The Isoscalar Monopole Resonance of ^{4}He

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Work done in collaboration with:

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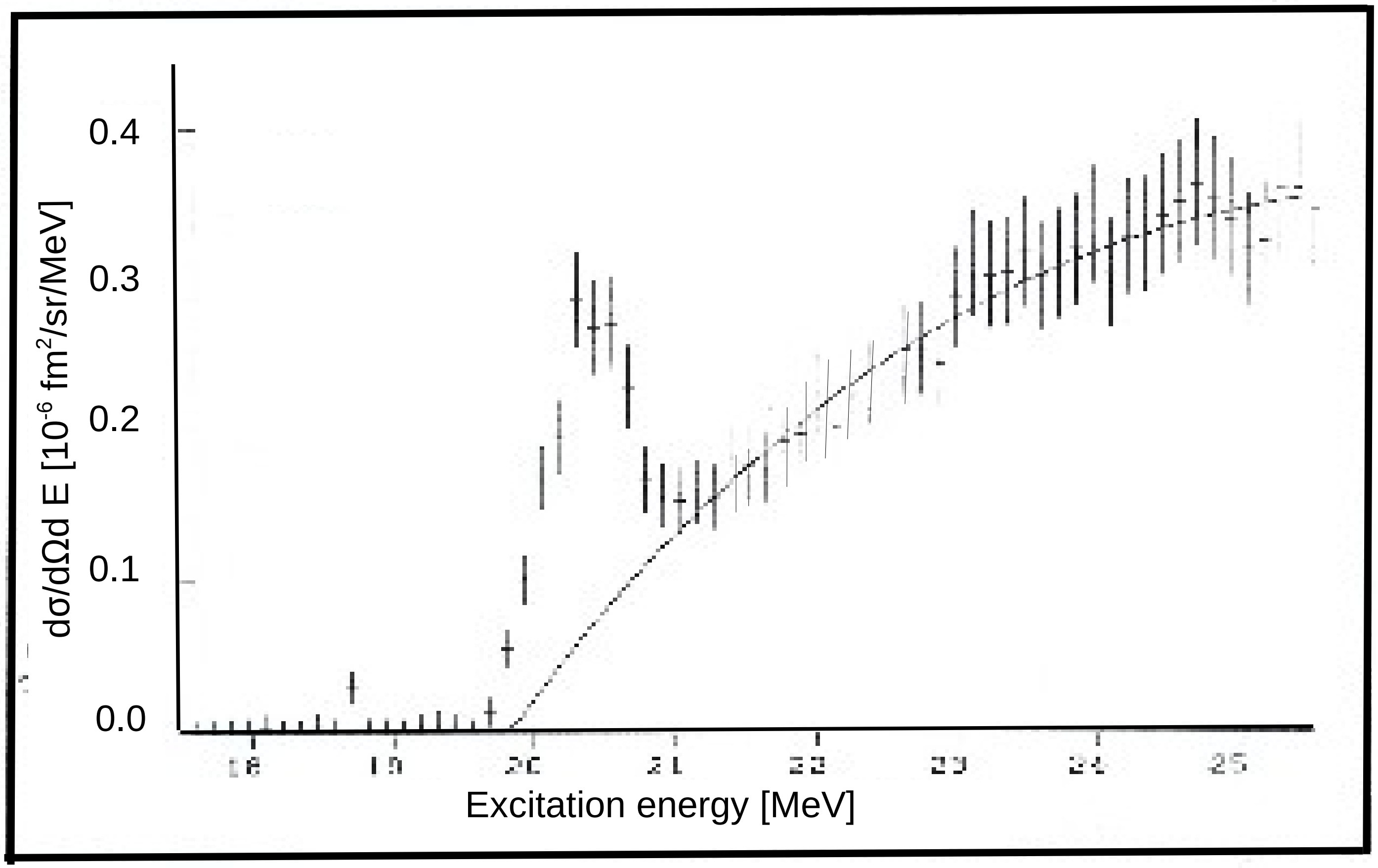
Nir **Barnea** (Jerusalem)
Winfried **Leidemann** (Trento)

0^+ Resonance in the ^{4}He compound system

Position at E_R = - 8.2 MeV, i.e. **above** the ^{3}H-p threshold

Γ = 270±70 keV - **Strong** evidence in electron scattering

G. Koebschall et al./ Quasi bound state in ^{4}He - Nucl. Phys. A405, 648 (1983)



With **electron scattering** one can study not only

the energy E_R of the resonance,

but also

the variation of the strength with q,

i.e. the momentum transferred from the electron to the nucleus

(different resolutions !)

In fact:

$$\left.\frac{d\sigma}{d\Omega\, dE_e}\right|_{Long} = \sigma_{Mott}\, q_\mu^4/q^4\, \mathbf{F_L}\,(\mathbf{q},\mathbf{E})$$

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at $E=E_R$ it is called "transition f.f."

The longitudinal form factor $\mathbf{F_L}$ **(q,E)** is given by

$$F_L(q, E) = \sum\!\!\!\!\!\!\int_n |\langle n | \rho(q) | 0 \rangle|^2 \, \delta(E - E_n + E_0)$$

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where

$\mathbf{H} \, |n\rangle = E_n \, |n\rangle$ and **H** is the nuclear Hamiltonian

$\rho(\mathbf{q}) = \sum_i^{A\;Z} \exp[\, i\, \mathbf{q} \cdot \mathbf{r}_i] \; (1 + \tau_i^3)/2$ is the **charge density** operator

$|n\rangle$ is in the continuum (N-body scattering state)

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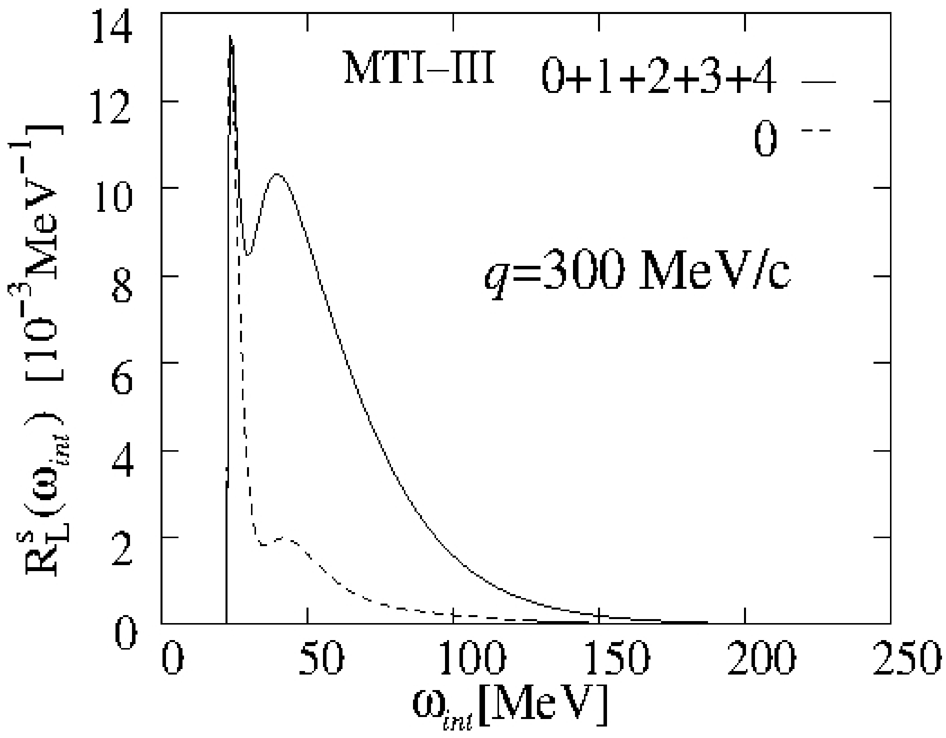
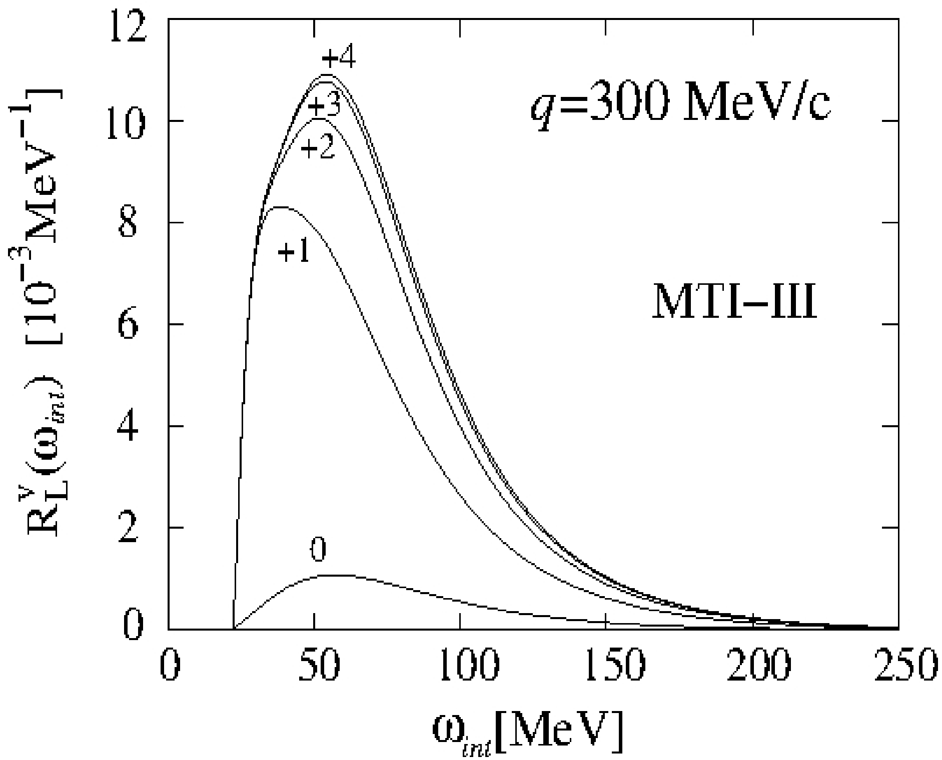
We have calculated the **isoscalar L=0 (monopole)** component i.e.

$$\rho_M(\mathbf{q}) = \sum_i^A j_0(qr_i)\, Y_{00}$$



Bacca et al.
Phys.Rev.C76:014003(2007)

isoscalar



isovector

Negligible !

