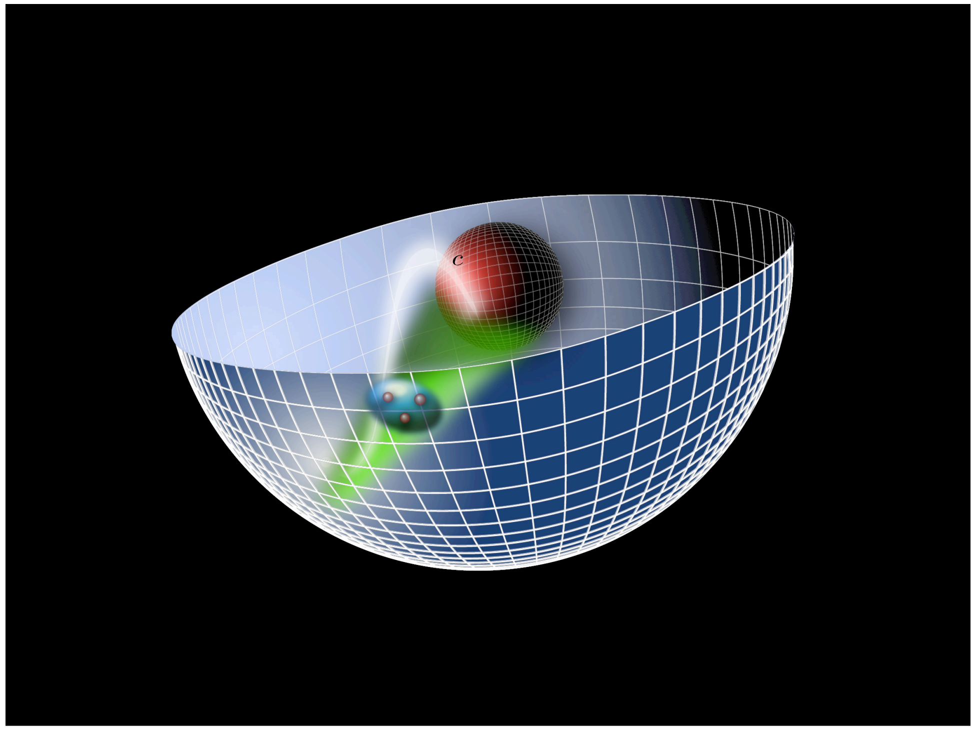
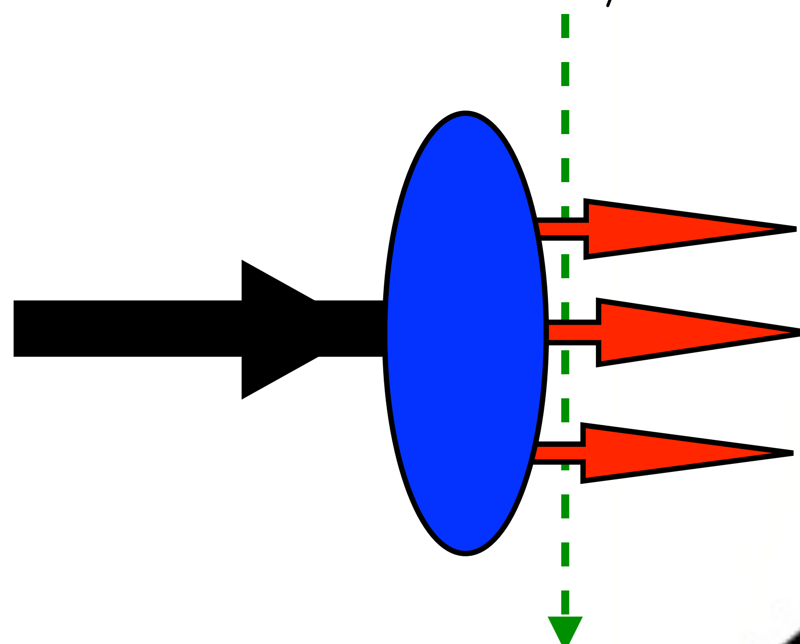
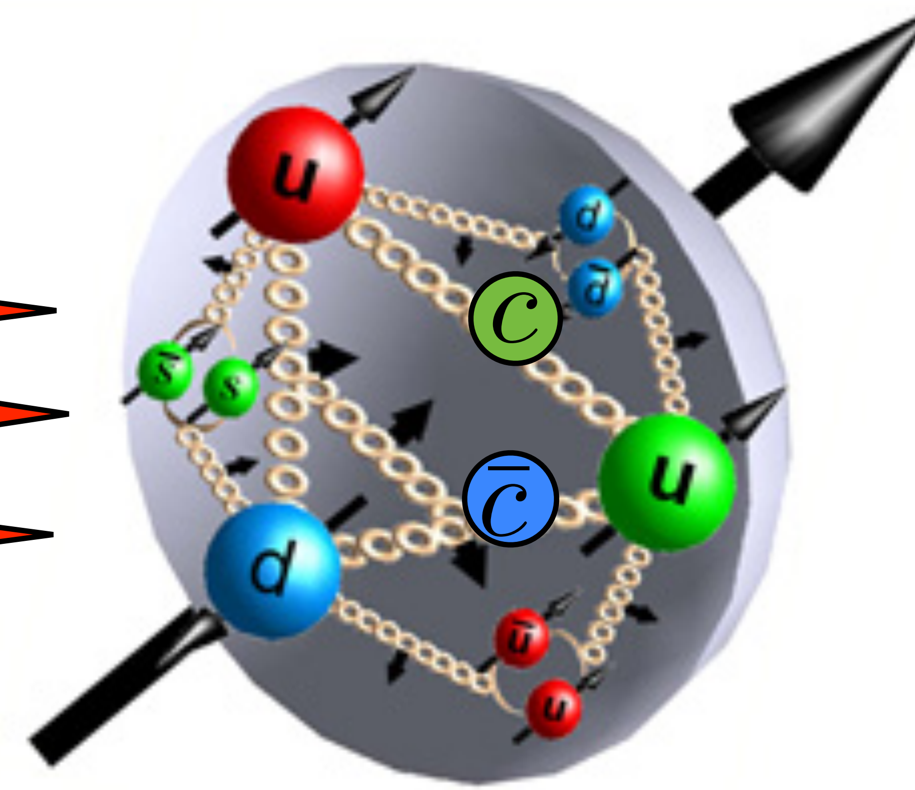
Light-Front Holography, Transversity and Quark Orbital Angular Momentum







INT Workshop

Orbital Angular Momentum in QCD

February 6 - 17, 2012

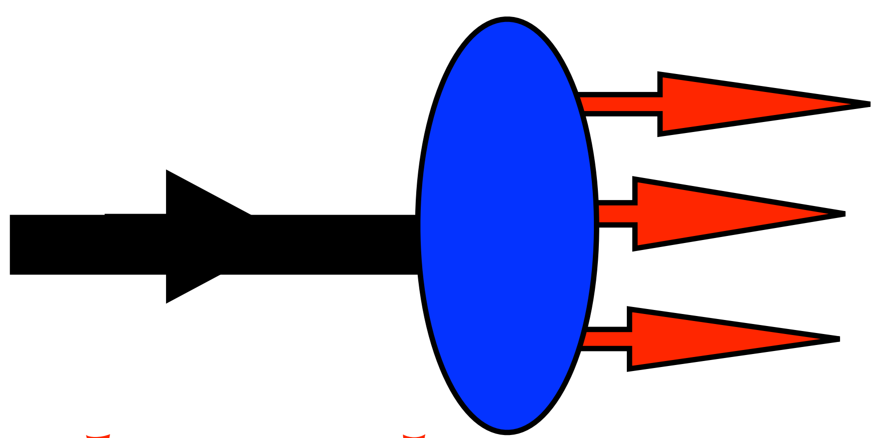
Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

- *Angular Momentum and Spin Phenomena in QCD*
- *Essentials of Spin on the Light Front*
- New Insights from higher space-time dimensions: *AdS/QCD*
- *Light-Front Holography*
- *Light Front Wavefunctions:* analogous to the Schrodinger wavefunctions of atomic physics

Dynamics plus Spectroscopy!

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

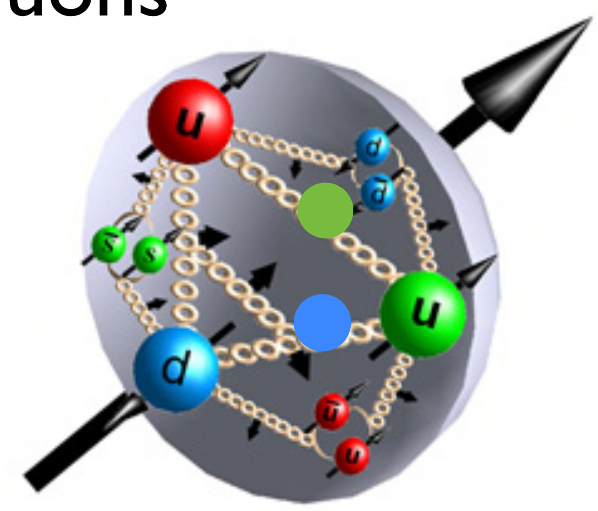


- *Hadronization at the Amplitude Level*

Transversity

Angular Momentum Structure, and the Spin Dynamics of Hadrons

- Test Fundamentals of Gauge Structure of QCD
- Fundamental Measures of Hadron Structure
- Angular Momentum of Confined Quarks and Gluons
- Breakdown of Conventional Wisdom
- Breakdown of Factorization Ideas
- Crucial Experiment Tests, Measurements



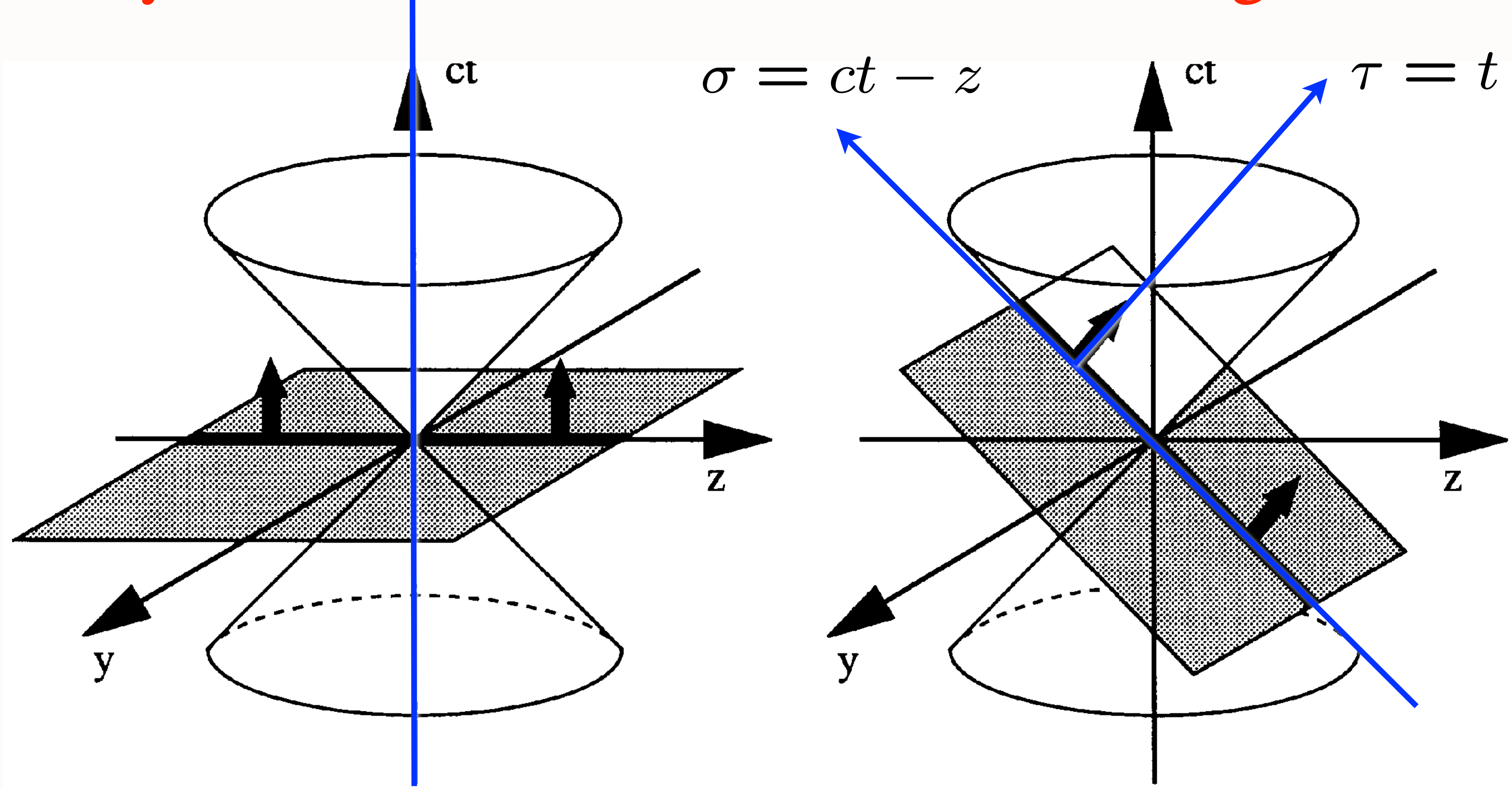
Remarkable array of theory and experimental topics

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

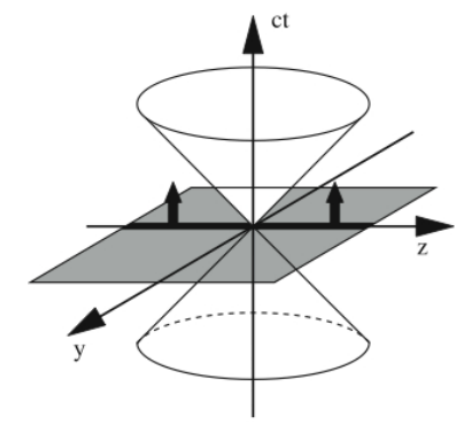
$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

Causally-Connected Domains

HELEN BRADLEY - PHOTOGRAPHY

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



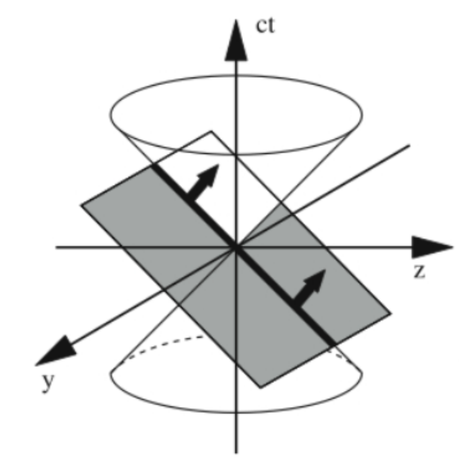
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

$k^+ = k^0 + k^3$ longitudinal momentum $(k^+ > 0)$

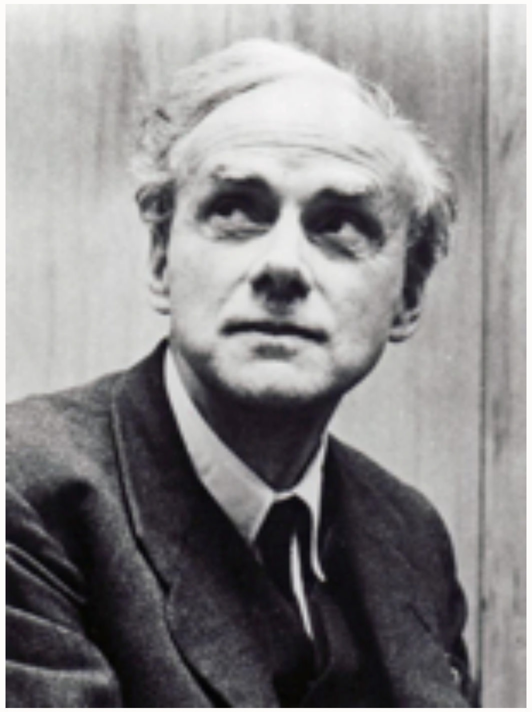
$k^- = k^0 - k^3$ light-front energy

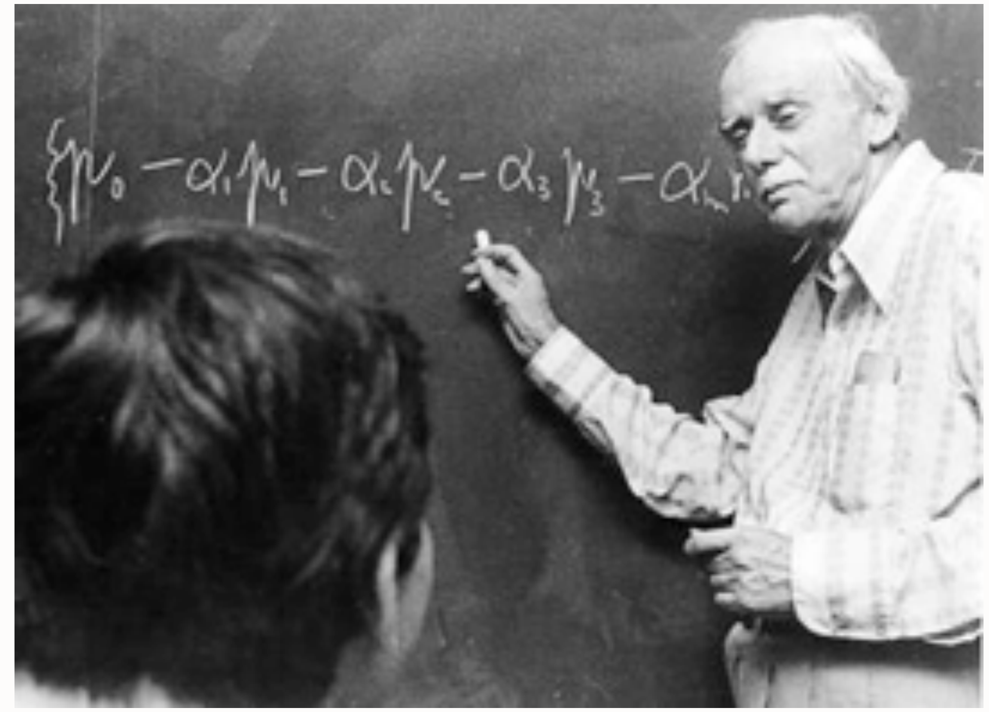


$$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

Quantum chromodynamics and other field theories on the light cone.
Stanley J. Brodsky (SLAC), Hans-Christian Pauli (Heidelberg, Max Planck Inst.),
Stephen S. Pinsky (Ohio State U.). SLAC-PUB-7484, MPIH-V1-1997. Apr 1997. 203 pp.
Published in **Phys.Rept. 301 (1998) 299-486**
e-Print: **hep-ph/9705477**





"Working with a front is a process that is unfamiliar to physicists.

But still I feel that the mathematical simplification that it introduces is all-important.

I consider the method to be promising and have recently been making an extensive study of it.

It offers new opportunities, while the familiar instant form seems to be played out. " -

P.A.M. Dirac (1977)

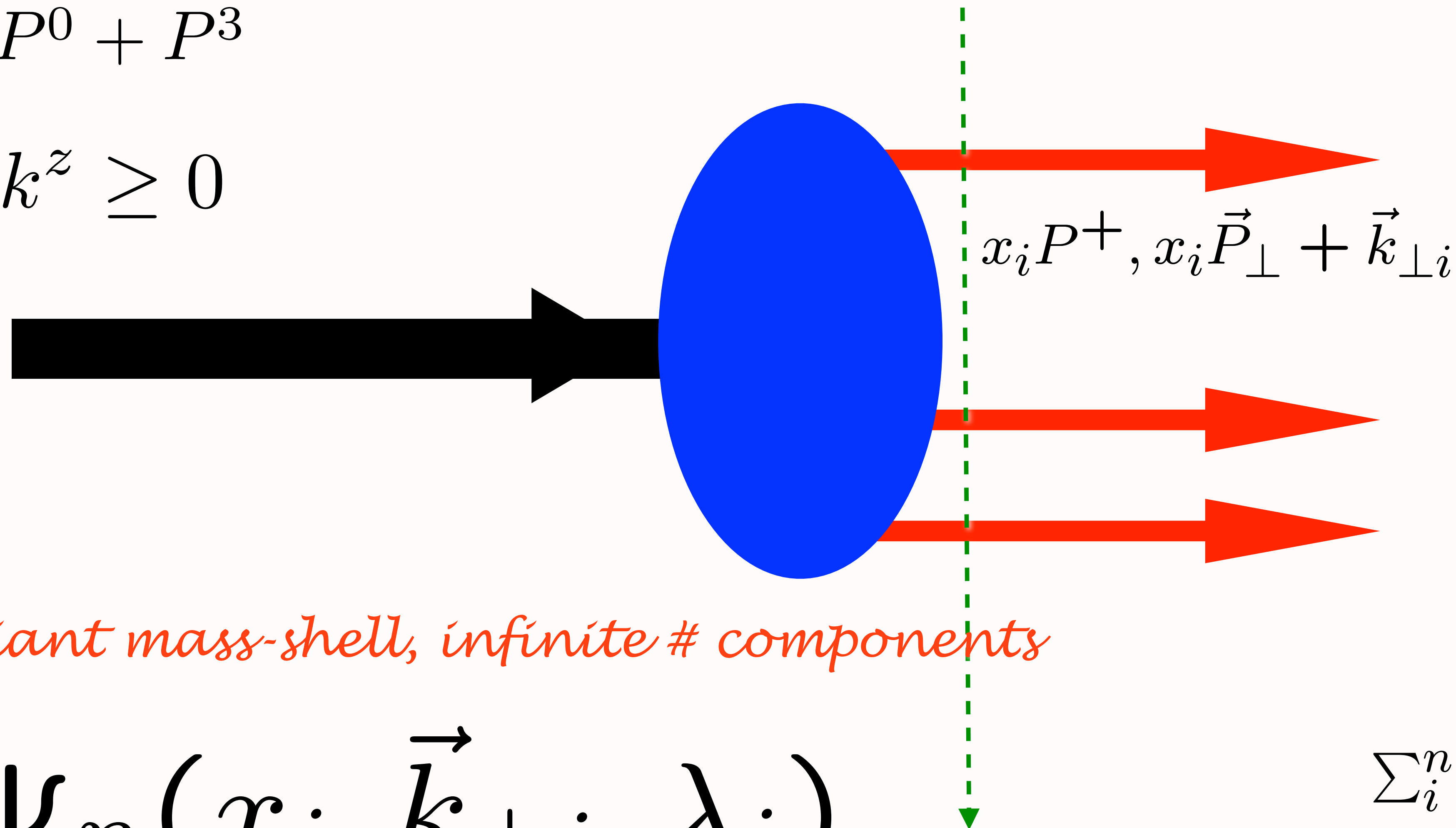
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$k^+ = k^0 + k^z \geq 0$$

Fixed $\tau = t + z/c$

$P^+, \vec{P}_\perp$



LFWFs: off invariant mass-shell, infinite # components

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

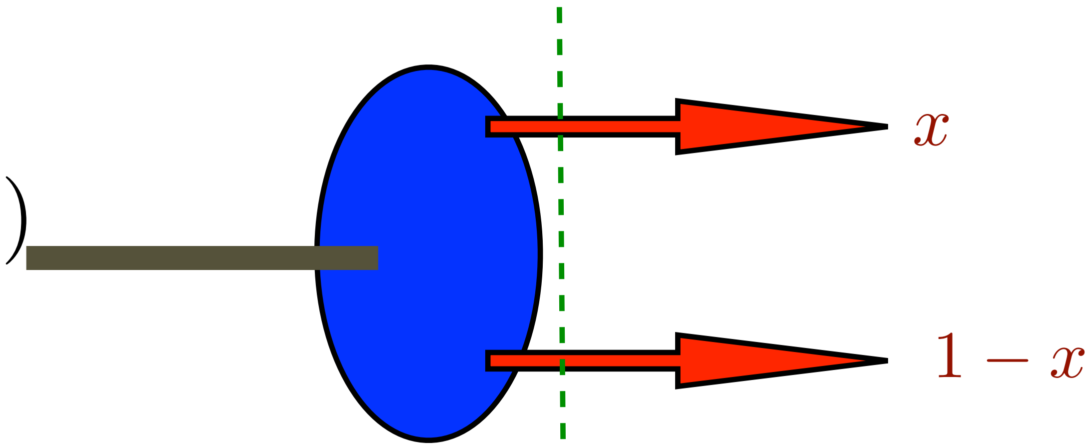
$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Hadron Distribution Amplitudes

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$$k_\perp^2 < Q^2$$

$$\sum_i x_i = 1$$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

- Evolution Equations from PQCD, OPE

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

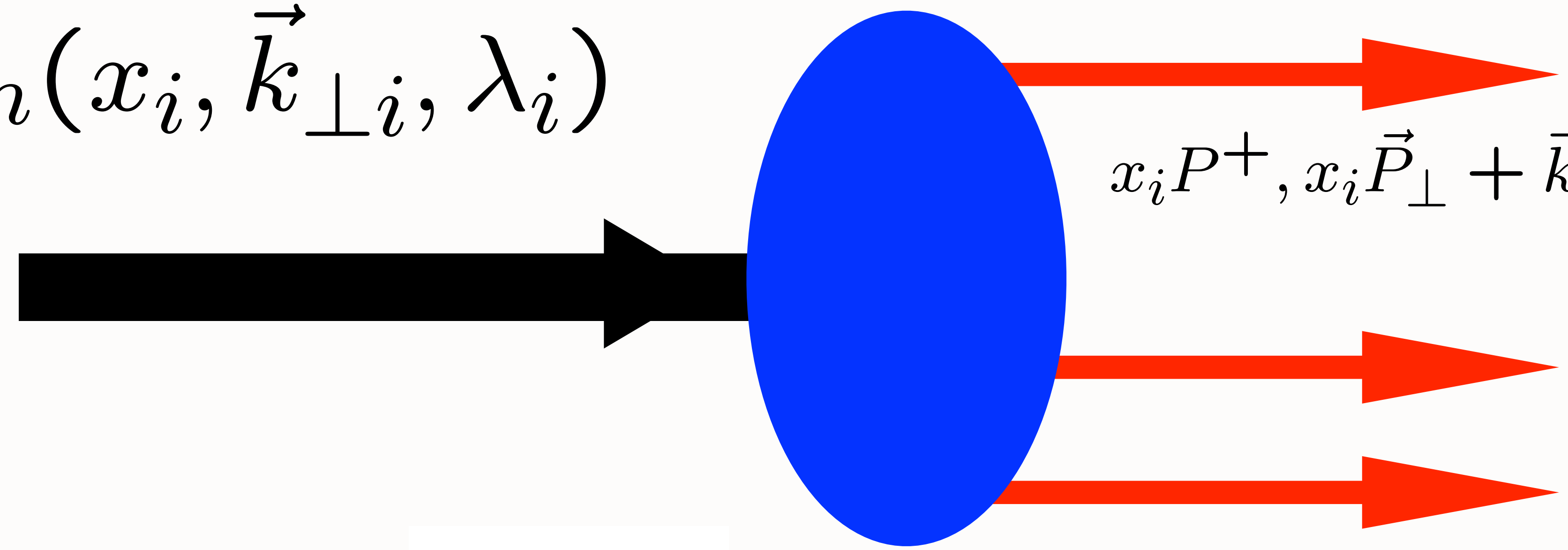
- Conformal Invariance
- Compute from valence light-front wavefunction in light-cone gauge

$\sum_i^n x_i = 1$ $\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$\sum_{i=1}^n k_i^+ = \sum_{i=1}^n x_i \vec{P}^+ = \vec{P}^+$ $\sum_{i=1}^n (x_i \vec{P}_\perp + \vec{k}_{\perp i}) = \vec{P}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

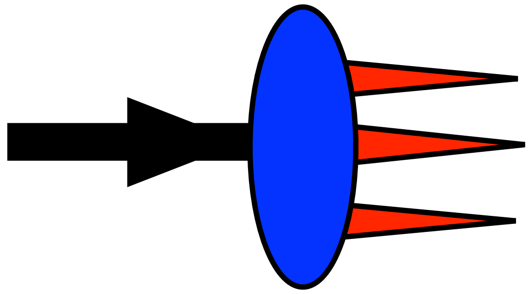
$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right) \qquad j = 1, 2, \cdots (n-1)$$

n-1 Intrinsic Orbital Angular Momenta
Frame Independent



$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

LF Spin Sum Rule

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

Orbital Angular Momentum in QFTH

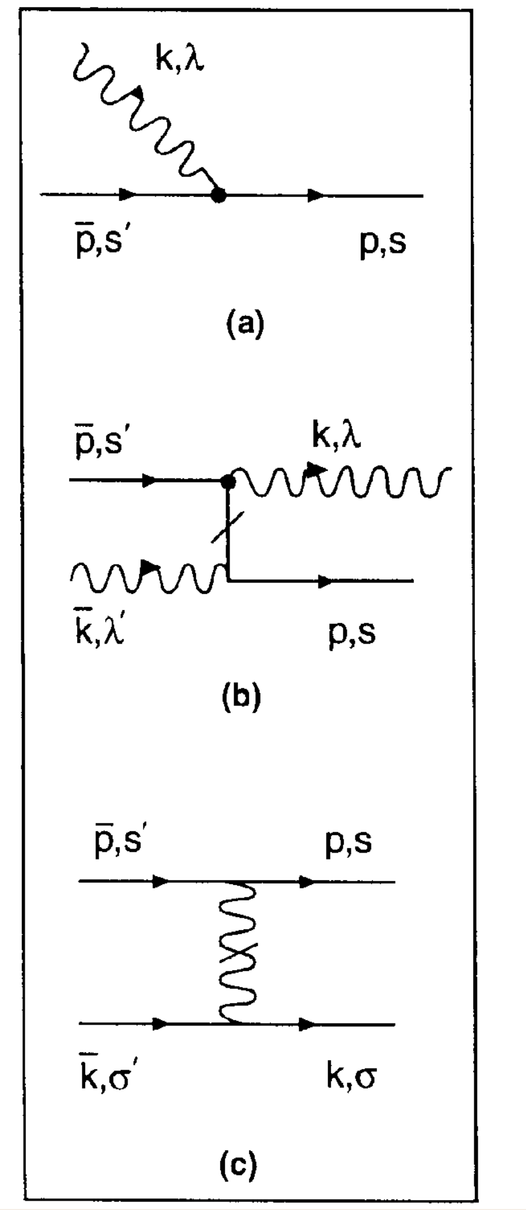
- *Rigorous boost-invariant definition of L^z from LF Theory*
- *Non-Zero Pauli Form Factor, Anomalous Moment and Sivers Effect require nonzero quark orbital angular momentum*
- *Sum of n L^z cancel in n-particle Fock state: overcounting*
- *Vanishing anomalous gravitomagnetic moment*
- *Wavefunctions in Instant Form do not determine current matrix elements!*
- *AdS/QCD: Spin J^z of Proton carried by quark L^z*

Light-Front QCD

Heisenberg Matrix Formulation

Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$



$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

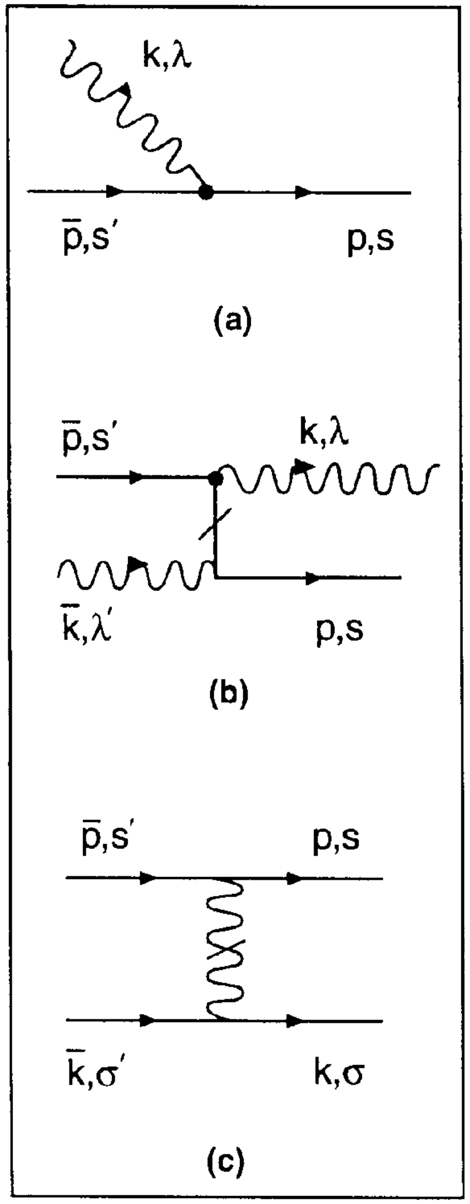
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

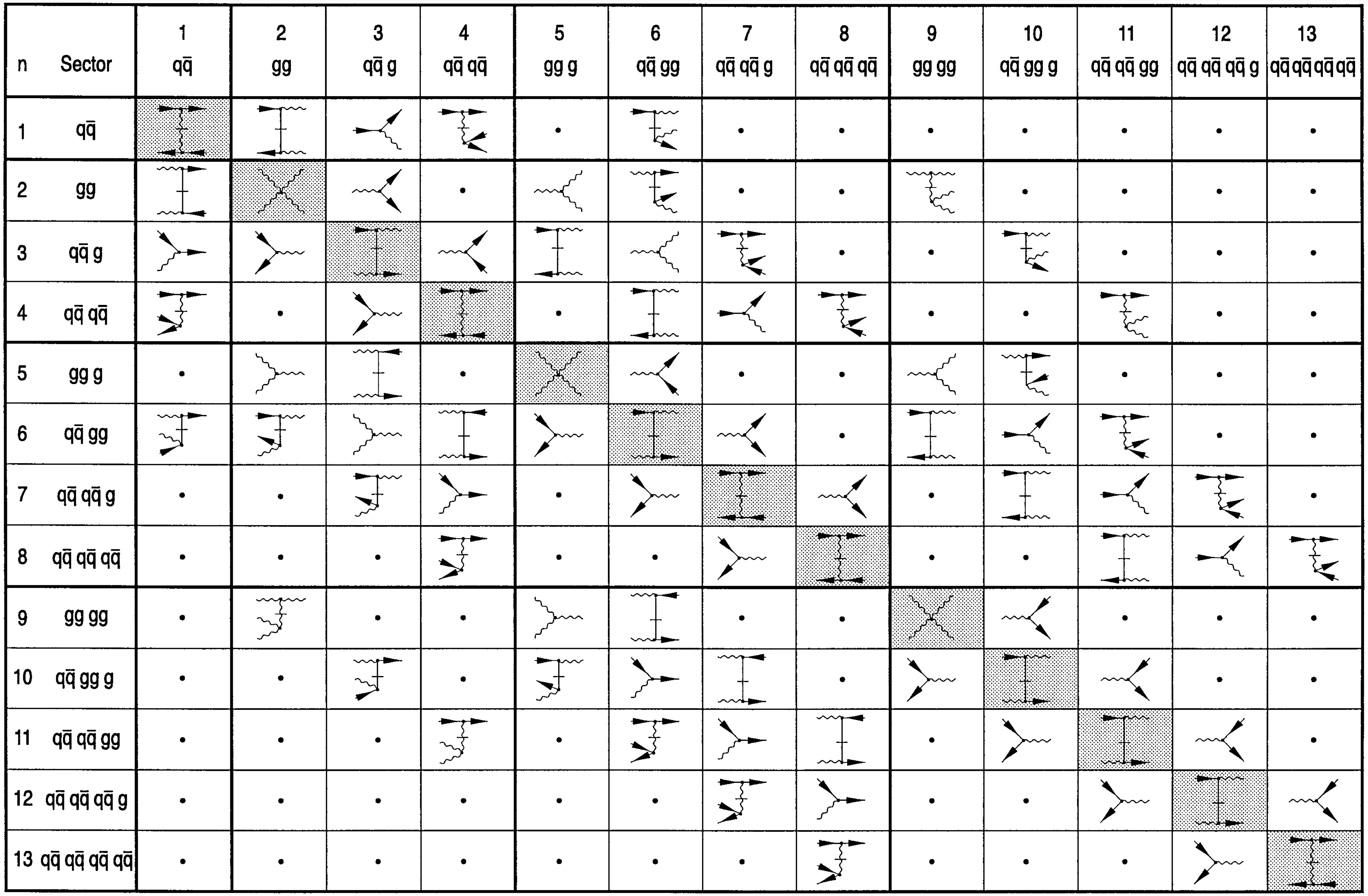
Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Light-Front QCD
Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

First Principle Solutions to QCD

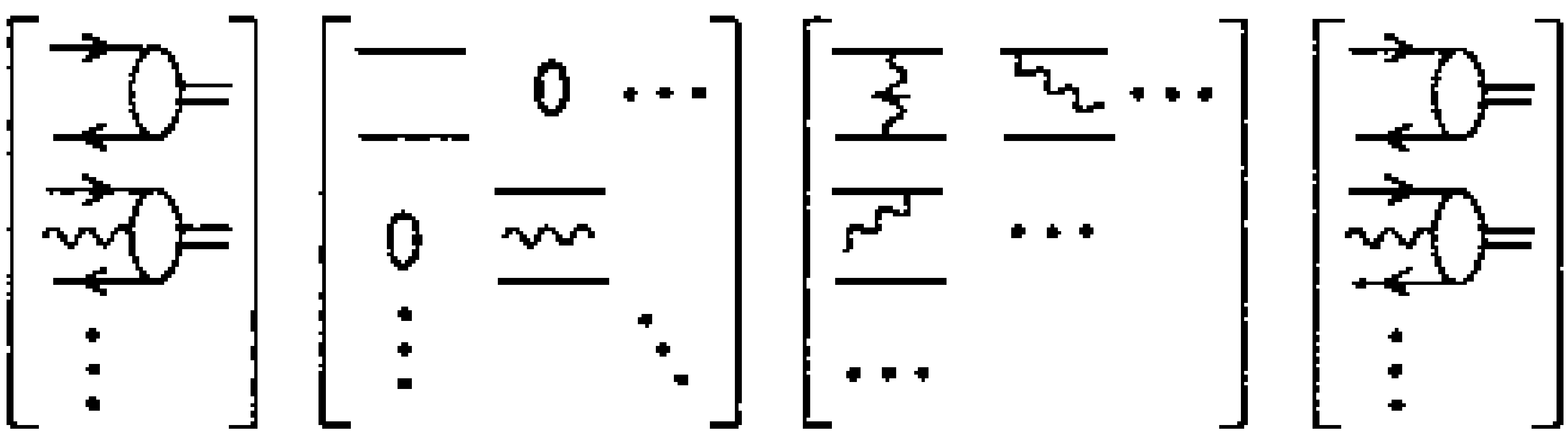




LIGHT-FRONT SCHRODINGER EQUATION

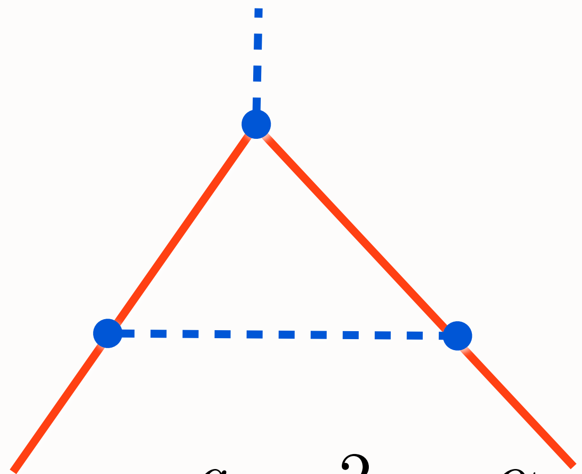
Direct connection to QCD Lagrangian

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right)\begin{bmatrix}\psi_{q\bar{q}/\pi}\\ \psi_{q\bar{q}g/\pi}\\ \vdots\end{bmatrix} = \begin{bmatrix}\langle q\bar{q}|\,V\,|q\bar{q}\rangle & \langle q\bar{q}|\,V\,|q\bar{q}g\rangle & \cdots\\ \langle q\bar{q}g|\,V\,|q\bar{q}\rangle & \langle q\bar{q}g|\,V\,|q\bar{q}g\rangle & \cdots\\ \vdots & \vdots & \ddots\end{bmatrix}\begin{bmatrix}\psi_{q\bar{q}/\pi}\\ \psi_{q\bar{q}g/\pi}\\ \vdots\end{bmatrix}$$

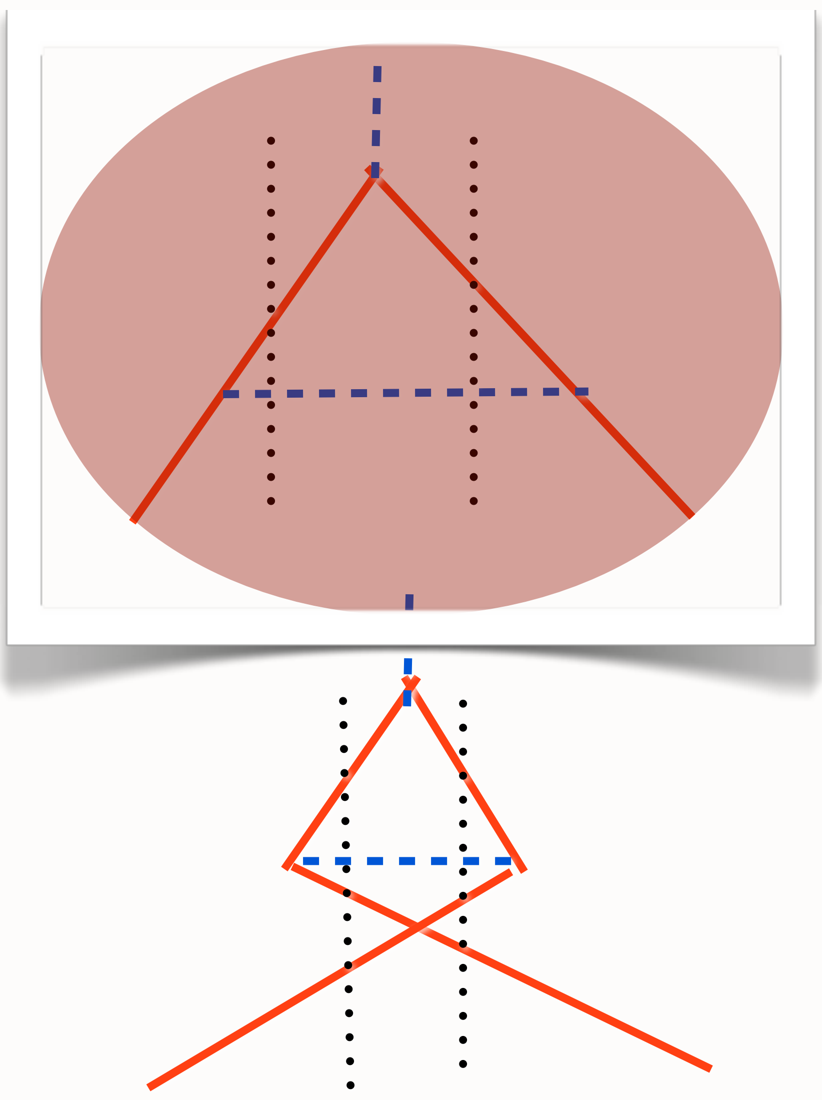


$A^+ = 0$

G.P. Lepage, sjb



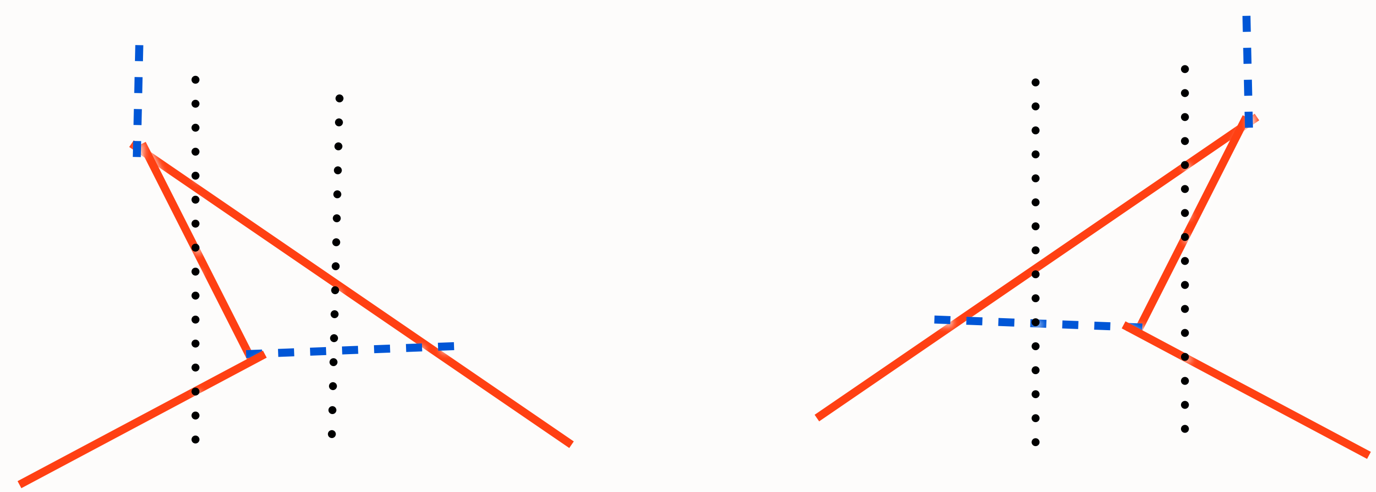
$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$



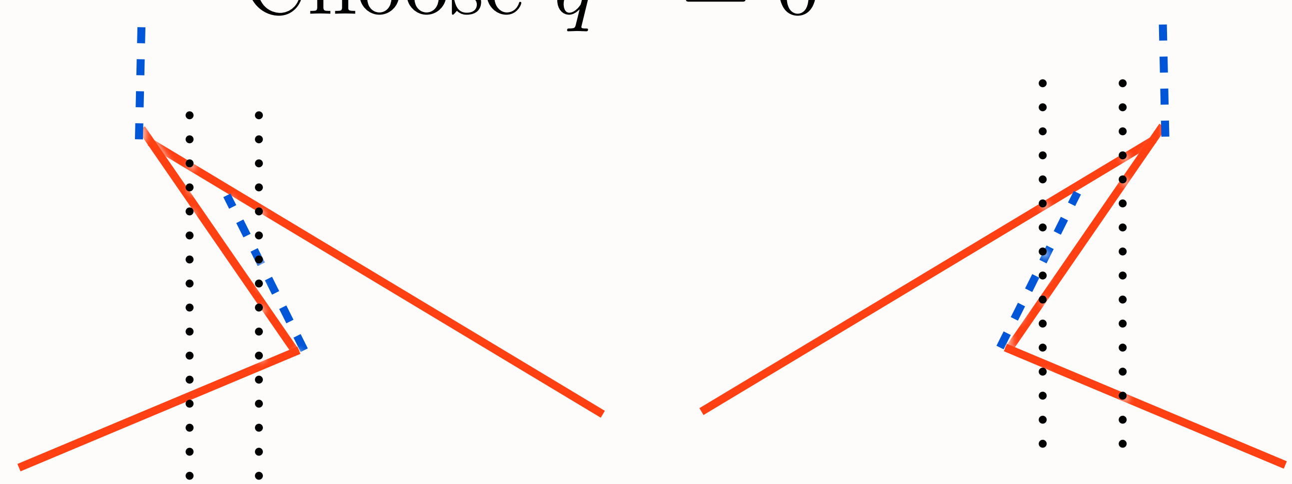
Wick Theorem

Feynman diagram =
single front-form time-ordered diagram!

Also $P \to \infty$ observer frame (Weinberg)

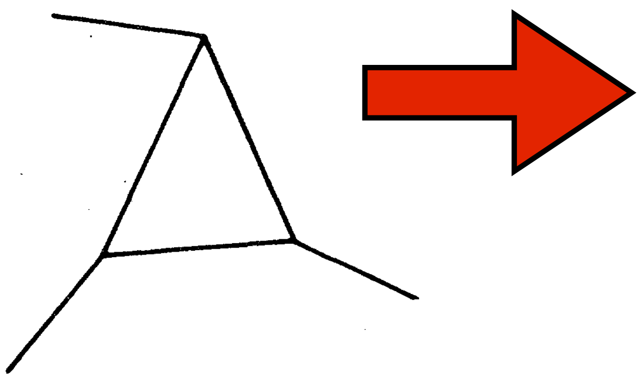


Choose $q^+ = 0$

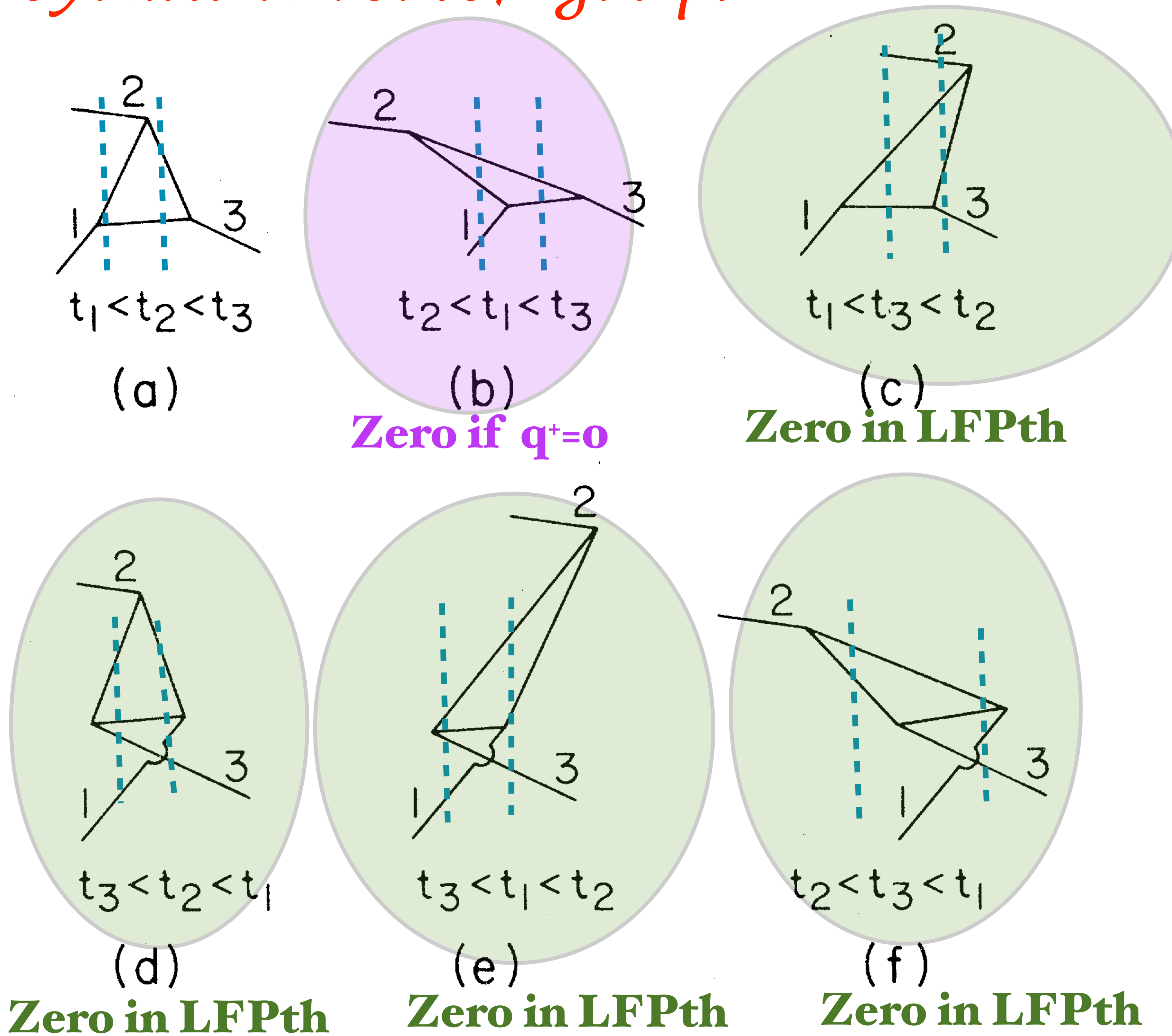


The surviving LF time-ordered contributions to the Feynman vertex graph

Feynman



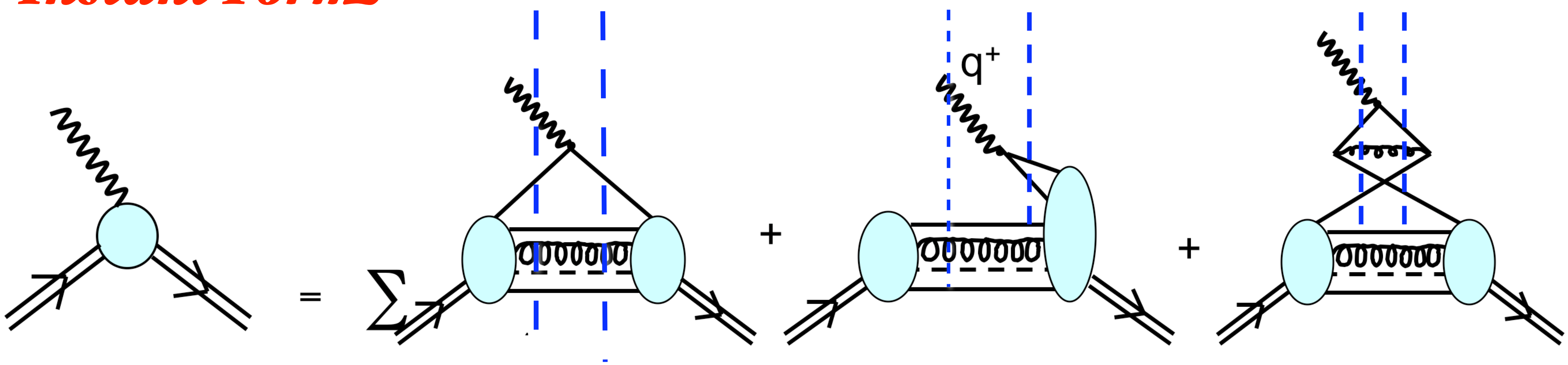
$$k^+ = k^0 + k^z \geq 0$$



Time flows from left to right

Calculation of Form Factors in Equal-Time Theory

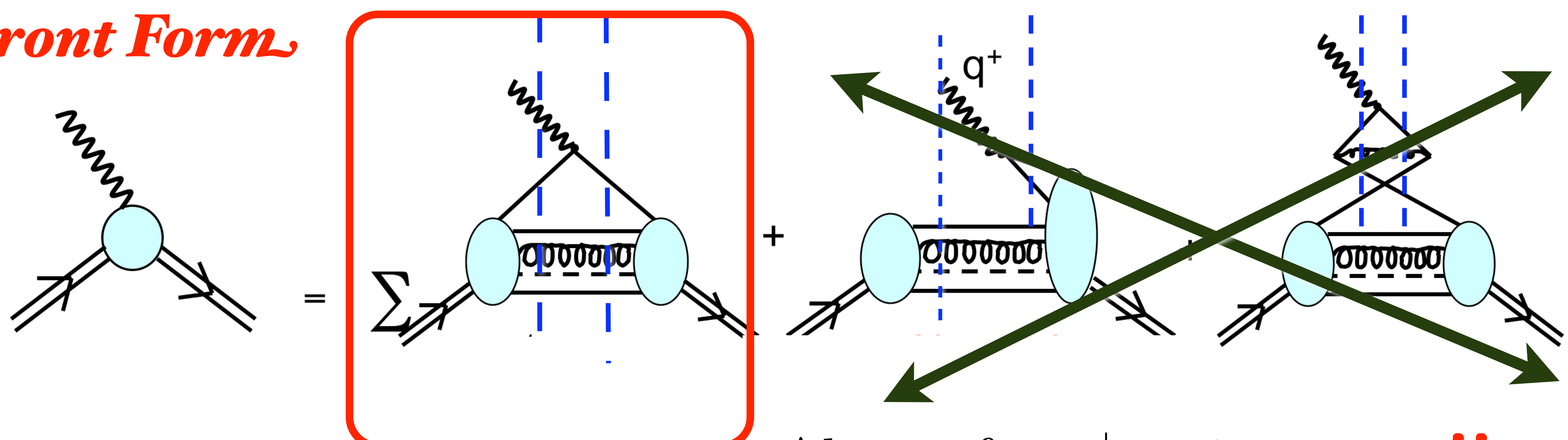
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

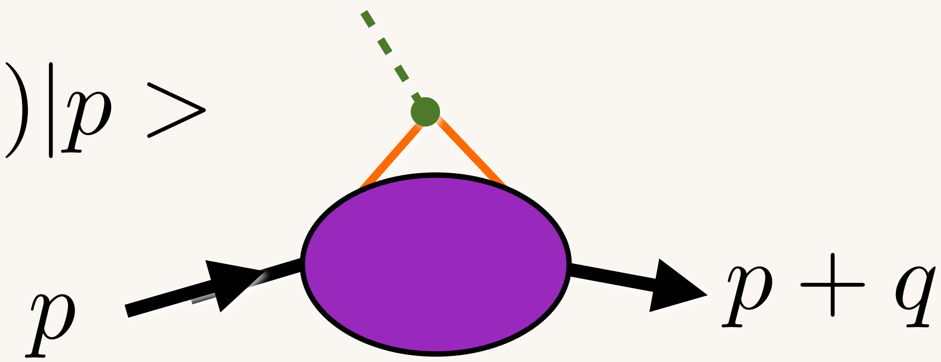
Front Form

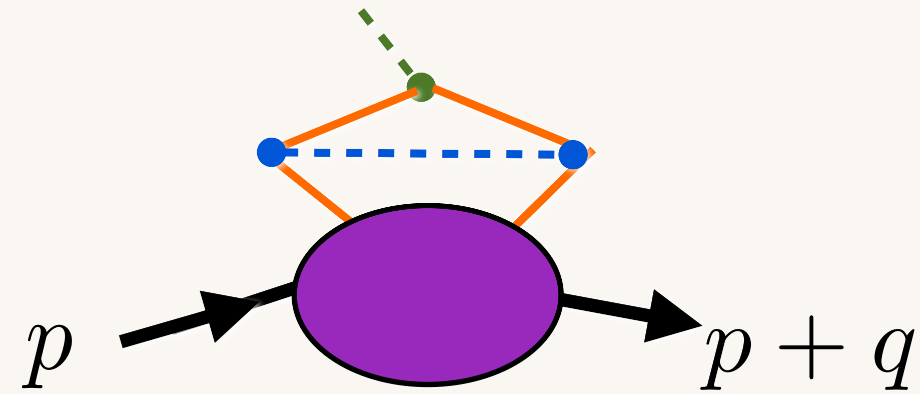


Absent for $q^+ = 0$ **zero !!**

Calculation of proton form factor in Instant Form

$$< p + q | J^{\mu}(0) | p >$$



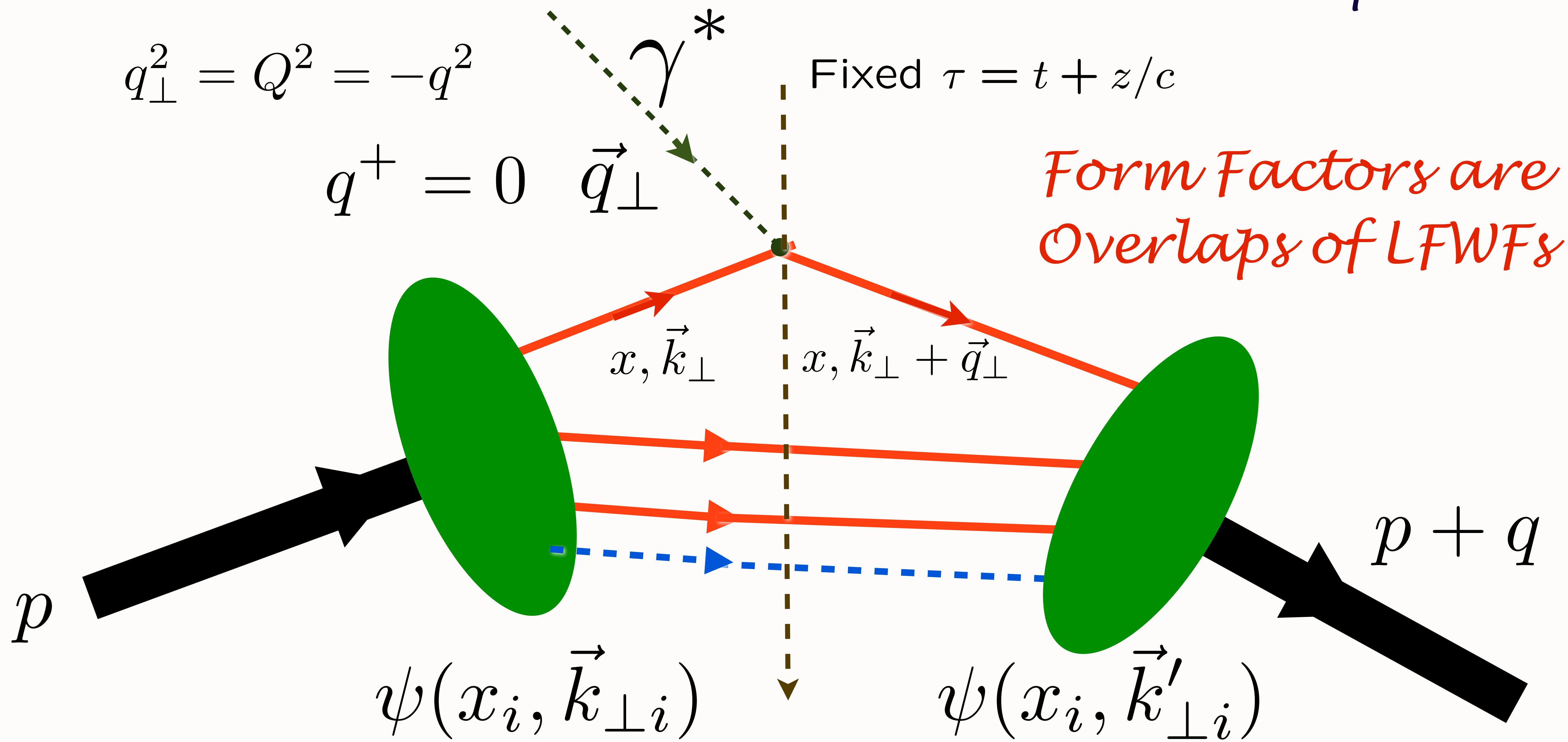


- **Need to boost proton wavefunction from p to p+q: Extremely complicated dynamical problem; particle number changes**
- **Need to couple to all currents arising from vacuum!!**
- **Wavefunction insufficient to compute matrix elements**
- **Each time-ordered contribution is frame-dependent**
- **States built on normal-ordered acausal vacuum**
- **Divide by disconnected vacuum diagrams**
- **Light-Front vacuum trivial! No conflict with cosmology**

Cosmological constant 10^{120} too large from QED?

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1-x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell, Yan; West

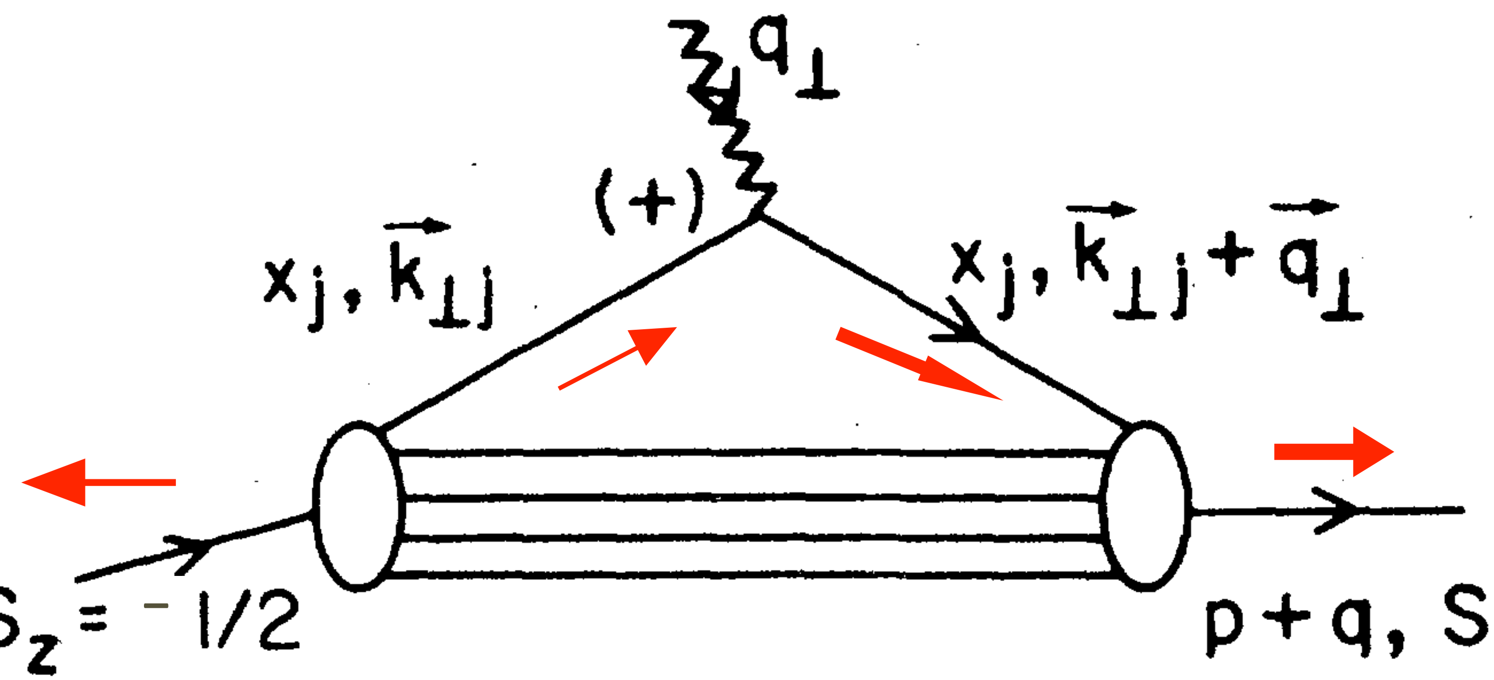
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

Drell, sjb

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Same matrix elements appear in Sivers effect
-- connection to quark anomalous moments

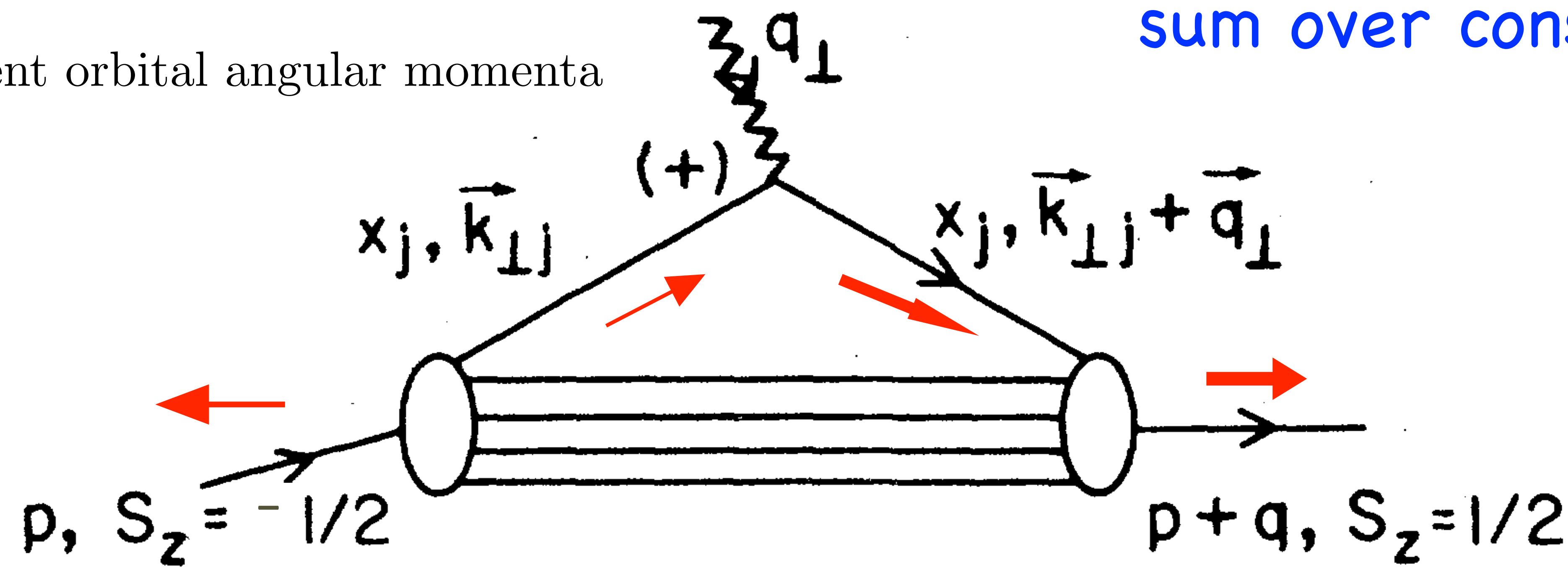
Anomalous gravitomagnetic moment B(0)

Terayev, Okun: *B(0) Must vanish because of Equivalence Theorem*

graviton

$\sum_{i=1}^{n} L_i = 0$
$n-1$ independent orbital angular momenta

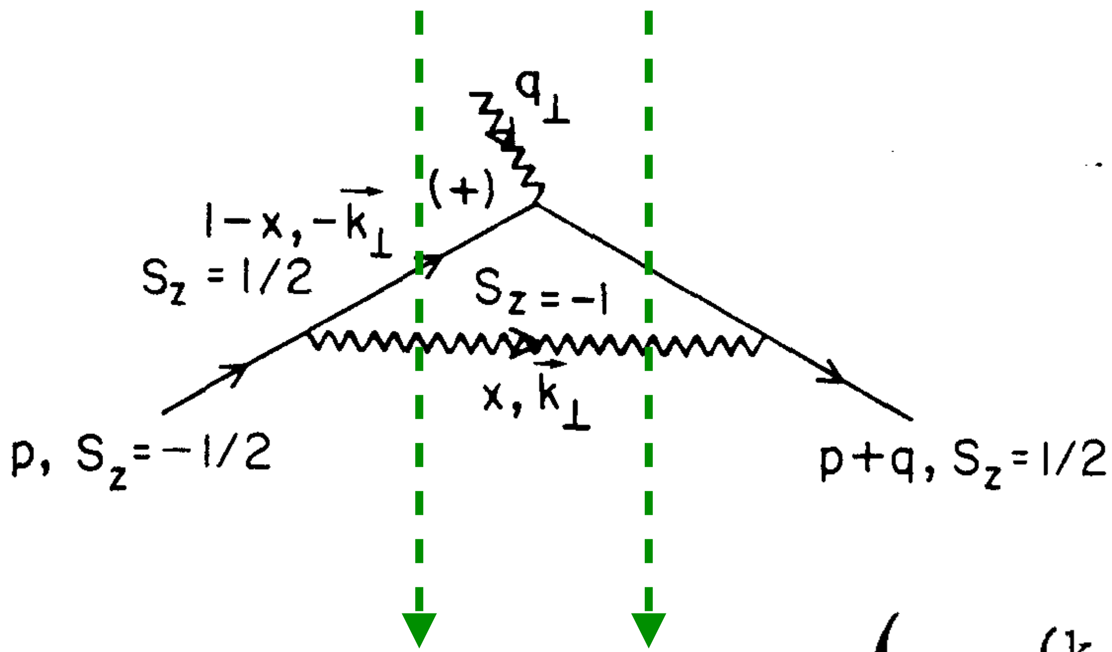
sum over constituents

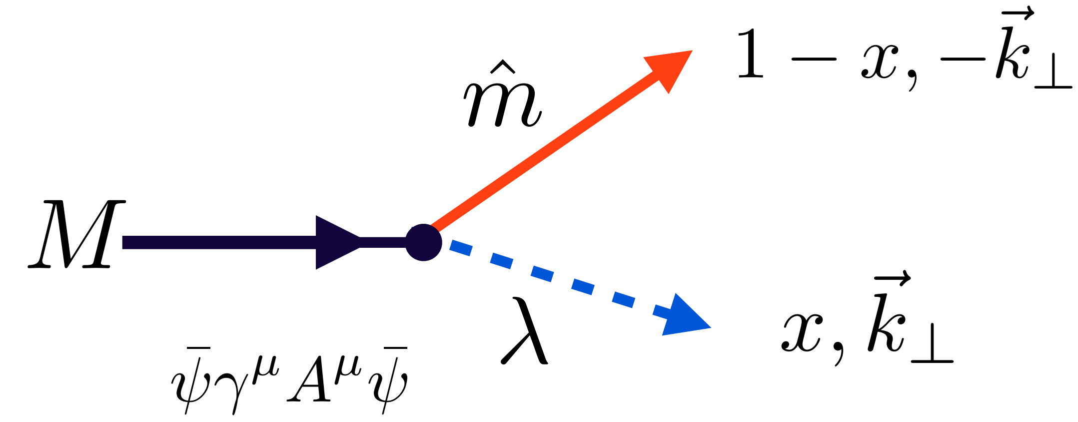


Hwang, Schmidt, sjb; Holstein et al

B(0) = 0

Each Fock State





$$\psi_{p\downarrow} = \frac{e/\sqrt{x}}{M^2 - \frac{k_\perp^2+\lambda^2}{x} - \frac{k_\perp^2+\hat{m}^2}{1-x}} \begin{cases} \sqrt{2}\,\frac{(k_1 - ik_2)}{x} & |-1/2\rangle \to |-1/2,\ 1\rangle \\ \sqrt{2}\,\frac{M(1-x) - \hat{m}}{1-x} & |-1/2\rangle \to |1/2,\ -1\rangle \end{cases}$$

$L^z = -1$

$L^z = 0$

and

$$\psi^*_{p\uparrow} = \frac{e/\sqrt{x}}{M^2 - \frac{k_\perp^2+\lambda^2}{x} - \frac{k_\perp^2+\hat{m}^2}{1-x}} \begin{cases} -\sqrt{2}\,\frac{M(1-x) - \hat{m}}{1-x} & |-1/2,\ 1\rangle \to |1/2\rangle \\ -\sqrt{2}\,\frac{(k_1 - ik_2)}{x} & |1/2,\ -1\rangle \to |1/2\rangle \end{cases}$$

$L^z = 0$

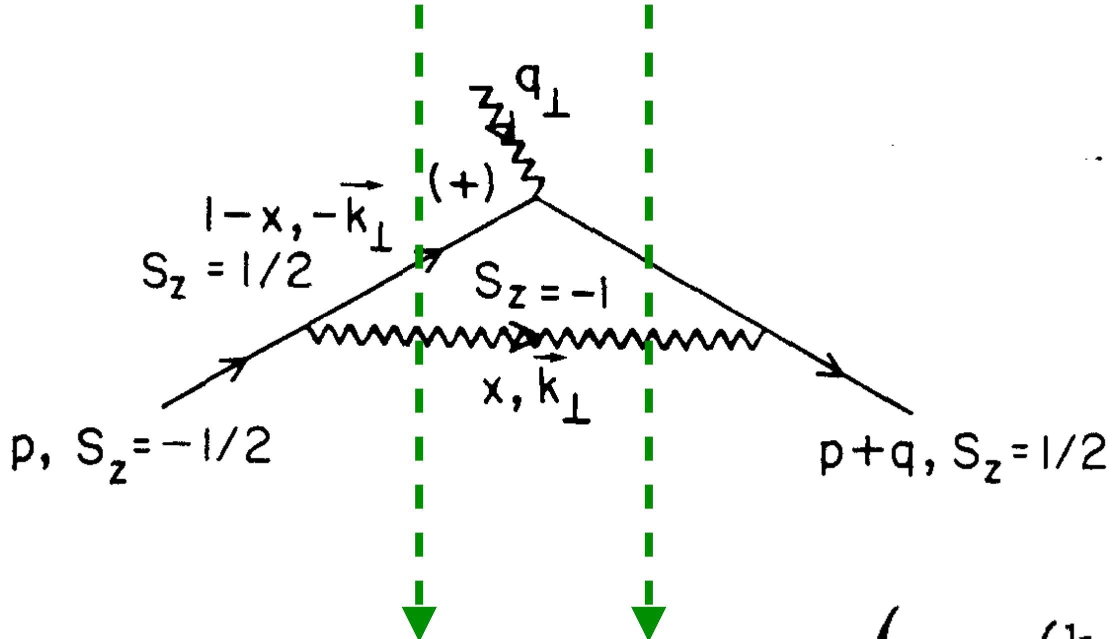
$L^z = +1$

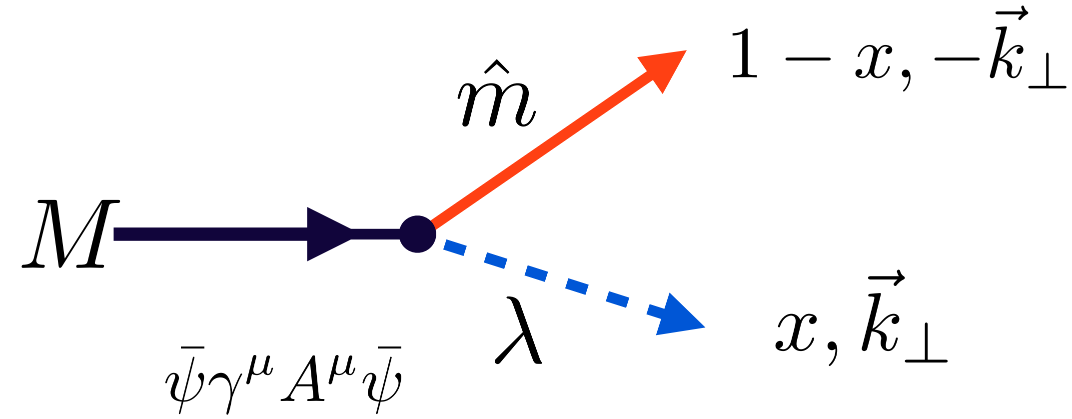
Light-Front Wavefunctions of Lepton in pQED

$$\frac{\bar{u}}{\sqrt{p^+ - k^+}}\ \gamma\cdot\varepsilon^*\ \frac{u}{\sqrt{p^+}} \quad \text{and} \quad \frac{\bar{u}}{\sqrt{p^+}}\ \gamma\cdot\varepsilon\ \frac{u}{\sqrt{p^+ - k^+}}$$

$$\hat{\varepsilon} = \hat{\varepsilon}_{\uparrow(\downarrow)} = \pm\frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y}),\quad \varepsilon\cdot k = 0,$$

$S_z = \pm 1$





$$\psi_{p\downarrow} = \frac{e/\sqrt{x}}{M^2 - \frac{k_\perp^2 + \lambda^2}{x} - \frac{k_\perp^2 + \hat{m}^2}{1-x}} \begin{cases} \sqrt{2}\,\frac{(k_1 - ik_2)}{x} & |{-1/2}\rangle \to |{-1/2},\ 1\rangle \\ \sqrt{2}\,\frac{M(1-x) - \hat{m}}{1-x} & |{-1/2}\rangle \to |1/2,\ {-1}\rangle \end{cases}$$

and

$$\psi^{*}_{p\uparrow} = \frac{e/\sqrt{x}}{M^2 - \frac{k_\perp^2 + \lambda^2}{x} - \frac{k_\perp^2 + \hat{m}^2}{1-x}} \begin{cases} -\sqrt{2}\,\frac{M(1-x) - \hat{m}}{1-x} & |{-1/2},\ 1\rangle \to |1/2\rangle \\ -\sqrt{2}\,\frac{(k_1 - ik_2)}{x} & |1/2,\ {-1}\rangle \to |1/2\rangle \end{cases}$$

$L^z = -1$

$L^z = 0$

$L^z = 0$

$L^z = +1$

Light-Front Wavefunctions of Lepton in pQED

Easily derive all form factors, Schwinger anomalous moment, B(0)=0.

