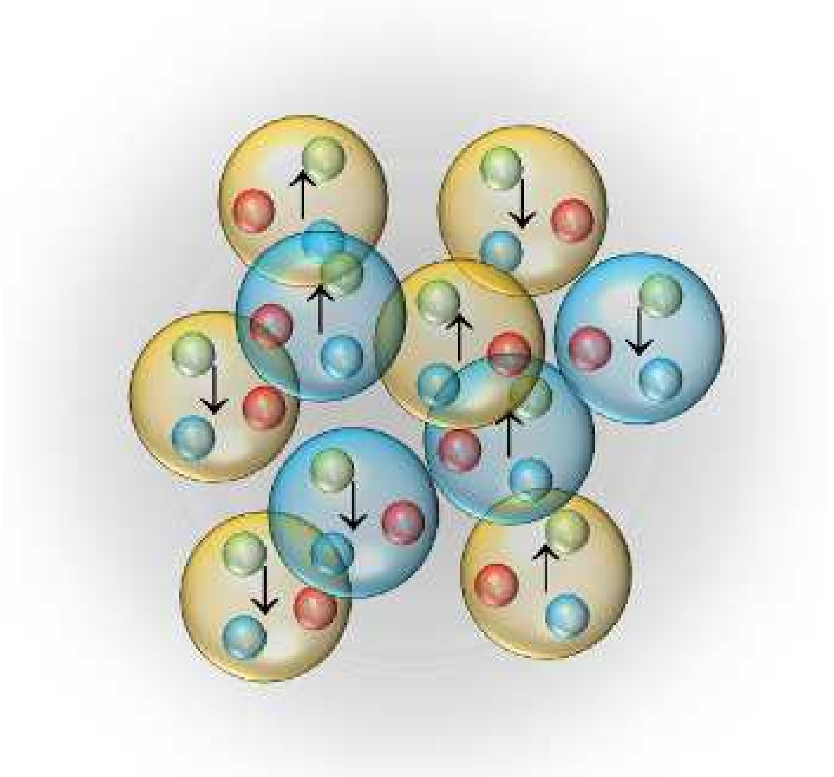
# Insight into the structure of light nuclei through short- and long-range correlations



**Thomas Neff**

**INT Workshop**
**“Structure of Light Nuclei”**

**Seattle, USA**

**October 8-12, 2012**

# Overview

**Two-body densities in (very) light Nuclei**

**Unitary Correlation Operator Method**

**Fermionic Molecular Dynamics**

**$^{3}$He($\alpha$,$\gamma$)$^{7}$Be Radiative Capture Reaction**

- **bound and scattering states**
- **astrophysical $S$-factor**

**Cluster States in $^{12}$C**

- **FMD and microscopic cluster model**
- **form factors, expansion in HO basis, two-body densities**

# Short-Range Correlations

**Two-body densities for A=2-4 nuclei from few-body calculations with AV8' interaction**

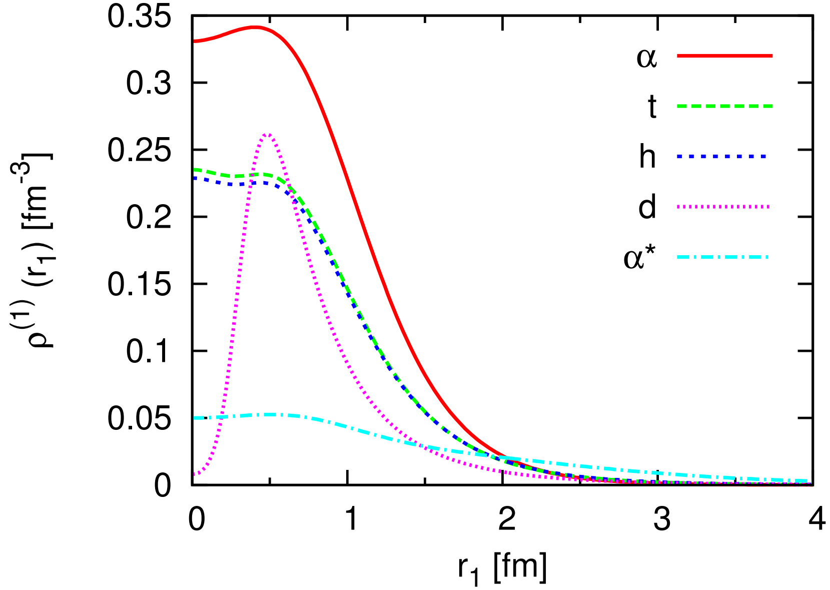
*preliminary:*

**Two-body densities for $^4$He using NCSM and SRG evolved AV18 interactions**

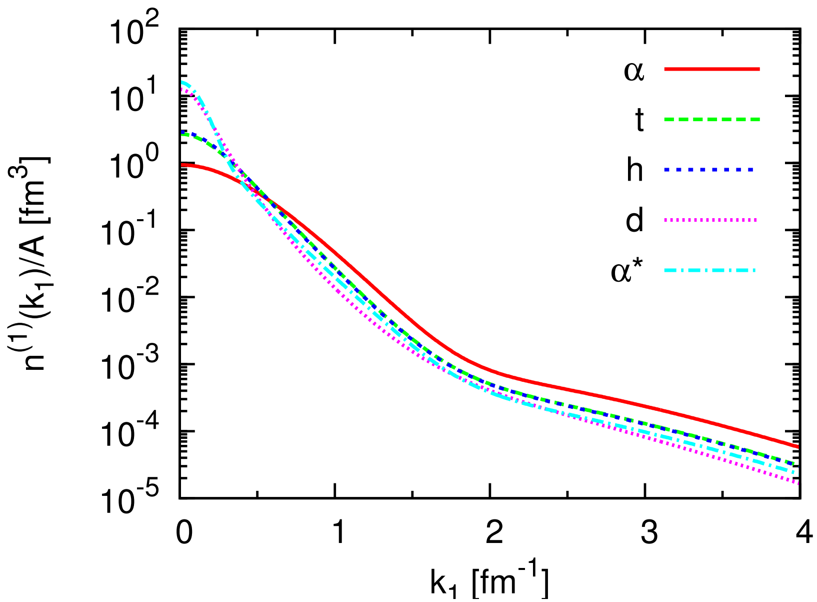
**Two-body densities for $^4$He using NCSM, SRG evolved AV18 and N3LO interactions and SRG transformed two-body density operators**

# One-body densities (AV8’)

## coordinate space




## momentum space



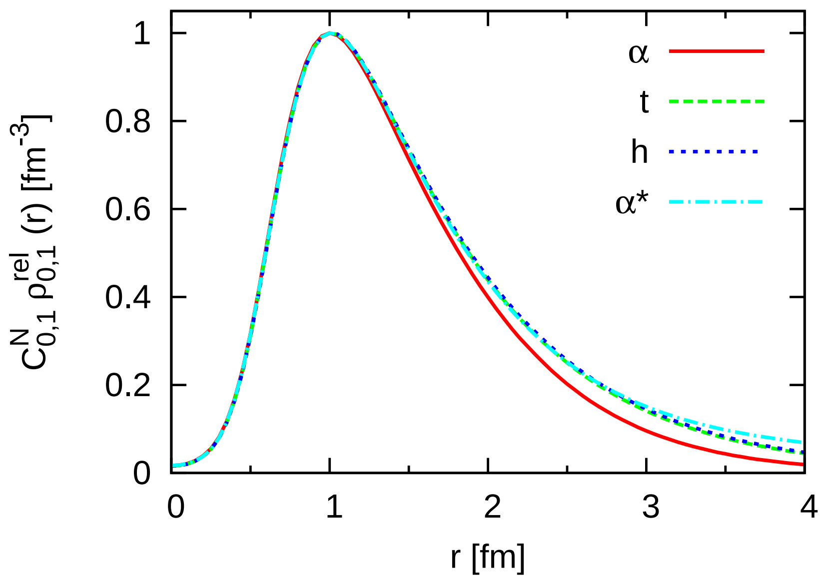

- one-body densities calculated from exact wave functions (Correlated Gaussians) for AV8’ interaction
- coordinate space densities reflect different sizes and densities of $^2$H, $^3$H, $^3$He, $^4$He and the $0_2^+$ state in $^4$He
- similar high-momentum tails in the one-body momentum distribution

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

## Short-range correlations

# Two-body densities in $A = 2, 3, 4$ Nuclei (AV8')

**coordinate space**
$S = 0$, $T = 1$




**momentum space**
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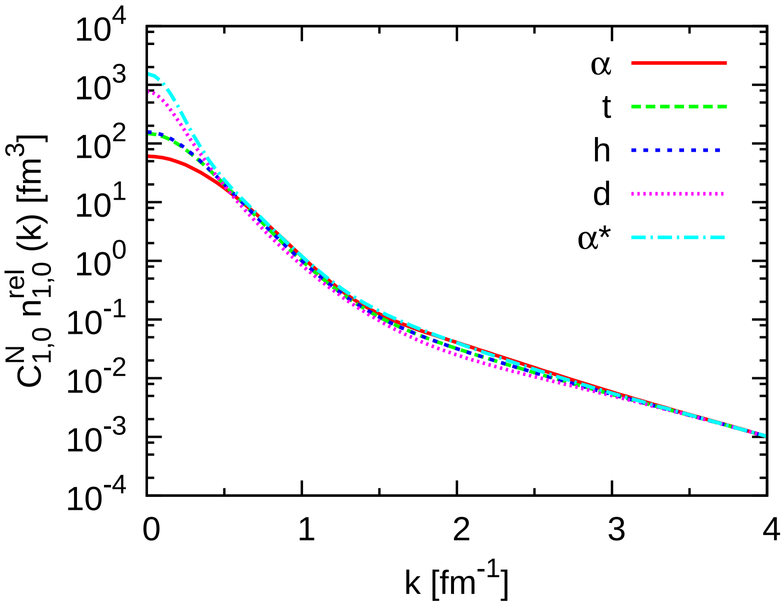
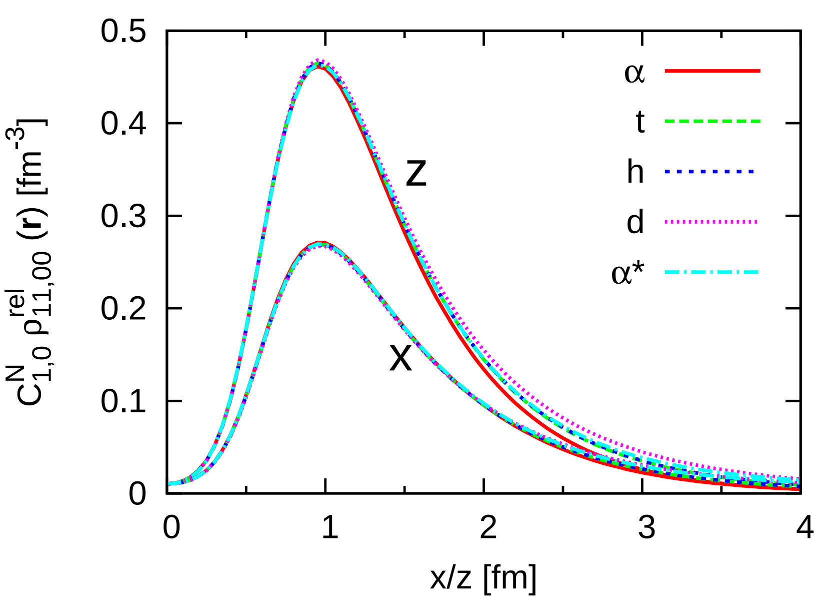



- normalize two-body density in coordinate space at $r$=1.0 fm
- normalized two-body densities in coordinate space are identical at short distances for all nuclei
- use the **same** normalization factor in momentum space – high momentum tails agree for all nuclei

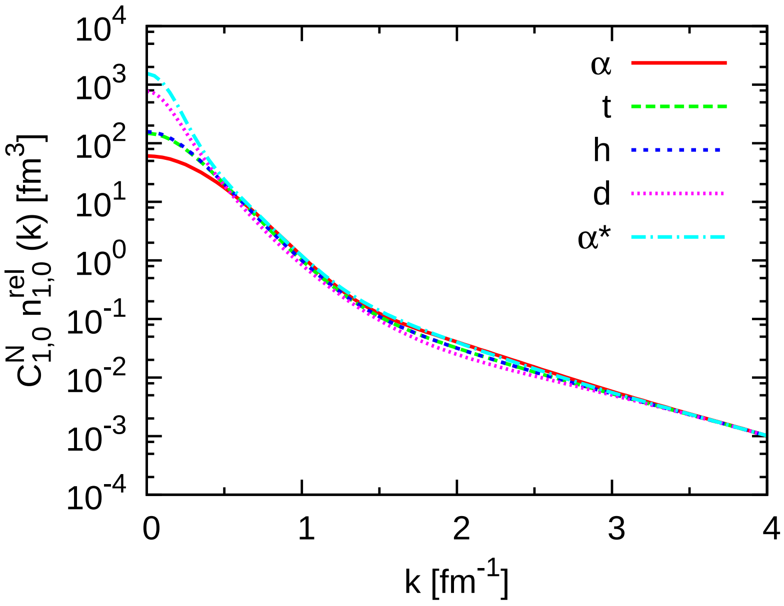
Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

# Two-body densities in $A = 2, 3, 4$ Nuclei (AV8')

**coordinate space**
$S = 1, M_S = 1, T = 0$




**momentum space**
$S = 1, T = 0$

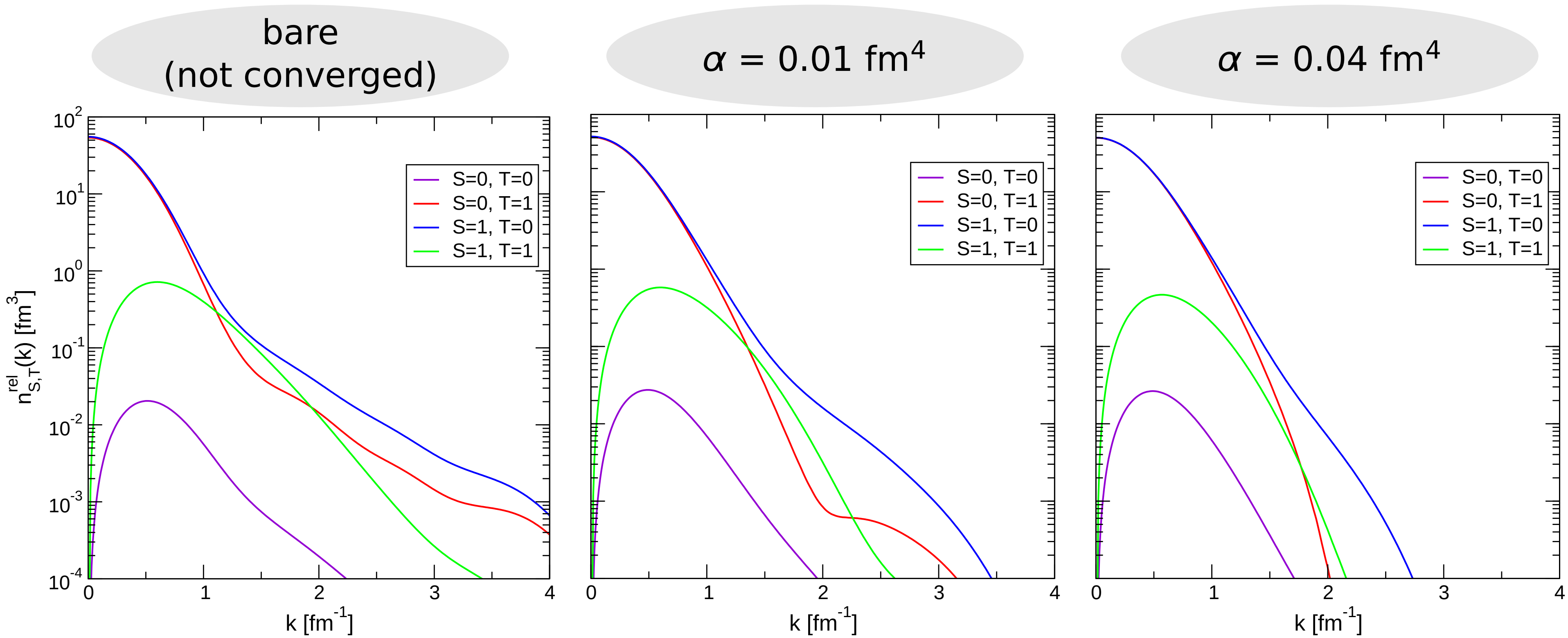



- normalize two-body density in coordinate space at $r$=1.0 fm averaged over all angles
- normalized two-body densities in coordinate space are identical at short distances for all nuclei
- use the **same** normalization factor in momentum space – high momentum tails agree for all nuclei

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

## Two-body densities

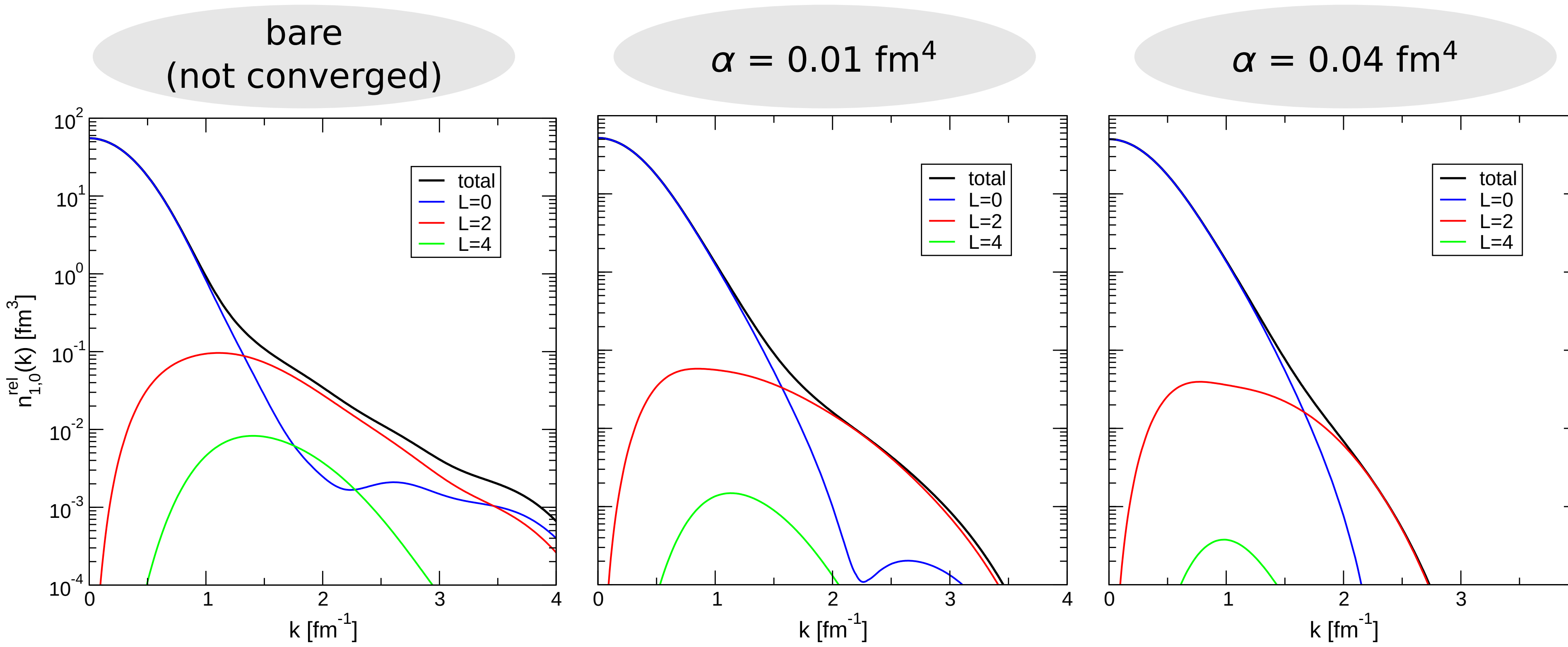
# $^4$He: SRG evolved AV18, unevolved density operator




- NCSM calculations with $N_{max} = 16$
- bare interaction: 2.99 pairs in $S = 1, T = 0$ channel, 2.57 pairs in $S = 0, T = 1$ channel and 0.43 pairs in $S = 1, T = 1$ channel — tensor force induces three-body correlations
- high-momentum components reduced for evolved interactions
- number of $S = 1, T = 1$ pairs reduced for evolved interactions — weaker three-body correlations

