Recent Developments and Applications of Integral Transforms in Few- and Many-Body Physics

Outline

- Introduction
- Compton scattering (A=2)
- Δ degrees of freedom in ^{3}He(e,e') (A=3)
- Role of 0^+ resonance in ^{4}He(e,e') (A=4)
- Density excitation response in bulk atomic ^{4}He at T=0

Introduction

Consider an observable R(E) and an integral transform $\Phi(\sigma)$:

$$\Phi(\sigma) = \int dE\, K(\sigma,E)\, R(E)$$

with some kernel $K(\sigma,E)$

Often it is easier to calculate $\Phi(\sigma)$ than R(E). Then the observable R(E) can be obtained via inversion of the integral transform.

In order to make the inversion sufficiently stable the kernel $K(\sigma,E)$ should resemble a kind of energy filter (Lorentzians, Gaussians, ...); best choice would be a δ-function.

In the following we will consider LITs (Lorentz integral transforms) with

$$K(\sigma,E) = [(E-\sigma_R)^2 + \sigma_I^2]^{-1}$$

and Sumudu transforms with

$$K_P(\sigma,E) = N\,(e^{-\mu E/\sigma} - e^{-\nu E/\sigma})^P$$

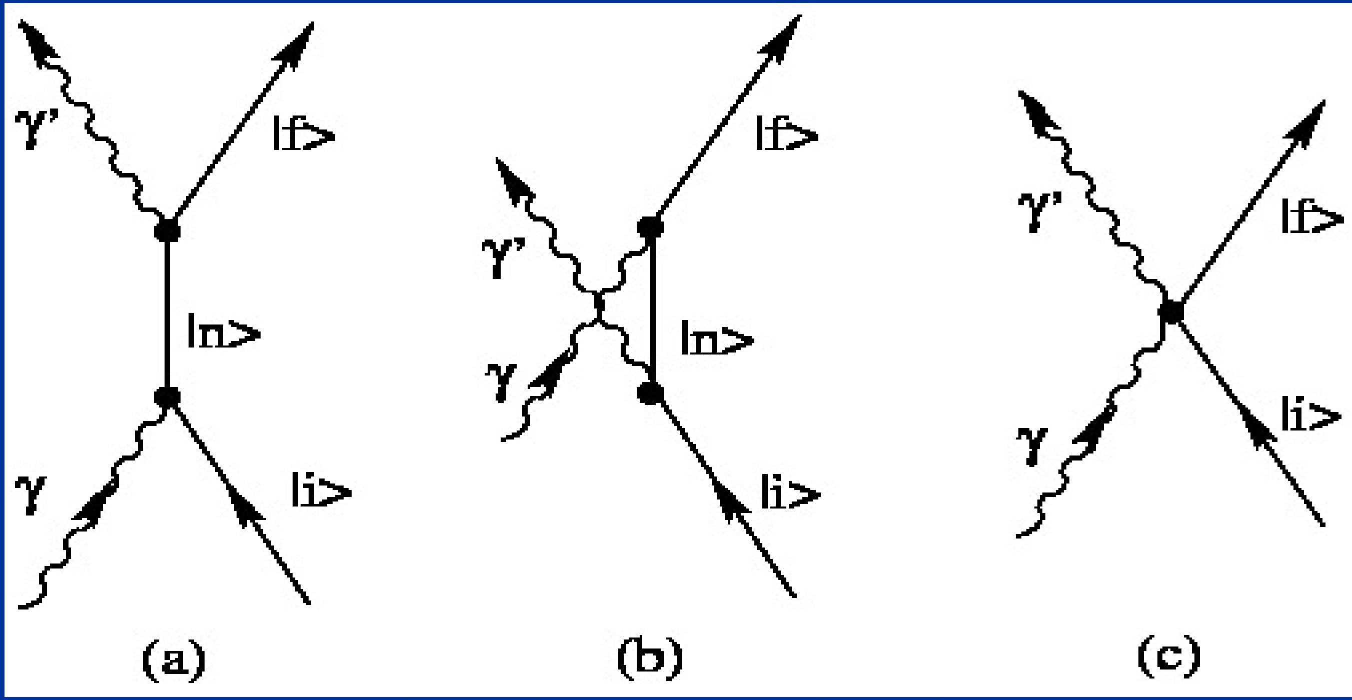
Photon Scattering with the LIT method

(more details in G. Bampa, WL, H. Arenhövel, PRC 84, 034005)

Photon scattering

The photon scattering amplitude is given by two terms:

The contact or two photon amplitude (TPA) $B_{\lambda'\lambda}(\mathbf{k}',\mathbf{k})$ and the resonance amplitude (RA) $R_{\lambda'\lambda}(\mathbf{k}',\mathbf{k})$



total scattering amplitude:

$$T^{fi}_{\lambda'\lambda}(\vec{k}\,',\vec{k}) = B^{fi}_{\lambda'\lambda}(\vec{k}\,',\vec{k}) + R^{fi}_{\lambda'\lambda}(\vec{k}\,',\vec{k}),$$

TPA has the form:

$$B^{fi}_{\lambda'\lambda}(\vec{k}',\vec{k}) = -\langle f| \int d^3x d^3y e^{i\vec{k}'\cdot\vec{x}} e^{-i\vec{k}\cdot\vec{y}} \vec{e}^{\,\prime *}_{\lambda'} \cdot \overleftrightarrow{B}(\vec{x},\vec{y}) \cdot \vec{e}_\lambda |i\rangle,$$

RA is given by

$$R^{fi}_{\lambda'\lambda}(\vec{k}\,',\vec{k}) = \langle f| \Big[\vec{e}^{\,\prime *}_{\lambda'} \cdot \vec{J}(-\vec{k}\,', 2\vec{P}_f + \vec{k}\,') G(k + i\epsilon) \vec{e}_\lambda \cdot \vec{J}(\vec{k}, 2\vec{P}_i + \vec{k}) + \vec{e}_\lambda \cdot \vec{J}(\vec{k}, 2\vec{P}_f - \vec{k}) G(-k' + i\epsilon) \vec{e}^{\,\prime *}_{\lambda'} \cdot \vec{J}(-\vec{k}', 2\vec{P}_i - \vec{k}\,') \Big] |i\rangle,$$

with intermediate propagator

$$G(z) = (H - E_i - z)^{-1}.$$

Cartesian tensor operator B of rank 2 represents the second order term of the e.m. interaction