Notice!

$$F_L(\mathbf{q},E) = \sum_n |\langle n|\rho(\mathbf{q})|0\rangle|^2 \delta(E - E_n + E_0)$$

can be rewritten as

$$F_L(\mathbf{q},E) = \mathrm{Im}\{\langle 0|\, \rho_M^{\dagger}(\mathbf{q})\, (E+E_0-H+i\,\varepsilon)^{-1}\, \rho_M(\mathbf{q})\, |0\rangle\}$$

Calculation with “continuum discretization”

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$\Sigma_n |m\rangle\langle m| = I$

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where $|n\rangle$ **is a square integrable** basis

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Approximation: continuum has been discretized!

We have calculated **F_L (q ,E)** with the **Lorentz Integral Transform (LIT)** method, which allows to reduce the **continuum** problem to a **bound state-like** problem, **rigorously**

Illustration of the LIT

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σ_I **finite!**

$$\Phi_L^{\sigma_I}(\mathbf{q},\sigma_R)$$

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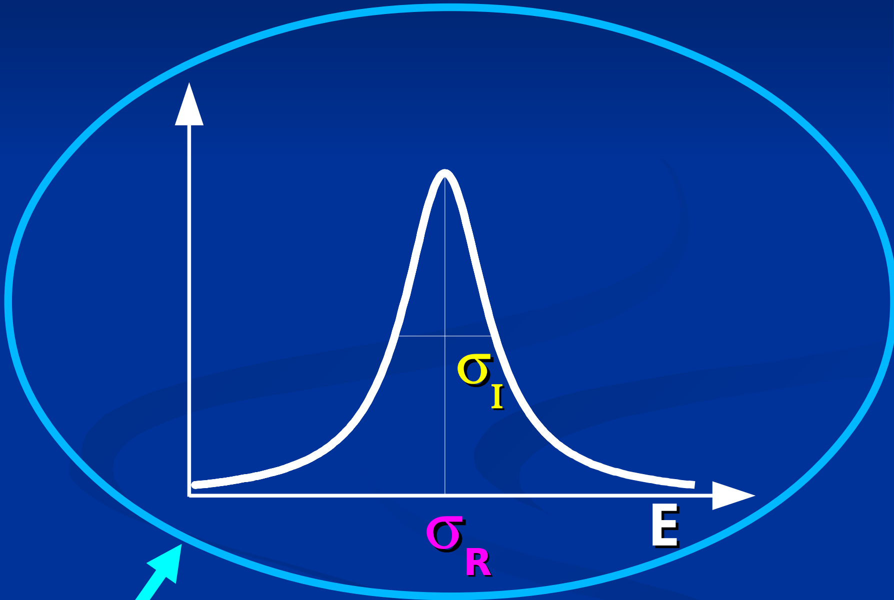

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$\Phi_L(\mathbf{q},\sigma_R)$ **is a bound state like problem !!!**

$$\Phi_L^{\sigma_I}(\mathbf{q},\sigma_R) = \int dE\, F_L(\mathbf{q},E)\, L(E, \sigma_R, \sigma_I)$$

To get $F_L(q, E)$ one has to invert the transform

$$\Phi_L^{\sigma_I}(q, \sigma_R) = \int dE\, F_L(q, E)\, L(E, \sigma_R, \sigma_I)$$

We have calculated

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$\Phi_L^{\sigma_I}(\mathbf{q}, \sigma_R)$ is NOT $F_L(\mathbf{q}, E)$

even if σ_I is very very small

Let' s test this statement on deuteron photodisintegration

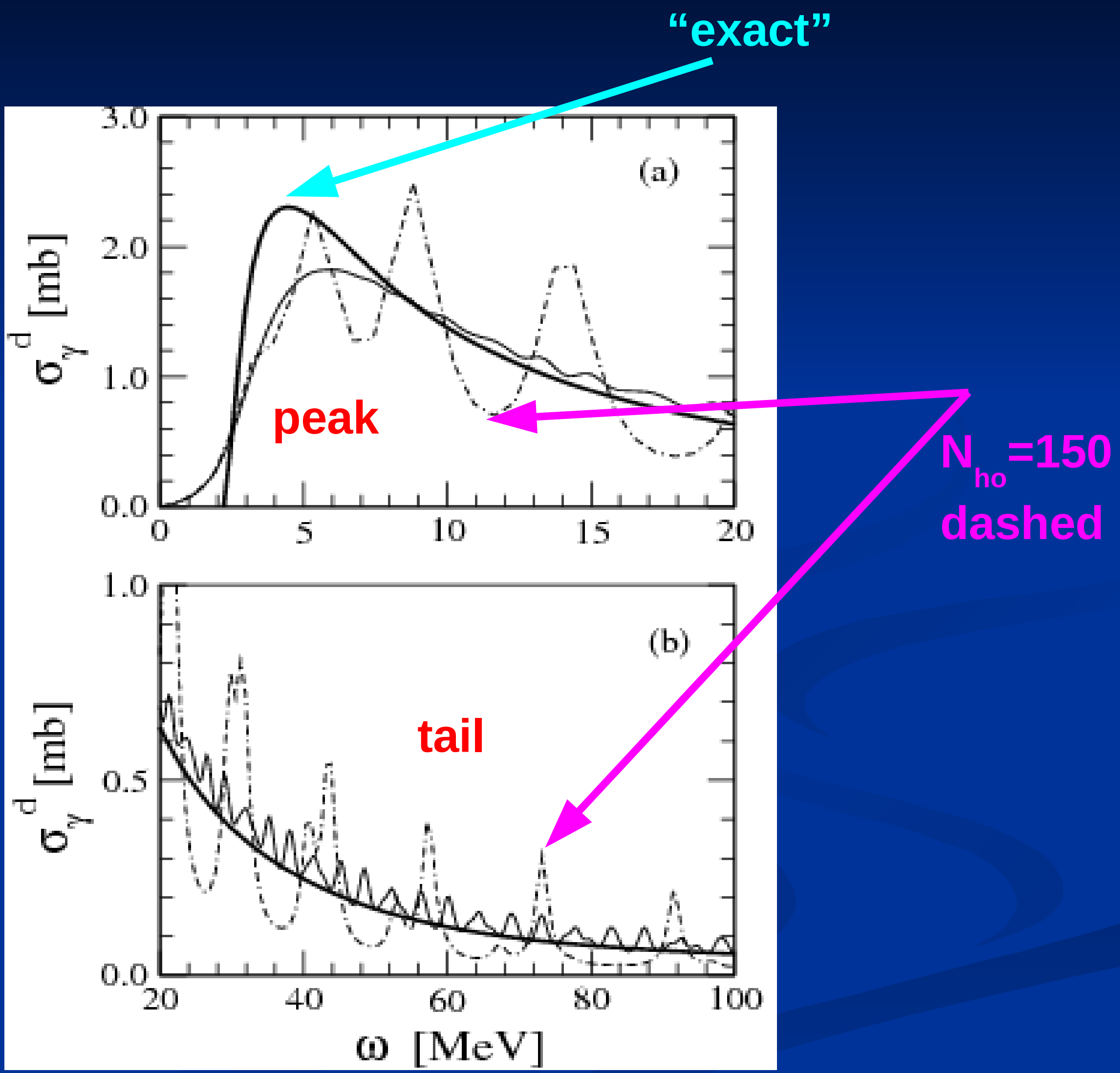
$$q = E = \omega \qquad F_L(q, E) \longrightarrow \sigma_\gamma(\omega)$$

$$\rho_M(q) \longrightarrow D$$

Let' s test this statement on deuteron photodisintegration

|n> = h.o. basis:

