Weak reactions in few nucleon systems: pp and muon captures

M. Viviani

INFN, Sezione di Pisa &
Department of Physics, University of Pisa
Pisa (Italy)

Electroweak properties of light nuclei, November 5-9, 2012

INT, Seattle
Program INT-12-3
Light nuclei from first principles
September 17 - November 16, 2012

Outline

Collaborators

- L.E. Marcucci - *INFN & Pisa University, Pisa (Italy)*
- L. Girlanda - *INFN & Salento University, Lecce (Italy)*
- R. Schiavilla & M. Piarulli- *Jefferson Lab. & ODU, Norfolk (VA, USA)*
- A. Kievsky - *INFN & Pisa University, Pisa (Italy)*

Reactions of interest ($A \leq 4$)

Weak captures of astrophysical interest:

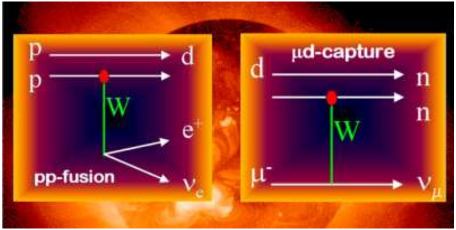
- $p + p \rightarrow {}^2\mathrm{H} + e^+ + \nu_e$
- $p + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + e^+ + \nu_e$

Muon catpures:

- $\mu^- + d \rightarrow n + n + \nu_\mu$
- $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu$ (70%)
- $\mu^- + {}^3\mathrm{He} \rightarrow n + d + \nu_\mu$ (20%)
- $\mu^- + {}^3\mathrm{He} \rightarrow n + n + p + \nu_\mu$ (10%)

Motivations

- Calculations of reaction rates at astrophysical energies
- Muon captures: test of the theoretical models of weak current (MuSun experiment)
- Extraction of $G_{PS}(q^2) \rightarrow$ validate χPT

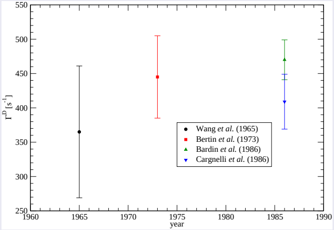


$$j^\mu = \overline{u_p}\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N} - G_A(q^2)\gamma^\mu\gamma^5 - G_{PS}(q^2)\frac{q^\mu\gamma^5}{2M_N}\right]u_n$$

Experimental situation

$\mu^- + d \to n + n + \nu_\mu$

- Two hyperfine states: $f = 1/2$ and $3/2$
- Dominant capture from $f = 1/2$ $\to \Gamma^D$



- New measurement in progress: MuSun
- $\to$ Kammel's talk

$\mu^- + {}^3\mathrm{He} \to {}^3\mathrm{H} + \nu_\mu$

- Hyperfine states: $(f, f_z) = (1, \{\pm 1, 0\})$ and $(0, 0)$

$$\frac{d\Gamma}{d(\cos\theta)} = \frac{1}{2}\Gamma_0 \Big[1 + A_v P_v \cos\theta + A_t P_t \Big(\frac{3\cos^2\theta - 1}{2}\Big) + A_\Delta P_\Delta\Big]$$

$$P_v = P_{1,1} - P_{1,-1}$$
$$P_t = P_{1,1} + P_{1,-1} - 2P_{1,0}$$
$$P_\Delta = 1 - 4P_{0,0}$$

- $\Gamma_0 =$ total capture rate, $A_{v,t,\Delta} =$ angular correlation parameters
- Ackerbauer *et al.*, (1998):

$$\boxed{\Gamma_0 = 1496(4)\ \mathrm{s}^{-1}}$$

- Souder *et al.*, (1998): $A_v = 0.63 \pm 0.09$ (stat.)$^{+0.11}_{-0.14}$ (syst.)

