

Quantum Monte Carlo study of the Hyperon-Nucleon interaction

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- ★ P. Armani (Trento, Italy)
- ★ S. Gandolfi (LANL, US-NM)
- ★ K. E. Schmidt (ASU, US-AZ)
- ★ G. Co' (Lecce, Italy)

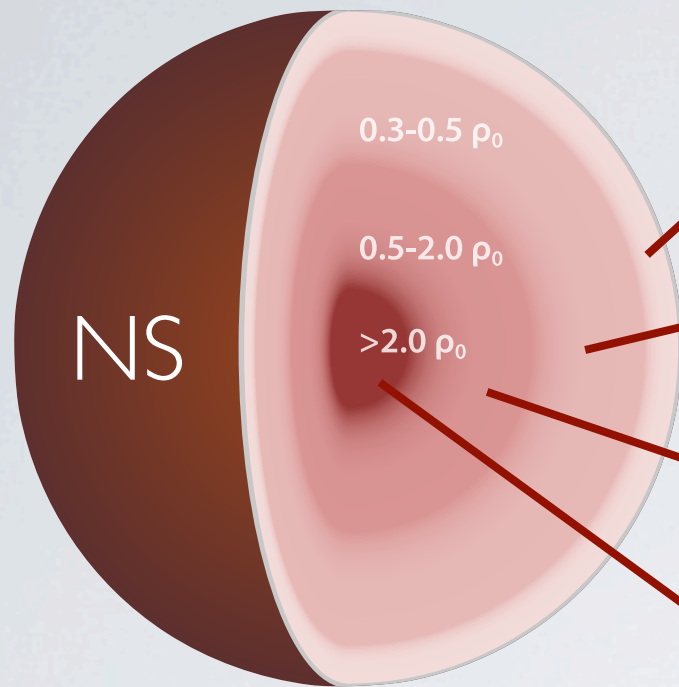


INT, Seattle - October 25, 2012

Outline

- ✓ Motivations:
 - theoretical & experimental interest
 - the idea of the project
- ✓ The method: Auxiliary Field Diffusion Monte Carlo
- ✓ The interaction: Usmani ΛN & ΛNN
- ✓ Results: B_Λ , ρ_N , ρ_Λ
- ✓ Conclusions & Perspectives

Motivations: theoretical interest



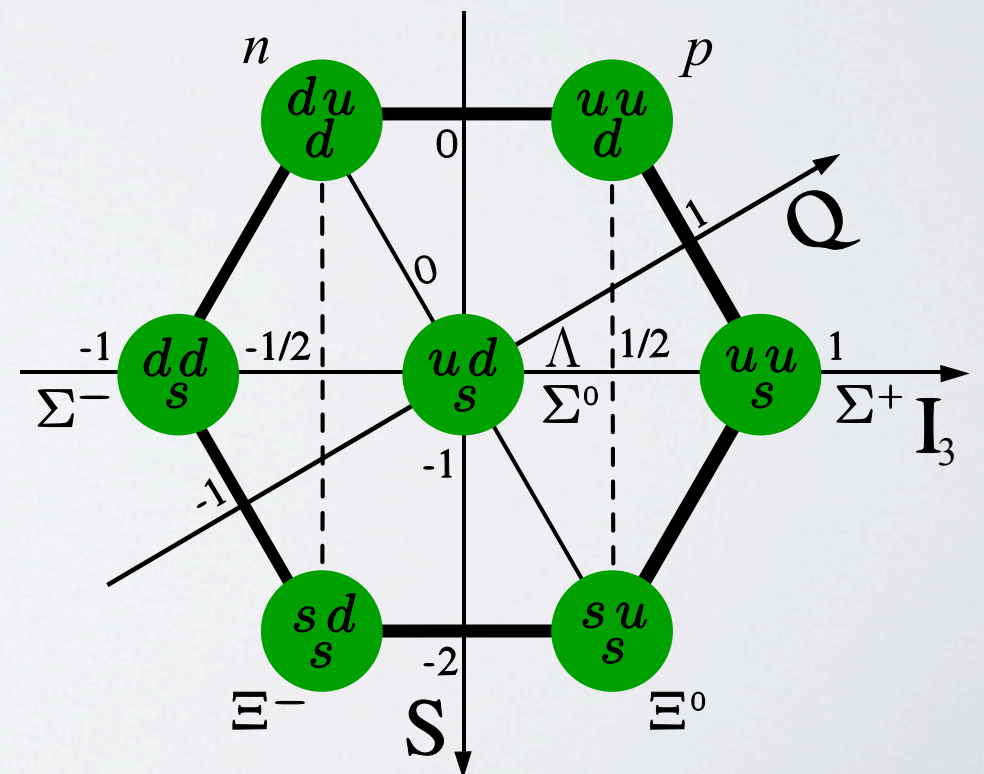
$R \sim 10 \text{ km}$
 $M \sim 1.4 M_{\odot}$

outer crust: $Z e$
 (0.3 ÷ 0.5 km)

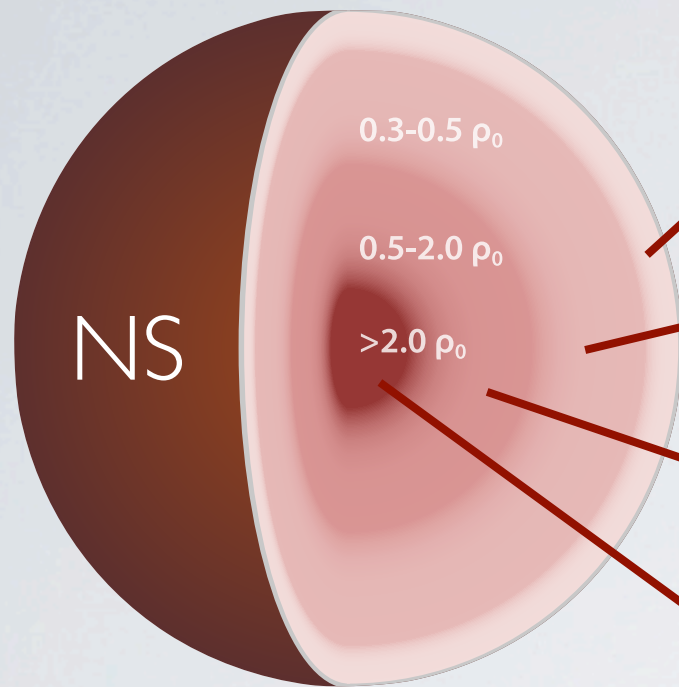
inner crust: $Z n e$
 (1 ÷ 2 km)

outer core: $n p e \mu$
 (~ 9 km)

inner core: $n p e \mu \Lambda \Sigma \Xi \pi_c K_c q_p ?$



Motivations: theoretical interest



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inner crust: $Z n e$
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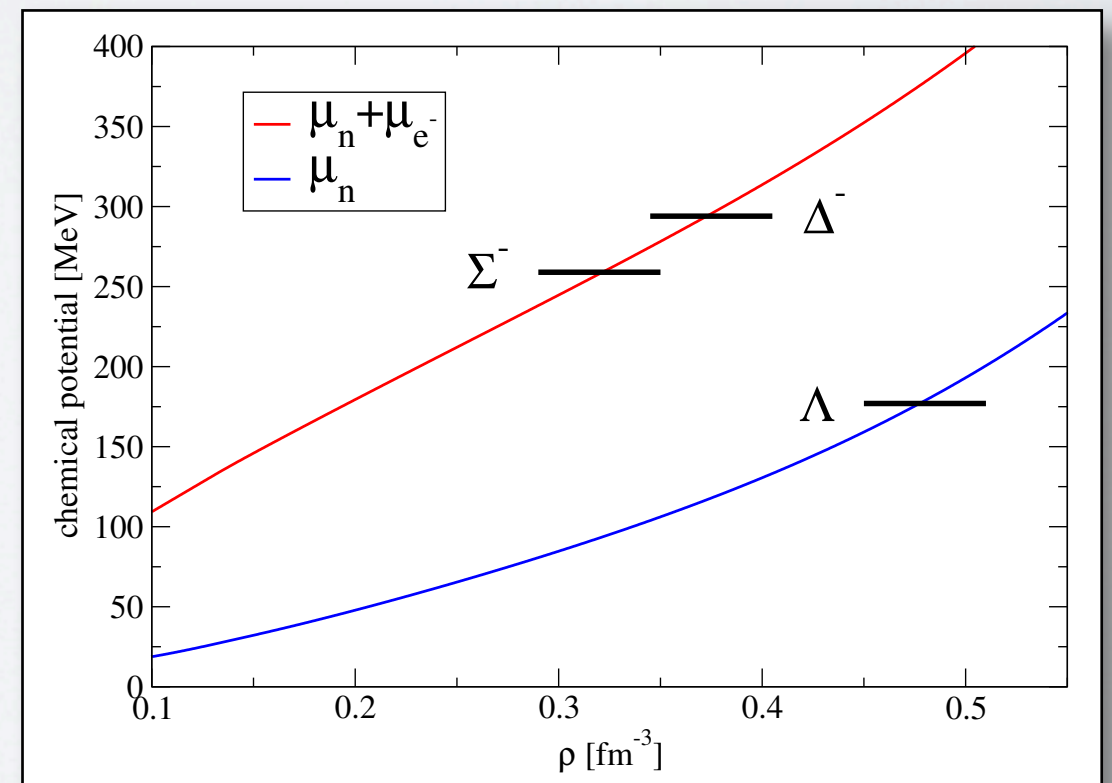
outer core: $n p e \mu$
 ($\sim 9 \text{ km}$)

inner core: $n p e \mu \Lambda \Sigma \Xi \pi_c K_c q_p ?$

$$Q = -1 : \mu_{Y^-} = \mu_n + \mu_e$$

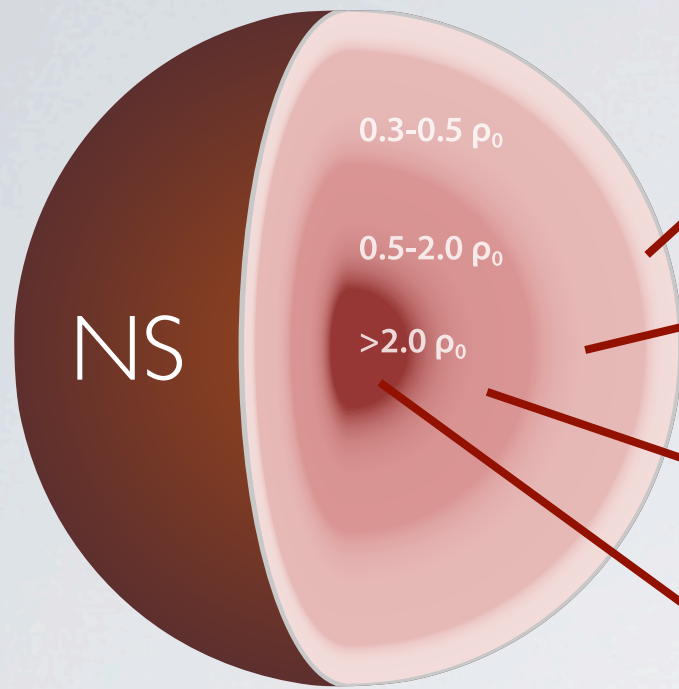
$$Q = 0 : \mu_{Y^0} = \mu_n$$

$$Q = +1 : \mu_{Y^+} = \mu_n - \mu_e$$



courtesy of Stefano Gandolfi

Motivations: theoretical interest



$R \sim 10 \text{ km}$
 $M \sim 1.4 M_{\odot}$

outer crust:
 (0.3 ÷ 0.5 km)

$Z e$

inner crust:
 (1 ÷ 2 km)

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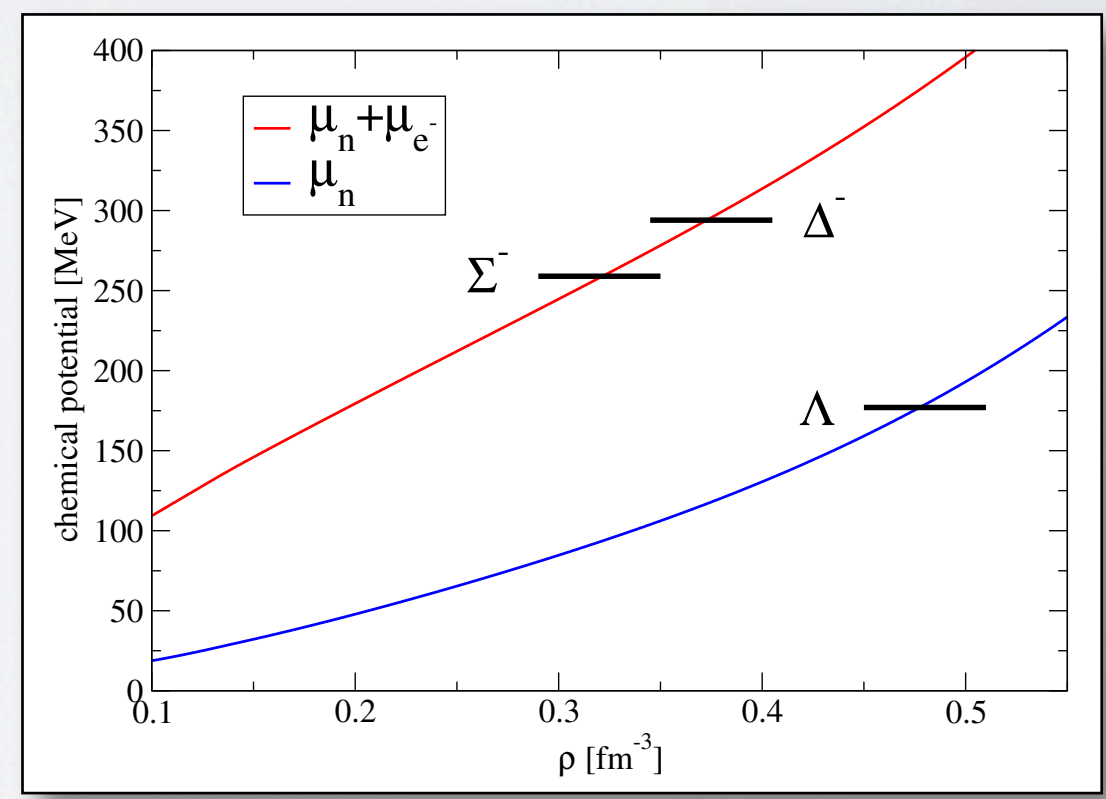
inner core:
 (0 ÷ 3 km)

$n p e \mu \Lambda \Sigma \Xi \pi_c K_c q_p ?$

composition strongly affects the properties of the neutron star



EOS & M(R) relation



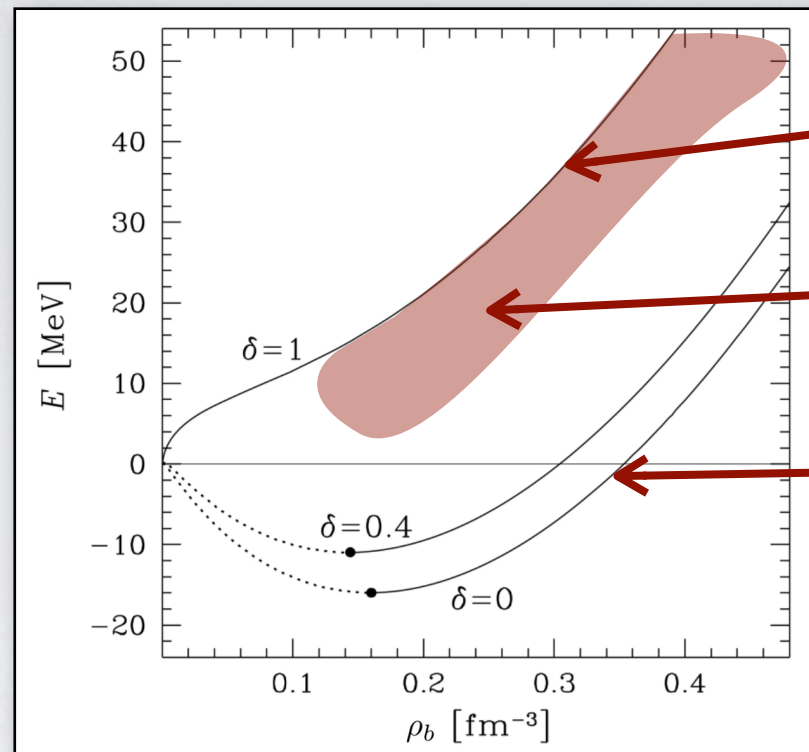
courtesy of Stefano Gandolfi

Motivations: theoretical interest

$$\begin{cases} E \equiv E(\rho_b, \delta) \\ P = \rho_b^2 \frac{\partial E(\rho_b, \delta)}{\partial \rho_b} \end{cases}$$

$$\rho_b = \rho_p + \rho_n = A/V$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_b}$$