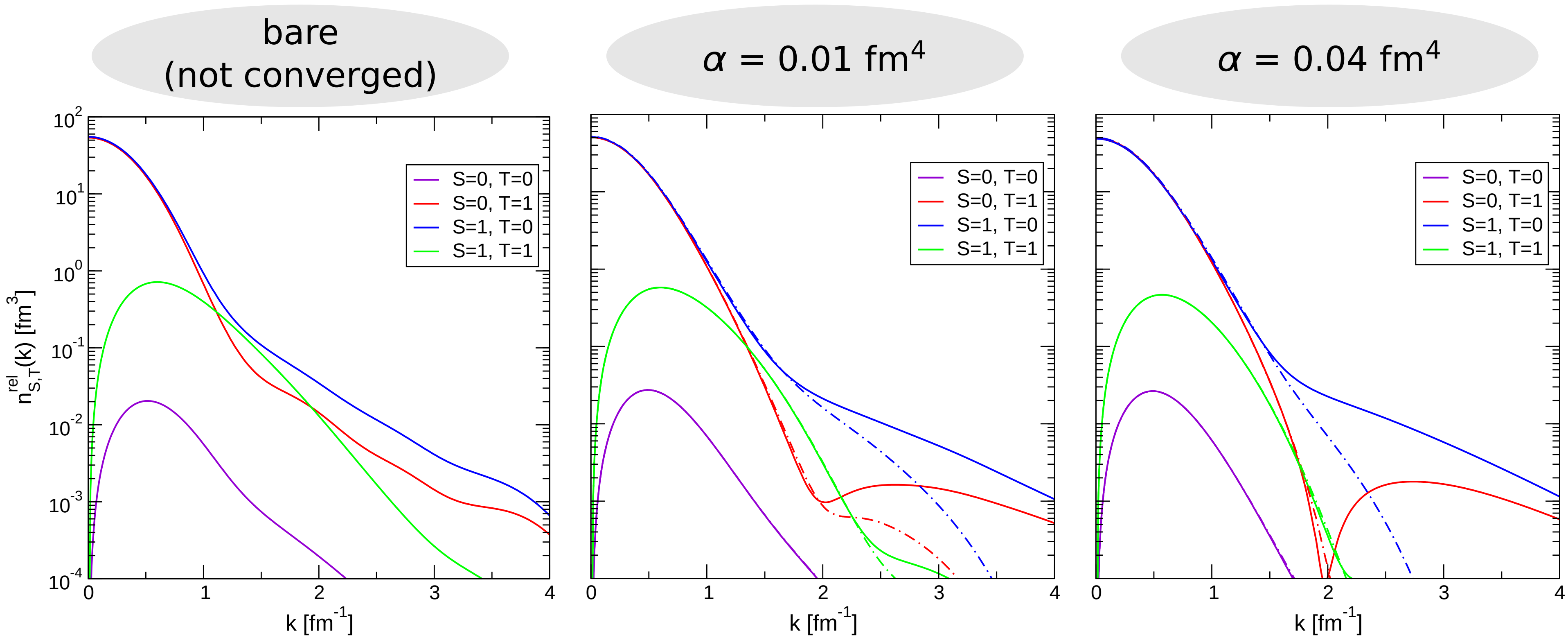
# Two-body densities $S = 1, T = 0$

# $^4$He: SRG evolved AV18, unevolved density operator




- in the intermediate momentum region, momentum distribution dominated by *D*-wave contributions
- caused by tensor force, explains the enhancement of *np*-pairs versus *pp*-pairs above the Fermi momentum
- *D*-wave contributions reduced for evolved interactions – tensor force no longer connects to high momenta

# $^4$He: SRG evolved AV18, evolved density operator




➨ use SRG transformed two-body density

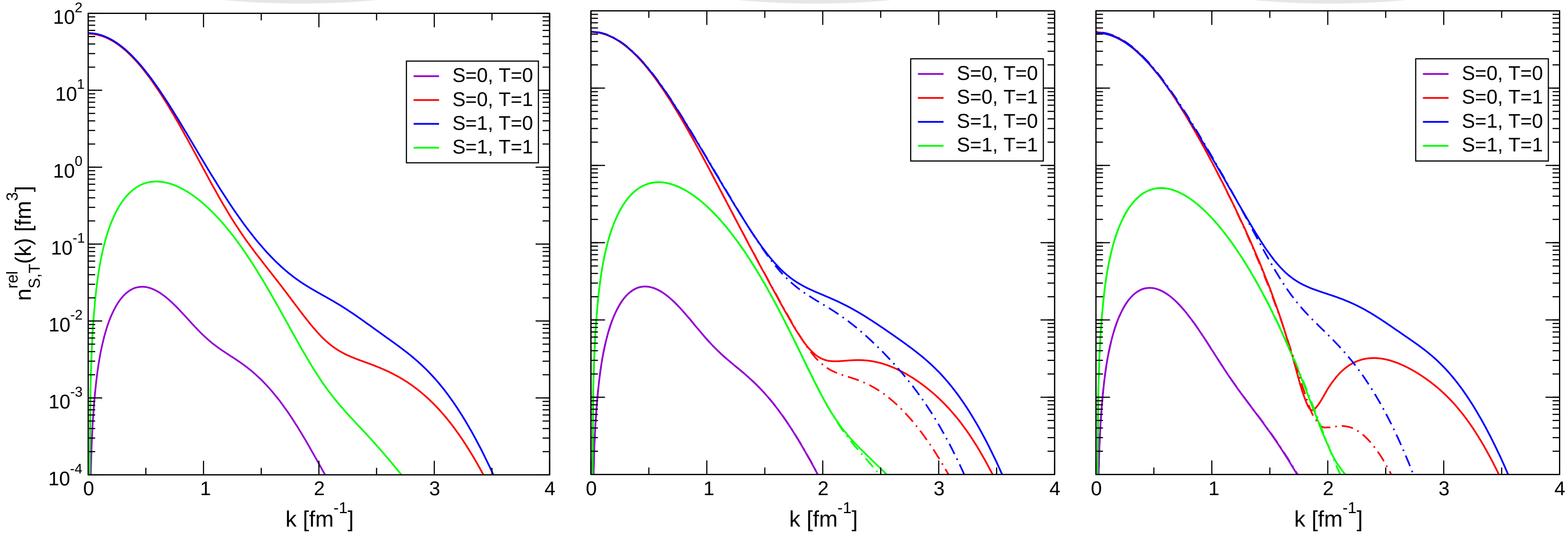
- high-momentum components recovered
- significant differences for medium momenta — SRG done on two-body level for Hamiltonian and two-body density, three-body correlations important

# $^4$He: SRG evolved N3LO, evolved density operator

bare

$\alpha$ = 0.01 fm$^4$

$\alpha$ = 0.04 fm$^4$

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# Unitary Correlation Operator Method

## Short-range Correlations

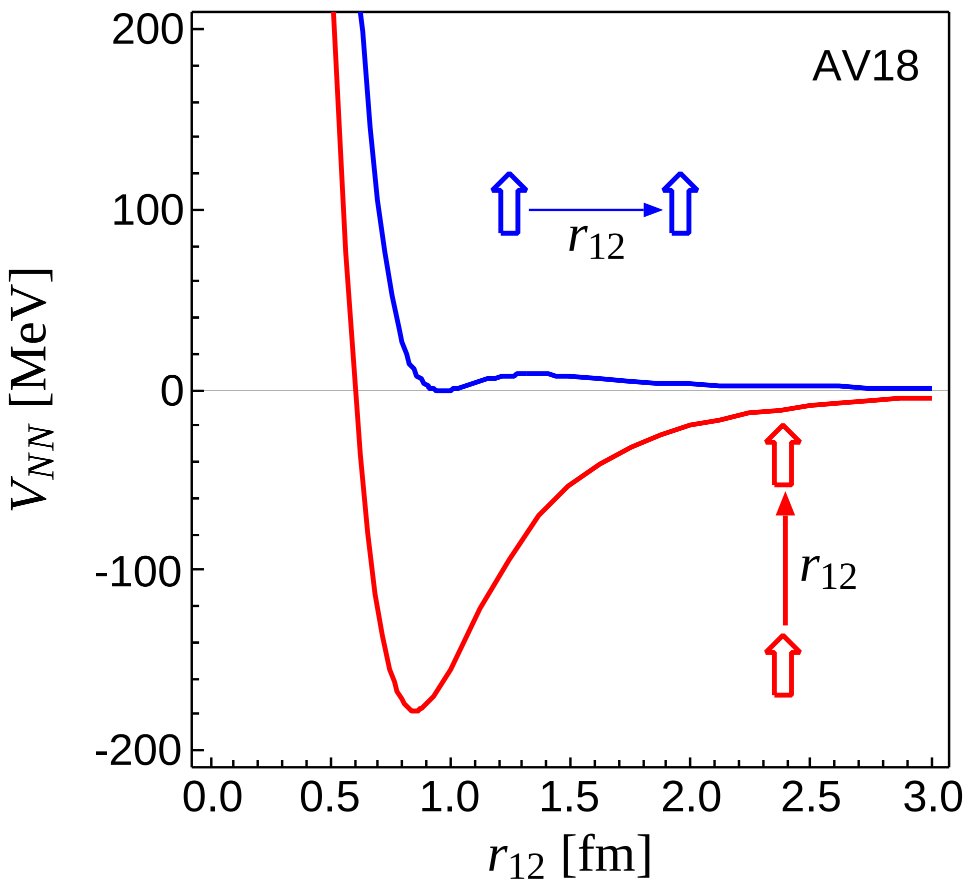
## Unitary Correlation Operator Method

- Unitary Transformation
- Central and Tensor Correlations
- Interaction in Momentum Space
- Few-body Calculations

# Nuclear Force

Argonne V18 (T=0)

spins aligned parallel or perpendicular to the relative distance vector




- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm

➨ **central correlations**

- strong dependence on the orientation of the spins due to the tensor force

➨ **tensor correlations**

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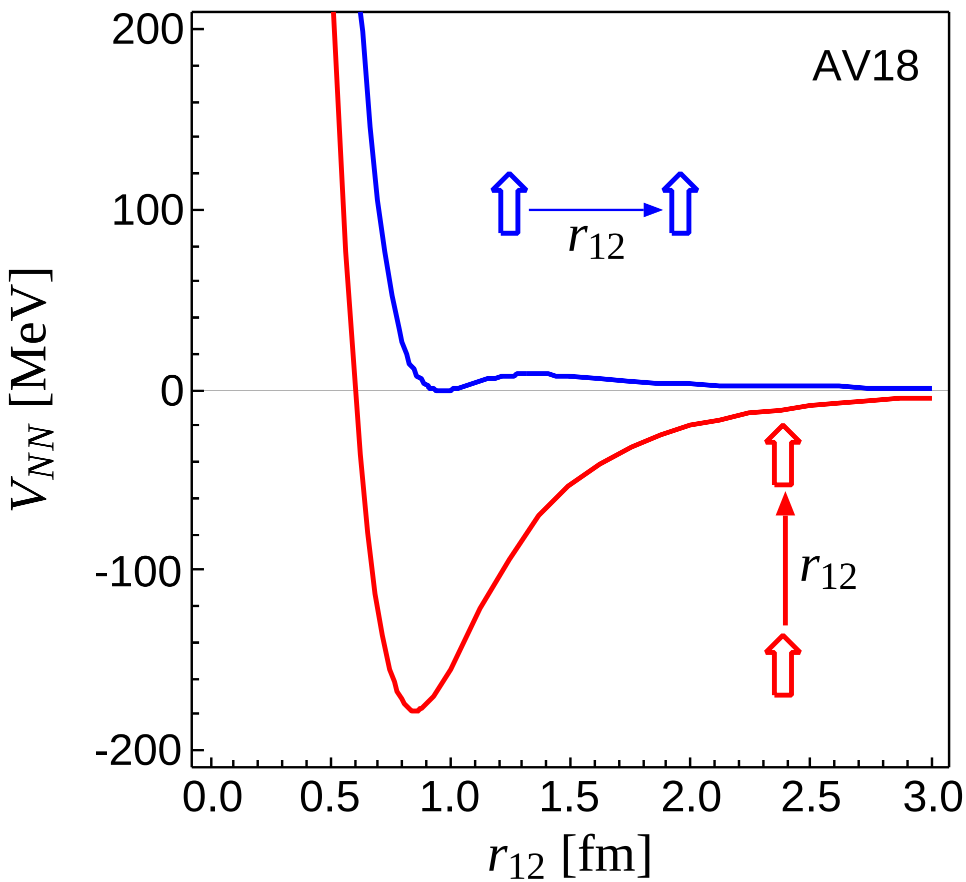


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➨ **central correlations**

- strong dependence on the orientation of the spins due to the tensor force

➨ **tensor correlations**

the nuclear force will induce **strong short-range correlations** in the nuclear wave function

# Unitary Correlation Operator Method

## Correlation Operator

- induce short-range (two-body) central and tensor correlations into the many-body state

$$\underset{\sim}{C} = \underset{\sim}{C}_\Omega \underset{\sim}{C}_r = \exp\Big[-i \sum_{i<j} \underset{\sim}{g}_{\Omega,ij}\Big] \exp\Big[-i \sum_{i<j} \underset{\sim}{g}_{r,ij}\Big] \quad , \quad \underset{\sim}{C}^\dagger \underset{\sim}{C} = \underset{\sim}{1}$$

- correlation operator should conserve the symmetries of the Hamiltonian and should be of finite-range, correlated interaction **phase shift equivalent** to bare interaction by construction

## Correlated Operators

- correlated operators will have contributions in higher cluster orders

$$\underset{\sim}{C}^\dagger \underset{\sim}{O} \underset{\sim}{C} = \underset{\sim}{\hat{O}}^{[1]} + \underset{\sim}{\hat{O}}^{[2]} + \underset{\sim}{\hat{O}}^{[3]} + \ldots$$

- two-body approximation: correlation range should be small compared to mean particle distance

## Correlated Interaction

$$\underset{\sim}{C}^\dagger (\underset{\sim}{T} + \underset{\sim}{V}) \underset{\sim}{C} = \underset{\sim}{T} + \underset{\sim}{V}_{\mathrm{UCOM}} + \underset{\sim}{V}^{[3]}_{\mathrm{UCOM}} + \ldots$$

# Central and Tensor Correlations

$$\underset{\sim}{C} = \underset{\sim}{C}_{\Omega} \underset{\sim}{C}_{r}$$

$$\mathbf{p} = \mathbf{p}_r + \mathbf{p}_\Omega$$

$$\mathbf{p}_r = \frac{1}{2}\left\{\frac{\mathbf{r}}{r}\left(\frac{\mathbf{r}}{r}\mathbf{p}\right) + \left(\mathbf{p}\frac{\mathbf{r}}{r}\right)\frac{\mathbf{r}}{r}\right\}, \qquad \mathbf{p}_\Omega = \frac{1}{2r}\left\{\mathbf{l} \times \frac{\mathbf{r}}{r} - \frac{\mathbf{r}}{r} \times \mathbf{l}\right\}$$

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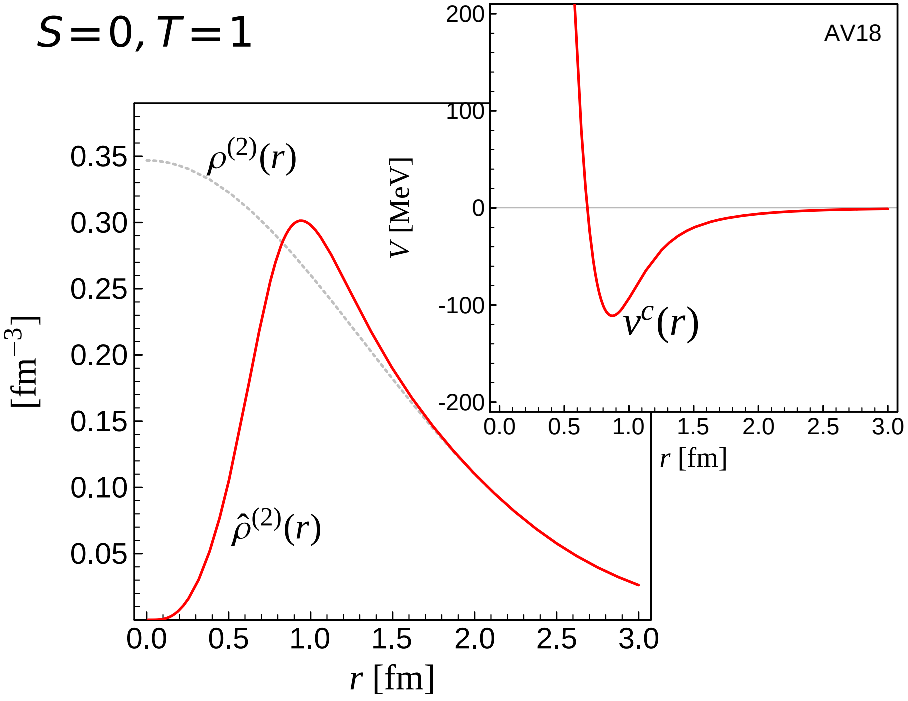
## Central Correlations

$$\underset{\sim}{c}_r = \exp\left\{-\frac{i}{2}\{p_r s(r) + s(r) p_r\}\right\}$$

➨ probability density shifted out of the repulsive core

$S=0, T=1$



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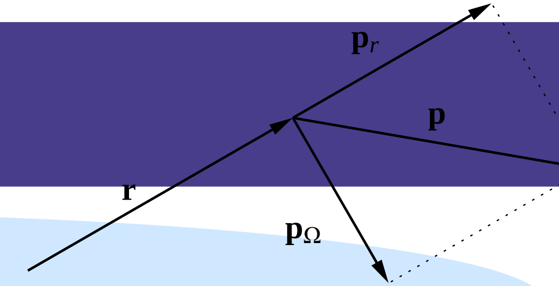


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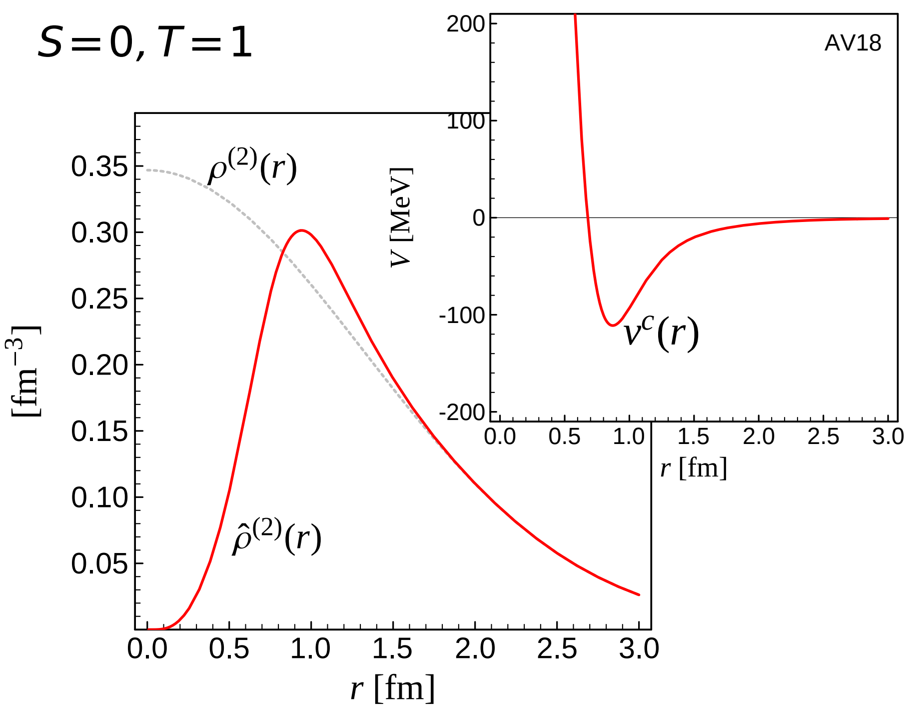
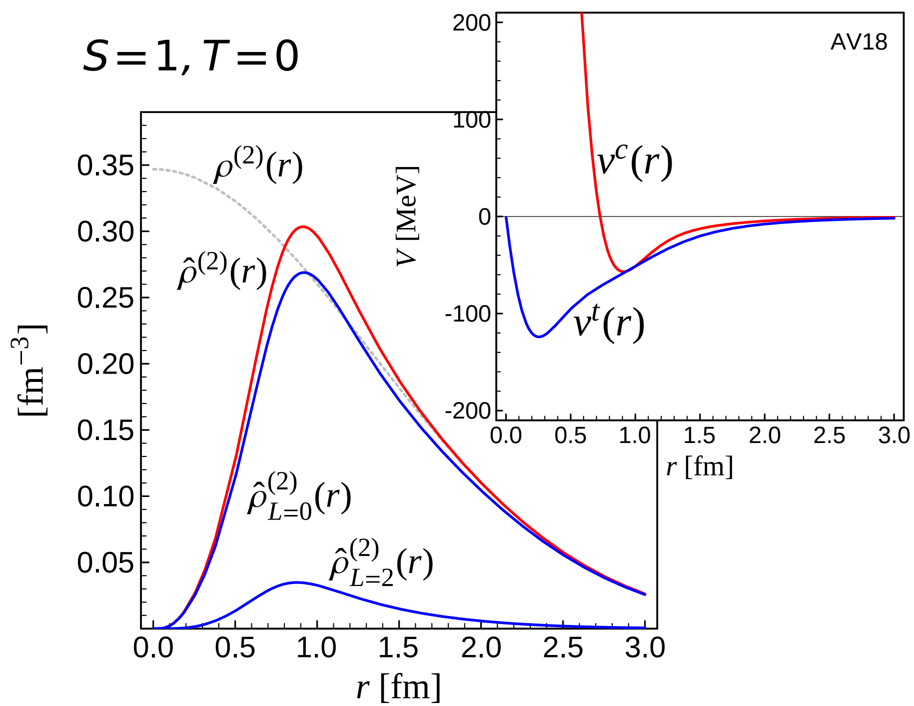