Renormalization using alternate denominators

Special Features of LF Spin

- 'LF Helicity' refers to z direction, not the particle's 3-momentum p
- LF spinors are eigenstates of $S^z = \pm \frac{1}{2}$
- Gluon polarization vectors are eigenstates of $S^z = \pm 1$

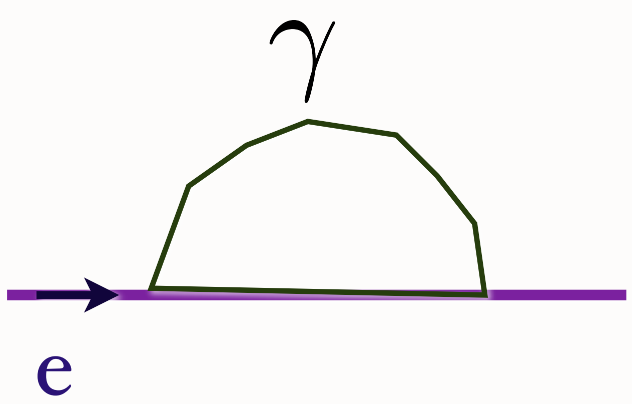
$$\epsilon^\mu = (\epsilon^+, \epsilon^-, \vec{\epsilon}_\perp) = (0, 2\frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k^+}, \vec{\epsilon}_\perp)$$

$$\vec{\epsilon}^{\,\pm}_\perp = \mp \frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y}) \qquad k^\mu \epsilon_\mu = 0 \qquad A^+ = 0$$

Light-cone gauge

Orbital angular momentum of electron carried by photon at LO in QED

$$< L^z >_{\Lambda^2} = -\frac{\alpha}{4\pi}[\frac{4}{3}\log\frac{\Lambda^2}{m^2} - \frac{2}{9}]$$



$$\frac{d}{d\log Q^2} < L^z >_{Q^2} = -\frac{\alpha}{3\pi}$$

Evolution of OAM

Angular Momentum Decomposition for an Electron.
Matthias Burkardt, Hikmat BC (New Mexico State U.) . JLAB-THY-08-920, Dec 2008. 7pp.
e-Print: **arXiv:0812.1605** [hep-ph]

Light cone representation of the spin and orbital angular momentum of relativistic composite systems.
Stanley J. Brodsky (SLAC) , Dae Sung Hwang (Sejong U.) , Bo-Qiang Ma (CCAST World Lab, Beijing & Peking U. & Beijing, Inst. High Energy Phys.) , Ivan Schmidt (Santa Maria U., Valparaiso) . SLAC-PUB-8392, USM-TH-90, Mar 2000. 28pp.
Published in **Nucl.Phys.B593:311-335,2001**.
e-Print: **hep-th/0003082**

G. P. Lepage and sjb

$$\left.\begin{matrix} u_{\uparrow}(p) \\ u_{\downarrow}(p) \end{matrix}\right\} = \frac{1}{(p^+)^{1/2}} (p^+ + \beta m + \alpha_\perp \cdot \mathrm{p}_\perp) \times \left\{\begin{matrix} \chi(\uparrow) \\ \chi(\downarrow) \end{matrix}\right. ,$$

$$\left.\begin{matrix} v_{\uparrow}(p) \\ v_{\downarrow}(p) \end{matrix}\right\} = \frac{1}{(p^+)^{1/2}} (p^+ - \beta m + \vec{\alpha}_\perp \cdot \vec{\mathrm{p}}_\perp) \times \left\{\begin{matrix} \chi(\downarrow) \\ \chi(\uparrow) \end{matrix}\right.$$

$$\chi(\uparrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi(\downarrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix},$$

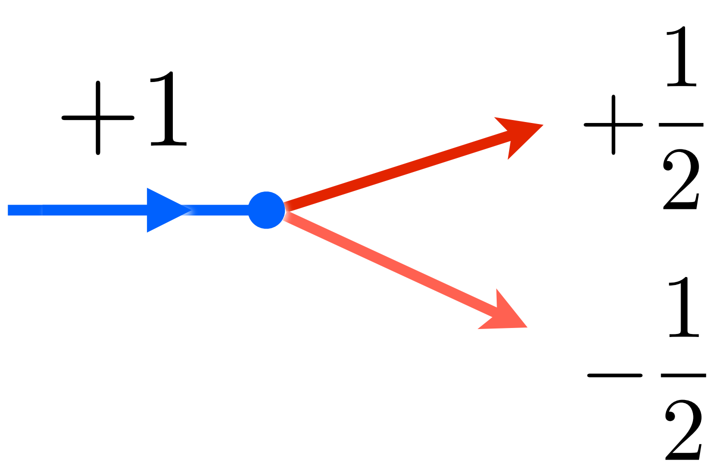
Matrix element $\bar{v}_{\lambda'} \cdots u_\lambda$	Helicity ($\lambda \to \lambda'$) $\uparrow \to \uparrow$, $\downarrow \to \downarrow$	$\uparrow \to \downarrow$, $\downarrow \to \uparrow$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma^+ \frac{u(q)}{(q^+)^{1/2}}$	0	2
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma^- \frac{u(q)}{(q^+)^{1/2}}$	$\mp \frac{2m}{p^+q^+}[(p^1 \pm ip^2) + (q^1 \pm iq^2)]$	$\frac{2}{p^+q^+}(p_\perp \cdot q_\perp \pm ip_\perp \times q_\perp - m^2)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma_\perp^i \frac{u(q)}{(q^+)^{1/2}}$	$\mp m\left(\frac{p^+ + q^+}{p^+q^+}\right)(\delta^{i1} \pm i\delta^{i2})$	$\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+} + \frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \frac{u(q)}{(q^+)^{1/2}}$	$\mp\left(\frac{p^1 \pm ip^2}{p^+} - \frac{q^1 \pm iq^2}{q^+}\right)$	$m\left(\frac{p^+ - q^+}{p^+q^+}\right)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma^-\gamma^+\gamma^- \frac{u(q)}{(q^+)^{1/2}}$	$\mp \frac{8m}{p^+q^+}[(p^1 \pm ip^2) + (q^1 \pm iq^2)]$	$\frac{8}{p^+q^+}(p_\perp \cdot q_\perp \pm ip_\perp \times q_\perp - m^2)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma^-\gamma^+\gamma_\perp^i \frac{u(q)}{(q^+)^{1/2}}$	$\mp \frac{4m}{p^+}(\delta^{i1} \pm i\delta^{i2})$	$4\left(\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+}\right)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma_\perp^i\gamma^+\gamma^- \frac{u(q)}{(q^+)^{1/2}}$	$\mp \frac{4m}{q^+}(\delta^{i1} \pm i\delta^{i2})$	$4\left(\frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}\right)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma_\perp^i\gamma^+\gamma_\perp^j \frac{u(q)}{(q^+)^{1/2}}$	0	$2(\delta^{ij} \pm i\epsilon^{ij})$

G. P. Lepage and sjb

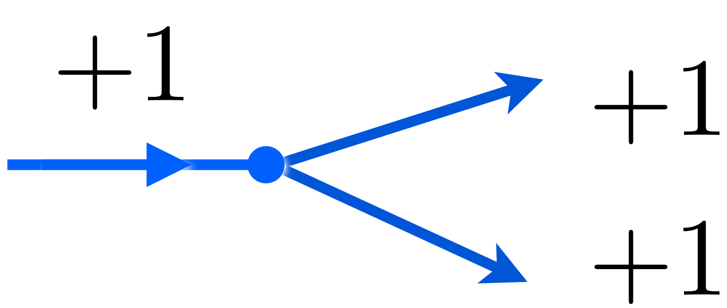
Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} S_i^z + \sum_{i=1}^{n-1} L_i^z \qquad L_j^z = -i\left(k_j^x \frac{\partial}{\partial k_j^y} - k_j^y \frac{\partial}{\partial k_j^x}\right)$$

chiral conserving decay of spin 1



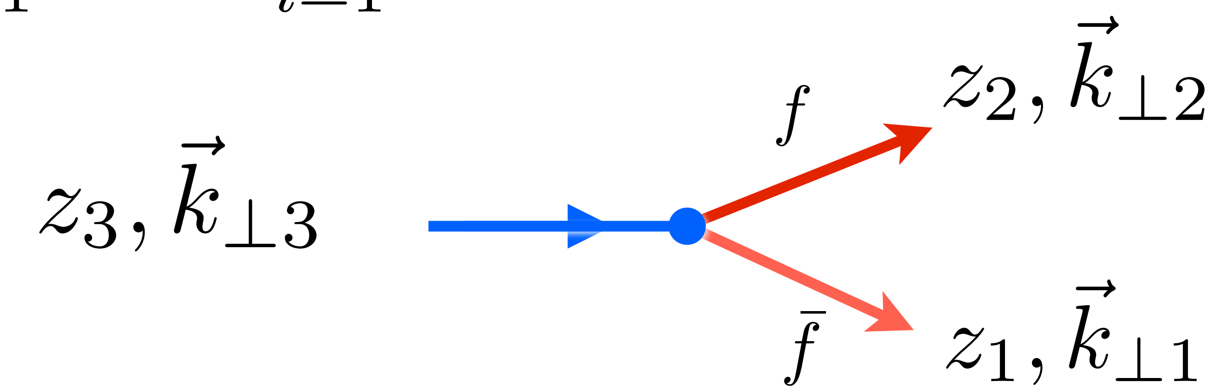
$$L^z = +1 \qquad \vec{\epsilon}_\perp^{\,(+)} \cdot \frac{\vec{k}_{\perp 2}}{z_2} - \vec{\epsilon}_\perp^{\,(-)} \cdot \frac{\vec{k}_{\perp 3}}{z_3},$$



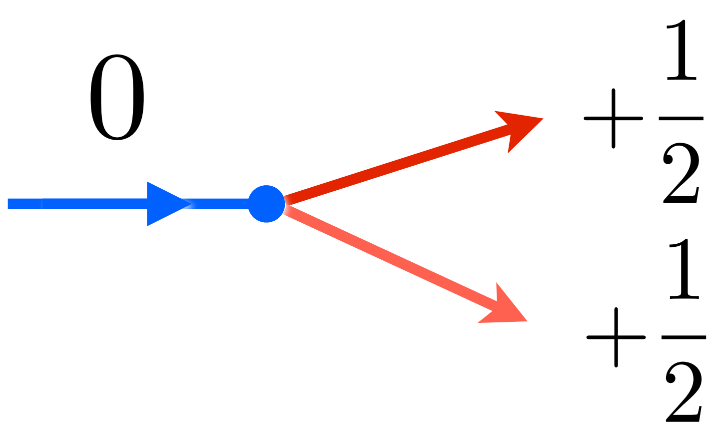
$$L^z = -1 \qquad \vec{\epsilon}_\perp^{\,(-)} \cdot \frac{\vec{k}_{\perp 2}}{z_2} - \vec{\epsilon}_\perp^{\,(+)} \cdot \frac{\vec{k}_{\perp 3}}{z_3}$$

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} S_i^z + \sum_{i=1}^{n-1} L_i^z \qquad L_j^z = -i\left(k_j^x \frac{\partial}{\partial k_j^y} - k_j^y \frac{\partial}{\partial k_j^x}\right)$$

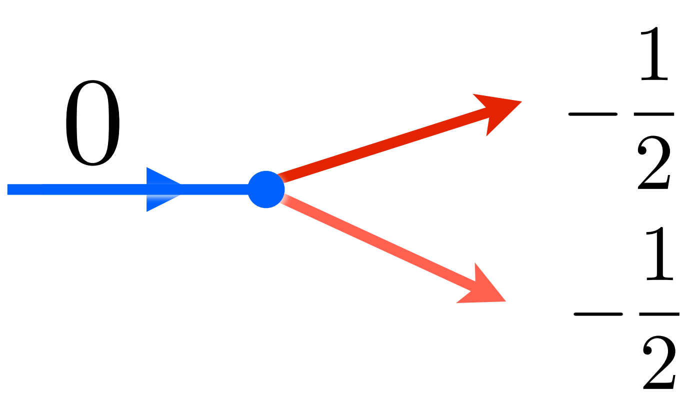


Spin-0 coupling to fermion pair



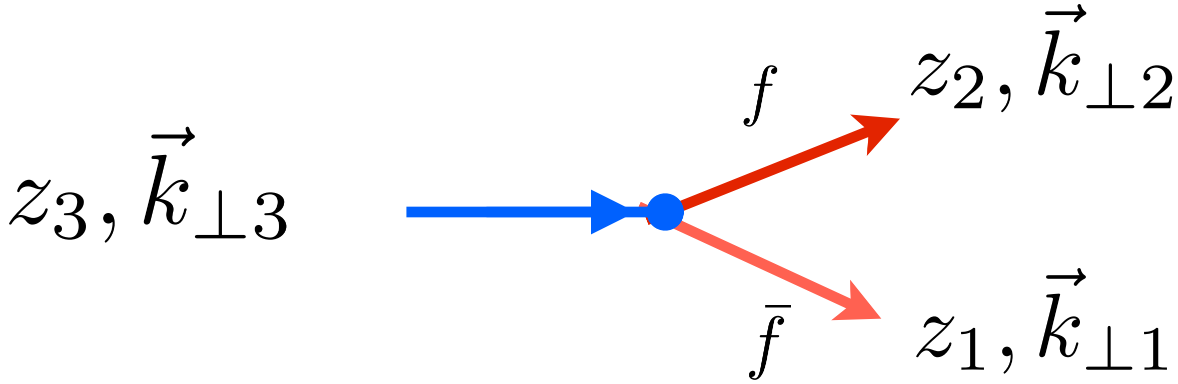
$$L^z = -1 \qquad < ij >= -\sqrt{2 z_i z_j} \vec{\epsilon}_\perp^{\,+} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

spinor overlap

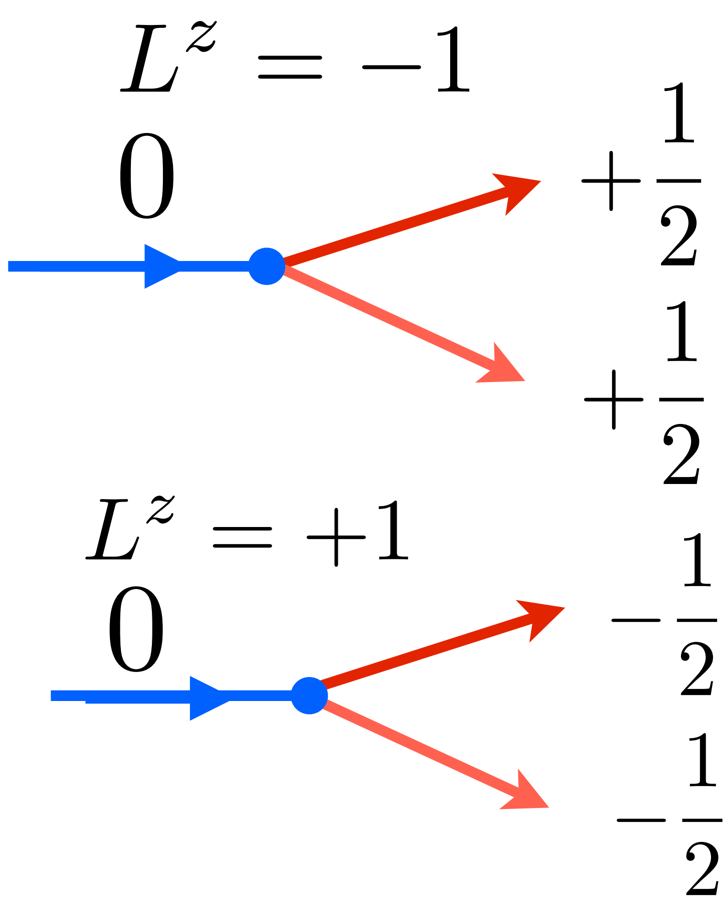


$$L^z = +1 \qquad [ij] = \sqrt{2 z_i z_j} \vec{\epsilon}^{\,(-)} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

Angular Momentum on the Light-Front



P-Wave Decay
Spin-0 coupling
to fermion pair



spinor overlap

$$< ij > = < i - | j + > = -\sqrt{2 z_i z_j} \vec{\epsilon}_\perp^{\,+} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

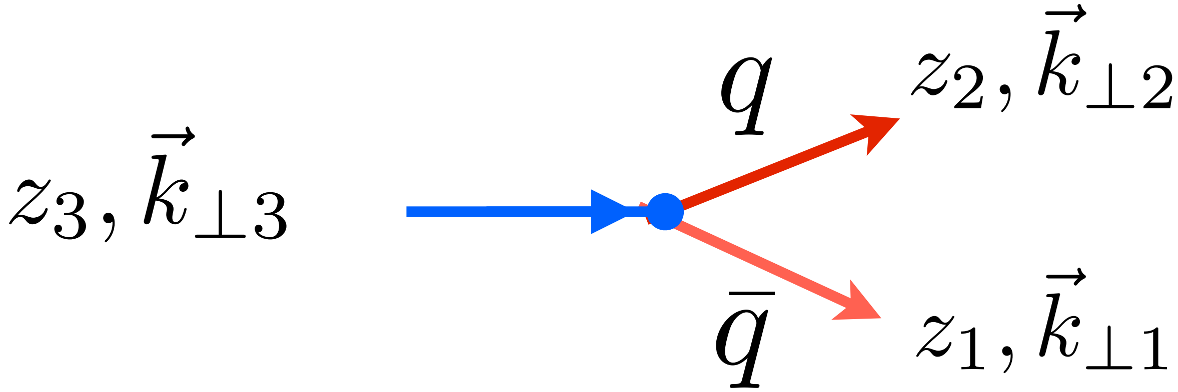
$$[ij] = < i^+ | j - > = \sqrt{2 z_i z_j} \vec{\epsilon}^{\,(-)} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

$$< ij > [ij] = z_i z_j \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)^2 = \mathcal{M}^2_{ij}$$

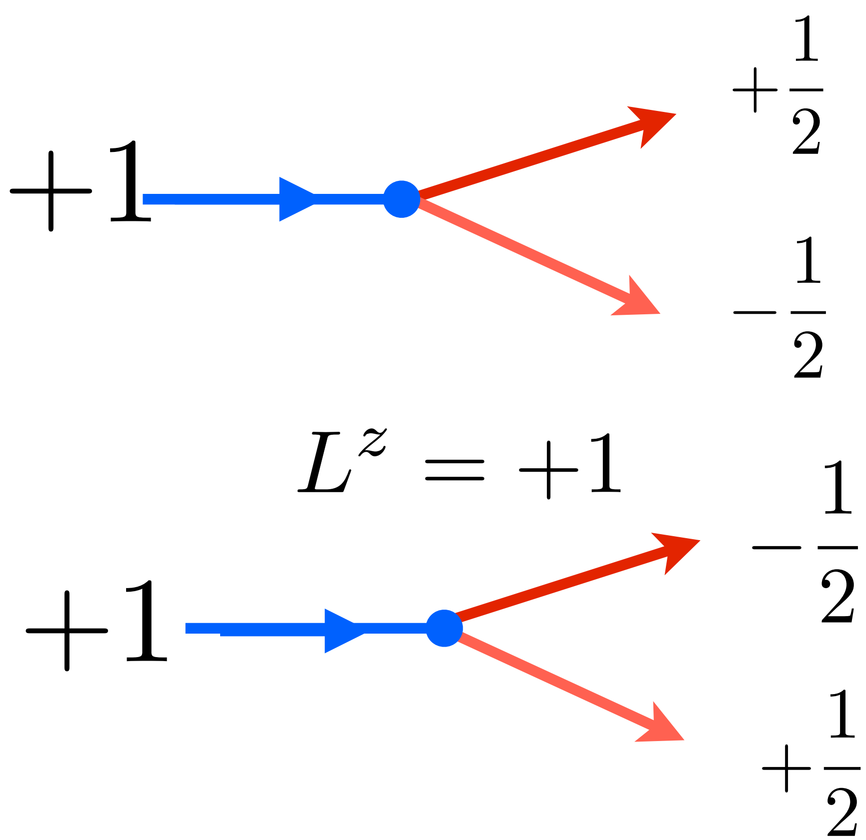
Identity

Connection to Penrose-Witten twistors?

Angular Momentum on the Light-Front



Spin-1 coupling
to massless fermion pair



$$\vec{\epsilon}_\perp^{(+)} \cdot \frac{\vec{k}_{\perp 2}}{z_2} - \vec{\epsilon}_\perp^{(-)} \cdot \frac{\vec{k}_{\perp 3}}{z_3},$$

P-Wave Decay

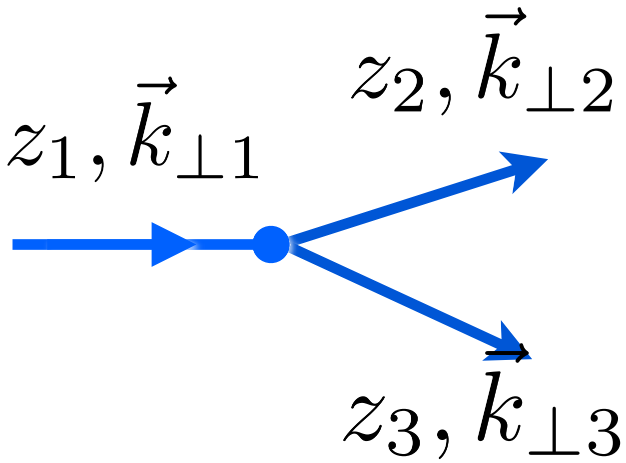
$$\vec{\epsilon}_\perp^{(-)} \cdot \frac{\vec{k}_{\perp 2}}{z_2} - \vec{\epsilon}_\perp^{(+)} \cdot \frac{\vec{k}_{\perp 3}}{z_3}$$

Compare CM distribution $1 + \cos^2\theta_{CM}$

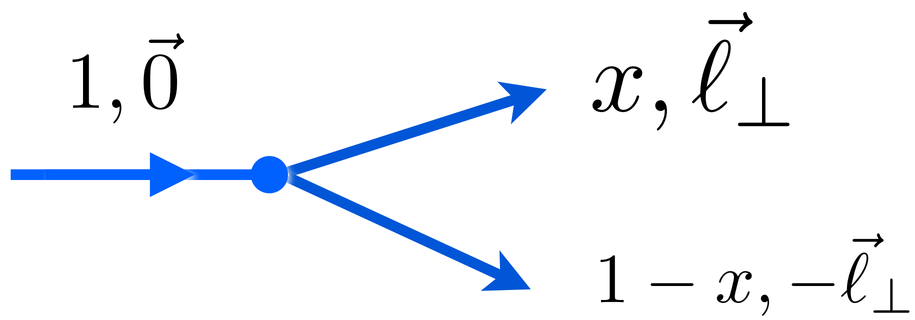
Mimics S and D-Wave Decay

Angular Momentum on the Light-Front

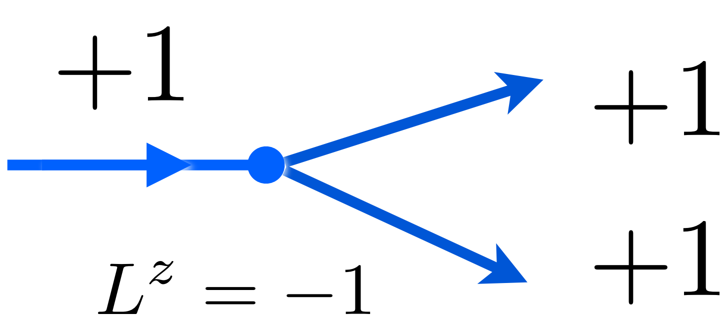
Triple-Gluon Coupling



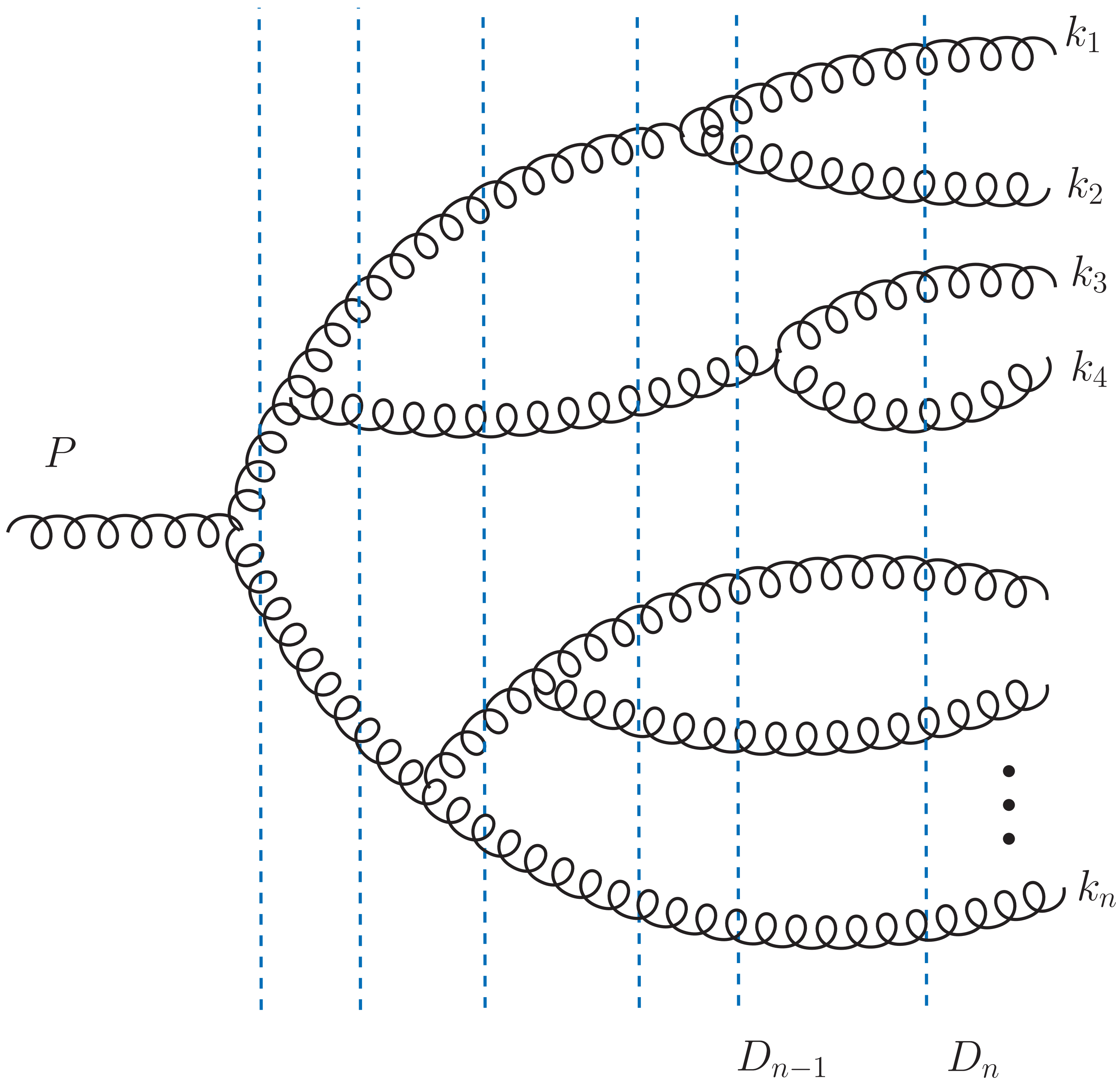
$$g z_1 \vec{\epsilon}^{\,+}_{\perp} \cdot \vec{v}_{23} = g z_1 \vec{\epsilon}^{\,+}_{\perp} \cdot \left(\frac{\vec{k}_{\perp 2}}{z_2} - \frac{\vec{k}_{\perp 3}}{z_3}\right)$$



$$g z_1 \vec{\epsilon}^{\,+}_{\perp} \cdot \vec{v}_{23} = g \vec{\epsilon}^{\,+}_{\perp} \cdot \frac{\vec{\ell}_\perp}{x(1-x)}$$



$$< ij > = -\sqrt{2 z_i z_j}\, \vec{\epsilon}^{\,+}_{\perp} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

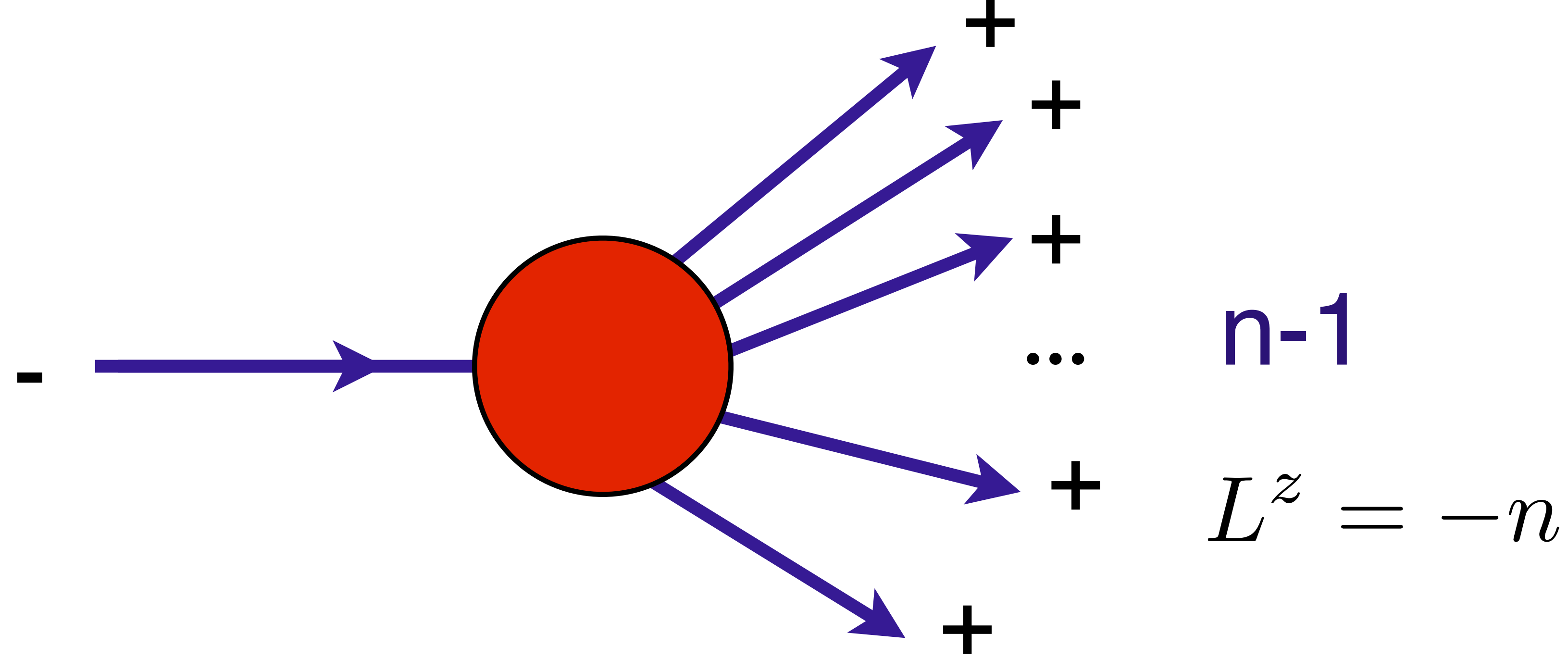


Exact kinematics in the small x evolution of the color dipole and gluon cascade.
Leszek Motyka (Hamburg U. & Jagiellonian U.) , Anna M. Stasto (Penn State U. & RIKEN BNL & Cracow, INP) . Jan 2009. 37pp.
e-Print: **arXiv:0901.4949** [hep-ph]

G. de Teramond and sjb

$$M(-1 \to +1 + 1 \cdots + 1) \propto g^{n-2} = 0$$

$$J^z = -1 = \sum_{i=1}^{n} S_i^z + L^z = (n-1) + L^z$$



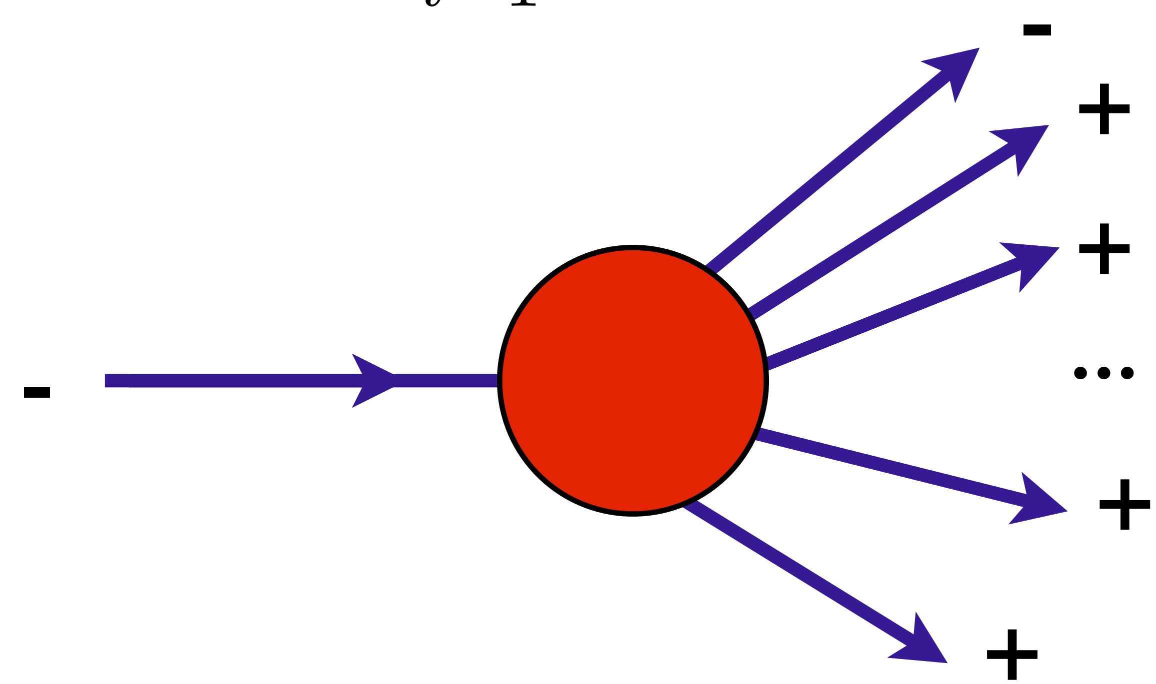
Vanishes Because Maximum $|L^z| = n - 2$

Renormalizability

G. de Teramond and sjb

$$M(-1 \to -1 + 1 + 1 + 1 \cdots + 1) \propto g^{n-2} = 0$$

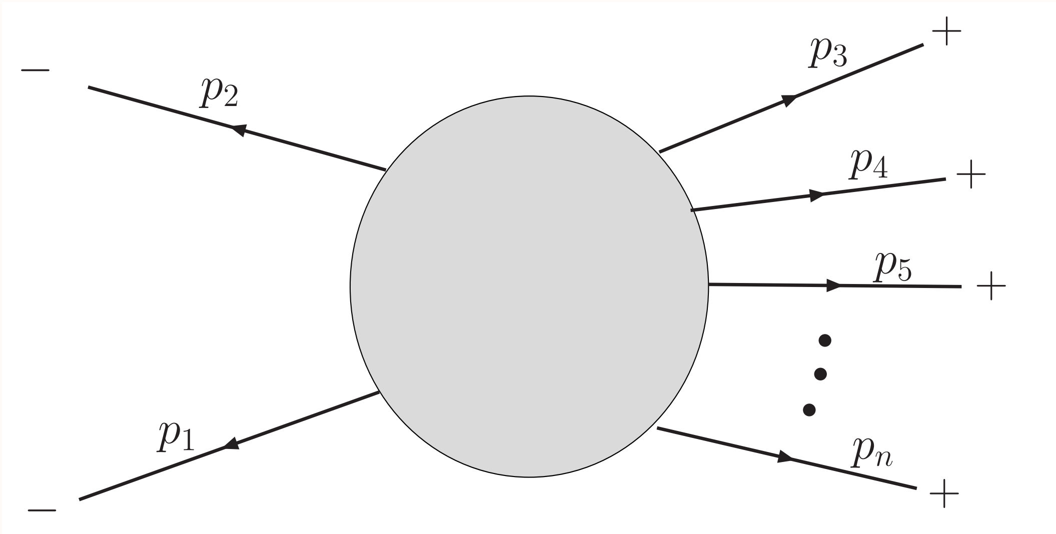
$$J^z = -1 = \sum_{i=1}^{n} S_i^z + L^z = (n-2) + L^z$$

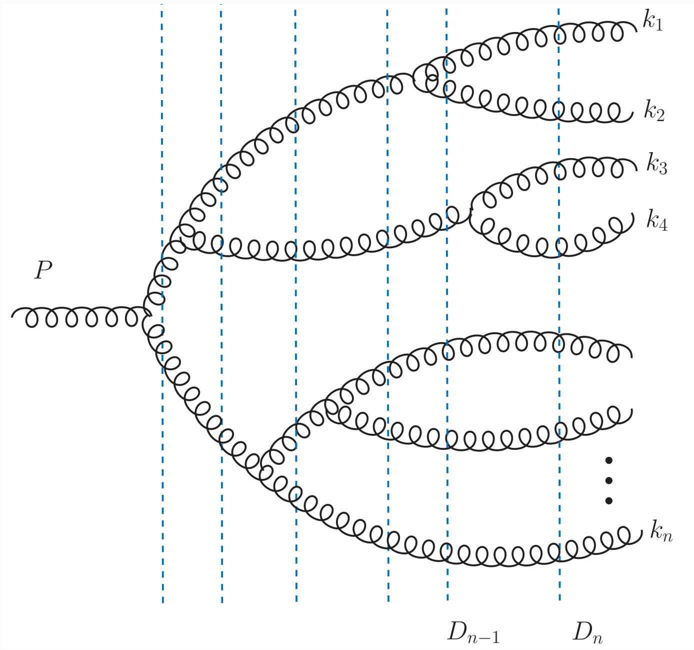


Vanishes Because Maximum $|L^z| = n - 2$

Light Front Analog of MHV rules

LF Proof of Parke-Taylor



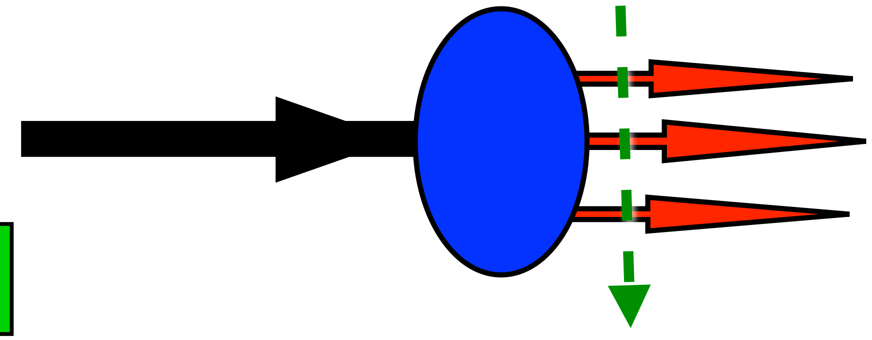


$$m(1^-,2^-,3^+,\ldots,n^+) = ig^{n-2}\,\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n-2\;n-1\rangle\langle n-1\;n\rangle\langle n1\rangle}\,,$$

$$m(\pm,\pm,\ldots,\pm) = m(\mp,\pm,\pm,\ldots,\pm) = 0\,.$$

Exact kinematics in the small x evolution of the color dipole and gluon cascade.
Leszek Motyka (Hamburg U. & Jagiellonian U.) , Anna M. Stasto (Penn State U. & RIKEN BNL & Cracow, INP) . Jan 2009. 37pp.
e-Print: **arXiv:0901.4949** [hep-ph]

Frame-Independent Hadron Eigenfunctions of the QCD Hamiltonian



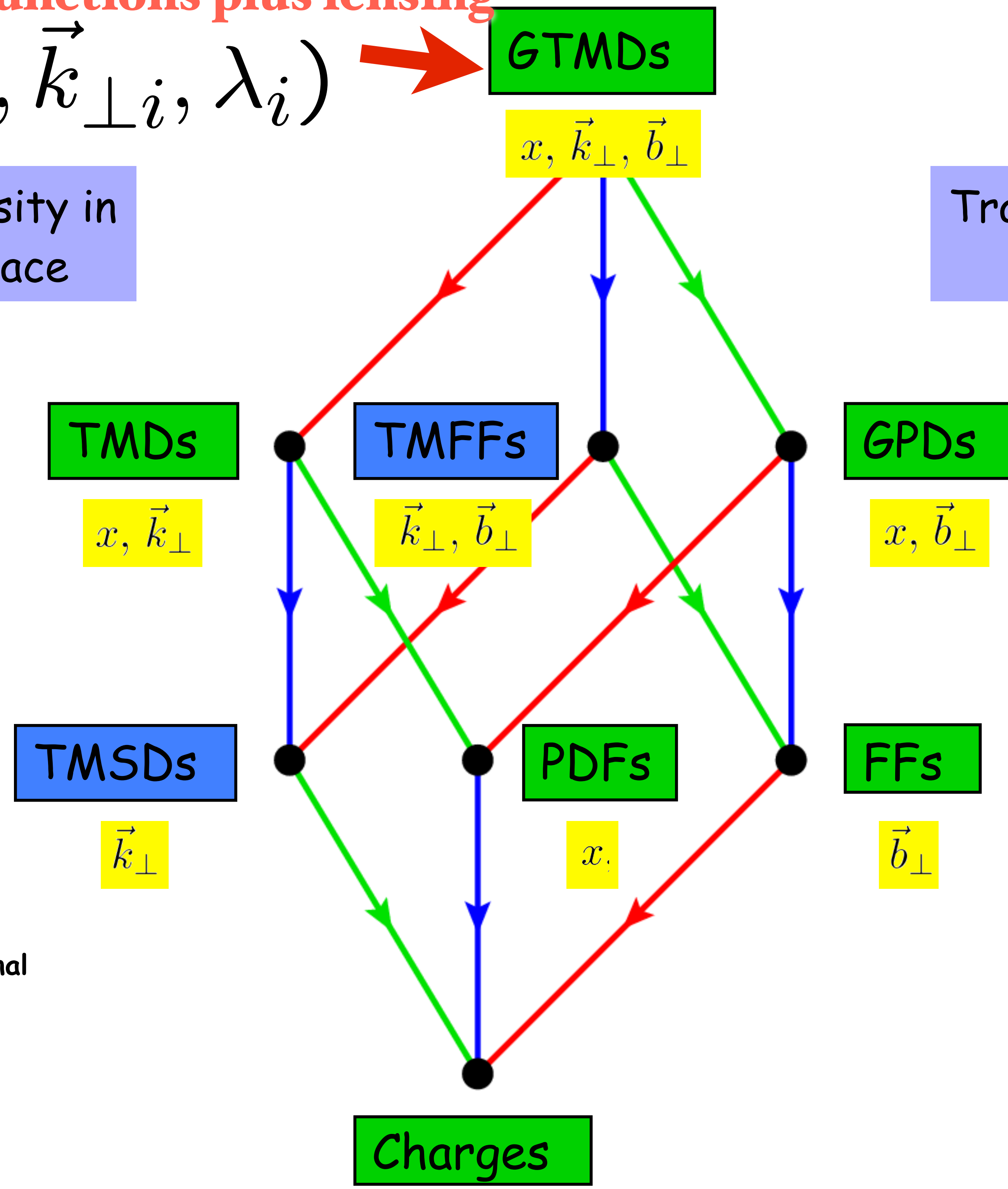
Light-Front Wavefunctions plus lensing

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

Transverse density in position space

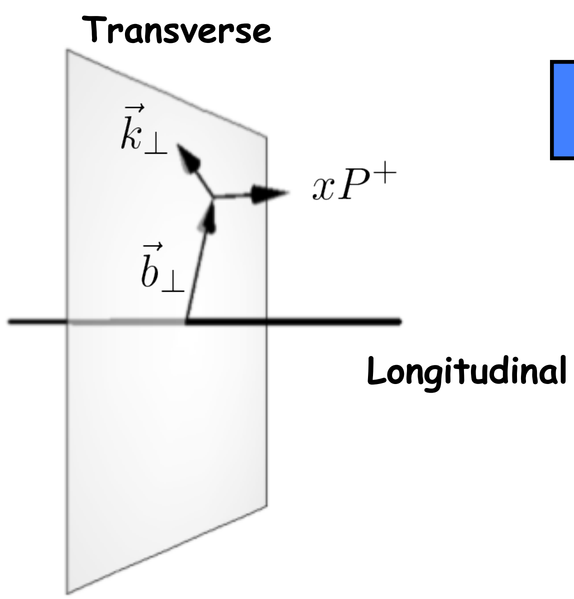
Lorce



$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$

$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

$\xi = 0$



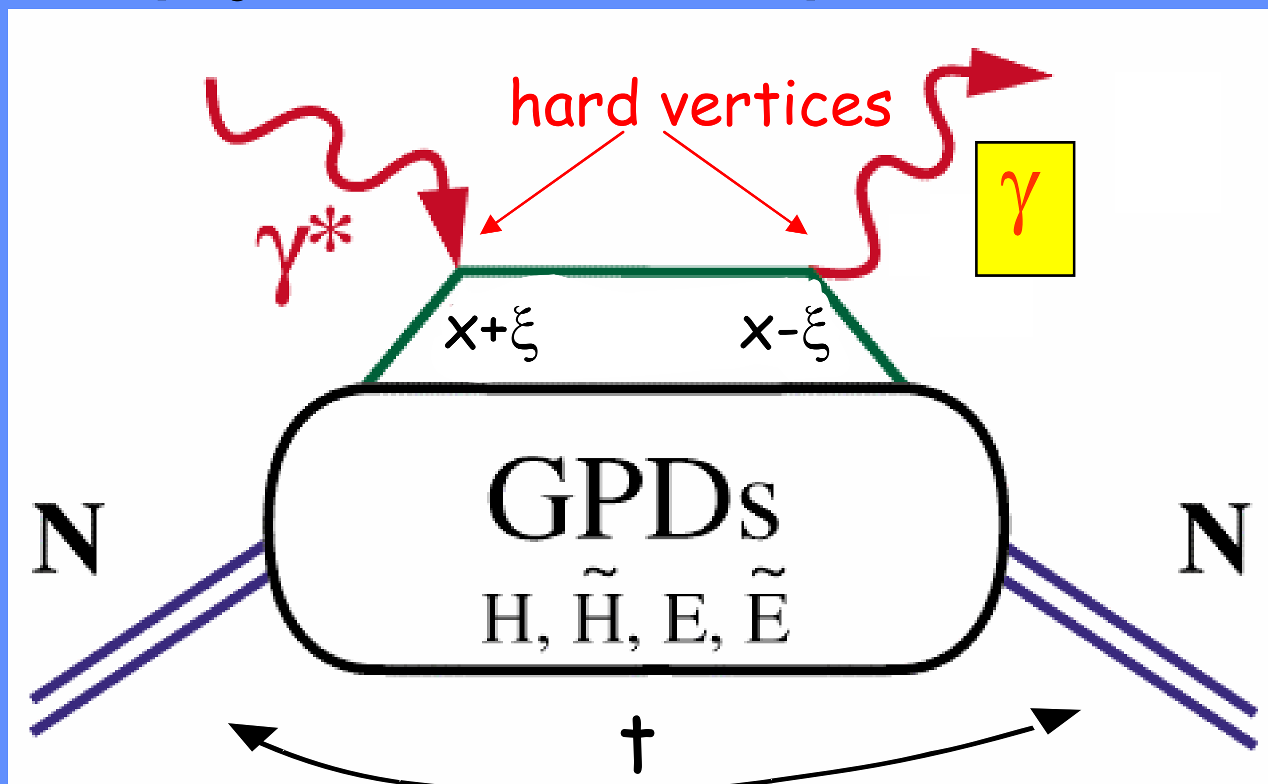
$\int d^2 b_\perp$ (red)

$\int dx$ (blue)

$\int d^2 k_\perp$ (green)

GPDs & Deeply Virtual Exclusive Processes
- New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)

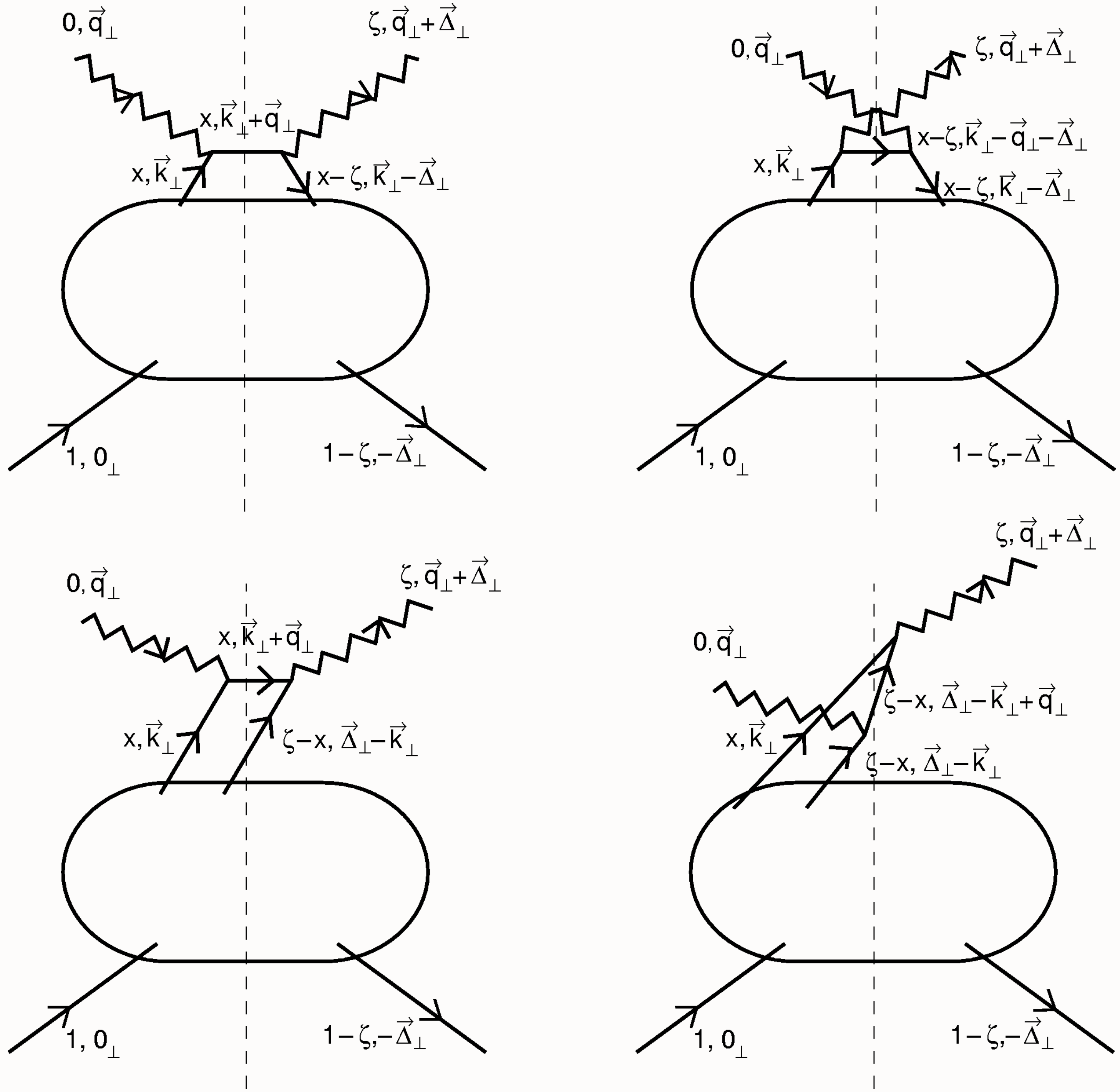


x – quark momentum fraction

ξ– longitudinal momentum transfer

$\sqrt{-t}$ – Fourier conjugate to transverse impact parameter

H(x,ξ,t), E(x,ξ,t), . . . "Generalized Parton Distributions"



Light-cone wavefunction representation of deeply virtual Compton scattering ☆

Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

Example of LFWF representation of GPDs (n => n)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\,\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x-x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

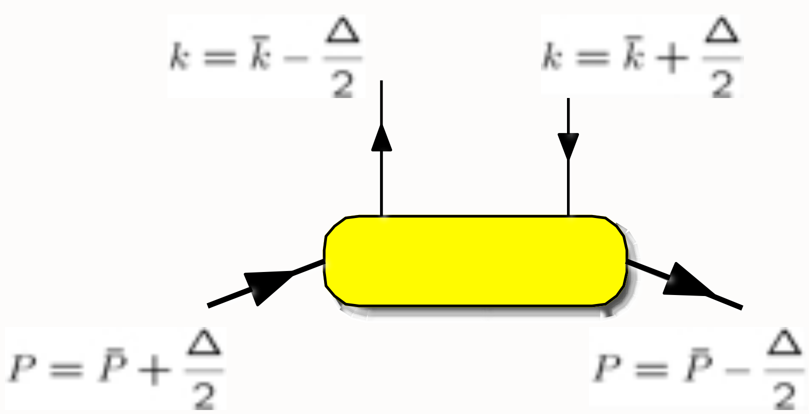
$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

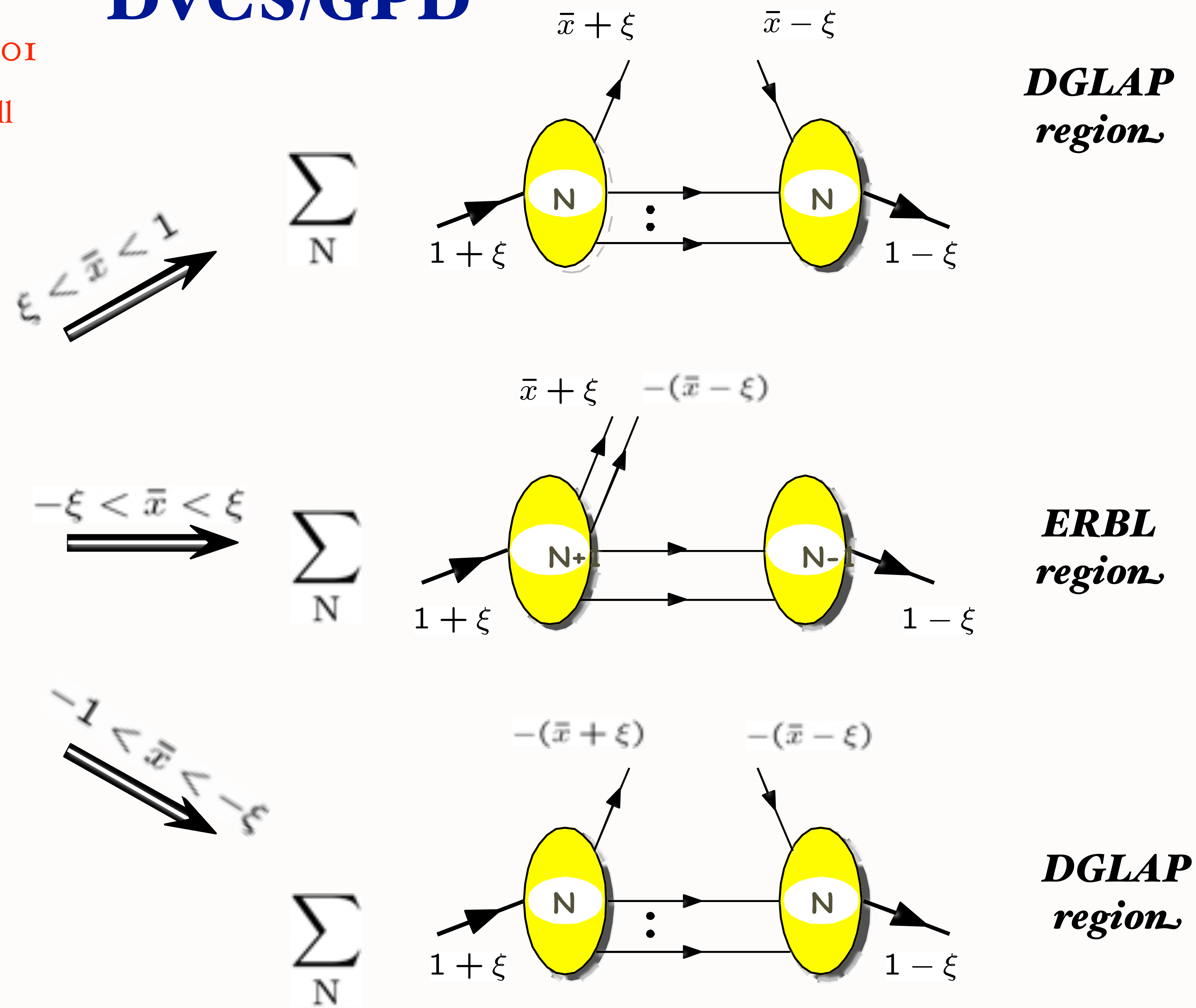
Light-Front Wave Function Overlap Representation

DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll





Bakker & JI
Lorce

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\tilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$

$$\int^1 dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

Verified using LFWFs

Diehl, Hwang, sjb

$$H^q, E^q, \tilde{H}^q, \tilde{E}^q(x,\xi,t)$$

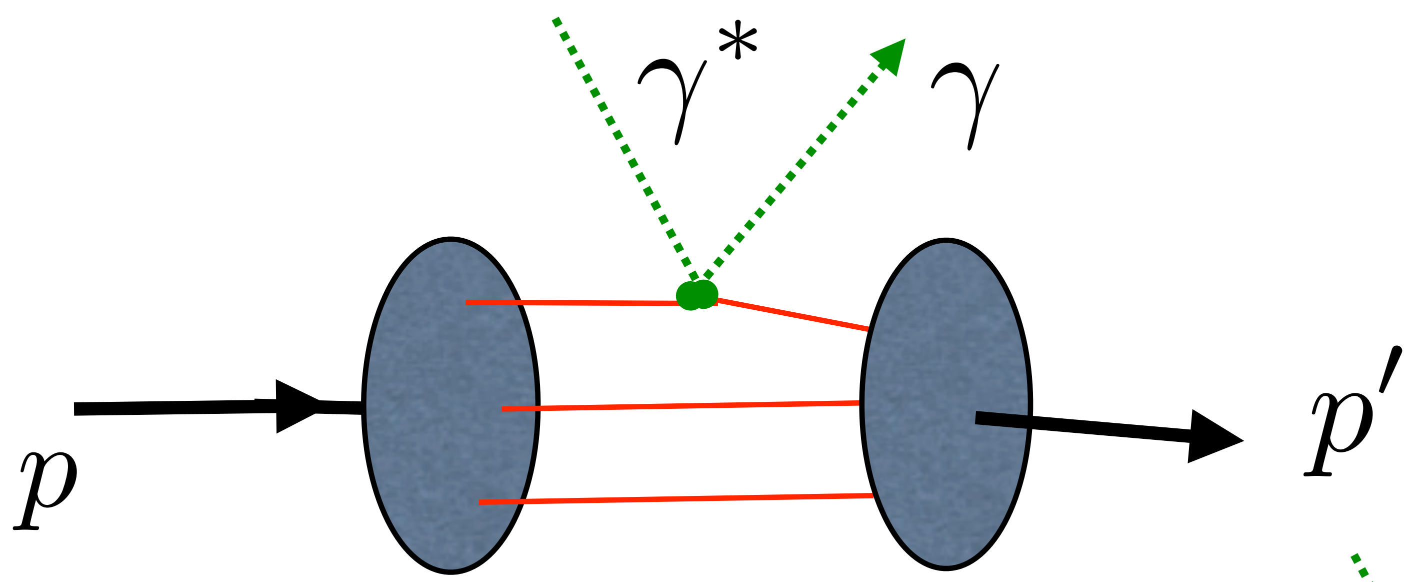
Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

J=0 Fixed Pole Contribution to DVCS

- **J=0 fixed pole -- direct test of QCD locality -- from seagull or instantaneous contribution to Feynman propagator**

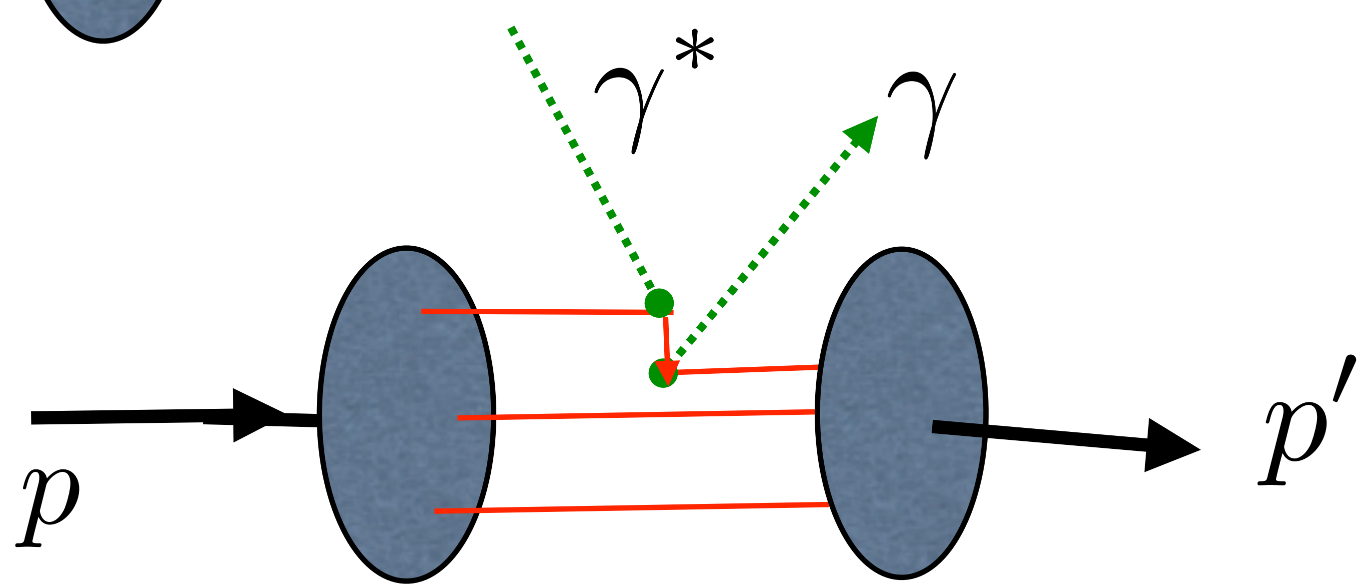


Szczepaniak, Llanes-Estrada, sjb

Close, Gunion, sjb

Important for muonic hydrogen!

Dirac-Feynman propagator invalid for composite spin-1/2 state!