pure n matter

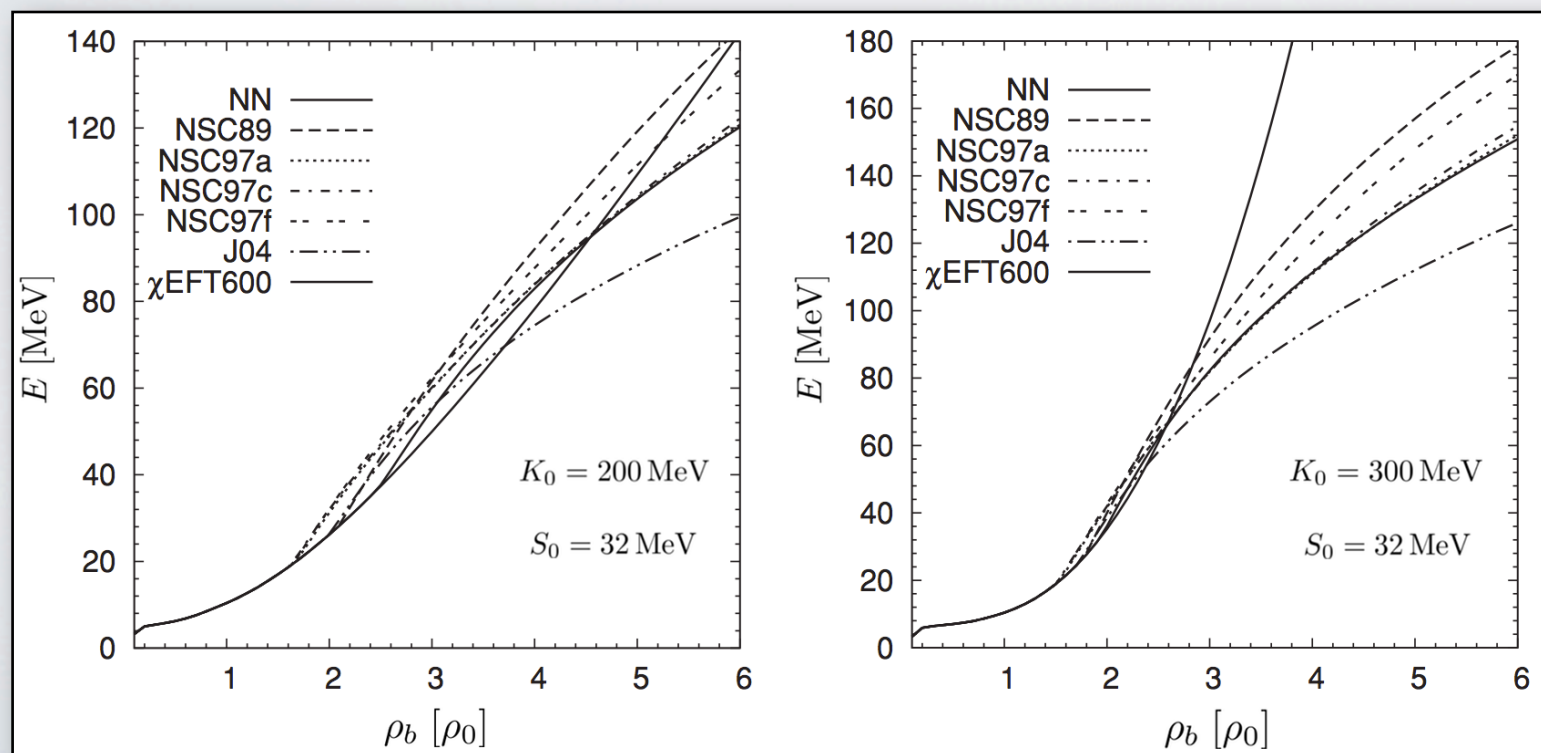
NS core

symmetric matter

$$\rho_0 = 0.16 \text{ fm}^{-3}$$

$$E_0 = -16 \text{ MeV}$$

P. Haensel, A.Y. Potekhin, D.G. Yakovlev,
Neutron Stars I, Springer 2007



hyperons: softening
of the EOS

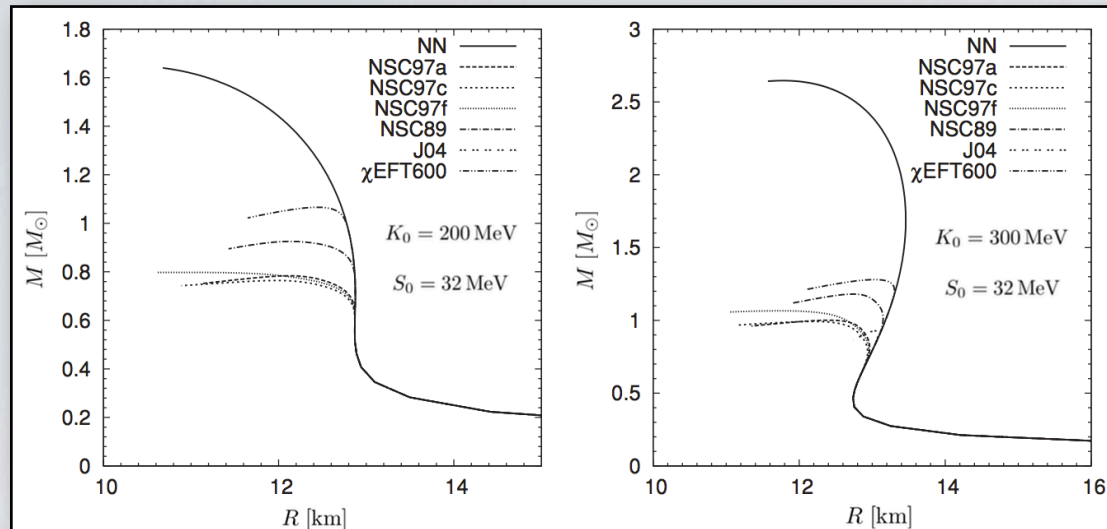


model
dependent

H. Dapo, B.-J. Schaefer, and J. Wambach, Phys. Rev. C 81 (2010) 035803

Motivations: theoretical interest

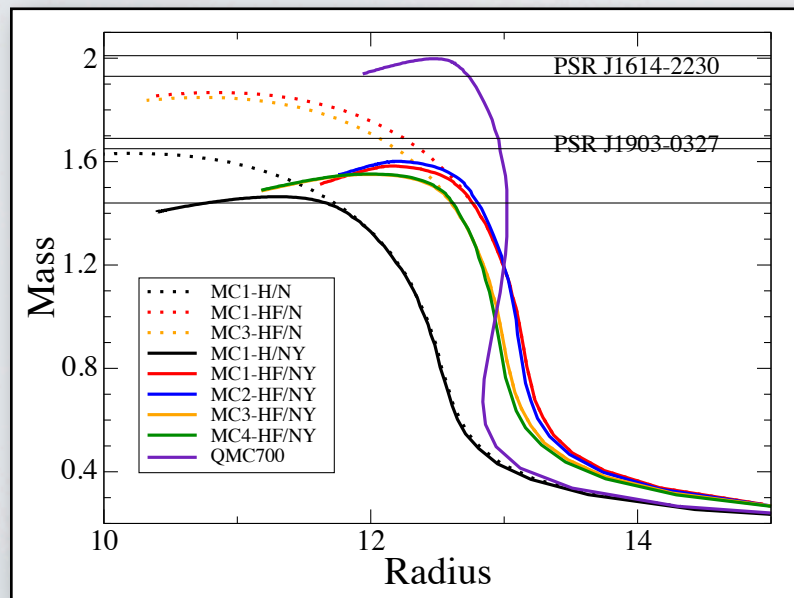
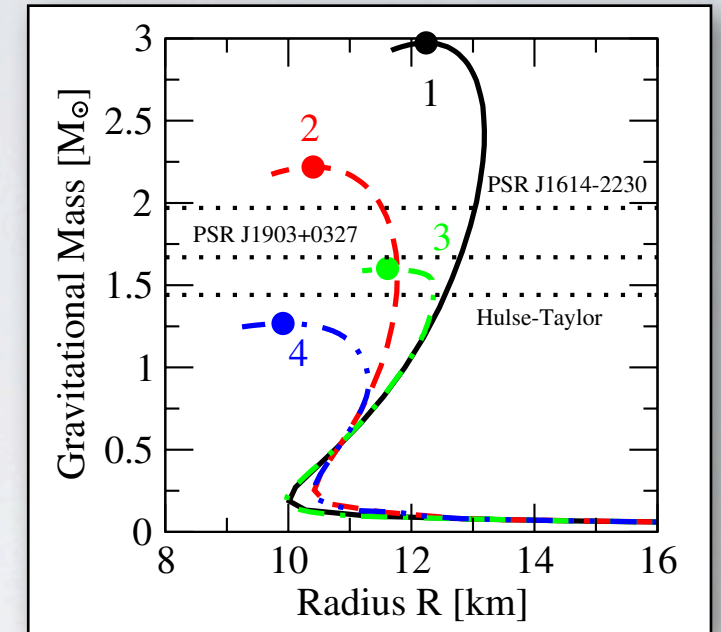
TOV equations \longrightarrow mass-radius relation & maximum mass



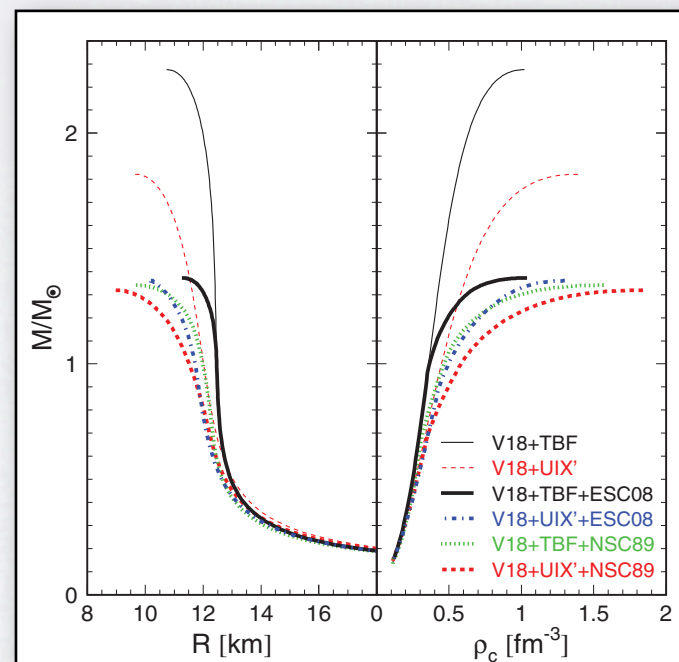
H. Ćapo, B.-J. Schaefer, J. Wambach,
Phys. Rev. C 81 (2010) 035803

I. Vidaña, D. Logoteta,
C. Providência,
A. Polls, I. Bombaci,
EPL 94 (2011) 11002

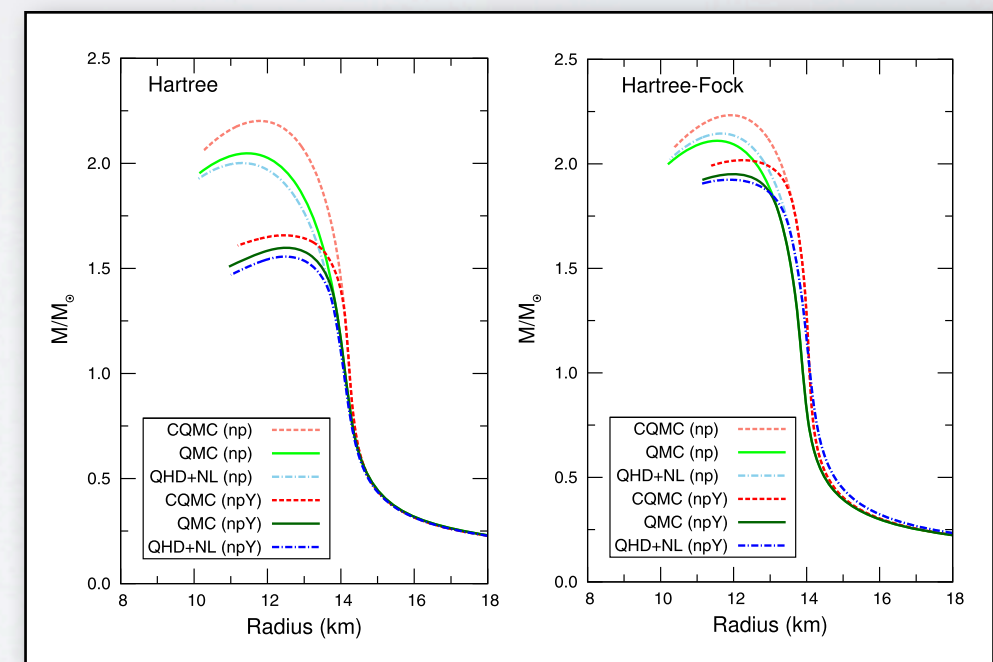
NS obs: $2 M_{\odot}$



É. Massot, J. Margueron, G. Chanfray,
EPL 97 (2012) 39002



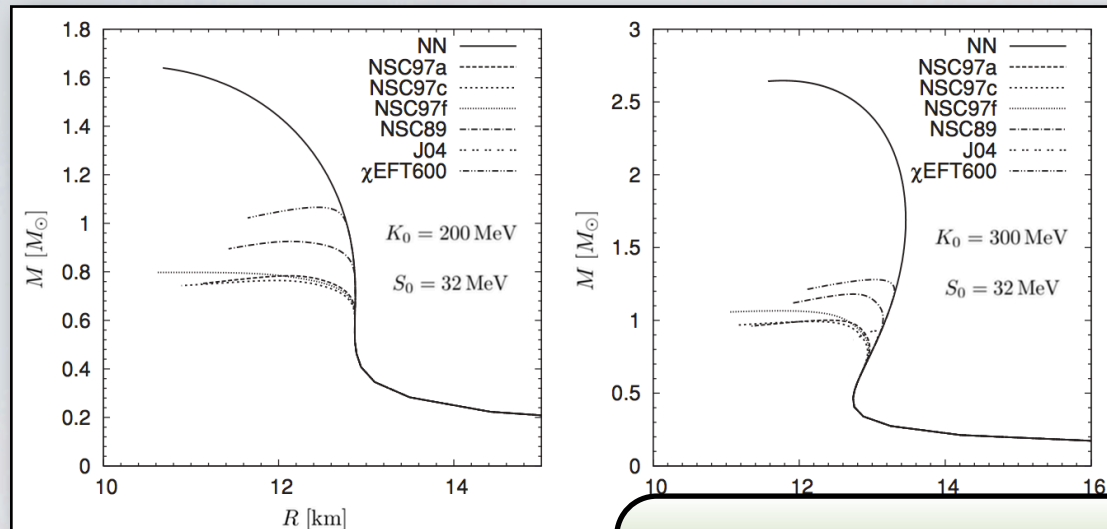
H.-J. Schulze, T. Rijken,
Phys. Rev. C 84 (2011) 035801



