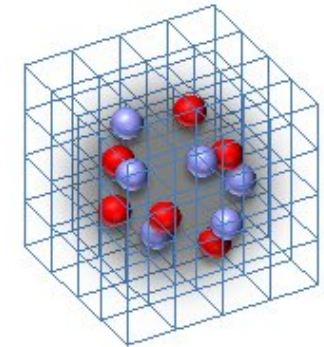




**Testing the Anthropic Principle  
with Lattice Simulations**  
Ulf-G. Meißner, Univ. Bonn & FZ Jülich



NLEFT

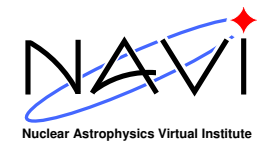
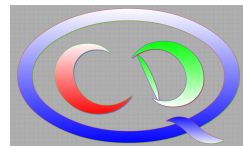
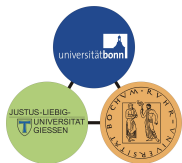
Supported by DFG, SFB/TR-16

and by DFG, SFB/TR-110

and by EU, I3HP EPOS

and by BMBF 06BN9006

and by HGF VIQCD VH-VI-417



- **Nuclear Lattice Effective Field Theory collaboration**

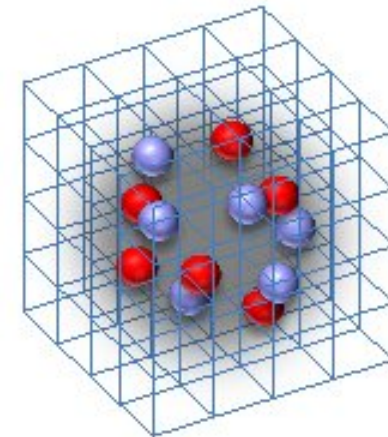
*Evgeny Epelbaum (Bochum)*

*Hermann Krebs (Bochum)*

*Timo Lähde (Jülich)*

*Dean Lee (NC State)*

*Ulf-G. Meißner (Bonn/Jülich)*



# CONTENTS

- Introduction I: The Anthropic Principle & the Hoyle state
- Introduction II: Effective Field Theory for Nuclear Physics
- Nuclear lattice simulations: methods
- Nuclear lattice simulations: results
- How anthropic is the Hoyle state?
- Summary & outlook

# Introduction I: The Anthropic Principle & the Hoyle state

# THE ANTHROPIIC PRINCIPLE

- The anthropic principle:

“The observed values of all physical and cosmological quantities are not equally probable but they take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirements that the Universe be old enough for it to have already done so.”

Carter 1974, Barrow & Tipler 1988, ...

⇒ can this be tested? / have physical consequences?

- Ex. 1: “Anthropic bound on the cosmological constant” Weinberg (1987) [505 cites]
- Ex. 2: “The anthropic string theory landscape” Susskind (2003) [681 cites]









# The RELEVANT QUESTION

Date: Sat, 25 Dec 2010 20:03:42 -0600  
 From: Steven Weinberg <weinberg@zipy.ph.utexas.edu>  
 To: Ulf-G. Meissner <meissner@hiskp.uni-bonn.de>  
 Subject: Re: Hoyle state in  $^{12}\text{C}$

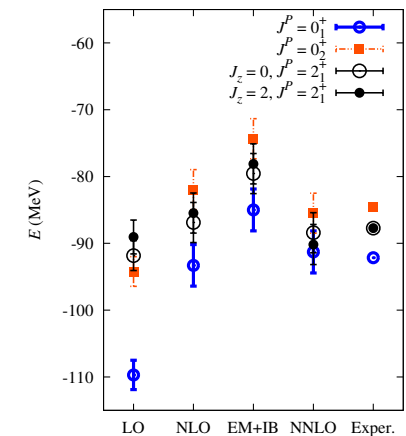
Dear Professor Meissner,

Thanks for the colorful graph. It makes a nice Christmas card. But I have a detailed question. Suppose you calculate not only the energy of the Hoyle state in  $^{12}\text{C}$ , but also of the ground states of  $^4\text{He}$  and  $^8\text{Be}$ . How sensitive is the result that the energy of the Hoyle state is near the sum of the rest energies of  $^4\text{He}$  and  $^8\text{Be}$  to the parameters of the theory? I ask because I suspect that for a pretty broad range of parameters, the Hoyle state can be well represented as a nearly bound state of  $^8\text{Be}$  and  $^4\text{He}$ .

All best,

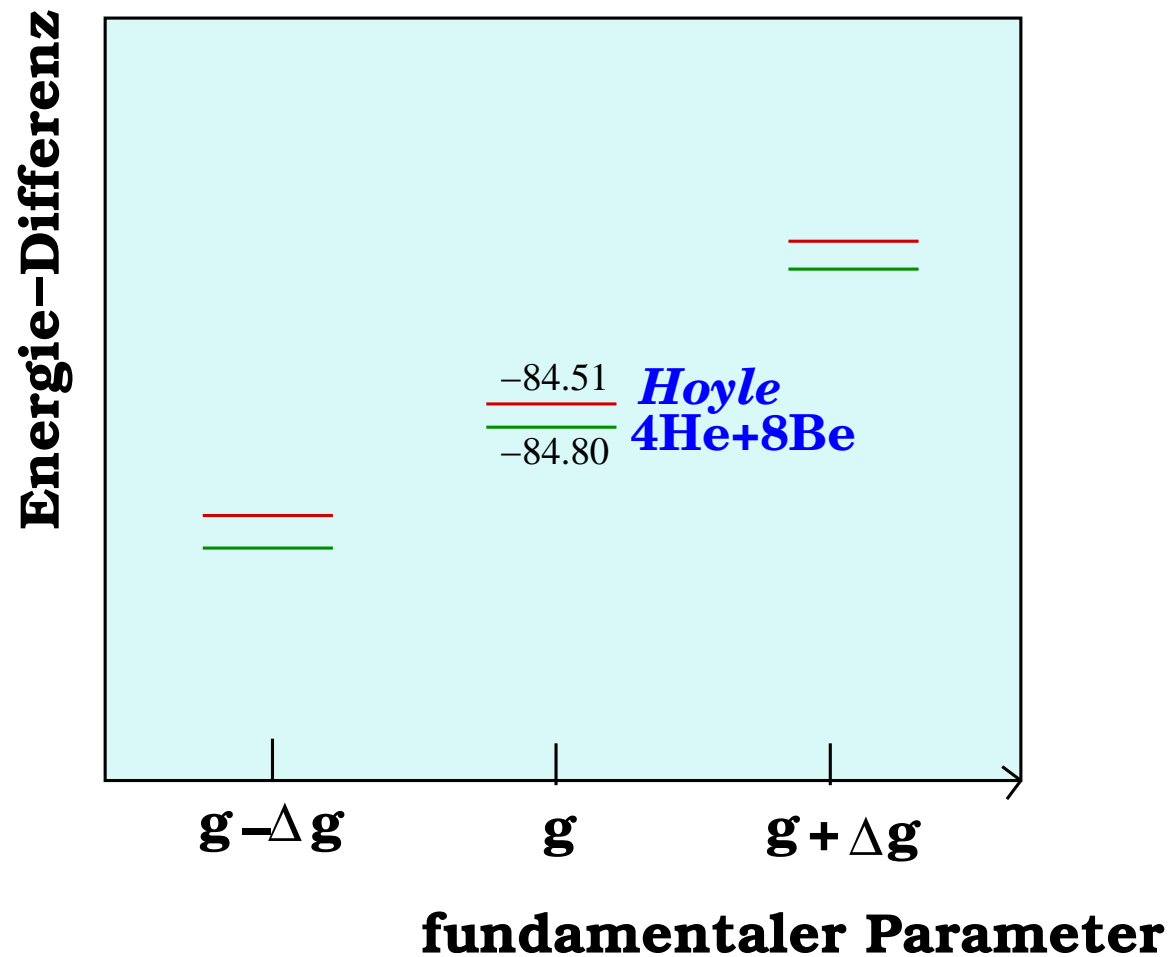
Steve Weinberg

- How does the Hoyle state relative to the  $4\text{He}+8\text{Be}$  threshold, if we change the fundamental parameters of QCD+QED?
- not possible in nature, *but on a high-performance computer!*



# The NON-ANTHROPIC SCENARIO

- Weinberg's assumption: The Hoyle state stays close to the  $4\text{He}+8\text{Be}$  threshold







# Introduction II: Effective Field Theory for Nuclear Physics

only a brief reminder → details in

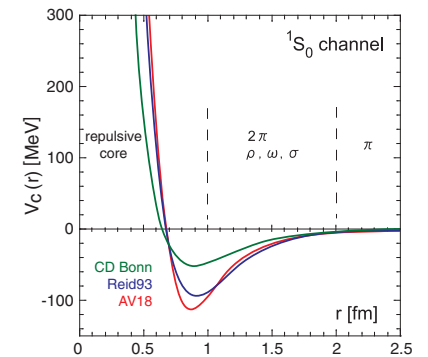
E. Epelbaum, H.-W. Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773  
[arXiv:0811.1338 [nucl-th]]

Gasser, Leutwyler, Weinberg, van Kolck, Epelbaum, Bernard, Kaiser, UGM, . . .