## Tensor Correlations

$$\underset{\sim}{c}_\Omega = \exp\left\{-i\vartheta(r)\left\{\frac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{p}_\Omega)(\boldsymbol{\sigma}_2\cdot\mathbf{r}) + \frac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p}_\Omega)\right\}\right.$$

- tensor force admixes other angular momenta



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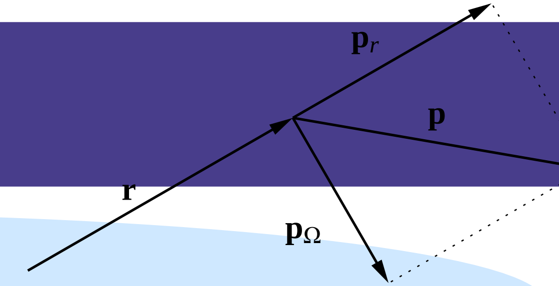


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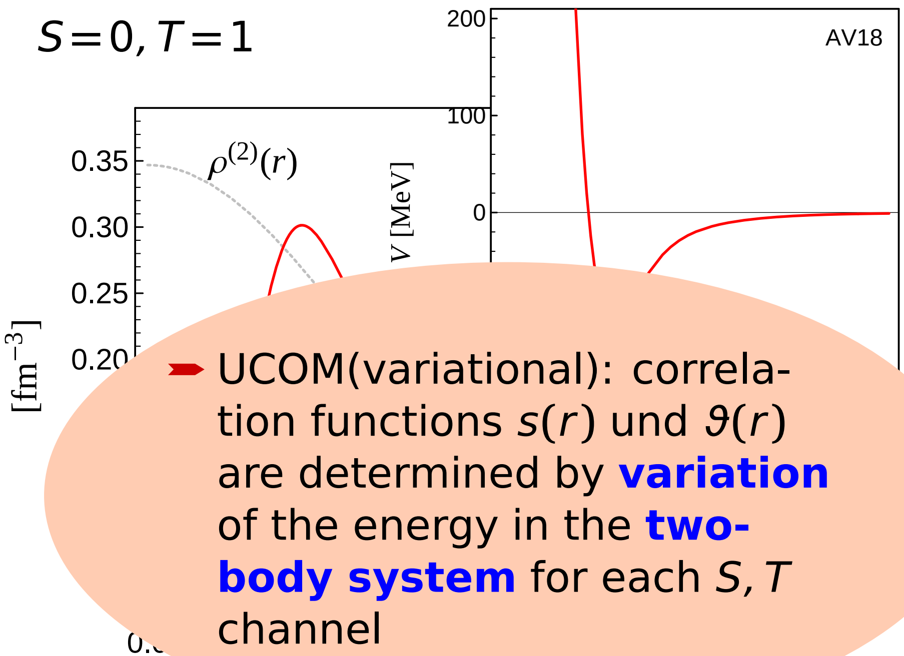


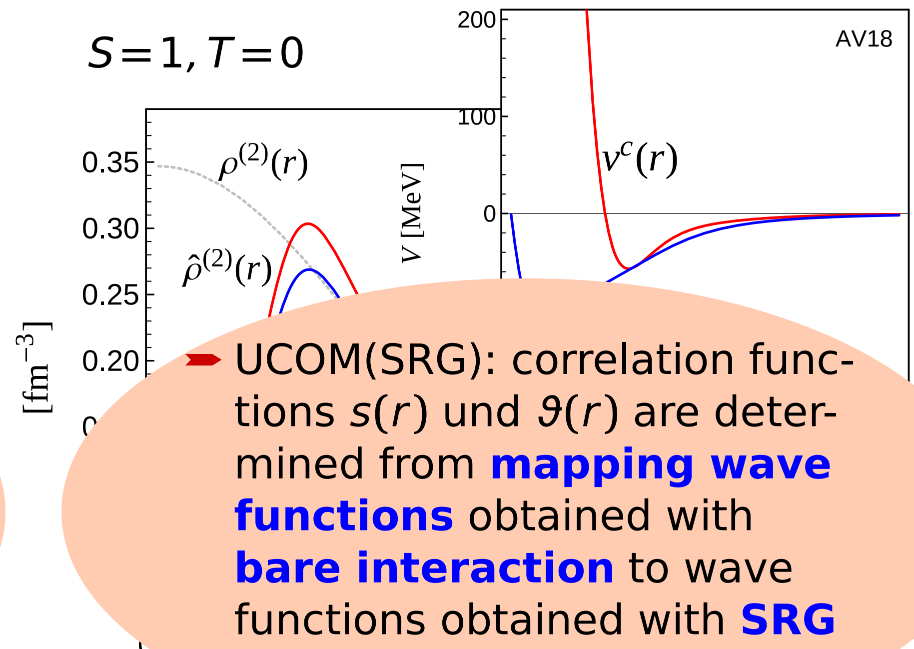

- UCOM(variational): correlation functions $s(r)$ und $\vartheta(r)$ are determined by **variation** of the energy in the **two-body system** for each $S, T$ channel

## Tensor Correlations

$$\underset{\sim}{c}_\Omega = \exp\left\{-i\vartheta(r)\left\{\frac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{p}_\Omega)(\boldsymbol{\sigma}_2\cdot\mathbf{r}) + \frac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p}_\Omega)\right\}\right\}$$

- tensor force admixes other angular momenta

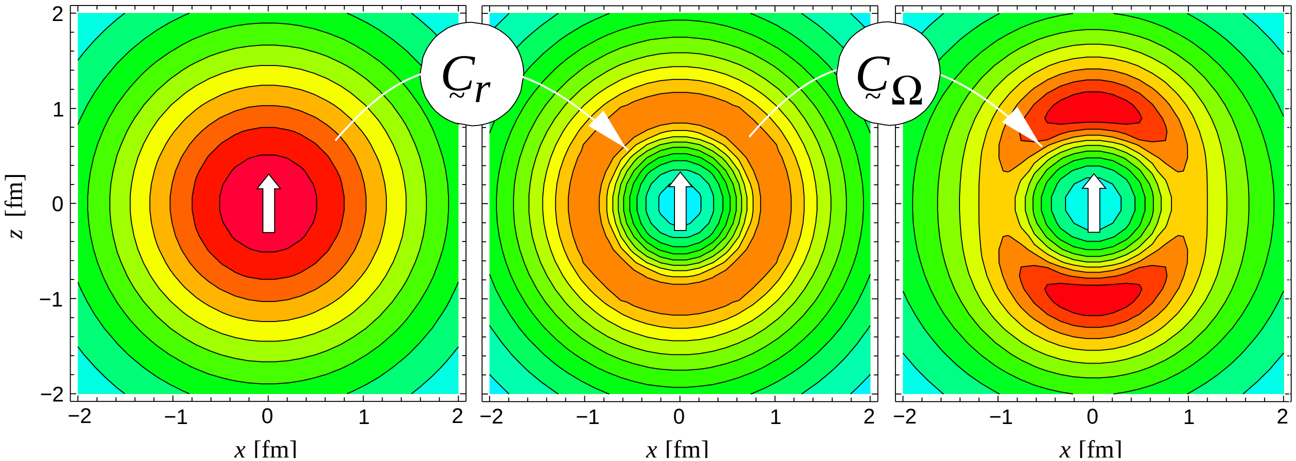
$S=1, T=0$




- UCOM(SRG): correlation functions $s(r)$ und $\vartheta(r)$ are determined from **mapping wave functions** obtained with **bare interaction** to wave functions obtained with **SRG interaction**

# Correlations and Energies

**two-body densities**




$\rho^{(2)}_{S,T}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0$

**central correlator** $\underset{\sim}{C}_{\mathbf{r}}$
shifts density out of the repulsive core

**tensor correlator** $\underset{\sim}{C}_{\mathbf{\Omega}}$
aligns density with spin orientation

Neff and Feldmeier, Nucl. Phys. **A713** (2003) 311

# Correlations and Energies

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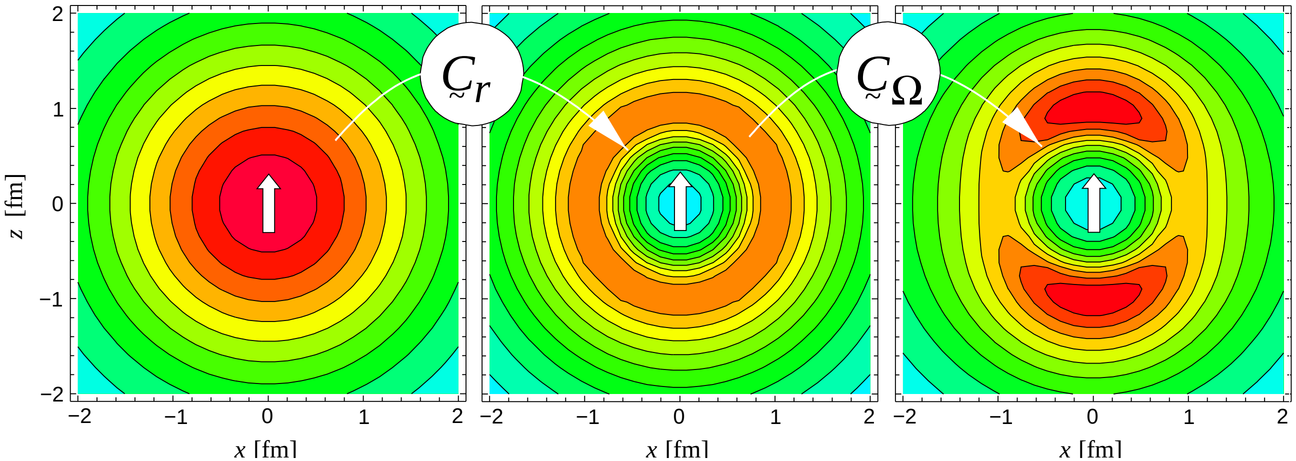


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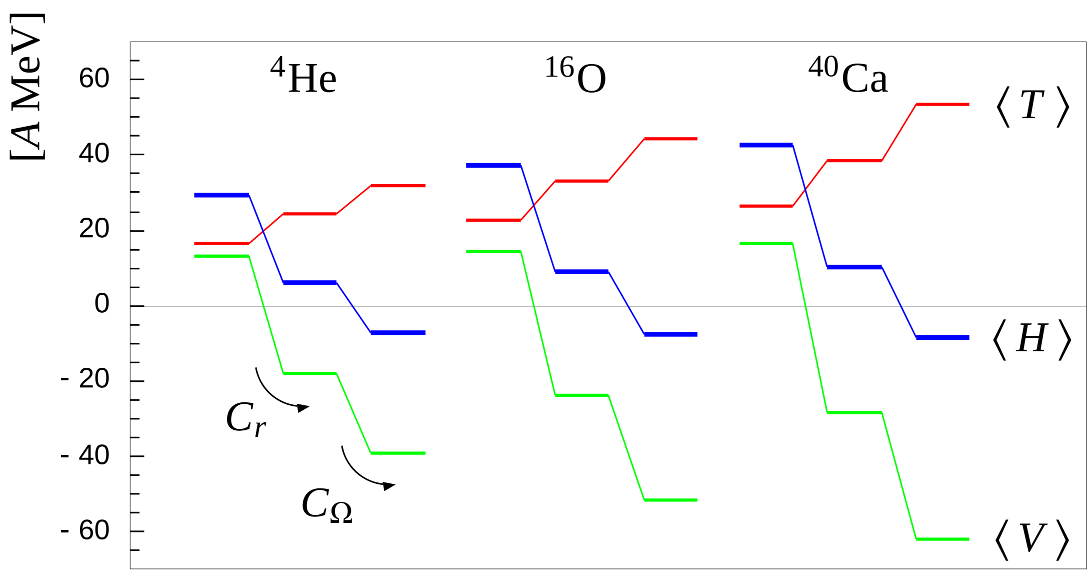
**energies**

$0\hbar\omega$ Harmonic Oscillator

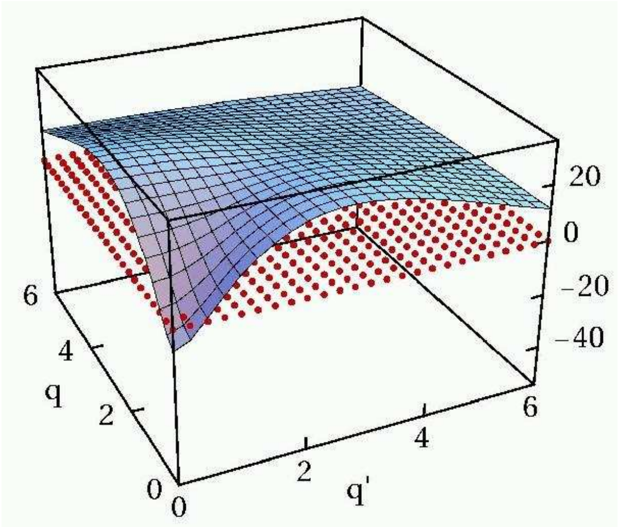



both central and tensor correlations are essential for binding

Neff and Feldmeier, Nucl. Phys. **A713** (2003) 311

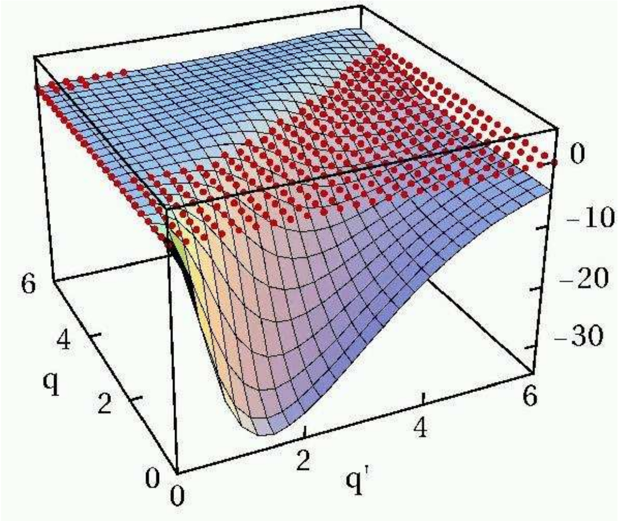
# Correlated Interaction in Momentum Space

$^3S_1$ bare




bare interaction has **strong off-diagonal** matrix elements connecting to high momenta

$^3S_1$ - $^3D_1$ bare




Roth, Hergert, Papakonstaninou, Neff, Feldmeier, Phys. Rev. C **72**, 034002 (2005)

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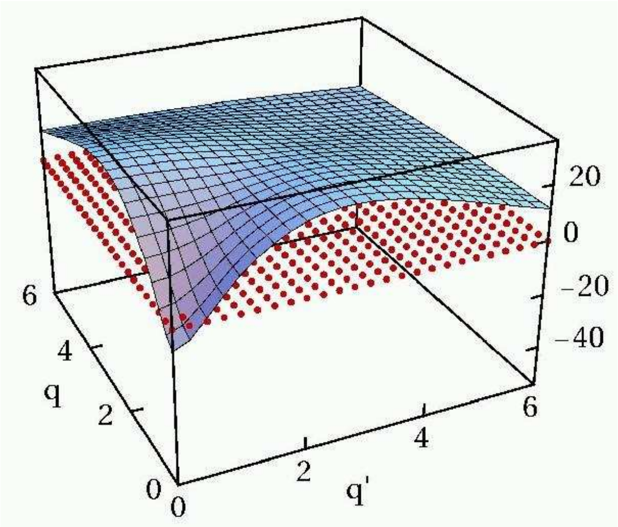


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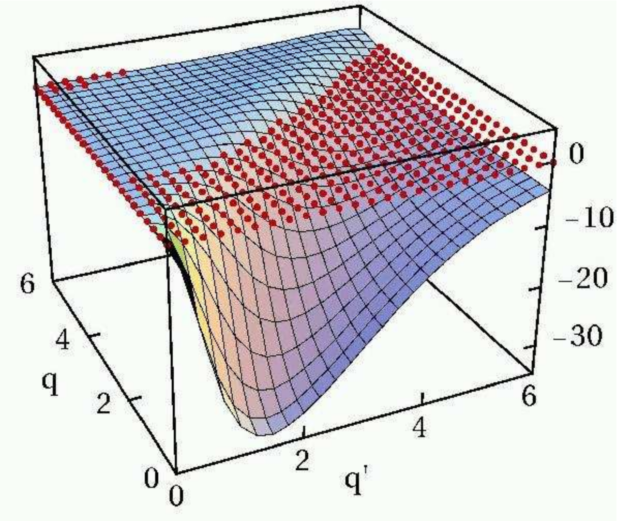
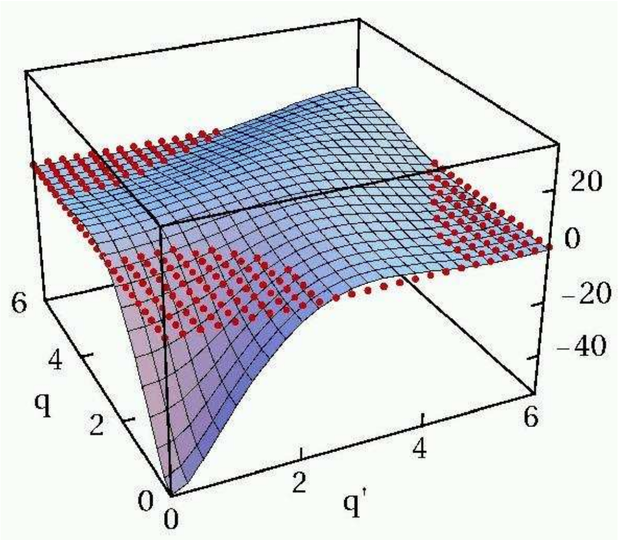



bare interaction has **strong off-diagonal** matrix elements connecting to high momenta

correlated interaction is **more attractive** at low momenta

**off-diagonal matrix elements** connecting low- and high- momentum states are **strongly reduced**

$^3S_1$ correlated

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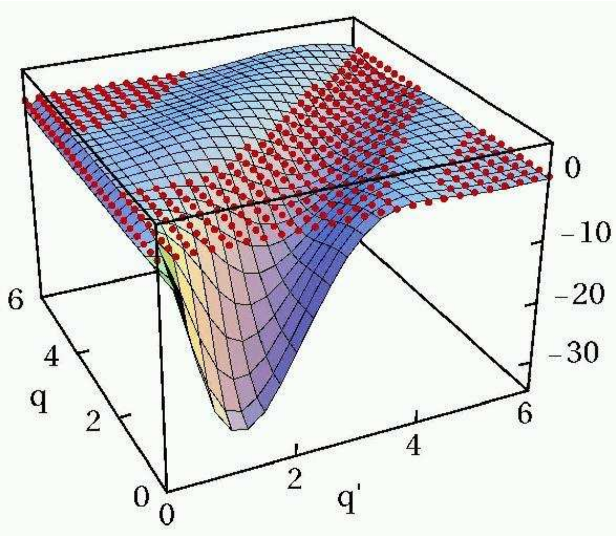


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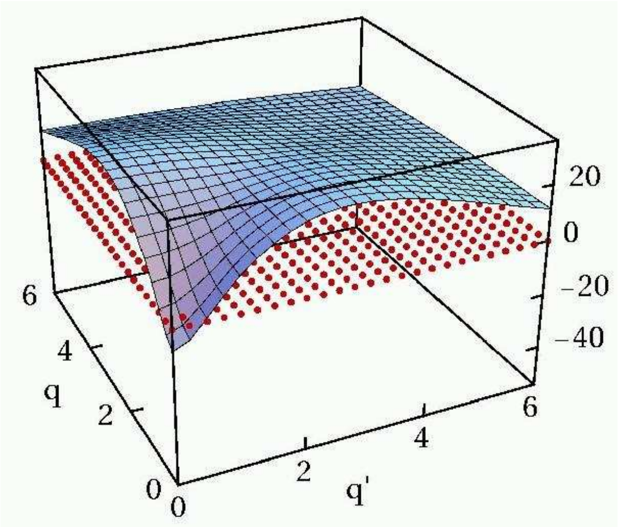


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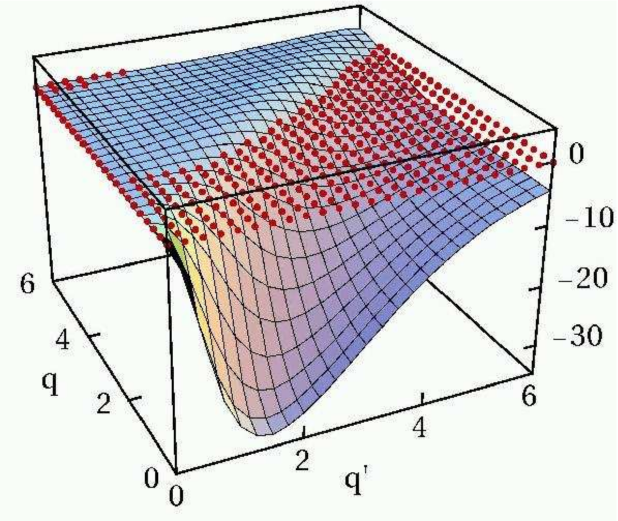


