QUARK ANGULAR MOMENTUM FROM TMDS

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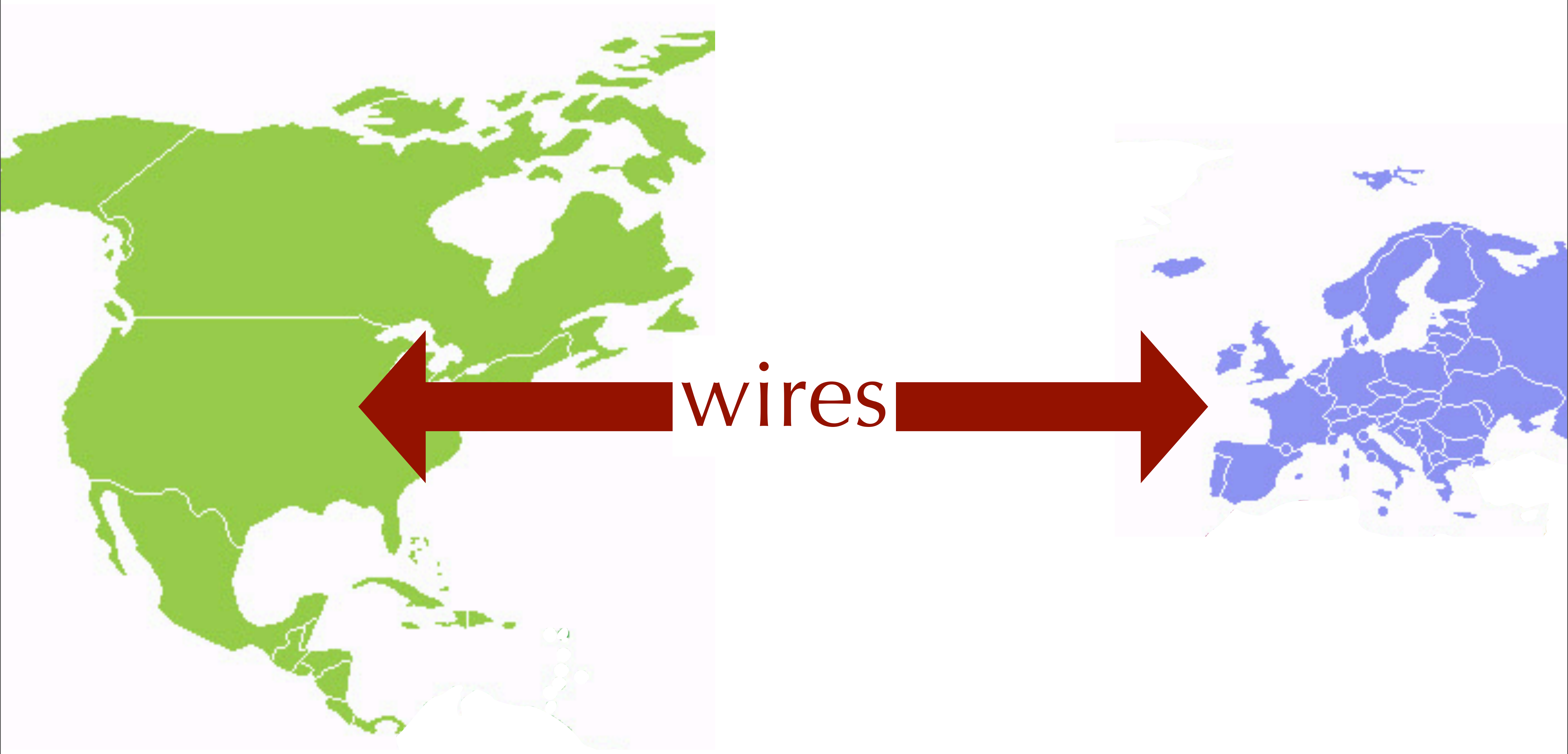
PRL 107 (2011) 212001

Outline

- Introduction
- The lensing function
- Extraction of Sivers and E(x,0,0)
- Final results on angular momenta


wires

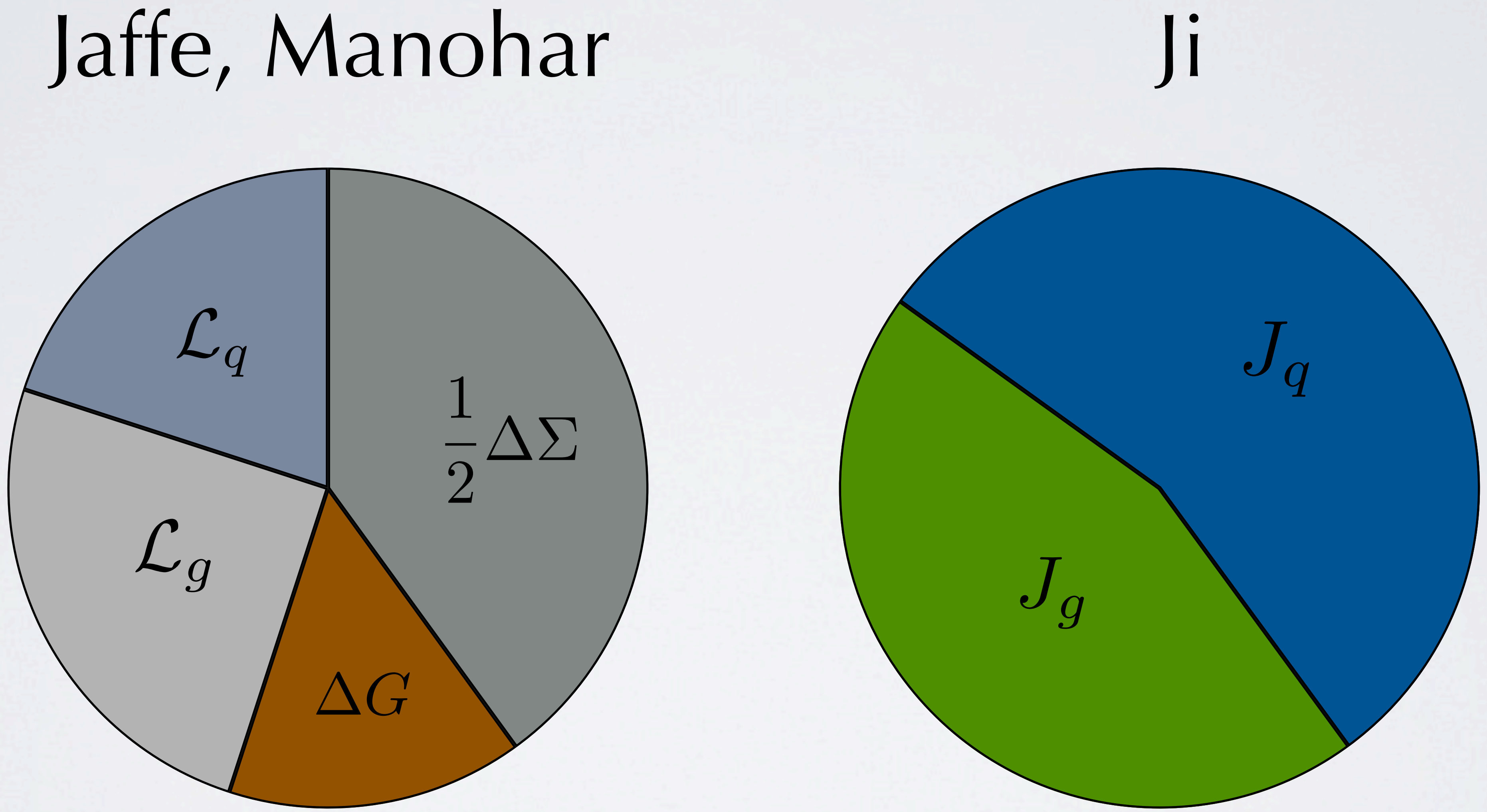
GPDs TMDs

GPDs ⟵ lensing ⟶ **TMDs**

GPDs ⟵ lensing ⟶ TMDs

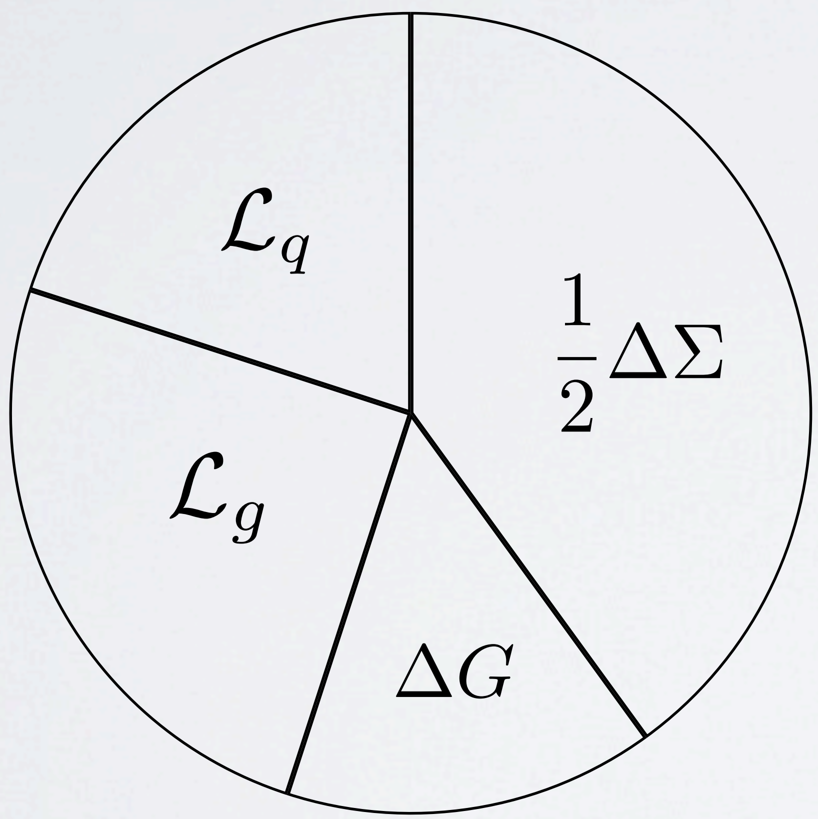
- LIMITATIONS: the connection is model-inspired and not general
- ADVANTAGES: it is possible to give an estimate of angular momentum for the first time using also TMD data

Angular momenta

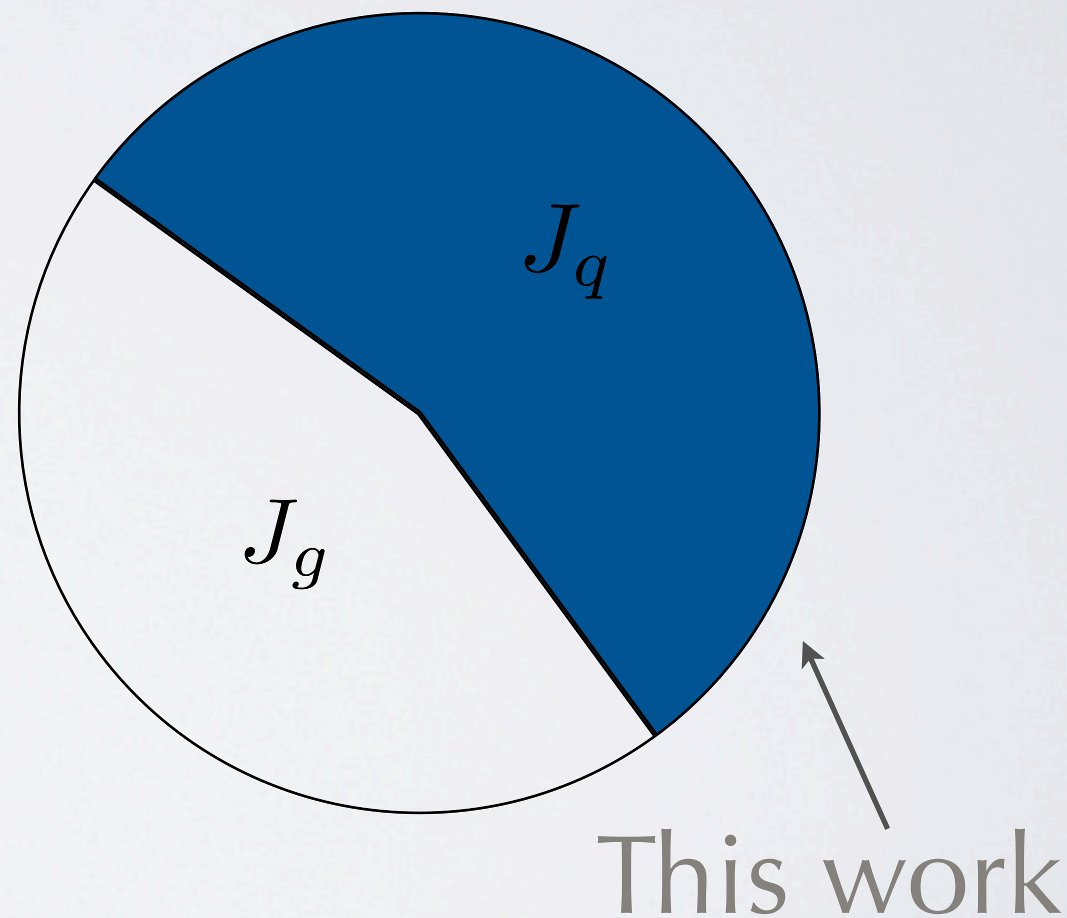


Angular momenta

Jaffe, Manohar



Ji



Ji's total angular mom.

$$J^q = \frac{1}{2}\int_0^1 dx\, x\left(H^q(x,0,0) + E^q(x,0,0)\right)$$

Ji's total angular mom.

Forward limits of GPDs

$$J^q = \frac{1}{2}\int_0^1 dx\, x\left(H^q(x,0,0) + E^q(x,0,0)\right)$$

Ji's total angular mom.

Forward limits of GPDs

$$J^q = \frac{1}{2}\int_0^1 dx\, x\left(H^q(x,0,0) + E^q(x,0,0)\right)$$

$q(x)$

Well-known unpolarized PDF

Impossible
to measure directly

The only "data" on E(x,0,0)

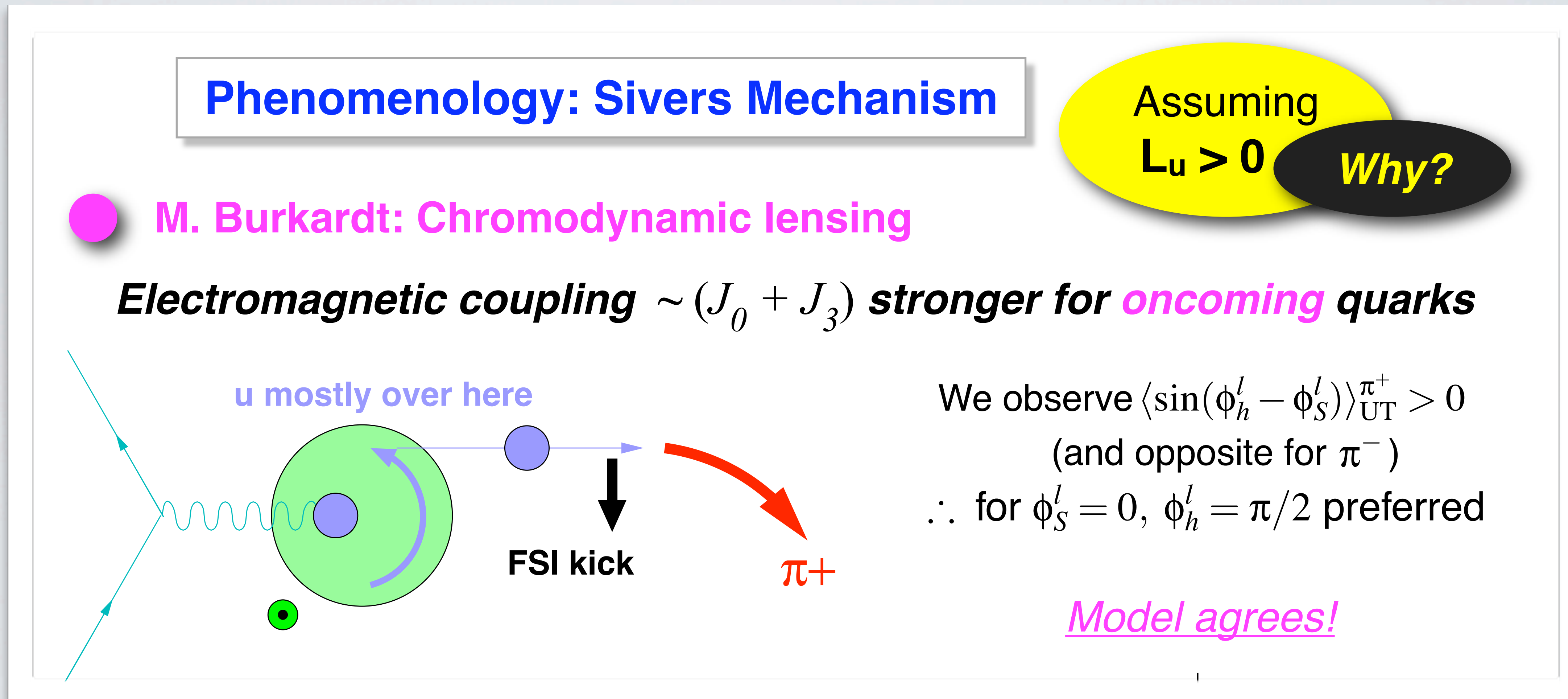
Anomalous magnetic moments

$$\kappa^p = \int_0^1 \frac{dx}{3}\Big[2E^{u_v}(x,0,0) - E^{d_v}(x,0,0) - E^{s_v}(x,0,0)\Big],$$

