Electroweak response of ^{4}He and its level structure

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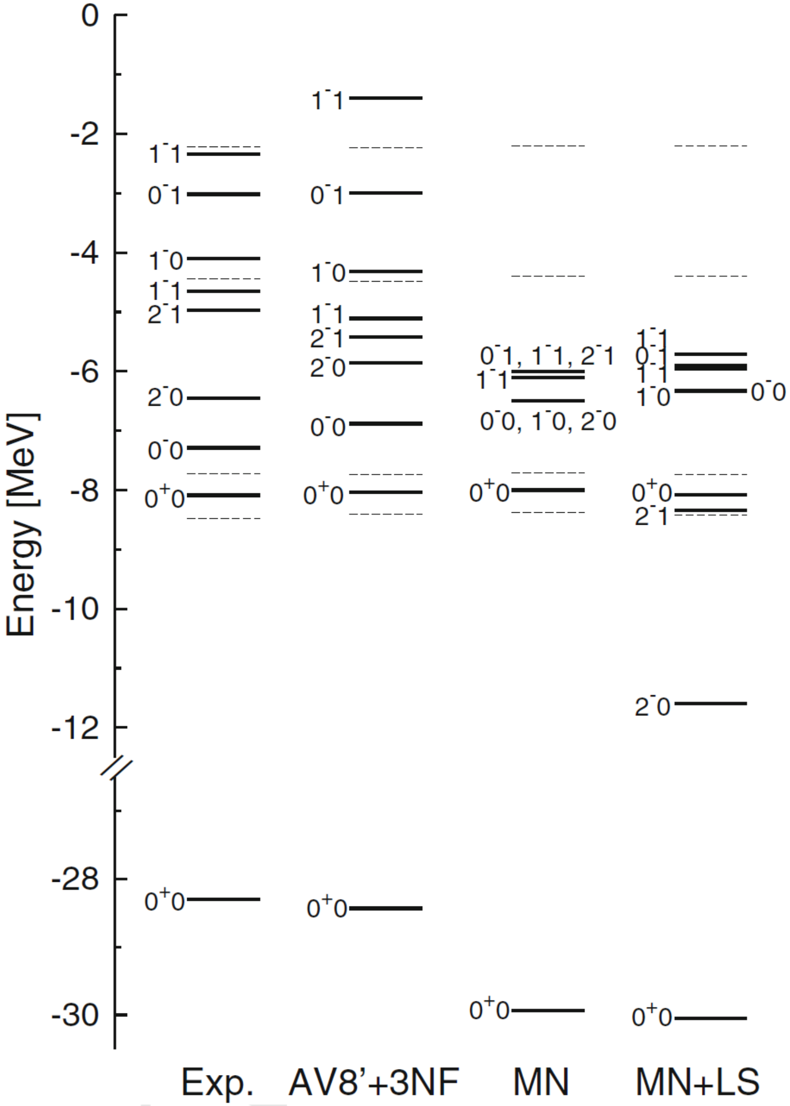
Outline

- Spectrum of ^{4}He: Correlated Gaussians
- Application of square-integrable basis to continuum problems
- Photoabsorption and spin-dipole response of ^{4}He

In collaboration with W. Horiuchi (Hokkaido) and K. Arai (Nagaoka)

INT workshop Nov.5-9 2012: Electroweak properties of light nuclei

Spectrum of ^{4}He



- The first excited state is 0^+ but not a negative parity
- A variational calculation with realistic forces reproduces the spectrum fairly well
- Tensor force is crucial to account for the level splitting
- Most levels are broad resonances that can be excited by spin-dipole and electric-dipole operators
  Study of EW response is interesting

W.Horiuchi, Y.S., PRC78 (2008)

Basis functions

LS coupling

$$\Psi^{\pi}_{JM_J,TM_T} = \sum_{LS} C_{LS,T}\, \Phi^{\pi}_{(LS)JM_J,TM_T}$$

$$\Phi^{\pi}_{(LS)JM_J,TM_T} = \mathcal{A}\left[\phi^{\pi}_{L}\,\chi_S\right]_{JM_J}\eta_{TM_T}$$

Spin part

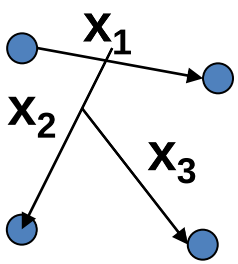
$$\chi_{(S_{12}S_{123}\ldots)SM_S} = [\ldots[[\chi_{\frac{1}{2}}(1)\chi_{\frac{1}{2}}(2)]_{S_{12}}\chi_{\frac{1}{2}}(3)]_{S_{123}}\ldots]_{SM_S}$$

Orbital part

$$\phi^{\pi}_{(L_1L_2)LM_L}(A,u_1,u_2) = \exp(-\tilde{\boldsymbol{x}}A\boldsymbol{x})[\mathcal{Y}_{L_1}(\tilde{u}_1\boldsymbol{x})\mathcal{Y}_{L_2}(\tilde{u}_2\boldsymbol{x})]_{LM_L}$$

$\boldsymbol{x}=(\boldsymbol{x}_i)$ **A set of relative coordinates**

$$F_{\ell m}(\boldsymbol{r}) \approx \sum_a C_a \underline{\exp(-ar^2)}\underline{r^\ell Y_{\ell m}(\hat{\boldsymbol{r}})}$$



Extension to N-particle system

Explicitly correlated Gaussian (ECG)

$$\exp(-ar^2) \rightarrow \exp\left[-\sum_{i<j} a_{ij}(\boldsymbol{r}_i - \boldsymbol{r}_j)^2\right] = \exp\left(-\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)$$

$$\boldsymbol{r}_i - \boldsymbol{r}_j = c_{ij}^{(1)}\boldsymbol{x}_1 + \ldots c_{ij}^{(N-1)}\boldsymbol{x}_{N-1}$$

$$\tilde{\boldsymbol{x}}A\boldsymbol{x} = \sum_{i,j} A_{ij}\boldsymbol{x}_i \cdot \boldsymbol{x}_j \qquad A_{ji} = A_{ij}$$

S.F. Boys K. Singer
Proc. R. Soc. London, Ser. A258 (1960)

spherical motion

Angular functions with global vectors (GV)

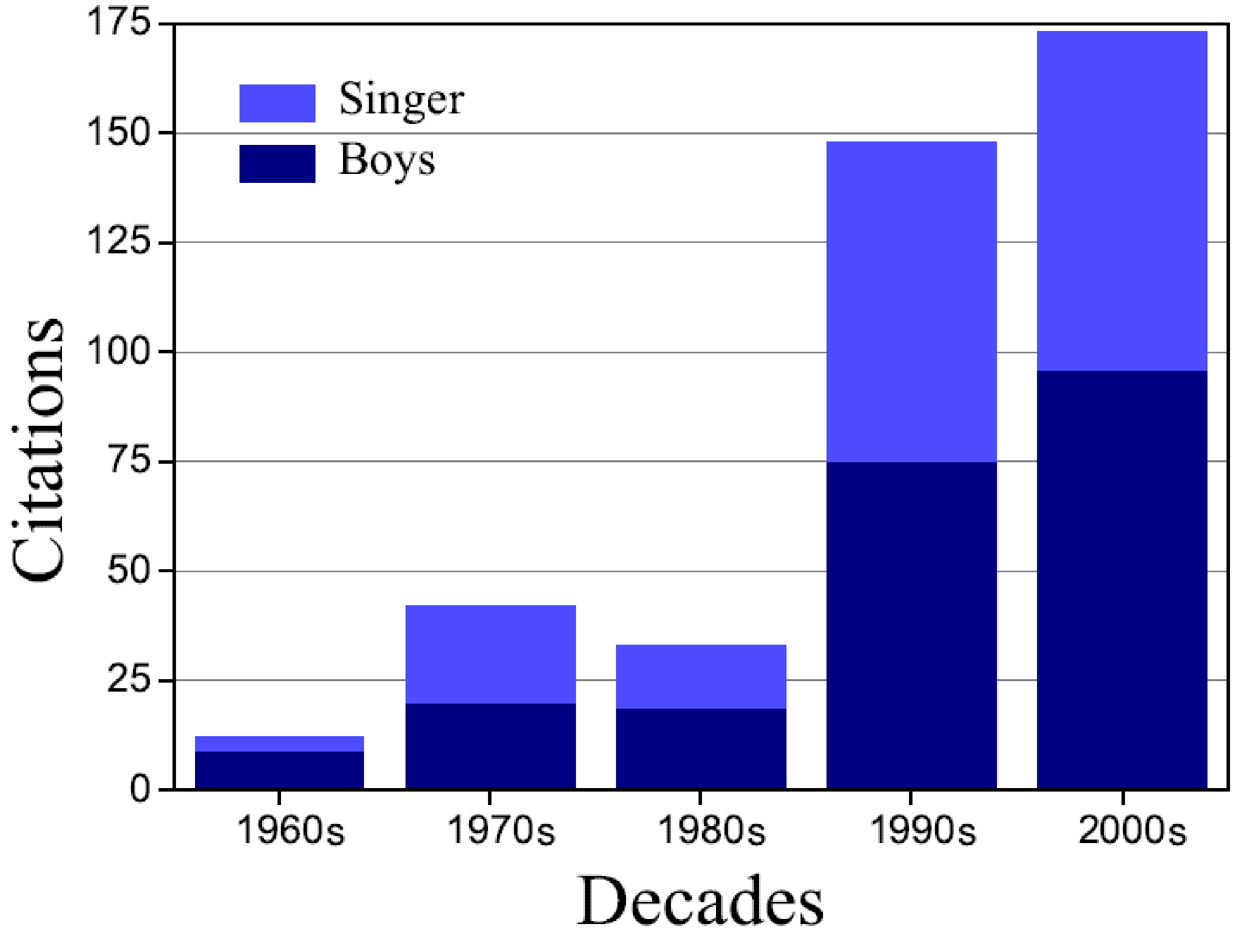
$$\boldsymbol{r} \rightarrow u_1\boldsymbol{x}_1 + u_2\boldsymbol{x}_2 + \ldots + u_{N-1}\boldsymbol{x}_{N-1} = \widetilde{u}\boldsymbol{x}$$

$$r^\ell Y_{\ell m}(\hat{\boldsymbol{r}}) \rightarrow |\widetilde{u}\boldsymbol{x}|^L Y_{LM}(\widehat{\widetilde{u}\boldsymbol{x}}) = \mathcal{Y}_{LM}(\widetilde{u}\boldsymbol{x})$$

parameters A_{ij} u_i

Y. S. and K. Varga, *Stochastic variational approach to quantum-mechanical few-body problems*, Lecture Notes in Physics 54 (Springer, 1998).
K. Varga and Y. S., Phys. Rev. C52, 2885 (1995).

