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THREE-BODY FORCE IN HALO NUCLEI

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Outline

- EFT
- Nucleon-alpha system
- ~~Alpha-alpha system~~
- ^{6}He
- ~~Other systems~~
- Outlook

Nuclear physics scales

$\ln Q$

expansion in α_s

perturbative QCD

~1 GeV $M_{QCD} \sim m_N, m_\rho, 2\pi F_\pi, \ldots$

unknown;
use brute force
(lattice, …)

hadronic theory

Chiral EFT

~100 MeV $M_{nuc} \sim F_\pi, 1/r_{NN}, \ldots \sim m_\pi$

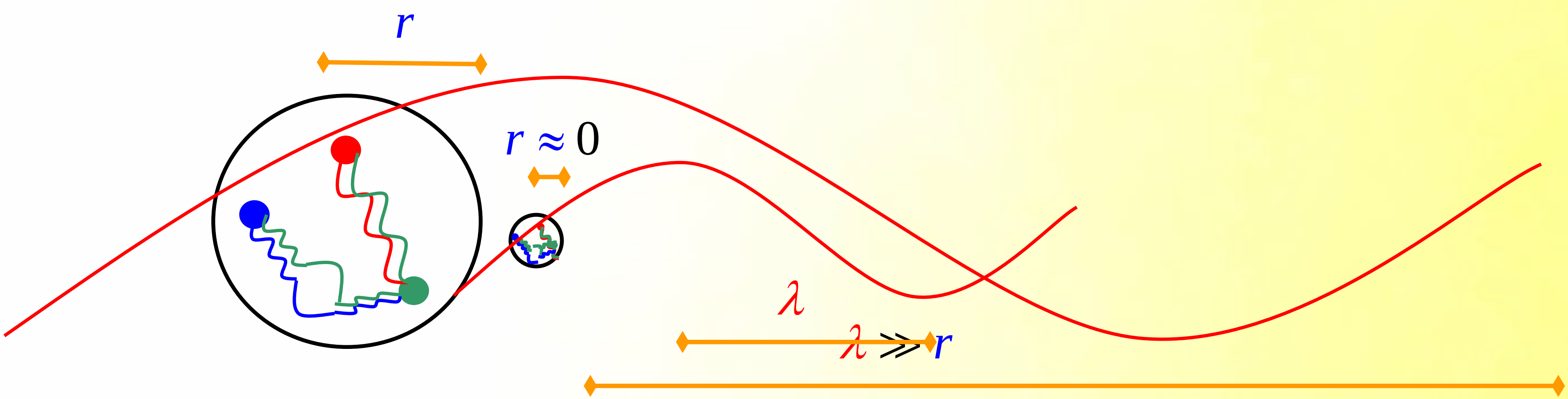
Q/M_{QCD}

~30 MeV $\aleph \sim 1/a_{NN}$

Q/M_{nuc}

Contact EFT

no small coupling constants!



$$V=\int d^3r\,\rho(\vec r)\,A^0(\vec r)$$

$$=A^0(0)\int d^3r\,\rho(\vec r)+\left[\partial_i A^0\right](0)\cdot\int d^3r\,r_i\,\rho(\vec r)+\left[\partial_i\partial_j A^0\right](0)\int d^3r\,r_i r_j\,\rho(\vec r)+\ldots$$

$$=qA^0(0)\qquad\qquad =-\vec D\cdot\vec E(0)\qquad\qquad =-\frac{1}{3}\left(q\left\langle r^2\right\rangle\delta_{ij}+Q_{ij}\right)\partial_i E_j(0)$$

$$\mathcal{O}\left(\frac{r}{\lambda}\right)\qquad\qquad \mathcal{O}\left(\frac{r^2}{\lambda^2}\right)$$

- All interactions for point charge allowed by gauge invariance
- Expansion in powers of $\dfrac{r}{\lambda}$ — r: distance scale of underlying distribution; λ: distance scale of interest

$Q \sim \aleph \ll M_{nuc}$

pionless EFT

- degrees of freedom: nucleons
- symmetries: Lorentz, $\not{B}$, $\not{P}$, $\not{T}$
- expansion in:

$$\frac{Q}{M_{nuc}} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \ldots & \text{multipole} \end{cases}$$

Kaplan '97
v.K. '99

simplest formulation: auxiliary field for two-nucleon bound states

$$N + N \begin{cases} d\left({}^3S_1\right) & \mapsto \text{vector field } \vec{d} \\ d^*\left({}^1S_0\right) & \mapsto \text{isovector field } \mathbf{d}^* \end{cases}$$

$$\mathcal{L}_{EFT} = N^+\left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_N}\right)N + d^+(-\Delta)\,d + \frac{g}{\sqrt{2}}\left[d^+NN + N^+N^+d\right] + h\,d^+d\,N^+N$$

$$+ N^+\left(\frac{\vec{\nabla}^4}{8m_N^3}\right)N + \sigma d^+\left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_N}\right)d + \ldots$$

sign

(omitting spin, isospin)

Bedaque, Hammer + v.K. '98, '99, '00
Hammer, Platter + Meissner '04
Stetcu, Barrett + v.K. '07
Rotureau, Stetcu, Barrett + v.K. '11
...

➡ $A \lesssim 6$

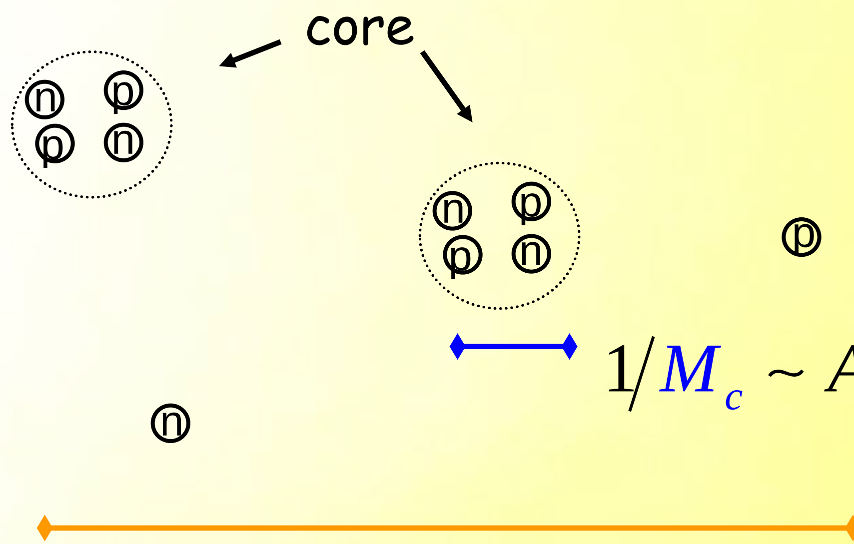
- ✓ describes structure and reactions of bound states --
  deuteron, triton, alpha particle
- ✓ can be extended to p-shell nuclei with No-Core Shell Model
- ✓ makes evident new phenomena --
  from one-parameter three-body force at LO:
  SU(4) invariance, limit-cycle behavior, Phillips line, Efimov spectrum, Tjon line

First orders apply also to atoms $M_{nuc} \to 1/l_{vdW}$ from $V(r) = -\frac{l_{vdW}^4}{2mr^6} + \ldots$

- many-body systems get complicated rapidly, just as for models new scale leads to proliferation of shallow states (near driplines):

loosely bound nucleons around tightly bound cores ("normal" nuclei)

<u>*halo/cluster states*</u>



$1/M_c \sim A_c^{1/3}/M_{nuc}$

$1/\aleph$

separation energy $E_{sep} = \mathcal{O}\left(\frac{\aleph^2}{2m_N}\right) \ll E_{core} = \mathcal{O}\left(\frac{M_c^2}{2m_N}\right) \lesssim \mathcal{O}\left(\frac{m_\pi^2}{2m_N}\right)$ core excitation energy *e.g.* alpha particle family

^{4}He $E_{t+p} \cong$ 20 MeV > 5 MeV $\simeq \dfrac{M_c^2}{2m_N}$ ⇐ $M_c \sim F_\pi / 4^{1/3} \simeq 100$ MeV

