Two nucleons in a harmonic-oscillator trap with chiral EFT

With focus on new power counting of chiral EFT

Chieh-Jen (Jerry) Yang

University of Arizona

INT 2012

Collaborators: Bingwei Long, Bruce Barrett, U. van Kolck, J. Rotureau

From QCD to nuclear structure

Effective field theory for low energy $\pi\pi$, πN, NN interaction.

Some open issues in NN sector

Propose a new power counting scheme

Nuclear structure Strong short-range interaction

4^+ nucleons Difficulty: Model space grow combinatorial.

Low E

Many-body

Reasonable truncation of model space

+

Unitary transformation of NN, NNN forces

SRG

Other bypass ?

No-core shell model (NCSM)

Chiral EFT at NN sector

- Infinitely many diagrams contribute, most of them require renormalization.
- Need to arrange a way to include them based on their importance (there maybe more than one consistent way).
- Pure perturbation doesn't work.

Conventional power counting

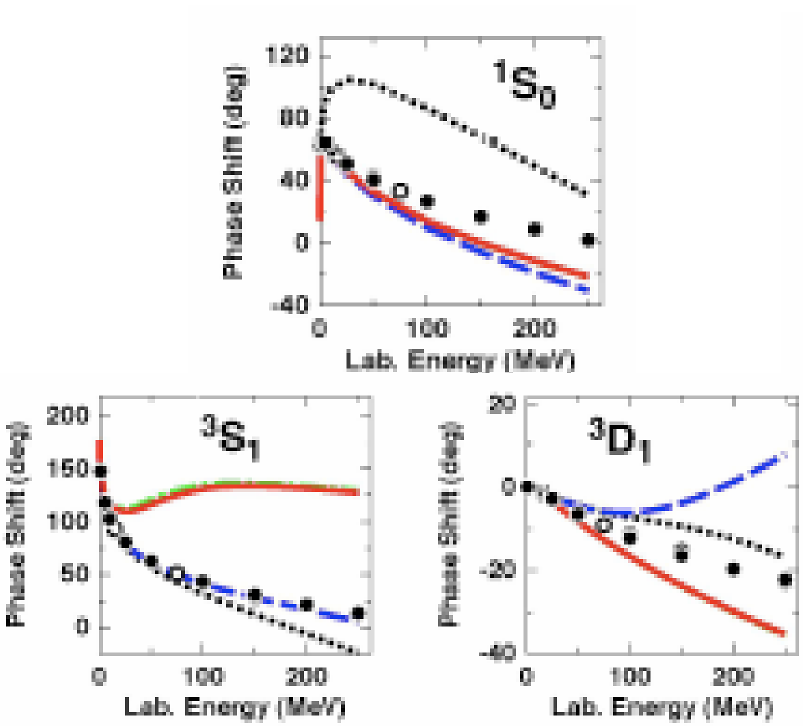
- **Arrange diagrams base on Weinberg's power counting (WPC):** each derivative on the Lagrangian terms is always suppressed by the underlying scale of chiral EFT, $M_{hi} \sim m_{\sigma}$.
- **Iterate potential to all order (L.S. or Schrodinger eq.), with an ultraviolet Λ.**

Carried out to $N^3LO(Q^4/M^4_{hi})$

Epelbaum, Entem, Machleidt, Kaiser, Valderrama, … etc.

Problems

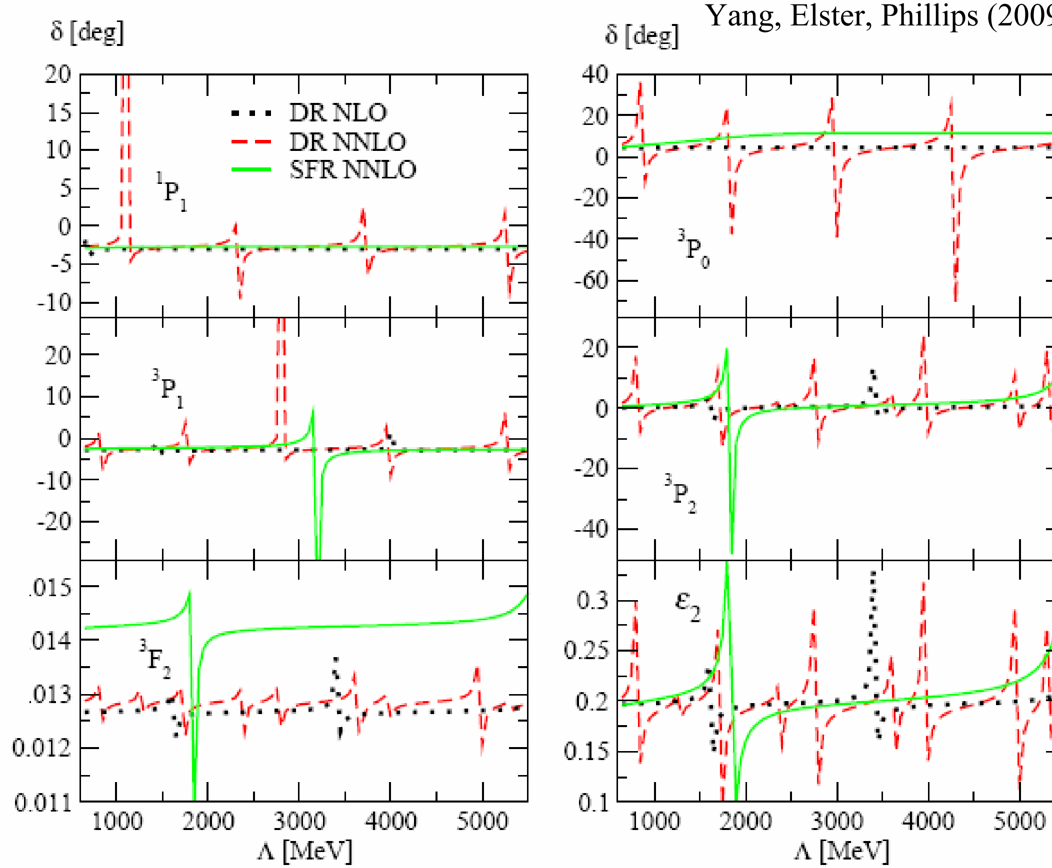
- Singular attractive potentials demand contact terms. (Nogga, Timmermans, van Kolck (2005))
- Beyond LO: Has RG problem at Λ>1 GeV (due to iterate to all order)