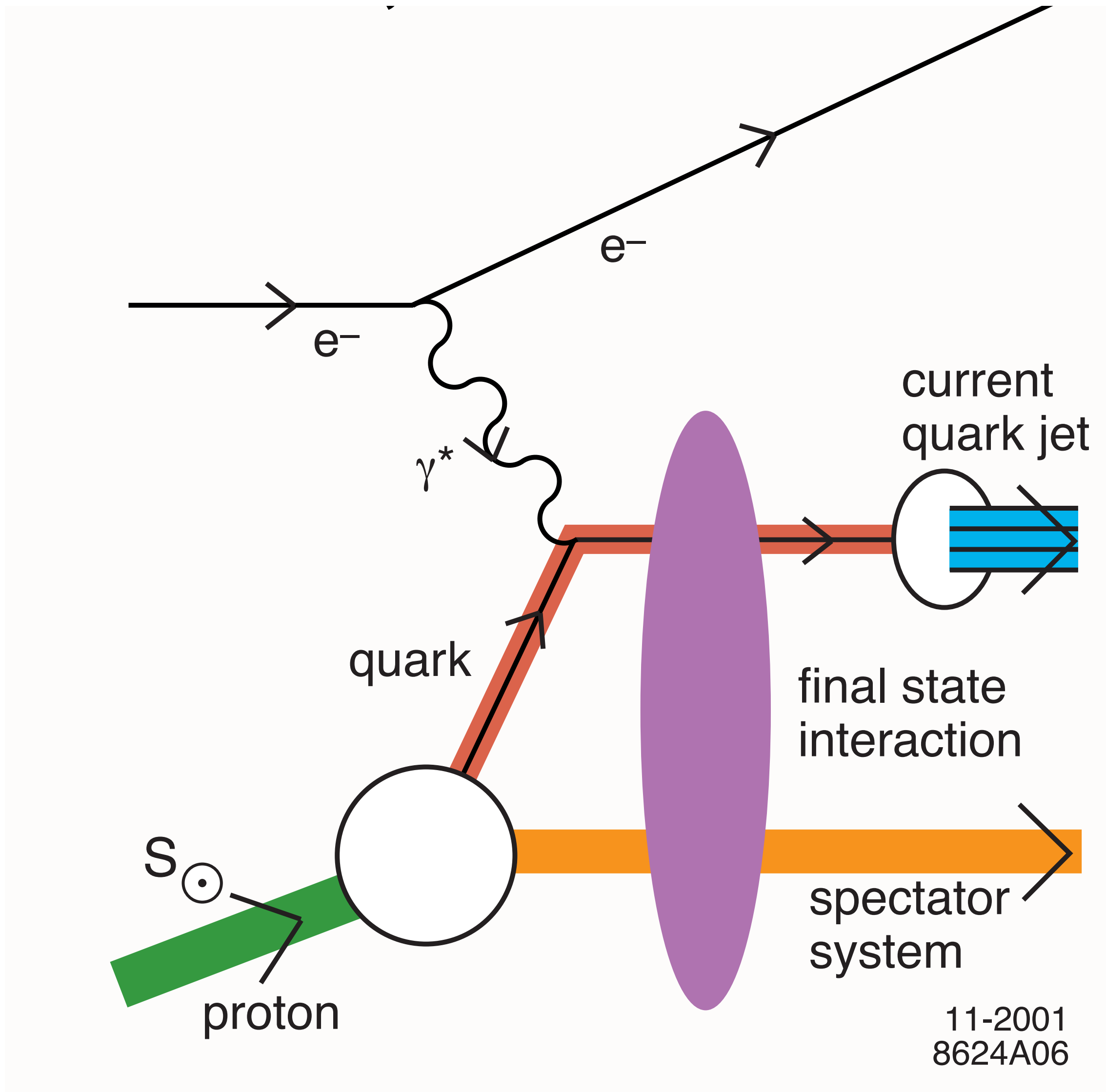
$$\kappa^n = \int_0^1 \frac{dx}{3}\Big[2E^{d_v}(x,0,0) - E^{u_v}(x,0,0) - E^{s_v}(x,0,0)\Big].$$

The lensing function

Naomi Makins, last week



Stan Brodsky, Transversity2011



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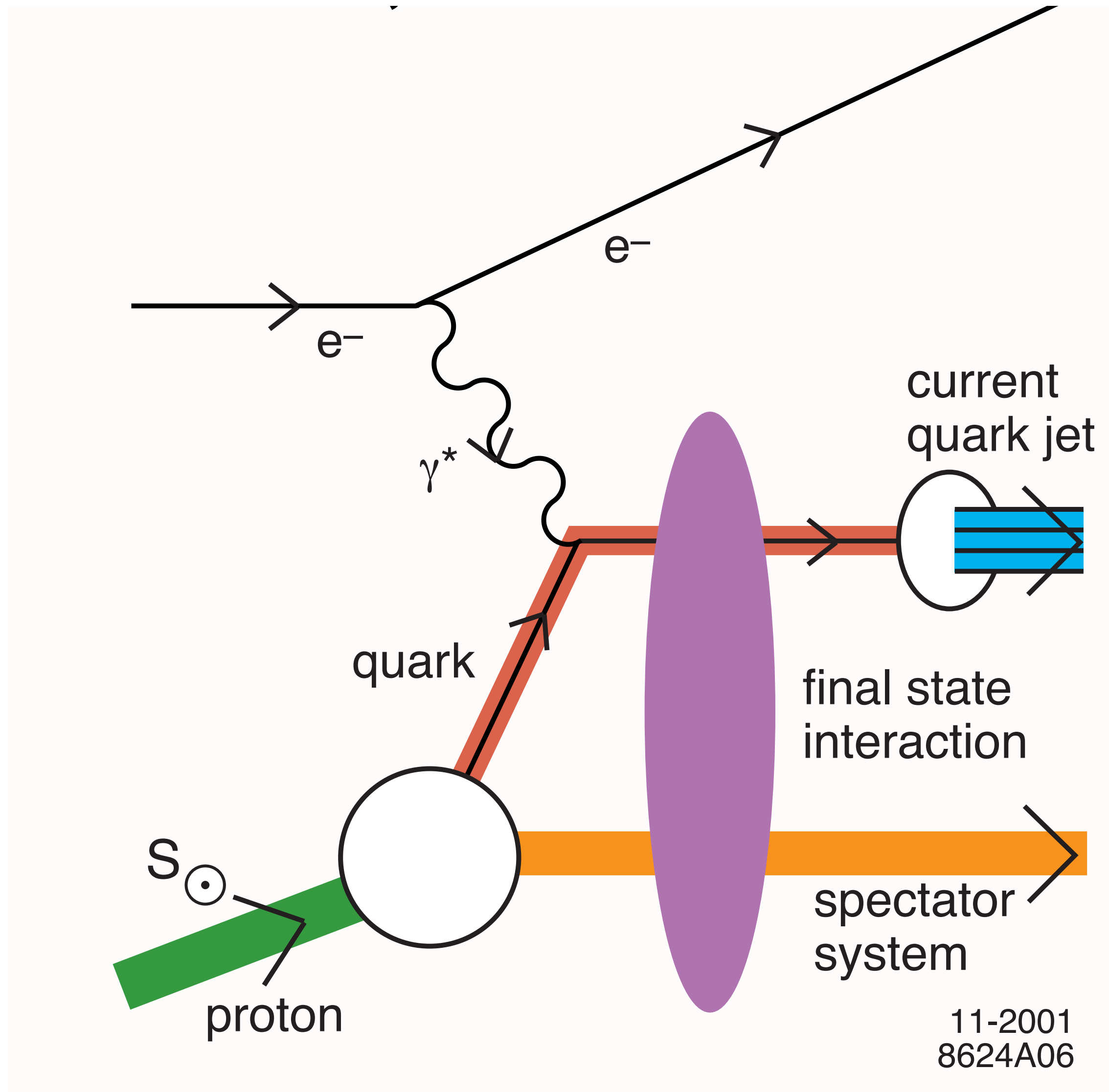
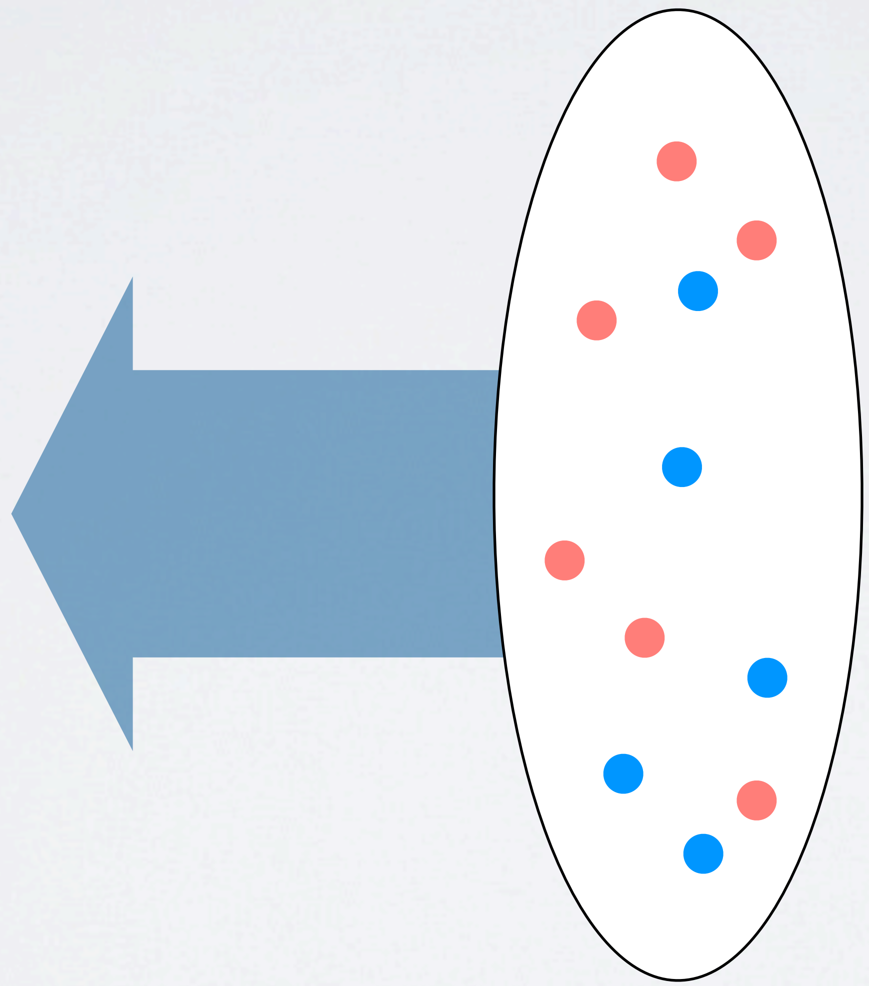


This image occurred **6** times in Stan's talk.

The word "lensing" occurred **8** times.

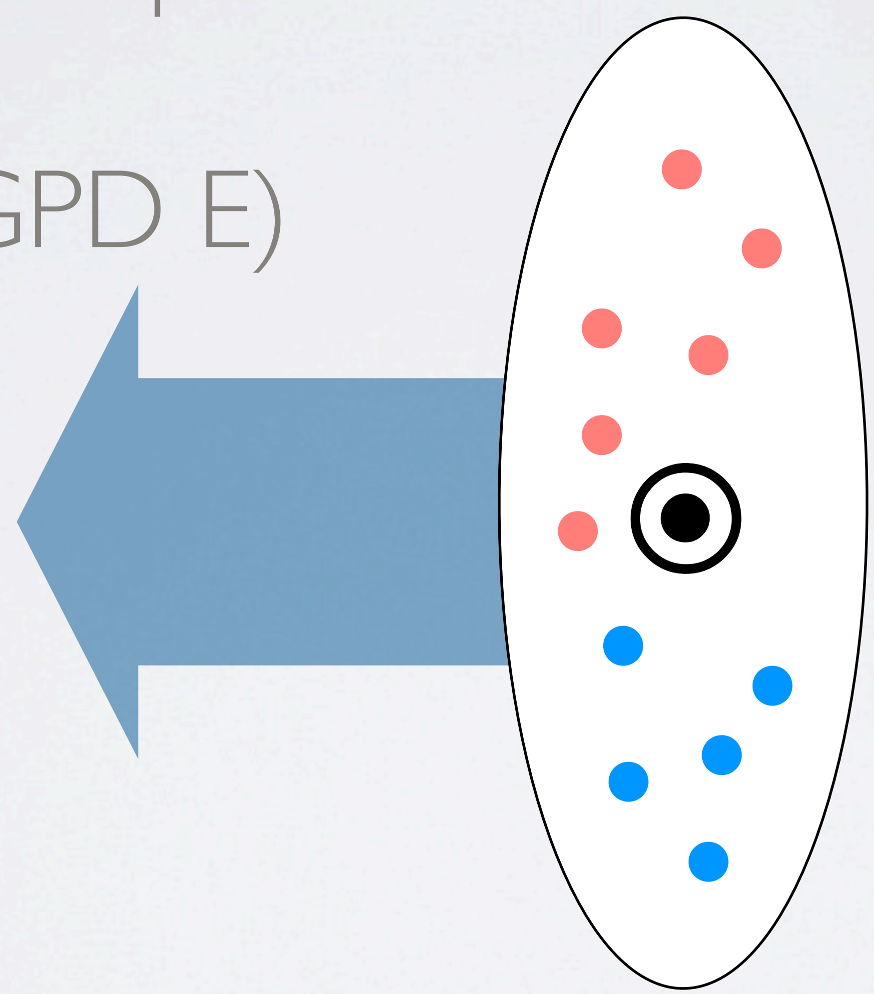
The physical picture



- Burkardt, PRD66 (02)

The physical picture

Distortion in impact parameter
(related to GPD E)



- Burkardt, PRD66 (02)