^{5}He $p_{3/2}$ resonance $E_{n\alpha} \simeq 0.8$ MeV ⇒ $k_R \simeq \sqrt{2m_N E_{n\alpha}} \simeq 38$ MeV

^{5}Li $p_{3/2}$ resonance $E_{p\alpha} \simeq 1.7$ MeV ⇒ $k_R \simeq \sqrt{2m_N E_{p\alpha}} \simeq 56$ MeV

^{6}He s_0 bound state $E_{nn\alpha} \simeq 0.97$ MeV

^{6}Be s_0 resonance $E_{pp\alpha} \simeq 1.4$ MeV

^{8}Be s_0 resonance $E_{\alpha\alpha} \simeq 0.09$ MeV ⇒ $k_R = \sqrt{m_\alpha E_{\alpha\alpha}} \simeq 18$ MeV

^{9}Be $p_{3/2}$ bound state $E_{n\alpha\alpha} \simeq 1.6$ MeV

^{9}B $p_{3/2}$ resonance $E_{p\alpha\alpha} \simeq 0.19$ MeV

^{12}C s_0 resonance $E_{\alpha\alpha\alpha} \simeq 0.38$ MeV

$\sim \aleph$

etc.

$Q \sim \aleph \ll M_c$

halo/cluster EFT

- degrees of freedom: nucleons, cores
- symmetries: Lorentz, $\not{B}$, $\not{P}$, $\not{T}$
- expansion in:

$$\frac{Q}{M_c} = \begin{cases} Q/m_N \,,\; Q/m_c & \text{non-relativistic} \\ Q/m_\pi \,, \cdots & \text{multipole} \end{cases}$$

simplest formulation: auxiliary fields for cores + nucleon states

e.g.

$$^4\mathrm{He} \mapsto \text{scalar field } \varphi$$

$$^4\mathrm{He} + N \quad \begin{cases} s_{1/2} \equiv 0+ \mapsto \text{spin-0 field } s \\ p_{1/2} \equiv 1- \mapsto \text{spin-1/2 field } T_1 \\ p_{3/2} \equiv 1+ \mapsto \text{spin-3/2 field } T_3 \\ \vdots \end{cases}$$

Bertulani, Hammer + v.K. '02
Bedaque, Hammer + v.K. '03

$$\mathcal{L}_{EFT} = N^{+}\left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_N}\right)N + \varphi^{+}\left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_\alpha}\right)\varphi$$

$$+ T_3^{+}\left[\sigma_3\left(i\partial_0 + \frac{\vec{\nabla}^2}{2(m_\alpha + m_N)}\right) - \Delta_3\right]T_3$$

$$+ \frac{g_3}{\sqrt{2}}\left[T_3^{+}\vec{S}^{+}\cdot(N\vec{\nabla}\varphi - \varphi\vec{\nabla}N) + \text{H.c.}\right]$$

spin transition operator

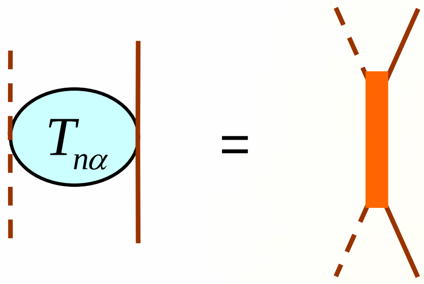
$$+ s^{+}(-\Delta_0)s + \frac{g_0}{\sqrt{2}}\left[s^{+}N\varphi + \text{H.c.}\right]$$

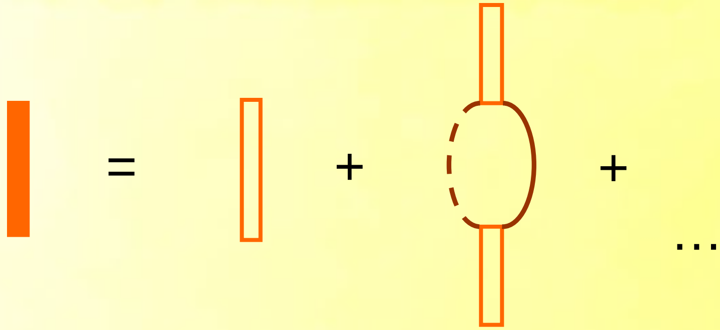
$$+ \ldots$$

$$+ T_1^{+}(-\Delta_1)T_1 + \frac{g_1}{\sqrt{2}}\left[T_1^{+}\vec{\sigma}\cdot(N\vec{\nabla}\varphi - \varphi\vec{\nabla}N) + \text{H.c.}\right]$$

$$+ \ldots$$

$n\alpha$





$p_{3/2}$

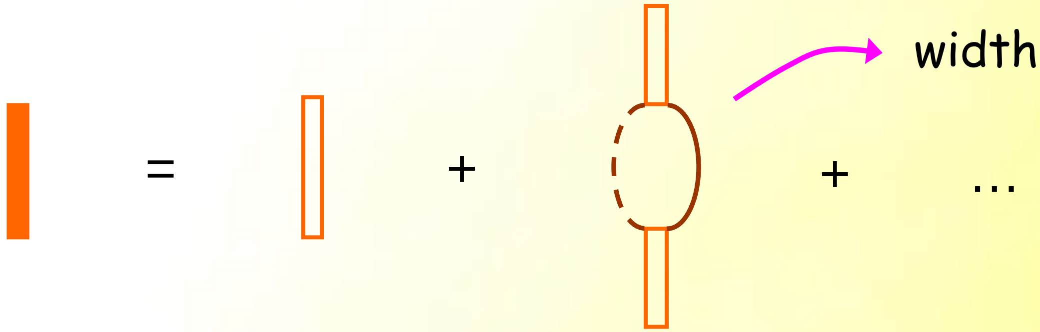
$$= \frac{i\sigma_3}{E - \sigma_3 \Delta_3} = \frac{i\sigma_3 2\mu}{k^2 - \sigma_3 2\mu \Delta_3}$$

reduced mass

resonance at $Q \sim \pm \aleph$

if $\sigma_3 \Delta_3 > 0$ and

$$\Delta_3 \sim \frac{\aleph^2}{\mu}, \quad \frac{g_3^{\ 2}}{4\pi} \sim \frac{1}{\mu^2 M_c}, \ \ldots \quad \Rightarrow \quad a_{1+} \sim \frac{1}{\aleph^2 M_c}, \quad r_{1+} \sim M_c, \ \ldots$$



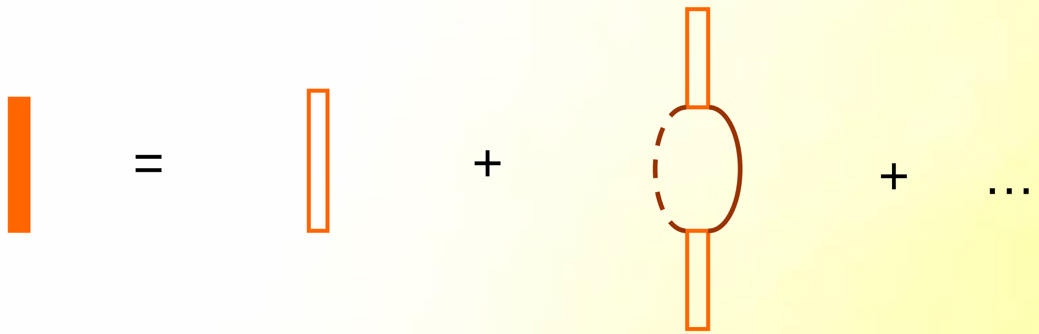
$$\ll 1$$

$$\sim \frac{\mu}{Q^2 - \aleph^2} \sim \left(\frac{\mu}{Q^2 - \aleph^2} \right)^2 \frac{4\pi Q^2}{\mu^2 M_c} \frac{Q^3}{4\pi} \frac{\mu}{Q^2} \sim \frac{\mu}{Q^2 - \aleph^2} \frac{Q^2}{Q^2 - \aleph^2} \frac{Q}{M_c}$$

other waves:

$$\Delta_0 \sim \Delta_1 \sim \ldots \sim M_c\,,$$
$$\frac{g_0^{\,2}}{4\pi} \sim \frac{1}{\mu}, \quad \frac{g_1^{\,2}}{4\pi} \sim \frac{1}{\mu^2 M_c}, \quad \ldots$$

$$\Rightarrow \begin{cases} a_{0+} \sim \dfrac{1}{M_c}, \quad r_{0+} \sim \dfrac{1}{M_c}, \quad \ldots \\ a_{1-} \sim \dfrac{1}{M_c^{\,3}}, \quad r_{1-} \sim M_c\,, \quad \ldots \\ \vdots \end{cases}$$