Theoretical studies: "main ingredients" (I)

NN potentials

- "Old models": Argonne V18, CD-Bonn, Nijmengen ($\chi^2 \approx 1$)
- Fit of 3N data using non-locality in P-waves (INOY [Doleschall, 2008])
- Effective field theory (EFT)
  - J-N3LO – [Epelbaum and Coll, 1998-2006]
  - N3LO – [Entem & Machleidt, 2003]

3N potentials

- "Old models": Tucson-Melbourne [Coon *et al*, 1979, Friar *et al*, 1999]; Brazil [Robilotta & Coelho, 1986]; Urbana [Pudliner *et al*, 1995]
- Effective field theory
  - at N2LO [Epelbaum *et al*, 2002], [Navratil, 2007]
- Illinois [Pieper *et al*, 2001]
- Under progress: N3LO, N4LO

Accurate nuclear wave functions

- Methods for $A \geq 3$: Faddeev-Yakubovsky Equations, GFMC, Variational methods (Gaussians, NCSM)
  - HH method [J. Phys. G **35**, 063101 (2008)]

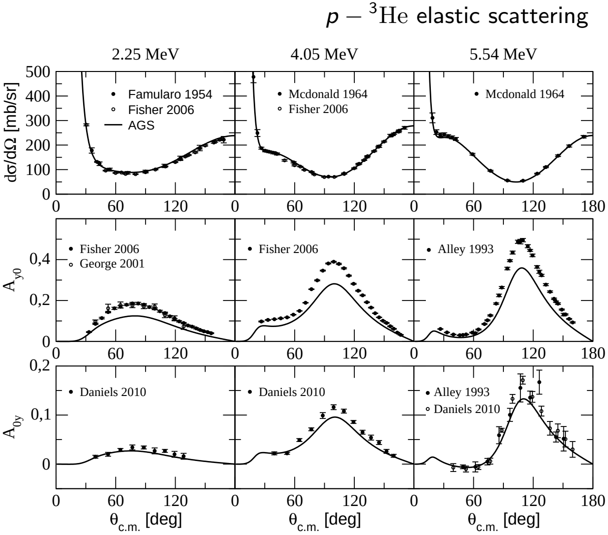
Some results for $A = 2$–4

$A = 2$	AV18	N3LO	Exp.
B_d (MeV)	2.22457	2.22456	2.224574(9)
a_{nn} (fm)	–18.487	–18.900	–18.9(4)
${}^1a_{np}$ (fm)	–23.732	–23.732	–23.740(20)
${}^3a_{np}$ (fm)	5.412	5.417	5.419(7)
$A = 3$	AV18/UIX	N3LO/N2LO	Exp.
$B_{^3\mathrm{H}}$ (MeV)	8.479	8.474	8.482
$B_{^3\mathrm{He}}$ (MeV)	7.750	7.733	7.718
${}^2a_{nd}$ (fm)	0.590	0.675	0.645(10)
${}^4a_{nd}$ (fm)	6.343	6.342	6.35(2)
$A = 4$	AV18/UIX	N3LO/N2LO	Exp.
$B_{^4\mathrm{He}}$ (MeV)	28.45	28.36	28.30
${}^0a_{n^3\mathrm{He}}$ (fm)	7.81	7.61	7.57(3)
${}^1a_{n^3\mathrm{He}}$ (fm)	3.39	3.37	3.36(1)

Accuracy of the calculation tested in several benchmarks

Bound states: [Kamada *et al.*, 2001] – Scattering states [MV *et al.*, 2011]

Benchmark test of 4N scattering calculations [PRC **84**, 054010 (2011)]

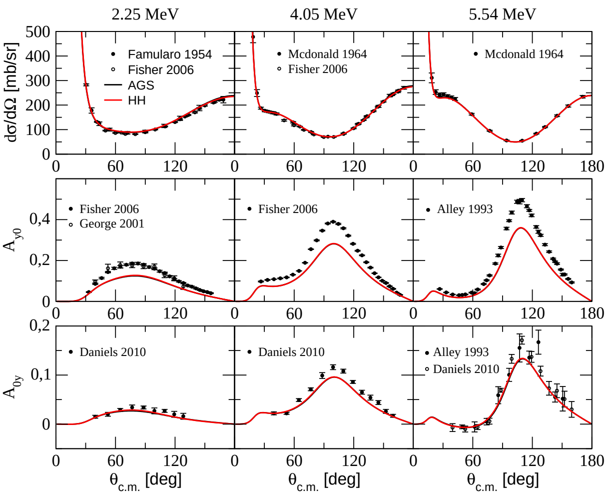


N3LO potential

AGS= Deltuva & Fonseca
FY= Lazauskas & Carbonell

Benchmark test of 4N scattering calculations [PRC **84**, 054010 (2011)]

