Electromagnetic structure and reactions of light nuclei from χEFT $*$

Saori Pastore @ INT - Seattle, WA - November 2012

Argonne NATIONAL LABORATORY

$*$ in collaboration with:
Steven Pieper, Rocco Schiavilla, Bob Wiringa,
Maria Piarulli, Luca Girlanda, Michele Viviani,
Laura E. Marcucci, Alejandro Kievsky, José L. Goity

PRC**78**, 064002 (2008) - PRC**80**, 034004 (2009) - PRC**81**, 034005 (2010) - PRL**105**, 232502, (2010) - PRC**84**, 024001 (2011)

- EM currents I: Standard Nuclear Physics Approach (SNPA)
- EM currents II: Nuclear χEFT approach
- EM observables in $A \leq 9$ systems
- Summary
- Outlook

The Basic Model

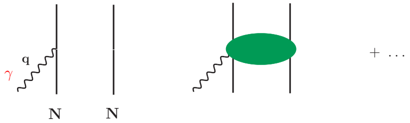
- The nucleus is a system made of A interacting nucleons, its energy is given by

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

where υ_{ij} and V_{ijk} are 2- and 3-nucleon interaction operators

- Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of 1−, 2−, ... nucleon operators:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \dots\,, \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$



- EM current operator $\mathbf{j}$ satisfies the current conservation relation (CCR) with the nuclear Hamiltonian, hence V, ρ, $\mathbf{j}$ need to be derived consistently

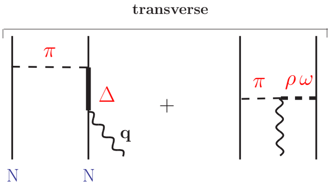
$$\mathbf{q} \cdot \mathbf{j} = [H, \rho]$$

CCR does not constrain transverse (orthogonal to **q**) currents

Currents from nuclear interactions $*$- Marcucci *et al.* PRC**72**, 014001 (2005)

- Current operator **j** constructed so as to satisfy the continuity equation with a realistic Hamiltonian
- Short- and intermediate-behavior of the EM operators inferred from the nuclear two- and three-body potentials

$$\mathbf{j} = \mathbf{j}^{(1)} + \mathbf{j}^{(2)}(v) + \mathbf{j}^{(3)}(V)$$



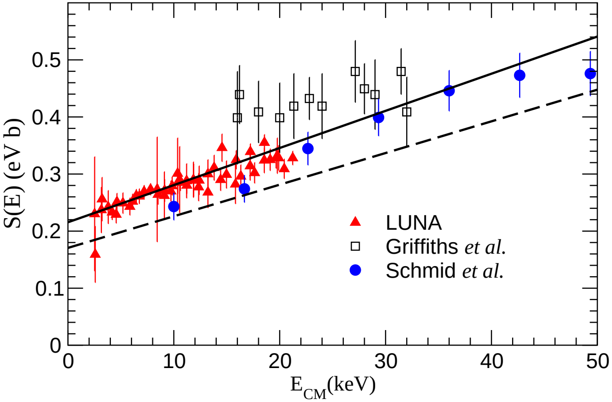
$*$ also referred to as Standard Nuclear Physics Approach (SNPA) currents

- Long range part of $\mathbf{j}(\upsilon)$ corresponds to OPE seagull and pion-in-flight EM currents

Currents from nuclear interactions - Marcucci *et al.* PRC**72**, 014001 (2005)

Satisfactory description of a variety of nuclear EM properties [see Marcucci *et al.* (2005) and (2008)]

^{2}H(p,γ)^{3}He capture



- Isoscalar magnetic moments are a few % off (10% in A=7 nuclei)

Currents and nuclear electroweak properties:

- Park, Rho *et al.* (1996–2009);
  hybrid studies in A=2–4 by Song *at al.* (2009-2011)
- Meissner *et al.* (2001), Kölling *et al.* (2009–2011);
  applications to d and ^{3}He photodisintegration by Rozpedzik *et al.* (2011);
  applications to d and $A = 3$ magnetic f.f.'s by Kölling, Epelbaum, Phillips (2012)
- Phillips (2003);
  applications to deuteron static properties and f.f.'s