Number of citations by decade to the original works (Boys, 1960) and (Singer, 1960)



Theory and application of explicitly correlated Gaussians

submitted to RMP **atomic, molecular, condensed matter, nuclear**

AV8' + Coulomb+3NF

$$V_q = \sum_{i<j} v^{(q)}(r_{ij})\mathcal{O}_{ij}^{(q)}$$

q=4,6
OPEP

$$1,\ \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j,\ \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\ \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$

$$S_{ij},\ S_{ij}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\ (\boldsymbol{L}\cdot\boldsymbol{S})_{ij},\ (\boldsymbol{L}\cdot\boldsymbol{S})_{ij}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$

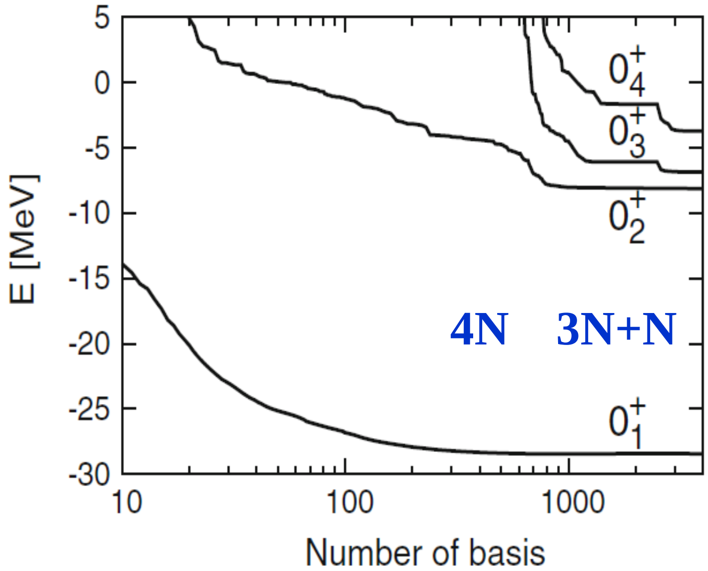
Phenomenological 3NF

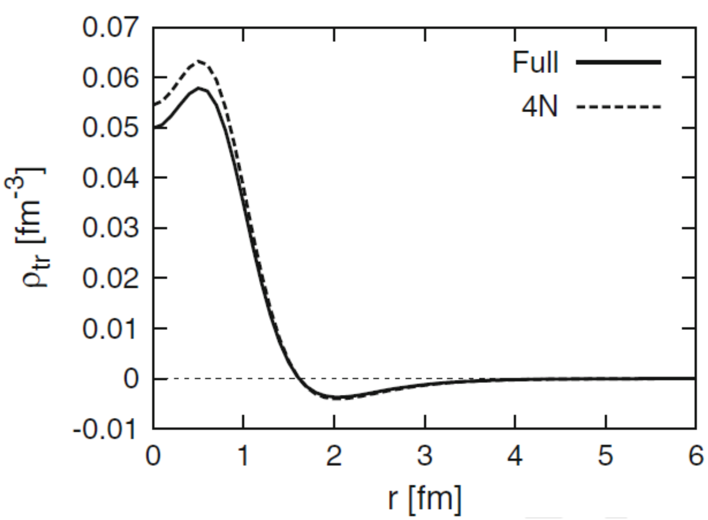
$0_1 \rightarrow 0_2$ Transition density & monopole ME

$$\rho_{\rm tr}(r) = \frac{1}{4\pi}\langle\Psi(0_2^+)|\sum_{i=1}^{4}\frac{\delta(|\boldsymbol{r}_i-\boldsymbol{x}_4|-r)}{r^2}|\Psi(0_1^+)\rangle$$

$$\int_0^\infty \rho_{\rm tr}(r)r^2dr = 0$$

Convergence with SVM





$$\int_0^\infty \rho_{\rm tr}(r)r^4dr$$

$$\propto \langle 0_2^+|r_p^2|0_1^+\rangle$$

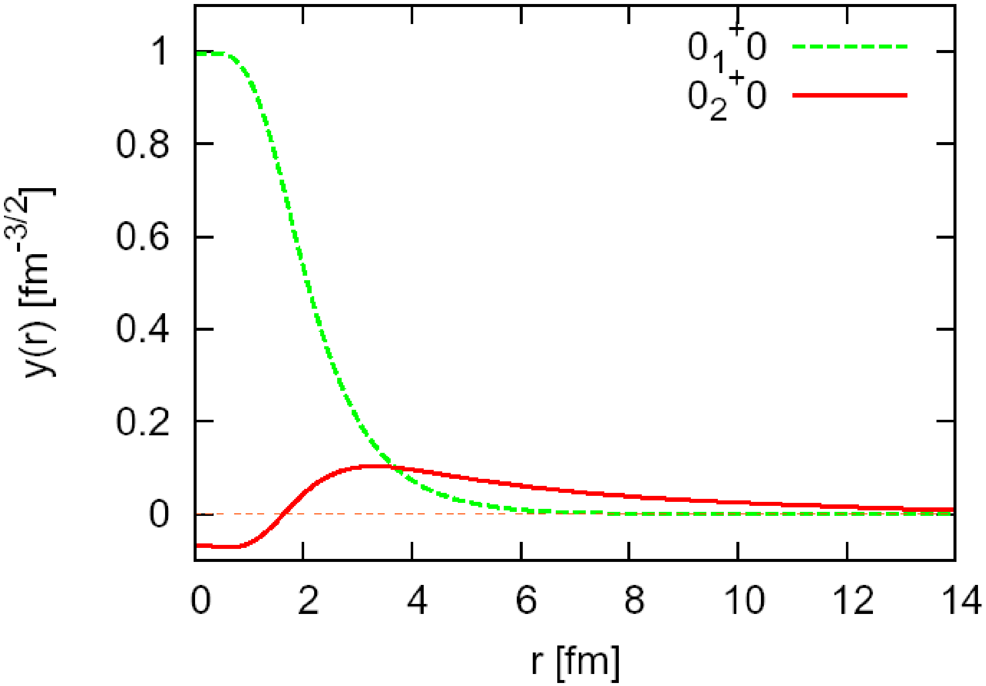
$$= 1.45\ (1.57)\ {\rm fm}^2$$

contributed from
$r \gtrsim 3$ fm

Spectroscopic amplitudes for (3N)+N decay

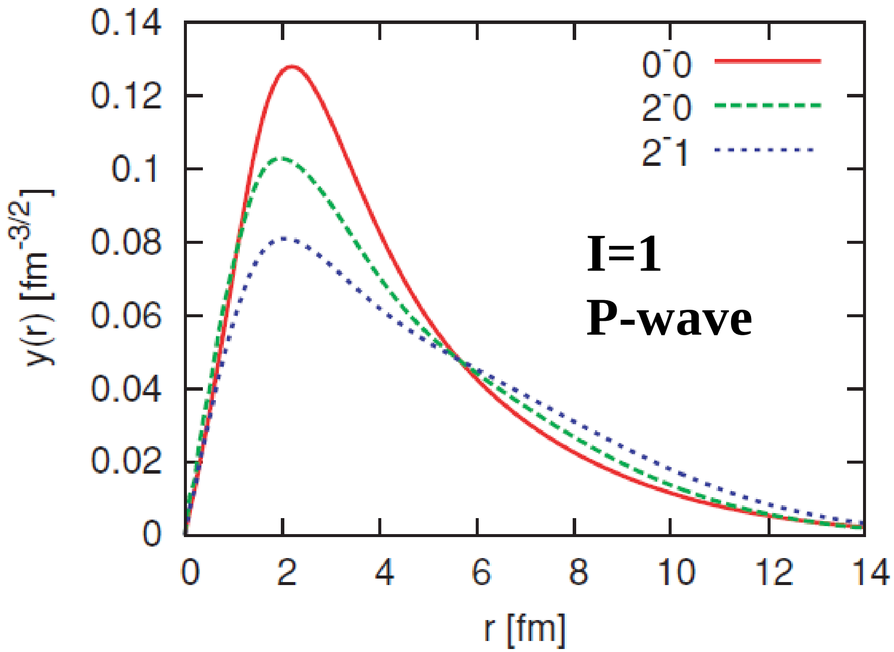
$$\left\langle \left[\left[\Psi_{\frac{1}{2},\frac{1}{2}m_t}(3N)\phi_{\frac{1}{2},\frac{1}{2}-m_t}(N)\right]_I Y_\ell(\hat{\boldsymbol{R}})\right]_{JM_J} \middle| \Psi_{JM_JT0}(^4\mathrm{He})\right\rangle$$

channel spin I=0, ℓ=0 (S-wave)



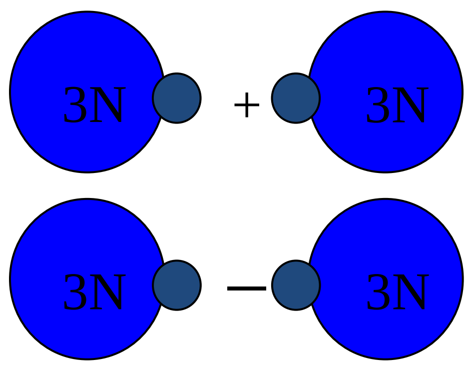
The first excited 0^+ state has 3N+N cluster structure

Spectroscopic amplitudes for P-wave decay of three lowest negative-parity states



3N+N cluster structure

Inversion doublet



Channel spin I=0
S-wave

Channel spin I=1
P-wave

Application of square-integrable basis to continuum problems

Problems including continuum states

- **Decay of resonance**

  A* $\longrightarrow$ B+b,   C+d+e

- **Strength (response) function due to perturbation W**

  A+W $\longrightarrow$ A*,   B+b,   C+d+e

- **Radiative capture reactions**

  A+a $\longrightarrow$ C+γ

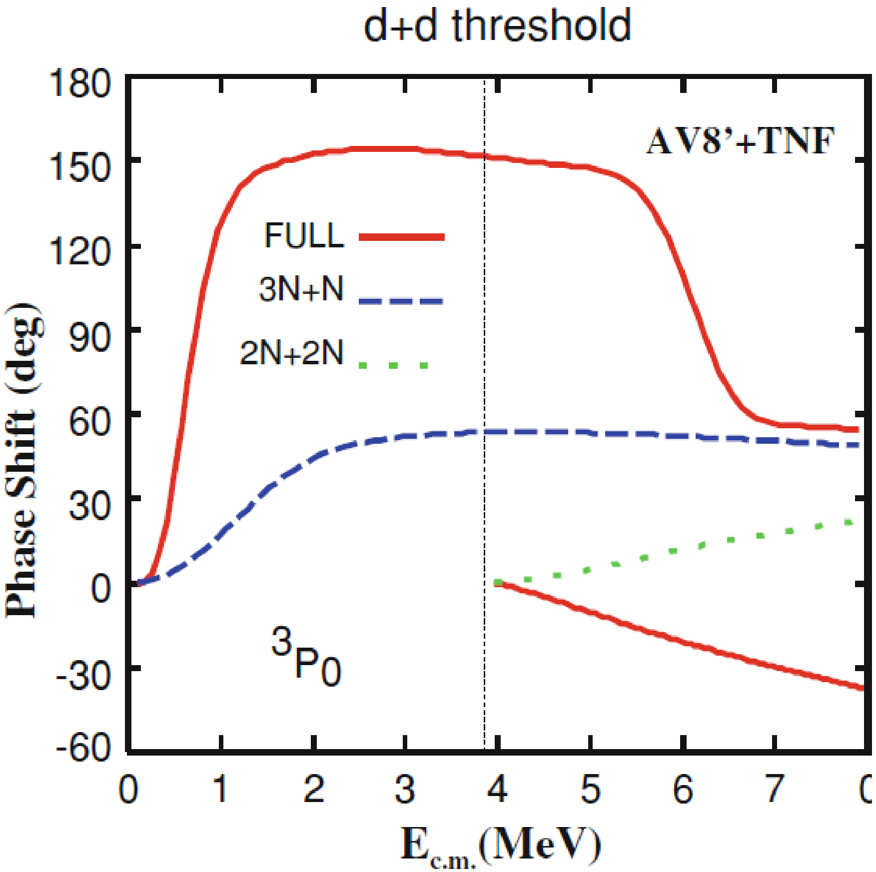
  (Inverse process (photodisintegration):   C+γ $\longrightarrow$ A+a)