Current operator

$$\vec{J}(\vec{k},\vec{P}) = \vec{j}(\vec{k}) + \frac{\vec{P}}{2AM}\rho(\vec{k})$$

Intrinsic current j plus a term taking into account the convection current of the separated cm-motion (M: nucleon mass, A: mass number of nucleus)

The intrinsic charge and current operators consist of one- and two- body parts

$$\rho(\vec{k}) = \rho_{[1]}(\vec{k}) + \rho_{[2]}(\vec{k}),$$
$$\vec{j}(\vec{k}) = \vec{j}_{[1]}(\vec{k}) + \vec{j}_{[2]}(\vec{k}),$$

$$\rho_{[1]}(\vec{k}) = \sum_l e_l\, e^{-i\vec{k}\cdot\vec{r}_l},$$
$$\vec{j}_{[1]}(\vec{k}) = \frac{1}{2M}\sum_l \left(e_l\{\vec{p}_l, e^{-i\vec{k}\cdot\vec{r}_l}\} + \mu_l \vec{\sigma}_l \times \vec{k}\, e^{-i\vec{k}\cdot\vec{r}_l}\right).$$

e_l, m_l, p_l, and σ_l: charge, magnetic moment, internal momentum, and spin operator of l-th particle

Low-energy limits

$$\vec{j}(0) = [H, \vec{D}] \,,$$

$$B^{ii}_{[1],\lambda'\lambda}(0,0) = -\vec{e}^{\,\prime *}_{\lambda'} \cdot \vec{e}_{\lambda} \frac{Ze^2}{M} \,,$$

$$B^{ii}_{[2],\lambda'\lambda}(0,0) = -\langle i | [\vec{e}^{\,\prime *}_{\lambda'} \cdot \vec{D}, [V, \vec{e}_{\lambda} \cdot \vec{D}]] | i \rangle \,,$$

$$R^{ii}_{\lambda'\lambda}(0,0) = \vec{e}^{\,\prime *}_{\lambda'} \cdot \vec{e}_{\lambda} \frac{NZe^2}{AM} - B^{ii}_{[2],\lambda'\lambda}(0,0) \,,$$

Resulting in the low-energy limit for the total scattering amplitude:

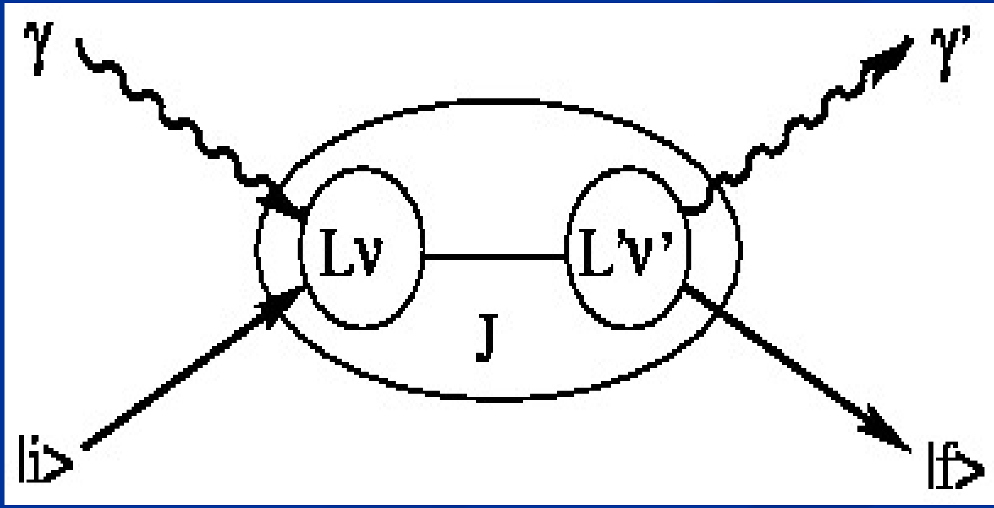
$$T^{ii}_{\lambda'\lambda}(0,0) = -\vec{e}^{\,\prime *}_{\lambda'} \cdot \vec{e}_{\lambda} \frac{(Ze)^2}{AM} \,,$$

which is the classical Thomson limit

Reaction strength is described by polarizabilities

$$P_{if,J}^{L'L\lambda'\lambda}(k',k) = \sum_{\nu'\nu=0,1} \lambda'^{\nu'} \lambda^{\nu} P_{if,J}(M^{\nu'}L', M^{\nu}L, k', k) \,,$$

M^0: electric multipole, M^1: magnetic multipole
incoming photon transfers angular momentum L
scattered photon transfers angular momentum L'
total momentum transfer J to the nucleus with |L-L'|≤ J ≤ L+L'



Expansion of total scattering amplitude in terms of polarizabilities

$$T^{fi}_{\lambda'\lambda}(\vec{k}',\vec{k}) = (-)^{1+\lambda'+I_f-M_i} \sum_{L',M',L,M,J} (-)^{L+L'}(2J+1)\begin{pmatrix} I_f & J & I_i \\ -M_f & m & M_i \end{pmatrix}\begin{pmatrix} L & L' & J \\ M & M' & -m \end{pmatrix}$$
$$\times P^{L'L\lambda'\lambda}_{if,J}(k',k) D^L_{M,\lambda}(R) D^{L'}_{M',-\lambda'}(R'),$$

where (I_i, M_i) and (J_f, M_f) refer to the angular momenta and their projections on the quantization axis of the initial and final states.

The polarizabilities can be separated in a TPA and a RA contribution

$$P_{if,J}(M^{\nu'}L', M^{\nu}L, k', k) = P^{TPA}_{if,J}(M^{\nu'}L', M^{\nu}L, k', k) + P^{\mathrm{res}}_{if,J}(M^{\nu'}L', M^{\nu}L, k', k),$$

Polarizability contribution for resonance amplitude:

$$P^{\mathrm{res}}_{if,J}(M^{\nu'}L', M^{\nu}L, k', k) = 2\pi(-)^{L+J}\frac{\hat{L}\hat{L}'}{\hat{J}} \tag{29}$$
$$\times\langle I_f E_f \| \left(\left[M^{\nu',L'}(k')G(k+i\varepsilon)M^{\nu,L}(k) \right]^J + \left[M^{\nu,L}(k)G(-k'+i\varepsilon)M^{\nu',L'}(k') \right]^J \right) \| I_i E_i \rangle\,.$$

(small cm current contribution neglected)