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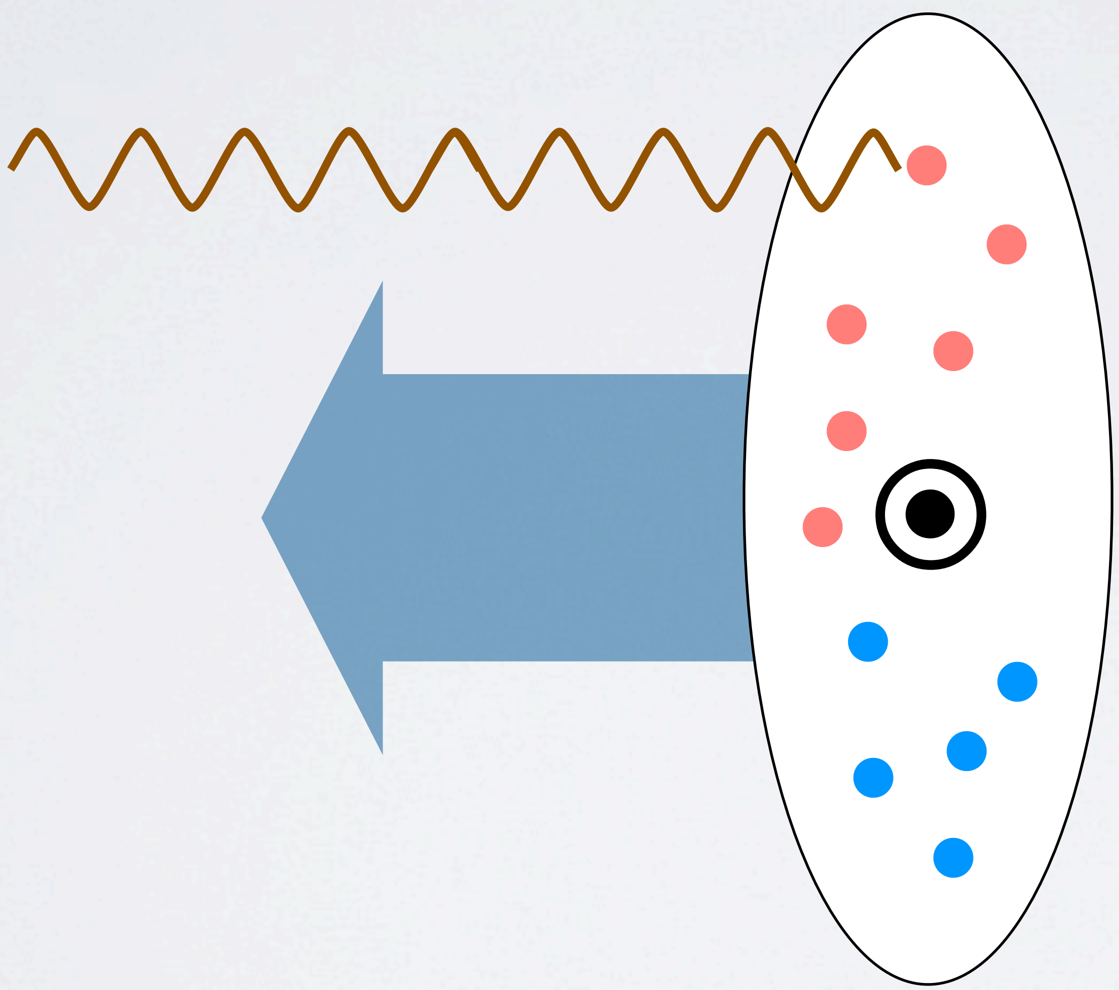
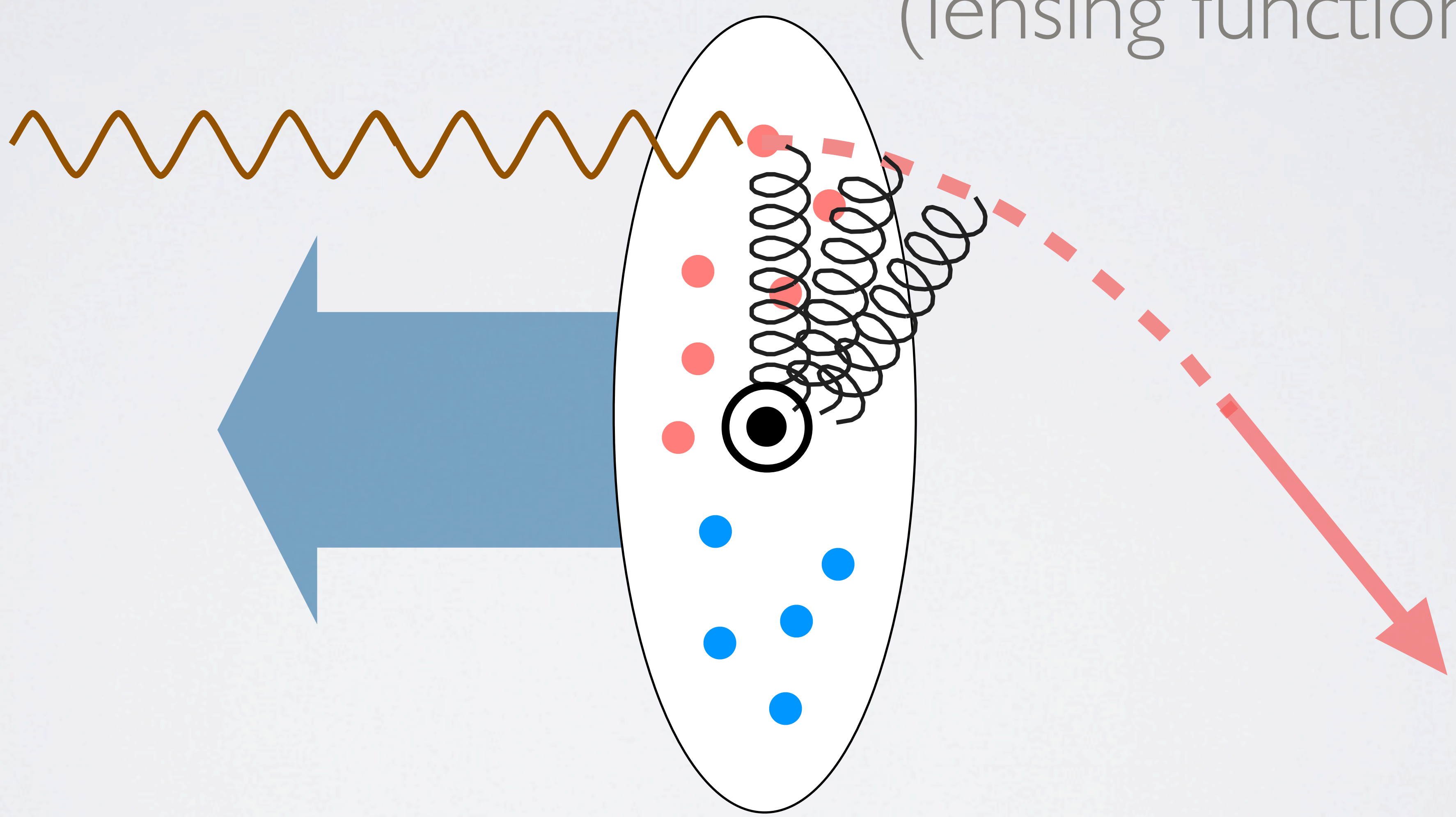


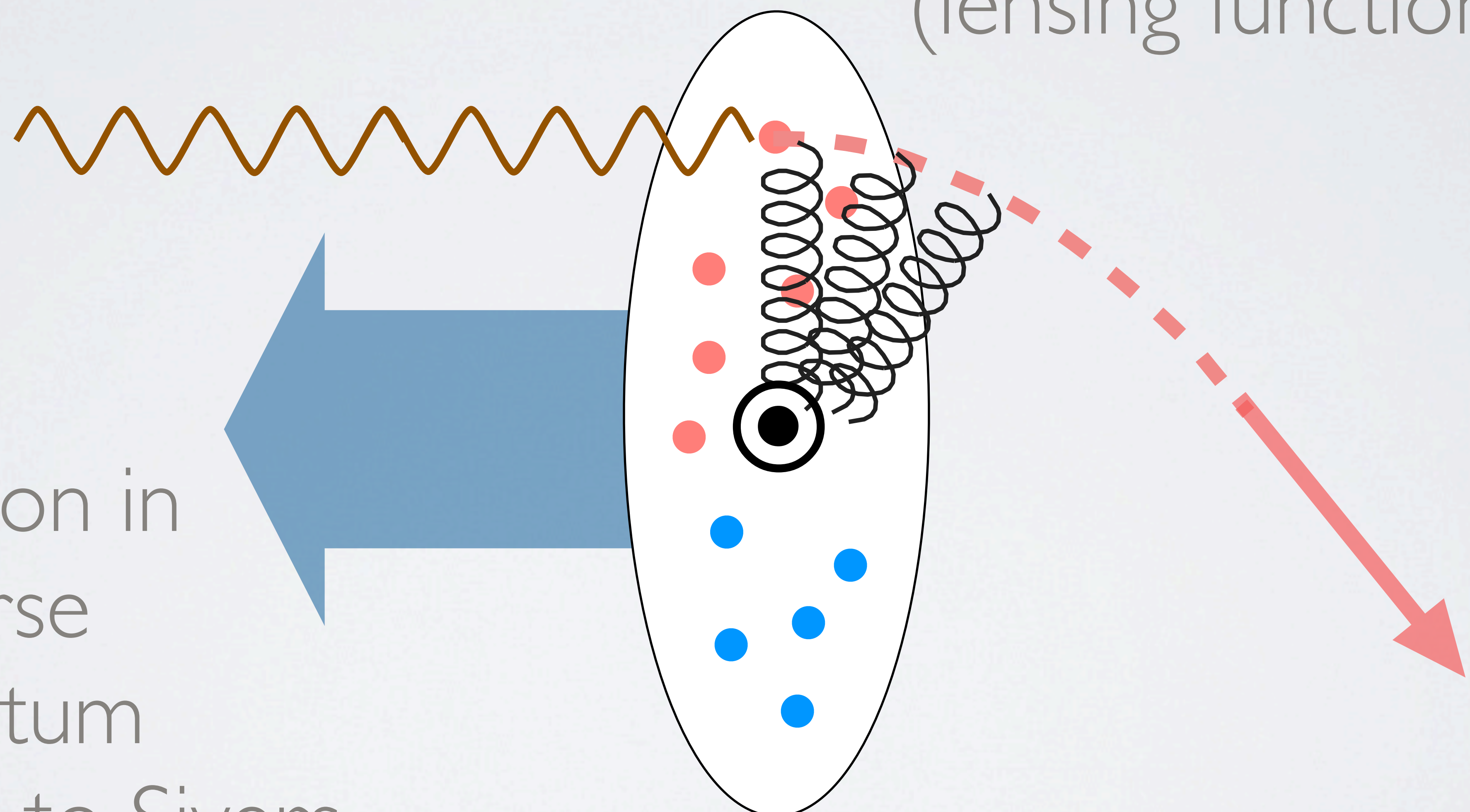
- Burkardt, PRD66 (02)

The physical picture



- Burkardt, PRD66 (02)

The physical picture

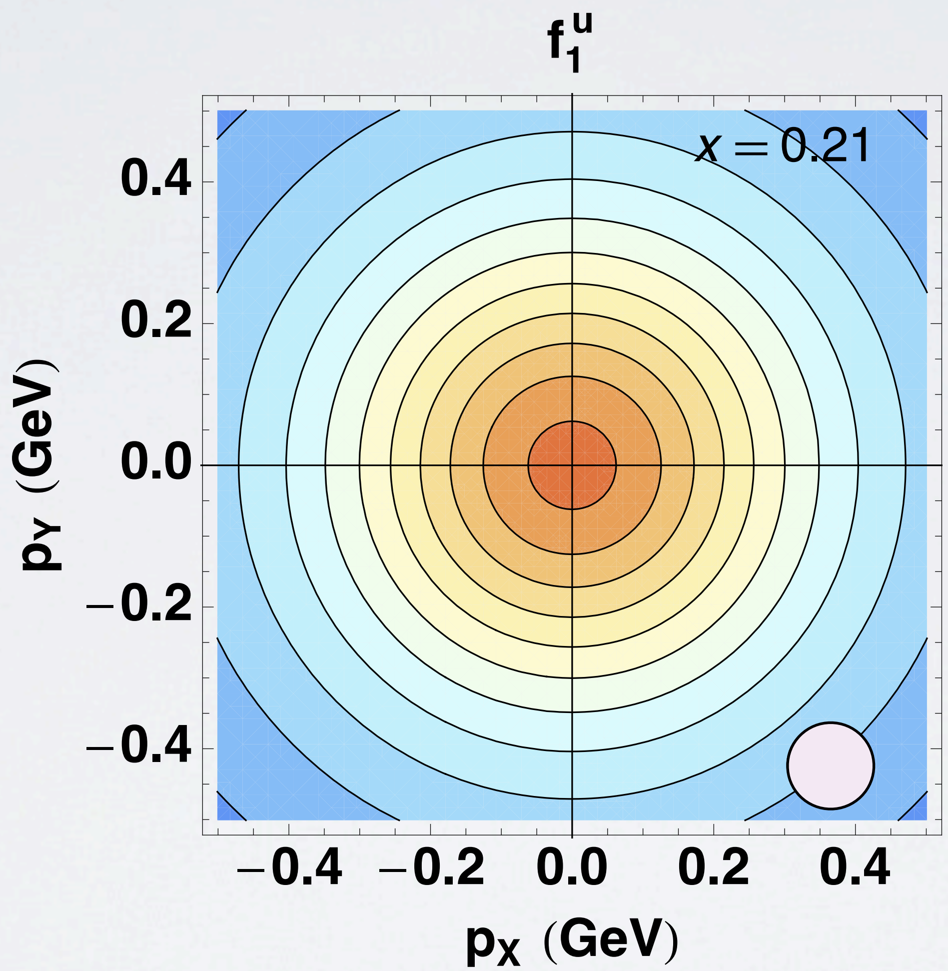


Final-state interaction (lensing function)

Distortion in transverse momentum (related to Sivers function)

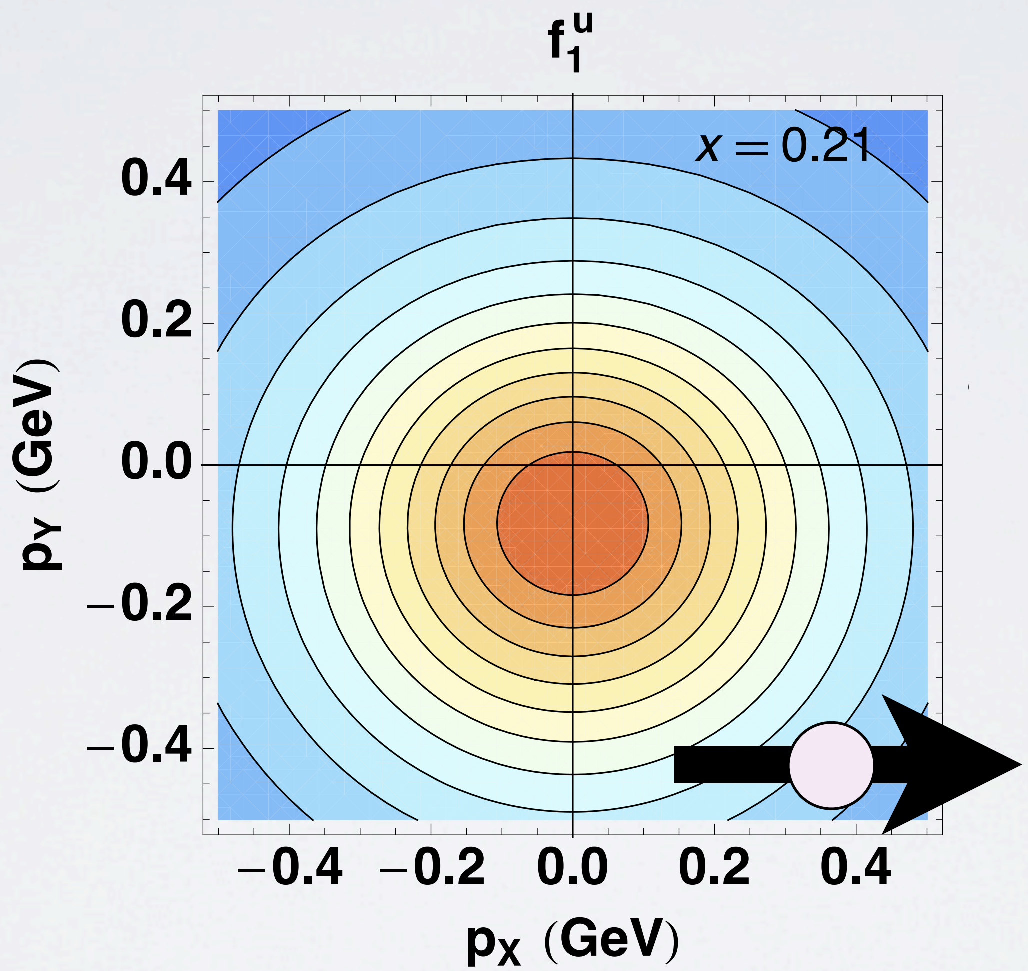
- Burkardt, PRD66 (02)

The Sivers function



- Based on model calculation A.B., Conti, Guagnelli, Radici, EPJ A45 (2010)