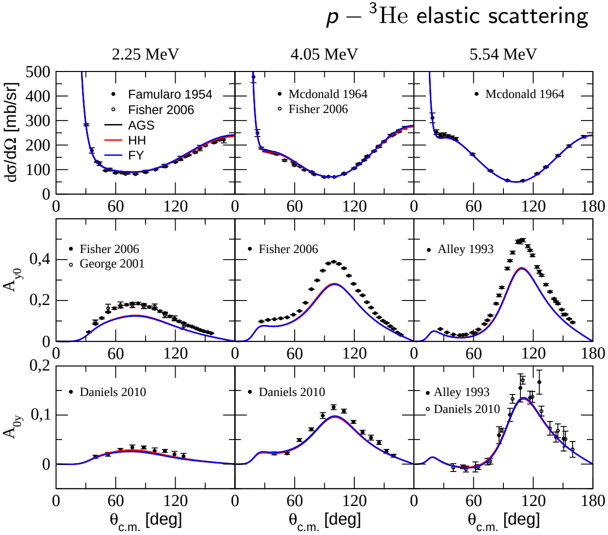
$p - {}^{3}\mathrm{He}$ elastic scattering



N3LO potential

AGS= Deltuva & Fonseca
FY= Lazauskas & Carbonell

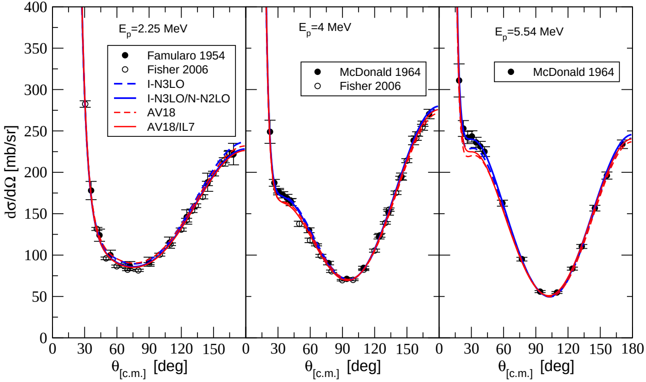
Benchmark test of 4N scattering calculations [PRC **84**, 054010 (2011)]



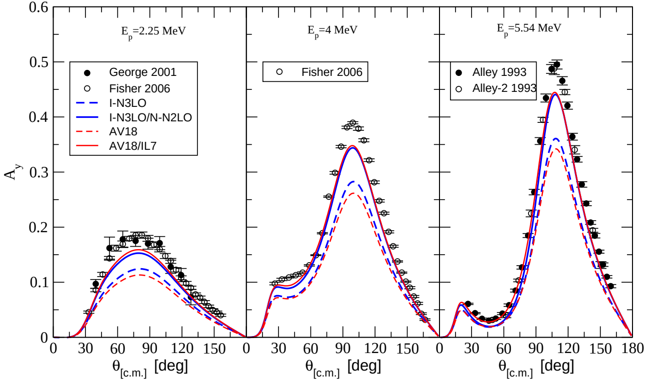
N3LO potential

AGS= Deltuva & Fonseca
FY= Lazauskas & Carbonell

$p - {}^3\mathrm{He}$ differential cross section



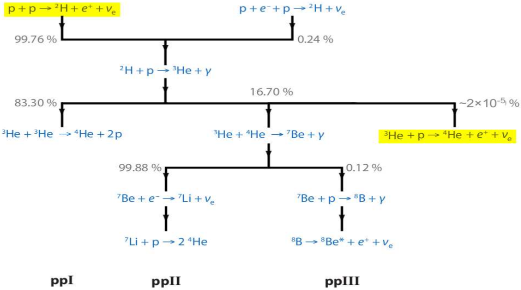
$p - {}^3\mathrm{He}\ A_y$



Theoretical studies: “main ingredients” (II)