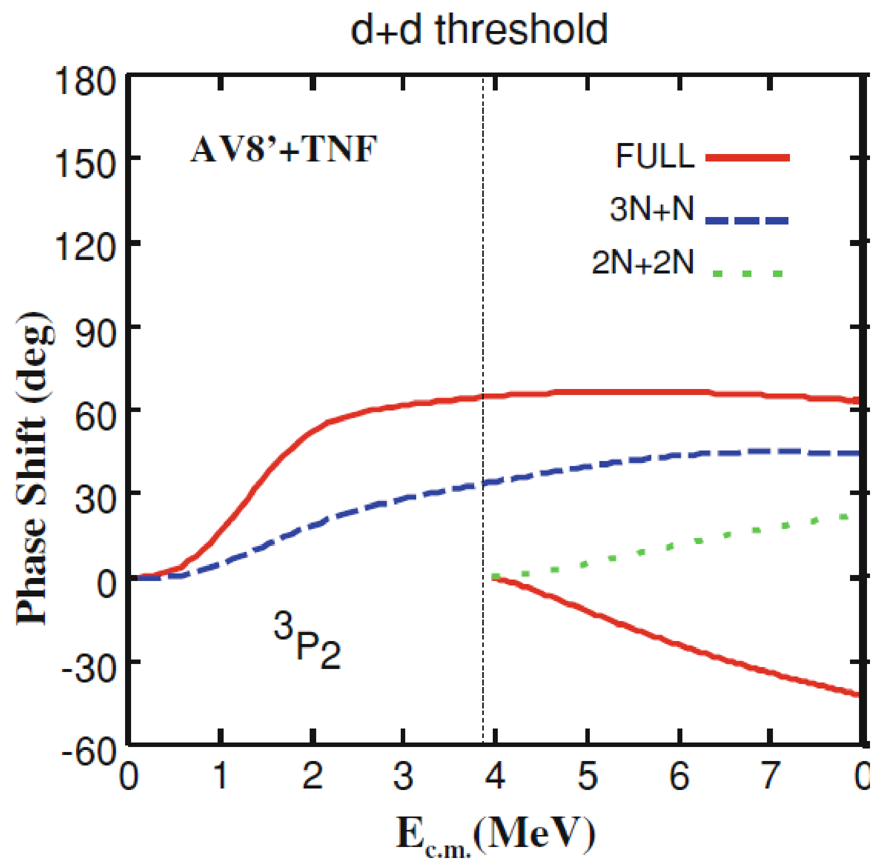
- **Two-body scattering and reactions**

  A+a $\longrightarrow$ B+b

Four-nucleon scattering with ECG + MRM

(Microscopic R-matrix method)

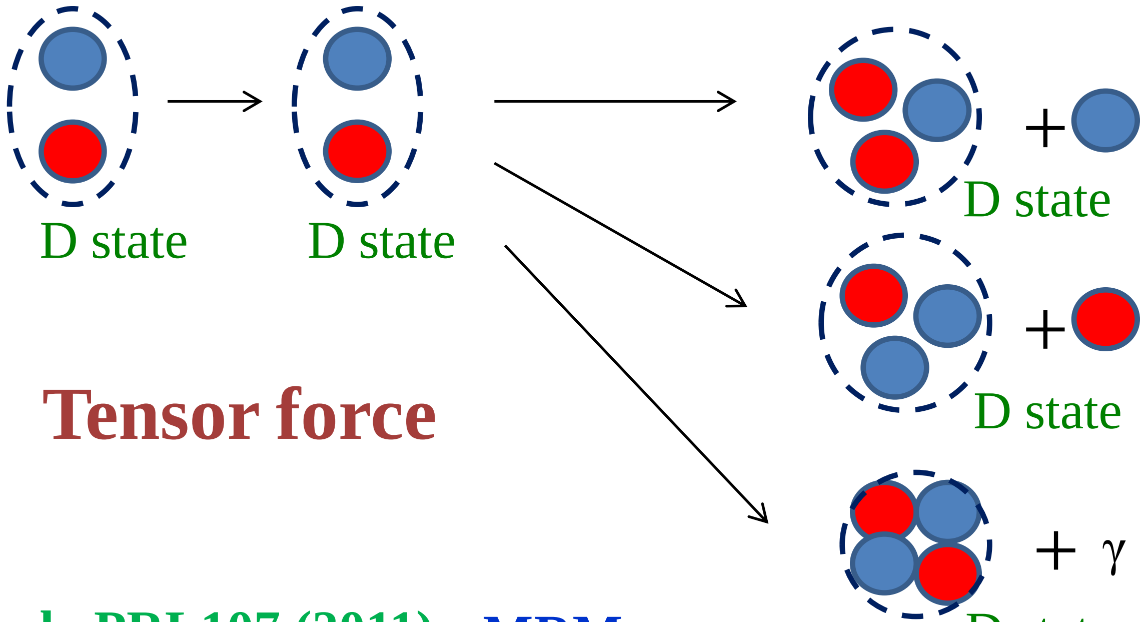




0^- resonance is clearly seen but 2^- resonance is not that clear

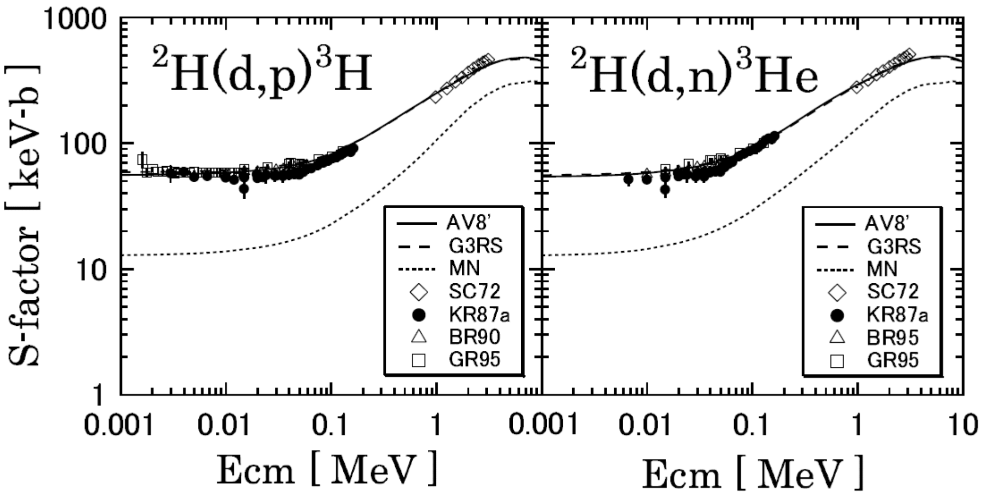
S.Aoyama, K.Arai, Y.S., P.Descouvemont, D.Baye, FBS52(2012)

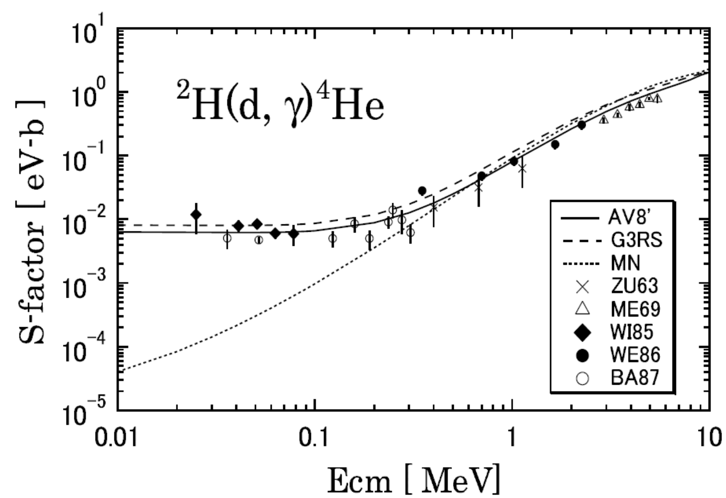
Recombination of constituent particles



K.Arai et al., PRL107 (2011) **MRM**

Reactions at astrophysical energy



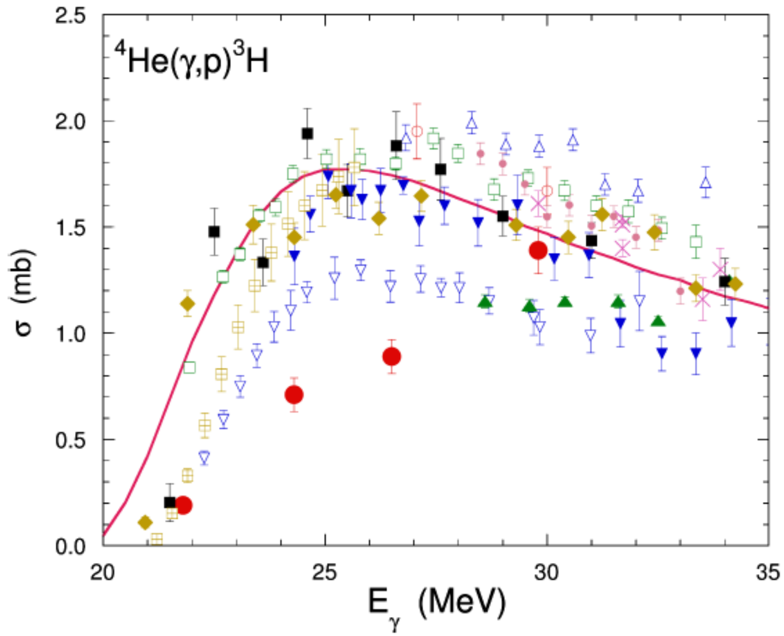


Photoabsorption cross section

Questions

Experiments in discrepancy
Peak position of the giant resonance
E1 sum rules
Charge symmetry breaking effects in (γ,p) and (γ,n)

(1^- T=1 states: 23.64MeV Γ=6.20MeV
25.95MeV Γ=12.66MeV)



Photoabsorption cross section

$$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$

Strength function for E1

$$S(E) = \mathcal{S}_{\mu f} |\langle \Psi_f | \mathcal{M}_{1\mu} | \Psi_0 \rangle|^2 \delta(E_f - E_0 - E)$$
$$= -\frac{1}{\pi} \mathrm{Im} \sum_\mu \langle \Psi_0 | \mathcal{M}_{1\mu}^\dagger \frac{1}{E - H + E_0 + i\epsilon} \mathcal{M}_{1\mu} | \Psi_0 \rangle$$

Continuum states are involved
Use of square-integrable basis to compute S(E)
- **Lorentz integral transform method (Leidemann's talk)**
- **Complex scaling method (CSM)**

Complex scaling method

$$U(\theta) \qquad \boldsymbol{x} \to \mathrm{e}^{i\theta}\boldsymbol{x} \qquad \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}} \to \mathrm{e}^{(-\sin\theta + i\cos\theta)\boldsymbol{k}\cdot\boldsymbol{x}}$$

Continuum is made to damp asymptotically

$$S(E) = -\frac{1}{\pi}\frac{1}{2J_i+1}\sum_{M_i\mu}\mathrm{Im}\langle\Psi_{J_iM_i}|\mathcal{M}^{\dagger}_{\lambda\mu}U^{-1}(\theta)R(\theta)U(\theta)\mathcal{M}_{\lambda\mu}|\Psi_{J_iM_i}\rangle$$

$$R(\theta) = U(\theta)\frac{1}{E-H+i\varepsilon}U^{-1}(\theta) = \frac{1}{E-H(\theta)+i\varepsilon}$$

$$= \sum_{\lambda}\frac{1}{E-E^{\lambda}(\theta)+i\varepsilon}|\Psi^{\lambda}(\theta)\rangle\langle\tilde{\Psi}^{\lambda}(\theta)|$$

$$H(\theta) = U(\theta)HU^{-1}(\theta)$$

$$\boxed{H(\theta)\Psi^{\lambda}(\theta) = E^{\lambda}(\theta)\Psi^{\lambda}(\theta)}$$

$$\tilde{\Psi}^{\lambda}(\theta) = (\Psi^{\lambda}(\theta))^*$$

H(θ) can be diagonalized in square-integrable basis
Continuous strength function is obtained
Stability of S(E) wrt θ is to be examined