Polarizability contribution for two-photon amplitude:

$$P^{TPA}_{if,J}(M^{\nu',L'}, M^{\nu,L}, k', k) = 2\pi(-)^{L+J+1}\frac{\hat{L}\hat{L}'}{\hat{J}}\langle I_f E_f \| \int d^3x\, d^3y \left[\vec{A}^{L'}(M^{\nu'}; k, \vec{x}) \cdot \overleftrightarrow{B}(\vec{x}, \vec{y}) \cdot \vec{A}^{L}(M^{\nu}; k, \vec{y}) \right]^J \| I_i E_i \rangle\,.$$

Evaluation of the TPA contribution is straight forward once the TPA operator B(x,y) is given

For the RA contribution one finds by evaluating the reduced matrix element in standard fashion

$$P^{\rm res}_{if,J}(M^{\nu'}L', M^{\nu}L, k', k) = 2\pi(-)^{L+I_f+I_i}\hat{L}\hat{L}' \times \sum_{E_n, I_n} \left[\begin{Bmatrix} L & L' & J \\ I_f & I_i & I_n \end{Bmatrix} \frac{\langle I_f E_f || M^{\nu',L'}(k') || I_n E_n \rangle \langle I_n E_n || M^{\nu,L}(k) || I_i E_i \rangle}{E_n - E_i - k - i\varepsilon} + (-)^{L+L'+J} \begin{Bmatrix} L' & L & J \\ I_f & I_i & I_n \end{Bmatrix} \frac{\langle I_f E_f || M^{\nu,L}(k) || I_n \rangle \langle I_n E_n || M^{\nu',L'}(k') || I_i E_i \rangle}{E_n - E_i + k' - i\varepsilon} \right].$$

Calculation of the RA part is more involved !

One has to sum over all possible intermediate states $|I_n>$ and energies E_n

For k=0 only the scalar E1-E1 polarizability is nonvanishing:

$$P_J(E1, E1)|_{k=0} = -\delta_{J0}\, \widehat{I}\, \sqrt{3}\, \frac{e^2 Z^2}{M_A} \,,$$

(I is ground-state spin)

The scattering cross section

$$\frac{d\sigma}{d\Omega} = \frac{k'}{k} \frac{c(\vec{k},\vec{p}_i,k')}{2(2I_i+1)} \sum_{\lambda,\lambda',M_i,M_f} |T^{fi}_{\lambda'\lambda,M_f,M_i}(\vec{k}',\vec{k})|^2 ,$$

with

$$c(\vec{k},\vec{p}_i,k') = \frac{\omega + E_i - \omega'}{(\omega + E_i)|\frac{\vec{k}}{\omega} - \frac{\vec{p}_i}{E_i}|} .$$

E1 transitions only:

$$\frac{d\sigma(E1)}{d\Omega} = \frac{k'}{k} \frac{c(\vec{k},\vec{p}_i,k')}{(2I_i+1)} \sum_J |P_{if,J}(E1,E1)|^2 \, g_J^{E1}(\theta) ,$$

with

$$\begin{aligned} g_0^{E1}(\theta) &= \frac{1}{6}(1+\cos^2\theta) , \\ g_1^{E1}(\theta) &= \frac{1}{4}(2+\sin^2\theta) , \\ g_2^{E1}(\theta) &= \frac{1}{12}(13+\cos^2\theta) . \end{aligned}$$

Application of the LIT method

Introduction of a polarizability strength function

$$F^{I_f I_i}_{(\nu' L', \nu L) J}(k', k, E) = \frac{(-)^{J+I_f+I_i}}{\hat{J}} \langle I_f E_f || \left[M^{\nu', L'}(k') \times \delta(H-E)\, M^{\nu, L}(k) \right]^J || I_i E_i \rangle \, .$$

In general the strength funcion is off-energy shell: $E \neq E_i + k$.

One finds:

$$F^{I_f I_i}_{(\nu' L', \nu L) J}(k', k, E) = \sum_{I_n} \rho(I_n, E) \begin{Bmatrix} L & L' & J \\ I_f & I_i & I_n \end{Bmatrix} \langle I_f E_f || M^{\nu', L'}(k') || I_n, E \rangle \langle I_n, E || M^{\nu, L}(k) || I_i E_i \rangle \, ,$$

ρ(I,E) is density of states for energy E and angular momentum J

Polarizability becomes

$$P^{\rm res}_{if,J}(M^{\nu'} L', M^{\nu} L, k', k) = 2\pi (-)^{L+I_f+I_i} \hat{L} \hat{L}' \times \int_{E_0}^{\infty} dE \left[\frac{F^{I_f I_i}_{(\nu' L', \nu L) J}(k', k, E)}{E - E_i - k - i\varepsilon} + (-)^{L+L'+J} \frac{F^{I_f I_i}_{(\nu L, \nu' L') J}(k, k', E)}{E - E_i + k' - i\varepsilon} \right] .$$

Consider a fixed intermediate total angular momentum state $|I_n M_n>$

$$F^{I_f I_i;I_n}_{\nu' L',\nu L}(k',k,E) = \rho(I_n,E)\langle I_f E_f \| M^{\nu',L'}(k') \| I_n,E\rangle\langle I_n,E \| M^{\nu,L}(k) \| I_i E_i\rangle \,.$$

leads to following polarization strength

$$F^{I_f I_i}_{(\nu' L',\nu L)J}(k',k,E) = \sum_{I_n} \begin{Bmatrix} L & L' & J \\ I_f & I_i & I_n \end{Bmatrix} F^{I_f I_i;I_n}_{\nu' L',\nu L}(k',k,E) \,.$$

The partial strength function can be calculated with the LIT method

$$L^{I_f,I_i;I_n}_{\nu' L',\nu L}(k',k,\sigma) = \int_{E_0}^{\infty} dE \, \frac{F^{I_f,I_i;I_n}_{\nu' L',\nu L}(k',k,E)}{(E-\sigma)(E-\sigma^*)} \, .$$

One finds

$$L^{I_f,I_i;I_n}_{\nu' L',\nu L}(k',k,\sigma) = (-)^{I_n-I_i+L-L'+\nu'} \rho(I_n,\sigma) \sum_{M_n} \langle \widetilde{\psi}^{\nu',L'}_{I_f;I_n M_n}(k',\sigma) | \widetilde{\psi}^{\nu,L}_{I_i;I_n M_n}(k,\sigma) \rangle \, ,$$

where the LIT state is obtained from

$$(H-\sigma^*) | \widetilde{\psi}^{\nu,L}_{I_i;I_n M_n}(k,\sigma) \rangle = | (M^{\nu,L}(k) \times \psi^{I_i}) I_n M_n \rangle \, .$$

