# Field theoretic vs. partonic formulation of angular momentum

Matthias Burkardt

New Mexico State University

February 7, 2012

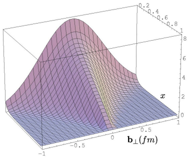
# Outline

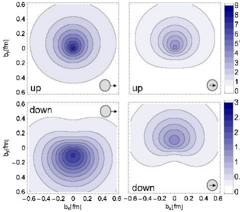


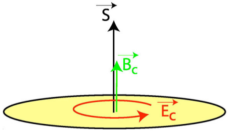
- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
    - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
    - $\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
    - $E(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs when the target is $\perp$ polarized

$\hookrightarrow$ Ji relation (poor man's derivation)

- comparison Jaffe$\leftrightarrow$Ji decomposition
- $\int dx \bar{g}_2(x) x^2 \quad \Rightarrow \quad \perp$ force in DIS
- Summary



- form factors: $\stackrel{FT}{\longleftrightarrow} \rho(\vec{r})$
- $GPDs(x, \vec{\Delta})$: form factor for quarks with momentum fraction $x$

$\hookrightarrow$ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction $x$

- careful: cannot measure longitudinal momentum ($x$) and longitudinal position simultaneously (Heisenberg)

$\hookrightarrow$ consider purely transverse momentum transfer

**Impact Parameter Dependent Quark Distributions**

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, \xi = 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

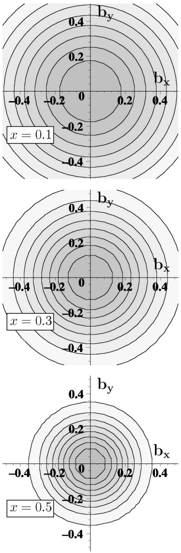
$q(x, \mathbf{b}_\perp)$ = parton distribution as a function of the separation $\mathbf{b}_\perp$ from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i\in q,g} \mathbf{r}_{\perp,i} x_i$
MB, Phys. Rev. D62, 071503 (2000)

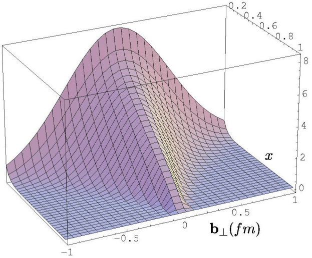
- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free of relativistic corrections ($\rightarrow$G.Miller)

- probabilistic interpretation

$q(x, \mathbf{b}_\perp)$ for unpol. p

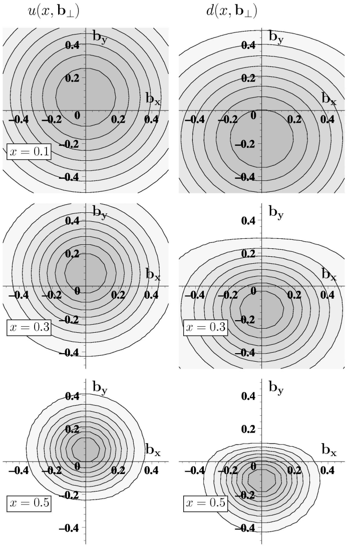


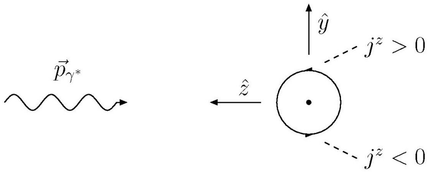




## unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$
- $x$ = momentum fraction of the quark
- $\vec{b} = \perp$ distance of quark from $\perp$ center of momentum
- small $x$: large 'meson cloud'
- larger $x$: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$




## proton 'polarized in $+\hat{x}$ direction'

no axial symmetry!

