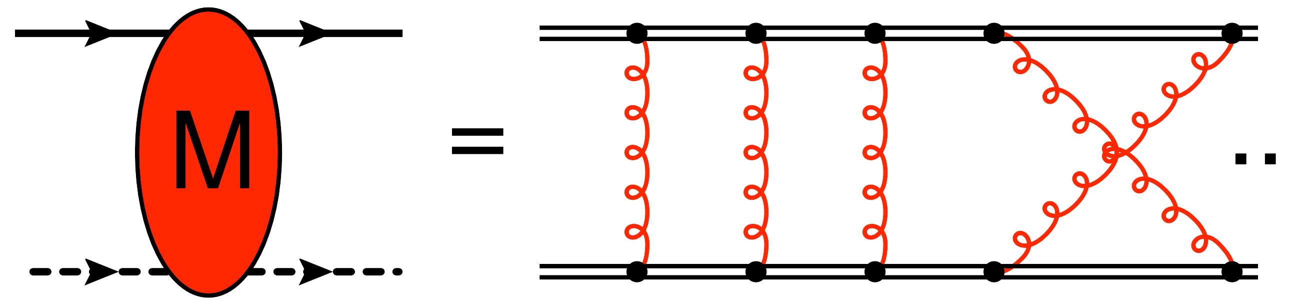
- **Relativistic Eikonal models: Treat FSI non-perturbatively.**

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & in prep for Sivers...AIP 1374 (2011) 309-313

- **Relativistic Eikonal models: Treat FSI non-perturbatively.**





We calc "W" again....

$$\epsilon_T^{ij} k_T^i S_T^j f_{1T}^{\perp}(x, \vec{k}_T^2) = -\frac{M}{8(2\pi)^3(1-x)P^+}\left(\bar{W}\gamma^+ W\Big|_{S_T} - \bar{W}\gamma^+ W\Big|_{-S_T}\right)$$

$$\Delta W(P,k) = \int \frac{d^4q}{(2\pi)^4} g_N[(P-q)^2] \frac{[(\not{P} - \not{q} + m_q)u(P,S)]_i \mathcal{M}_{bc}^{ab}(q, P-k)}{[n\cdot(P-k-q)+i\varepsilon][(P-q)^2+m_q^2+i\varepsilon][q^2-m_s^2+i\varepsilon]}$$

- **Step 1: Integration over q^-:**

Assume no q^- & q^+ poles in M.

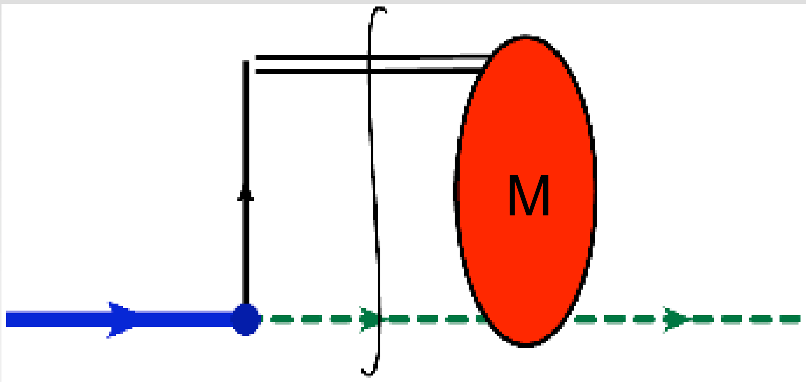
q^- - poles at one loop for higher twist T-odd TMDs [Gamberg, Hwang, Metz, MS, PLB 639, 508]

- **Step 2: Integration over q^+:**

$$\frac{1}{(1-x)P^+ - q^+ + i\varepsilon} = \mathrm{P}\frac{1}{(1-x)P^+ - q^+} - i\pi\delta((1-x)P^+ - q^+)$$

Fix the q+ - pole
emphasizes a "natural" picture of FSI
equivalent to Cutkosky cut, assumptions of Step 1 valid in Eikonal models

Lensing Function



Assume a non-perturbative scattering amplitude M +

Separate GPD and FSI via contour integration

Contour integration → cut diagram → enforces *"natural"* picture of FSI

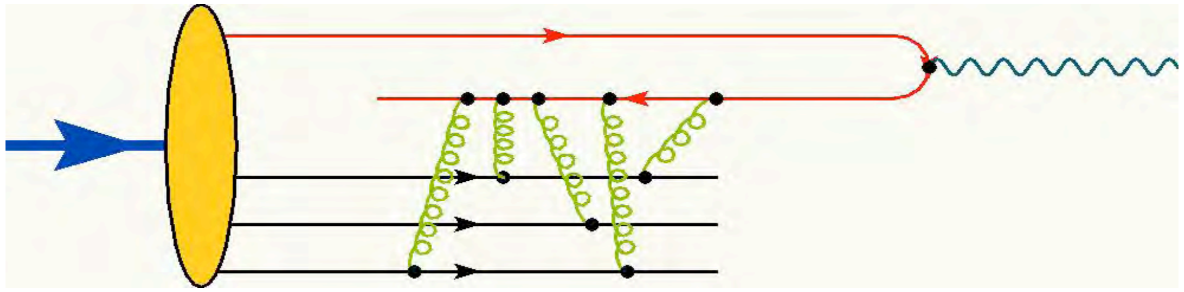
$$f_{1T}^{\perp,(1)u}(x) = -\frac{1}{2(1-x)M^2}\int \frac{d^2q_T}{(2\pi)^2}\, q_T^y\, I^y(x,\vec{q}_T)\, E^u\left(x,0,-\frac{\vec{q}_T^{\,2}}{(1-x)^2}\right)$$

$$I^i(x,\vec{q}_T) = \int \frac{d^2p_T}{(2\pi)^2}\,(2p_T-q_T)^i\,\Im M_{bc}^{ab}(|\vec{p}_T|)\Big((2\pi)^2\delta^{ac}\delta^{(2)}(\vec{p}_T-\vec{q}_T)+\Re M_{da}^{cd}(|p_T-q_T|)\Big)$$