Deuteron Case for elastic scattering

Calculation is made in the cm-system, where one has k=k'

only the dominant E1 transistions are considered taking the long wave length approximation (Siegert form)

$$E^1_M = i[H, D^1_M], \text{ where } D^1_M = \frac{\sqrt{\alpha}}{3\sqrt{2}}\, r\, Y_{1M}(\Omega)$$

Thus only the polarizabilities $P_J(E1,E1,k)$ with $J = 0, 1, 2$ contribute

E1-E1 polarization strength function:

$$\begin{aligned}\widetilde{F}^{11;j}_{E1,E1}(E) &= \frac{F^{11;j}_{E1,E1}(E)}{(E-E_0)^2} \\ &= (-)^{j-1}\sum_m \langle (D^1 \times \psi^1_d) jm |\, \delta(H-E)\, | (D^1 \times \psi^1_d) jm \rangle\,.\end{aligned}$$

LIT equation $$(H-\sigma^*)|\widetilde{\psi}_{jm}(\sigma)\rangle = |(D^1\times\psi_d^1)jm\rangle\,,$$

Expansion of LIT state $$\langle r,\Omega|\widetilde{\psi}_{jm}(\sigma)\rangle = \frac{\sqrt{\alpha}}{r}\sum_{l=|j-1|}^{j+1}\Phi_{jl}(\sigma,r)\langle\Omega|(l1)j\,m\rangle\,,$$

leads to following radial equations:

$$\left[-\frac{\hbar^2}{M}\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)-\sigma^*\right]\Phi_{jl}(\sigma,r)+\sum_{l'}V_{jl,jl'}\Phi_{jl'}(\sigma,r)=\frac{\sqrt{2}}{6}\,r\,f_{jl}(r)$$

with

$$f_{jl}(r)=\delta_{l1}\,u(r)+\;(-)^{j+1}\,3\sqrt{5}\,\hat{l}\begin{pmatrix}2&1&l\\0&0&0\end{pmatrix}\begin{Bmatrix}2&1&1\\j&1&l\end{Bmatrix}w(r)\,,$$

resulting in three LITs

$$L_j(\sigma):=(-)^{j-1}\frac{4\pi}{2j+1}\widetilde{L}^{11;j}_{E1,E1}(\sigma)=\frac{4\pi}{2j+1}\sum_m\langle\widetilde{\psi}_{jm}(\sigma)|\widetilde{\psi}_{jm}(\sigma)\rangle=\alpha\sum_l\int_0^\infty|\Phi_{jl}(\sigma,r)|^2dr\,,$$

Inversion of LIT $L_j(\sigma)$ gives function $F_j(E)$ and leads to polarization

strength function

$$F^{11;j}_{E1,E1}(E) = \frac{(E-E_0)^2}{4\pi} \sum_j (-)^{j+1} \begin{Bmatrix} 1 & 1 & J \\ 1 & 1 & j \end{Bmatrix} F_j(E)\,,$$

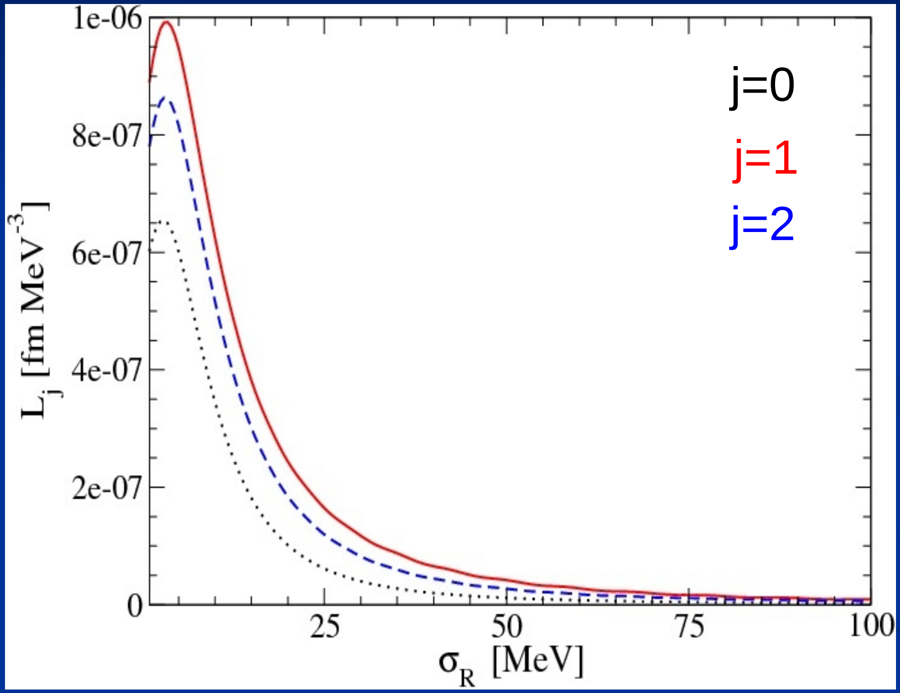
Then one has the following polarizabilities