Real amplitude, independent of Q^2 at fixed t

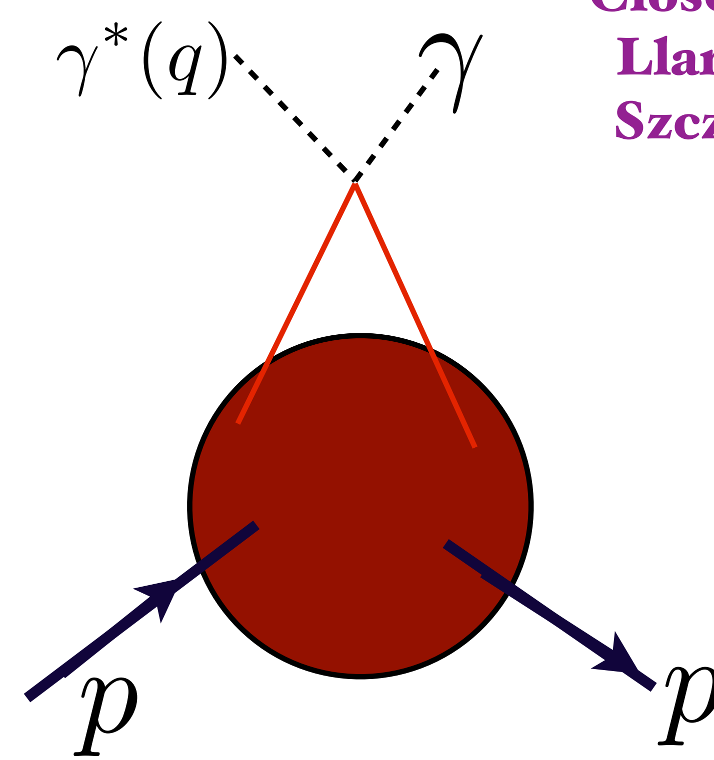
J=0 Fixed pole in real and virtual Compton scattering

Damashek, Gilman;
Close, Gunion, sjb
Llanes-Estrada,
Szczepaniak, sjb

- Effective two-photon contact term
- Seagull for scalar quarks
- Real phase

$$M = s^0 \sum e_q^2 F_q(t)$$

- Independent of Q^2 at fixed t
- <1/x> Moment: Related to Feynman-Hellman Theorem
- Fundamental test of local gauge theory



No ambiguity in D-term

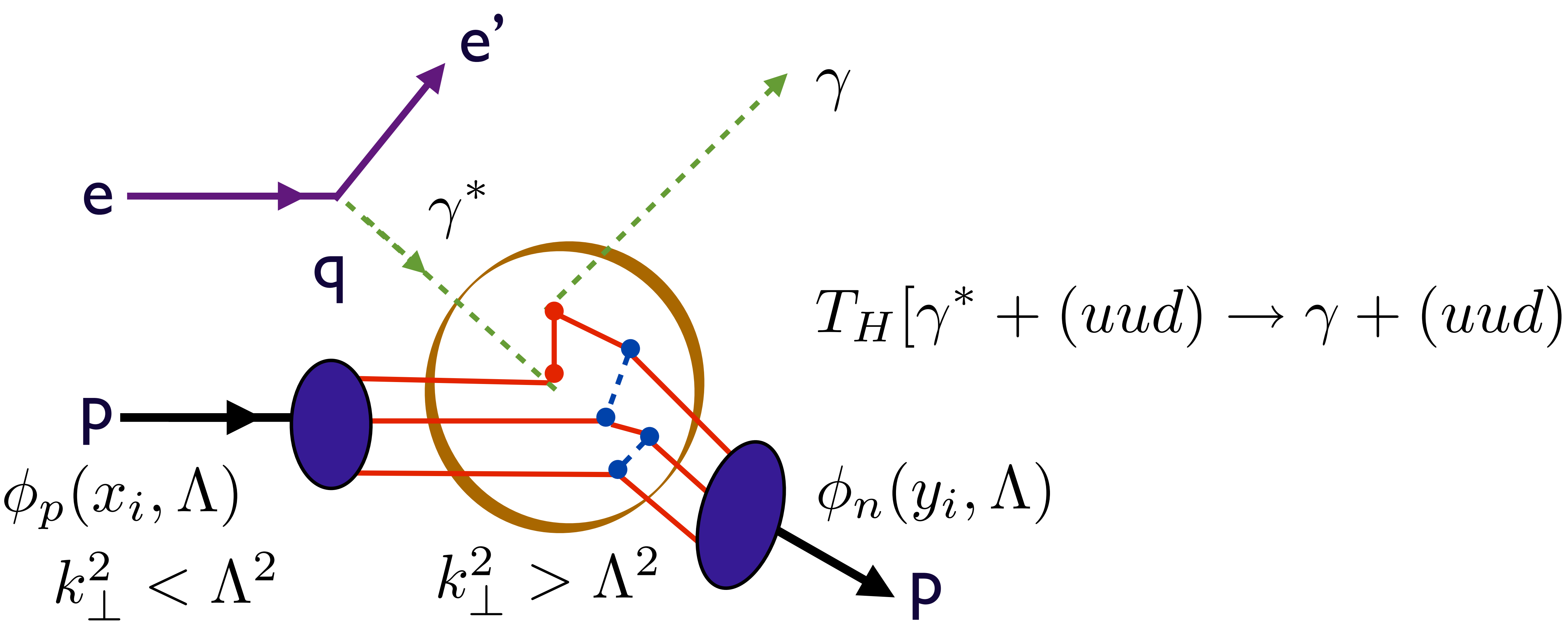
Q^2-independent contribution to Real DVCS amplitude at fixed t

$$s^2 \frac{d\sigma}{dt}(\gamma^* p \to \gamma p) = F^2(t)$$

QCD Factorization
DVCS in hard-scattering domain

Lepage, sjb

$$ep \to e'\gamma p$$



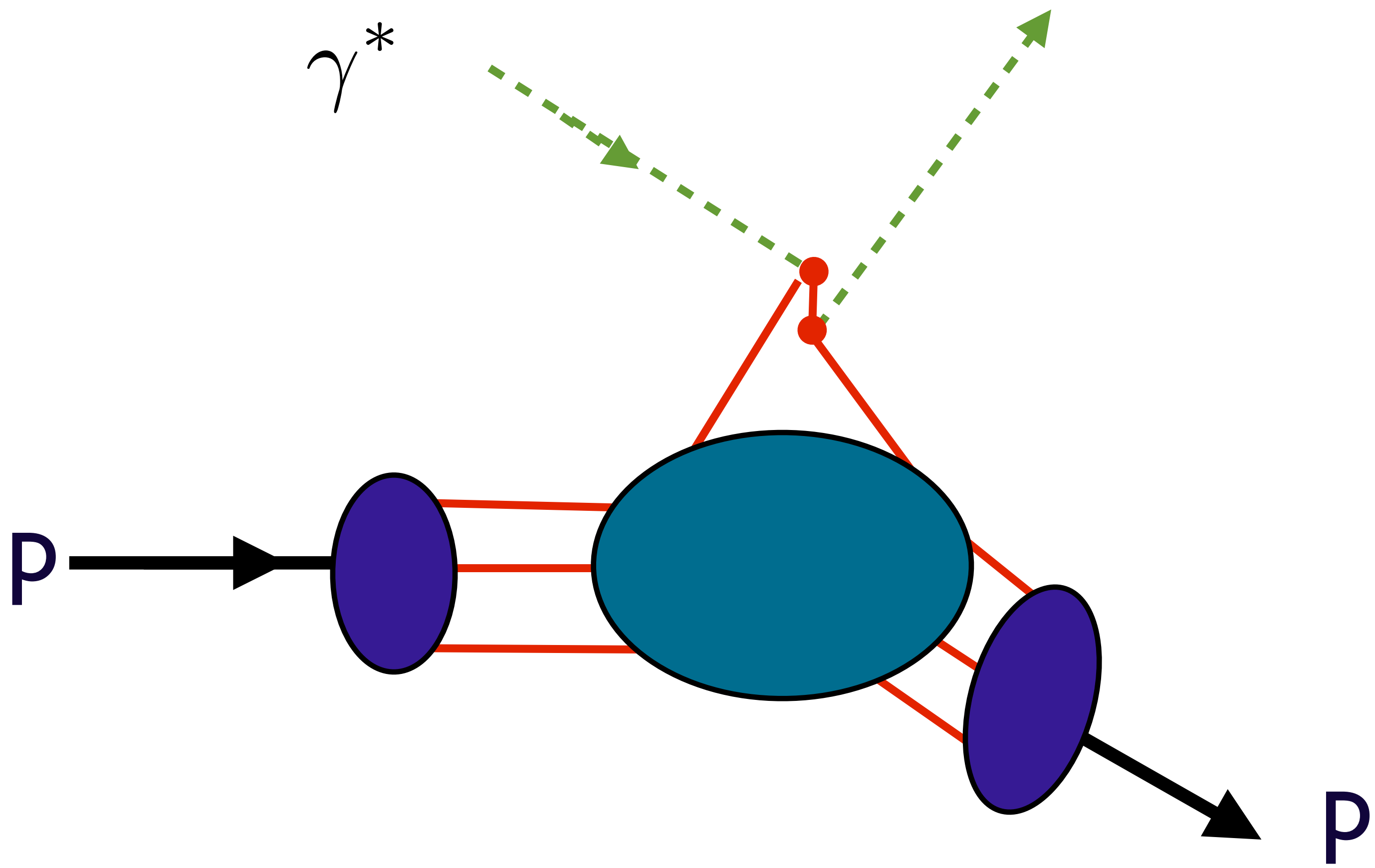
$$T = \int_0^1 dx \int_0^1 dy \int_0^1 dx \; \phi_p(x,\Lambda) T_H(x,y,z; Q^2, s, t; \Lambda) \phi_n(y,\Lambda) \phi_\pi^+(z,\Lambda)$$

Universal distribution amplitudes.
Renormalization Group Invariance:
Renormalization scale is unambiguous -- BLM

J=0 Fixed pole from instantaneous quark

Deeply Virtual Compton Scattering

$$\gamma^* p \to \gamma p$$



Seagull interaction (instantaneous quark exchange or Z-graph)

$$s >> -t, Q^2 >> \Lambda^2_{QCD}$$

Hard Reggeon Domain

$$T(\gamma^*(q)p \to \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s^{\alpha}_R(t)\beta_R(t)$$

$$\alpha_R(t) \to 0$$

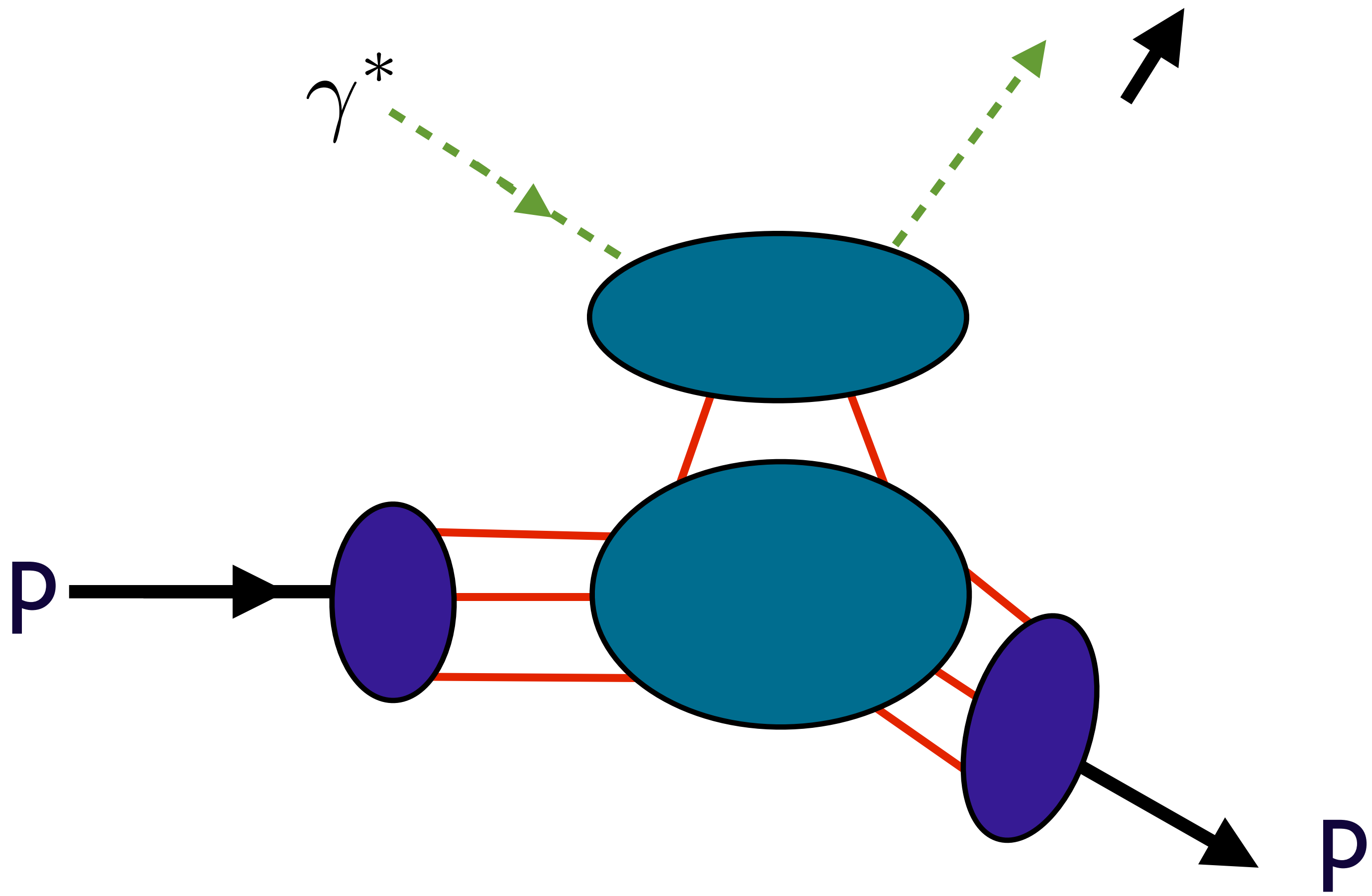
Reflects elementary coupling of two photons to quarks

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}\frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

Deeply Virtual Compton Scattering

$$\gamma^* p \to \gamma p$$



Hard Reggeon Domain

$$s >> -t, Q^2 >> \Lambda^2_{QCD}$$

$$T(\gamma^*(q)p \to \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s_R^\alpha(t) \beta_R(t)$$

$$\alpha_R(t) \to 0$$

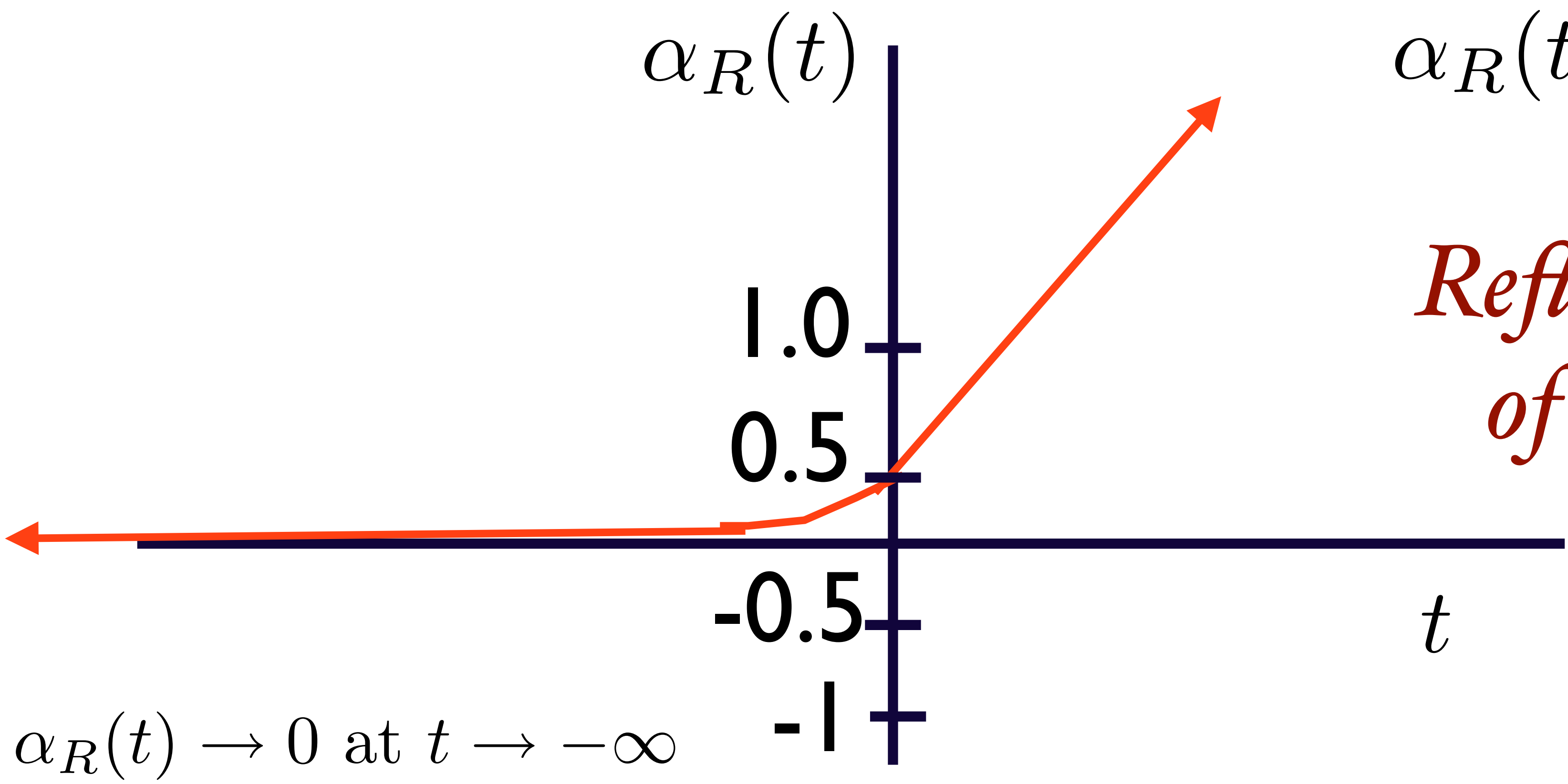
Reflects elementary coupling of two photons to quarks

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}\frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

Regge domain

$$T(\gamma^* p \to \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha}(t) \beta_R(t) \qquad s >> -t, Q^2$$



$\alpha_R(t) \to 0$ at $t \to -\infty$

J=0 fixed pole

Reflects elementary coupling of two photons to quarks

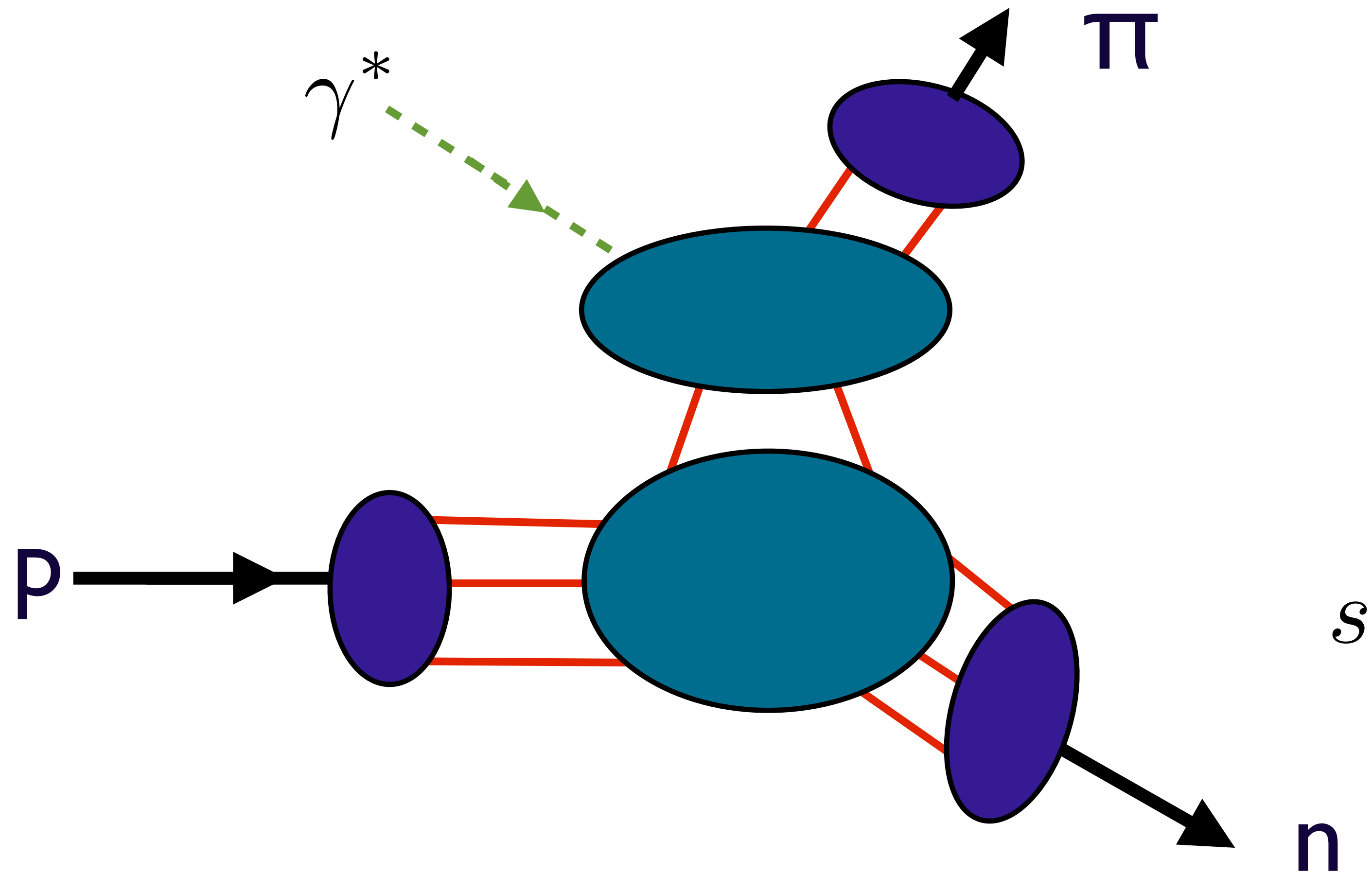
$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt}(\gamma^* p \to \gamma p) \to \frac{1}{s^2}\beta_R^2(t) \sim \frac{1}{s^2 t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{t}{s}, \frac{Q^2}{s}$$

Fundamental test of QCD

Exclusive Electroproduction

$$ep \rightarrow e'\pi^+ n$$



Hard Reggeon Domain

$$s >> -t, Q^2 >> \Lambda^2_{QCD}$$

$$T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha}(t)\beta_R(t)$$

$$\alpha_R(t) \rightarrow -1$$

Reflects elementary exchange of quarks in t-channel

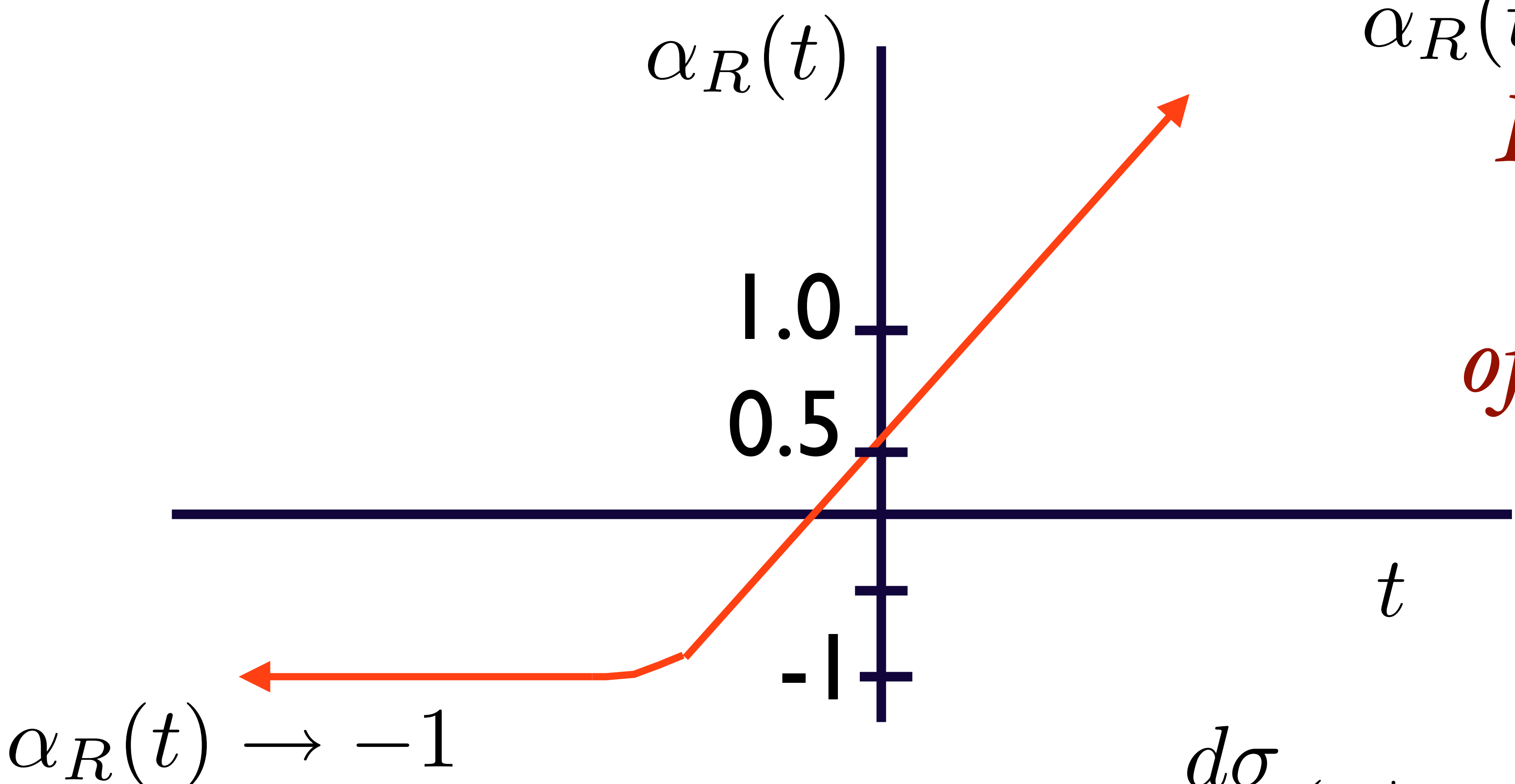
$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

Gunion, Blankenbecler, Savit, sjb

Regge domain

$$T(\gamma^* p \to \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha}(t) \beta_R(t) \qquad s >> -t, Q^2$$



$\alpha_R(t) \to -1$ at $t \to -\infty$

Reflects elementary exchange of quarks in t-channel

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt}(\gamma^* p \to \pi^+ n) \to \frac{1}{s^3} \beta_R^2(t)$$

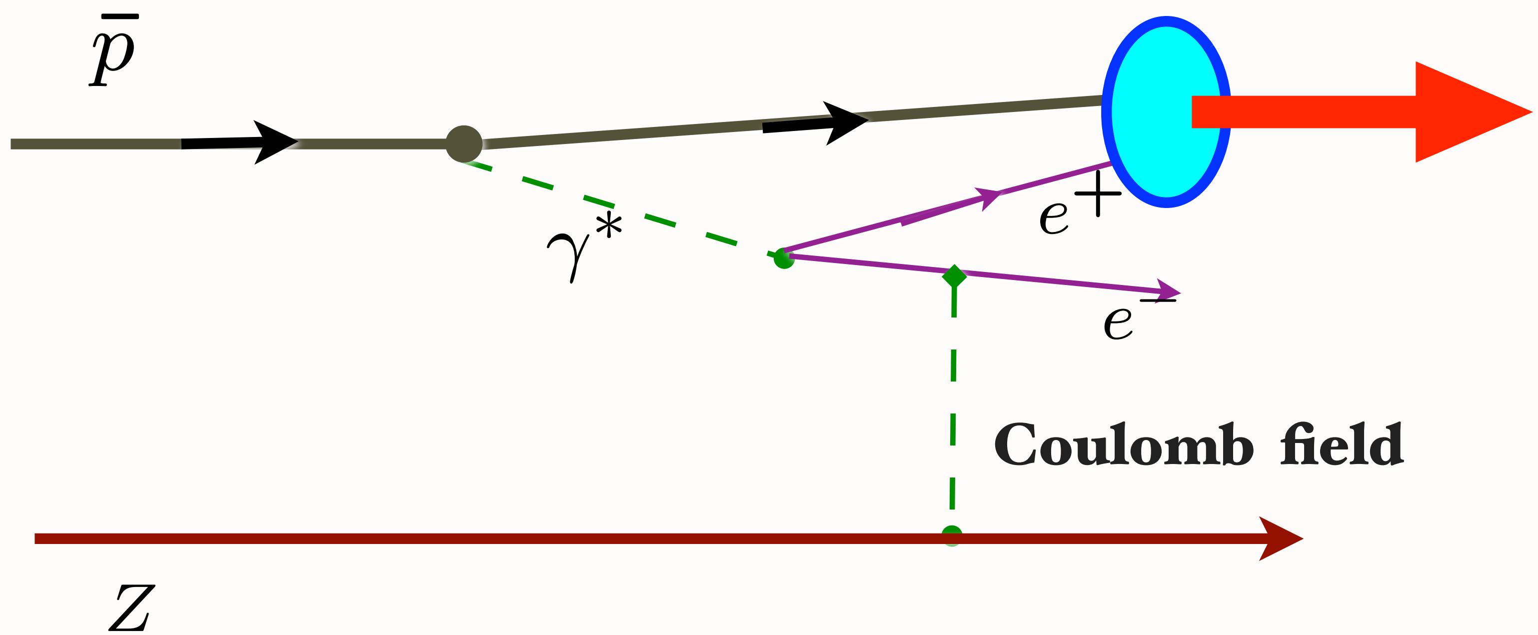
$$\frac{d\sigma}{dt} \sim \frac{1}{s^3} \frac{1}{t^4} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

Fundamental test of QCD

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$\bar{H}(\bar{p}e^+)$

$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

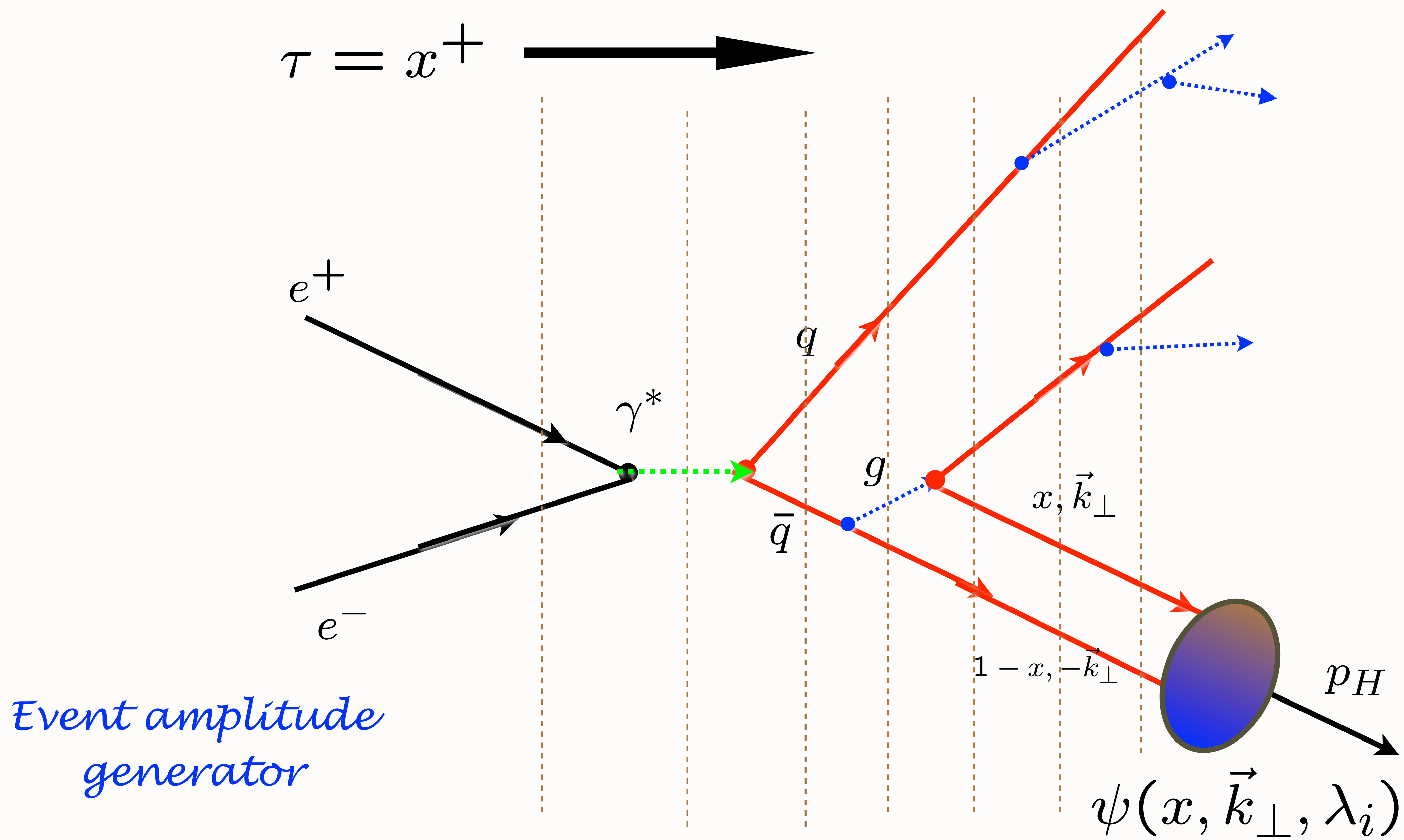
$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of** off-shell **co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

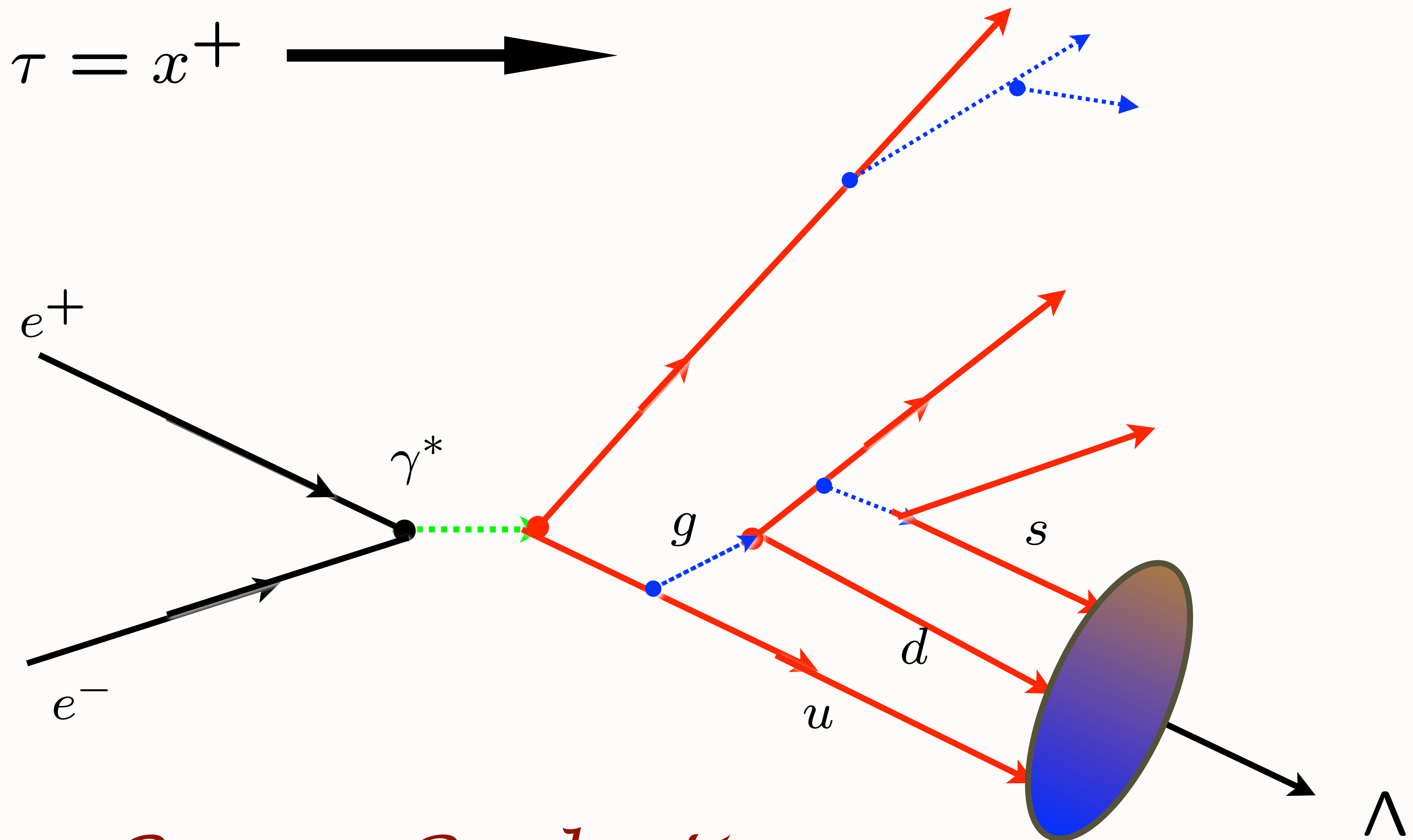
"Hadronization" at the Amplitude Level

Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Baryon Production

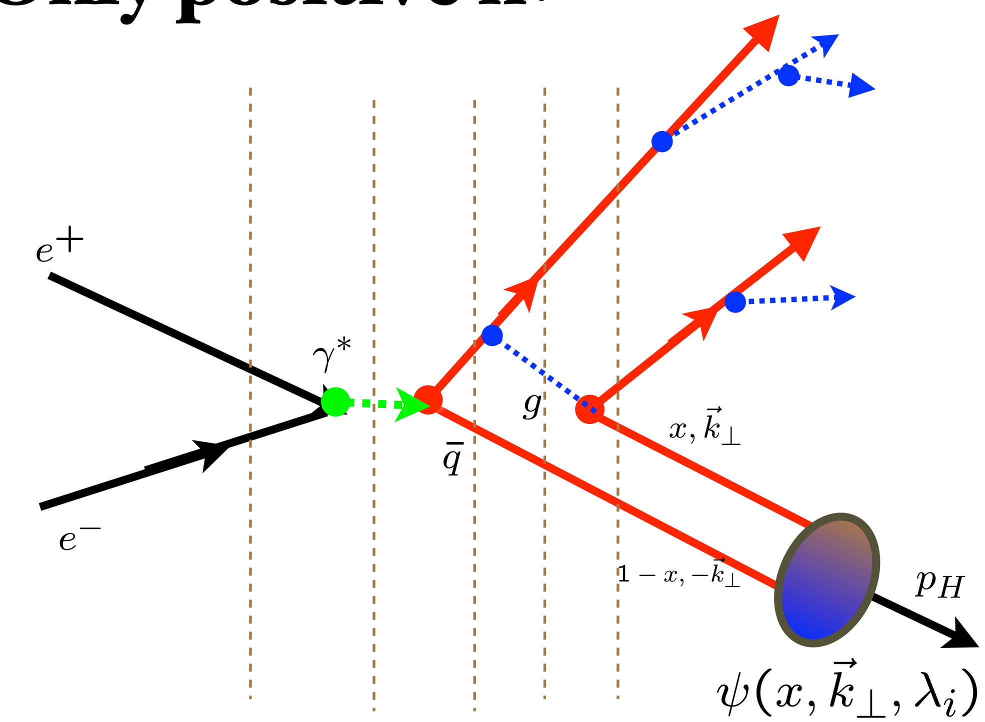
$\psi(x, \vec{k}_\perp, \lambda_i)$

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Off -Shell T-Matrix

Event amplitude generator

- **Quarks and Gluons Off-Shell**
- **LFPth: Minimal Time-Ordering Diagrams-Only positive k+**
- **J^z Conservation at every vertex**
- **Frame-Independent**
- **Cluster Decomposition** Ji, sjb
- **"History"-Numerator structure universal**
- **Renormalization- alternate denominators**
- **LFWF takes Off-shell to On-shell**
- **Tested in QED: g-2 to three loops**



Roskies, Suaya, sjb

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

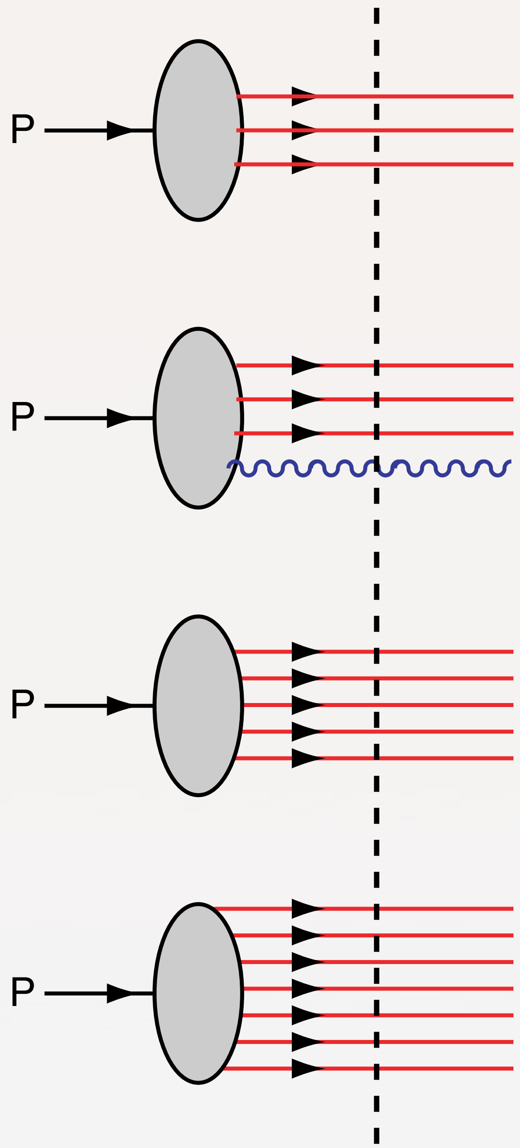
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time
Coupled. infinite set

Intrinsic heavy quarks
c(x), b(x) at high x !

$$\bar{s}(x) \neq s(x)$$
$$\bar{u}(x) \neq \bar{d}(x)$$

Mueller: gluon Fock states: BFKL Pomeron

Deuteron: Hidden Color

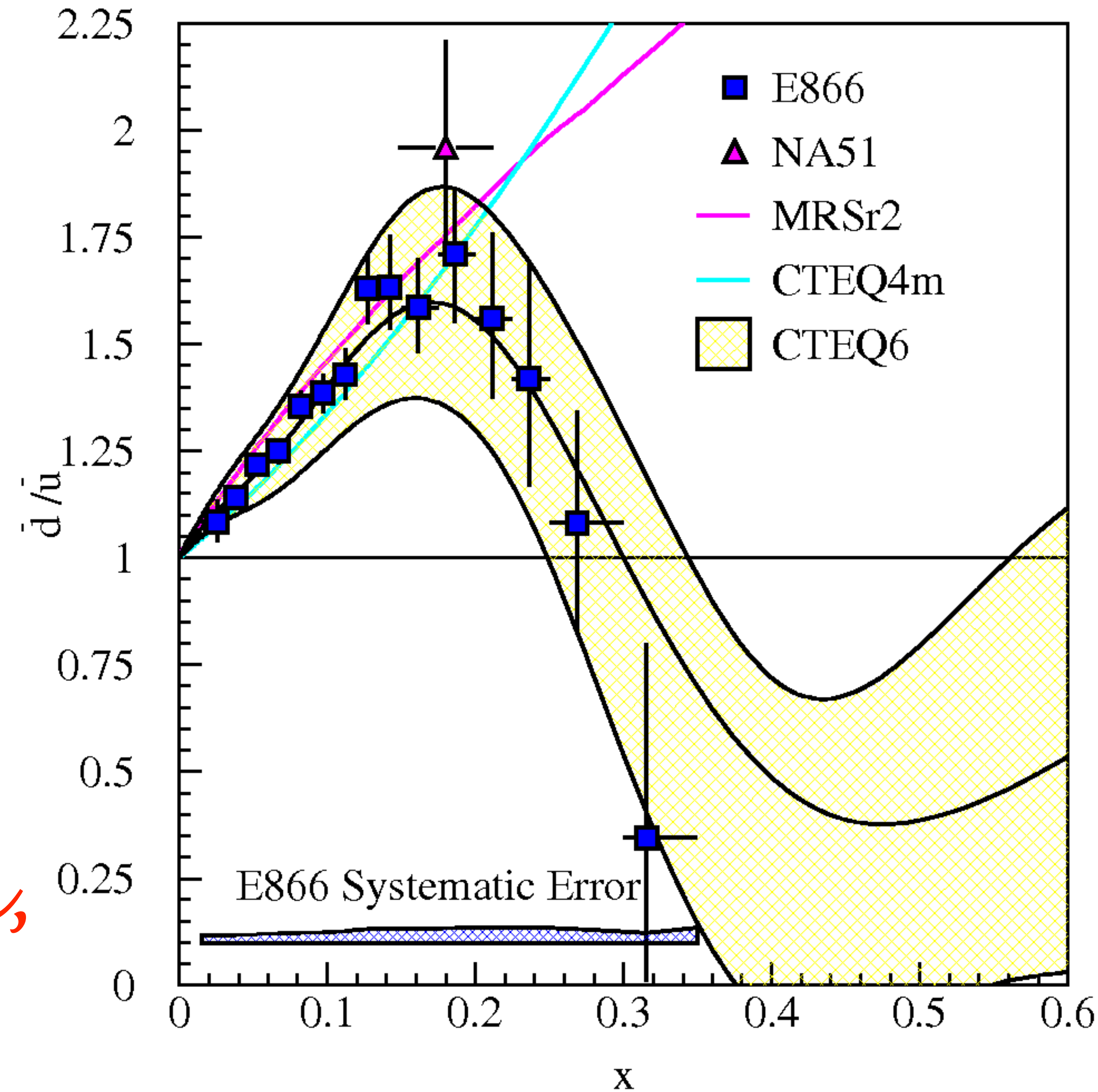
- E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$

$$s(x) \neq \bar{s}(x)$$

Intrinsic glue, sea, heavy quarks

$\bar{d}(x)/\bar{u}(x)$ for $0.015 \leq x \leq 0.35$



Remarkable Features of Hadron Structure

- **Valence quark helicity represents less than half of the proton's spin and momentum**
- **Non-zero quark orbital angular momentum!**
- **Asymmetric sea:** $\bar{u}(x) \neq \bar{d}(x)$
- **Non-symmetric strange and anti-strange sea**

$$\bar{s}(x) \neq s(x)$$
$$\Delta s(x) \neq \Delta \bar{s}(x)$$

- **Intrinsic charm and bottom at high x**

Hoyer, Peterson, Sakai, sjb

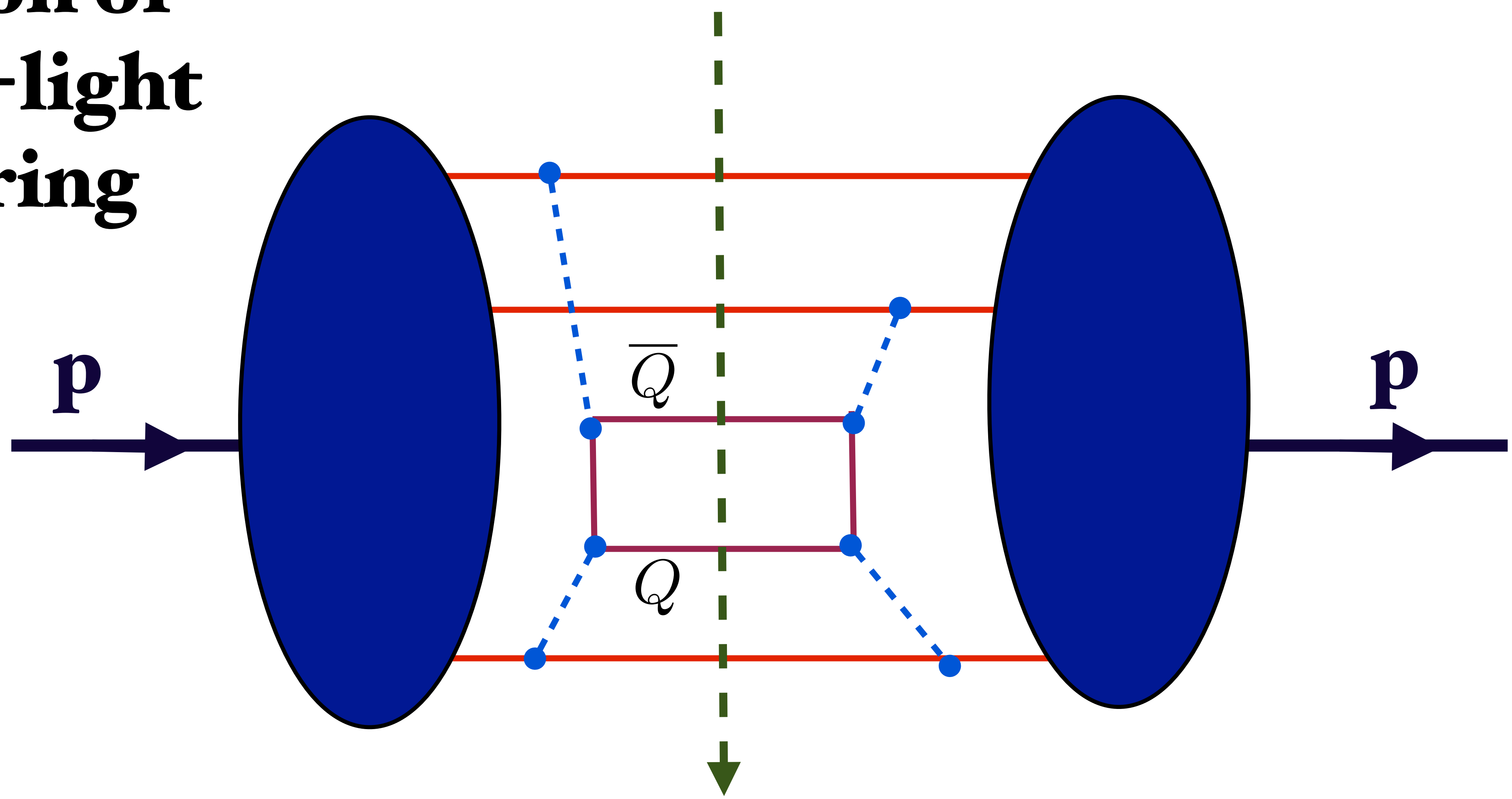
- **Hidden-Color Fock states of the Deuteron**

Lepage, Ji, sjb

Proton Self Energy

QCD predicts Intrinsic Heavy Quarks!

Insertion of light-by-light scattering

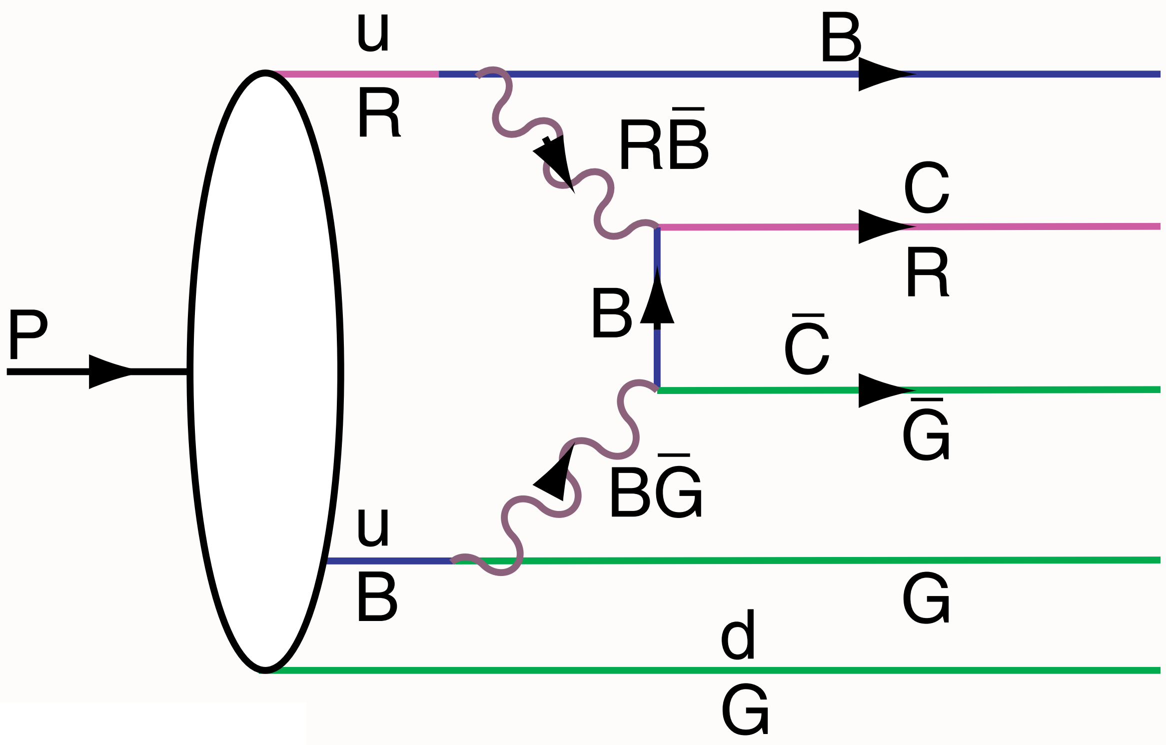


$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4} \qquad \text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov, et al.

Hoyer, Peterson, Sakai, sjb



$|uudc\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p >$ vs. $< p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p >$

$c\bar{c}$ in Color Octet

Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

High x charm! *Charm at Threshold*

Action Principle: Minimum KE, maximal potential

HERMES: Two components to s(x,Q²)!

W. C. Chang and J.-C. Peng
arXiv:1105.2381

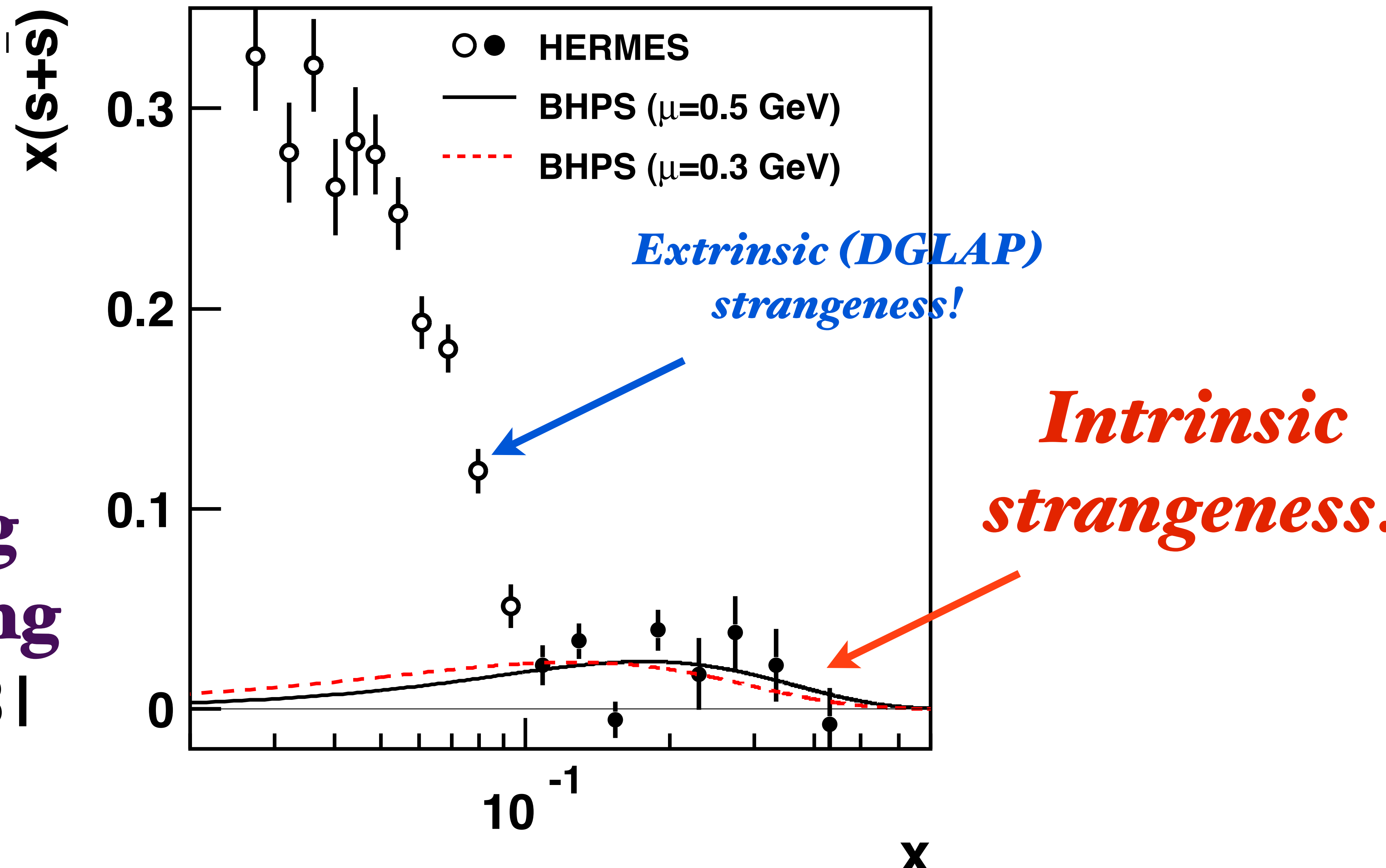
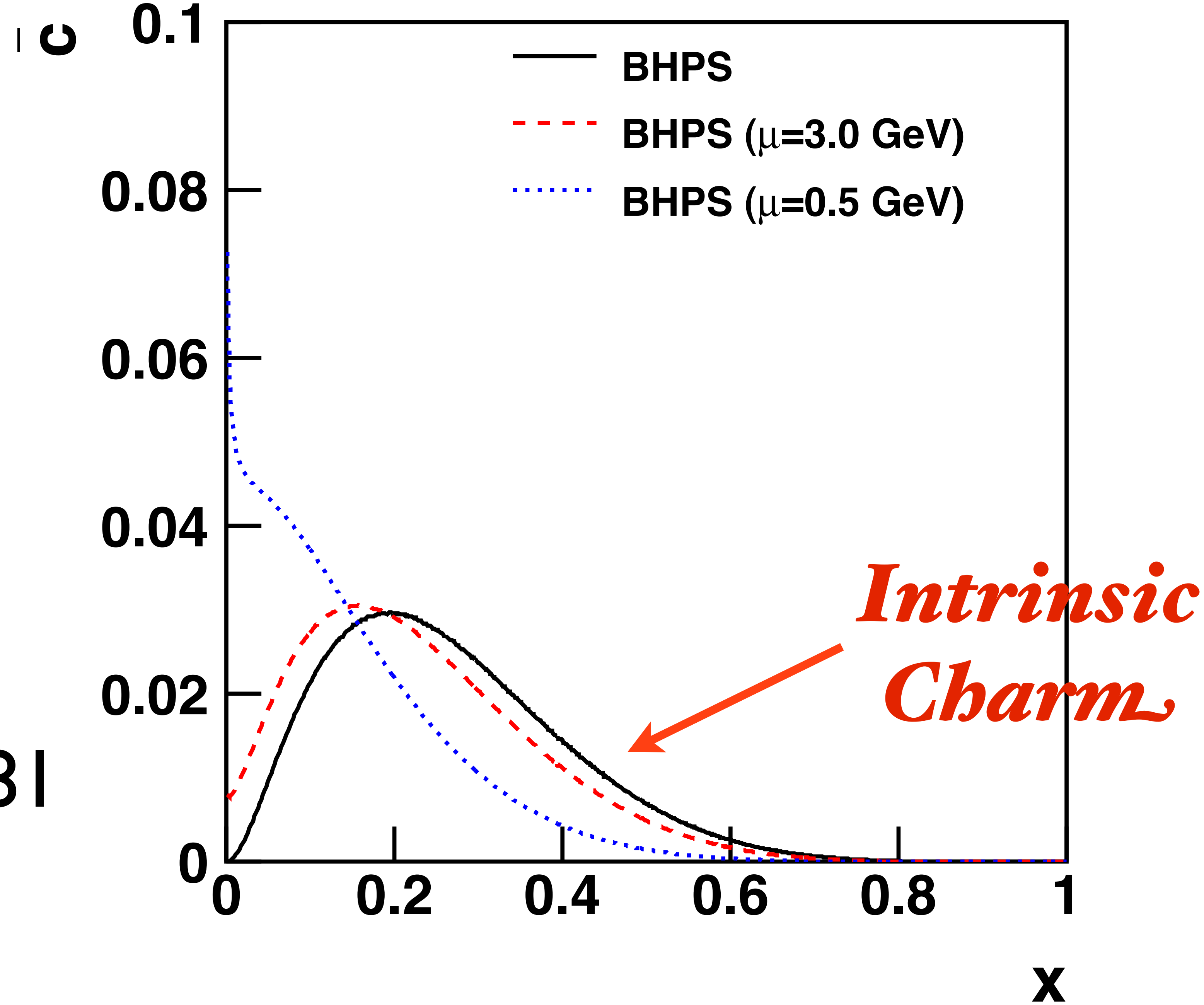


Figure 2: Comparison of the HERMES $x(s(x)+\bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV^2 using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

$$s(x,Q^2) = s(x,Q^2)_{\text{extrinsic}} + s(x,Q^2)_{\text{intrinsic}}$$

Scale intrinsic strangeness by $\frac{1}{m_Q^2}$

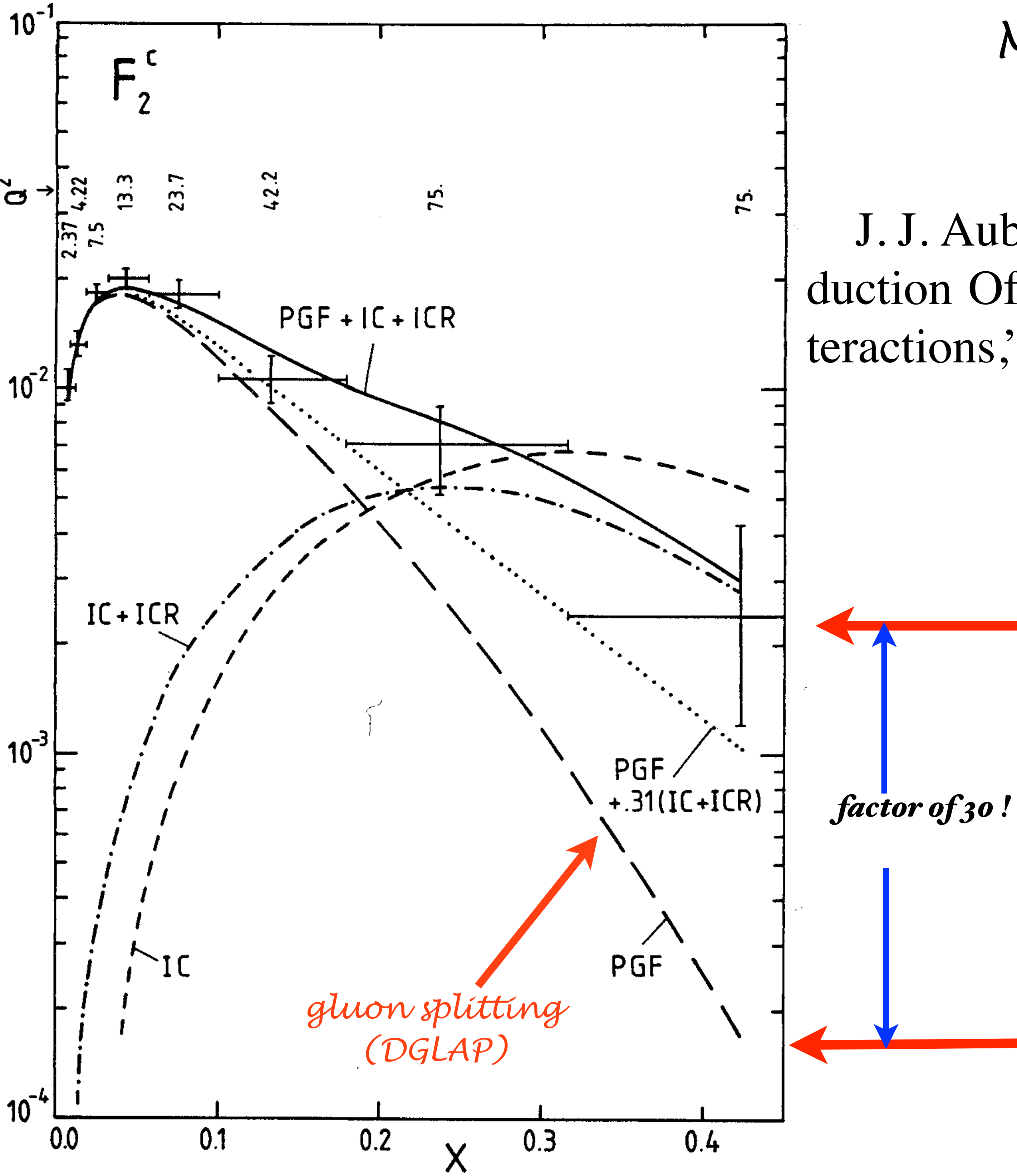


W. C. Chang
J.-C. Peng

arXiv:1105.2381

Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75$ GeV^2 using $\mu = 3.0$ GeV, and $\mu = 0.5$ GeV, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.

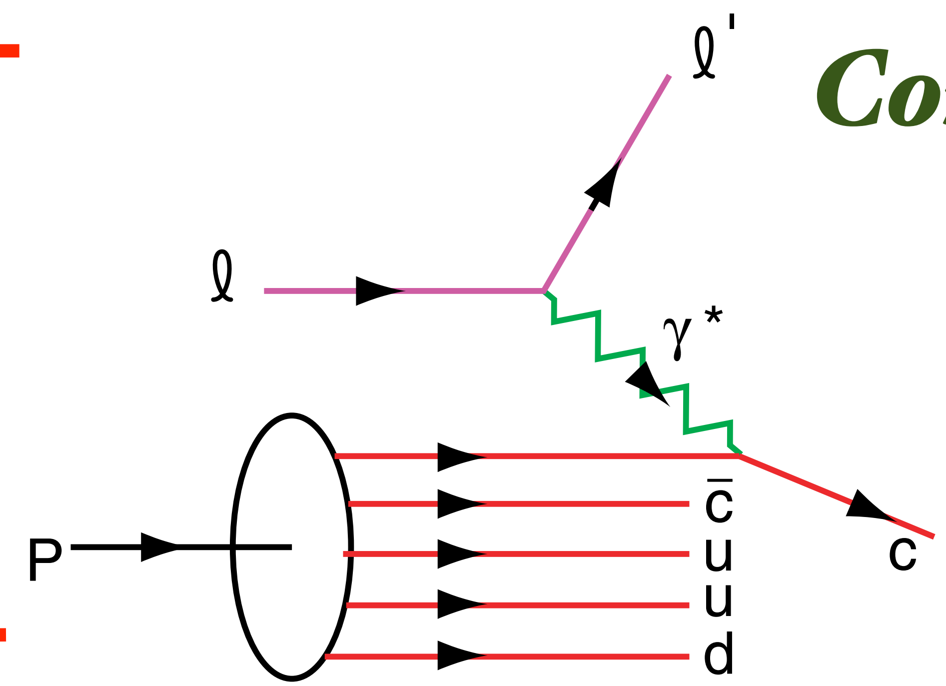
Measurement of Charm Structure Function



J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm

Compass!



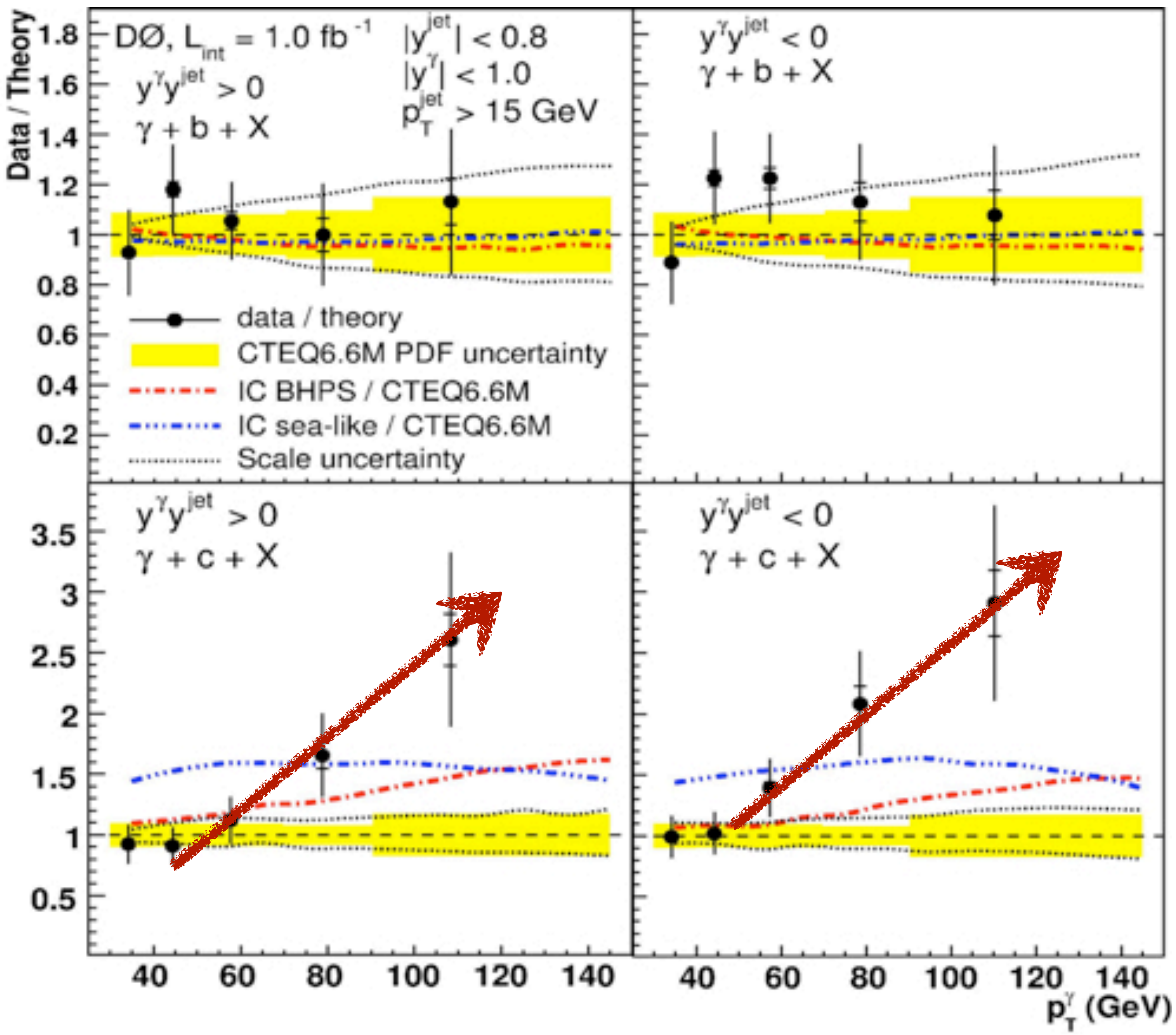
DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

PRL **102,** 192002 (2009) PHYSICAL REVIEW LETTERS week ending 15 MAY 2009

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV



$$\frac{\Delta\sigma(\bar{p}p \to \gamma c X)}{\Delta\sigma(\bar{p}p \to \gamma b X)}$$

Ratio insensitive to gluon PDF, scales

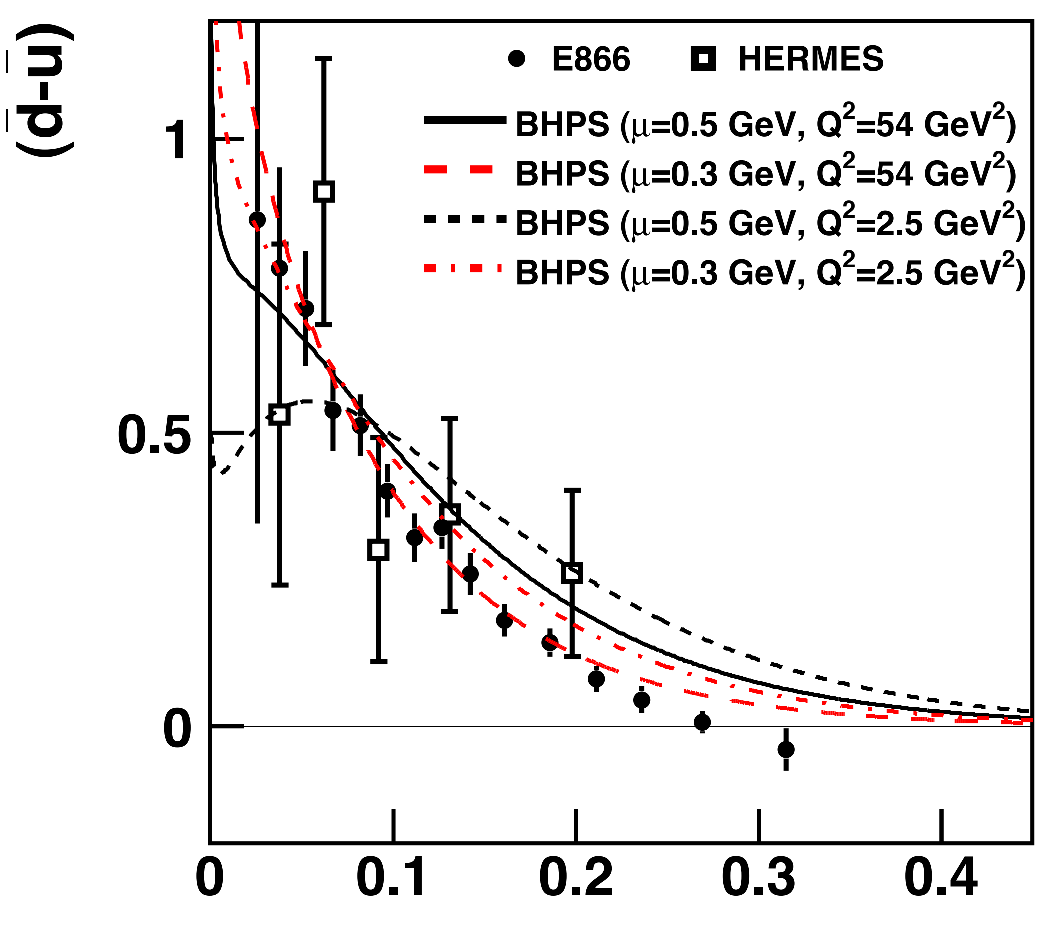
Signal for significant IC at x > 0.1 ?

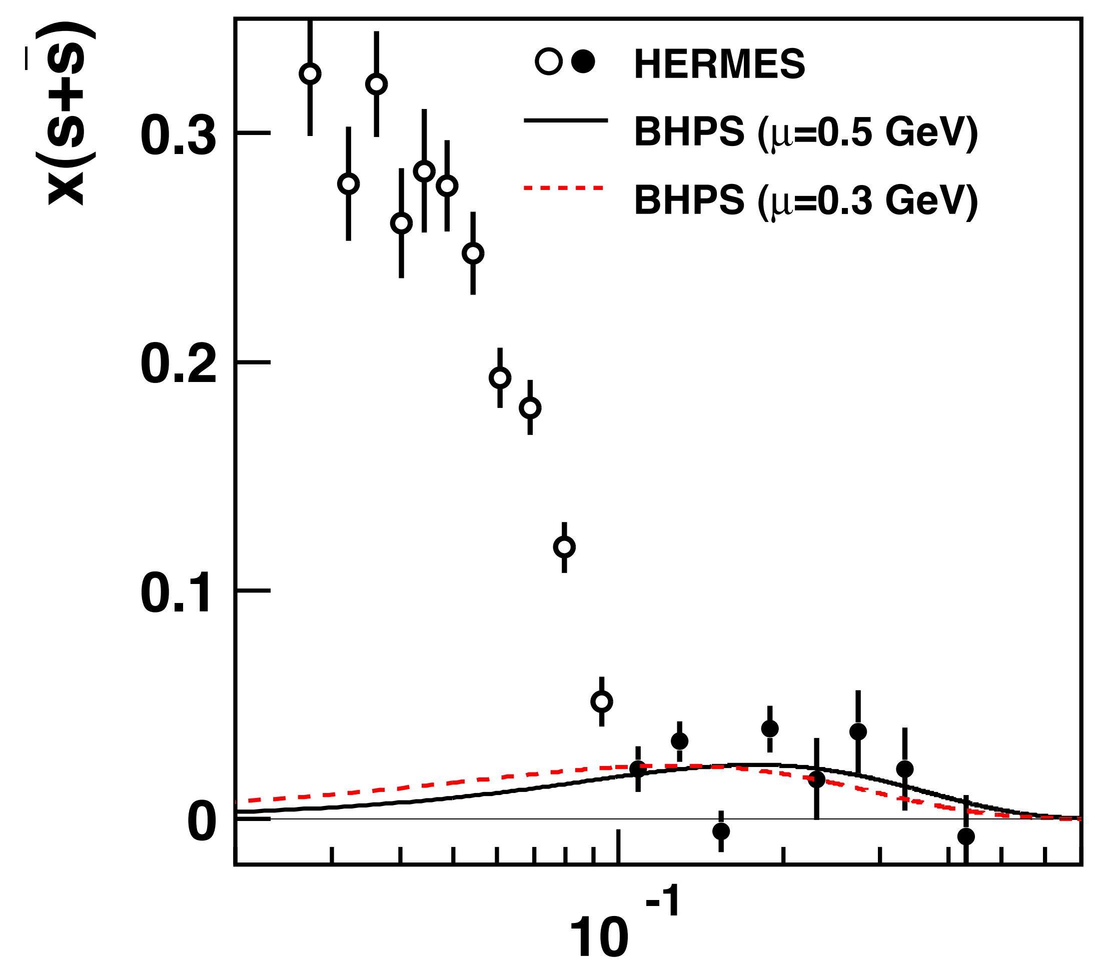
DGLAP evolution issues?