N³LO(Q⁴)

Ch. Zeoli R. Machleidt D. R. Entem (2012)

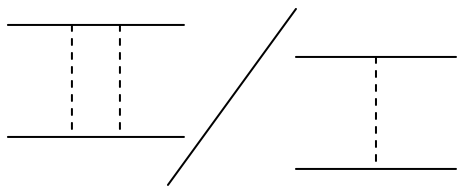
Yang, Elster, Phillips (2009)



New power counting Long & Yang, (2010-2012)

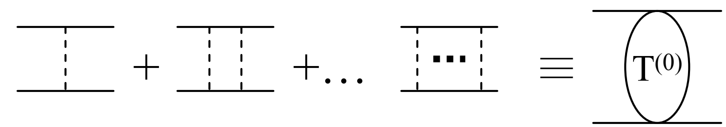
LO: Still iterate to all order (at least for $l<2$).

Reason: van Kolck, Bedaque,… etc.



$\sim \frac{g_A^2 M}{8\pi f_\pi^2} Q$

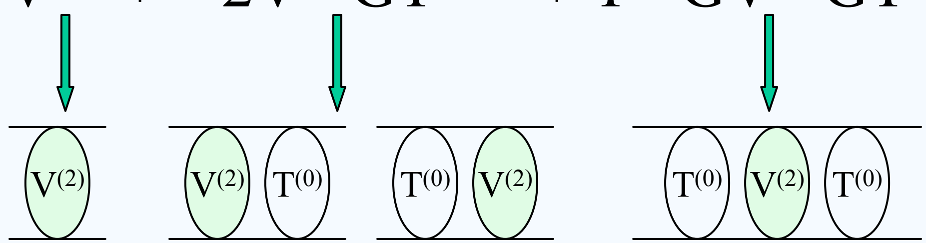
Thus, $O(Q^0)$:



Start at NLO, do perturbation. $(T = T^{(0)}+T^{(1)}+T^{(2)}+T^{(3)}+\ldots)$

If $V^{(1)}$ is absent:

$$T^{(2)} = V^{(2)} + 2V^{(2)}GT^{(0)} + T^{(0)}GV^{(2)}GT^{(0)}.$$



$$G \equiv \frac{2M_N}{\pi}\int_0^{\Lambda} \frac{p^2 dp}{p_0^2 - p^2 + i\varepsilon}$$

$$T^{(3)} = V^{(3)} + 2V^{(3)}GT^{(0)} + T^{(0)}GV^{(3)}GT^{(0)}.$$

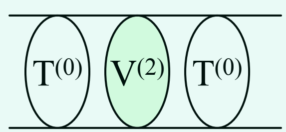
$$V^{(n)} = V^{(n)}_{Long} + V^{(n)}_{Short};$$

$V^{(n)}_{Long}$: pion-exchange at $O\left((\frac{\mathrm{Q}}{\mathrm{M_{hi}}})^{\mathrm{n}} \right)$

$V^{(n)}_{Short}$: counter terms, $\underbrace{\color{red}{C_0} + \color{red}{C_2} q^2 + \color{red}{C_4} q^4 +}$ ...

value of C's decided from renormalization

3 types of counter terms

1. Primordial: Those renormalize the pion-exchange diagrams.
   (always there if survived from partial-wave decomposition)
2. Distorted –wave counter terms: Required due to the divergence of $< \phi_{\mathrm{LO}} | V^{(\mathrm{sub})} | \phi_{\mathrm{LO}} >$, e.g., 

could diverge more than Q^2
3. Residual counter terms: Decided by the requirement from RG.

e.g., if $|T^{(n)}(k;\Lambda) - T^{(n)}(\infty;\Lambda)| \geq O(\frac{Q^{n+2}}{M_{hi}^{n+2}})$, then need V^{n+1}_{Short} at order n+1.

Results (up to O(Q^3))

1. If V_{Long} at LO is repulsive, then primordial counter terms is enough (WPC).
2. If V_{Long} at LO is attractive:

a. Need to promote a counter term to LO if it's absent originally.

b. Due to the divergence of the distorted-wave matrix element, all counter terms are promoted one order earlier starting at NLO. (distorted-wave counter term)

e.x. $\psi_{LO}(r) \sim (\frac{\lambda}{r})^{1/4}\left[u_0 + k^2 r^2 u_1 + O(k^4)\right],\ u_{0,1}:$ oscillatory wave, amplitude < 1.