- Nuclear weak transition operators: $[\rho^{(A,V)}, \mathbf{j}^{(A,V)}]$
  - Standard Nuclear Physics Approach - SNPA [Schiavilla *et al.*, PRC **58**, 1263 (1998); Marcucci *et al.*, PRC **63**, 015801 (2000)]
  - Chiral Effective Field Theory “hybrid” Approach - χEFT* [Park, Min, & Rho, Phys. Rep. **233**, 341 (1993); Park *et al.*, PRC **67**, 055206 (2003)]

  SNPA and χEFT* used for $p + p \to d + e^+ + \nu_e$ and $p + {}^3\mathrm{He} \to {}^4\mathrm{He} + e^+ + \nu_e$



  - “Less hybrid” Chiral Effective Field Theory Approach - χEFT [Marcucci *et al.*, PRL **108**, 052502 (2012)]

Nuclear transition operators: SNPA

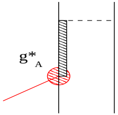
- One-body operators: NRR of $j_i^\mu \to O(1/m^2)$
- Two-body operators: $\pi-$, $\rho-$, $\omega-$, $\ldots$ exchanges + Δ d.o.f. excitations
  - $\rho^{(V)}$ and $\mathbf{j}^{(V)}$: CVC $\to$ EM operators (constructed to verify current conservation)

	$\mu(^3\mathrm{H})$	$\mu(^3\mathrm{He})$
1b	2.5745	-1.7634
Full	**2.9525**	**-2.1299**
Exp.	2.9790	-2.1276

AV18/UIX, $\Rightarrow$ Full=1b+2b+3b
[Marcucci *et al.*, PRC **72**, 014001 (2005)]

  - Two-body $\rho^{(A)}$: PCAC + low-energy theorem $\to$ π-exchange and short-range terms
  - Two-body $\mathbf{j}^{(A)}$: π- and ρ-exchange, $\pi\rho$ mechanism, and $\mathbf{j}^{(A)}(\Delta)$

Largest contribution to $\mathbf{j}^{(A)}(\Delta)$ from



g_A^* fit to observable: GT_{exp} of tritium β-decay

Nuclear transition operators: χEFT at N^{3}LO

- One-body operators $\equiv$ SNPA
- Two-body operators: from [Park, Min, & Rho, Phys. Rep. **233**, 341 (1993)], [Song *et al.*, PRC **79**, 064002 (2009)]
  - Two-body $\rho^{(A)}$: soft π-exchange dominant
  - Two-body $\rho^{(V)} = 0$ at N^{3}LO
  - Two-body $\mathbf{j}^{(V)}$: CVC $\rightarrow$EM current
    here $1\pi + 2\pi$ + CT $\rightarrow$ two LECs (g_{4S} & g_{4V}) $\Rightarrow$ from $\mu(^3\mathrm{H} - {}^3\mathrm{He})$
  - Two-body $\mathbf{j}^{(A)}$: 1π + CT $\rightarrow$ one LEC (d_R) $\Rightarrow$ from GT_{exp} of ^{3}H β-decay

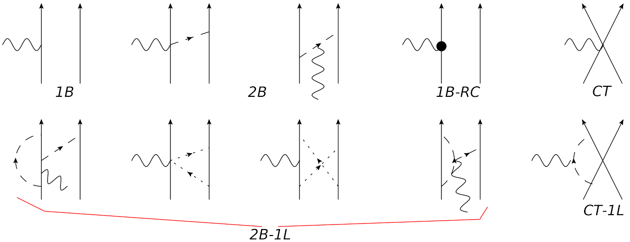
"hybrid" χEFT* $\Rightarrow$ AV18/UIX [N3LO/N2LO] $\Rightarrow$ current and potentials "uncorrelated"