Ab initio study of the photoabsorption

W.Horiuchi, Y.S., K.Arai, PRC85 (2012)

The same approach as the previous spectrum calculation

(1)Use realistic interactions

(2)Consider final-state interactions as well as sum rule in basis construction

(3)Check CSM results with MRM radiative capture

Photoabsorption and radiative capture

Detailed balance $\dfrac{v_1\sigma_{1\to 2}}{\rho_2} = \dfrac{v_2\sigma_{2\to 1}}{\rho_1}$

From radiative capture cross section to photoabsorption cross section

$$\sigma_{\rm cap}(E) = \frac{2J_f+1}{(2I_1+1)(2I_2+1)}\frac{8\pi}{\hbar}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1}\frac{(\lambda+1)}{\lambda(2\lambda+1)!!^2} \times \sum_{J_iI_i\ell_i}\frac{1}{(2\ell_i+1)}\left|\langle \Psi^{J_f\pi_f}||\mathcal{M}^E_\lambda||\Psi^{J_i\pi_i}_{\ell_iI_i}(E)\rangle\right|^2,$$

$$\rightarrow \quad \sigma_\gamma(E)$$

Photoabsorption

$\gamma + {}^4He \rightarrow {}^3H + p$

${}^3He + n$

${}^2H + p + n$

Radiative capture

${}^3H + p \rightarrow {}^4He + \gamma$

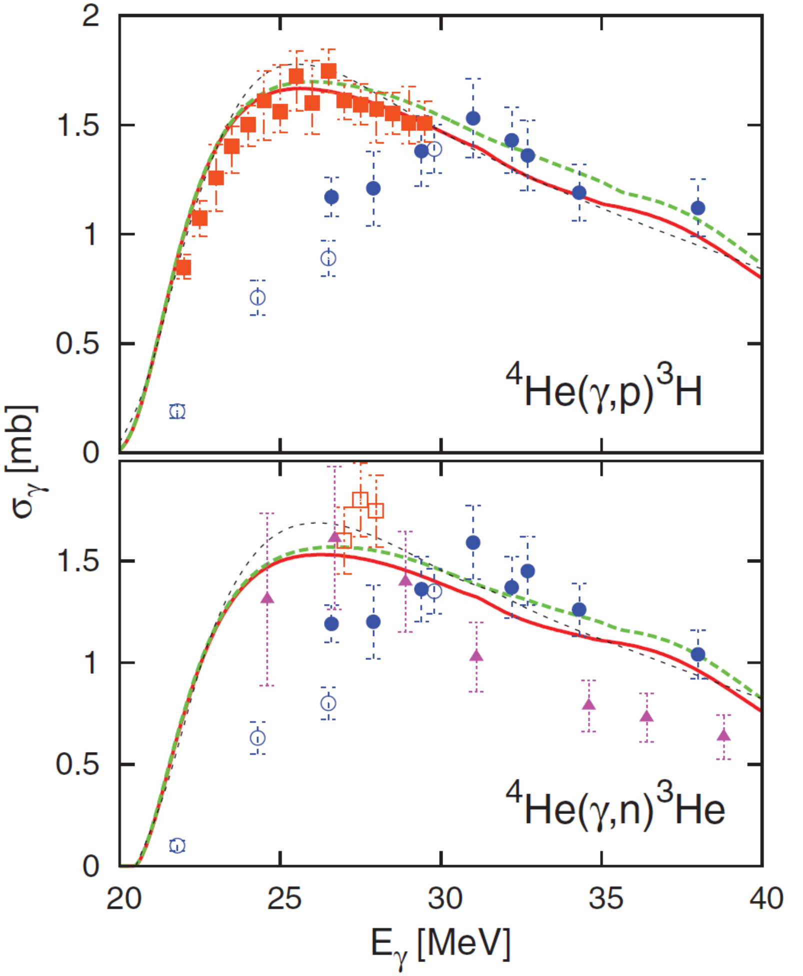
${}^3He + n \rightarrow {}^4He + \gamma$

${}^2H + p + n \rightarrow {}^4He + \gamma$

(difficult to evaluate)

calculable in MRM

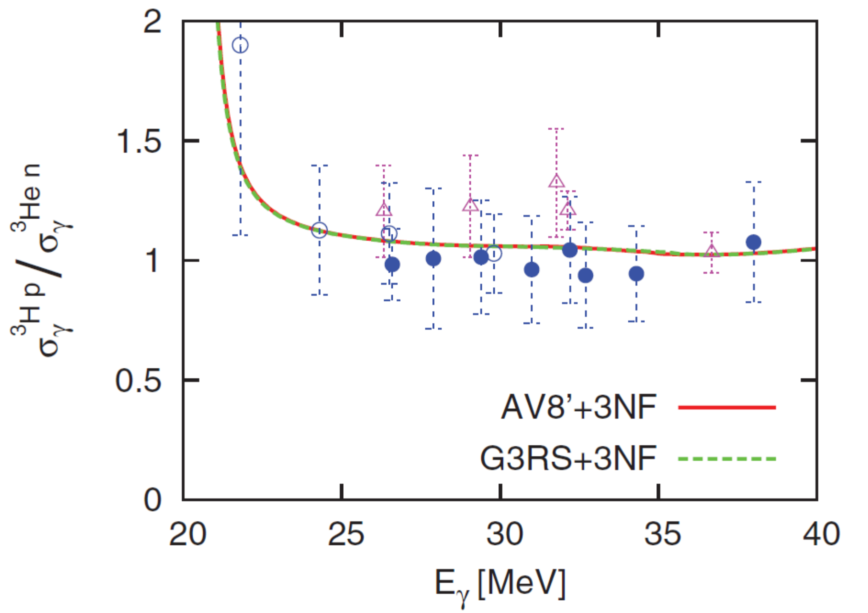
Comparison with experiment



Thin dotted curve: LIT with Malfliet-Tjon pot. (Quaglioni et al.)

MRM result

Difficult to go to high E

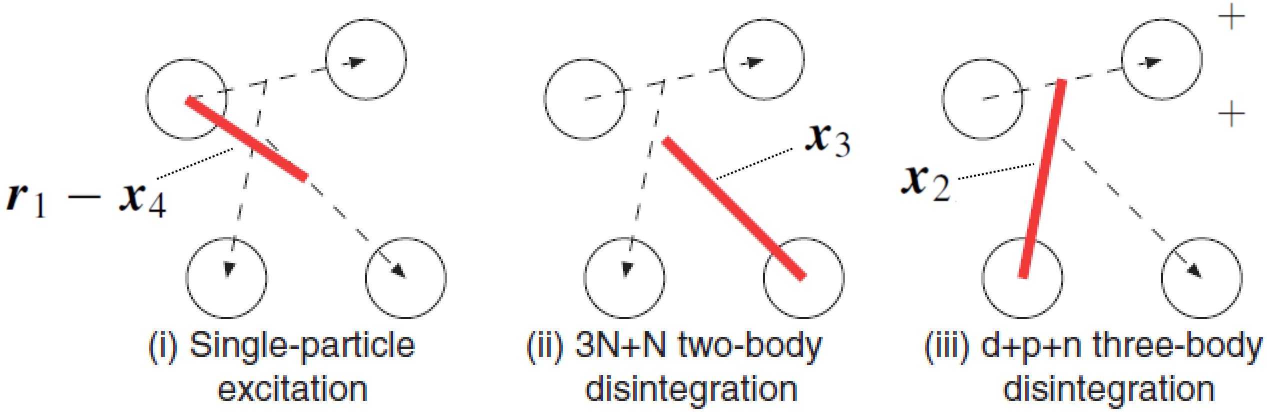


Construction for continuum discretized basis

$$\mathcal{M}_{1\mu} = \sum_{i=1}^{4} \frac{e}{2}(1-\tau_{3i})(\boldsymbol{r}_i - \boldsymbol{x}_4)_\mu$$
$$\sim \frac{1}{2}(\tau_{3_1}-\tau_{3_2})\mathcal{Y}_{1\mu}(\boldsymbol{x}_1)$$
$$+ \frac{1}{3}(\tau_{3_1}+\tau_{3_2}-2\tau_{3_3})\mathcal{Y}_{1\mu}(\boldsymbol{x}_2)$$
$$+ \frac{1}{4}(\tau_{3_1}+\tau_{3_2}+\tau_{3_3}-3\tau_{3_4})\mathcal{Y}_{1\mu}(\boldsymbol{x}_3)$$

Basis states for 1$^-$

- ‘Goldhaber –Teller’ type (ED)
  (E1 sum rule)



1200 bases | 3000 | 3200

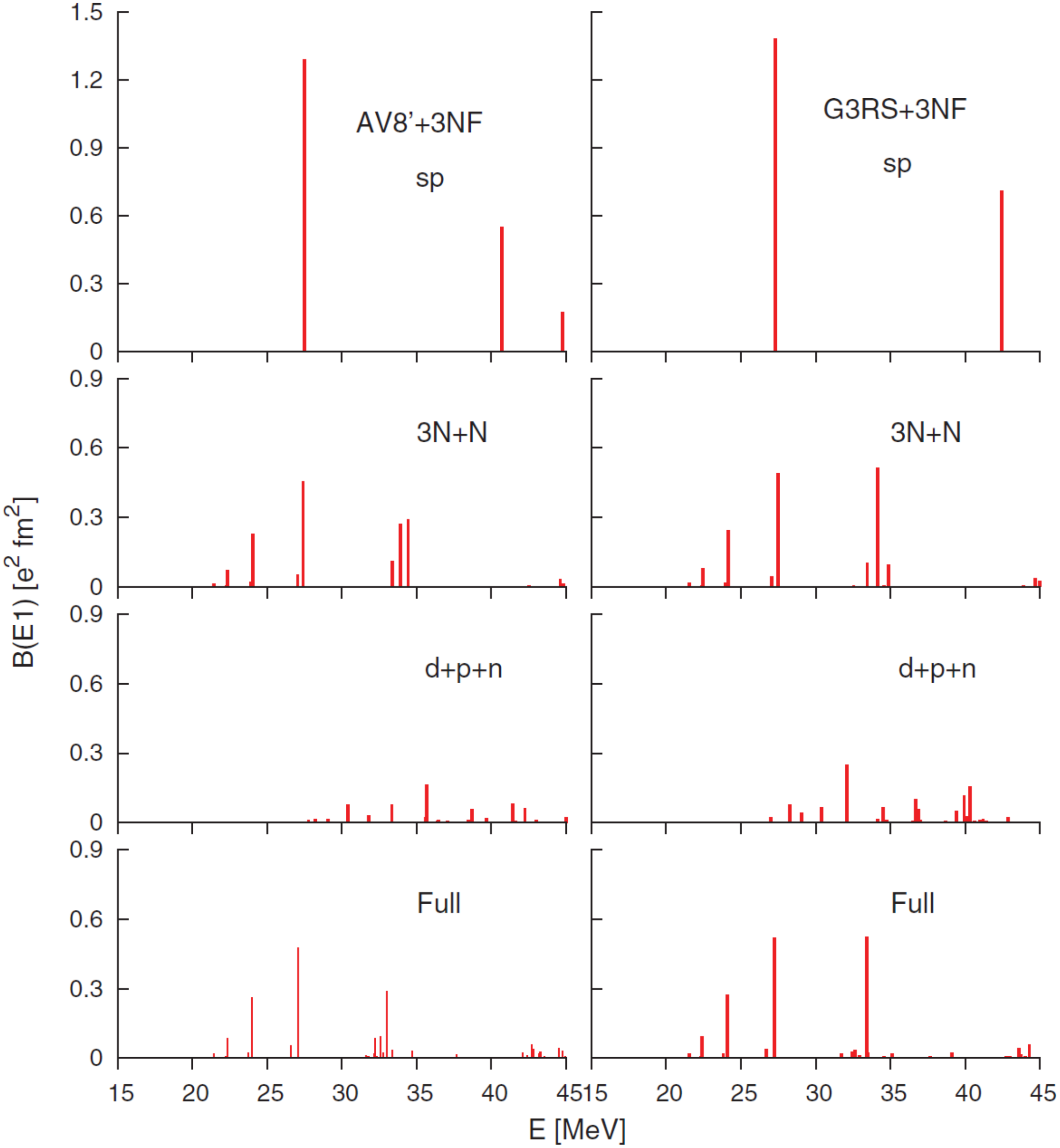
- 3N + N cluster type
- 3N* + N cluster type
  (Final state asymptotics)

$$\mathcal{A}\big[\Phi_0^{(4)}(i)\mathcal{Y}_1(\boldsymbol{r}_1 - \boldsymbol{x}_4)\big]_{1M}\eta^{(4)}_{T_{12}T_{123}10}$$

$$\mathcal{A}\big[\Phi_{J_3}^{(3)}(i)\exp\big(-a_3x_3^2\big)[\mathcal{Y}_1(\boldsymbol{x}_3)\chi_{\frac{1}{2}}(4)]_j\big]_{1M}\big[\eta^{(3)}_{T_{12}\frac{1}{2}}\eta_{\frac{1}{2}}(4)\big]_{10}$$