Extraction of Various Five-Quark Components of the Nucleons

Wen-Chen Chang[a], Jen-Chieh Peng[a,b]

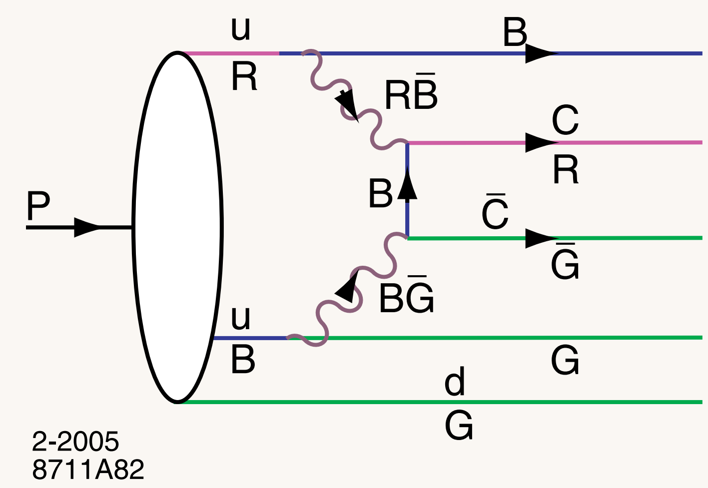
[a] *Institute of Physics, Academia Sinica, Taipei 11529, Taiwan*
[b] *Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*





Hoyer, Peterson, Sakai, sjb

Intrinsic Heavy-Quark Fock States



- ***Rigorous*** prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x!
- **Greatly increases kinematics of colliders such as Higgs production** (***Kopeliovich, Schmidt, Soffer, sjb***)
- Severely underestimated in conventional parameterizations of heavy quark distributions (***Pumplin, Tung***)
- Slow evolution compared to extrinsic quarks from gluon splitting!
- Many empirical tests

- EMC data: $c(x, Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV^2, $x = 0.42$

- High x_F $pp \to J/\psi X$

Color-octet IC explains anomalous nuclear dependence

- High x_F $pp \to J/\psi J/\psi X$

- High x_F $pp \to \Lambda_c X$

- High x_F $pp \to \Lambda_b X$

Charm near threshold: JLab 12 GeV!

- High x_F $pp \to \Xi(ccd) X$ (SELEX)

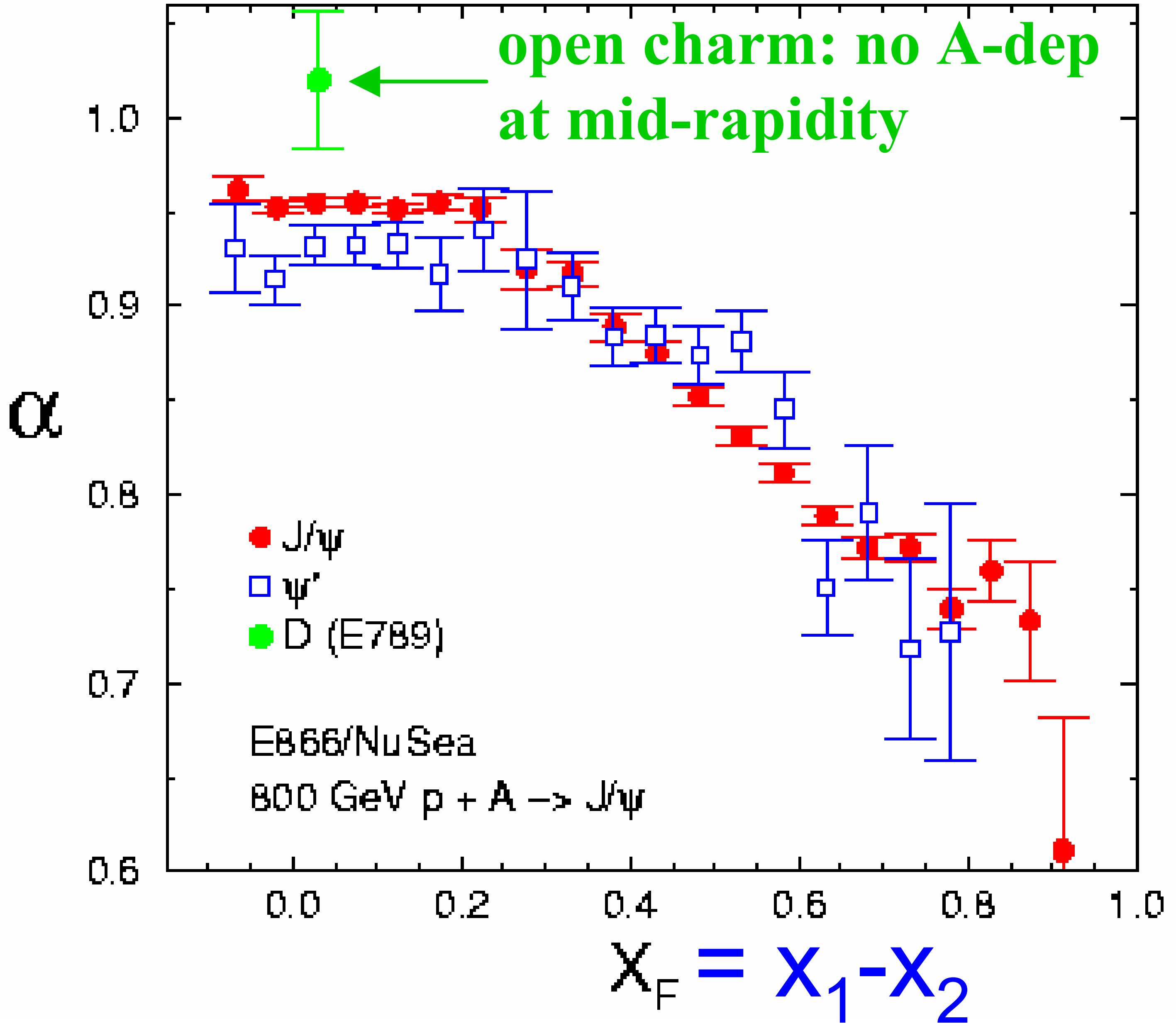
IC Structure Function: Critical Measurement for EIC

Many interesting spin, charge asymmetry, spectator effects

M. Leitch

800 GeV p-A (FNAL) $\sigma_A = \sigma_p * A^{\alpha}$

PRL 84, 3256 (2000); PRL 72, 2542 (1994)



$$\frac{d\sigma}{dx_F}(pA \to J/\psi X)$$

Remarkably Strong Nuclear Dependence for Fast Charmonium

Violation of PQCD Factorization!

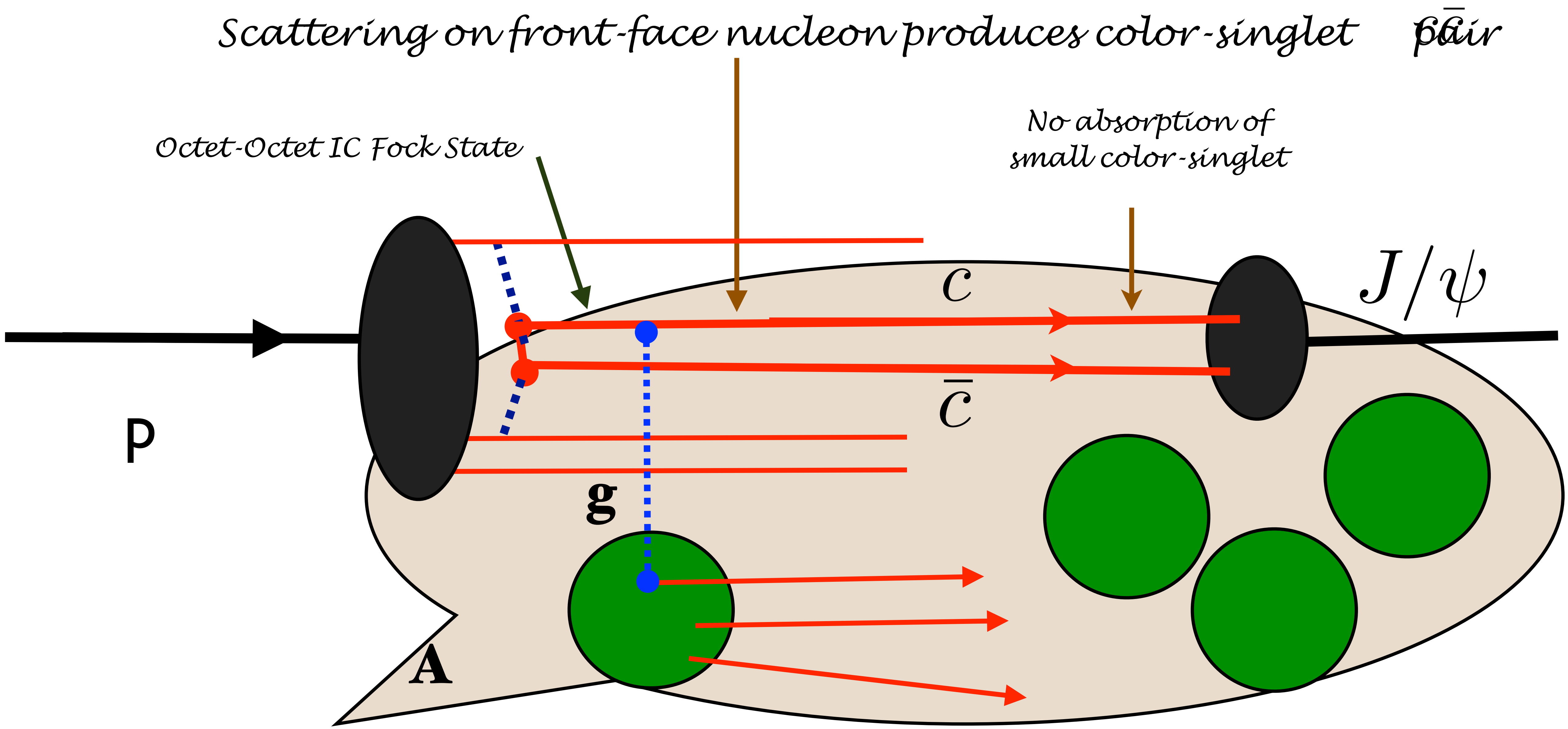
Violation of factorization in charm hadroproduction.
P. Hoyer, M. Vanttinen (Helsinki U.) , U. Sukhatme (Illinois U., Chicago) . HU-TFT-90-14, May 1990. 7pp.
Published in Phys.Lett.B246:217-220,1990

IC Explains large excess of quarkonia at large x_F, A-dependence

Anomalous Energy Loss? Test in $\gamma^* A \to J/\psi X$

Kopeliovich, Schmidt, Soffer, sjb

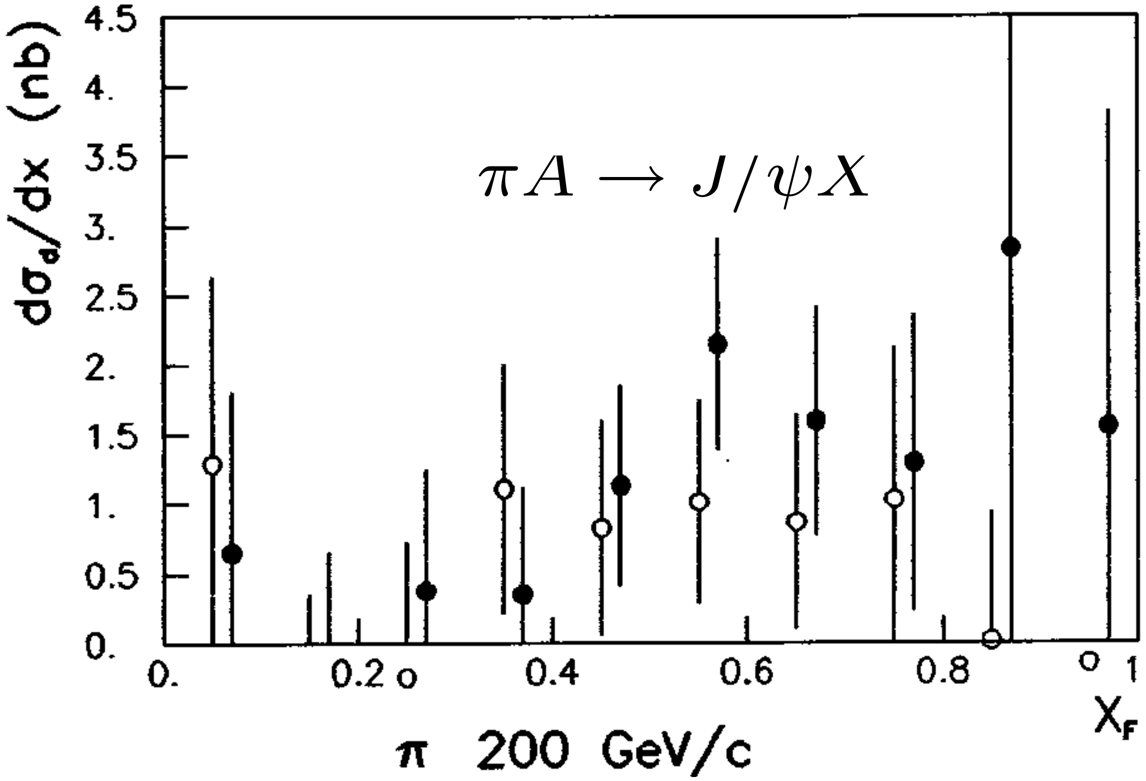
Color-Opaque IC Fock state interacts on nuclear front surface



$$\frac{d\sigma}{dx_F}(pA \to J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \to J/\psi X)$$

J. Badier et al, NA3

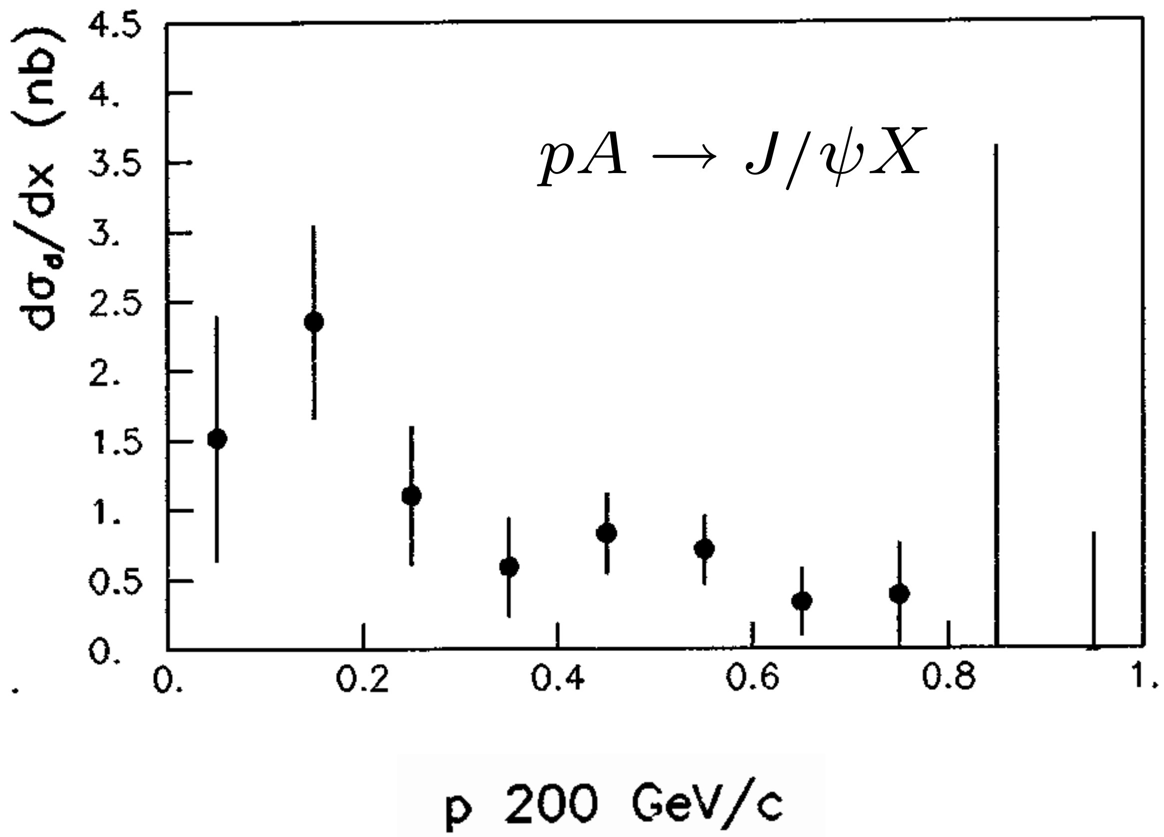
$$\frac{d\sigma}{dx_F}(pA \to J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$



$A^{2/3}$ component

High x_F:

Consistent with color-octet intrinsic charm

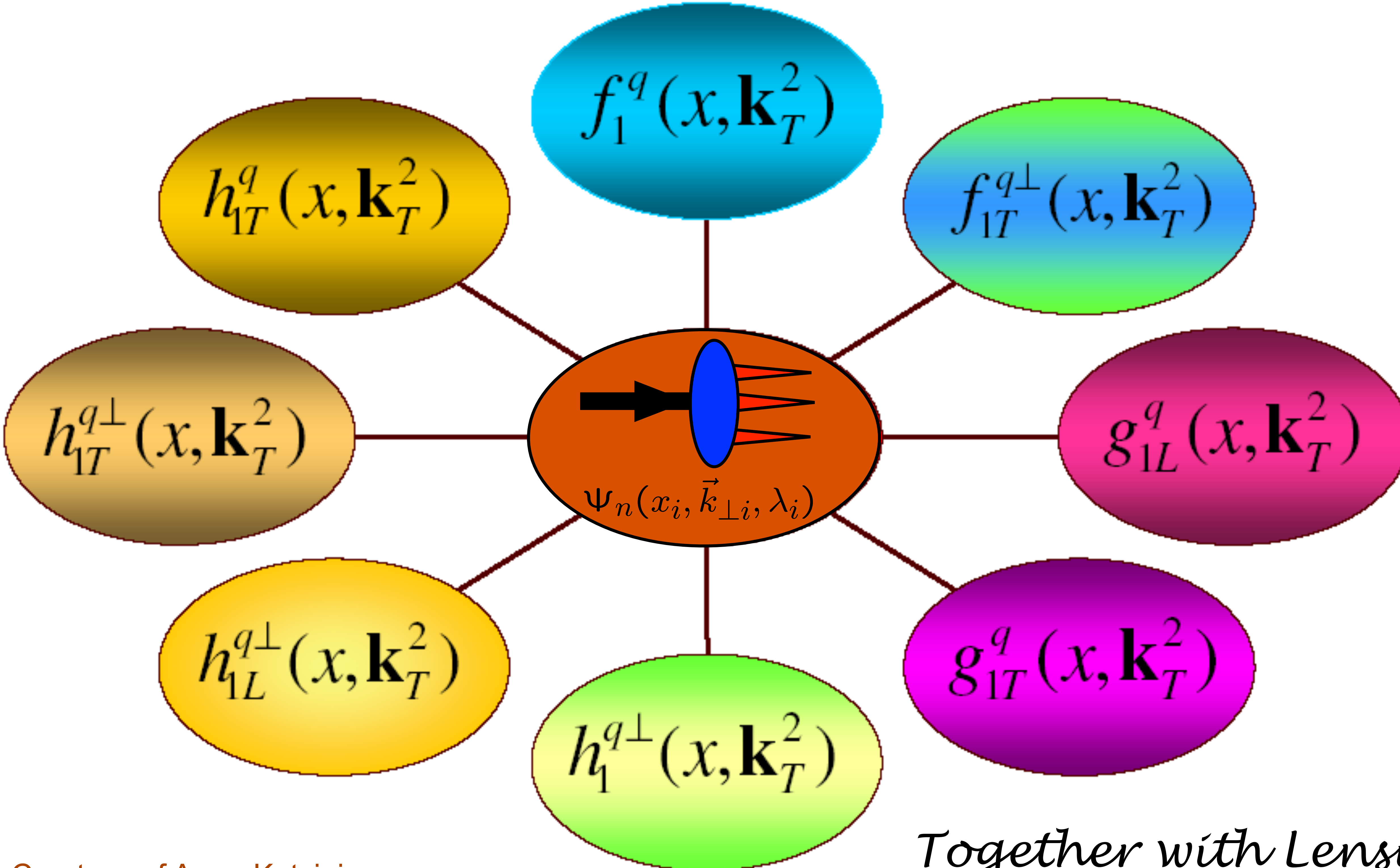


Excess beyond conventional gluon-splitting PQCD subprocesses

Why is IQ Important for Flavor Physics?

- **New perspective on fundamental nonperturbative hadron structure**
- **Charm structure function at high x**
- **Dominates high x_F charm and charmonium production**
- **Hadroproduction of new heavy quark states such as ccu, ccd, bcc, bbb, at high x_F**
- **Intrinsic charm -- long distance contribution to penguin mechanisms for weak decay** *Gardner, sjb*
- $J/\psi \to \rho\pi$ **puzzle explained** *Karliner , sjb*
- **Novel Nuclear Effects from color structure of IC, Heavy Ion Collisions**
- **New mechanisms for high x_F Higgs hadroproduction**
- **Dynamics of b production: LHCb** *New Multi-lepton Signals*
- **Fixed target program at LHC: produce bbb states**
- **Intrinsic strangeness effects at JLab: charm at threshold!**
- **φ photo- and electroproduction**

8 leading-twist spin-$k_\perp$ dependent distribution functions



Courtesy of Aram Kotzinian

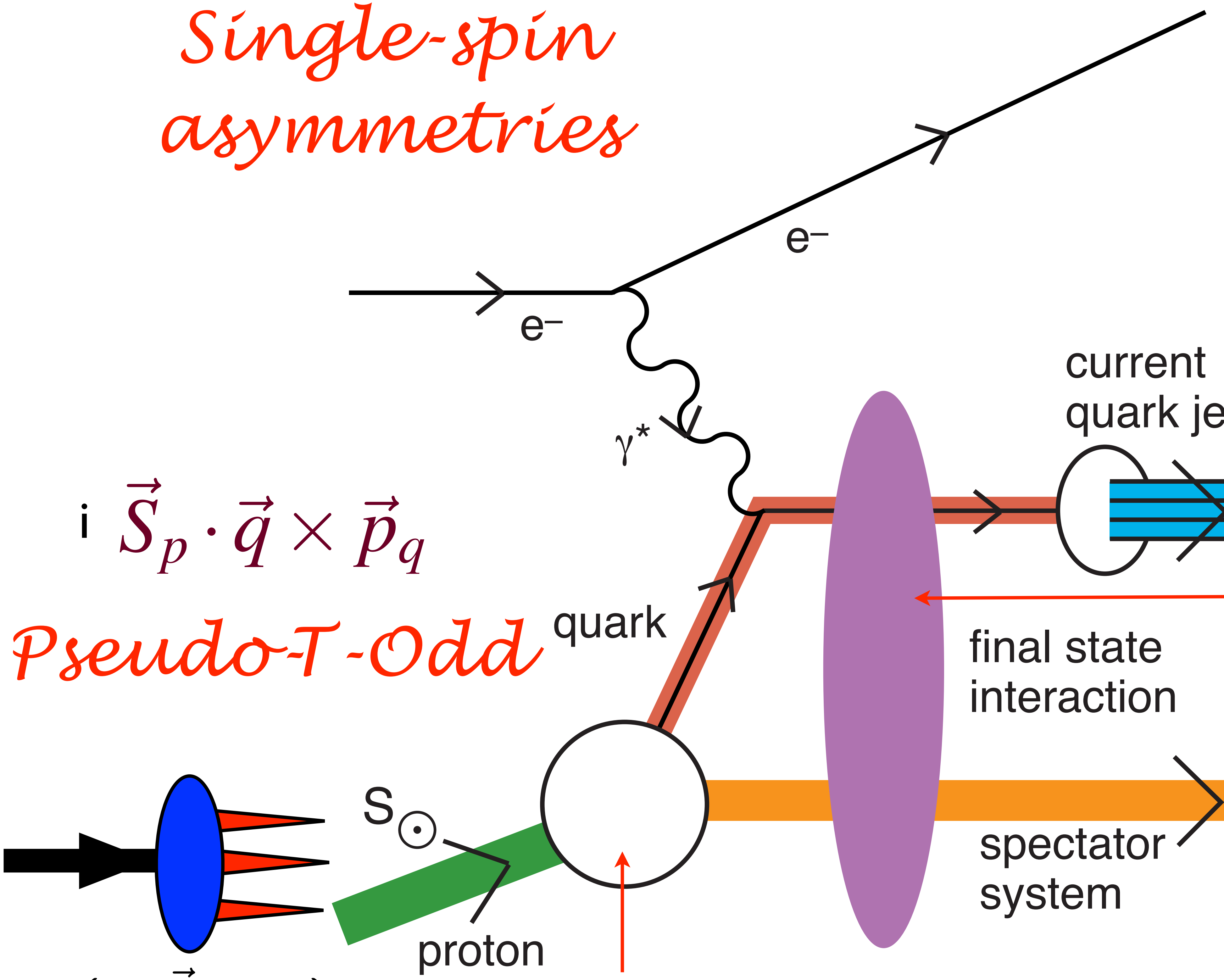
Together with Lensing

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...



i $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd

QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

Light-Front Wavefunction S and P- Waves!

Leading-Twist Rescattering Violates pQCD Factorization!

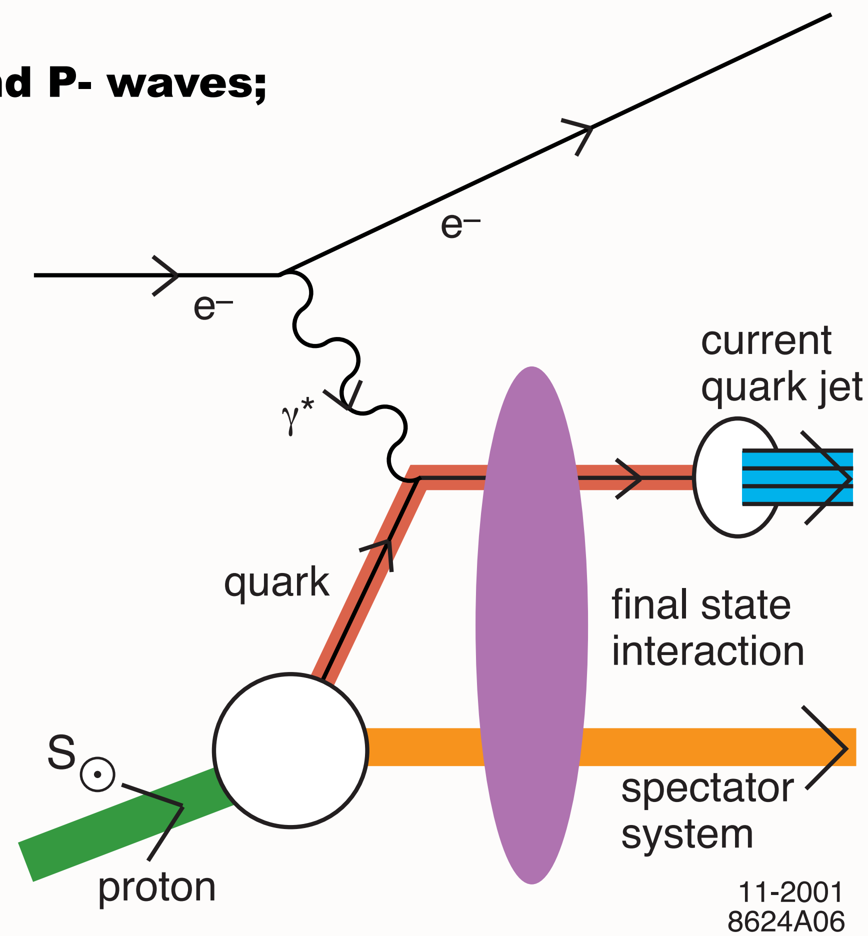
Sign reversal in DY!

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

Hwang, Schmidt, sjb
Collins

- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark!**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves;**
- **Wilson line effect -- lc gauge prescription**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**
- **Alternate: Retarded and Advanced Gauge: Augmented LFWFs**

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

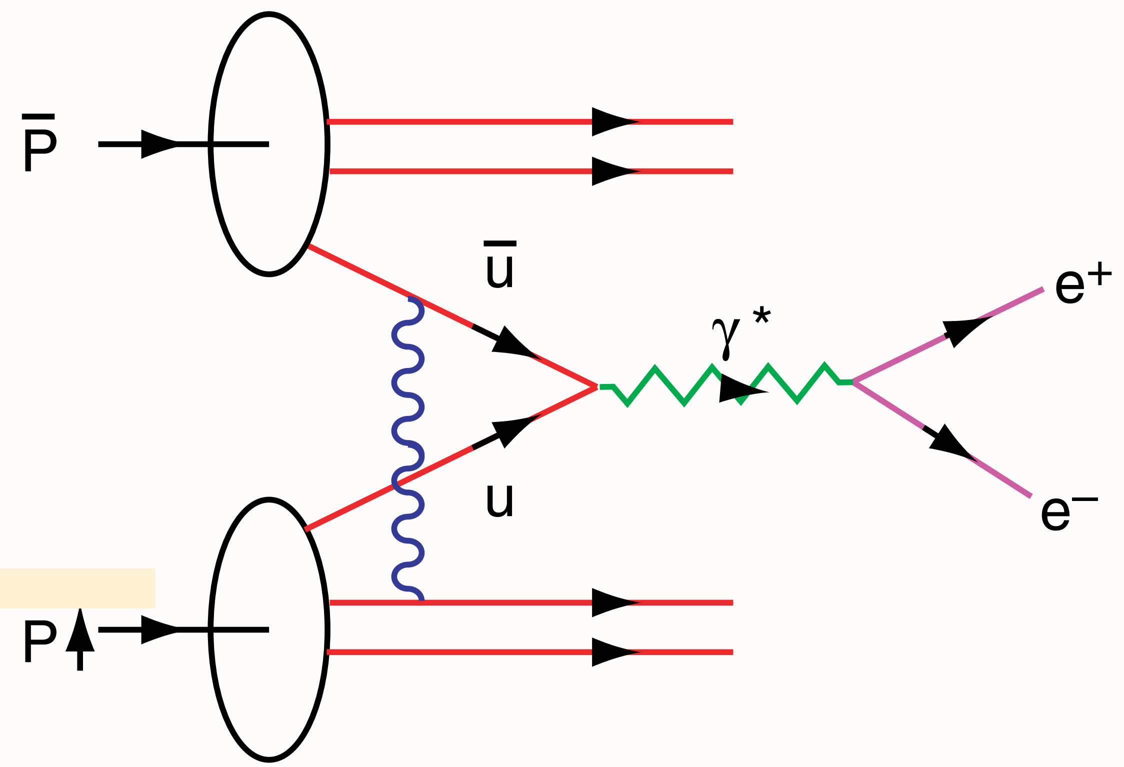


Pasquini, Xiao, Yuan, sjb
Mulders, Boer   Qiu, Sterman

Measure single-spin asymmetry A_N
in Drell-Yan reactions

Leading-twist Bjorken-scaling A_N
from S, P-wave
initial-state gluonic interactions

Predict: $A_N(DY) = -A_N(DIS)$
Opposite in sign!

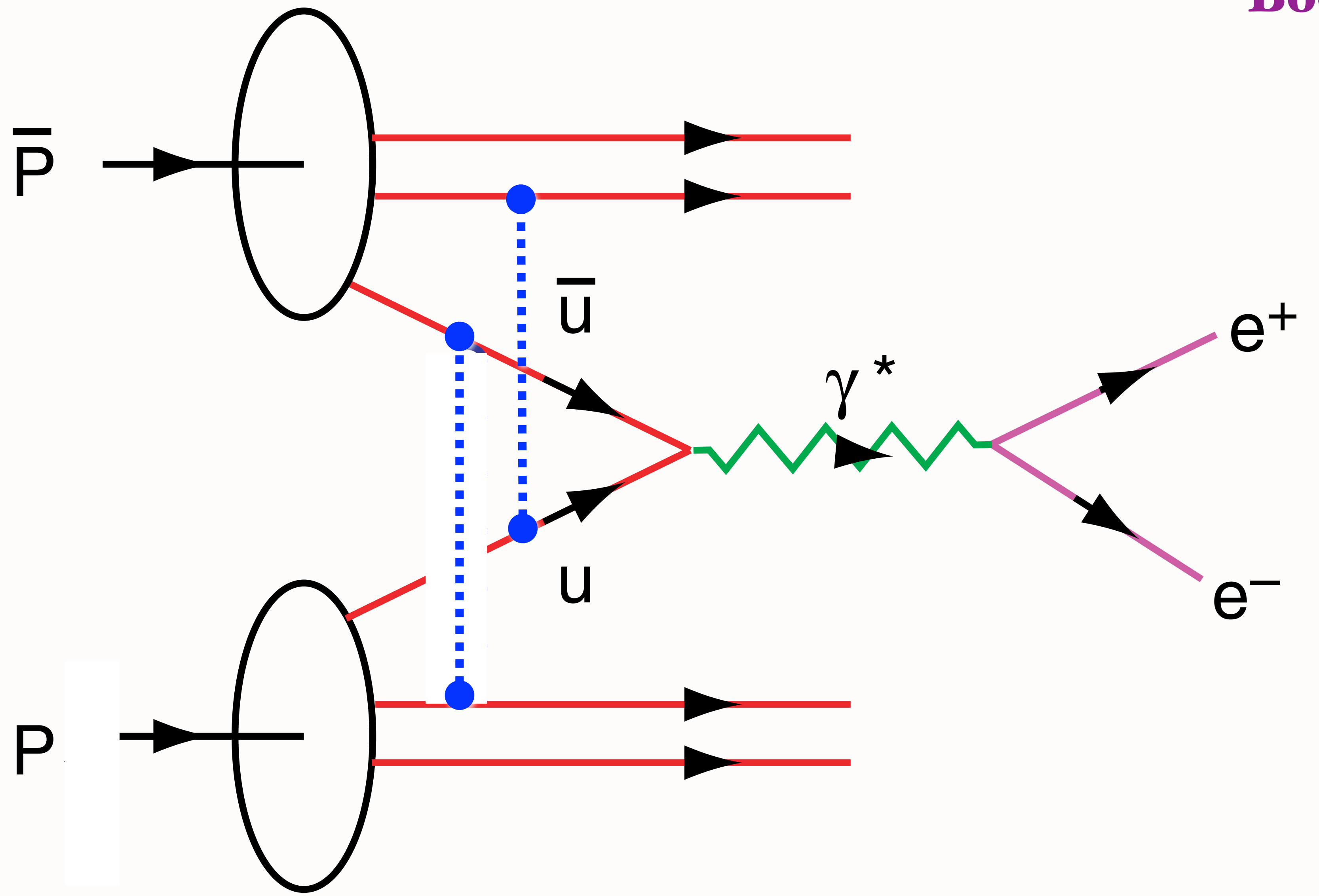


$$\bar{p}p_\uparrow \rightarrow \ell^+\ell^- X$$

$\vec{S} \cdot \vec{q} \times \vec{p}$ correlation

Collins;
Hwang, Schmidt. sjb

Boer, Hwang, sjb



DY $\cos 2\phi$ correlation at leading twist from double ISI

Product of Boer - Mulders Functions

$$h_1^{\perp}(x_1, \boldsymbol{p}_\perp^2) \times \bar{h}_1^{\perp}(x_2, \boldsymbol{k}_\perp^2)$$

Double Initial-State Interactions generate anomalous $\cos 2\phi$

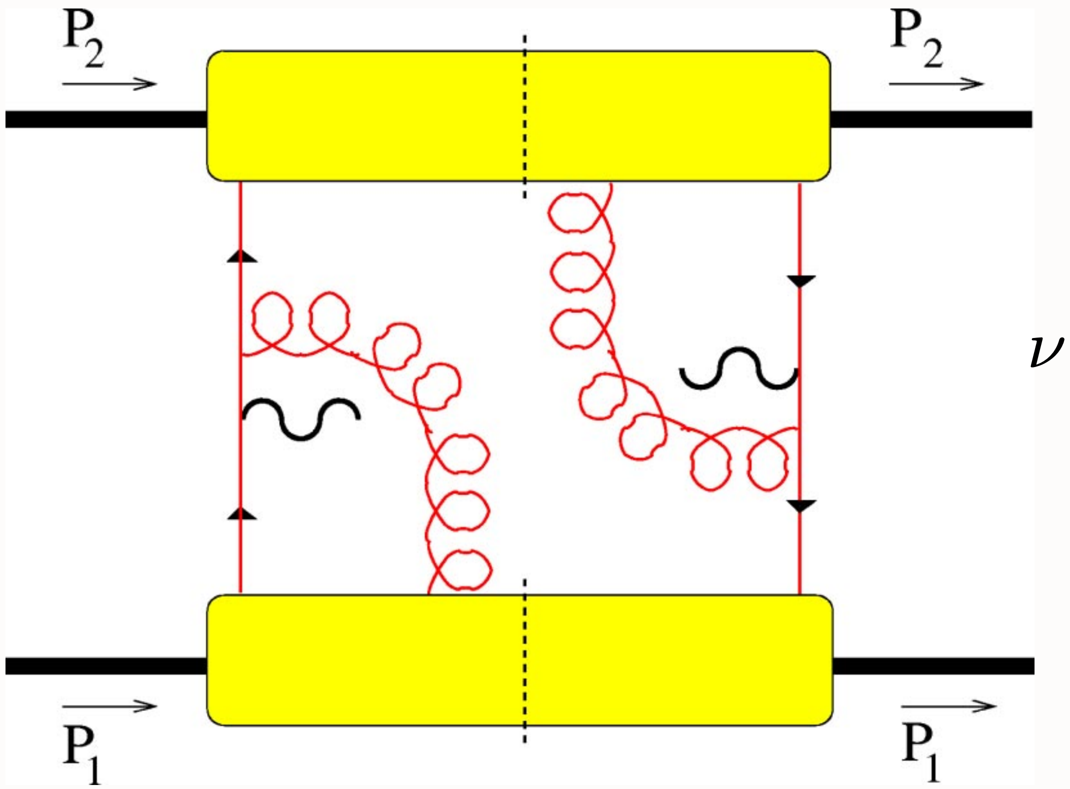
Boer, Hwang, sjb

Drell-Yan planar correlations

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \, \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

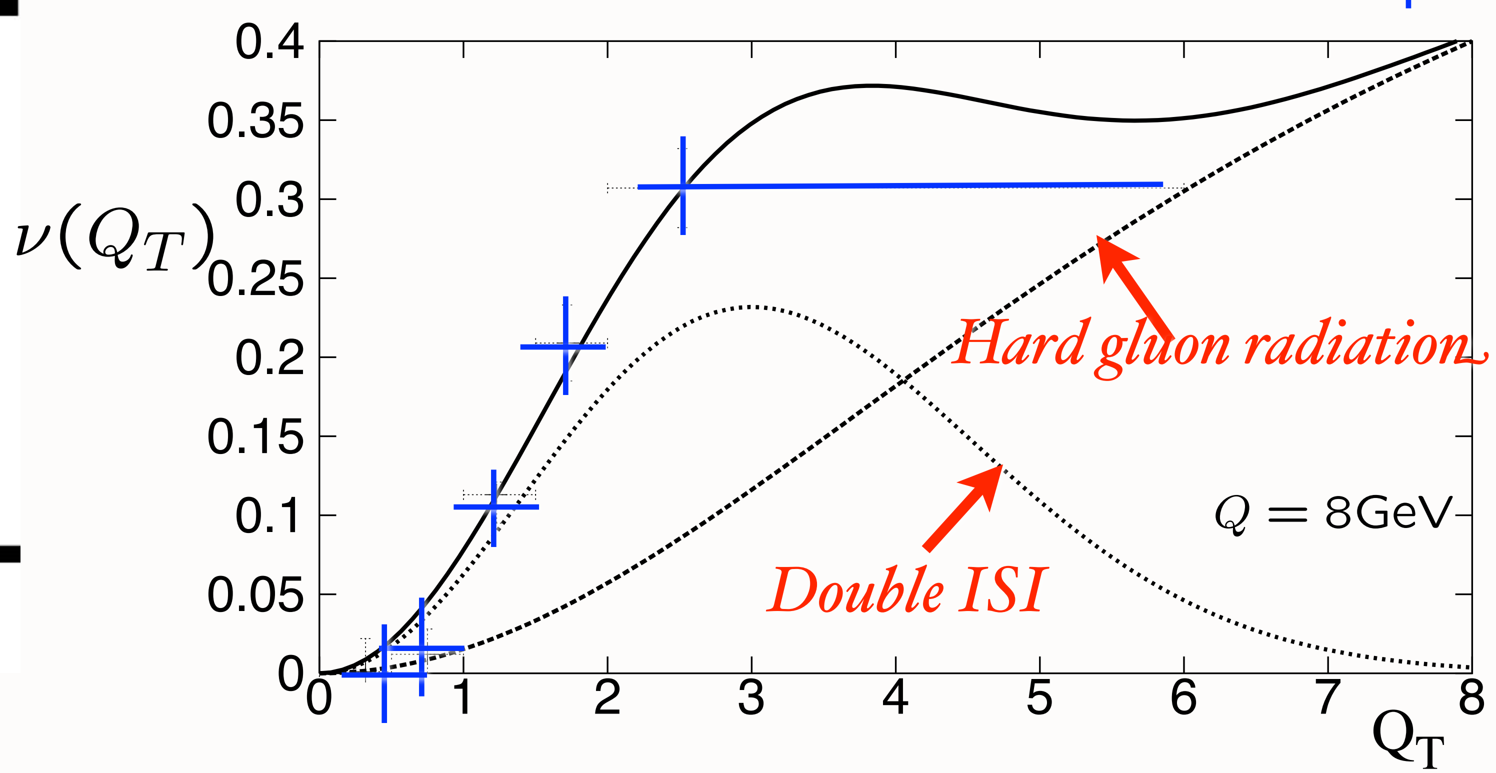
PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^\perp(\pi)\, h_1^\perp(N)$$



Violates Lam-Tung relation!

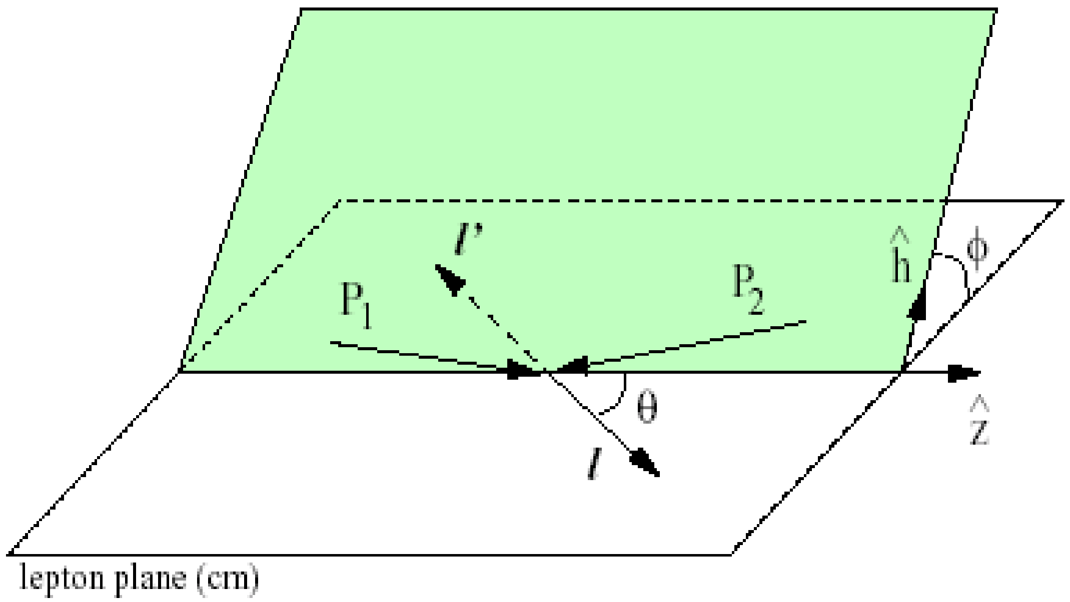
$\pi N \rightarrow \mu^+\mu^- X$ NA10



Model: Boer,

Drell-Yan angular distribution

Unpolarized DY



- Experimentally, a violation of the Lam-Tung sum rule is observed by sizeable cos2Φ moments
- Several model explanations
  - higher twist
  - spin correlation due to non-triva QCD vacuum
  - Non-zero Boer Mulders function

$$\text{Lam} - \text{Tung SR}: \; 1 - \lambda = 2\nu$$

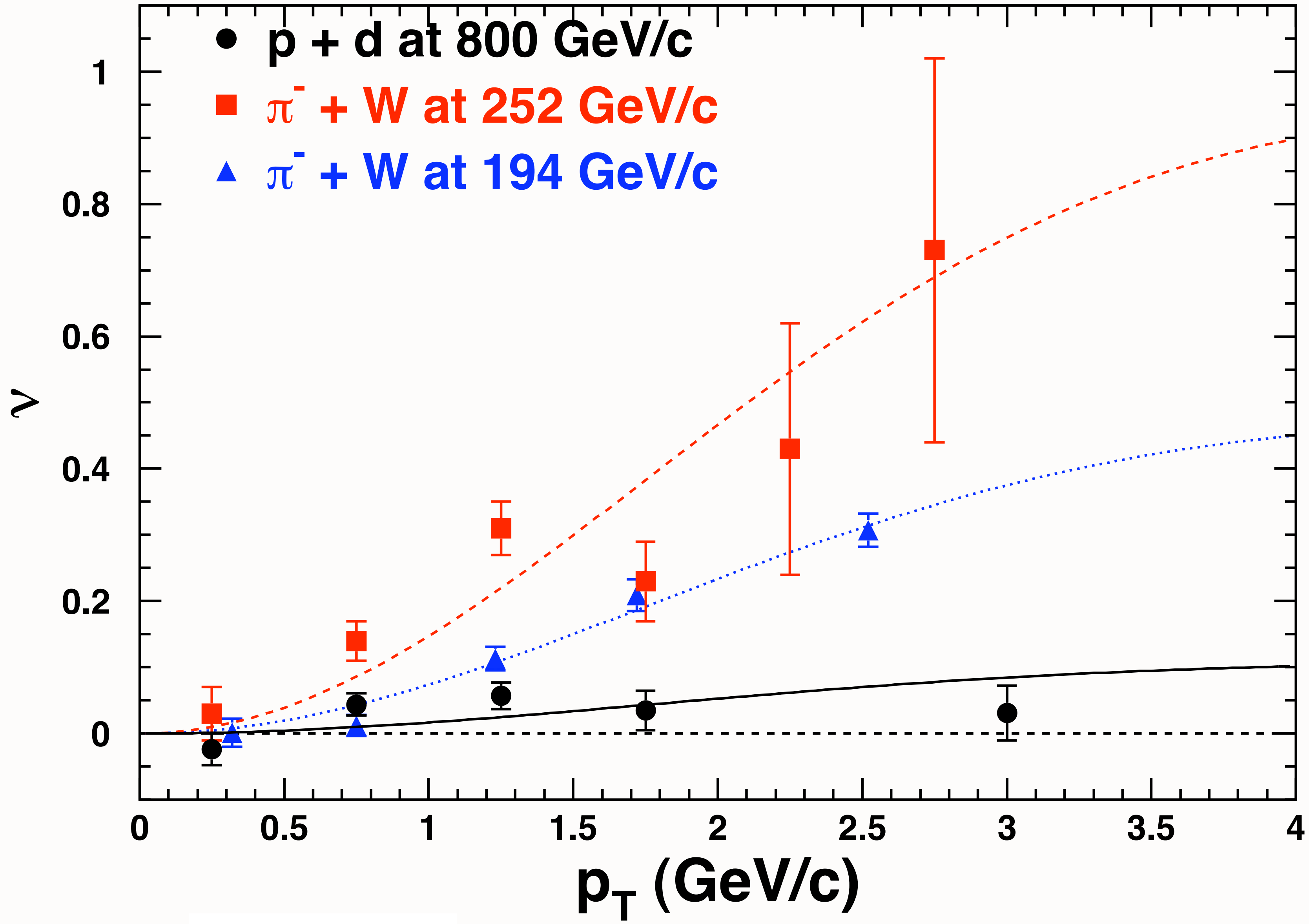
$$\text{NLO pQCD}: \lambda \approx 1 \; \mu \approx 0 \; \nu \approx 0$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

$$\text{Experiment:} \; \nu \simeq 0.6$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta\, \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

Measurement of Angular Distributions of Drell-Yan Dimuons in $p + d$ Interaction at 800 GeV/c



Huge Effect in $\pi W \to \mu^+\mu^- X$

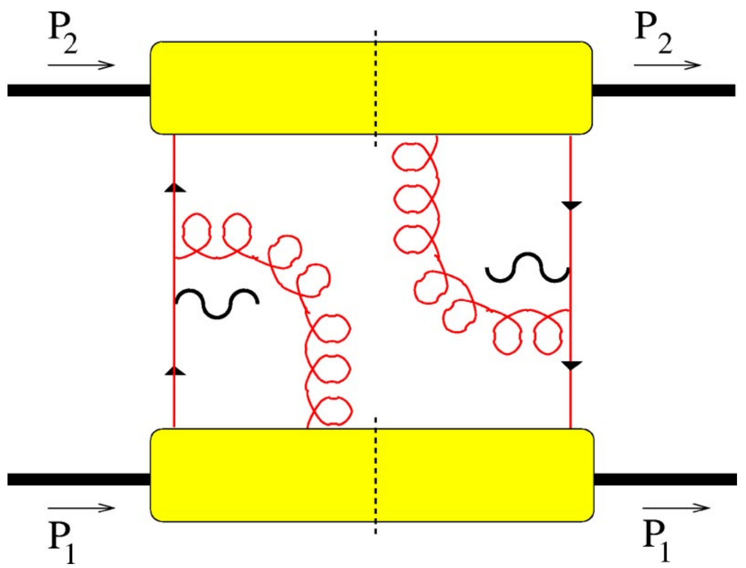
Negligible Effect $pd \to \mu^+\mu^- X$

(FNAL E866/NuSea Collaboration

Parameter ν vs. p_T in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and $M_C = 2.4$ GeV/c^2 are also shown.

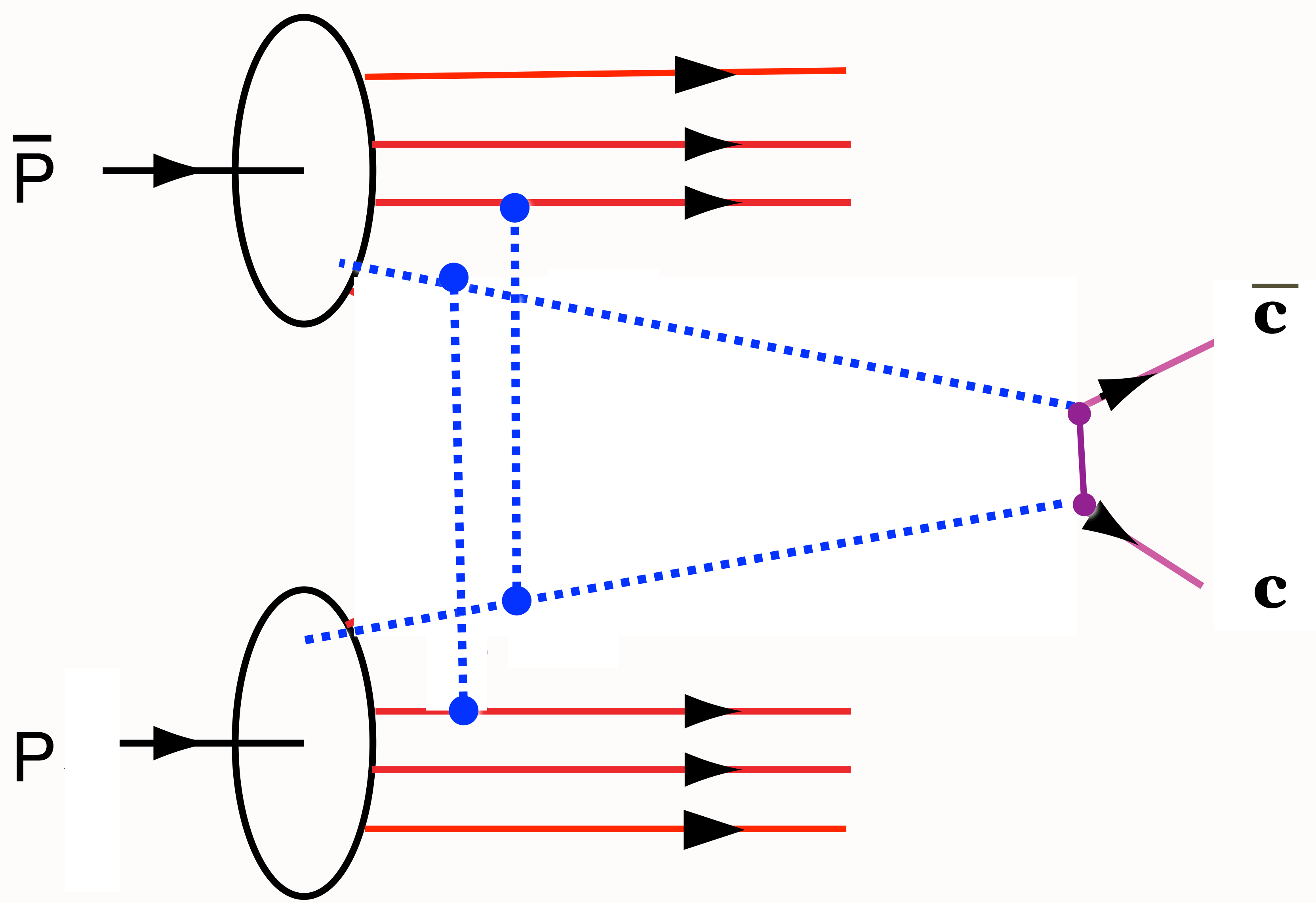
Anomalous effect from Double ISI in Massive Lepton Production

$\cos 2\phi$ correlation



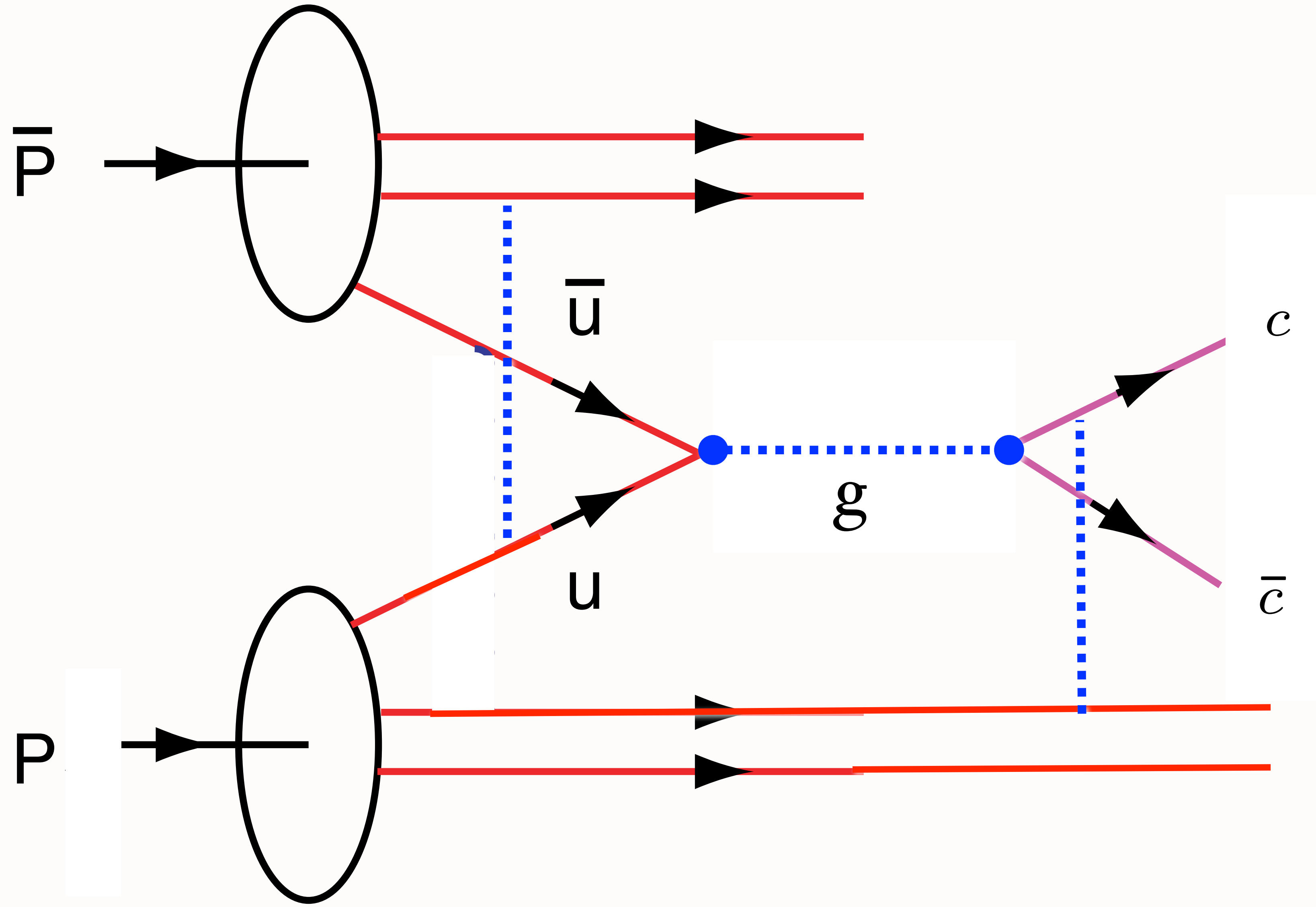
- **Leading Twist, valence quark dominated**
- **Violates Lam-Tung Relation!**
- **Not obtained from standard PQCD subprocess analysis**
- **Normalized to the square of the single spin asymmetry in semi-inclusive DIS**
- **No polarization required**
- **Challenge to standard picture of PQCD Factorization**

Boer, Hwang, sjb



$\cos 2\phi$ **correlation for quarkonium production at leading twist from double ISI**

Enhanced by gluon color charge

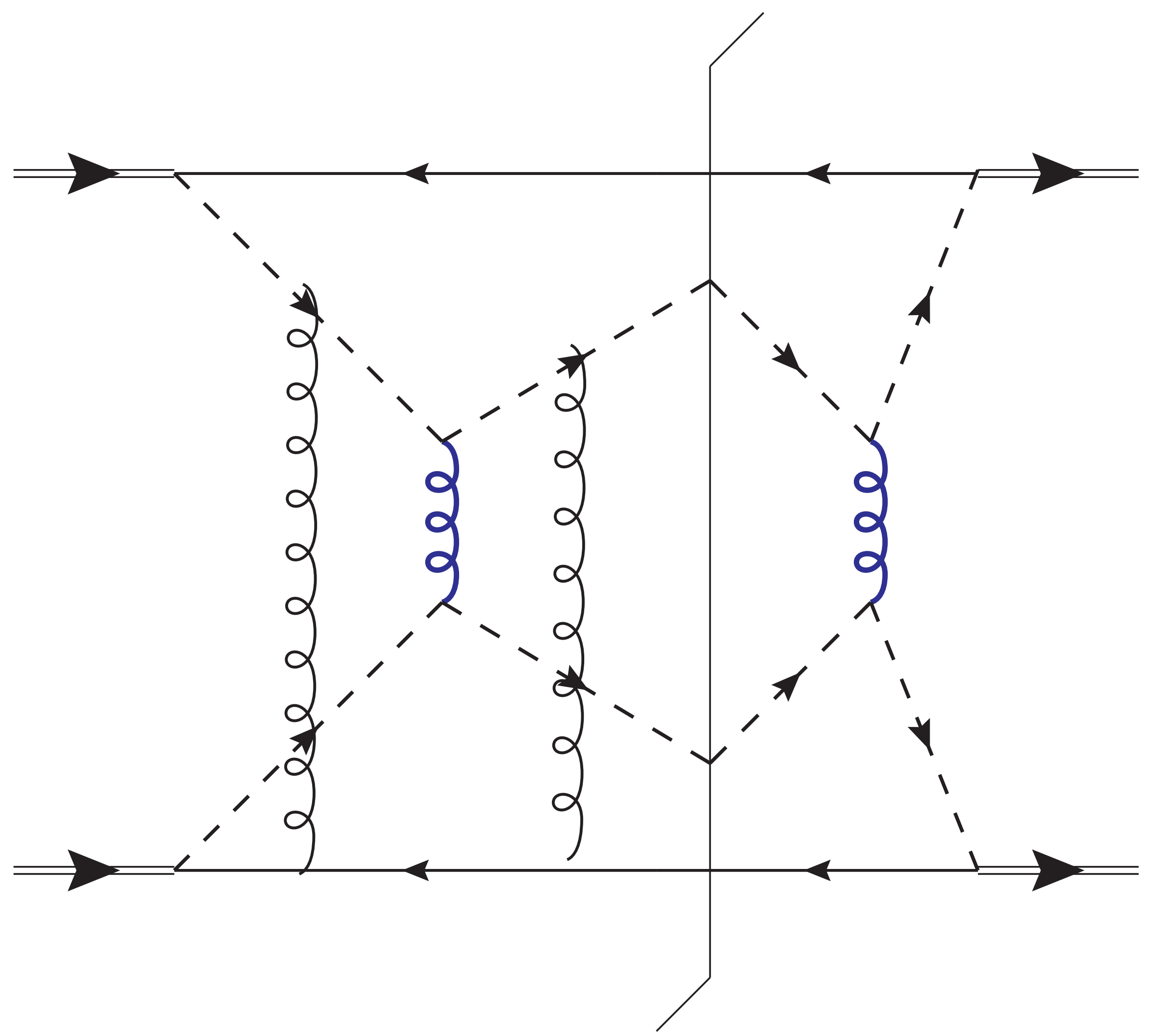


Problem for factorization when both ISI and FSI occur

Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

John Collins, Jian-Wei Qiu . ANL-HEP-PR-07-25, May 2007.

e-Print: **arXiv:0705.2141** [hep-ph]



The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.