fix σ_I=1 MeV



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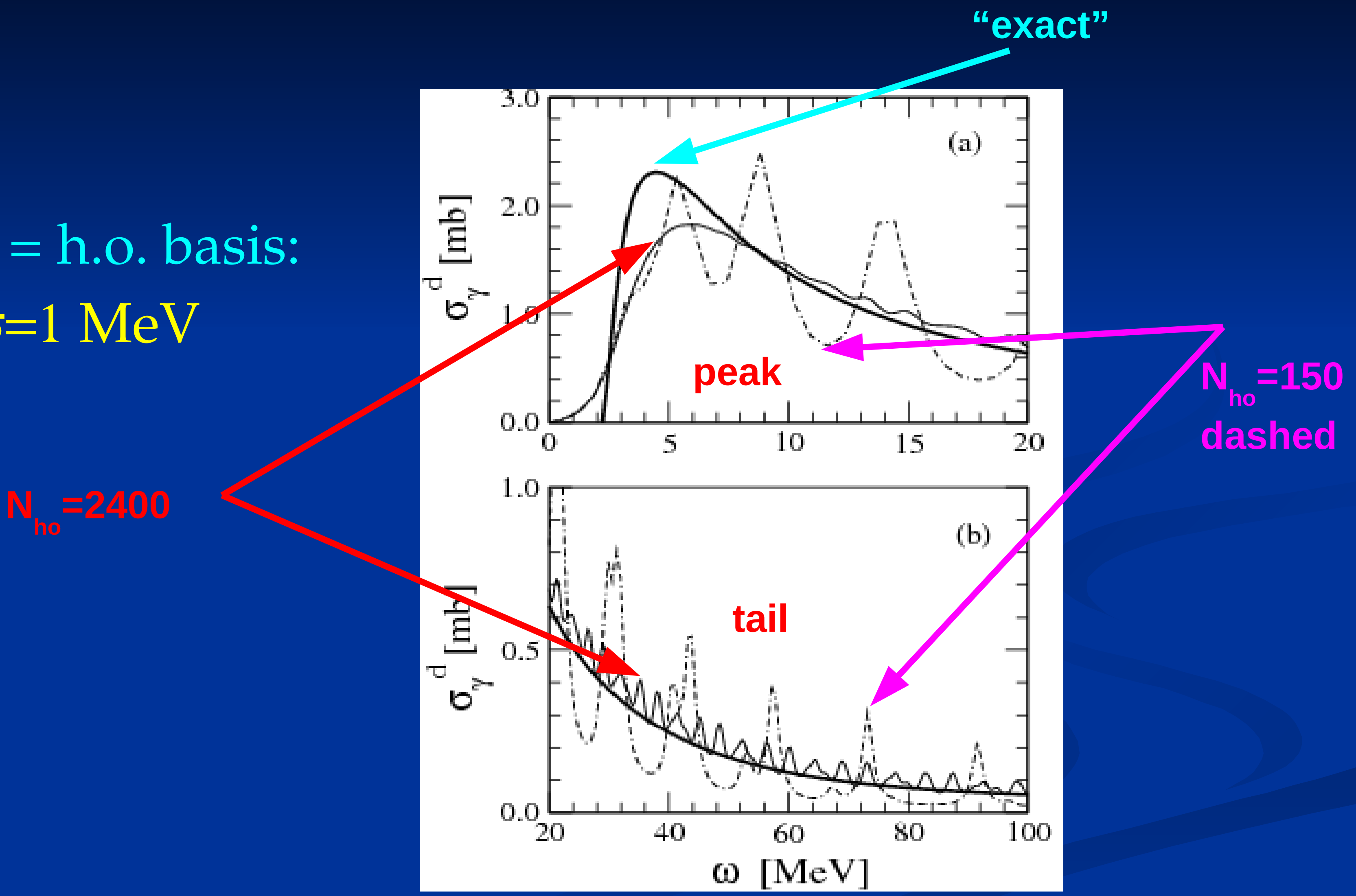

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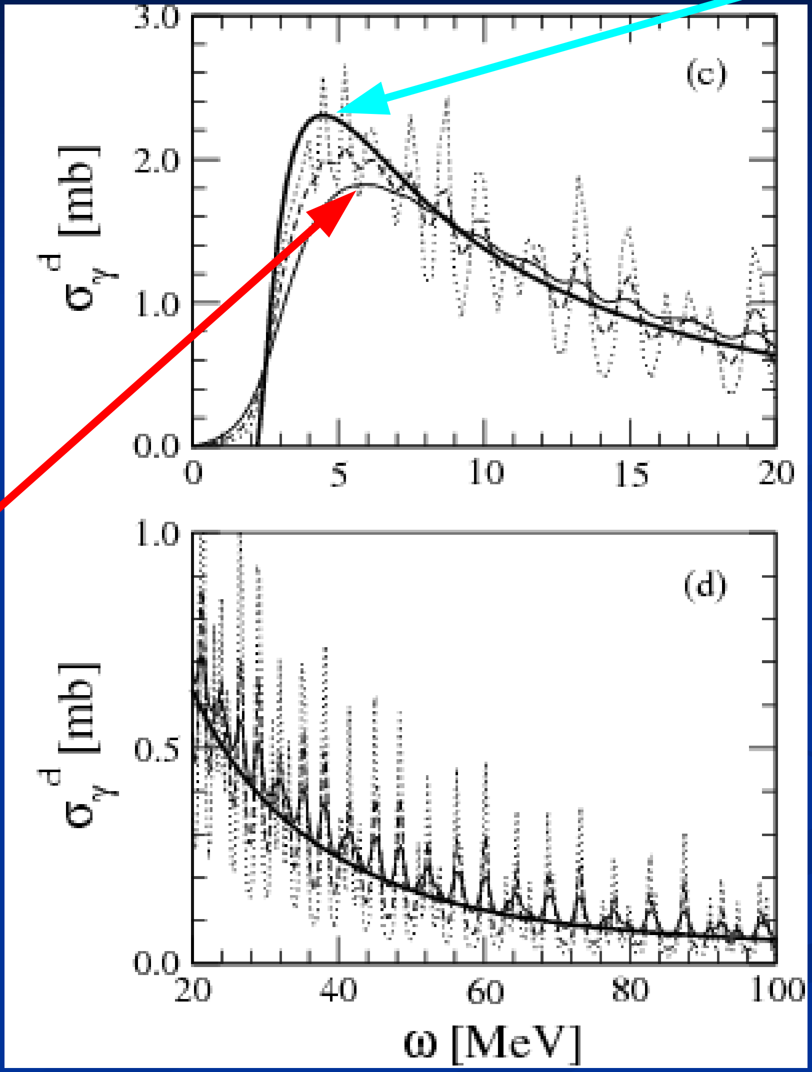
Fix a high

N_{ho}=2400

σ_I=1 MeV

“exact”



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$\sigma_I = 0.5$ MeV

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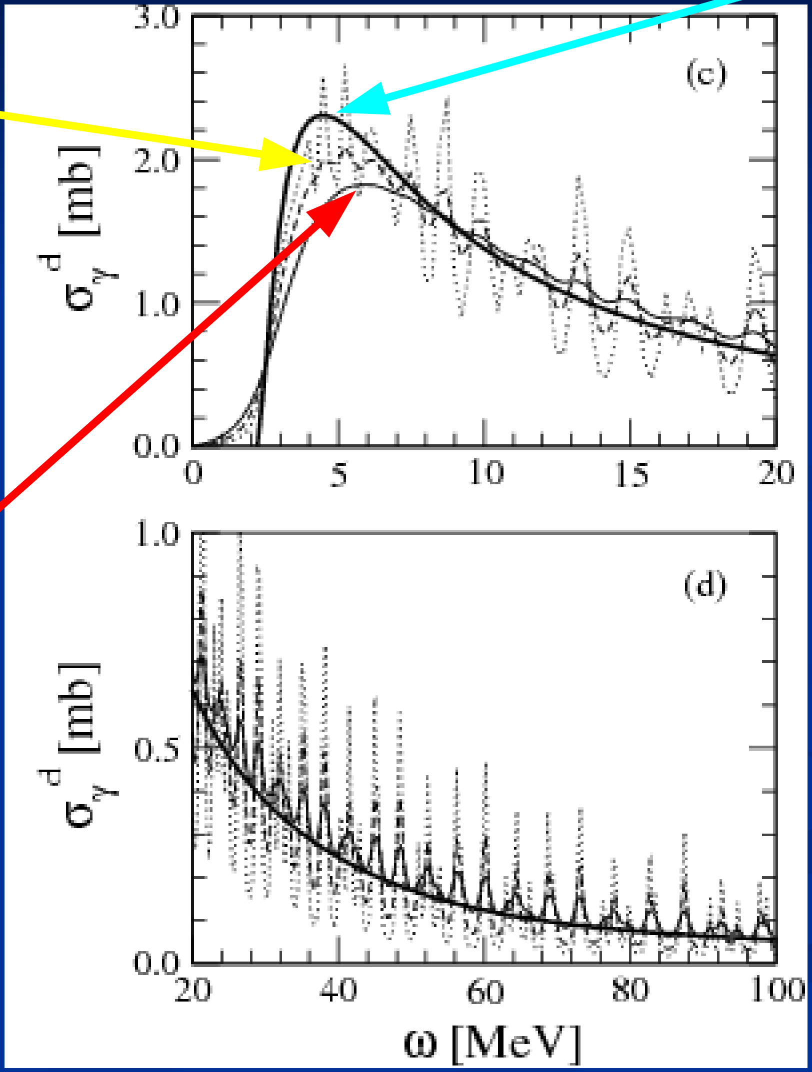
Fix a high