$s_{1/2}$ $\quad \sim \dfrac{1}{M_c} \quad \sim \left(\dfrac{1}{M_c}\right)^2 \dfrac{4\pi}{\mu}\dfrac{Q^3}{4\pi}\dfrac{Q}{\mu^2} \sim \dfrac{1}{M_c}\dfrac{Q}{M_c}$

$p_{1/2}$ $\quad \sim \dfrac{1}{M_c} \quad \sim \left(\dfrac{1}{M_c}\right)^2 \dfrac{4\pi Q^2}{\mu^2 M_c}\dfrac{Q^3}{4\pi}\dfrac{\mu}{Q^2} \sim \dfrac{1}{M_c}\dfrac{Q^3}{\mu M_c^{\,2}}$

$$\nu = 0 \qquad\qquad \nu = 1 \qquad\qquad \nu = 2$$

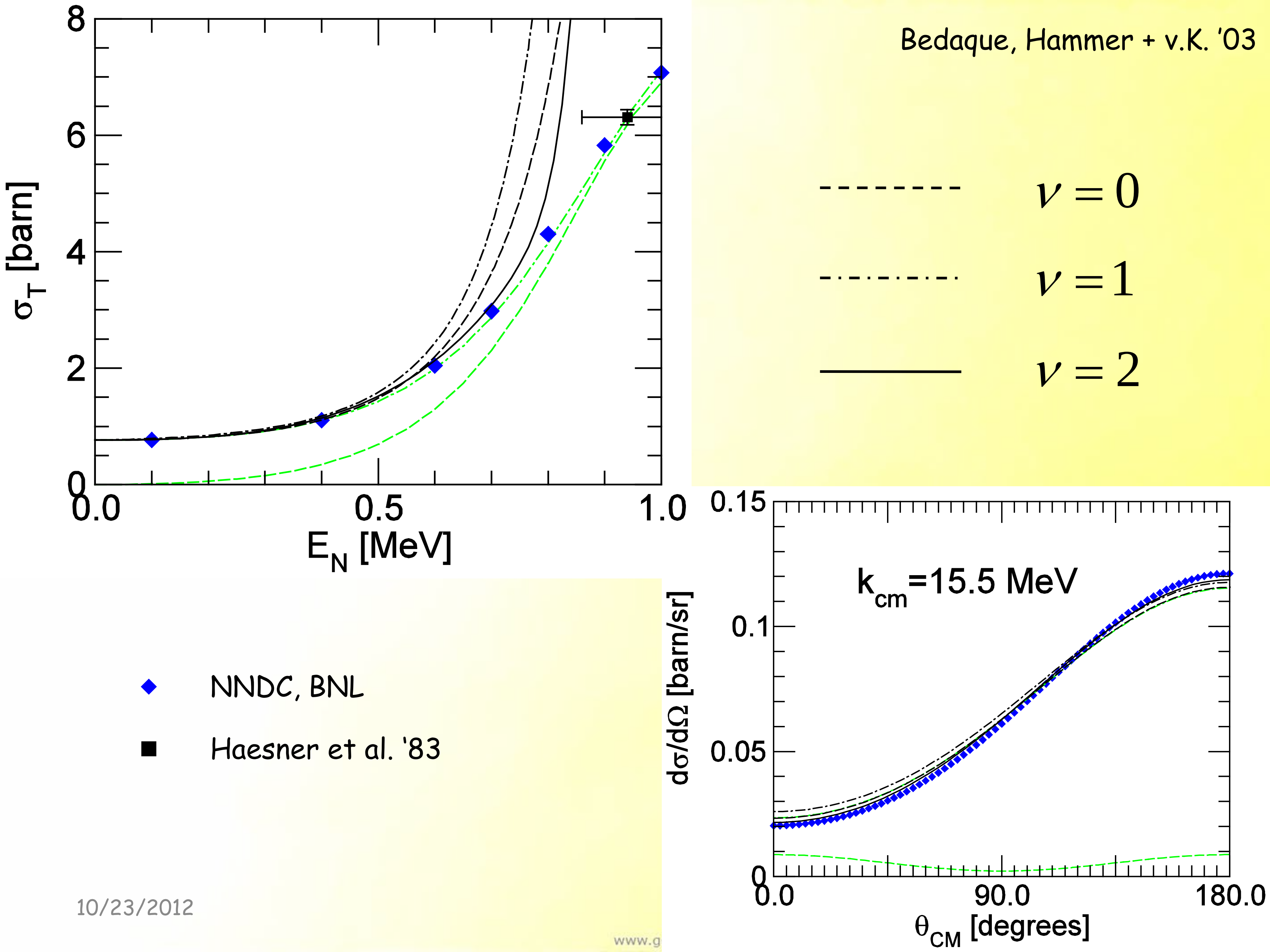
$$T_{n\alpha} \sim \frac{4\pi}{\mu M_c}\left\{\begin{array}{l} \dfrac{Q^2}{Q^2-\aleph^2} + \dfrac{Q}{M_c}\left(\dfrac{Q^2}{Q^2-\aleph^2}\right)^2 + \left(\dfrac{Q}{M_c}\right)^2\left(\dfrac{Q^2}{Q^2-\aleph^2}\right)^3 + \ldots \\ 1 \quad + \quad 0 \quad + \quad \left(\dfrac{Q}{M_c}\right)^2 \quad + \ldots \\ 0 \quad + \quad 0 \quad + \quad 0 \quad + \ldots \\ \ldots \end{array}\right\} \begin{array}{l} p_{3/2} \\ s_{1/2} \\ p_{1/2} \end{array}$$

$$\Rightarrow \quad \left\{\begin{array}{l} a_{1+} \sim \aleph^{-2} M_c^{-1} \\ r_{1+} \sim M_c \end{array}\right. \qquad \mathcal{P}_{1+} \sim M_c^{-1} \qquad a_{1-} \sim M_c^{-3}$$

$$a_{0+} \sim M_c^{-1} \qquad r_{0+} \sim M_c^{-1}$$

etc.

$$T_l(k,\theta) = \frac{2\pi}{\mu}(2l+1)k^{2l}P_l(\cos\theta)\left[-\frac{1}{a_l} + \frac{r_l}{2}k^2 + \frac{\mathcal{P}_l}{4}k^4 + \ldots - ik^{2l+1}\right]^{-1}$$

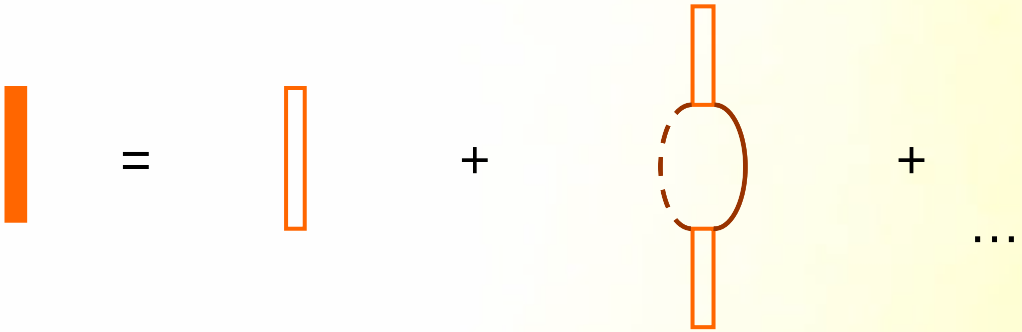


Bedaque, Hammer + v.K. '03
ν = 0
ν = 1
ν = 2
NNDC, BNL
Haesner et al. '83
σ_T [barn]
E_N [MeV]
0
2
4
6
8
0.0
0.5
1.0
k_cm=15.5 MeV
dσ/dΩ [barn/sr]
θ_CM [degrees]
0
0.05
0.1
0.15
0.0
90.0
180.0