Physics of Rescattering

- Sivers Asymmetry and Diffractive DIS: New Insights into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions!
- T-odd SSAs, Shadowing, Antishadowing *Not in LFWFs*
- Diffractive dijets/ trijets, doubly diffractive Higgs
- Novel Effects: Color Transparency, Color Opaqueness, Intrinsic Charm, Odderon
- **CT: Kawtar Hafidi**

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

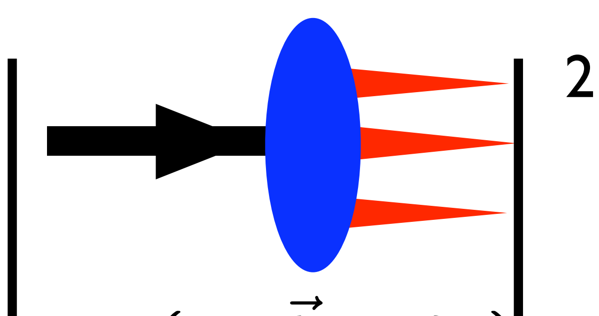
Hwang, Schmidt, sjb,

Mulders, Boer

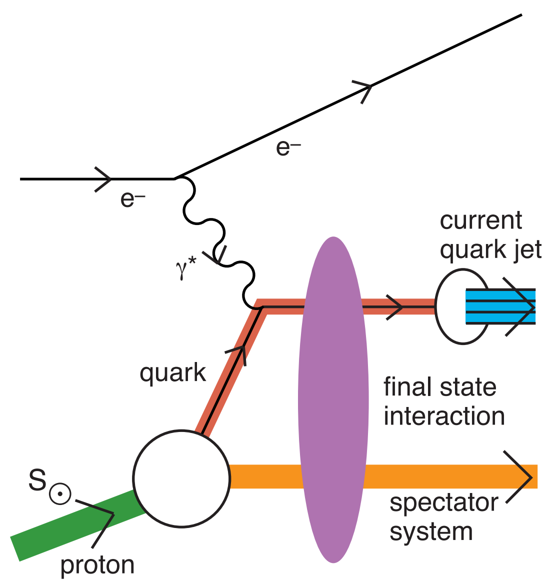
Qiu, Sterman

Collins, Qiu

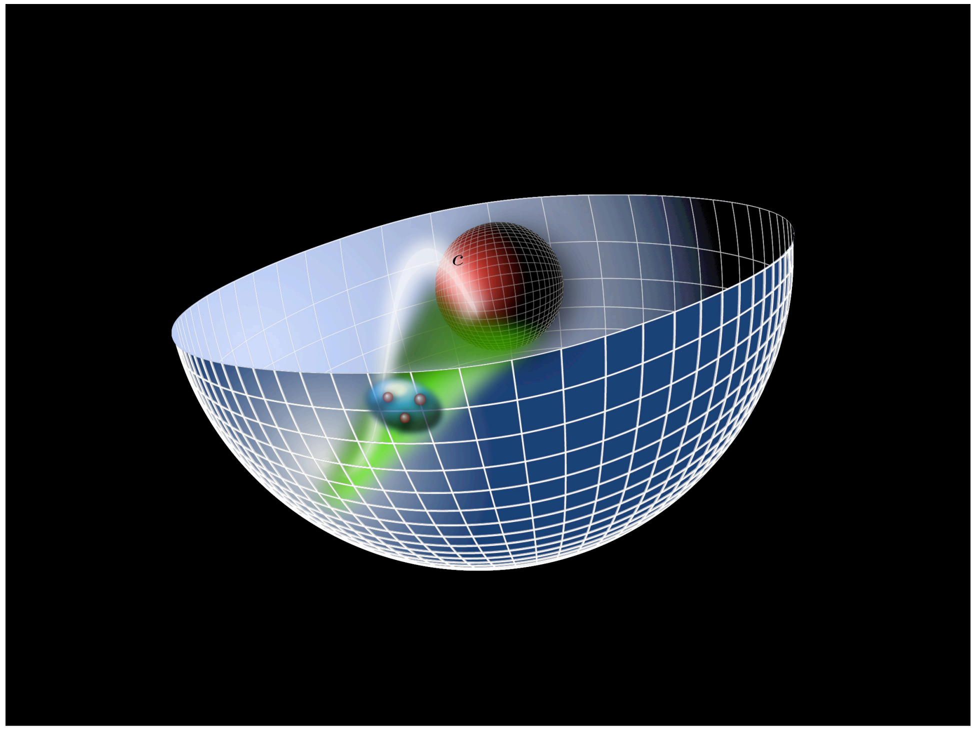
Pasquini, Xiao, Yuan, sjb

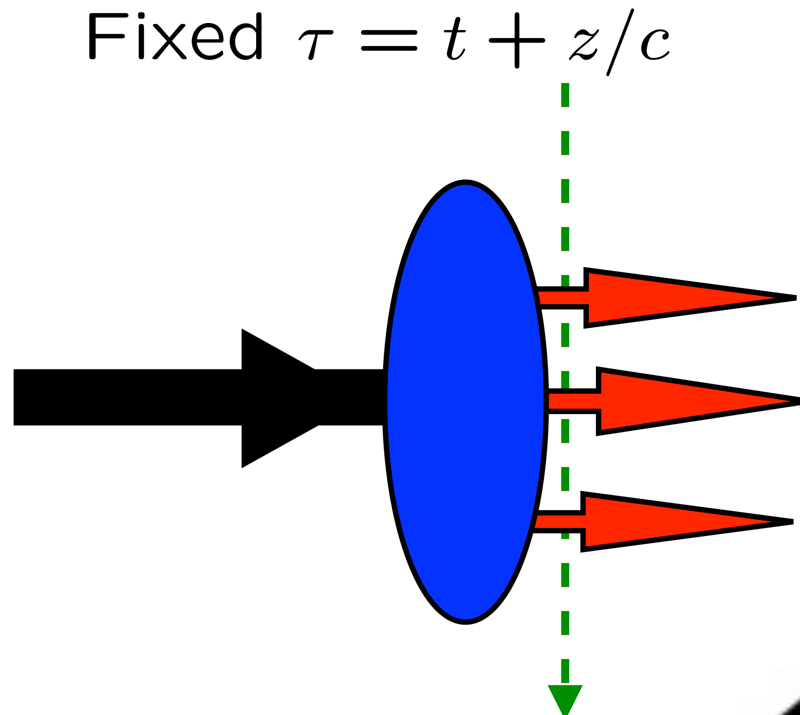


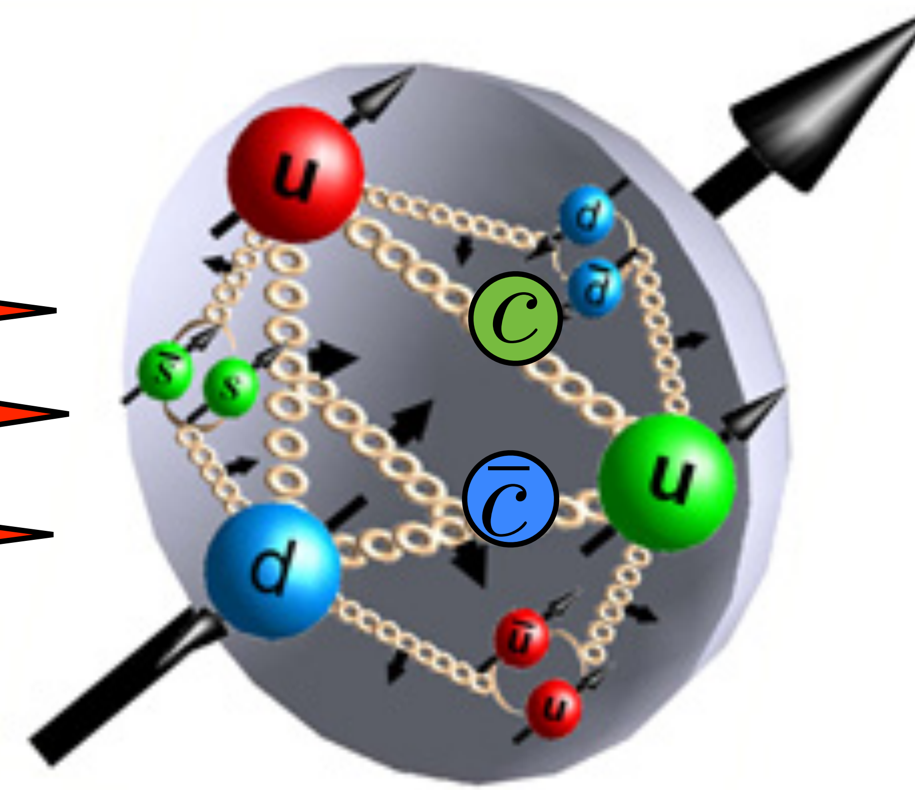
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Light-Front Holography, Transversity and Quark Orbital Angular Momentum







Part II

INT Workshop

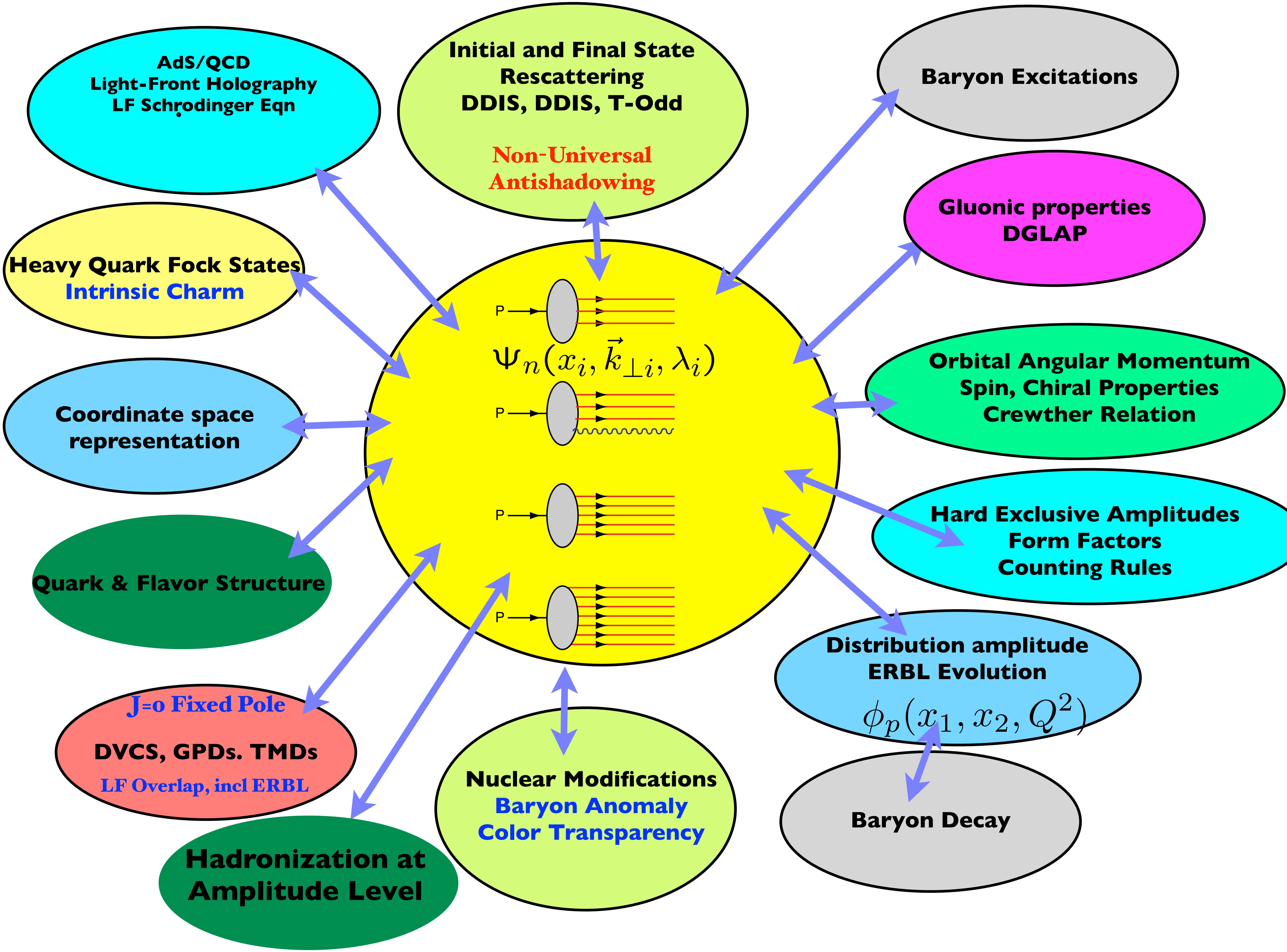
Orbital Angular Momentum in QCD

February 6 - 17, 2012

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

QCD and LF Hadron Wavefunctions

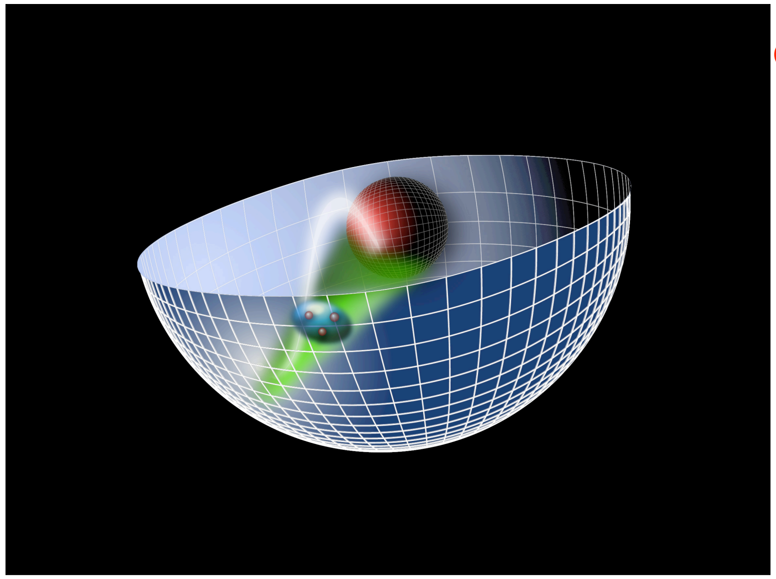


Goal: an analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

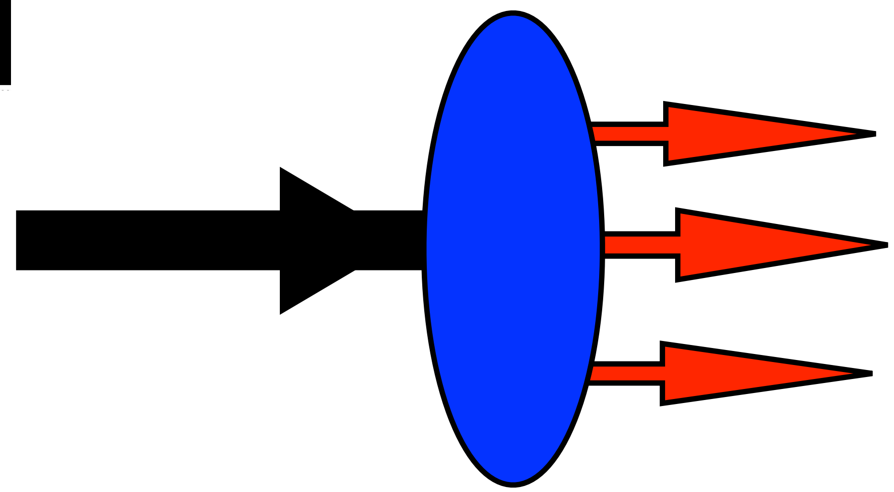
Guy de Teramond, sjb

$\phi(z)$

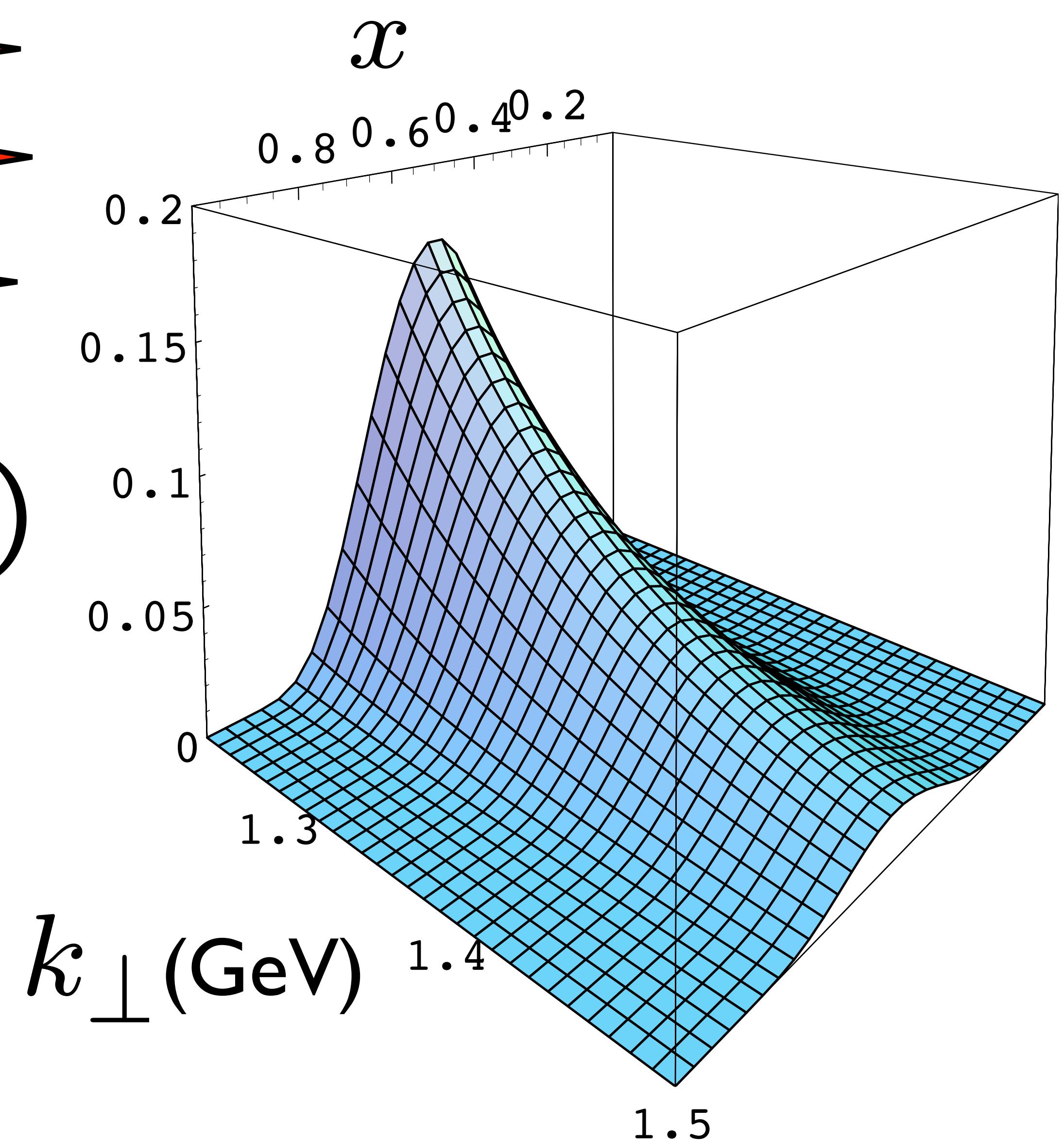


- *Light-Front Holography*

Remarkable new insights from AdS/CFT



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



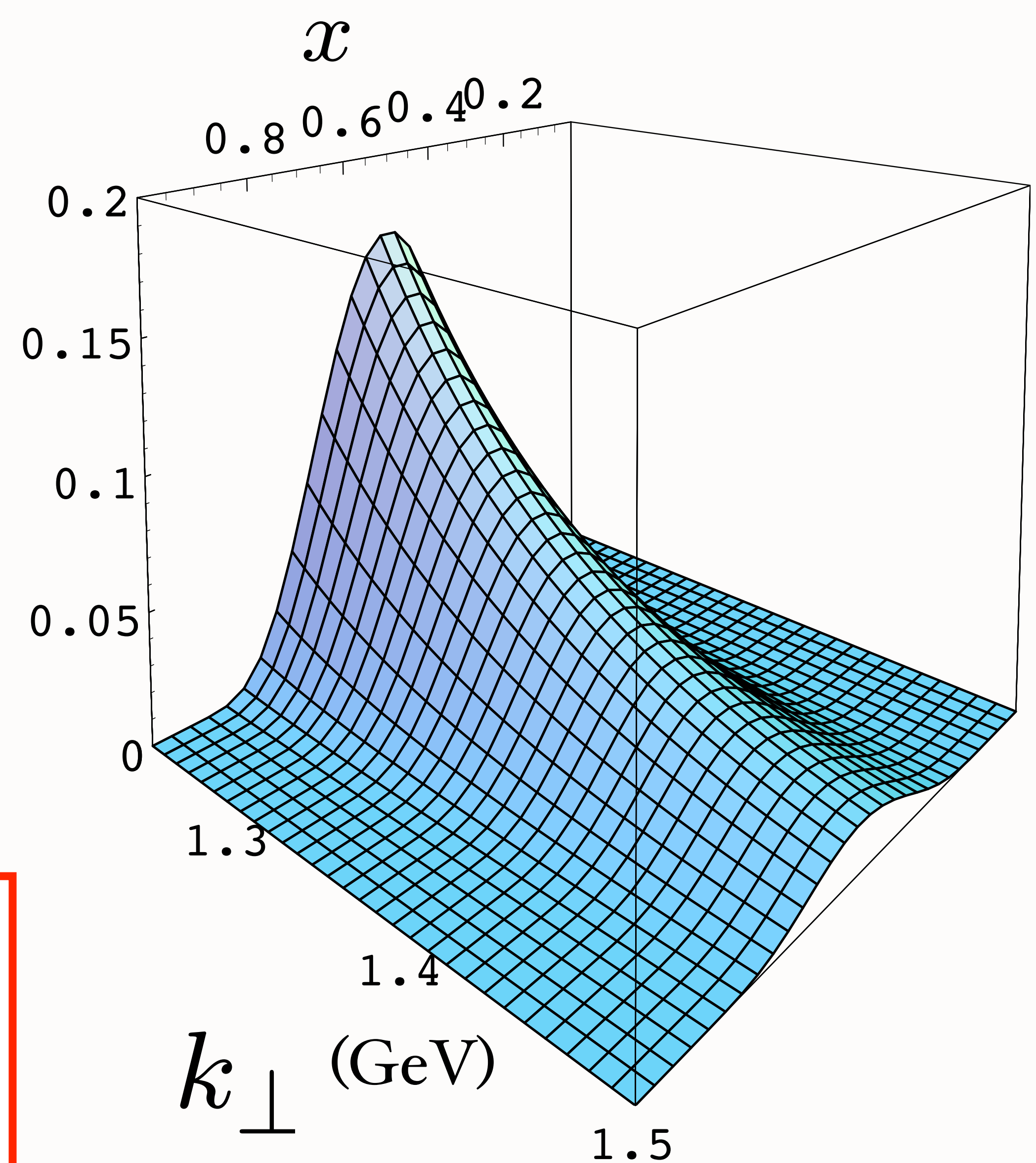
- *Light Front Wavefunctions:*

Schrödinger Wavefunctions
of Hadron Physics

Prediction from AdS/QCD: Meson LFWF

de Teramond, sjb

$$\psi_M(x, k_\perp^2)$$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

Generalized parton distributions in AdS/QCD

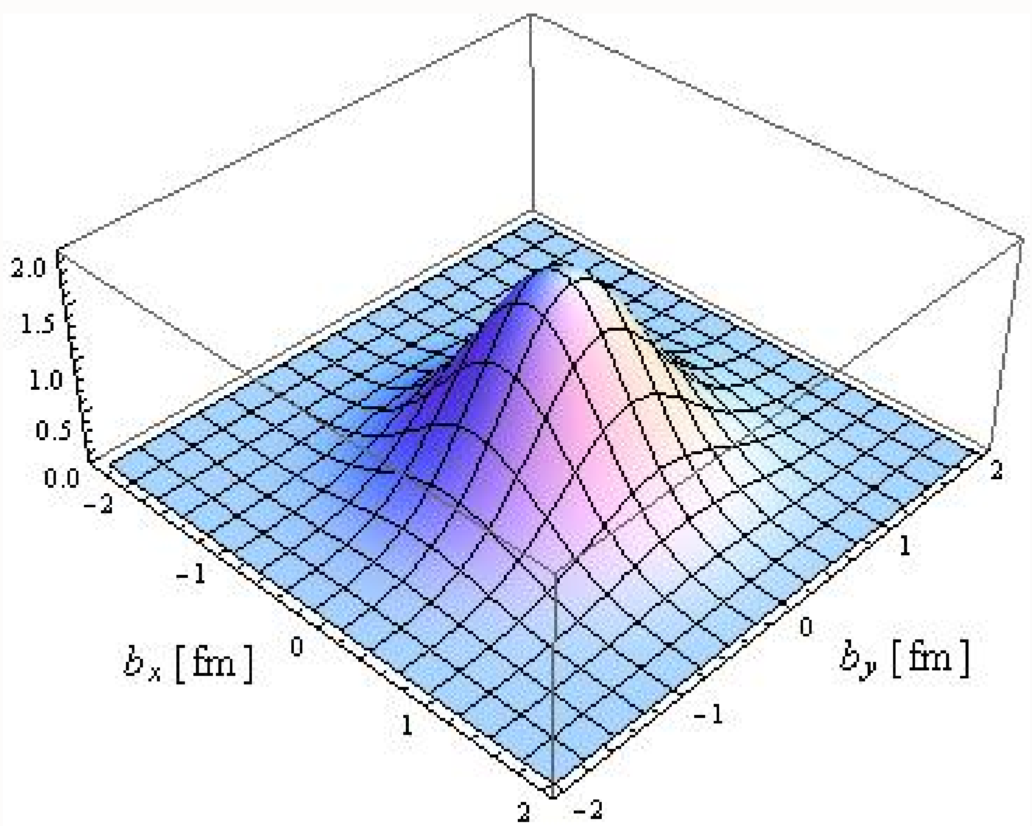
Alfredo Vega[1], Ivan Schmidt[1], Thomas Gutsche[2], Valery E. Lyubovitskij[2]*

[1] *Departamento de Física y Centro Científico y Tecnológico de Valparaíso,
Universidad Técnica Federico Santa María,
Casilla 110-V, Valparaíso, Chile*

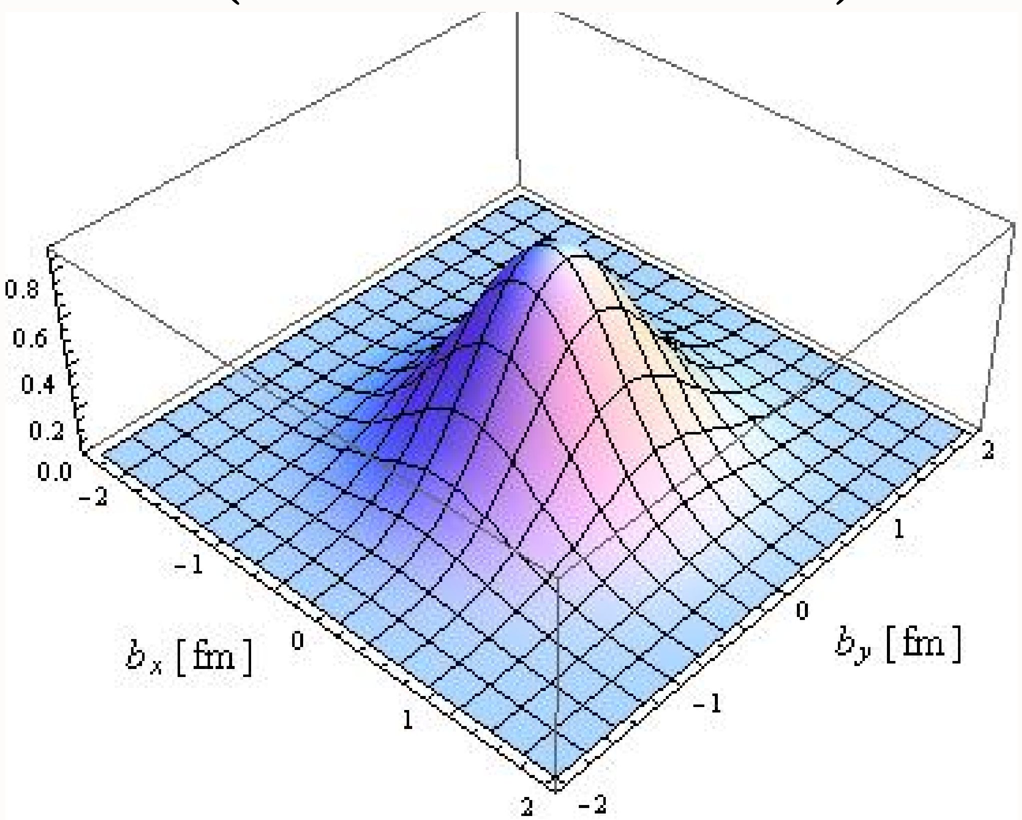
[2] *Institut für Theoretische Physik, Universität Tübingen,
Kepler Center for Astro and Particle Physics,
Auf der Morgenstelle 14, D–72076 Tübingen, Germany*

(Dated: January 19, 2011)

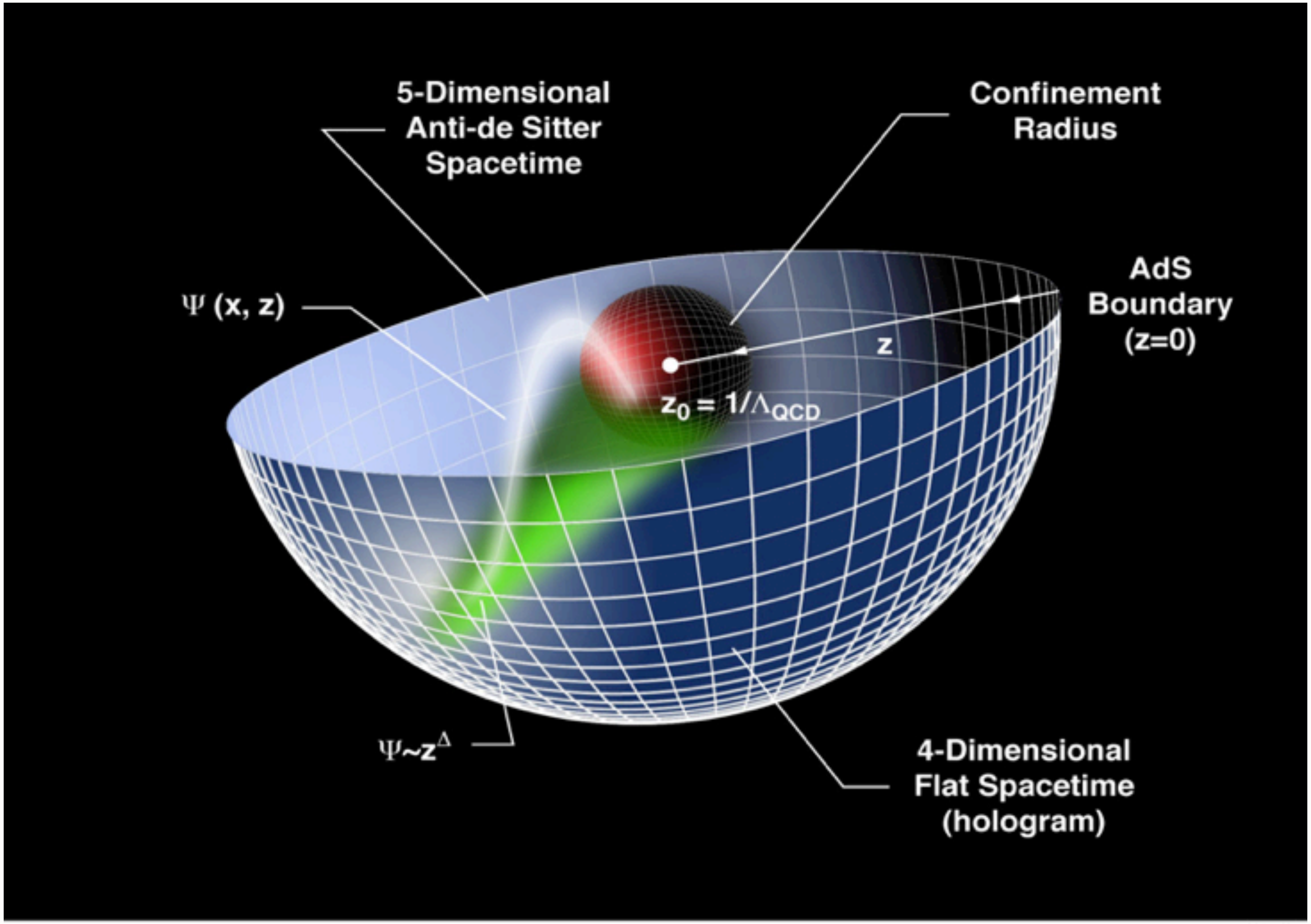
$$u(x = 0.1, \vec{b}_\perp)$$



$$d(x = 0.1, \vec{b}_\perp)$$



Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Soft-Wall Model

$$S = \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L}, \qquad \varphi(z) = \pm \kappa^2 z^2$$

Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field

Karch, Katz, Son and Stephanov (2006)]

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\right)$

$$\left[z^2 \partial_z^2 - \left(3 \mp 2\kappa^2 z^2\right) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$.

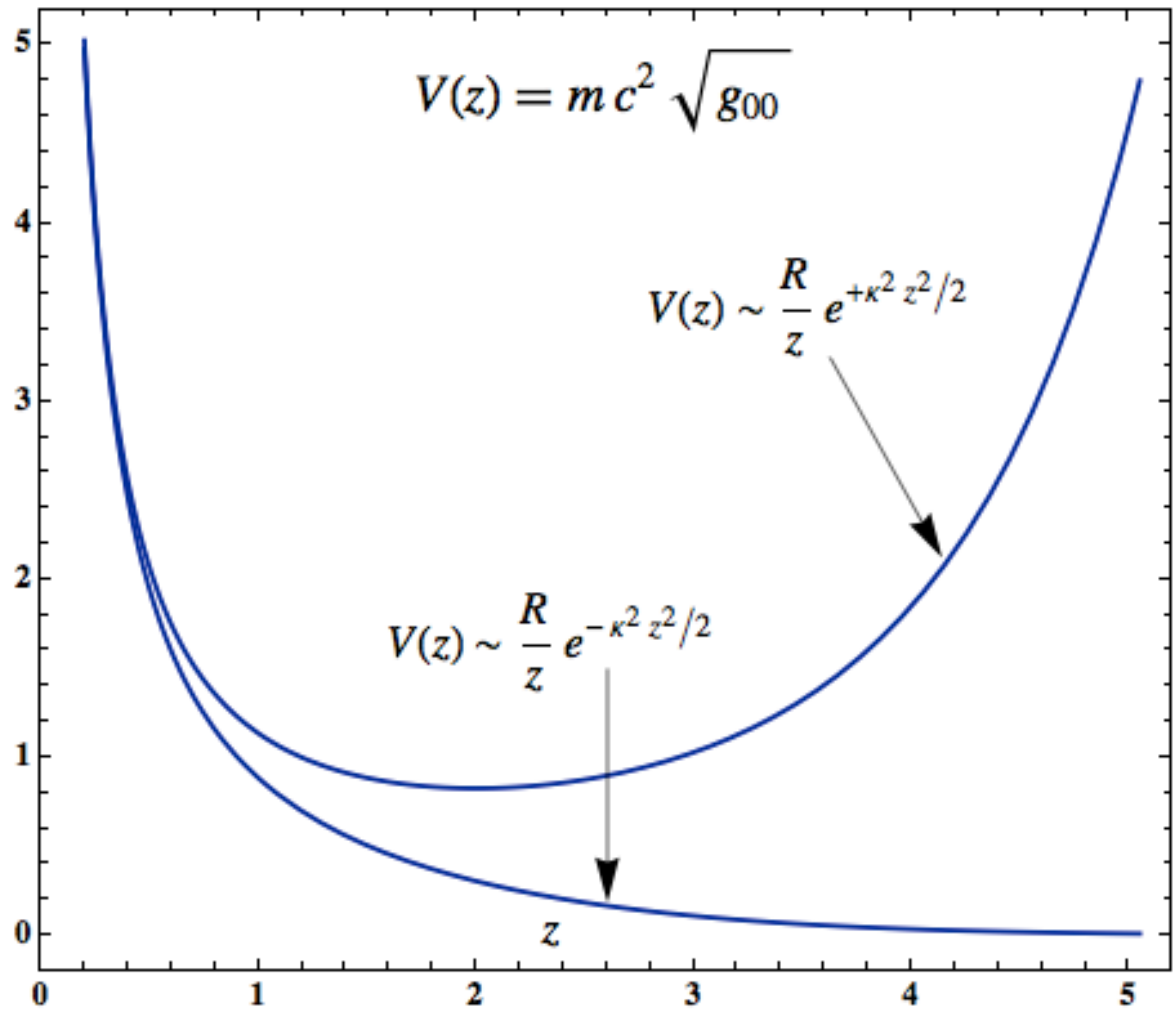
- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

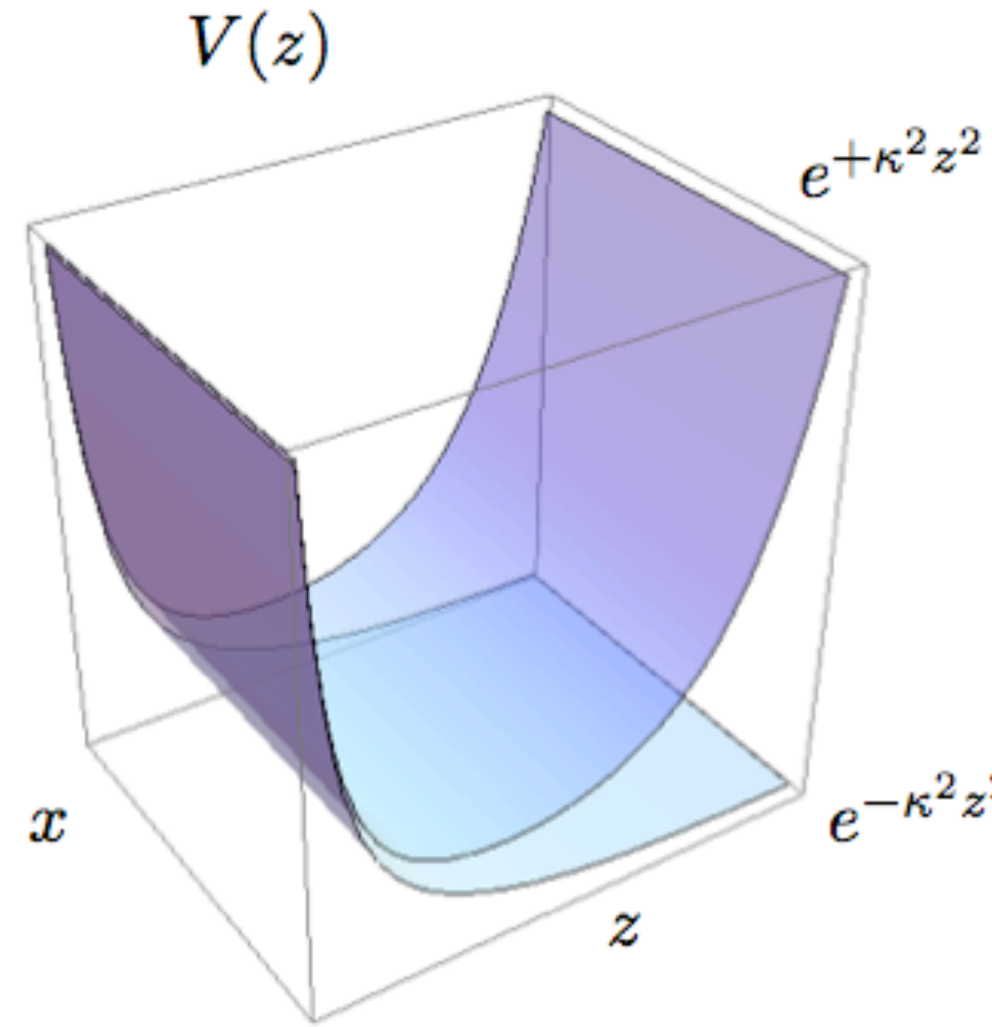
A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion

$$ds^2 = e^{\kappa^2 z^2}\frac{R^2}{z^2}(dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2)$$



Agrees with Klebanov and Maldacena for positive-sign exponent of dilaton



- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2} e^{2A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $A(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{A(z)}}{z}$$

- Consider warp factor $\exp(\pm \kappa^2 z^2)$
- Plus solution $e^{\kappa^2 z^2}$: $V(z)$ increases exponentially confining any object to distances $\langle z \rangle \sim 1/\kappa$
- Minus solution $e^{-\kappa^2 z^2}$: does not provides area law for the Wilson loop

- Erlich, Karch, Katz, Son, Stephanov
- de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\phi(z) = \mathcal{M}^2\phi(z)$$

Identify L from twist of interpolating operator at z=0

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action
Dilaton-Modified AdS_5

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

Quark separation increases with L

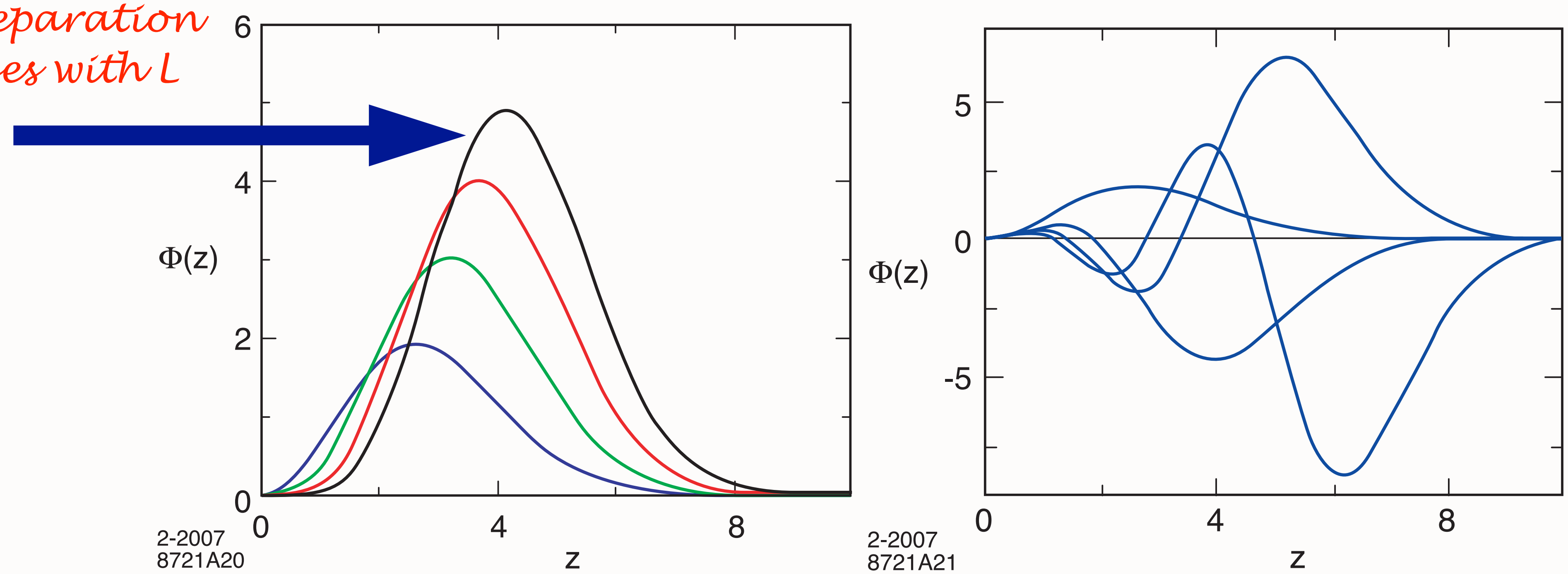


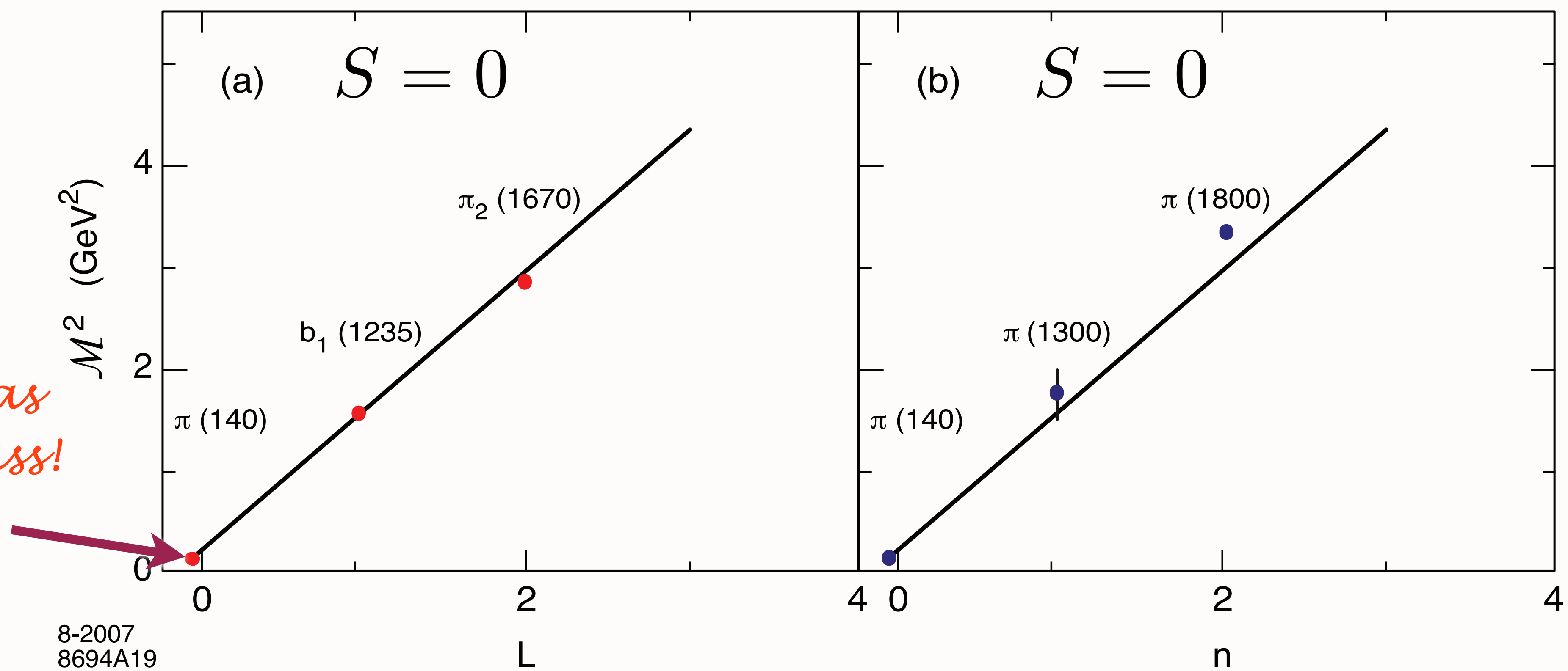
Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Pion mass automatically zero!

$$m_q = 0$$

Pion has zero mass!



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

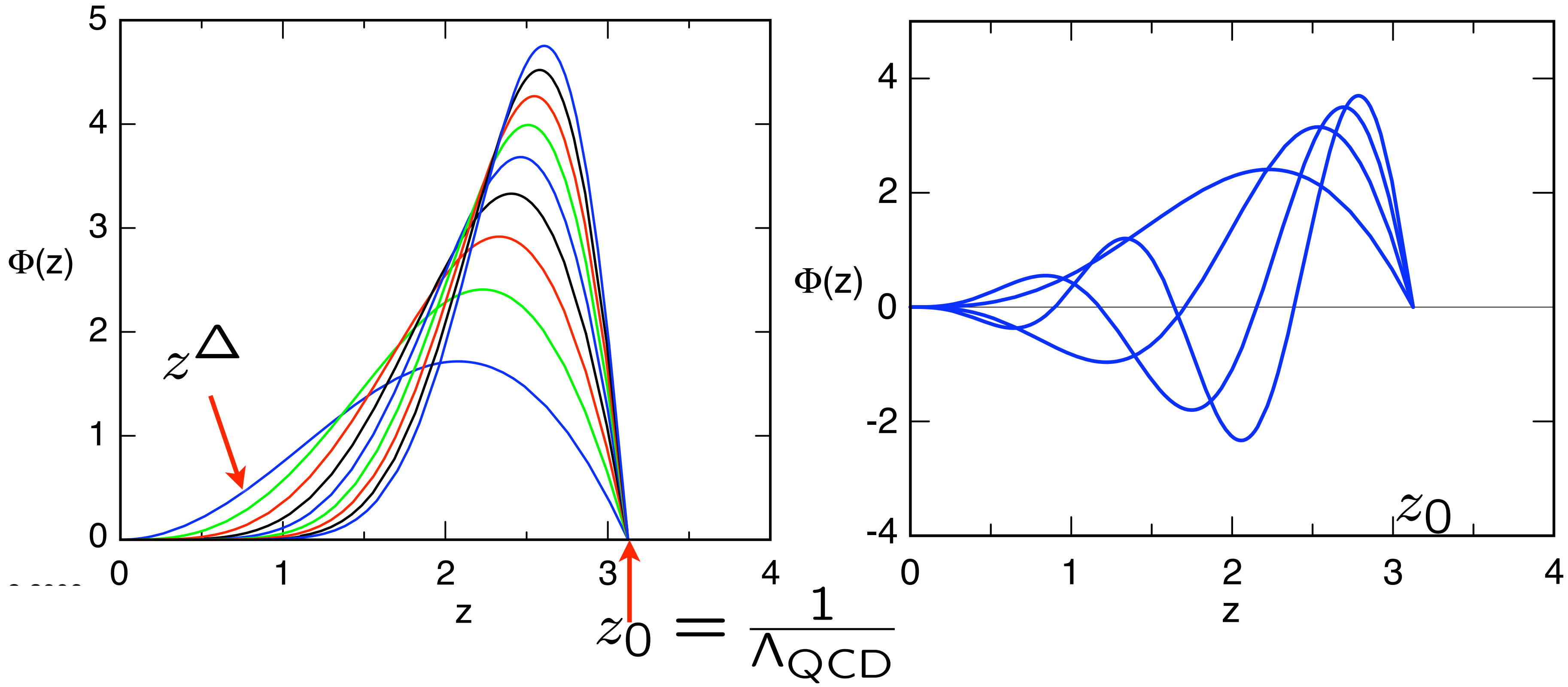
Match fall-off at small z to conformal twist-dimension at short distances

twist

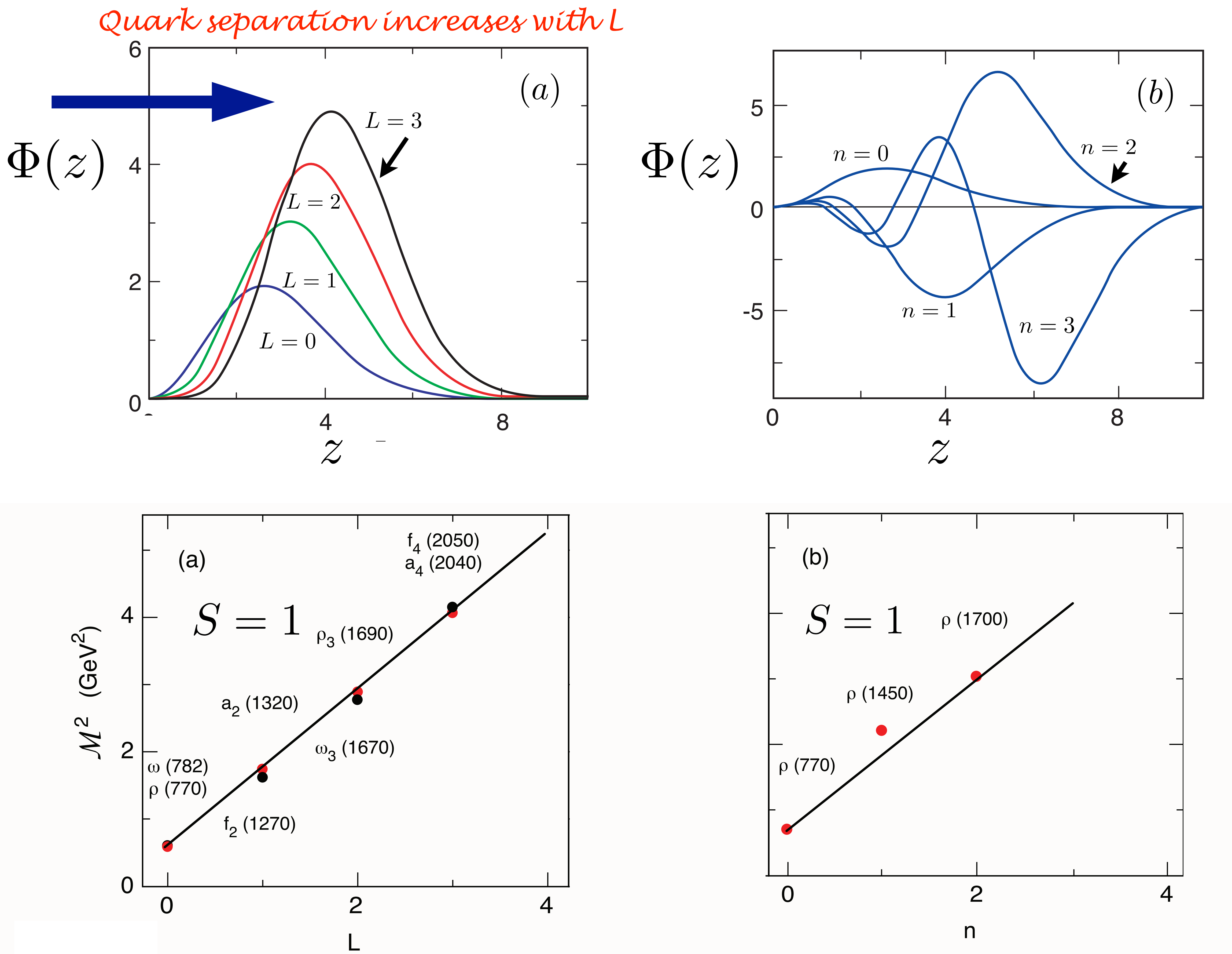
- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

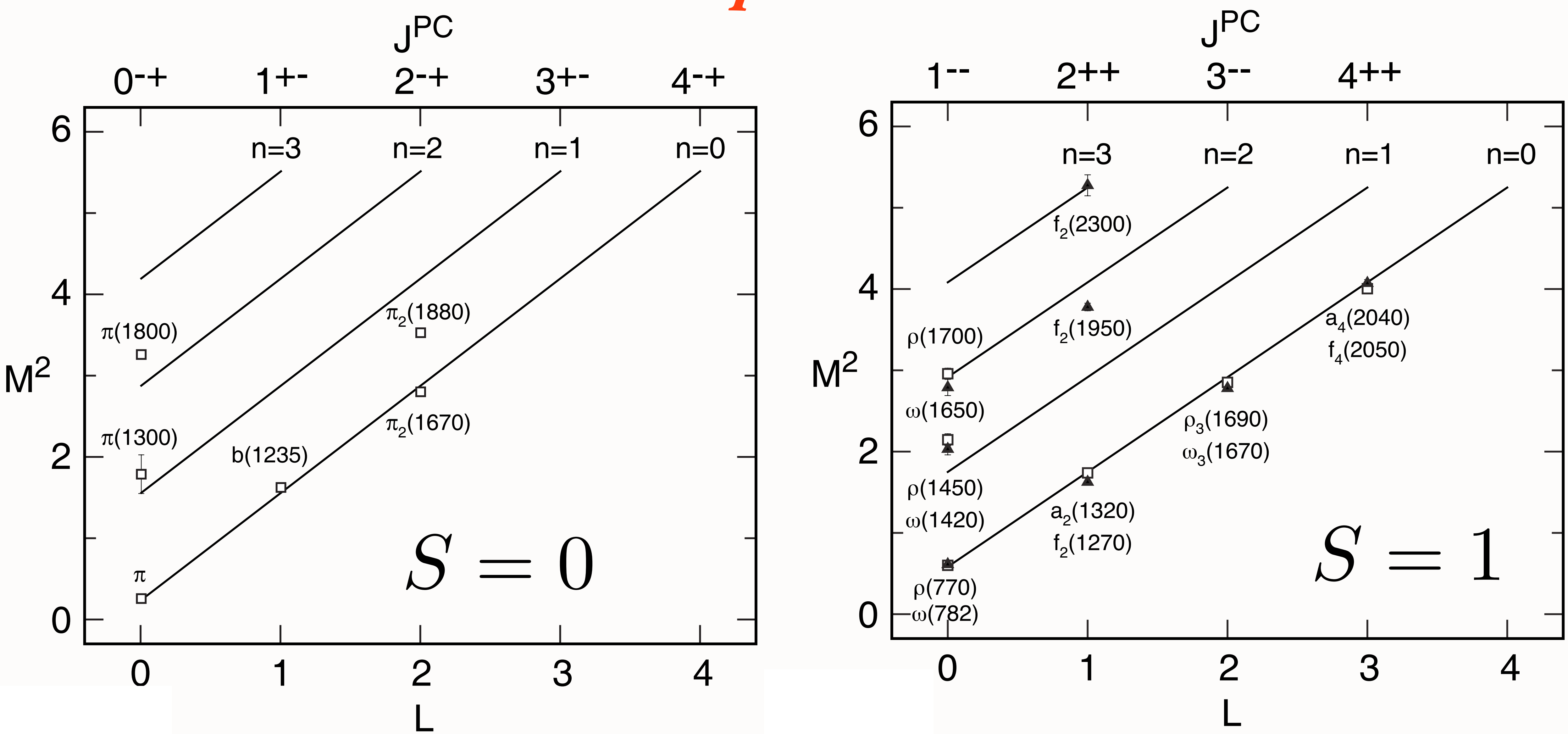

Quark separation increases with L
Φ(z)
(a)
L = 3
L = 2
L = 1
L = 0
z
(b)
n = 0
n = 1
n = 2
n = 3
(a)
S = 1
f4 (2050)
a4 (2040)
ρ3 (1690)
a2 (1320)
ω3 (1670)
ω (782)
ρ (770)
f2 (1270)
M² (GeV²)
L
(b)
ρ (1700)
ρ (1450)
ρ (770)
n

Bosonic Modes and Meson Spectrum

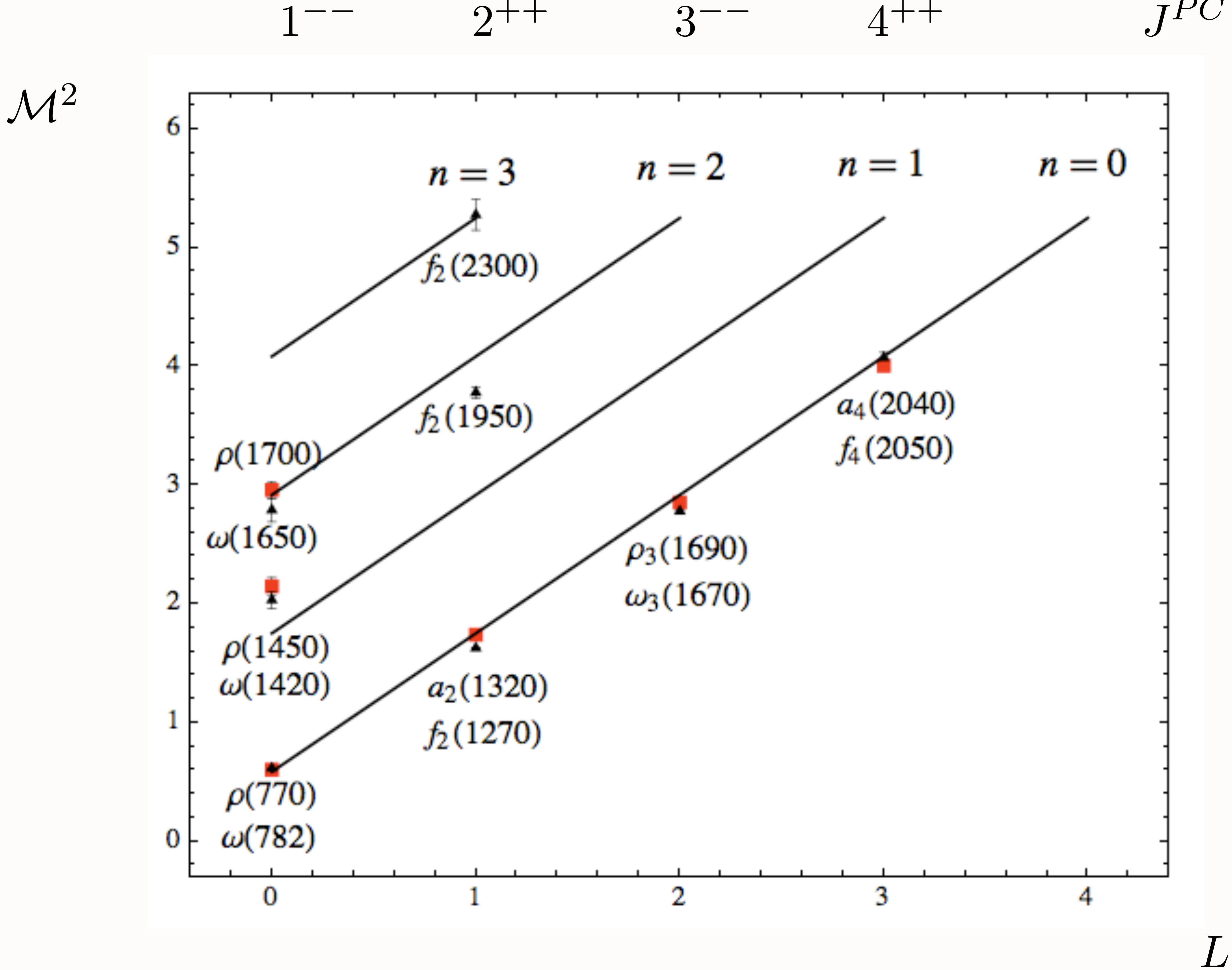
$$\mathcal{M}^2 = 4\kappa^2(n + J/2 + L/2) \to 4\kappa^2(n + L + S/2)$$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Same slope in n and L



Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I = 1$ ρ-meson and $I = 0$ ω-meson families ($\kappa = 0.54$ GeV)



Parent and daughter Regge trajectories for the $I=1$ ρ-meson family (red) and the $I=0$ ω-meson family (black) for $\kappa = 0.54$ GeV

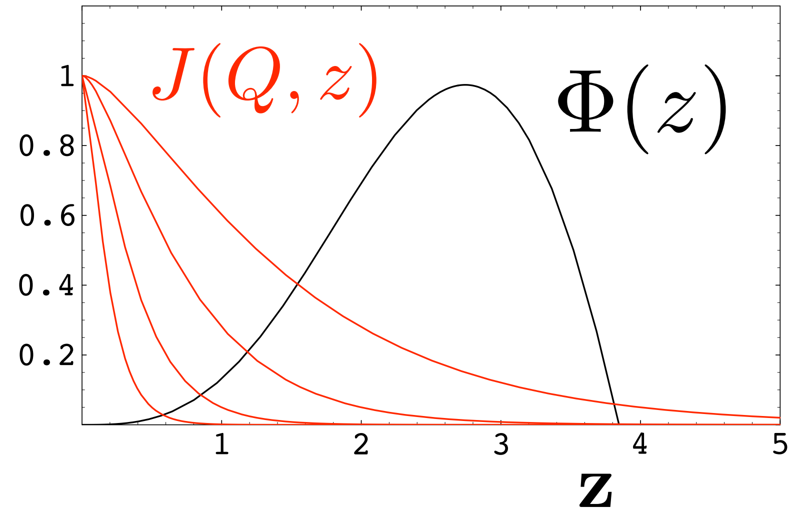
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

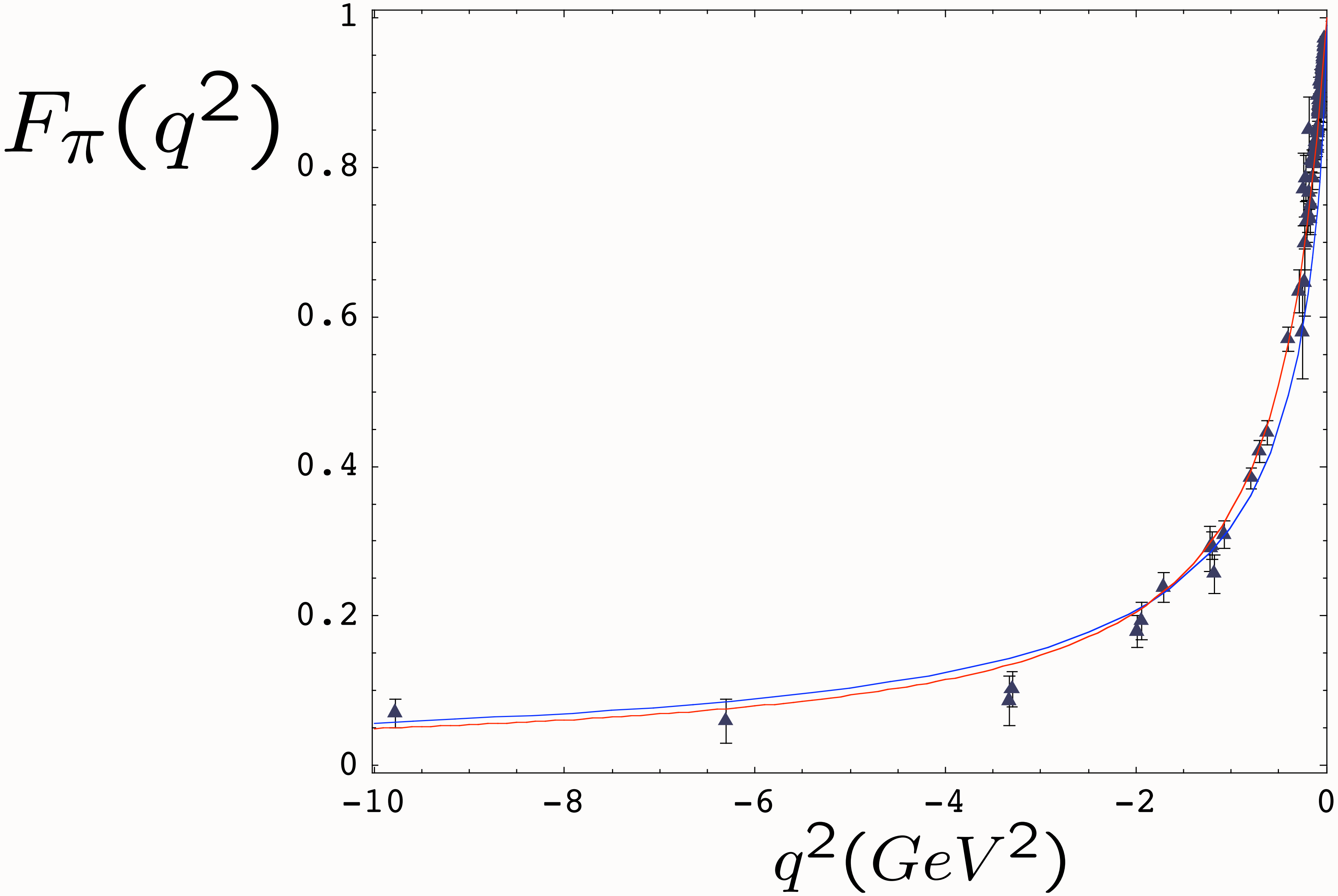
- Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^{\,2} = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x,b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\left(bqx\right)\, \left|\widetilde{\psi}(x,b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\boxed{\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

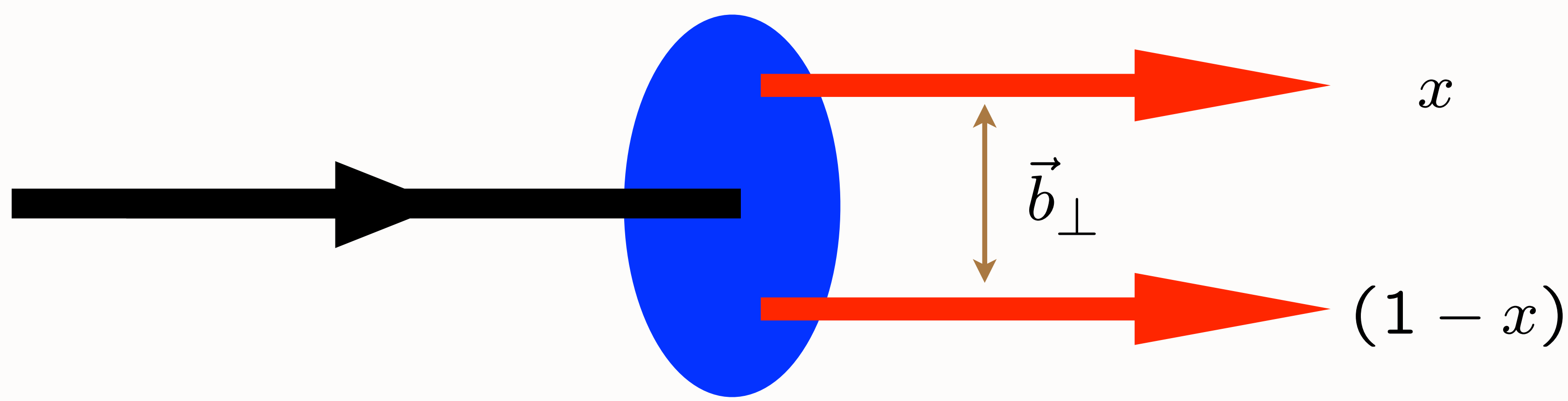
$$F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q,z) \Phi_I(z)$$

de Teramond, sjb

LF(3+1) ⟷ AdS_5

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}} \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) \left|\Phi_\pi(z)\right|^2 ,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0\left(zQ \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\left|\tilde\psi_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},$$

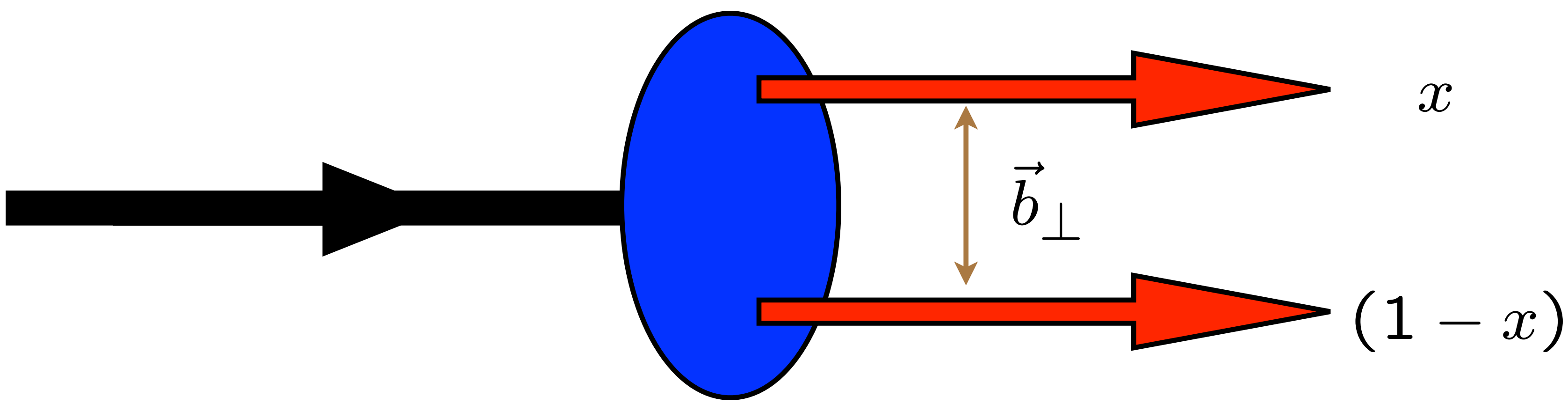
Identical to LF Holography obtained from electromagnetic current

Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

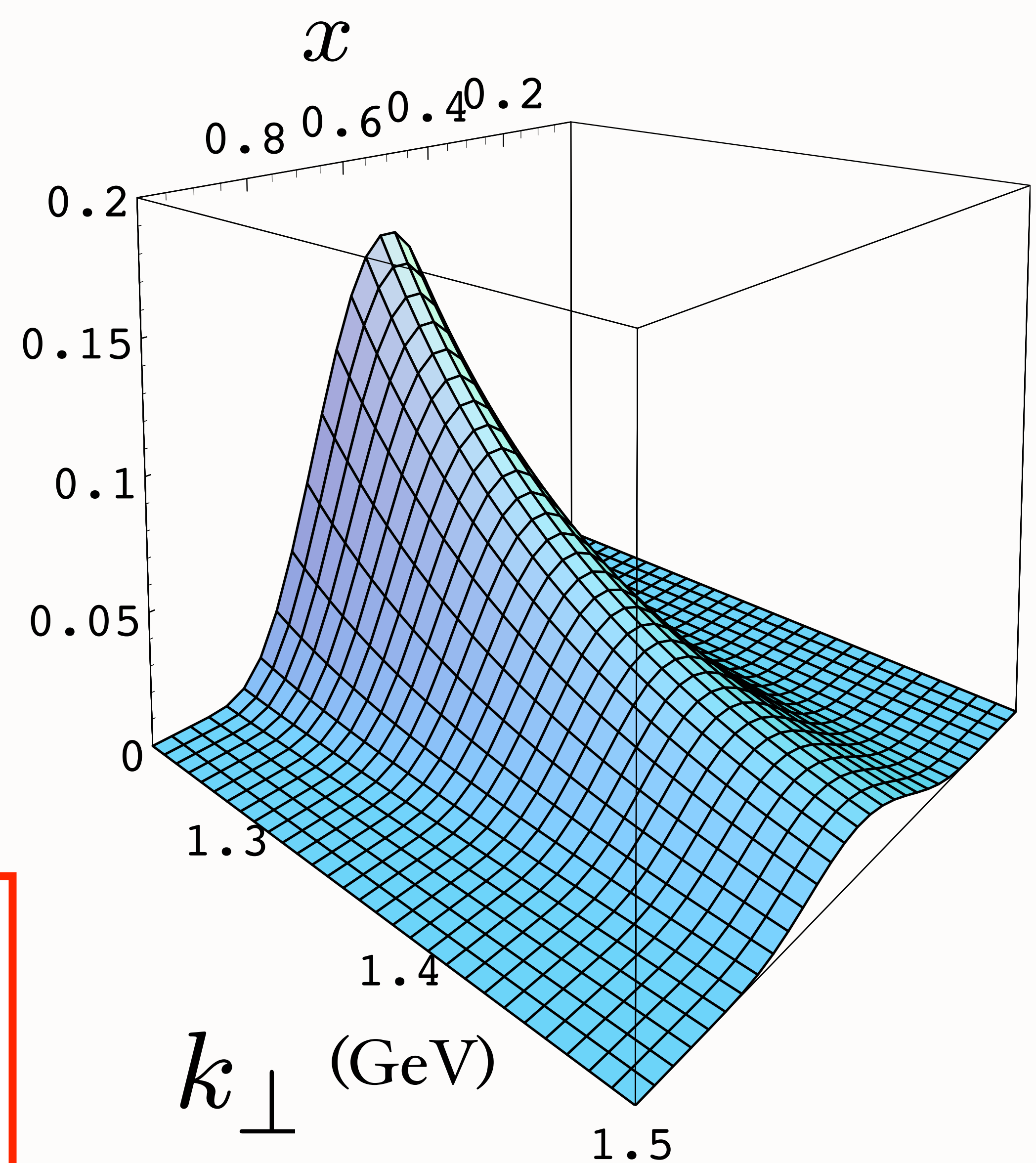
soft wall confining potential:

G. de Teramond, sjb

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

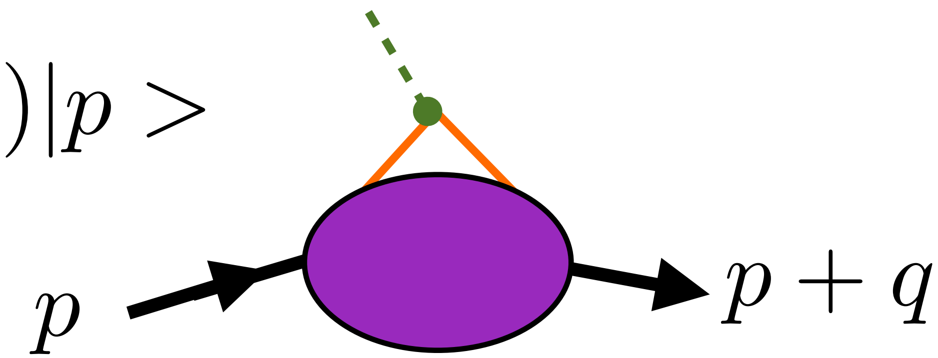
Light-Front Holography

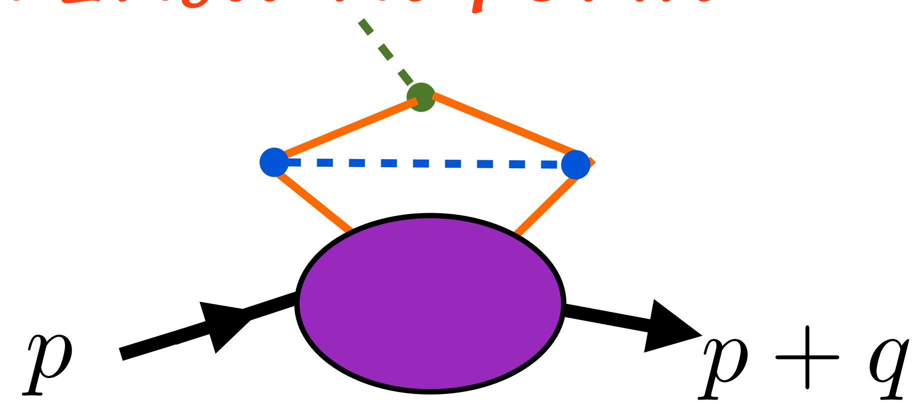
AdS Space matches 3+1 spacetime at fixed Light-Front Time!

- *Matching of AdS and LF Expressions for EM and Gravitational Form Factors*
- *Overlap of LFWFs Only -- No Vacuum Currents*
- *No Instant-Time formula*
- *Matches Equations of LF Hamiltonian Theory*
- *Matches LF Kinetic Energy*
- *Angular Momentum Matches to AdS Mass*

Calculation of proton form factor in Instant Form

$$< p+q|J^\mu(0)|p >$$





- **Need to boost proton wavefunction from p to p+q: Extremely complicated dynamical problem; particle number changes**
- **Need to couple to all currents arising from vacuum!!**
- **Each time-ordered contribution is frame-dependent**
- **States built on normal-ordered acausal vacuum**
- **Divide by disconnected vacuum diagrams**

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

QCD Meson Spectrum

$$H_{QCD}^{LF}$$

$$(H_{LF}^0 + H_{LF}^I)|\Psi> = M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\rm eff}^{LF}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$[-\frac{d^2}{d\zeta^2} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

de Teramond, sjb

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables $(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

$$+ \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

- To first approximation LF dynamics depend only on the invariant variable ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x,\zeta,\varphi) = e^{iL^z\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular φ, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$ $(P^+, \mathbf{P}_\perp, J_z$ commute with $P^-)$

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $(L = |L^z|)$

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta)\, U(\zeta)\, \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential $U(\zeta)$

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\Big(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \Big) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable
- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time

Baryons in AdS/QCD

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\, \psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

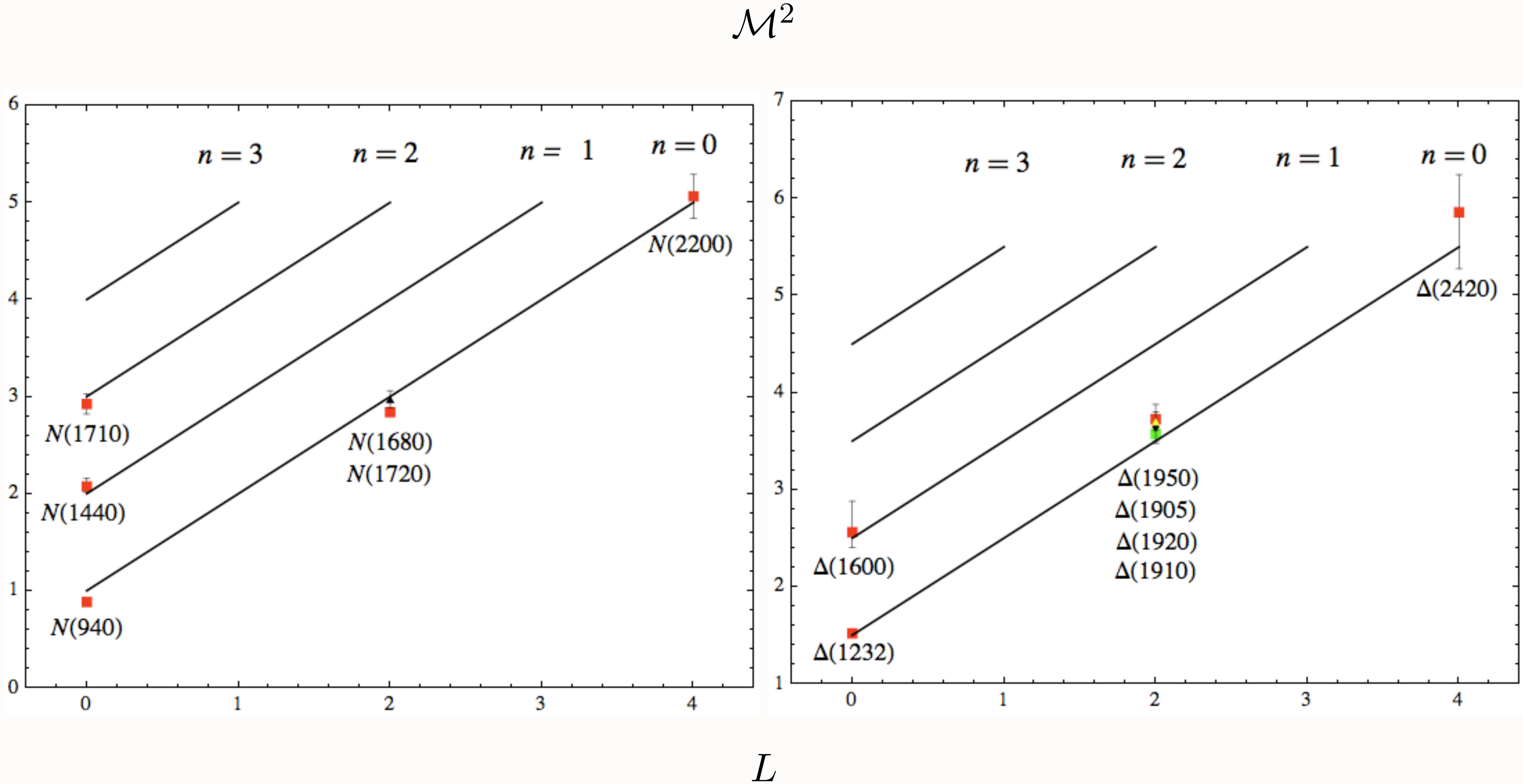
- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

- **Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)**

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Same multiplicity of states for mesons and baryons!