$N_{ho} = 2400$

$\sigma_I = 1$ MeV



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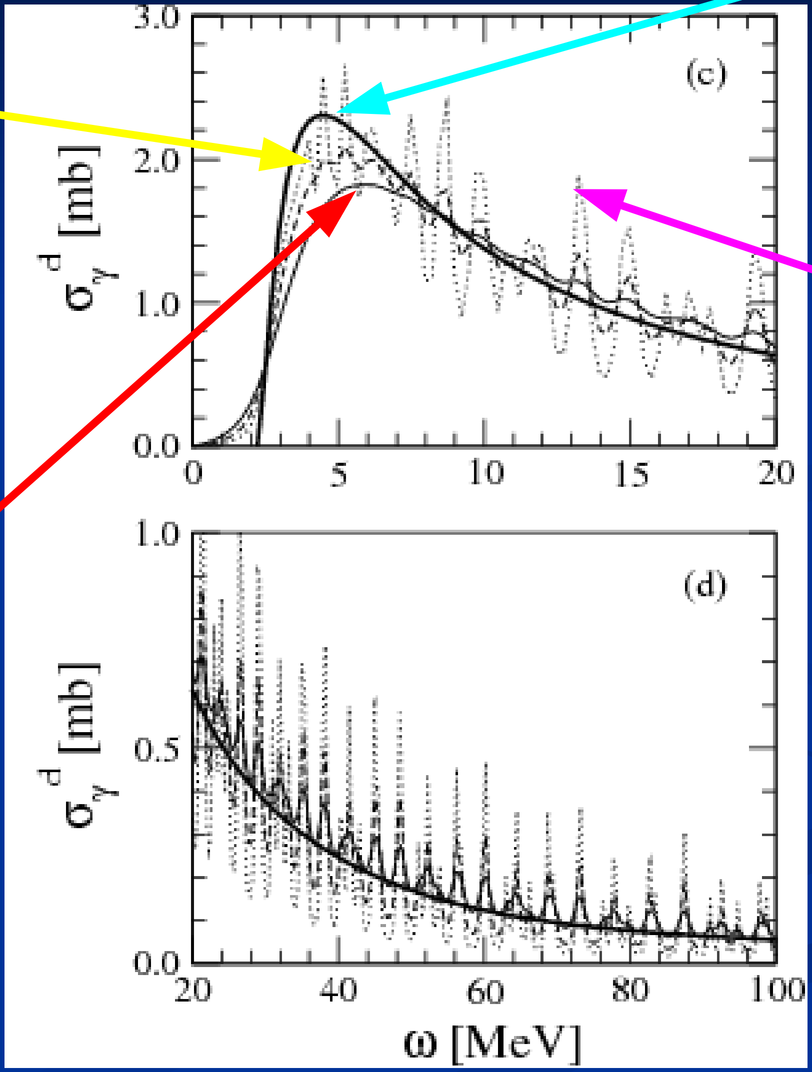


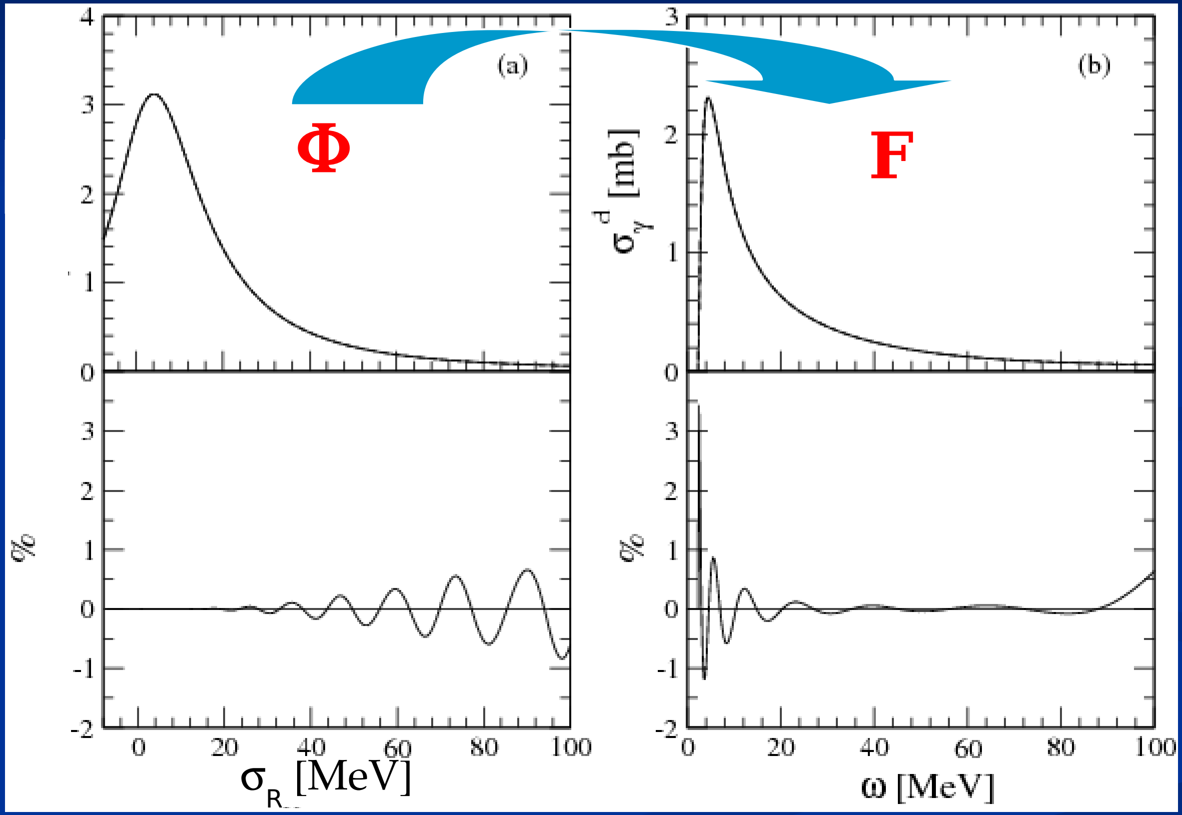
"exact"

σ_I=0.25 MeV

with a large σ_I=10 MeV
+ inversion (regularization)

Two almost equal curves at N_{ho}=150 and N_{ho}=2400

Per cent difference



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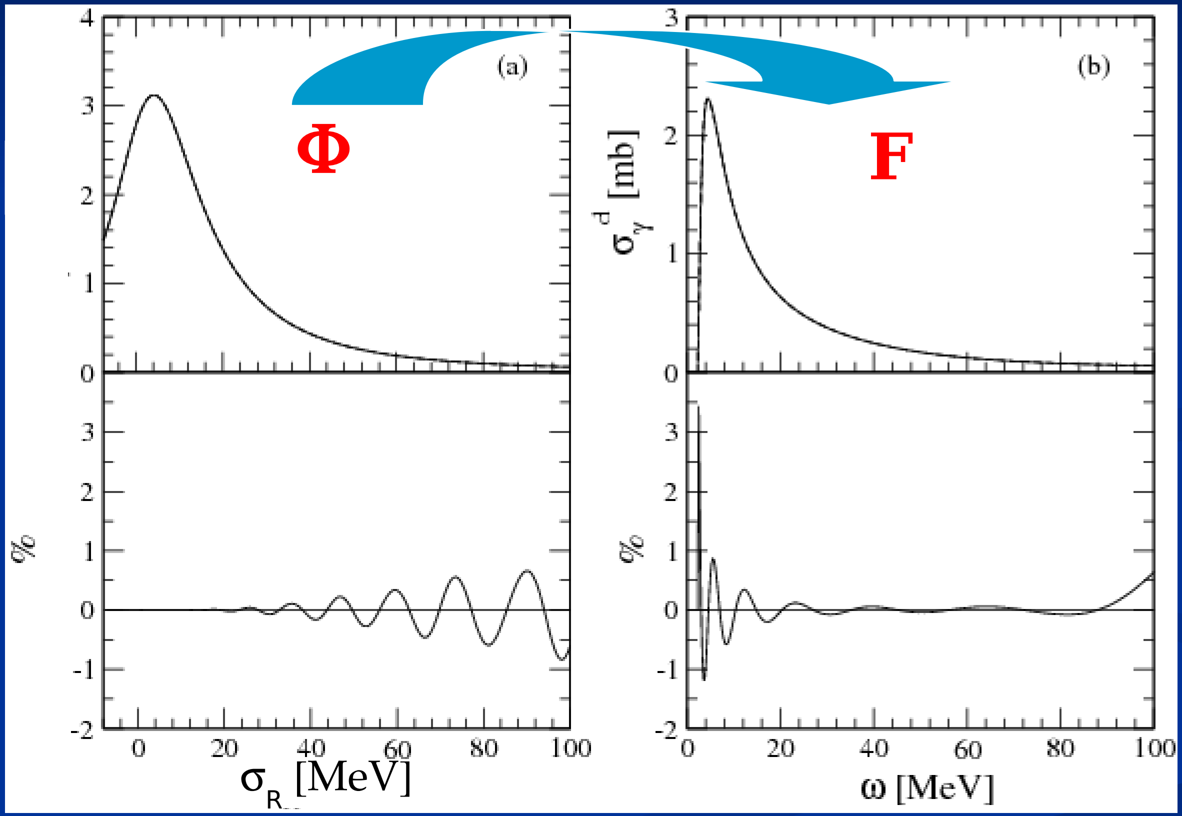


N_{ho}=150 is enough for accuracies at the % level!!