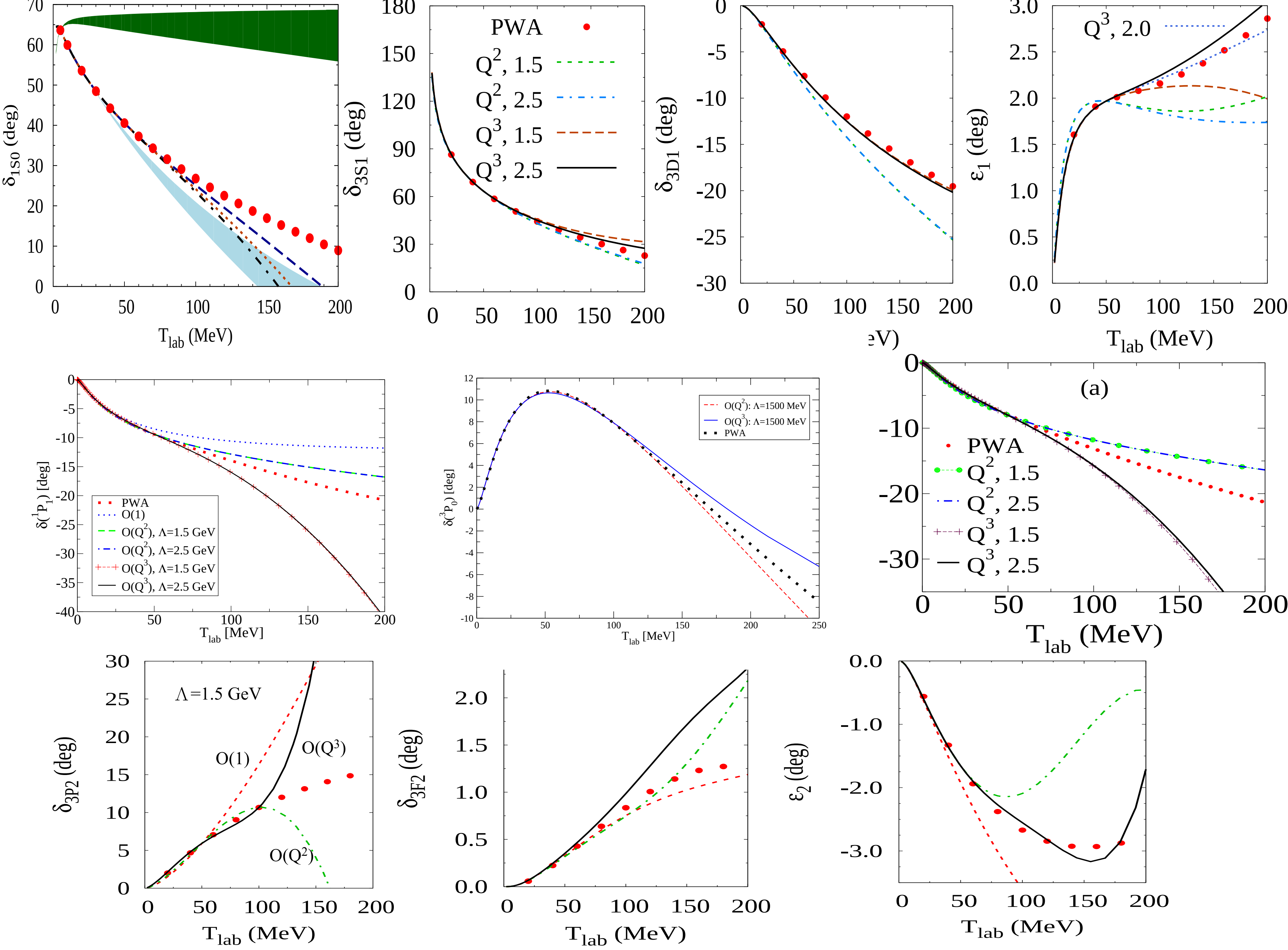
$$\langle \psi_{LO} | V^{(2)}_{Long} | \psi_{LO} \rangle \sim \int_{\sim 1/\Lambda} dr r^2 |\psi_{LO}(r)|^2 \frac{1}{r^5} \sim \underbrace{\alpha(\Lambda)\Lambda^{5/2} + \beta(\Lambda)k^2}_{\text{2 divergent terms !}} + O(k^4 \Lambda^{-5/2}),$$

($\alpha(\Lambda), \beta(\Lambda)$ are oscillatory function diverging slower than Λ.)