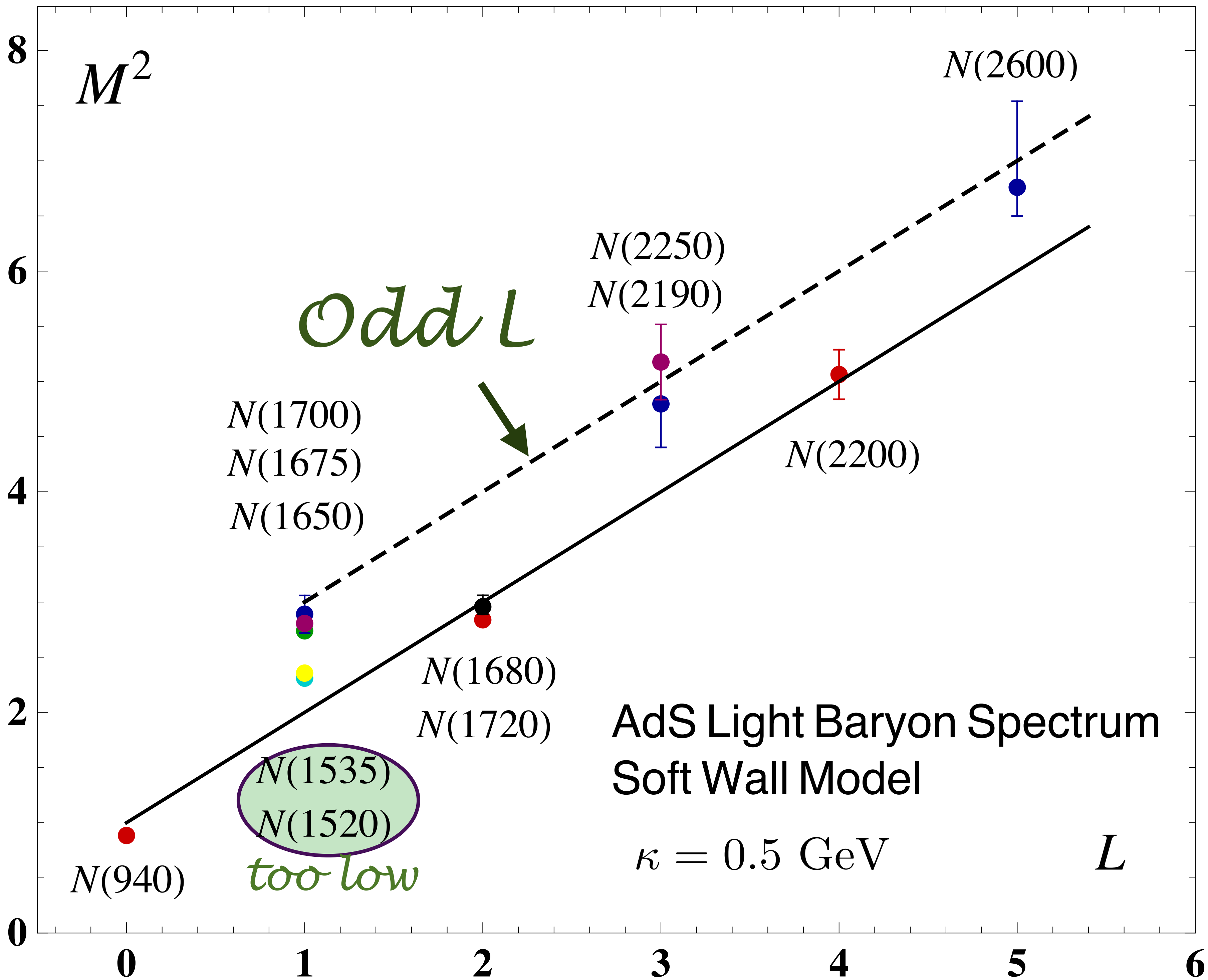
Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

Positive Parity Nucleons

Negative Parity Nucleons

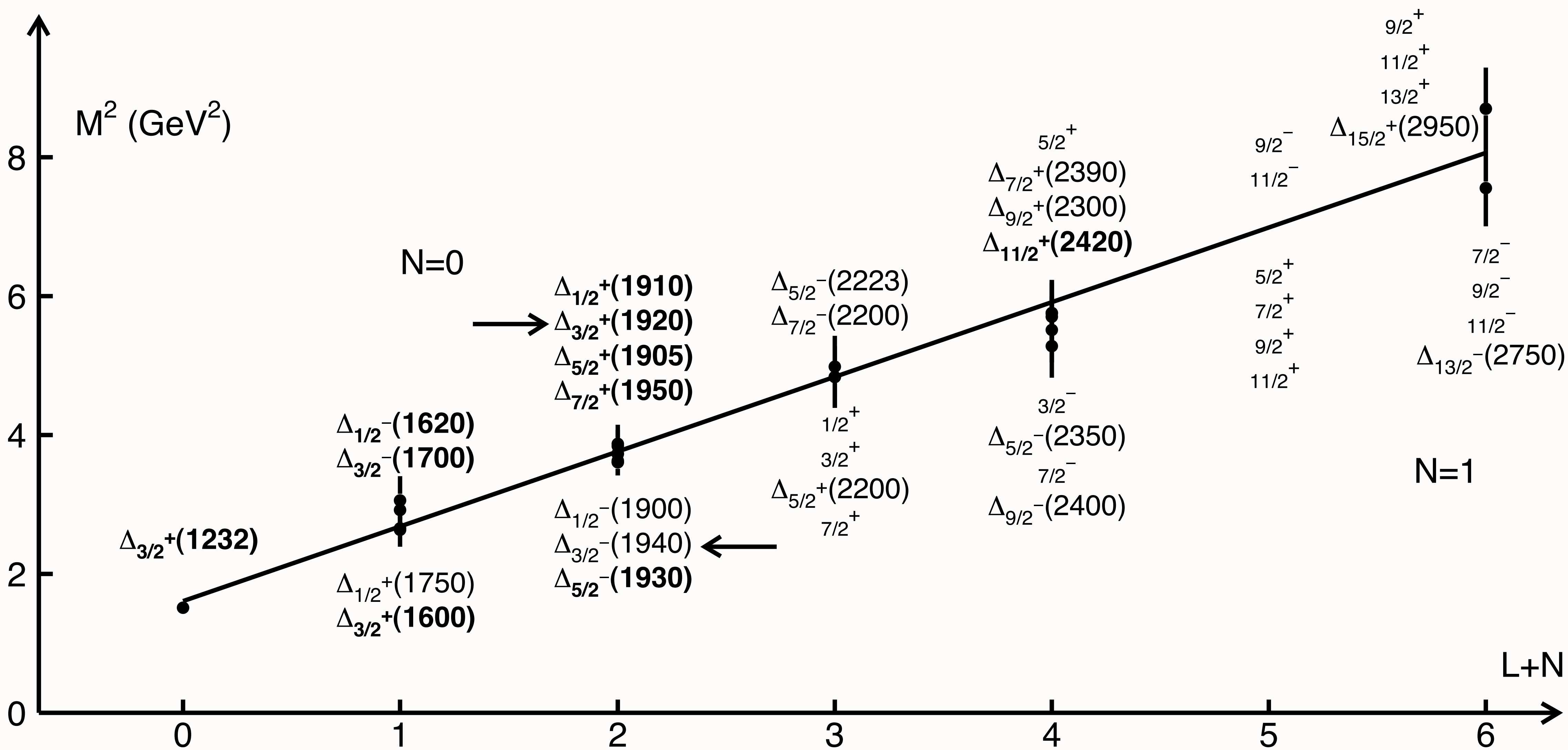
$$M^2 = 4\kappa^2(n + L + 1)$$

$$M^2 = 4\kappa^2(n + L + 2)$$



$L + 1 = \nu = \mu R - 1/2$ **(even P)**

$L + 1 = \nu = \mu R + 1/2$ **(odd P)**



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

Chiral Features of Soft-Wall AdS/QCD Model

Proton spin carried by quark angular momentum!

- **Boost Invariant**
- **Trivial LF vacuum.**
- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S^z_q = 0 >$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

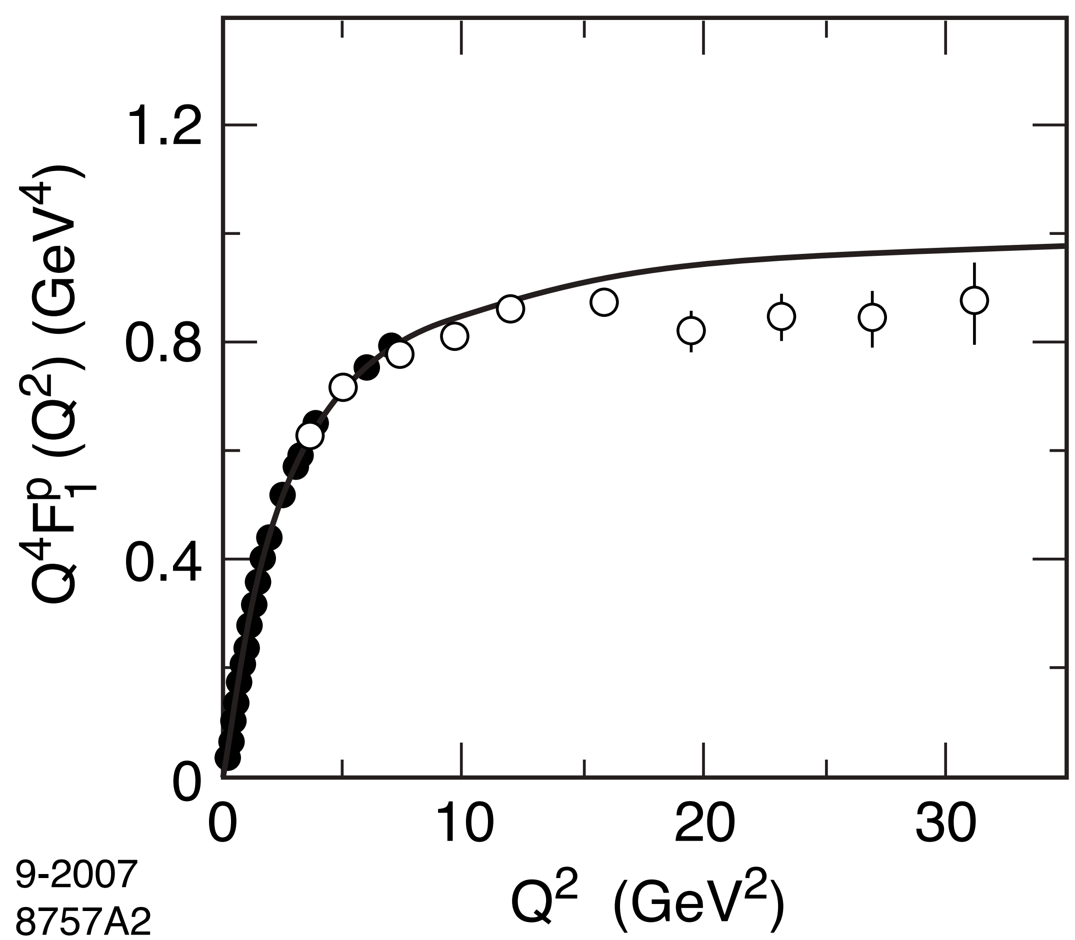
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

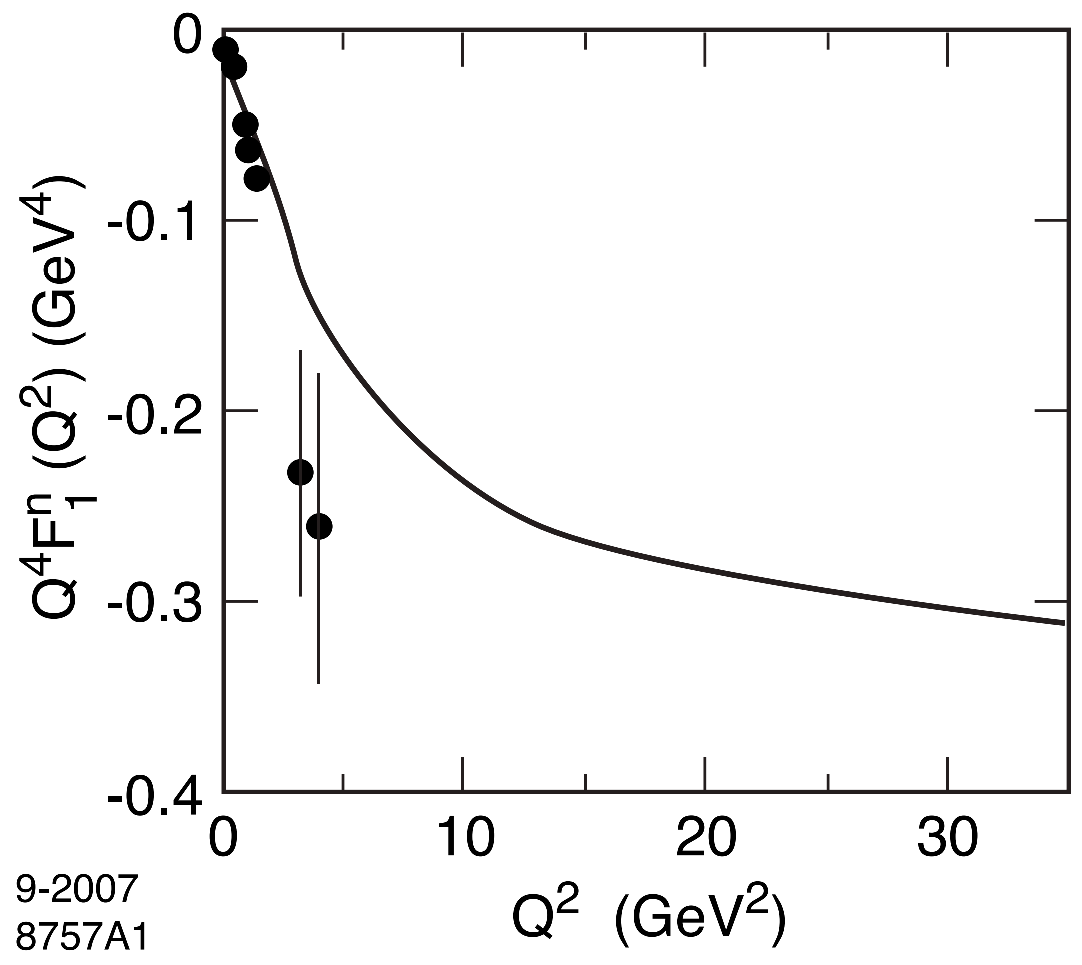
where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

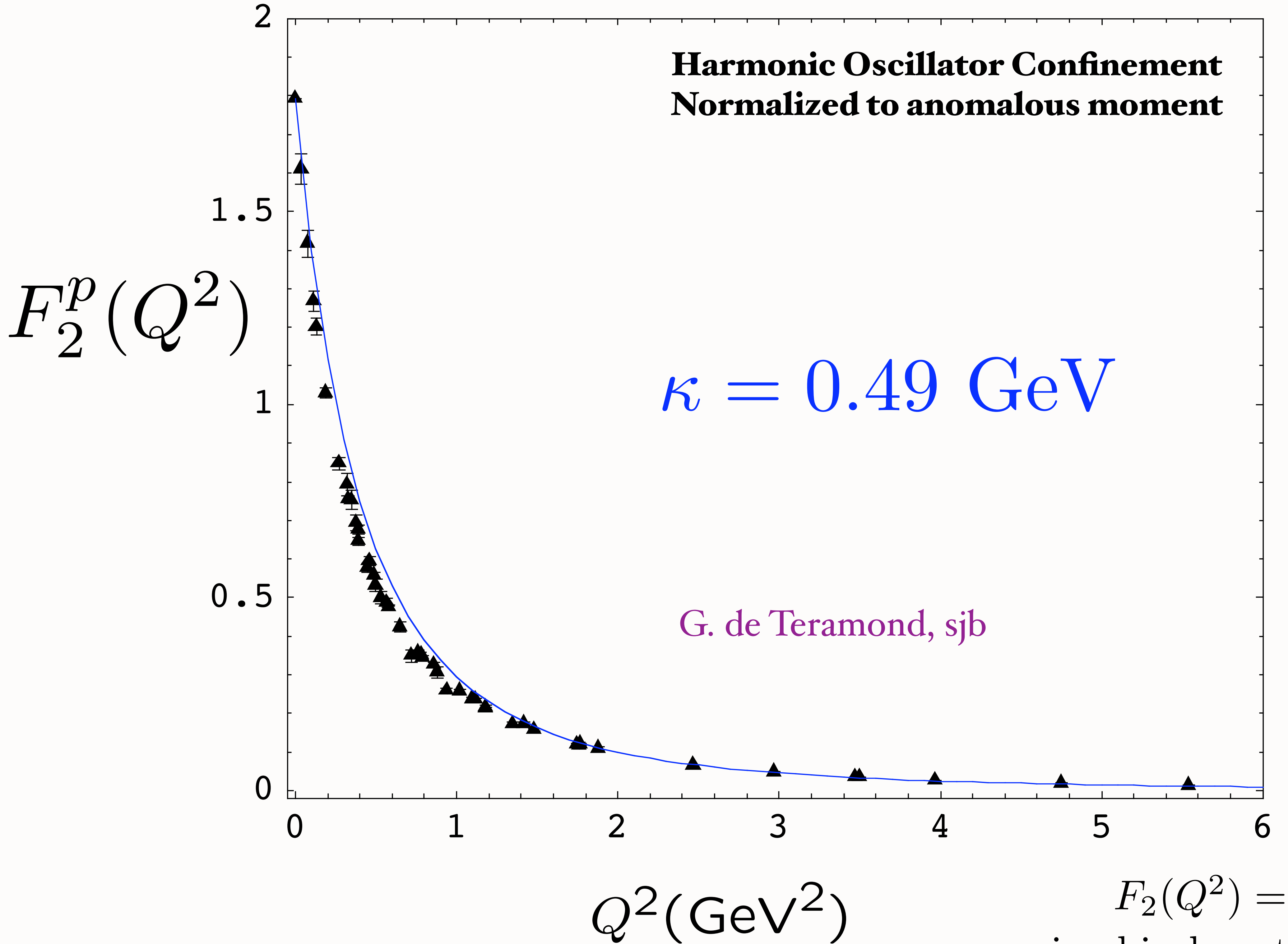


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



AdS/QCD No chiral divergence!

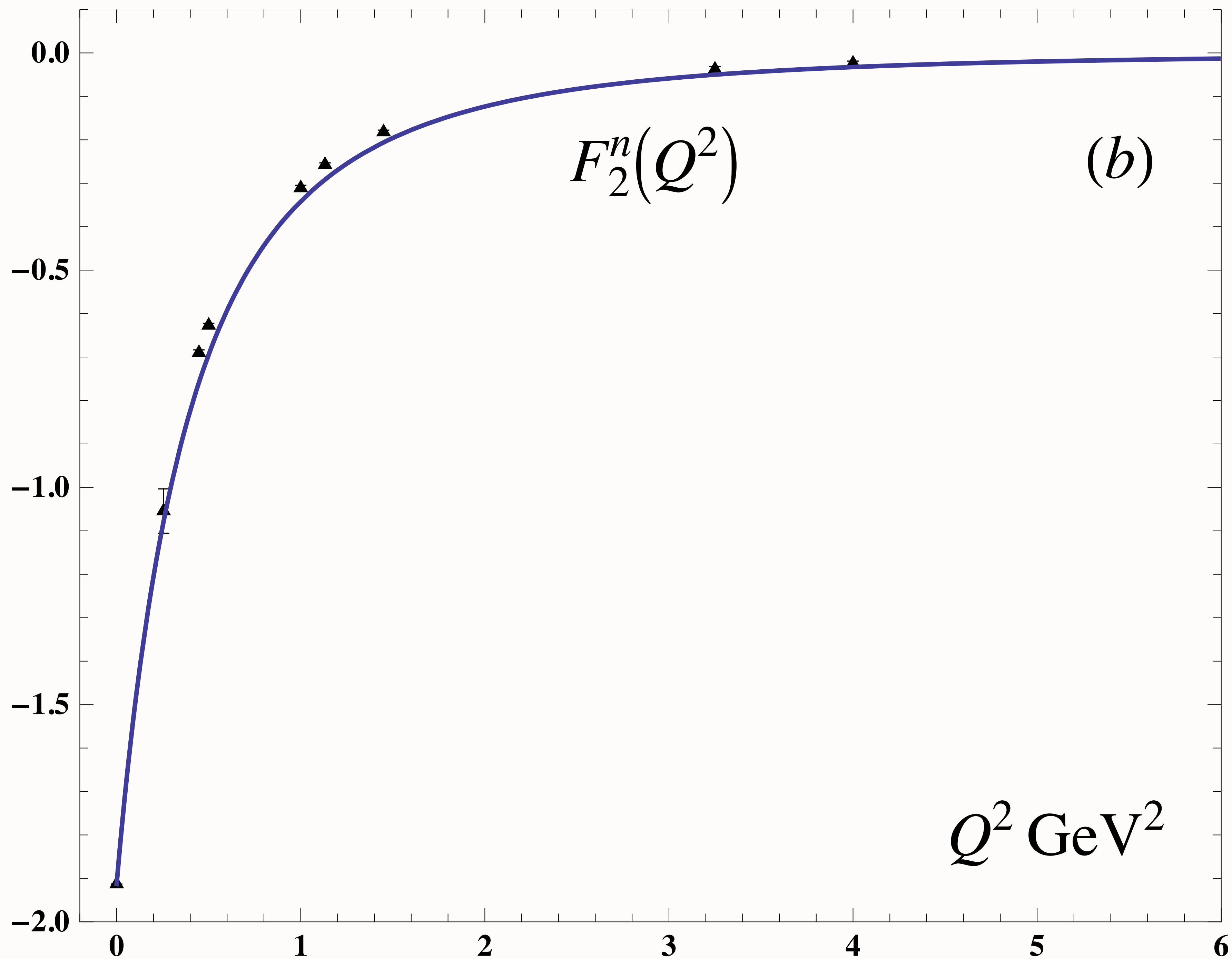
$$F_2(Q^2) = 1 + \mathcal{O}\frac{Q^2}{m_\pi m_p}$$

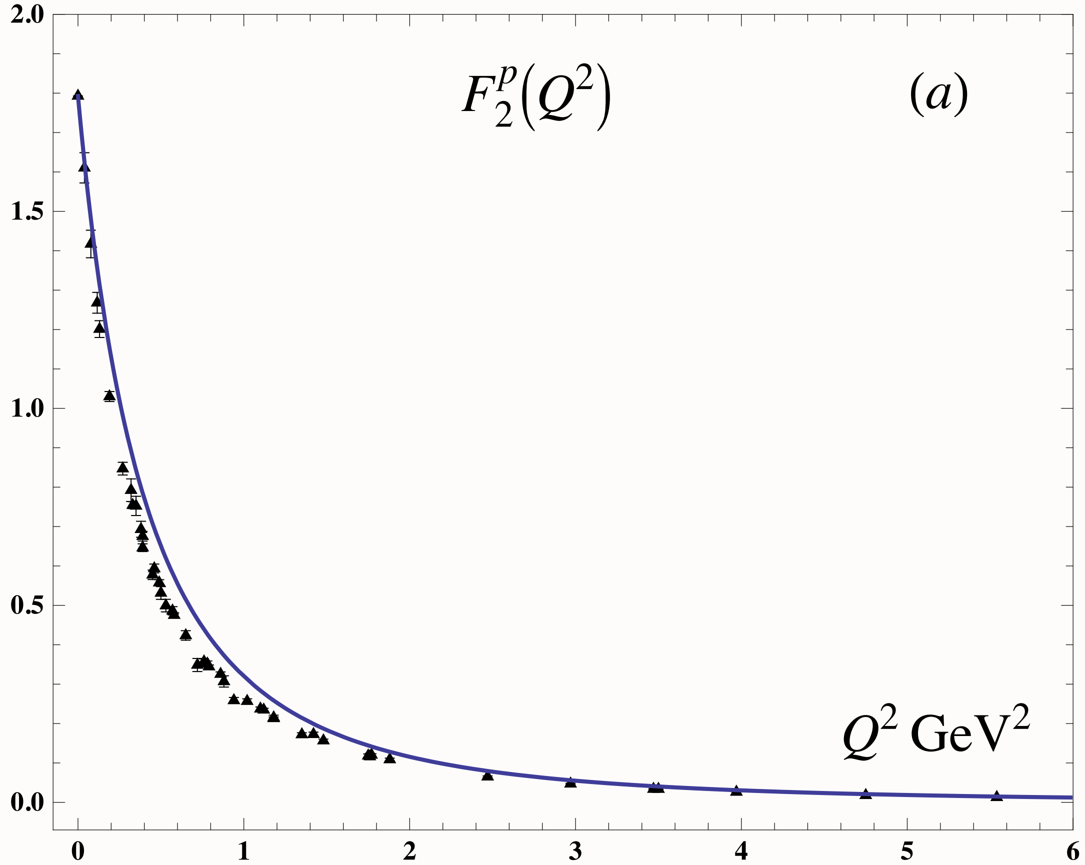
in chiral perturbation theory

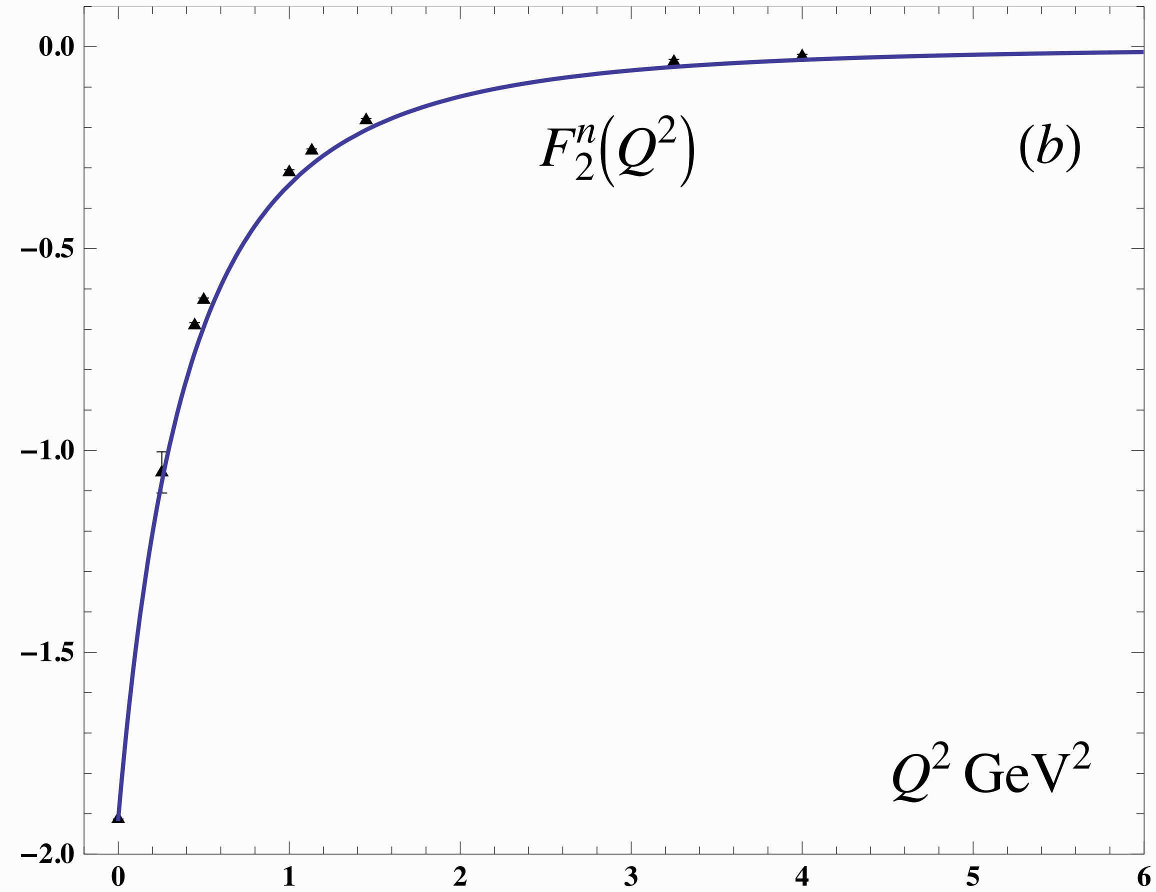
Spacelike Neutron Pauli Form Factor

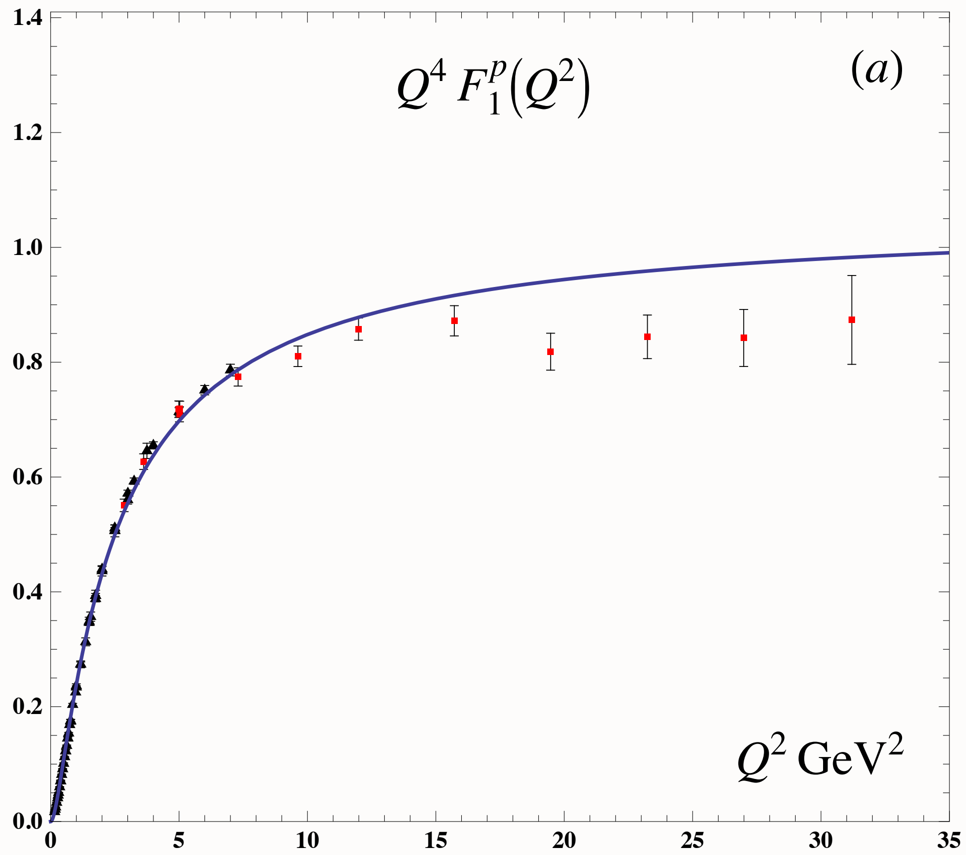
Preliminary

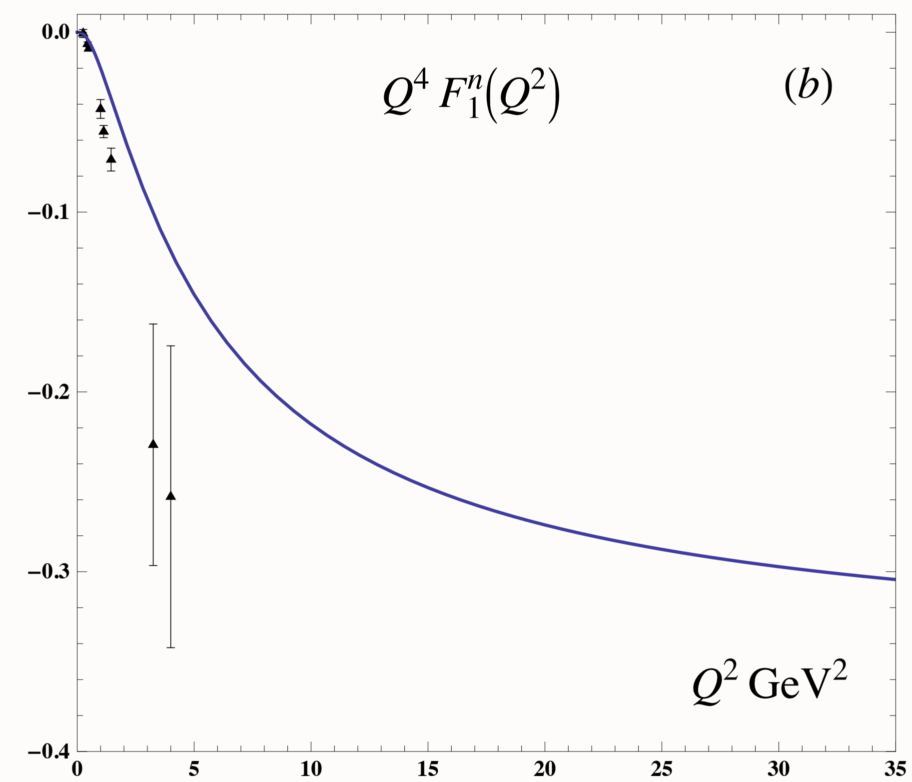
From overlap of $L = 1$ and $L = 0$ LFWFs




$F_2^p(Q^2)$
(a)
Q^2 GeV^2
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6


$F_2^n(Q^2)$
(b)
Q^2 GeV^2
0.0
−0.5
−1.0
−1.5
−2.0
0
1
2
3
4
5
6


$Q^4 F_1^p(Q^2)$
(a)
Q^2 GeV^2
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35


$Q^4 F_1^n(Q^2)$
(b)
Q^2 GeV^2
0.0
−0.1
−0.2
−0.3
−0.4
0
5
10
15
20
25
30
35

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

- Transition form factor

$$F_{1\,N\to N^*}^{\,p}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_{1\,N\to N^*}^{\,p}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

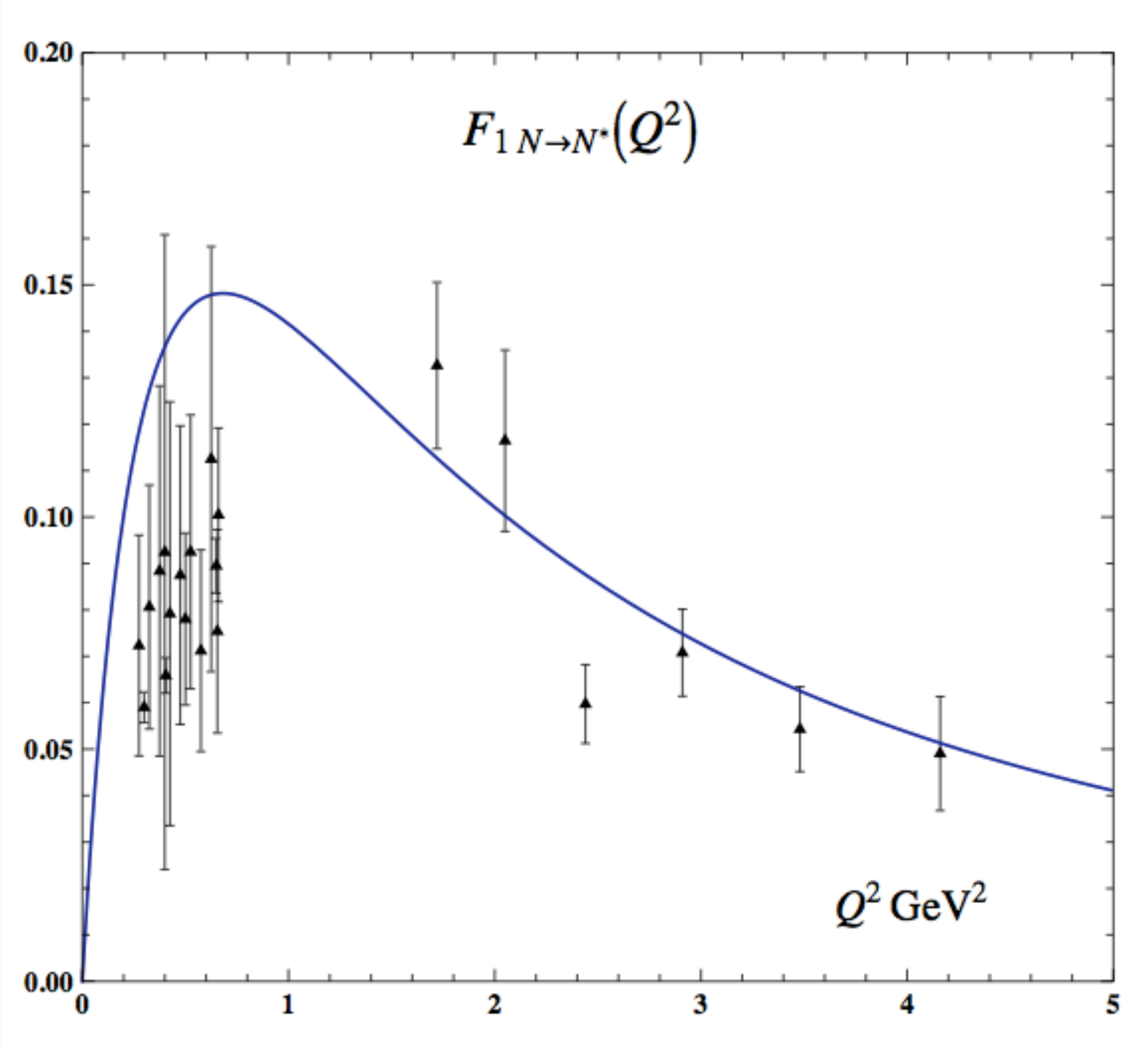
$$F_{1\,N\to N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

$$N(940) \to N^*(1440): \quad \Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$$



Data from I. Aznauryan, *et al.* CLAS (2009)

$$F_{1\,N\to N^*}^{p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^{\;2} \to 4\kappa^2(n+1/2)$

Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

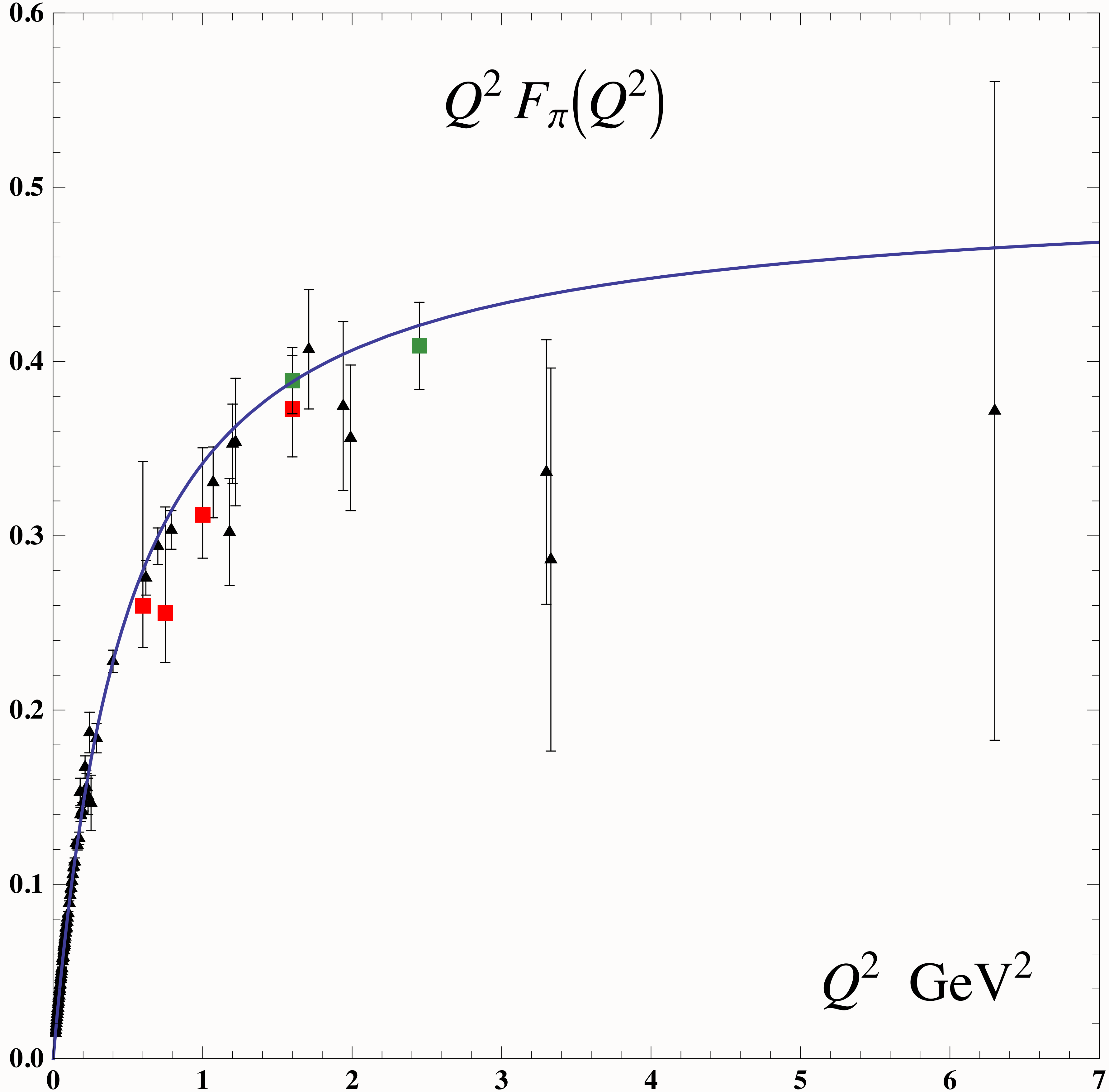
- For $\tau = N$, $\quad \Gamma(N + z) = (N - 1 + z)(N - 2 + z) \ldots (1 + z)\Gamma(1 + z)$.

- Form factor expressed as $N - 1$ product of poles

$$\begin{aligned}
F(Q^2) &= \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2, \\
F(Q^2) &= \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3, \\
&\cdots \\
F(Q^2) &= \frac{(N-1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right) \cdots \left(N - 1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N.
\end{aligned}$$

- For large Q^2:

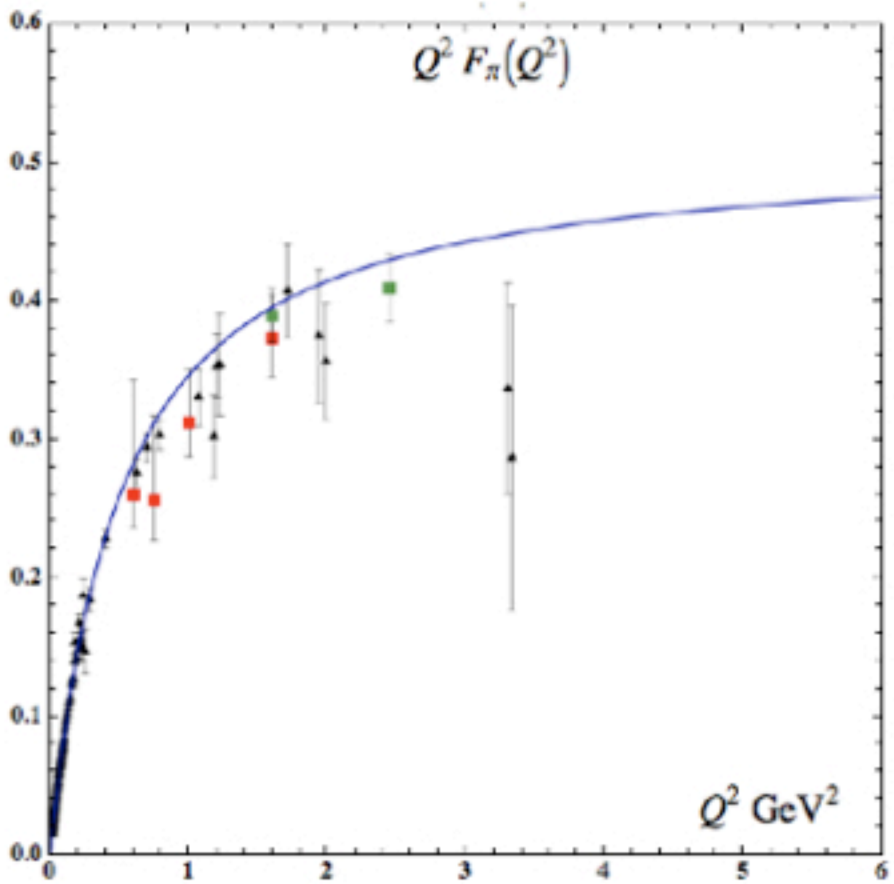
$$F(Q^2) \to (N-1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

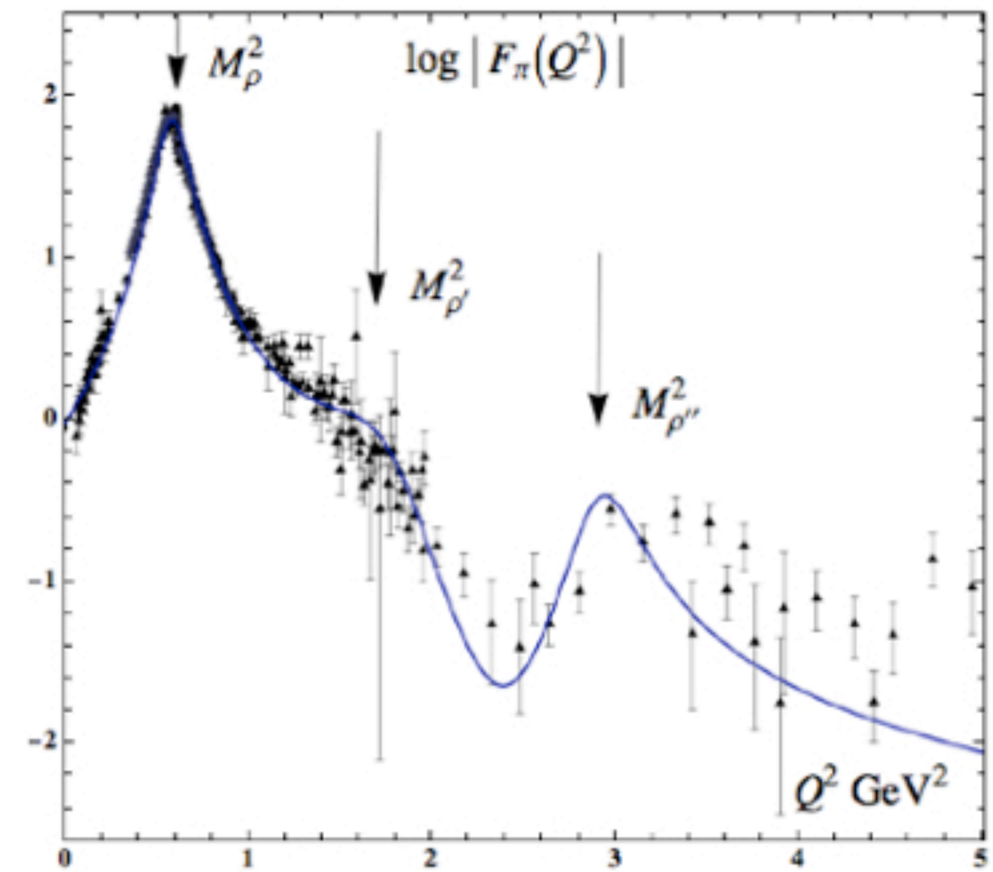


$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Space- and Time Like Pion Form-Factor (HFS)

PRELIMINARY





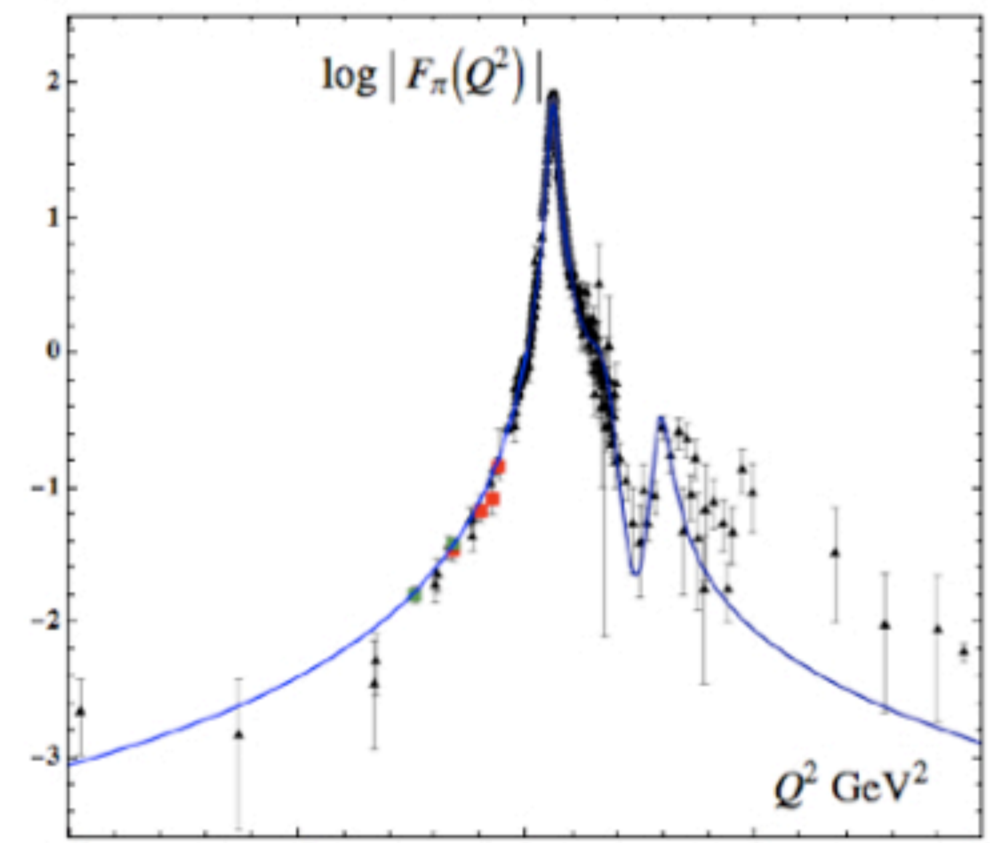
$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle$$

$$\mathcal{M}^2 \rightarrow 4\kappa^2(n + 1/2)$$

$\kappa = 0.54$ GeV

$$\Gamma_\rho = 130,\ \Gamma_{\rho'} = 400,\ \Gamma_{\rho''} = 300 \text{ MeV}$$

$P_{q\bar{q}q\bar{q}} = 13$ %



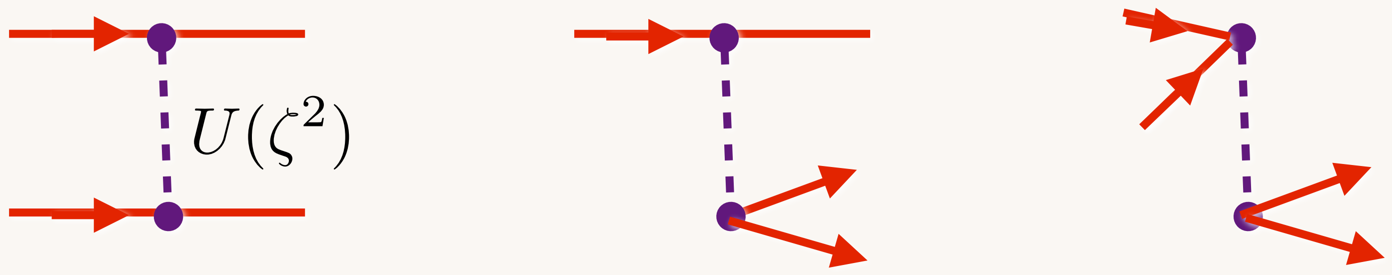
AdS/QCD predicts Higher Fock States

- Exposed by timelike form factor through dressed current.

- Created by confining interaction

$$P^-_{\text{confinement}} \simeq \kappa^4 \int dx^- d^2\vec{x}_\perp \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+} \frac{1}{(\partial/\partial_\perp)^4} \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+}$$

- Similar to QCD(1+1) in lcg



de Teramond, sjb

Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)} \left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Take $A_z \propto \Phi_\pi(z)/z, \quad \Phi_\pi(z) = \sqrt{2P_{q\bar{q}}}\, \kappa\, z^2 e^{-\kappa^2 z^2/2}, \quad \langle \Phi_\pi | \Phi_\pi \rangle = P_{q\bar{q}}$
- Find $\left(\phi(x) = \sqrt{3} f_\pi x(1-x), \quad f_\pi = \sqrt{P_{q\bar{q}}}\, \kappa / \sqrt{2}\pi\right)$

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{-P_{q\bar{q}} Q^2 (1-x)/4\pi^2 f_\pi^2 \, x}\right]$$

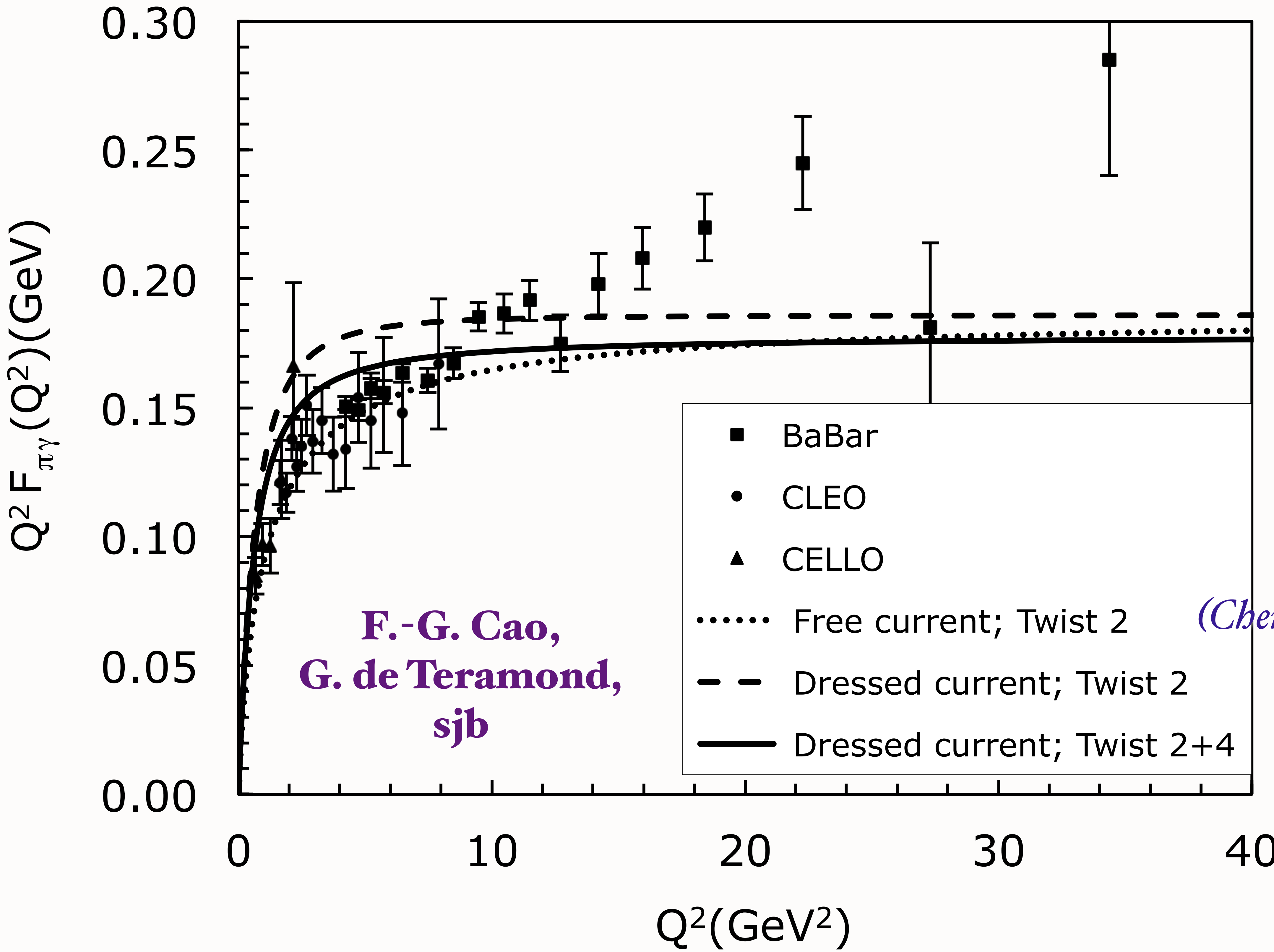
normalized to the asymptotic DA $[P_{q\bar{q}} = 1 \rightarrow$ Musatov and Radyushkin (1997)]

G.P. Lepage, sjb

- Large Q^2 TFF is identical to first principles asymptotic QCD result $Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2 f_\pi$
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA

Photon-to-pion transition form factor

$$Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$$



(Chern-Simons)

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

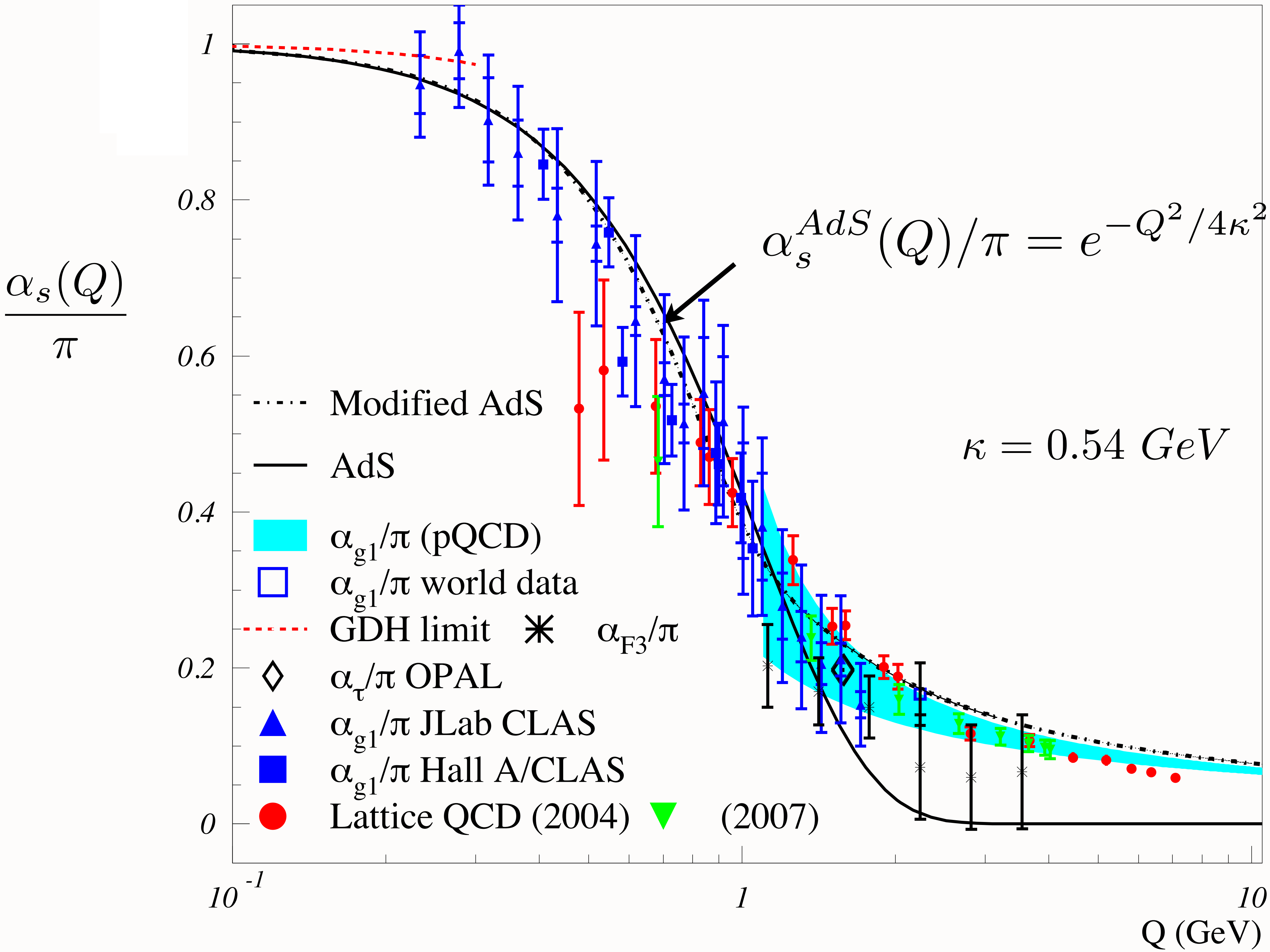
- Solution

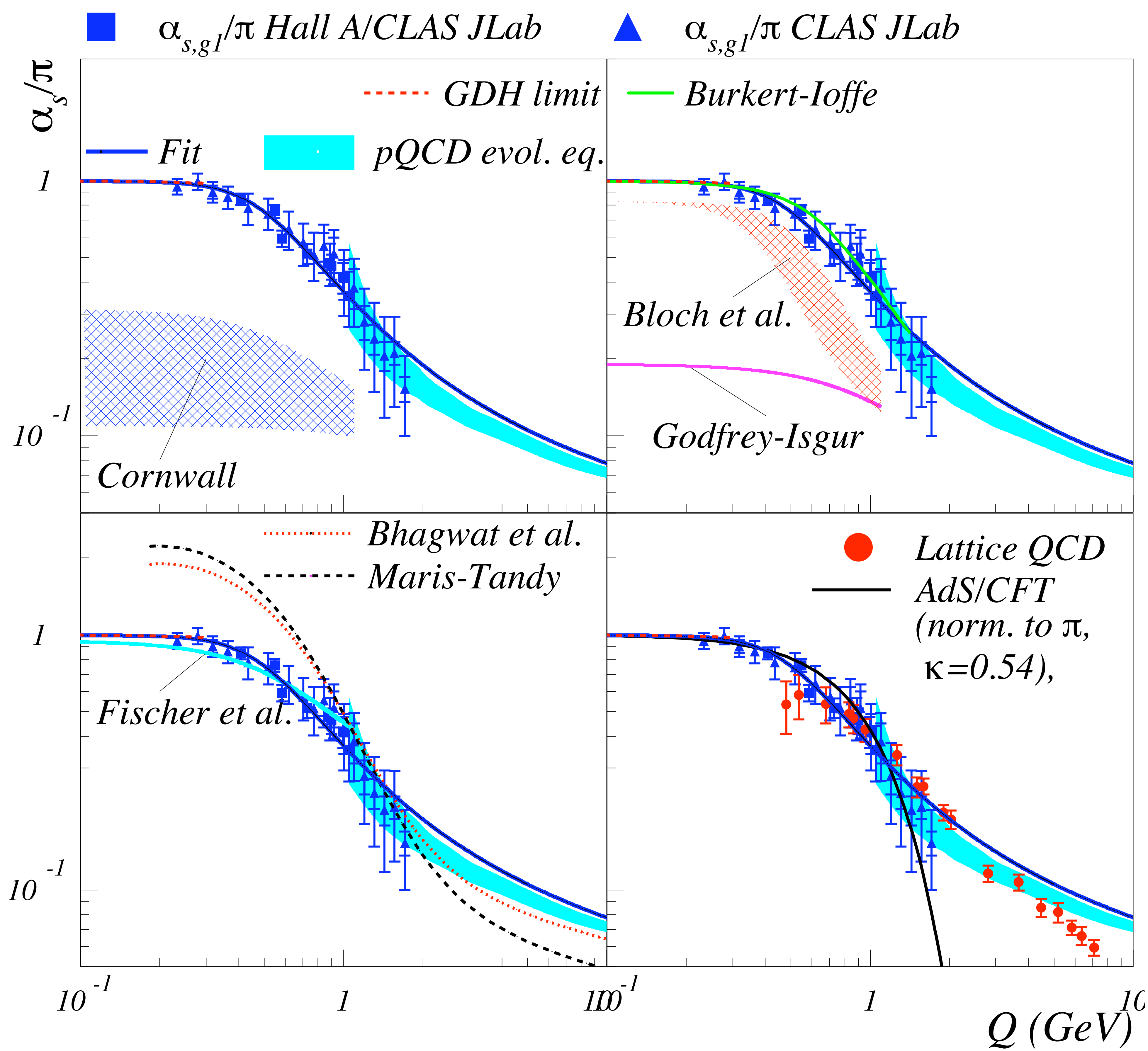
$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from Light-Front Holography and AdS/QCD

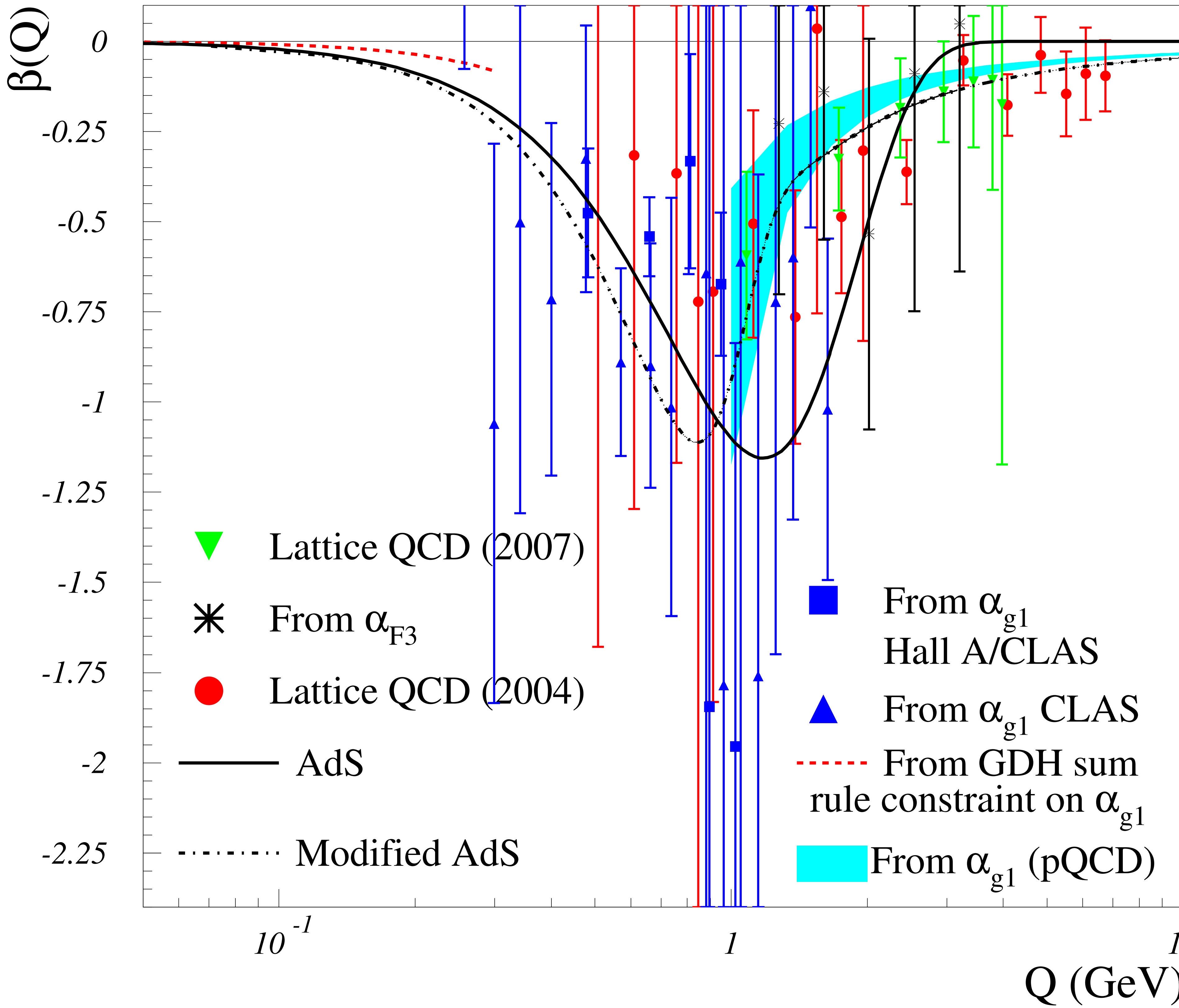
Analytic, defined at all scales, IR Fixed Point





Deur, de Teramond, sjb, (preliminary)

$$\beta^{AdS}(Q^2) = \frac{d}{d\log Q^2}\alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2}e^{-Q^2/4\kappa^2}$$



Deur, de Teramond, sjb

New Way to Solve

- Maldacena Correspondence
- Mathematical Representation of Lorentz Invariant and Conformal (Scale-Free) Theories
- Add new 5th space dimension to 3+1 space-time
- Holographic Model with Color Confinement and Quark Counting Rules

de Teramond, sjb

String Theory

Mapping of Poincare' and Conformal SO (4,2) symmetries of 3+1 space to AdS5 space

AdS/CFT

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering

Regge Trajectories

Conformal behavior at short distances + Confinement at large distance

AdS/QCD

QCD at the Amplitude Level

Semi-Classical QCD / Wave Equations

Holography

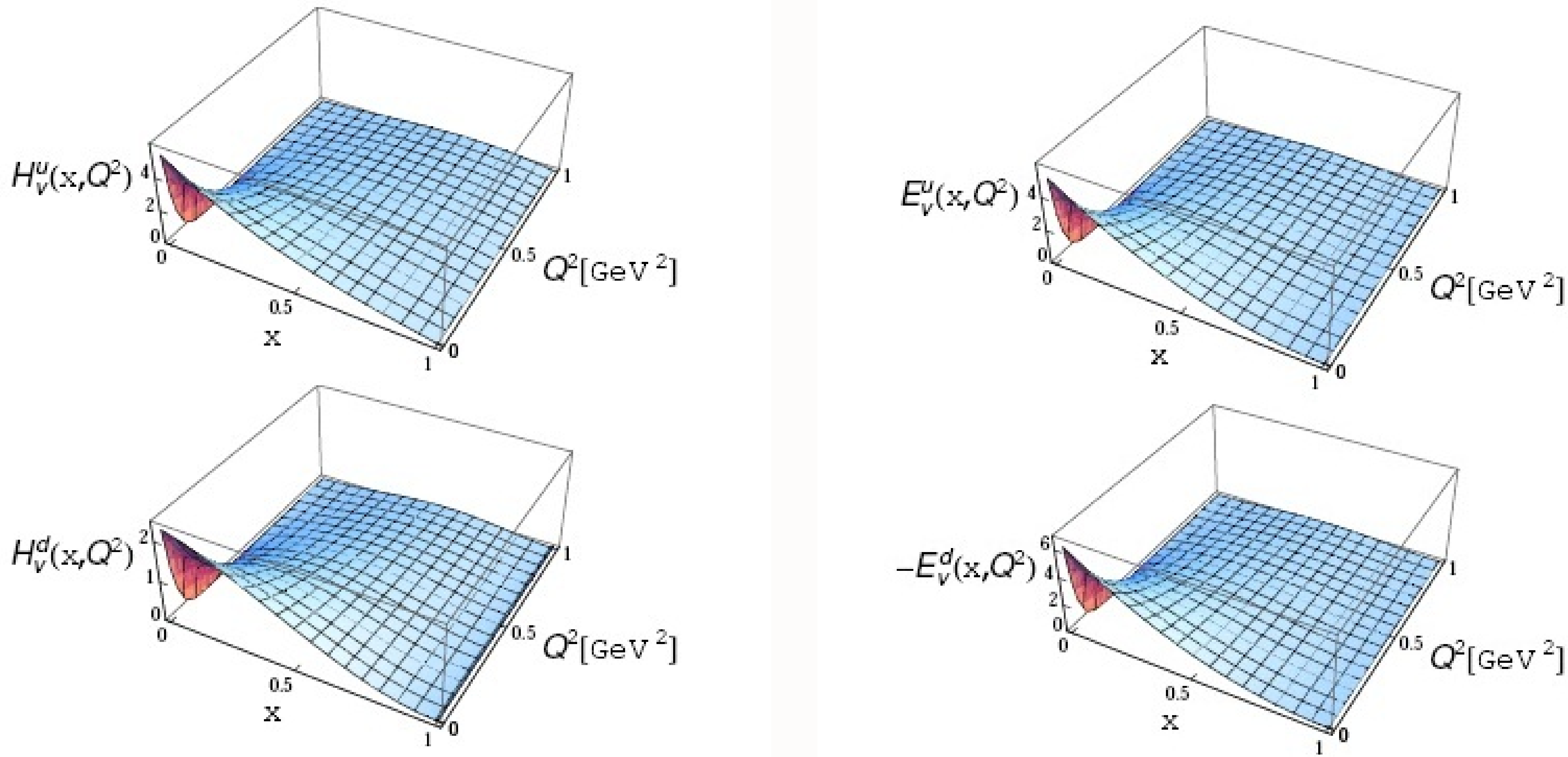
Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

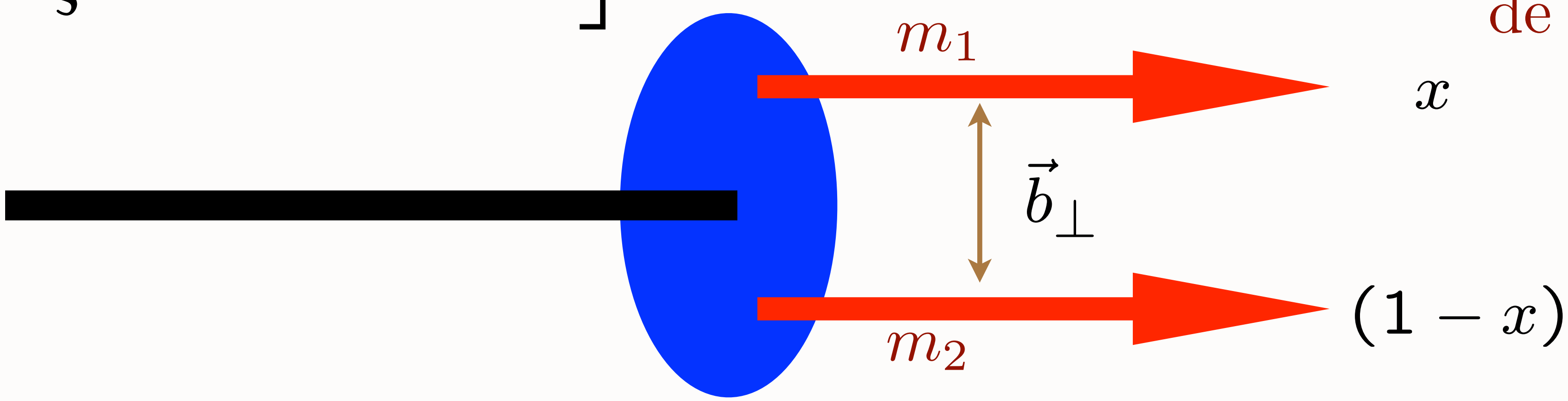
Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt, Alfredo Vega



GPDs $H_v^q(x,Q^2)$ and $E_v^q(x,Q^2)$ calculated in the holographical model.

