How Solving Light Nuclei Can Help Solve Quantum Field Theory

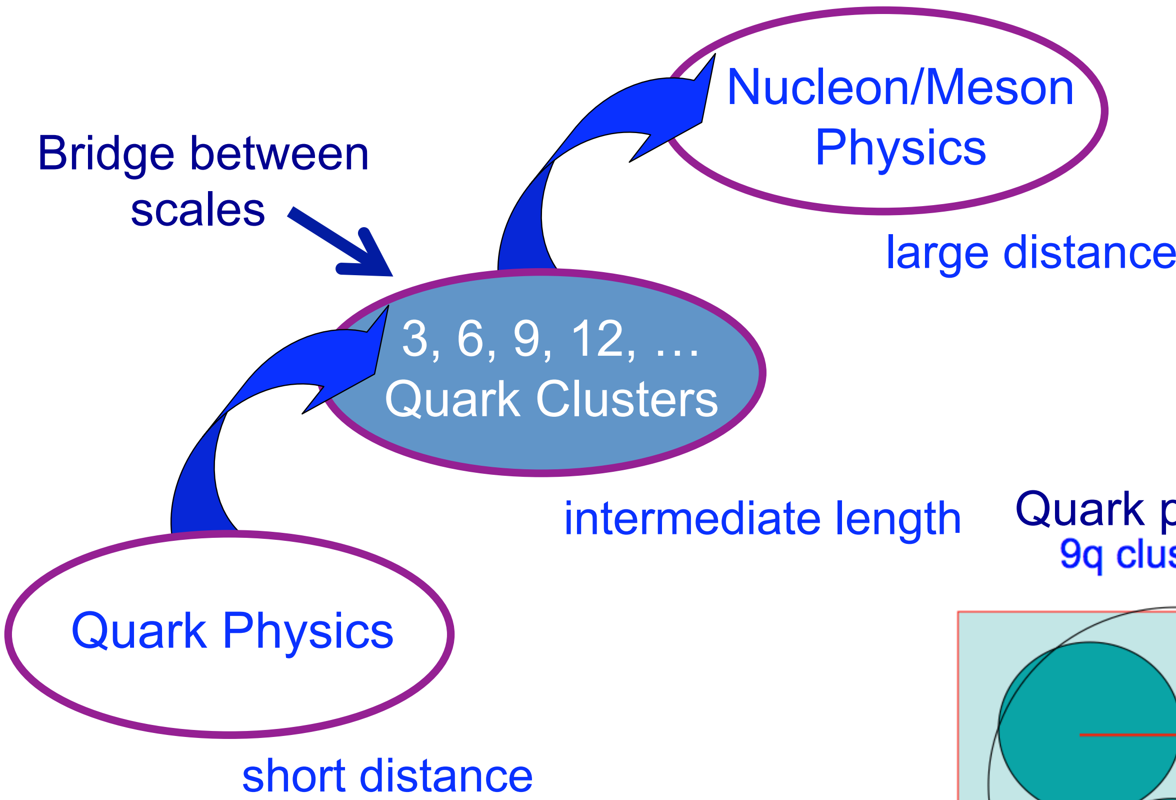
James P. Vary

With Xingbo Zhao, Anton Ilderton,
Heli Honkanen, Pieter Maris,
Stan J. Brodsky

Department of Physics and Astronomy
Iowa State University
Ames, USA

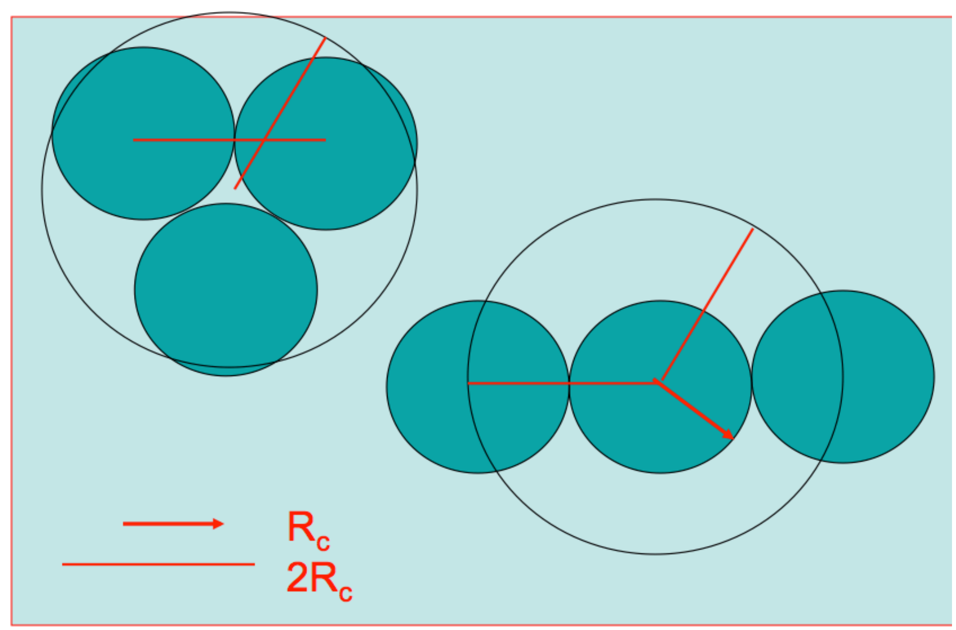
INT Program 12-3 Workshop, October 8-12, 2012

Under what conditions do we require a quark-based description on nuclear structure?



Probes with Q > 1 GeV/c
Spin content of the proton
Nuclear form factors at high Q
DIS on nuclei: EMC, Bjorken x > 1
Nuclear EOS at high T and/or ρ

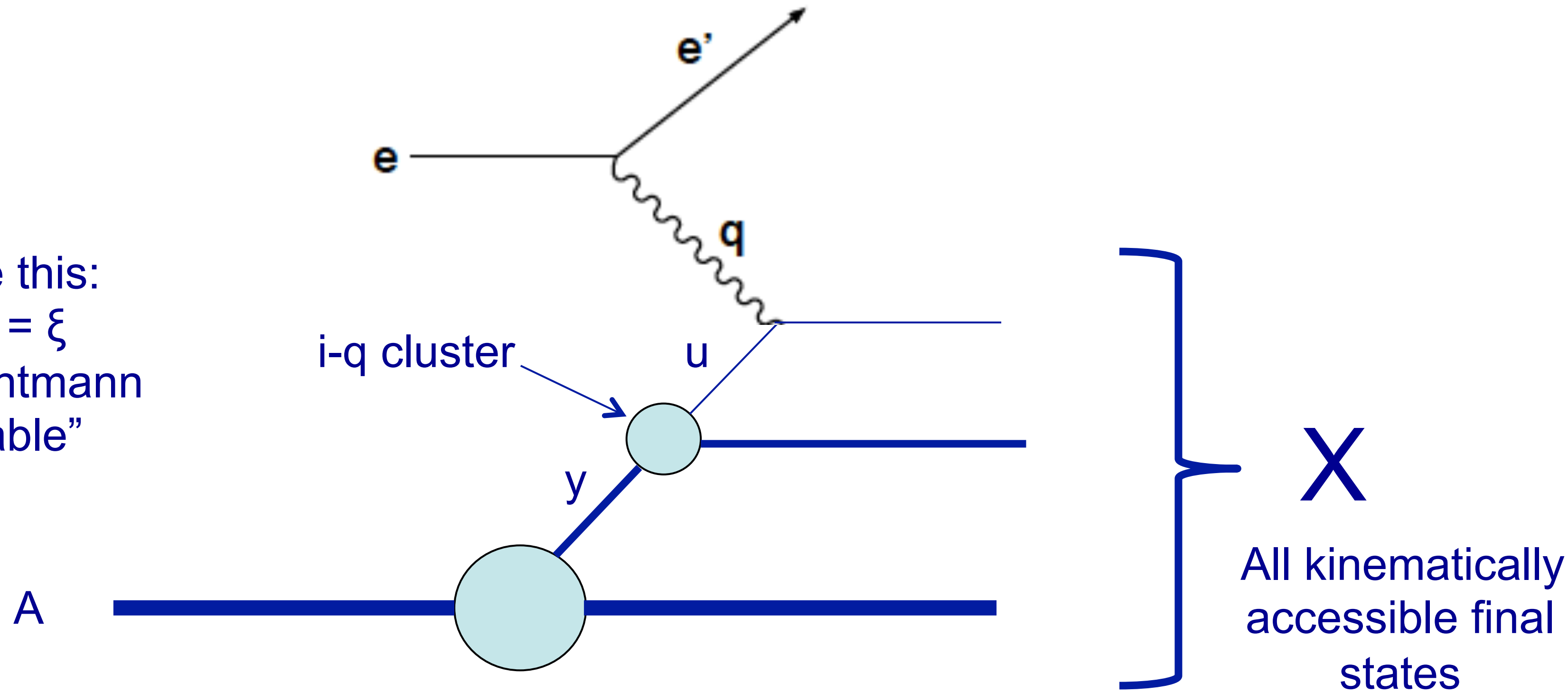
Quark percolation in cold and hot nuclei
9q cluster at geometrical limits of formation



H.J. Pirner and J.P. Vary,
Phys. Rev. Lett. **46**, 1376 (1981);
Phys. Rev. C. **84**, 015201(2011);
arXiv: nucl-th/1008.4962;

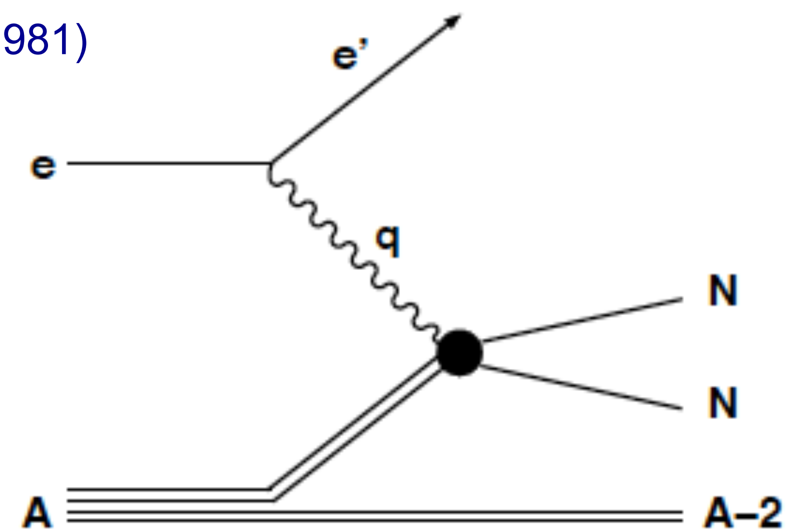
DIS from nuclei at high Q in the Quark Cluster Model

Like this:
uy = ξ
"Nachtmann variable"



H.J. Pirner and J.P. Vary,
Phys. Rev. Lett. **46**, 1376 (1981)

Not like this:



DIS in the quark cluster model
Convolution model based on ab initio structure (assumes scale separation)

$$\frac{\nu}{\sigma_M}\frac{d^2\sigma}{d\Omega dE'} = \nu W_2\left(\nu,Q^2\right) + \nu W_1\left(\nu,Q^2\right)\tan^2\left(\theta/2\right)$$

$$\nu W_2\left(\nu,Q^2\right) = \nu W_2^{in}\left(\nu,Q^2\right) + \nu W_2^{q-el}\left(\nu,Q^2\right)$$

$$\nu W_2^{in}\left(\nu,Q^2\right) = \sum_{quarks-j} e_j^2 \xi \mathrm{P}\left(\xi\right)$$

$$\mathrm{P}\left(\xi\right) = \sum_{clusters-i} p_i \bar{P}_i\left(\xi\right)$$

$$\bar{P}_i\left(\xi\right) = \int_0^{\xi_{i/A}^{th}} dy \int_0^{\xi_{q/i}^{th}} du\ \bar{n}_{q/i}\left(u\right) N_{i/A}\left(y\right)\delta\left(uy-\xi\right)$$

Nachtmann variable:

$$\xi_{i/A}^{th} = \left\{\left(1+\frac{m_i^2}{M^2}\frac{Q^2}{\nu^2}\right)^{1/2}+1\right\} \Bigg/ \left\{\left(1+\frac{Q^2}{\nu^2}\right)^{1/2}+1\right\}$$

$$\xi_{q/i}^{th} = 2 \Bigg/ \left\{\left(1+\frac{4m_i^2}{Q^2}\right)^{1/2}+1\right\}$$

$\bar{n}_{q/i}$ from Regge behavior and counting rules (phase space)

$N_{i/A}$ from non-relativistic wave functions (NRWFs)

p_i quark cluster probabilities evaluated from NRWFs

based on critical separation of $2R_c \sim 1\,fm$

Ab initio NRWF inputs

H.J. Pirner and J.P. Vary,
Phys. Rev. Lett. **46**, 1376 (1981)

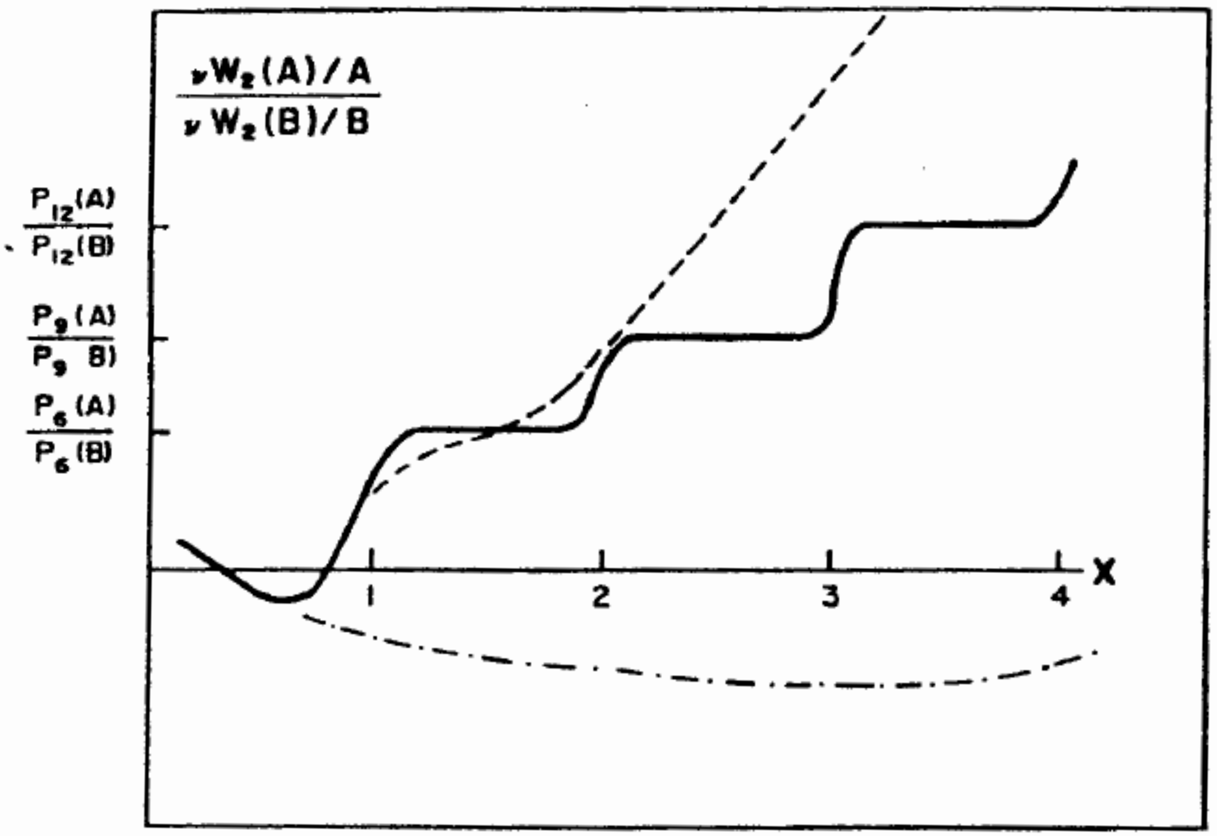
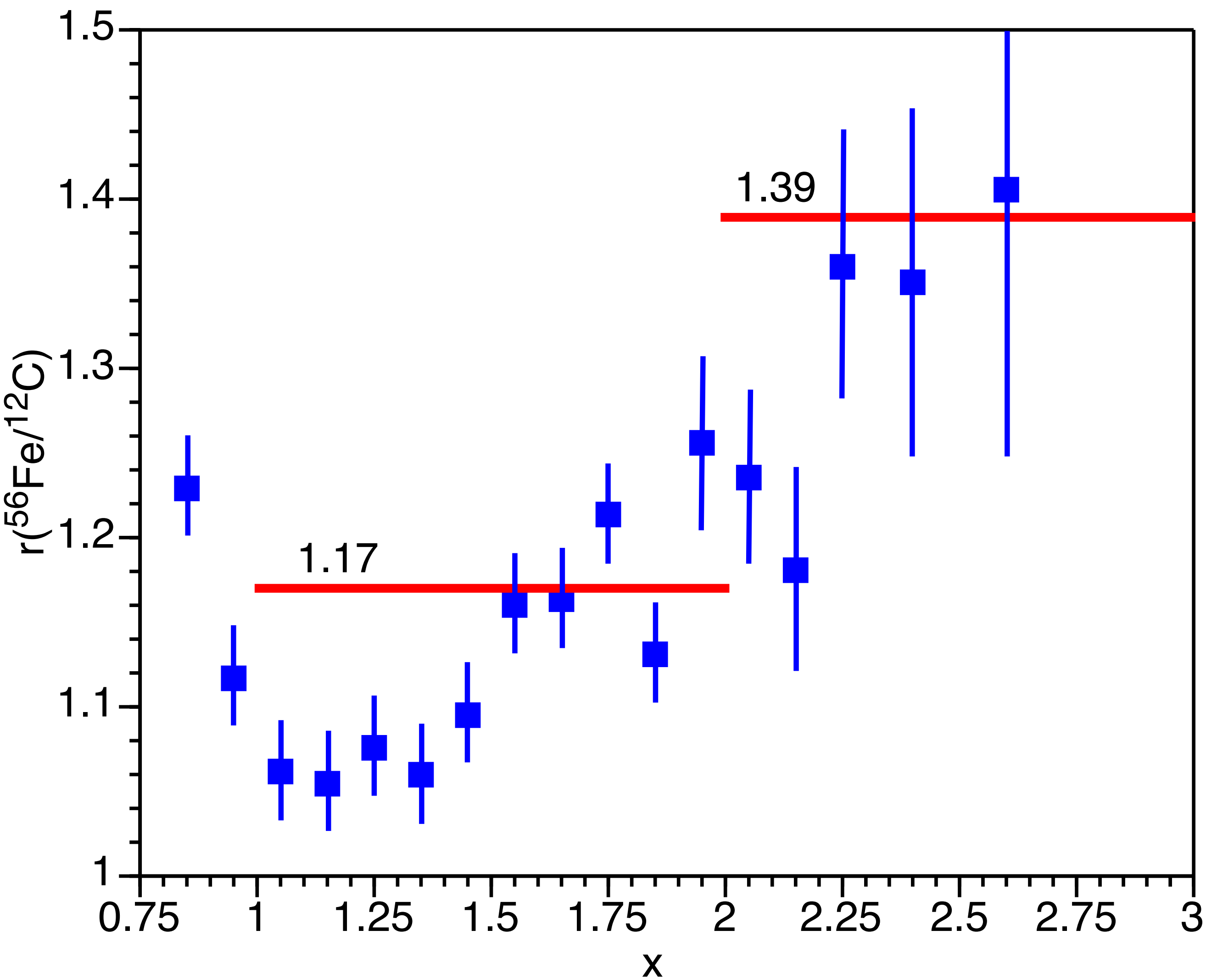


Fig. 2. Characteristic behaviour of the ratio of nuclear structure functions per nucleon for different models over a wide kinematic range of x. The QCM gives the solid curve. The dashed curve is due to the model of reference 22. The dashed-dot curve approximates the predictions of references 23 and 24.

J.P. Vary, Proc. VII Int'l Seminar on High Energy Physics Problems, "Quark Cluster Model of Nuclei and Lepton Scattering Results," Multiquark Interactions and Quantum Chromodynamics, V.V. Burov, Ed., Dubna #D-1, 2-84-599 (1984) 186 [staircase function for x > 1]

See also: Proceedings of HUGS at CEBAF1992, & many conf. proceedings

Comparison between Quark-Cluster Model and JLAB data



Data: K.S. Egiyan, et al., Phys. Rev. Lett. **96**, 082501 (2006)

Theory: H.J. Pirner and J.P. Vary, Phys. Rev. Lett. **46**, 1376 (1981) and Phys. Rev. C **84**, 015201 (2011); nucl-th/1008.4962;
M. Sato, S.A. Coon, H.J. Pirner and J.P. Vary, Phys. Rev. C **33**, 1062 (1986)

New Measurements of High-Momentum Nucleons and Short-Range Structures in Nuclei

N. Fomin,[1,2,3] J. Arrington,[4] R. Asaturyan,[5,*] F. Benmokhtar,[6] W. Boeglin,[7] P. Bosted,[8] A. Bruell,[8] M. H. S. Bukhari,[9] M. E. Christy,[8] E. Chudakov,[8] B. Clasie,[10] S. H. Connell,[11] M. M. Dalton,[3] A. Daniel,[9] D. B. Day,[3] D. Dutta,[12,13] R. Ent,[8] L. El Fassi,[4] H. Fenker,[8] B. W. Filippone,[14] K. Garrow,[15] D. Gaskell,[8] C. Hill,[3] R. J. Holt,[4] T. Horn,[6,8,16] M. K. Jones,[8] J. Jourdan,[17] N. Kalantarians,[9] C. E. Keppel,[8,18] D. Kiselev,[17] M. Kotulla,[17] R. Lindgren,[3] A. F. Lung,[8] S. Malace,[18] P. Markowitz,[7] P. McKee,[3] D. G. Meekins,[8] H. Mkrtchyan,[5] T. Navasardyan,[5] G. Niculescu,[19] A. K. Opper,[20] C. Perdrisat,[21] D. H. Potterveld,[4] V. Punjabi,[22] X. Qian,[13] P. E. Reimer,[4] J. Roche,[20,8] V. M. Rodriguez,[9] O. Rondon,[3] E. Schulte,[4] J. Seely,[10] E. Segbefia,[18] K. Slifer,[3] G. R. Smith,[8] P. Solvignon,[8] V. Tadevosyan,[5] S. Tajima,[3] L. Tang,[8,18] G. Testa,[17] R. Trojer,[17] V. Tvaskis,[18] W. F. Vulcan,[8] C. Wasko,[3] F. R. Wesselmann,[22] S. A. Wood,[8] J. Wright,[3] and X. Zheng[3,4]

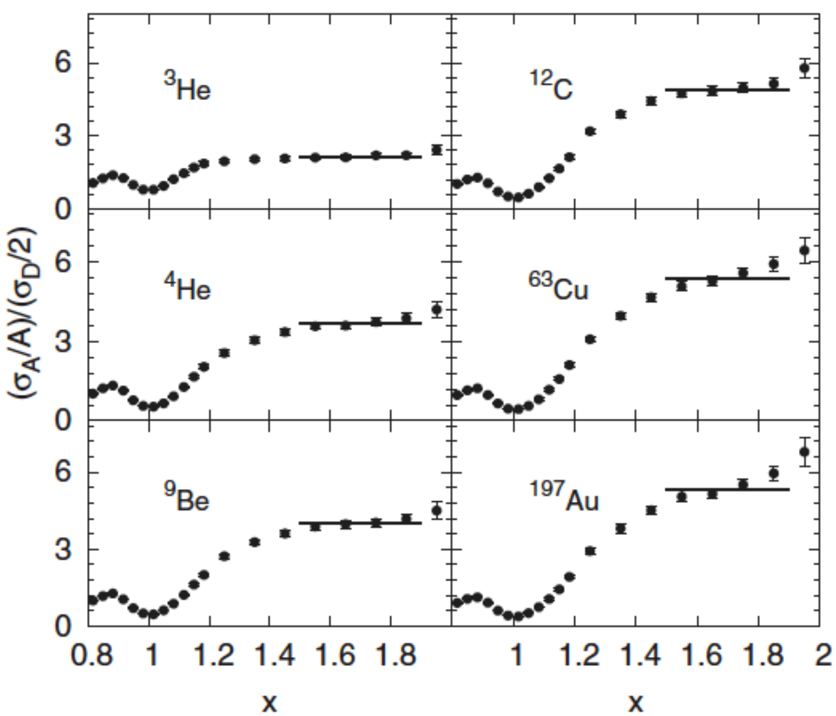


FIG. 2. Pernucleon cross section ratios vs x at $\theta_e = 18°$.

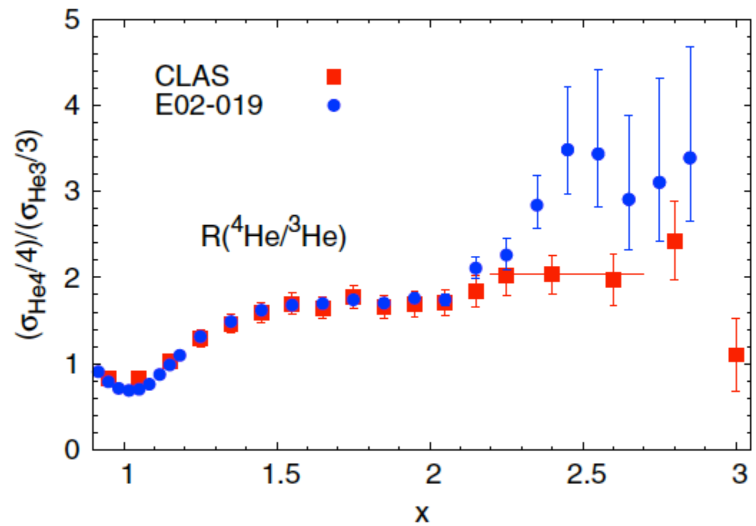


FIG. 3 (color online). The ^{4}He/^{3}He ratios from E02-019 ($Q^2 \approx 2.9$ GeV^2) and CLAS ($\langle Q^2 \rangle \approx 1.6$ GeV^2); errors are combined statistical and systematic uncertainties. For $x > 2.2$, the uncertainties in the ^{3}He cross section are large enough that a one-sigma variation of these results yields an asymmetric error band in the ratio. The error bars shown for this region represent the central 68% confidence level region.


JOINT INT/JLAB WORKSHOP ON
Nuclear Structure & Dynamics
at SHORT DISTANCES
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SEATTLE, WA
FEBRUARY 11-22, 2013
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SHORT-RANGE NUCLEON-NUCLEON CORRELATIONS
DEEP-INELASTIC LEPTON NUCLEUS SCATTERING
AB-INITIO AND MODEL NUCLEAR CALCULATIONS
ORGANIZING COMMITTEE:
ROLF ENT, JLAB
GERALD MILLER, U. WASHINGTON
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Beyond Model Building

- Central problems in hadron physics:
  - Structure of hadron -> Parton distribution?
  - Spin structure of hadron -> Where does proton spin come from?

- These problems involve the non-perturbative aspects of QCD

  ⇨ not well understood so far

- Lattice QCD set up in imaginary time

  ⇨ limited ability in extracting hadron structure

- A reliable non-perturbative approach in real time needed.
- Basis Light-Front Quantization (BLFQ) approach!
  - Solve quantum field theory in the Hamiltonian framework

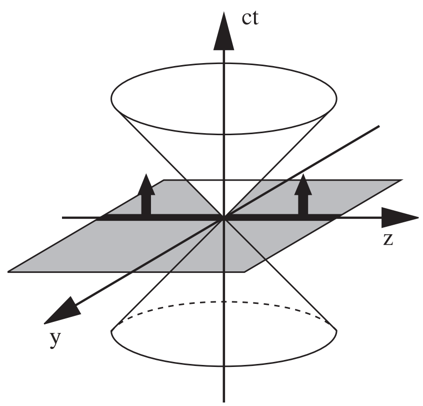
Basis Light-Front Quantization Approach

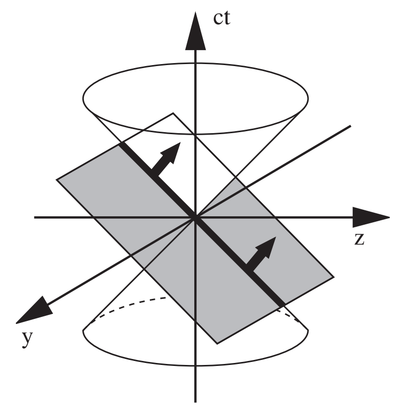
[Dirac 1949]

- Basic idea: solve generalized wave eq. for quantum field evolution

equal time quantization	vs	light front quantization
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$		$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = P_+\lvert\varphi(x^+)\rangle$

Boost-Invariant





- Time: $t \equiv x^0$ $\quad$ $t \equiv x^+ = x^0 + x^3$
- Hamiltonian: $H \equiv P^0$ $\quad$ $H \equiv P_+ = \frac{P^0 - P^3}{2}$
- On-shell condition: $P^0 = \sqrt{m^2 + P_\perp^2 + P_3^2}$ $\quad$ $P_+ = \frac{m^2 + P_\perp^2}{2P^+}$

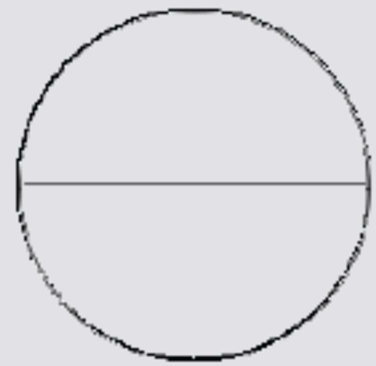
- p^+, conjugate to x^-, is the longitudinal momentum
- p^-, conjugate to x^+, is the light-front energy

$$p^{\pm} \equiv p^0 \pm p^3$$

$$p^{\pm} \geq 0$$

- Dispersion Relation:

$$p^- = \frac{\left(p^{\perp}\right)^2 + m^2}{p^+} \quad \Rightarrow \hat{\mathbf{H}} \propto P^-$$

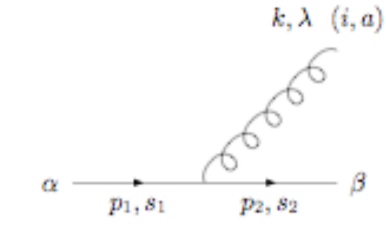


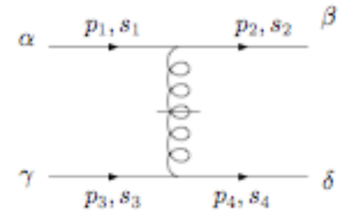
- Neglect "zero modes"

$$p^+ \neq 0 \Rightarrow p^+ > 0$$

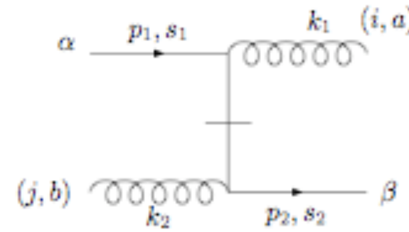
- Vacuum bubble kinematically forbidden!
  - Momentum conservation requires: $\sum_i p_i^+ = 0$
- Light Front vacuum is trivial if zero modes are excluded
  - Fock vacuum $|0\rangle$

Elementary vertices in LF gauge

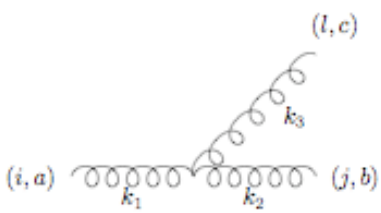


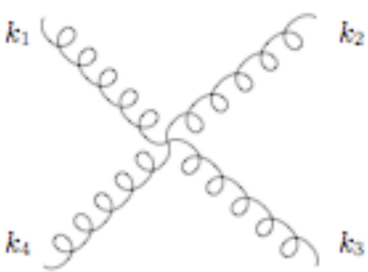


H_{qqgg1}

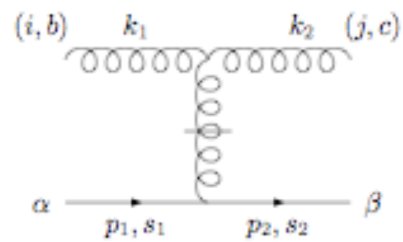


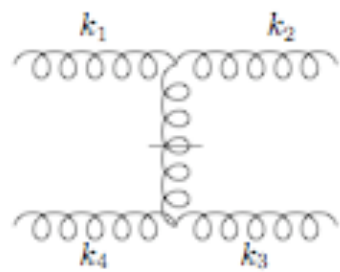
QED & QCD





H_{qqgg2}





QCD

$$
\begin{aligned}
H = &\frac{1}{2}\int d^3x \overline{\widetilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\widetilde{\psi} - A_a^i(\mathrm{i}\partial^\perp)^2 A_{ia} \\
&- \frac{1}{2}g^2\int d^3x \mathbf{Tr}\left[\widetilde{A}^\mu, \widetilde{A}^\nu\right]\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right] \\
&+ \frac{1}{2}g^2\int d^3x \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi}\frac{1}{(\mathrm{i}\partial^+)^2}\overline{\widetilde{\psi}}\gamma^+ T^a\widetilde{\psi} \\
&- g^2\int d^3x \overline{\widetilde{\psi}}\gamma^+\left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right)\widetilde{\psi} \\
&+ g^2\int d^3x \mathbf{Tr}\left(\left[\mathrm{i}\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right) \\
&+ \frac{1}{2}g^2\int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\frac{\gamma^+}{\mathrm{i}\partial^+}\widetilde{\not{A}}\widetilde{\psi} \\
&+ g\int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\widetilde{\psi} \\
&+ 2g\int d^3x \mathbf{Tr}\left(\mathrm{i}\partial^\mu\widetilde{A}^\nu\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right]\right)
\end{aligned}
$$

Discretized Light Cone Quantization (c1985)

⇩

Basis Light Front Quantization*

$$\phi(\vec{x}) = \sum_{\alpha} \left[f_\alpha(\vec{x}) a_\alpha^+ + f_\alpha^*(\vec{x}) a_\alpha \right]$$

where $\{a_\alpha\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_\alpha(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $\int f_\alpha(\vec{x}) f_{\alpha'}^*(\vec{x}) d^3x = \delta_{\alpha\alpha'}$

Complete: $\sum_\alpha f_\alpha(\vec{x}) f_\alpha^*(\vec{x}') = \delta^3(\vec{x} - \vec{x}')$

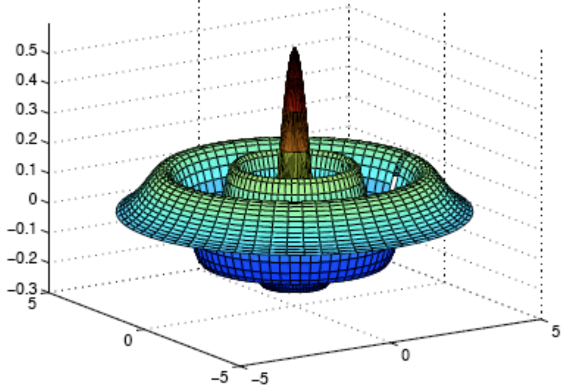
=> Wide range of choices for $f_a(\vec{x})$ and our initial choice is

$$f_\alpha(\vec{x}) = N e^{ik^+x^-} \Psi_{n,m}(\rho,\varphi) = N e^{ik^+x^-} f_{n,m}(\rho) \chi_m(\varphi)$$

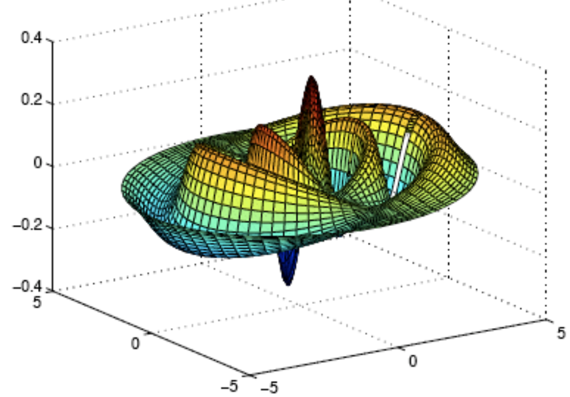
*J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010). ArXiv:0905:1411

Set of transverse 2D HO modes for n=4

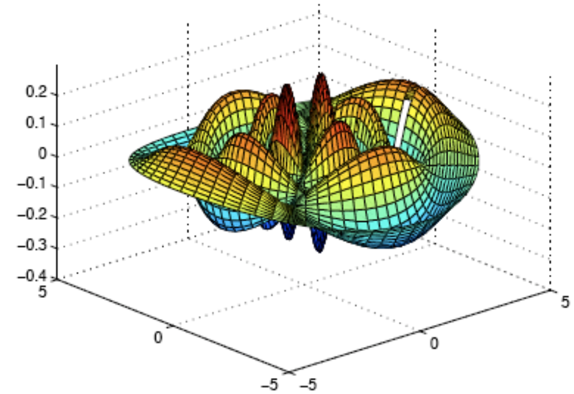
m=0



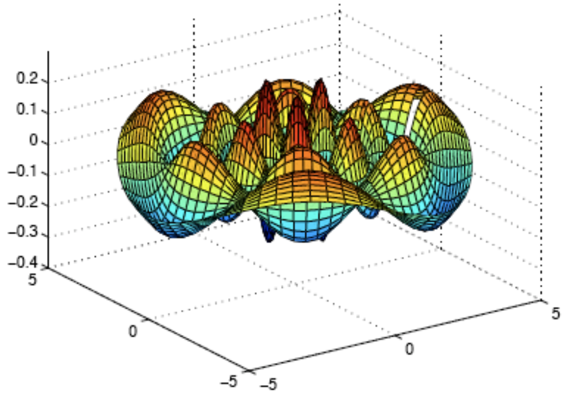
m=1



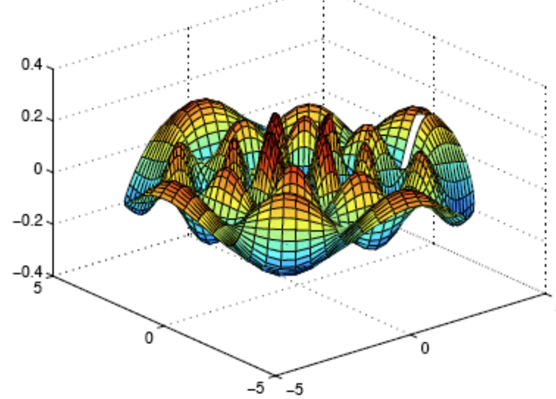
m=2



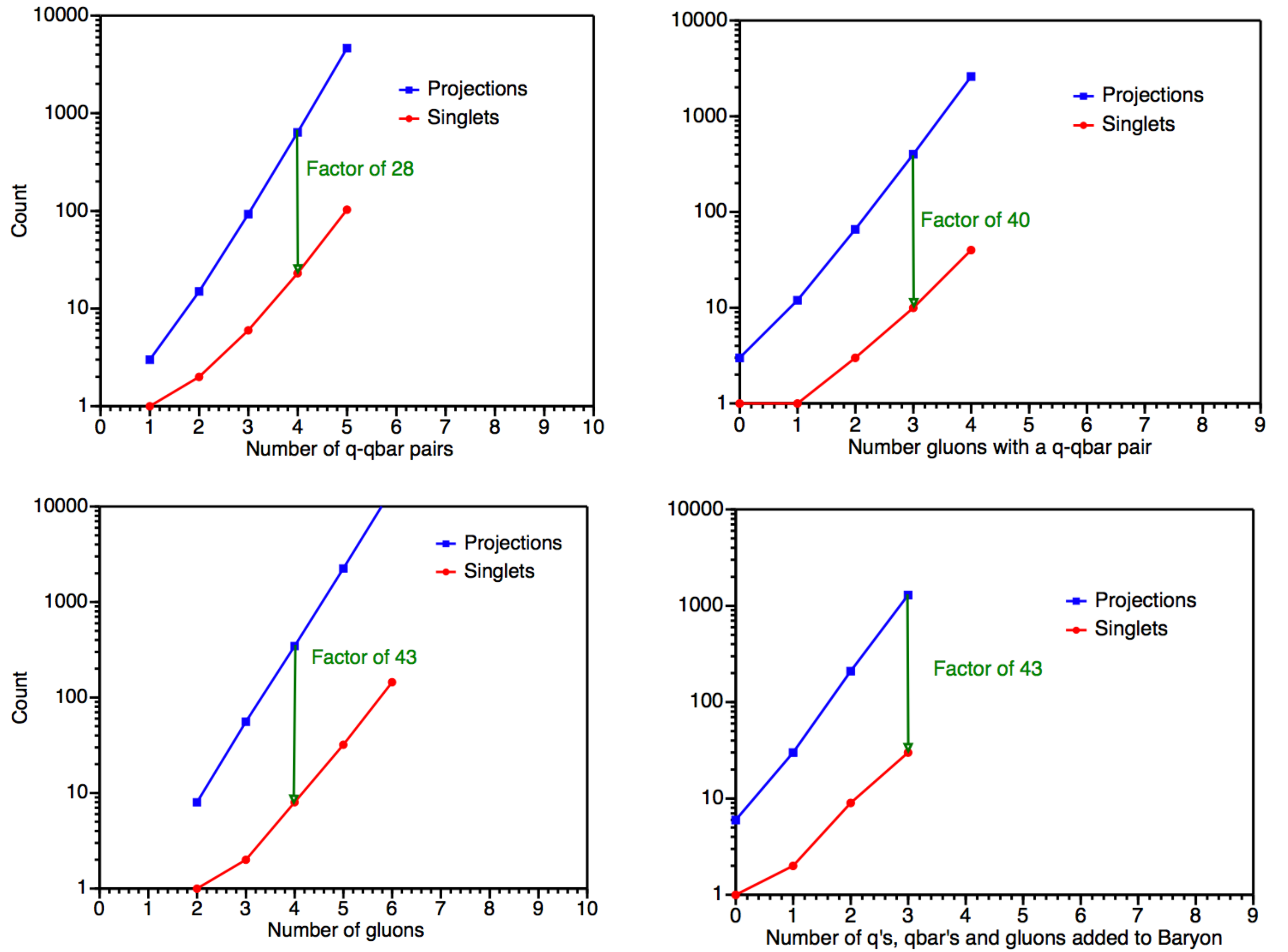
m=3



m=4



J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath,
G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010).
ArXiv:0905:1411



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G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010). ArXiv:0905:1411

Steps to implement BLFQ

- Enumerate Fock-space basis subject to symmetry constraints
- Evaluate/renormalize/store H in that basis (it is very sparse!)
- Diagonalize (Lanczos)
- Iterate previous two steps for sector-dep. renormalization
- Evaluate observables using eigenvectors (LF amplitudes)
- Repeat previous 4 steps for new regulator(s)
- Extrapolate to infinite matrix limit – remove all regulators
- Compare with experiment or predict new experimental results

Above achieved for QED test case – electron in a trap
H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky,
Phys. Rev. Lett. 106, 061603 (2011)

Improvements: trap independence, (m,e) renormalization, . . .
X. Zhao, H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky, in prep'n

Symmetries & Constraints

$$\sum_i b_i = B$$

$$\sum_i (m_i + s_i) = J_z$$

$$\sum_i k_i = K$$

$$\sum_i [2n_i + |m_i| + 1] \le N_{\max}$$

Global Color Singlets (QCD)

Light Front Gauge

Optional - Fock space cutoffs

Finite basis regulators

Regularization and Renormalization Schemes

1. Basis space regulators (2-D HO params, K)
2. Additional Fock space truncations (if any)
3. Counterterms identified/tested*
4. Sector-dependent renormalization**
5. SRG, OLS, . . . Adapted to BLFQ

*D. Chakrabarti, A. Harindranath and J.P. Vary,
"A Study of q-qbar States in Transverse Lattice QCD
Using Alternative Fermion Formulations,"
Phys. Rev. D **69**, 034502 (2004); hep-ph/0309317

**V. A. Karmanov, J.-F. Mathiot, and A. V. Smirnov,
Phys. Rev. D **77**, 085028 (2008);
and new paper - arXiv:1204.3257

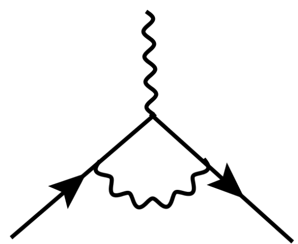
Evaluate Electron g-2 with BLFQ Approach

- Electron anomalous magnetic moment

$$a_e \equiv \frac{g-2}{2}$$

- Leading contribution to a_e is from QED [Schwinger 1948]

$$a_e = \frac{\alpha}{2\pi} \quad \left(\alpha = \frac{1}{137}\right)$$

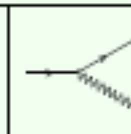
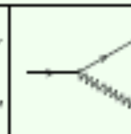
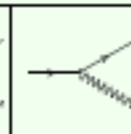
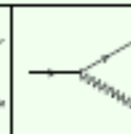
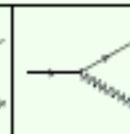
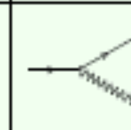
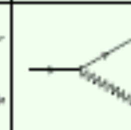
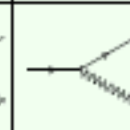
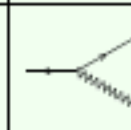
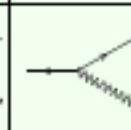
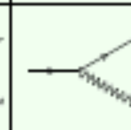
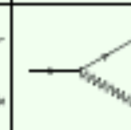
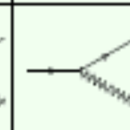
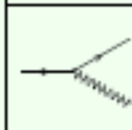
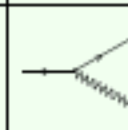
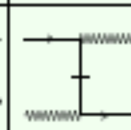
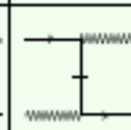
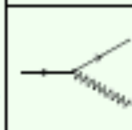
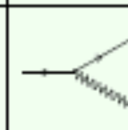
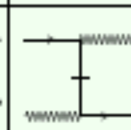
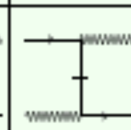
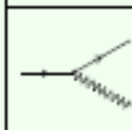
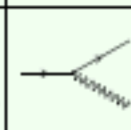
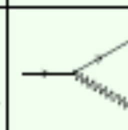
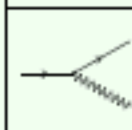
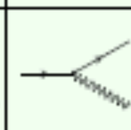
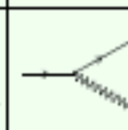
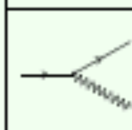
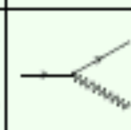
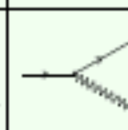


- a_e is electron Pauli form factor at zero-moment transfer limit:

$$a_e = F_2(q^2 \to 0)$$

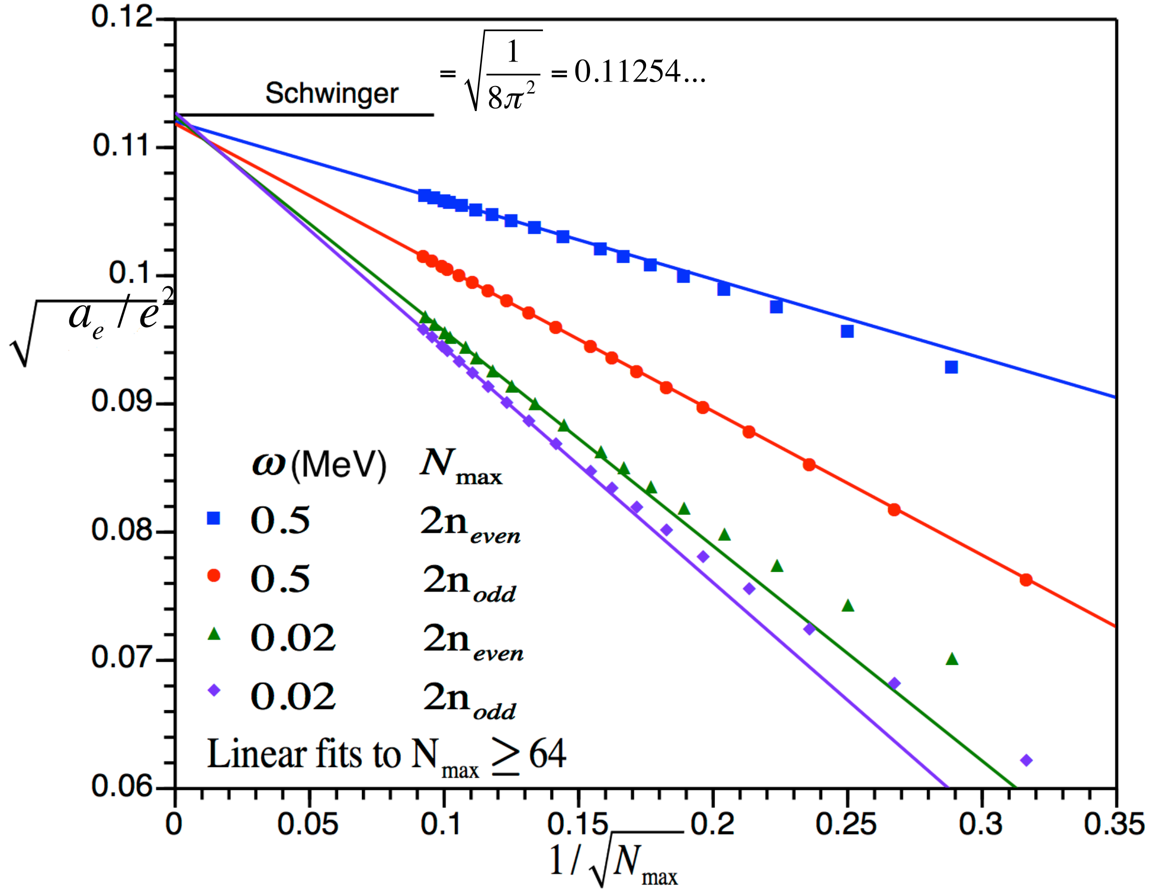
- In BLFQ, $a_e = \left\langle e_{physical} \middle| \hat{F}_2(q^2 \to 0) \middle| e_{physical} \right\rangle$

Matrix Example: $N_{max} = 3, K = 2$

e: ↑, n=m=0	0	0	0					
e: ↓, n=0, m=1	0	0	0	0	0			
e: ↑, n=1, m=0	0	0	0					
e: ↑, n=0, m=-1 γ: ↑, n=m=0		0				0	0	0
e: ↑, n=m=0 γ: ↑, n=0, m=-1		0				0	0	0
e: ↓, n=m=0 γ: ↑, n=m=0				0	0	0	0	0
e: ↑, n=m=0 γ: ↓, n=0, m=1				0	0	0	0	0
e: ↑, n=0, m=1 γ: ↓, n=m=0				0	0	0	0	0

Numerical Results for Electron g-2

Major update to: H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky, Phys. Rev. Lett. 106, 061603 (2011)



- As Nmax → ∞, results approach Schwinger result
- Less than 1% deviation from Schwinger's result (by linear extrapl.)
- Convergence over wide range of ω's (by a factor of 25!)

X. Zhao, H. Honkanen, J.P. Vary, P. Maris and S.J. Brodsky, in preparation

* * * Featured next * * *

Preliminary investigations of ISU PhD students:

Paul Wiecki

Yang Li

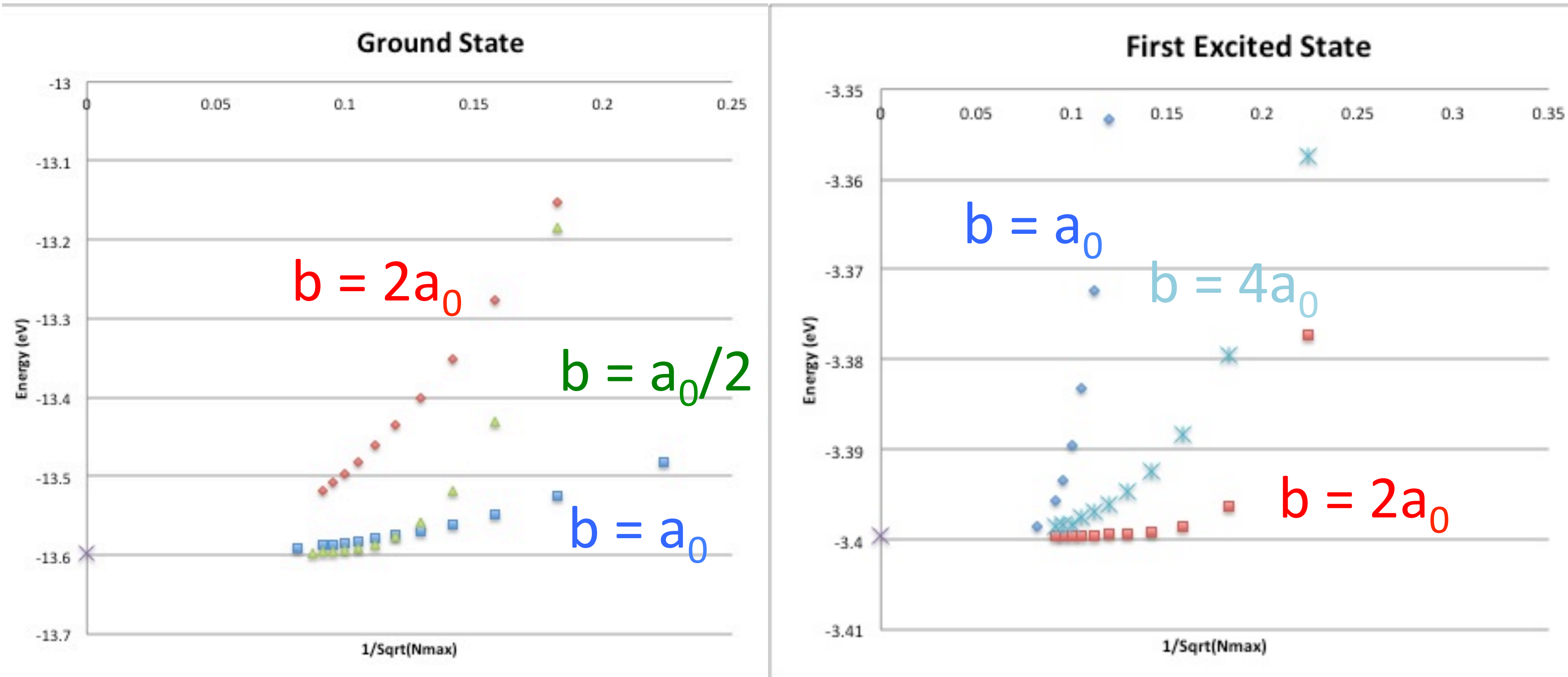
Mentoring Team:

JPV

Xingbo Zhao

Pieter Maris

Toy problem – Hydrogen atom in 3D HO Basis



Exact result marked by “X” on the vertical scale

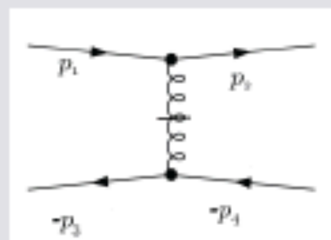
Convergence is fastest when HO length (b) is on the scale of the bound state WF

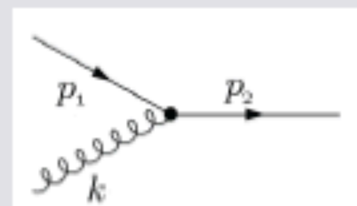
Sample convergence study of quarkonia spectra

- **Fock Sectors**

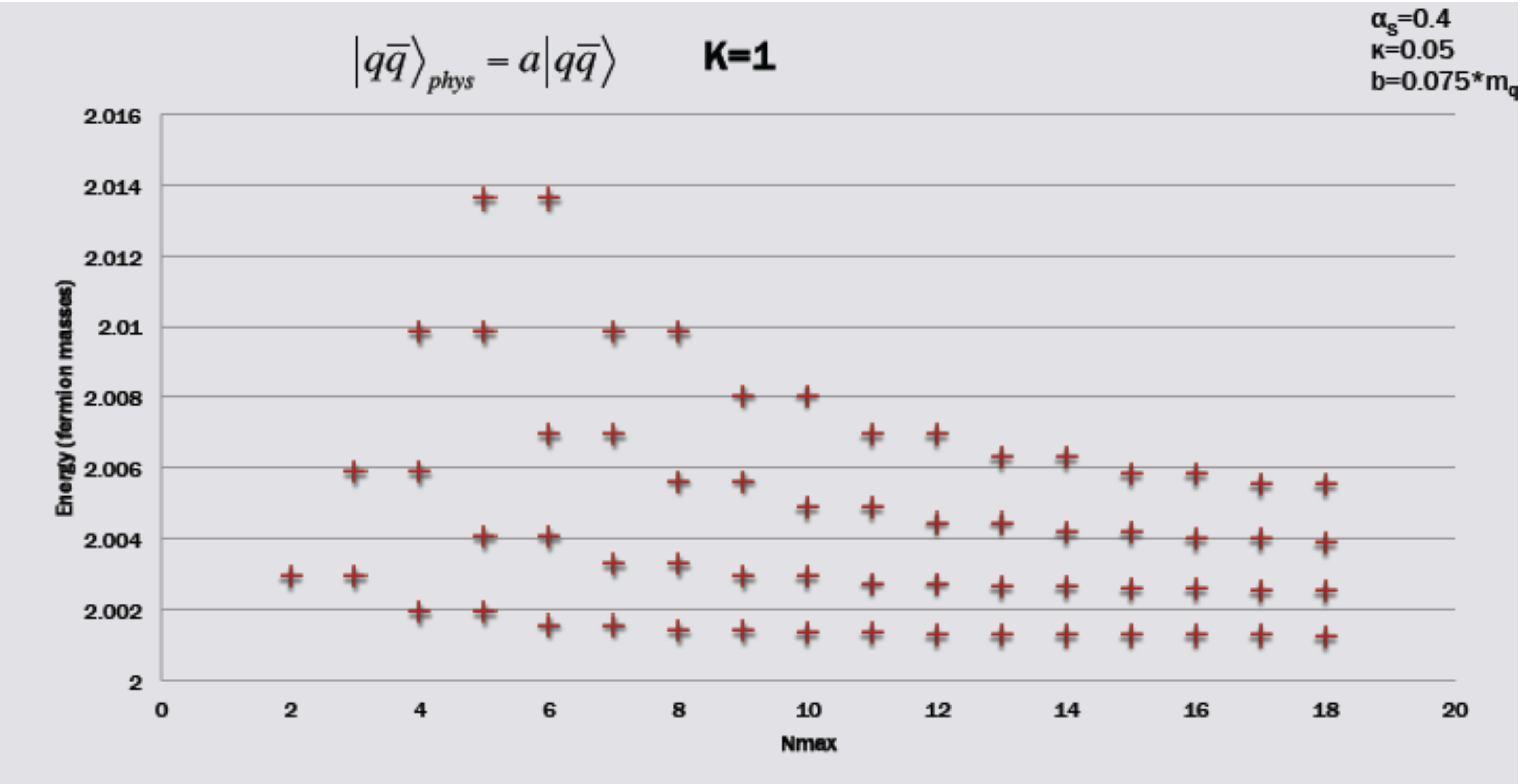
$$\left|q\bar{q}\right\rangle_{phys} = a\left|q\bar{q}\right\rangle + b\left|q\bar{q}g\right\rangle + \cdots$$

- **Light Front Hamiltonian Interactions**
  - **Instantaneous Exchange**
  - **“Vertex”**





- **Phenomenological Confinement** $V = \kappa^4 x_1 x_2 \left(\mathbf{r}_1 - \mathbf{r}_2\right)^2$



$|q\bar{q}\rangle_{phys} = a|q\bar{q}\rangle$
K=1
α_s=0.4
κ=0.05
b=0.075*m_q
Energy (fermion masses)
2.016
2.014
2.012
2.01
2.008
2.006
2.004
2.002
2
0
2
4
6
8
10
12
14
16
18
20
Nmax

Factorization of Center-of-Mass Motion

- In complete basis space, CM motion is factorizable if $\left[\hat{H}, \mathbf{P}_{\rm CM}\right] = 0$:

$$|\psi\rangle = |\psi_{\rm CM}\rangle \otimes |\psi_{\rm INT}\rangle$$

- In **finite truncated basis space**, this is not true anymore;
  - Can be checked by $\left\langle \psi_i | \hat{H}_{\rm CM} | \psi_i \right\rangle$ where $\hat{H}_{\rm CM} = \frac{1}{2Am}\mathbf{P}^2_{\rm CM} + \frac{1}{2}Am\omega^2\mathbf{R}^2_{\rm CM}$;
  - Spurious states emerge in the spectrum;
  - Diagonalize for much more eigenstates than interested in;
    - With Lanczos algorithm, it is much faster to only evaluate the first few eigenstates;
- In HO basis, with $N_{\rm max}$ truncation, the exact factorization holds;
  - $N_{\rm max}$ truncation is the total energy truncation:
    take all basis states $\sum_i 2n_i + |m_i| + 1 \leq N_{\rm max}$ (2D HO basis);
  - Introduce a Lawson term (Lagrange multiplier):

    $\hat{H}' = \hat{H} + \lambda(\hat{H}_{\rm CM} - \frac{d}{2}\hbar\omega)$, where λ is some large number.

    Lawson term effectively lifts states without vanishing CM motion.

Light-Front QFT Bound States

$$\hat{\mathcal{M}}^2 = \sum_i \frac{\mathbf{p}_i^2 + m_i^2}{x_i} - \mathbf{P}_{\mathrm{CM}}^2 + \sum_{i<j} \mathcal{V}_{ij} \qquad (2)$$

$\hat{\mathcal{M}}^2 = P^+_{\mathrm{CM}} \hat{P}^-$ invariant mass squared operator, $x_i = \frac{p_i^+}{P^+}$ longitudinal momentum fractions, $\mathbf{p}$ is a $(d-1)$-D transverse vector.

- Eigenvalue problem: $\hat{\mathcal{M}}^2 |\mathcal{M}\rangle = \mathcal{M}^2 |\mathcal{M}\rangle$.
- Basis Light-Front QFT (BLFQ) uses CI method in LFQFT;
- Transverse HO basis + N_{max} truncation does NOT provide exact factorization of CM motion in finite basis space;

Introduce new coordinates: $\mathbf{q} = \frac{\mathbf{p}}{\sqrt{x}}, \mathbf{s} = \sqrt{x}\mathbf{r}$ and a new basis:

$$|n, m, x\rangle = \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2} \phi_n^m(\mathbf{q}) |\mathbf{q}, x\rangle$$

where $\phi_n^m(\mathbf{q})$ is HO wavefunction in the new coordinates.

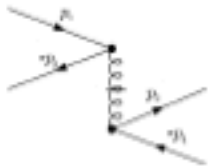
Properties of the New Basis

- Completeness and orthonormality:

$$\begin{aligned} \langle n, m, x | n', m', x' \rangle &= \delta_{n,n'}\delta_{m,m'}\delta_{x,x'} \\ \sum_{n,m,x} |n, m, x\rangle \langle n, m, x| &= \mathrm{Id}_3 \end{aligned} \tag{3}$$

- CM momentum $\mathbf{P}_{\mathrm{CM}} = \sum_i \sqrt{x_i}\mathbf{q}_i$;
- Exact factorization in finite basis space holds;
  - Can be checked by $\langle \psi_i | \hat{\mathcal{H}}_{\mathrm{CM}} | \psi_i \rangle$ where $\hat{\mathcal{H}}_{\mathrm{CM}} = \mathbf{P}^2_{\mathrm{CM}} + \omega^2 R_{CM}{}^2$
  - Lawson term: $\lambda \left(\hat{\mathcal{H}}_{\mathrm{CM}} - (d-1)\,\hbar\omega \right)$
- (generalized) Talmi-Moshinsky transform;
- Possible connection with AdS/QCD:
  - In 2-body case, $\mathbf{s}_{\mathrm{rel}} = \sqrt{x(1-x)}(\mathbf{r}_1 - \mathbf{r}_2)$ is equal to the impact paramter "ζ" in LF Holography, which is mapped to the fifth dimension coordinate z of AdS/QCD.

Factorization of Center-of-Mass Motion

$\mathcal{M}^2 = \mathcal{T} +$  $+ \kappa^4 s_{\text{rel}}^2 + \lambda(\mathcal{H}_{\text{CM}} - 2\hbar^2\omega^2)$, where

$m_e = 0.511\text{MeV}$, $\alpha = 0.3, \omega = 0.075 m_e$, $\kappa = 0.05 m_e$.

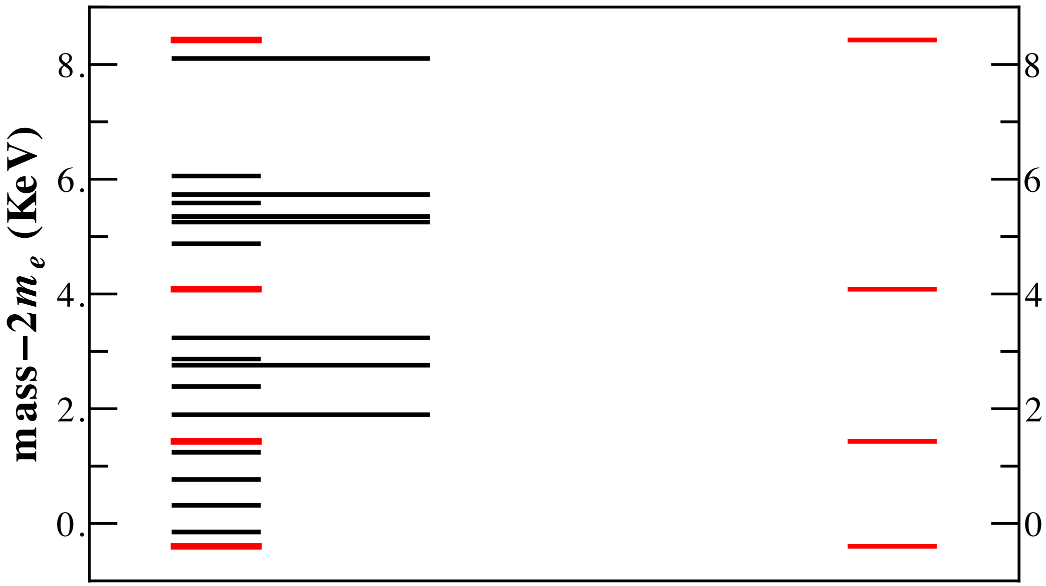
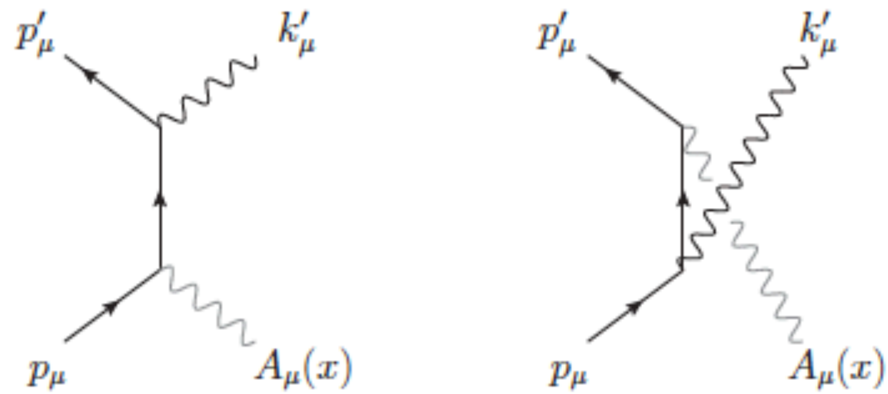


Figure: BLFQ postronium spectra, $N_{\text{max}} = 8, K = 1$. Left, $\lambda = 0$; Right $\lambda = 50$.

'Nonlinear Compton scattering'

- Simplest laser-particle scattering process.
- $e^- \to e^- + \gamma$ within a laser field. Reiss, Nikishov, Ritus, Kibble...
- 10^{20} photons in a laser: model as a background field.

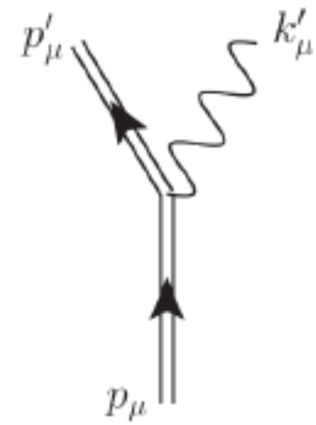
- Perturbation theory:
- Looks like ordinary Compton
- $\sigma \propto$ Klein-Nishina $\times \tilde{A}^2$



- But! High intensity $\Longrightarrow$
  background should be treated nonperturbatively.

Xingbo Zhao and Anton Ilderton lead this project

At high intensity



- Fermions become 'dressed' by the background.
-  $=$ exact propagator in background.
- Most analytic progress for plane waves. Volkov, 1935

  Harvey, Heinzl, Ilderton PRA 79 (2009) 063407

  Heinzl, Seipt, Kämpfer, PRA 81 (2010) 022125

$\ddot{\frown}$ Few analytic results for realistic background fields.
(Need solution of Dirac equation in background.)

$\ddot{\frown}$ Few results for $1 \to 3$, $2 \to 2 \ldots$ scattering. Ilderton, PRL 106 (2011) 020404
(Even plane wave calculations become very complex.)

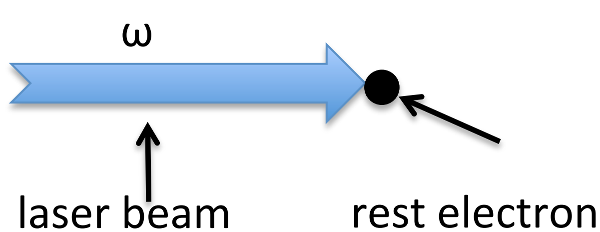
? A different approach needed ?

Electron in Strong Laser Field
Nonperturbative Approach

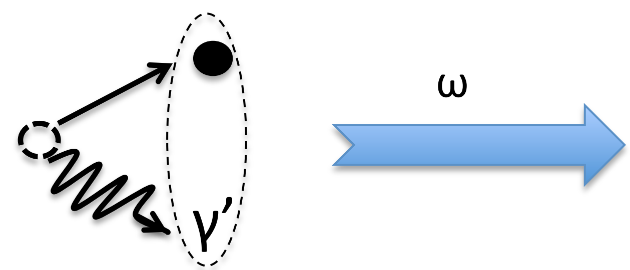
- Nonlinear Compton effect:
  - Electron absorbs multiple photons $n\gamma$ with frequency ω and emit a single photon γ' with frequency ω'

$$e + n\gamma \rightarrow e' + \gamma'$$

- Initial state:



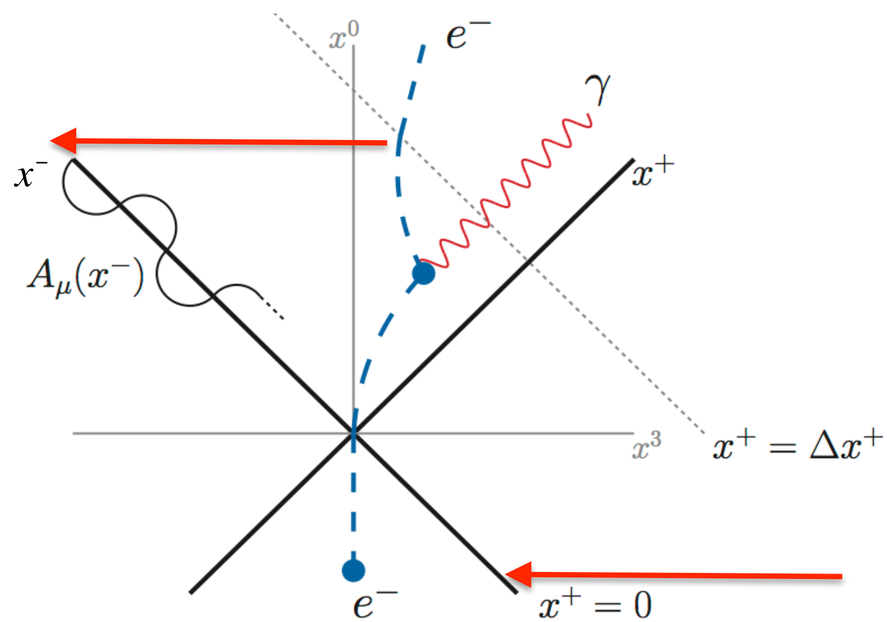
Final state:



- Important observable characterizing the energy transfer: invariant mass of the final electron and photon pair: "$\sqrt{s}$"

Electron in Strong Laser Field
Nonperturbative Approach

- Nonlinear Compton scattering: simplest laser-particle scattering
  - $e + n\gamma(laser) \rightarrow e' + \gamma'$
- Space-time structure



- Two effects: electron acceleration and photon emission

Electron in Strong Laser Field
Nonperturbative Approach

- Nonlinear Compton scattering: simplest laser-particle scattering
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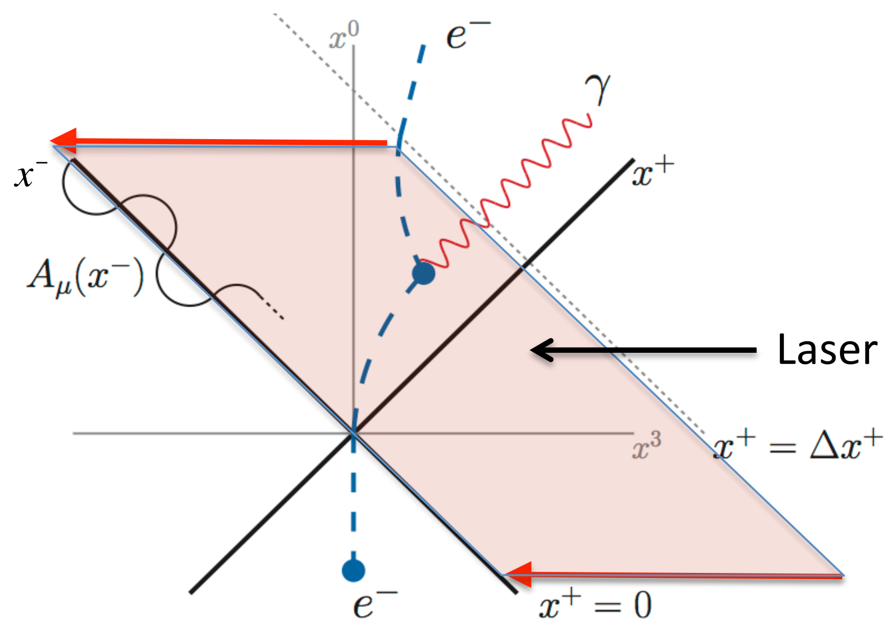


- Two effects: electron acceleration and photon emission

Nonlinear Compton Process in BLFQ

1. Write down the Hamiltonian P_+:

$$P_+ = P_+^{QED} + P_+^{Laser}$$

$$P_+ = P_+^{kinetic} + V^Q + V^L \qquad (P_+^{kinetic} \text{ for laser is absent})$$

2. Prepare initial state $|\Psi(0)\rangle$ out of eigenstates of QED Hamiltonian P_+^{QED}
   - e.g., a static electron is from solving $P_+^{QED}|\Psi(0)\rangle = m_e|\Psi(0)\rangle$
3. Apply the time evolution operator to $|\Psi(0)\rangle$ and obtain $|\Psi(x^+)\rangle$

$$|\Psi(x^+)\rangle = U(x^+,0)|\Psi(0)\rangle = T\exp\left(-i\int_0^{x^+} V_I^L(x^{+\prime})dx^{+\prime}\right)|\Psi(0)\rangle$$

$$\rightarrow (1-iV_I^L(x^+)\Delta x^+)\cdots(1-iV_I^L(x_2^+)\Delta x^+)(1-iV_I^L(x_1^+)\Delta x^+)|\Psi(0)\rangle$$

   - Work in interaction picture : $V_I^L(x^+) = e^{iP_+^{QED}x^+}V^L(x^+)e^{-iP_+^{QED}x^+}$
4. Extract observables: $O(x^+) = \langle\Psi(x^+)|\hat{O}|\Psi(x^+)\rangle$

Solving Nonlinear Compton Scattering in tBLFQ

1. Write down the Hamiltonian P_+:

$$P_+(x^+) = P_+^{QED} + V^{LAS}(x^+)$$

2. Solve $P_+^{QED}|\Phi_i\rangle = \tilde{P}_+^i|\Phi_i\rangle$ for the tBLFQ basis $|\Phi_i\rangle$

3. Prepare initial state $|\varphi(0)\rangle$
   - physical electron: the ground state of P_+^{QED} with n_f=1

4. Calculate matrix elements for V^{LAS}

$$\langle\Phi_j|V^{LAS}(x^+)|\Phi_i\rangle_I = e^{i(P_+^j - P_+^i)x^+}\langle\Phi_j|V^{LAS}(x^+)|\Phi_i\rangle$$

5 . Solve for the generalized wave-equation numerically

$$i\frac{\partial}{\partial x^+}\langle\Phi_i|\varphi(x^+)\rangle_I = \sum_j \langle\Phi_i|V^{LAS}|\Phi_j\rangle_I \langle\Phi_j|\varphi(x^+)\rangle_I$$