$$(P^{\rm res}_J(E1,k))_{Im} \;=\; -6\pi^2\, F^{11;j}_{E1,E1}(k+E_0)$$

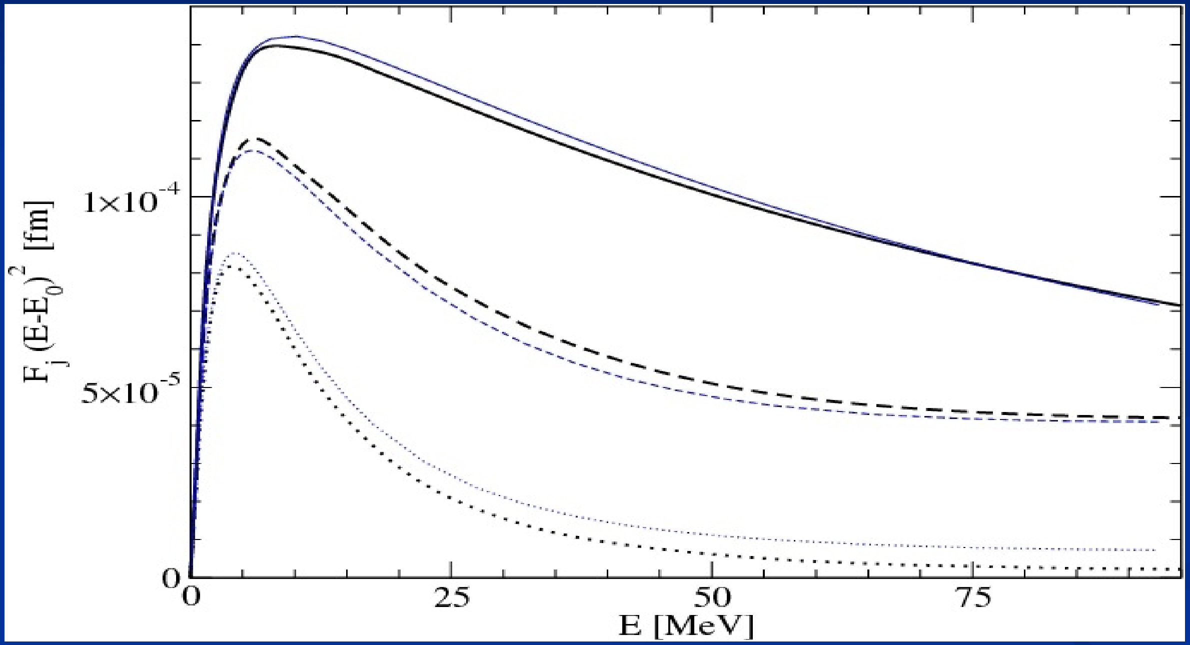
$$(P^{\rm res}_J(E1,k))_{Re} \;=\; \frac{1}{\pi}\mathcal{P}\int dk'\,(P^{\rm res}_J(E1,k'))_{Im}\left(\frac{1}{k'-k}+\frac{(-)^J}{k'+k}\right).$$

Following results are obtained with Argonne v18 potential

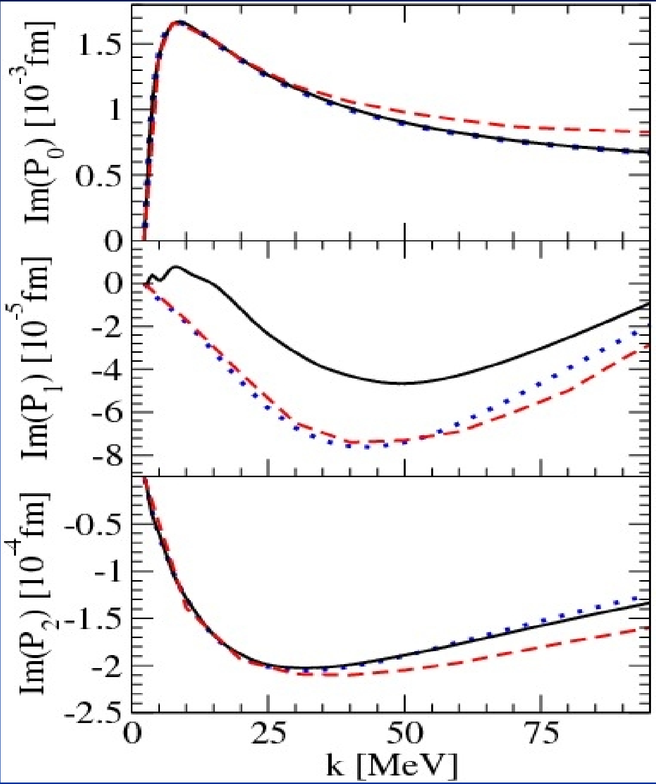
Results for the LITs with $\sigma_I = 5$ MeV