T. Miyatsu, T. Katayama, K. Saito,
Phys. Lett. B 709 (2012) 242-246

Motivations: theoretical interest

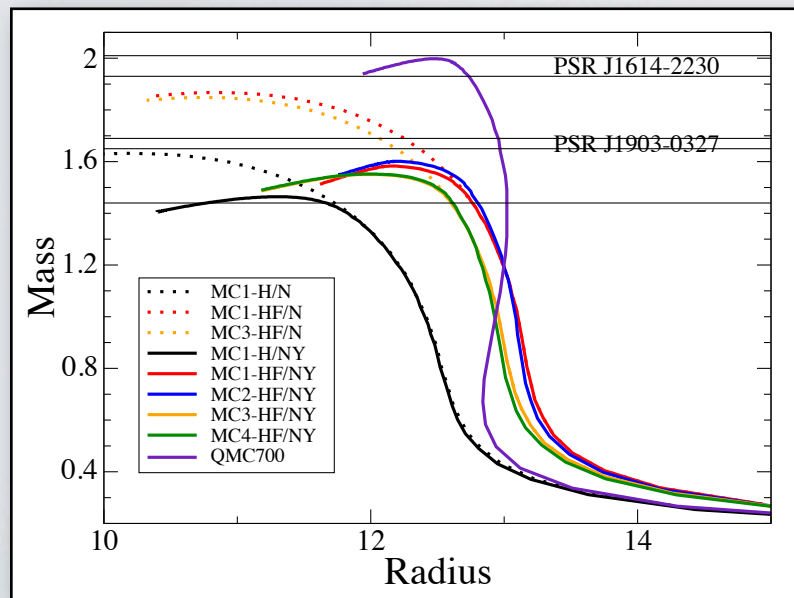
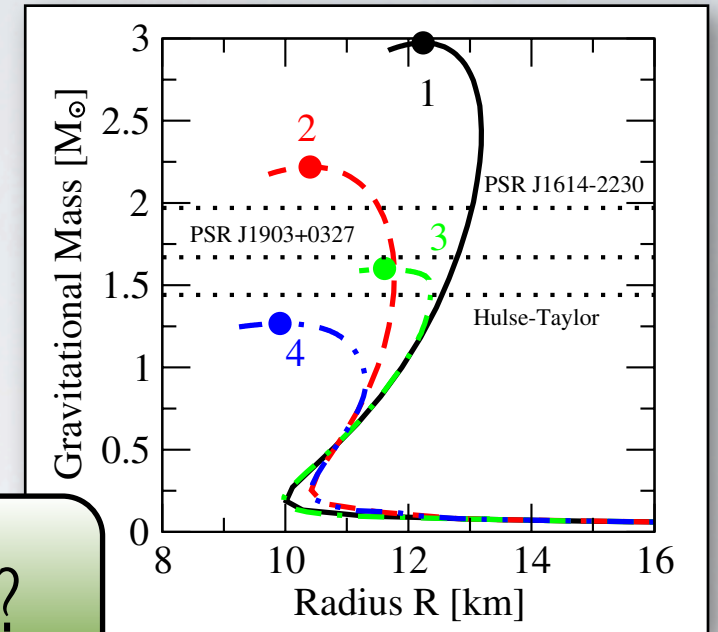
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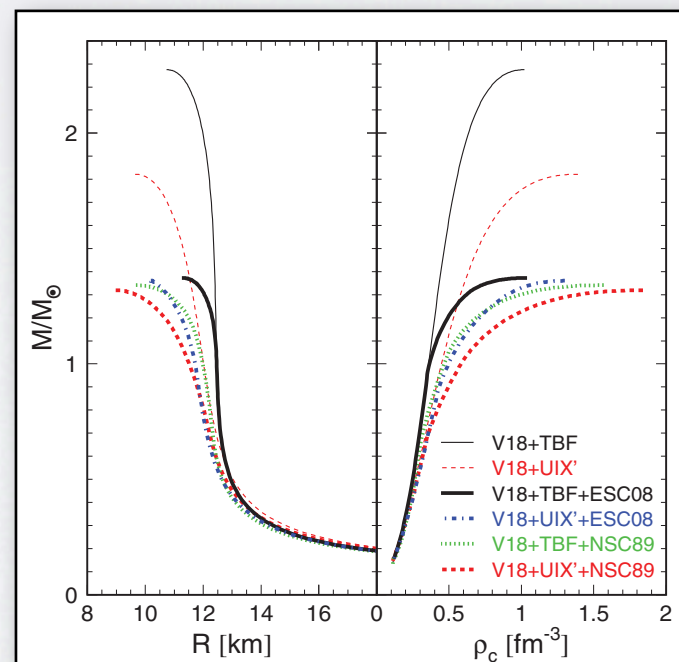
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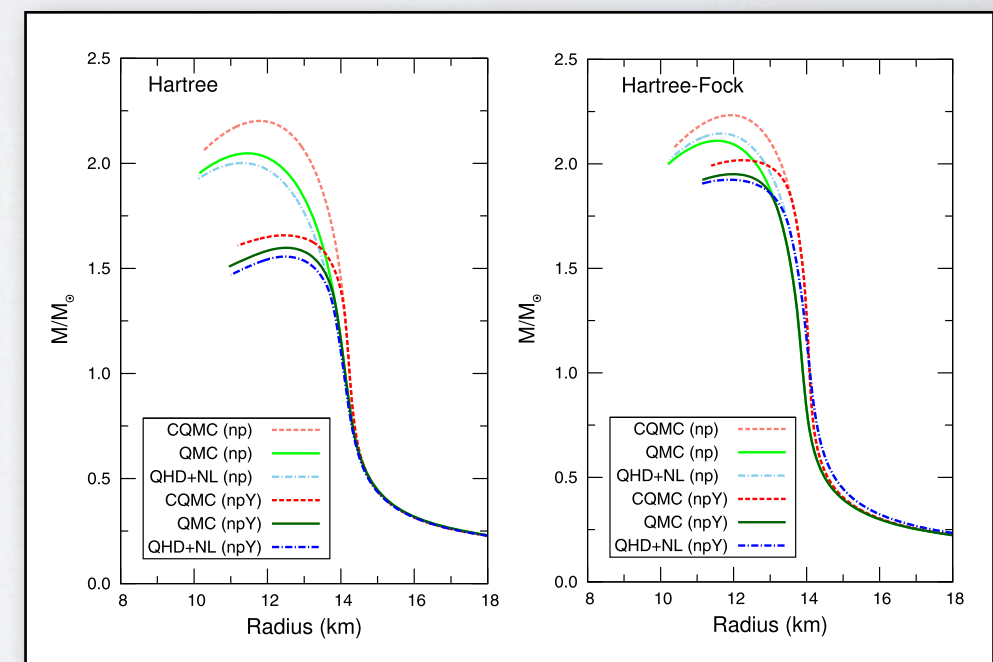
hyperon-nucleon interaction ?



É. Massot, J. Margueron, G. Chanfray,
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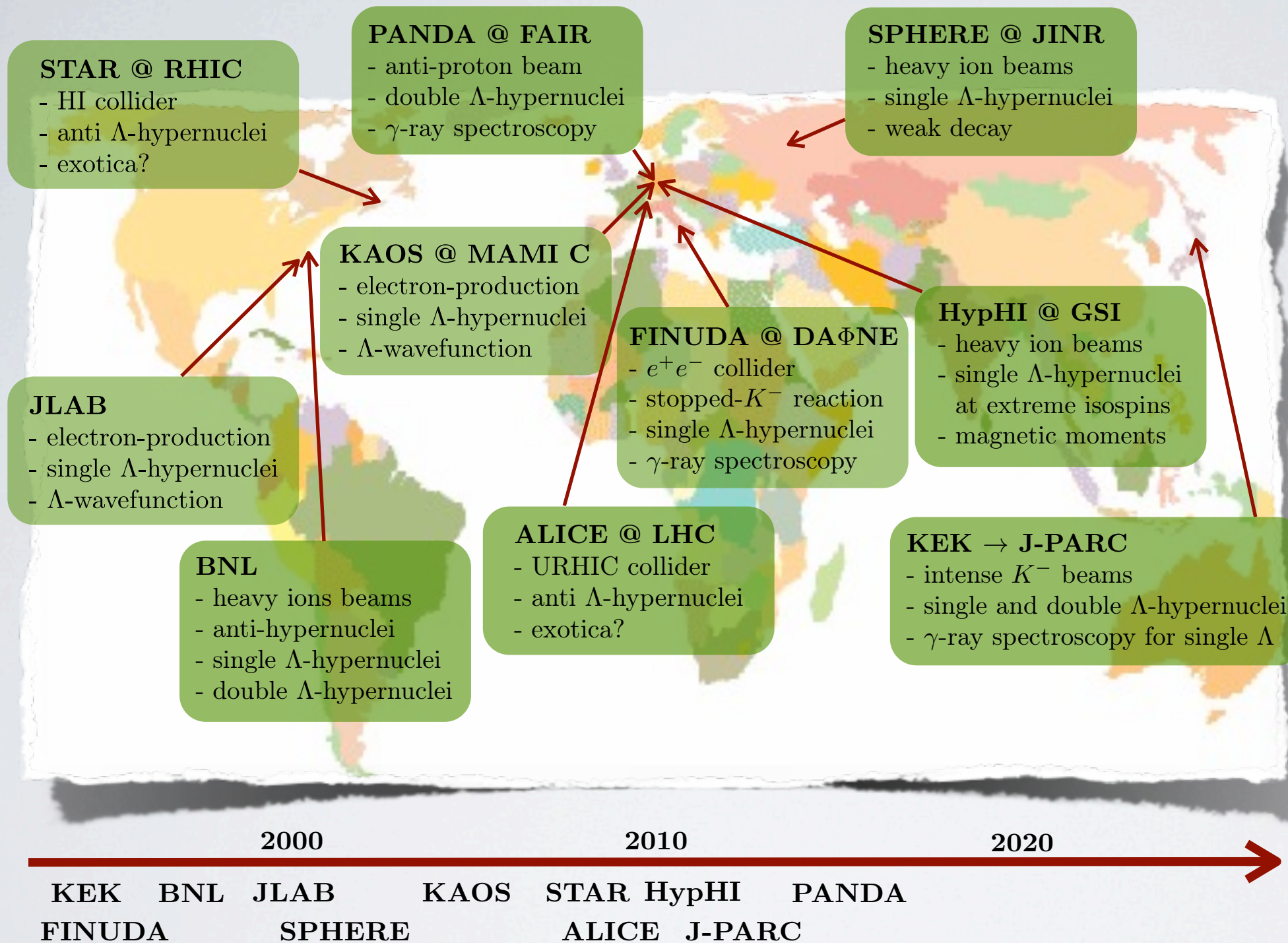


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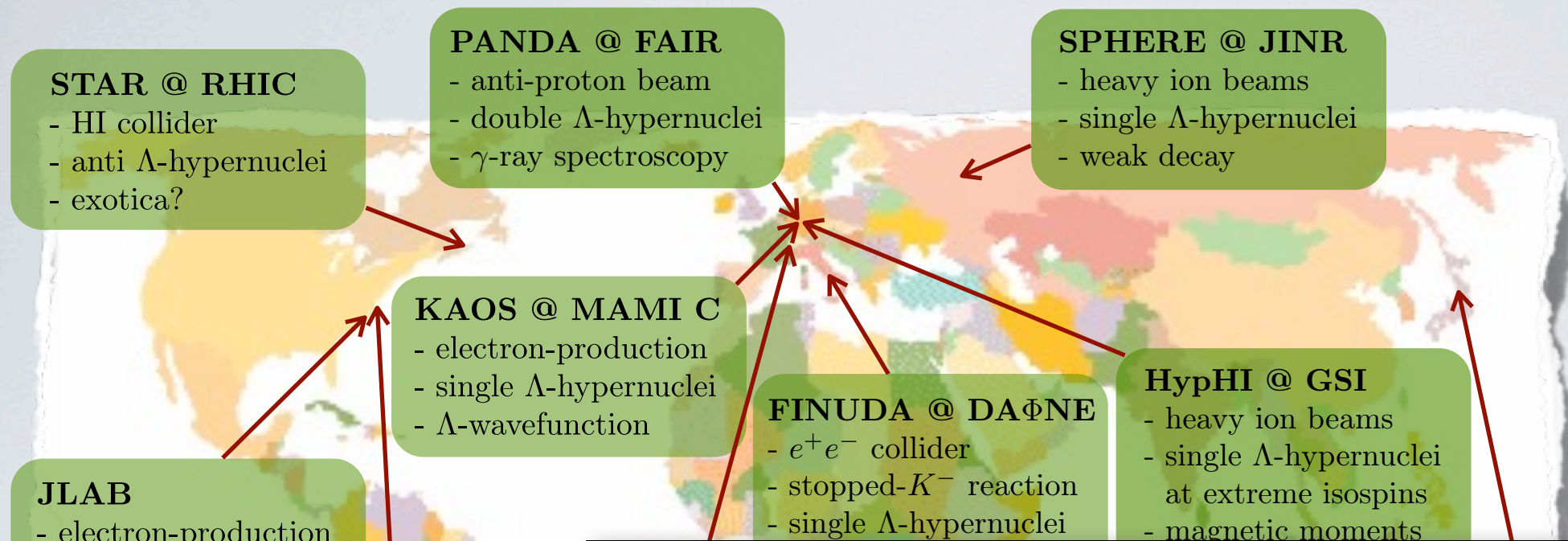


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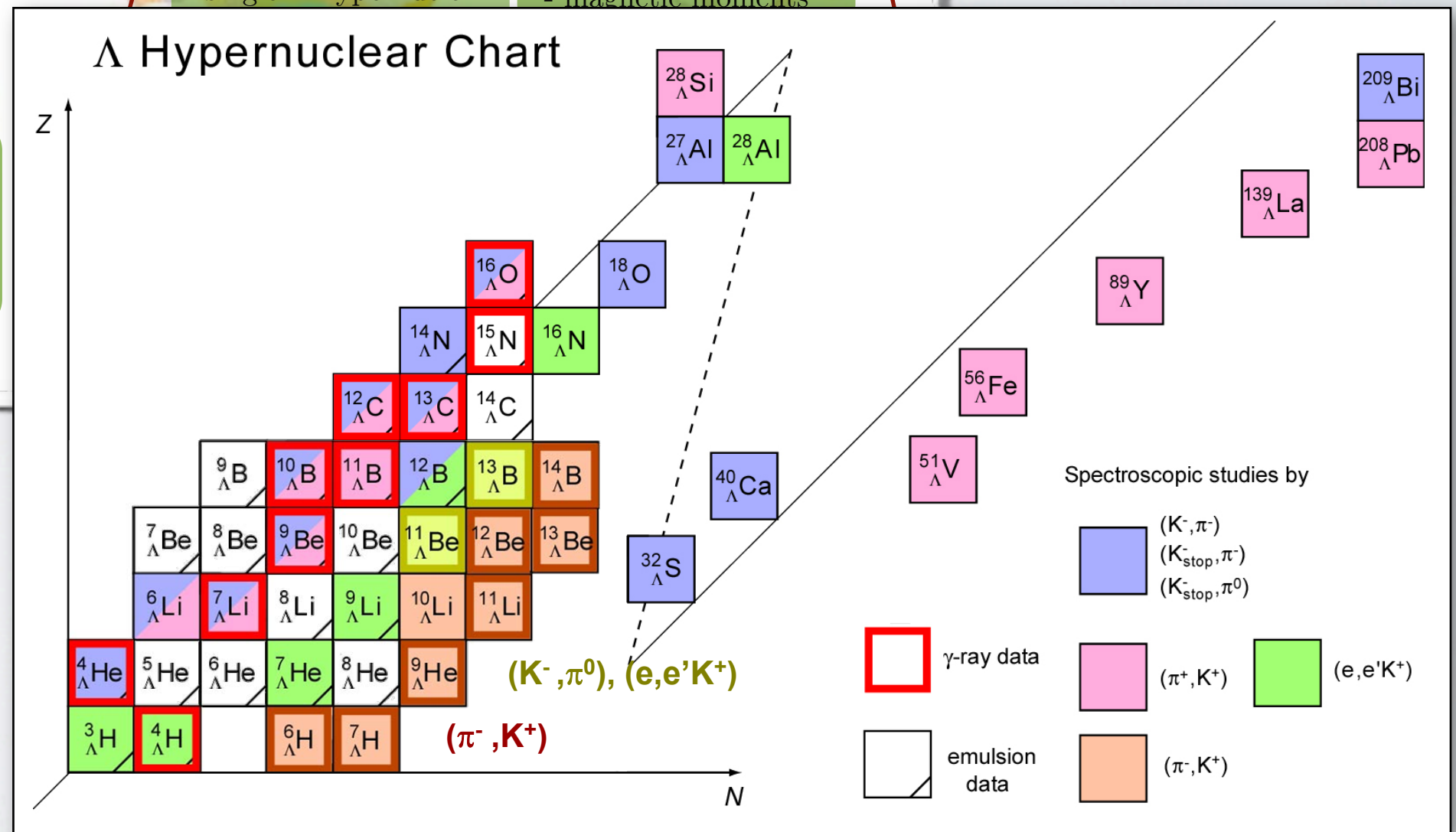
Motivations: experimental interest



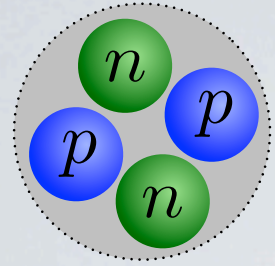
Motivations: experimental interest



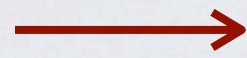
few data
 ↓
 interactions
 poorly known



Motivations: the idea

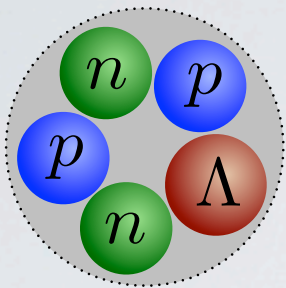


nucleus



$$BE_{nuc} = \frac{\langle \psi_{nuc} | \mathcal{H}_N | \psi_{nuc} \rangle}{\langle \psi_{nuc} | \psi_{nuc} \rangle}$$

ab-initio
method



Λ -hypernucleus



$$BE_{hyp} = \frac{\langle \psi_{hyp} | \mathcal{H}_{N+\Lambda} | \psi_{hyp} \rangle}{\langle \psi_{hyp} | \psi_{hyp} \rangle}$$



$$B_\Lambda = BE_{nuc} - BE_{hyp}$$

Hyp.: nuclear effects cancel at most

$$\mathcal{H}_N + \mathcal{H}_\Lambda$$



information about the hyperon-nucleon interaction

The method: Auxiliary Field DMC

stochastic ab-initio method with microscopic interaction

Auxiliary Field Diffusion Monte Carlo (AFDMC)

$$\tau = \frac{it}{\hbar} \quad \rightarrow \quad -\frac{\partial}{\partial \tau} \psi(\mathbf{R}, \mathbf{S}, \tau) = \mathcal{H} \psi(\mathbf{R}, \mathbf{S}, \tau)$$

$$\begin{aligned} \psi(\mathbf{R}, \mathbf{S}, \tau) &= e^{-(\mathcal{H}-E_0)\tau} \psi(\mathbf{R}, \mathbf{S}, 0) \\ &= e^{-(E_0-E_0)\tau} c_0 \varphi_0(\mathbf{R}, \mathbf{S}) \\ &\quad + \sum_{n>0} e^{-(E_n-E_0)\tau} c_n \varphi_n(\mathbf{R}, \mathbf{S}) \end{aligned}$$