10/23/2012

except at $Q = \aleph \pm \mathcal{O}\left(\frac{\aleph^2}{M_c}\right)$ where

$p_{3/2}$

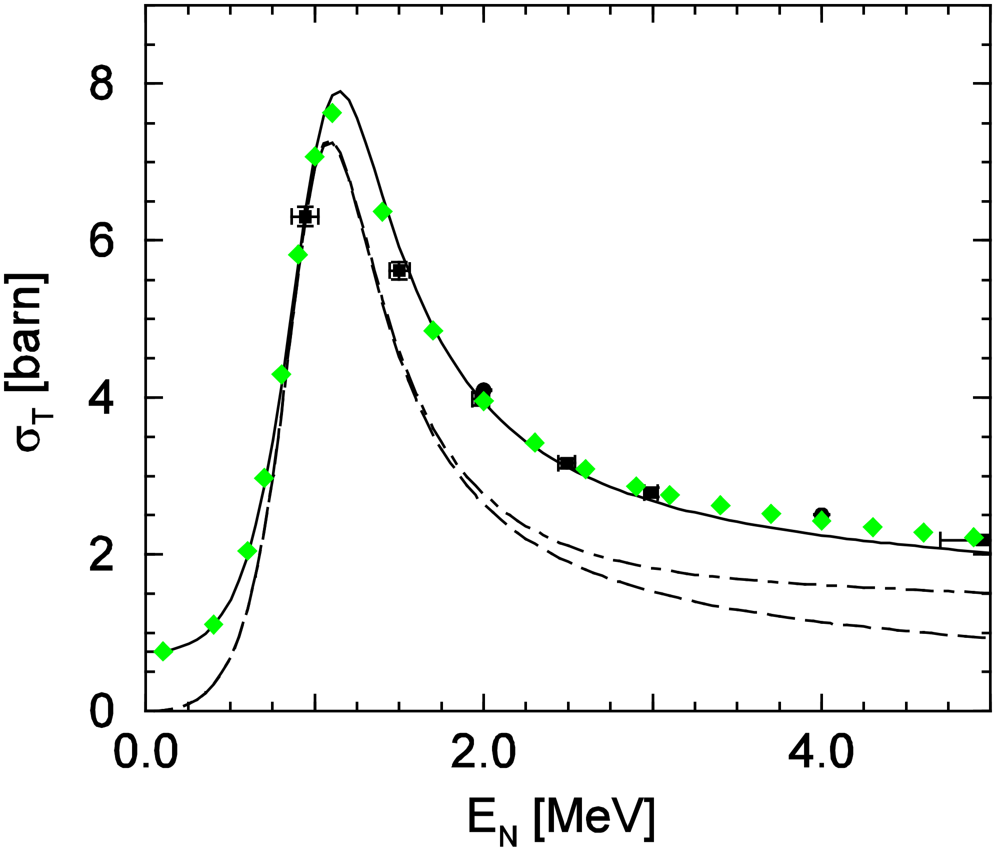


$$\sim \frac{\mu M_c}{\aleph^3} \qquad \sim \left(\frac{\mu M_c}{\aleph^3}\right)^2 \frac{4\pi \aleph^2}{\mu^2 M_c} \frac{\aleph^3}{4\pi} \frac{\mu}{\aleph^2} \sim \frac{\mu M_c}{\aleph^3}$$

⇨ enhanced by $\frac{M_c}{\aleph}$ ⇨ resum self-energy

$$T_{1+}^{(-1)} = \frac{2\pi}{\mu\sqrt{2\mu E}} \frac{i\,\Gamma(E)/2}{E - E_R + i\,\Gamma(E)/2}$$

Bertulani, Hammer + v.K. '02



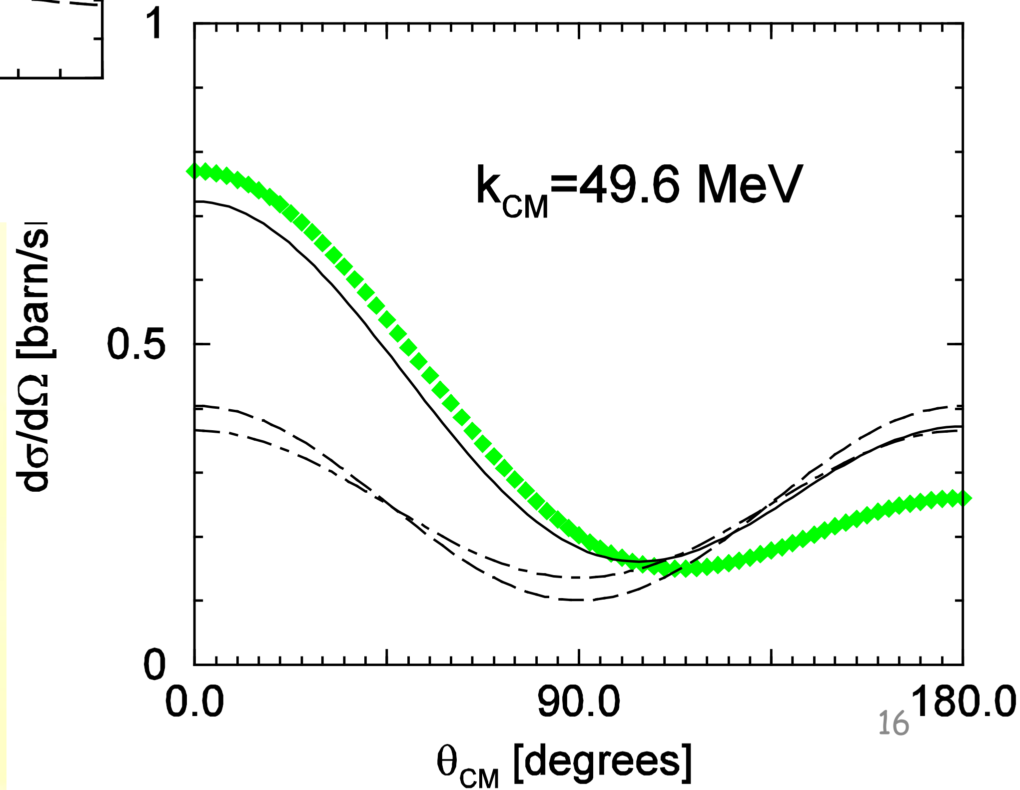
$\nu = -1$ (dashed line)

$\nu = 0$ (solid line)

$\nu = -1\,\mathrm{mod}$ (dash-dotted line)