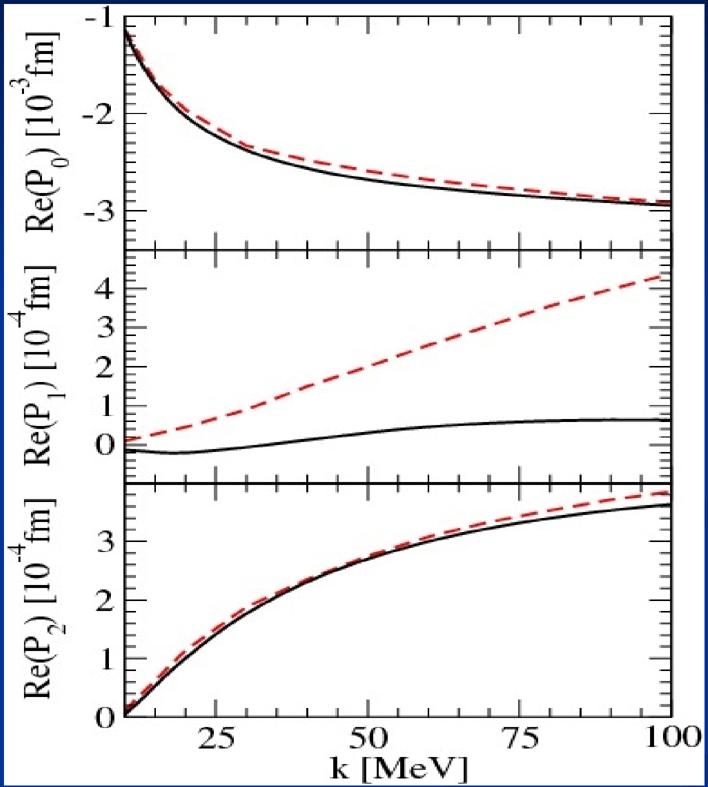


Comparison of functions F_j with standard calculation for full E1-operator



Results for E1-E1 polarizabilities

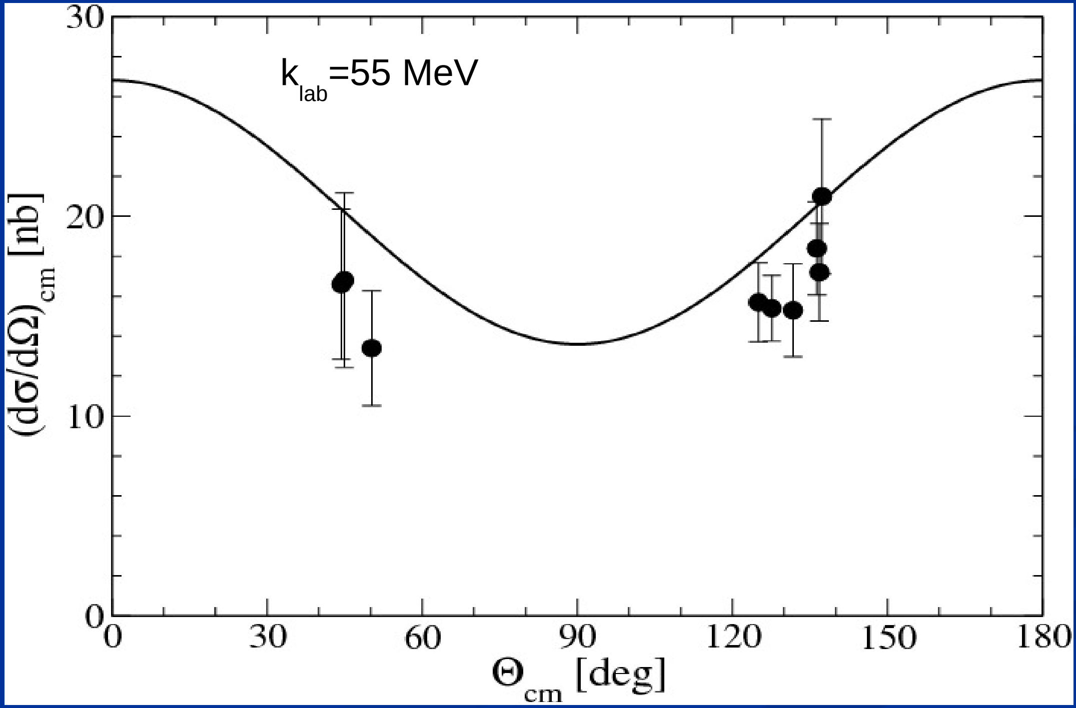




solid black lines: LIT results
dotted blue line: full E1-operator
dashed red line: inclusion of MEC (M. Weyrauch, H. Arenhövel, NPA 408, 425 (1983))

Cross section result

Note real parts of polarizabilities are normalized for k = 0 to obtain the correct low-energy result, i.e. classical Thomson limit for $J = 0$ and $\mathrm{Re}(P_2(k{=}0)) = 0$ (implicit consideration of MEC contribution in both cases)



Δ degrees of freedom in ^{3}He(e,e')

With the LIT method

more details in

L. Yuan, WL, V.D. Efros, G. Orlandini, E.L. Tomusiak, PLB 706, 90

L. Yuan, V.D. Efros, WL, E.L. Tomusiak, PRC 82, 054003

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta \quad (*)$$

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$

$$= H_\Delta$$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ $(V_{N\Delta})$ transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$\Psi_\Delta = -(H_\Delta - E)^{-1}\,V_{N\Delta,NN}\,\Psi_N \quad \textbf{(IA)}$$

$$(T_N + V_{NN} - V_{NN,N\Delta}(H_\Delta - E)^{-1}\,V_{N\Delta,NN} - E)\,\Psi_N = 0 \quad \textbf{(**)}$$

$$\cong V_{NN}^{realistic}$$

Step 1: solve (**) with realistic V_{NN} +3NF

Step 2: solve Ψ_Δ in IA

LIT equation with Δ degrees of freedom

$$\tilde{\Psi} = \tilde{\Psi}_N + \tilde{\Psi}_\Delta$$

$$(T_N + V_{NN} - \sigma)\, \tilde{\Psi}_N = - V_{NN,N\Delta}\, \tilde{\Psi}_\Delta + O_{NN}\, \Psi_{0,N} + O_{N\Delta}\, \Psi_{0,\Delta}$$

$$(\delta m + T_\Delta + V_{N\Delta} - \sigma)\, \tilde{\Psi}_\Delta = - V_{N\Delta,NN}\, \tilde{\Psi}_N + O_{\Delta N}\, \Psi_{0,N} + O_{\Delta\Delta}\, \Psi_{0,\Delta}$$

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

LIT equation with Δ degrees of freedom

$$\tilde\Psi = \tilde\Psi_N + \tilde\Psi_\Delta$$

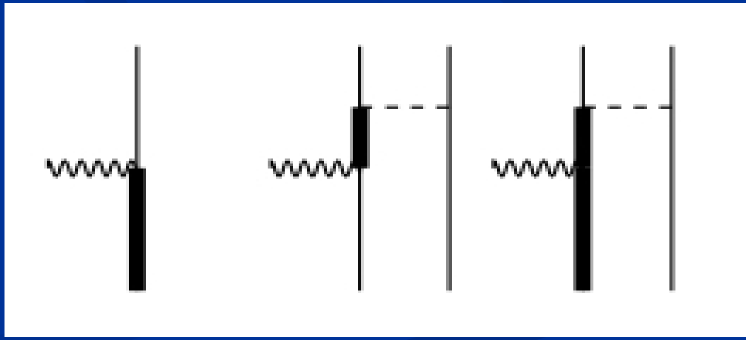
$$(T_N + V_{NN} - \sigma)\,\tilde\Psi_N = -V_{NN,N\Delta}\,\tilde\Psi_\Delta + O_{NN}\,\Psi_{0,N} + O_{N\Delta}\,\Psi_{0,\Delta}$$

$$(\delta m + T_\Delta + V_{N\Delta} - \sigma)\,\tilde\Psi_\Delta = -V_{N\Delta,NN}\,\tilde\Psi_N + O_{\Delta N}\,\Psi_{0,N} + O_{\Delta\Delta}\,\Psi_{0,\Delta}$$

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

We take into account electromagnetic operators with the Δ (Δ-IC) represented by the following graphs



LIT equation with Δ degrees of freedom

$$\widetilde{\Psi} = \widetilde{\Psi}_N + \widetilde{\Psi}_\Delta$$

$$(T_N + V_{NN} - \sigma)\,\widetilde{\Psi}_N = -V_{NN,N\Delta}\,\widetilde{\Psi}_\Delta + O_{NN}\,\Psi_{0,N} + O_{N\Delta}\,\Psi_{0,\Delta}$$

$$(\delta m + T_\Delta + V_{N\Delta} - \sigma)\,\widetilde{\Psi}_\Delta = -V_{N\Delta,NN}\,\widetilde{\Psi}_N + O_{\Delta N}\,\Psi_{0,N} + O_{\Delta\Delta}\,\Psi_{0,\Delta}$$