Message:

- Calculate the LIT where discretization is **correct**
- Convergence is **much faster**
- Invert the result using regularization with **continuum functions**

We have calculated

$$\Phi_L(\mathbf{q},\sigma_R) = \mathrm{Im}\{\langle 0|\, \rho_M(\mathbf{q})\, (\sigma_R + E_0 - H + i\,\sigma_I)^{-1}\, \rho_M(\mathbf{q})\, |0\rangle\}$$

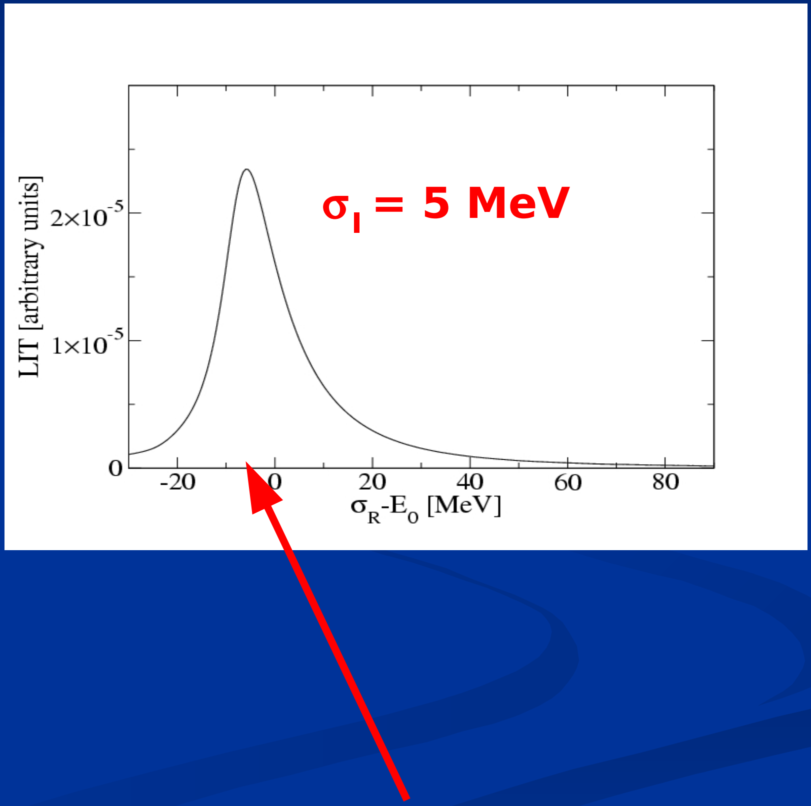
Because of a finite **σ_I,** now it is perfectly **legitimate** to solve the problem on a **square integrable** basis

We have used the **Hyperspherical Harmonics** basis and the Suzuki-Lee unitary transformation to speed up the convergence **(EIHH)**

As potentials we have used either

AV18+UIX or N3LO+N2LO

Some results:



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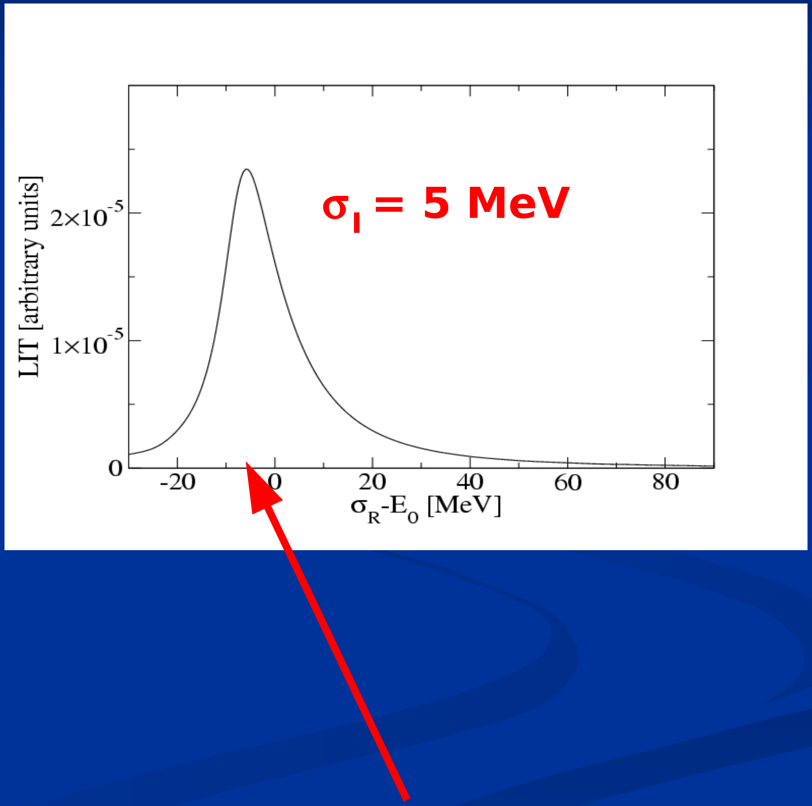

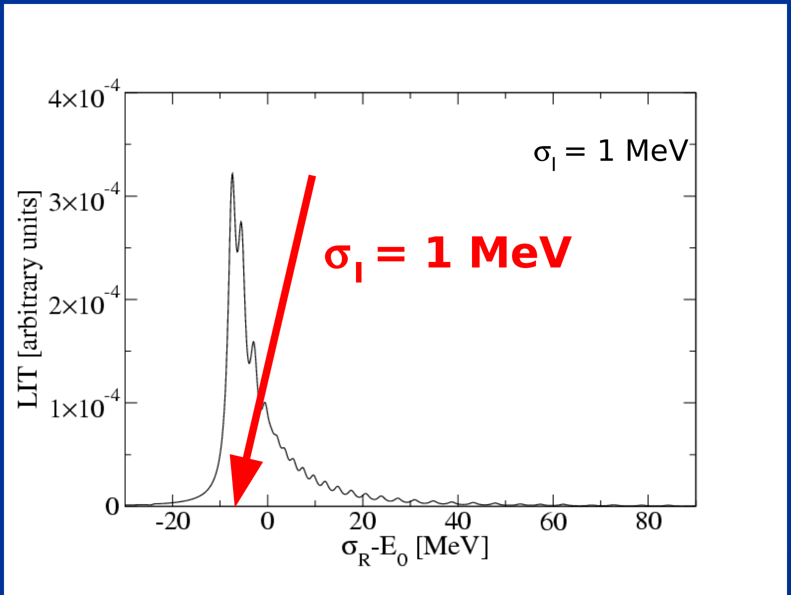

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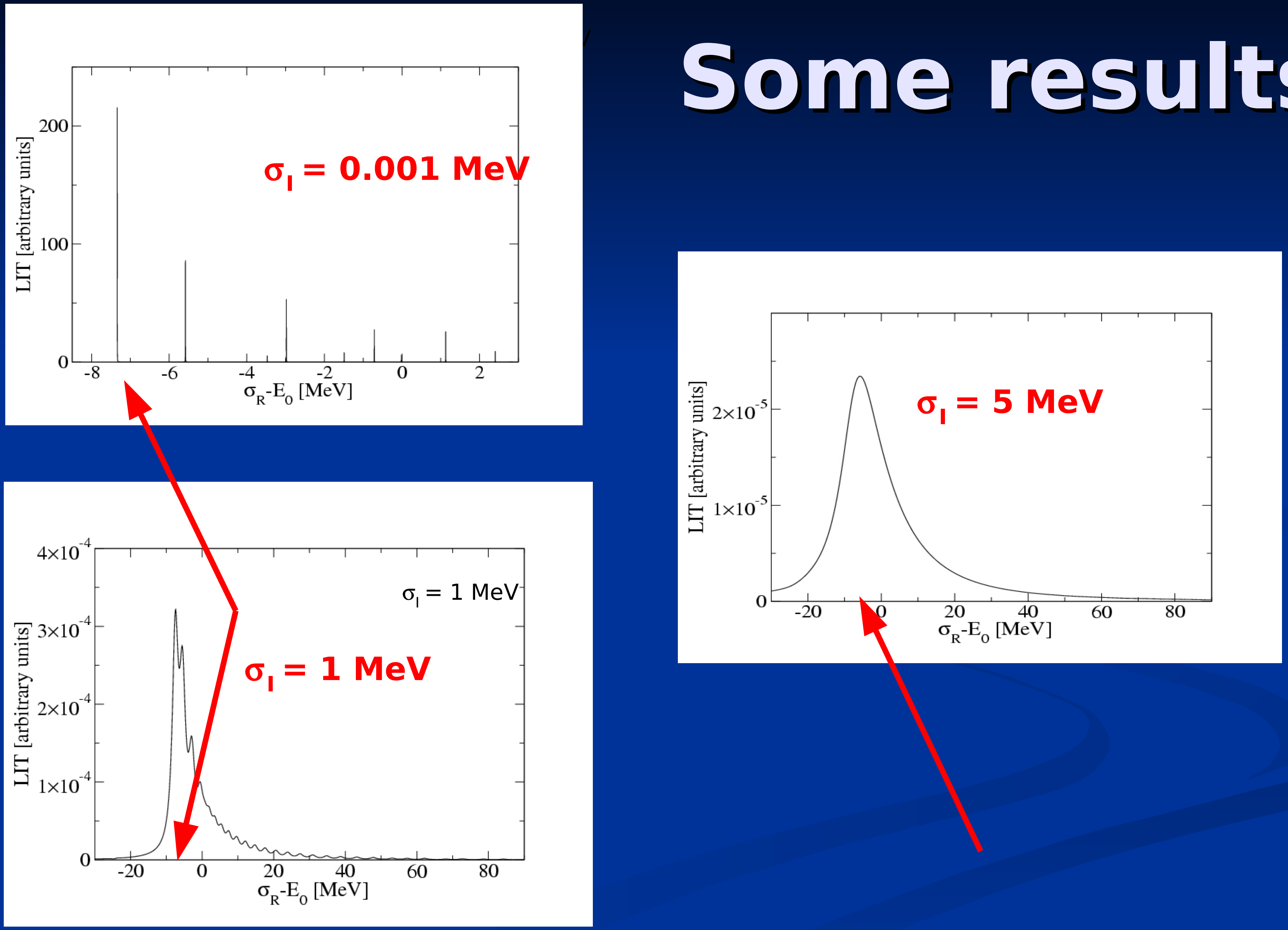