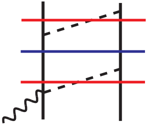
Transition amplitude in time-ordered perturbation theory

$$T_{\rm fi} = \langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle$$

$$= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \ldots$$

- A contribution with N interaction vertices and L loops scales as

$$\underbrace{e \left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1 \,\text{scaling}} \times \underbrace{Q^{-(N-N_K-1)} \, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$



α_i = number of derivatives in H_1 and β_i = number of π's at each vertex

N_K = number of pure nucleonic intermediate states

- $(N - N_K - 1)$ energy denominators expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$\frac{1}{E_i - E_I} |I\rangle = \frac{1}{E_i - E_N - \omega_\pi} |I\rangle \sim - \left[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \ldots \right] |I\rangle$$

- Due to the chiral expansion, the transition amplitude $T_{\rm fi}$ can be expanded as

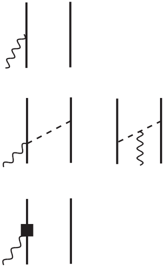
$$T_{\rm fi} = T^{\rm LO} + T^{\rm NLO} + T^{\rm N2LO} + \ldots \quad \text{and} \quad T^{\rm NnLO} \sim (Q/\Lambda_\chi)^n T^{\rm LO}$$

χEFT EM current up to $n = 1$ (or up to N3LO)

LO : $\mathbf{j}^{(-2)} \sim eQ^{-2}$

NLO : $\mathbf{j}^{(-1)} \sim eQ^{-1}$

N^{2}LO : $\mathbf{j}^{(-0)} \sim eQ^{0}$

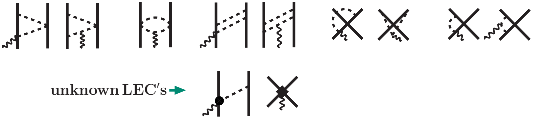


- $n = -2, -1, 0$, and 1-(loops only): depend on known LECs namely g_A, F_π, and proton and neutron μ
- $n = 0$: $(Q/m_N)^2$ relativistic correction to $\mathbf{j}^{(-2)}$
- unknown LECs enter the $n = 1$ contact and tree-level currents (the latter originates from a $\gamma\pi N$ vertex of order eQ^2)

- divergencies associated with loop integrals are reabsorbed by renormalization of contact terms
- loops contributions lead to purely isovector operators
- $\mathbf{j}^{(n\leq 1)}$ satisfies the CCR with χEFT two-nucleon potential $\upsilon^{(n\leq 2)}$

N^{3}LO: $\mathbf{j}^{(1)} \sim eQ$

unknown LEC's



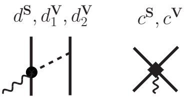
χEFT EM current up to $n = 1$ (or up to N3LO)

- LECs of contact interactions at Q^0 and 'minimal' contact interactions at Q^2 fixed from fits to *np* phases shifts: LECs taken from Q^4 NN potential of D.R. Entem, R.Machleidt—PRC**68**, 041001 (2003)
- LECs from 'non-minimal' interactions fixed by reproducing EM observables: Different parameterizations are possible
- No three-body currents at N3LO

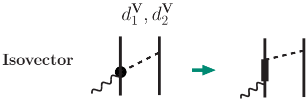
* Note:

- * currents associated with one loop corrections to the OPE are missing in our calculations; renormalization of OPE currents has been carried out in Kölling 2011
- * We revised derivation of current of involving CT interaction + pion loop (more on this issue on extra slides if interested)
- * The N3LO MIN contact current is in agreement with that of Kölling 2011 after Fierz-reordering, apart from differences in the term $\propto C_5$ (more on this issue on extra slides if interested)

* Piarulli *et al.* in preparation, **PRC**80**, 034004 (2009)



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



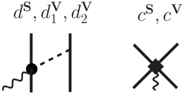
d_2^V and d_1^V are known assuming Δ-resonance saturation ($d_2^V/d_1^V = 1/4$)

Left with 5 LECs: Fixed in the $A = 2-3$ nucleons' sector

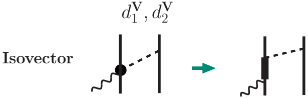
- Isoscalar sector:
  - d^S and c^S from EXPT μ_d and $\mu_S(^3\text{H}/^3\text{He})$

Λ	NN/NNN	$10 \times d^S$	c^S
600	AV18/UIX (N3LO/N2LO)	–2.033 (3.231)	5.238 (11.38)

χEFT EM currents at N3LO: fixing LECs p.2/2 – Piarulli *et al.* in prep.