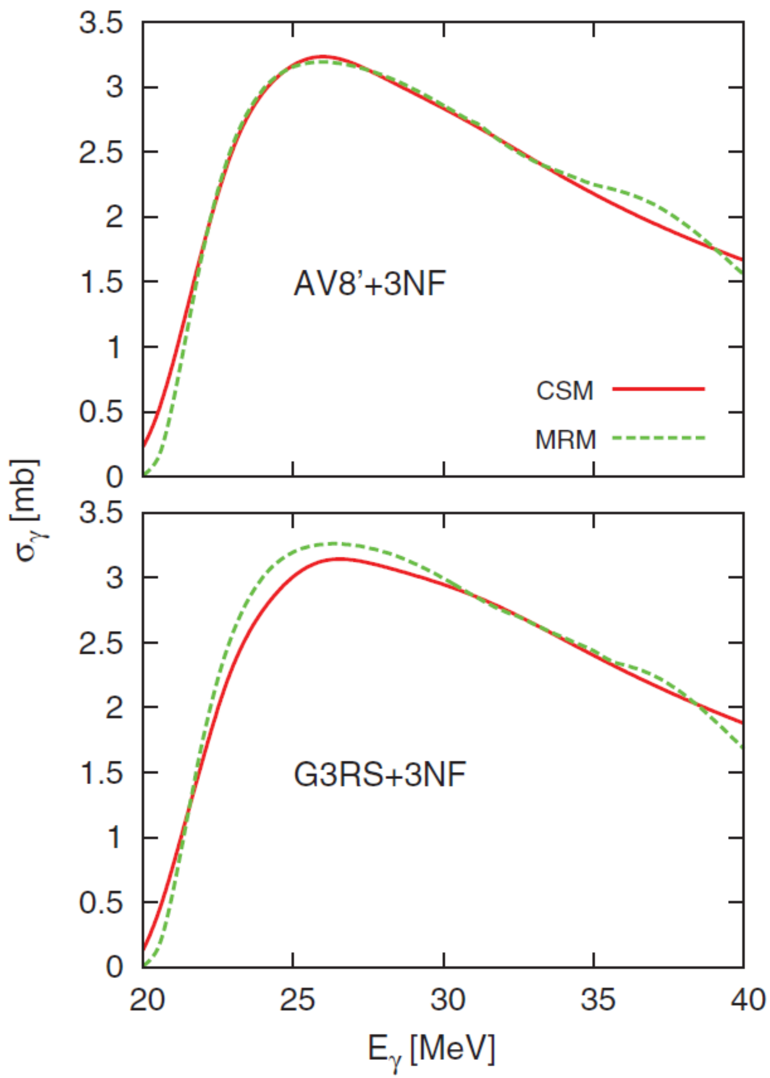
Discretized E1 strength

$$B(E1,\lambda) = \sum_{M\mu} \left| \langle \Psi_\lambda^{1M-}(\theta=0) | \mathcal{M}_{1\mu} | \Psi_0 \rangle \right|^2$$



Comparison between CSM and MRM

MRM results: Sum of two-body channels of ^{3}H+p and ^{3}He+n

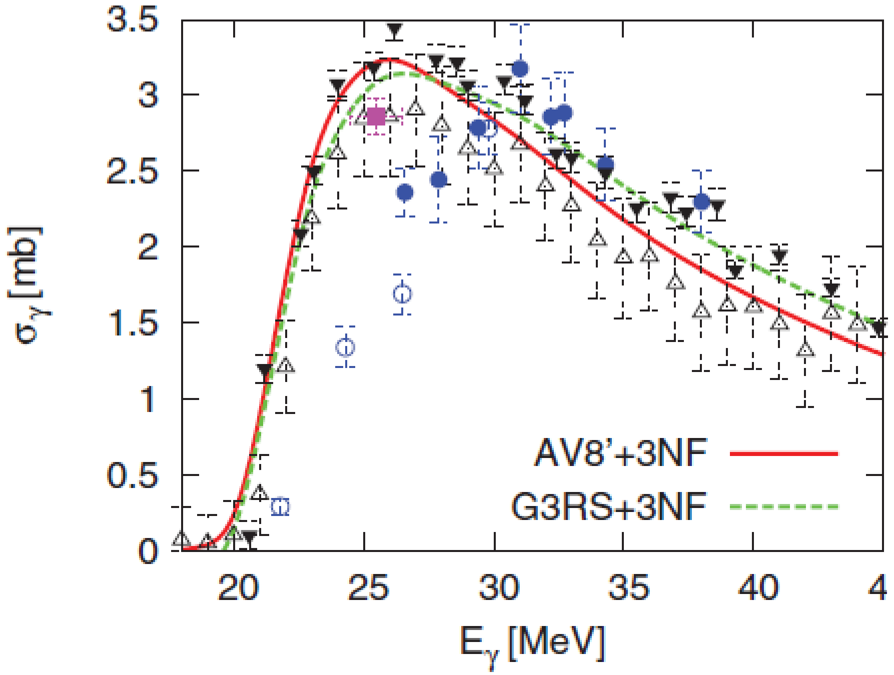


$$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$

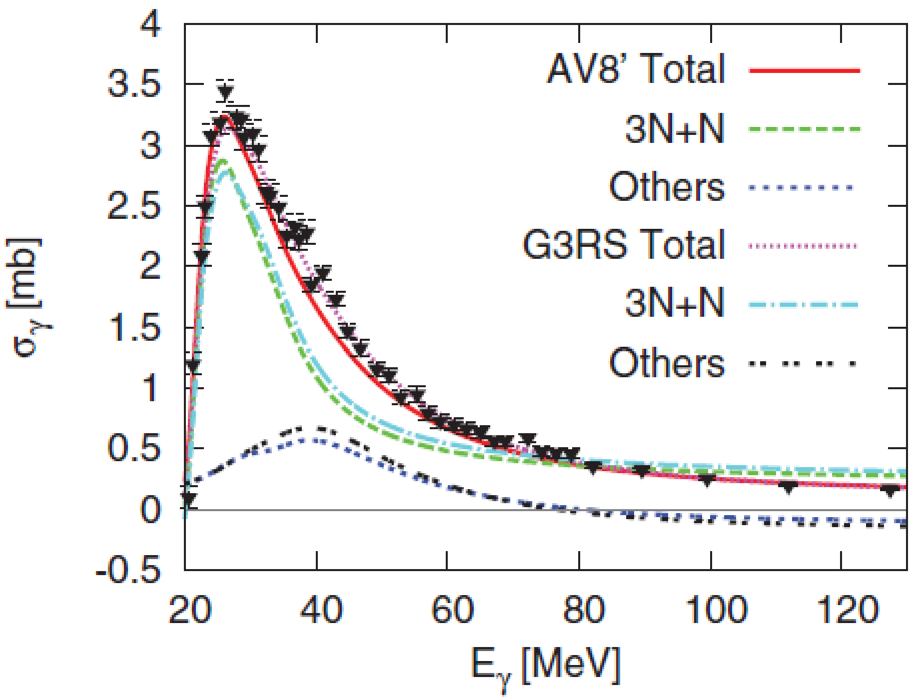
Fair agreement

CSM near threshold

Comparison between CSM and experiment



Good agreement with most data except for low-energy data of Shima et al.
Possible to go to high E



cf. Bacca's talk

Photonuclear sum rules

$$m_\kappa(E_{\max}) = \int_0^{E_{\max}} E_\gamma^\kappa \sigma_\gamma(E_\gamma)\, dE_\gamma \qquad \sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$

$$m_{-1}(\infty) = \mathcal{G}\left(Z^2 \langle r_p^2 \rangle - \frac{Z(Z-1)}{2} \langle r_{pp}^2 \rangle \right) \qquad \mathcal{G} = 4\pi^2 e^2 / 3\hbar c$$

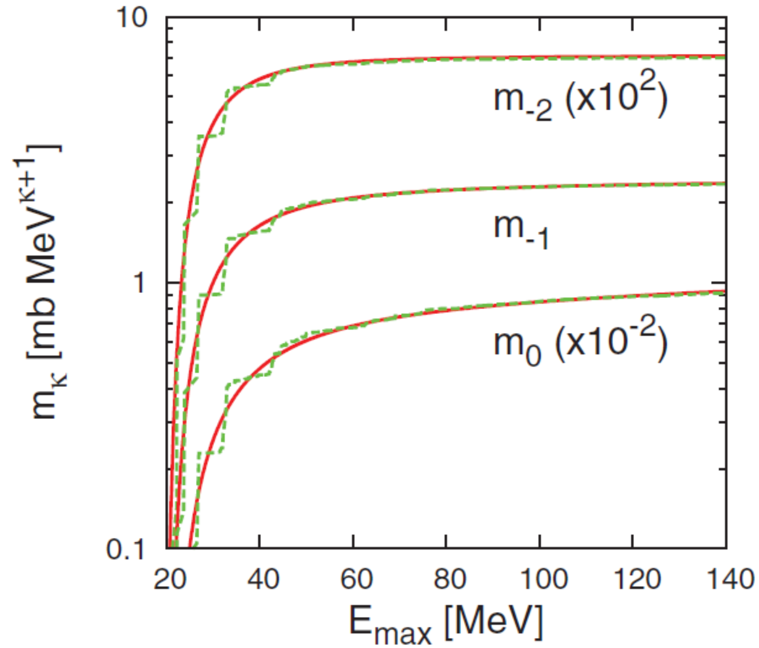
99.6%

$$m_0(\infty) = \mathcal{G} \frac{3NZ\hbar^2}{2Am_N}(1+K)$$

TRK sum rule

$K = \sum_{q=1}^{8} K_q$ **Enhancement factor**

$$K_q = \frac{2Am_N}{3NZ\hbar^2 e^2} \frac{1}{2} \sum_\mu \langle \Psi_0 | [\mathcal{M}_{1\mu}^\dagger, [V_q, \mathcal{M}_{1\mu}]] | \Psi_0 \rangle$$



Value of K

1.11 AV8'+3NF (q=4,6→93%)

1.29 AV14+UVII R. Schiavilla et al. (1987)

1.44 AV18+UIX D. Gazit et al. (2006)

Discretized strength gives good approx. at E_{max}=60 MeV

Spin-dipole excitations of ^{4}He

$$\sum_{i=1}^{N} \left[(\boldsymbol{r}_i - \boldsymbol{x}_N) \times \boldsymbol{\sigma}_i \right]_{\lambda\mu} \begin{pmatrix} 1 \\ \tau_{0_i} \\ t_{\pm_i} \end{pmatrix}$$