↓ $\tau \rightarrow \infty$

$$c_0 \varphi_0(\mathbf{R}, \mathbf{S})$$

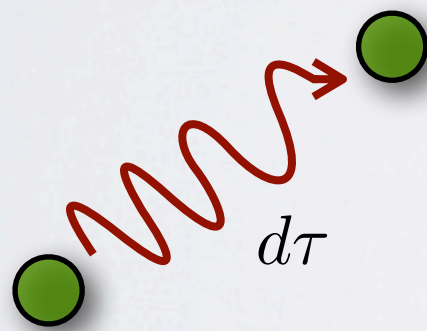
The method: Auxiliary Field DMC

$$\psi(\mathbf{R}, \mathbf{S}, \tau + d\tau) = \int \langle \mathbf{S} \mathbf{R} | e^{-(\mathcal{H} - E_0)d\tau} | \mathbf{R}' \mathbf{S}' \rangle \langle \mathbf{S}' \mathbf{R}' | \psi(\tau) \rangle d\mathbf{R}' d\mathbf{S}'$$

walkers

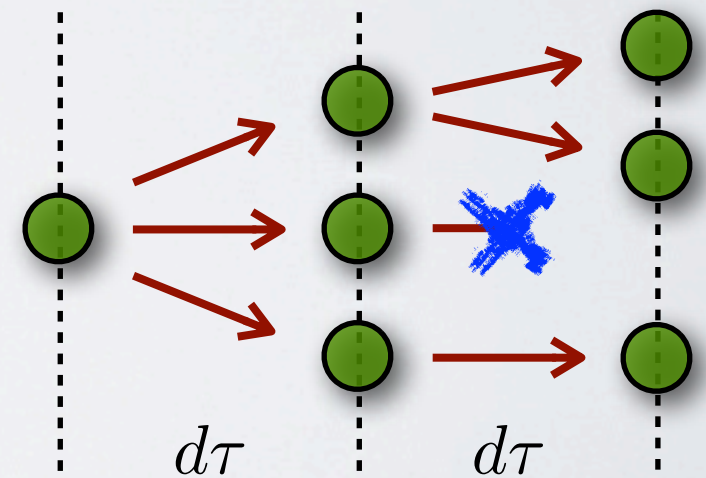
$$\mathcal{D} = \hbar^2 / 2m$$

$$\left(\frac{1}{4\pi\mathcal{D}d\tau} \right)^{\frac{3N}{2}} e^{-\frac{(\mathbf{R} - \mathbf{R}')^2}{4\mathcal{D}d\tau}} e^{-\left(\frac{\hat{V}(\mathbf{R}') + \hat{V}(\mathbf{R})}{2} - E_0 \right) d\tau}$$



kinetic
term

potential
term



problem

The method: Auxiliary Field DMC

$$\mathcal{P} \sim e^{-\frac{1}{2}\lambda d\tau \mathcal{O}^2} \quad \Rightarrow \quad \psi \sim \sum 2^A \frac{A!}{(A-Z)!Z!} \text{ terms}$$

high computational cost \rightarrow GFMC: $A \leq 12$

Idea: Hubbard-Stratonovich transformation

$$e^{-\frac{1}{2}\lambda d\tau \mathcal{O}^2} = \frac{1}{\sqrt{2\pi}} \int dx e^{-\frac{x^2}{2} + \sqrt{-\lambda d\tau} x \mathcal{O}}$$

auxiliary field

rotation over spin-isospin configurations



computational cost: $\sim A!$ \rightarrow $\sim A^3$

The method: Auxiliary Field DMC

AFDMC for nuclei & hypernuclei

- wave function
 - SD made of nucleon single particle orbitals (Skyrme & BI) \times Λ single particle orbital
 - Jastrow correlation functions
 - Note 1*: Λ single particle orbital
 - Note 2*: center of mass corrections
- observables
 - nucl. & hyp. binding energy
 - Λ separation energy
 - Λ & N single particle density
- interactions ?
 - local interactions in coordinate space

The interaction

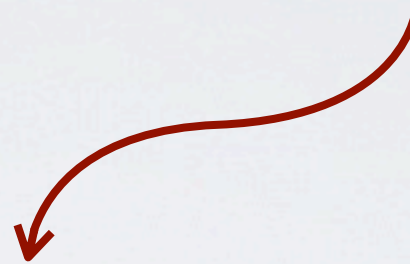
- nuclear potentials
 - Argonne V4', V6', V8'(6)
 - Minnesota

$$V_{NN}(\text{AV6}) : \mathcal{O}_{ij}^{p=1,6} = \underbrace{\{1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}\}}_{V_{NN}^{sd} + V_{NN}^{si}} \otimes \{1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\}$$

$$\begin{aligned}
 V_{NN}^{sd} &= \frac{1}{2} \sum_{ij} \sum_{\gamma} \tau_i^{\gamma} \left(\mathcal{A}_{ij}^{[\tau]} \right) \tau_j^{\gamma} && A \times A \\
 &+ \frac{1}{2} \sum_{ij} \sum_{\alpha\beta} \sigma_i^{\alpha} \left(\mathcal{A}_{i\alpha, j\beta}^{[\sigma]} \right) \sigma_j^{\beta} && 3A \times 3A \\
 &+ \frac{1}{2} \sum_{ij} \sum_{\alpha\beta\gamma} \tau_i^{\gamma} \sigma_i^{\alpha} \left(\mathcal{A}_{i\alpha, j\beta}^{[\sigma\tau]} \right) \tau_j^{\gamma} \sigma_j^{\beta} && 3A \times 3A
 \end{aligned}$$

The interaction

- nuclear potentials
 - Argonne V4', V6', V8' (6)
 - Minnesota
- hypernuclear potential
 - Usmani interaction



- ✓ diagrammatic contributions due to pion exchange
- ✓ 2-body ΛN and 3-body ΛNN terms

A. Bodmer, Q. N. Usmani, J. Carlson, Phys. Rev. C 29 (1984) 684-687

A. Bodmer, Q. N. Usmani, Nucl. Phys. A 477 (1988) 621-651

A. A. Usmani, S. C. Pieper, Q. N. Usmani, Phys. Rev. C 51 (1995) 2347

A. A. Usmani, Phys. Rev. C 52 (1995) 1773-1777

A. A. Usmani, S. Murtaza, Phys. Rev. C 68 (2003) 024001

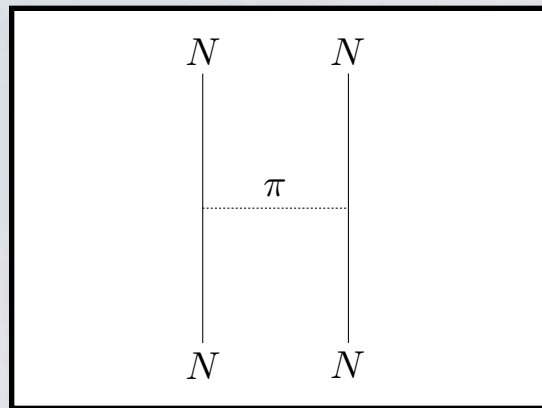
A. A. Usmani, Phys. Rev. C 73 (2006) 011302

A. A. Usmani, F. C. Khanna, J. Phys. G: Nucl. Part. Phys. 35 (2008) 025105

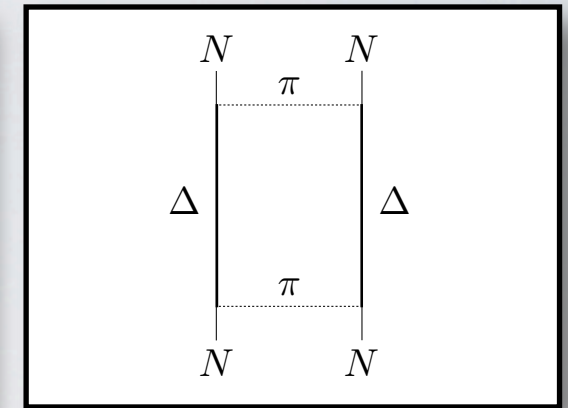
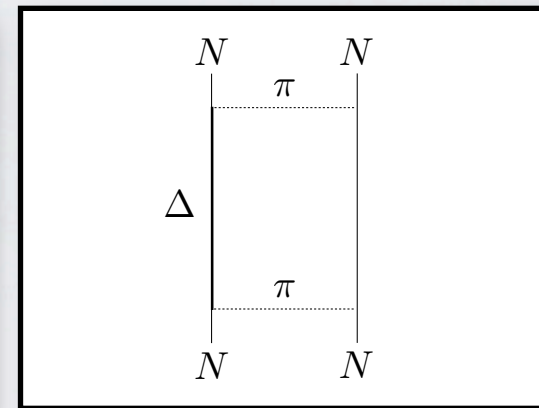
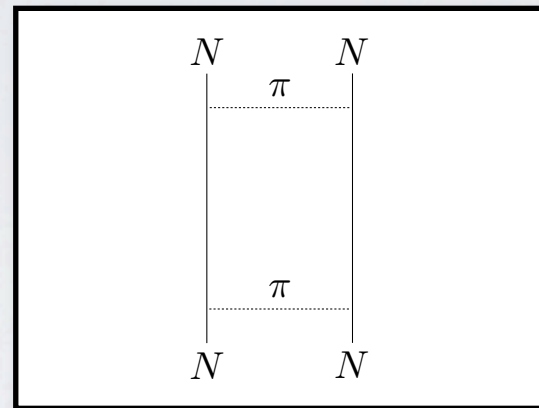
The interaction

2-body

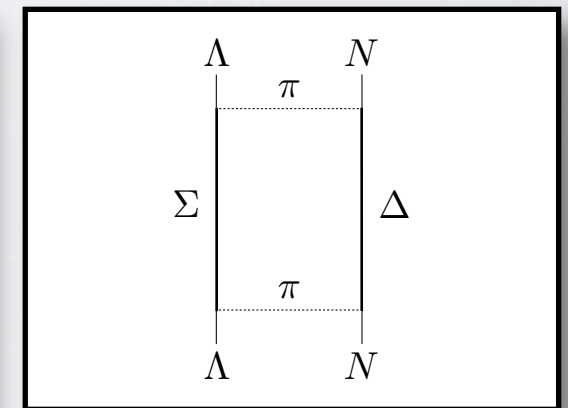
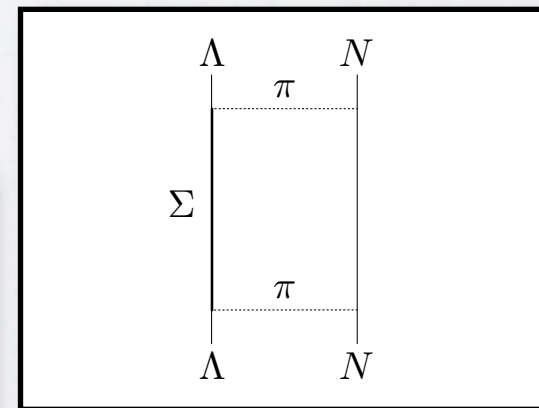
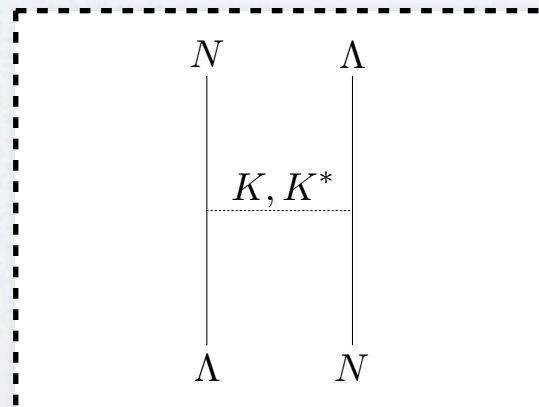
1π



2π



forbidden



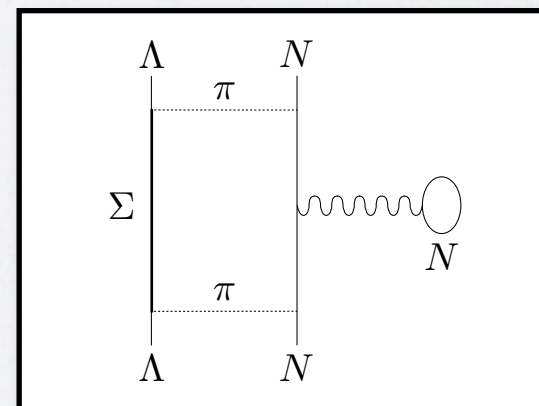
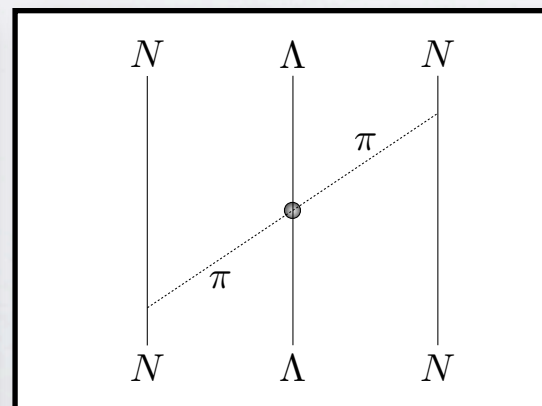
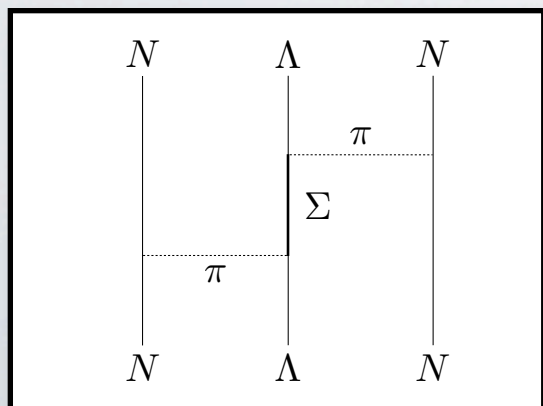
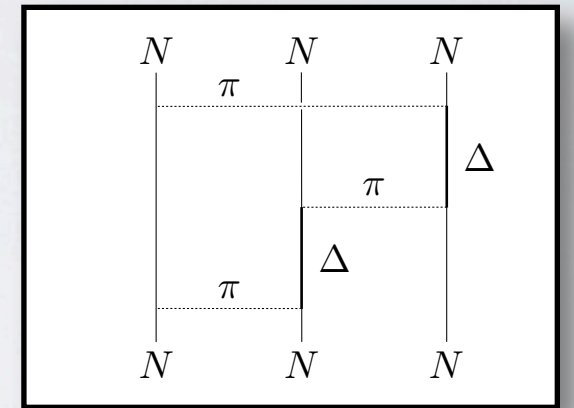
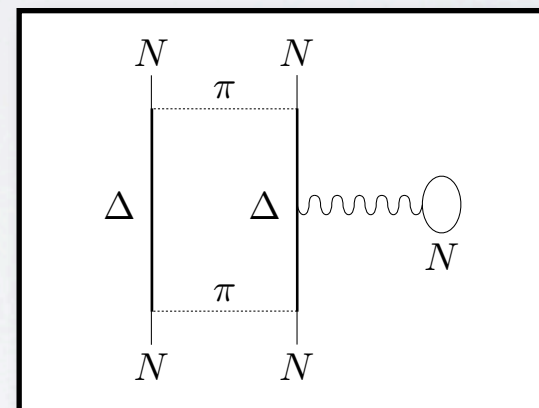
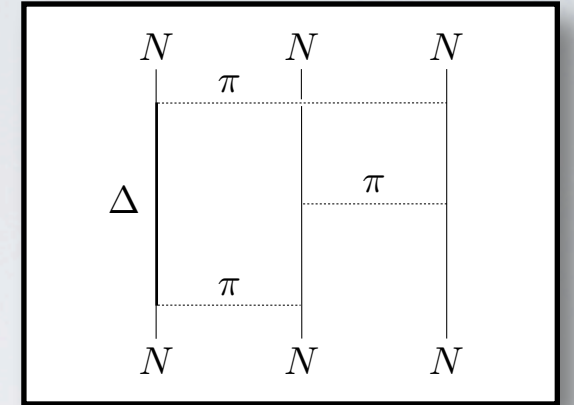
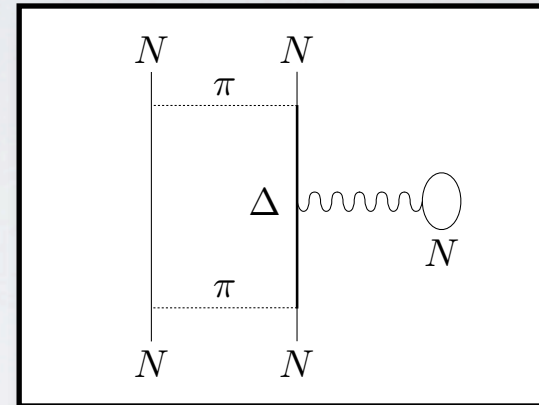
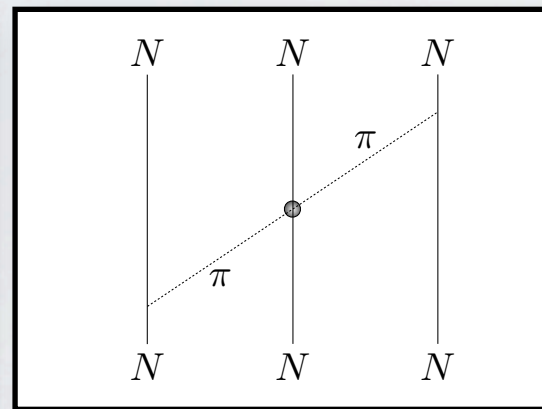
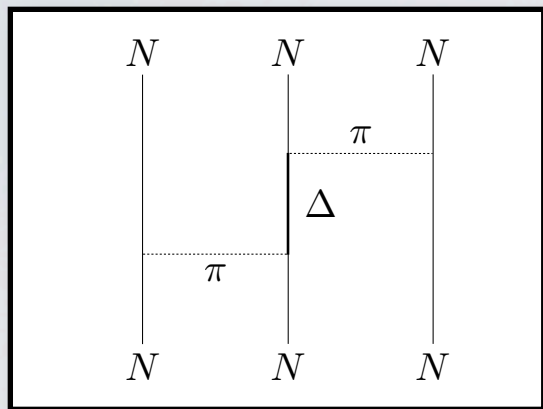
The interaction