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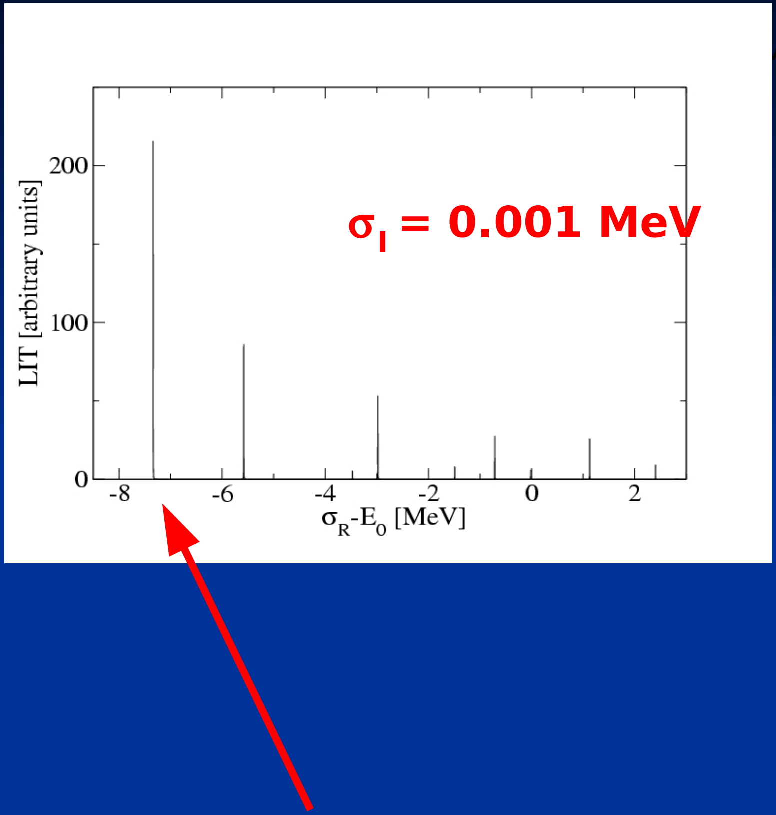

Of 200,000 “states” only very few are close to threshold

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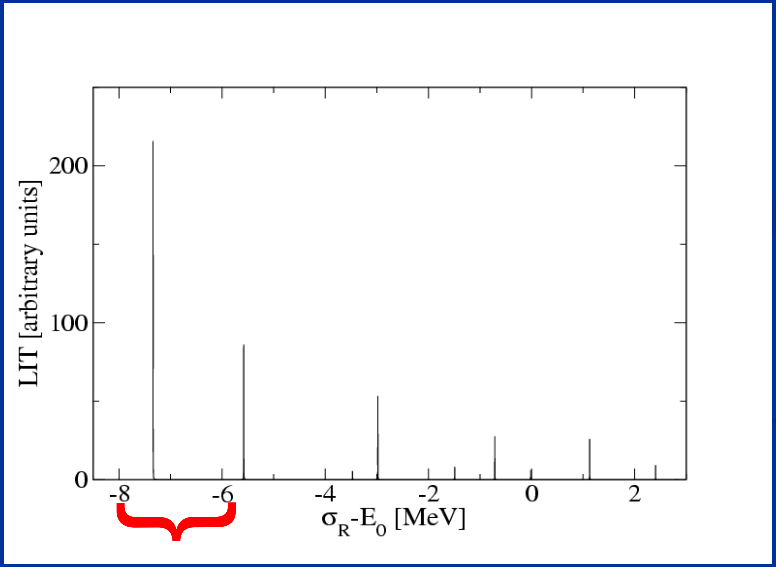

too few states!

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However, the strength of the resonance **can be determined**!

Of course **not** by taking the strength of the state $|\xi_\nu\rangle$!!

but by arranging **the inversion** in a suitable way:

LIT - Inversion

Standard LIT inversion method

1) Take the following ansatz for the response function $F_L(q, E)$

$$F_L(q, E) = \sum_{m=1}^{M} c_m \chi_m(q, E, \alpha_i)$$

with given set of functions χ_m , and unknown coefficients c_m

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2) Calculate: $\phi_L(q, \sigma_R) = \int dE \; \chi_m(q, E, \alpha_i) \, L(E, \sigma_R, \sigma_I)$

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4) Determine c_m and α_i by best fit on $\Phi_L(q,\sigma_R)$

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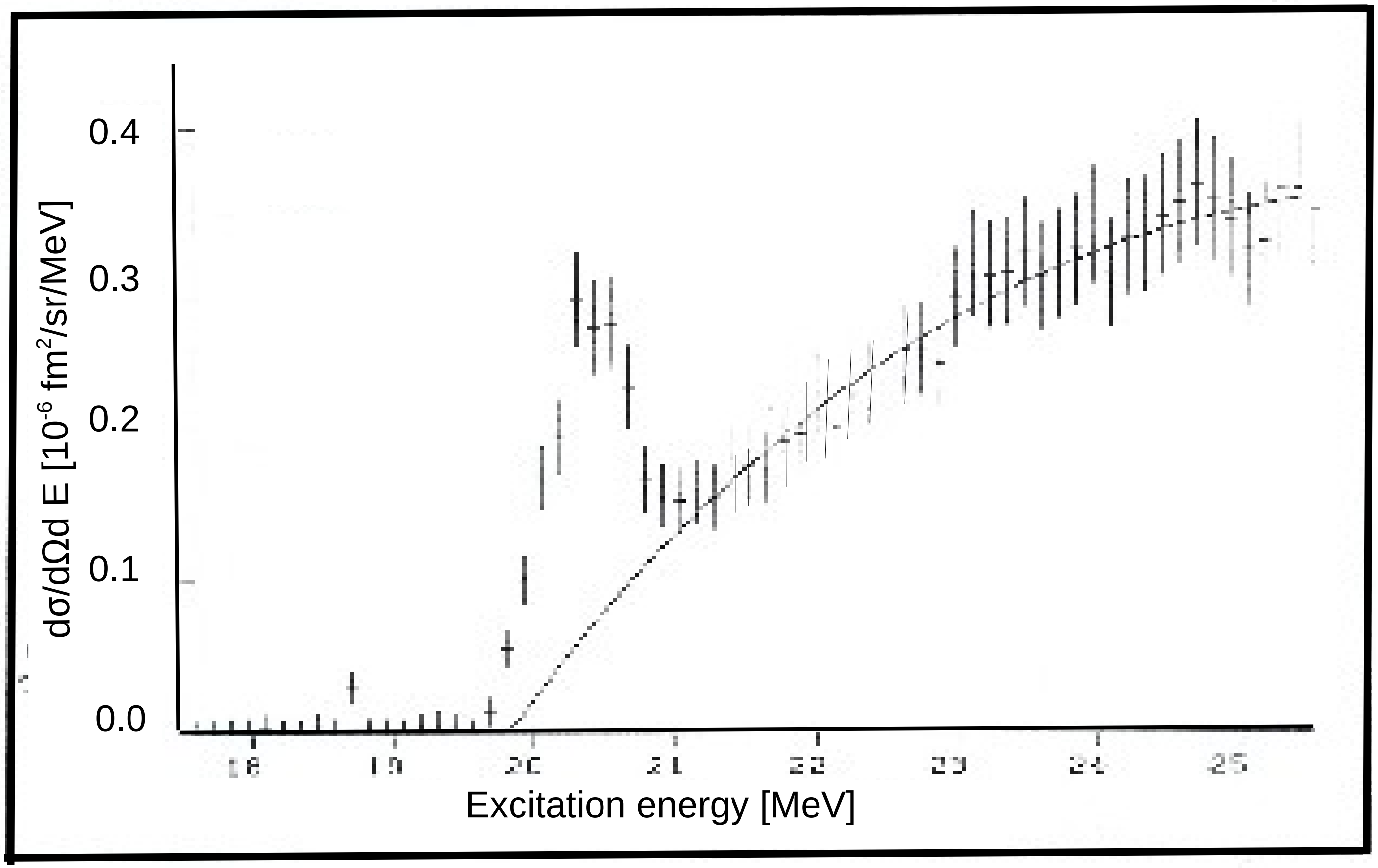

Inversion in the case of an (unresolved) resonance

1) Subtract a Lorentzian centered in E_R = energy of the big peak close to threshold, with parameter f_R, i.e.