The Sivers function

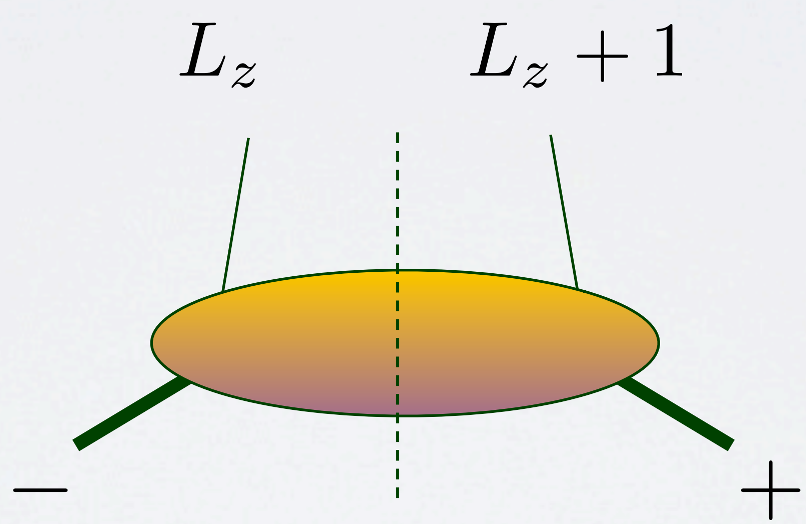


Distortion in transverse-momentum space

- Based on model calculation A.B., Conti, Guagnelli, Radici, EPJ A45 (2010)

Origin of Sivers function

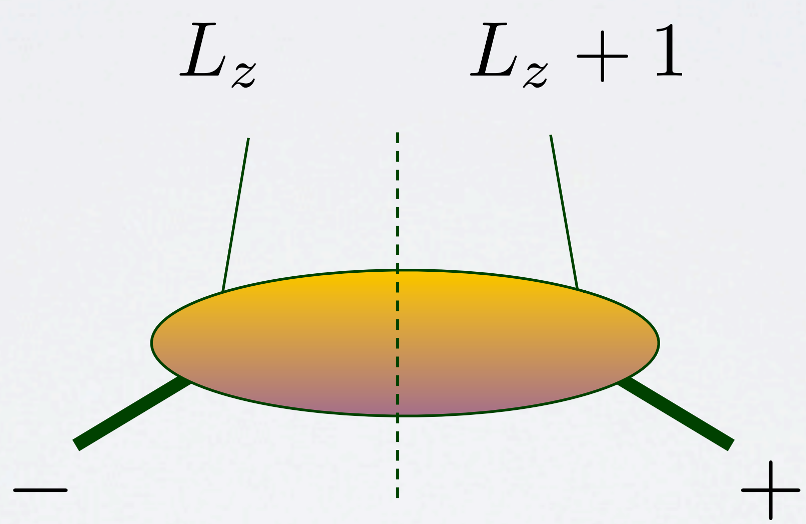
$$\frac{\epsilon_T^{jk} k_j S_k}{M} f_{1T}^{\perp} = \frac{1}{16\pi^3} \, \mathrm{Im}\Big[\big(\psi^+(x,k_T)\big)^* \psi^-(x,k_T)\Big]$$



- cf. Brodsky, Pasquini, Xiao, Yuan, PLB687 (2010)

Origin of Sivers function

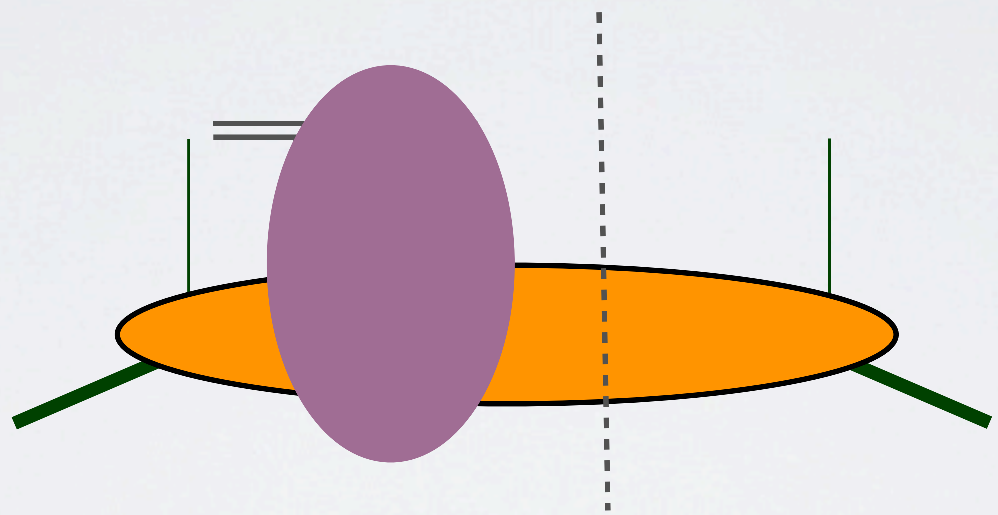
$$\frac{\epsilon_T^{jk} k_j S_k}{M} f_{1T}^{\perp} = \frac{1}{16\pi^3}\,\mathrm{Im}\Big[\big(\psi^{+}(x,k_T)\big)^{*}\psi^{-}(x,k_T)\Big]$$



$$E(x,0,0) = \lim_{q_T\to 0}\left(-\frac{1}{q_x - i q_y}\,\frac{1}{16\pi^3}\Big[\big(\psi^{+}(x,k_T+(1-x)q_T\big)^{*}\psi^{-}(x,k_T)\Big]\right)$$

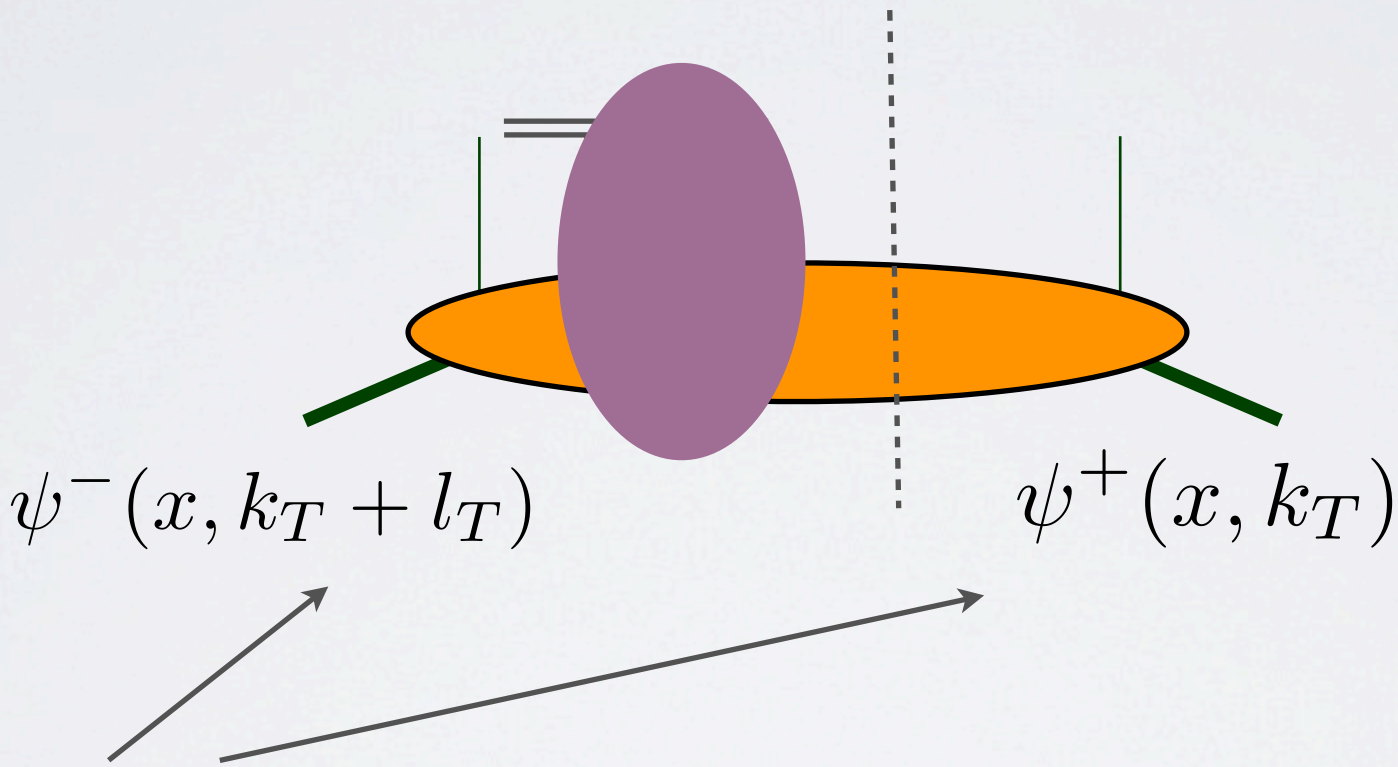
- cf. Brodsky, Pasquini, Xiao, Yuan, PLB687 (2010)

Origin of Sivers function



- Brodsky, Hwang, Schmidt PLB 530 (02)
- Ji, Yuan, PLB 543 (02)
- Gamberg, Schlegel, PLB 685 (2010)

Origin of Sivers function

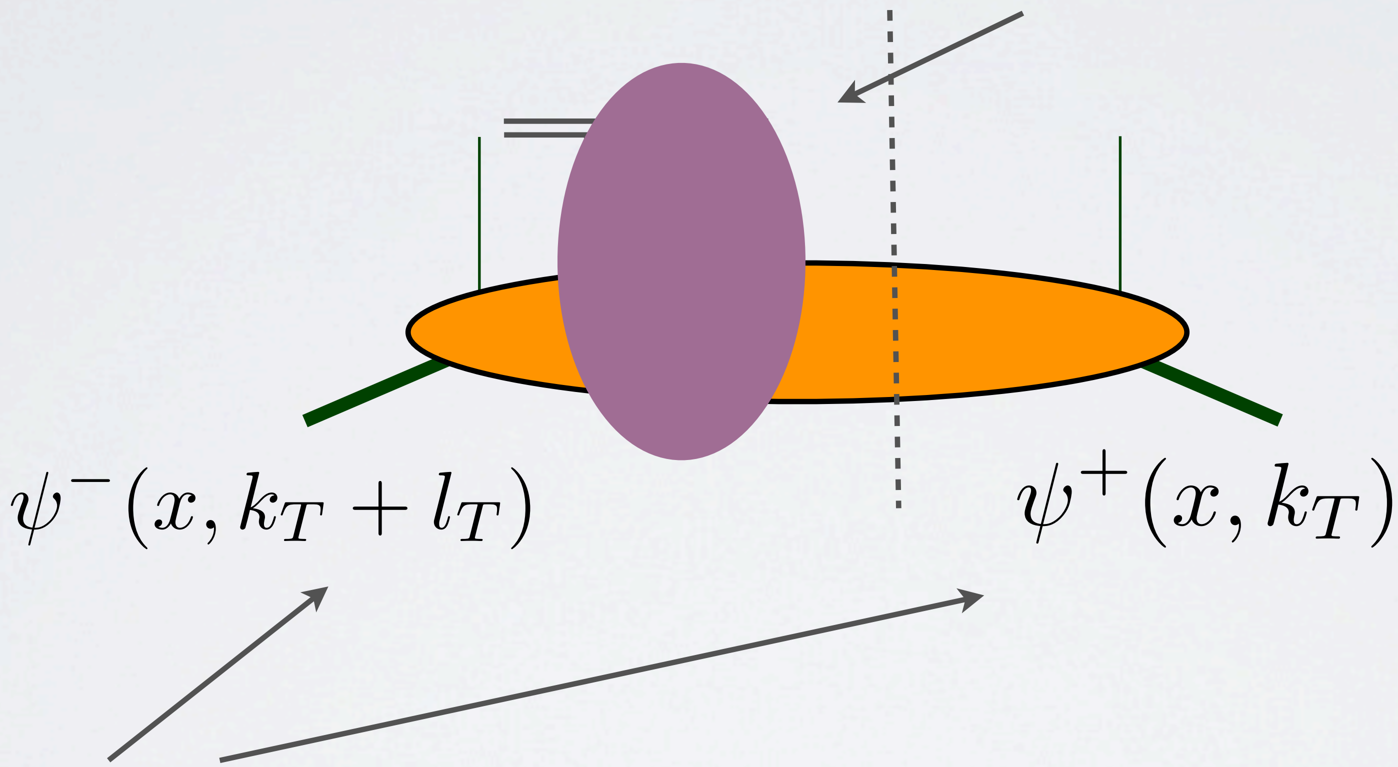


Light-front wavefunctions, same as in E(x,0,0)

- Brodsky, Hwang, Schmidt PLB 530 (02)
- Ji, Yuan, PLB 543 (02)
- Gamberg, Schlegel, PLB 685 (2010)

Origin of Sivers function

Final-state color interaction



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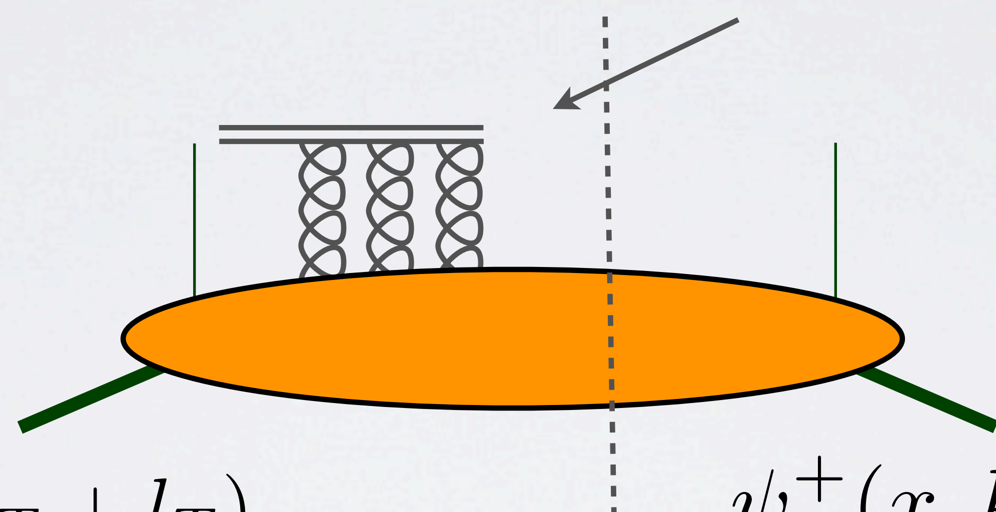
Origin of Sivers function

Final-state color interaction



$\psi^{-}(x, k_T + l_T)$ $\psi^{+}(x, k_T)$

Light-front wavefunctions, same as in E(x,0,0)

- Brodsky, Hwang, Schmidt PLB 530 (02)
- Ji, Yuan, PLB 543 (02)
- Gamberg, Schlegel, PLB 685 (2010)

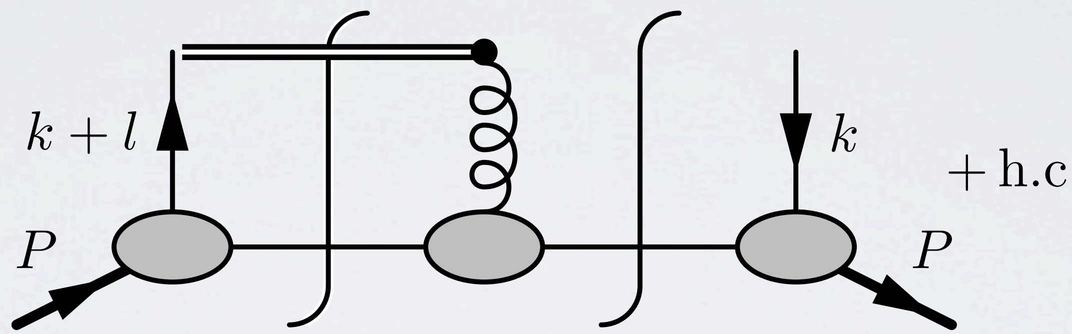
The lensing function

$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^2) \;\simeq\; \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

Sivers function — Lensing function — F.T. of E(x,0,t)

- Burkardt, PRD66 (02)
- Meissner, Metz, Goeke, PRD76 (07)