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

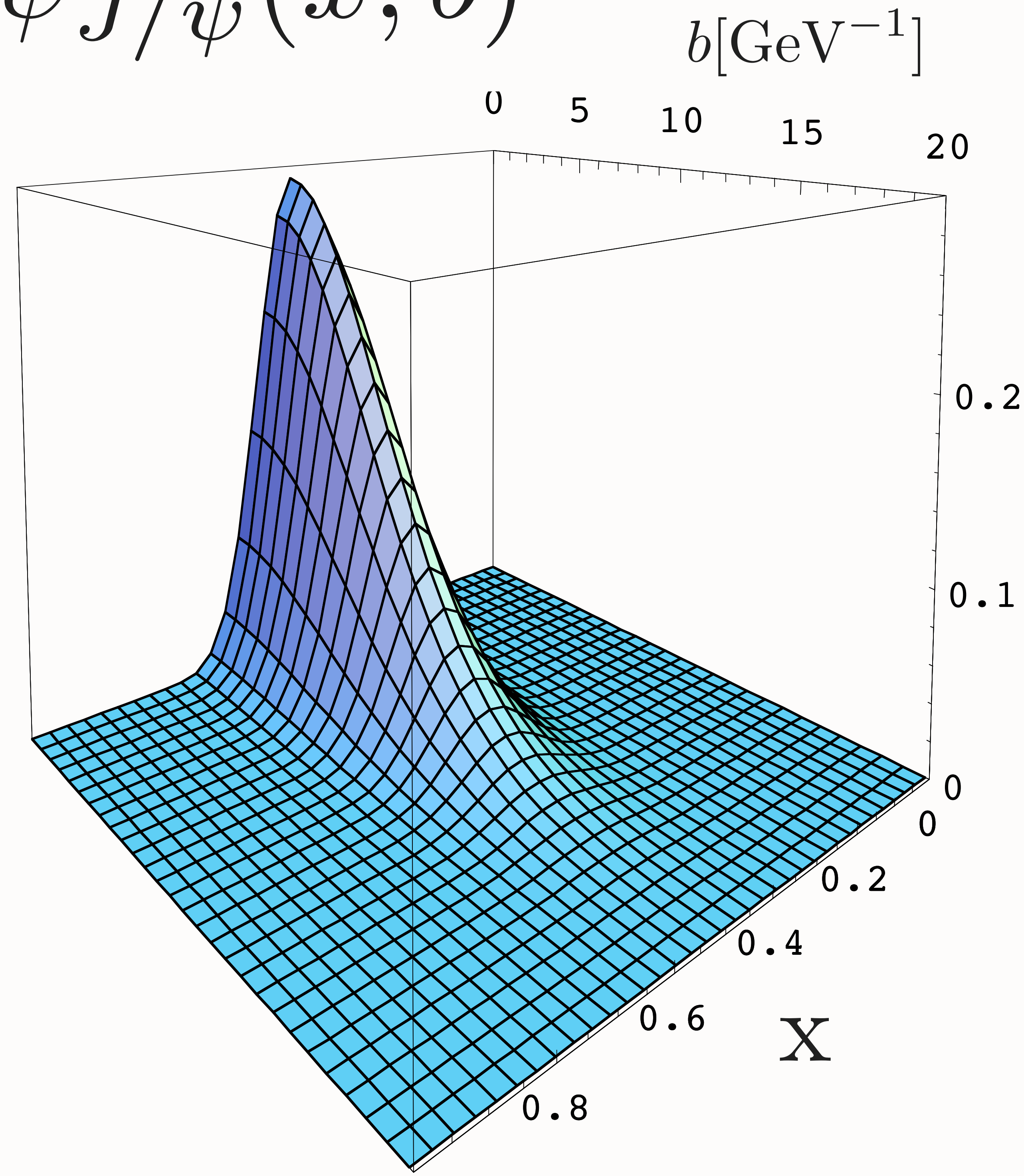
where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

$$\psi_{J/\psi}(x, b)$$



$$m_a = m_b = 1.25 \text{ GeV}$$

Light and heavy mesons in a soft-wall holographic model

Valery E. Lyubovitskij[*1†], **Tanja Branz**[1], **Thomas Gutsche**[1], **Ivan Schmidt**[2], **Alfredo Vega**[2]

[1] *Institut für Theoretische Physik, Universität Tübingen,*
Kepler Center for Astro and Particle Physics,
Auf der Morgenstelle 14, D–72076 Tübingen, Germany

[2]*Departamento de Física y Centro Científico Tecnológico de Valparaíso (CCTVal),*
Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

We study the spectrum and decay constants of light and heavy mesons in a soft-wall holographic approach, using the correspondence of string theory in Anti-de Sitter space and conformal field theory in physical space-time.

$|\pi^+ >= |u\bar{d} >$

$m_u = 2\ MeV$

$m_d = 5\ MeV$

$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d} >$

$m_c = 1.25\ GeV$

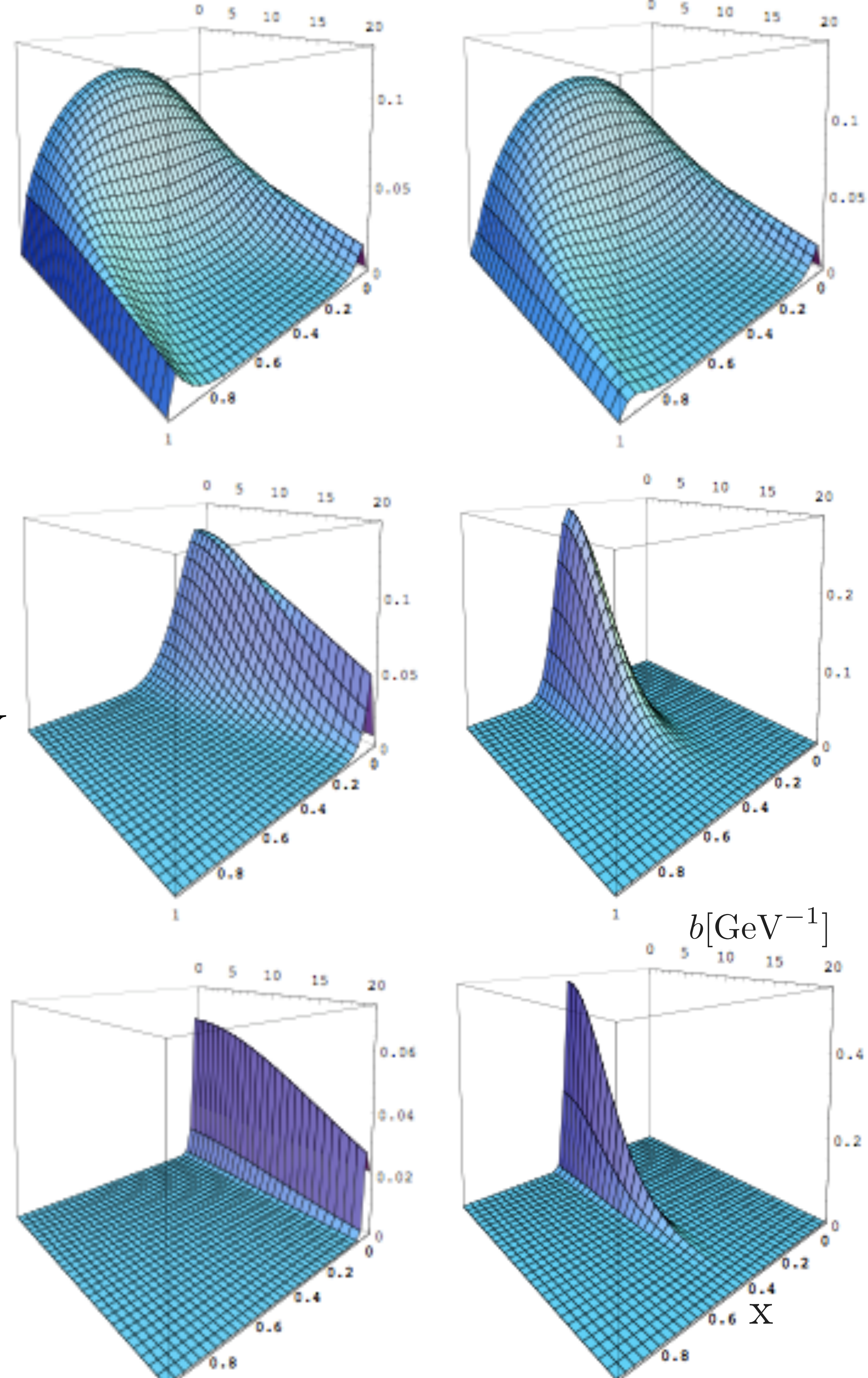
$|\eta_c >= |c\bar{c} >$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$

$|\eta_b >= |b\bar{b} >$

$\kappa = 375\ MeV$



Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum.**

Proton spin
carried by quark angular momentum!

- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z = 0 >$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

AdS/QCD and Light-Front Holography

- Hadrons are composites of quark and anti-quark constituents
- Soft gluons absent-- absorbed into confinement potential
- Higher Fock states with extra quark/anti-quark pairs created by confining potential
- Dominance of Quark Interchange in Hard Exclusive Reactions
- Short-distance behavior matches twist of interpolating operator at short distance -- guarantees dimensional counting rules --

Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS_5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite $N_c = 3$: Baryons built on 3 quarks -- Large N_c limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**
  J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,

  G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, sjb

Pauli, Hornbostel, Hiller, McCartor, Chabysheva, sjb

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{QCD} \propto < 0|q\bar{q}|0 >^4$$

QCD Problem Solved if quark and gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb Proc.Nat.Acad.Sci. 108 (2011) 45-50 "Condensates in Quantum Chromodynamics and the Cosmological Constant"

C. Roberts, R. Shrock, P. Tandy, sjb Phys.Rev. C82 (2010) 022201 "New Perspectives on the Quark Condensate"

Gell-Mann Oakes Renner Formula in QCD

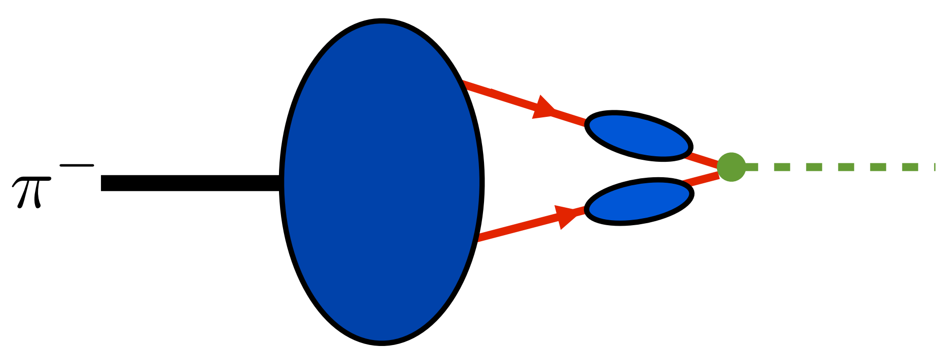
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"



$$< 0|\bar{q}\gamma_5 q|\pi >$$

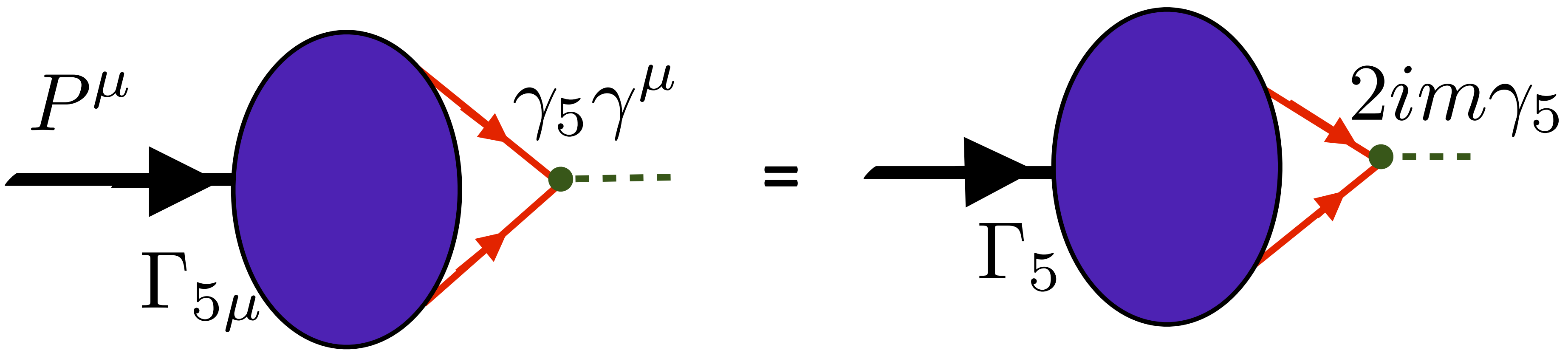
Maris, Roberts, Tandy

GK/GL pion electroproduction Phenomenology

Ward-Takahashi Identity for axial current

$$P^{\mu}\Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k+P/2)i\gamma_5 + i\gamma_5 S^{-1}(k-P/2)$$

$$S^{-1}(\ell) = i\gamma\cdot\ell A(\ell^2) + B(\ell^2) \qquad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



plus non-pole

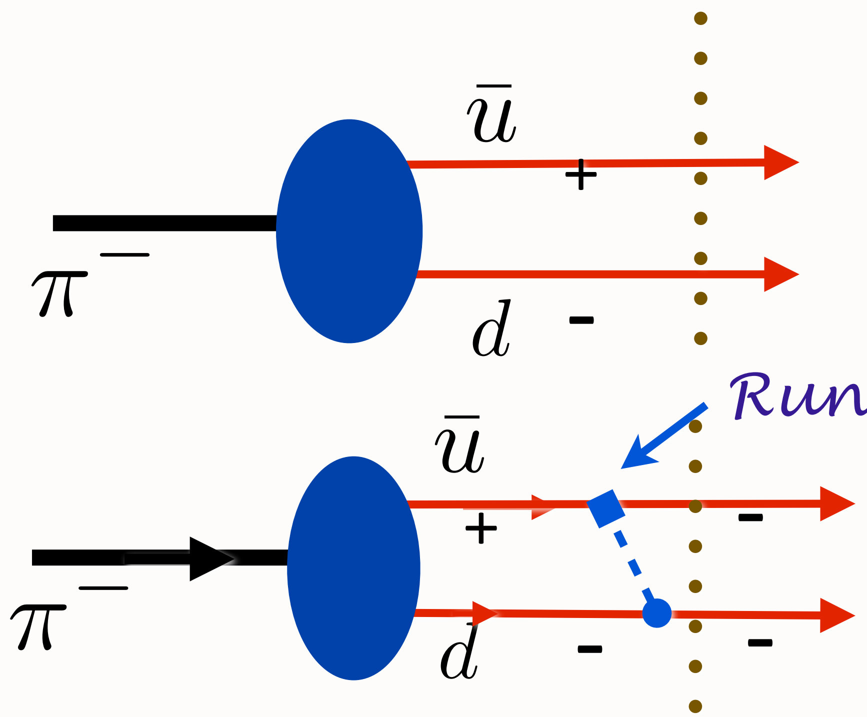
Identify pion pole at $P^2 = m_\pi^2$

$$P^{\mu} < 0|\bar{q}\gamma_5\gamma^{\mu}q|\pi >= 2m < 0|\bar{q}i\gamma_5 q|\pi >$$

$$f_\pi m_\pi^2 = -(m_u + m_d)\rho_\pi$$

Light-Front Pion Valence Wavefunctions

$S^z_{\bar u} + S^z_d = +1/2 - 1/2 = 0$



Couples to

$$L^z = 0, S^z = 0 \quad < \pi|\bar\gamma^\mu q \gamma_5 q|0 > \sim f_\pi$$

Running constituent mass at vertex

$$L^z = +1, S^z = -1 \quad < \pi|\bar q \gamma_5 q|0 > \sim \rho_\pi$$

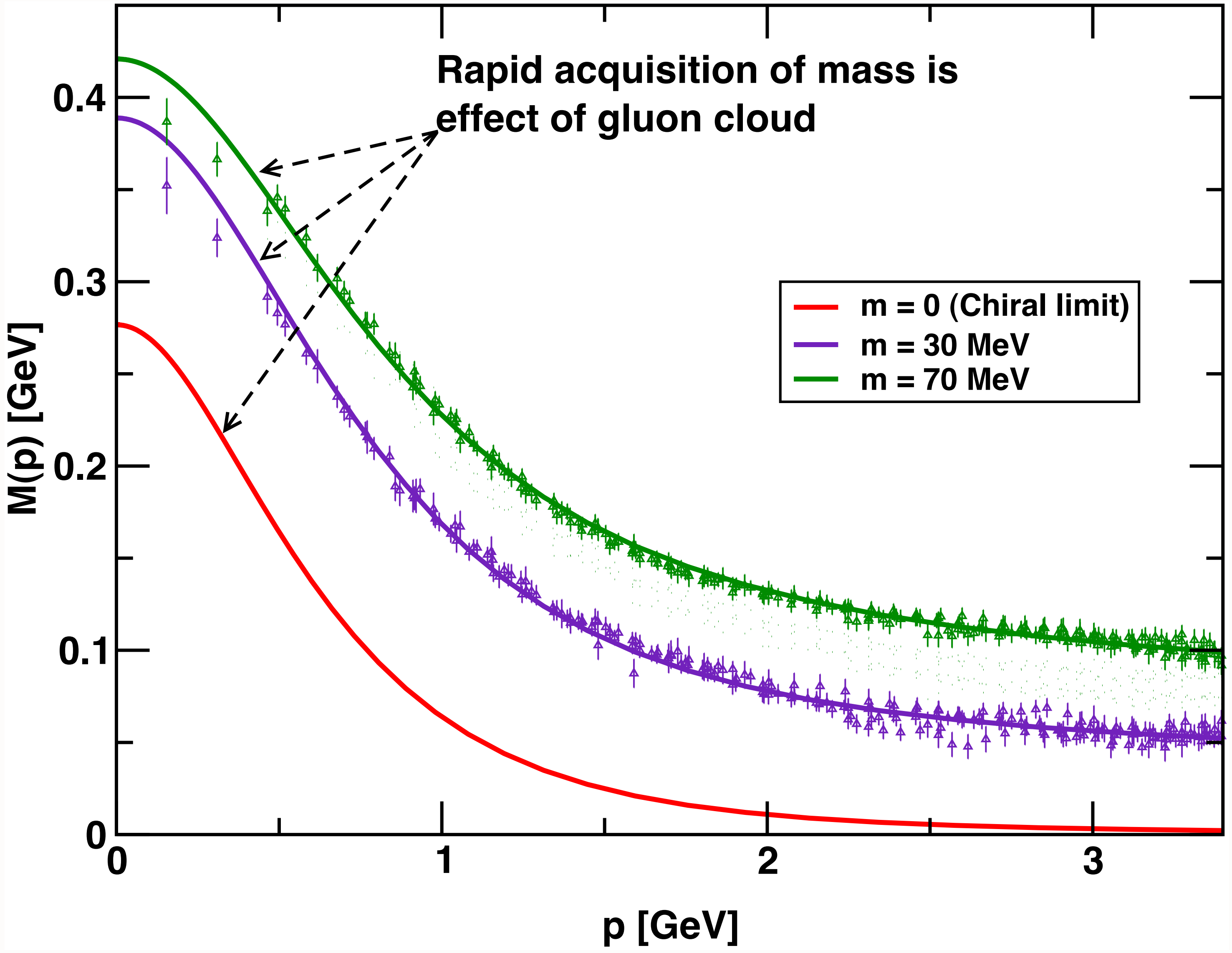
$S^z_{\bar u} + S^z_d = -1/2 - 1/2 = -1$

Angular Momentum Conservation

$$J^z = \sum_i^n S^z_i + \sum_i^{n-1} L^z_i$$

Running quark mass in QCD

$$S^{-1}(p) = i\gamma \cdot p \; A(p^2) + B(p^2) \qquad m(p^2) = \frac{B(p^2)}{A(p^2)}$$



Dyson-Schwinger

Chang, Cloet, El-Bennich Klahn, Roberts

Consistent with EW input at high p^2

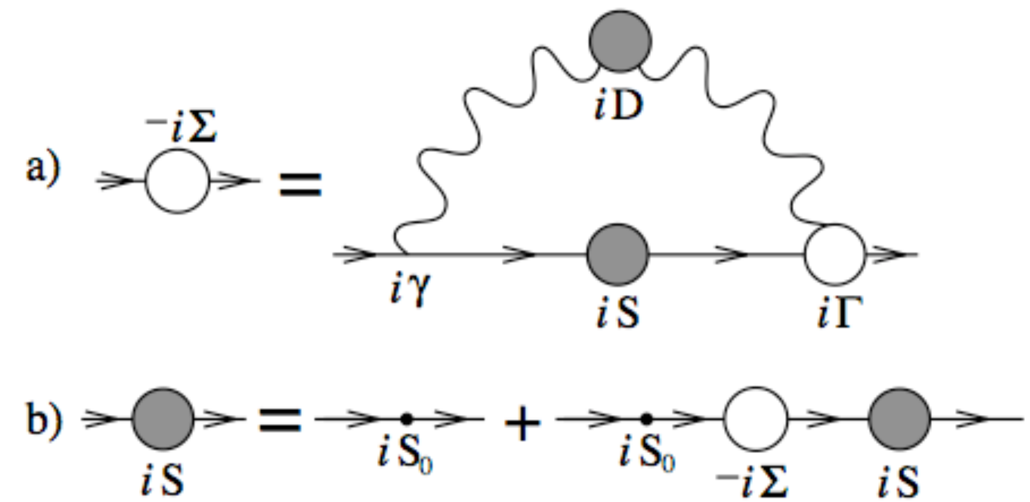
Survives even at m=0!

Spontaneous Chiral Symmetry Breaking!

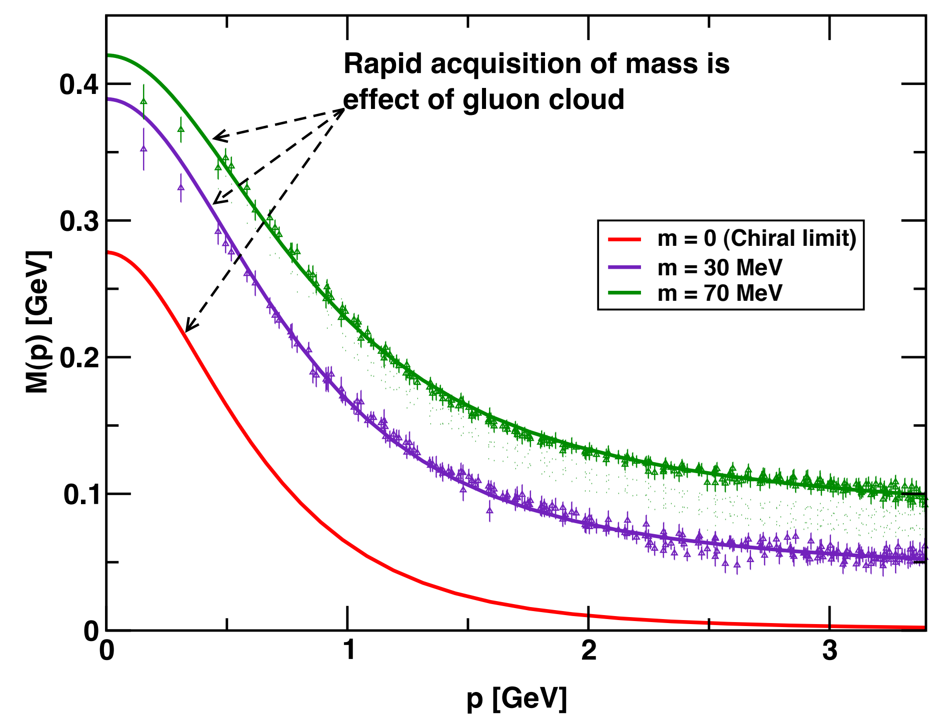
Running mass enhanced within Hadron Wavefunction

$$S^{-1}(p) = i\gamma \cdot p\ A(p^2) + B(p^2)$$

$$m(p^2) = \frac{B(p^2)}{A(p^2)}$$



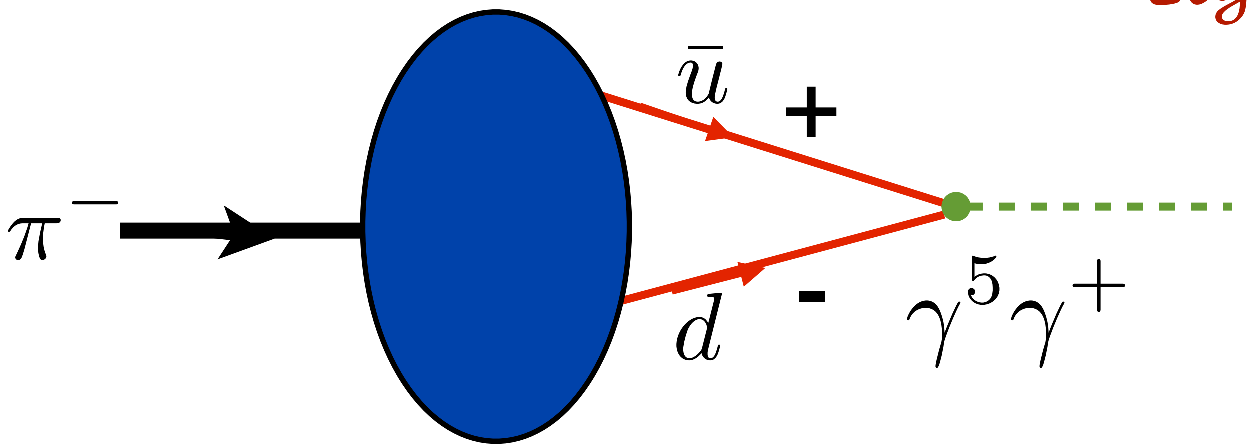
- QCD gluon loop corrections increase running mass
- Dyson-Schwinger model predictions Alkofer, Roberts et al.
- Effects of higher Fock states: Casher & Susskind
  spontaneous chiral symmetry breaking
- All effects within confinement domain
- IR cutoff from confinement/bound state Shrock, sjb



Lei Chang, et al.

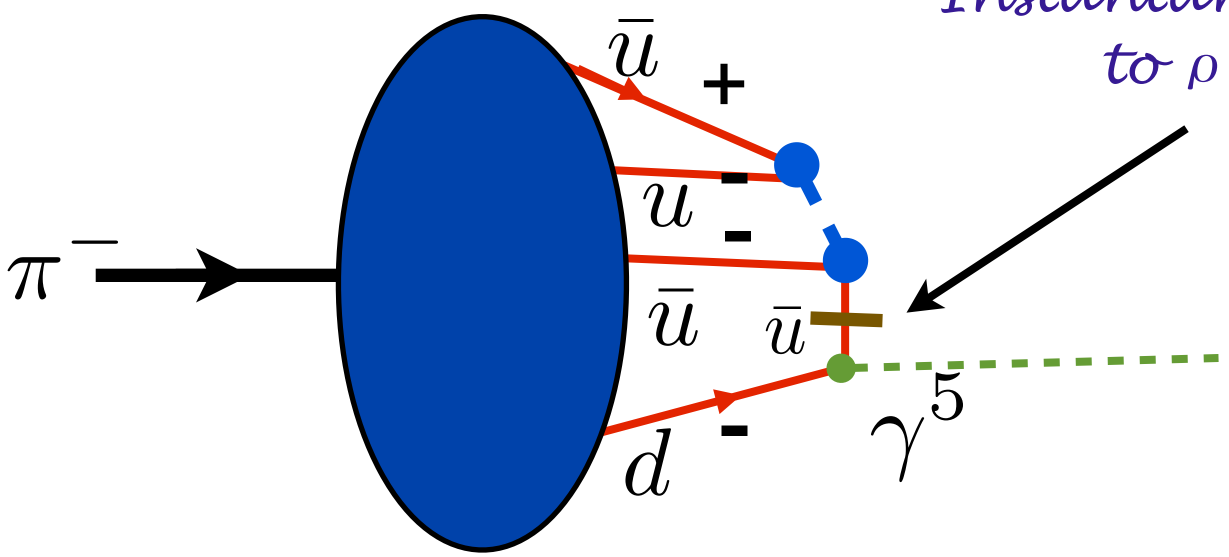
Higher Light-Front Fock State of Pion Simulates DCSB

Light Front Fock state Analysis

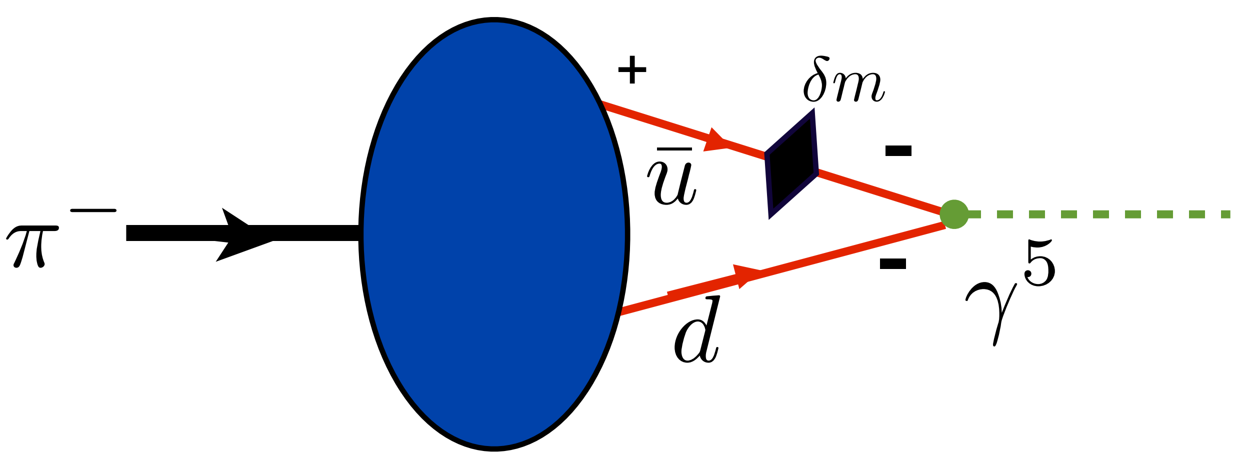


$$f_\pi P^+ =< 0|\bar{q}\gamma^5\gamma^+ q|\pi >$$

Instantaneous quark propagator contribution to ρ_π derived from higher Fock state



$$i\rho_\pi =< 0|\bar{q}\gamma^5 q|\pi >$$



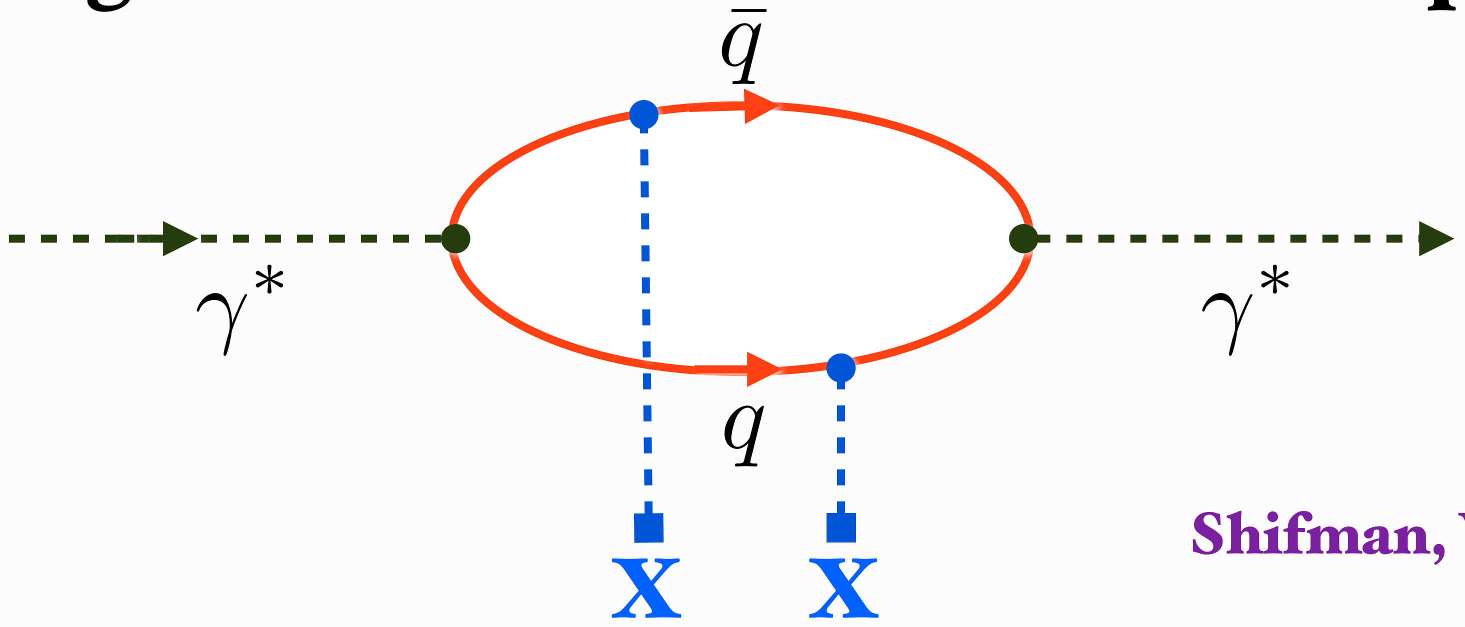
Higher Fock state acts like mass insertion

Roberts, Tandy, Shrock, sjb

Is there evidence for a gluon vacuum condensate?

$$< 0|\frac{\alpha_s}{\pi}G^{\mu\nu}(0)G_{\mu\nu}(0)|0 >$$

Look for higher-twist correction to current propagator



Shifman, Vainshtein, Zakharov

$$e^+e^- \to X, \ \tau \text{ decay}, \ Q\bar{Q} \text{ phenomenology}$$

$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \frac{\alpha_s}{\pi}\frac{\Lambda^4_{\rm QCD}}{s^2} + \cdots)$$

Determinations of the vacuum Gluon Condensate

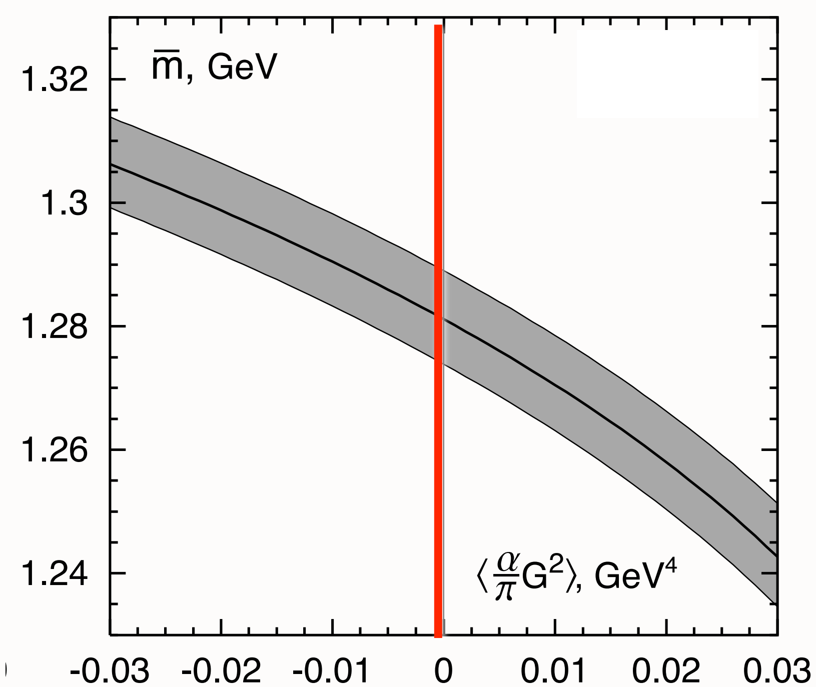
$$< 0|\frac{\alpha_s}{\pi} G^2|0 > [\text{GeV}^4]$$

-0.005 ± 0.003 from τ decay. Davier et al.

$+0.006 \pm 0.012$ from τ decay. Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$ from charmonium sum rules

Ioffe, Zyablyuk

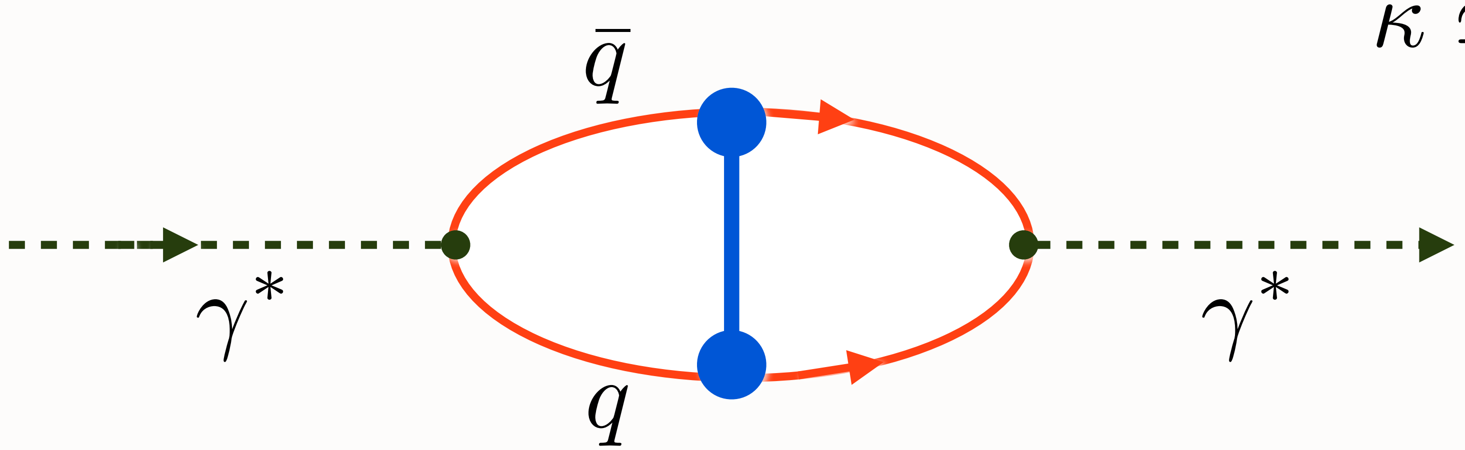


Consistent with zero vacuum condensate

Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator

$$M^2 = 4\kappa^2(n+L+S/2)$$ light-quark meson spectra

$\kappa \simeq 0.5$ GeV



$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \mathcal{O}\frac{\kappa^4}{s^2} + \cdots)$$

mimics dimension-4 gluon condensate $< 0|\frac{\alpha_s}{\pi}G^{\mu\nu}(0)G_{\mu\nu}(0)|0 >$ in

$e^+e^- \to X$, τ decay, $Q\bar{Q}$ phenomenology

PHYSICAL REVIEW C **82**, 022201(R) (2010)

New perspectives on the quark condensate

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]

[1] *SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*

[2] *Centre for Particle Physics Phenomenology: CP^3-Origins, University of Southern Denmark, Odense 5230 M, Denmark*

[3] *Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

[4] *Department of Physics, Peking University, Beijing 100871, China*

[5] *C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*

[6] *Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 25 May 2010; published 18 August 2010)

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

- **Eliminates 45 orders of magnitude conflict**

Fock vacuum $|0\rangle$ eigenstate of the full Hamiltonian

$$
\begin{aligned}
P^- \quad = \quad & \frac{1}{2}\int dx_+ d^2x_\perp \left(\overline{\Psi}\gamma^+ \frac{\bar{m}^2 + (i\nabla_\perp)^2}{i\partial^+}\Psi + A_a^\mu (i\nabla_\perp)^2 A_\mu^a\right) \quad \text{free} \\
+ \quad & g\int dx_+ d^2x_\perp J_a^\mu A_\mu^a \quad \text{vertex interaction} \\
+ \quad & \frac{g^2}{4}\int dx_+ d^2x_\perp B_a^{\mu\nu} B_{\mu\nu}^a \quad 4-\text{point gluon} \\
+ \quad & \frac{g^2}{2}\int dx_+ d^2x_\perp J_a^+ \frac{1}{(i\partial^+)^2} J_a^+ \quad \text{instantaneous gluon interaction} \\
+ \quad & \frac{g^2}{2}\int dx_+ d^2x_\perp \overline{\Psi}\gamma^\mu T^a A_\mu^a \frac{\gamma^+}{i\partial^+}\left(\gamma^\nu T^b A_\nu^b \Psi\right), \quad \text{instantaneous fermion interaction}
\end{aligned}
$$

where

$$J_a^\mu = \bar{\Psi}\gamma^\mu T^a \Psi \chi_a^\mu + f^{abc}\partial^\mu A_b^\nu A_\nu .$$

- Light-Front Vacuum: Frame-independent, causal, trivial, no normal ordering needed, zero cosmological constant!

- Instant-Form Vacuum: Frame-dependent, acausal, non-trivial, normal ordering needed, vacuum contributions to all matrix elements

Two Different Vacua!!

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon. Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame. A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.

Light-Front Formalism

Summary on QCD `Condensates'

- Condensates do not exist as space-time-independent phenomena

- Property of hadron wavefunctions: Bethe-Salpeter or Light-Front: "In-Hadron Condensates"

- Find: $$\frac{< 0|\bar{q}q|0 >}{f_\pi} \rightarrow - < 0|i\bar{q}\gamma_5 q|\pi >= \rho_\pi$$

$$< 0|\bar{q}i\gamma_5 q|\pi > \text{ similar to } < 0|\bar{q}\gamma^\mu\gamma_5 q|\pi >$$

- Zero contribution to cosmological constant! Included in hadron mass

- ρ_π survives for small m_q -- enhanced running mass from gluon loops / multiparton Fock states --measured in pion electroproduction (GK, GL)

- Light-Front Vacuum: Causal, frame-independent, trivial, no normal ordering needed

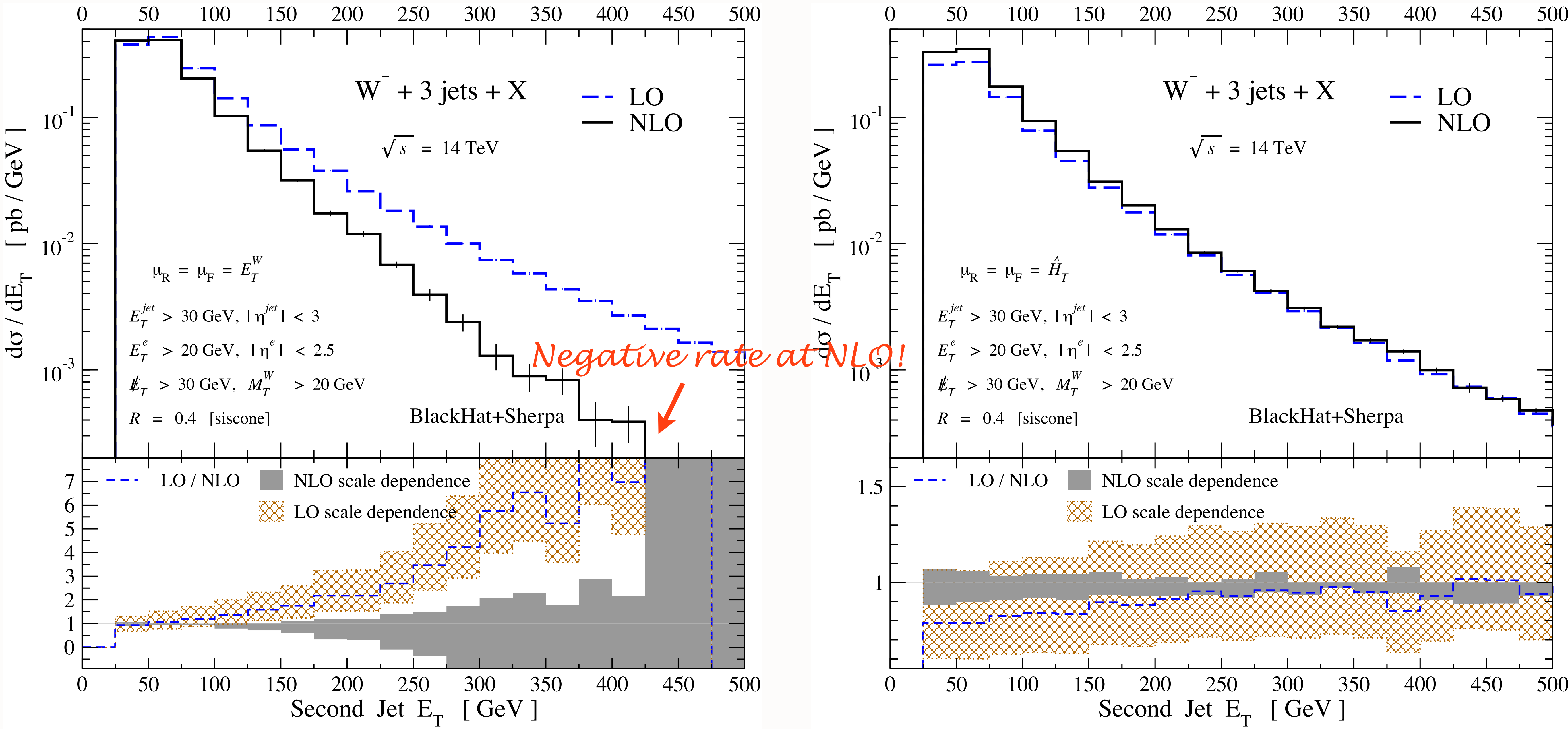
Goals

- Test QCD to maximum precision
- High precision determination of $\alpha_s(Q^2)$ at all scales
- Relate observable to observable --no scheme or scale ambiguity
- Eliminate renormalization scale ambiguity in a scheme-independent manner
- Relate renormalization schemes without ambiguity
- Maximize sensitivity to new physics at the colliders

Next-to-Leading Order QCD Predictions for W + 3-Jet Distributions at Hadron Colliders

Black Hat

$$\mu_R = \mu_F = E_T^W \qquad\qquad \mu_R = \mu_F = \hat{H}_T$$



F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower, and D. Maître

Electron-Electron Scattering in QED

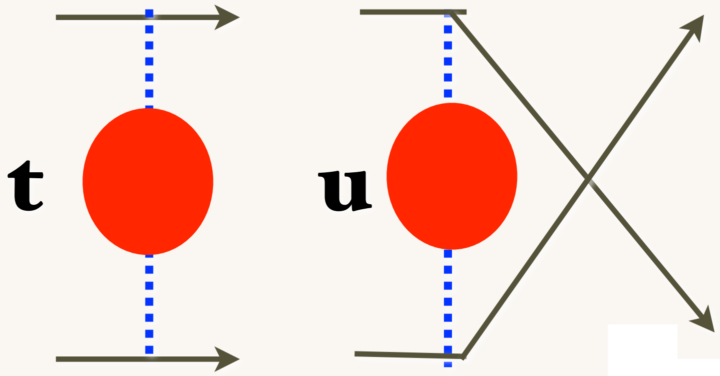
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

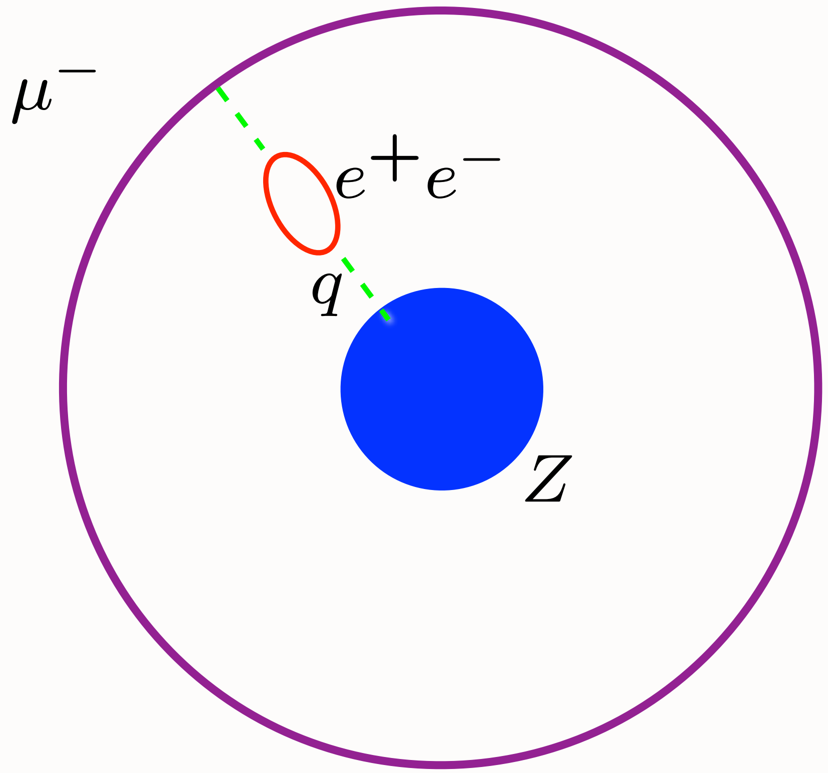
Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



- Two separate physical scales: t, u = photon virtuality
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!
- If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- No renormalization scale ambiguity!

Another Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

Relation between scales of the Gell-Mann--Low and $\overline{MS}$ schemes

$$\log \frac{\mu_0^2}{m_\ell^2} = 6 \int_0^1 x(1-x) \log \frac{m_\ell^2 + Q_0^2 x(1-x)}{m_\ell^2}$$

$$\log \frac{\mu_0^2}{m_\ell^2} = \log \frac{Q_0^2}{m_\ell^2} - 5/3$$

$$\boxed{\mu_0^2 = Q_0^2 \; e^{-5/3}} \quad \text{when } Q_0^2 >> m_\ell^2$$

D. S. Hwang, sjb

M. Binger

Can use $\overline{MS}$ scheme in QED; answers are scheme independent
Analytic extension: coupling is complex for timelike argument

Features of PMC/BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb Phys.Rev.D28:228,1983

- **"Principle of Maximum Conformality"** Di Giustino, Wu, sjb
- **All terms associated with nonzero beta function summed into running coupling; scheme independent**
- **Standard procedure in QED**
- **Resulting series identical to conformal series**
- **Renormalon n! growth of PQCD coefficients from beta function eliminated!**
- **Scheme Independent!!!**
- **In general, BLM/PMC scales depend on all invariants**

di Giustino, Wu
sjb

Principle of Maximum Conformality

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Scale-Free Conformal Series

Running Coupling Effects

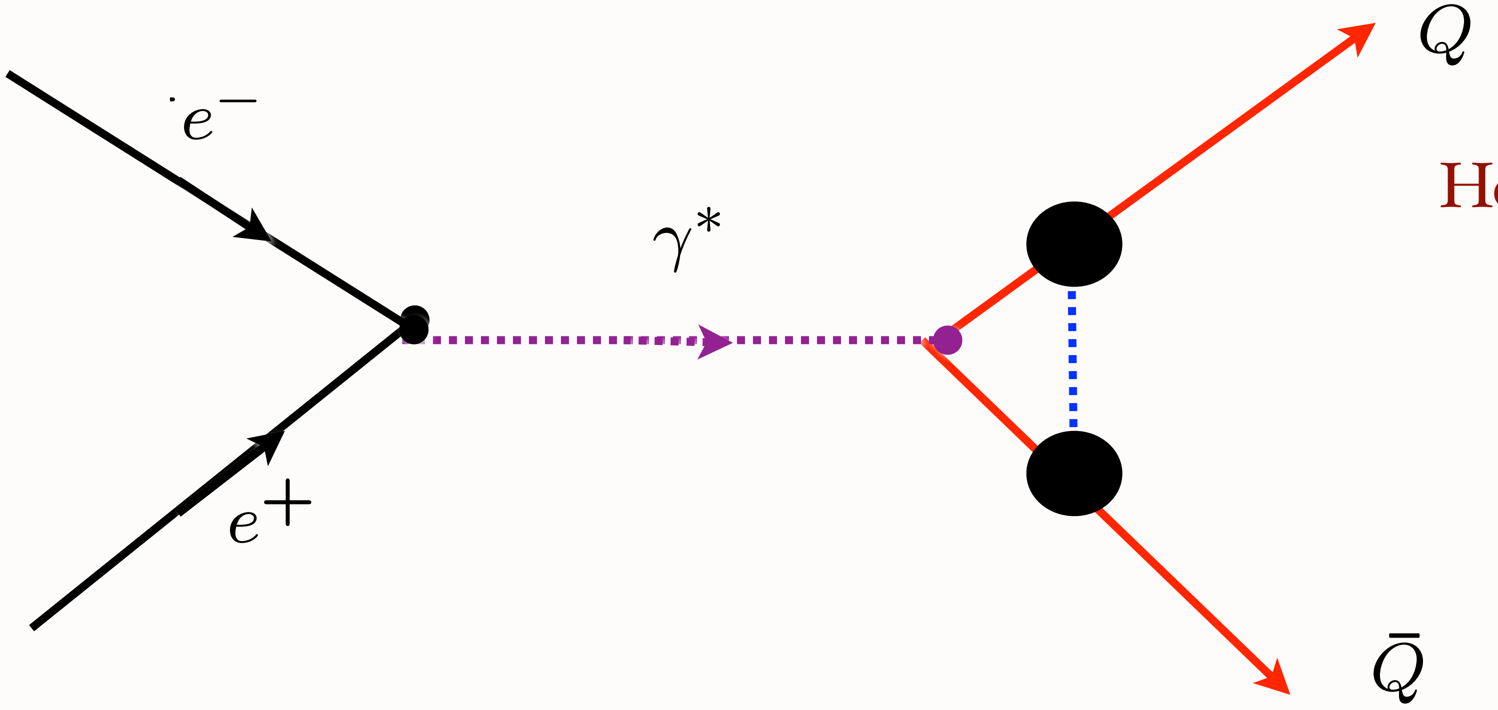
Higher Twist from Hadron Dynamics

Intrinsic Heavy Quarks

Light by Light Loops

PMC/BLM: Absorb β terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$



Hoang, Kuhn, Teubner, sjb

$$F_1 + F_2 = [1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}] \times [1 + \frac{\pi\alpha_s(sv^2)}{4v}]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity v

Conformal symmetry: Template for QCD

Principle of Maximal Conformality

- **Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses** Frishman, Lepage, Mackenzie, Sachrajda, Gardi, Braun, di Giustino, sjb
- **Eigensolutions of ERBL evolution equation for distribution amplitudes**
- **Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation** Gardi, Grunberg, Rathsman, Gabadadze, Kataev, Lepage, Lu, Mackenzie, sjb
- **PMC: Scheme-Independent Predictions for Observables**
- **IR Fixed Point -- A Conformal Domain**
- **Use AdS/CFT**

Future Directions

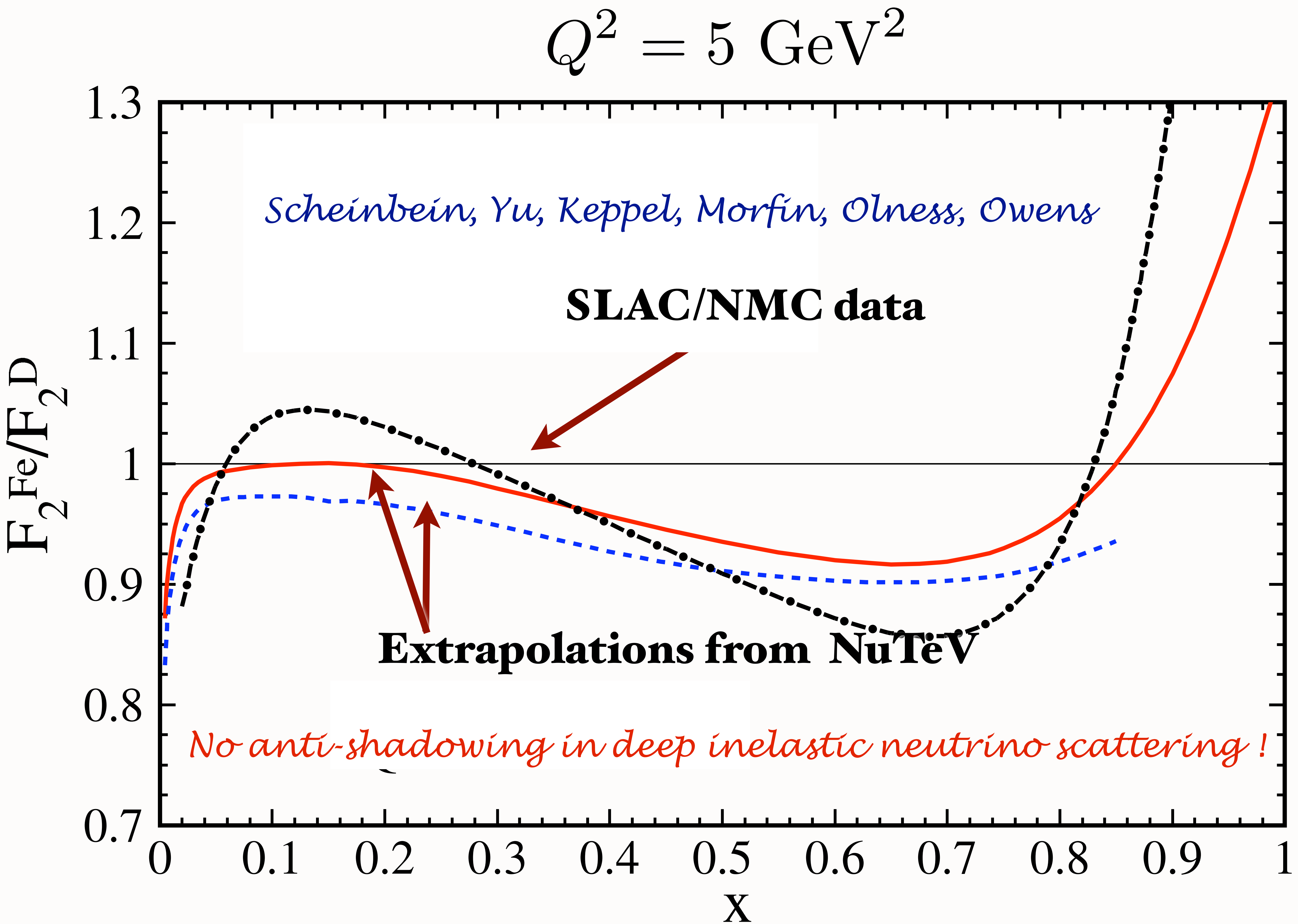
- **BLFQ -- use AdS/QCD basis to diagonalize H_{LF}** *Vary Honkanen et al.*
- **Lippmann-Schwinger -- perturbatively generate higher Fock States and systematically approach QCD** *Hiller and Chabysheva*
- **Transverse Lattice** *Burkardt Dalley Hiller*
- **Hadronization at the Amplitude Level -- Off-Shell T-matrix convoluted with AdS/QCD LFWFs**
- **Hidden Color** *C. Ji , Lepage, sjb*
- **Intrinsic Heavy Quarks from confinement interaction**
- **BLM/PMC -- Automatic Scale Setting -- pinch scheme** *Binosi, Cornwall, Popavassiliu Binger di Giustino sjb*
- **Direct Processes at the LHC**
- **Dynamic vs. Static Structure Functions**
- **AdS/QCD for DVCS, Hadrons with Heavy Quarks**

Novel JLab-12 Topics

- DVCS, DVMS, Hard Exclusive Processes at the Amplitude Level
- J=0 Fixed Pole
- Diffractive DIS
- Hidden Color in Deuteron
- $x > 1$ in Nuclei
- Nuclear Form Factors, Exclusive Amplitudes at large Q^2
- Shadowing, antishadowing, EMC
- Jet Energy Loss, LPM Non-Abelian Effect

Novel QCD Phenomena and Perspectives

- Hadroproduction at large transverse momentum does not derive exclusively from 2 to 2 scattering subprocesses: Baryon Anomaly at RHIC Sickles, sjb
- Color Transparency Mueller, sjb; Diffractive Di-Jets and Tri-jets Strikman et al
- Heavy quark distributions do not derive exclusively from DGLAP or gluon splitting -- component intrinsic to hadron wavefunction. Hoyer, et al
- Higgs production at large x_F from intrinsic heavy quarks Kopeliovitch, Goldhaber, Schmidt, Soffer, sjb
- Initial and final-state interactions are not always power suppressed in a hard QCD reaction: Sivers Effect, Diffractive DIS, Breakdown of Lam Tung PQCD Relation Schmidt, Hwang, Hoyer, Boer, sjb; Collins
- LFWFS are universal, but measured nuclear parton distributions are not universal -- antishadowing is flavor dependent Schmidt, Yang, sjb
- Renormalization scale is not arbitrary; multiple scales, unambiguous at given order. Disentangle running coupling and conformal effects, Skeleton expansion: Gardi, Grunberg, Rathsman, sjb
- Quark and Gluon condensates reside within hadrons: Shrock, sjb


$Q^2 = 5\ \mathrm{GeV}^2$
Scheinbein, Yu, Keppel, Morfin, Olness, Owens
SLAC/NMC data
Extrapolations from NuTeV
No anti-shadowing in deep inelastic neutrino scattering !
F_2^{Fe}/F_2^D
x
1.3
1.2
1.1
1
0.9
0.8
0.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Shadowing and Antishadowing in Lepton-Nucleus Scattering

- Shadowing: Destructive Interference
of Two-Step and One-Step Processes
Pomeron Exchange

- Antishadowing: Constructive Interference
of Two-Step and One-Step Processes!
Reggeon and Odderon Exchange

- Antishadowing is Not Universal!
Electromagnetic and weak currents:
different nuclear effects !

Can explain NuTeV result!

Jian-Jun Yang

Ivan Schmidt

Hung Jung Lu
sjb

Bjorken, Kogut, Soper; Blankenbecler, Gunion, sjb; Blankenbecler, Schmidt

Crucial Test of Leading -Twist QCD: Scaling at fixed x_T

$$E \frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{cm})}{p_T^{n_{\rm eff}}} \qquad x_T = \frac{2p_T}{\sqrt{s}}$$

Parton model: $n_{\rm eff} = 4$

As fundamental as Bjorken scaling in DIS

scaling law: $n_{\rm eff} = 2\, n_{\rm active} - 4$

Dimensional analysis

Scattering amplitude $1\ 2 \cdots \to \ldots n$ has dimension

$$\mathcal{M} \sim [\mathrm{length}]^{n-4}$$

Consequence

In a conformal theory (no intrinsic scale), scaling of inclusive particle production

$$E\,\frac{d\sigma}{d^3p}(A\ B\ \to C\ X) \sim \frac{|\mathcal{M}|^2}{s^2} = \frac{F(x_\perp, \vartheta^{\mathrm{cm}})}{p_\perp^{2n_{\mathrm{active}}-4}}$$

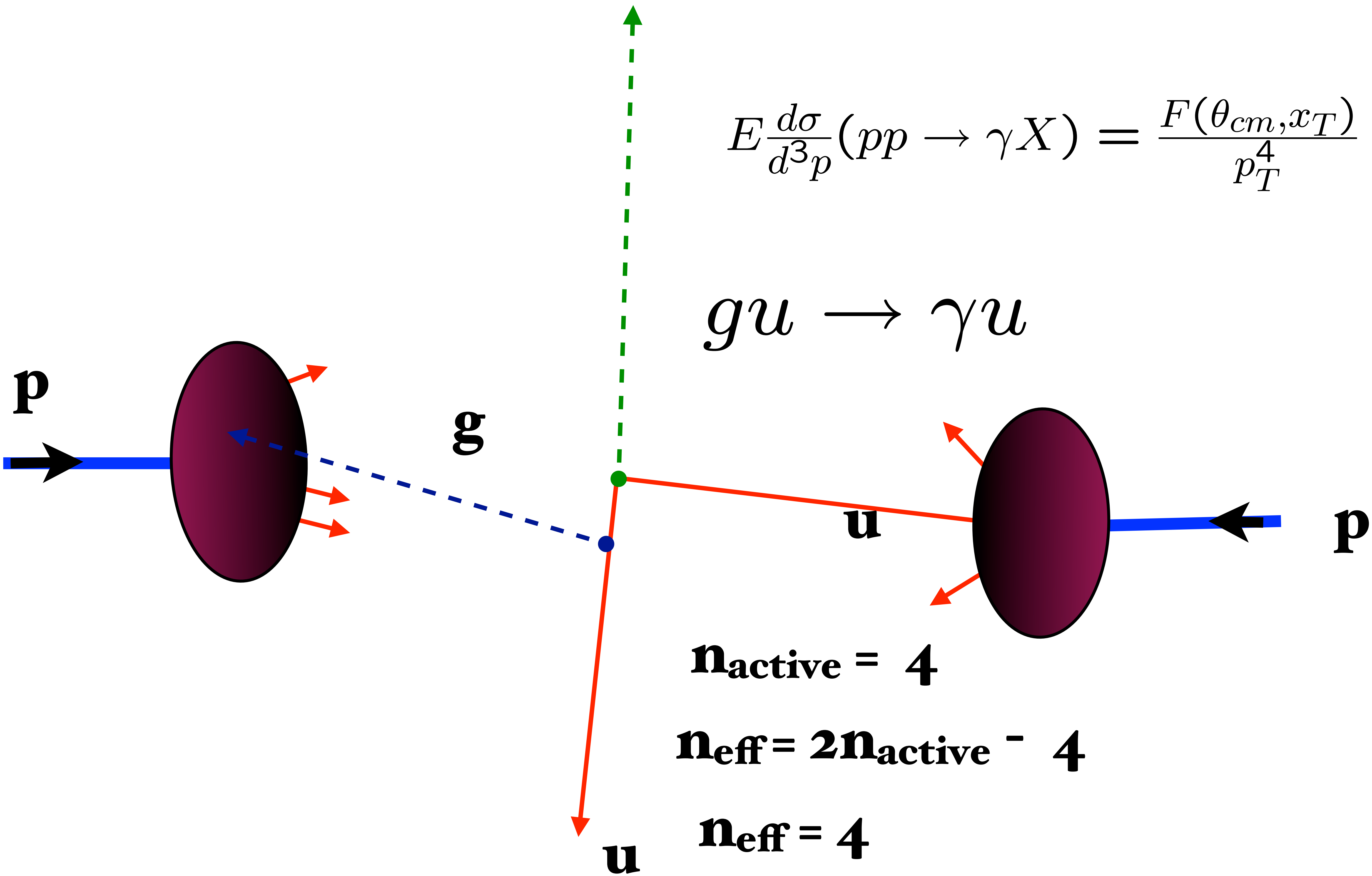
where n_{active} is the number of fields participating to the hard process

$x_\perp = 2p_\perp/\sqrt{s}$ and ϑ^{cm}: ratios of invariants

$$n_{active} = 4 \to n_{eff} = 4$$

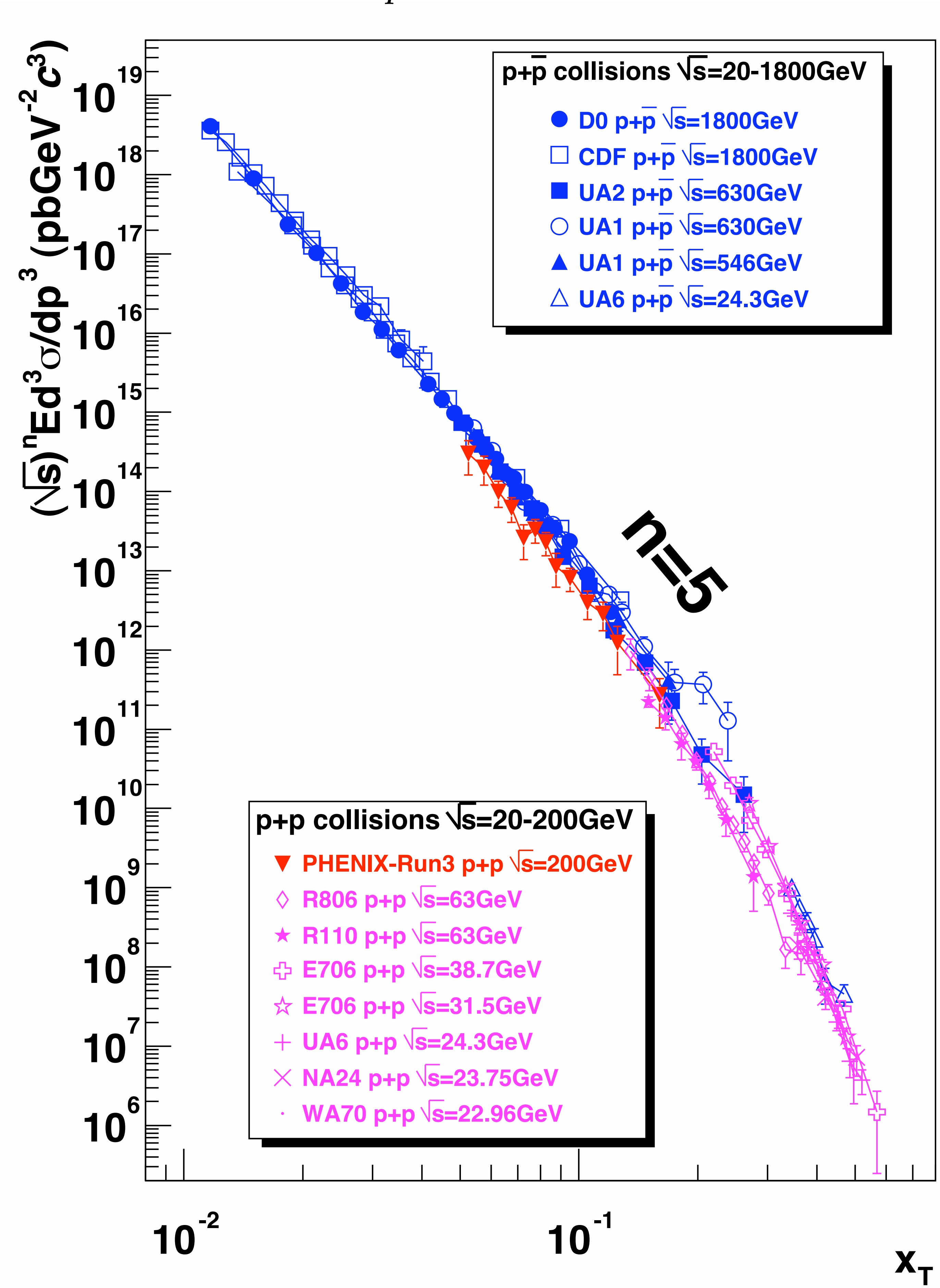
$$pp \longrightarrow \gamma X$$

$$E\frac{d\sigma}{d^3p}(pp \rightarrow \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$$