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$(T_N + V^{\text{realistic}} - \sigma)\,\widetilde{\Psi}_{\mathbf{N}} = -V_{NN,N\Delta}(H_\Delta - \sigma)^{-1}(O_{\Delta N}\,\Psi_{0,N} + O_{\Delta\Delta}\,\Psi_{0,\Delta})$$

$$+ O_{NN}\,\Psi_{0,N} + O_{N\Delta}\,\Psi_{0,\Delta}$$

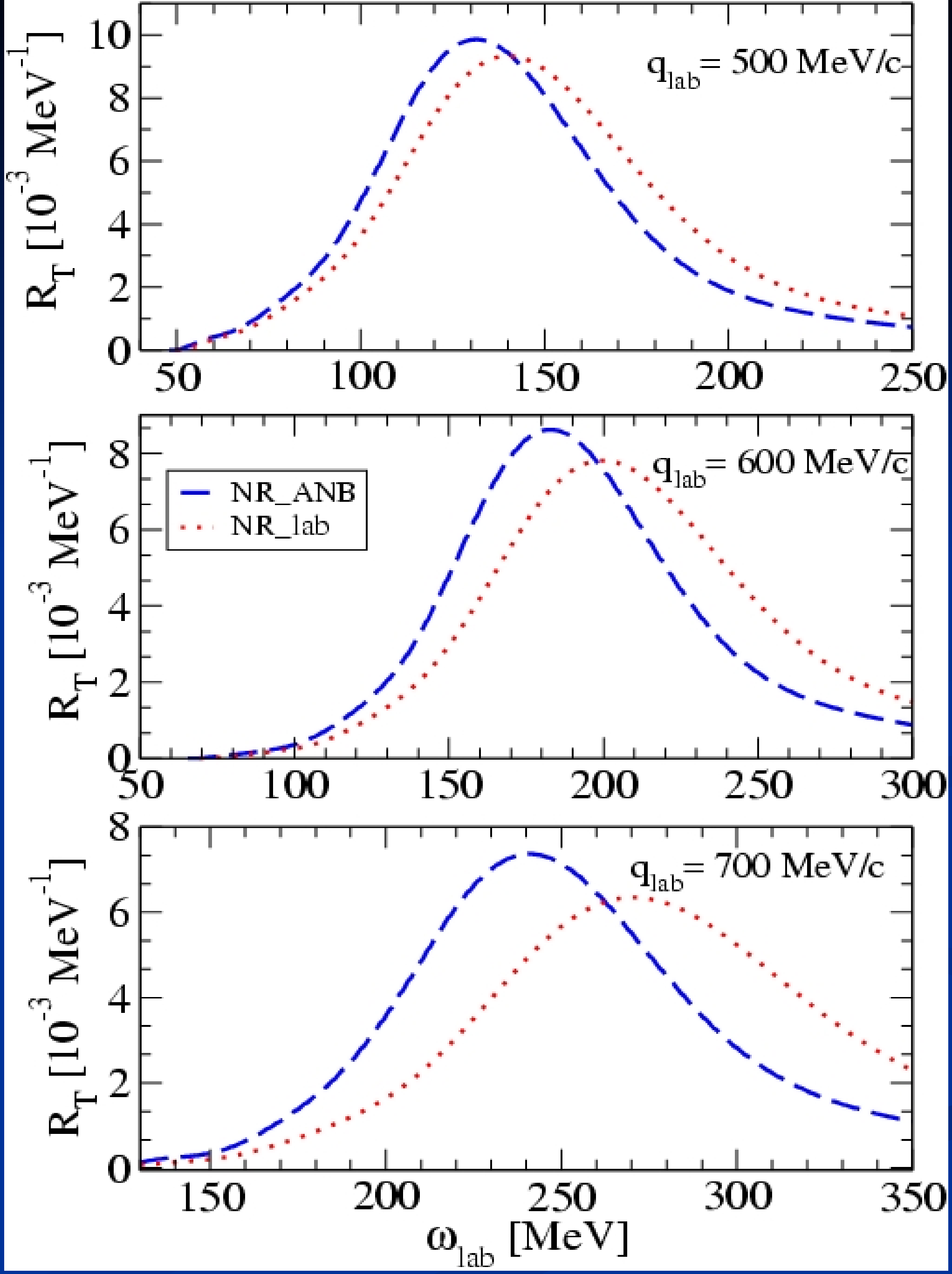
Details for the R_T calculation

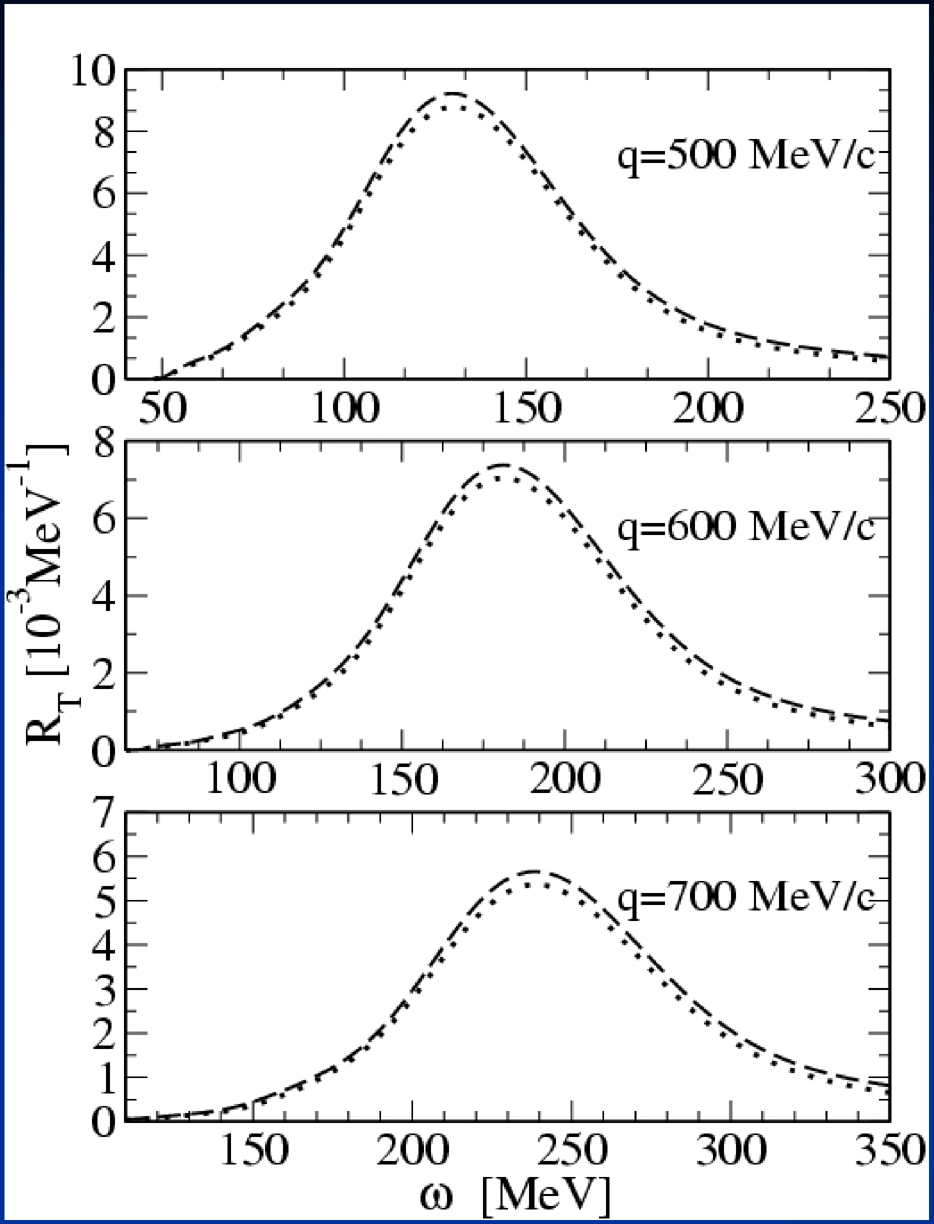
- Full consideration of final state interaction via LIT method
- Nuclear Force model: Argonne V18 two-nucleon potential and Urbana IX three-nucleon force
- Calculation of bound state wave function and solution of LIT equation with help of expansions in correlated hyperspherical harmonics
- Consideration of isovector meson exchange currents consistent with AV18 potential