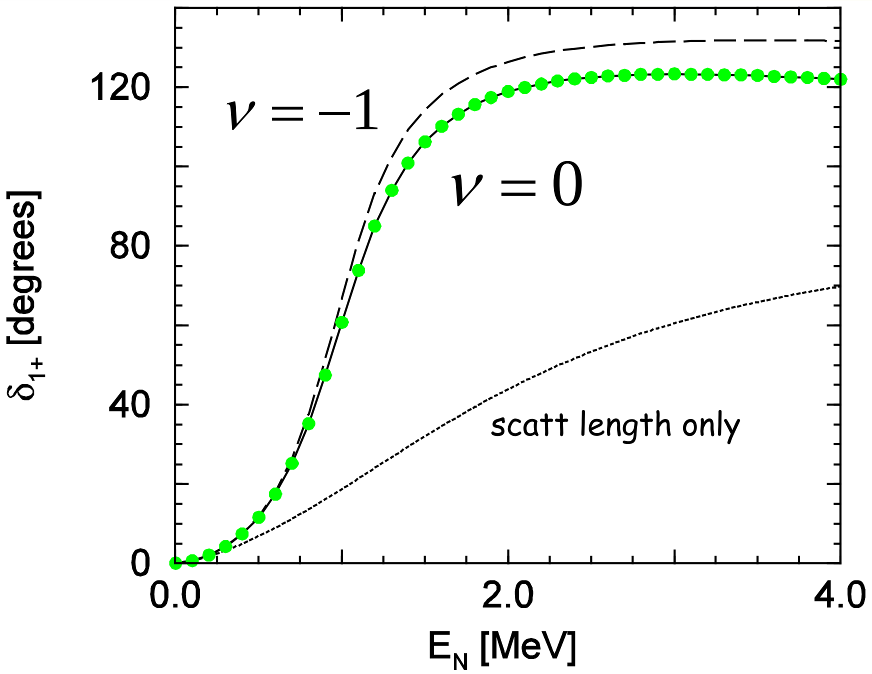
◆ NNDC, BNL

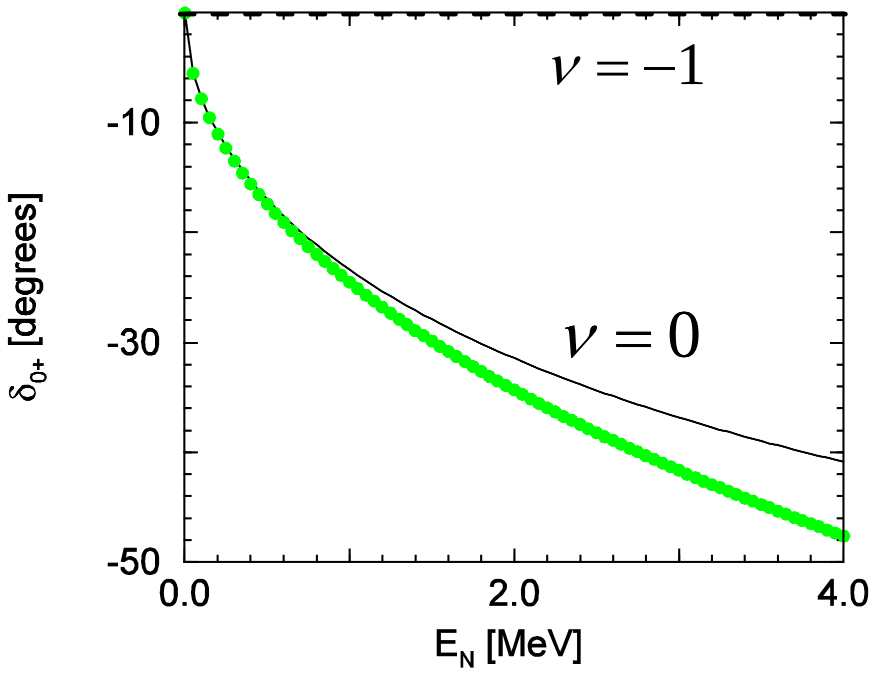
■ Haesner et al. '83

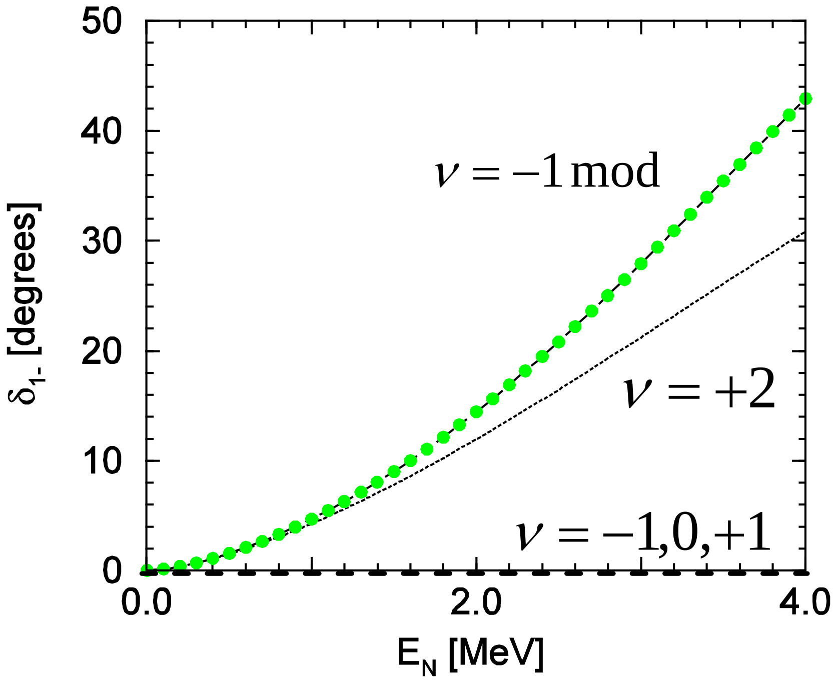


Bertulani, Hammer + v.K. '02

● PSA, Arndt et al. '73







$$E_R \cong 0.80 \text{ MeV}$$

$$\Gamma(E_R) \cong 0.55 \text{ MeV}$$

$$^5\text{He} = \text{res.}\left(^4\text{He} + n\right)$$

Arndt et al '73

Partial wave $l_\pm$	$a_{l\pm}$ [fm^{1+2l}]	$r_{l\pm}$ [fm^{1-2l}]	$\mathcal{P}_{l\pm}$ [fm^{3-2l}]
0+	2.4641(37)	1.385(41)	–
1−	−13.821(68)	−0.419(16)	–
1+	−62.951(3)	−0.8819(11)	−3.002(62)

$$a_{0+} \sim M_c^{-1} \qquad r_{0+} \sim M_c^{-1}$$

cf.

$$a_{1-} \sim M_c^{-3} \qquad r_{1-} \sim M_c$$

$$a_{1+} \sim \aleph^{-2} M_c^{-1} \qquad r_{1+} \sim M_c \qquad \mathcal{P}_{1+} \sim M_c^{-1}$$

$$\Rightarrow \begin{cases} M_c \simeq 100\,\text{MeV} \\ \aleph \simeq 30\,\text{MeV} \end{cases}$$

consistent…

Other two-body states

$$\begin{cases} {}^{5}\mathrm{Li} = \mathrm{res.}\left({}^{4}\mathrm{He} + p\right) \\ {}^{8}\mathrm{Be} = \mathrm{res.}\left({}^{4}\mathrm{He} + {}^{4}\mathrm{He}\right) \end{cases}$$

Higa, Bertulani + v.K. in progress

Higa, Hammer + v.K. '08

Main issues:
role of Coulomb, further fine-tuning…

Next: three-body states

$$\begin{cases} {}^{6}\mathrm{He} = \mathrm{b.s.}\left({}^{4}\mathrm{He} + n + n\right) \\ {}^{6}\mathrm{Be} = \mathrm{res.}\left({}^{4}\mathrm{He} + p + p\right) \\ {}^{9}\mathrm{Be} = \mathrm{b.s.}\left({}^{4}\mathrm{He} + {}^{4}\mathrm{He} + n\right) \\ {}^{9}\mathrm{B} = \mathrm{res.}\left({}^{4}\mathrm{He} + {}^{4}\mathrm{He} + p\right) \\ {}^{12}\mathrm{C} = \mathrm{b.s.}\left({}^{4}\mathrm{He} + {}^{4}\mathrm{He} + {}^{4}\mathrm{He}\right) \end{cases}$$

cf.

Bedaque, Hammer + v.K. '98

$${}^{3}\mathrm{H} = \mathrm{b.s.}\left(p + n + n\right)$$

Ando + Birse '10
König + Hammer '11

$${}^{3}\mathrm{He} = \mathrm{b.s.}\left(p + p + n\right)$$

in pionless EFT

Main issue:
three-body force in LO as in pionless EFT?

Michel, Nazarewicz, Ploszajczak + Bennaceur '02
...