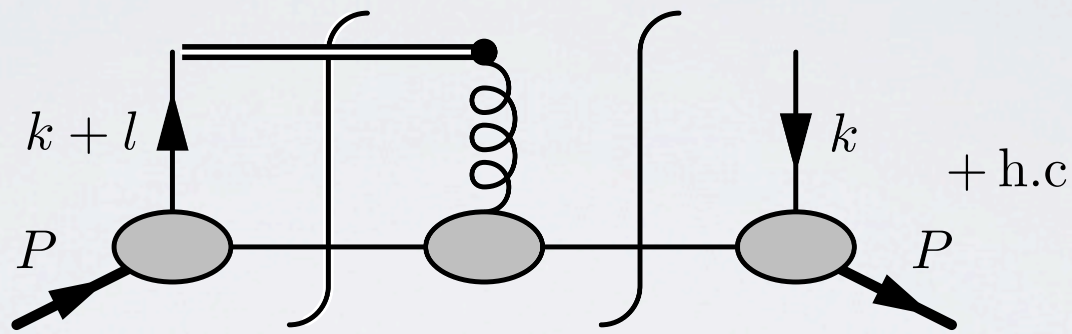
Spectator model results



$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -\frac{3MC_F\alpha_S}{2(1-x)}\ E^a(x,0,0;Q_L^2)$$

- Burkardt, Hwang, PRD69 (04)
  Lu, Schmidt, PRD75 (07)
  A.B., F. Conti, M. Radici, PRD 78 (08)

Spectator model results



$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -\frac{3MC_F\alpha_S}{2(1-x)}\;E^a(x,0,0;Q_L^2)$$

Lensing function (flavor independent)

- Burkardt, Hwang, PRD69 (04)
  Lu, Schmidt, PRD75 (07)
  A.B., F. Conti, M. Radici, PRD 78 (08)

Our assumption

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\,E^a(x,0,0;Q_L^2)$$

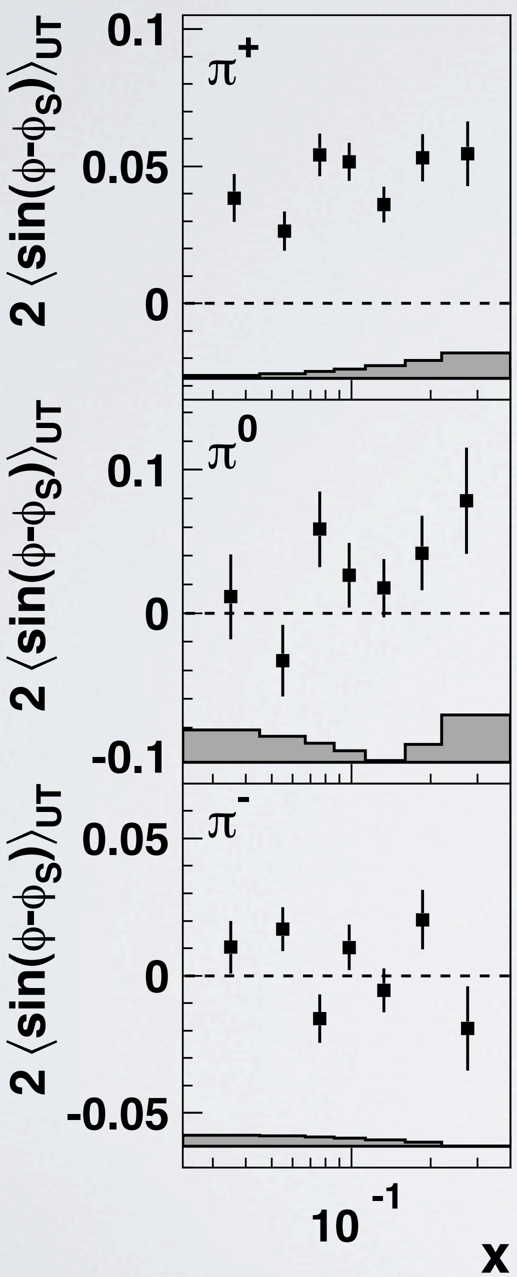
Sivers TMD

Lensing function

Using available data

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\,E^a(x,0,0;Q_L^2),$$

Using available data

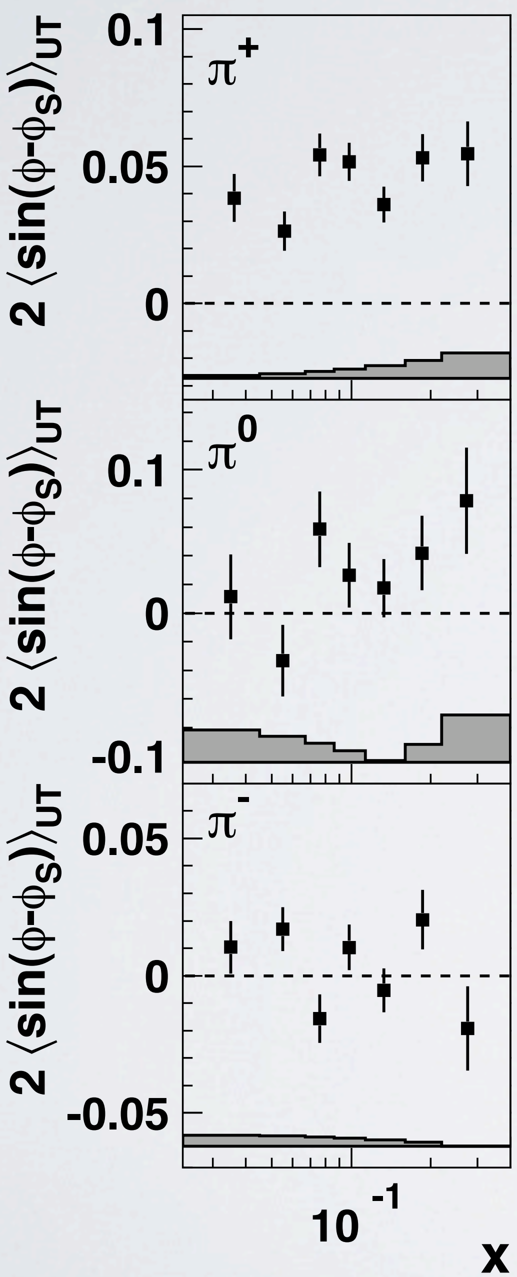


$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\,E^a(x,0,0;Q_L^2),$$

Use SIDIS Sivers asymmetry data to constrain shape

hermes COMPASS Jefferson Lab

Using available data



$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\, E^a(x,0,0;Q_L^2),$$

Use SIDIS Sivers asymmetry data to constrain shape

$$\kappa^p = \int_0^1 \frac{dx}{3}\left[2E^{u_v}(x,0,0) - E^{d_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$
$$\kappa^n = \int_0^1 \frac{dx}{3}\left[2E^{d_v}(x,0,0) - E^{u_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$

Use anomalous magnetic moments to constrain integral

Data fitting

Choice of functional form

- Sivers function

- Lensing function

Choice of functional form

- Sivers function

$$f_{1T}^{\perp q_v}(x, p_T^2) \propto C^{q_v}\,(1 - x/\alpha^{q_v})\,(1-x) f_1^{q_v}(x)\, e^{-p_T^2/M_1^2}\, e^{-p_T^2/\langle p_T^2\rangle}$$

- Lensing function

Choice of functional form

- Sivers function

possible node

Transverse-momentum Gaussian

$$f_{1T}^{\perp q_v}(x, p_T^2) \propto C^{q_v}\,(1 - x/\alpha^{q_v})\,(1-x) f_1^{q_v}(x)\, e^{-p_T^2/M_1^2}\, e^{-p_T^2/\langle p_T^2\rangle}$$

fixed large-x behavior

unpolarized PDF

- Lensing function

Choice of functional form

- Sivers function

possible node — Transverse-momentum Gaussian

$$f_{1T}^{\perp q_v}(x, p_T^2) \propto C^{q_v}\,(1 - x/\alpha^{q_v})\,(1-x) f_1^{q_v}(x)\, e^{-p_T^2/M_1^2}\, e^{-p_T^2/\langle p_T^2 \rangle}$$

fixed large-x behavior — unpolarized PDF

- Lensing function

$$L(x) = \frac{K}{(1-x)^{\eta}}$$

Some details on the fit

- 221 SIDIS data points + 2 anomalous magnetic moments
- 10 parameters
- χ^2/dof=1.32
- no TMD evolution

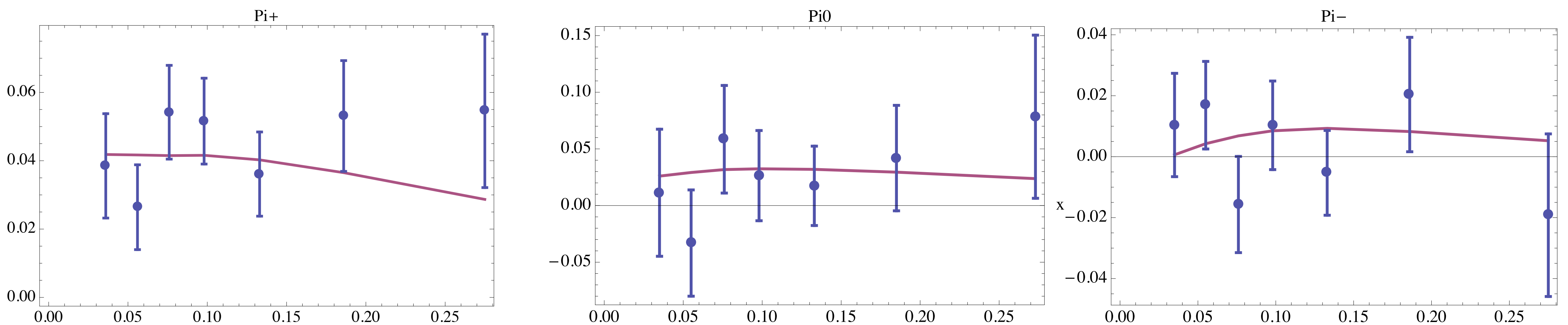
Best-fit parameters

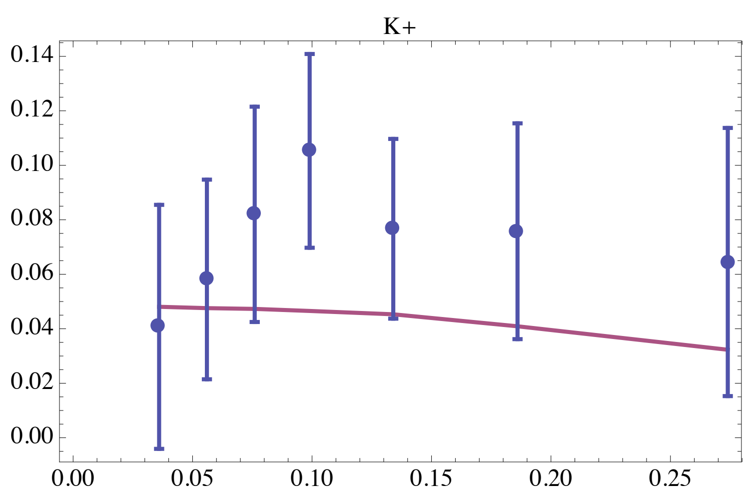
$$f_{1T}^{\perp q_v}(x, p_T^2) \propto C^{q_v}\,(1 - x/\alpha^{q_v})\,(1-x) f_1^{q_v}(x)\, e^{-p_T^2/M_1^2}\, e^{-p_T^2/\langle p_T^2 \rangle}$$

$$L(x) = \frac{K}{(1-x)^\eta}$$

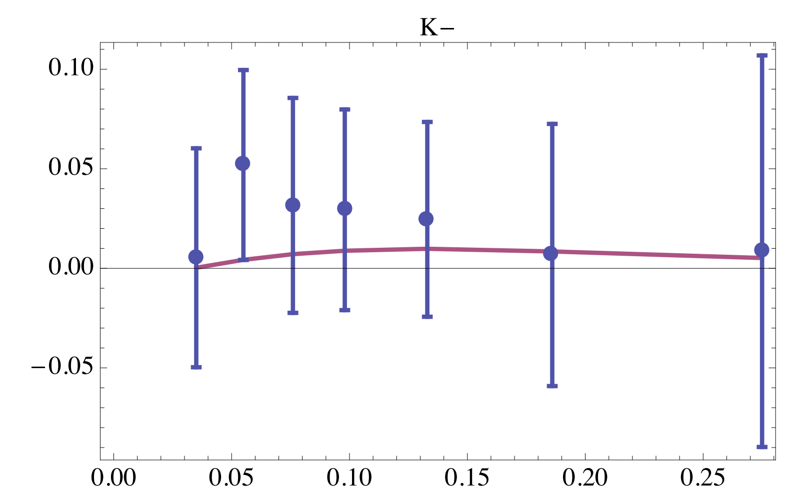
C^{u_v}	C^{d_v}	$C^{\bar{u}}$	$C^{\bar{d}}$
-0.229 ± 0.002	1.591 ± 0.009	0.054 ± 0.107	-0.083 ± 0.122
M_1 [GeV]	K [GeV]	η	α^{u_v}
0.346 ± 0.015	1.888 ± 0.009	0.392 ± 0.040	0.783 ± 0.001

Fit vs. data (example)