- Scales in nuclear physics:

Natural:  $\lambda_\pi = 1/M_\pi \simeq 1.5 \text{ fm}$  (Yukawa 1935)

Unnatural:  $|a_{np}(^1S_0)| = 23.8 \text{ fm}$ ,  $a_{np}(^3S_1) = 5.4 \text{ fm} \gg 1/M_\pi$

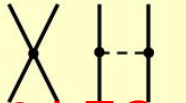
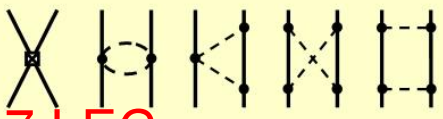
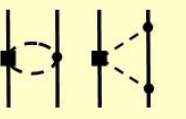
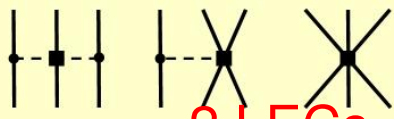
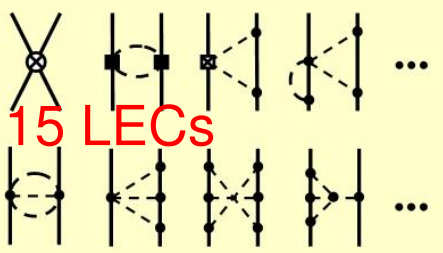
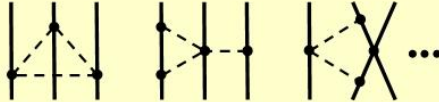



- this can be analyzed in a suitable EFT based on

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{EFF}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \dots$$

- pion and pion-nucleon sectors are perturbative in  $Q/\Lambda_\chi \rightarrow$  chiral perturbation th'y
- $\mathcal{L}_{NN}$  collects short-distance contact terms, to be fitted
- NN interaction requires non-perturbative resummation  
 $\rightarrow$  chirally expand  $V_{NN(N)}$ , use in regularized LS/FY equation

# CHIRAL POTENTIAL and NUCLEAR FORCES

	Two-nucleon force	Three-nucleon force	Four-nucleon force	
LO	 2 LECs	—	—	$\mathcal{O}((Q/\Lambda_\chi)^0)$
NLO	 7 LECs	—	—	$\mathcal{O}((Q/\Lambda_\chi)^2)$
N <sup>2</sup> LO		 2 LECs	—	$\mathcal{O}((Q/\Lambda_\chi)^3)$
N <sup>3</sup> LO	 15 LECs			$\mathcal{O}((Q/\Lambda_\chi)^4)$

- explains naturally the observed hierarchy of nuclear forces
- MANY successful tests in few-nucleon systems (continuum calc's)

# Nuclear lattice simulations – Formalism –

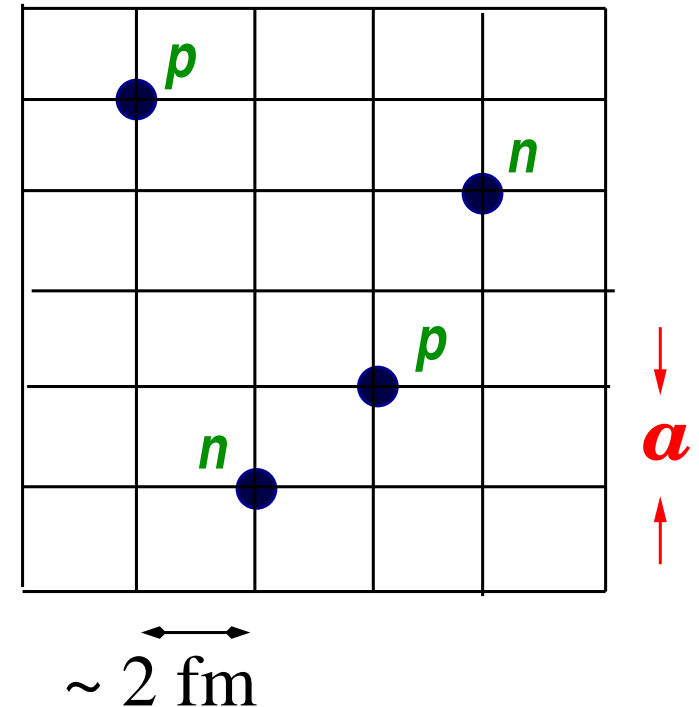


# NUCLEAR LATTICE SIMULATIONS

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000), Lee, Schäfer (2004), . . .  
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

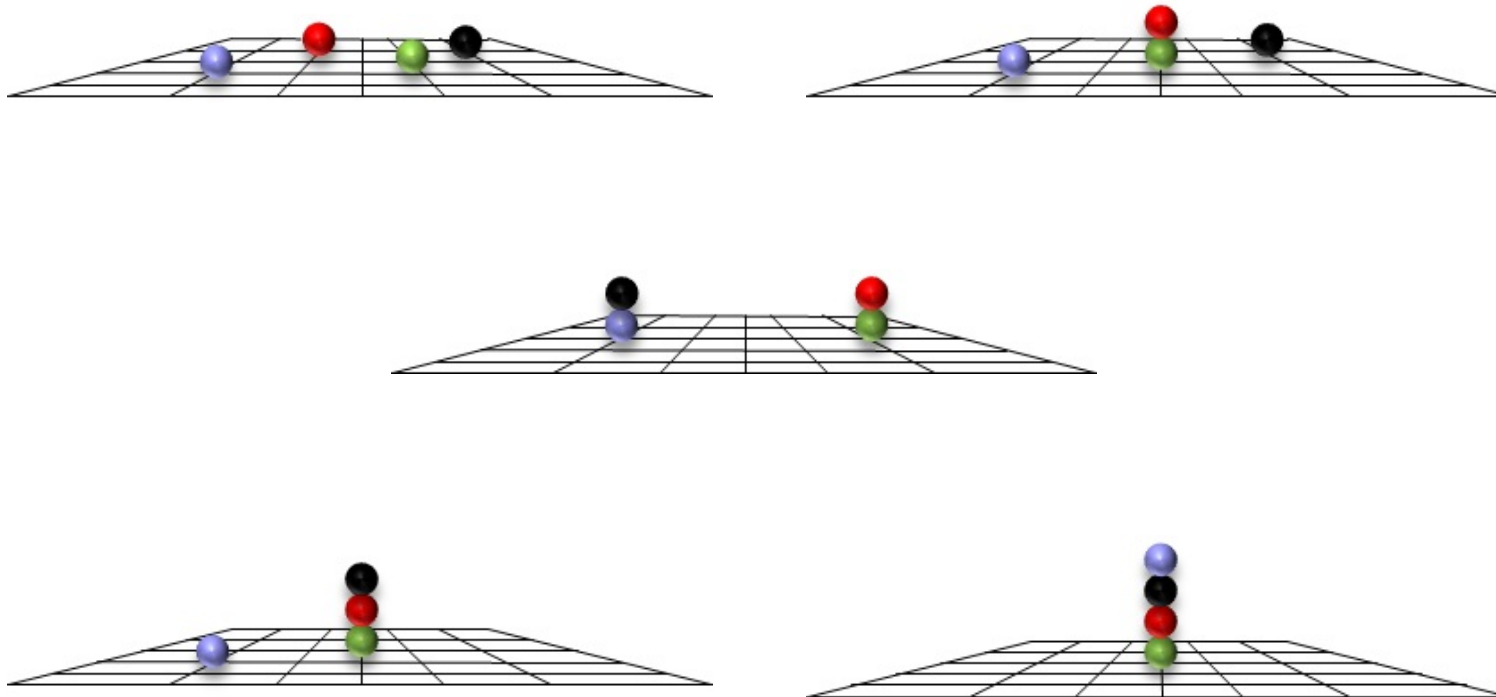
- *new method* to tackle the nuclear many-body problem
- discretize space-time  $V = L_s \times L_s \times L_s \times L_t$ :  
nucleons are point-like fields on the sites
- discretized chiral potential w/ pion exchanges  
and contact interactions
- typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300 \text{ MeV [UV cutoff]}$$



- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry
- J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302
- hybrid Monte Carlo & transfer matrix (similar to LQCD)