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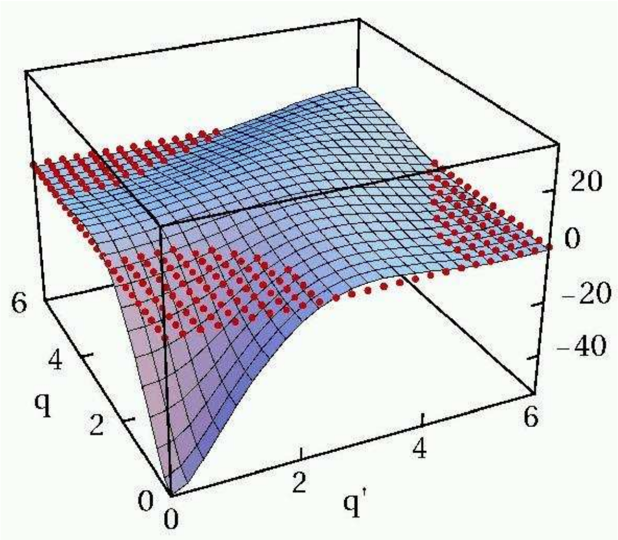


**off-diagonal matrix elements** connecting low- and high- momentum states are **strongly reduced**

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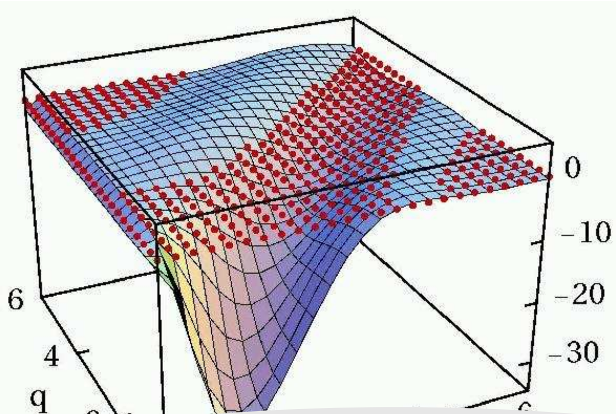
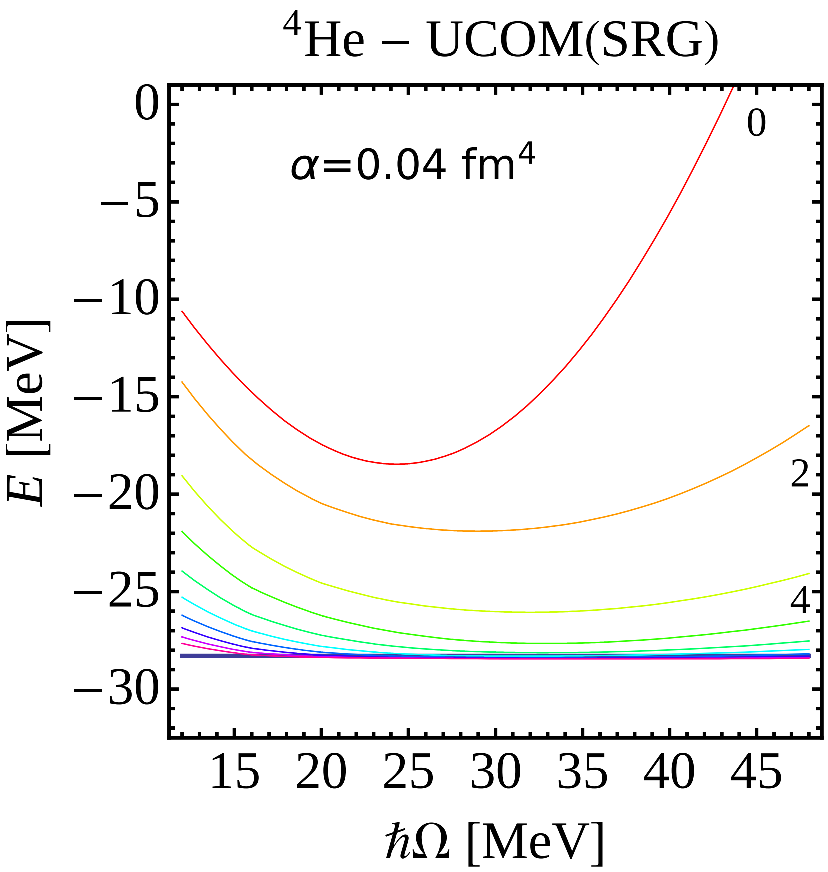
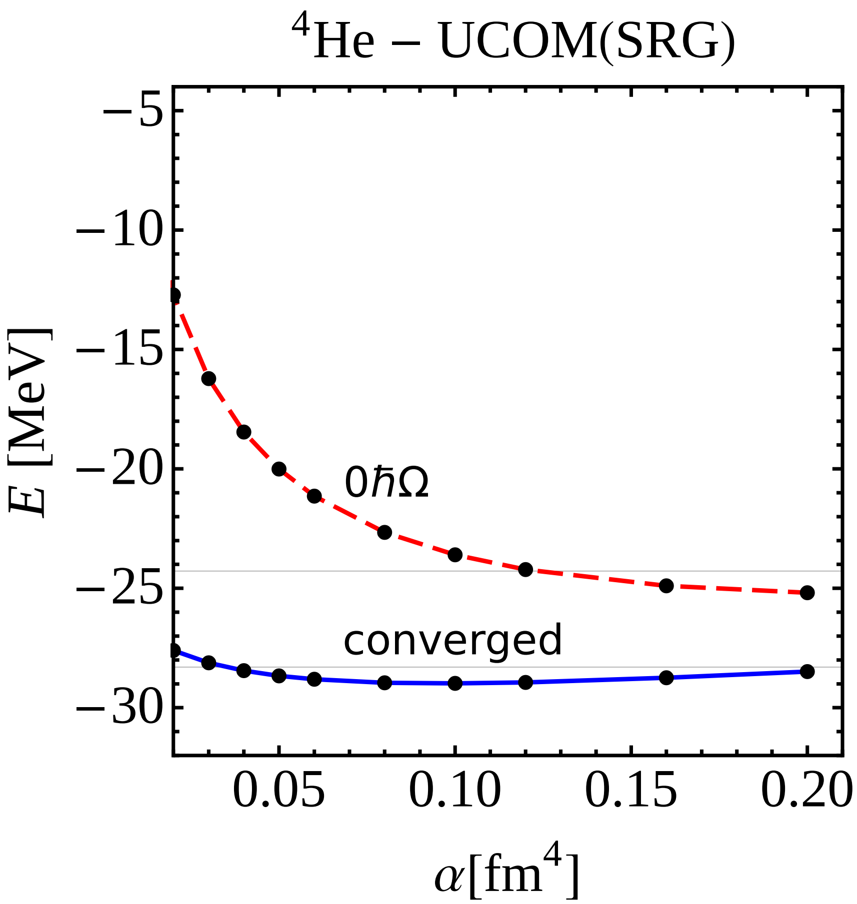



similar to $V_{low\text{-}k}$, **SRG**

Roth, Hergert, Papakonstaninou, Neff, Feldmeier, Phys. Rev. C **72**, 034002 (2005)

# No-Core Shell Model Calculations




- convergence much improved compared to bare interaction
- effective interaction – in two-body approximation – converges to different energy then bare interaction
- transformed interaction can be tuned to obtain simultaneously (almost) exact $^3$He and $^4$He binding energies

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

# Fermionic Molecular Dynamics

## Fermionic

Slater determinant

$$\left| Q \right\rangle = \underset{\sim}{\mathcal{A}} \Big( \left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle \Big)$$

- antisymmetrized $A$-body state

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655

Neff, Feldmeier, Nucl. Phys. **A738** (2004) 357

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## Molecular

single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \sum_i c_i \exp\left\{ -\frac{(\mathbf{x} - \mathbf{b}_i)^2}{2a_i} \right\} \otimes \left| \chi^\uparrow_i, \chi^\downarrow_i \right\rangle \otimes \left| \xi \right\rangle$$

- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width $a_i$ is an independent variational parameter for each wave packet
- use one or two wave packets for each single particle state

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655

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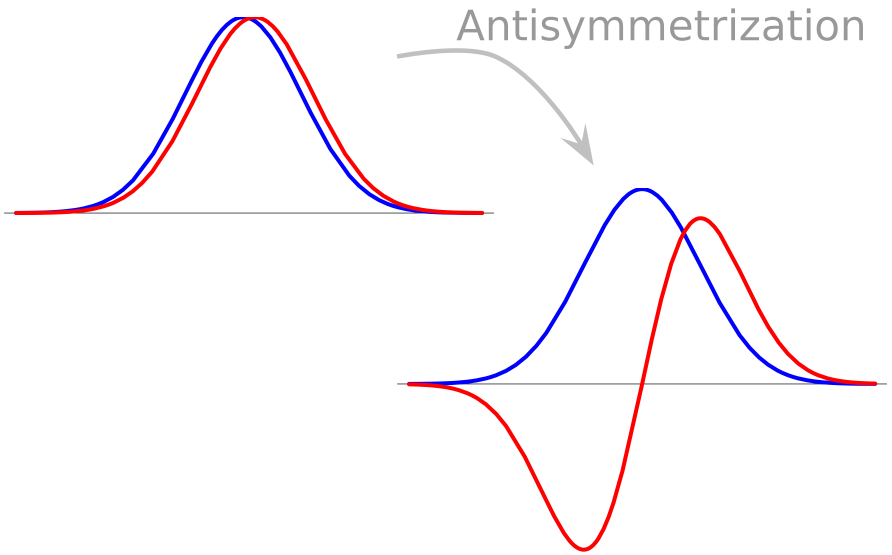


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- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
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- use one or two wave packets for each single particle state

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655

Neff, Feldmeier, Nucl. Phys. **A738** (2004) 357

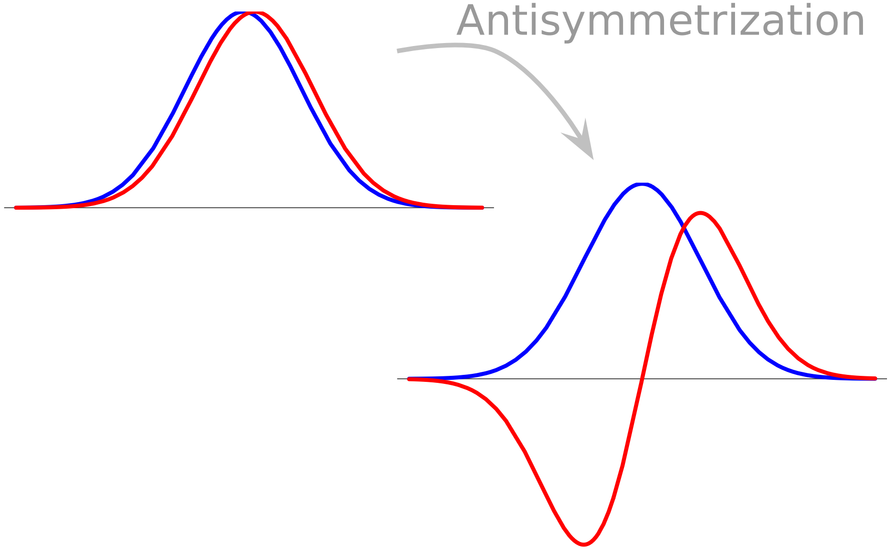
# Fermionic Molecular Dynamics

## Fermionic

Slater determinant

$$\left| Q \right\rangle = \mathcal{A}\left( \left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle \right)$$

- antisymmetrized $A$-body state




## Molecular

single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \sum_i c_i \exp\left\{ -\frac{(\mathbf{x} - \mathbf{b}_i)^2}{2a_i} \right\} \otimes \left| \chi^{\uparrow}_i, \chi^{\downarrow}_i \right\rangle \otimes \left| \xi \right\rangle$$

- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width $a_i$ is an independent variational parameter for each wave packet
- use one or two wave packets for each single particle st[ate]

see also
**Antisymmetrized Molecular Dynamics**
Horiuchi, Kanada-En'yo, Kimura, . . .

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655

Neff, Feldmeier, Nucl. Phys. **A738** (2004) 357

# Interaction Matrix Elements

## (One-body) Kinetic Energy

$$\langle\, q_k \,|\, \underset{\sim}{T} \,|\, q_l \,\rangle = \langle\, a_k \mathbf{b}_k \,|\, \underset{\sim}{T} \,|\, a_l \mathbf{b}_l \,\rangle \langle\, \chi_k \,|\, \chi_l \,\rangle \langle\, \xi_k \,|\, \xi_l \,\rangle$$

$$\langle\, a_k \mathbf{b}_k \,|\, \underset{\sim}{T} \,|\, a_l \mathbf{b}_l \,\rangle = \frac{1}{2m} \left( \frac{3}{a_k^\star + a_l} - \frac{(\mathbf{b}_k^\star - \mathbf{b}_l)^2}{(a_k^\star + a_l)^2} \right) R_{kl}$$

$$\alpha_{klmn} = \frac{a_k{}^\star a_m}{a_k{}^\star + a_m} + \frac{a_l{}^\star a_n}{a_l{}^\star + a_n}$$

$$\rho_{klmn} = \frac{a_m \mathbf{b}_k{}^\star + a_k{}^\star \mathbf{b}_m}{a_k{}^\star + a_m} - \frac{a_n \mathbf{b}_l{}^\star + a_l{}^\star \mathbf{b}}{a_l{}^\star + a_n}$$

$$R_{km} = \langle\, a_k \mathbf{b}_k \,|\, a_m \mathbf{b}_m \,\rangle$$

## (Two-body) Potential

- fit radial dependencies by (a sum of) Gaussians

$$G(\mathbf{x}_1 - \mathbf{x}_2) = \exp\left\{ -\frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{2\kappa} \right\}$$

- Gaussian integrals

$$\langle\, a_k \mathbf{b}_k, a_l \mathbf{b}_l \,|\, \underset{\sim}{G} \,|\, a_m \mathbf{b}_m, a_n \mathbf{b}_n \,\rangle = R_{km} R_{ln} \left( \frac{\kappa}{\alpha_{klmn} + \kappa} \right)^{3/2} \exp\left\{ -\frac{\rho_{klmn}^2}{2(\alpha_{klmn} + \kappa)} \right\}$$

- analytical expressions for matrix elements

# Operator Representation of $V_{\mathrm{UCOM}}$

$$\underset{\sim}{C}^{\dagger}(\underset{\sim}{T}+\underset{\sim}{V})\underset{\sim}{C} \quad = \quad \underset{\sim}{T}$$

one-body kinetic energy

$$+\sum_{ST}\hat{V}_{c}^{ST}(r)+\frac{1}{2}\left(\underset{\sim}{p_r}^{2}\,\hat{V}_{p^2}^{ST}(r)+\hat{V}_{p^2}^{ST}(r)\,\underset{\sim}{p_r}^{2}\right)+\hat{V}_{l^2}^{ST}(r)\,\underset{\sim}{\mathbf{l}}^{2}$$

**central** potentials

$$+\sum_{T}\hat{V}_{ls}^{T}(r)\,\underset{\sim}{\mathbf{l}}\cdot\underset{\sim}{\mathbf{s}}+\hat{V}_{l^2ls}^{T}(r)\,\underset{\sim}{\mathbf{l}}^{2}\,\underset{\sim}{\mathbf{l}}\cdot\underset{\sim}{\mathbf{s}}$$

**spin-orbit** potentials

$$+\sum_{T}\hat{V}_{t}^{T}(r)\,\underset{\sim}{S}_{12}(\mathbf{r},\mathbf{r})+\hat{V}_{trp_\Omega}^{T}(r)\,\underset{\sim}{p_r}\,\underset{\sim}{S}_{12}(\mathbf{r},\mathbf{p_\Omega})+\hat{V}_{tll}^{T}(r)\,\underset{\sim}{S}_{12}(\mathbf{l},\mathbf{l})+$$
$$\hat{V}_{tp_\Omega p_\Omega}^{T}(r)\,\underset{\sim}{S}_{12}(\mathbf{p_\Omega},\mathbf{p_\Omega})+\hat{V}_{l^2tp_\Omega p_\Omega}^{T}(r)\,\underset{\sim}{\mathbf{l}}^{2}\underset{\sim}{S}_{12}(\mathbf{p_\Omega},\mathbf{p_\Omega})$$

**tensor** potentials

bulk of tensor force mapped onto central part of correlated interaction

tensor correlations also change the spin-orbit part of the interaction

Nucl. Phys. **A745** (2004) 3

# PAV, VAP and Multiconfiguration

## Projection After Variation (PAV)

- mean-field may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by projection on parity, linear and angular momentum

$$\underset{\sim}{P}^{\pi} = \frac{1}{2}(1 + \pi \underset{\sim}{\Pi})$$

$$\underset{\sim}{P}^{J}_{MK} = \frac{2J+1}{8\pi^2} \int d^3\Omega \, {D^{J}_{MK}}^{\star}(\Omega) \, \underset{\sim}{R}(\Omega)$$

$$\underset{\sim}{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3} \int d^3X \; \exp\{-i(\underset{\sim}{\mathbf{P}} - \mathbf{P})\cdot\mathbf{X}\}$$

# PAV, VAP and Multiconfiguration

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## Variation After Projection (VAP)

- effect of projection can be large
- full **Variation after Angular Momentum and Parity Projection** (VAP) for light nuclei

$$\underset{\sim}{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3} \int d^3X \; \exp\{-i(\underset{\sim}{\mathbf{P}} - \mathbf{P}) \cdot \mathbf{X}\}$$

- perform VAP in GCM sense by applying **constraints** on radius, dipole moment, quadrupole moment or octupole moment and minimizing the energy in the projected energy surface for heavier nuclei

# PAV, VAP and Multiconfiguration

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- perform VAP in GCM sense by applying **constraints** on radius, dipole moment, quadrupole moment or octupole moment and minimizing the energy in the projected energy surface for heavier nuclei

## Multiconfiguration Calculations

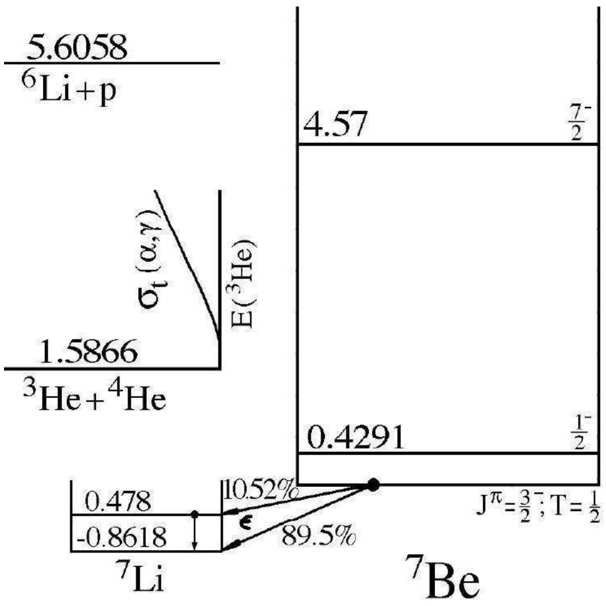
- **diagonalize** Hamiltonian in a set of projected intrinsic states

$$\left\{ \left| Q^{(a)} \right\rangle , \quad a = 1, \ldots, N \right\}$$

$$\sum_{K'b} \left\langle Q^{(a)} \right| \underset{\sim}{H} \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b} = E^{J^\pi \alpha} \sum_{K'b} \left\langle Q^{(a)} \right| \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b}$$

# $^{3}$He$(\alpha,\gamma)^{7}$Be radiative capture

**one of the key reactions in the solar pp-chains**




**Effective Nucleon-Nucleon interaction:**

**UCOM(SRG)** $\alpha = 0.20\ \mathrm{fm}^4$ – $\lambda \approx 1.5\ \mathrm{fm}^{-1}$

**Many-Body Approach:**

## Fermionic Molecular Dynamics

- **Internal region: VAP configurations with radius constraint**
- **External region: Brink-type cluster configurations**
- **Matching to Coulomb solutions: Microscopic $R$-matrix method**

**Results:**

- $^{7}$Be **bound and scattering states**
- **Astrophysical $S$-factor**

T. Neff, Phys. Rev. Lett. **106** (2011) 042502

# FMD model space

## Frozen configurations

- antisymmetrized wave function built with $^4$He and $^3$He FMD clusters up to channel radius $a$=12 fm