$$\Phi'_L(q, \sigma_R, f_R) \equiv \Phi_L(q, \sigma_R) - f_R \,/\, [(E_R - \sigma_R)^2 + \sigma_I^2]$$

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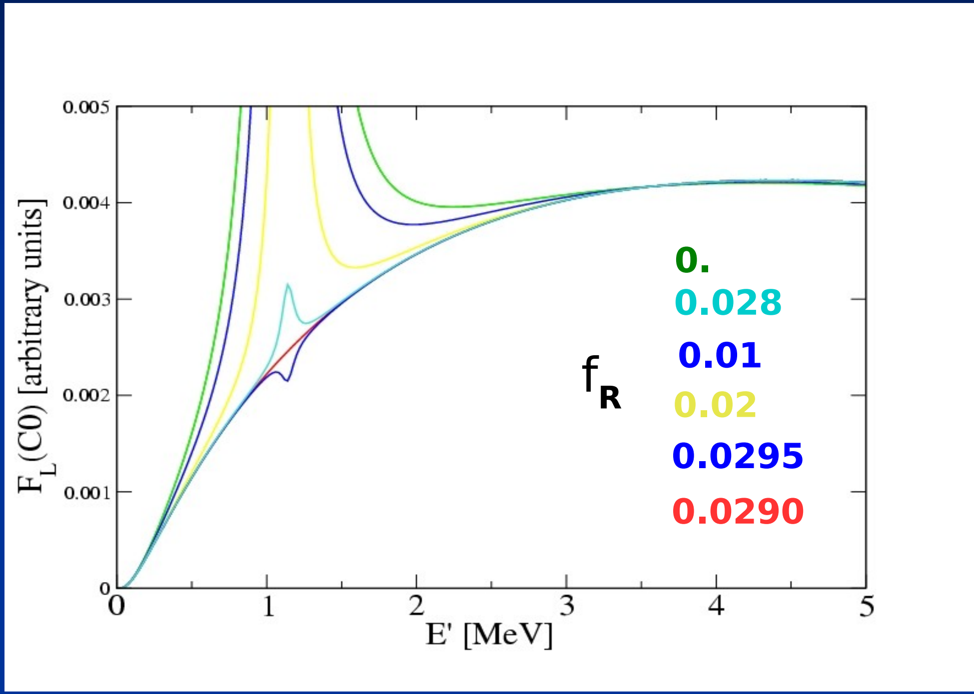
3) Reduce the strength f_R up to the point that the inversion does not show any resonant structure at the resonance energy E_R

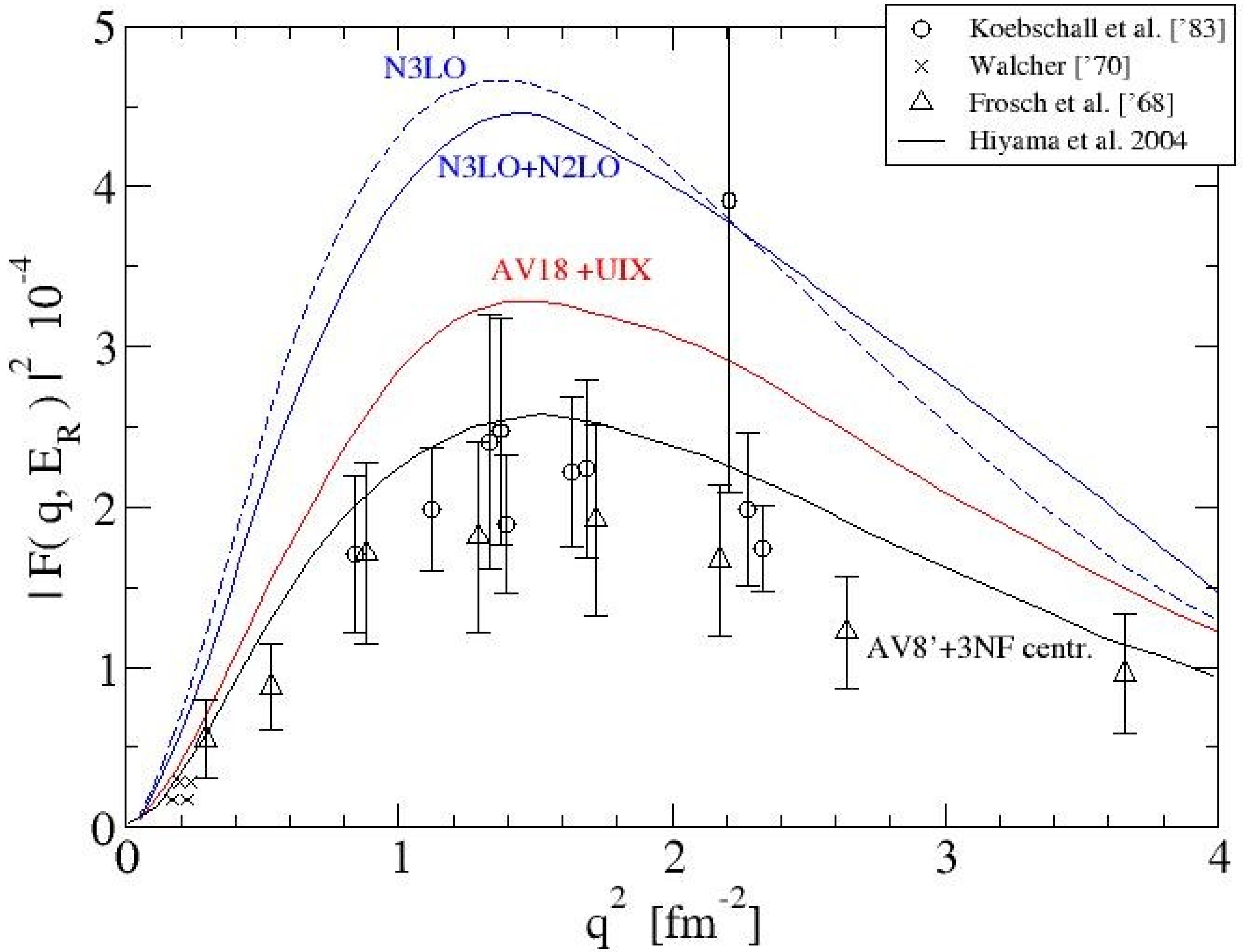


Inversion results with different f_R values

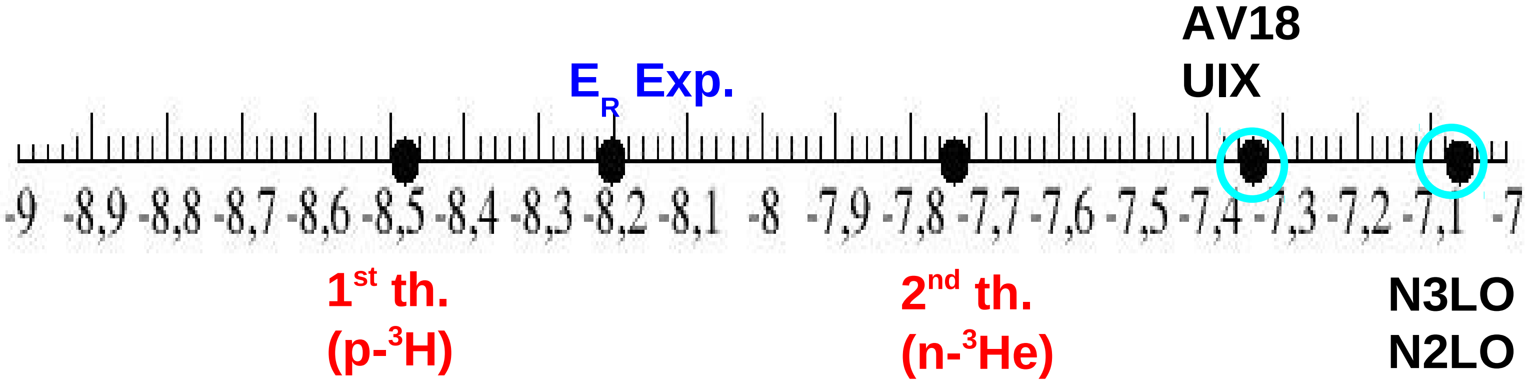
AV18+UIX, q=300 MeV/c

(σ_I = 5 MeV)

preliminary



ENERGIES


AV18
UIX
E_R Exp.
-9 -8,9 -8,8 -8,7 -8,6 -8,5 -8,4 -8,3 -8,2 -8,1 -8 -7,9 -7,8 -7,7 -7,6 -7,5 -7,4 -7,3 -7,2 -7,1 -7
1st th.
(p-3H)
2nd th.
(n-3He)
N3LO
N2LO

Sum Rules and Collectivity

$$m_0(q) = \int F(q,E)\, dE = \langle 0|\, \rho_M^\dagger(\mathbf{q})\ \rho_M(\mathbf{q})\, |0\rangle$$

$$m_1(q) = \int F(q,E)\, E\, dE = \langle 0|\, \rho_M^\dagger(\mathbf{q})\, H\ \rho_M(\mathbf{q})\, |0\rangle$$

$$m_{-1}(q) = \int F(q,E)/E\, dE = \alpha_M = \text{compressibility}$$

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In many-body theories the fraction of **total strength (m_0)** exhausted by the strength of a resonance is considered an index of how much a resonance is the result of a collective motion (typical example: GDR).