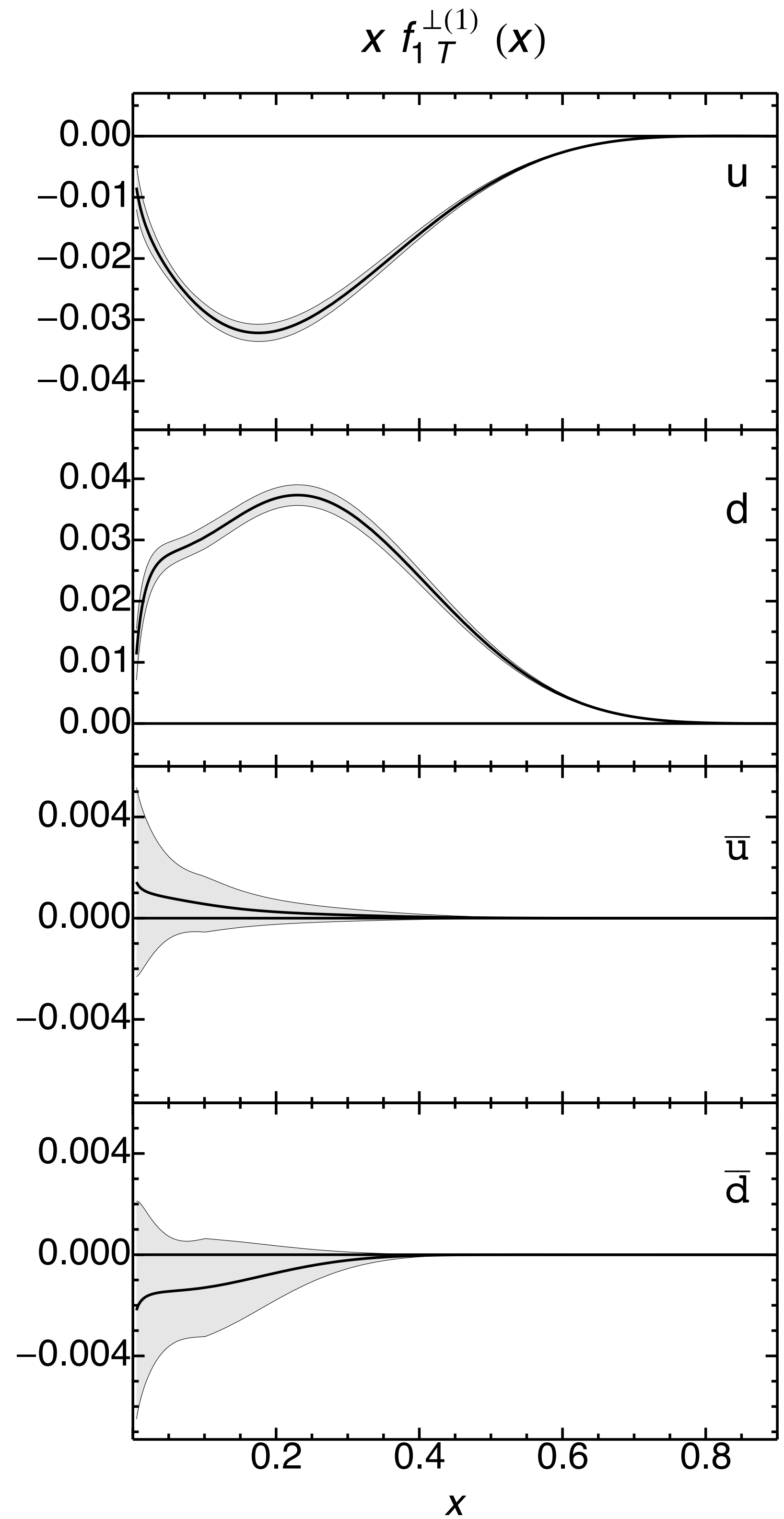




$$A_{UT}^{\sin(\phi_h - \phi_S)}$$

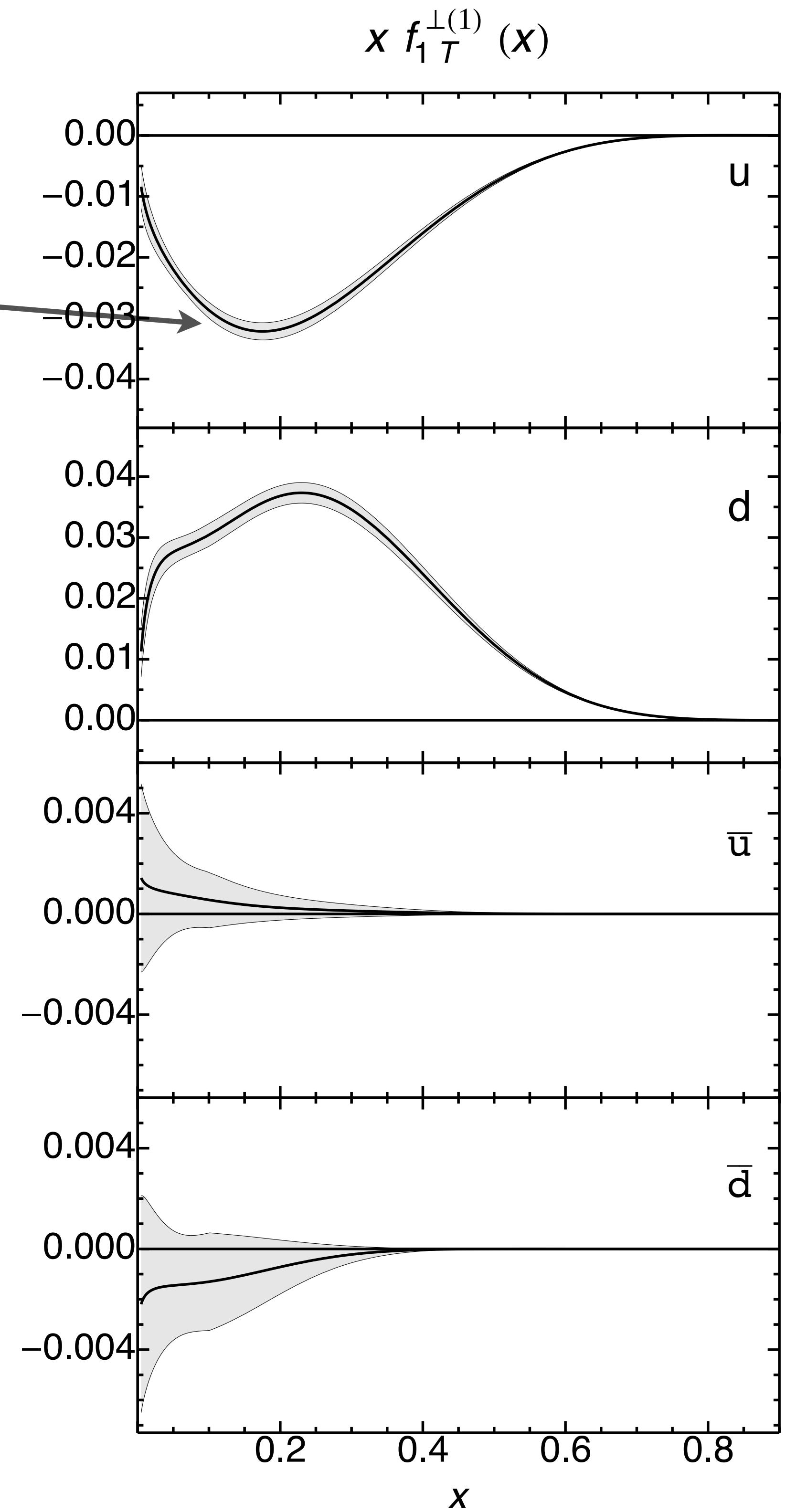


Resulting Sivers function



Resulting Sivers function

$\Delta\chi^2 = 1$



No estimate of error due to choice of functional form

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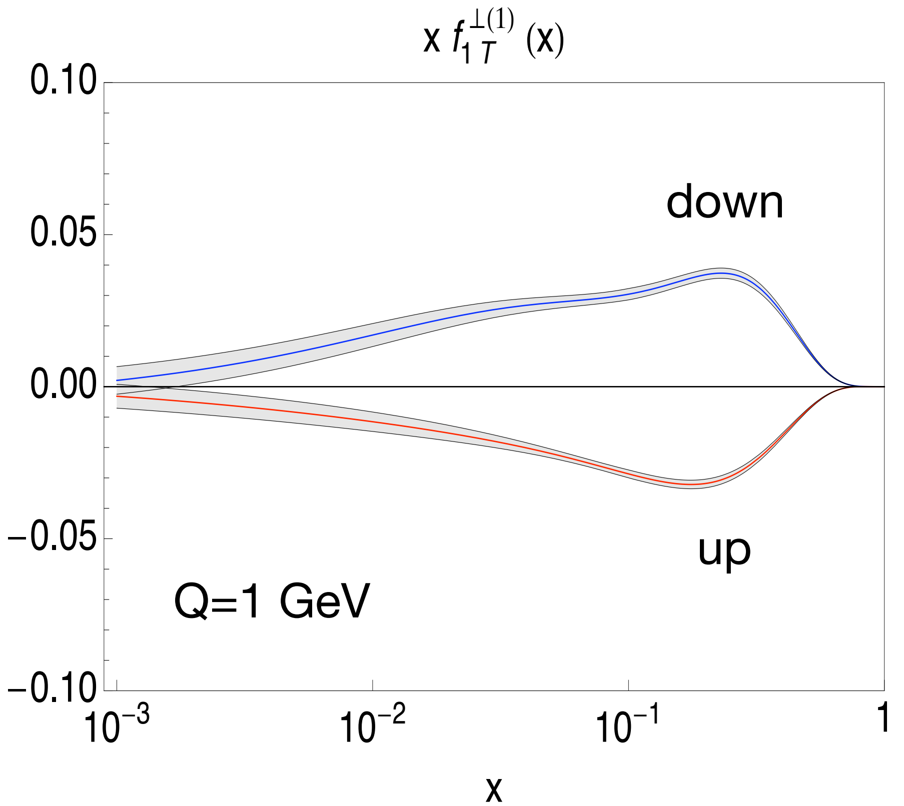
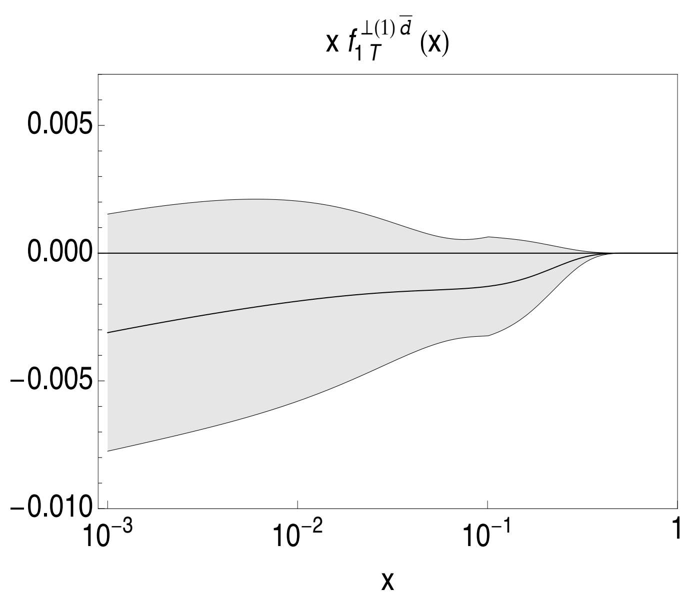
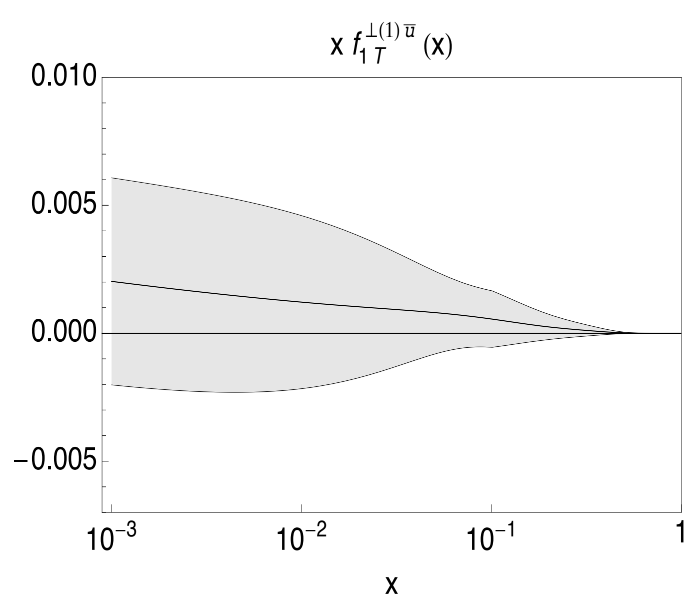

$x\, f_{1T}^{\perp(1)}(x)$
0.10
0.05
0.00
−0.05
−0.10
down
up
Q=1 GeV
10^{-3}
10^{-2}
10^{-1}
1
x


$x\, f_{1T}^{\perp(1)\bar{d}}(x)$
0.005
0.000
−0.005
−0.010
10^{-3}
10^{-2}
10^{-1}
1
x


$x\, f_{1T}^{\perp(1)\bar{u}}(x)$
0.010
0.005
0.000
−0.005
10^{-3}
10^{-2}
10^{-1}
1
x

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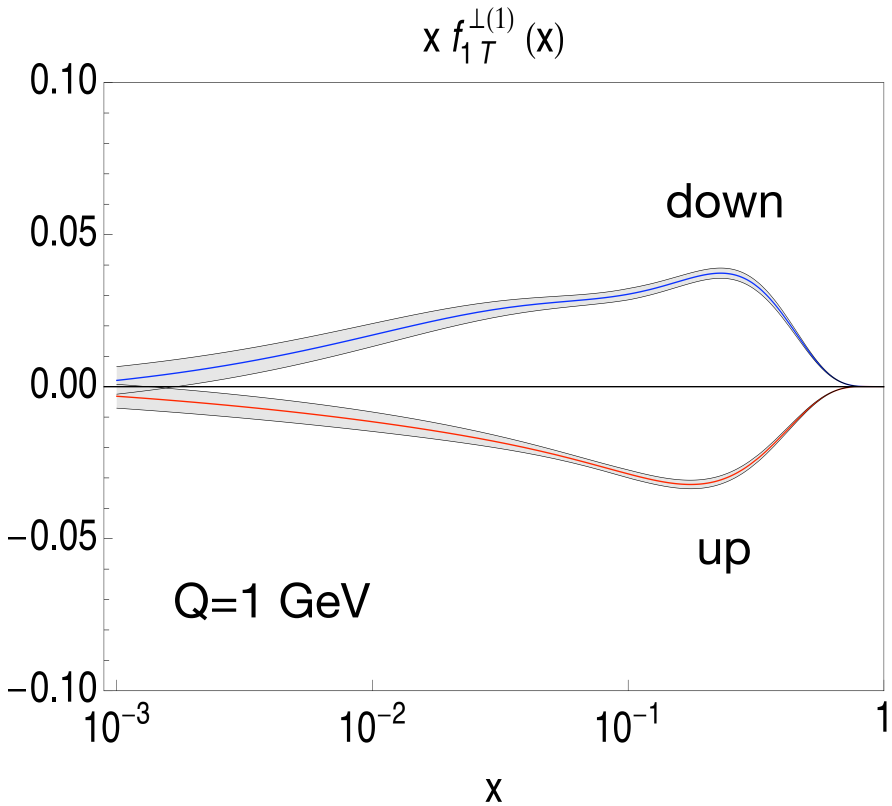

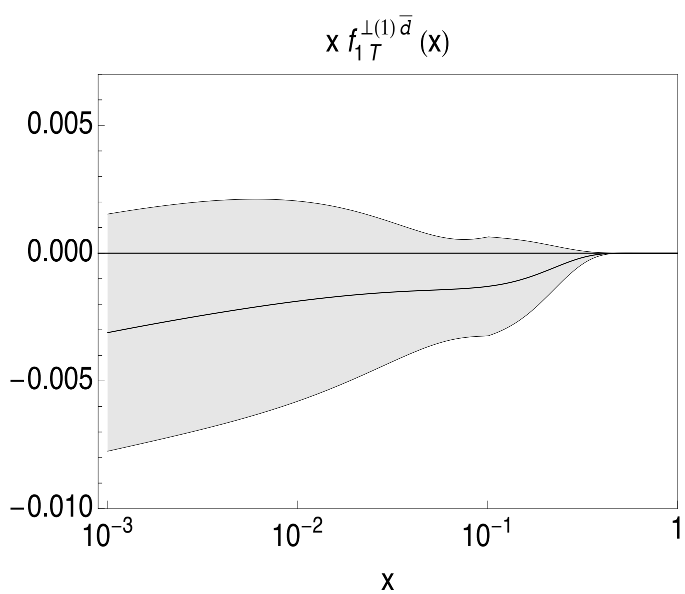

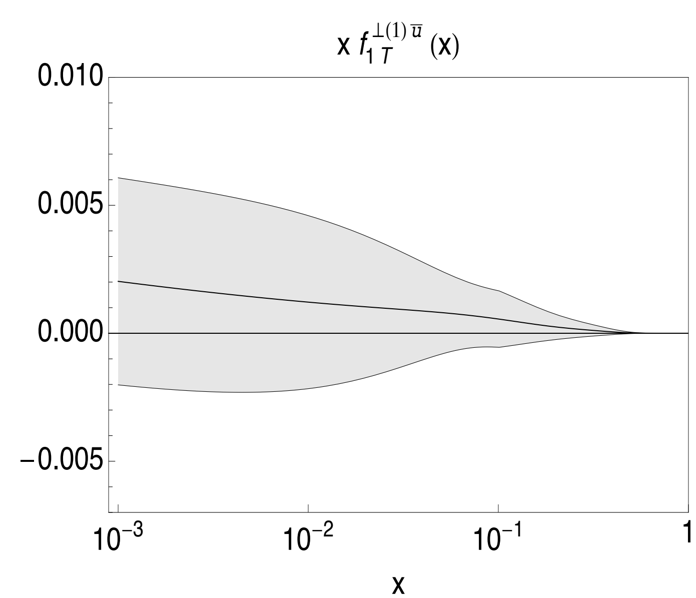

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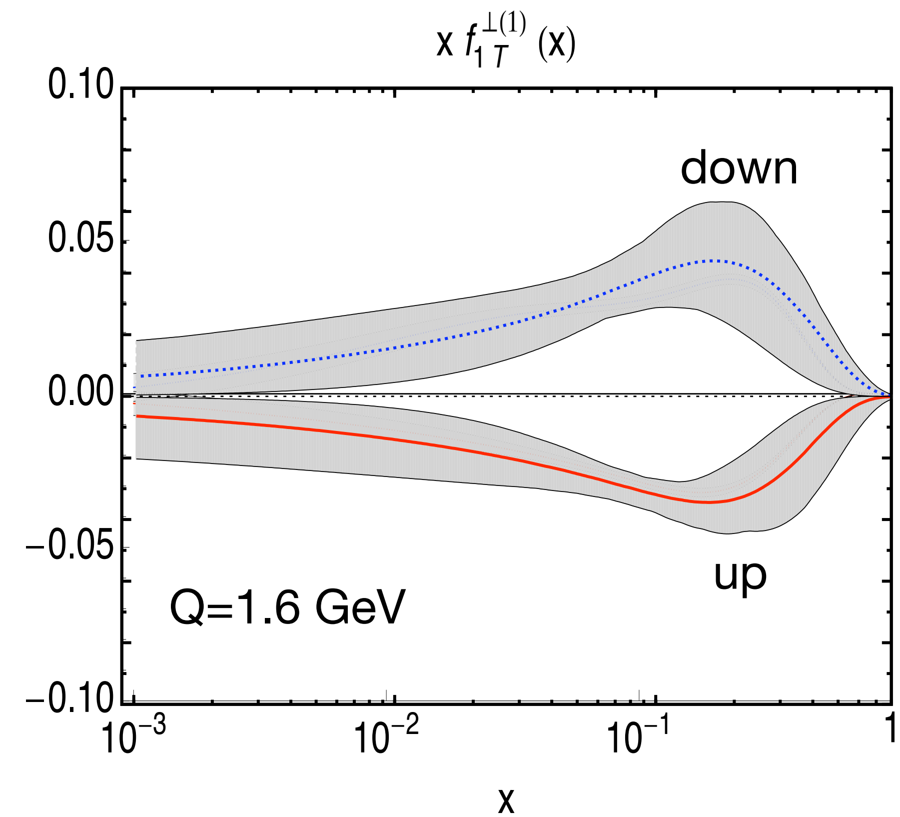