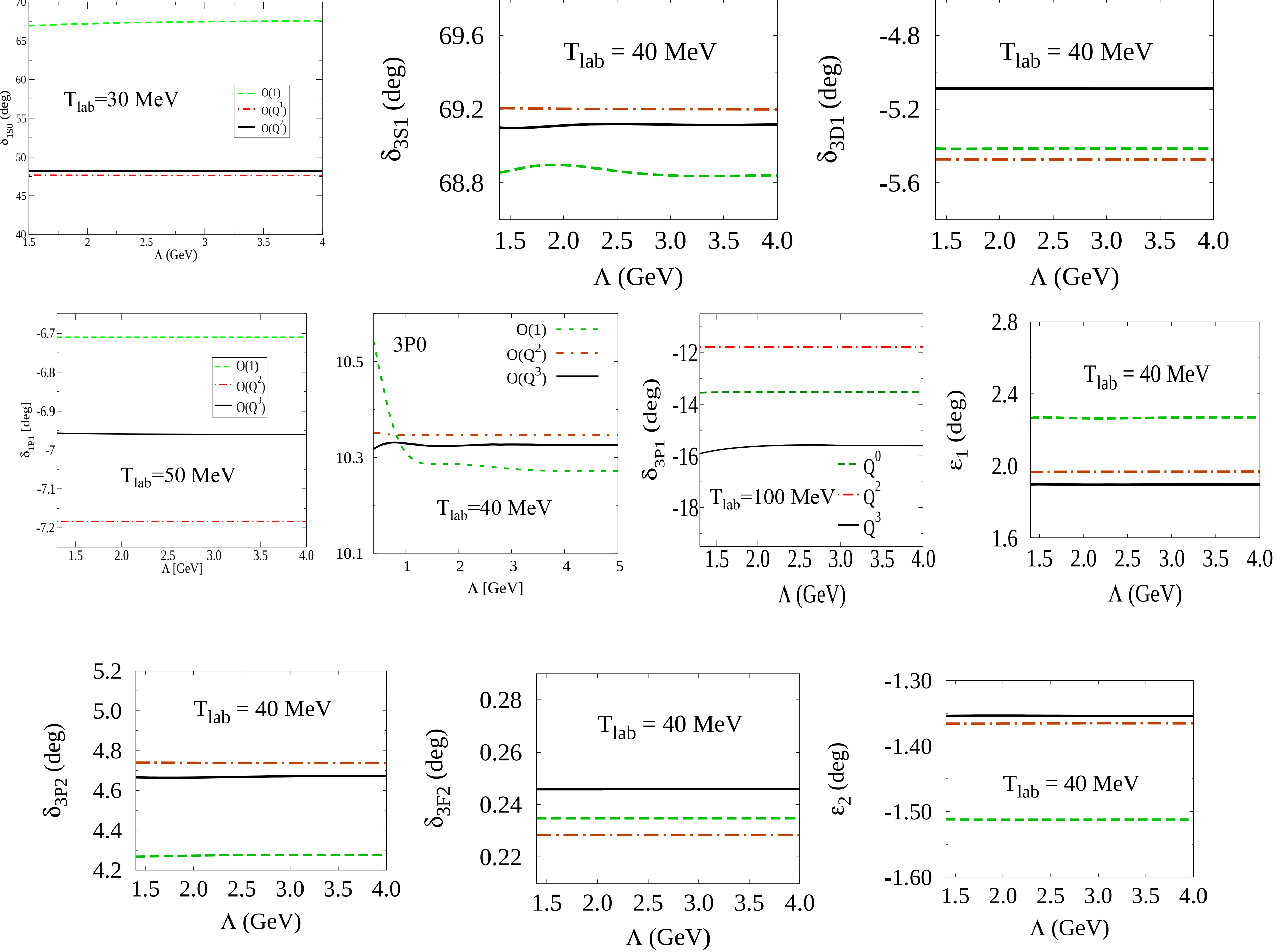
3. Residue counter term enters in $^1S_0$ channel at O(Q)

$\mathcal{O}(1)$	OPE, $C_{^1S_0}$, $\begin{pmatrix} C_{^3S_1} & 0 \\ 0 & 0 \end{pmatrix}$, $C_{^3P_0}p'p$, $\begin{pmatrix} C_{^3P_2}p'p & 0 \\ 0 & 0 \end{pmatrix}$
$\mathcal{O}(Q)$	$D_{^1S_0}(p'^2+p^2)$
$\mathcal{O}(Q^2)$	TPE0, $E_{^1S_0}p'^2p^2$, $\begin{pmatrix} D_{^3S_1}(p'^2+p^2) & E_{\mathrm{SD}}\,p^2 \\ E_{\mathrm{SD}}\,p'^2 & 0 \end{pmatrix}$, $D_{^3P_0}\,p'p(p'^2+p^2)$, $p'p\begin{pmatrix} D_{^3P_2}(p'^2+p^2) & E_{\mathrm{PF}}\,p^2 \\ E_{\mathrm{PF}}\,p'^2 & 0 \end{pmatrix}$, $C_{^1P_1}p'p$, $C_{^3P_1}p'p$
$\mathcal{O}(Q^3)$	TPE1, $F_{^1S_0}p'^2p^2(p'^2+p^2)$

Table: Power counting of counter terms



δ1S0 (deg)
Tlab (MeV)
PWA
Q2, 1.5
Q2, 2.5
Q3, 1.5
Q3, 2.5
δ3S1 (deg)
δ3D1 (deg)
ε1 (deg)
Q3, 2.0
Tlab (MeV)
δ(1P1) [deg]
PWA
O(1)
O(Q2), Λ=1.5 GeV
O(Q2), Λ=2.5 GeV
O(Q3), Λ=1.5 GeV
O(Q3), Λ=2.5 GeV
Tlab [MeV]
δ(3P0) [deg]
O(Q2): Λ=1500 MeV
O(Q3): Λ=1500 MeV
PWA
Tlab [MeV]
(a)
PWA
Q2, 1.5
Q2, 2.5
Q3, 1.5
Q3, 2.5
Tlab (MeV)
δ3P2 (deg)
Λ=1.5 GeV
O(1)
O(Q3)
O(Q2)
Tlab (MeV)
δ3F2 (deg)
Tlab (MeV)
ε2 (deg)
Tlab (MeV)