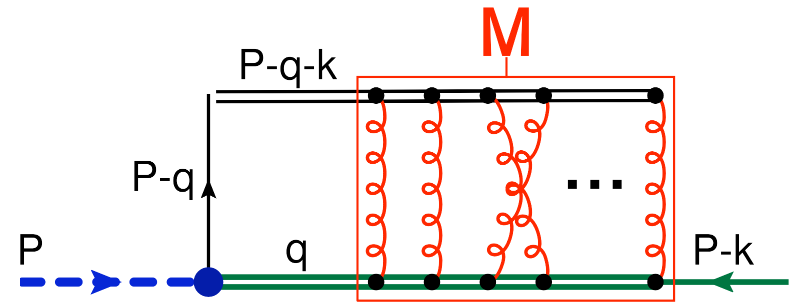
- **More or less "realistic" model for M → allows for numerical comparison**

- **Sivers function from HERMES/COMPASS data,**
  GPD E from models or parameterizations

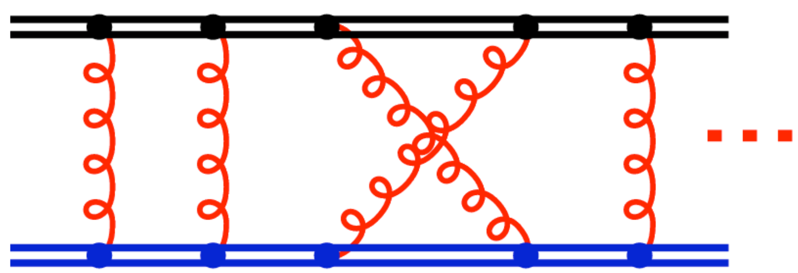
Calculation of *M*



L.G. and M. Schlegel Phys. Lett B 10 and in prepr



Eikonal Color calculation and path ordered gauge link
Color Structure



Abarabanel Itzykson PRL 1970, L.G, Milton PRD 1999, Fried et al. 2000

$$G^{ab}_{\rm eik}(x,y|A) = -i\int_0^\infty ds\, \mathrm{e}^{-is(m_q-i0)}\,\delta^{(4)}(x-y-sv)\left(\mathrm{e}^{-ig\int_0^s d\beta\, v\cdot A^\alpha(y+\beta v)\, t^\alpha}\right)^{ab}_{+}$$

Trick to disentangle the A-field and the color matrices t: Functional FT

$$\left(\mathrm{e}^{-ig\int_0^s d\beta\, v\cdot A^\alpha(y+\beta v)\, t^\alpha}\right)^{ab}_{+} = \mathcal{N}'\int \mathcal{D}\alpha \int \mathcal{D}u\, \mathrm{e}^{i\int d\tau\, \alpha^\beta(\tau) u^\beta(\tau)}\mathrm{e}^{ig\int d\tau\, \alpha^\beta(\tau)\, v\cdot A^\beta(y+\tau v)}\left(\mathrm{e}^{i\int_0^s d\tau\, t^\beta u^\beta(\tau)}\right)^{ab}_{+}$$

FLOW CHART for calculation of Boer Mulders

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

$$2m_\pi^2 h_1^{\perp(1)}(x) \simeq \int d^2 b_T\, \vec{b}_T \cdot \vec{\mathcal{I}}(x, \vec{b}_T)\, \frac{\partial}{\partial \vec{b}_T^2} \mathcal{H}_1^\pi(x, \vec{b}_T^2),$$

$$I^i(x, \vec{q}_T) = \frac{1}{N_c} \int \frac{d^2 p_T}{(2\pi)^2} (2p_T - q_T)^i \left(\Im[\bar{\mathrm{M}}^{\mathrm{eik}}]\right)_{\delta\beta}^{\alpha\delta} (|\vec{p}_T|)$$
$$\left((2\pi)^2 \delta^{\alpha\beta} \delta^{(2)}(\vec{p}_T - \vec{q}_T) + \left(\Re[\bar{\mathrm{M}}^{\mathrm{eik}}]\right)_{\gamma\alpha}^{\beta\gamma} (|\vec{p}_T - \vec{q}_T|)\right).$$

Non-pertb FSIs in here

$$\left(\mathrm{M}^{\mathrm{eik}}\right)_{\delta\beta}^{\alpha\delta} (x, |\vec{q}_T + \vec{k}_T|) = \frac{(1-x)P^+}{m_s} \int d^2 z_T\, \mathrm{e}^{-i\vec{z}_T \cdot (\vec{q}_T + \vec{k}_T)} \quad (20)$$
$$\times \left[\int d^{N_c^2-1} \alpha \int \frac{d^{N_c^2-1} u}{(2\pi)^{N_c^2-1}}\, \mathrm{e}^{-i\alpha \cdot u} \left(\mathrm{e}^{i\chi(|\vec{z}_T|) t \cdot \alpha}\right)_{\alpha\delta} \left(\mathrm{e}^{it \cdot u}\right)_{\delta\beta} - \delta_{\alpha\beta} \right].$$