3-body

2π

dispersive

3π



forbidden

The interaction

$$\Lambda N \quad v_{\Lambda i}(r) = v_0(r) + v_0(r)\varepsilon(\mathcal{P}_x - 1) + \frac{1}{4}v_\sigma T_\pi^2(m_\pi r)\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i$$

$$\left\{ \begin{array}{l} v_0(r) = v_c(r) - v_{2\pi}(r) \\ v_c(r) = W_c \left[1 + e^{\frac{r-\bar{r}}{a}} \right]^{-1} \\ v_{2\pi}(r) = \bar{v} T_\pi^2(m_\pi r) \end{array} \right. \quad \left\{ \begin{array}{l} v_\sigma = v_s - v_t \\ \bar{v} = \frac{1}{4}(v_s + 3v_t) \end{array} \right.$$

parameters fitted on Λp scattering data

The interaction

$$\Lambda N \quad v_{\Lambda i}(r) = v_0(r) + v_0(r)\varepsilon(\mathcal{P}_x - 1) + \frac{1}{4}v_\sigma T_\pi^2(m_\pi r)\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i$$

$$\Lambda NN \quad v_{\Lambda ij} = v_{\Lambda ij}^{2\pi} + v_{\Lambda ij}^D = v_{\Lambda ij}^{PW} + v_{\Lambda ij}^{SW} + v_{\Lambda ij}^D$$

$$\left\{ \begin{array}{l} v_{\Lambda ij}^{PW} = -\frac{1}{6}C^P \{X_{i\Lambda}, X_{\Lambda j}\} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\Lambda ij}^{SW} = C^S Z_\pi(m_\pi r_{\Lambda i}) Z_\pi(m_\pi r_{\Lambda j}) (\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{i\Lambda} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{j\Lambda}) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\Lambda ij}^D = W^D T_\pi^2(m_\pi r_{\Lambda i}) T_\pi^2(m_\pi r_{\Lambda j}) \left[1 + \frac{1}{6}\boldsymbol{\sigma}_\Lambda \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \right] \end{array} \right.$$

parameters not yet fixed !!



fitting of the parameters to reproduce experimental separation energies

The method: Auxiliary Field DMC

$$V_{\Lambda N} + V_{\Lambda NN}$$



$$V_{\Lambda N} = \sum_i v_0(r_{\Lambda i})(1 - \epsilon + \epsilon P_x) + \sum_i \sum_{\alpha} \sigma_{\Lambda}^{\alpha} \left(\mathcal{B}_i \right) \sigma_i^{\alpha} \quad 1 \times A$$

$$V_{\Lambda NN}^D = \frac{1}{2} \sum_{i \neq j} W^D T_{\Lambda i}^2 T_{\Lambda j}^2 + \frac{1}{2} \sum_{i \neq j} \sum_{\alpha} \sigma_{\Lambda}^{\alpha} \left(\mathcal{C}_{ij} \right) \sigma_i^{\alpha} \quad A \times A$$

$$V_{\Lambda NN}^{2\pi} = \frac{1}{2} \sum_{i \neq j} \sum_{\alpha \beta \gamma} \tau_i^{\gamma} \sigma_i^{\alpha} \left(\mathcal{D}_{i\alpha, j\beta} \right) \tau_j^{\gamma} \sigma_j^{\beta} \quad 3A \times 3A$$



good for Hubbard-Stratonovich

The interaction work in progress

possible charge symmetry breaking term:

$$v_{\Lambda i}^{\text{CSB}} = \tau_i^3 v_0^{\text{CSB}} T_\pi^2(m_\pi r_{\Lambda i})$$

$$v_0^{\text{CSB}} = -0.050(5) \text{ MeV}$$



from $A = 4$
hypernuclei

Q. N. Usmani, A. R. Bodmer, Phys. Rev. C 60 (1998) 055215

$\left\{ \begin{array}{l} \Lambda p : \text{more attractive} \\ \Lambda n : \text{less attractive} \end{array} \right.$



important for light isobar
hypernuclei



significant effect for heavy
hypernuclei, with large n excess

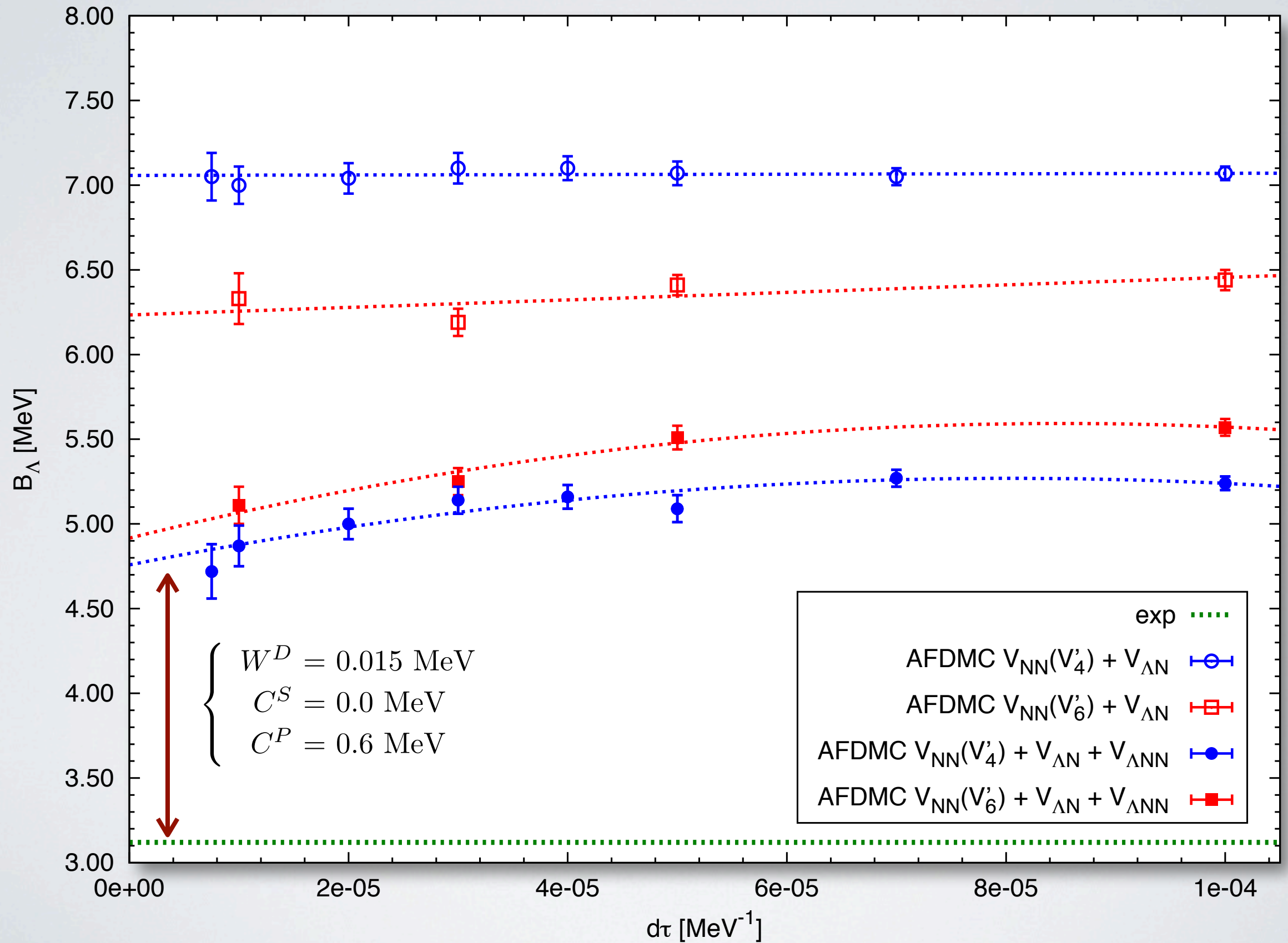
no quadratic
operator



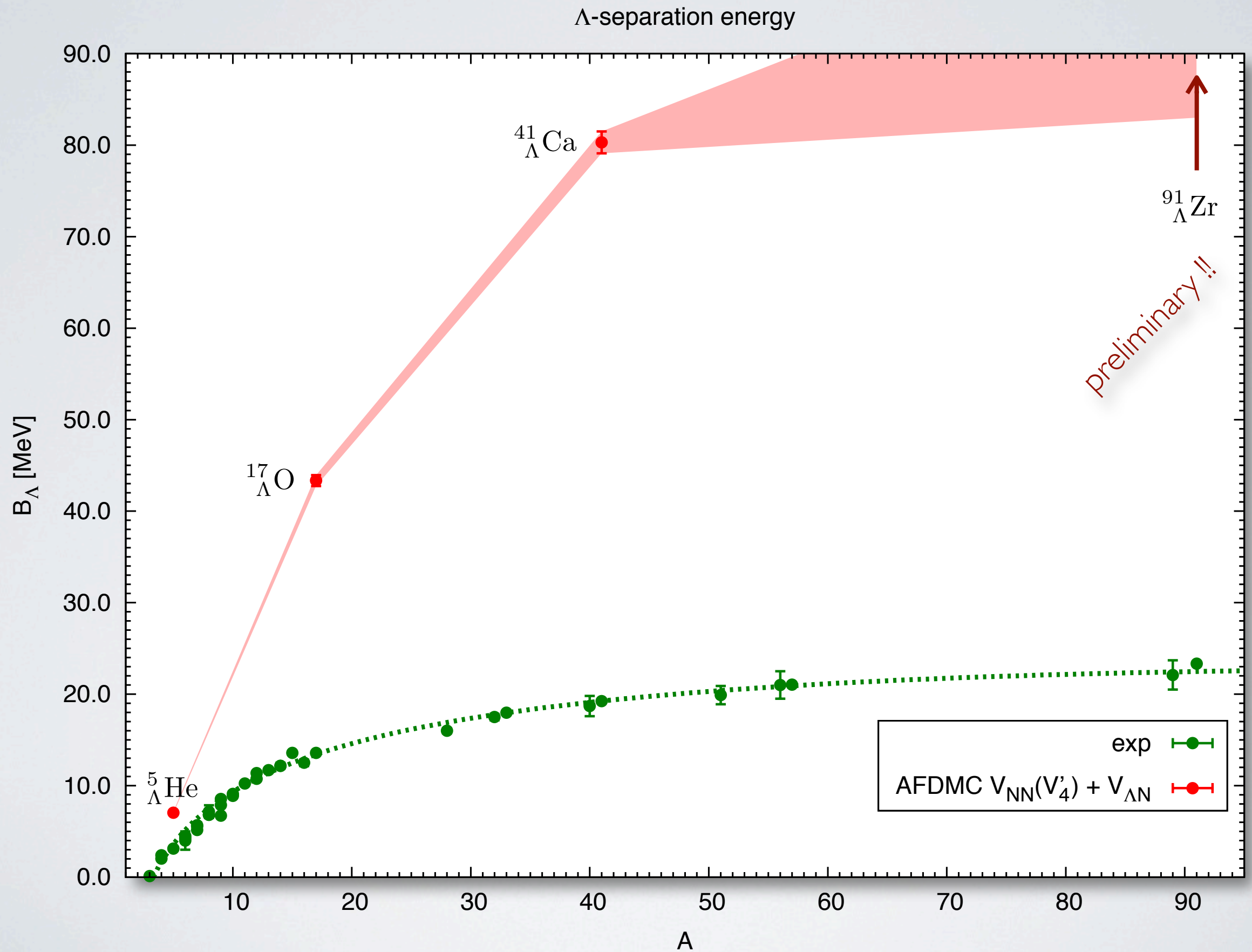
simple implementation
in the AFDMC code

Results

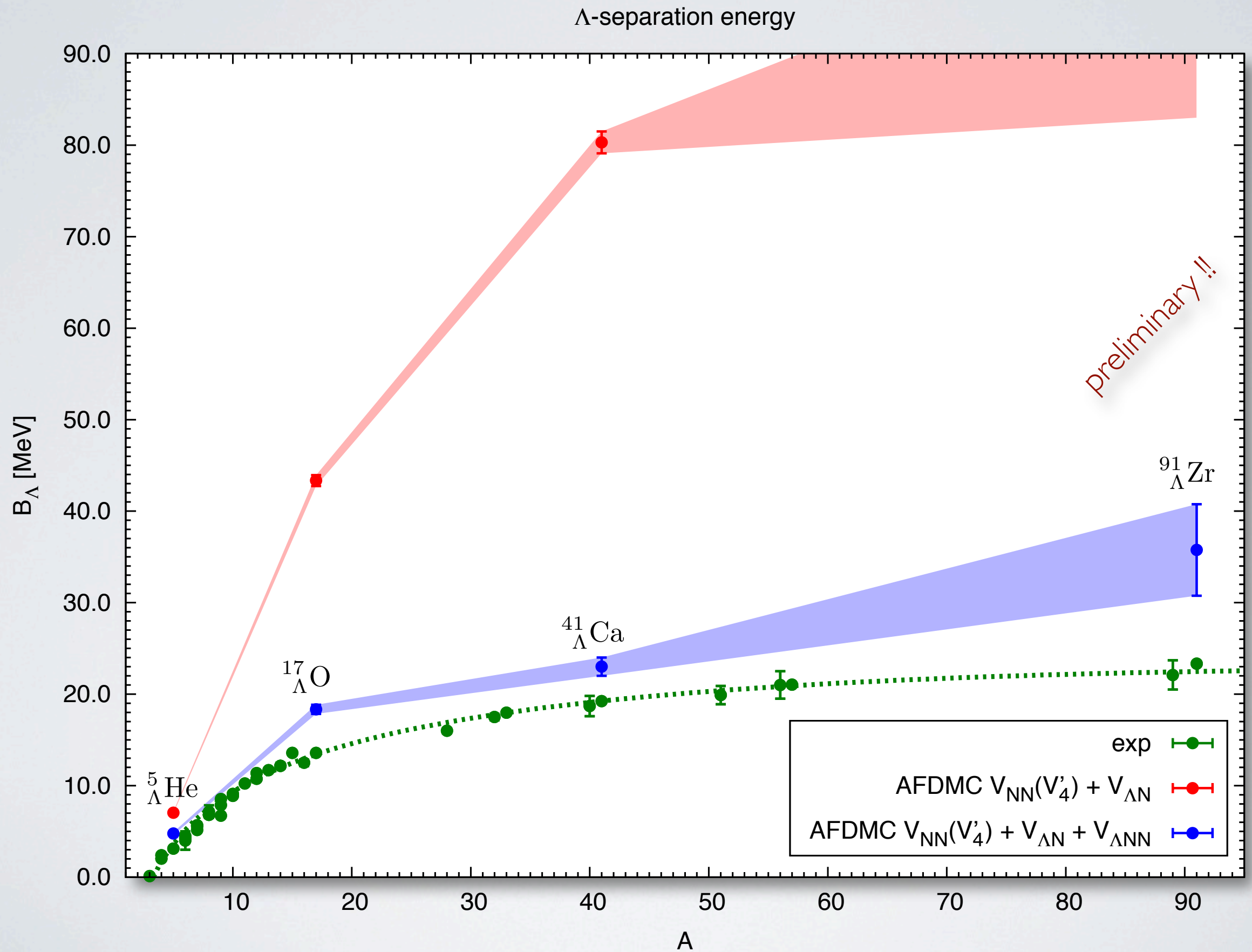
${}^5_{\Lambda}\text{He}$ Λ -separation energy