	$\Lambda = 500$ MeV	$\Lambda = 600$ MeV	$\Lambda = 800$ MeV
d_R	0.97(7)	1.75(8) [1.00(9)]	3.89(10)
g_{4S}	0.69(1)	0.55(1) [0.11(1)]	0.25(2)
g_{4V}	2.065(6)	0.793(6) [3.124(6)]	-1.07(1)

χ charge & current: chiral counting

Contributions from each type of current at $\mathbf{q} = \mathbf{p}_e + \mathbf{p}_\nu = 0$.

J^μ	LO	NLO	N^{2}LO	N^{3}LO	N^{4}LO
$\mathbf{j}^A$	1B	–	1B-RC	2B	1B-RC, 2B-1L and 3B
ρ^A	–	1B	2B	1B-RC	1B-RC, 2B-1L
$\mathbf{j}^V$	–	1B	2B	1B-RC	1B-RC, 2B-1L
ρ^V	1B	–	–	2B	1B-RC, 2B-1L and 3B



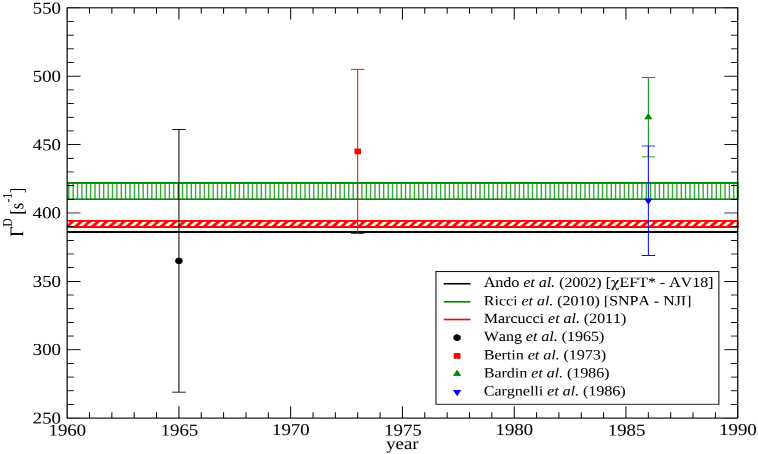
Results: $\Gamma^D(\mu^- + d)$ (SNPA and χEFT*)

SNPA(AV18)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
g_A=1.2654(42)	246.6(7)	20.1	46.7	71.6	4.5	0.9	390.4(7)
g_A=1.2695(29)	246.8(5)	20.1	46.8	71.8	4.5	0.9	390.9(7)
χEFT*(AV18)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
$\Lambda = 500$ MeV	250.0(8)	19.9	46.2	71.2	4.5	0.9	392.7(8)
$\Lambda = 600$ MeV	250.0(8)	19.8	46.3	71.1	4.5	0.9	392.6(8)
$\Lambda = 800$ MeV	249.7(7)	19.8	46.4	71.1	4.5	0.9	392.4(7)
χEFT*(N3LO)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
$\Lambda = 600$ MeV	250.5(7)	19.9	46.4	71.5	4.4	0.9	393.6(7)

$\Rightarrow$ $\Gamma^D = 390 \div 394\ s^{-1}$

Marcucci *et al.*, PRC **83**, 014002 (2011)

Comparison with data and previous calculations



Results: $\Gamma_0(\mu^- + {}^3\mathrm{He})$ (SNPA and χEFT*)

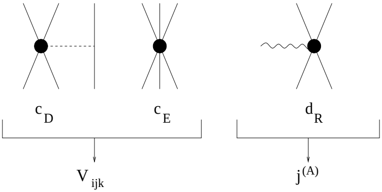
SNPA(AV18/UIX)	Γ_0
g_A=1.2654(42)	1486(8)
g_A=1.2695(29)	1486(5)
χEFT*(AV18/UIX)	Γ_0
$\Lambda = 500$ MeV	1487(8)
$\Lambda = 600$ MeV	1488(9)
$\Lambda = 800$ MeV	1488(8)
χEFT*(N3LO/N2LO; Λ=600 MeV)	1480(9)