# CONFIGURATIONS



⇒ all *possible* configurations are sampled  
⇒ *clustering* emerges *naturally*

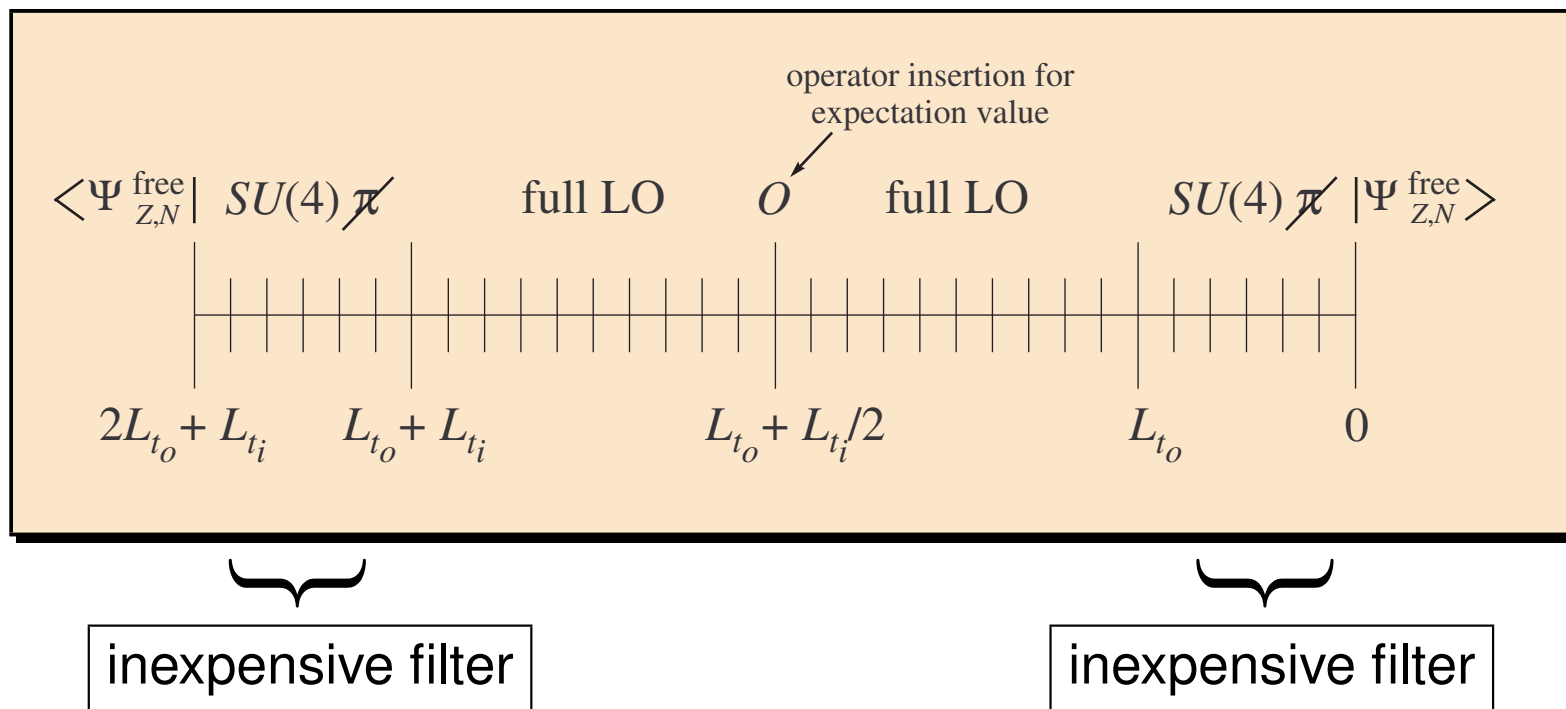


# TRANSFER MATRIX CALCULATION

- Expectation value of any normal-ordered operator  $\mathcal{O}$

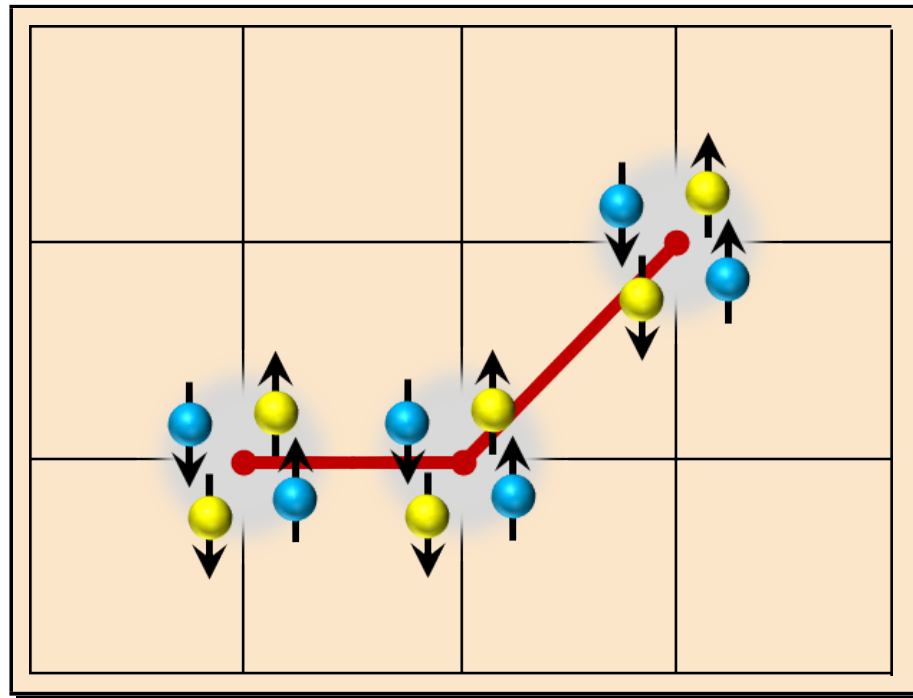
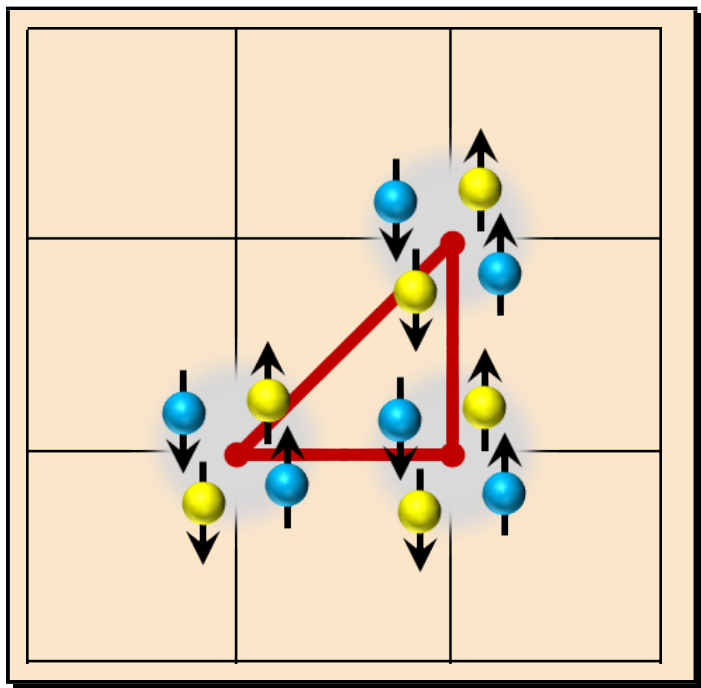
$$\langle \Psi_A | \mathcal{O} | \Psi_A \rangle = \lim_{t \rightarrow \infty} \frac{\langle \Psi_A | \exp(-tH/2) \mathcal{O} \exp(-tH/2) | \Psi_A \rangle}{\langle \Psi_A | \exp(-tH) | \Psi_A \rangle}$$

- Anatomy of the transfer matrix



# PROJECTION MONTE CARLO TECHNIQUE

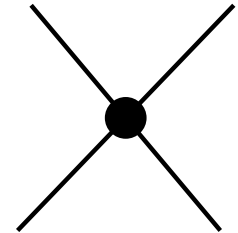
- Insert clusters of nucleons at initial/final states (spread over some time interval)
  - allows for all type of wave functions (shell model, clusters, ...)
  - removes directional bias
- Example: two basic configurations in the spectrum of  $^{12}\text{C}$



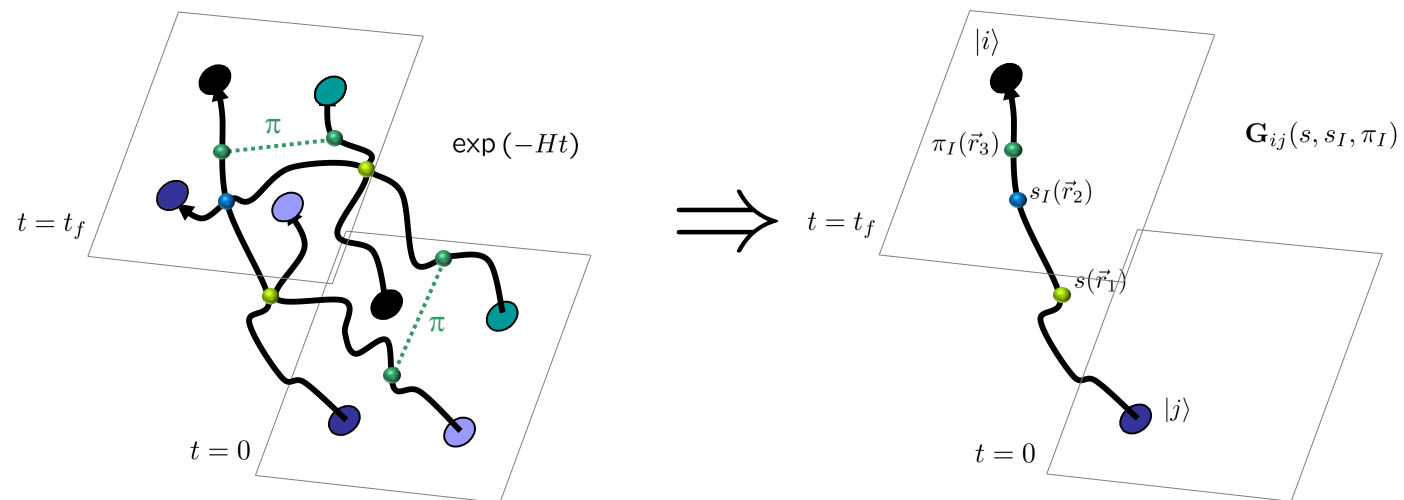
# MONTE CARLO with AUXILIARY FIELDS

- Contact interactions represented by auxiliary fields  $s, s_I$

$$\exp(\rho^2/2) \propto \int_{-\infty}^{+\infty} ds \exp(-s^2/2 - s\rho), \quad \rho \sim N^\dagger N$$



- Correlation function = path-integral over pions & auxiliary fields



# COMPUTATIONAL EQUIPMENT

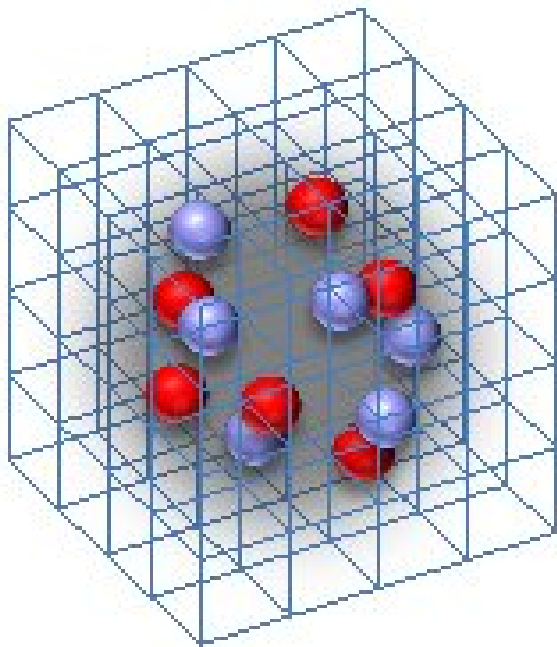
- Past = JUGENE (BlueGene/P)
- Present = JUQUEEN (BlueGene/Q)