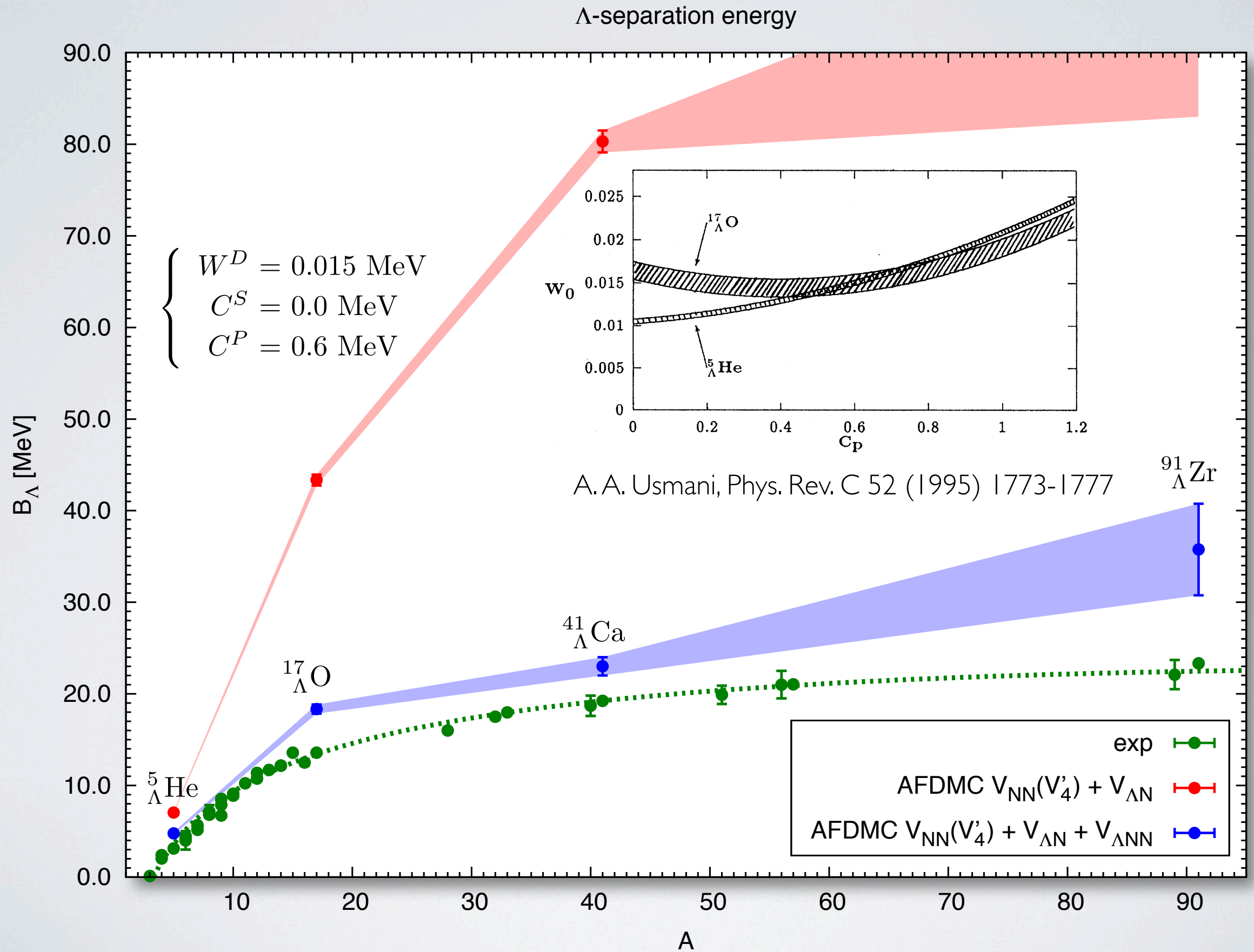
Results



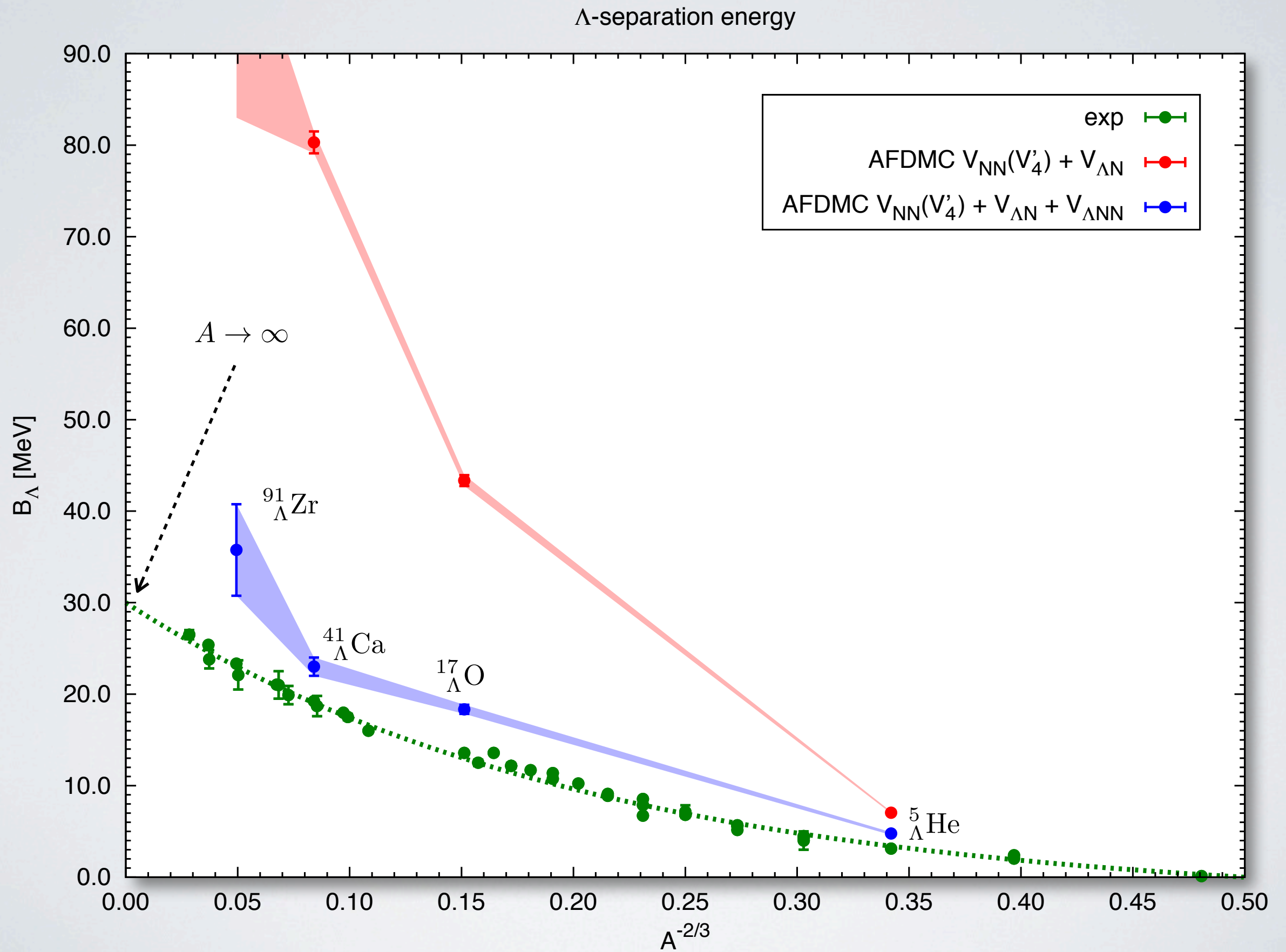
Results



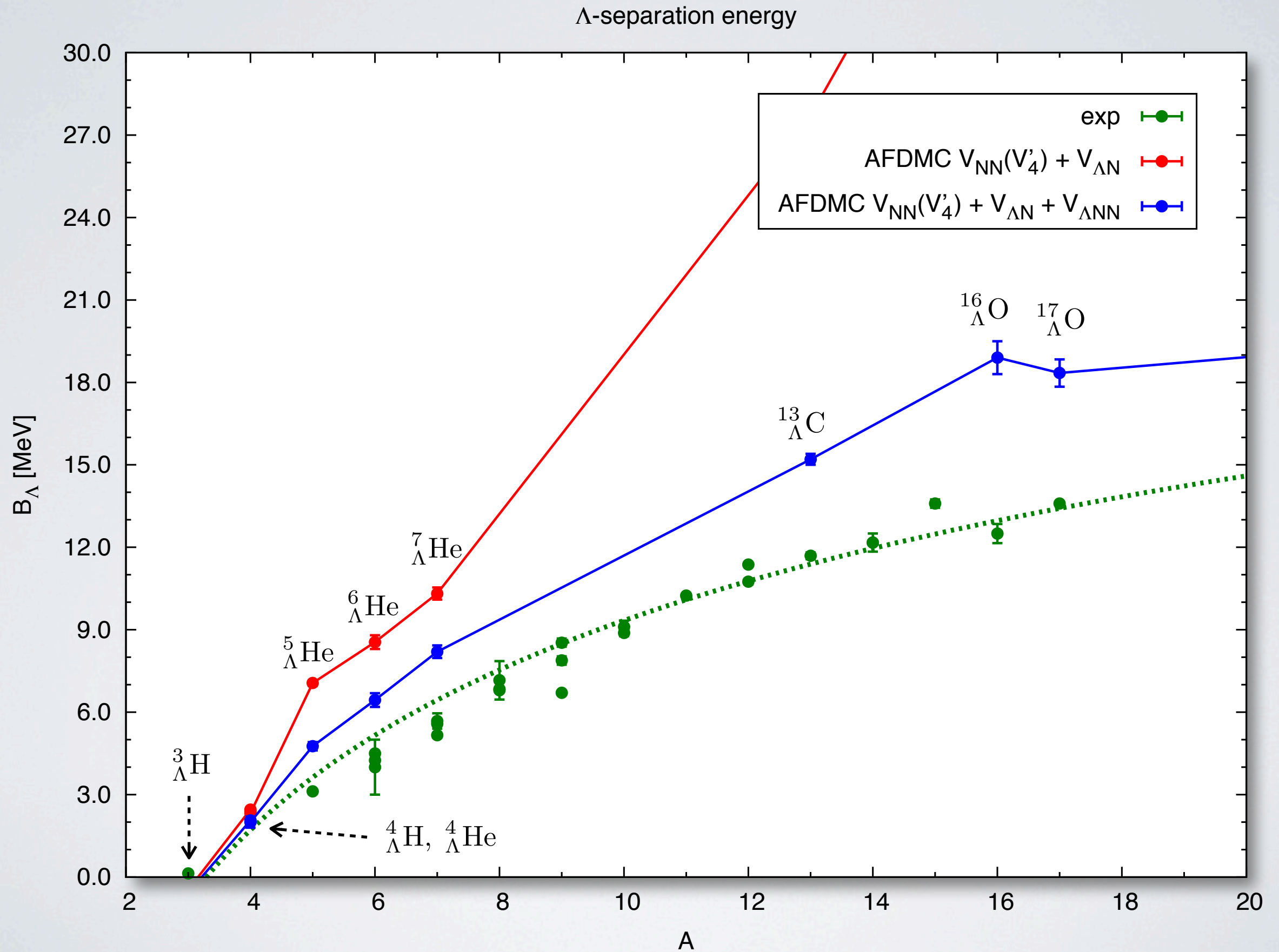
Results



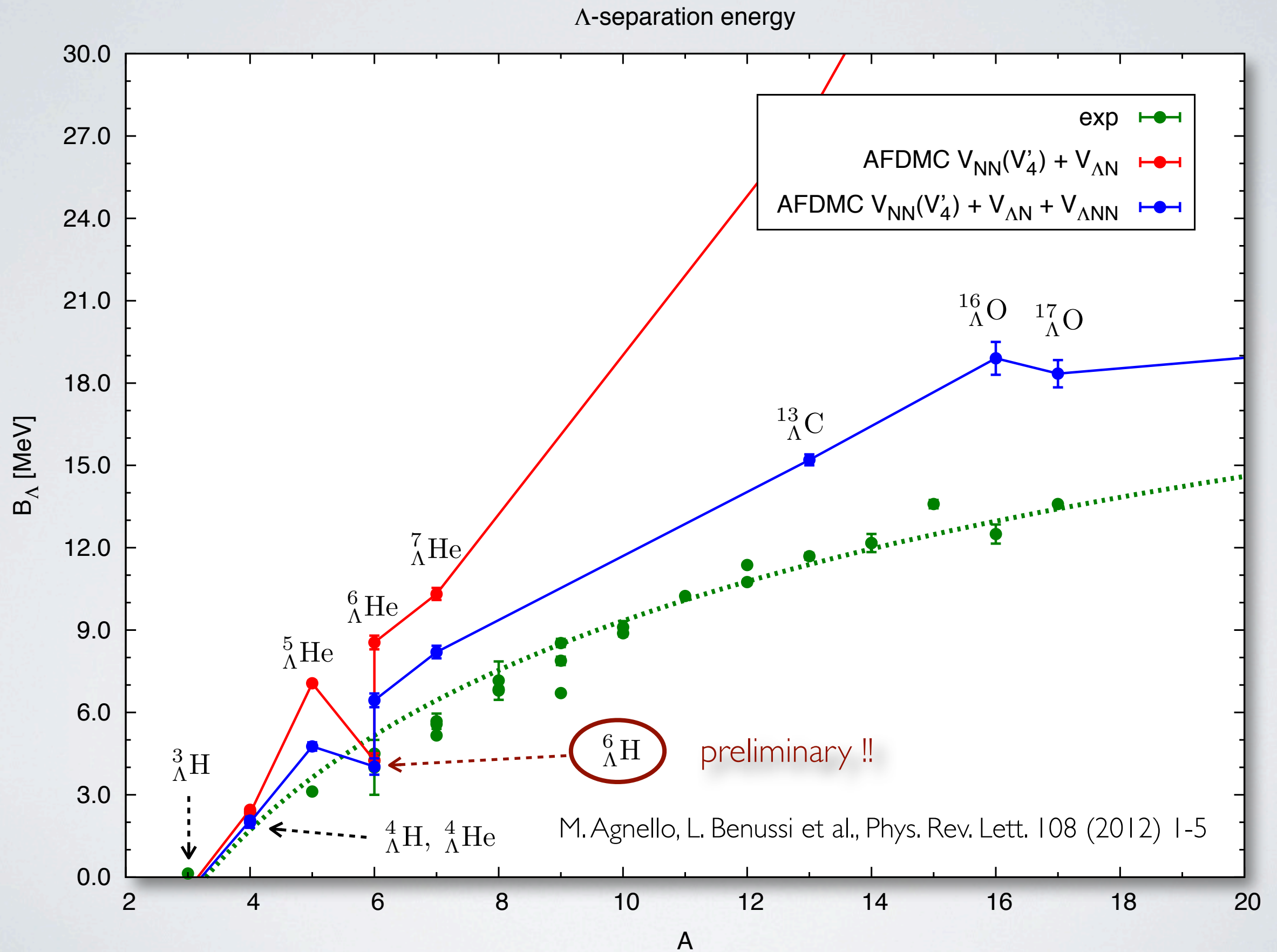
Results



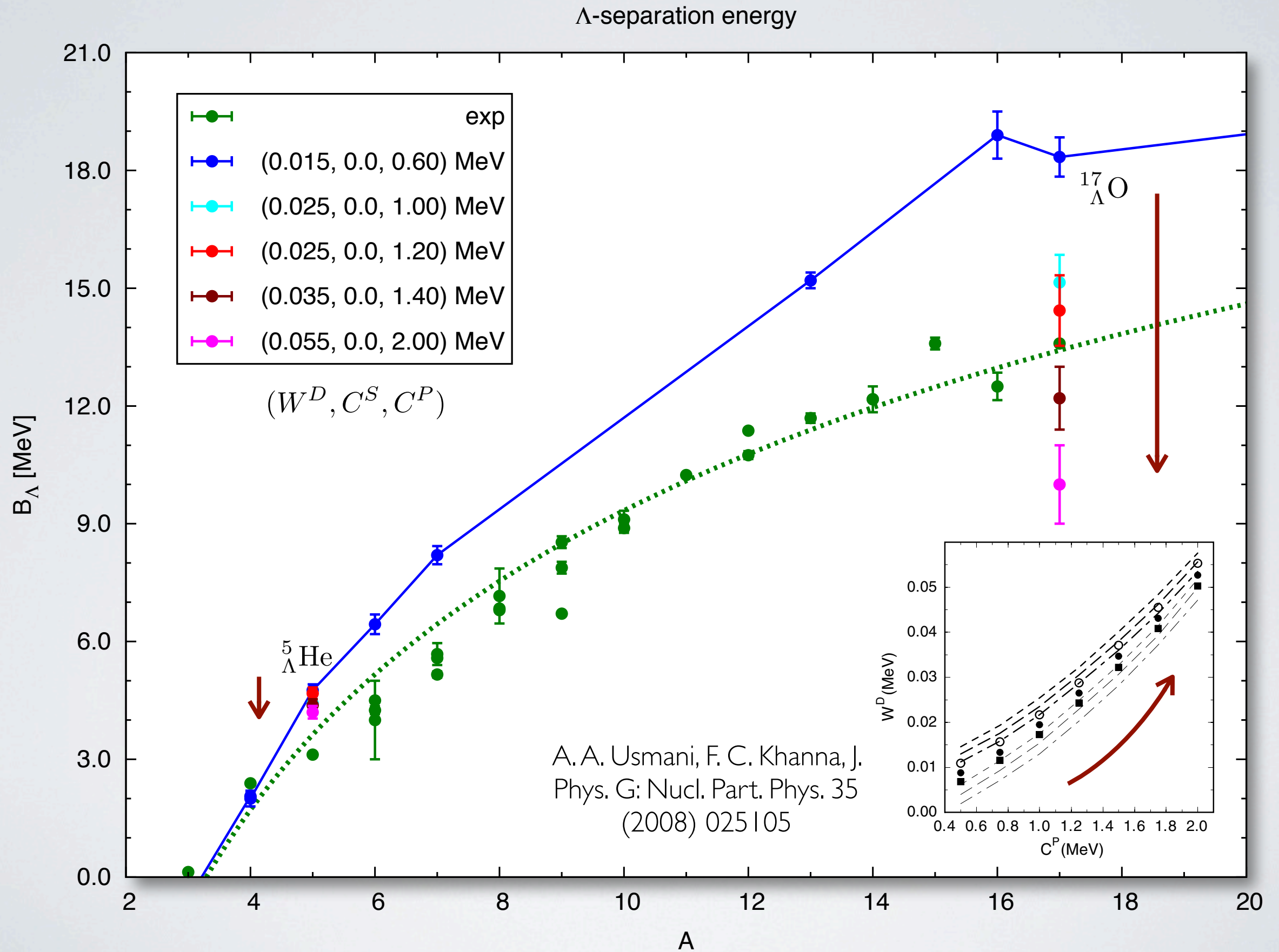
Results



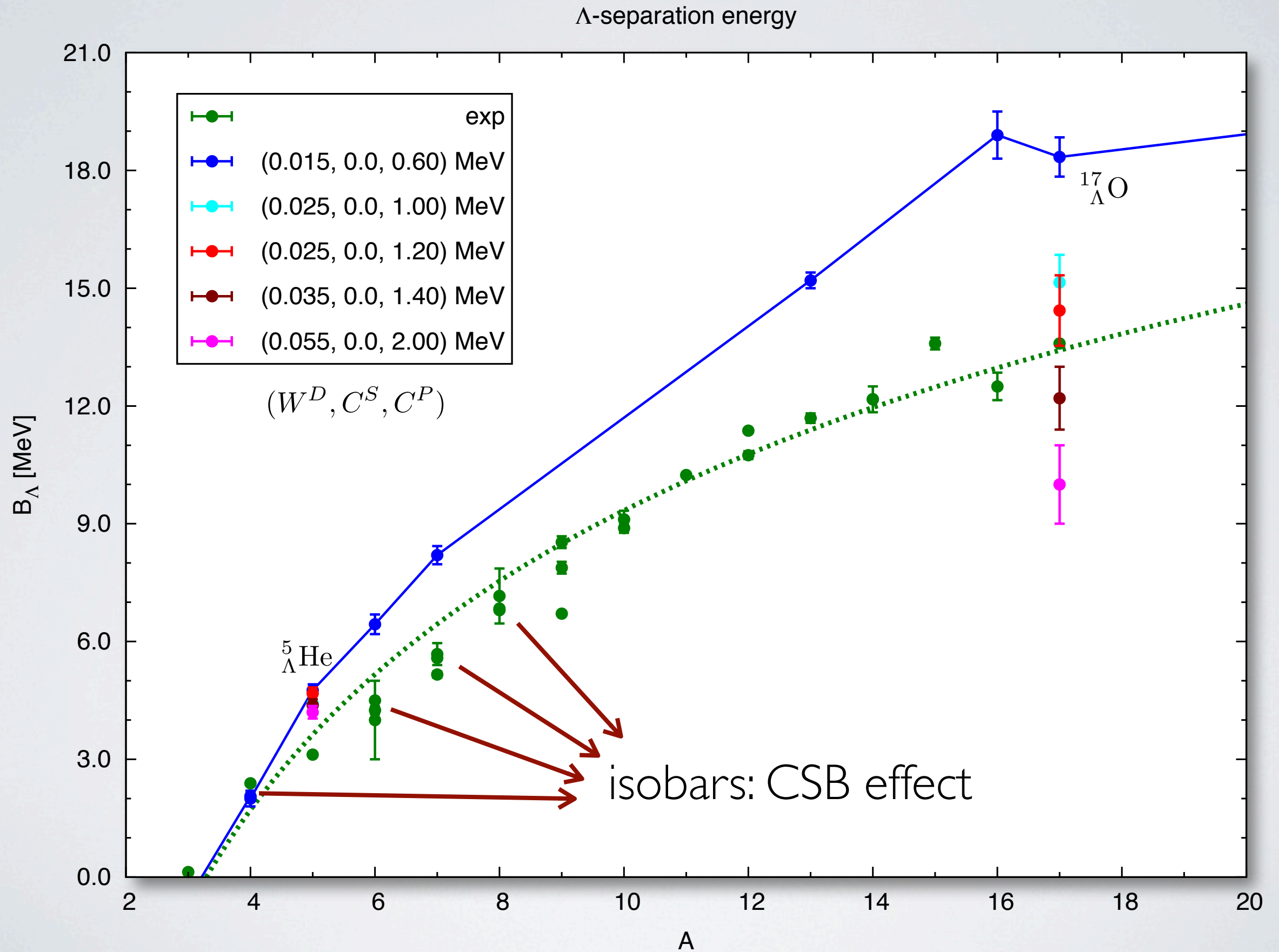
Results



Results



Results



Results

charge symmetry breaking effect \longrightarrow $A = 4 \dots 17$

preliminary !!	${}^4_{\Lambda}\text{H}$	${}^4_{\Lambda}\text{He}$	${}^5_{\Lambda}\text{He}$	${}^6_{\Lambda}\text{H}$	${}^6_{\Lambda}\text{He}$	${}^{17}_{\Lambda}\text{O}$
B_{Λ}^{noCSB}	2.06(8)	2.0(1)	5.24(4)	4.0(2)	6.5(2)	18.3(5)
B_{Λ}^{CSB}	1.95(8)	2.1(1)	5.21(7)	3.7(2)	6.4(2)	—