$\Rightarrow$ Γ_0= 1484(13) s^{-1}

To be compared with:

- Γ_0(exp)=1496(4) s^{-1}
- Marcucci *et al.*, PRC **66**, 054003 (2002) [SNPA–AV18/UIX] $\rightarrow$ 1484(8) s^{-1}
- Gazit, PLB **666**, 472 (2008) [χEFT*–AV18/UIX] $\rightarrow$ 1499(16) s^{-1}

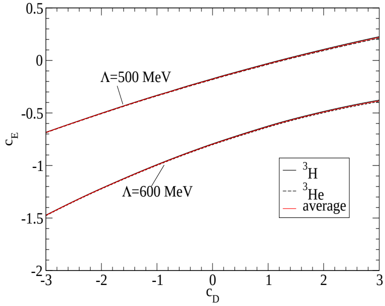
"Less hybrid" χEFT calculation

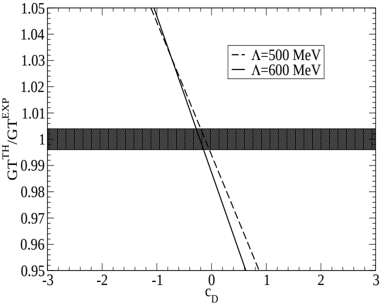


$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

Gardestig and Phillips, PRL **96**, 232301 (2006)
Gazit *et al.*, PRL **103**, 102502 (2009)

fit c_D and c_E to $B(A=3)$ and GT_{exp} with N3LO/N2LO
$\Rightarrow$ $\{c_D; c_E\}_{\mathrm{MAX}}$ and $\{c_D; c_E\}_{\mathrm{MIN}}$





Remaining LEC's: g_{4S} and g_{4V} in the vector current $\Rightarrow$ fit to the $A = 3$ magnetic moments

	$\{c_D; c_E\}$	g_{4S}	g_{4V}
Λ=500 MeV	$\{-0.20; -0.208\}$	0.207 ± 0.007	0.765 ± 0.004
	$\{-0.04; -0.184\}$	0.200 ± 0.007	0.771 ± 0.004
Λ=600 MeV	$\{-0.32; -0.857\}$	0.146 ± 0.008	0.585 ± 0.004
	$\{-0.19; -0.833\}$	0.145 ± 0.008	0.590 ± 0.004

Radiative corrections[1] ARE included

[1] Czarnecki *et al.*, PRL **99**, 032003 (2007)

Results: $\Gamma^D(\mu^- + d)$ and $\Gamma_0(\mu^- + {}^3\mathrm{He}) \to \chi$EFT

	${}^1S_0$	${}^3P_0$	${}^3P_1$	${}^3P_2$	Γ^D	Γ_0
IA – $\Lambda = 500$ MeV	238.8	21.1	44.0	72.4	381.7	1362
IA – $\Lambda = 600$ MeV	238.7	20.9	43.8	72.0	380.8	1360
FULL – $\Lambda = 500$ MeV	254.4(9)	20.5	46.8	72.1	399.2(9)	1488(9)
FULL – $\Lambda = 600$ MeV	255(1)	20.3	46.6	71.6	399(1)	1499(9)