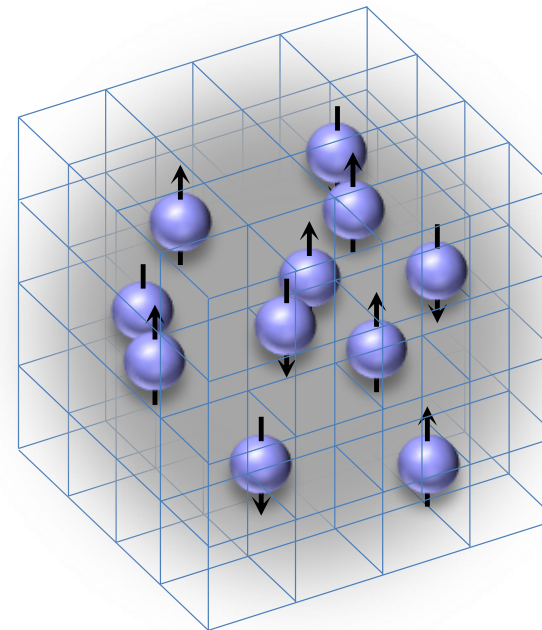
# Nuclear lattice simulations

## – Results –

nuclei



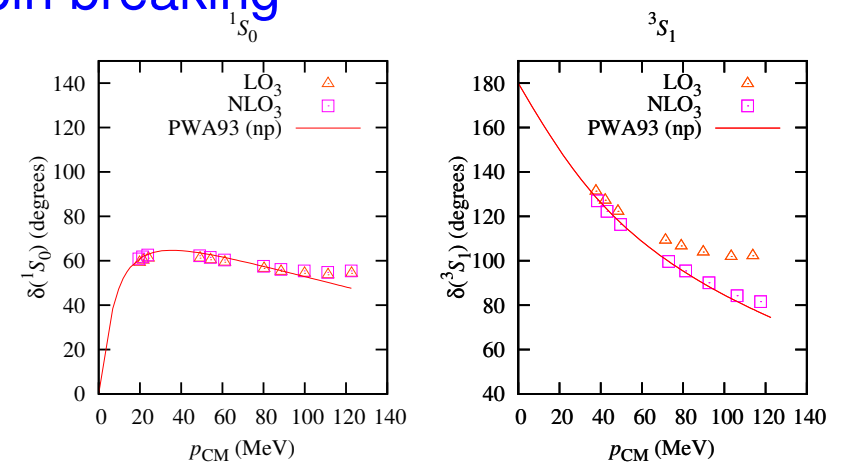
neutron matter



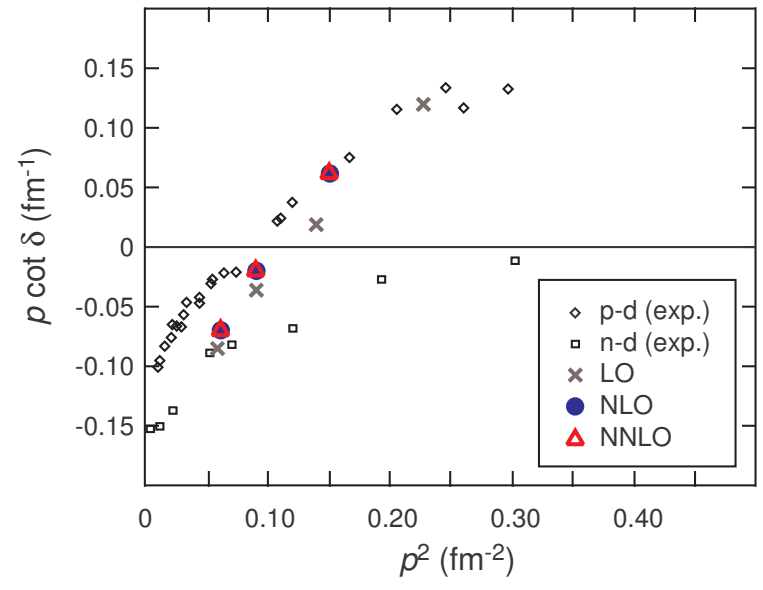
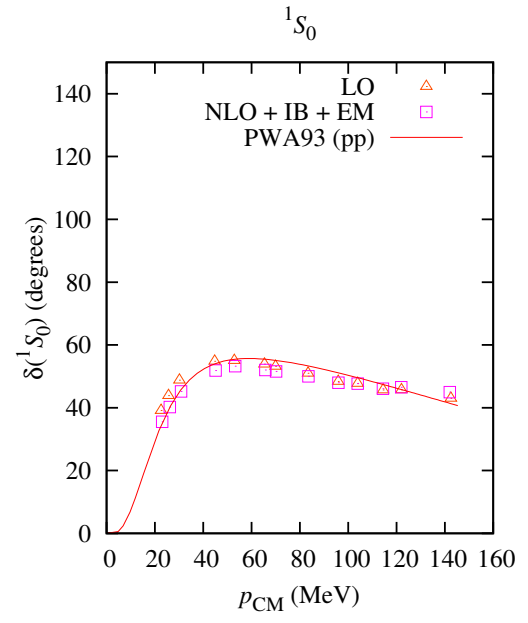


# FIXING PARAMETERS & FIRST PREDICTIONS

- work at NNLO including strong and em isospin breaking
  - 9 NN LECs from  $np$  scattering and  $Q_d$
  - 2 LECs for isospin-breaking ( $np, pp, nn$ )
  - 2 LECs  $D, E$  related to the leading 3NF
- ⇒ make predictions



- $pp$  vs  $np$  scattering
- nd spin-3/2 quartet channel
- ...