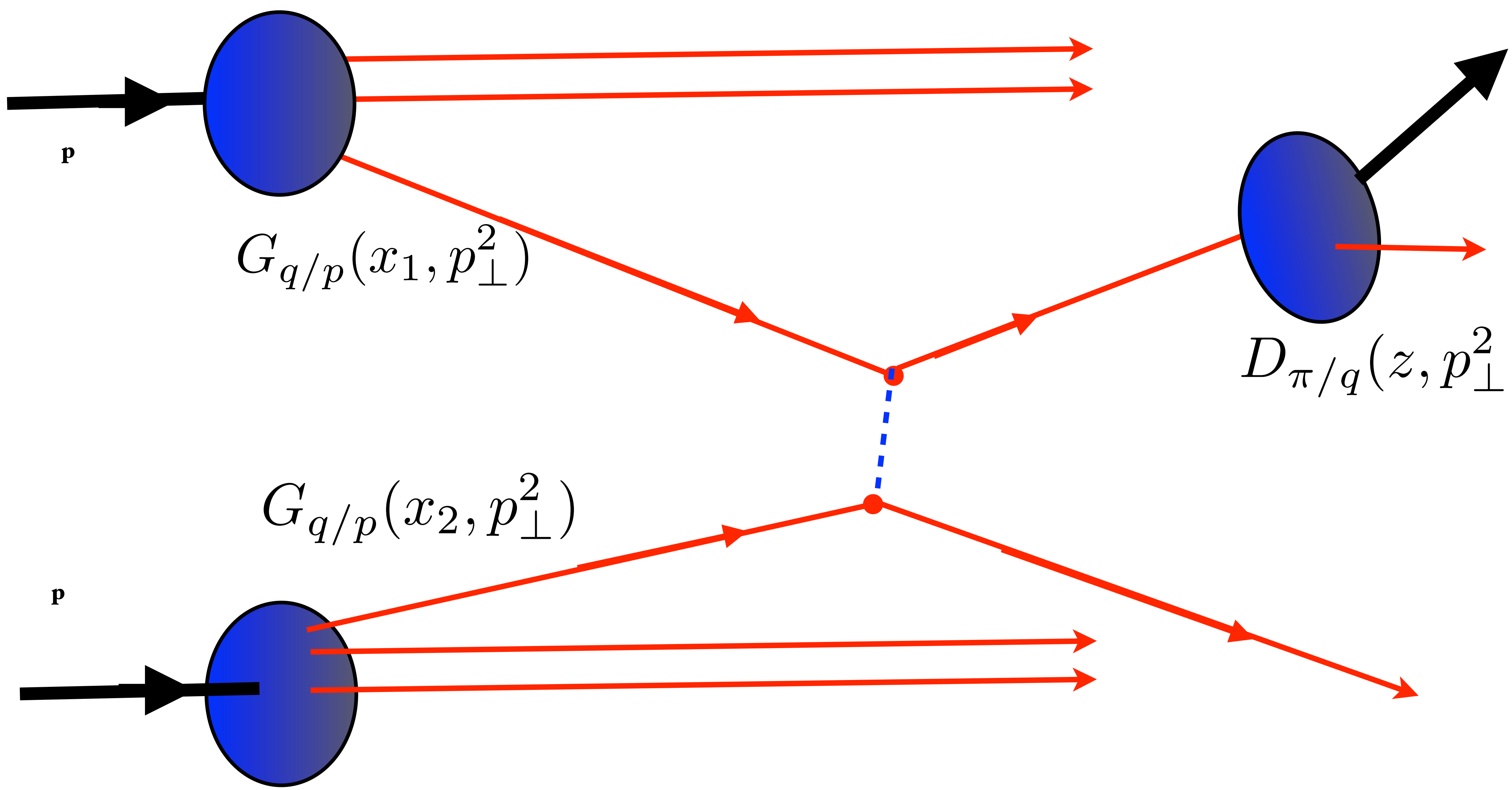
Tannenbaum

$\sqrt{s}^n E \frac{d\sigma}{d^3p}(pp \to \gamma X)$ at fixed x_T



x_T-scaling of direct photon production: consistent with PQCD

Leading-Twist Contribution to Hadron Production

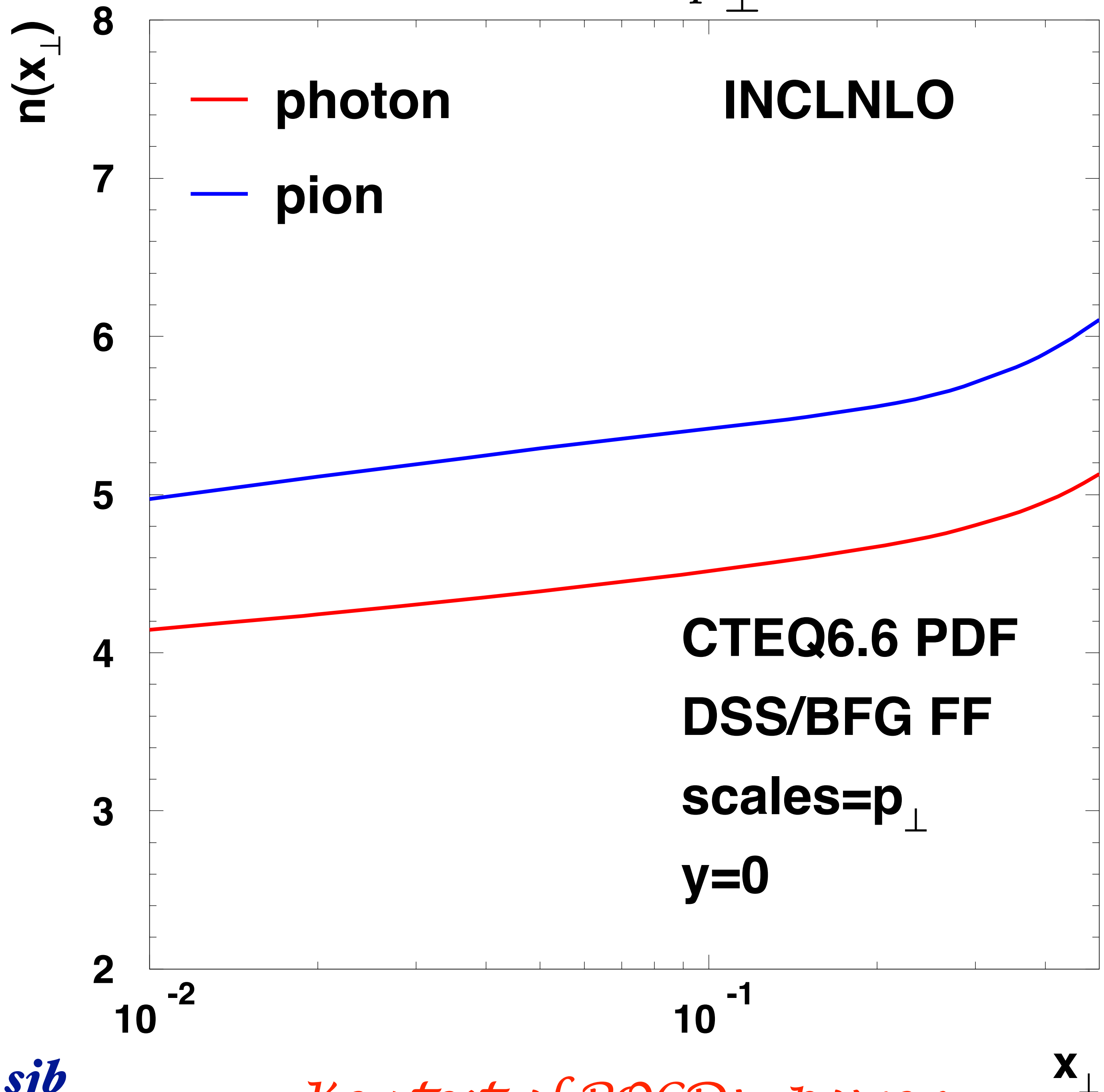


Parton model and Conformal Scaling:

$$\frac{d\sigma}{d^3p/E} = \alpha_s^2 \frac{F(x_\perp, y)}{p_\perp^4}$$

QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling

$$\frac{d\sigma}{d^3p/E} = \frac{F(x_\perp, y)}{p_\perp^{n(x_\perp)}}$$



$pp \to \pi X$

$pp \to \gamma X$

$5 < p_\perp < 20\ GeV$

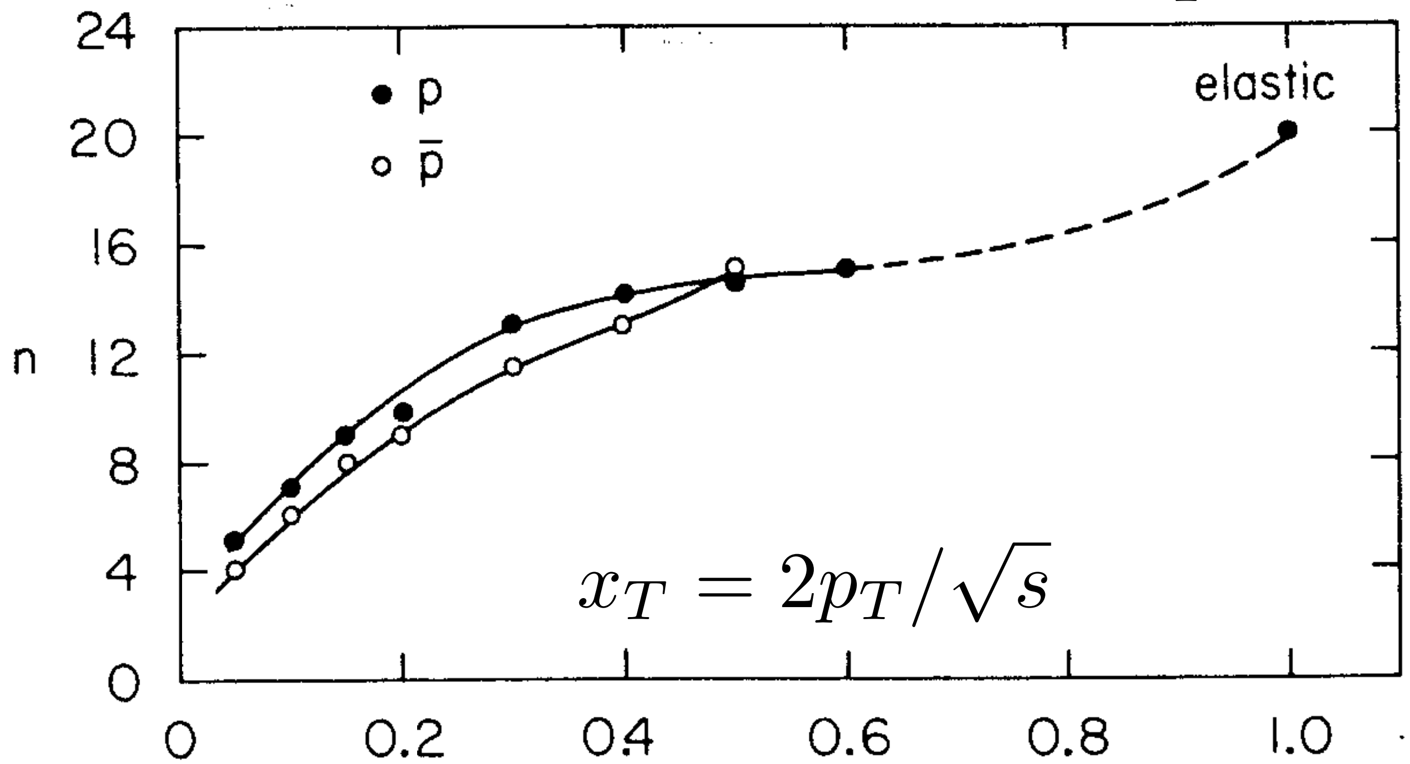
$70\ GeV < \sqrt{s} < 4\ TeV$

Key test of PQCD: power-law fall-off at fixed x_T

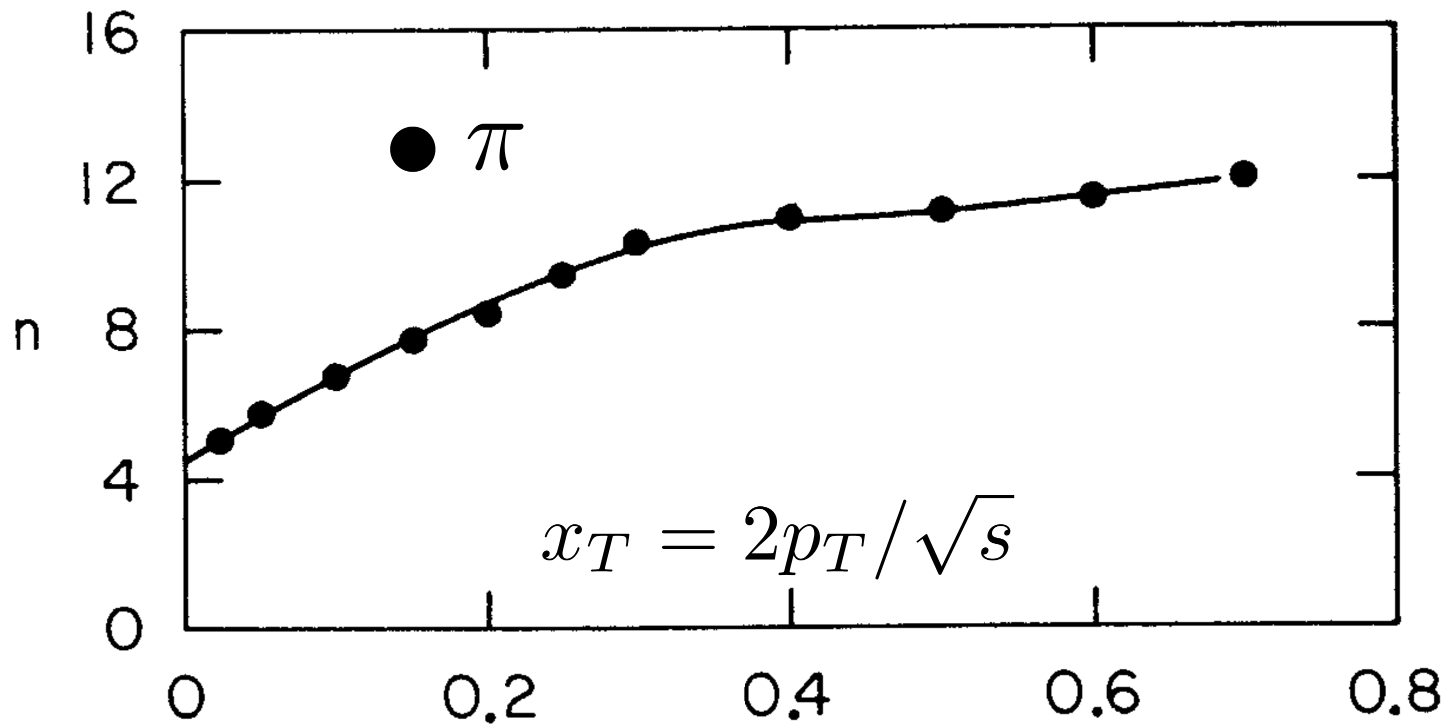
Arleo, Hwang, Sickles, sjb

Pirner, Raufeisen, sjb

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^n}$$

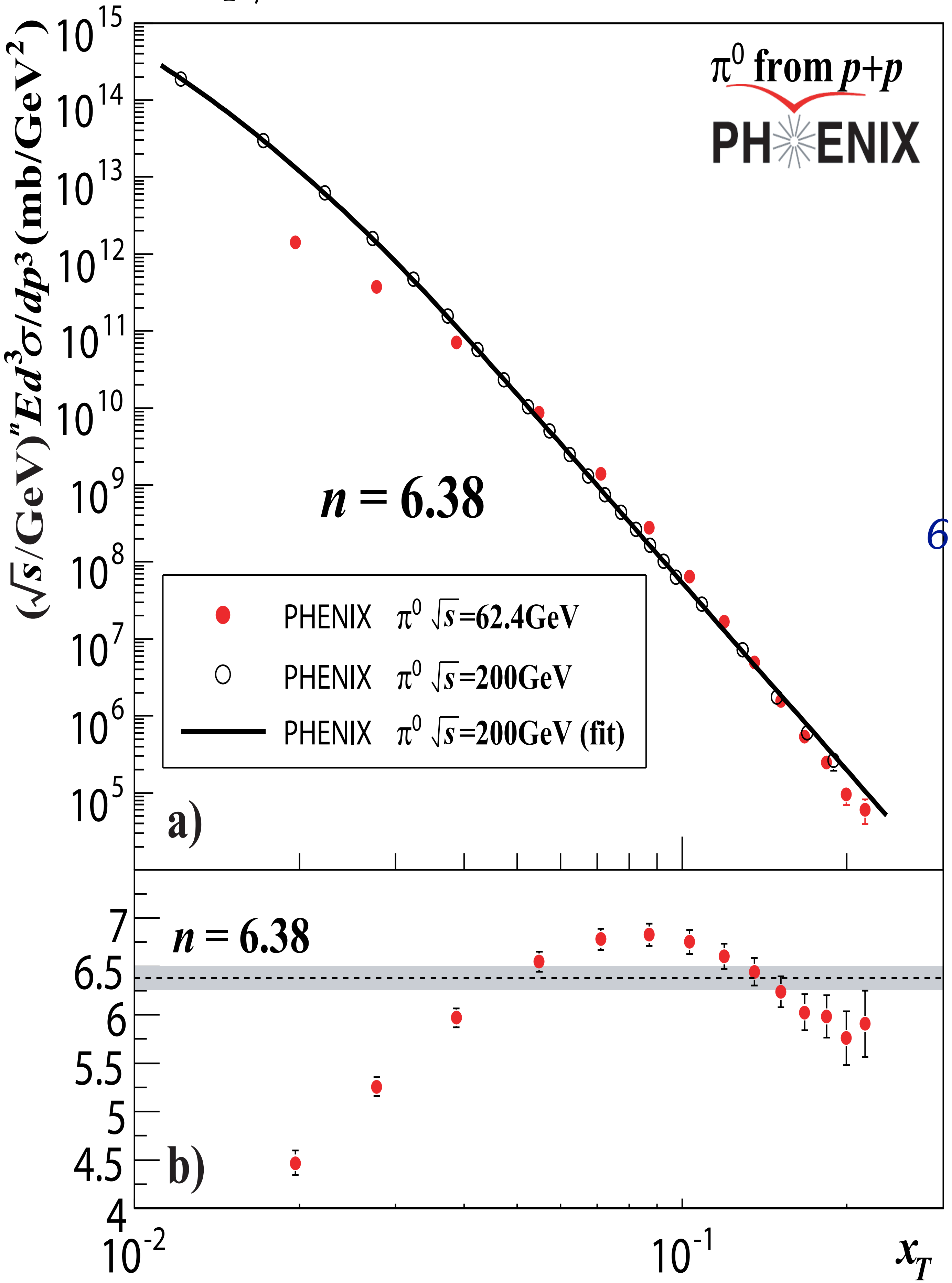


Clear evidence for higher-twist contributions



Chicago-Princeton FNAL **J. W. Cronin, SSI 1974**

$$[\sqrt{s}]^n \frac{d\sigma}{d^3p/E}(pp \to \pi^0 X) \text{ at fixed } x_T = \frac{2p_T}{\sqrt{s}}$$



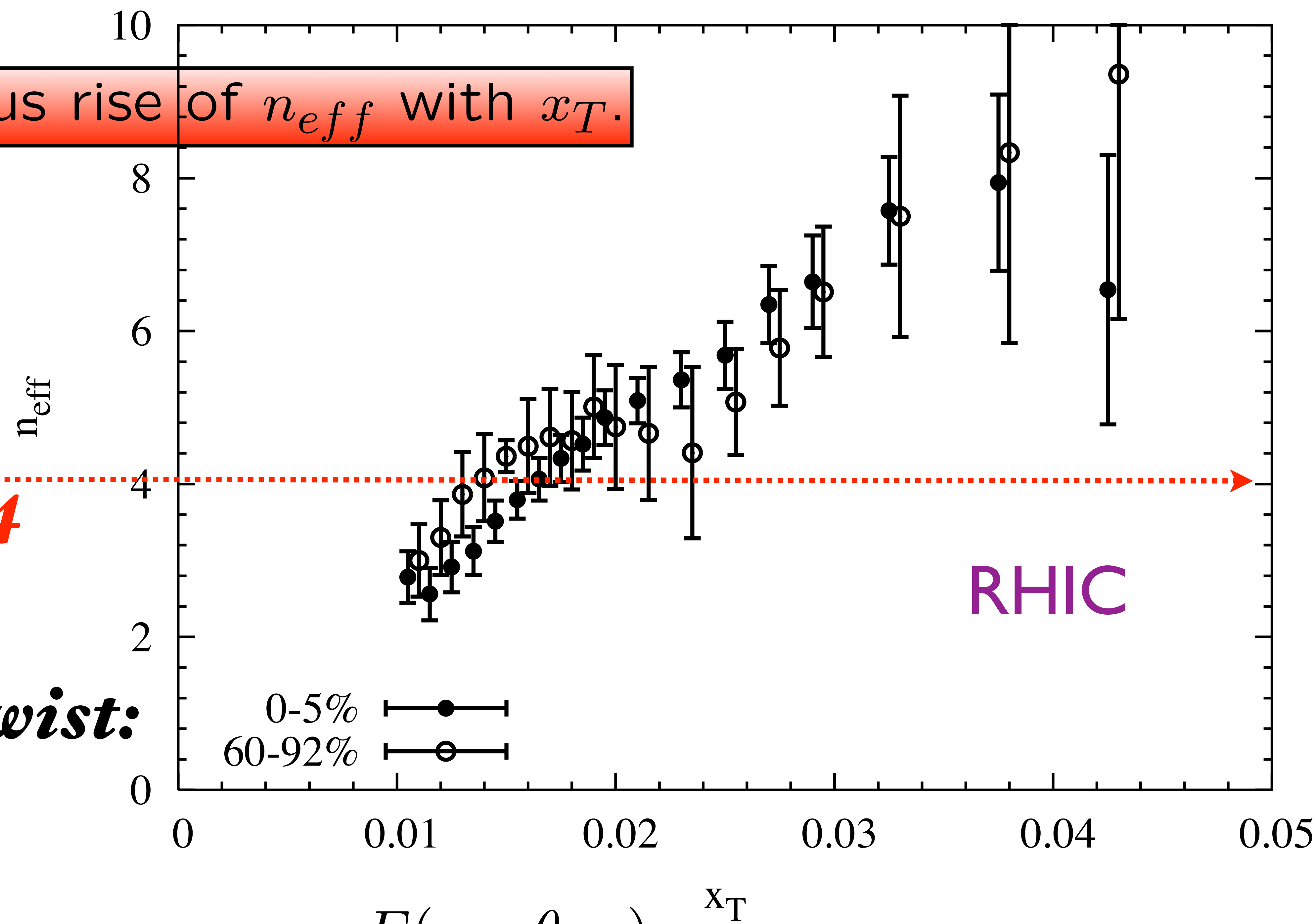
n

M. J. Tannenbaum

PHENIX
62.4 and 200 GeV data

Protons produced in AuAu collisions at RHIC do not exhibit clear scaling properties in the available p_T range. Shown are data for central $(0-5\%)$ and for peripheral $(60-90\%)$ collisions.

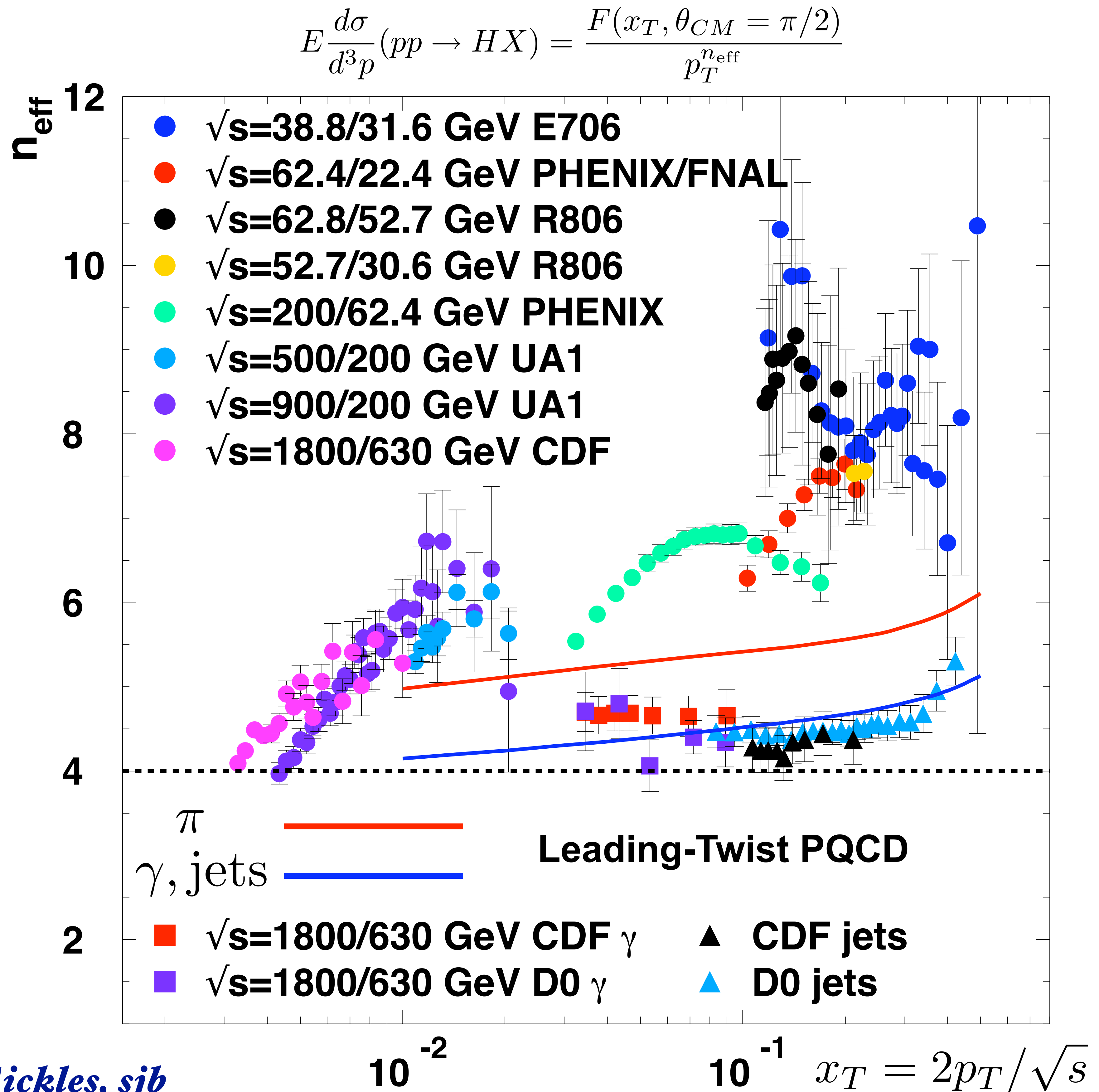
Continuous rise of n_{eff} with x_T.



$n_{eff} = 4$

Leading twist:

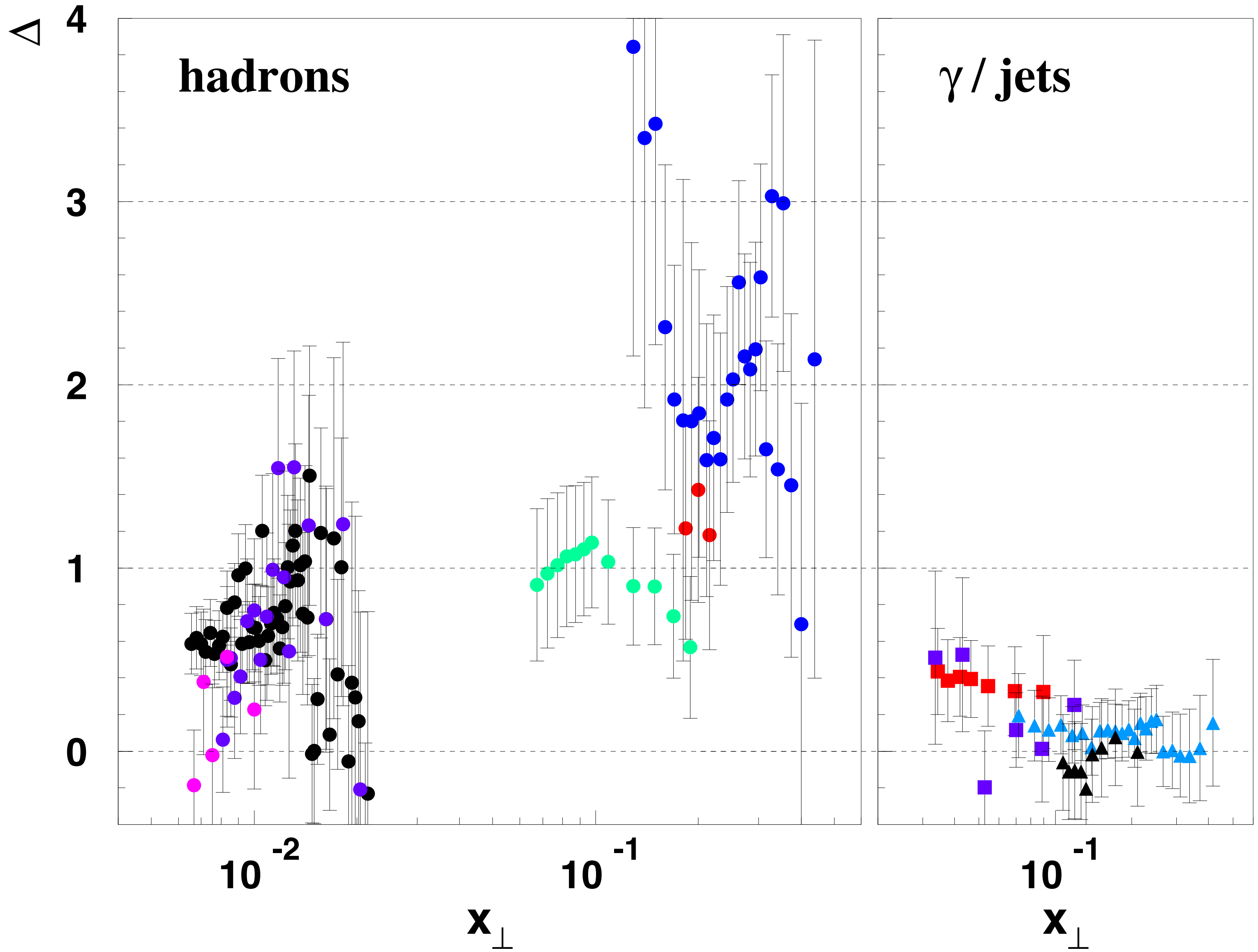
$$E\frac{d\sigma}{d^3p}(AA \to pX) = \frac{F(x_T, \theta_{cm})}{p_T^{n_{\rm eff}}}$$



Arleo, Hwang, Sickles, sjb

Comparing to QCD

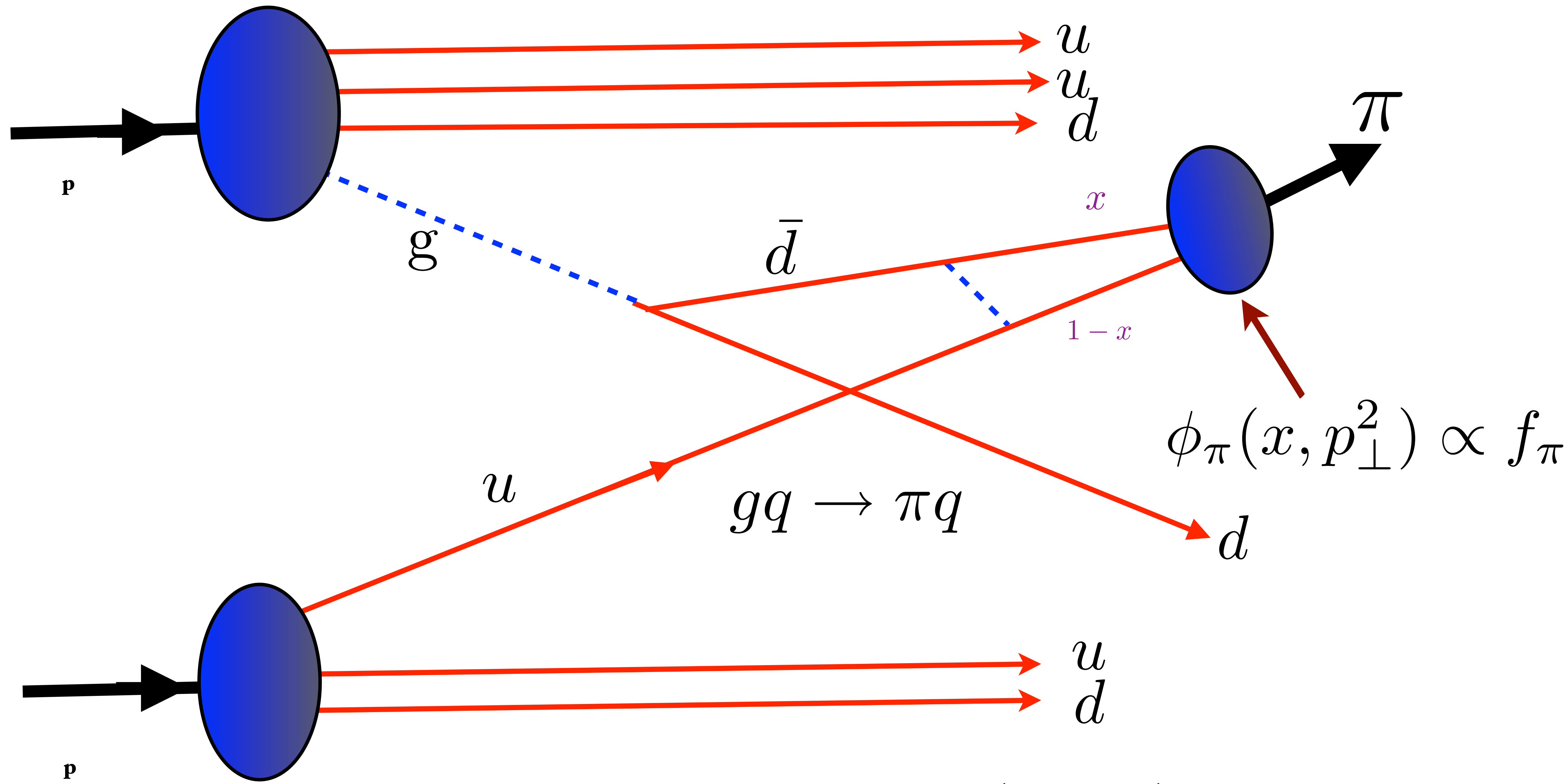
Arleo, Hwang, Sickles, sjb



Clear hierarchy

Tevatron	$x_\perp \sim 10^{-2}$	$\Delta \simeq 0.5$
RHIC	$x_\perp \sim 10^{-1}$	$\Delta \simeq 1$
fixed target	$x_\perp \sim$ few times 10^{-1}	$\Delta \simeq 2$

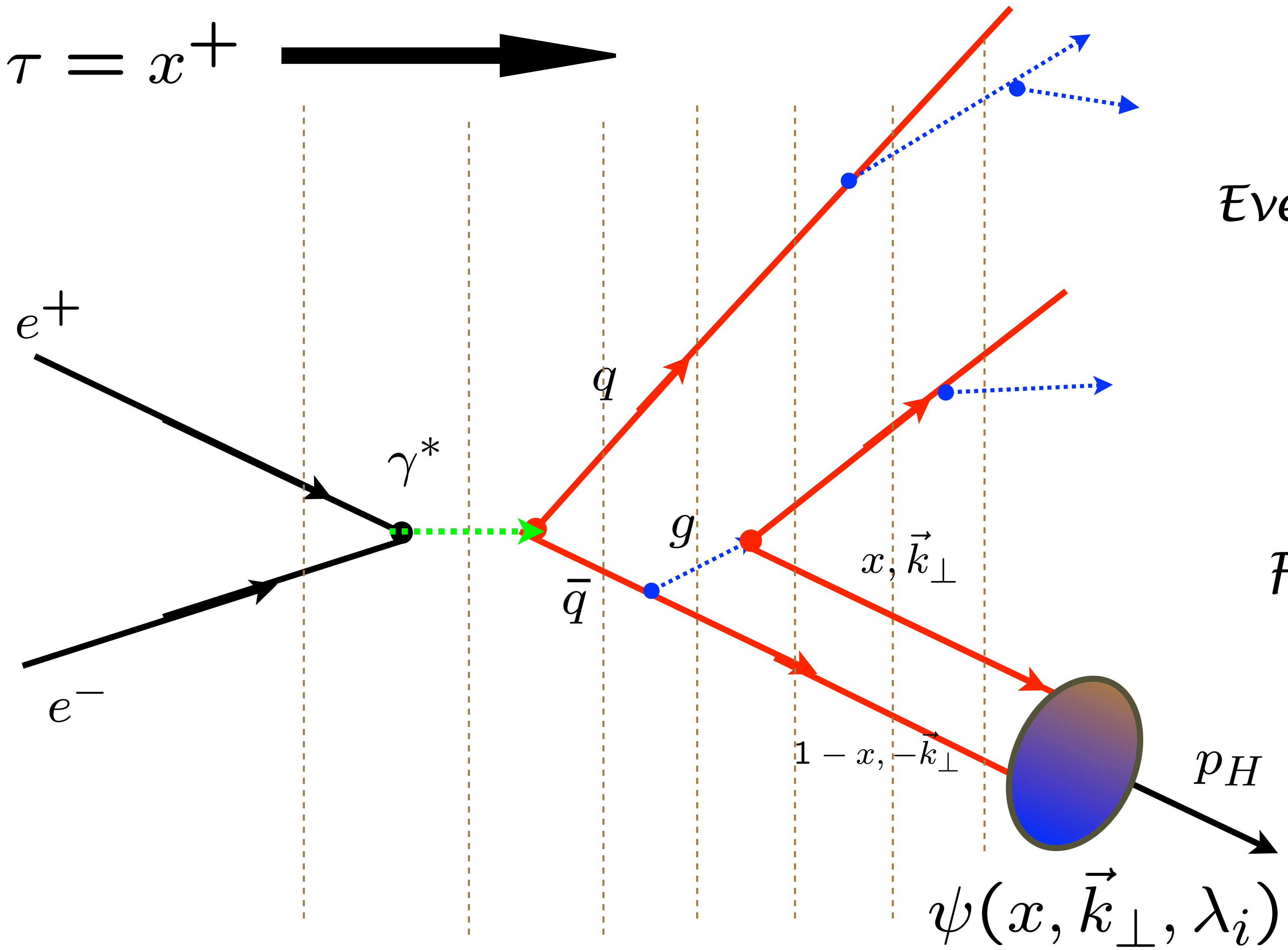
Direct Contribution to Hadron Production



$$\frac{d\sigma}{d^3p/E} = \alpha_s^3 f_\pi^2 \frac{F(x_\perp, y)}{p_\perp^6}$$

No Fragmentation Function

Hadronization at the Amplitude Level



Event amplitude generator

Cluster Factorization

Capture if $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda^2_{QCD}}$

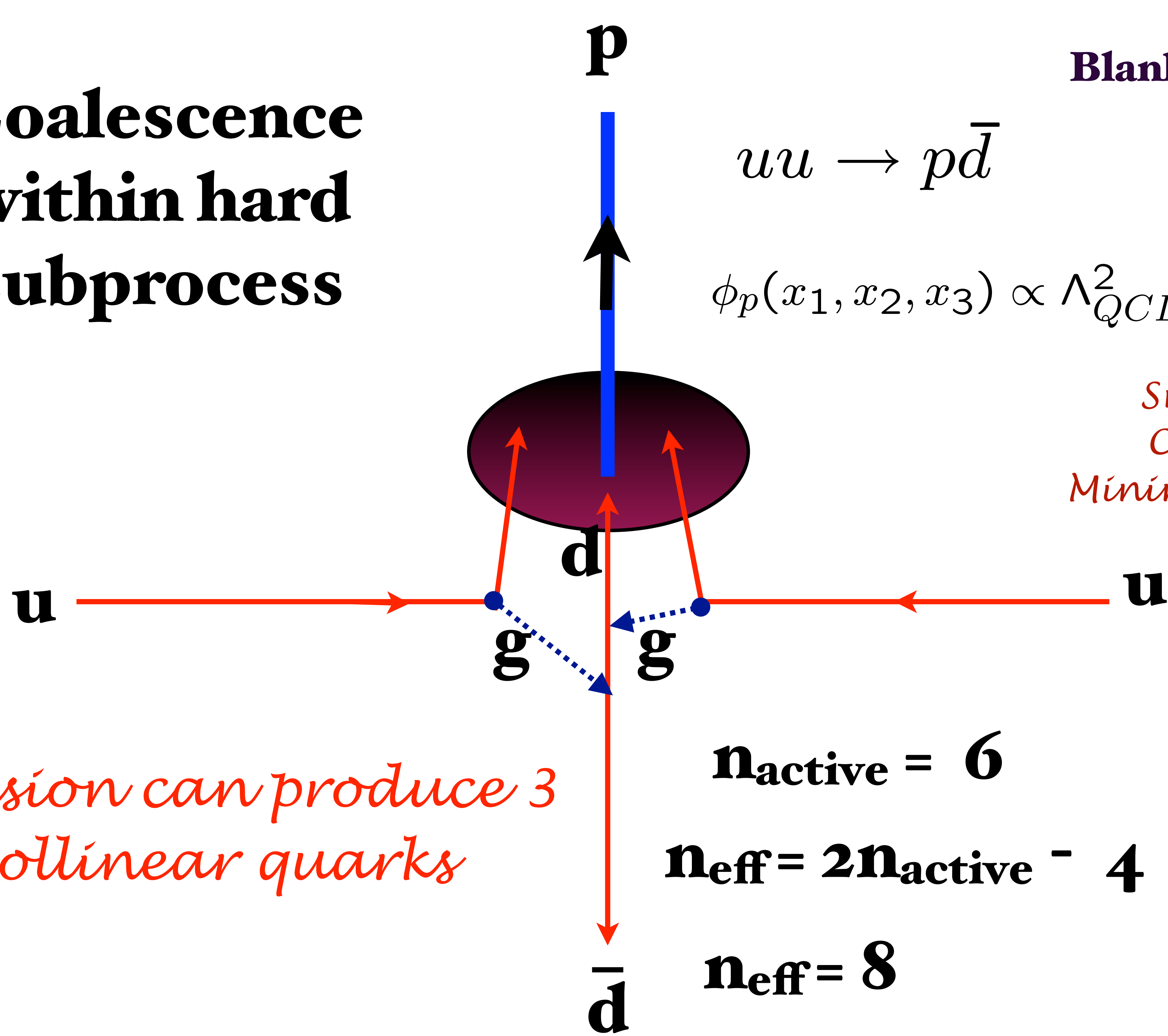
i.e.,

$\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda^2_{QCD}$

AdS/QCD Hard Wall Confinement:

Baryon can be made directly within hard subprocess

Coalescence within hard subprocess



$uu \rightarrow p\bar{d}$

$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Sickles, Sjb

Small color-singlet
Color Transparent
Minimal same-side energy

Collision can produce 3 collinear quarks

$\mathbf{n_{active} = 6}$

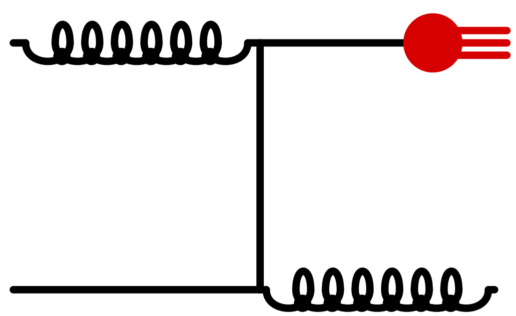
$\mathbf{n_{eff} = 2n_{active} - 4}$

$\mathbf{n_{eff} = 8}$

$qq \rightarrow B\bar{q}$

Scaling laws in inclusive pion production

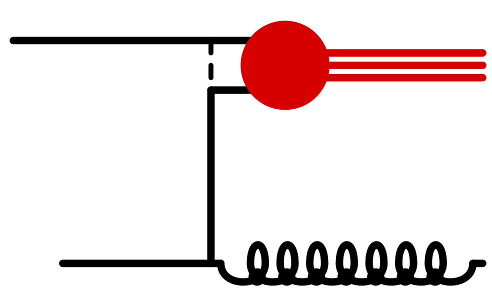
- Conventional pQCD picture (leading twist): $2 \to 2$ process followed by fragmentation into a pion on long time scales



$$n_{\mathrm{active}} = 4 \to n = 4 \; (= 2 \times 4 - 4)$$

$$E \, \frac{d\sigma}{d^3p}(p \; p \; \to \pi \; X) \sim \frac{F(x_\perp, \vartheta^{\mathrm{cm}})}{p_\perp^4}$$

- Direct higher-twist picture: pion produced directly in the hard process



$$n_{\mathrm{active}} = 5 \to n = 6 \; (= 2 \times 5 - 4)$$

$$E \, \frac{d\sigma}{d^3p}(p \; p \; \to \pi \; X) \sim \frac{F'(x_\perp, \vartheta^{\mathrm{cm}})}{p_\perp^6}$$

Scale dependence

Pion scaling exponent extracted vs. $p_\perp$ at fixed $x_\perp$

2-component toy-model

$$\sigma^{\mathrm{model}}(pp \to \pi\ \mathrm{X}) \propto \frac{A(x_\perp)}{p_\perp^4} + \frac{B(x_\perp)}{p_\perp^6}$$

Define effective exponent

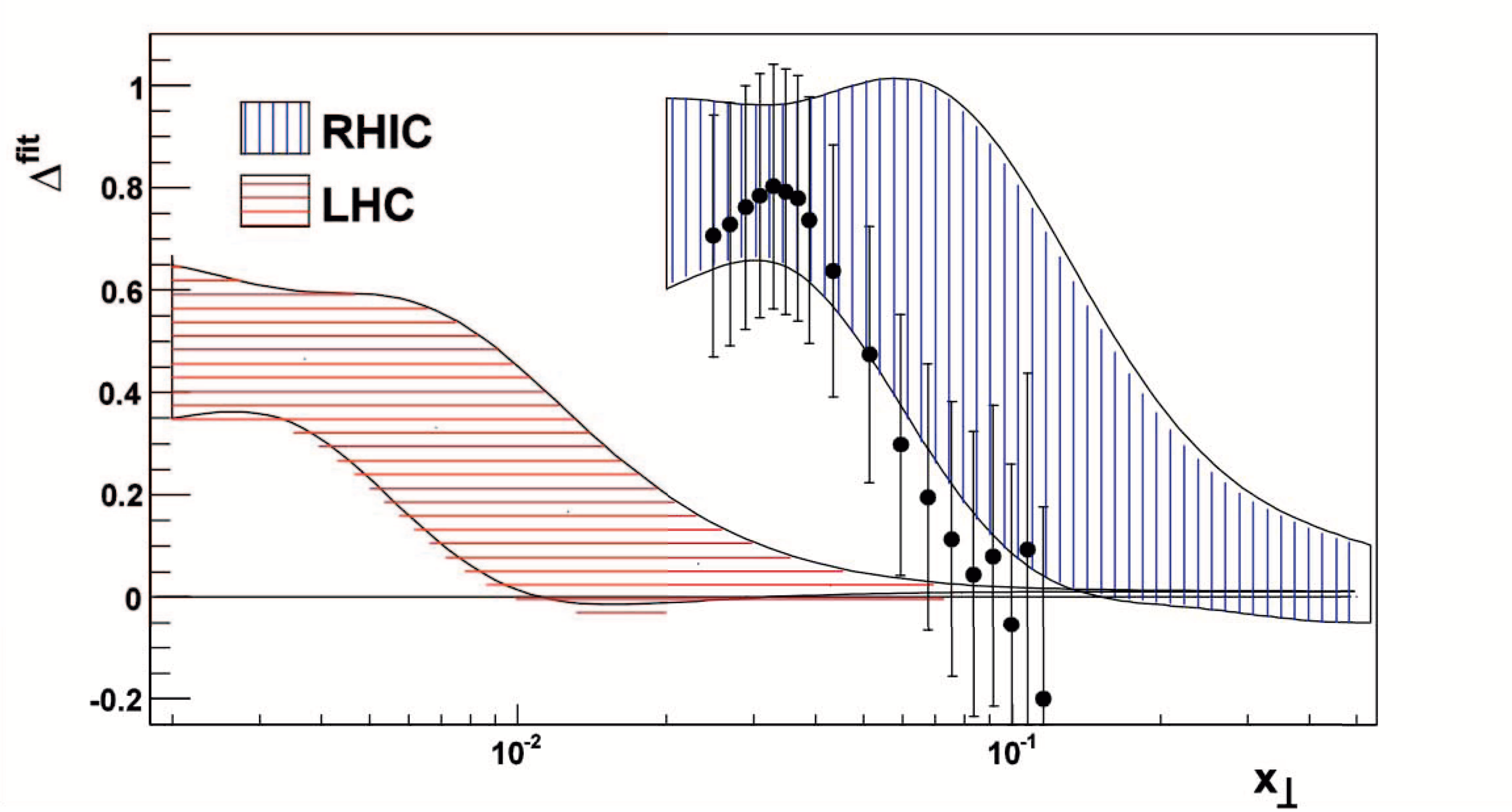
$$\begin{aligned} n_{\mathrm{eff}}(x_\perp, p_\perp, B/A) &\equiv -\frac{\partial \ln \sigma^{\mathrm{model}}}{\partial \ln p_\perp} + n^{\mathrm{NLO}}(x_\perp, p_\perp) - 4 \\ &= \frac{2B/A}{p_\perp^2 + B/A} + n^{\mathrm{NLO}}(x_\perp, p_\perp) \end{aligned}$$

RHIC/LHC predictions

PHENIX results

Scaling exponents from $\sqrt{s} = 500$ GeV preliminary data

[A. Bezilevsky, APS Meeting]



- Magnitude of Δ and its $x_\perp$-dependence consistent with predictions

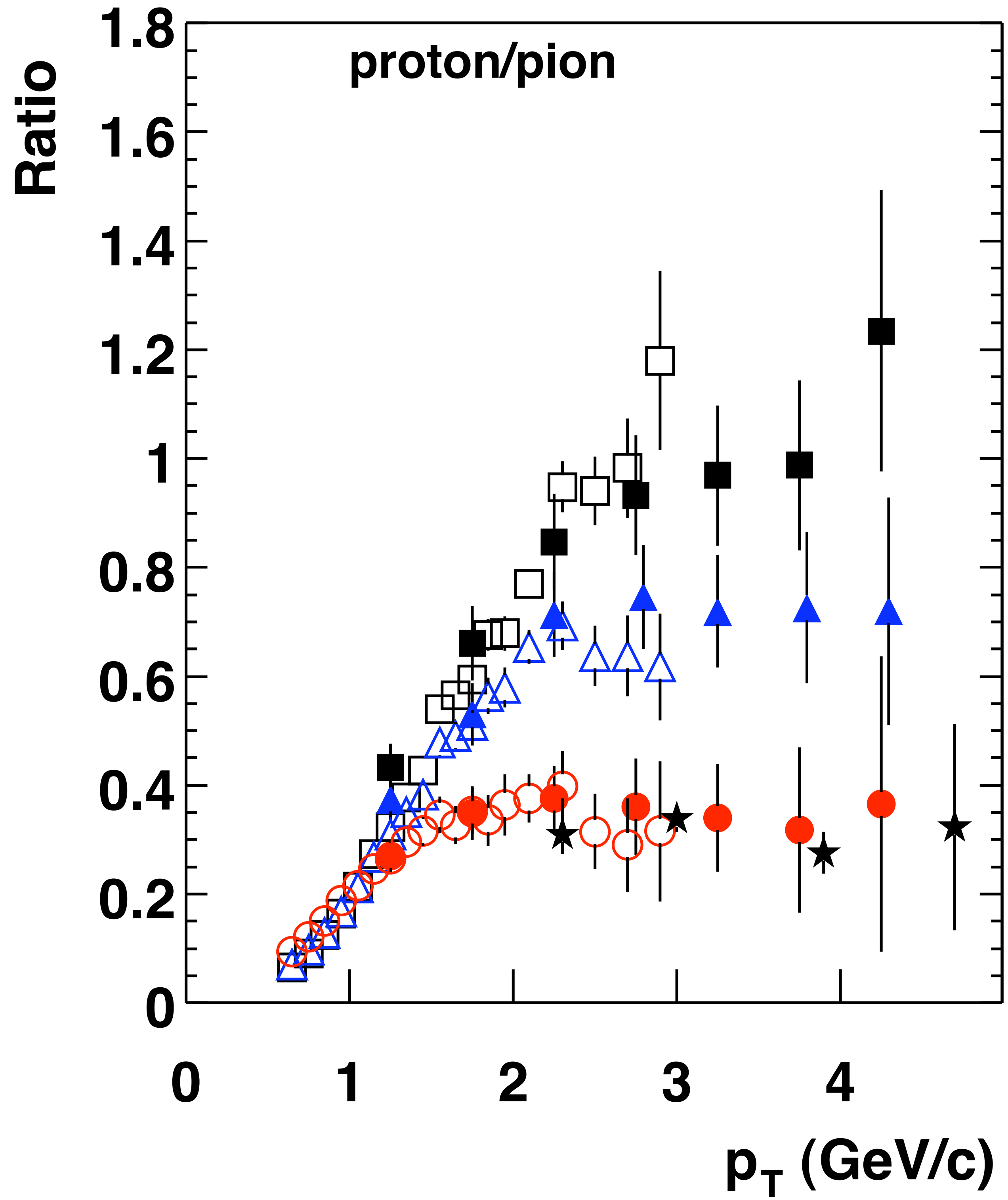
Higher Twist at the LHC

- **Fixed x_T: powerful analysis of PQCD**
- **Insensitive to modeling**
- **Higher twist terms energy efficient since no wasted fragmentation energy**
- **Evaluate at minimal x_1 and x_2 where structure functions are maximal**
- **Higher Twist competitive despite faster fall-off in p_T**
- **Direct processes can confuse new physics searches**
- **Related to Quarkonium Processes -- Jian-wei Qiu**
- **Bound-state production: Light-Front Wavefunctions, Distribution amplitudes, ERBL evolution.**

S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett*. **91**, 172301 (2003).

Particle ratio changes with centrality!

Protons less absorbed in nuclear collisions than pions because of dominant color transparent higher twist process



← **Central**

Au+Au 0-10%

Au+Au 20-30%

Au+Au 60-92%

p+p, $\sqrt{s}$ = 53 GeV, ISR

e^+e^-, gluon jets, DELPHI

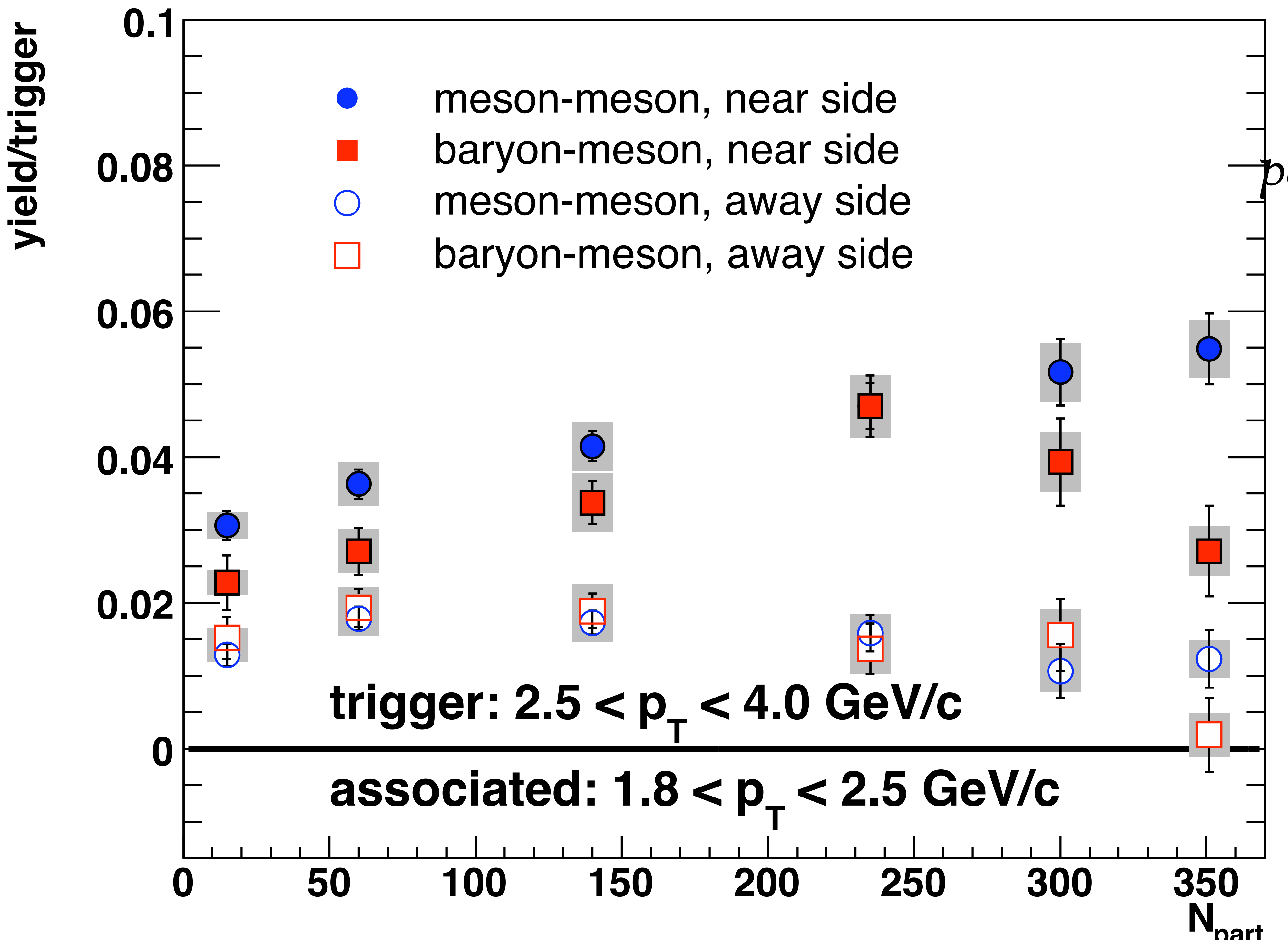
e^+e^-, quark jets, DELPHI

← **Peripheral**

Tannenbaum:

Baryon Anomaly:

Anne Sickles

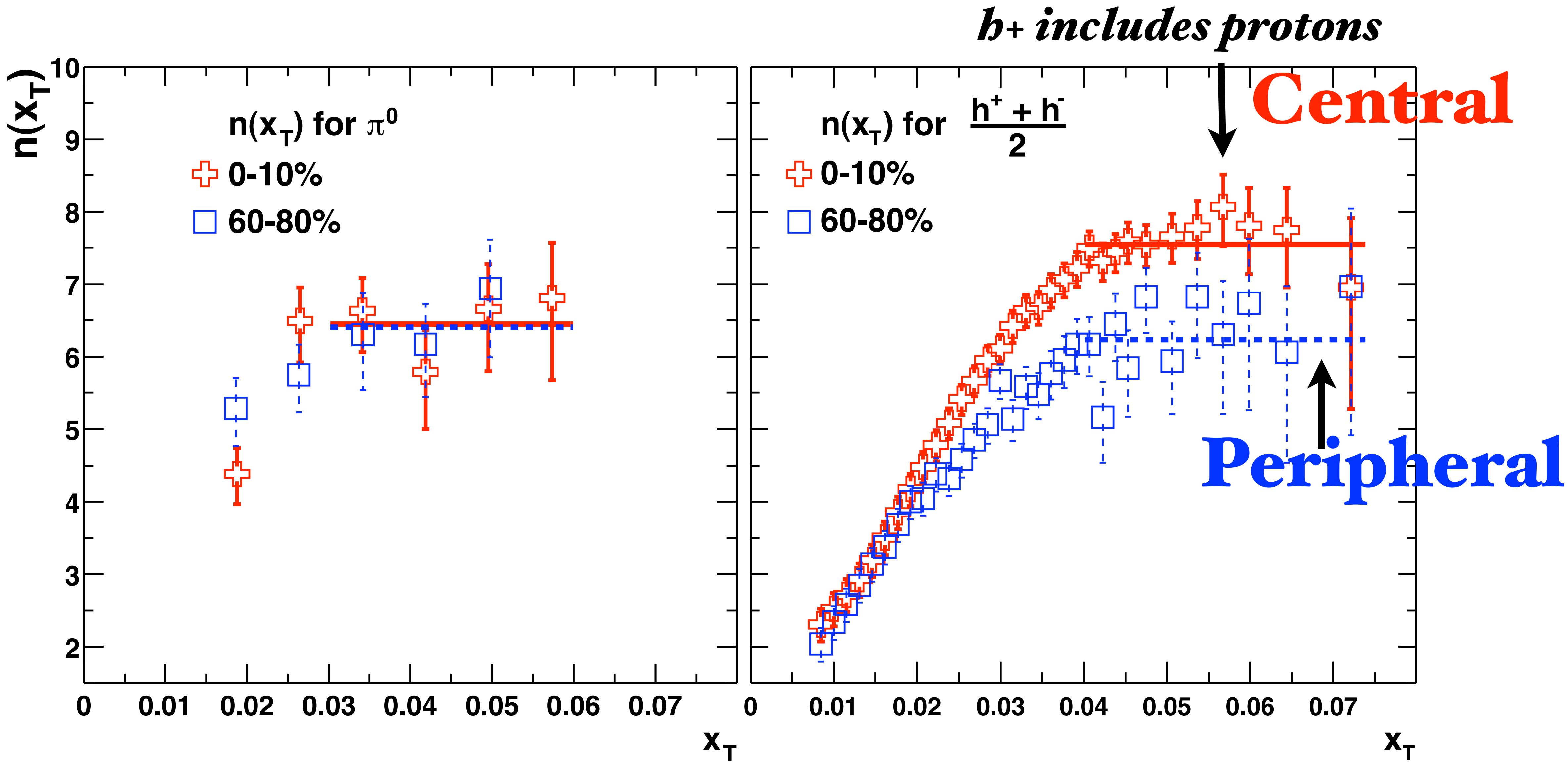


proton trigger: # same-side particles decreases with centrality

Proton production more dominated by color-transparent direct high-n_{eff} subprocesses

Power-law exponent $n(x_T)$ for π^0 and h spectra in central and peripheral Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV

S. S. Adler, *et al.*, PHENIX Collaboration, *Phys. Rev.* C **69**, 034910 (2004) [`nucl-ex/0308006`].



Proton power changes with centrality !

Proton production dominated by
color-transparent direct high n_{eff} subprocesses

Baryon Anomaly: Evidence for Direct, Higher-Twist Subprocesses

- **Explains anomalous power behavior at fixed x_T**
- **Protons more likely to come from direct higher-twist subprocess than pions**
- **Protons less absorbed than pions in central nuclear collisions because of color transparency**
- **Predicts increasing proton to pion ratio in central collisions**
- **Proton power n_{eff} increases with centrality since leading twist contribution absorbed**
- **Fewer same-side hadrons for proton trigger at high centrality**
- **Exclusive-inclusive connection at $x_T = 1$**

Anne Sickles, sjb

Higher Twist at the LHC

- **Fixed x_T: powerful analysis of PQCD**
- **Insensitive to modeling**
- **Higher twist terms energy efficient since no wasted fragmentation energy**
- **Evaluate at minimal x_1 and x_2 where structure functions are maximal**
- **Higher Twist competitive despite faster fall-off in p_T**
- **Direct processes can confuse new physics searches**
- **Related to Quarkonium Processes -- Jian-wei Qiu**
- **Bound-state production: Light-Front Wavefunctions, Distribution amplitudes, ERBL evolution.**

Orbital Angular Momentum in QFTH

- *Rigorous boost-invariant definition of L^z from LF Theory*
- *Non-Zero Pauli Form Factor, Anomalous Moment and Sivers Effect require nonzero quark orbital angular momentum*
- *Sum of n L^z cancel in n-particle Fock state: overcounting*
- *Vanishing anomalous gravitomagnetic moment*
- *Wavefunctions in Instant Form do not determine current matrix elements!*
- *AdS/QCD: Spin J^z of Proton carried by quark L^z*

Fock vacuum $|0\rangle$ eigenstate of the full Hamiltonian

$$P^- = \frac{1}{2}\int dx_+ d^2x_\perp \left(\overline{\Psi}\gamma^+ \frac{\bar{m}^2 + (i\nabla_\perp)^2}{i\partial^+}\Psi + A^\mu_a (i\nabla_\perp)^2 A^a_\mu\right) \quad \text{free}$$
$$+ \; g\int dx_+ d^2x_\perp J^\mu_a A^a_\mu \quad \text{vertex interaction}$$
$$+ \; \frac{g^2}{4}\int dx_+ d^2x_\perp B^{\mu\nu}_a B^a_{\mu\nu} \quad 4-\text{point gluon}$$
$$+ \; \frac{g^2}{2}\int dx_+ d^2x_\perp J^+_a \frac{1}{(i\partial^+)^2} J^+_a \quad \text{instantaneous gluon interaction}$$
$$+ \; \frac{g^2}{2}\int dx_+ d^2x_\perp \overline{\Psi}\gamma^\mu T^a A^a_\mu \frac{\gamma^+}{i\partial^+}\left(\gamma^\nu T^b A^b_\nu \Psi\right), \quad \text{instantaneous fermion interaction}$$

where

$$J^\mu_a = \bar{\Psi}\gamma^\mu T^a \Psi \chi^\mu_a + f^{abc}\partial^\mu A^\nu_b A_\nu .$$

- Light-Front Vacuum: Frame-independent, causal, trivial, no normal ordering needed, zero cosmological constant!

- Instant-Form Vacuum: Frame-dependent, acausal, non-trivial, normal ordering needed, vacuum contributions to all matrix elements

Two Different Vacua!!

QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- heavy quarks only from gluon splitting
- renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary

Dirac-Feynman propagator invalid for nucleon!

- Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.

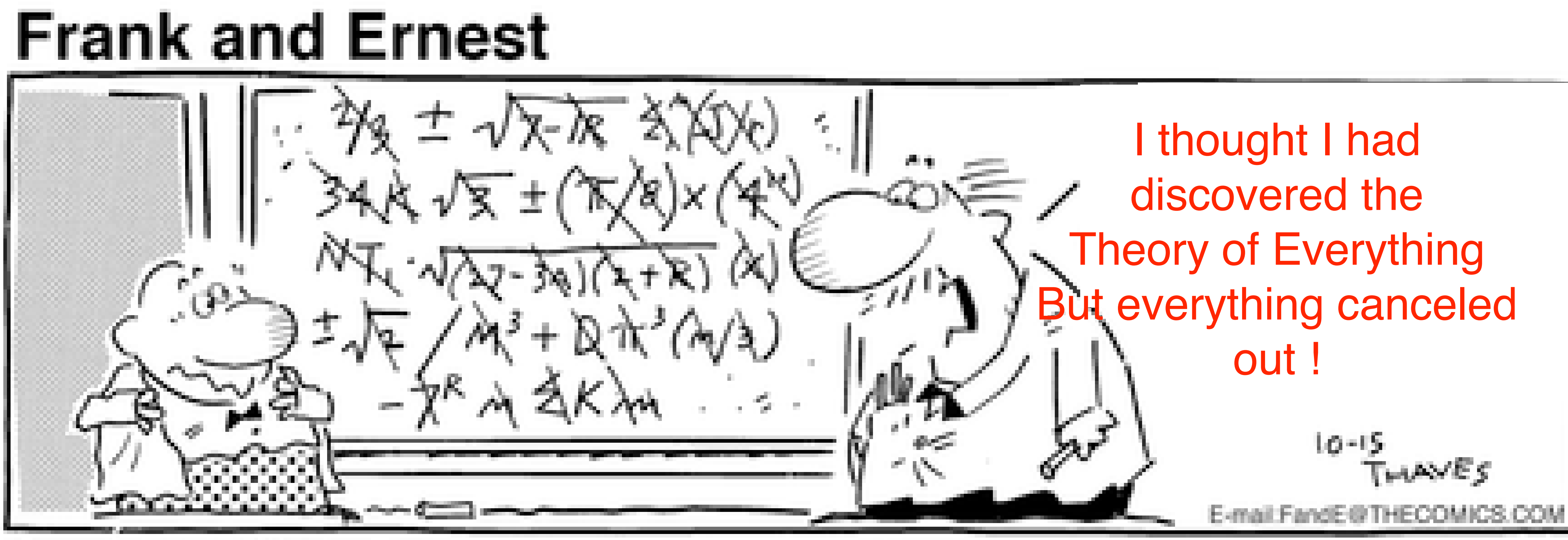
- Novel QCD Phenomena: hidden color, color transparency, strangeness asymmetry, intrinsic charm, anomalous heavy quark phenomena, anomalous spin effects, single-spin asymmetries, odderon, diffractive deep inelastic scattering, dangling gluons, shadowing, antishadowing, quark-gluon plasma, ...

Truth is stranger than fiction, but it is because Fiction is obliged to stick to possibilities. —Mark Twain

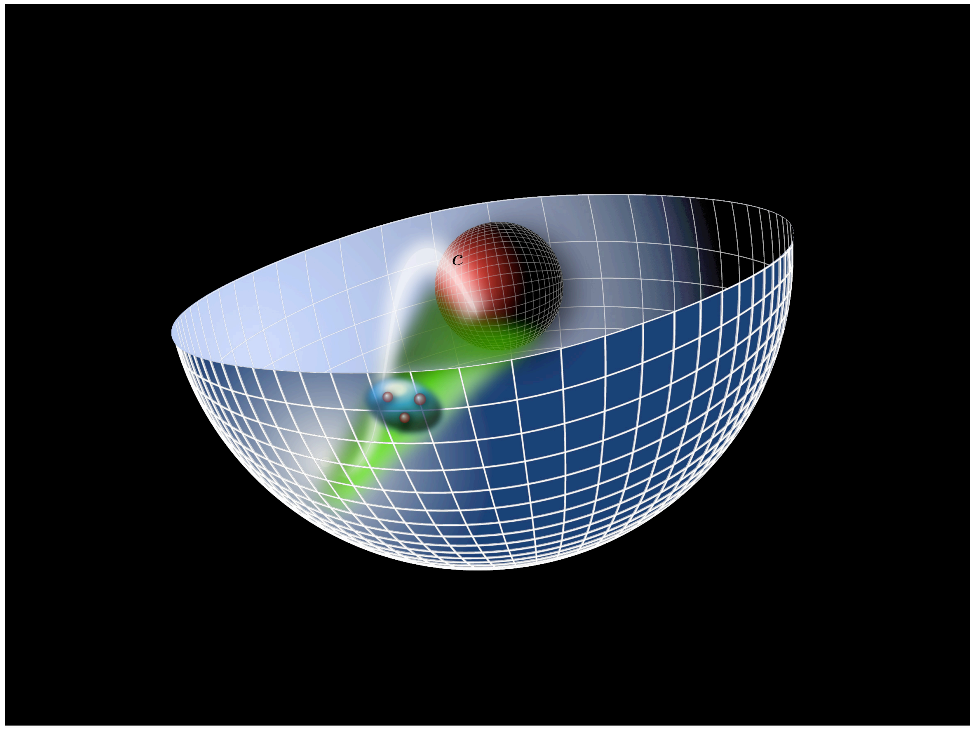
SCIENCE VOL 265 15 SEPTEMBER 1995

A Theory of Everything Takes Place

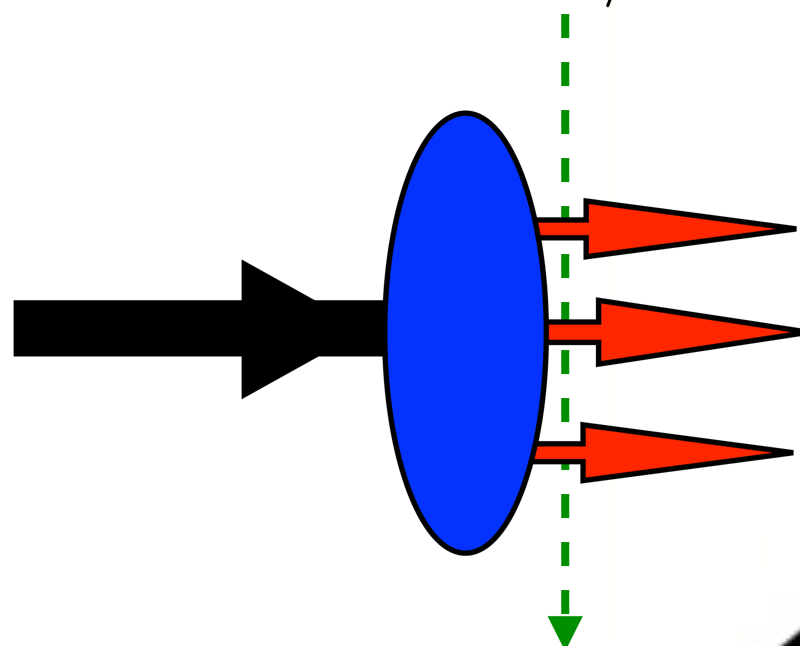
String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.

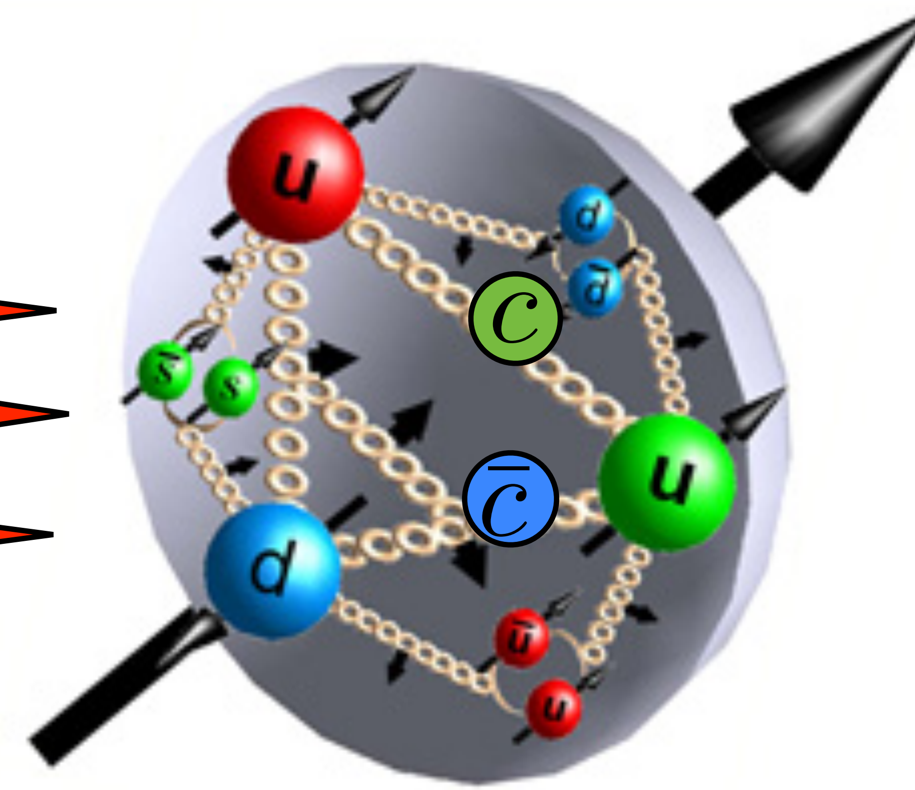


Light-Front Holography, Transversity and Orbital Angular Momentum



Fixed $\tau = t + z/c$





INT Workshop

Orbital Angular Momentum in QCD

February 6 - 17, 2012

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY