Parity-Violation in Neutron Deuteron Scattering in Pionless Effective Field Theory

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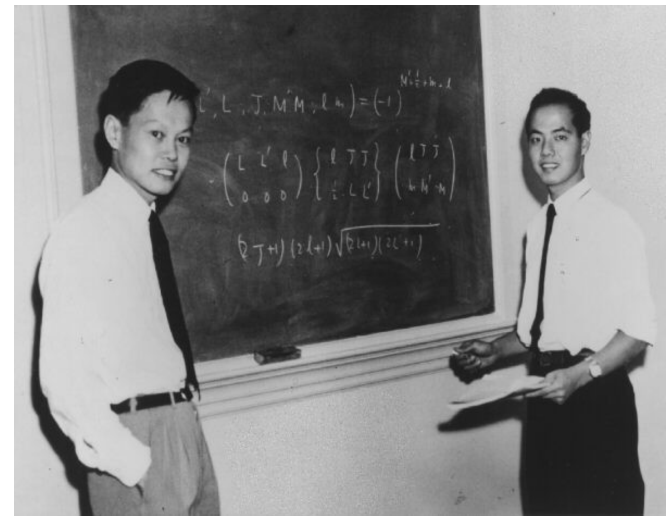
Parity Violation

Parity violation in weak force proposed by Lee and Yang to explain ▲– puzzle



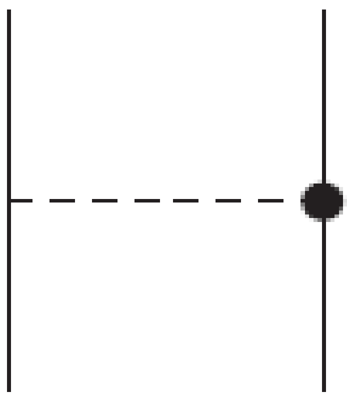
$$\tau \longrightarrow \pi^- + \pi^0 + \pi^0$$

$$\theta \longrightarrow \pi^- + \pi^0$$

Madame Wu discovered parity-violation in the beta decay of ^{60}Co

$$^{60}_{27}\mathrm{Co} \longrightarrow {}^{60}_{28}\mathrm{Ni} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$$

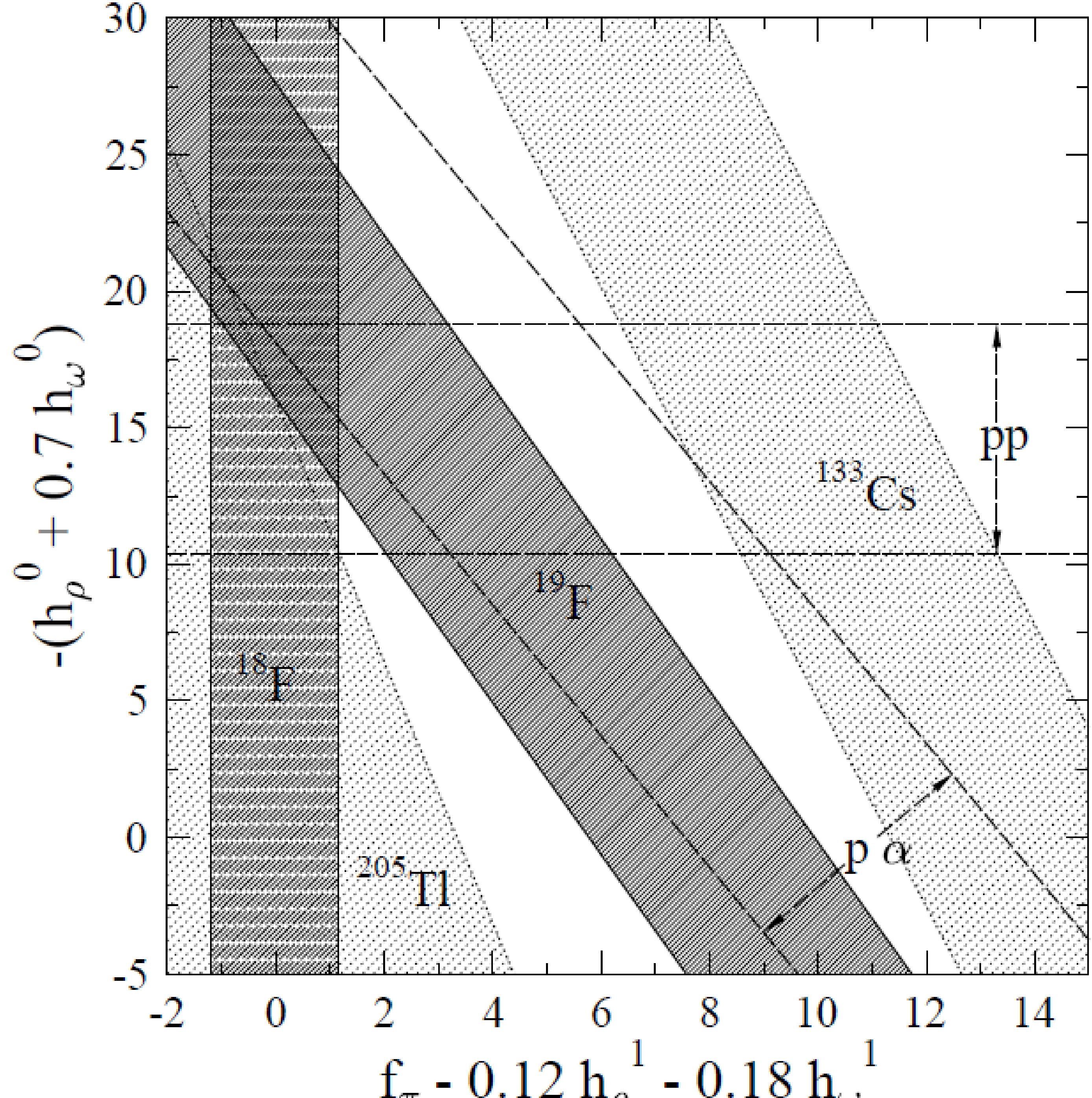
DDH Potential



$$\mathcal{H}_{\rm st} = i g_{\pi NN} \bar{N} \gamma_5 \tau \cdot \pi N + g_\rho \bar{N} \left(\gamma_\mu + i \frac{\chi_\rho}{2M_N} \sigma_{\mu\nu} k^v \right) \tau \cdot \rho^\mu N$$
$$+ g_\omega \bar{N} \left(\gamma_\mu + i \frac{\chi_\omega}{2M_N} \sigma_{\mu\nu} k^\nu \right) \omega^\mu N$$

$$\mathcal{H}_{\rm wk} = i \frac{h_\pi}{\sqrt{2}} \bar{N} (\tau \times \pi)_3 N + \bar{N} \left(h_\rho^0 \tau \cdot \rho^\mu + h_\rho^1 \rho_3^\mu + \frac{h_\rho^2}{2\sqrt{6}} (3\tau_3 \rho_3^\mu - \tau \cdot \rho^\mu) \right) \gamma_\mu \gamma_5 N$$
$$+ \bar{N} (h_\omega^0 \omega^\mu + h_\omega^1 \tau_3 \omega^\mu) \gamma_\mu \gamma_5 N - h_\rho'^1 \bar{N} (\tau \times \rho^\mu)_3 \frac{\sigma_{\mu\nu} k^\nu}{2M_N} \gamma_5 N$$

$$
\begin{aligned}
V_{DDH}^{PV}(\vec{\mathbf{r}}) =& i\frac{f_\pi g_{\pi NN}}{\sqrt{2}}\left(\frac{\tau_1\times\tau_2}{2}\right)_z(\vec{\boldsymbol{\sigma}}_1+\vec{\boldsymbol{\sigma}}_2)\cdot\left[\frac{\vec{\mathbf{p}}_1-\vec{\mathbf{p}}_2}{2M_N},w_\pi(r)\right] \\
& - g_\rho\left(h_\rho^0\tau_1\cdot\tau_2+h_\rho^1\left(\frac{\tau_1+\tau_2}{2}\right)_z+h_\rho^2\frac{(3\tau_1^z\tau_2^z-\tau_1\cdot\tau_2)}{2\sqrt{6}}\right) \\
& \times\left((\vec{\boldsymbol{\sigma}}_1-\vec{\boldsymbol{\sigma}}_2)\cdot\left\{\frac{\vec{\mathbf{p}}_1-\vec{\mathbf{p}}_2}{2M_N},w_\rho(r)\right\}\right. \\
& \left.+i(1+\chi_V)\vec{\boldsymbol{\sigma}}_1\times\vec{\boldsymbol{\sigma}}_2\cdot\left[\frac{\vec{\mathbf{p}}_1-\vec{\mathbf{p}}_2}{2M_N},w_\rho(r)\right]\right) \\
& - g_\omega\left(h_\omega^0+h_\omega^1\left(\frac{\tau_1+\tau_2}{2}\right)_z\right) \\
& \times\left((\vec{\boldsymbol{\sigma}}_1-\vec{\boldsymbol{\sigma}}_2)\cdot\left\{\frac{\vec{\mathbf{p}}_1-\vec{\mathbf{p}}_2}{2M_N},w_\omega(r)\right\}\right. \\
& \left.+i(1+\chi_S)\vec{\boldsymbol{\sigma}}_1\times\vec{\boldsymbol{\sigma}}_2\cdot\left[\frac{\vec{\mathbf{p}}_1-\vec{\mathbf{p}}_2}{2M_N},w_\omega(r)\right]\right) \\
& - (g_\omega h_\omega^1-g_\rho h_\rho^1)\left(\frac{\tau_1-\tau_2}{2}\right)_z(\vec{\boldsymbol{\sigma}}_1+\vec{\boldsymbol{\sigma}}_2)\cdot\left\{\frac{\vec{\mathbf{p}}_1-\vec{\mathbf{p}}_2}{2M_N},w_\rho(r)\right\} \\
& - g_\rho h_\rho^{1'} i\left(\frac{\tau_1\times\tau_2}{2}\right)_z(\vec{\boldsymbol{\sigma}}_1+\vec{\boldsymbol{\sigma}}_2)\cdot\left[\frac{\vec{\mathbf{p}}_1-\vec{\mathbf{p}}_2}{2M_N},w_\rho(r)\right]
\end{aligned}
$$



-(h_ρ^0 + 0.7 h_ω^0)
f_π - 0.12 h_ρ^1 - 0.18 h_ω^1
30
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15
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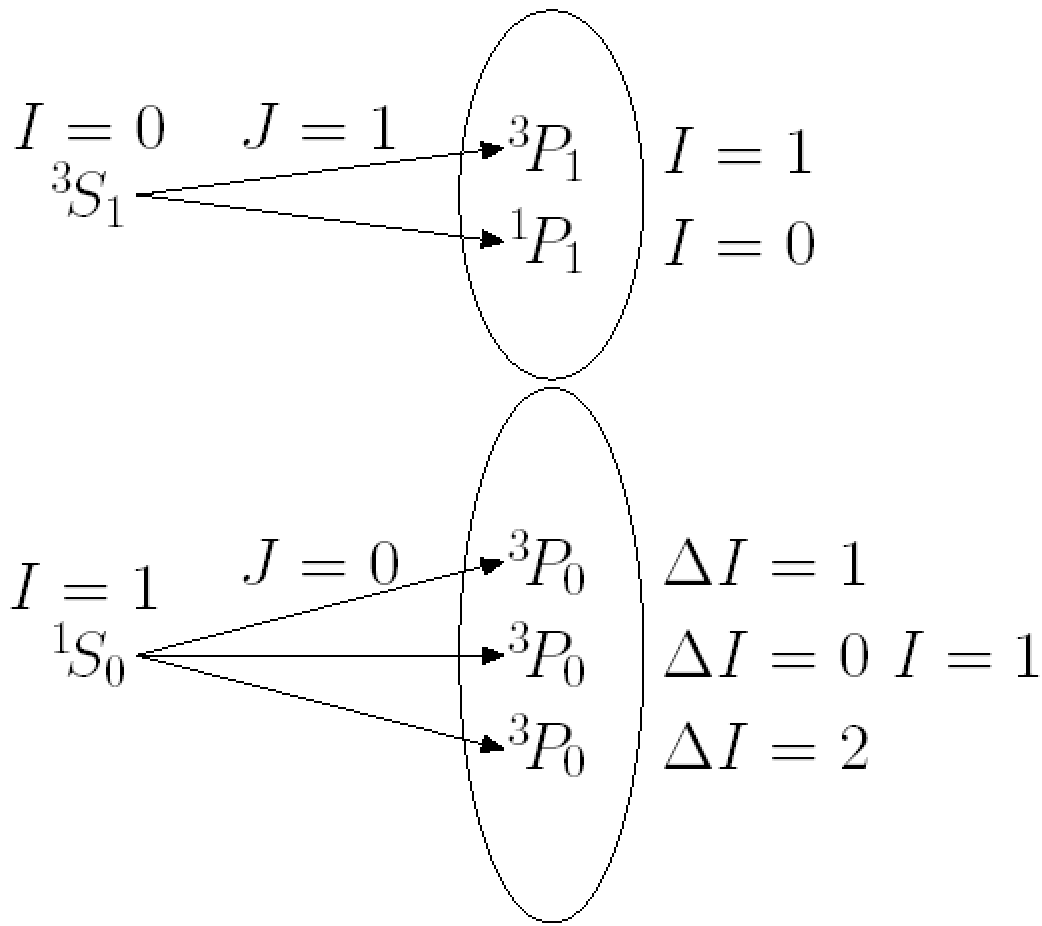
10
12
14
pp
133Cs
19F
18F
205Tl
p α

Low Energy Nucleon-Nucleon Parity-Violation

Conservation of angular momentum, and possible isospin transitions tell us



EFT Ingredients

- Identity degrees of freedom
- Write down all possible operators that obey symmetry of underlying theory using degrees of freedom
- Obtain power counting scheme to order terms in powers of $(p/\Lambda)^n$
- Calculate all terms up to a given order $(p/\Lambda)^N$

The Lagrangian for Pionless EFT is

$$\mathcal{L}_2 = - C_0^{(^3S_1)} (N^T P_i N)^\dagger (N^T P_i N)$$
$$+ C_2^{(^3S_1)} \frac{1}{8} \left[(N^T P_i N)^\dagger (N^T P_i \overleftrightarrow{\nabla}^2 N) + h.c. \right]$$
$$- \frac{1}{16} C_4^{(^3S_1)} \left(N^T P_i \overleftrightarrow{\nabla}^2 N \right)^\dagger \left(N^T P_i \overleftrightarrow{\nabla}^2 N \right)$$

The Lagrangian can be rewritten in the more useful form

$$\mathcal{L}_{PC}^d = N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2M_N} \right) N - t_i^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{4M_N} - \Delta_{(-1)}^{(^3S_1)} - \Delta_{(0)}^{(^3S_1)} \right) t_i + y_t \left[t_i^\dagger N^T P_i N + h.c. \right]$$

$$- s_a^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{4M_N} - \Delta_{(-1)}^{(^1S_0)} - \Delta_{(0)}^{(^1S_0)} \right) s_a + y_s \left[s_a^\dagger N^T \bar{P}_a N + h.c. \right]$$

Spin and Isospin Projectors are

$$P_i = \frac{1}{\sqrt{8}} \sigma_2 \sigma_i \tau_2 \qquad \bar{P}_a = \frac{1}{\sqrt{8}} \sigma_2 \tau_2 \tau_a$$

Effective Range Expansion

Partial wave expansion gives

$$f = \sum_{\ell} (2\ell + 1) f_\ell P_\ell(\cos(\theta)) = \sum_{\ell} \frac{1}{k\cot(\delta_\ell(k)) - ik} (2\ell + 1) P_\ell(\cos(\theta))$$

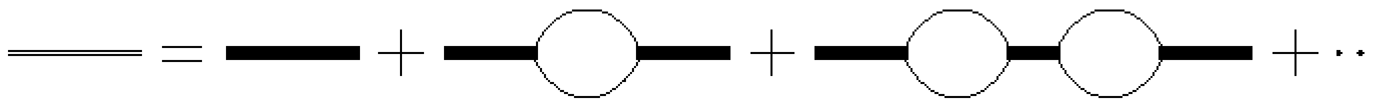
Effective range expansion (ERE) is

$$k\cot(\delta_0(k)) = -\frac{1}{a} + \frac{1}{2} r k^2 + s k^4 + \ldots$$

Reexpanding about deuteron pole ERE becomes

$$k\cot(\delta_0(k)) = -\gamma_t + \frac{1}{2}\rho_t(k^2 + \gamma_t^2) + w_0(k^2 + \gamma_t)^2 + \cdots$$

Dressed deuteron propagator is



Single nucleon bubble gives

$$i\Sigma(p) = -i\frac{y_t^2 M_N}{4\pi}\left(\mu - \sqrt{\frac{1}{4}\vec{\mathrm{p}}^2 - M_N p_0 - i\epsilon}\right)$$

Resulting dressed deuteron propagator is

$$i\Delta_t(p_0, \vec{\mathrm{p}}) = -\frac{4\pi i}{M_N y_t^2}\frac{1}{\frac{4\pi\Delta_{(-1)}^{(^3S_1)}}{M_N y_t^2} - \mu + \sqrt{\frac{\vec{\mathrm{p}}^2}{4} - M_N p_0 - i\epsilon}}$$

Matching onto the ERE we find

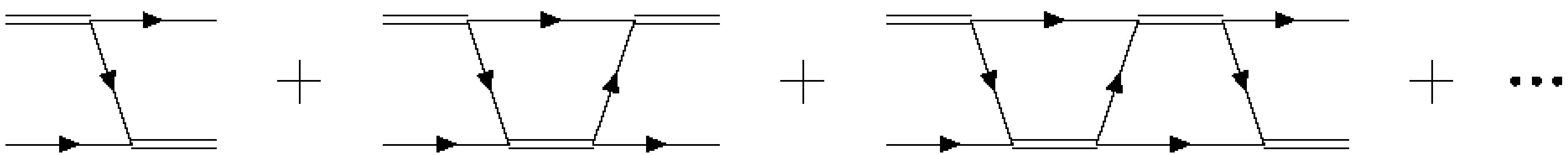
$$\frac{y_t^2}{\Delta_{(-1)}^{(^3S_1)}} = -\frac{4\pi}{M_N}\frac{1}{\gamma_t - \mu} \qquad i\Delta_t(p_0, \vec{\mathrm{p}}) = -\frac{4\pi i}{M_N y_t^2}\frac{1}{-\gamma_t + \sqrt{\frac{\vec{\mathrm{p}}^2}{4} - M_N p_0 - i\epsilon}}$$

Parity Conserving nd Scattering

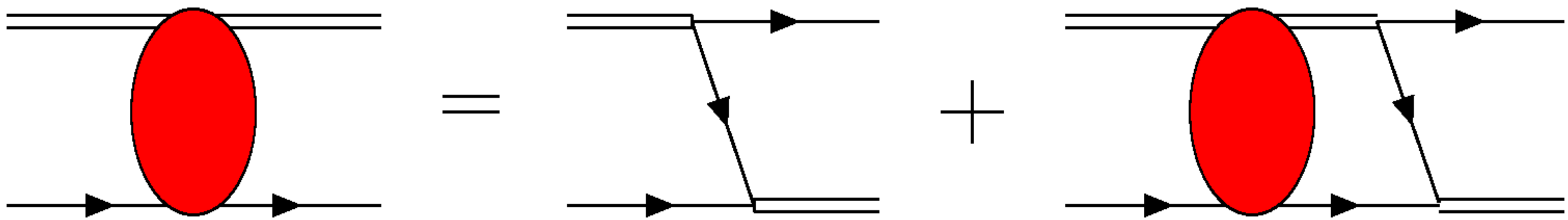
Deuteron has spin 1 and neutron is spin ½, thus nd scattering has a Quartet and Doublet channel

$$1 \otimes {}^{1}\!/_{2} = {}^{1}\!/_{2} \oplus {}^{3}\!/_{2}$$

At LO and infinite sum of diagrams in needed in the Quartet channel



Quartet Channel



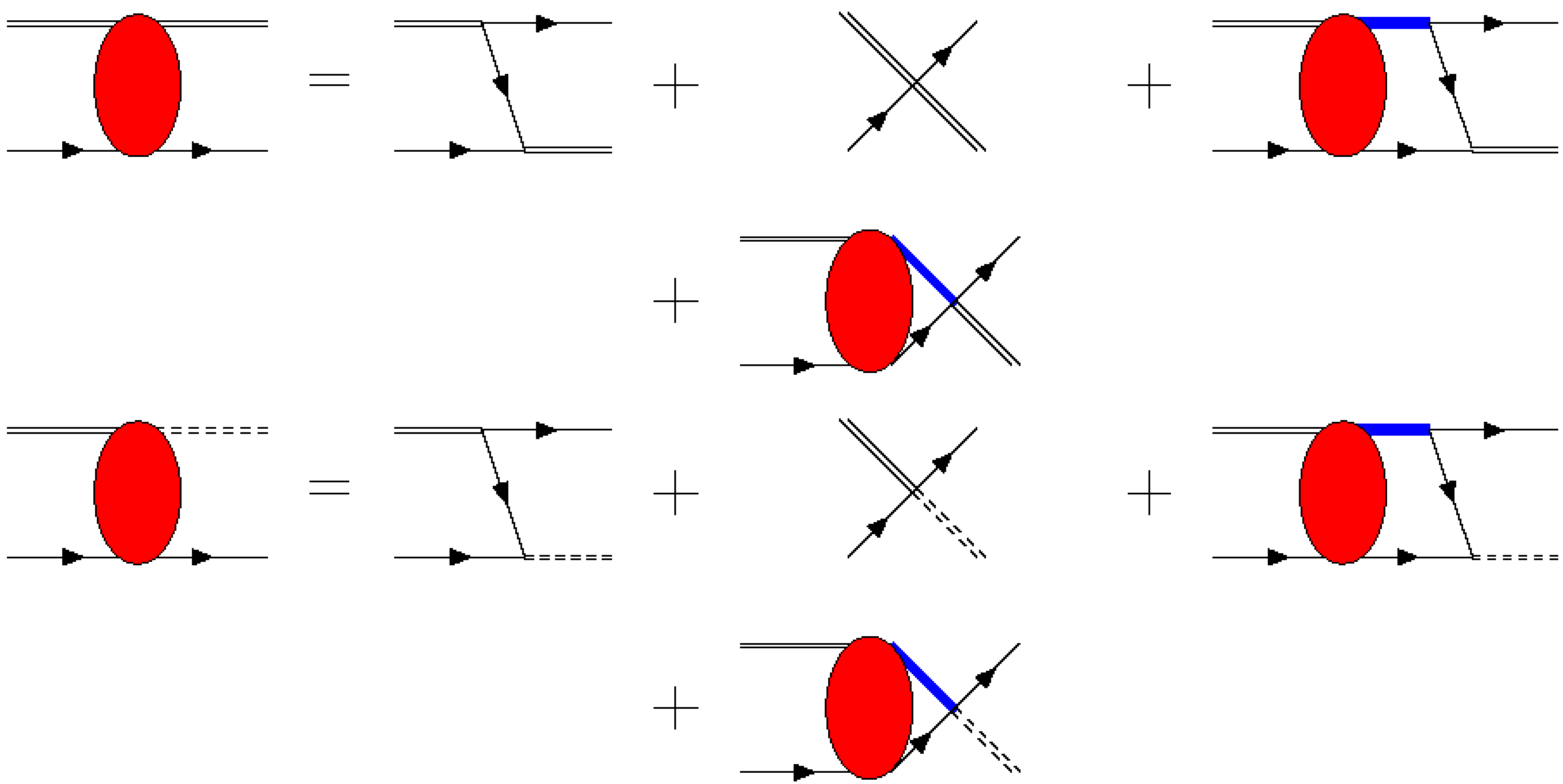
After projection integral equation is

$$t_{0q}^{(l)}(k,p) = -\frac{y_t^2 M_N}{pk} Q_l\left(\frac{p^2+k^2-M_N E-i\epsilon}{pk}\right) - \\ -\frac{2}{\pi}\int_0^\infty dq q^2 t_{0q}^{(l)}(k,q)\frac{1}{\sqrt{\frac{3q^2}{4}-M_N E-i\epsilon}-\gamma_t}\frac{1}{qp}Q_l\left(\frac{p^2+q^2-M_N E-i\epsilon}{pq}\right)$$

Where $Q_l(a)$ is related to Legendre polynomial of second kind up to factor $(-1)^l$

$$Q_l(a) = \frac{1}{2}\int_{-1}^{1}\frac{P_l(x)}{x+a}dx$$

Doublet Channel

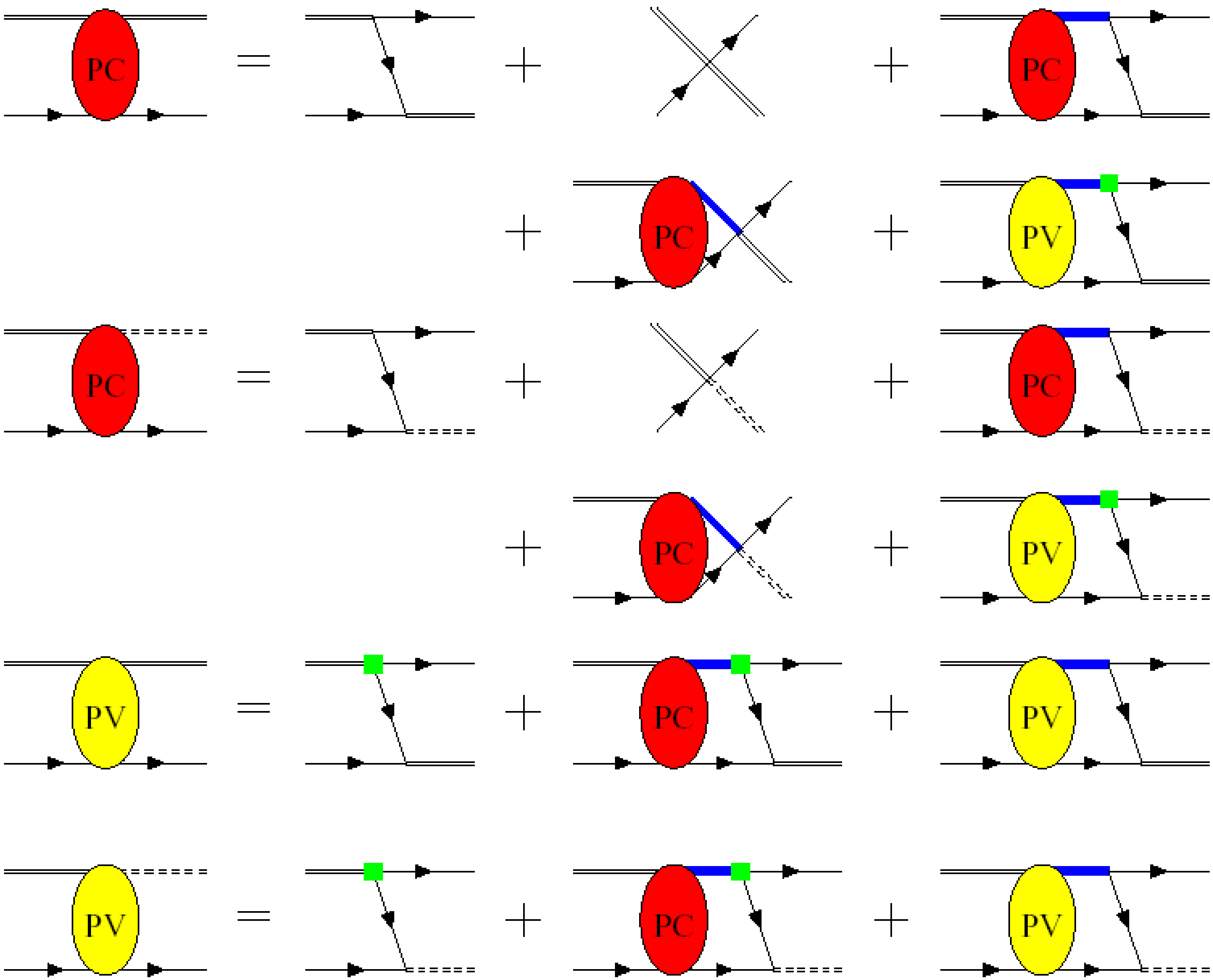


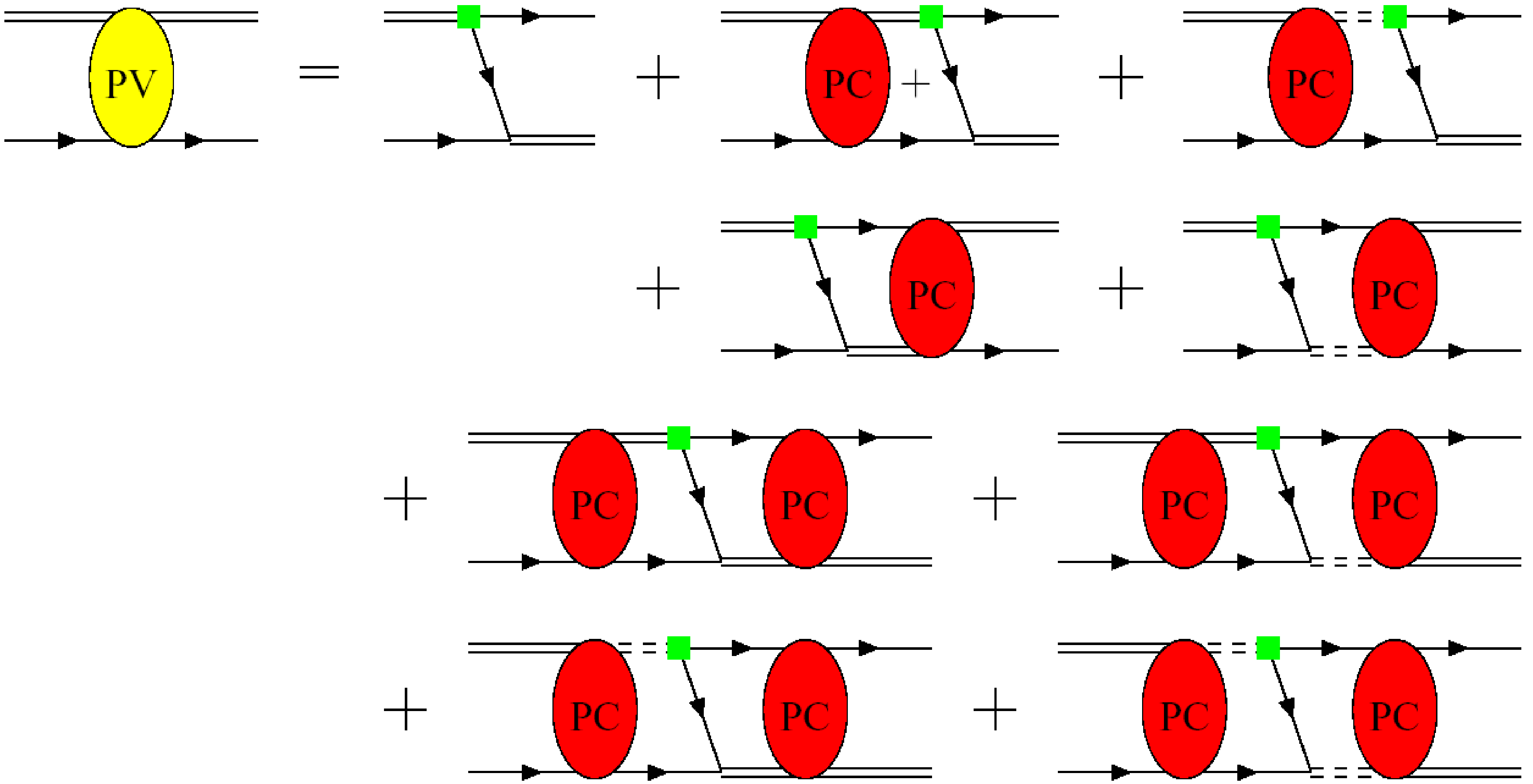
Parity-Violation in EFT

The parity-violating Lagrangian contains five terms and is given by

$$\begin{aligned}
\mathcal{L}^{d}_{\rm PV} = -\Big[& g^{(^3S_1-^1P_1)} t_i^\dagger \left(N^T \sigma_2 \tau_2 i \overleftrightarrow{\nabla}_i N \right) \\
& + g^{(^1S_0-^3P_0)}_{(\Delta I=0)} s_a^\dagger \left(N^T \sigma_2 \vec{\boldsymbol{\sigma}} \cdot \tau_2 \tau_a i \overleftrightarrow{\nabla} N \right) \\
& + g^{(^1S_0-^3P_0)}_{(\Delta I=1)} \epsilon^{3ab} \left(s^a \right)^\dagger \left(N^T \sigma_2 \vec{\boldsymbol{\sigma}} \cdot \tau_2 \tau^b \overleftrightarrow{\nabla} N \right) \\
& + g^{(^1S_0-^3P_0)}_{(\Delta I=2)} \mathcal{I}^{ab} \left(s^a \right)^\dagger \left(N^T \sigma_2 \vec{\boldsymbol{\sigma}} \cdot \tau_2 \tau^b i \overleftrightarrow{\nabla} N \right) \\
& + g^{(^3S_1-^3P_1)} \epsilon^{ijk} \left(t^i \right)^\dagger \left(N^T \sigma_2 \sigma^k \tau_2 \tau_3 \overleftrightarrow{\nabla}^j N \right) \Big] + h.c.
\end{aligned}$$

Parity-Violating nd Scattering




PV
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PC

Focus on single diagram

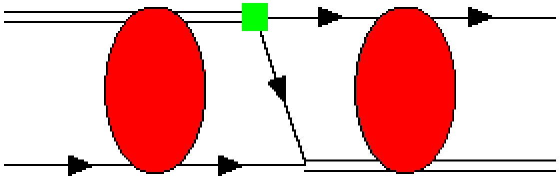


Diagram is given by

$$i\frac{2\sqrt{2}M_N\pi}{\gamma_t y_t}\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3\ell}{(2\pi)^3}\left(it^{ki}_{Nt\to Nt}(\vec{\mathbf{k}},\vec{\mathbf{q}})\right)^{\gamma c}_{\alpha a}\left(it^{jl}_{Nt\to Nt}(\vec{\mathbf{p}},\vec{\boldsymbol{\ell}})\right)^{\beta b}_{\delta d}$$

$$\frac{1}{\sqrt{\frac{3\vec{\mathbf{q}}^2}{4}-M_NE-i\epsilon}-\gamma_t}\frac{1}{\sqrt{\frac{3\vec{\boldsymbol{\ell}}^2}{4}-M_NE-i\epsilon}-\gamma_t}$$

$$\frac{1}{\vec{\mathbf{q}}^2+\vec{\mathbf{q}}\cdot\vec{\boldsymbol{\ell}}+\vec{\boldsymbol{\ell}}^2-M_NE-i\epsilon}\left({K^{11}_{PV}}^{lk}\right)^{\delta d}_{\gamma c}(\vec{\mathbf{q}},\vec{\boldsymbol{\ell}})$$

where

$$\left({K^{11}_{PV}}^{lk}\right)^{\delta d}_{\gamma c}(\vec{\mathbf{q}},\vec{\boldsymbol{\ell}})=g^{{}^3S_1-{}^1P_1}(\sigma^l)^\delta_\gamma\delta^d_c(\vec{\mathbf{q}}+2\vec{\boldsymbol{\ell}})^k+ig^{{}^3S_1-{}^3P_1}\epsilon^{ijk}(\sigma^j\sigma^l)^\delta_\gamma(\tau_3)^d_c(\vec{\mathbf{q}}+2\vec{\boldsymbol{\ell}})^i$$

$$+g^{{}^3S_1-{}^1P_1}(\sigma^k)^\delta_\gamma\delta^d_c(2\vec{\mathbf{q}}+\vec{\boldsymbol{\ell}})^l-ig^{{}^3S_1-{}^3P_1}\epsilon^{ijl}(\sigma^k\sigma^j)^\delta_\gamma(\tau_3)^d_c(2\vec{\mathbf{q}}+\vec{\boldsymbol{\ell}})^i$$

Partial wave series is

$$t_{PV}(\vec{\mathbf{k}},\vec{\mathbf{p}}) = \sum_{J=0}^{\infty}\sum_{M=-J}^{M=J}\sum_{L=|J-S|}^{J+S}\sum_{L'=|J-S'|}^{J+S'}\sum_{S,S'} 4\pi t^{JM}_{L'S',LS}(k,p)\mathscr{Y}^{M}_{J,L'S'}(\hat{\mathbf{p}})\left(\mathscr{Y}^{M}_{J,LS}(\hat{\mathbf{k}})\right)$$

Where the spin angle functions are given by

$$\mathscr{Y}^{M}_{J,LS}(\hat{\mathbf{k}}) = \sum_{m_L,m_S} C^{m_L,m_S,M}_{L,S;J} Y^{m_L}_{L}(\hat{\mathbf{k}})\chi^{m_S}_{S}$$

Terms are projected out by

$$t^{JM}_{L'S',LS}(k,p) = \frac{1}{4\pi}\int d\Omega_k \int d\Omega_p \left(\mathscr{Y}^{M}_{J,L'S'}(\hat{\mathbf{p}})\right)^{*} t_{PV}(\vec{\mathbf{k}},\vec{\mathbf{p}})\mathscr{Y}^{M}_{J,LS}(\hat{\mathbf{k}})$$

Projecting amplitude leads to integral

$$\frac{1}{4\pi}\int d\Omega_k \int d\Omega_p \left(\mathscr{Y}^M_{J,L'S'}(\hat{\boldsymbol{\ell}})\right)^* \frac{1}{a+\hat{\mathbf{q}}\cdot\hat{\boldsymbol{\ell}}} \left(\mathcal{K}^{11}_{PV}{}^{lk}\right)^{\delta d}_{\gamma c}(\vec{\mathbf{q}},\vec{\boldsymbol{\ell}})\mathscr{Y}^M_{J,LS}(\hat{\mathbf{q}})$$

where

$$a = \frac{\vec{\mathbf{q}}^2+\vec{\boldsymbol{\ell}}^2 - M_N E - i\epsilon}{|\vec{\mathbf{q}}||\vec{\boldsymbol{\ell}}|}$$

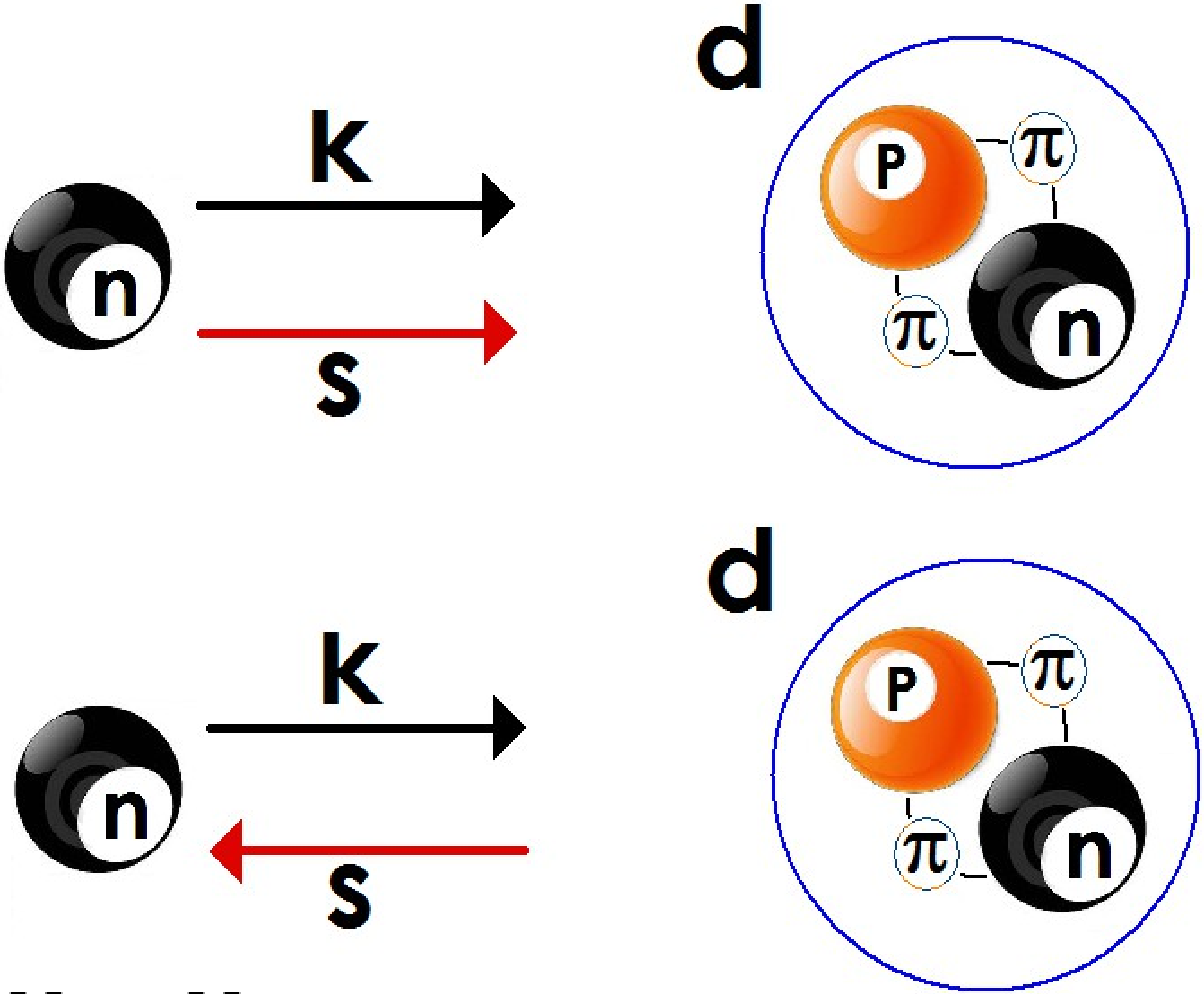
Focusing on first term we find

$$\frac{1}{4\pi}\sum_{m_L,m_S}\sum_{m_{L'},m_{S'}}\sum_{m_1,m_2}\sum_{m_1',m_2'} C^{m_L,m_S,M}_{L,S,J} C^{m_{L'},m_{S'},M}_{L',S',J} C^{m_1,m_2,m_S}_{1,1/2,S} C^{m_1',m_2',m_{S'}}_{1,1/2,S'}$$
$$\int d\Omega k \int d\Omega p \frac{1}{a+\hat{\mathbf{q}}\cdot\hat{\boldsymbol{\ell}}}\sqrt{\frac{4\pi}{3}}\left(qY^{m_1}_1(\hat{\mathbf{q}}) + 2\ell Y^{m_1}_1(\hat{\boldsymbol{\ell}})\right) Y^{m_L}_L(\hat{\mathbf{q}})\left(Y^{m_{L'}}_{L'}(\hat{\boldsymbol{\ell}})\right)^*$$
$$(-1)^{m_1'}\langle 1/2, m_2'|\sigma^{-m_1'}|1/2, m_2\rangle$$

Expression has simple solution

$$-4\pi\sqrt{3}(-1)^{3/2+2S+L-J}\delta_{S'1/2}\sqrt{\bar{S}\bar{L}}C^{0,0,0}_{L,1,L'}\left\{\begin{array}{ccc} 1/2 & 1 & S \\ L & J & L' \end{array}\right\}(qQ_{L'}(a)+2\ell Q_L(a))$$

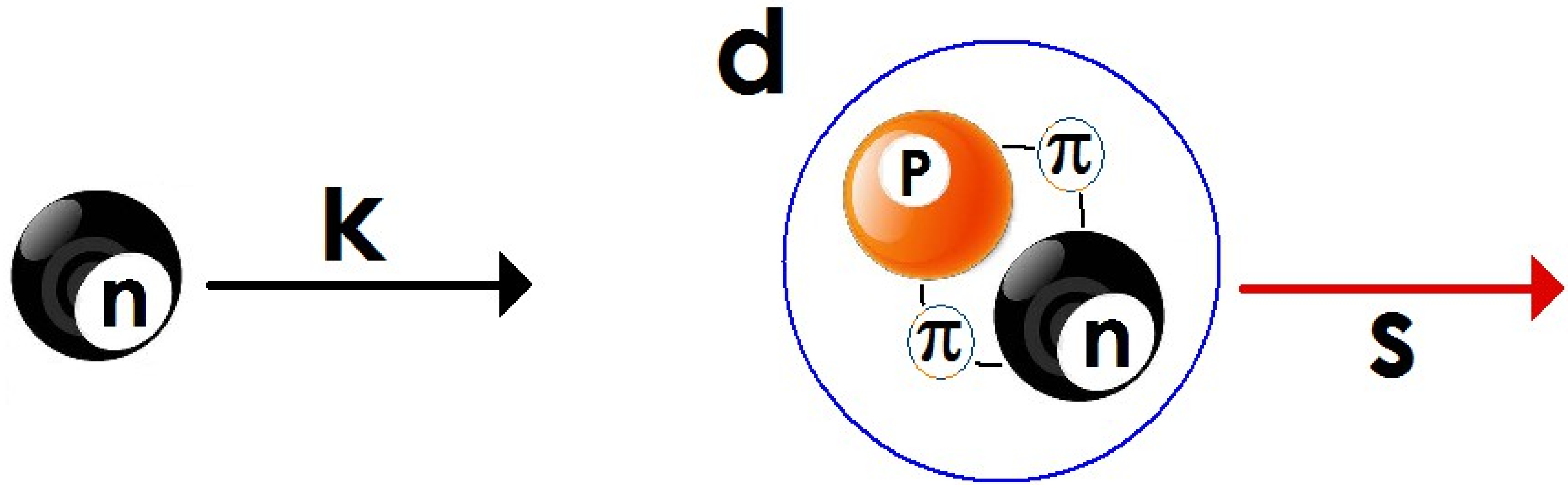
Beam Asymmetry nd Scattering

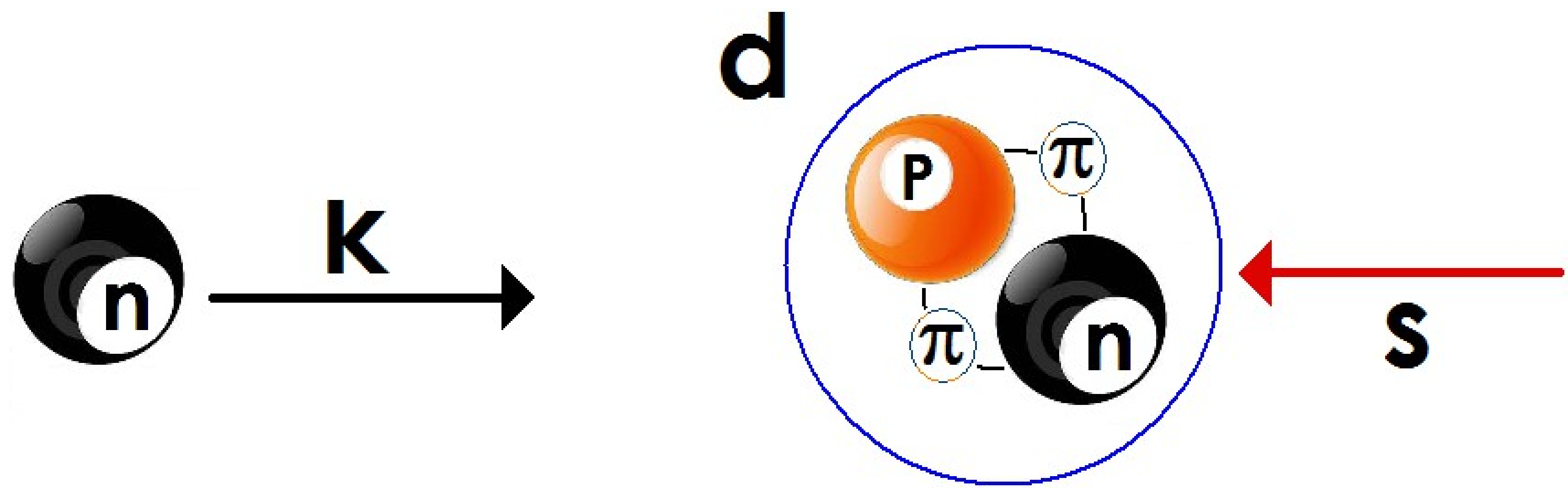


$$A_{nd} = \frac{N_+ - N_-}{N_+ + N_-}$$

Vanasse

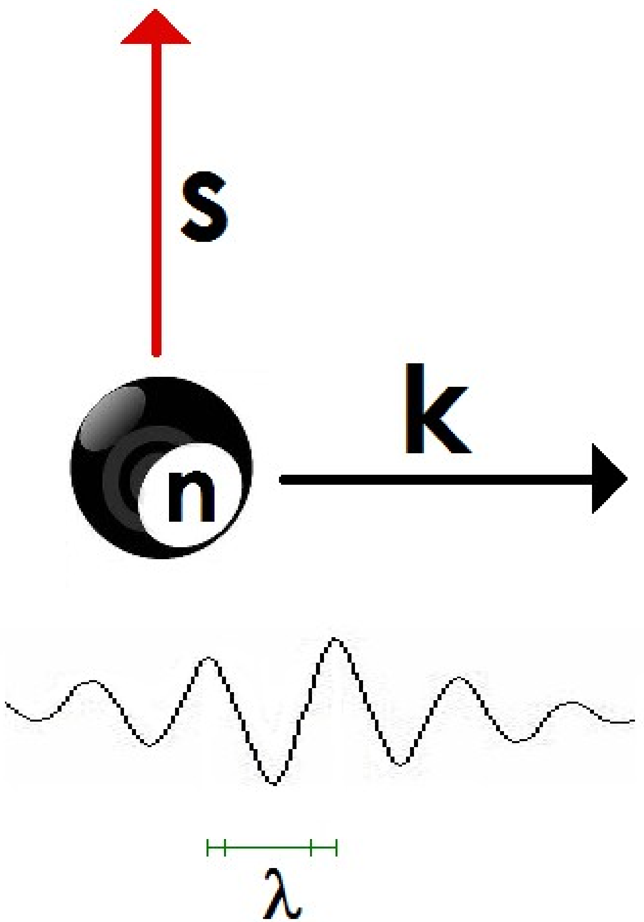
Target Asymmetry nd Scattering

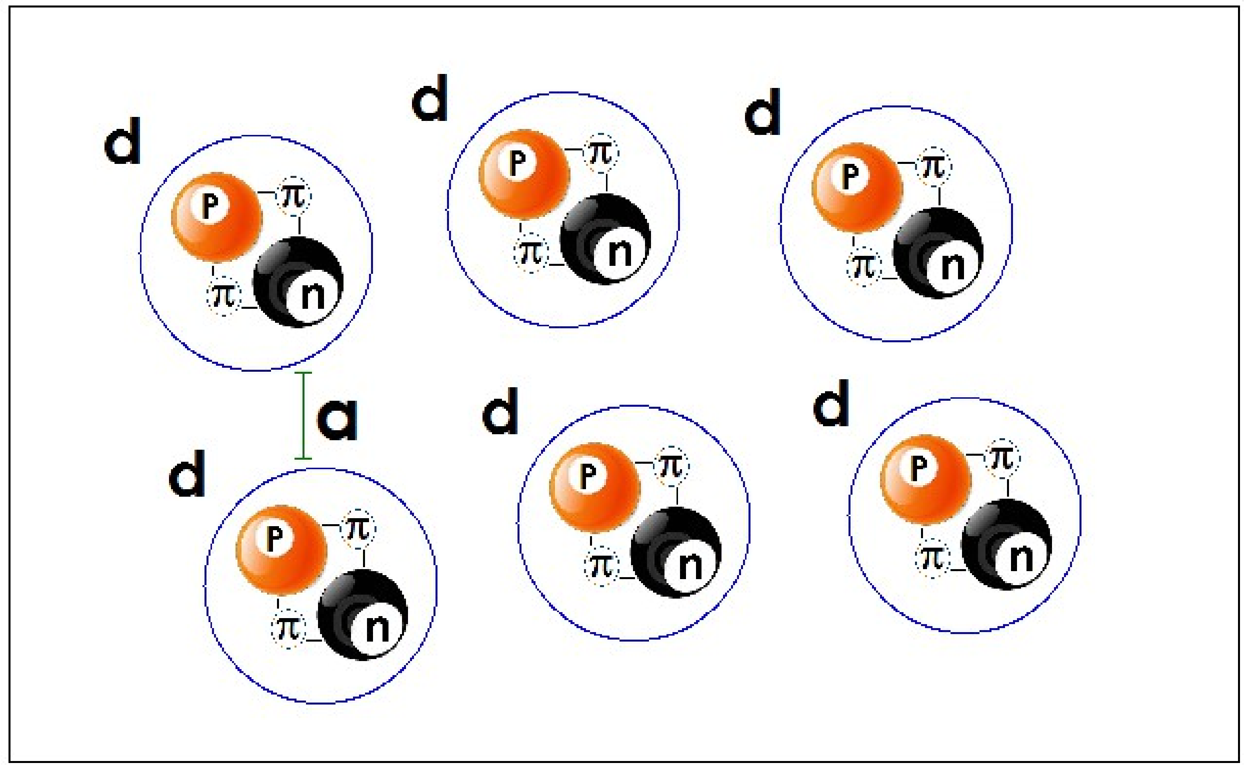




Vanasse

nd Spin Rotation

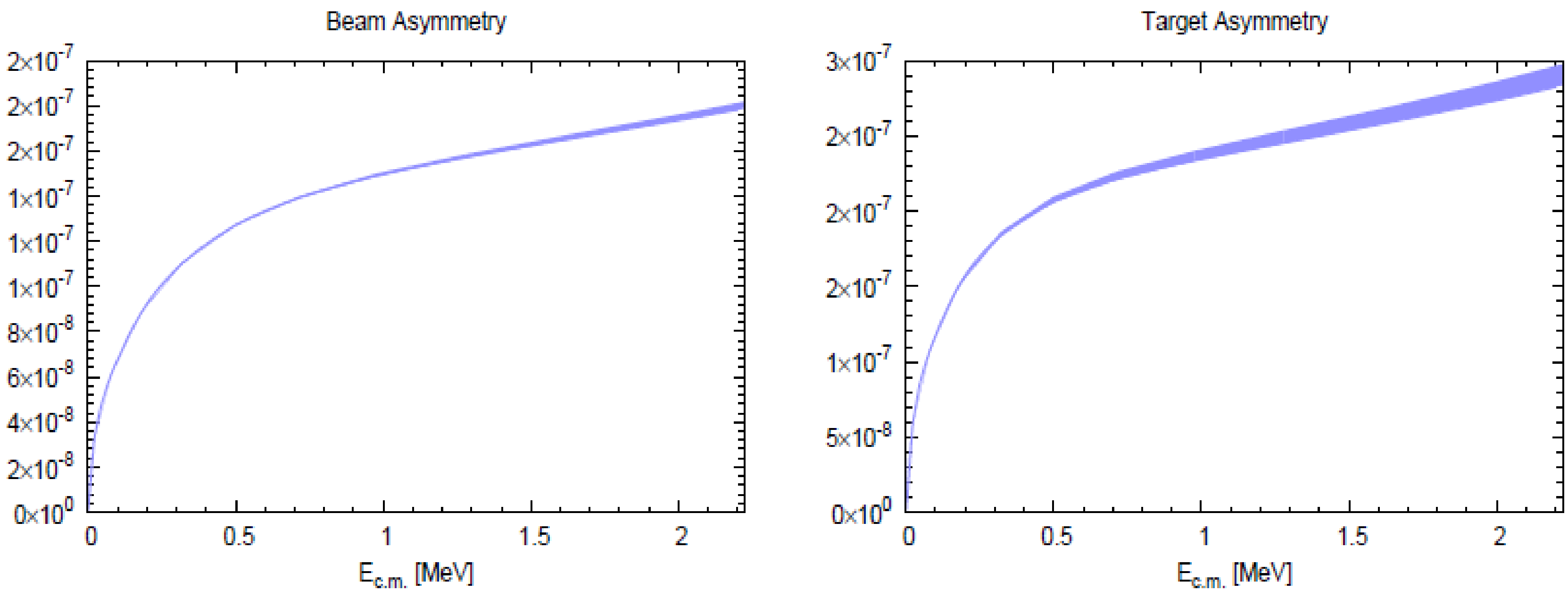




Griesshammer, Schindler, and Springer

Vanasse

Asymmetry Results



Spin Rotation Results

Spin rotation

$$\frac{1}{N}\frac{d\phi}{dz} = \sum_{n=1}^{5} c_n^{Gir} I_n^{Gir}$$

$$1.83 \times 10^{-8} \text{ rad cm}^{-1} \text{ to } 1.84 \times 10^{-8} \text{ rad cm}^{-1}$$

Comparison of different results for spin rotation

I_n^{Gir}	$\mathrm{EFT}_{\not\pi}$-I/AV18		$\mathrm{EFT}_{\not\pi}$-I/AV18+UIX		$\mathrm{EFT}_{\not\pi}$	
$n =$	Song	Schiavilla	Song	Schiavilla	LO	NLO
1	61.6	65.6	60.0	63.2	129.3 - 135.7	98.5 - 120.3
2	60.6	62.3	58.8	57.8	35.0 - 57.1	33.4 - 51.9
4	-76.1	-77.9	-75.7	-75.2	-59.6 - -77.2	-48.2 - -67.2
5	-9.46	-9.89	-6.62	-6.12	7.16 - -8.66	-1.85 - -10.6

Future Directions

- Resolve factor of two
- Carry calculation to higher orders
- Calculate pd scattering
- Ay puzzle
- ^{3}He photodisintegration

Summary

- We can calculate nd scattering PV amplitudes to LO.
- Estimates for the PV LEC's can be obtained by matching onto DDH estimates.
- Using LEC estimates and PV amplitudes we can predict any PV observable in nd scattering.

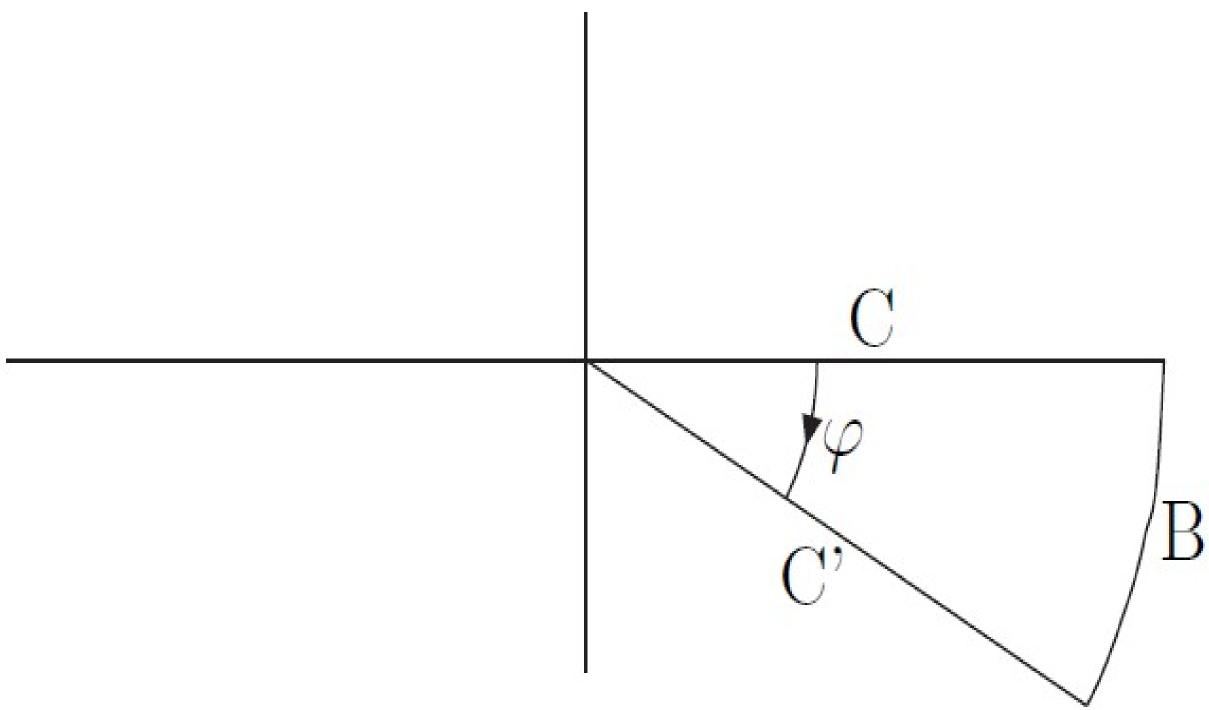

Thank You

Numerical Techniques

Integrals are discretized using quadrature. Gives set of linear equations to solve.

$$t_i = B_i + \sum_j w_j K_{ij} t_j \qquad t_i = \left(\delta_{ij} - \sum_j w_j K_{ij}\right)^{-1} B_j$$

In order to avoid singularities amplitude is evaluated along path C'B.



Girlanda Reduction

Girlanda showed nucleon-nucleon parity-violation is described by only five LEC's.

There are twelve possible unique relativistic parity-violating nucleon-nucleon terms

$$\mathcal{O}_1 = \bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\gamma_5\psi \qquad \tilde{\mathcal{O}}_1 = \bar{\psi}\gamma^\mu\gamma_5\psi\partial^\nu(\bar{\psi}\sigma_{\mu\nu}\psi)$$

$$\mathcal{O}_2 = \bar{\psi}\gamma^\mu\psi\bar{\psi}\tau_3\gamma_\mu\gamma_5\psi \qquad \tilde{\mathcal{O}}_2 = \bar{\psi}\gamma^\mu\gamma_5\psi\partial^\nu(\bar{\psi}\tau_3\sigma_{\mu\nu}\psi)$$

$$\mathcal{O}_3 = \bar{\psi}\tau_a\gamma^\mu\psi\bar{\psi}\tau^a\gamma_\mu\gamma_5\psi \qquad \tilde{\mathcal{O}}_3 = \bar{\psi}\tau_a\gamma^\mu\gamma_5\psi\partial^\nu(\bar{\psi}\tau^a\sigma_{\mu\nu}\psi)$$

$$\mathcal{O}_4 = \bar{\psi}\tau_3\gamma^\mu\psi\bar{\psi}\gamma_\mu\gamma_5\psi \qquad \tilde{\mathcal{O}}_4 = \bar{\psi}\tau_3\gamma^\mu\gamma_5\psi\partial^\nu(\bar{\psi}\sigma_{\mu\nu}\psi)$$

$$\mathcal{O}_5 = \mathcal{I}_{ab}\bar{\psi}\tau_a\gamma^\mu\psi\bar{\psi}\tau_b\gamma_\mu\gamma_5\psi \qquad \tilde{\mathcal{O}}_5 = \mathcal{I}_{ab}\bar{\psi}\tau_a\gamma^\mu\gamma_5\psi\partial^\nu(\bar{\psi}\tau_b\sigma_{\mu\nu}\psi)$$

$$\mathcal{O}_6 = i\epsilon_{ab3}\bar{\psi}\tau_a\gamma^\mu\psi\bar{\psi}\tau_b\gamma_\mu\gamma_5\psi \qquad \tilde{\mathcal{O}}_6 = i\epsilon_{ab3}\bar{\psi}\tau_a\gamma^\mu\gamma_5\psi\partial^\nu(\bar{\psi}\tau_b\sigma_{\mu\nu}\psi)$$

Using Fierz identities and equations of motion one can find six relations between operators

$$\mathcal{O}_3 = \mathcal{O}_1 \qquad \tilde{\mathcal{O}}_2 + \tilde{\mathcal{O}}_4 = M_N(\mathcal{O}_2 + \mathcal{O}_4)$$

$$\mathcal{O}_2 - \mathcal{O}_4 = 2\mathcal{O}_6 \qquad \tilde{\mathcal{O}}_2 - \tilde{\mathcal{O}}_4 = -2M_N\mathcal{O}_6 - \tilde{\mathcal{O}}_6$$

$$\tilde{\mathcal{O}}_3 + 3\tilde{\mathcal{O}}_1 = 2M_N(\mathcal{O}_1 + \mathcal{O}_3) \qquad \tilde{\mathcal{O}}_5 = M_N\mathcal{O}_5$$

Taking the non-relativistic reduction we find two operators are the same and get Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{PV}}^{\text{Gir}} = \mathcal{G}_1(N^\dagger\vec{\boldsymbol{\sigma}}N \cdot N^\dagger i \overset{\leftrightarrow}{\nabla} N - N^\dagger N N^\dagger i \overset{\leftrightarrow}{\nabla} \cdot \vec{\boldsymbol{\sigma}}N) - \tilde{\mathcal{G}}_1\epsilon_{ijk}N^\dagger\sigma_i N\nabla_j(N^\dagger\sigma_k N) \\ - \mathcal{G}_2\epsilon_{ijk}[N^\dagger\tau_3\sigma_i N\nabla_j(N^\dagger\sigma_k N) + N^\dagger\sigma_i N\nabla_j(N^\dagger\tau_3\sigma_k N)] \\ - \tilde{\mathcal{G}}_5\mathcal{I}_{ab}\epsilon_{ijk}N^\dagger\tau_a\sigma_i N\nabla_j(N^\dagger\tau_b\sigma_k N) + \mathcal{G}_6\epsilon_{ab3}\overset{\rightarrow}{\nabla}(N^\dagger\tau_a N)\cdot N^\dagger\tau_b\vec{\boldsymbol{\sigma}}N\end{aligned}$$

LEC Estimates

$$g_1 = -\frac{M_N(\frac{1}{a_t} - \mu)}{8\sqrt{2}\pi}\left[\frac{g_\omega \chi_\omega}{M_N m_\omega^2} h_\omega^0 - \frac{3 g_\rho \chi_\rho}{M_N m_\rho^2} h_\rho^0\right] \sim 1.75 \times 10^{-10}\mathrm{MeV}^{-1}$$

$$g_2 = \frac{M_N(\frac{1}{a_t} - \mu)}{8\sqrt{2}\pi}\left[\frac{g_{\pi NN}}{\sqrt{2} M_N m_\pi^2} f_\pi + \frac{g_\rho}{M_N m_\rho^2} h_\rho^1 - \frac{g_\omega}{M_N m_\omega^2} h_\omega^1\right] \sim -6.34 \times 10^{-10}\mathrm{MeV}^{-1}$$

$$g_3 = \frac{M_N(\gamma_s - \mu)}{8\sqrt{2}\pi}\left[\frac{g_\omega(2+\chi_\omega)}{M_N m_\omega^2} h_\omega^0 + \frac{g_\rho(2+\chi_\rho)}{M_N m_\rho^2} h_\rho^0\right] \sim 1.50 \times 10^{-10}\mathrm{MeV}^{-1}$$

$$g_4 = \frac{M_N(\gamma_s - \mu)}{8\sqrt{2}\pi}\left[\frac{g_\rho(2+\chi_\rho)}{M_N m_\rho^2} h_\rho^1 + \frac{g_\omega(2+\chi_\omega)}{M_N m_\omega^2} h_\omega^1\right] \sim 1.47 \times 10^{-11}\mathrm{MeV}^{-1}$$

$$g_5 = \frac{M_N(\gamma_s - \mu)}{8\sqrt{2}\pi}\left[\frac{g_\rho(2+\chi_\rho)}{\sqrt{6} M_N m_\rho^2} h_\rho^2\right] \sim 4.39 \times 10^{-11}\mathrm{MeV}^{-1}$$