T_lab=30 MeV
O(1)
O(Q^1)
O(Q^2)
δ_1S0 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3S1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3D1 (deg)
Λ (GeV)
O(1)
O(Q^2)
O(Q^3)
δ_1P1 [deg]
T_lab=50 MeV
Λ [GeV]
3P0
O(1)
O(Q^2)
O(Q^3)
T_lab=40 MeV
Λ [GeV]
δ_3P1 (deg)
Q^0
T_lab=100 MeV
Q^2
Q^3
Λ (GeV)
T_lab = 40 MeV
ε_1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3P2 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3F2 (deg)
Λ (GeV)
T_lab = 40 MeV
ε_2 (deg)
Λ (GeV)

Chiral EFT approach to NCSM

Motivation

No-core shell model is great, but requires unitary transform.

Whenever a model space is truncated, (artificial) higher body forces arise

$$V_{ij} \xrightarrow[\textit{unitary}\ \text{trans.}]{} \underbrace{V_{ij}^{'}}_{keep} \underbrace{+V_{ijk}^{'} + V_{ijkl}^{'} + \ldots}_{truncated\,?}$$

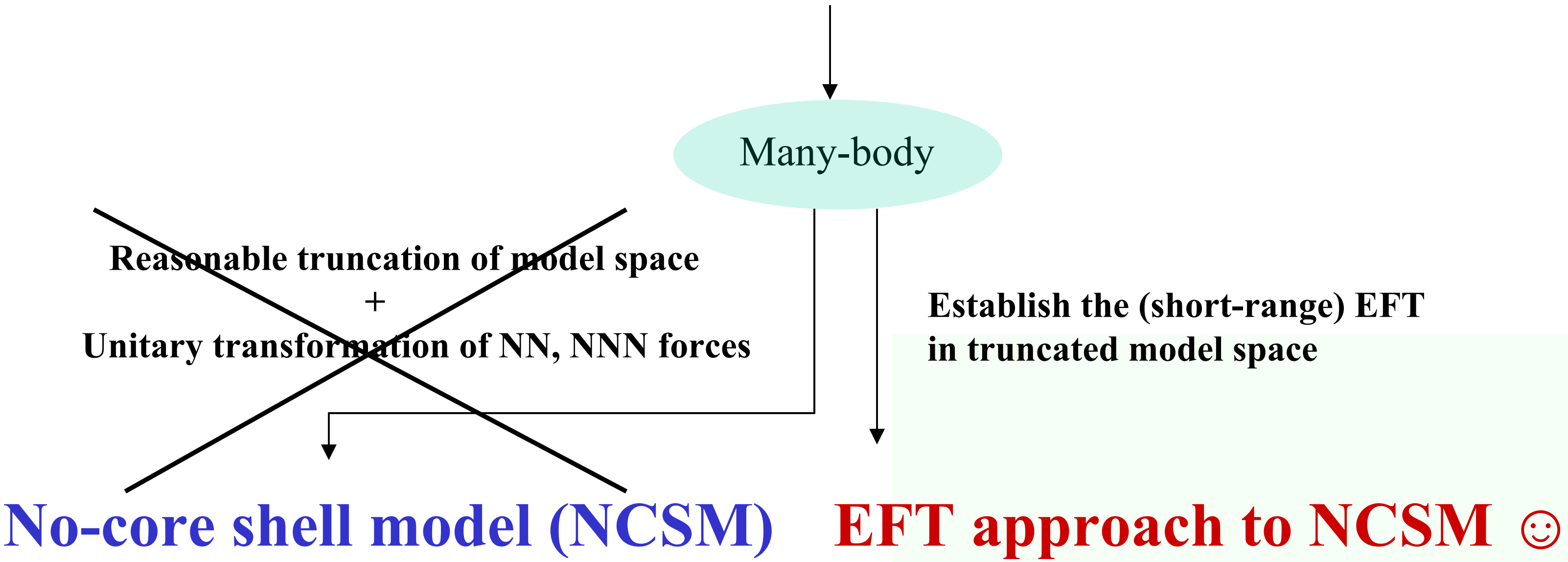
$O(1)$ | $O(Q^1)$ | $O(Q^2)$ | $O(Q^3)$ … $\xrightarrow[\text{Unitary trans.}]{}$ Well-organized power counting in EFT could be destroyed! Especially when $V^{subleading}$ need to be treated perturbatively.

EFT: separation of scale

From QCD to nuclear structure

Nuclear structure — Strong short-range interaction

4^+ nucleons — Difficulty: Model space grow combinatorial.



Idea: Directly perform renormalization of EFT in the truncated model space

$$V = \underbrace{C_0 \delta(\vec{r})}_{LO} - \underbrace{C_2 [\nabla^2 \delta(\vec{r}) + 2(\vec{\nabla}\delta(\vec{r}))\cdot\vec{\nabla} + 2\delta(\vec{r})\nabla^2]}_{NLO} + \ldots$$

Adjust $C_{0,2,\ldots}$ so that $<\psi^{\text{truncated}} | V | \psi^{\text{truncated}}>$ fit some physical observables.