Gamow Shell Model

$$H = \sum_{i=1}^{K} h_i + H_{\text{int}}$$

$$H\left|\Psi\right\rangle = E\left|\Psi\right\rangle$$

$$\left|\Psi\right\rangle = \sum_{n=1}^{N_{sh}} c_n \left|\psi_n\right\rangle$$

single-particle basis

$$h\left|\psi_n\right\rangle = e_n\left|\psi_n\right\rangle$$

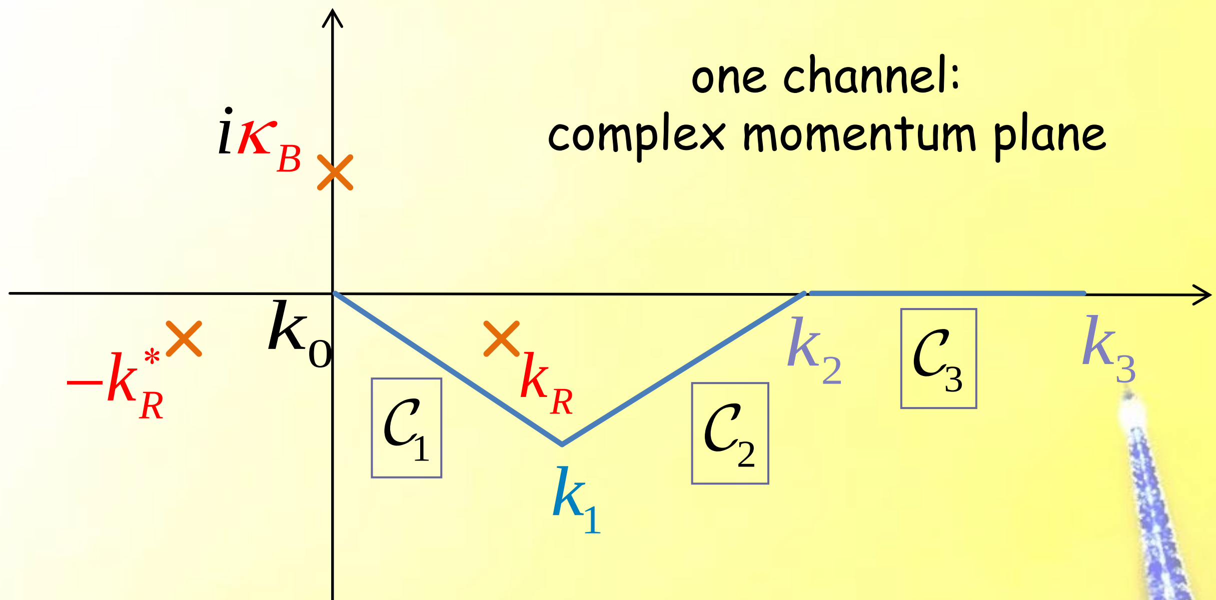
$$\int_{\mathcal{C}} dk\, k^2 \left|k\right\rangle\left\langle k\right| = 1$$

$$\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3$$

discretized

$$N_{sh} = N_1 + N_2 + N_3$$

one channel:
complex momentum plane



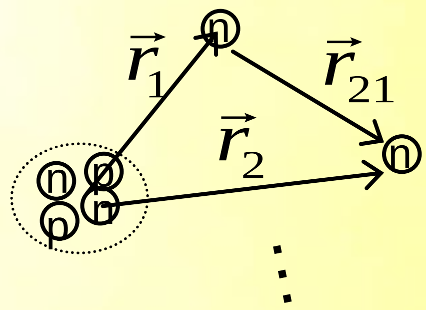
$$\sum_{B,R} \left|\Psi_{B,R}\right\rangle\left\langle\tilde{\Psi}_{B,R}\right| + \int_{\mathcal{C}} dk\, k^2 \left|\Psi(k)\right\rangle\left\langle\tilde{\Psi}(k)\right| = 1$$

Berggren '68

$$\left\langle\tilde{\Psi}\middle|r\right\rangle \equiv \left\langle r\middle|\Psi\right\rangle$$

Rotureau + v.K. '12

$$^{5+\ldots}\mathrm{He} = \text{b.s. or res.}\ \left(^4\mathrm{He} + n + \ldots\right)$$



$$\begin{cases} h_i = \dfrac{\vec{p}_i^{\,2}}{2\mu} + V_{\alpha i} \\[2ex] H_{\text{int}} = \dfrac{1}{2}\displaystyle\sum_{i\neq j=1}^{N-2}\left(\dfrac{\vec{p}_i\cdot\vec{p}_j}{m_\alpha} + V_{ij} + V_{\alpha ij}\right) \end{cases}$$

$$V_{\alpha i}^{(-1)}\left(\vec{p}_i', \vec{p}_i, k_i\right) = \frac{\vec{p}_i'\cdot\vec{p}_i}{A\left(\Lambda_{\alpha n}\right) + B\left(\Lambda_{\alpha n}\right)k_i^2} F\left(p_i'^2/\Lambda_{\alpha n}^2\right)F\left(p_i^2/\Lambda_{\alpha n}^2\right)$$

$$V_{ij}^{(-1)}\left(p_{ij}', p_{ij}\right) = C\left(\Lambda_{nn}\right)F\left(p_{ij}'^2/\Lambda_{nn}^2\right)F\left(p_{ij}^2/\Lambda_{nn}^2\right)$$

$$V_{\alpha ij}^{(-1)}\left(\vec{p}_i', \vec{p}_j', \vec{p}_i, \vec{p}_j\right) = D\left(\Lambda_{\alpha nn}\right)\vec{p}_i'\cdot\vec{p}_i\ \vec{p}_j'\cdot\vec{p}_j$$
$$\times F\left(p_i'^2/\Lambda_{\alpha nn}^2\right)F\left(p_j'^2/\Lambda_{\alpha nn}^2\right)F\left(p_i^2/\Lambda_{\alpha nn}^2\right)F\left(p_j^2/\Lambda_{\alpha nn}^2\right)$$

$F(0) = 1$
$F(x \gg 1) \ll 1$
Here, $F(x) = \exp(-x)$

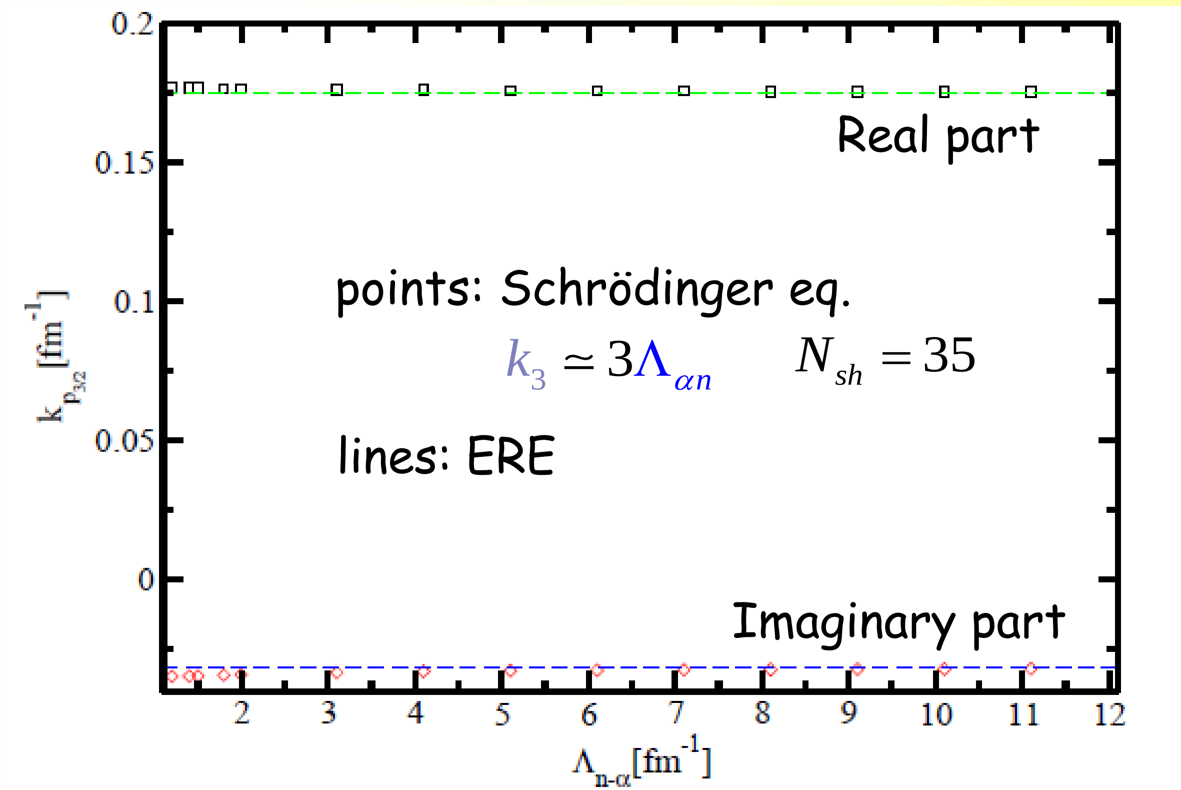
- $A\left(\Lambda_{\alpha n}\right), B\left(\Lambda_{\alpha n}\right), C\left(\Lambda_{nn}\right)$ known functions of ERE parameters and cutoffs
- $D\left(\Lambda_{\alpha nn}\right)$ 3BF: similar, but not quite the same as changing V_{ij}