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$$-\frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from $j^3$

## proton 'polarized in $+\hat{x}$ direction' & localized in the $\perp$ direction

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

## spin + relativity = weirdness ($\rightarrow$ Naomi Makins)

above $q(x,\mathbf{b}_\perp)$ calculated in $\perp$ localized state
$|'\hat{x}'\rangle \equiv |p^+,\mathbf{R}_\perp\!=\!0,+\rangle + |p^+,\mathbf{R}_\perp\!=\!0,-\rangle$ which is not eigenstate of $\perp$ nucleon spin

- due to presence of $\mathbf{p}_\perp \neq 0$
- $\pm$ refers to light-front helicity states (issue when $\mathbf{p}_\perp \neq 0$)

## distribution in delocalized wave packet

MB, PRD72, 094020 (2005)
$q_\psi(x,\mathbf{b}_\perp) = \int d^2r_\perp q(x,b_\perp - r_\perp)\left(|\psi(\mathbf{r}_\perp)|^2 - \frac{1}{2M}\frac{\partial}{\partial r}_\perp|\psi(\mathbf{r}_\perp)|^2\right)$
two contributions to $\perp$ shift

- intrinsic shift relative to center of momentum $\mathbf{R}_\perp$
- overall shift of $\mathbf{R}_\perp$ for $\perp$ polarized nucleon

## spherically symmetric wave packet has center of momentum off-center:

- illustrate this relativistic effect using bag model wave functions:

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M_N} f(r) \end{pmatrix} \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

  $\int d^3r f^2(r) = 1$, take limit of large 'radius' $R$ for wave packet
- evaluate $T_q^{0z} = \frac{i}{2}\bar{q}\left(\gamma^0\partial^z + \gamma^z\partial^0\right)q$ in this state
- $\psi^\dagger\partial_z\psi$ even under $y \to -y$, i.e. no contribution to $\langle yT_q^{0z}\rangle$
- use $i\psi^\dagger\gamma^0\gamma^z\partial^0\psi = E\psi^\dagger\gamma^0\gamma^z\psi$

$$\begin{aligned} \langle T^{0z}y\rangle &= E\int d^3r\psi^\dagger\gamma^0\gamma^z\psi y = E\int d^3r\psi^\dagger \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix}\psi y \\ &= \frac{2E}{E+M_N}\int d^3r\chi^\dagger\sigma^z\sigma^y\chi f(r)(-i)\partial^y f(r) y \\ &= \frac{E}{E+M_N}\int d^3r f^2(r) \stackrel{R\to\infty}{\longrightarrow} \frac{1}{2} \end{aligned}$$

$\hookrightarrow$ $p$ pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2M_N}$ in $+\hat{y}$ direction!

## spherically symmetric wave packet has center of momentum off-center:

- illustrate this relativistic effect using bag model wave functions:

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M_N} f(r) \end{pmatrix} \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

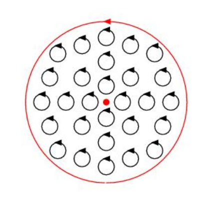
$$\langle T^{0z} y \rangle \stackrel{R\to\infty}{\longrightarrow} \frac{1}{2}$$

$\hookrightarrow$ $p$ pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2M_N}$ in $+\hat{y}$ direction!

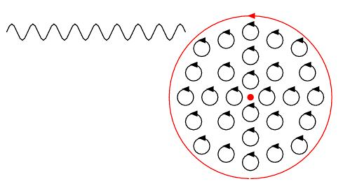
## origin of 'shift' of CoM

- nucleon polarization: $\odot$
- counterclockwise momentum density from lower component
- $p \sim \frac{1}{R}$, but $y \sim R$