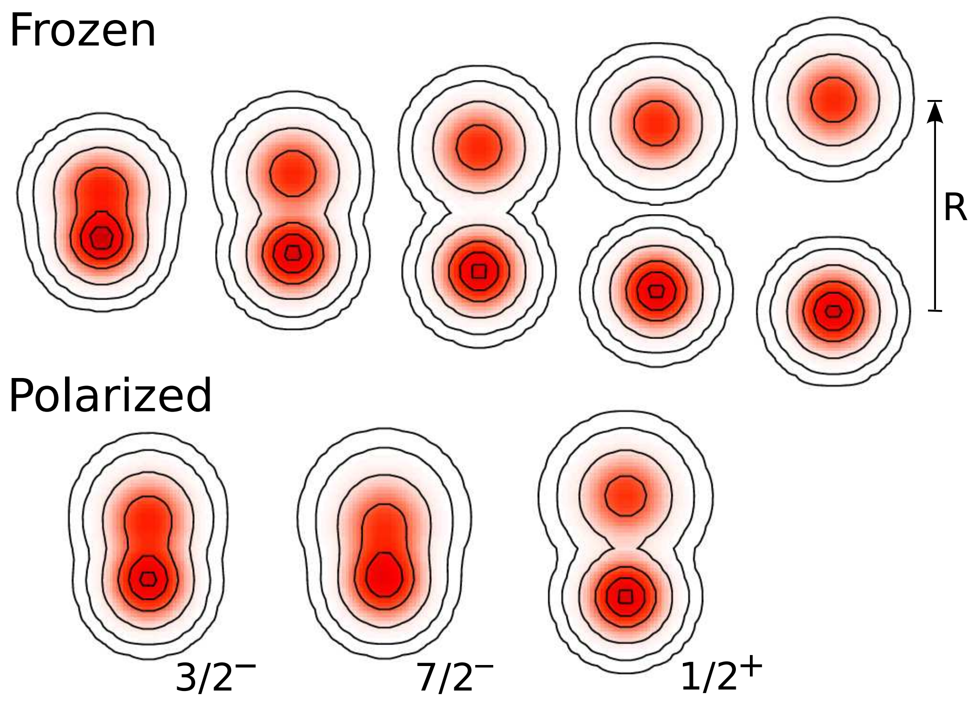
## Polarized configurations

- FMD wave functions obtained by VAP on $1/2^-$, $3/2^-$, $5/2^-$, $7/2^-$ and $1/2^+$, $3/2^+$ and $5/2^+$ combined with radius constraint in the interaction region

## Boundary conditions

- Match relative motion of clusters at channel radius to Whittaker/Coulomb functions with the **microscopic *R*-matrix** method of the Brussels group

  D. Baye, P.-H. Heenen, P. Descouvemont



# $p$-wave Bound and Scattering States

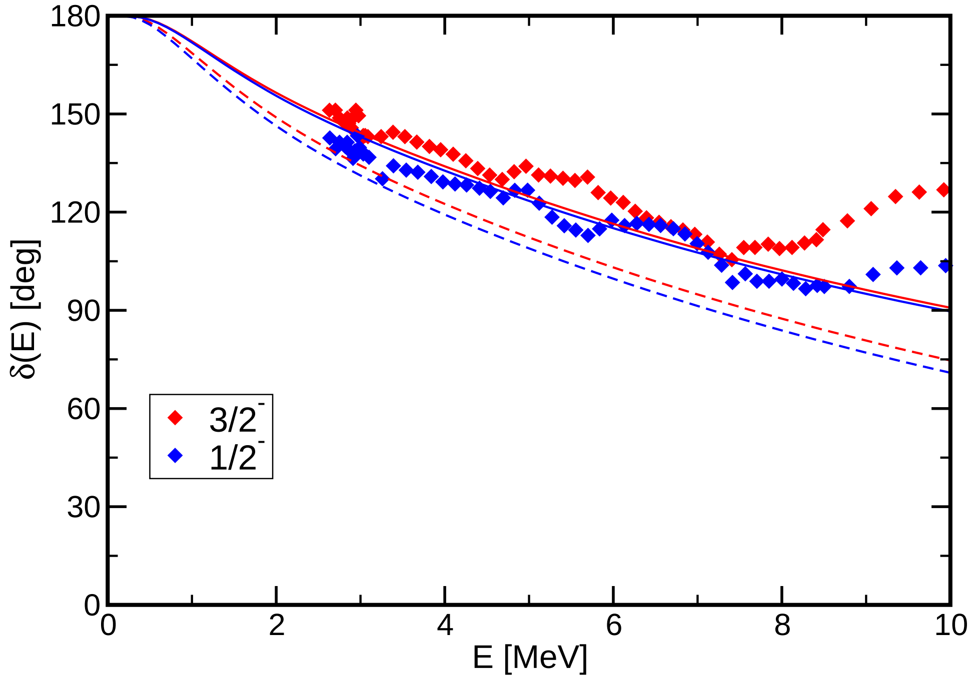
## Bound states

| | | Experiment | FMD |
|---|---|---|---|
| $^7Be$ | $E_{3/2-}$ | -1.59 MeV | -1.49 MeV |
| | $E_{1/2-}$ | -1.15 MeV | -1.31 MeV |
| | $r_{ch}$ | 2.647(17) fm | 2.67 fm |
| | $Q$ | – | -6.83 $e$ fm$^2$ |
| $^7Li$ | $E_{3/2-}$ | -2.467 MeV | -2.39 MeV |
| | $E_{1/2-}$ | -1.989 MeV | -2.17 MeV |
| | $r_{ch}$ | 2.444(43) fm | 2.46 fm |
| | $Q$ | -4.00(3) $e$ fm$^2$ | -3.91 $e$ fm$^2$ |

- centroid of bound state energies well described if polarized configurations included
- tail of wave functions tested by charge radii and quadrupole moments

Phase shift analysis:

Spiger and Tombrello, PR **163**, 964 (1967)

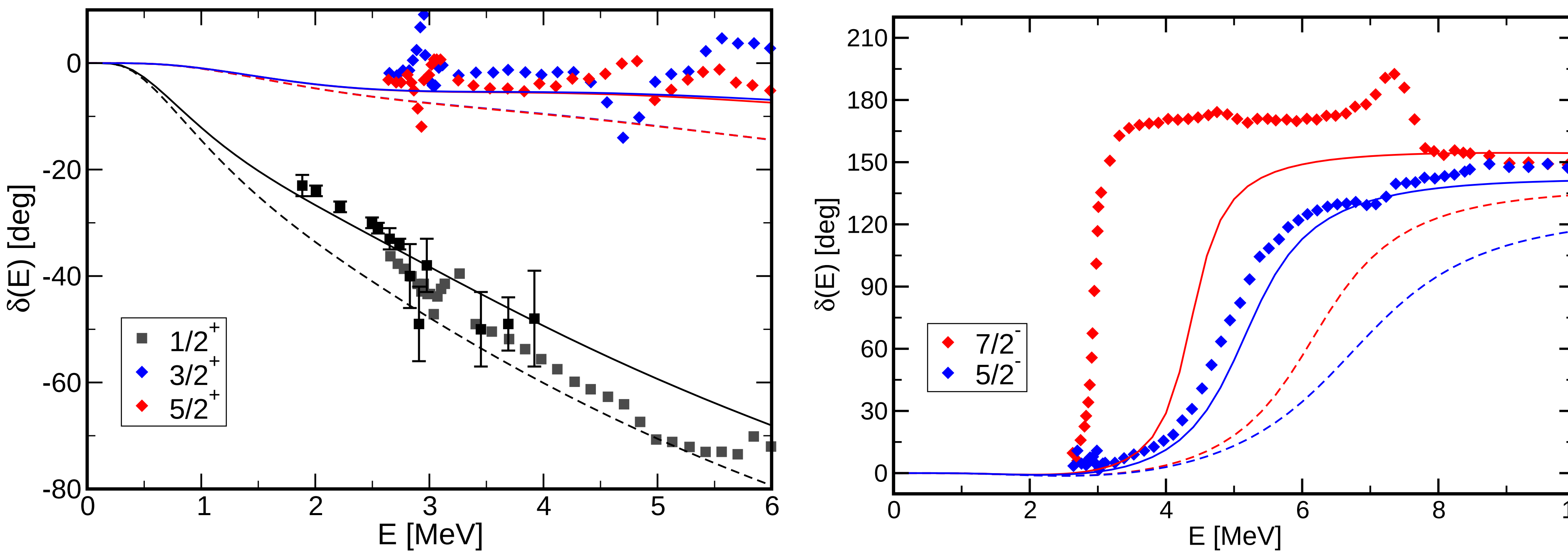



dashed lines – frozen configurations only

solid lines – polarized configurations in interaction region included

- Scattering phase shifts well described, polarization effects important

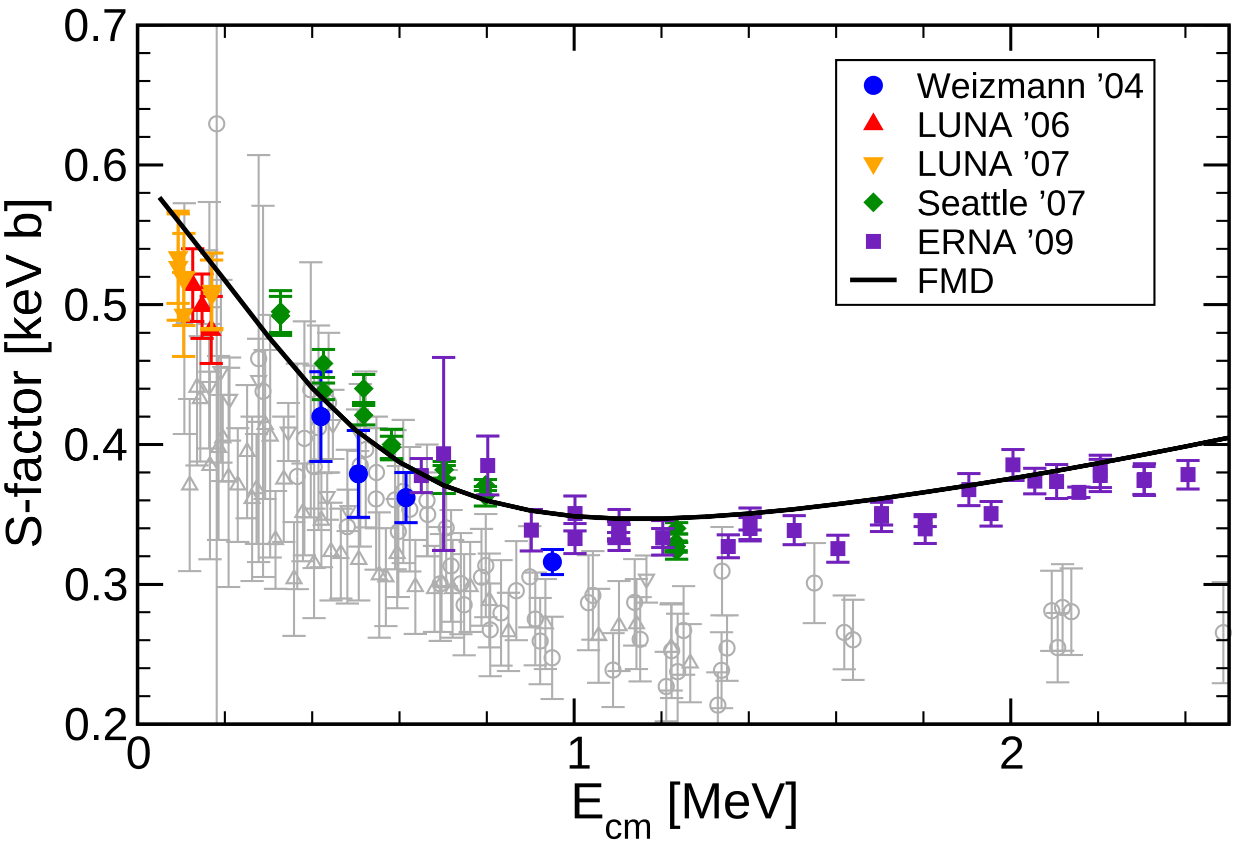
# *s*-, *d*- and *f*-wave Scattering States




dashed lines – frozen configurations only – solid lines – FMD configurations in interaction region included

- polarization effects important
- *s*- and *d*-wave scattering phase shifts well described
- $7/2^-$ resonance too high, $5/2^-$ resonance roughly right, consistent with no-core shell model calculations

# S-Factor




**S-factor:**

$$S(E) = \sigma(E) E \exp\{2\pi\eta\}$$

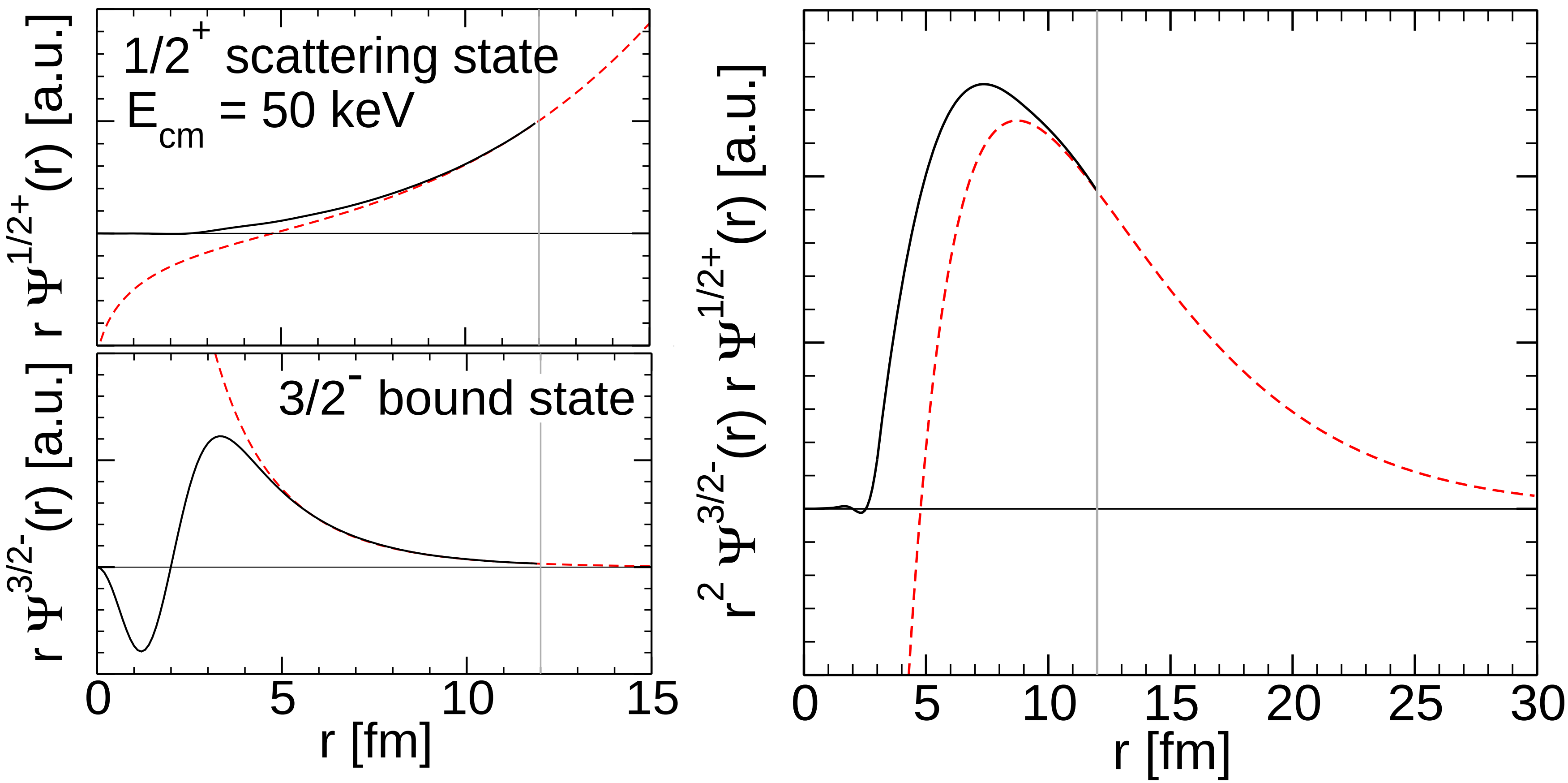
$$\eta = \frac{\mu Z_1 Z_2 e^2}{k}$$


Nara Singh *et al.*, PRL **93**, 262503 (2004)
Bemmerer *et al.*, PRL **97**, 122502 (2006)
Confortola *et al.*, PRC **75**, 065803 (2007)
Brown *et al.*, PRC **76**, 055801 (2007)
Di Leva *et al.*, PRL **102**, 232502 (2009)


- dipole transitions from $1/2^+$, $3/2^+$, $5/2^+$ scattering states into $3/2^-$, $1/2^-$ bound states
- ➨ FMD is the only model that describes well the energy dependence and normalization of new high quality data
- ➨ fully microscopic calculation, bound and scattering states are described consistently

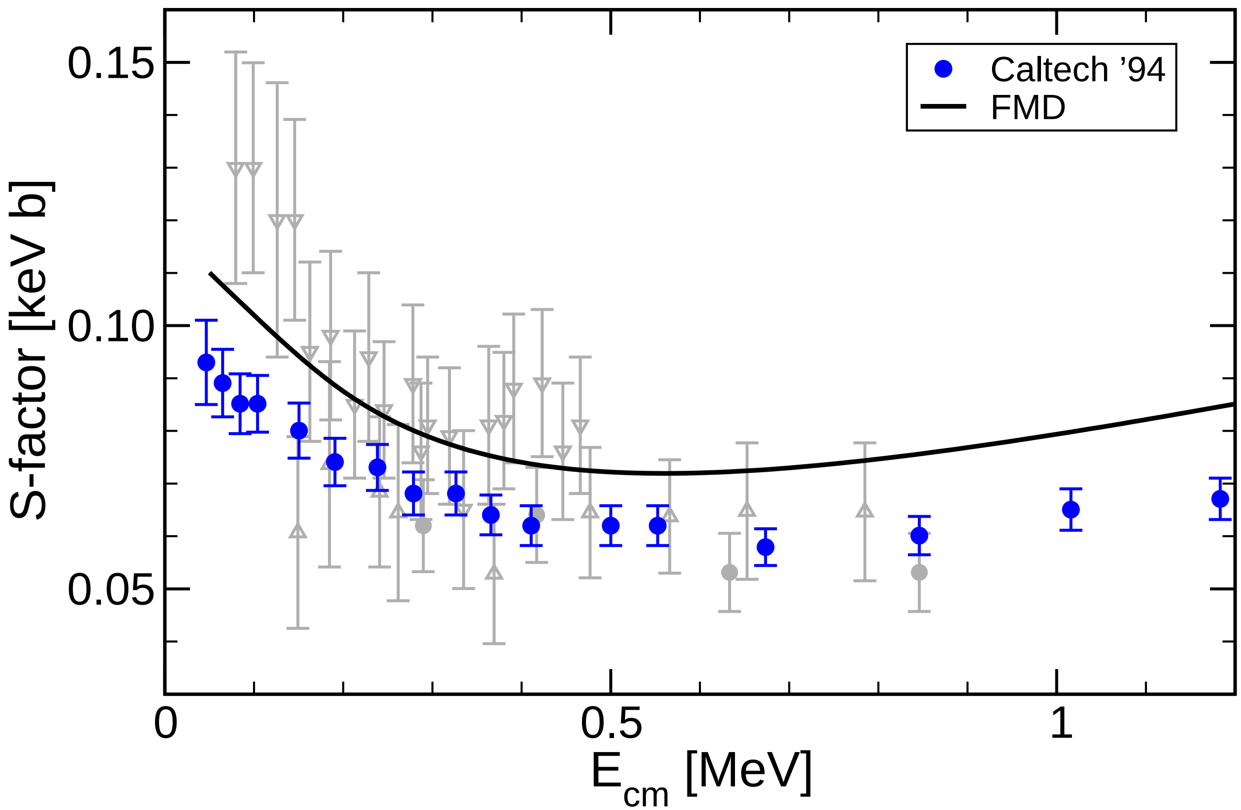
T. Neff, Phys. Rev. Lett. **106** (2011) 042502

# Overlap Functions and Dipole Matrixelements




- Overlap functions from projection on RGM-cluster states
- Coulomb and Whittaker functions matched at channel radius $a$=12 fm
- Dipole matrix elements calculated from overlap functions reproduce full calculation within 2%
- cross section depends significantly on internal part of wave function, description as an “external” capture is too simplified

# $^3H(\alpha,\gamma)^7Li$
## S-Factor




**S-factor:**

$$S(E) = \sigma(E) E \exp\{2\pi\eta\}$$

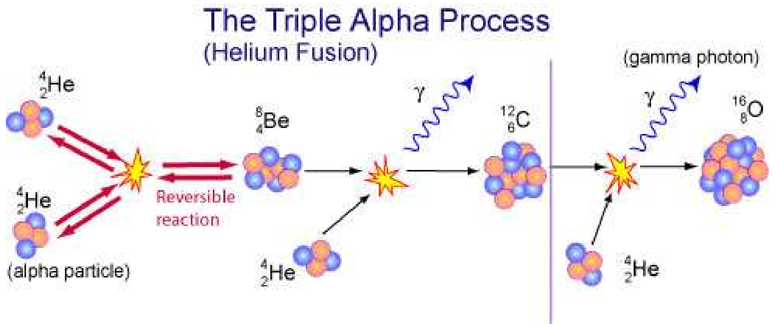
$$\eta = \frac{\mu Z_1 Z_2 e^2}{k}$$

Brune *et al.*, PRC **50**, 2205 (1994)

- isospin mirror reaction of $^3He(\alpha,\gamma)^7Be$
- $^7Li$ bound state properties and phase shifts well described

➡ FMD calculation describes energy dependence of Brune *et al.* data but cross section is larger by about 15%