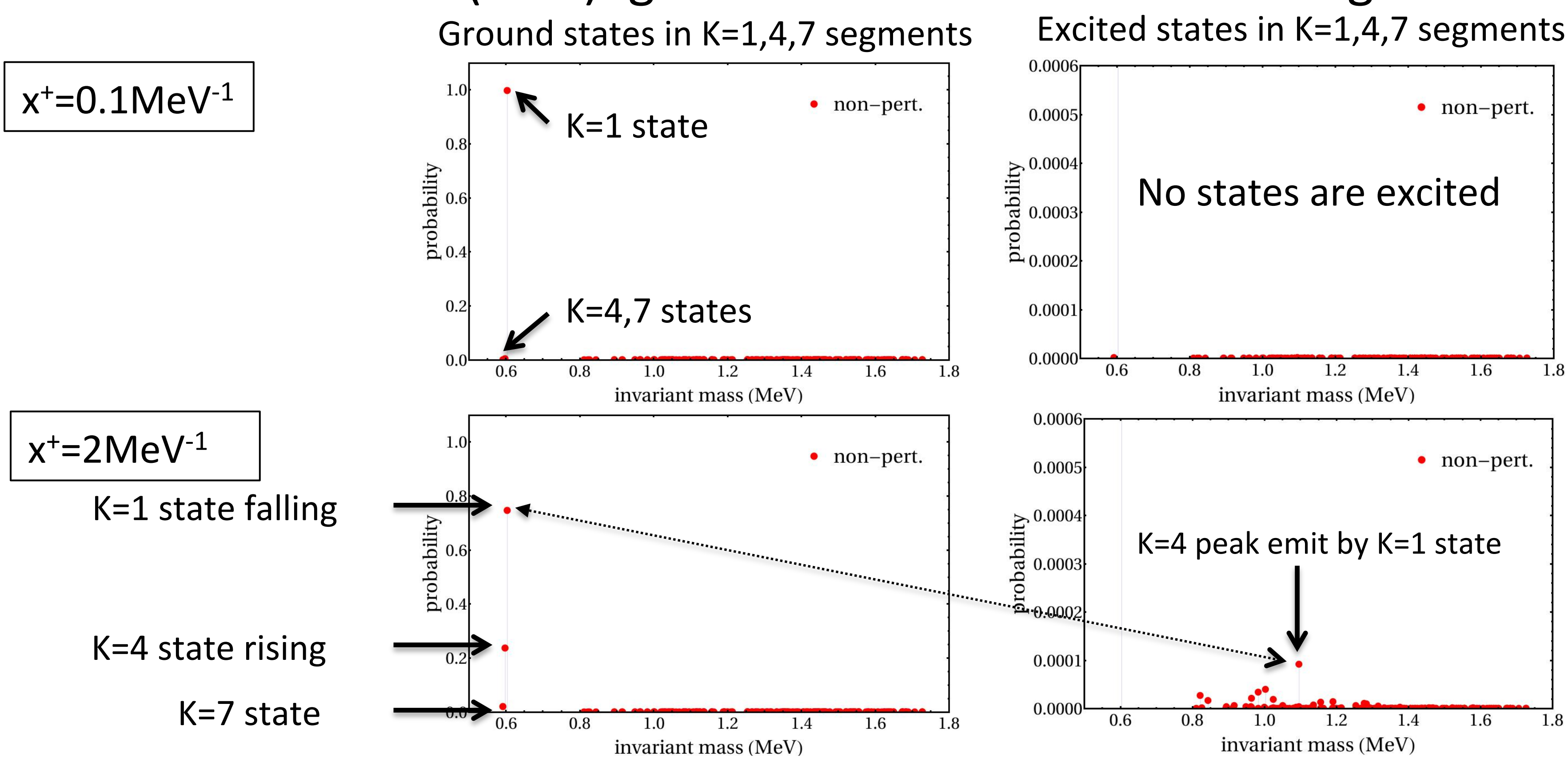
A Simple Laser Field Profile

$$V^{LAS} = e\bar{\psi}\gamma^{+}\psi\mathcal{A}_{+} \quad \text{with} \quad \mathcal{A}_{+}(x^{-}) = a_0 \exp\left(-\frac{i}{2}\omega x^{-}\right)$$

- Key properties:
  - $\mathcal{A}_{+}$ is treated classically
  - $\mathcal{A}_{+}$ is in lightcone gauge, $\mathcal{A}^{+} = 0$
  - $\mathcal{A}_{+}$ is uniform in $x^{1,2}$ and light=front time x^{+}
  - $\mathcal{A}_{+}$ depends on $x^{-} \longrightarrow \mathcal{F}^{+-}$: electric field in longitudinal (x^{-}) direction
  - a_0 describes the field strength
  - ω describes the laser field's spatial frequency in x^{-}

In Nonperturbative Regime: A Test Case

- Parameters: $\alpha = \frac{1}{137}, \quad a_0 = 0.5 m_e, \quad \omega = 3 MeV$
- Basis space: Nmax=6, K: 1+4+7 three segments
- Initial condition (x^+=0): ground state electron in K=1 segment



In Nonperterbative Regime: A Test Case

Ground states in K=1,4,7 segments

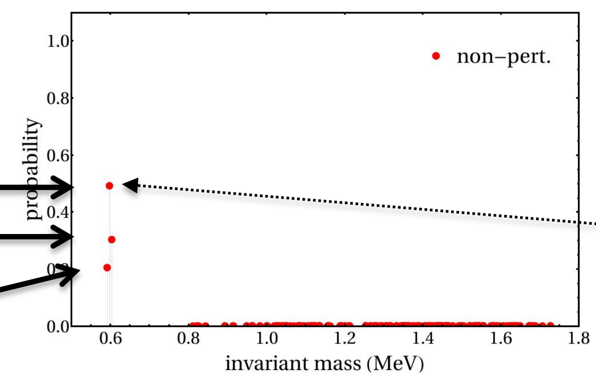
Excited states in K=1,4,7 segments

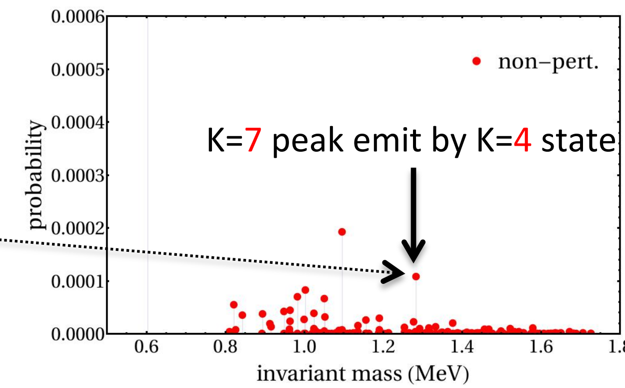
$x^+=4MeV^{-1}$

K=4 state rising

K=1 state falling

K=7 state rising



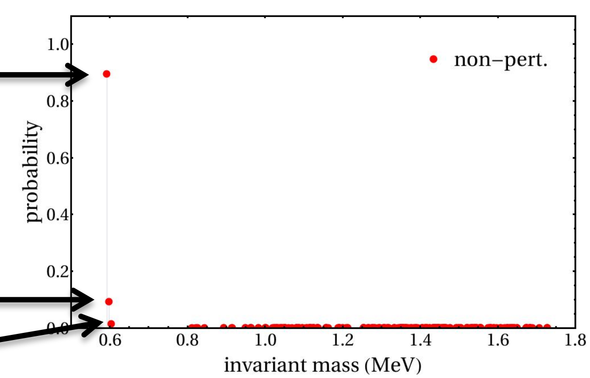


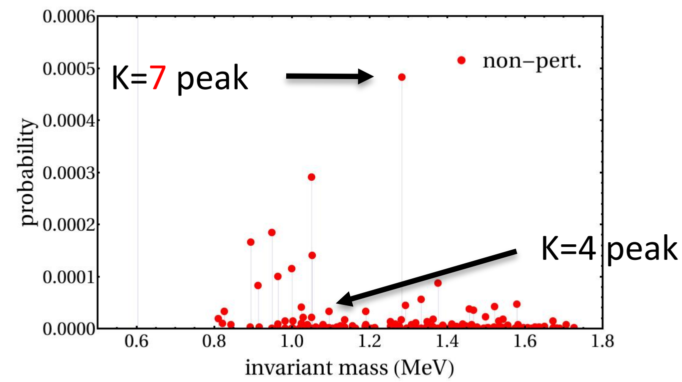
$x^+=7.5MeV^{-1}$

K=7 state rising

K=4 state falling

K=1 state



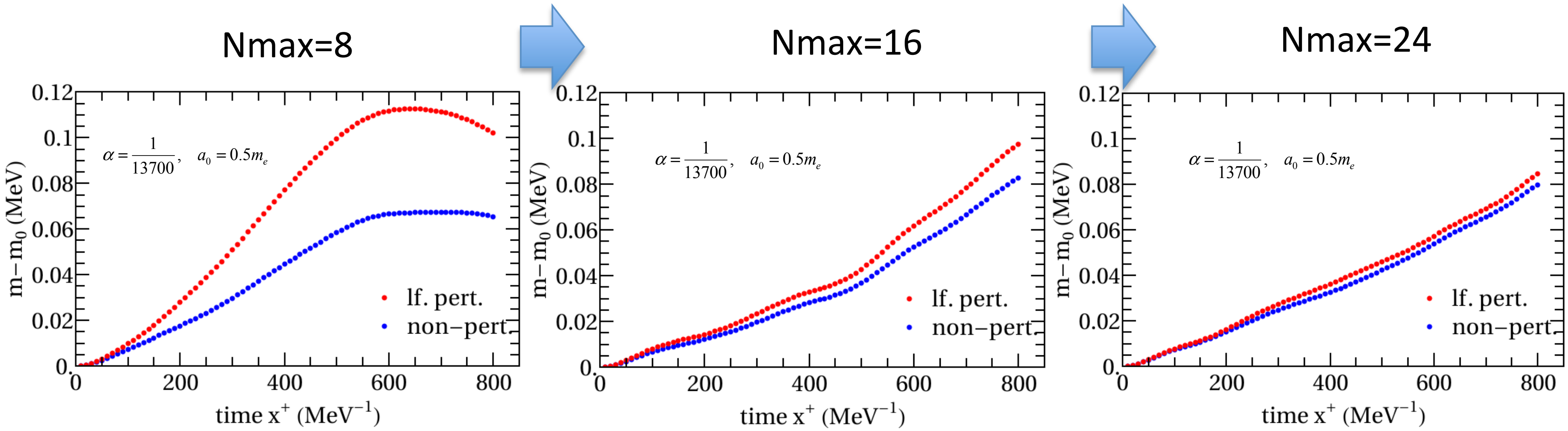


- Electron is accelerating and radiating
- Acceleration process is nonperturbative (initial state almost gone!)
- Faster electrons radiate eγ pair with larger invariant mass

Challenges and Solution

- Challenges
  - Covariant perturbation theory calculates S-matrix between in- and out-states with infinite evolution time in-between
  - Nontrivial transform between results in BLFQ basis and momentum basis (often used in perturbative calculation):
    1. Integration over HO wave function needed
    2. Different normalization for basis states,
       Kronecker delta (BLFQ basis) vs. Dirac delta (momentuem basis)
    3. Nmax truncation exclusive for BLFQ basis
- Solution -- Lightfront (LF) perturbation theory in BLFQ basis
  - Able to calculate transition amplitude per unit time
  - Allows for comparison with nonpert. calculation on the level of transition matrix element of the laser field $\langle \Psi' | V^L | \Psi \rangle$, where $| \Psi \rangle$ and $| \Psi' \rangle$ are eigenstates of P_+^{QED} (adopt the interaction picture)

Evolution of Invariant Mass of the System



- Invariant mass increases with time as laser field "pumps" energy in
- As Nmax increases better agreements are achieved between calculations based on laser matrix elements from LF. pert. and nonpert. methods, intermediate truncation effects are removed gradually in the nonperturbative case
- Quasi-linear dependence on x^+ is expected in the perturbative regime

Conclusions

We have entered an era of first principles, high precision, nuclear structure and nuclear reaction theory facilitated by leadership computational facilities and good resources

New insights into the UV and IR properties of finite basis results are emerging

Applications are underway to Light Front QCD and strong time-dependent QED

Pioneering collaborations between Physicists, Computer Scientists and Applied Mathematicians have become essential to progress