$\hookrightarrow$ $\langle T^{++} y \rangle = \mathcal{O}(1)$



## $q$ with polarization $\odot$



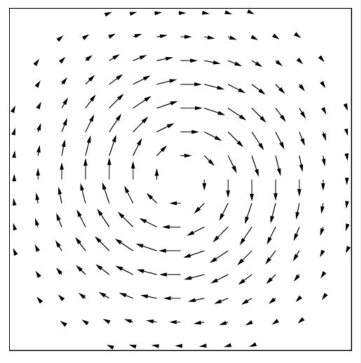
- $q$ in ground state orbit

↪ counterclockwise current from lower component

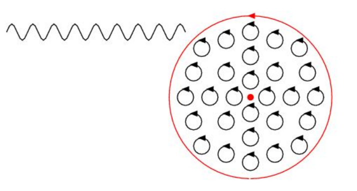
↪ $q$ distribution shifted to top

## unpolarized target

- all $q$ polns. equally likely



## $q$ with polarization $\odot$



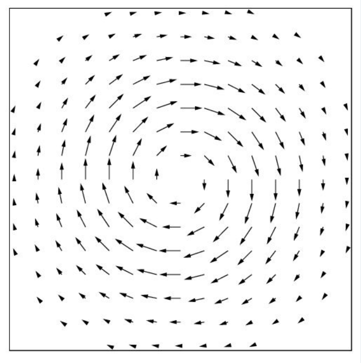
- $q$ in ground state orbit

$\hookrightarrow$ counterclockwise current from lower component

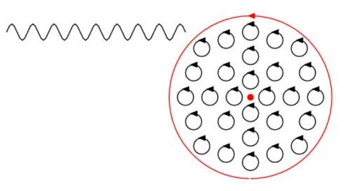
$\hookrightarrow$ $q$ distribution shifted to top

## unpolarized target

- $q$ with pol. $\uparrow$ shifted to left



## $q$ with polarization $\odot$



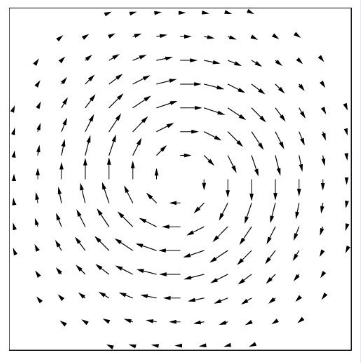
- $q$ in ground state orbit

$\hookrightarrow$ counterclockwise current from lower component

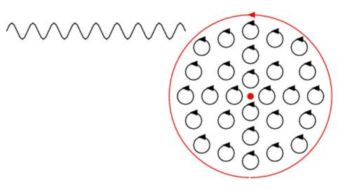
$\hookrightarrow$ $q$ distribution shifted to top

## unpolarized target

- $q$ with pol. $\downarrow$ shifted to right



## $q$ with polarization $\odot$



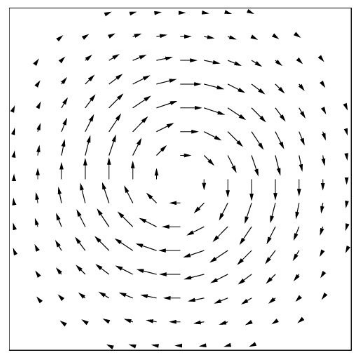
- $q$ in ground state orbit

$\hookrightarrow$ counterclockwise current from lower component

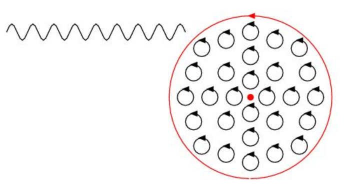
$\hookrightarrow$ $q$ distribution shifted to top

## unpolarized target

- $q$ with pol. $\rightarrow$ shifted to top



### $q$ with polarization $\odot$



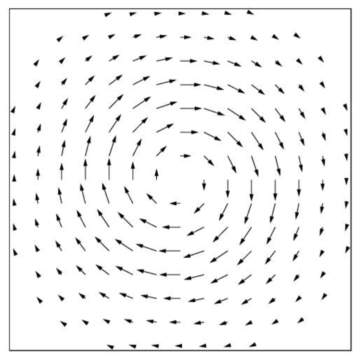
- $q$ in ground state orbit

$\hookrightarrow$ counterclockwise current from lower component

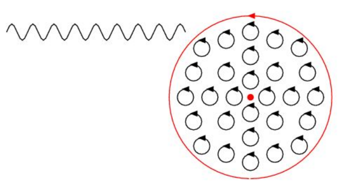
$\hookrightarrow$ $q$ distribution shifted to top

### unpolarized target

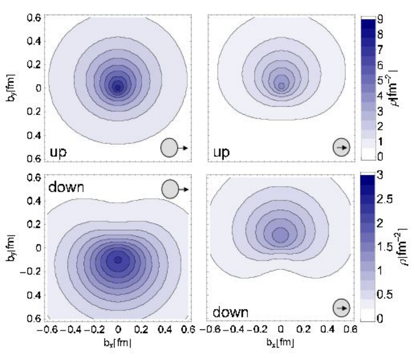
- $q$ with pol. $\leftarrow$ shifted to bottom



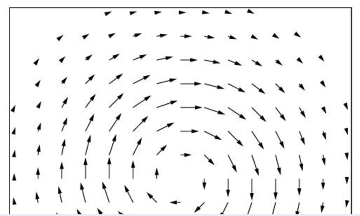
## $q$ with polarization $\odot$



## lattice calculations (QCDSF)




## unpolarized target



- transversity distribution in unpol. target described by chirally odd GPD $\bar{E}_T$
- $\bar{E}_T > 0$ for $u$ & $d$ (QCDSF)
- connection $h_1^\perp(x, \mathbf{k}_\perp) \leftrightarrow \bar{E}_T$ similar to $f_{1T}^\perp(x, \mathbf{k}_\perp) \leftrightarrow E$.