Then can use $\psi^{\text{truncated}}$ and V to make predictions.

- Converge with increasing N_{max} and decreasing ω.
- Extrapolation is needed (unavoidable).

Not enough bound-state to decide LECs.

↓

Add physical HO trap and use Busch formula

↓

fit to scattering data.

Generalization of Busch formula

Uncoupled channels:

$$\frac{\Gamma(\frac{2l+3}{4}-\frac{E(\infty)}{2\omega})}{\Gamma(\frac{1-2l}{4}-\frac{E(\infty)}{2\omega})}=(-1)^{l+1}(\frac{bk}{2})^{2l+1}\cot\delta_l(k)$$

Exact only for $N_{max}\to\infty$, l=0 and zero-range interaction case.

Otherwise has error ~ $O(\mu\omega R^2)$. R : range of potential

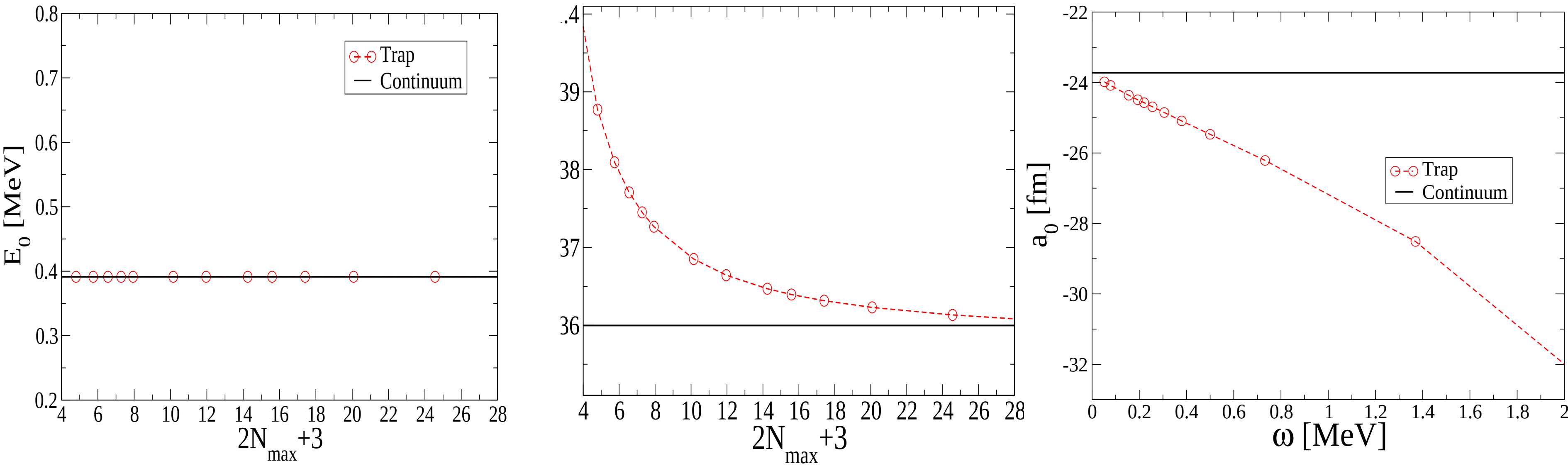
Coupled channels:

$$\cot\delta_1=R_1-\tan^2\delta_3[\frac{\cot\delta_1-R_2}{\cot\delta_2-R_2}](R_1+\cot\delta_2),$$

$$R_1=-2\frac{\Gamma(\frac{3}{4}-\frac{\varepsilon}{2})}{\Gamma(\frac{1}{4}-\frac{\varepsilon}{2})}\frac{1}{bk},R_2=-32\frac{\Gamma(\frac{7}{4}-\frac{\varepsilon}{2})}{\Gamma(-\frac{3}{4}-\frac{\varepsilon}{2})}(bk)^{-5}$$