Γ^D **=399(3) s^{-1} & Γ_0 =1494(21) s^{-1}**

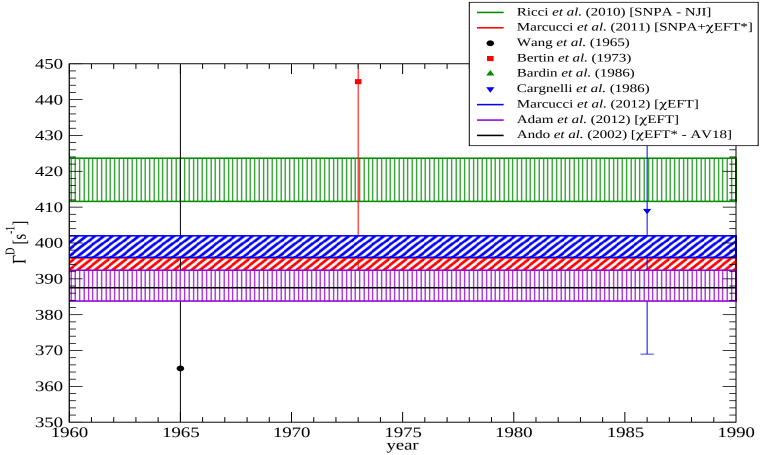
vs. Γ^D(exp) $\cdots$ & Γ_0(exp)=1496(4) s^{-1}

Comparison between Γ_0 and Γ_0(exp) $\to$

G_{PS} =8.2 $\pm$0.7 vs. $G_{PS}^{\chi\mathrm{PT}}$ =7.99$\pm$0.20

Marcucci *et al.*, PRL **108**, 052502 (2012)

Comparison with data and previous calculations



$p + p \to d + e^+ + \nu_e$ astrophysical factor

$$\sigma(E) = \frac{1}{(2\pi)^3} \frac{G_V^2}{v} m_e^5 f(E) \sum_M |\langle d, M | \mathbf{A}_- | pp \rangle|^2 \qquad S_{11}(E) = S_{11}(0) + S'_{11}(0)E + \frac{1}{2} S''_{11}(0)E^2 + \ldots$$

Goal: $< 1\%$ accuracy

- Dominant contribution from the $^1S_0$ wave
- P-wave contribution: $\sim 1\%$
- Two-body contribution: $\sim 1\%$

pp wave function

- EM interaction: $V_{C1} + V_{C2} + V_{DF} + V_{VP} + \cdots$
- $V_{VP} \sim \exp(-2m_e r)$: sizeable effect at low energies
- Necessity to solve the Schroedinger equation up to $1,000$ fm
- 1% effect

SNPA: [Schiavilla *et al.*, 1998], χEFT*: [Park *et al.*, 2003]
$S_{11}(0) = 3.94(1 \pm 0.0015 \pm 0.0010 \pm \epsilon)$ (only the $^1S_0$ wave)
errors from uncertainties in g_A, fit of the tritium β-decay, etc; ϵ "systematic error" (?)

See also the review paper: [E. G. Adelberger *et al.*, Rev. Mod. Phys. **83**, 195 (2011) [arXiv:1004.2318]]

Results with the 'Less hybrid" χEFT

N3LO potential – PRELIMINARY

d_R, g_{4S}, g_{4V} fixed using the N3LO/N2LO wave functions - only V_{C1} EM int.

calculation performed between $0 < E < 10$ keV

	$S_{11}(0)$ [$\times 10^{-25}$ MeV b]		$S'_{11}(0)/S_{11}(0)$ [MeV^{-1}]	
	$^1S_0$	$S+P$	$^1S_0$	$S+P$
IA(500)	3.96	3.98	11.16	11.68
IA(600)	3.94	3.96	11.17	11.68
FULL(500)	4.025(5)	4.052(5)	11.17	11.68
FULL(600)	4.007(5)	4.033(5)	11.17	11.68

Summary:

$^1S_0$

$$S_{11}(0) = 4.00 \div 4.03 \times 10^{-25}\,\mathrm{MeV\ b}$$
$$\frac{S'_{11}(0)}{S_{11}(0)} = 11.17\,\mathrm{MeV}^{-1}$$

All waves

$$S_{11}(0) = 4.03 \div 4.06 \times 10^{-25}\,\mathrm{MeV\ b}\ \ (1\%)$$
$$\frac{S'_{11}(0)}{S_{11}(0)} = 11.68\,\mathrm{MeV}^{-1}\ \ (4\%)$$