$\hookrightarrow$ $h_1^\perp(x, \mathbf{k}_\perp) < 0$ for $u/p$, $d/p$, $u/\pi$, $\bar{d}/\pi$, ..(MB+BH, 2008)

- different valence quarks add coherently $|h_1^\perp| > |f_1^\perp|$ (MB+BH; Musch)

## Total (Spin+Orbital) Quark Angular Momentum

$$J_q^x = L_q^x + S_q^x = \int d^3r \left[ y T_q^{0z}(\vec{r}) - z T_q^{0y}(\vec{r}) \right]$$

- $T_q^{\mu\nu}(\vec{r})$ energy momentum tensor $(T_q^{\mu\nu}(\vec{r}) = T_q^{\nu\mu}(\vec{r}))$
- $T_q^{0i}(\vec{r})$ momentum density $[P_q^i = \int d^3r T_q^{0i}(\vec{r})\ ]$
- think: $(\vec{r} \times \vec{p})^x = yp^z - zp^y$

## relate to impact parameter dependent quark distributions $q_\psi(x, \mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

- eigenstate under rotations about $x$-axis

↪ both terms in $J_q^x$ equal:
$J_q^x = 2\int d^3r\, y T_q^{0z}(\vec{r}) = \int d^3r\, y \left[ T_q^{0z}(\vec{r}) + T_q^{z0}(\vec{r}) \right]$

- $\int d^3r\, y T_q^{00}(\vec{r}) = 0 = \int d^3r\, y T_q^{zz}(\vec{r})$

$$\Rightarrow \quad J_q^x = \int d^3r\, y T_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

## relate to impact parameter dependent quark distributions $q_\psi(x, \mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

- eigenstate under rotations about $x$-axis

$\hookrightarrow$ both terms in $J_q^x$ equal:
$J_q^x = 2\int d^3r\, yT_q^{0z}(\vec{r}) = \int d^3r\, y\left[T_q^{0z}(\vec{r}) + T_q^{z0}(\vec{r})\right]$

- $\int d^3r\, yT_q^{00}(\vec{r}) = 0 = \int d^3r\, yT_q^{zz}(\vec{r})$

$$\Rightarrow \quad J_q^x = \int d^3r\, yT_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

- $\int dx\, xq(x, \mathbf{r}_\perp) = \frac{1}{2m_N}\int dr^z T^{++}(\vec{r})$
(note: here $x$ is momentum fraction and not $r^x$)

$\hookrightarrow$ $\langle\psi|J_q^x|\psi\rangle = m_N \int dx \int d^2b_\perp\, xb^y q_\psi(x, \mathbf{b}_\perp)$

## distribution in delocalized wave packet (pol. in $+\hat{x}$ direction)

$q_\psi(x,\mathbf{b}_\perp) = \int d^2 r_\perp q(x, b_\perp - r_\perp) \left( |\psi(\mathbf{r}_\perp)|^2 - \frac{1}{2M}\frac{\partial}{\partial r_y}|\psi(\mathbf{r}_\perp)|^2 \right)$ with

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

two contributions to $\perp$ shift

- intrinsic shift relative to center of momentum $\mathbf{R}_\perp$
- overall shift of $\mathbf{R}_\perp$ for $\perp$ polarized nucleon

## insert into $\langle\psi|J_q^x|\psi\rangle = \int dx \int d^2 b_\perp q_\psi(x,\mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)

$\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H_q(x,0,0) + E_q(x,0,0)\right]$ (here: derived for $\vec{p} = \vec{0}$ only!)

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- result for $J_q^z$ also applies to $p_z \neq 0$
- partonic interpretation ($\perp$ shift) exists only for $\perp$ components!
- not valid for $J_q^x$ when $p_z \neq 0$

**insert into $\langle\psi|J_q^x|\psi\rangle = \int dx \int d^2b_\perp q_\psi(x,\mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)**

$\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H_q(x,0,0)+E_q(x,0,0)\right]$ (here: derived for $\vec{p}=\vec{0}$ only!)