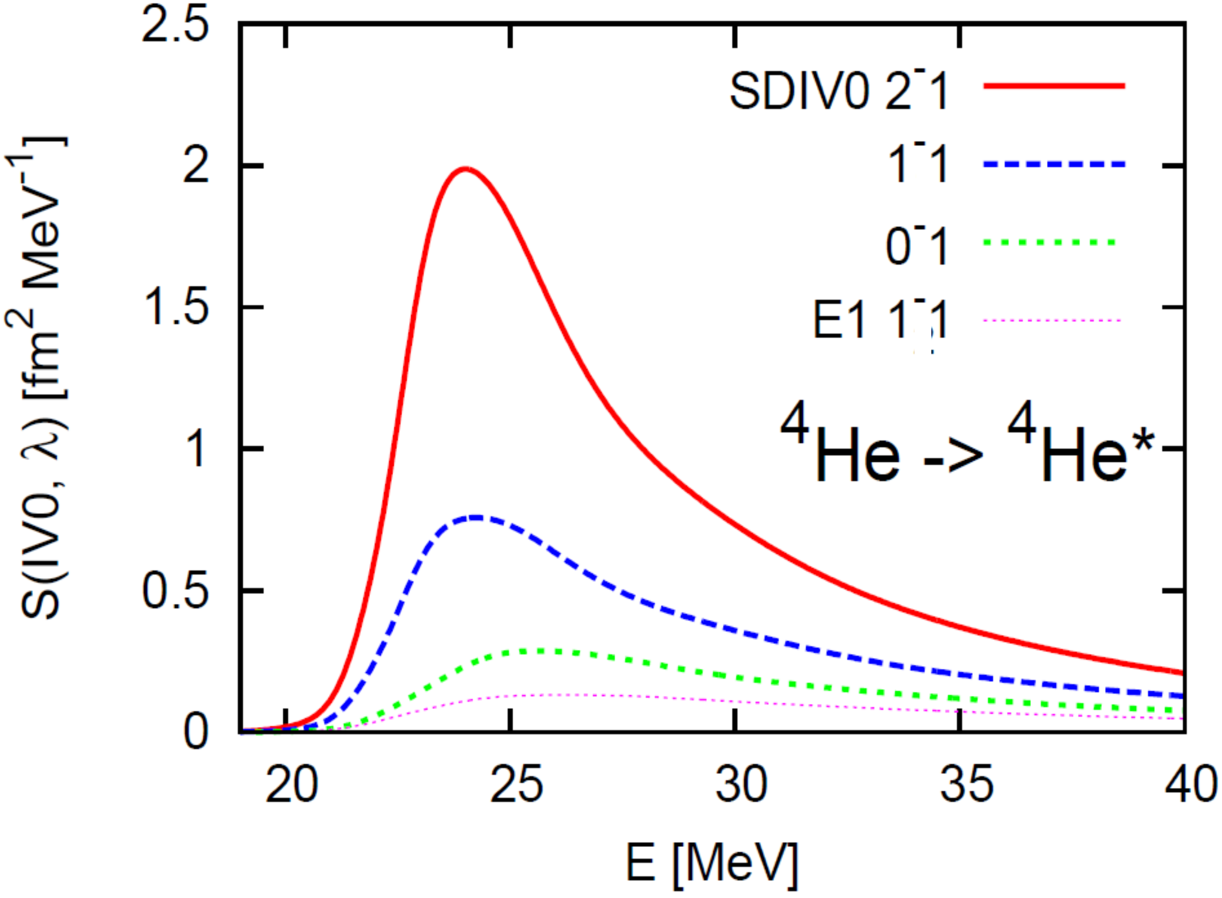
Isoscalar (IS SD)
Isovector (IV SD)
Charge-exchange (IV SD)

Multipoles: λ=0, 1, 2 T=0, 1

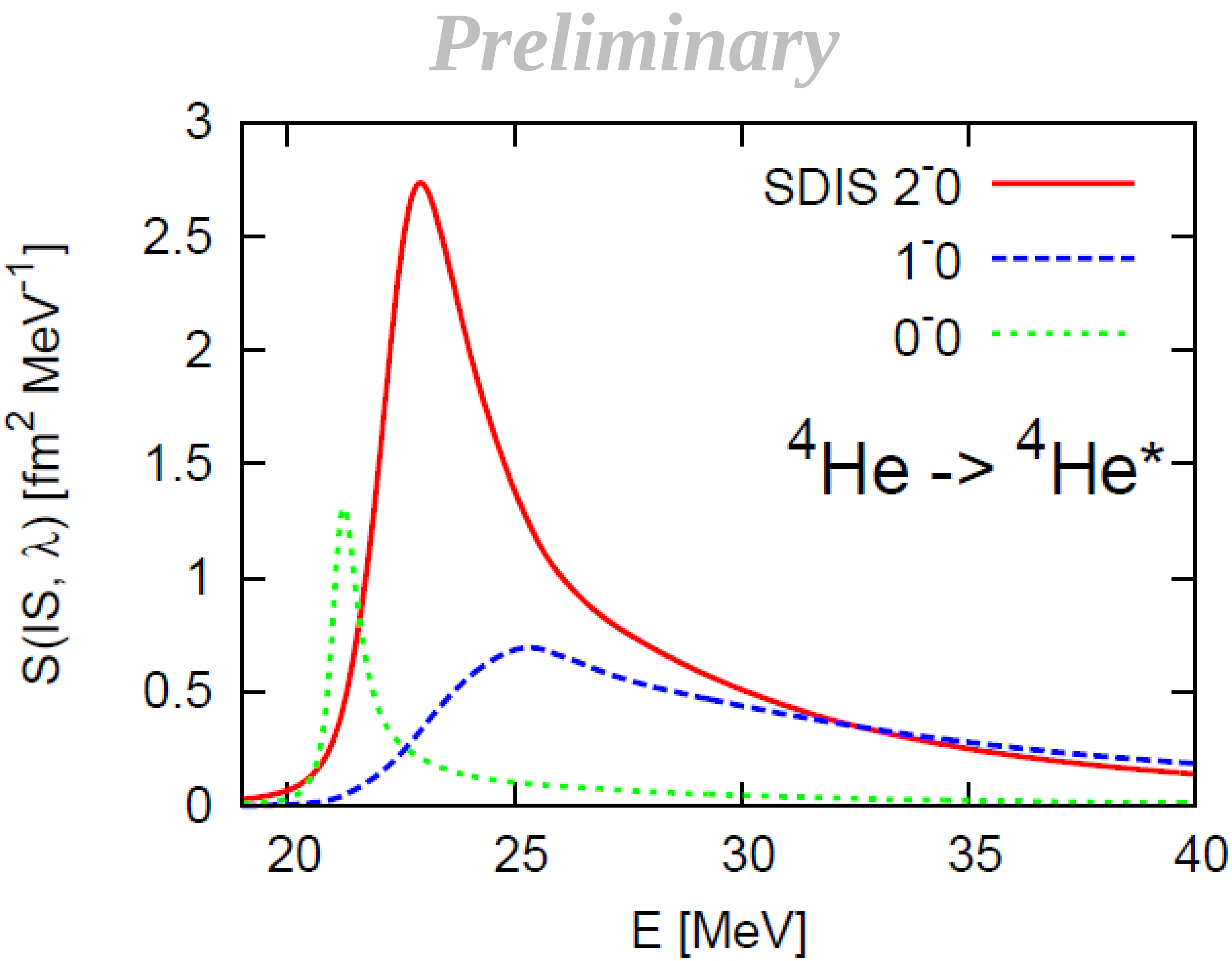
Spin-dipole (SD) operators excite states with $J^{\pi}=\lambda^{-}$
Response to SD operators are interesting for ν-nucleus reactions
D. Gazit, N. Barnea, PRL98 (2007)
Study akin to E1 is in progress using CSM
W. Horiuchi, Y.S.

IV SD strength functions

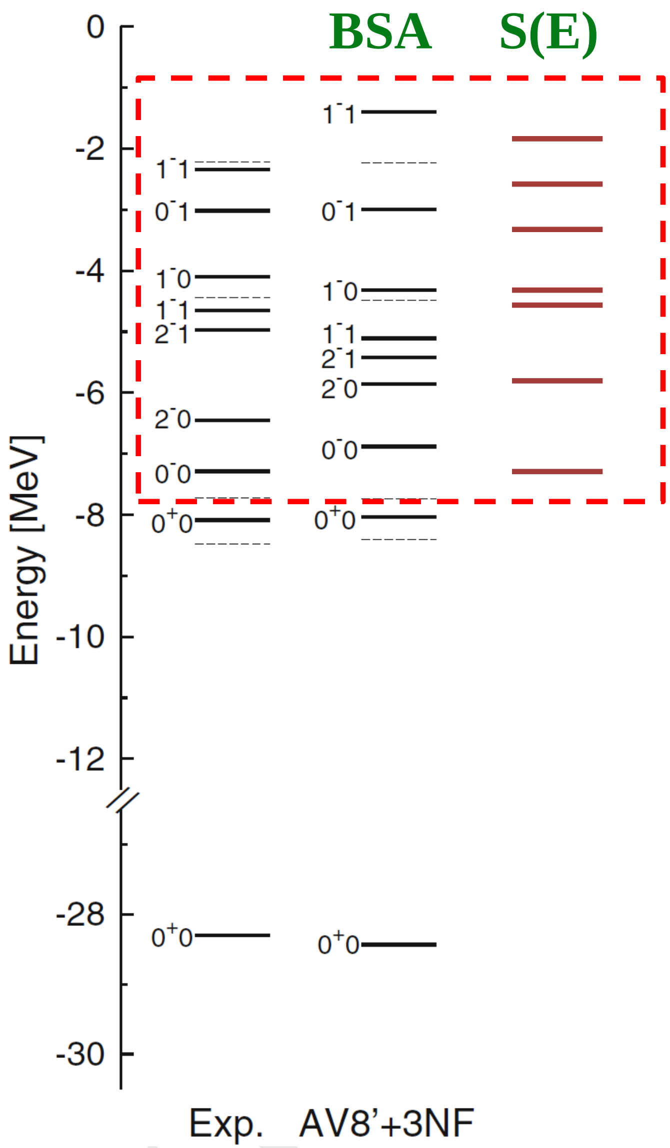
Preliminary



IS SD strength functions



Resonance properties of ^{4}He



$J^\pi T$	^{4}He E_R	Γ	
0^-0	21.21 (21.01)	0.8 (0.84)	
2^-0	22.94 (21.84)	3.1 (2.01)	
1^-0	25.34 (24.25)	9.5 (6.10)	
2^-1	23.99 (23.33)	5.6 (5.01)	
$1^-_1 1$	24.24 (23.64)	7.15 (6.20)	← **SD**
0^-1	25.59 (25.28)	9.95 (7.97)	
$1^-_2 1$	26.20 (25.95)	13.4 (12.66)	← **E1**

Calc. (Expt.)