- Anselmino et al. arXiv:1107.4446

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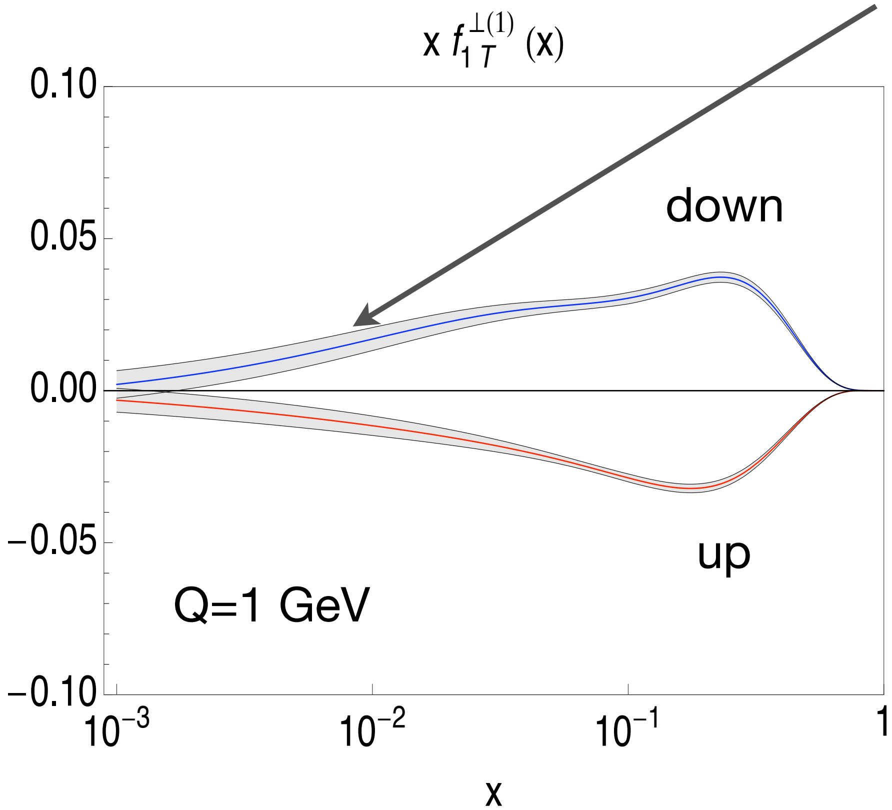
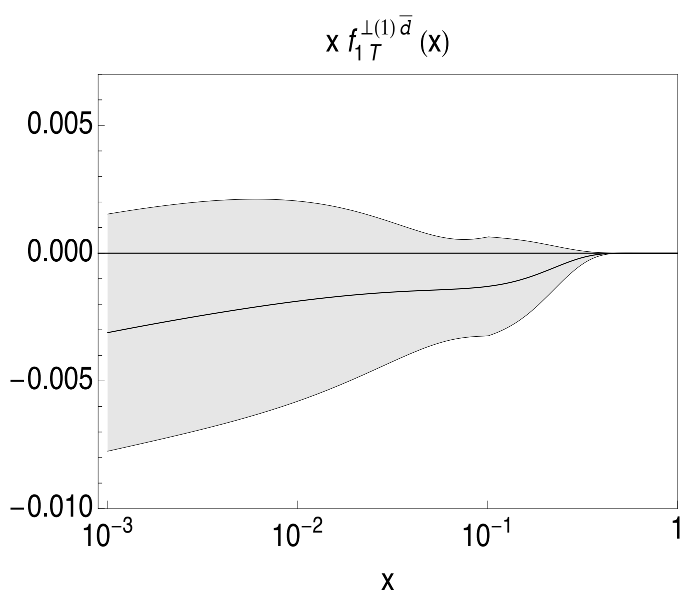
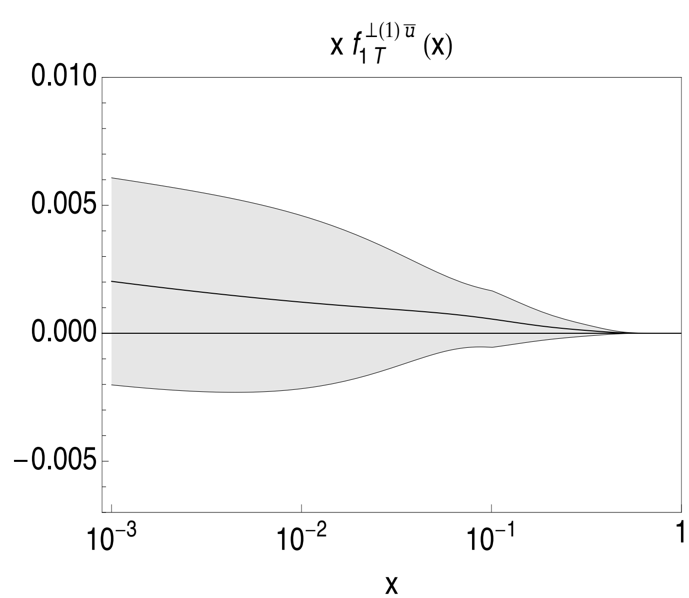
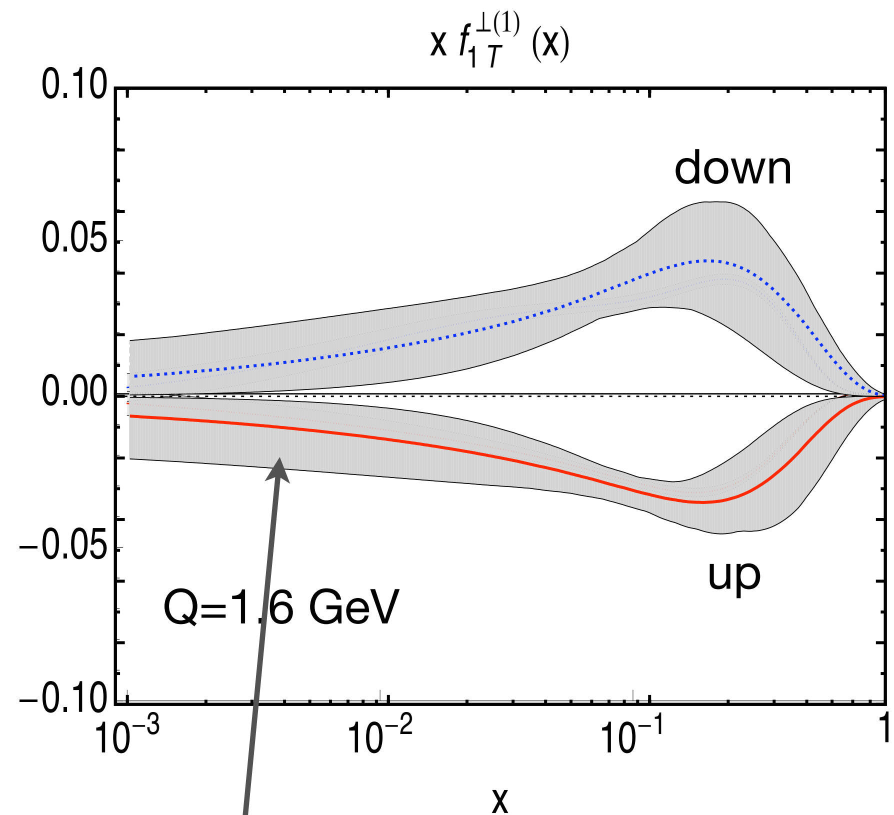
$\Delta\chi^2=1$







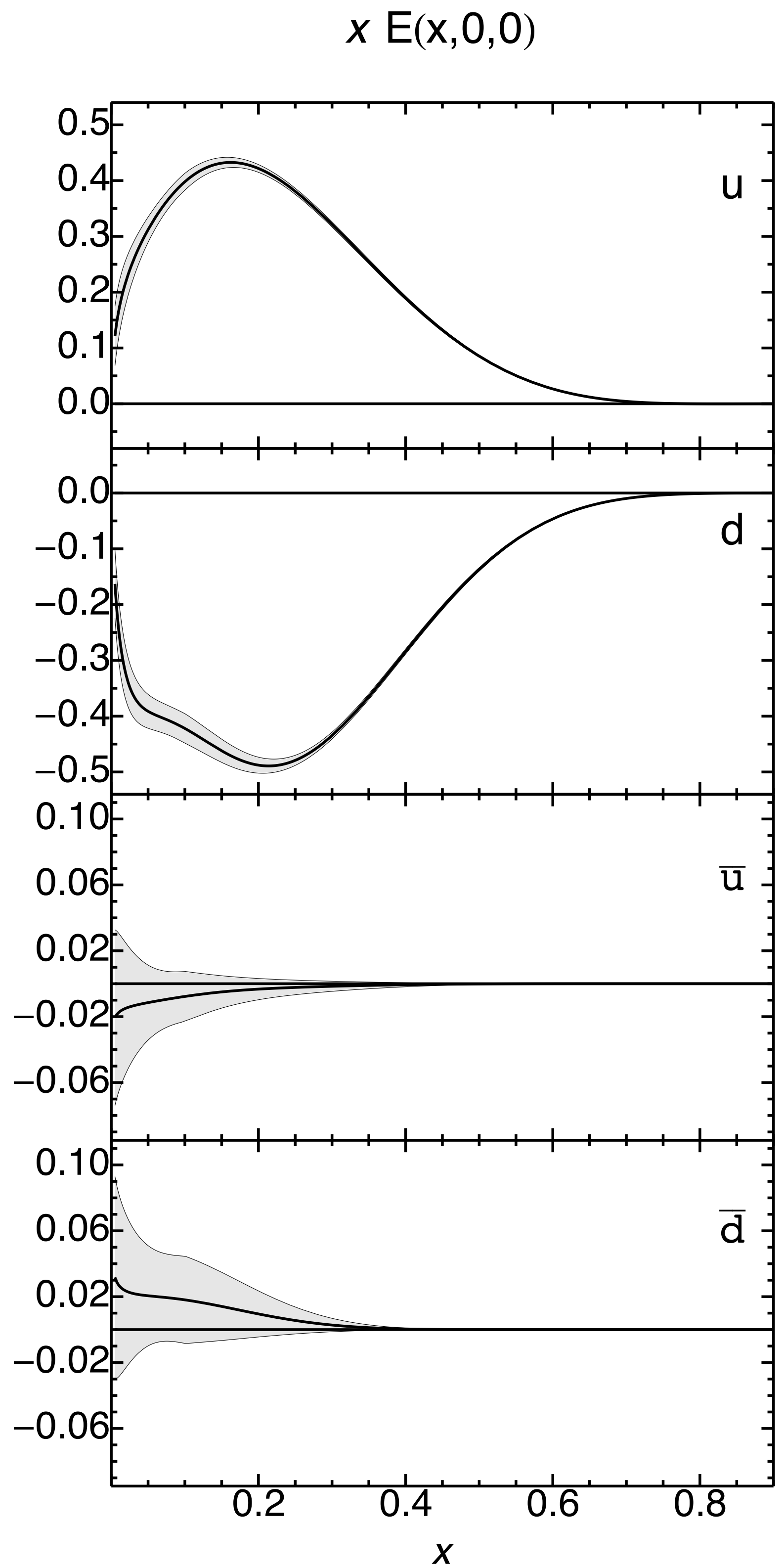
Torino 2011



$\Delta\chi^2\simeq15$

- Anselmino et al. arXiv:1107.4446

Resulting E(x,0,0) function



$$E^q(x,0,0;Q_L^2) \propto -\frac{C^q}{K}\,(1-x/\alpha^q)\,(1-x)^{1+\eta} f_1^q(x:Q_L^2)$$

Results on angular momenta

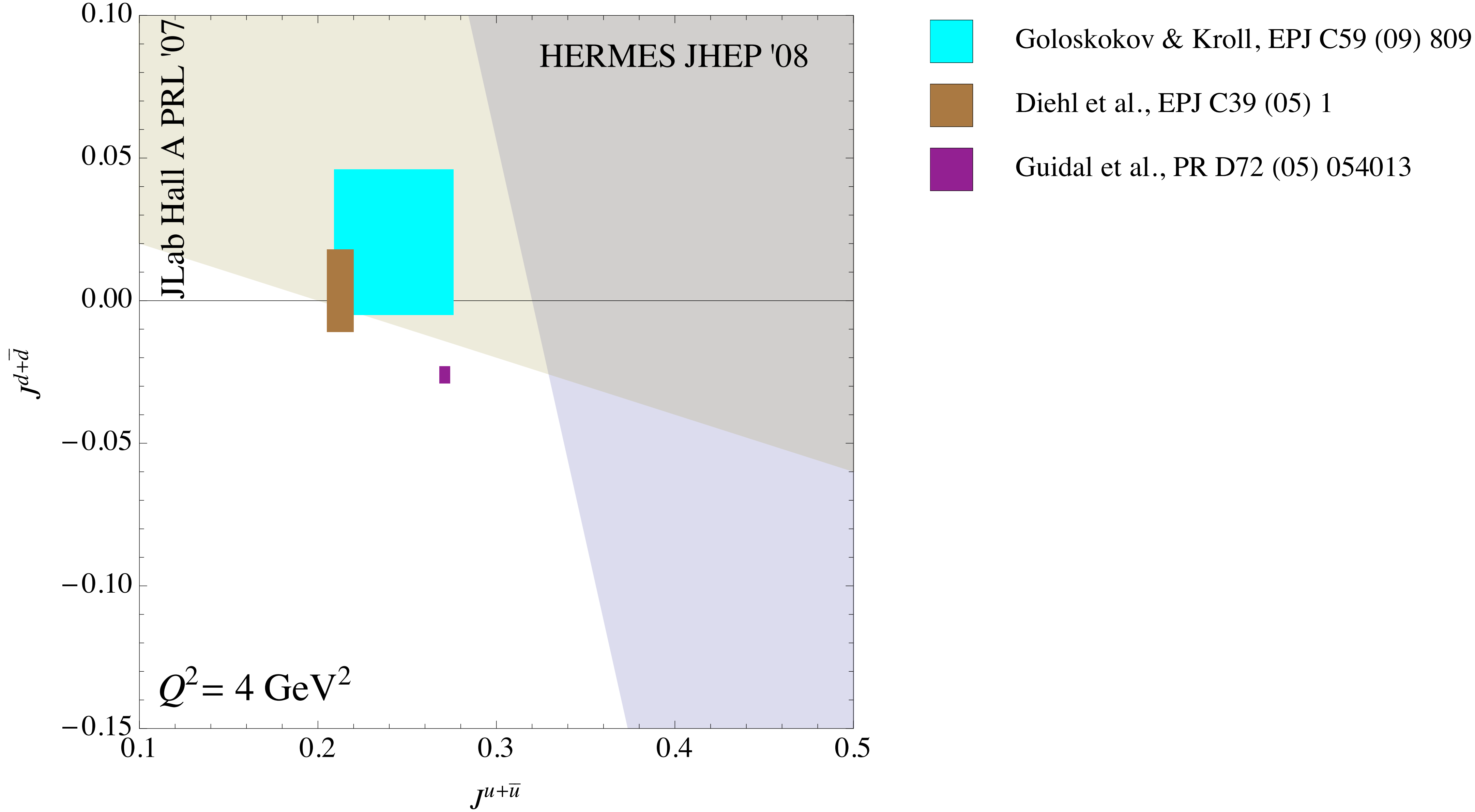
$$J^u = 0.229 \pm 0.002^{+0.008}_{-0.012}, \qquad J^{\bar{u}} = 0.015 \pm 0.003^{+0.001}_{-0.000},$$