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- result for $J_q^z$ also applies to $p_z \neq 0$
- partonic interpretation ($\perp$ shift) exists only for $\perp$ components!
- not valid for $J_q^x$ when $p_z \neq 0$

**gauge invariance**

- matrix element of $T_q^{++} = \bar{q}\gamma^+ i\partial^+ q$ in $A^+=0$ gauge same as that of $\bar{q}\gamma^+\left(i\partial^+ - gA^+\right)q$ in any gauge

$\hookrightarrow$ identify $\frac{1}{2}\int dx\, x\left[H(x,0,0)+E(x,0,0)\right]$ with $J_q$ in decomposition where $\vec{L}_q = \int d^3x \langle P,S|q^\dagger(\vec{x})\left(\vec{x}\times i\vec{D}\right)q(\vec{x})|P,S\rangle$

**insert into $\langle\psi|J_q^x|\psi\rangle = \int dx \int d^2b_\perp q_\psi(x,\mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)**

$\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H_q(x,0,0)+E_q(x,0,0)\right]$ (here: derived for $\vec{p}=\vec{0}$ only!)

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- result for $J_q^z$ also applies to $p_z \neq 0$
- partonic interpretation ($\perp$ shift) exists only for $\perp$ components!
- not valid for $J_q^x$ when $p_z \neq 0$

**caution!**

- made heavily use of rotational invariance

$\hookrightarrow$ itentification $\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H(x,0,0)+E(x,0,0)\right]$ does not apply to unintegrated quantities

- $\int d^2\Delta_\perp e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\frac{x}{2}\left[H(x,0,-\Delta_\perp^2)+E(x,0,-\Delta_\perp^2)\right]$ not equal to $J^z(\mathbf{b})_\perp$
- $J_q(x) \equiv \frac{x}{2}\left[H_q(x,0,0)+E_q(x,0,-\Delta_\perp^2)\right]$ not $x$-distribution of angular momentum $J_q^z(x)$ in long. pol. target

regardless whether one takes gauge covariant definition or not

## first: QED without electrons

- apply $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{b}(\vec{a}\cdot\vec{c})$ to $\vec{E}\times\left(\vec{\nabla}\times\vec{A}\right)$

$$\begin{aligned}\vec{J} &= \int d^3r\,\vec{x}\times\left(\vec{E}\times\vec{B}\right) = \int d^3r\,\vec{x}\times\left[\vec{E}\times\left(\vec{\nabla}\times\vec{A}\right)\right]\\ &= \int d^3r\left[E^j\left(\vec{x}\times\vec{\nabla}\right)A^j - \vec{x}\times(\vec{E}\cdot\vec{\nabla})\vec{A}\right]\end{aligned}$$

- integrate by parts (drop surface term)

$$\vec{J} = \int d^3r\left[E^j\left(\vec{x}\times\vec{\nabla}\right)A^j + \left(\vec{x}\times\vec{A}\right)\vec{\nabla}\cdot\vec{E} + \vec{E}\times\vec{A}\right]$$

- drop $2^{nd}$ term (eq. of motion $\vec{\nabla}\cdot\vec{E}=0$), yielding $\vec{J}=\vec{L}+\vec{S}$ with

$$\vec{L} = \int d^3r\,E^j\left(\vec{x}\times\vec{\nabla}\right)A^j \qquad \vec{S} = \int d^3r\,\vec{E}\times\vec{A}$$

- note: $\vec{L}$ and $\vec{S}$ not separately gauge invariant as written, but can be made so ($\rightarrow$ nonlocal)

## QED with electrons

$$\begin{aligned}\vec{J}_\gamma &= \int d^3r\,\vec{r}\times\left(\vec{E}\times\vec{B}\right) = \int d^3r\,\vec{r}\times\left[\vec{E}\times\left(\vec{\nabla}\times\vec{A}\right)\right] \\ &= \int d^3r\left[E^j\left(\vec{r}\times\vec{\nabla}\right)A^j - \vec{r}\times(\vec{E}\cdot\vec{\nabla})\vec{A}\right] \\ &= \int d^3r\left[E^j\left(\vec{r}\times\vec{\nabla}\right)A^j + \left(\vec{r}\times\vec{A}\right)\vec{\nabla}\cdot\vec{E} + \vec{E}\times\vec{A}\right]\end{aligned}$$