LO results: $^1S_0$

C_S renormalized by $E_0(\infty)$



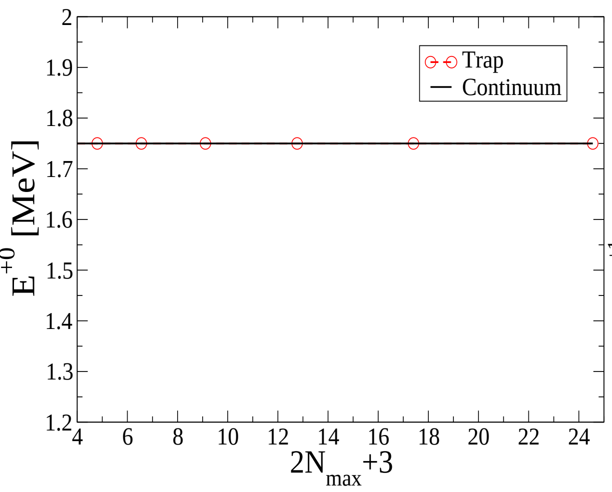
Fix ω (=0.5 MeV), increase N_{max}

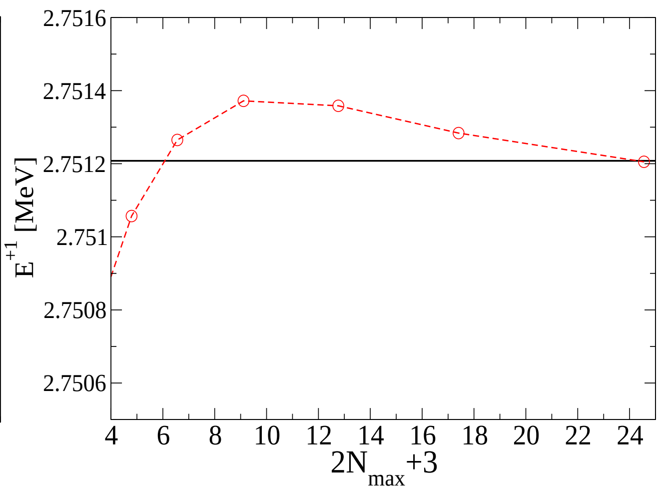
Fix Λ, decrease ω

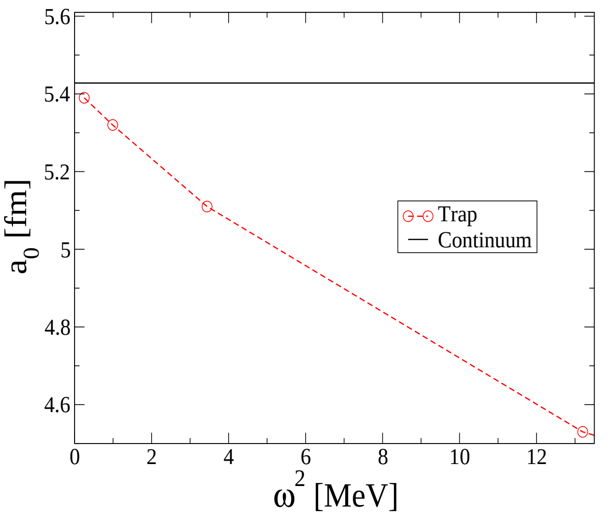
Converge to continuum limit !