# Ground states

Epelbaum, Krebs, Lähde, Lee, UGM, arxiv:1208.1328

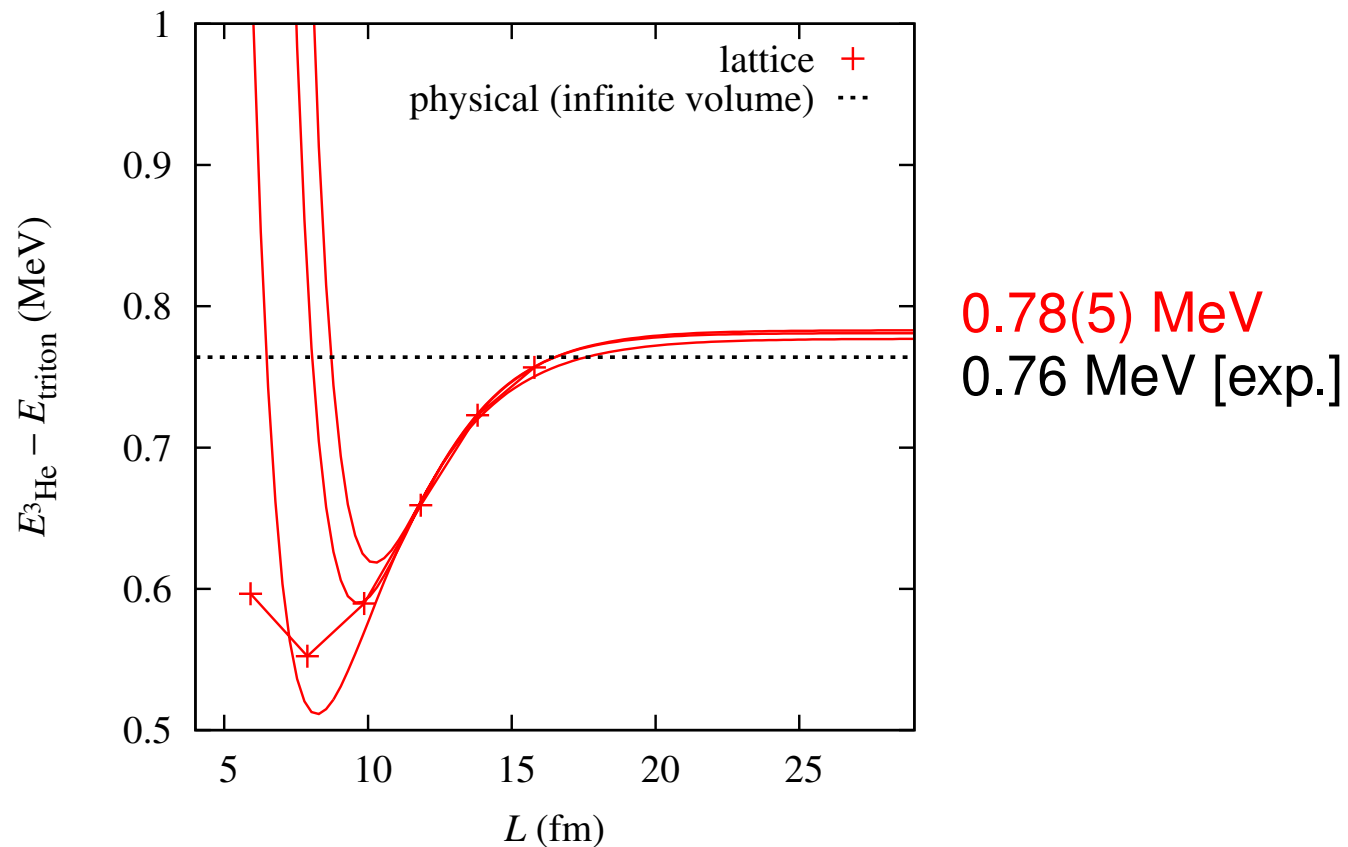
# PREDICTIONS: TRITON & HELIUM-3

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. **A 45** (2010) 335

- binding energies of 3N systems:  $E(L) = \text{B.E.} - \frac{a}{L} \exp(-bL)$

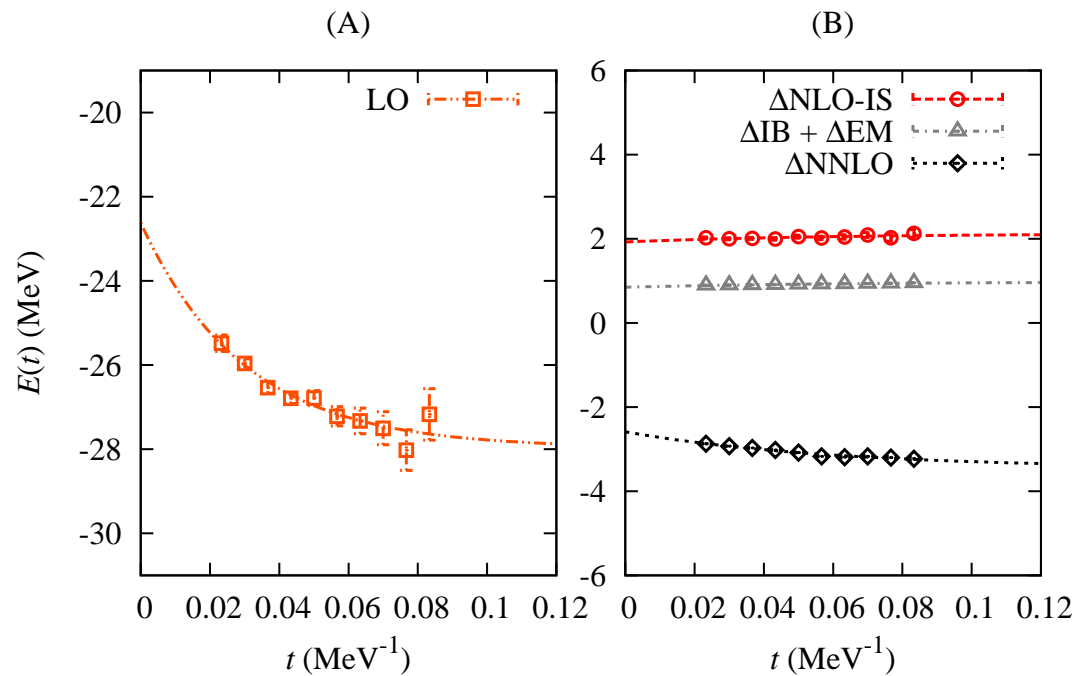
see also Hammer, Kreuzer (2011)

⇒ predict the energy difference  $E(^3\text{He}) - E(^3\text{H})$



# Ground state of ${}^4\text{He}$

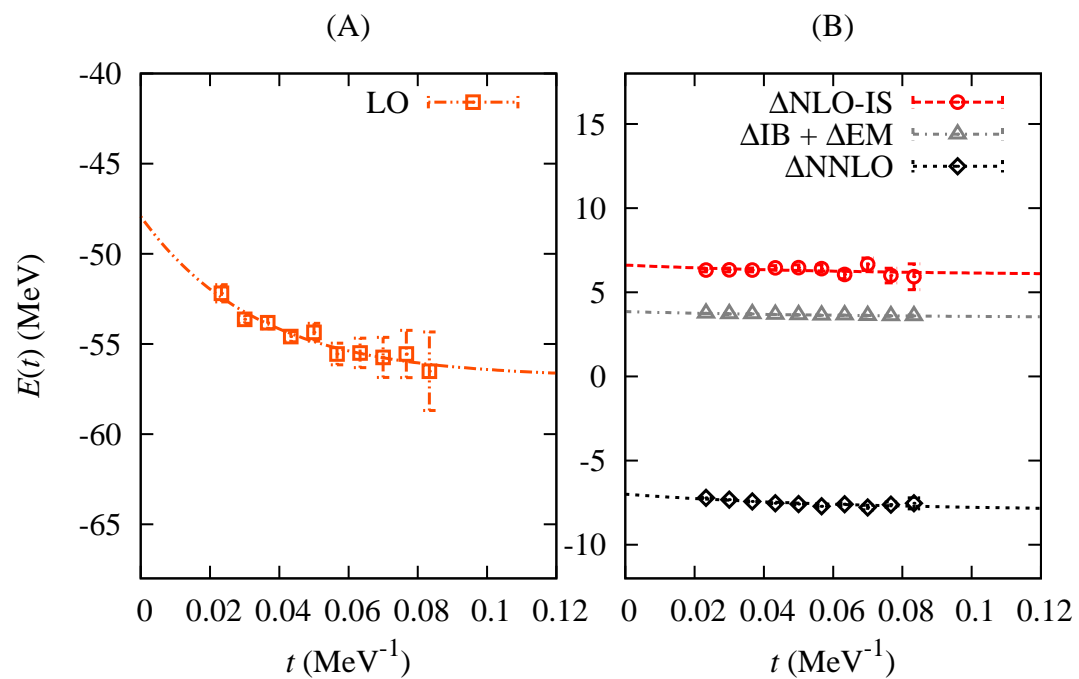
$L = 11.8 \text{ fm}$



LO ( $\mathcal{O}(Q^0)$ )	-28.0(3) MeV
NLO ( $\mathcal{O}(Q^2)$ )	-24.9(5) MeV
NNLO ( $\mathcal{O}(Q^3)$ )	-28.3(6) MeV
Exp.	-28.3 MeV

# Ground state of ${}^8\text{Be}$

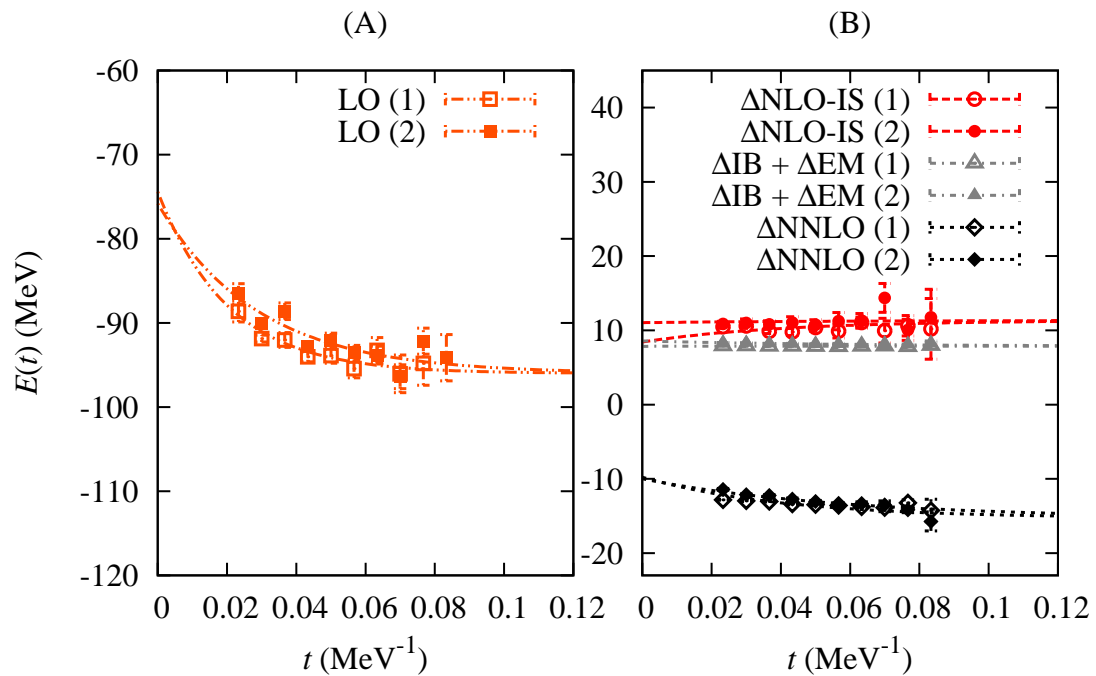
$L = 11.8 \text{ fm}$



LO ( $\mathcal{O}(Q^0)$ )	-57(2) MeV
NLO ( $\mathcal{O}(Q^2)$ )	-47(2) MeV
NNLO ( $\mathcal{O}(Q^3)$ )	-55(2) MeV
Exp.	-56.5 MeV

# Ground state of $^{12}\text{C}$

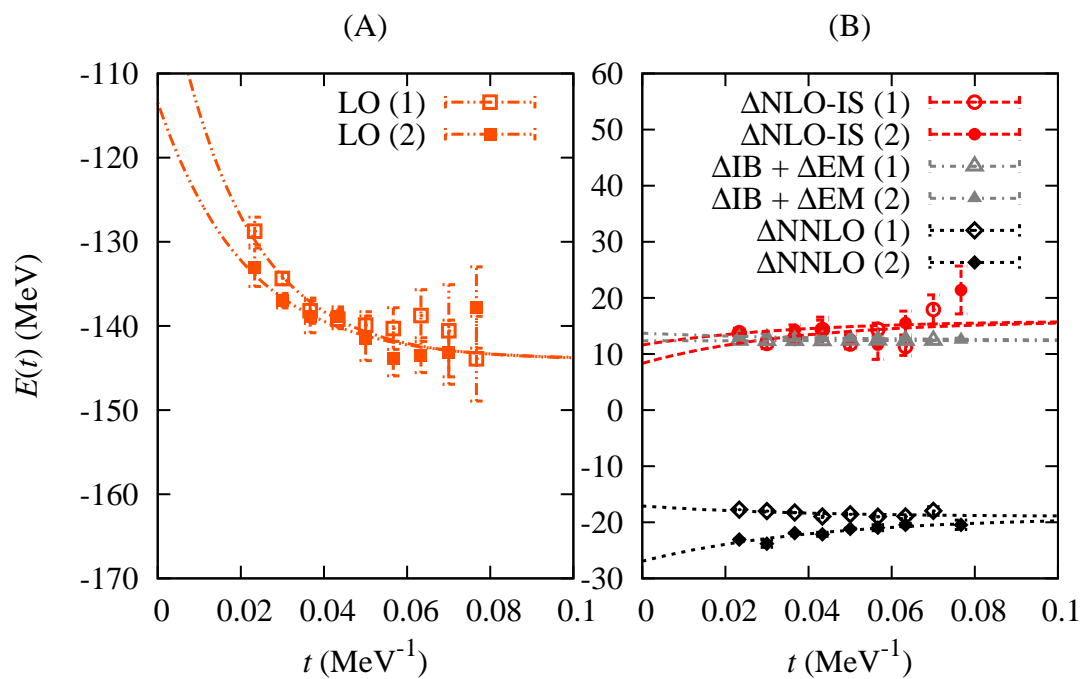
$L = 11.8 \text{ fm}$



LO ( $\mathcal{O}(Q^0)$ )	-96(2) MeV
NLO ( $\mathcal{O}(Q^2)$ )	-77(3) MeV
NNLO ( $\mathcal{O}(Q^3)$ )	-92(3) MeV
Exp.	-92.2 MeV