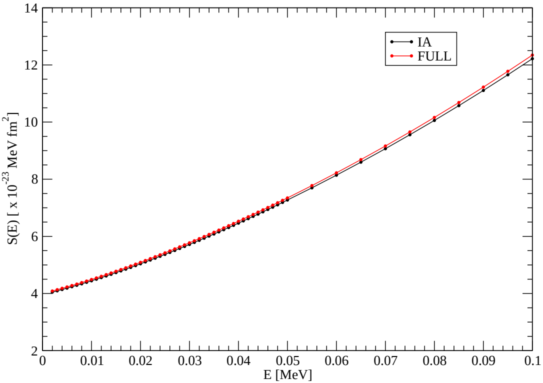
Pionless EFT at N^{2}LO [Wei *et al.*, 2012]

$S_{11}(0) = (3.99 \pm 0.14)10^{-25}$ MeV b, $S'_{11}(0)/S_{11}(0) = (11.3 \pm 0.1)$ MeV^{-1}

New interest: $S''_{11}(0)$

$S(E)$ calculated in the range $0 - 100$ keV

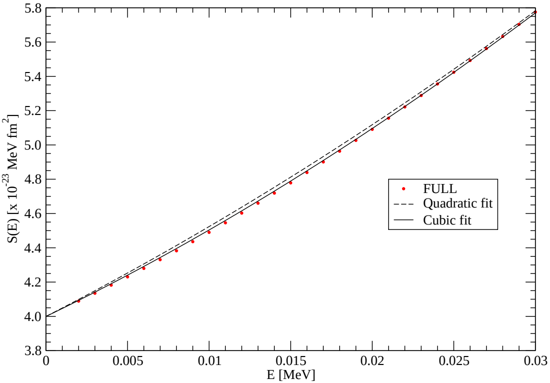
Effect of two-body currents



Calculation with AV18 - only the $^1S_0$ wave

$S(E)$ calculated in the range $0 - 100$ keV

Test of the quadratic approximation



Calculation with AV18 - only the $^1S_0$ wave

Small effects

Quadratic vs. cubic fit

$$S(E) = S(0) + a_0 E + a_1 E^2$$
$$S(E) = S(0) + a_0 E + a_1 E^2 + a_2 E^3$$

	$S(0) \times 10^{23}$ [MeV fm^2]	$S'(0)/S(0)$ [MeV^{-1}]	$S''(0)/S(0)$ [MeV^{-2}]
quadratic fit	4.00	12.23	175.0
cubic fit	4.00	11.47	233.3
Wei *et al.*	3.99 ± 0.14	11.3 ± 0.1	170 ± 2

Effect of the EM interactions $V_{C2} + V_{DF} + V_{VP} + \cdots$

Calculation performed so far only for the AV18 interaction

	$S(0) \times 10^{23}$ [MeV fm^2]	$S'(0)/S(0)$ [MeV^{-1}]	$S''(0)/S(0)$ [MeV^{-2}]
AV18+V_{C1}	4.03	11.59	226.5
AV18+V_{EM}	4.00	11.47	233.3

Conclusions and outlook

- Extensive theoretical work on muon capture in SNPA, χEFT* and χEFT
- $\Gamma_0(\mu^- + {}^3\mathrm{He})$: nice agreement theory vs. experiment
- $\Gamma^D(\mu^- + d)$:
  - some discrepancies among different theoretical works
  - more accurate experimental results $\rightarrow$ MuSun
- New refined calculation of the *pp* fusion up to 100 keV (in progress)
- In the future:
  - χEFT $\rightarrow$ $\mu^- + {}^3\mathrm{He} \rightarrow n + d + \nu_\mu$
    $\mu^- + {}^3\mathrm{He} \rightarrow n + n + p + \nu_\mu$
  - χEFT $\rightarrow$ reactions of astrophysical interest
    $p + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + e^+ + \nu_e$
    $p + d \rightarrow^3 \mathrm{He} + \gamma$
    $d + d \rightarrow^4 \mathrm{He} + \gamma$
    ....
- New model for $\mathbf{j}^A, \rho^A$ consistent with the EM charge/current derived by Pastore *et al.* ($\rightarrow$ Pastore's talk)