LO results: $^3S_1$-$^3D_1$

C_T renormalized by $E_0(\infty)$





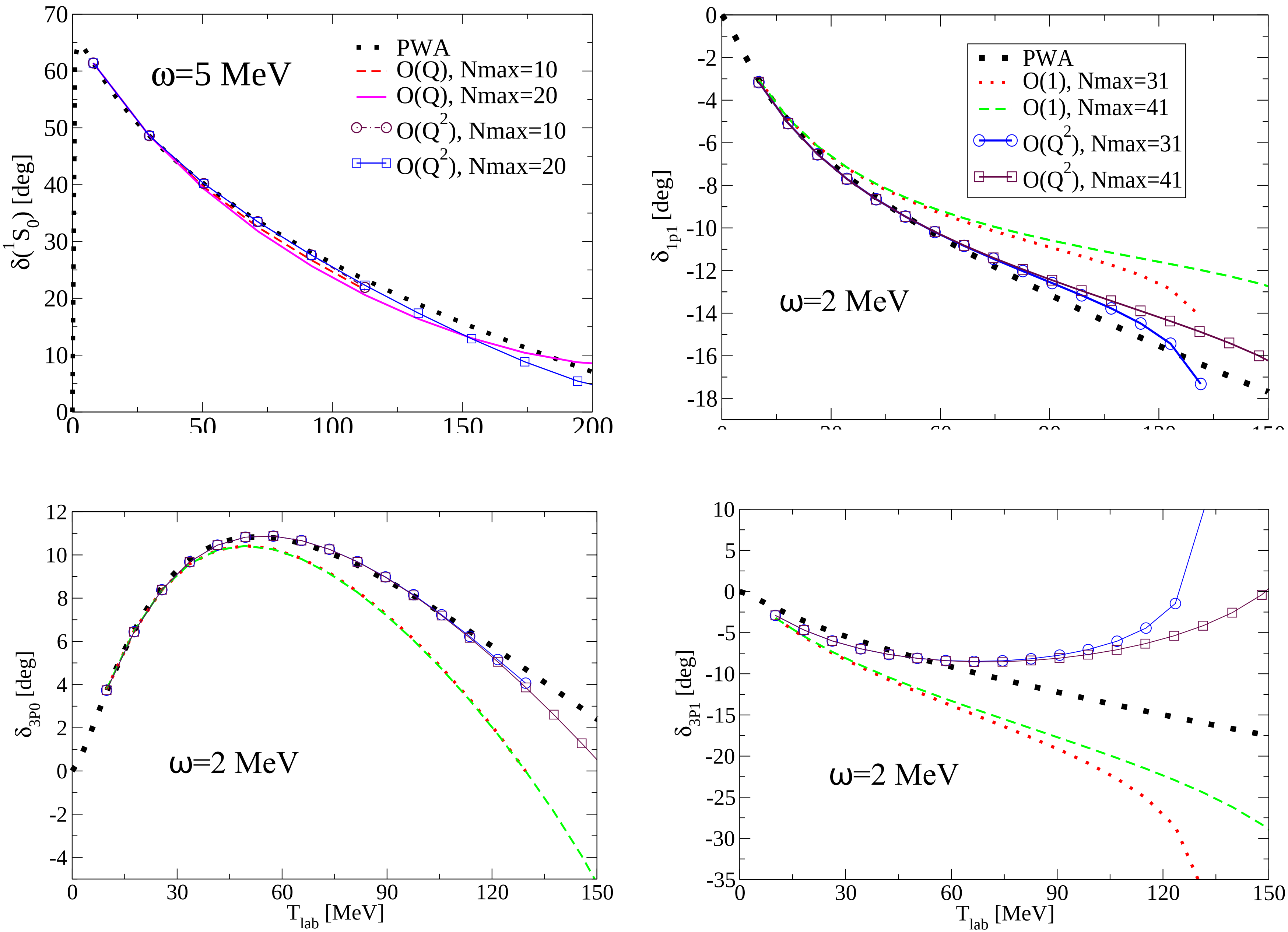


Fix ω(0.5 MeV), increase N_{max}

Fix Λ, decrease ω

Results up to $O(Q^2)$

Preliminary



PWA
O(Q), Nmax=10
O(Q), Nmax=20
O(Q²), Nmax=10
O(Q²), Nmax=20
ω=5 MeV
δ(¹S₀) [deg]
PWA
O(1), Nmax=31
O(1), Nmax=41
O(Q²), Nmax=31
O(Q²), Nmax=41
ω=2 MeV
δ_1p1 [deg]
ω=2 MeV
δ_3P0 [deg]
T_lab [MeV]
ω=2 MeV
δ_3P1 [deg]
T_lab [MeV]

Error analysis: $^1S_0$

ω	E_∞ adopted	E_n predicted	Corresponded E_∞	Relative error
0.5	5.3387	21.355	21.332	0.11%
1	4.6788	20.706	20.666	0.19%
2	5.3523	21.403	21.330	0.34%
4	2.6941	18.813	18.666	0.79%
8	5.3131	21.546	21.311	1.1%

Relative error scales as $O(\omega)$!

Summary

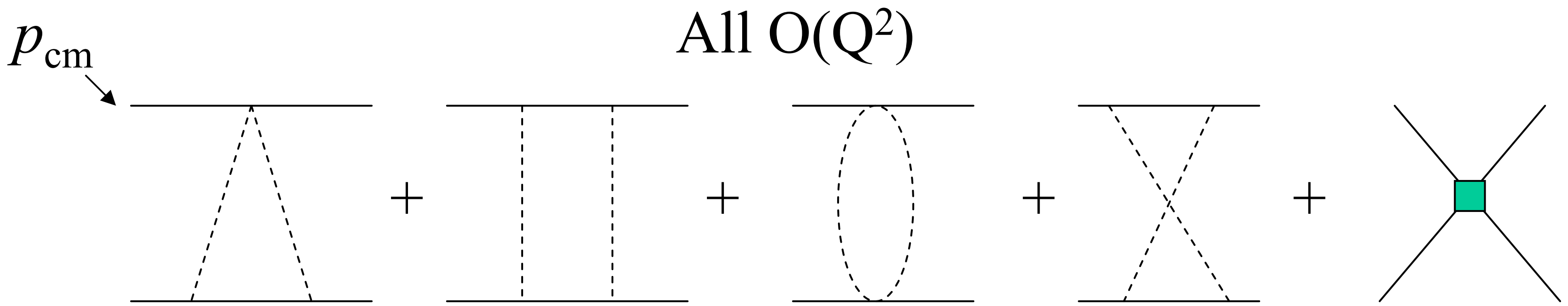
- New power counting of chiral EFT is ready!

- The EFT approach to NCSM for chiral potential is under work.
- With V^{NLO} included, results converge (1%) for n_{max} as small as 20 (ω-dependent).

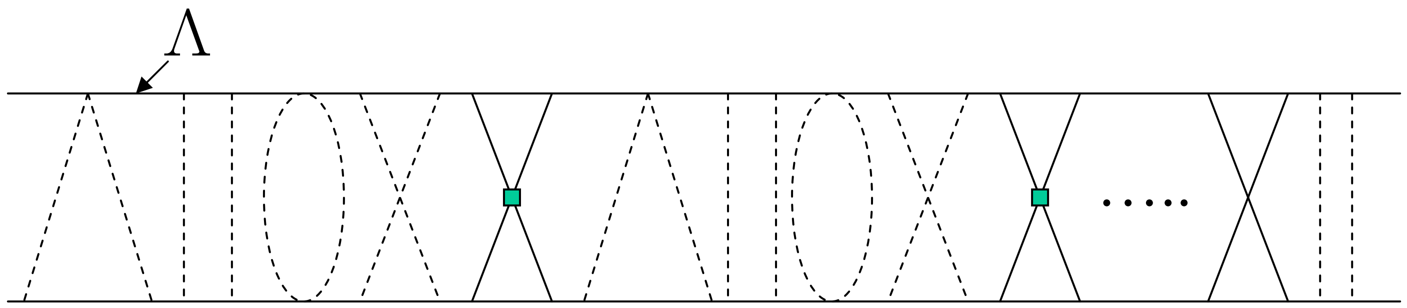
Thank you!

[back](#)

All O(Q^2)



O.k., as long as p_{cm} is small enough, so that $\frac{p_{cm}}{M_{hi}} < 1$



Has problem, as Λ-dependence enter here; has poles also (Baru, et al (2012)).

The expansion parameter is no longer $\frac{p_{cm}}{M_{hi}}$.

The counter terms are just not enough in this case, reflected in problem at Λ>1 GeV.