B_{Λ}^{exp}	2.04(4)	2.39(3)	3.12(2)	4(1)	4.2(1)	13.6(*)
ΔB_{Λ}	0.15(13)			2.7(3)		
ΔB_{Λ}^{exp}	0.35(5)			?		

Results

charge symmetry breaking effect \longrightarrow $A = 4 \dots 17$

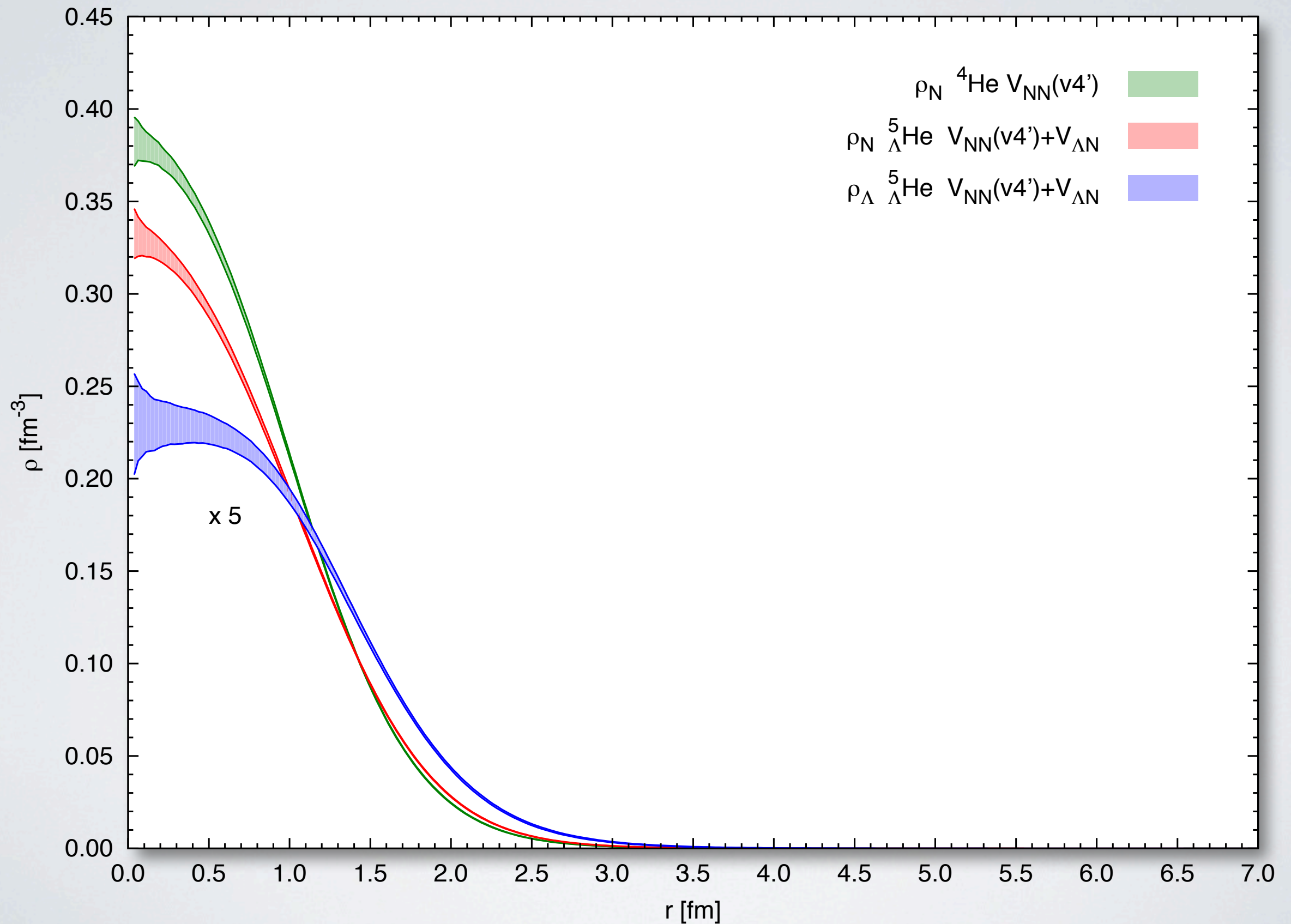
preliminary !!	${}^4_{\Lambda}\text{H}$	${}^4_{\Lambda}\text{He}$	${}^5_{\Lambda}\text{He}$	${}^6_{\Lambda}\text{H}$	${}^6_{\Lambda}\text{He}$	${}^{17}_{\Lambda}\text{O}$
B_{Λ}^{noCSB}	2.06(8)	2.0(1)	5.24(4)	4.0(2)	6.5(2)	18.3(5)
B_{Λ}^{CSB*}	1.88(9)	2.30(15)	5.21(7)	3.7(2)	6.4(2)	—
B_{Λ}^{exp}	2.04(4)	2.39(3)	3.12(2)	4(1)	4.2(1)	13.6(*)
ΔB_{Λ}^*	0.43(17)			2.7(3)		
ΔB_{Λ}^{exp}	0.35(5)			?		

consistent for the Λp & Λn channels

deeper investigation needed !!