- replace $2^{nd}$ term (eq. of motion $\vec{\nabla}\cdot\vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

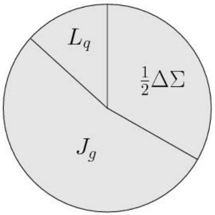
$$\vec{J}_\gamma = \int d^3r\left[\psi^\dagger\vec{r}\times e\vec{A}\psi + E^j\left(\vec{x}\times\vec{\nabla}\right)A^j + \vec{E}\times\vec{A}\right]$$

- $\psi^\dagger\vec{r}\times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger\vec{r}\times(\vec{p}-e\vec{A})\psi$

↪ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

- can also be done for only part of $\vec{A}$ → Chen/Goldman, Wakamatsu

## Ji decomposition




'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

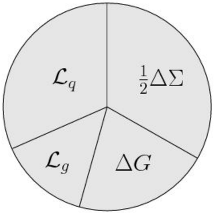
$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S| \, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \, |P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

## Jaffe decomposition




'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$$

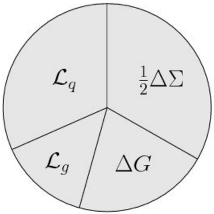
$$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S| \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j |P,S\rangle$$

## Jaffe decomposition

- $\Delta q$ from polarized DIS
- $\Delta G$ from $\Delta g(x)$
  $(\vec{p}\overleftarrow{p}$ & $\frac{d}{d\ln Q^2}\Delta q(x))$
- $\Delta G$ gauge invariant! Nonlocal for $A^+ \neq 0$
- no exp./lattice access to $\mathcal{L}_q$, $\mathcal{L}_g$
- only $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q$, by subtraction
  $\mathcal{L} = \frac{1}{2} - \Delta G - \sum_q \frac{1}{2}\Delta q$

## Jaffe decomposition




'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

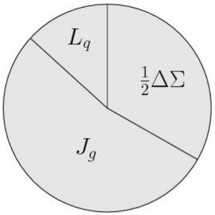
$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S|\, \bar{q}(\vec{r})\gamma^+\left(\vec{r}\times i\vec{\partial}\right)^z q(\vec{r})|P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij}\int d^3r\, \langle P,S|\, \mathrm{Tr} F^{+i}A^j\, |P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S|\mathrm{Tr} F^{+j}\left(\vec{x}\times i\vec{\partial}\right)^z A^j|P,S\rangle$$

## Ji decomposition



'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S| \, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \, |P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

## Ji decomposition

- $\Delta q$ from polarized DIS
- $J_q \equiv \frac{1}{2}\Delta q + L_q = \frac{1}{2}\int_0^1 dx \left[H_q(x,0,0) + E_q(x,0,0)\right]$ from DVCS
- $J_g$ in principle from gluon-GPDs; in practice $J_g = \frac{1}{2} - J_q$ easier

- $L_q$ matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- (for $\vec{p} = 0$) matrix element of $\bar{q}\gamma^z \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ vanishes (parity!)

↪ $L_q$ identical to matrix element of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ (nucleon at rest)

↪ even in light-cone gauge, $L_q^z$ and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger \left(r^x g A^y - r^y g A^x\right) q\Big|_{A^+=0}$

- how significant is that difference?

## scalar diquark model

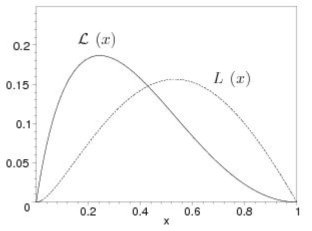
- LC wave functions $\psi_s^S(x,\mathbf{k}_\perp)$

$\hookrightarrow$ $\mathcal{L}_q$ from $|\psi_s^S(x,\mathbf{k}_\perp)|^2$

- GPDs from overlap integrals of $\psi^\dagger\psi$

$\hookrightarrow$ $L_q$ from Ji

- $L_q = \mathcal{L}_q$.
  No surprise since $L_q - \mathcal{L}_q \sim \langle q^\dagger \vec{r}\times\vec{A}q\rangle$ and no $\vec{A}$ in scalar diquark model
- $L_q(x) \neq \mathcal{L}_q(x)$