# Ground state of $^{16}\text{O}$

$L = 11.8 \text{ fm}$



to be published

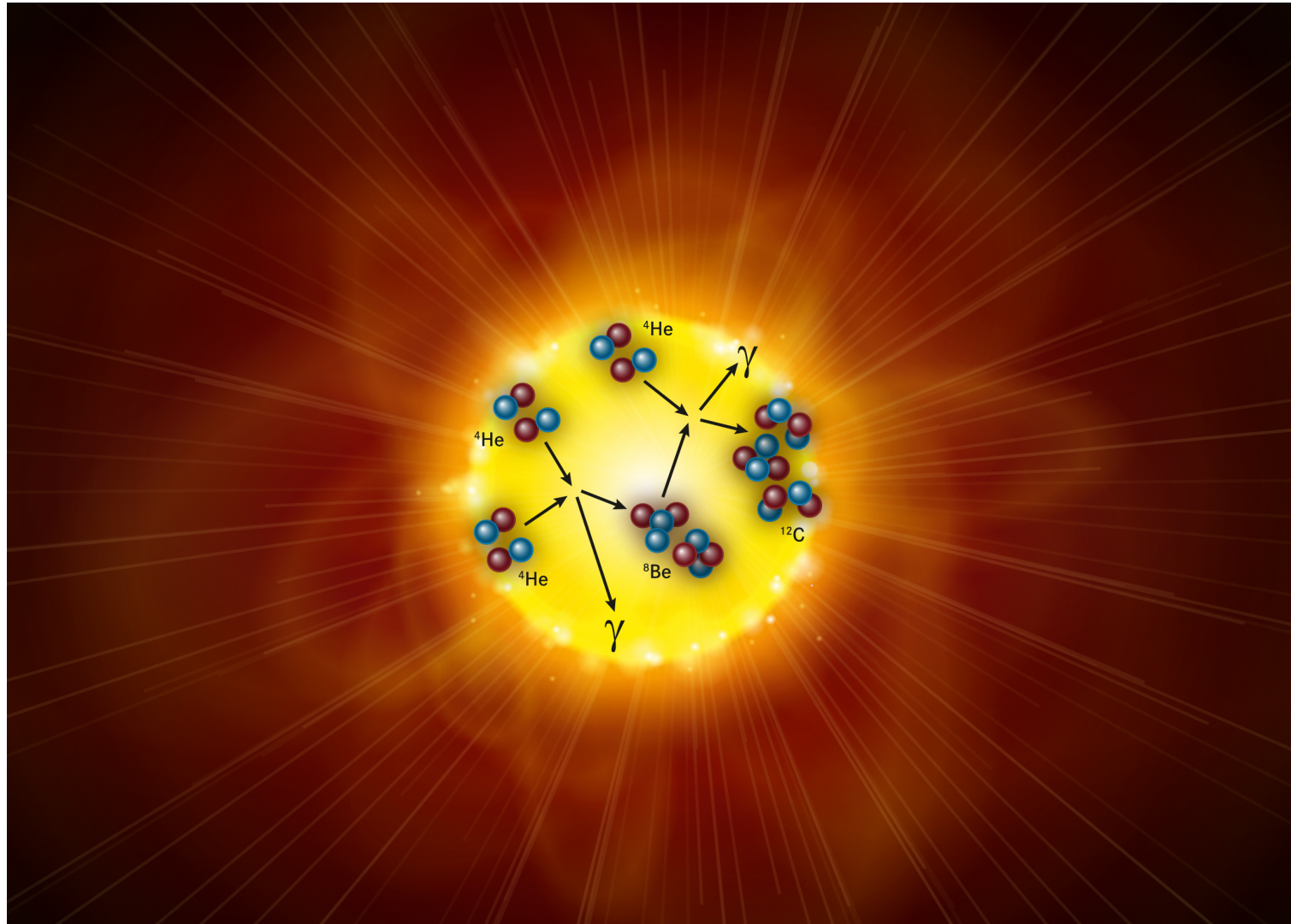
LO ( $\mathcal{O}(Q^0)$ )	-144(4) MeV
NLO ( $\mathcal{O}(Q^2)$ )	-116(6) MeV
NNLO ( $\mathcal{O}(Q^3)$ )	-135(6) MeV
Exp.	-127.6 MeV

# SPECTRUM OF $^{12}\text{C}$ & the HOYLE STATE

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501

Viewpoint: Hjorth-Jensen, Physics **4** (2011) 38

Epelbaum, Krebs, Lähde, Lee, UGM, arxiv:1208.1328 (numbers from this ref.)



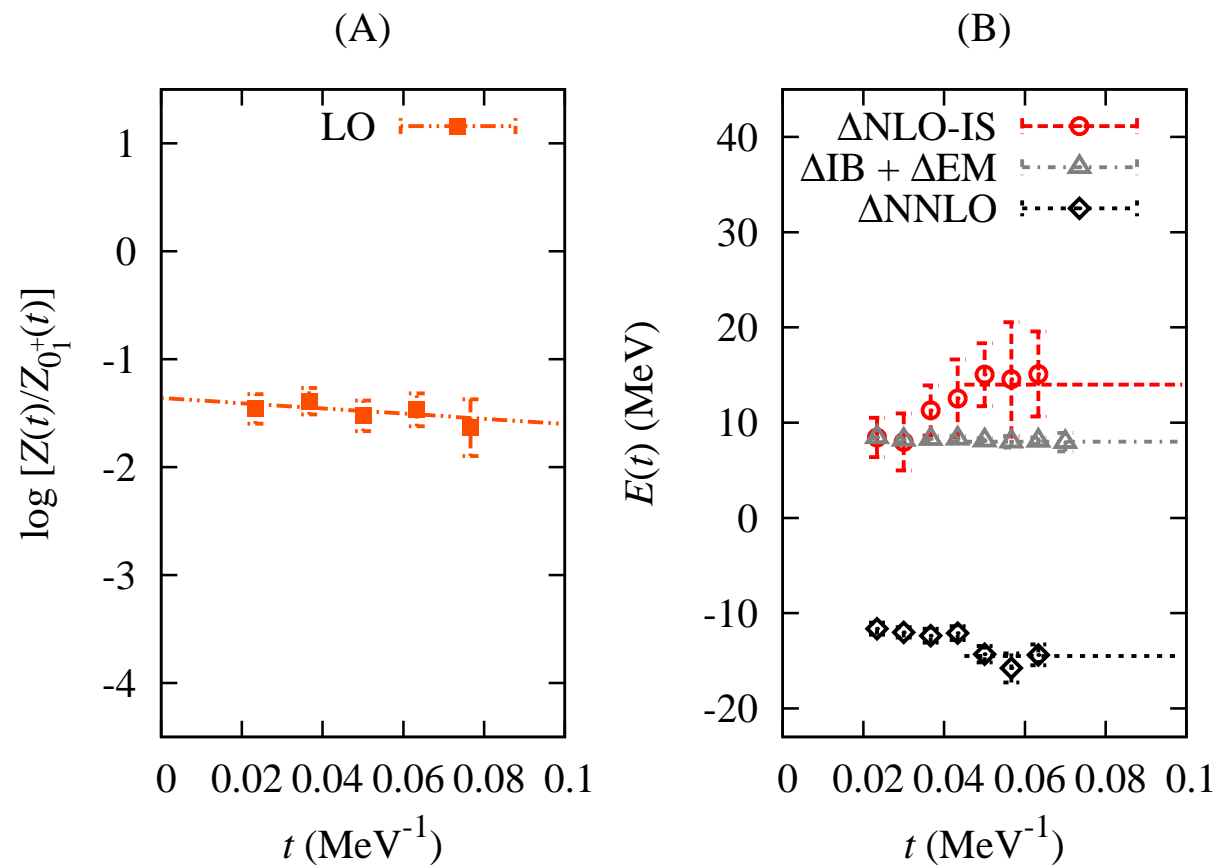


# EXCITED STATES of $^{12}\text{C}$

- Lowest excited state is  $2_1^+$  (as in nature)

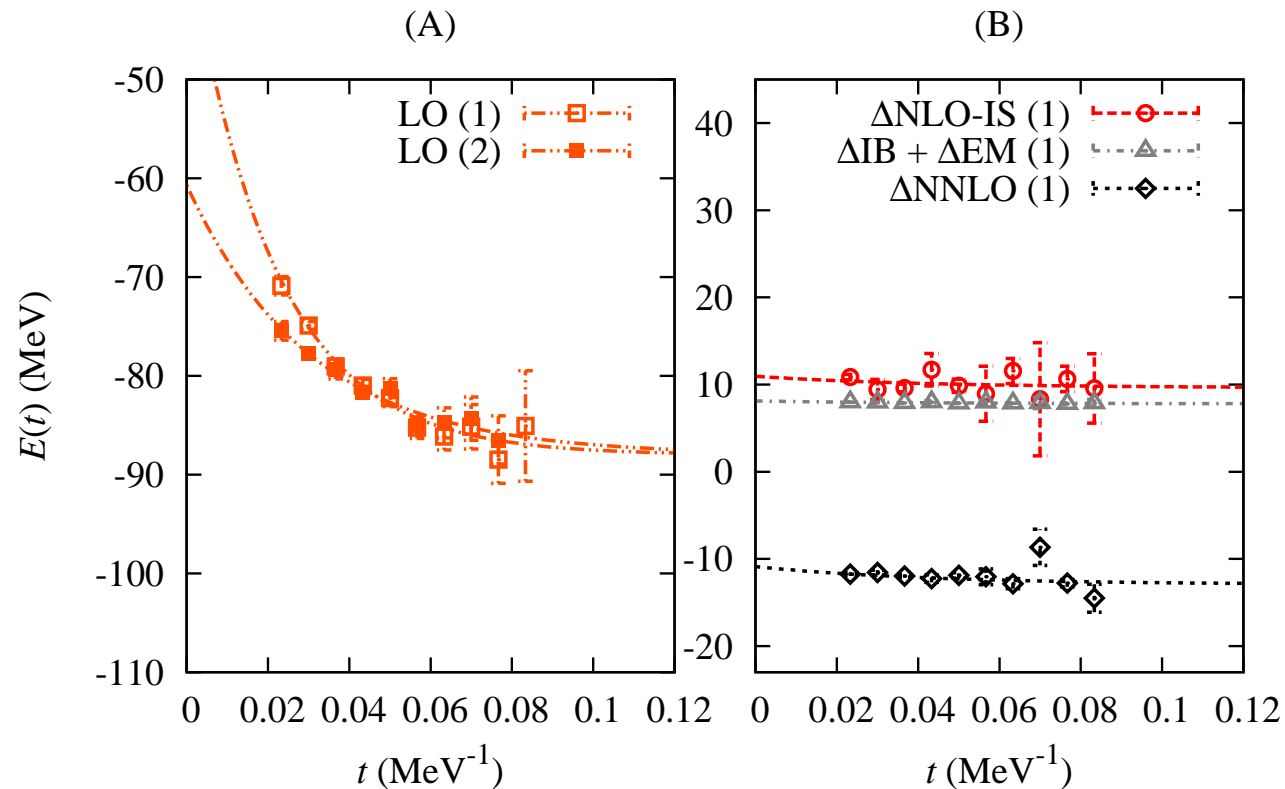
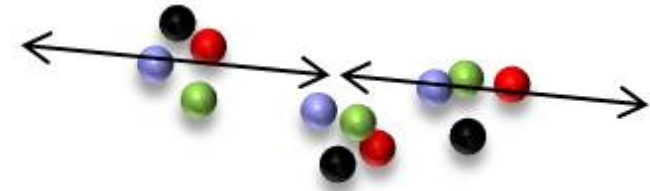
$$E(2_1^+) = -89(3) \text{ MeV}$$

$$[-87.7 \text{ MeV}]$$



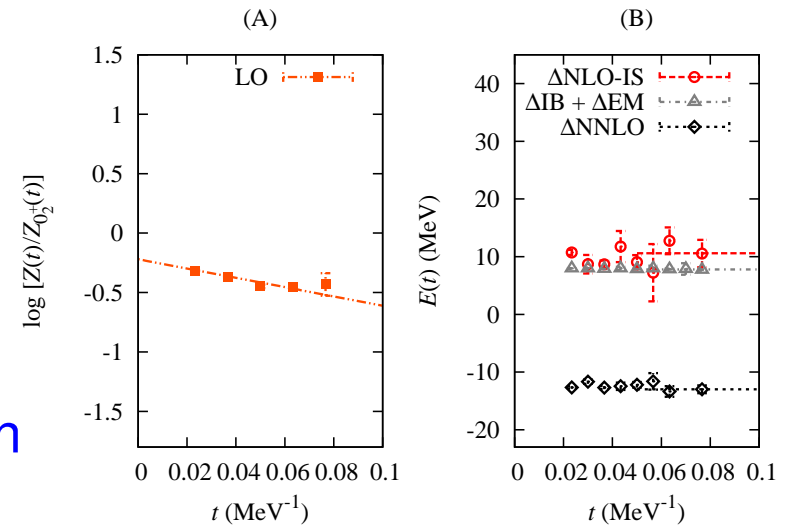
# THE HOYLE STATE ( $0_2^+$ )

- energy:  $E(0_2^+) = -85(3)$  MeV
- close to  $E(^4\text{He}) + E(^8\text{Be}) = -83.3(2.0)$  MeV
- structure: “bent” alpha-chain like (not “BEC”)



# A HOYLE STATE EXCITATION ( $2_2^+$ )

- a  $2^+$  state 2 MeV above the Hoyle state
- interpretation:
  - a rotational band of the Hoyle state
  - generated from excitations of the alpha-chain



- what's in the data ?

a  $2^+$  state 3.51 MeV above the Hoyle state seen in  $^{11}\text{B}(d, n)^{12}\text{C}$   
not included in the level scheme!

Ajzenberg-Selove, Nucl. Phys. A506 (1990) 1

a  $2^+$  state 3.8(4) MeV above the Hoyle state seen in  $^{12}\text{C}(\alpha, \alpha)^{12}\text{C}$

Bency John et al., Phys. Rev. C 68 (2003) 014305

- and much more, see next slide and: → talk by Henry Weller

⇒ ab initio prediction requires experimental confirmation

# SPECTRUM OF $^{12}\text{C}$

- Summarizing the results for carbon-12:

	$0_1^+$	$2_1^+$	$0_2^+$	$2_2^+$
LO	−96(2) MeV	−94(2) MeV	−89(2) MeV	−88(2) MeV
NLO	−77(3) MeV	−74(3) MeV	−72(3) MeV	−70(3) MeV
NNLO	−92(3) MeV	−89(3) MeV	−85(3) MeV	−83(3) MeV
Exp.	−92.16 MeV	−87.72 MeV	−84.51 MeV	−82.6(1) MeV [1,2] −82.32(6) MeV [3] −81.1(3) MeV [4] −82.13(11) MeV [5]

- [1] Freer et al., Phys. Rev. C 80 (2009) 041303
- [2] Zimmermann et al., Phys. Rev. C 84 (2011) 027304
- [3] Hyldegaard et al., Phys. Rev. C 81 (2010) 024303
- [4] Itoh et al., Phys. Rev. C 84 (2011) 054308
- [5] Weller et al., in preparation

- importance of consistent 2N & 3N forces
- good agreement w/ experiment, can be improved

# Testing the Anthropic Principle

# MC ANALYSIS of the AP

- consider QCD only → calculate  $\partial\Delta E/\partial M_\pi$
- relevant quantities (energy differences)

$$\Delta E_h \equiv E_{12}^* - E_8 - E_4, \quad \Delta E_b \equiv E_8 - 2E_4 \quad \Delta E_c \equiv E_{12}^* - E_{12}$$

- energy differences depend on parameters of QCD (LO analysis)

$$E_i = E_i \left( M_\pi^{\text{OPE}}, m_N(M_\pi), \tilde{g}_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi) \right)$$