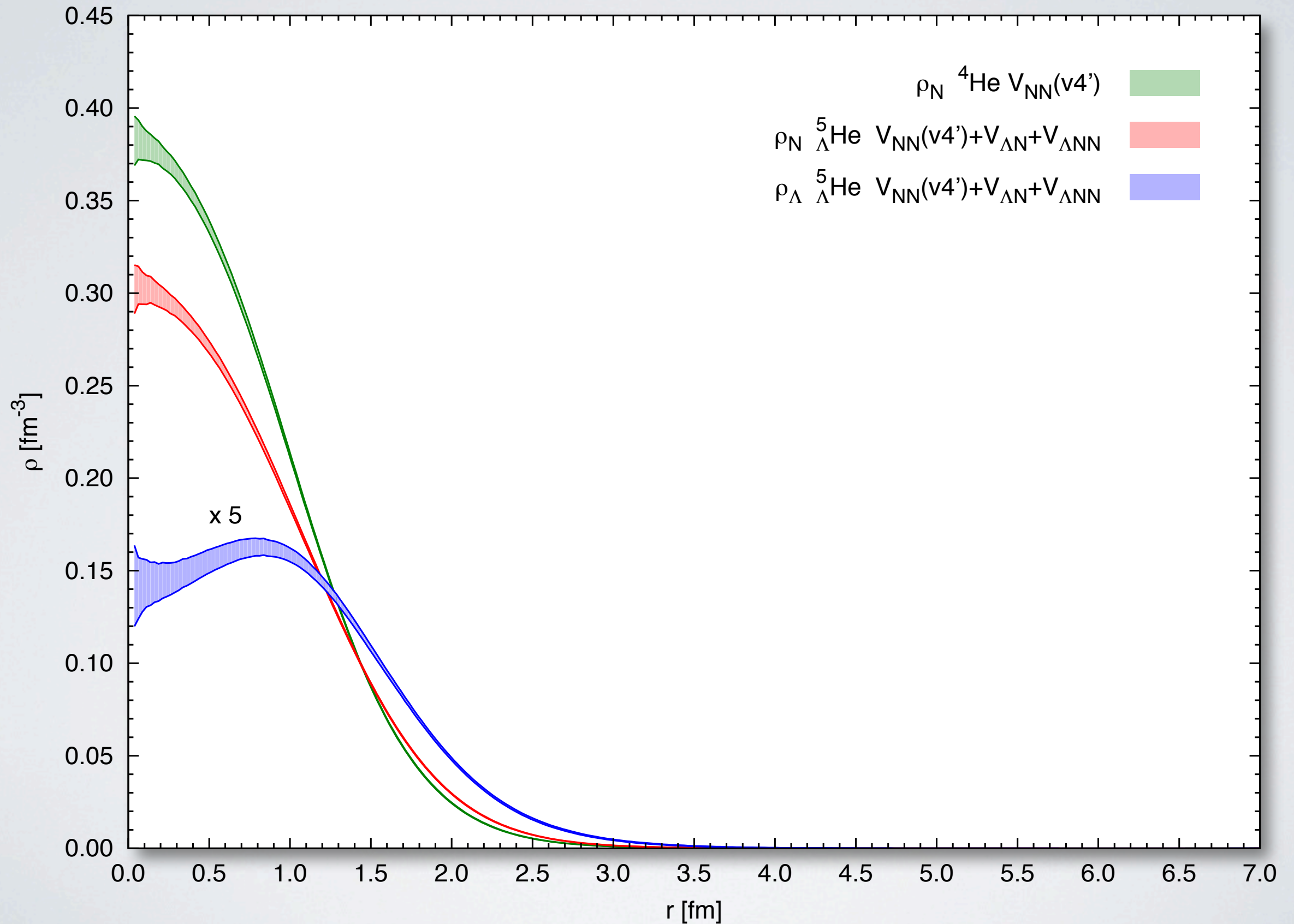
Results

density: ${}^4\text{He}$ vs ${}^5_{\Lambda}\text{He}$



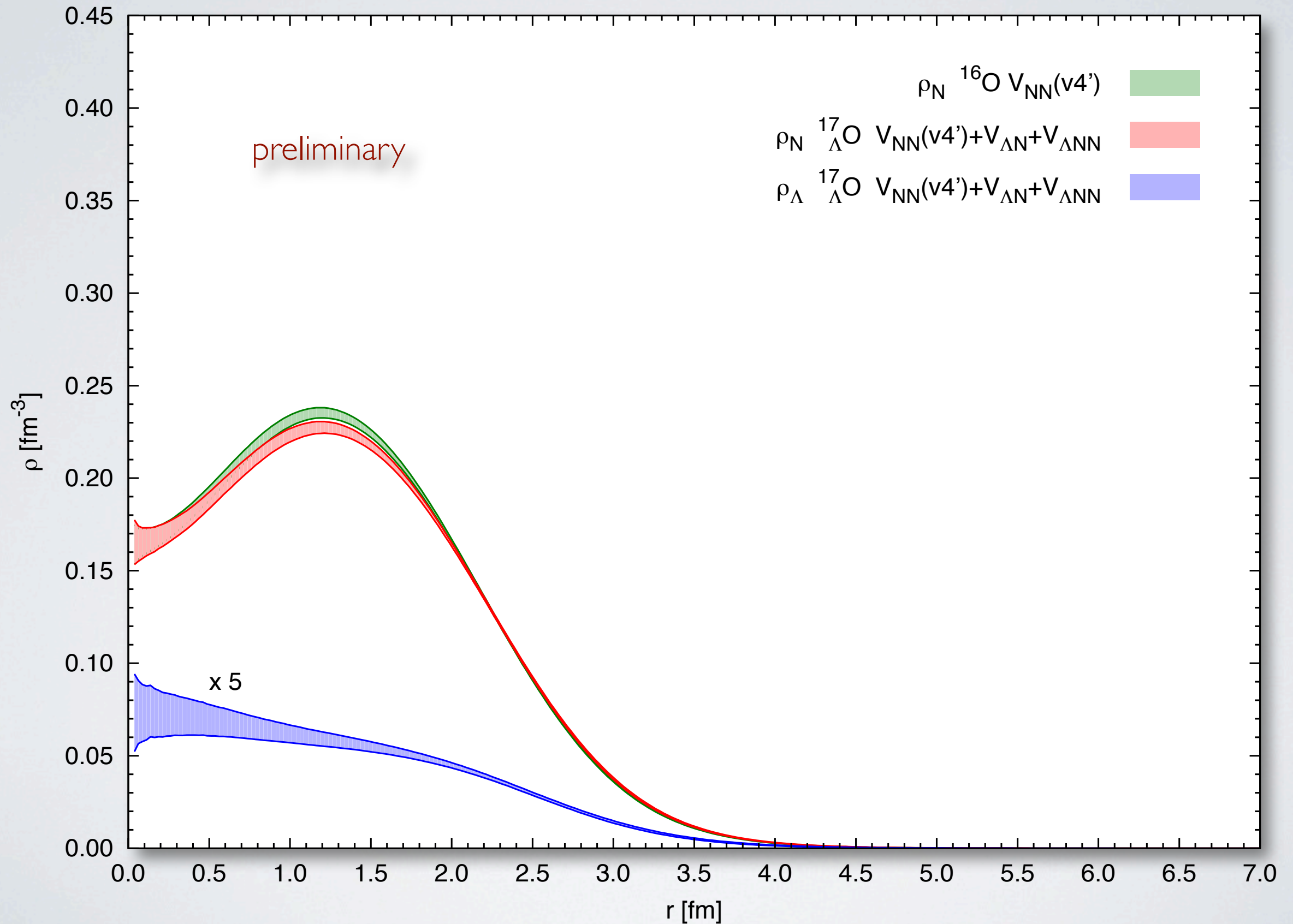
Results

density: ${}^4\text{He}$ vs ${}^5_{\Lambda}\text{He}$



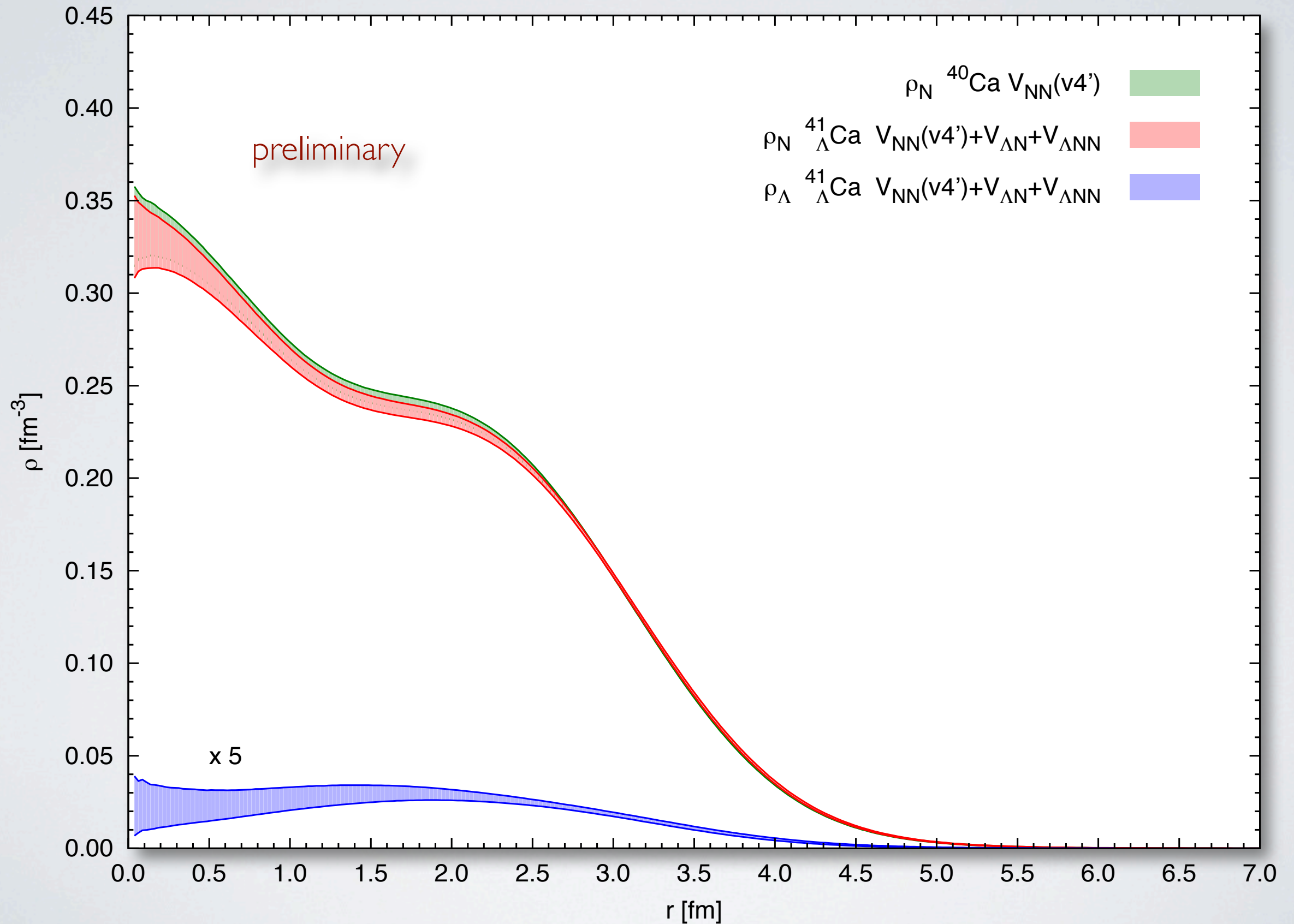
Results

density: ^{16}O vs $^{17}_{\Lambda}\text{O}$



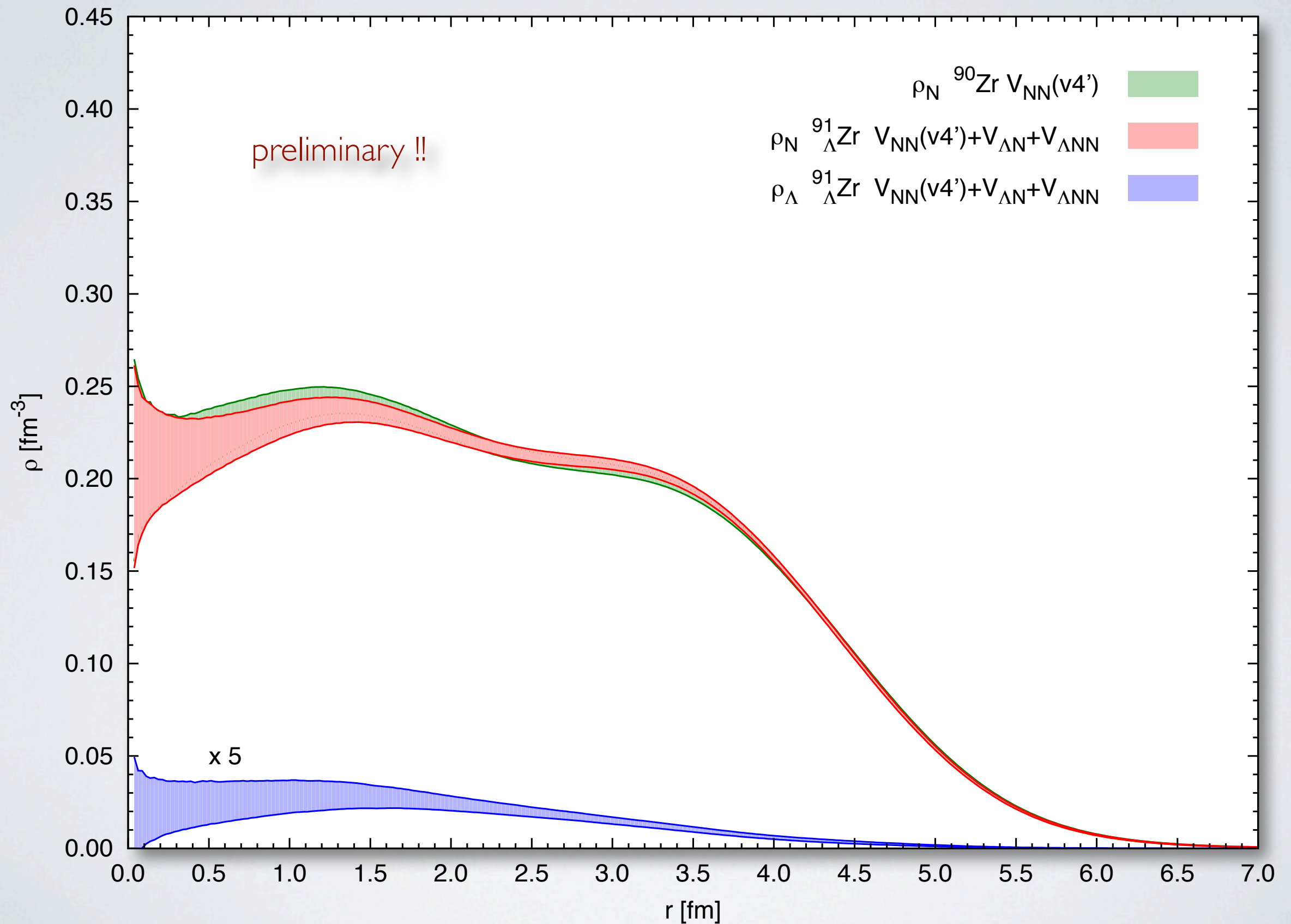
Results

density: ^{40}Ca vs $^{41}_{\Lambda}\text{Ca}$



Results

density: ^{90}Zr vs $^{91}_{\Lambda}\text{Zr}$



Conclusions & Perspectives

- AFDMC algorithm can be extended to hypernuclear systems: study of the hyperon-nucleon interaction
- 3-body ΛNN interaction fundamental for the computation of the hyperon separation energy
- CSB interaction needed
- 3-body ΛNN interaction repulsive in hypernuclei: extrapolation for nuclear matter \longrightarrow NS EOS with Λ
- possible inclusion of a $\Lambda\Lambda$ interaction: AFDMC hypernuclear code ready
- possible application to neutron drops: study of exotic systems

fine tuning of the parameters

work in progress

next step
(LANL)

future work

(future work)

Thank you for your attention

The interaction

$$T_{\pi}(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) Y_{\pi}(x) \xi(r)$$

$$Y_{\pi}(x) = \frac{e^{-x}}{x} \xi(r) \quad \xi(r) = 1 - e^{-cr^2}$$

$$Z_{\pi}(x) = \frac{x}{3} [Y_{\pi}(x) - T_{\pi}(x)]$$

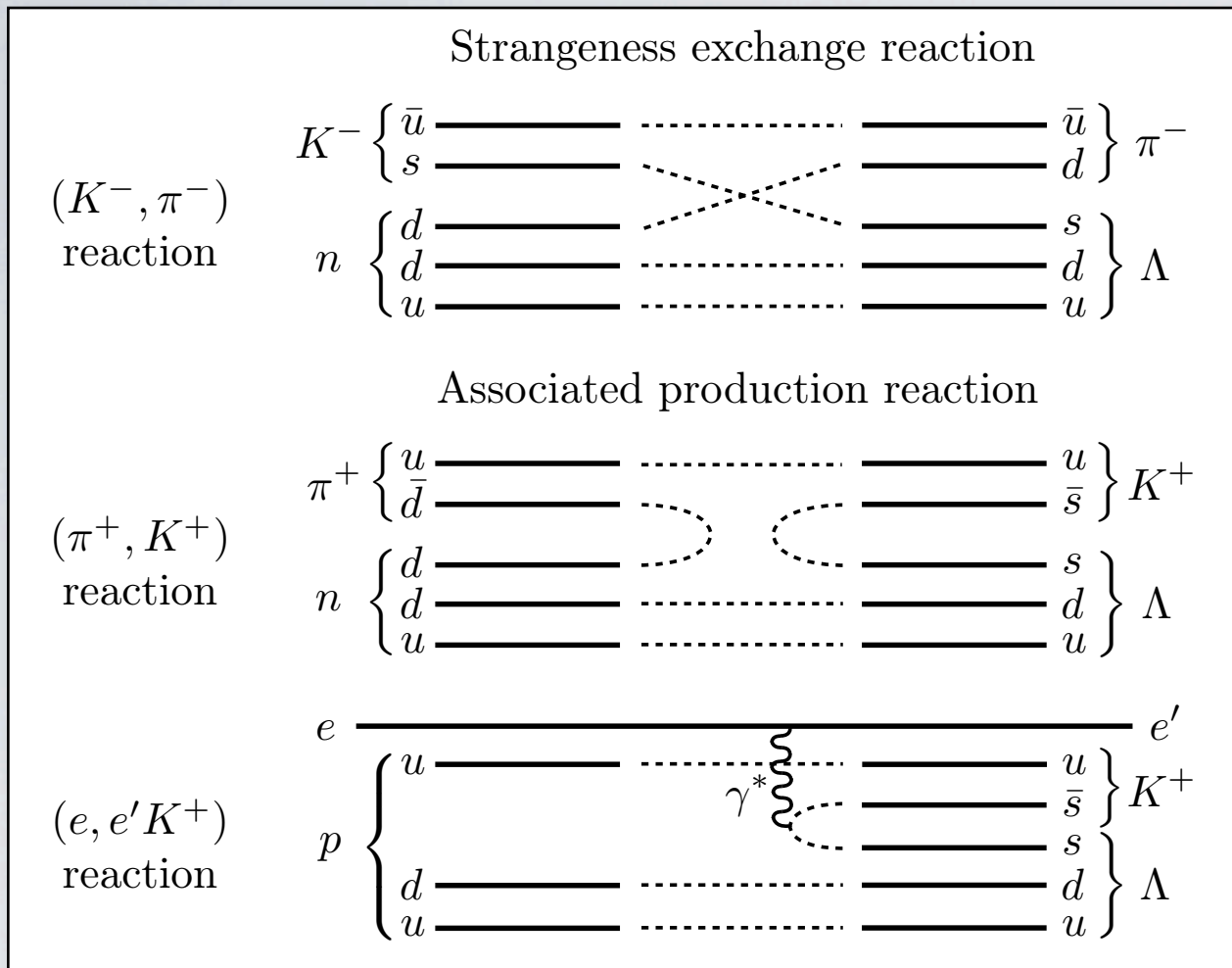
$$X_{\Lambda i} = (\boldsymbol{\sigma}_{\Lambda} \cdot \boldsymbol{\sigma}_i) Y_{\pi}(m_{\pi} r_{\Lambda i}) + S_{\Lambda i} T_{\pi}(m_{\pi} r_{\Lambda i})$$

$$S_{\Lambda i} = 3 (\boldsymbol{\sigma}_{\Lambda} \cdot \hat{\boldsymbol{r}}_{\Lambda i}) (\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{\Lambda i}) - \boldsymbol{\sigma}_{\Lambda} \cdot \boldsymbol{\sigma}_i$$

The interaction

Constant	Value	Unit
m_{π^\pm}	139.57018(35)	MeV
m_{π^0}	134.9766 (6)	MeV
W_c	2137	MeV
r_0	0.5	fm
a	0.2	fm
ε	0.1 \div 0.38	–
\bar{v}	6.15(5)	MeV
v_s	6.33, 6.28, 6.23	MeV
v_t	6.09, 6.04, 5.99	MeV
v_σ	0.24	MeV
c	2.0	fm ⁻²
W^D	0.01 \div 0.05	MeV
C^P	0.4 \div 2.0	MeV
C^S	\sim 1.5	MeV

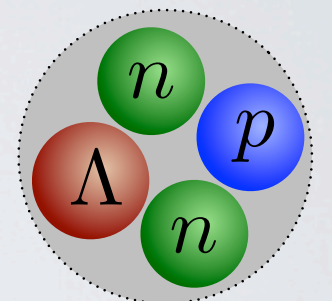
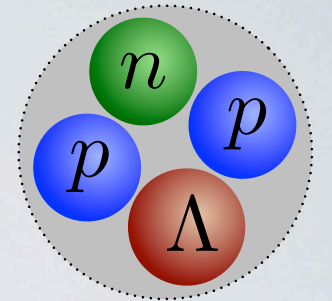
Motivations: experimental interest



$${}^A_Z(K^-, \pi^-) {}^A_\Lambda Z$$

$${}^A_Z(\pi^+, K^+) {}^A_\Lambda Z$$

$${}^A_Z(e, e' K^+) {}^A_\Lambda [Z - 1]$$



hypernuclear
fine structure



γ -ray spectroscopy