$$J^d = -0.007 \pm 0.003^{+0.020}_{-0.005}, \qquad J^{\bar{d}} = 0.022 \pm 0.005^{+0.001}_{-0.000},$$

$$J^s = 0.006^{+0.002}_{-0.006}, \qquad J^{\bar{s}} = 0.006^{+0.000}_{-0.005}.$$

$Q^2 = 4\ \mathrm{GeV}^2$

Available extractions



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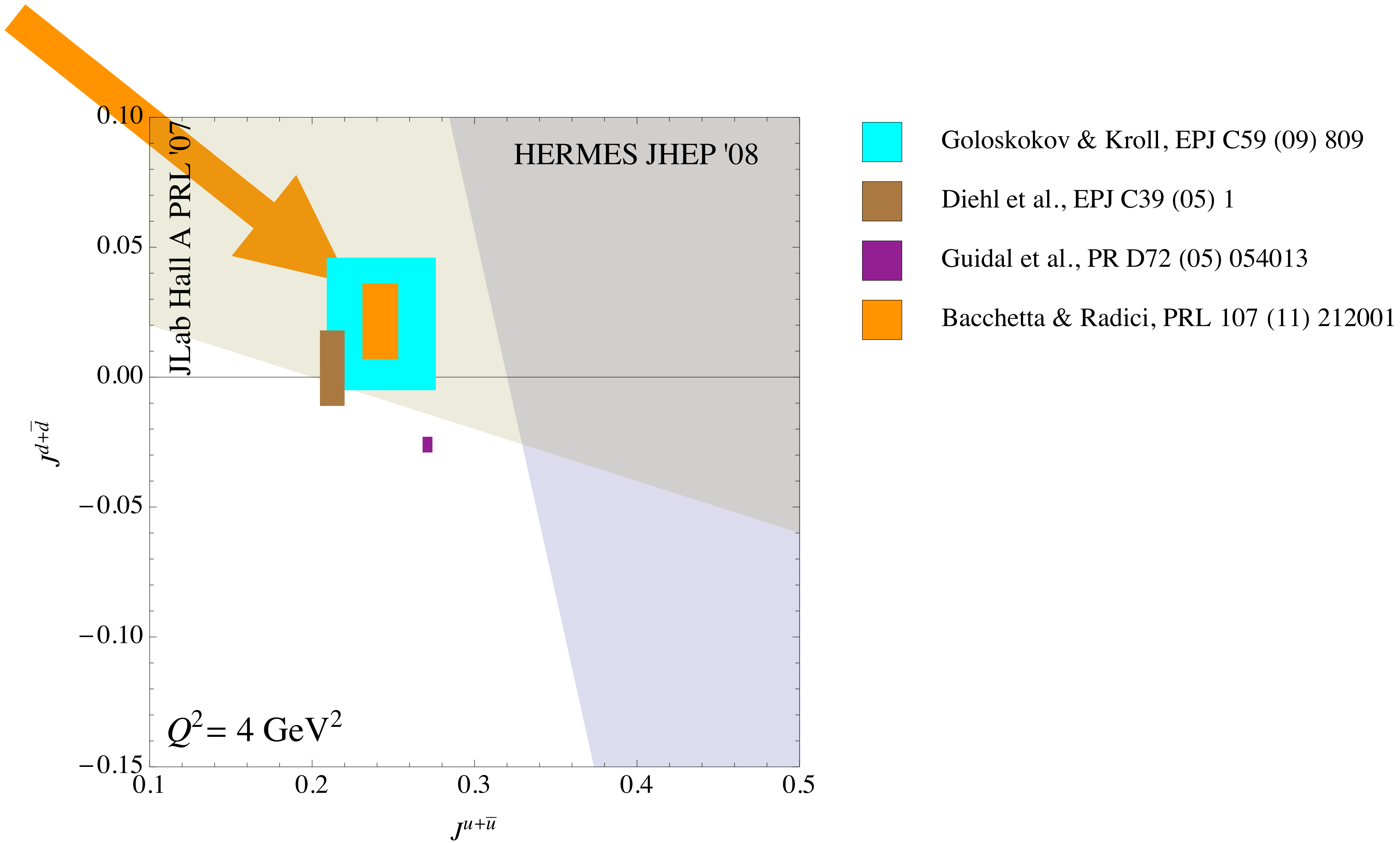
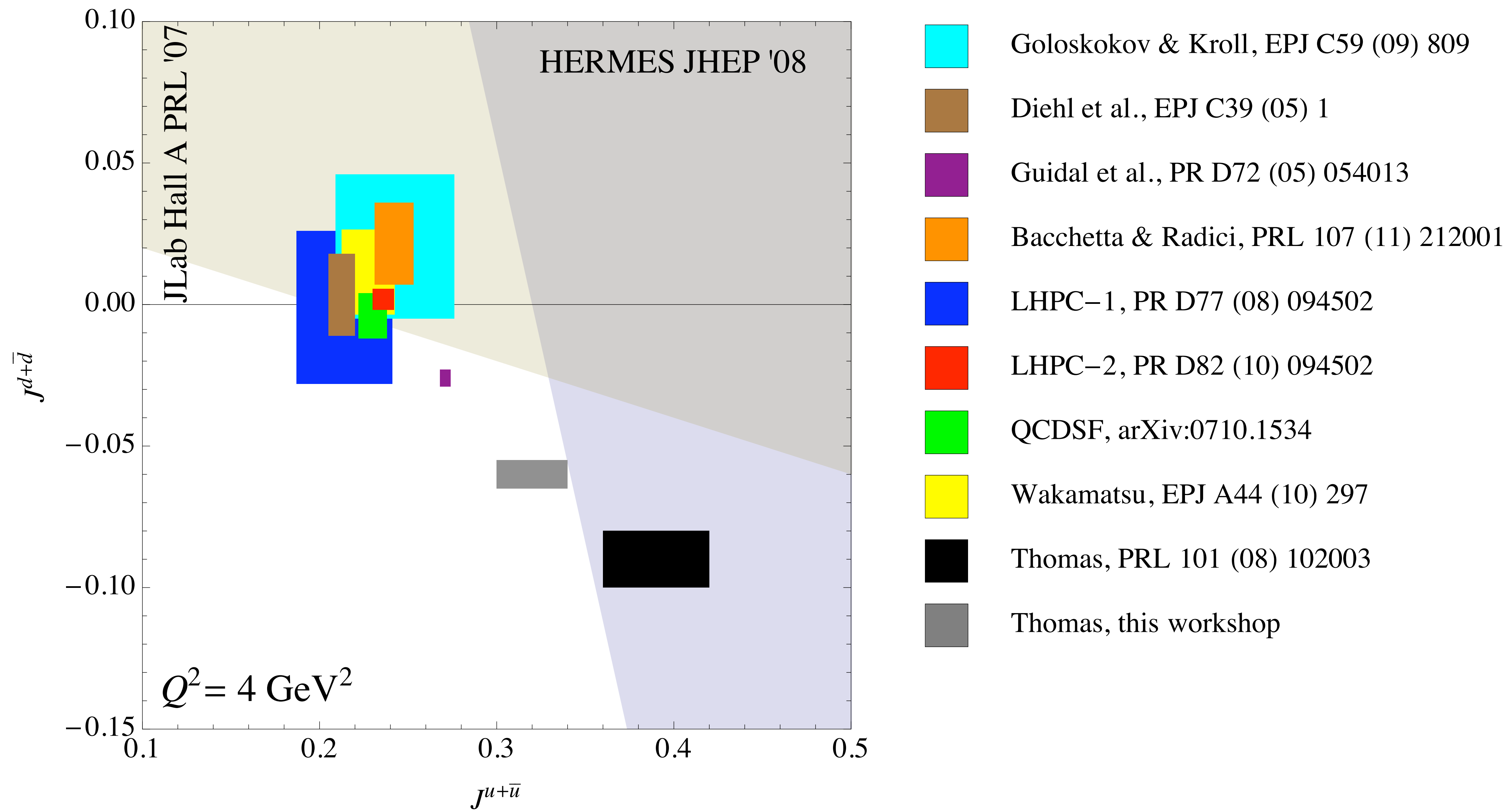
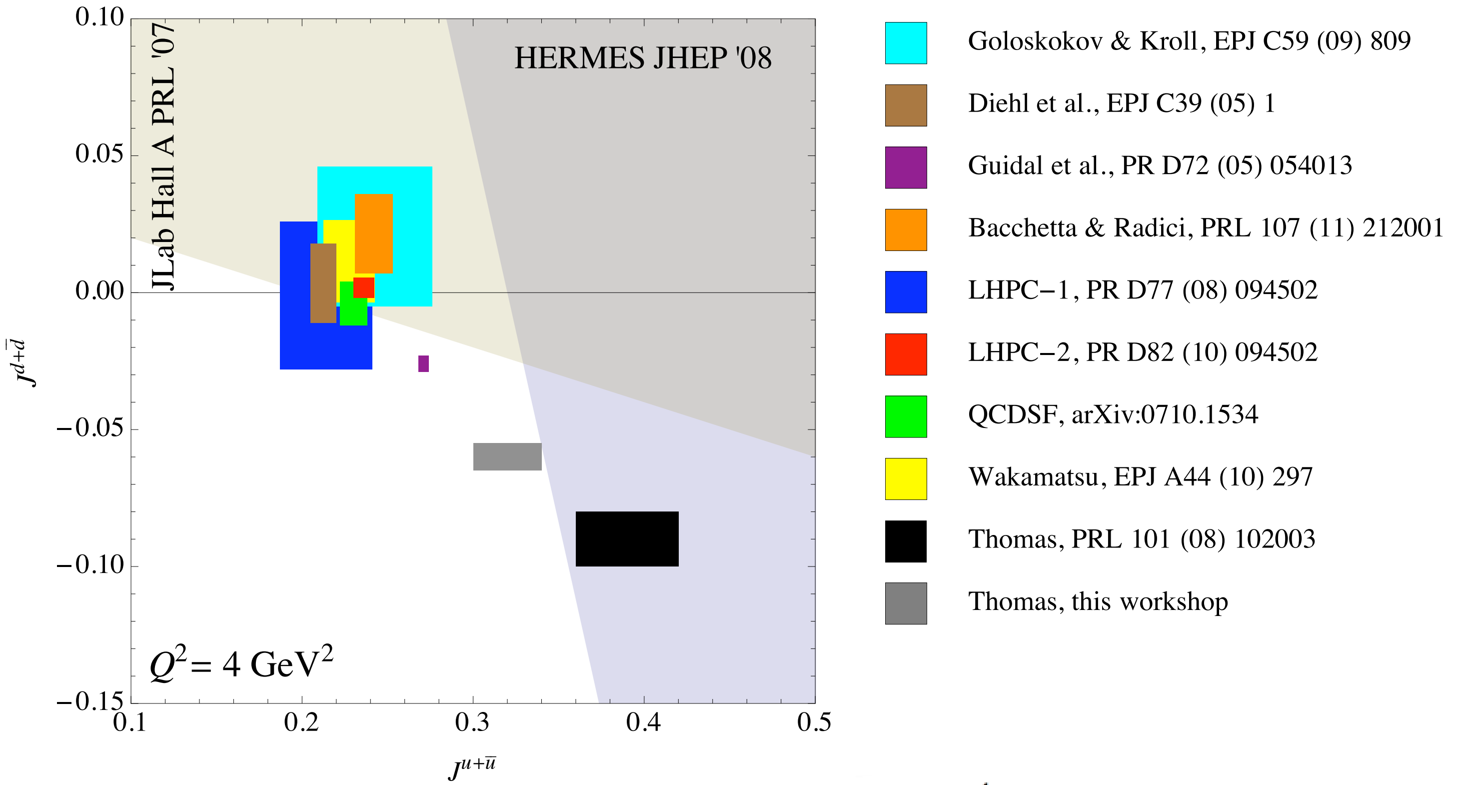
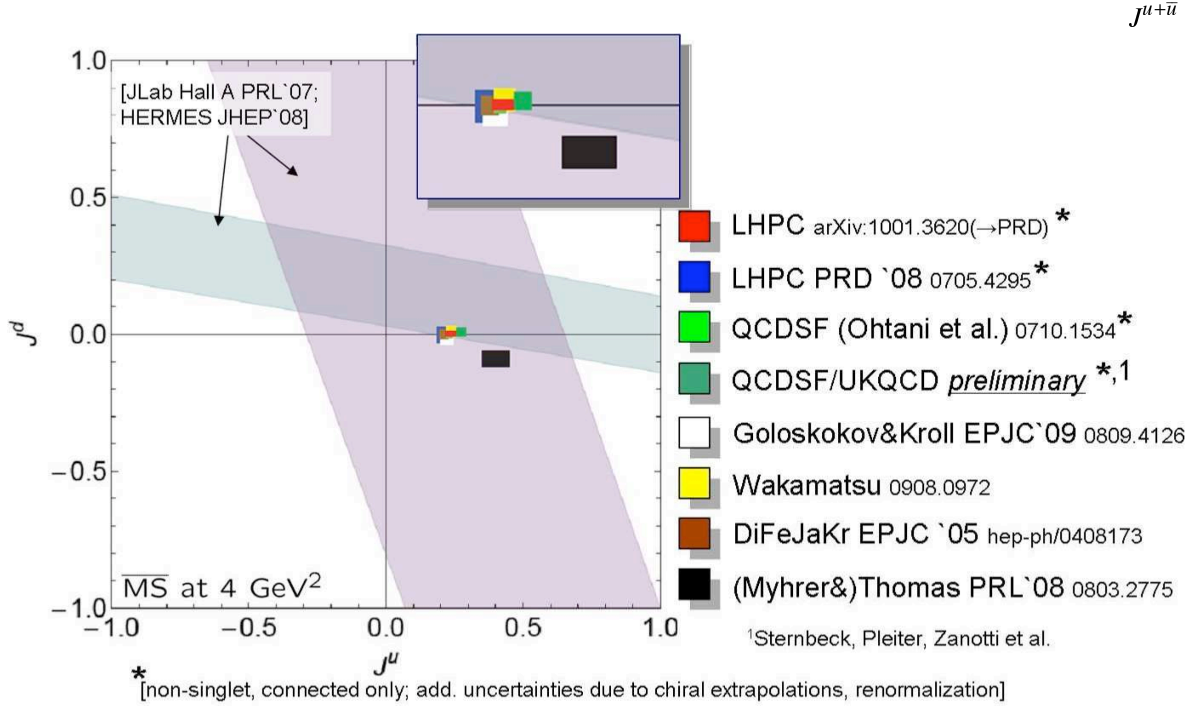


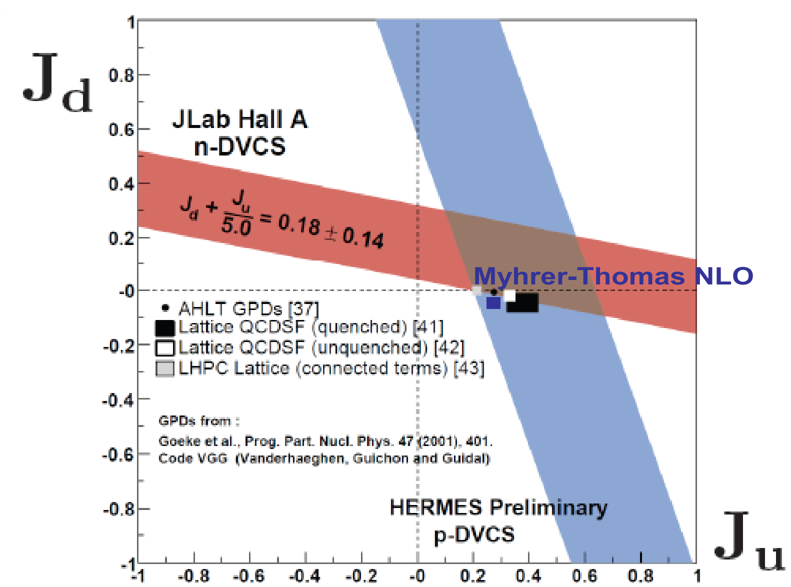
Extractions and lattice QCD



Other versions of the plot







- Boer et al., arXiv:1108.1713 (fig. 4.3)
- JLab HallA, PRL99 (2007) and T.Thomas' talk

Conclusions

- The data are compatible with our assumptions
- We obtain results for the Sivers function and for quark angular momenta in agreement with other extractions
- We can estimate also sea-quark angular momenta
- More work is needed, especially to assess the (probably large) errors due to the choice of functional form.
- Our approach can complement GPD studies