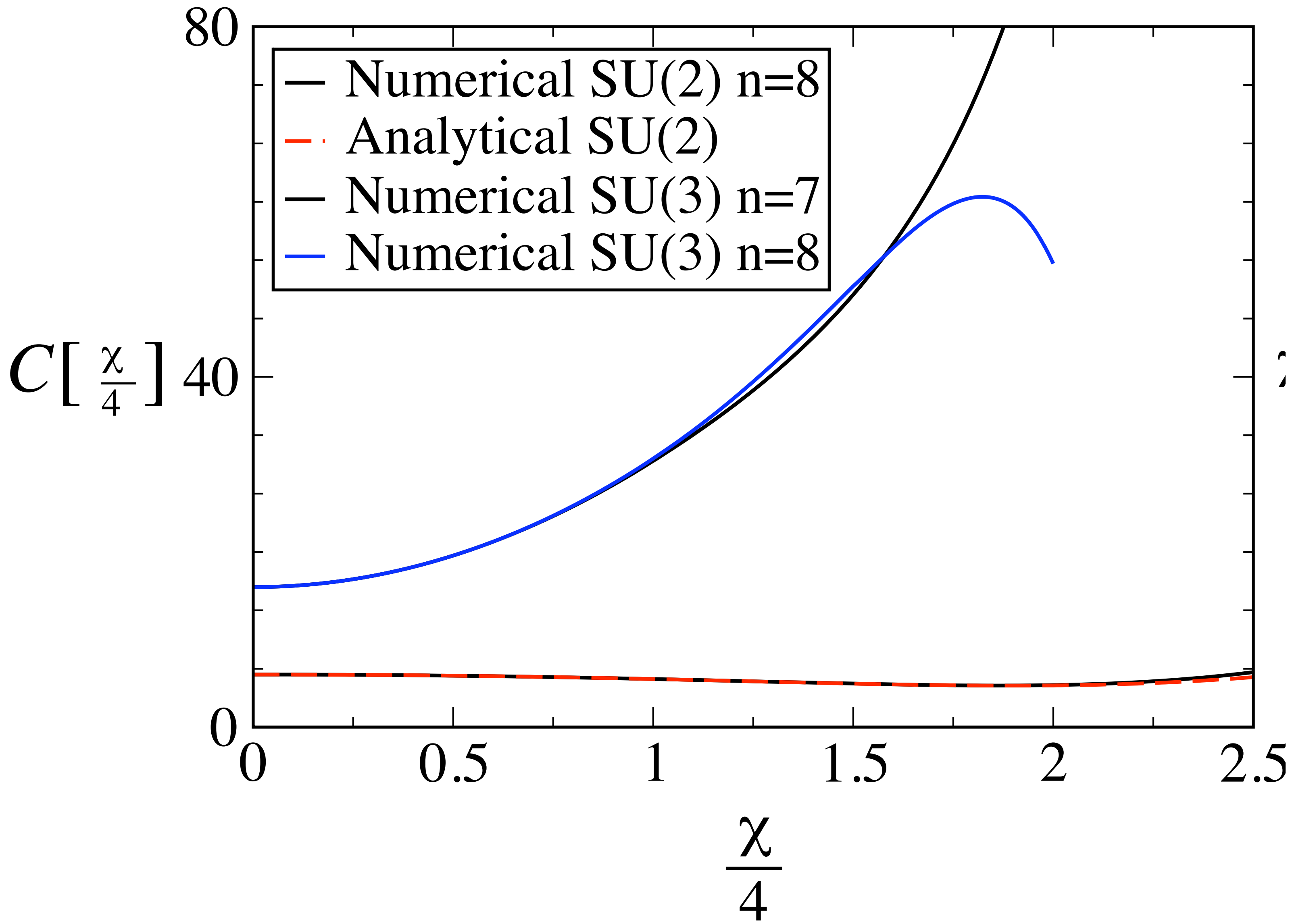
COLOR Integral

$$f_{\alpha\beta}(\chi) \equiv \int d^{N_c^2-1} \alpha \int \frac{d^{N_c^2-1} u}{(2\pi)^{N_c^2-1}}\, \mathrm{e}^{-i\alpha \cdot u} \left(\mathrm{e}^{i\chi(|\vec{z}_T|) t \cdot \alpha}\right)_{\alpha\delta} \left(\mathrm{e}^{it \cdot u}\right)_{\delta\beta} - \delta_{\alpha\beta}$$

$$f_{\alpha\beta}(\chi) = \sum_{n=1}^{\infty} \frac{(i\chi)^n}{(n!)^2} \sum_{a_1=1}^{N_c^2-1} \cdots \sum_{a_n=1}^{N_c^2-1} \sum_{P_n} \left(t^{a_1} \ldots t^{a_n} t^{a_{P_n(1)}} \ldots t^{a_{P_n(n)}}\right)_{\alpha\beta}.$$

Lensing Function & untangling the COLOR FACTOR

$$\mathcal{I}^i(x,\vec{b}_T) = \frac{(1-x)}{2N_c}\frac{b^i_T}{|\vec{b}_T|}\frac{\chi'}{4}C\left[\frac{\chi}{4}\right],$$



$$f_{\alpha\beta}(\chi) = \sum_{n=1}^{\infty}\frac{(i\chi)^n}{(n!)^2}\sum_{a_1=1}^{N_c^2-1}\cdots\sum_{a_n=1}^{N_c^2-1}\sum_{P_n}(t^{a_1}\ldots t^{a_n}t^{a_{P_n(1)}}\ldots t^{a_{P_n(n)}})_{\alpha\beta}\,.$$

Eikonal Phase and FSIs

$$\chi^{DS}(|\vec{z}_T|) = 2\int_0^\infty dk_T\, k_T \alpha_s(k_T^2) J_0(|\vec{z}_T| k_T) Z(k_T^2, \Lambda_{QCD}^2)/k_T^2.$$

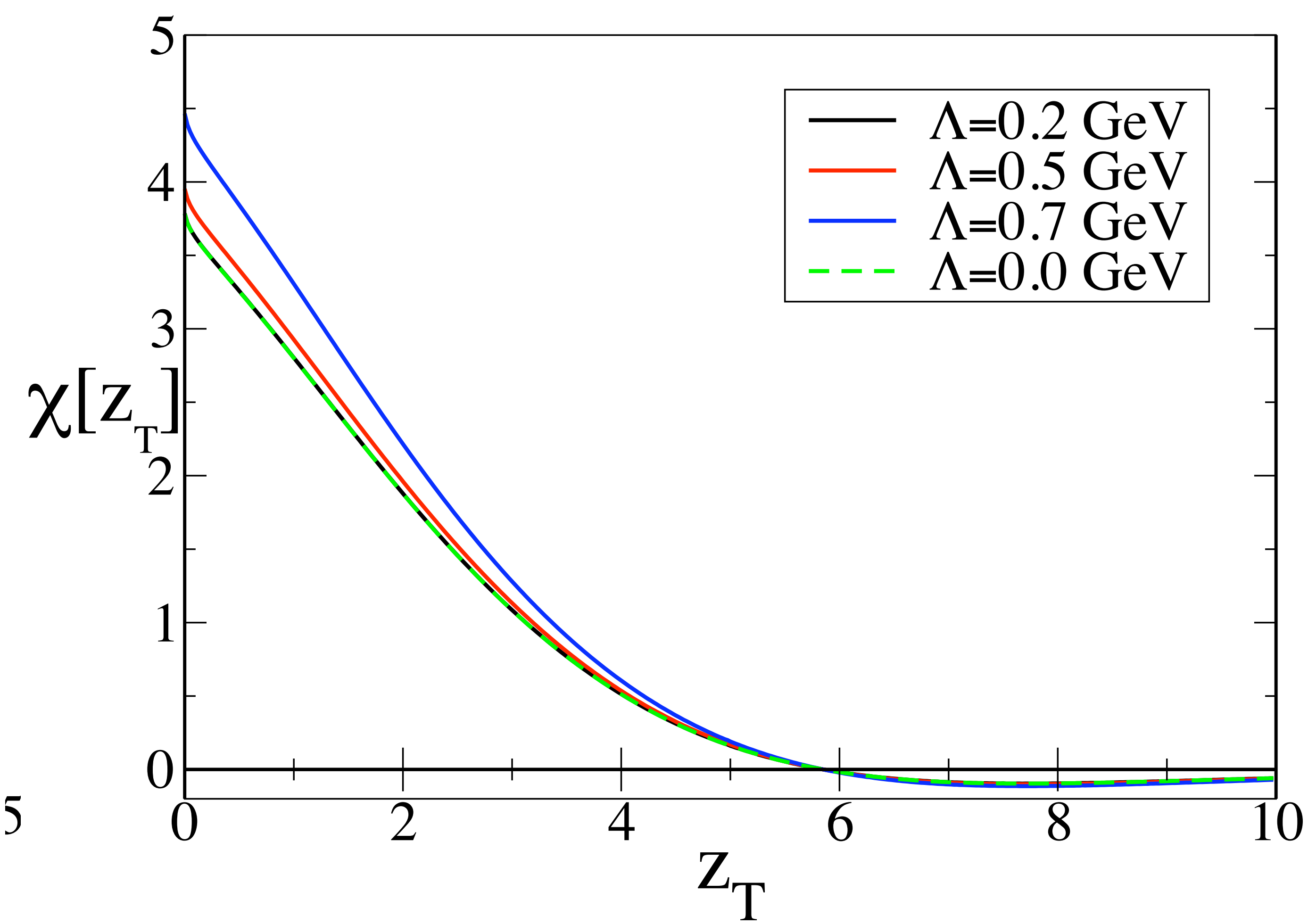
$$\alpha_s(\mu^2) = \frac{\alpha_s(0)}{\ln\left[e + a_1(\mu^2/\Lambda^2)^{a_2} + b_1(\mu^2/\Lambda^2)^{b_2}\right]}. \quad (35)$$

The values for the fit parameters are $\Lambda = 0.71$ GeV, $a_1 = 1.106$, $a_2 = 2.324$, $b_1 = 0.004$ and $b_2 = 3.169$.

$$Z(p^2,\mu^2) = p^2 \mathcal{D}^{-1}(p^2,\mu^2) = \left(\frac{\alpha_s(p^2)}{\alpha_s(\mu^2)}\right)^{1+2\delta} \left(\frac{c\left(\frac{p^2}{\Lambda^2}\right)^\kappa + d\left(\frac{p^2}{\Lambda^2}\right)^{2\kappa}}{1 + c\left(\frac{p^2}{\Lambda^2}\right)^\kappa + d\left(\frac{p^2}{\Lambda^2}\right)^{2\kappa}}\right)^2, \quad (36)$$

Fisher & Alkofer prd 03, Annal of phys. 09

with the parameters $c = 1.269$, $d = 2.105$, and $\delta = -\frac{9}{44}$.



- use running coupling extended to non-perturbative regime
- gluon non-perturbative gluon propagator

Lensing Function

Express Lensing Function in terms of Eikonal Phase:

$$\mathcal{I}^i_{(N=1)}(x,\vec{b}_T) = \frac{1}{4}\frac{b^i_T}{|\vec{b}_T|}\chi'\left(\frac{|\vec{b}_T|}{1-x}\right)\left[1+\cos\chi\left(\frac{|\vec{b}_T|}{1-x}\right)\right]$$

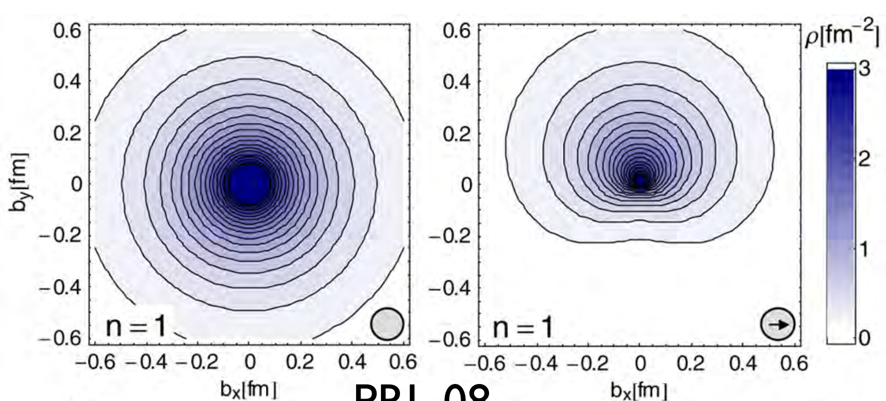
$$\mathcal{I}^i_{(N=2)}(x,\vec{b}_T) = \frac{1}{8}\frac{b^i_T}{|\vec{b}_T|}\chi'\left(\frac{|\vec{b}_T|}{1-x}\right)\left[3\left(1+\cos\frac{\chi}{4}\right)+\left(\frac{\chi}{4}\right)^2-\sin\frac{\chi}{4}\left(\frac{\chi}{4}-\sin\frac{\chi}{4}\right)\right]\left(\frac{|\vec{b}_T|}{1-x}\right)$$

$$\mathcal{I}^i_{(N=3)}(x,\vec{b}_T) = \text{numerics}$$

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

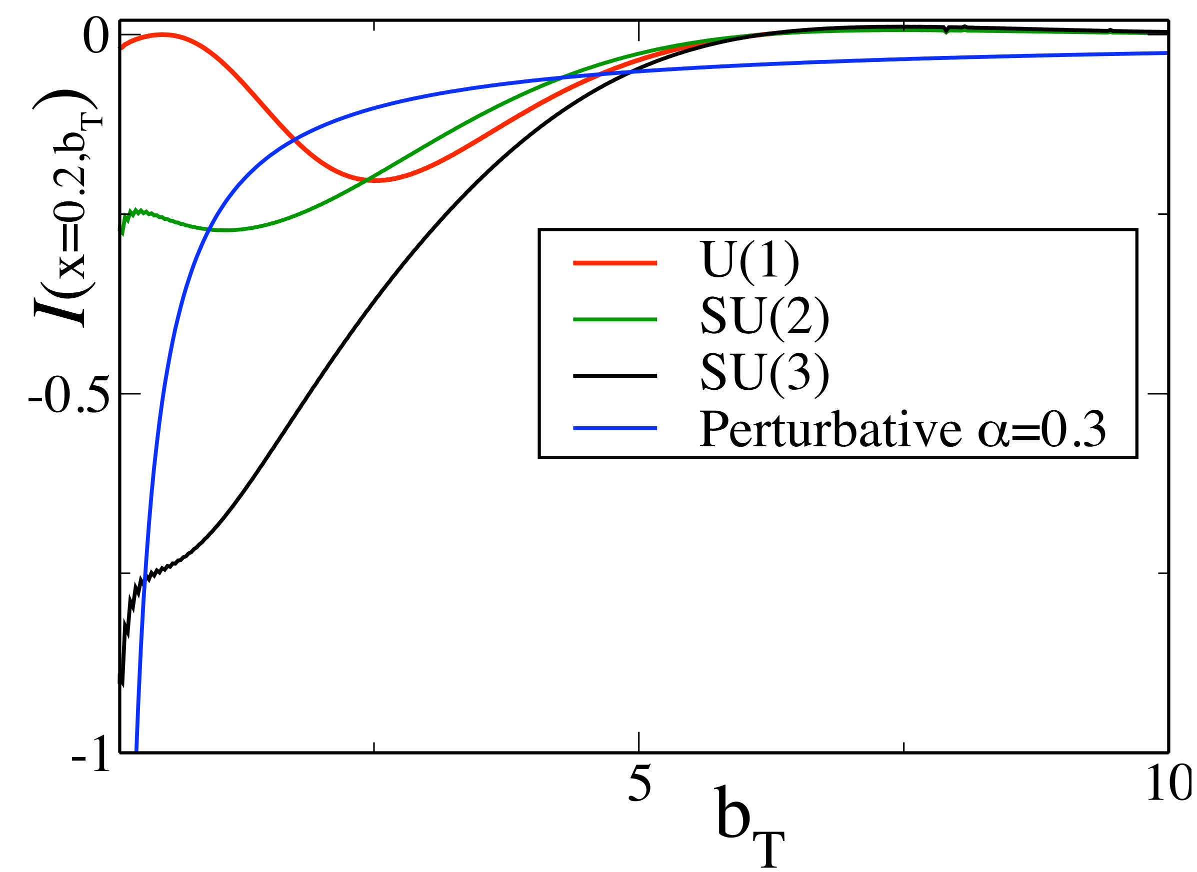
Spin Structure of the Pion



PRL 08

D. Brömmel,[1,2] M. Diehl,[1] M. Göckeler,[2] Ph. Hägler,[3]

FSI + distortion

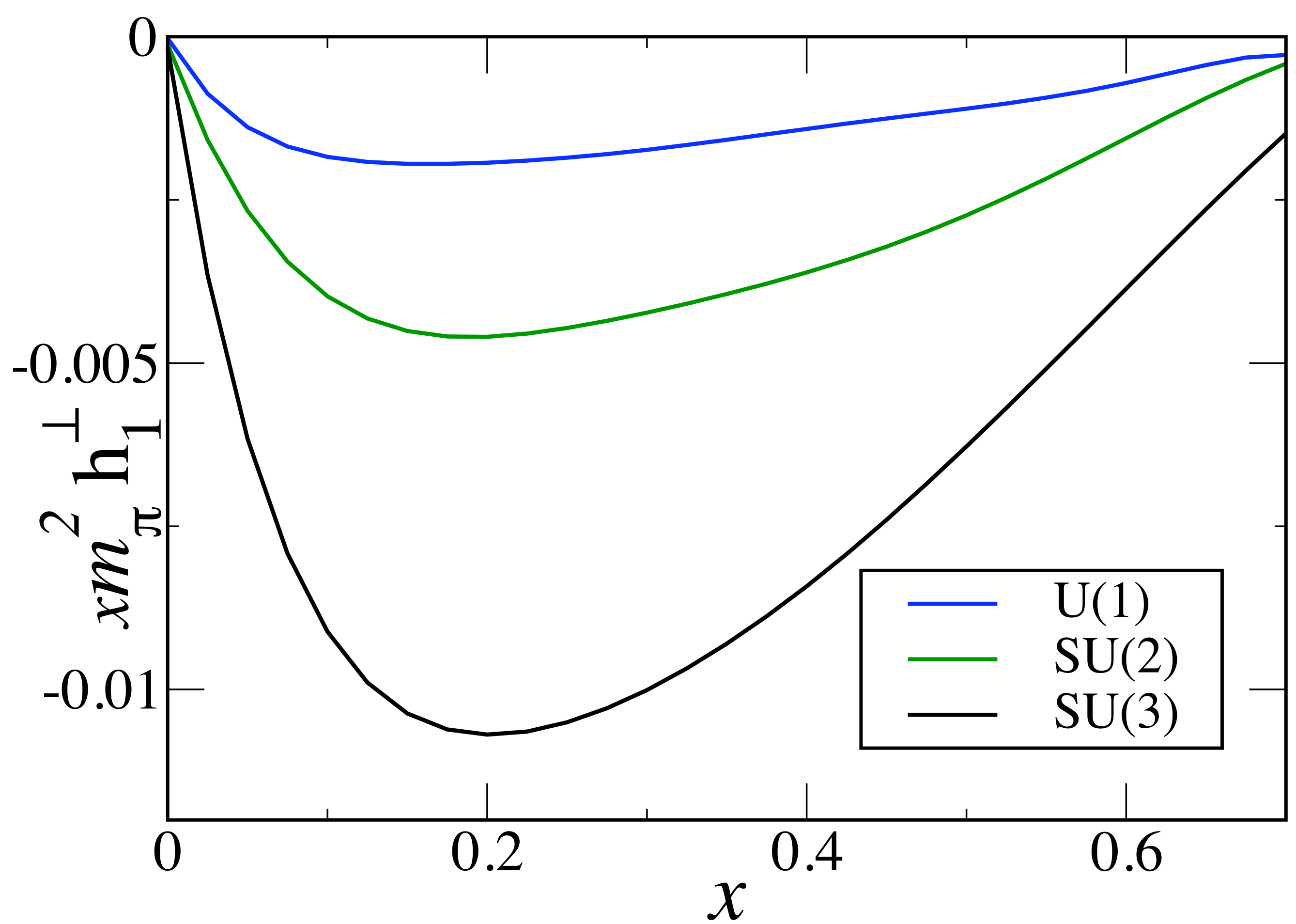


FSIs are negative and "grow" with Color!

Prediction for Boer-Mulders Function of PION

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

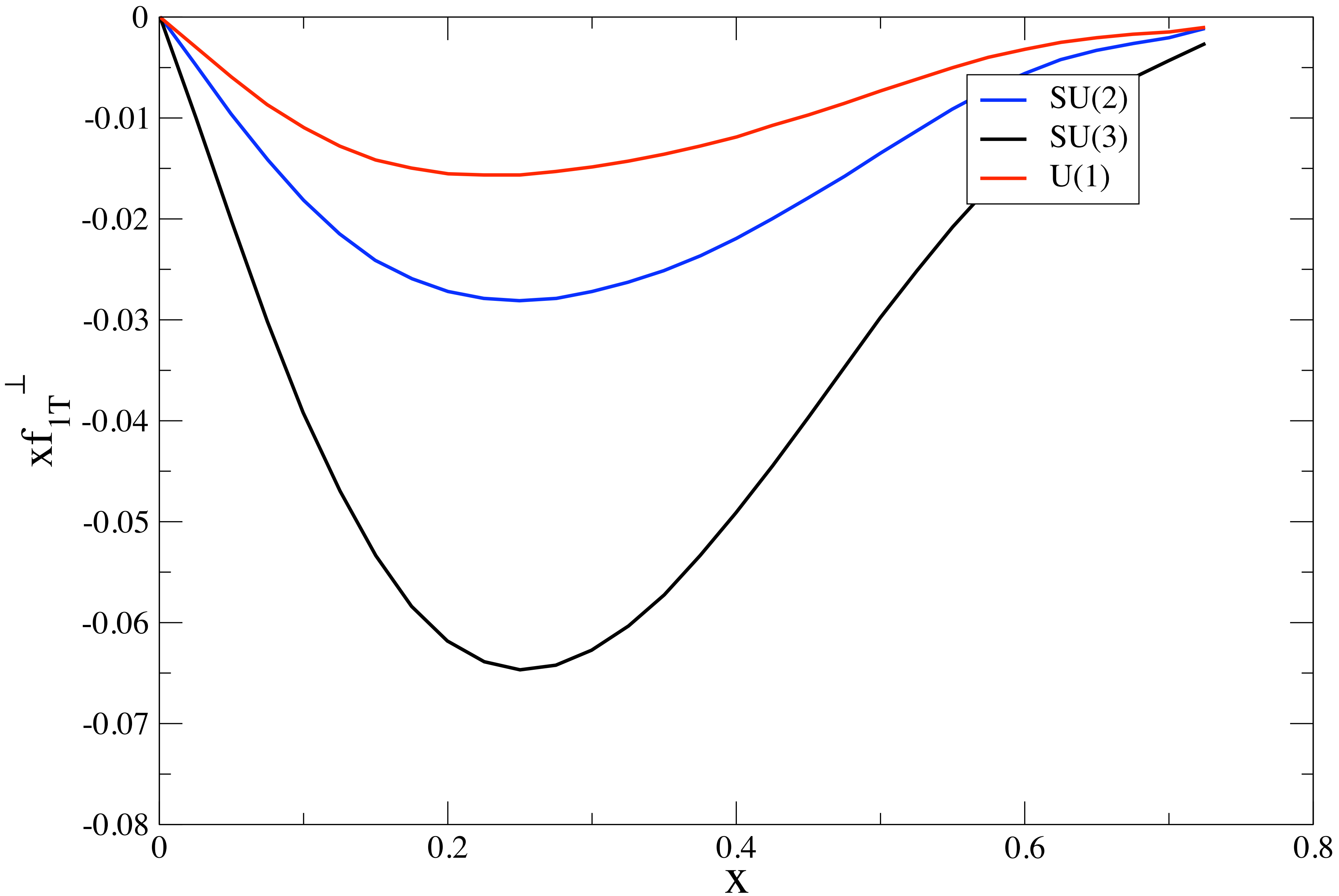


Relations produce a BM funct. approx equiv. to Sivers from HERMES

Expected sign i.e. FSI are negative

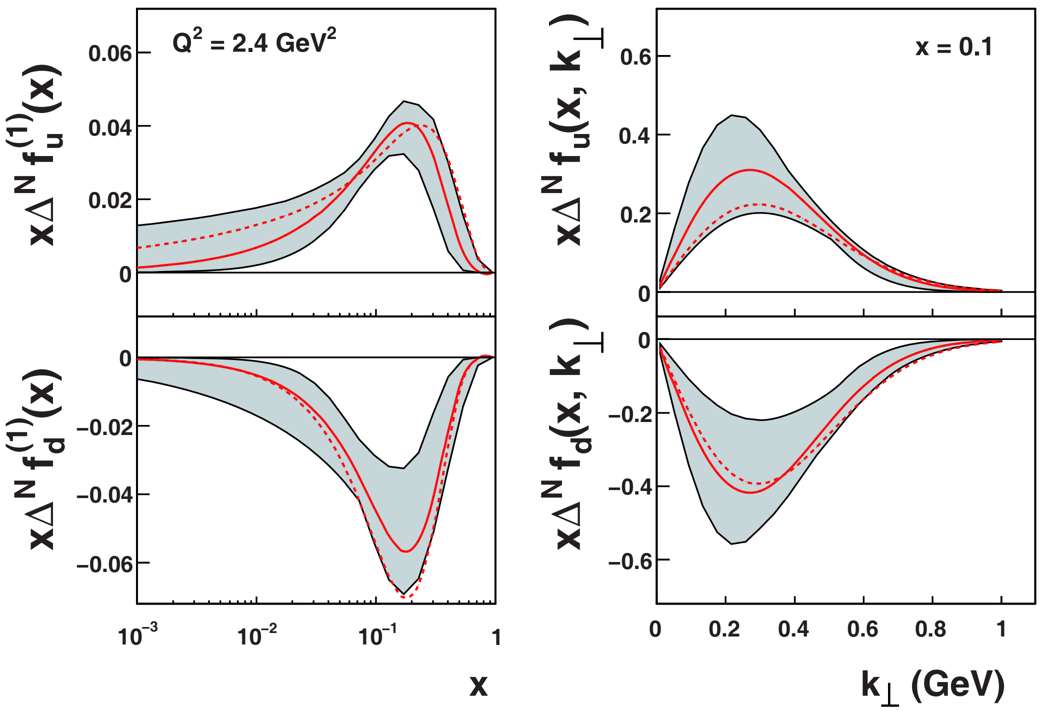
Answer will come from pion BM from COMPASS πN Drell Yan

Study how Sivers function scales with color

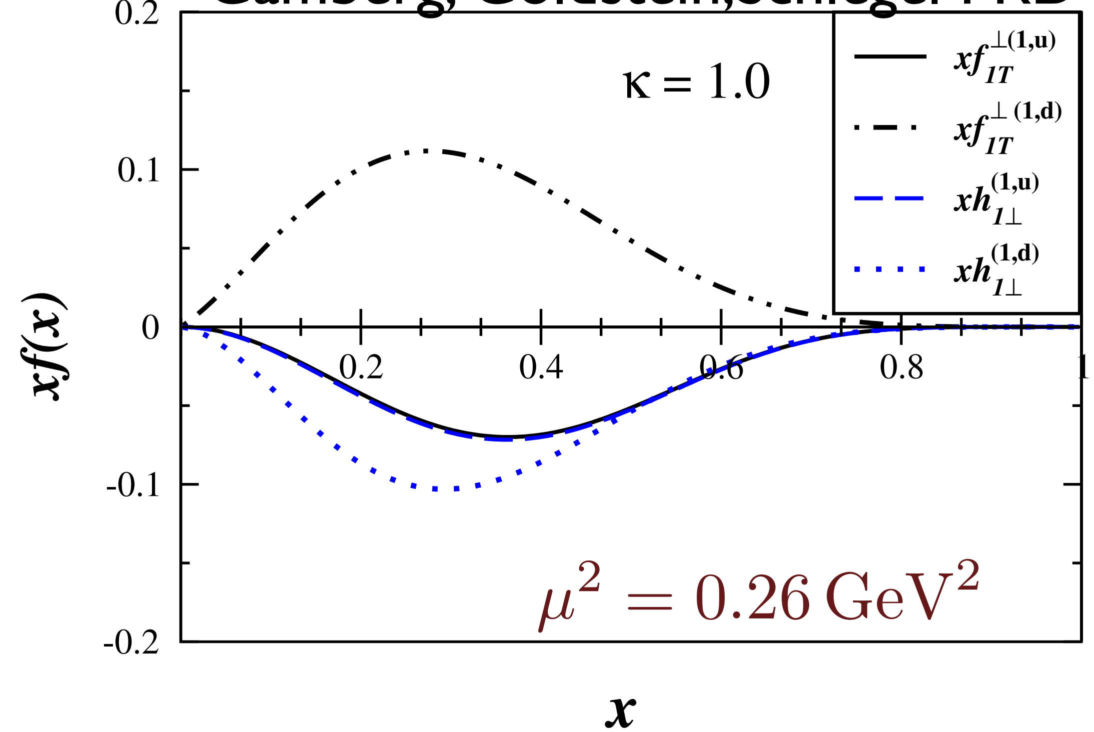


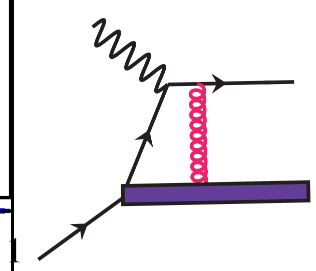
Sivers Modeling

Anselmino Prokudin et al. PRD 05, EPJA 08



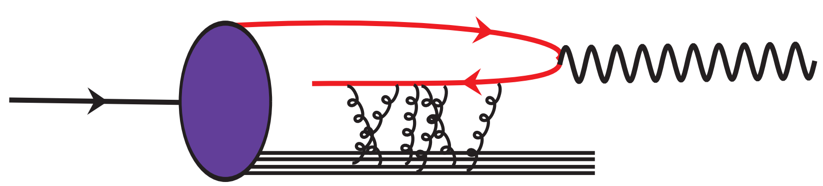
Gamberg, Goldstein,Schlegel PRD 77, 2008

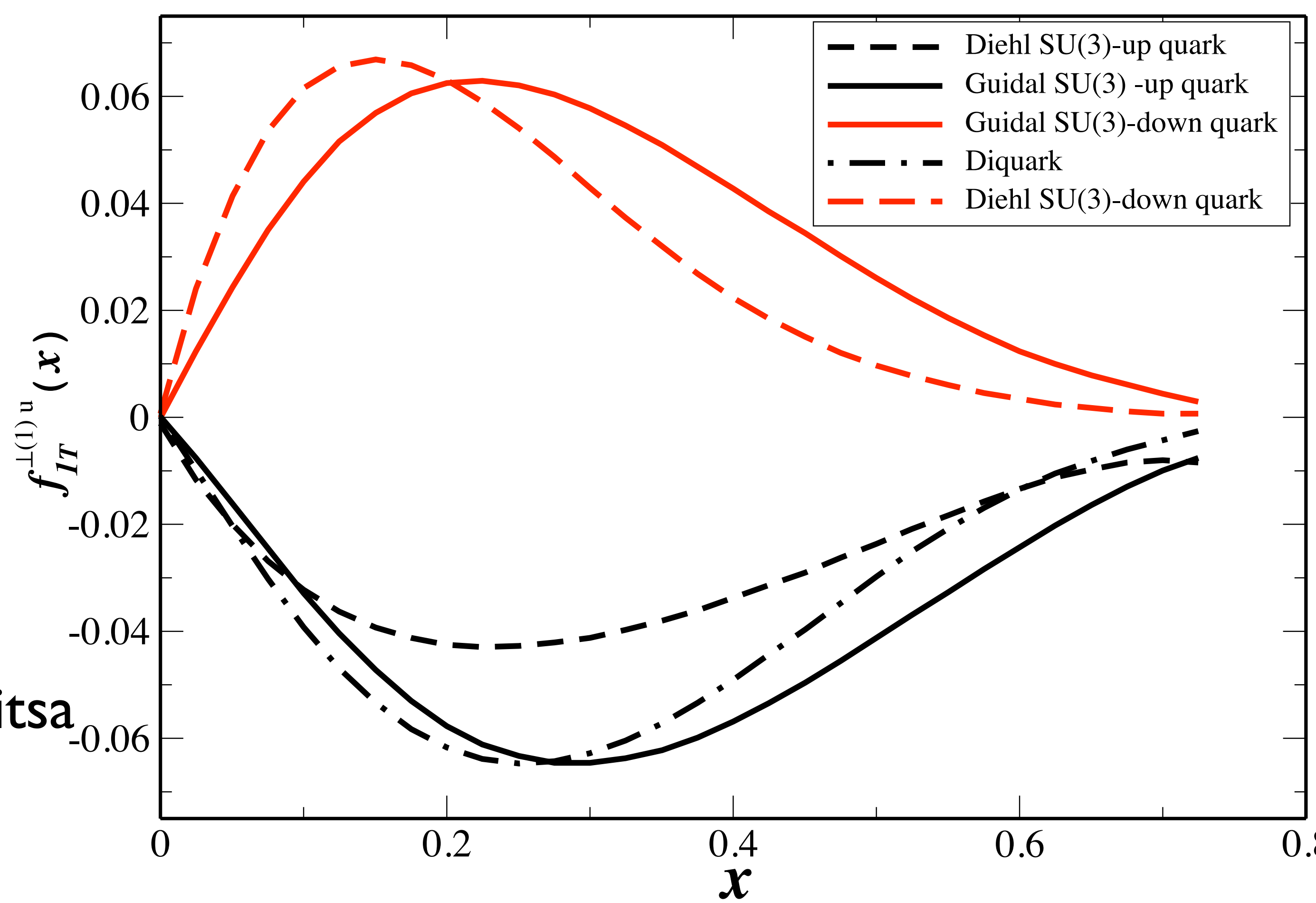




L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.





- Relations produce a Sivers effect ~0.05 N_c=3
- Torino extraction ~ 0.05
  SU(3)Chromo-lensing (Burkardt NPA 2003)
- Sivers effect increases with color
- Color tracing gives result of N_c counting of Pobylitsa