Rotureau + v.K. '12

$$ {}^{5}\mathrm{He} = \mathrm{res.}\left({}^{4}\mathrm{He} + n \right) $$

$p_{3/2}$

$$ \begin{cases} A(\Lambda_{\alpha n}) = -2\mu\left(-\dfrac{1}{a_{1+}} + \dfrac{\Lambda_{\alpha n}^{3}}{4\sqrt{2\pi}} \right) \\ B(\Lambda_{\alpha n}) = -2\mu\left(\dfrac{r_{1+}}{2} + \dfrac{2}{a_{1+}\Lambda_{\alpha n}^{2}} + \dfrac{\Lambda_{\alpha n}}{\sqrt{2\pi}} \right) \end{cases} $$



Complications with basis:

- eigenstates of *h* not orthogonal and Berggren relation does not hold
- a deep bound states appears for large cutoff

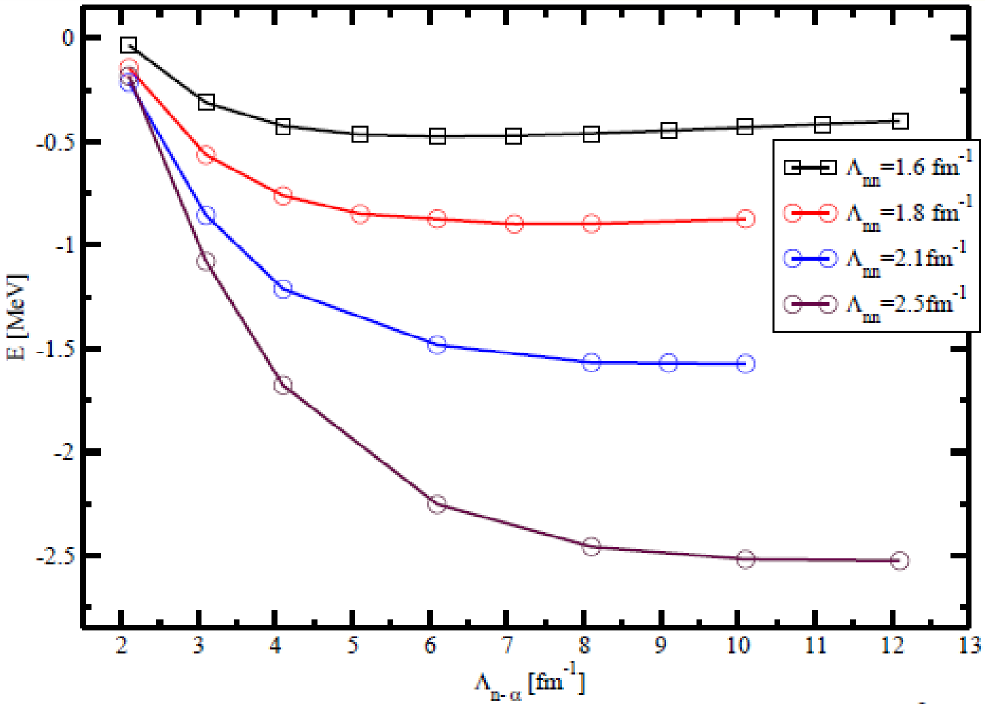
Way around: convert energy to momentum dependence

$$\langle k' | V'_{\alpha n} | \psi_n \rangle = \langle k' | V_{\alpha n}(k = \sqrt{2\mu e_n}) | \psi_n \rangle \qquad \text{with} \quad |\psi_n\rangle \neq |\psi_B\rangle$$

non -Hermitian

$$\longrightarrow \sum_n |\psi_n\rangle \langle \psi_n^{left}| = 1$$

Rotureau
+ v.K. '12

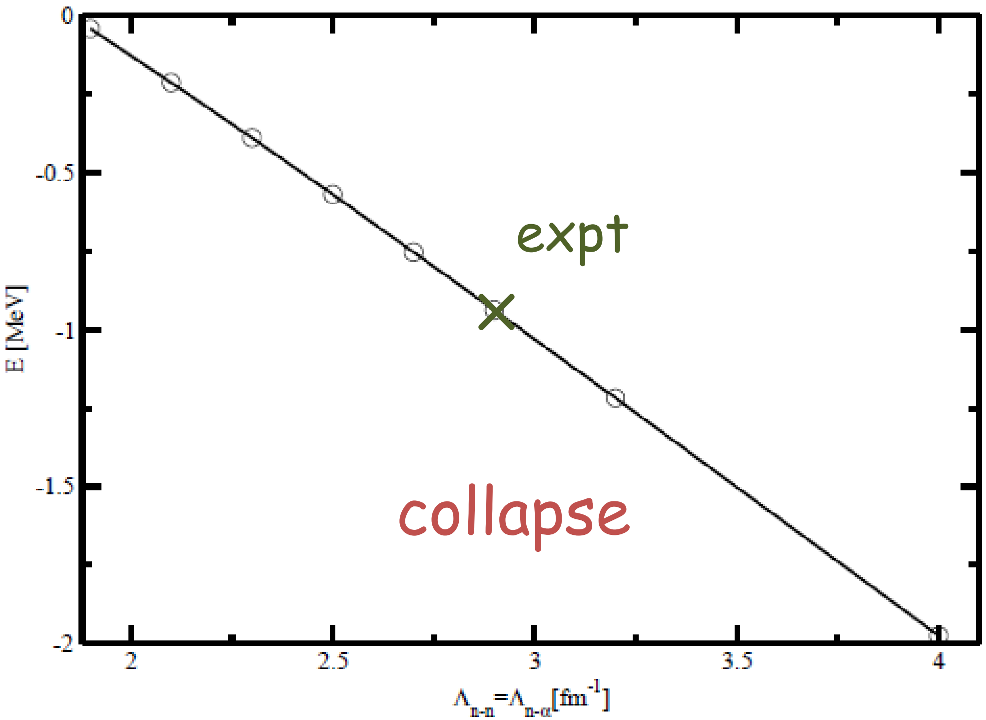


$$^{6}\text{He} = \text{b.s.}\left(^{4}\text{He} + n + n\right)$$

$$C^{-1}\left(\Lambda_{nn}\right) = -m_n\left(-\frac{1}{a_{nn}} + \frac{\Lambda_{nn}}{\sqrt{2\pi}}\right)$$