## scalar diquark model

- interpretation of $J_q(x) \equiv \frac{x}{2}\left[q(x) + E^q(x,0,0)\right]$ not that of distribution of AM in $x$
- FT of $J(t) \equiv \frac{x}{2}\left[q(x) + E^q(x,0,0)\right]$ not distribution of $J_q^z$ in $\mathbf{b}_\perp$

M.B. + Hikmat BC, PRD **79**, 071501 (2009)

## QED for dressed $e^-$ in QED

- LC wave functions $\psi_{sh}^S(x,\mathbf{k}_\perp)$

$\hookrightarrow$ $\mathcal{L}_q$ from $|\psi_{sh}^S(x,\mathbf{k}_\perp)|^2$

- GPDs from overlap integrals of $\psi^\dagger\psi$

$\hookrightarrow$ $L_q$ from Ji

- $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

## higher twist in polarized DIS

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $g_1 = \frac{1}{2}\sum_q e_q^2 g_1^q$ with $g_1^q = q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x)$
- $g_2$ involves quark-gluon correlations

$\hookrightarrow$ no parton interpret. as difference between number densities for $g_2$

- for $\perp$ pol. target, $g_1$ & $g_2$ contribute equally

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ 'clean' separation between $g_2$ and $\frac{1}{Q^2}$ corrections to $g_1$

## What can we learn from $g_2$?

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2M{P^+}^2 S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0)\gamma^+ q(0) \right| P,S \right\rangle$$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

$$\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x$$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

**color Lorentz force**

$$\sqrt{2}G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

**color Lorentz force**

$$\sqrt{2}G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

$\hookrightarrow$ $d_2 \leftrightarrow$ average **color Lorentz force** acting on quark moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by $\gamma^*$

$$\langle F^y\rangle = -2M^2 d_2 = -\frac{M}{P^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

*cf.* Qiu-Sterman matrix element $\langle k_\perp^y\rangle \equiv \int_0^1 dx \int d^2k_\perp\, k_\perp^2 f_{1T}^\perp(x,k_\perp^2)$

$$\langle k_\perp^y\rangle = -\frac{1}{2p^+}\left\langle P,S\left|\bar{q}(0)\int_0^\infty dx^- gG^{+y}(x^-)\gamma^+q(0)\right|P,S\right\rangle$$

semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

matrix element defining $d_2$ $\leftrightarrow$ $1^{st}$ integration point in QS-integral

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

## color Lorentz force

$$\sqrt{2}G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

↪ $d_2 \leftrightarrow$ average **color Lorentz force** acting on quark moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by $\gamma^*$

$$\langle F^y\rangle = -2M^2 d_2 = -\frac{M}{P^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

## sign of $d_2 \leftrightarrow \perp$ imaging

- $\kappa_q/p \longrightarrow$ sign of deformation

↪ direction of average force

↪ $d_2^u > 0,\ d_2^d < 0$

- *cf.* $f_{1T}^{\perp u} < 0,\ f_{1T}^{\perp u} < 0$

## lattice (Göckeler et al., 2005)

$d_2^u \approx 0.010,\ d_2^d \approx -0.0056$

## magnitude of $d_2$

- $\langle F^y\rangle = -2M^2 d_2 = -10\frac{GeV}{fm}d_2$
- expect partial cancellation of forces in SSA

↪ $|\langle F^y\rangle| \ll \sigma \approx 1\frac{GeV}{fm}$

↪ $d_2 = \mathcal{O}(0.01)$ (→ C.Weiss)

- $E^q(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- $\perp$ deformation $\leftrightarrow$ (sign of) SSA (Sivers; Boer-Mulders)
- parton interpretation for Ji-relation
- $L_q \neq \mathcal{L}_q$
- higher-twist $(\int dx\, x^2 \bar{g}_2(x), \int dx\, x^2 \bar{e}(x)) \leftrightarrow \perp$ force in DIS
- $\perp$ deformation $\leftrightarrow$ (sign of) quark-gluon correlations
  $(\int dx\, x^2 \bar{g}_2(x), \int dx\, x^2 \bar{e}(x)\ )$