# Cluster States in $^{12}$C

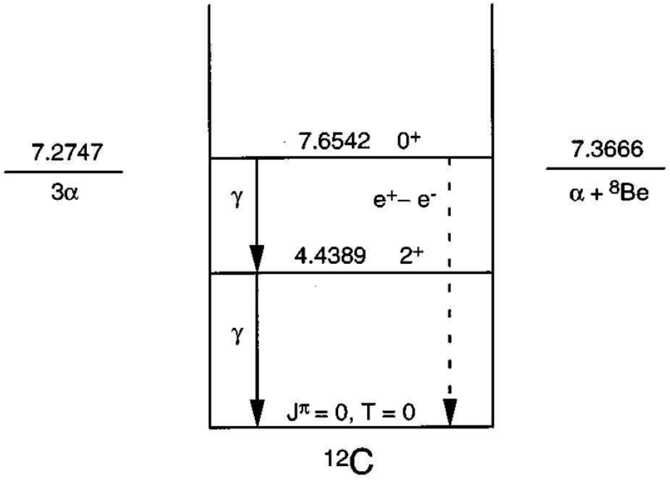



## Astrophysical Motivation

- **Helium burning: triple alpha-reaction**

## Structure

- **Is the Hoyle state a pure $\alpha$-cluster state ?**
- **Other excited $0^+$ and $2^+$ states**

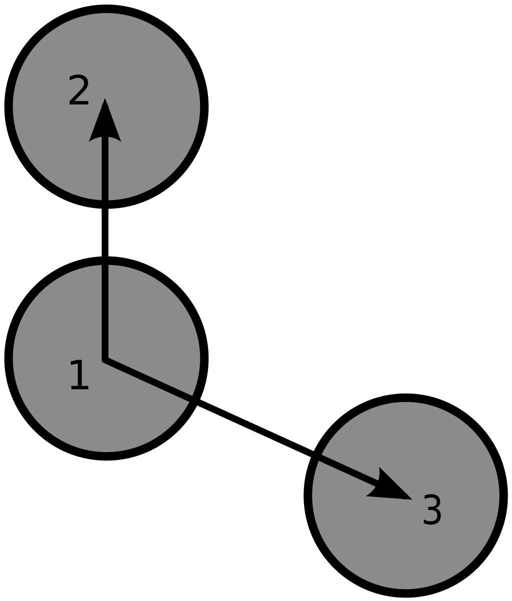



➨ **Compare FMD results to microscopic $\alpha$-cluster model**

➨ **Intrinsic structure from two-body densities**

➨ **Analyze wave functions in harmonic oscillator basis**

# Microscopic $\alpha$-Cluster Model




$$R_{12} = (2, 4, \ldots, 10)\,\text{fm}$$
$$R_{13} = (2, 4, \ldots, 10)\,\text{fm}$$
$$\cos(\vartheta) = (1.0, 0.8, \ldots, -1.0)$$

alltogether 165 configurations

Kamimura, Nuc. Phys. **A351** (1981) 456

Funaki et al., Phys. Rev. C **67** (2003) 051306(R)

## Basis States

- describe Hoyle State as a system of 3 $^4$He nuclei

$$\left| \Psi_{3\alpha}(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3); JMK\pi \right\rangle = P^J_{MK} P^\pi \mathcal{A} \left\{ \left| \psi_\alpha(\mathbf{R}_1) \right\rangle \otimes \left| \psi_\alpha(\mathbf{R}_2) \right\rangle \otimes \left| \psi_\alpha(\mathbf{R}_3) \right\rangle \right\}$$

## Volkov Interaction

- simple central interaction
- parameters adjusted to give reasonable $\alpha$ binding energy and radius, $\alpha - \alpha$ scattering data, adjusted to reproduce $^{12}$C ground state energy

✘ only reasonable for $^4$He, $^8$Be and $^{12}$C nuclei

## ‘BEC’ wave functions

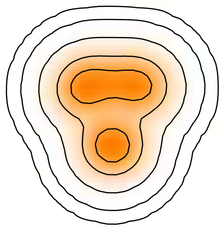
- interpretation of the Hoyle state as a Bose-Einstein Condensate of $\alpha$-particles by Funaki, Tohsaki, Horiuchi, Schuck, Röpke
- same interaction and $\alpha$-cluster parameters used

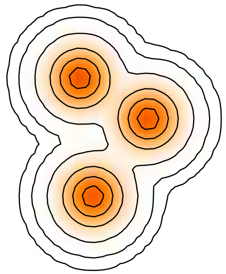
# FMD

## Basis States

- 20 FMD states obtained in Variation after Projection on $0^+$ and $2^+$ with constraints on the radius
- 42 FMD states obtained in Variation after Projection on parity with constraints on radius and quadrupole deformation
- 165 $\alpha$-cluster configurations

➨ projected on angular momentum and linear momentum

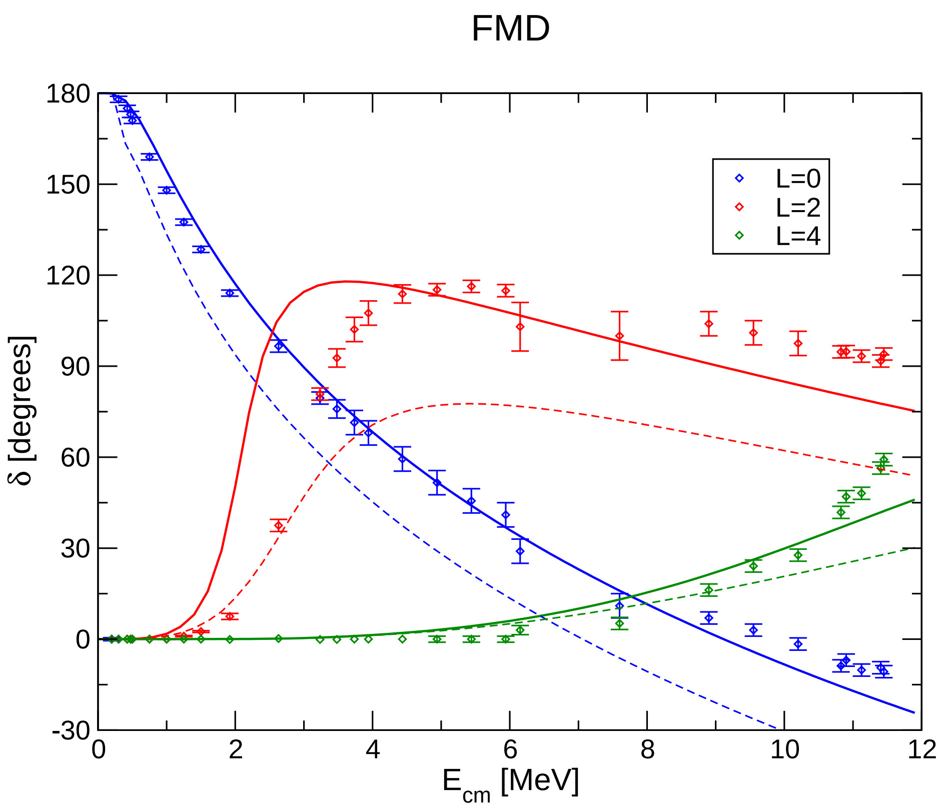


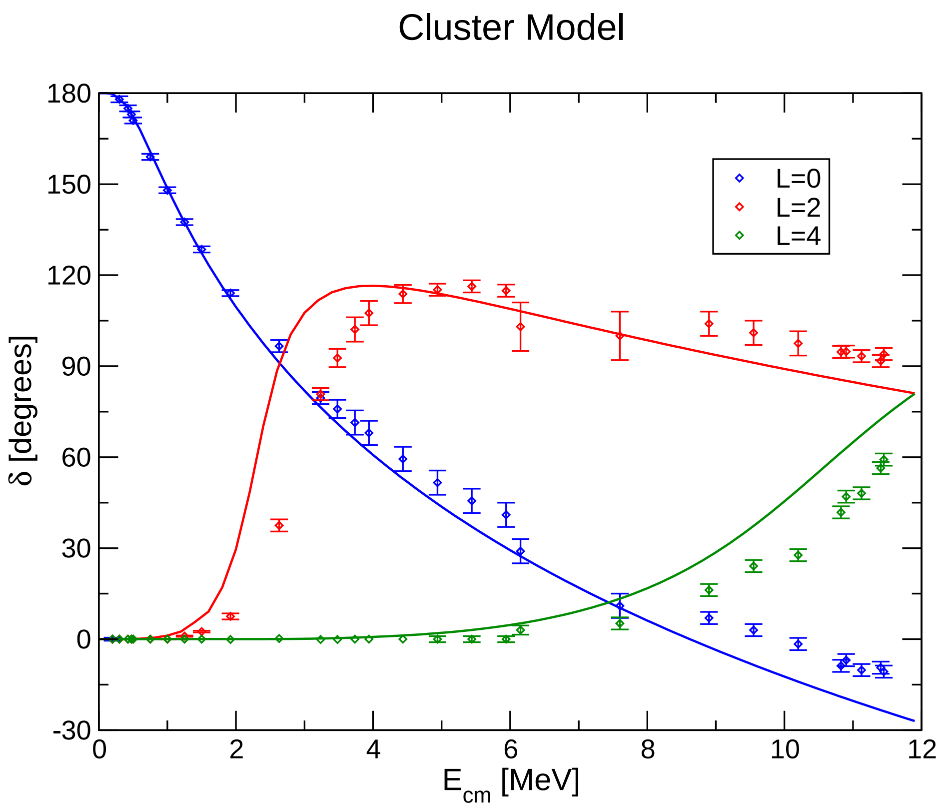


## Interaction

- UCOM interaction ($I_\vartheta$=0.30 fm$^3$ with phenomenological two-body correction term (momentum-dependent central and spin-orbit) fitted to doubly-magic nuclei
- not tuned for $\alpha$-$\alpha$ scattering or $^{12}$C properties

# $\alpha$-$\alpha$ Phaseshifts



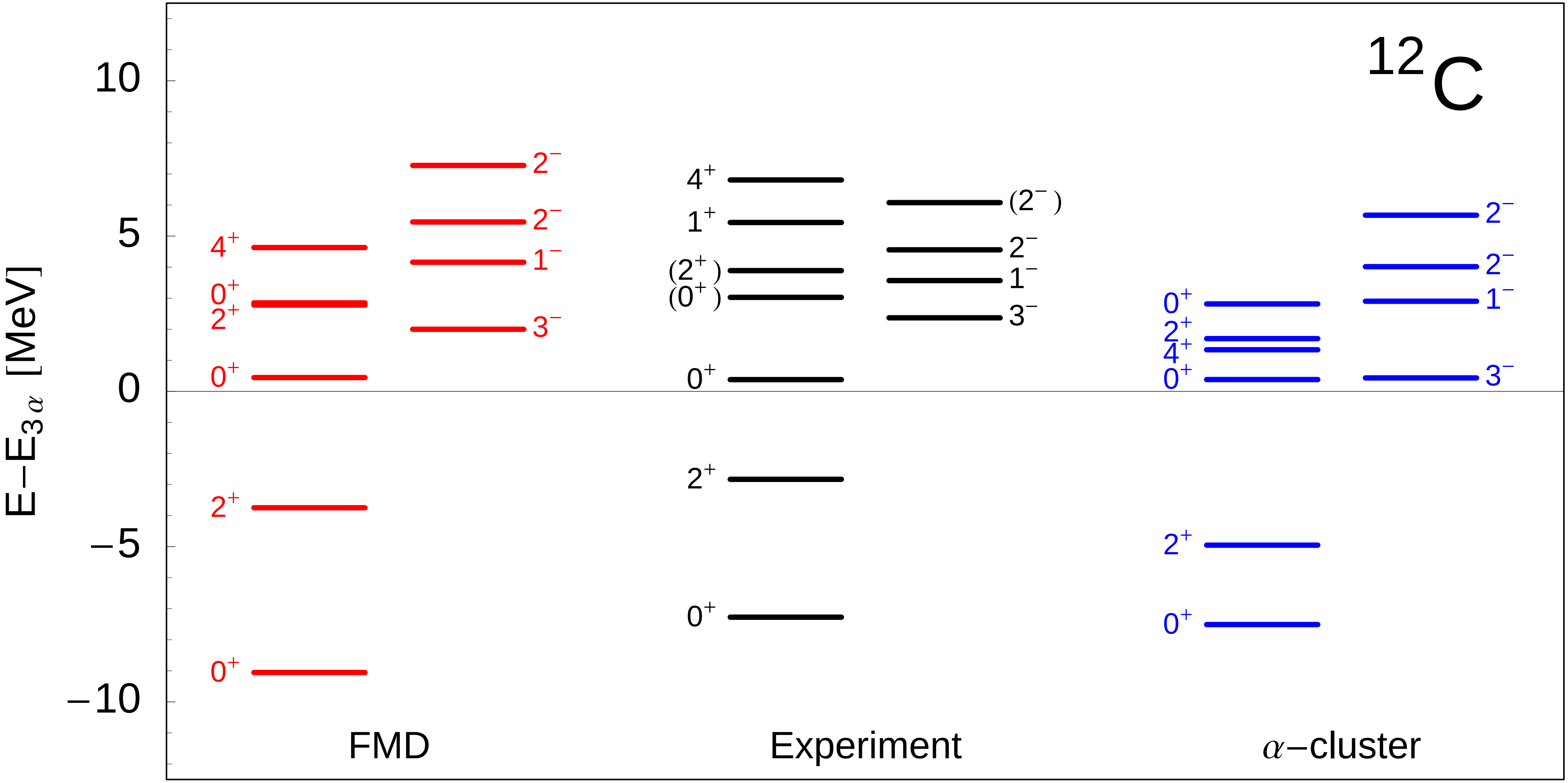




- Phaseshifts calculated with cluster configurations only (dashed lines)
- Phaseshifts calculated with additional FMD VAP configurations in the interaction region (solid lines)

- only cluster configurations included

➨ similar quality for description of $\alpha$-$\alpha$-scattering

# Comparison



$^{12}$C
E−E$_{3\alpha}$ [MeV]
10
5
0
−5
−10
FMD
Experiment
α−cluster
$2^-$
$2^-$
$1^-$
$3^-$
$4^+$
$0^+$
$2^+$
$0^+$
$2^+$
$0^+$
$4^+$
$1^+$
$(2^+)$
$(0^+)$
$0^+$
$2^+$
$0^+$
$(2^-)$
$2^-$
$1^-$
$3^-$
$0^+$
$2^+$
$4^+$
$0^+$
$2^+$
$0^+$
$2^-$
$2^-$
$1^-$
$3^-$

# Comparison

| | Exp[1] | Exp[2] | FMD | $\alpha$-cluster | 'BEC'[3] |
|---|---|---|---|---|---|
| $E(0_1^+)$ | -92.16 | | -92.64 | -89.56 | -89.52 |
| $E^*(2_1^+)$ | 4.44 | | 5.31 | 2.56 | 2.81 |
| $E(3\alpha)$ | -84.89 | | -83.59 | -82.05 | -82.05 |
| $E(0_2^+) - E(3\alpha)$ | 0.38 | | 0.43 | 0.38 | 0.26 |
| $E(0_3^+) - E(3\alpha)$ | (3.0) | 2.7(3) | 2.84 | 2.81 | |
| $E(2_2^+) - E(3\alpha)$ | (3.89) | 2.6(3) | 2.77 | 1.70 | |
| $r_{\mathrm{charge}}(0_1^+)$ | 2.47(2) | | 2.53 | 2.54 | |
| $r(0_1^+)$ | | | 2.39 | 2.40 | 2.40 |
| $r(0_2^+)$ | | | 3.38 | 3.71 | 3.83 |
| $r(0_3^+)$ | | | 4.62 | 4.75 | |
| $r(2_1^+)$ | | | 2.50 | 2.37 | 2.38 |
| $r(2_2^+)$ | | | 4.43 | 4.02 | |
| $M(E0, 0_1^+ \to 0_2^+)$ | 5.4(2) | | 6.53 | 6.52 | 6.45 |
| $B(E2, 2_1^+ \to 0_1^+)$ | 7.6(4) | | 8.69 | 9.16 | |
| $B(E2, 2_1^+ \to 0_2^+)$ | 2.6(4) | | 3.83 | 0.84 | |
| $B(E2, 2_2^+ \to 0_1^+)$ | ? | | 0.46 | 1.99 | |

experimental situation for $0_3^+$ and $2_2^+$ states still unsettled (?)

$2_2^+$ resonance at 1.8 MeV above treshold included in NACRE compilation

[1] Ajzenberg-Selove, Nuc. Phys. **A506**, 1 (1990)
[2] Itoh et al., Nuc. Phys. **A738**, 268 (2004)
[3] Funaki et al., Phys. Rev. C **67**, 051306(R) (2003)

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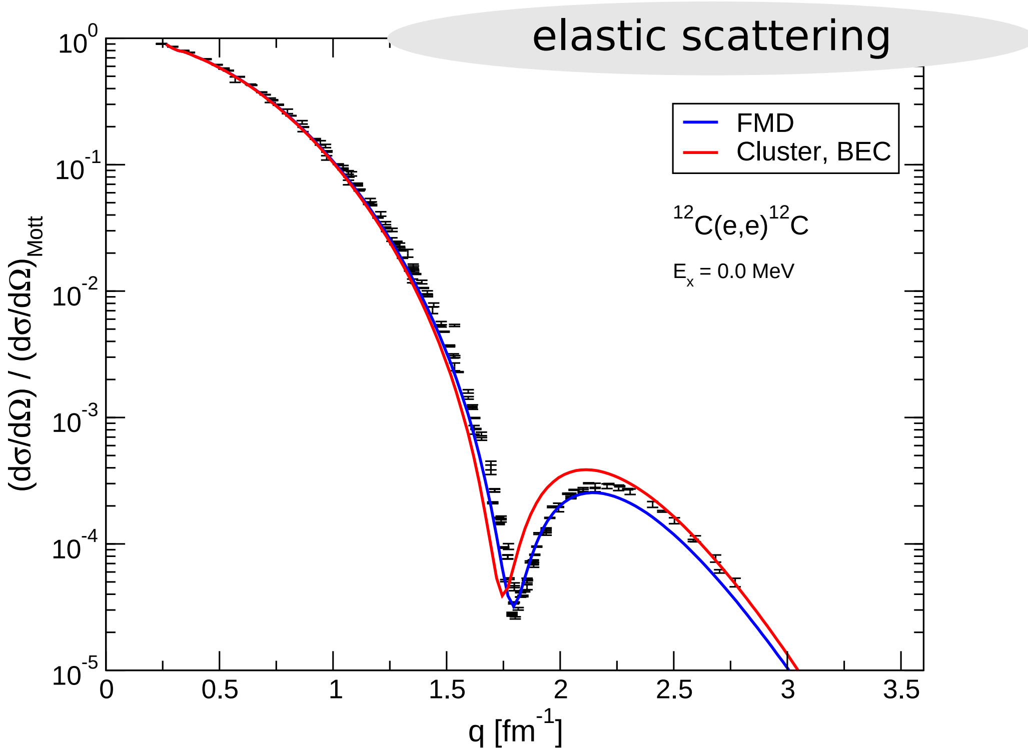
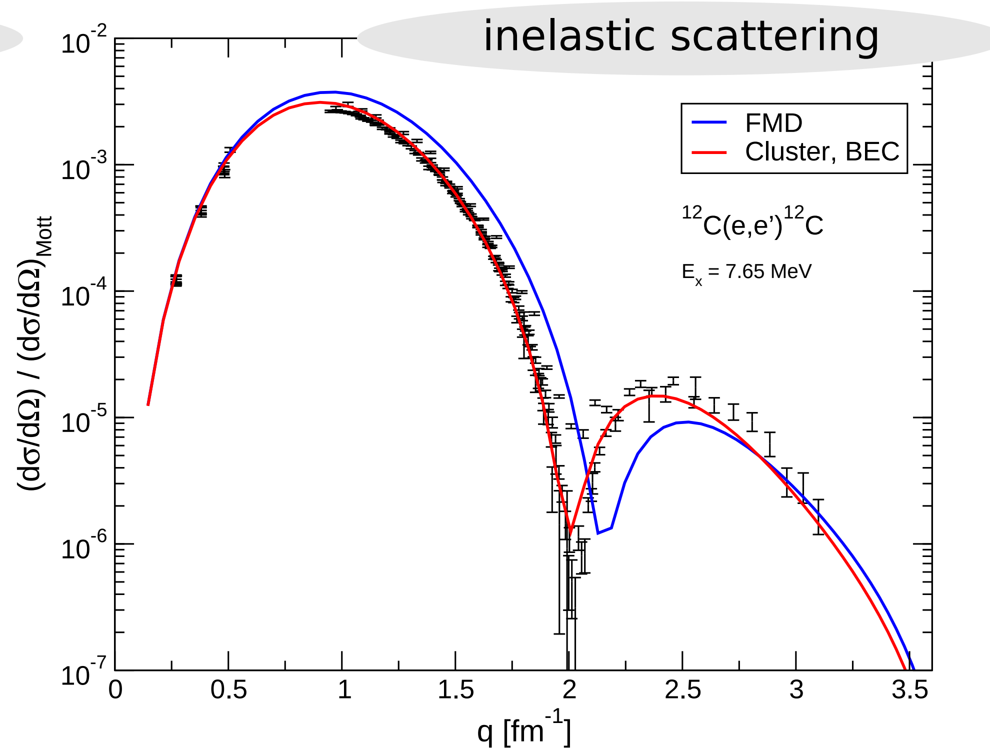
$2_2^+$ resonance at 1.8 MeV above treshold included in NACRE compilation

calculated in bound state approximation


[1] Ajzenberg-Selove, Nuc. Phys. **A506**, 1 (1990)
[2] Itoh et al., Nuc. Phys. **A738**, 268 (2004)
[3] Funaki et al., Phys. Rev. C **67**, 051306(R) (2003)