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



d_2^V and d_1^V are known assuming Δ-resonance saturation ($d_2^V/d_1^V = 1/4$)

Left with 4 LECs: Fixed in the $A = 2-3$ nucleons' sector

- Isovector sector:
  - I = c^V and d_1^V from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m. and EXPT $npd\gamma$ xsec.

    or
  - II = c^V from EXPT $npd\gamma$ xsec. and d_1^V from Δ-saturation*

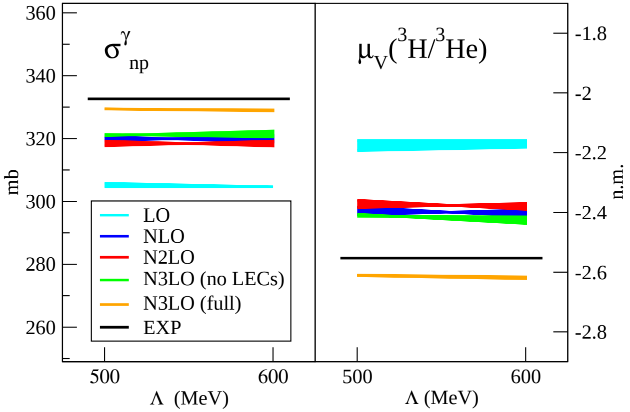
    or
  - III = c^V from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m. and d_1^V from Δ-saturation*

Λ	NN/NNN	Current	d_1^V	c^V
600	AV18/UIX (N3LO/N2LO)	I	75.0 (33.14)	257.5 (41.84)
		II	4.98 (4.98)	-11.57 (-22.31)
		III	4.98 (4.98)	-1.025 (-11.69)

* $d_1^V = 4\frac{\mu^* h_A}{9m(m_\Delta - m)}\Lambda^2$

Predictions with χEFT EM currents for $A = 2$–3 systems- Piarulli *et al.* in prep.

np capture xsec. (using model III) / μ_V of $A = 3$ nuclei (using model II)
bands represent nuclear model dependence (N3LO/N2LO – AV18/UIX)



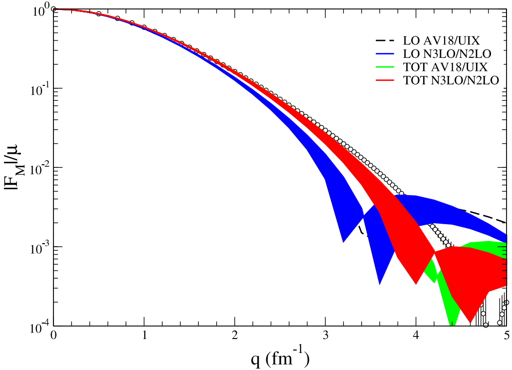
trinucleon w.f.'s from hyperspherical harmonics expansion

Kievsky *et al.*, FBS**22**, 1 (1997); Viviani *et al.*, FBS**39**, 59 (2006); Kievsky *et al.*, J. Phys. G: Nucl. Part. Phys. **35**, 063101 (2008)

Predictions with χEFT EM currents for $A = 2$–3 systems - Piarulli *et al.* in prep.

^{3}H magnetic f.f. using model III

bands represent cutoff dependence ($\Lambda = 500 - 600$ MeV)



trinucleon w.f.'s from hyperspherical harmonics expansion

Kievsky *et al.*, FBS**22**, 1 (1997); Viviani *et al.*, FBS**39**, 59 (2006); Kievsky *et al.*, J. Phys. G: Nucl. Part. Phys. **35**, 063101 (2008)

Minimize expectation value of H

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using trial function

$$|\Psi_V\rangle = \left[\mathscr{S} \prod_{i<j} \left(1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}\right) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

- single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
- central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
- pair correlation operators U_{ij} reflect influence of υ_{ij} (AV18)
- triple correlation operator U_{ijk} added when V_{ijk} (IL7) is present

Ψ_V are spin-isospin vectors in $3A$ dimensions with $\sim 2^A \binom{A}{Z}$ components

Lomnitz-Adler, Pandharipande, Smith, NP **A361**, 399 (1981) Wiringa, PRC **43**, 1585 (1991)