Fair agreement is obtained

BSA results are reasonable

More realistic 3NF

Summary

The spectrum and response of ^{4}He are studied on the same type of square-integrable basis functions

Correlated Gaussians + Global vectors

Complex scaling method presents virtually the same photoabsorption cross section as microscopic R-matrix method

More realistic 3NF has to be tested to see its effect on the resonance properties of ^{4}He

Experimental info on spin-dipole strength of ^{4}He is desired

E.g. NWESR for λ=0,1,2 → tensor correlation in the ground state

Stochastic variational method (SVM)

Trial and error search of parameters

Increase of the basis dimension

Let A_k be the parameter set defining the kth basis function, and assume that the sets $A_1, \ldots, A_{k-1}$ have already been selected. The next step is the following:

Competitive selection

s1. A number n of different sets of $(A_k^1, ..., A_k^n)$ are generated randomly.

s2. By solving the n eigenvalue problems of k-dimension, the corresponding energies $(E_k^1, ..., E_k^n)$ are determined.

s3. The parameter set A_k^m that produces the lowest energy from among the set $(E_k^1, ..., E_k^n)$ is selected to be the kth parameter set.

s4. Increase k to $k+1$.

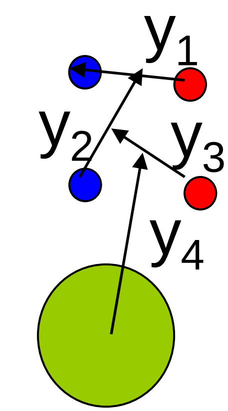
$A_1, \ldots, A_{k-1}, A^j_k$

Generate randomly
Select the best one
and include it as A_k

Y. S. and K. Varga, *Stochastic variational approach to quantum-mechanical few-body problems*, Lecture Notes in Physics 54 (Springer, 1998).

K. Varga and Y. S., Phys. Rev. C52, 2885 (1995).

Unifying various types of correlations with ECG

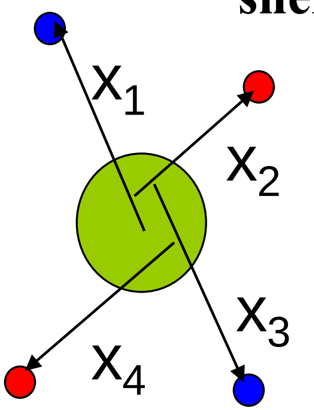


'cluster-like state'

$$\begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \\ \vdots \\ \boldsymbol{y}_{N-1} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} & \dots & T_{1N-1} \\ T_{21} & T_{22} & \dots & T_{2N-1} \\ \vdots & \vdots & \vdots & \vdots \\ T_{N1} & T_{N2} & \dots & T_{N-1N-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_{N-1} \end{pmatrix}$$

$$\boldsymbol{y} = T\boldsymbol{x}$$

'shell-model like state'



$$\exp\left(-\widetilde{\boldsymbol{y}}B\boldsymbol{y}\right)\mathcal{Y}_{LM}(\widetilde{v}\boldsymbol{y}) = \exp\left(-\widetilde{\boldsymbol{x}}A\boldsymbol{x}\right)\mathcal{Y}_{LM}(\widetilde{u}\boldsymbol{x})$$

$$A = \widetilde{T}BT$$
$$u = \widetilde{T}v$$

reduce to a choice of A, u in x coordinate

Both types of correlations are describable in a single coordinate set
Permutation also induces a linear transf. of coordinates
No need of coord. transf. Only suitable choice of A and u is needed

Characteristics of ECG

○ **Analytic evaluation of matrix elements**
◎ **Coordinate transf. & permutations keep ECG**
○ **Versatility in describing different shapes**
○ **Momentum rep. is again ECG**
× **Uneconomical to cope with SR repulsion**

Both natural and unnatural parities

$$\textbf{Single GV: } \mathcal{Y}_{LM}(\tilde{u}\boldsymbol{x}) \rightarrow \textbf{Parity} = (-1)^L$$

$$\textbf{Two GVs: } [\mathcal{Y}_L(\tilde{u}\boldsymbol{x})\mathcal{Y}_1(\tilde{v}\boldsymbol{x})]_{LM} \rightarrow \textbf{Parity} = (-1)^{L+1}$$

Y.S., W.Horiuchi, M.Orabi, K.Arai, Few-Body Syst. 42 (2008)
S.Aoyama, K.Arai, Y.S., P.Descouvemont, D.Baye, Few-Body Syst. 52 (2012)

Lorentz integral transform method

Lorentzian weight

V.D.Efros, W.Leidemann, G.Orlandini, PLB338 (1994)

$$\mathcal{L}(z) = \int_{E_{\min}}^{\infty} \frac{S(E)}{(E-z)(E-z^*)} dE = \int_{E_{\min}}^{\infty} \frac{S(E)}{(E-E_R)^2 + E_I^2} dE$$

$$z = E_R + iE_I$$

$$\mathcal{L}(z) = \frac{1}{2J_i+1} \sum_{M_i \mu} \langle \Psi_{M_i \mu}(z) | \Psi_{M_i \mu}(z) \rangle$$

$$\Psi_{M_i \mu}(z) = \frac{1}{H - E_i - z} \mathcal{M}_{\lambda\mu} \Psi_{J_i M_i}$$

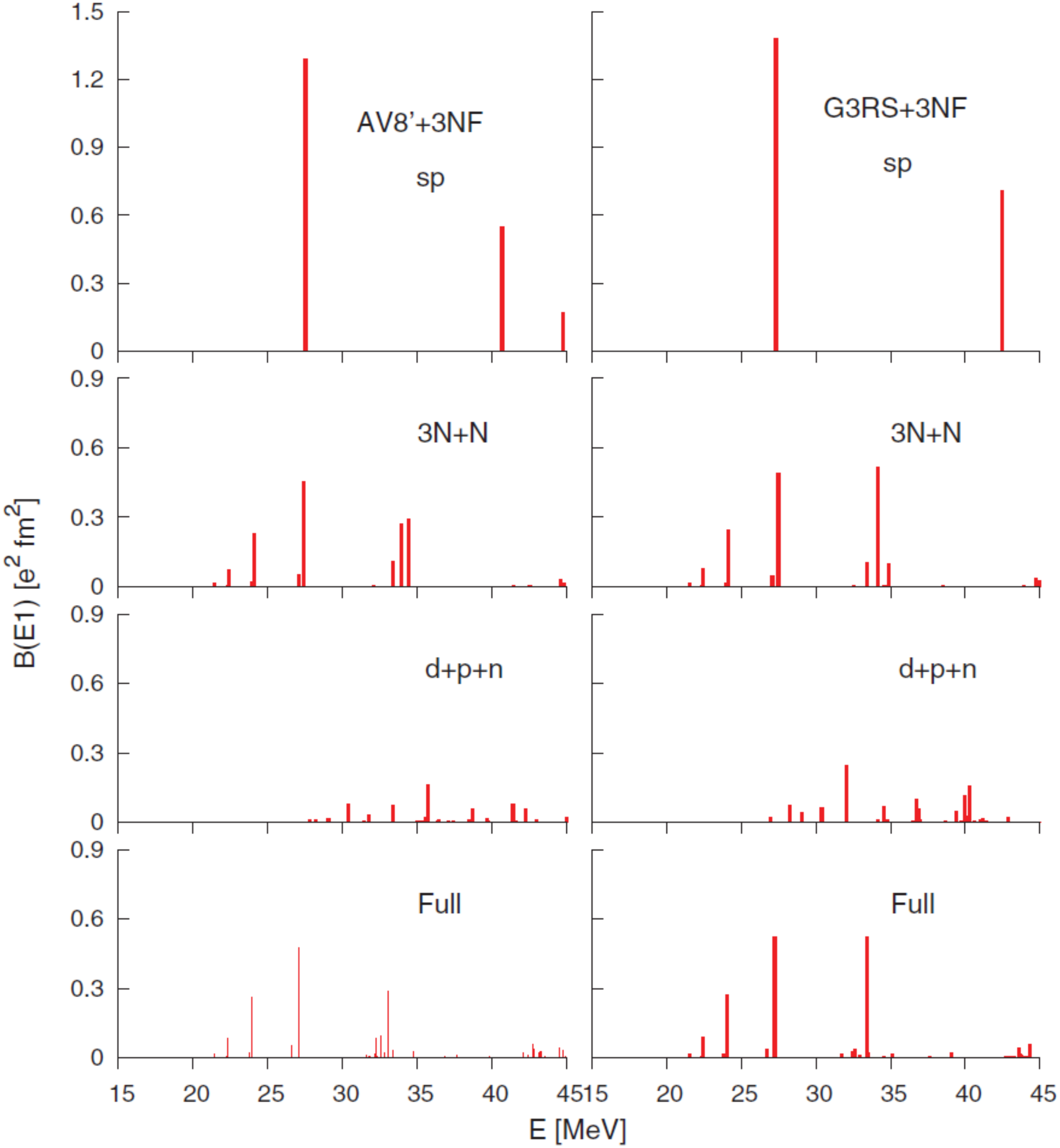
$$(H - E_i - z)\Psi_{M_i \mu}(z) = \mathcal{M}_{\lambda\mu} \Psi_{J_i M_i}$$

L(z) is finite, hence the norm of Ψ(z) is finite
Ψ(z) can be obtained in L^2-integrable basis
L(z) has to be computed for many z values (E_R varied, E_I fixed) to make the inversion possible

The inversion from L(z) to S(E) requires some skill

Discretized E1 strength

$$B(E1,\lambda) = \sum_{M\mu} \left|\langle \Psi_\lambda^{1M-}(\theta=0) \middle| \mathcal{M}_{1\mu} \middle| \Psi_0 \rangle\right|^2$$



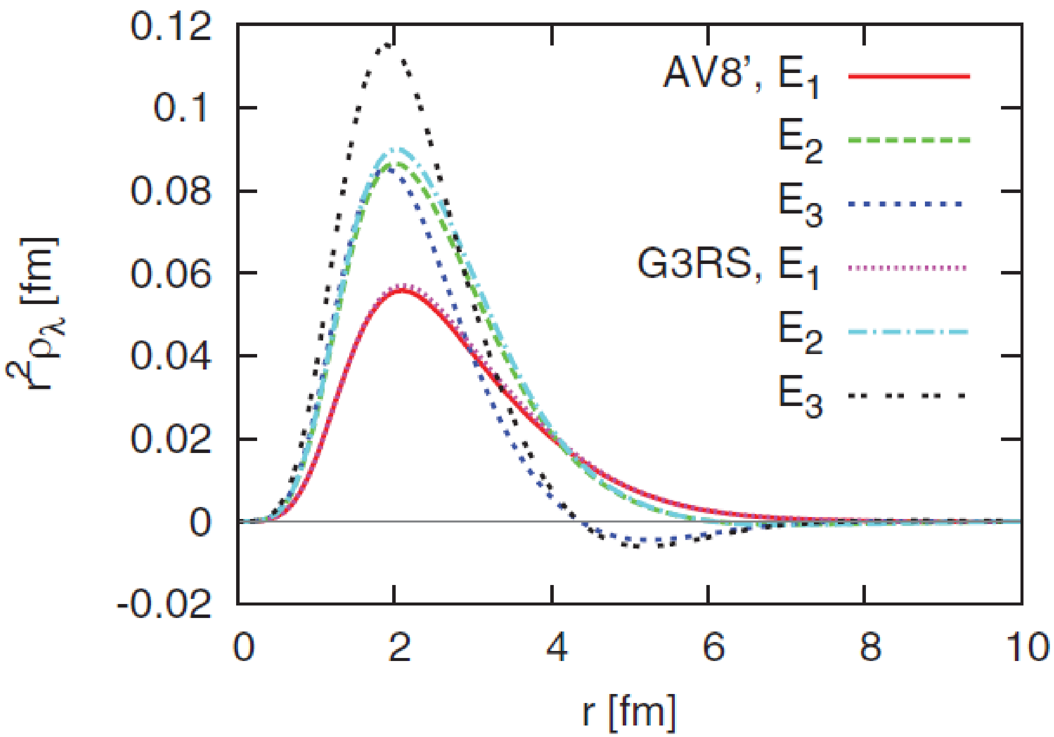
Properties of the three main states

E	AV8′ + 3NF		
	23.96	27.05	33.02
$\langle H \rangle$	−4.46	−1.38	4.60
$\langle T \rangle$	51.21	54.78	43.71
$\langle V_1 \rangle$	6.42	6.37	4.44
$\langle V_2 \rangle$	−3.41	−3.68	−1.61
$\langle V_3 \rangle$	−2.17	−2.15	−1.65
$\langle V_4 \rangle$	−23.83	−24.04	−16.09
$\langle V_5 \rangle$	0.22	0.22	0.14
$\langle V_6 \rangle$	−30.60	−30.51	−22.71
$\langle V_7 \rangle$	4.79	4.77	3.55
$\langle V_8 \rangle$	−6.76	−6.73	−4.96
$\langle V_{3NF} \rangle$	−0.74	−0.86	−0.55
$\langle V_{Coul} \rangle$	0.42	0.45	0.32
$P(1,0)$	87.18	84.58	82.70
$P(1,1)$	4.76	7.47	7.59
$P(2,1)$	0.16	0.25	0.22
$P(1,2)$	0.89	0.74	4.56
$P(2,2)$	2.17	1.99	1.40
$P(3,2)$	4.85	4.97	3.53