# Electron Scattering Data




- compare with precise electron scattering data up to high momenta in Distorted Wave Born Approximation
- use intrinsic density

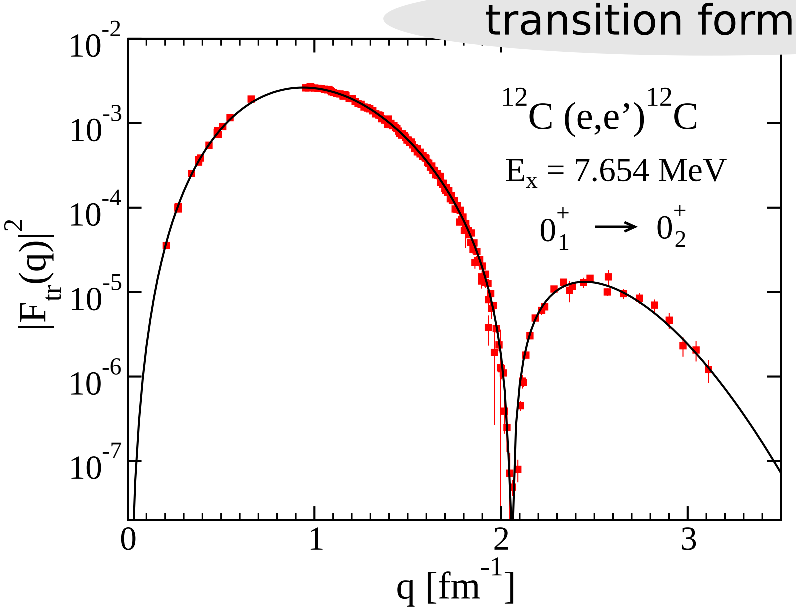
$$\rho(\mathbf{x}) = \sum_{k=1}^{A} \left\langle \Psi \middle| \delta(\underset{\sim}{\mathbf{x}}_k - \underset{\sim}{\mathbf{X}} - \mathbf{x}) \middle| \Psi \right\rangle$$

- ➡ elastic cross section described very well by FMD
- ➡ transition cross section better described by cluster model

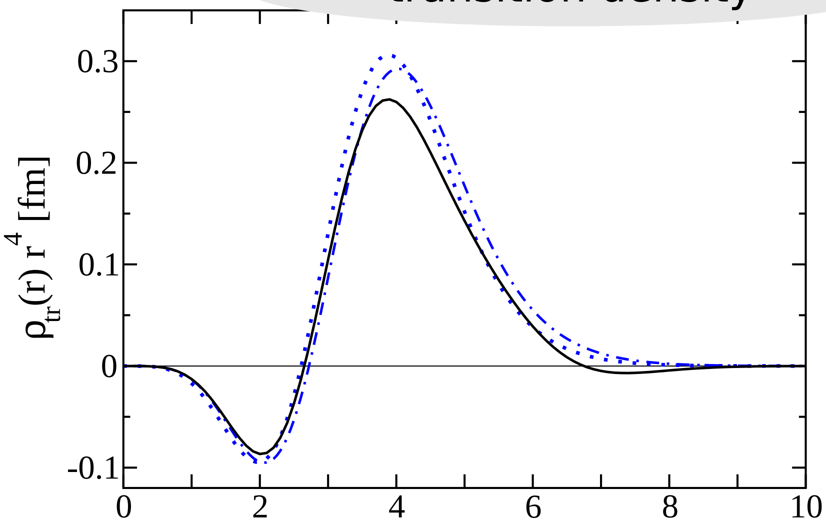
M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, and A. Richter, Phys. Rev. Lett. **98** (2007) 032501

# Monopole Matrix Element

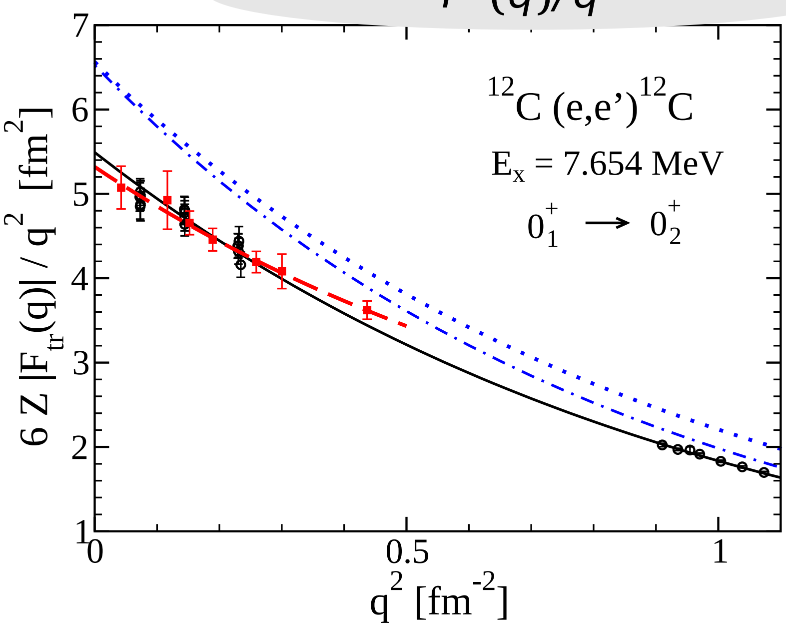
transition form factor




transition density




$F^2(q)/q^2$

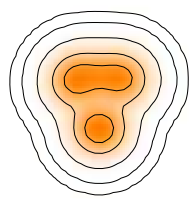



- $M(E0)$ determines the pair decay width
- model-independent self-consistent determination of transition form-factor/density in DWBA
- data at high momentum transfer necessary to constrain $M(E0)$ matrix element

M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, A. Richter, Phys. Rev. Lett. **105** (2010) 022501

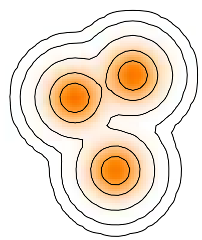
# Important Configurations

- Calculate the overlap with FMD basis states to find the most important contributions to the Hoyle state

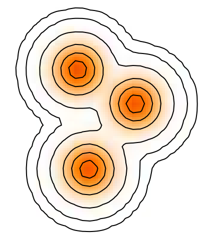


$\left|\left\langle \cdot \mid 0_1^+ \right\rangle\right| = 0.94$

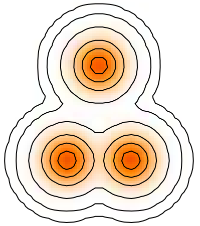
$\left|\left\langle \cdot \mid 2_1^+ \right\rangle\right| = 0.93$



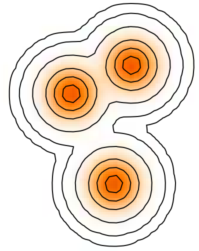
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.72$



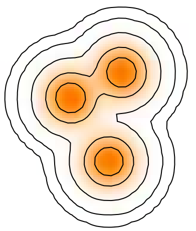
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.71$



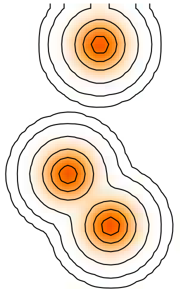
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$



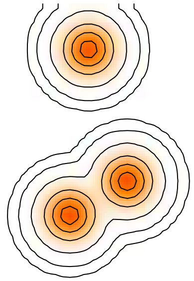
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$



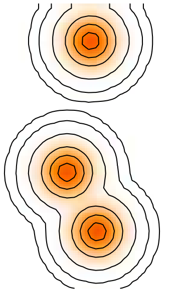
$\left|\left\langle \cdot \mid 3_1^- \right\rangle\right| = 0.83$



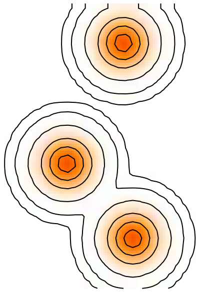
$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.50$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.49$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.44$

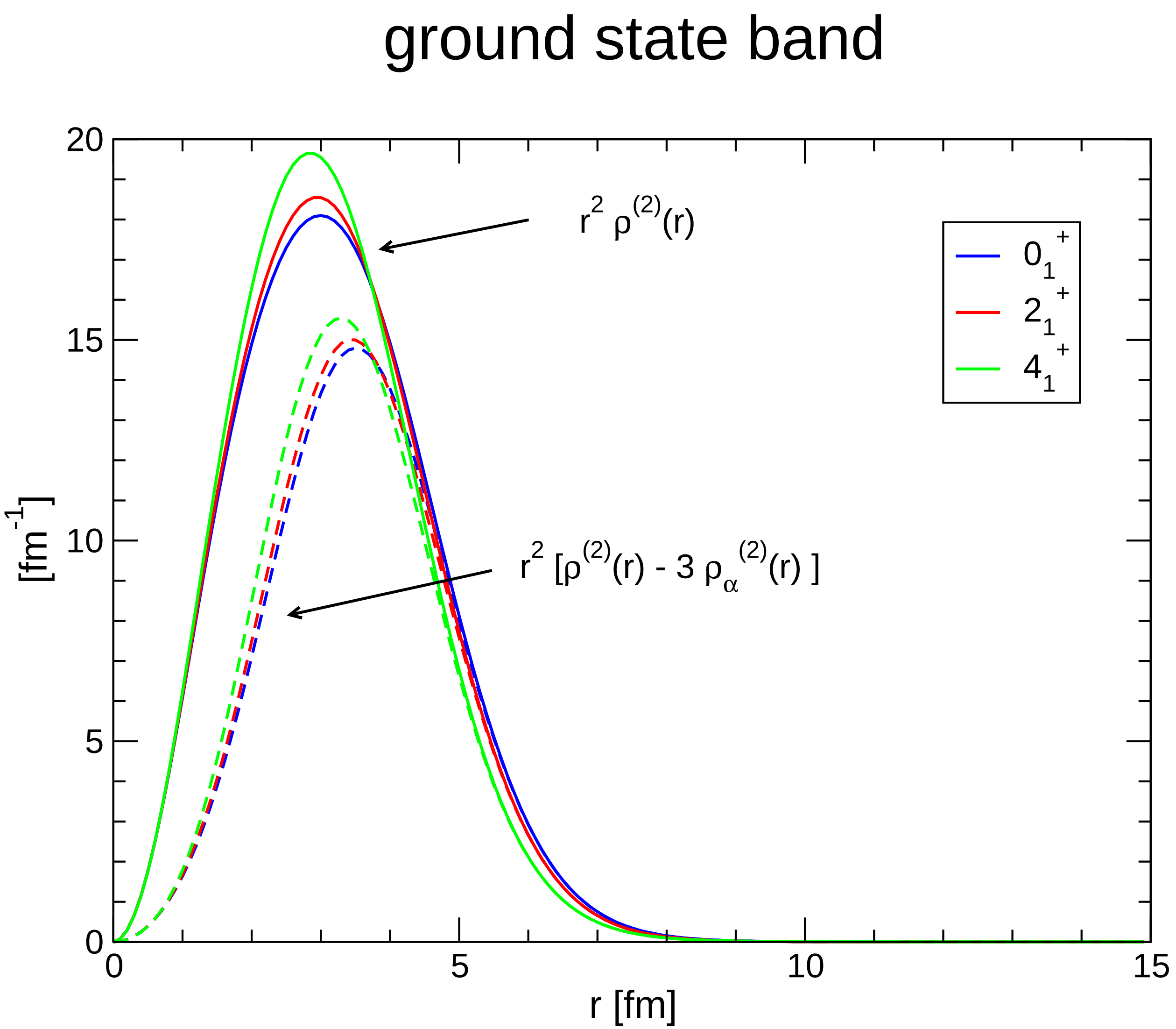


$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.41$

FMD basis states are not orthogonal!

$0_2^+$ and $0_3^+$ states have no rigid intrinsic structure

# Two-body Densities and Intrinsic Structure



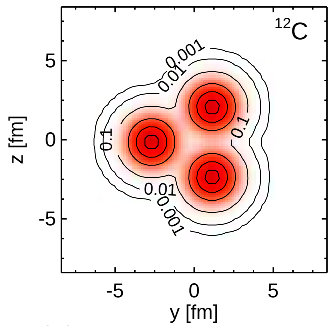

Cluster Model

$$\rho^{(2)}(r) = \langle \Psi \,|\, \sum_{i<j} \delta(\mathbf{r} - \underset{\sim}{\mathbf{r}}_{ij}) \,|\, \Psi \rangle$$

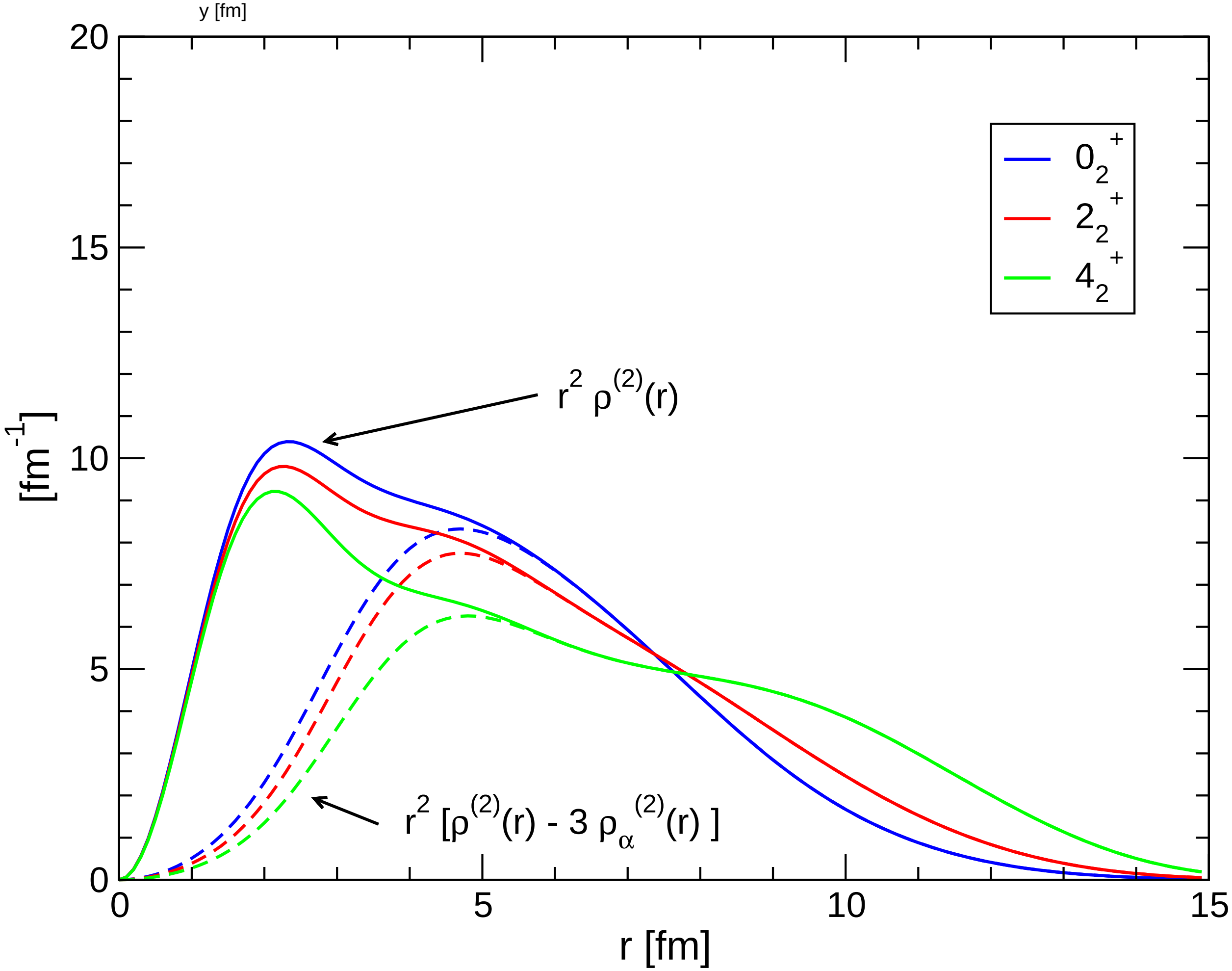
- substract contributions from $\alpha$'s to extract "$\alpha$-$\alpha$" correlations
- (substracted) two-body density peaks at 3.5 fm

➥ consistent with **compact triangular structure**

# Two-body Densities and Intrinsic Structure




Hoyle state "band"

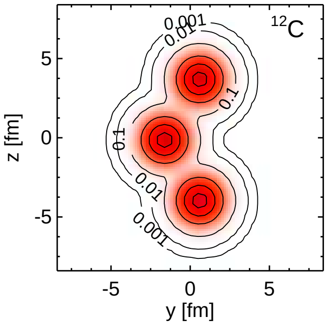



Cluster Model

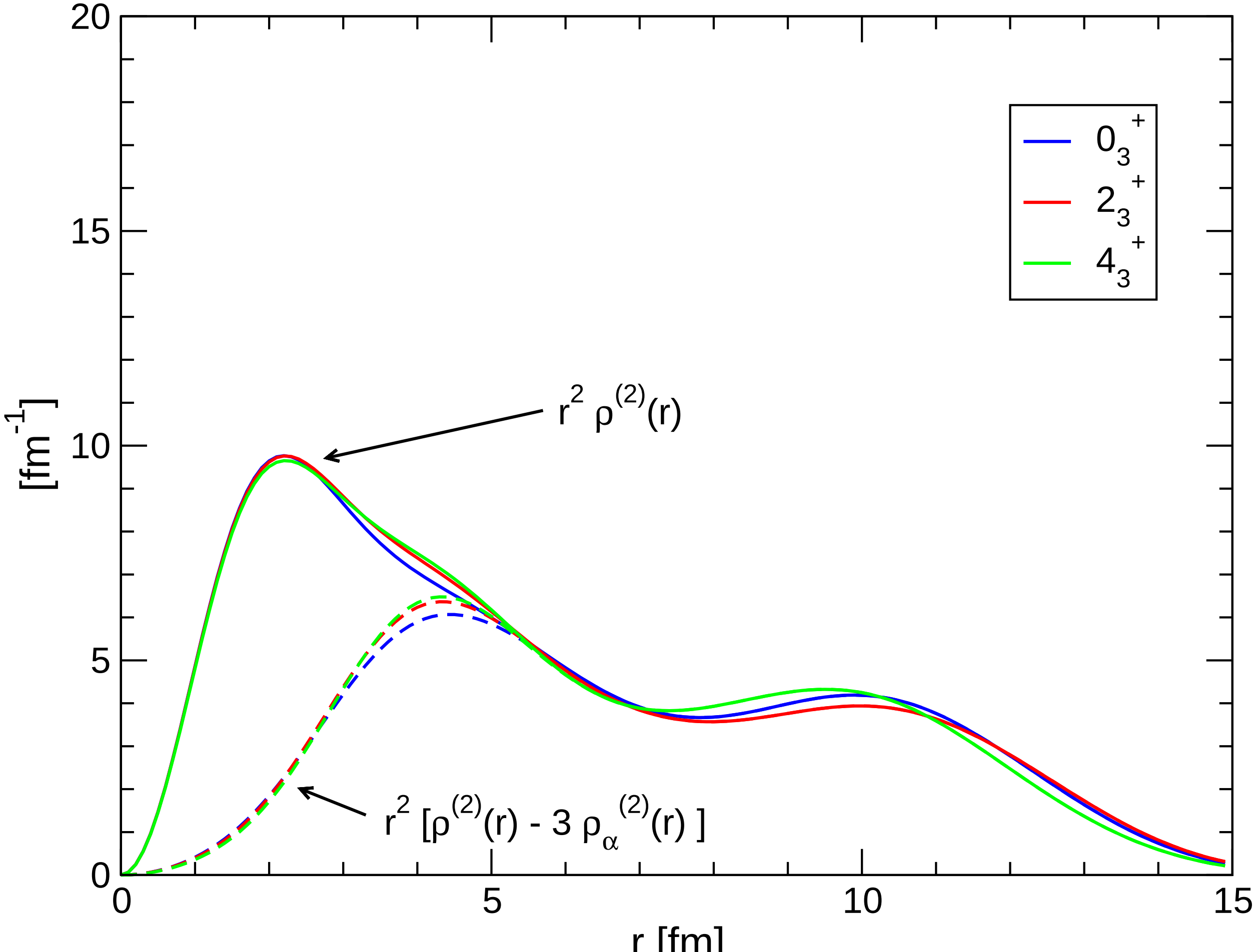
$$\rho^{(2)}(r) = \langle\, \Psi \,\big|\, \sum_{i<j} \delta(\mathbf{r} - \underset{\sim}{\mathbf{r}}_{ij}) \,\big|\, \Psi \,\rangle$$

- substract contributions from $\alpha$'s to extract "$\alpha$-$\alpha$ correlations"
- Hoyle state two-body density peaks at 5 fm, extended tail
- ➡ consistent with **triangular structure**
- tail in $2_2^+$ and $4_2^+$ states more pronounced
- ➡ admixture of open triangle configurations