GFMC Predictions $A = 6$–9 – Green's function Monte Carlo

Given a decent trial function Ψ_V, we can further improve it by "filtering" out the remaining excited state contamination:

$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n$$

$$\Psi(\tau \to \infty) = a_0 \psi_0$$

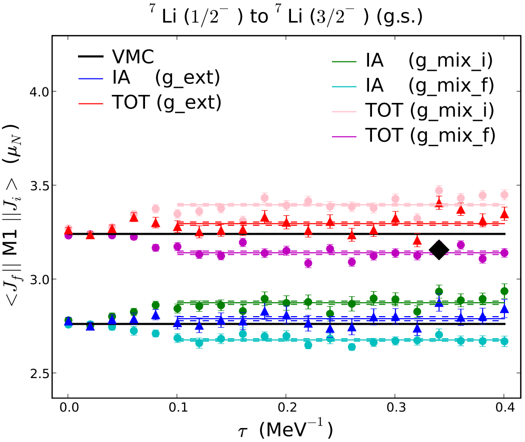
Evaluation of $\Psi(\tau)$ is done stochastically (Monte Carlo method) in small time steps $\Delta\tau$ using a Green's function formulation.
In practice, we evaluate a "mixed" estimates

$$\langle O(\tau)\rangle = \frac{{}_f\langle\Psi(\tau)|O|\Psi(\tau)\rangle_i}{\langle\Psi(\tau)|\Psi(\tau)\rangle} \approx \langle O(\tau)\rangle^i_{\text{Mixed}} + \langle O(\tau)\rangle^f_{\text{Mixed}} - \langle O\rangle_V$$

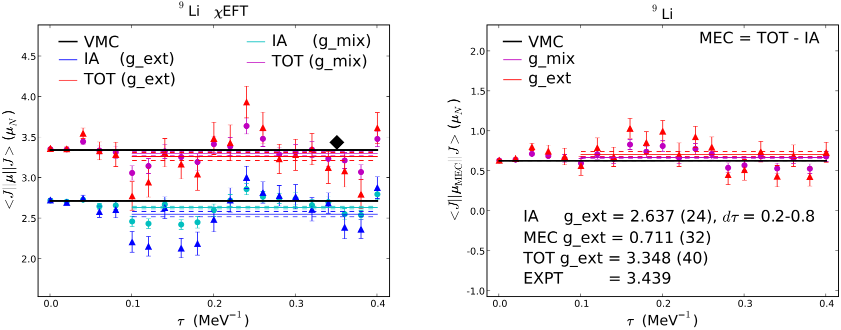
$$\langle O(\tau)\rangle^i_{\text{Mixed}} = \frac{{}_f\langle\Psi_V|O|\Psi(\tau)\rangle_i}{{}_f\langle\Psi_V|\Psi(\tau)\rangle_i} \;;\; \langle O(\tau)\rangle^f_{\text{Mixed}} = \frac{{}_f\langle\Psi(\tau)|O|\Psi_V\rangle_i}{{}_f\langle\Psi(\tau)|\Psi_V\rangle_i}$$

Pudliner, Pandharipande, Carlson, Pieper, & Wiringa, PRC **56**, 1720 (1997)
Wiringa, Pieper, Carlson, & Pandharipande, PRC **62**, 014001 (2000)
Pieper, Wiringa, & Carlson, PRC **70**, 054325 (2004)

Examples of GFMC propagation: M1 Transition in $A = 7$



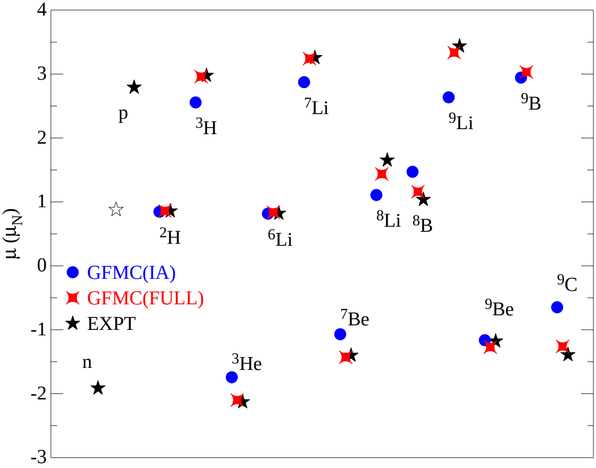
Examples of GFMC propagation: Magnetic moment in $A = 9$