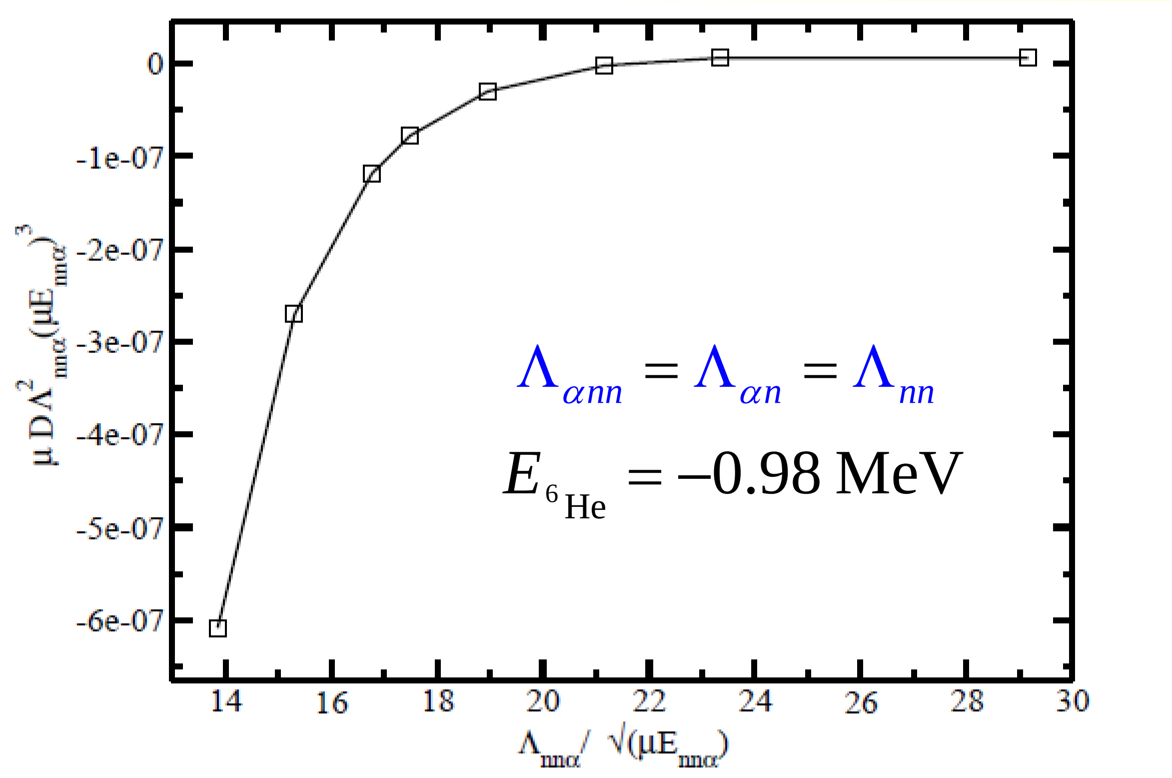
$$D\left(\Lambda_{\alpha nn}\right) = 0$$

no RG invariance = no good



Rotureau + v.K. '12

3BF is LO!



predictive power?

- Other ^{6}He observables
- $^8He = \text{b.s.}\left(^4He + n + n + n + n\right)$, *etc.*

Rotureau + v.K. in progress

Other cores

$^{8}\mathrm{Li} = \mathrm{b.s.}\left({}^{7}\mathrm{Li} + n\right)$

$^{7}\mathrm{Li}(n,\gamma)\,^{8}\mathrm{Li}$

Rupak + Higa '11
Fernando, Higa + Rupak '11

$^{11}\mathrm{Be} = \mathrm{b.s.}\left({}^{10}\mathrm{Be} + n\right)$

Coulomb dissociation of ^{11}Be

Hammer + Phillips '11

$^{A}\mathrm{Z} = \mathrm{b.s.}\left({}^{A-2}\mathrm{Z} + n + n\right)$

Efimov states?

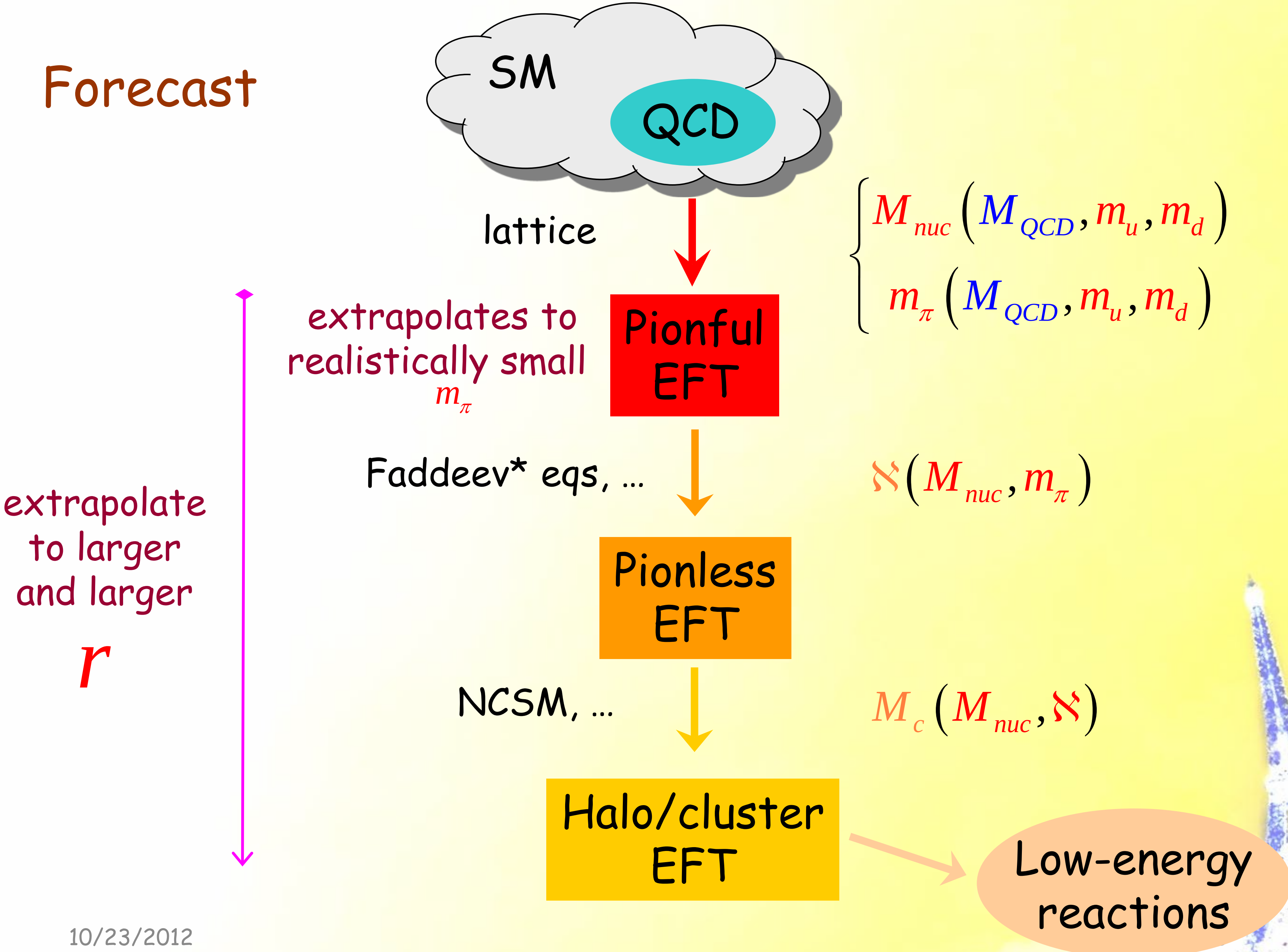
Canham + Hammer '08, '10

s-wave interaction

spin 0

$^{A-2}\mathrm{Z} = {}^{9}\mathrm{Li}, {}^{10}\mathrm{Be}, {}^{12}\mathrm{Be}, {}^{16}\mathrm{C}, {}^{18}\mathrm{C}, \ldots$

Forecast


SM
QCD
lattice
Pionful EFT
extrapolates to realistically small m_π
$M_{nuc}(M_{QCD}, m_u, m_d)$
$m_\pi(M_{QCD}, m_u, m_d)$
Faddeev* eqs, …
$\aleph(M_{nuc}, m_\pi)$
Pionless EFT
NCSM, …
$M_c(M_{nuc}, \aleph)$
Halo/cluster EFT
Low-energy reactions
extrapolate to larger and larger
r