# Two-body Densities and Intrinsic Structure




third $0^+$ state band




Cluster Model

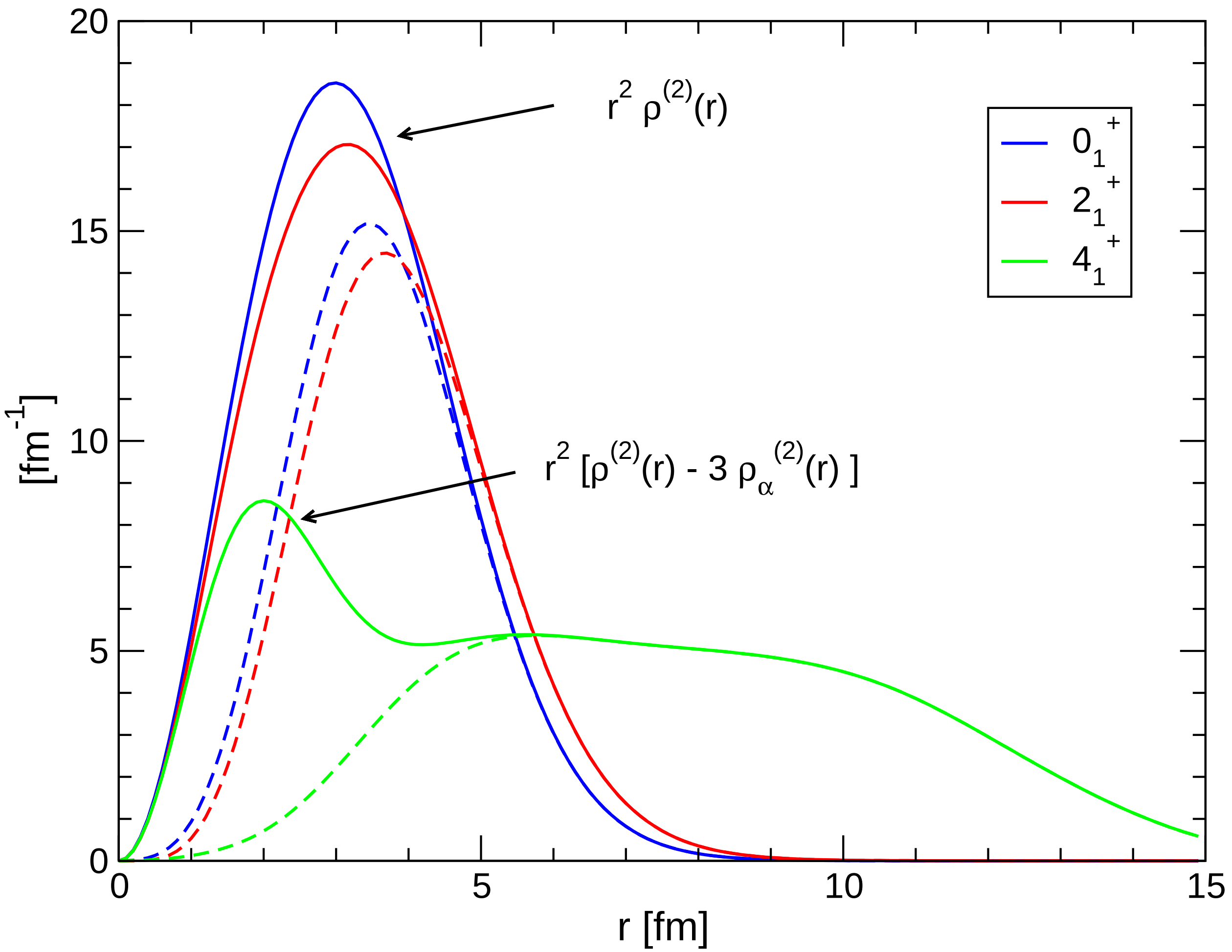
$$\rho^{(2)}(r) = \left\langle \Psi \middle| \sum_{i<j} \delta(\mathbf{r} - \mathbf{r}_{ij}) \middle| \Psi \right\rangle$$

- substract contributions from $\alpha$'s to extract "$\alpha$-$\alpha$" correlations
- two-body density peaks at 4.5 fm and 10 fm

➡ consistent with **open triangle/chain configuration**

# Two-body Densities and Intrinsic Structure

ground state band




FMD

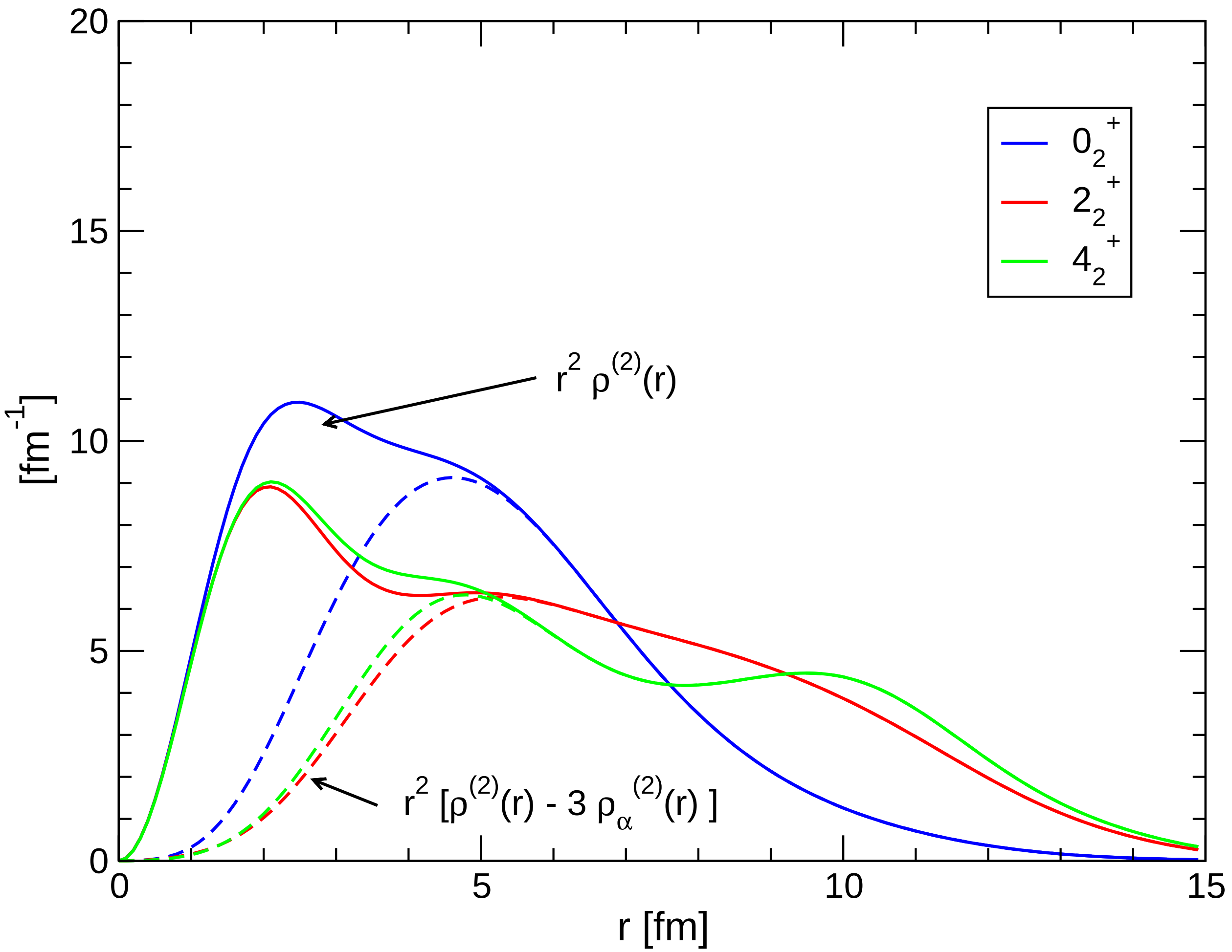
$$\rho^{(2)}(r) = \langle\, \Psi \,\big|\, \sum_{i<j} \delta(\mathbf{r} - \underset{\sim}{\mathbf{r}}_{ij}) \,\big|\, \Psi \,\rangle$$

- substract contributions from $\alpha$'s to extract $\alpha$-$\alpha$ correlations
- (corrected) two-body density peaks at 3.5 fm for $0^+$ and $2^+$
- $4^+$ state strongly mixed with cluster configurations

# Two-body Densities and Intrinsic Structure

FMD

Hoyle state "band"

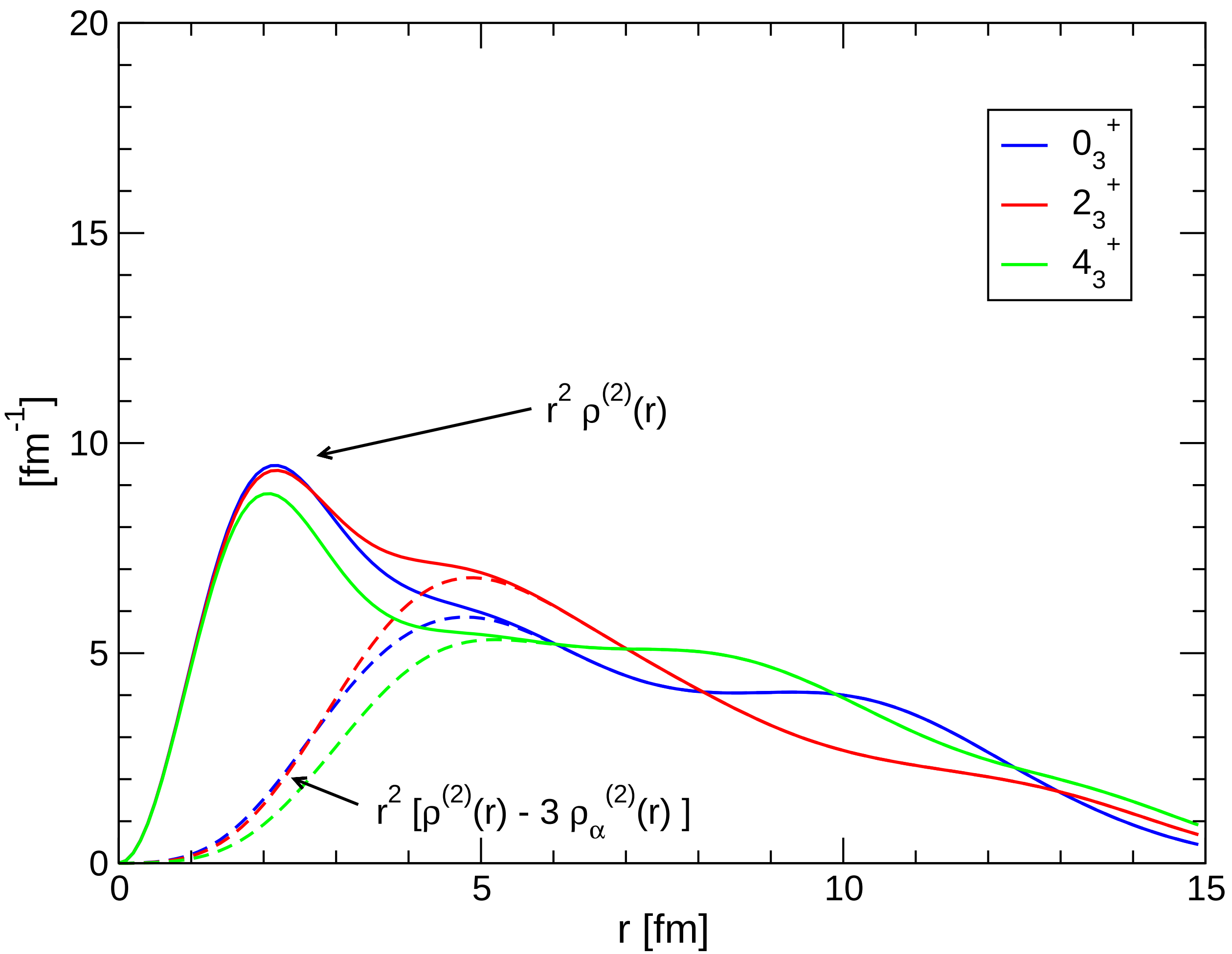



$$\rho^{(2)}(r) = \left\langle \Psi \middle| \sum_{i<j} \delta(\mathbf{r} - \underset{\sim}{\mathbf{r}}_{ij}) \middle| \Psi \right\rangle$$

- substract contributions from $\alpha$'s to extract $\alpha$-$\alpha$ correlations
- Hoyle state two-body density peaks at 5 fm, extended tail
- ➡ consistent with **extended triangular structure**
- $2_2^+$ and $4_2^+$ states have different intrinsic structure
- ➡ admixture of open triangle configurations

# Two-body Densities and Intrinsic Structure

third $0^+$ state band




FMD

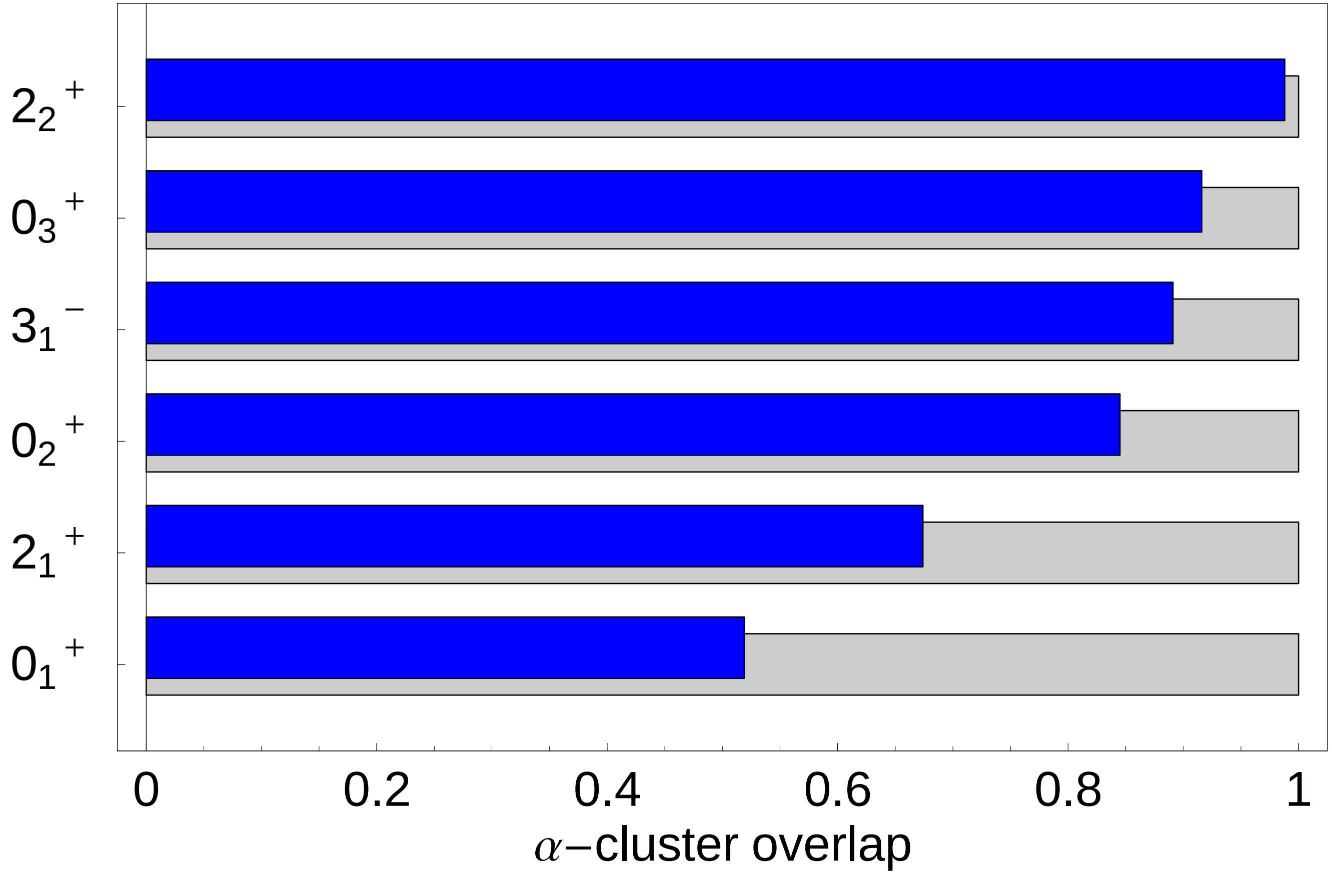
$$\rho^{(2)}(r) = \langle\, \Psi \,\big|\, \sum_{i<j} \delta(\mathbf{r} - \underset{\sim}{\mathbf{r}}_{ij}) \,\big|\, \Psi \,\rangle$$

- substract contributions from $\alpha$'s to extract $\alpha$-$\alpha$ correlations
- two-body density peaks at 4.5 fm and 10 fm
- consistent with **chain configuration**

# Overlap with Cluster Model Space

Calculate the overlap of FMD wave functions with pure $\alpha$-cluster model space

$$N_\alpha = \left\langle \Psi \middle| \underset{\sim}{P}_{3\alpha} \middle| \Psi \right\rangle$$



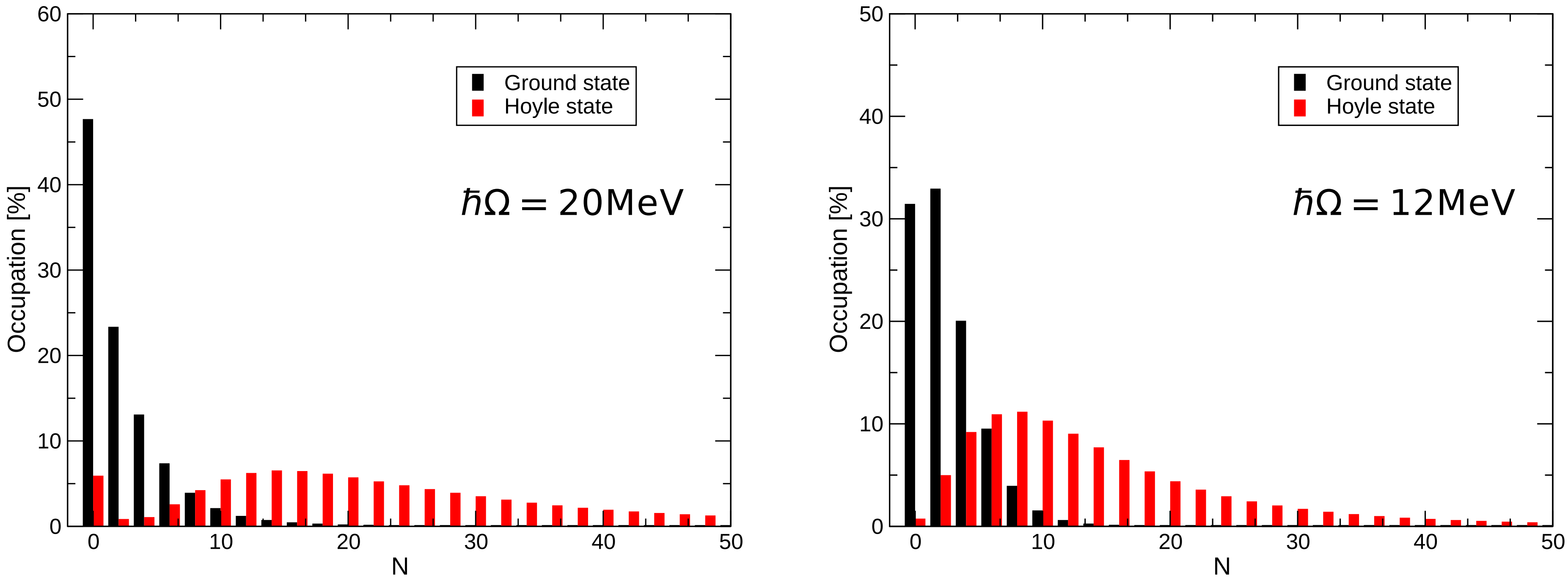

Hoyle state has 15% non-alpha admixtures

# Harmonic Oscillator $N\hbar\Omega$ Excitations

Y. Suzuki et al., Phys. Rev. C **54** 2073, (1996):

$$\mathrm{Occ}(N) = \left\langle \Psi \,\middle|\, \delta\left(\sum_i (\underset{\sim}{H}_i^{HO}/\hbar\Omega - 3/2) - N\right) \,\middle|\, \Psi \right\rangle$$

Cluster Model




- Hoyle state very difficult to converge in no-core shell model

T. Neff, H. Feldmeier, Few-Body Syst. **45**, 145 (2009)

# Summary

## Short-range correlations in light nuclei

- short-range and high-momentum behavior of two-body densities identical in A=2,3,4 nuclei
- Two-body densities in $^4$He with SRG evolved interactions

## $^3$He($\alpha$,$\gamma$)$^7$Be Radiative Capture

- Bound states, resonance and scattering wave functions
- S-Factor: energy dependence and normalization
- Analyzed in terms of overlap functions

## Cluster States in $^{12}$C

- Consistent description of ground state band and Hoyle state
- Investigate Hoyle state structure with electron scattering
- Two-body densities are a model independent tool for investigating structure
- Cluster states need tremendous model space in harmonic oscillator basis

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