Reduce noise by increasing the statistic for the IA results

GFMC calculation of magnetic moments in $A \leq 9$ nuclei: Summary

Predictions for $A > 3$ nuclei – AV18/IL7 + χEFT EM MEC



Preliminary results

$$\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

Magnetic moments in $A \leq 9$ nuclei: SNPA vs χEFT

	A	s.s.	IA	TOT SNPA	TOT χEFT*	EXP
IS	7	[43]	0.902 (3)	0.833 (12)	0.906 (7)	0.929
IV		[43]	– 3.944 (5)	– 4.587 (18)	– 4.670 (9)	– 4.654
IS	8	[431]	1.289 (8)	1.160 (15)	1.299 (9)	1.344
IV		[431]	0.182 (8)	– 0.129 (15)	– 0.139 (9)	– 0.310
IS	9	[432]	0.994 (15)	0.922 (32)	1.038 (21)	1.024
IV		[432]	– 1.095 (10)	– 1.371 (21)	– 1.532 (15)	– 1.610

Preliminary results

Overall improvement of isoscalar (IS) component of the magnetic moment

$$\mu = \mu_S + \tau_z \mu_V$$

Anomalous magnetic moment of ^{9}C

Mirror nuclei spin expectation value

- Charge Symmetry Conserving (CSC) picture ($p \longleftrightarrow n$) *

$$< \sigma_z >= \frac{\mu(T_z=+T)+\mu(T_z=-T)-J}{(g_s^p+g_s^n-1)/2} = \frac{2\mu(\mathrm{IS})-J}{0.3796}$$

- For $A=9$, $T=3/2$ mirror nuclei: ^{9}C and ^{9}Li
  EXP $<\sigma_z>= 1.44$ while THEORY $<\sigma_z> \sim 1$ (assuming CSC)
  possible cause: Charge Symmetry Breaking (CSB)
- Three different predictions for $<\sigma_z>$ with CSC w.f.'s (*) and CSB w.f.'s

$<\sigma_z>$	Symmetry	IA	TOT	EXP
CSB	$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2}),{}^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})$	1.29 (8)	1.52 (11)	1.44
CSC	$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2}),{}^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})^*$	0.95 (11)	1.00 (11)	
CSC	$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2})^*,{}^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})$	1.00 (11)	1.05 (9)	

Preliminary

- Need both CSB in the w.f.'s and MEC!

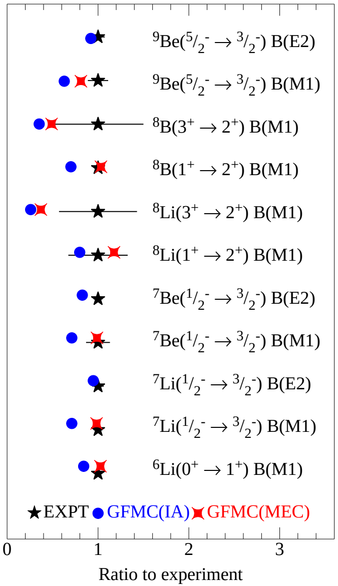
* Utsuno – PRC**70**, 011303(R) (2004)

GFMC calculation of M1 transitions in $A \leq 9$ nuclei: Summary

$$\mathrm{M1(IA)} = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

$$\mathrm{E2(IA)} = \sum_i e_{N,i}\, r_i^2\, Y_2(\hat{\mathbf{r}}_i)$$

Preliminary results



Summary

- SNPA and χEFT up to N3LO EM currents operators tested in the $A \leq 9$ nuclei
- Predictions from hybrid calculations of magnetic moment and M1 transitions in $A \leq 9$ nuclei are in good agreement with experimental data: Corrections beyond the IA are important to bring theory in agreement with experimental data
- Anomalous magnetic moment of ^{9}C is reproduced as a result of both CSB in the nuclear w.f.'s and χEFT two-body corrections

Outlook: electroweak properties of light nuclei

* EM structure of light nuclei
  - Extend hybrid calculations to different combinations of 2N and 3N potentials to study charge radii, charge and magnetic form factors of $A \leq 10$ systems (on going project)

* Weak structure of light nuclei
  - Extend hybrid calculations to weak properties of light nuclei