Sum Rules and Collectivity

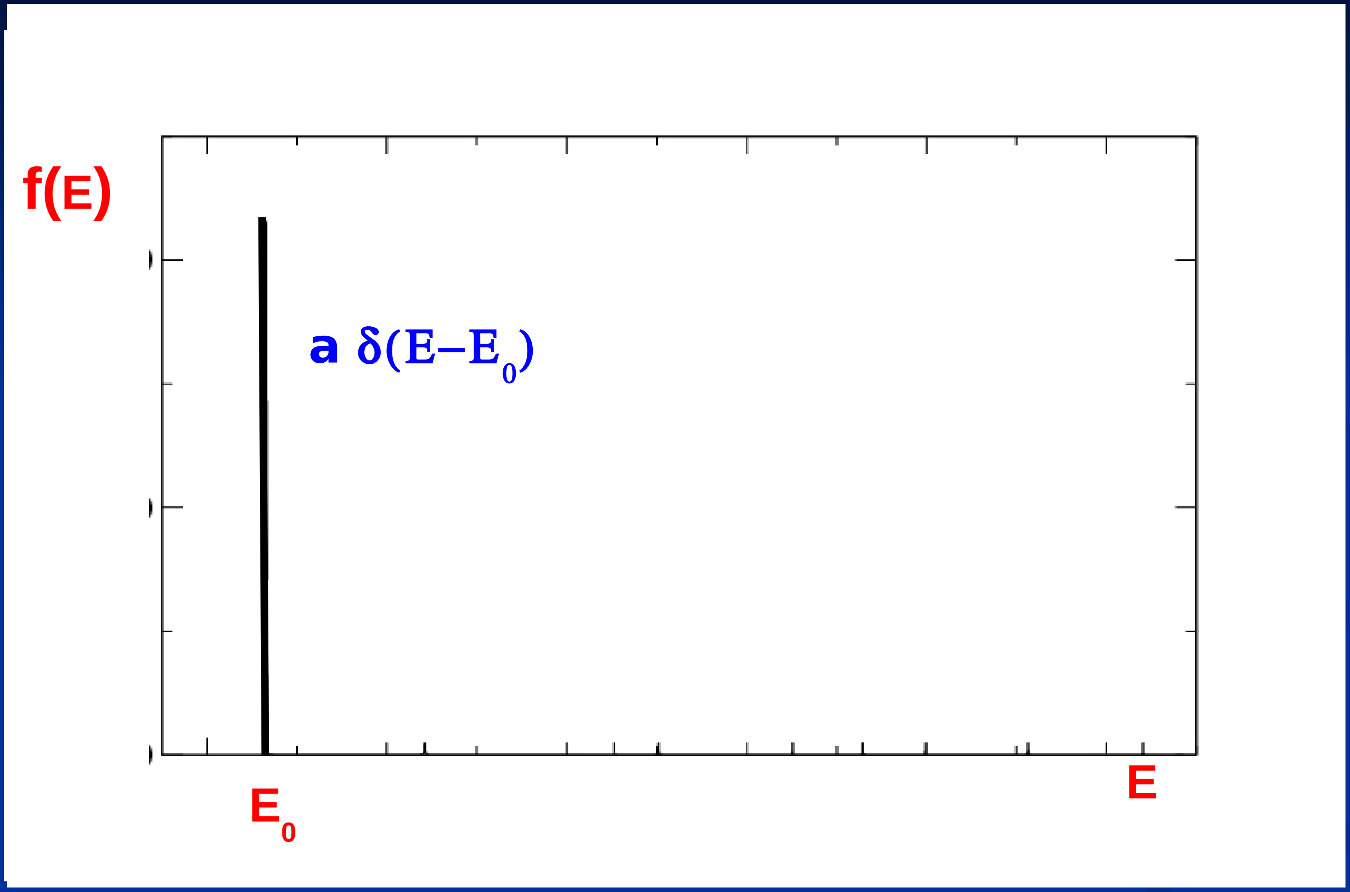
$$m_1(q) = \langle 0|\, \rho_M^{\dagger}(q)\, H\, \rho_M(q)\, |0\rangle =$$

$$= 1/2\, \langle 0|\, [\rho_M^{\dagger}(q)\, ,\, [H\, ,\, \rho_M(q)]]\, |0\rangle$$

In the limit q --> 0 $\rho_M(\mathbf{q})$ ---> $q^2 \sum_i r_i^{\,2}$ and $\mathbf{m_1}$ **---> $\mathbf{q^2 A / m <r^2>}$**

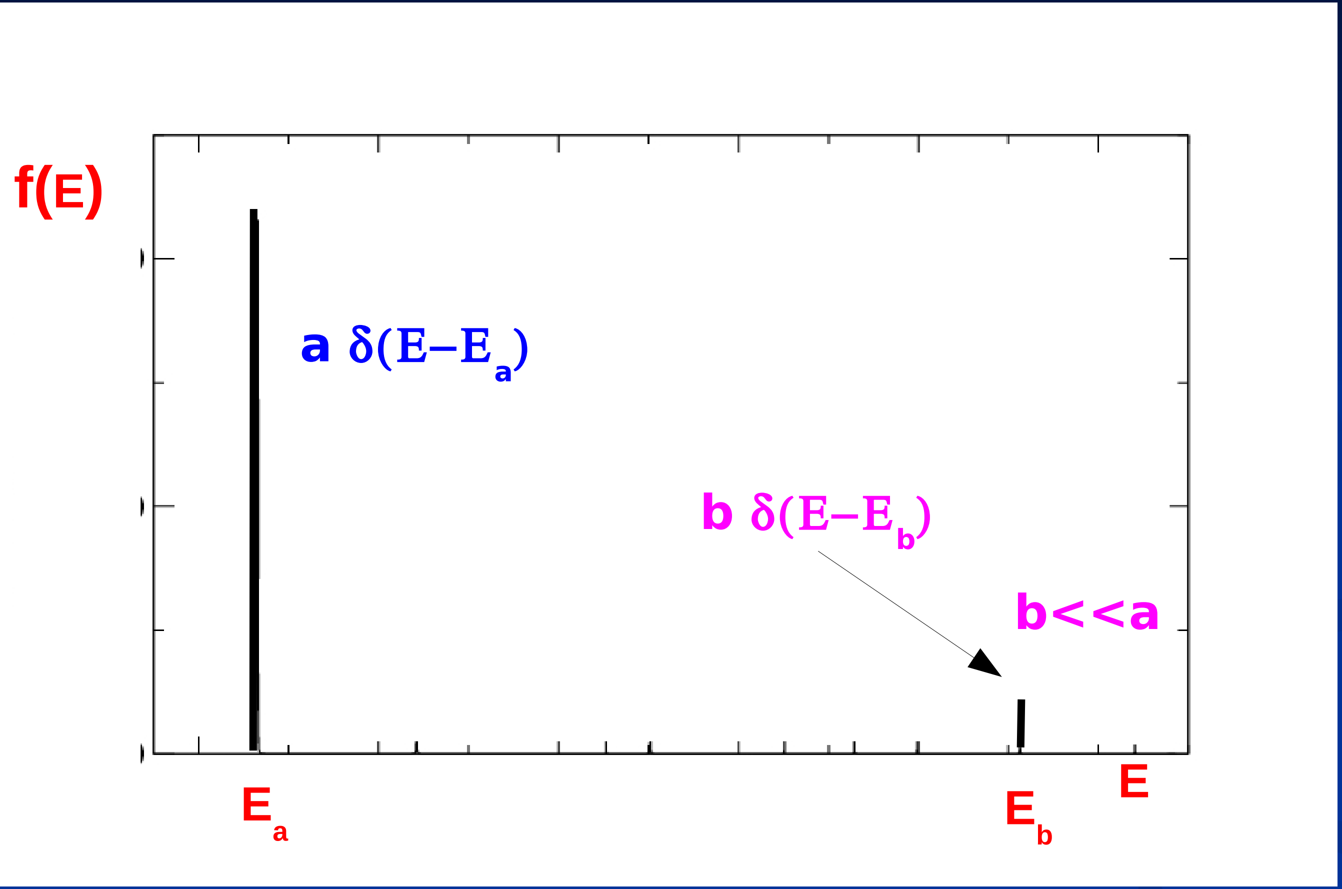
Since m_1 happens to be "model independent", in this case it is the fraction of $\mathbf{m_1}$ exhausted by the monopole resonance strength that is considered an index of how much a resonance is the result of a collective motion.

Sum Rules and Collectivity



$$\frac{\int E\,F(E)\,dE}{m_1} = \frac{\int F(E)\,dE}{m_0} = 1$$

Sum Rules and Collectivity



$$\frac{\int E\,F(E)\,dE}{m_1} << \frac{\int F(E)\,dE}{m_0} \sim 1$$

COLLECTIVITY ???

q [MeV/c]	$F(q,E_R)/m_0$ %	$E_R F(q,E_R)]/m_1$ %
50.	50	26
100.	45	29
150.	39	24
200.	32	18
300.	20	10

conclusions

- The form factor at the 0^+ resonant energy seems to be a good observable to **discriminate potential models.**

- Strength obtained from continuum discretization can be very different from the true continuum result. **LIT+regularization inversion** may give the good result

- Is the monopole resonance in ^{4}He a **collective** state?