$$\tilde{g}_{\pi N} \equiv \frac{g_A}{2F_\pi}$$

- remember:  $M_{\pi^\pm}^2 \sim (m_u + m_d)$

⇒ quark mass dependence  $\equiv$  pion mass dependence

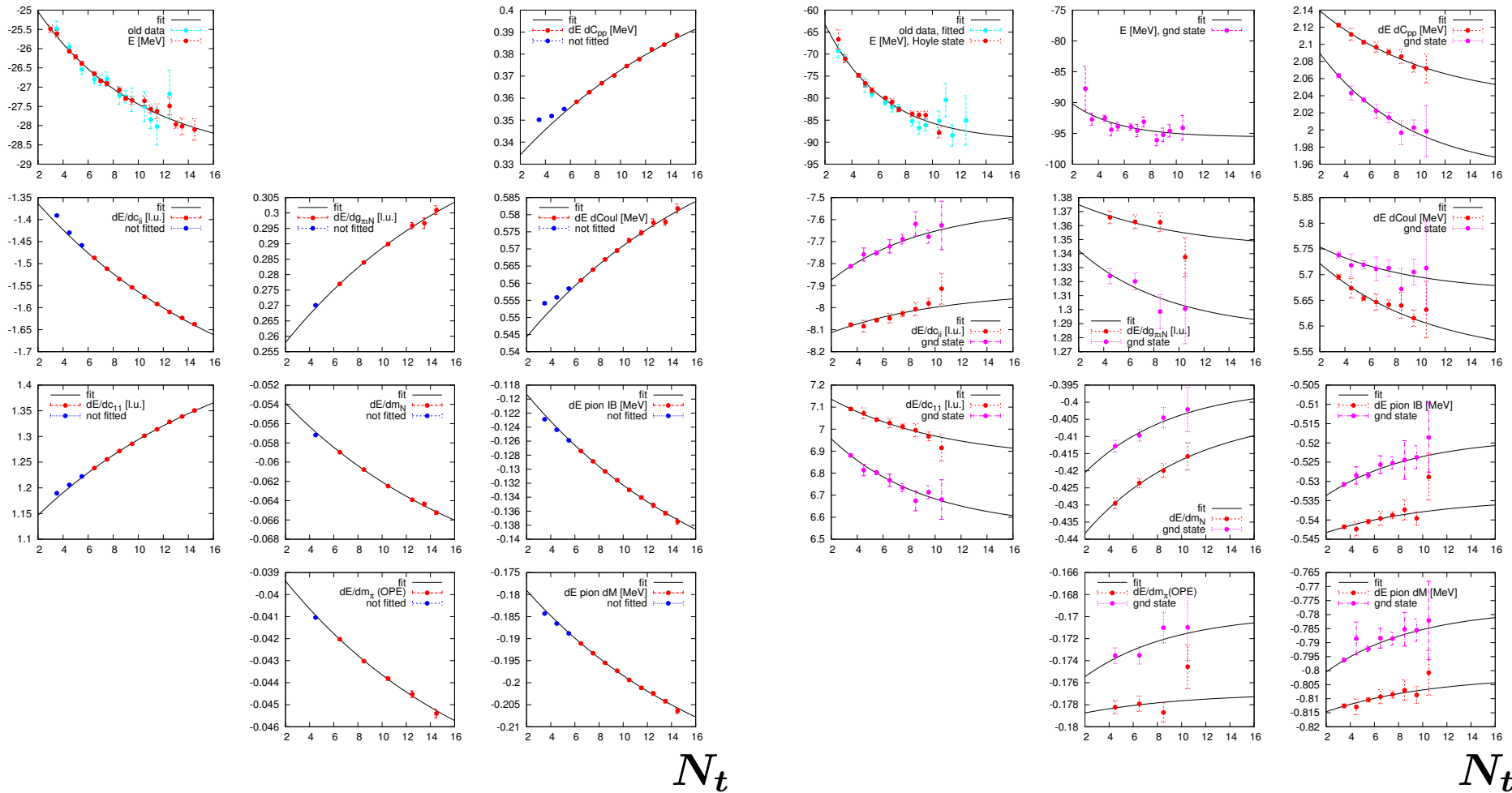


# AFQMC RESULTS for the DERIVATIVES

●  ${}^4\text{He}$

●  ${}^{12}\text{C}(0_2^+)$

$$E(N_t) = E(\infty) + \text{const} \exp(-N_t/\tau)$$





# DETERMINATION of the $x_i$

- $x_1$  from the quark mass expansion of the nucleon mass:  $x_1 \simeq 0.8 \pm 0.2$
- $x_2$  from the quark mass expansion of the pion decay constant and the nucleon axial-vector constant:  $x_2 \simeq -0.056 \dots 0.008$
- $x_3$  and  $x_4$  can be obtained from a two-nucleon scattering analysis & can be deduced from:

$$-\frac{\partial a^{-1}}{\partial M_\pi} \equiv \frac{A}{aM_\pi} = \frac{1}{\pi L} S'(\eta) \frac{\partial \eta}{\partial M_\pi}, \quad \eta \equiv m_N E \left( \frac{L}{2\pi} \right)^2$$

⇒ while this can straightforwardly be computed, we prefer to use a representation that substitutes  $x_3$  and  $x_4$  by:

$$\left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}}, \quad \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}}$$

⇒ we are ready to study the pertinent energy differences

# RESULTS

- putting pieces together:

$$\left. \frac{\partial \Delta E_h}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} = -0.455(35) \left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} - 0.744(24) \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} + 0.056(10)$$

$$\left. \frac{\partial \Delta E_b}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} = -0.117(34) \left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} - 0.189(24) \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} + 0.012(9)$$

$$\left. \frac{\partial \Delta E_c}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} = -0.07(3) \left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} - 0.14(2) \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} + 0.017(9)$$

- $x_1$  and  $x_2$  only affect the small constant terms
- also calculated the shifts of the individual energies (not shown here)

# INTERPRETATION

- $(\partial\Delta E_h/\partial M_\pi)/(\partial\Delta E_b/\partial M_\pi) \simeq 4$   
 $\Rightarrow \Delta E_h$  and  $\Delta E_b$  cannot be independently fine-tuned
- Within error bars,  $\partial\Delta E_h/\partial M_\pi$  &  $\partial\Delta E_b/\partial M_\pi$  appear unaffected by the choice of  $x_1$  and  $x_2 \rightarrow$  indication for  $\alpha$ -clustering
- For  $\Delta E_h$  &  $\Delta E_b$ , the dependence on  $M_\pi$  is small when

$$\partial a_s^{-1}/\partial M_\pi \simeq -1.6 \times \partial a_t^{-1}/\partial M_\pi$$

- the triple alpha process is controlled by :

$$\Delta E_{h+b} \equiv \Delta E_h + \Delta E_b = E_{12}^* - 3E_4$$

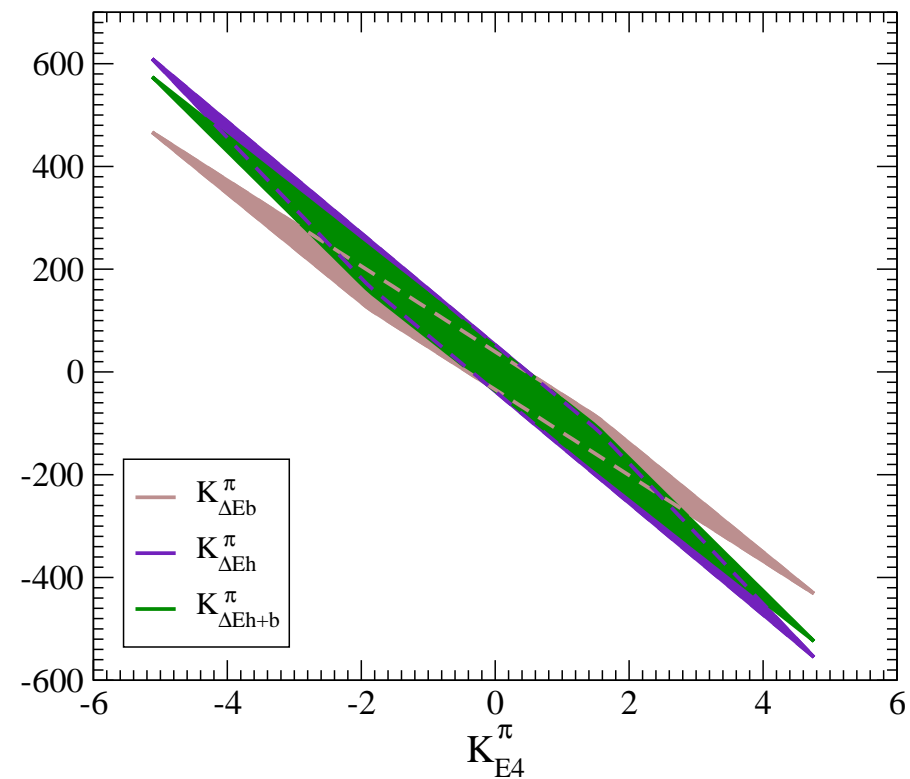
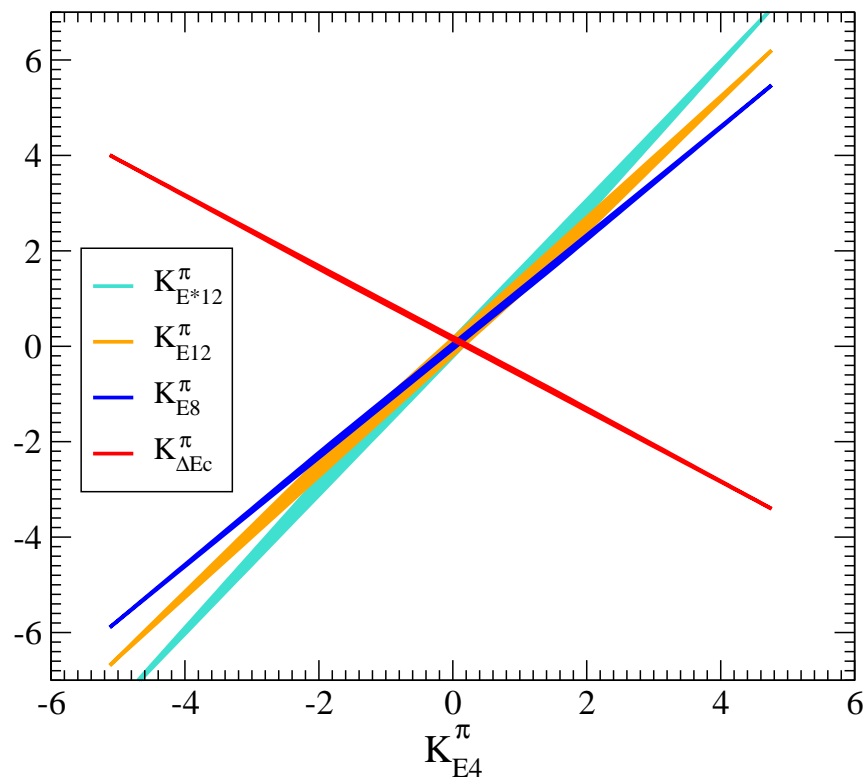
$$\left. \frac{\partial\Delta E_{h+b}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} = -0.571(14) \left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} - 0.934(11) \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} + 0.069(6)$$

$\Rightarrow$  so what can we say about the quark mass dependence of the scattering lengths?



# CORRELATIONS

- vary the quark mass derivatives of  $a_{s,t}^{-1}$  within  $-1, \dots, +1$ :



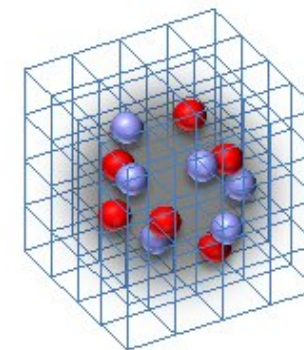
- clear correlations:  $\alpha$ -particle BE and the energies/energy differences

⇒ anthropic or non-anthropoc scenario depends on whether the  ${}^4\text{He}$  BE moves!



# SUMMARY & OUTLOOK

- Nuclear lattice simulations as a new quantum many-body approach
- Formulate continuum EFT on space-time lattice  $V = L_s \times L_s \times L_s \times L_t$
- New method to extract phase shifts & mixing angles
- Fix parameters in few-nucleon systems  $\rightarrow$  predictions
- Promising results for  $A = 2, 3, 4, 8, 12, 16$  at NNLO
- $^{12}\text{C}$  spectrum at NNLO  $\rightarrow$  **Hoyle state** &  $2^+$  excitation
- First ever ab initio MC calculation of  $^{16}\text{O}$
- Testing the anthropic principle  $\rightarrow$  strong correlations of  $\alpha$ -cluster type  
 $\Rightarrow$  the Hoyle state does not appear anthropic (Coulomb to be done)



$\Rightarrow$  **larger  $A$  and higher precision**