E1 transition density

$$\left\langle \Psi_\lambda^{10-}(\theta=0) \middle| \mathcal{M}_{10} \middle| \Psi_0 \right\rangle = \sqrt{\frac{4\pi}{3}} e \int_0^\infty \rho_\lambda(r) r^2 \, dr$$

$$\rho_\lambda(r) = \left\langle \Psi_\lambda^{10-}(\theta=0) \right| \sum_{i=1}^{4} \frac{\delta(|\boldsymbol{r}_i - \boldsymbol{x}_4| - r)}{r^2} \mathcal{Y}_{10}(\boldsymbol{r}_i - \boldsymbol{x}_4) \frac{1-\tau_{3_i}}{2} \left| \Psi_0 \right\rangle$$



Peak of $r^2\rho$ is at about 2 fm (much larger than 1.1 fm of that for $r^2\rho_{g.s.}$)

Extend to large distances due to 3N+N configurations

Constructive and destructive patterns in 2nd and 3rd states

Contributions of V_q to the enhancement factor K (cf. V_q to the ground-state energy)

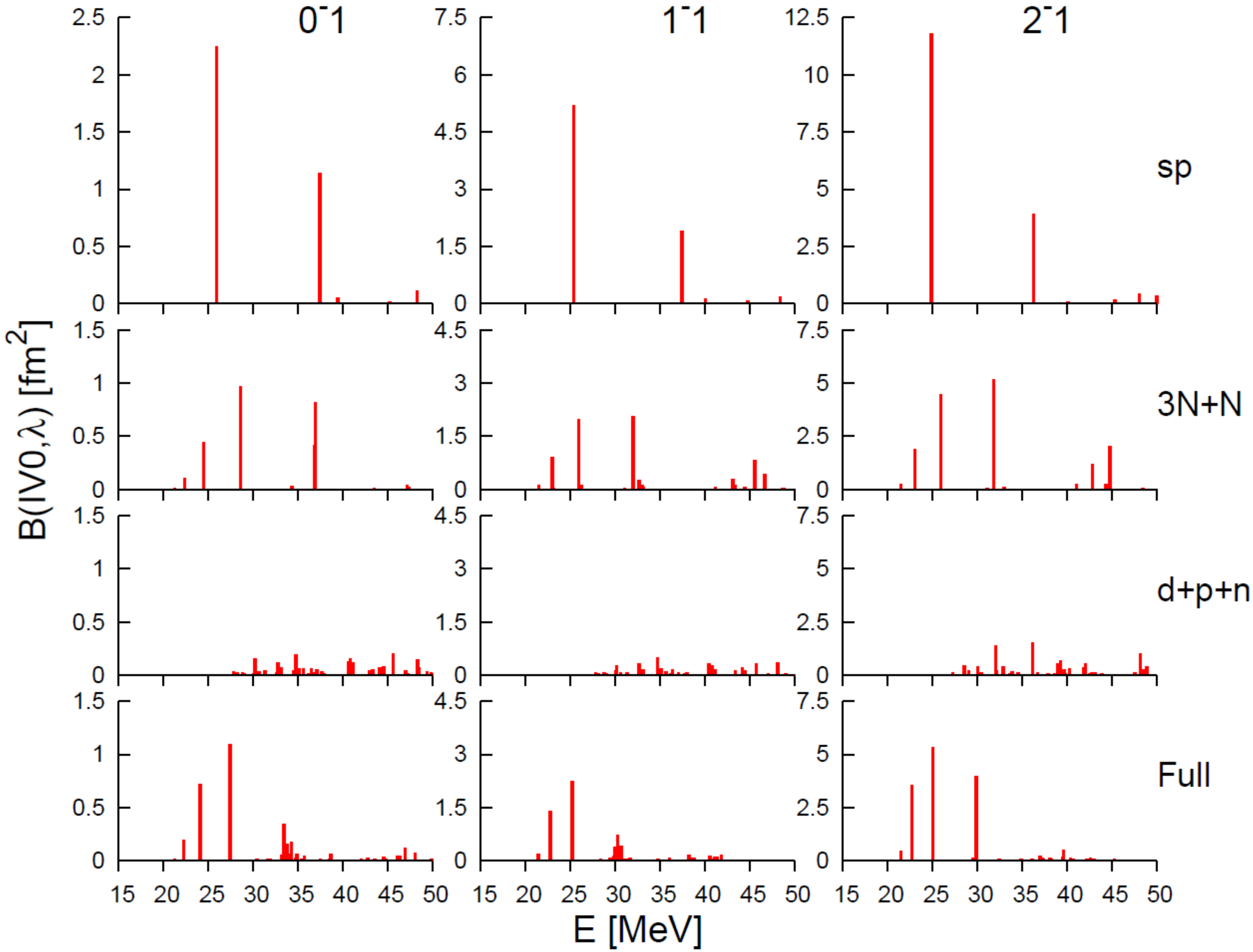
q	$\mathcal{O}_{ij}^{(q)}$	AV8′ + 3NF		G3RS + 3NF	
		$\langle V_q \rangle$	K_q	$\langle V_q \rangle$	K_q
1	1	17.39	0	1.07	0
2	$\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$	–9.59	0	–8.75	0
3	$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$	–5.22	0.011	–9.11	0.059
4	$\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$	–59.42	0.460	–51.80	0.474
		(–12.51)	(0.187)	(–12.50)	(0.191)
5	S_{ij}	0.75	0	–0.93	0
6	$S_{ij} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$	–70.93	0.574	–47.16	0.484
		(–68.65)	(0.667)	(–59.37)	(0.610)
7	$(\boldsymbol{L} \cdot \boldsymbol{S})_{ij}$	11.09	0	5.53	0
8	$(\boldsymbol{L} \cdot \boldsymbol{S})_{ij} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$	–15.93	0.061	–5.65	0.025
	Total	–131.9	1.11	–116.8	1.04

1.29 AV14+UVII R. Schiavilla et al. (1987)

1.44 AV18+UIX D. Gazit et al. (2006)

IV SD strengths of ^{4}He

Preliminary



Mechanism for splitting two 0^- states with different isospin

$$\Psi^{\pi}_{JM_J,TM_T} = \sum_{LS} C_{LS,T} \Phi^{\pi}_{(LS)JM_J,TM_T}$$

$$\Psi^{-}_{00,T0} = C_{1T}\Psi^{-}_{(11)00,T0} + C_{2T}\Psi^{-}_{(22)00,T0}$$

TABLE III: The Hamiltonian matrix elements, given in MeV, for the 0^-0 and 0^-1 states of ^{4}He. The column-row of the matrix is labeled by the channel (L^π, S), which is arranged in the order of $(1^-,1)$ and $(2^-,2)$. The C^2_{LT} values are C^2_{10}=0.945, C^2_{20}=0.055 for 0^-0 and C^2_{11}=0.963, C^2_{21}=0.037 for 0^-1. AV8$'$+TNF potential is used.

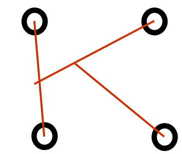
0^-0	H		T		$V_c+V_{\rm Coul}$	
	3.99	−45.00	51.16	0.0	−24.96	0.0
	−45.00	179.4	0.0	200.6	0.0	−13.58
	V_t		V_b		$V_{\rm TNF}$	
	−20.67	−44.19	−0.22	−0.81	−1.31	0.0
	−44.19	12.46	−0.81	−18.65	0.0	−1.49
0^-1	H		T		$V_c+V_{\rm Coul}$	
	3.89	−35.27	39.64	0.0	−21.25	0.0
	−35.27	177.8	0.0	189.5	0.0	−10.13
	V_t		V_b		$V_{\rm TNF}$	
	−13.39	−34.83	−0.23	−0.45	−0.88	0.0
	−34.83	18.23	−0.45	−18.73	0.0	−1.02

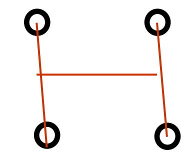
Coupling due to tensor force!

PWE $e^{-a_1\boldsymbol{x}_1^2 - a_2\boldsymbol{x}_2^2 - a_3\boldsymbol{x}_3^2 - \cdots} \; [[[\mathcal{Y}_{L_1}(\boldsymbol{x}_1) \times \mathcal{Y}_{L_2}(\boldsymbol{x}_2)]_{L_{12}} \times \mathcal{Y}_{L_3}(\boldsymbol{x}_3)]_{L_{123}} \ldots]_{LM}$

(Product form of 's.p.' orbits Rearrangement channels must be included)

	Potential	MN	G3RS		AV8′		
	Method	GVR	GVR	PWE	GVR	PWE	ref. [26]
$^3H(\frac{1}{2}^+)$	E	−8.38	−7.73	−7.72	−7.76	−7.76	−7.767
	$\langle T \rangle$	27.21	40.24	40.22	47.59	47.57	47.615
	$\langle V_c \rangle$	−35.59	−26.80	−26.79	−22.50	−22.49	−22.512
	$\langle V_t \rangle$	–	−21.13	−21.13	−30.85	−30.84	−30.867
	$\langle V_b \rangle$	–	−0.03	−0.03	−2.00	−2.00	−2.003
	$\sqrt{\langle r^2 \rangle}$	1.71	1.79	1.79	1.75	1.75	
$P(L,S)$ (%)	$P(0,\frac{1}{2})$	100	92.95	92.94	91.38	91.37	91.35
	$P(2,\frac{3}{2})$	–	7.01	7.02	8.55	8.57	8.58
	$P(1,\frac{1}{2})$	–	0.03	0.03	0.04	0.04	} 0.07
	$P(1,\frac{3}{2})$	–	0.02	0.02	0.02	0.02	
$^4He(0^+)$	E	−29.94	−25.29	−25.29	−25.09	−25.05	
	$\langle T \rangle$	58.08	86.93	86.90	101.62	101.41	
	$\langle V_c \rangle$	−88.86	−66.24	−66.19	−54.93	−54.76	
	$\langle V_{Coul} \rangle$	0.83	0.76	0.76	0.77	0.77	
	$\langle V_t \rangle$	–	−46.62	−46.65	−67.89	−67.82	
	$\langle V_b \rangle$	–	−0.13	−0.12	−4.66	−4.66	
	$\sqrt{\langle r^2 \rangle}$	1.41	1.51	1.51	1.49	1.49	
	$P(0,0)$	100	88.46	88.45	85.76	85.79	
	$P(2,2)$	–	11.30	11.30	13.88	13.85	
	$P(1,1)$	–	0.25	0.24	0.36	0.36	





Y.S., W.Horiuchi, M.Orabi, K. Arai, FBS42(2008)