- Calculation in active nucleon Breit (ANB) frame ($P_T = -Aq/2$) and subsequent transformation to laboratory system
- One-body current operator includes all relativistic corrections up to the order M^{-3} (leading order M^{-1}) as made for deuteron electrodisintegration (F. Ritz et al, PRC 55, 02214)
- Multipole expansion of current (maximal j_f q dependent, e.g, $j_f = 35/2$ for q=700 MeV/c)
- Δ-currents (Δ-IC)

Results

Frame dependence
can be "cured"
in a two-fragment model

- Comparison of ANB and LAB calculation: strong shift of peak to lower energies! (8.7, 16.7, 29.3 MeV at q=500, 600, 700 MeV/c)

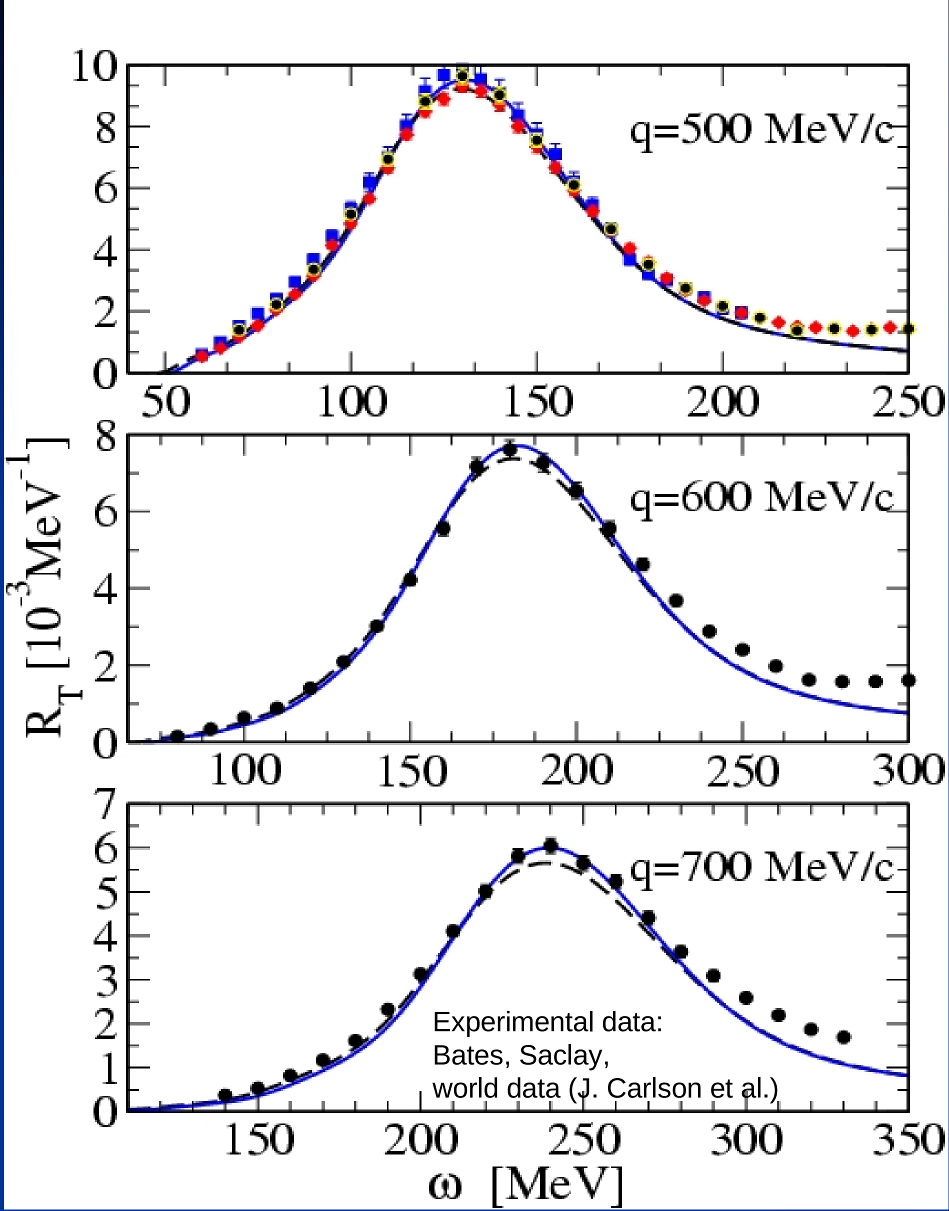




Δ-IC contribution

Dotted: without Δ

Dashed with Δ



Effect of two-fragment model

Dashed: with Δ (as before)

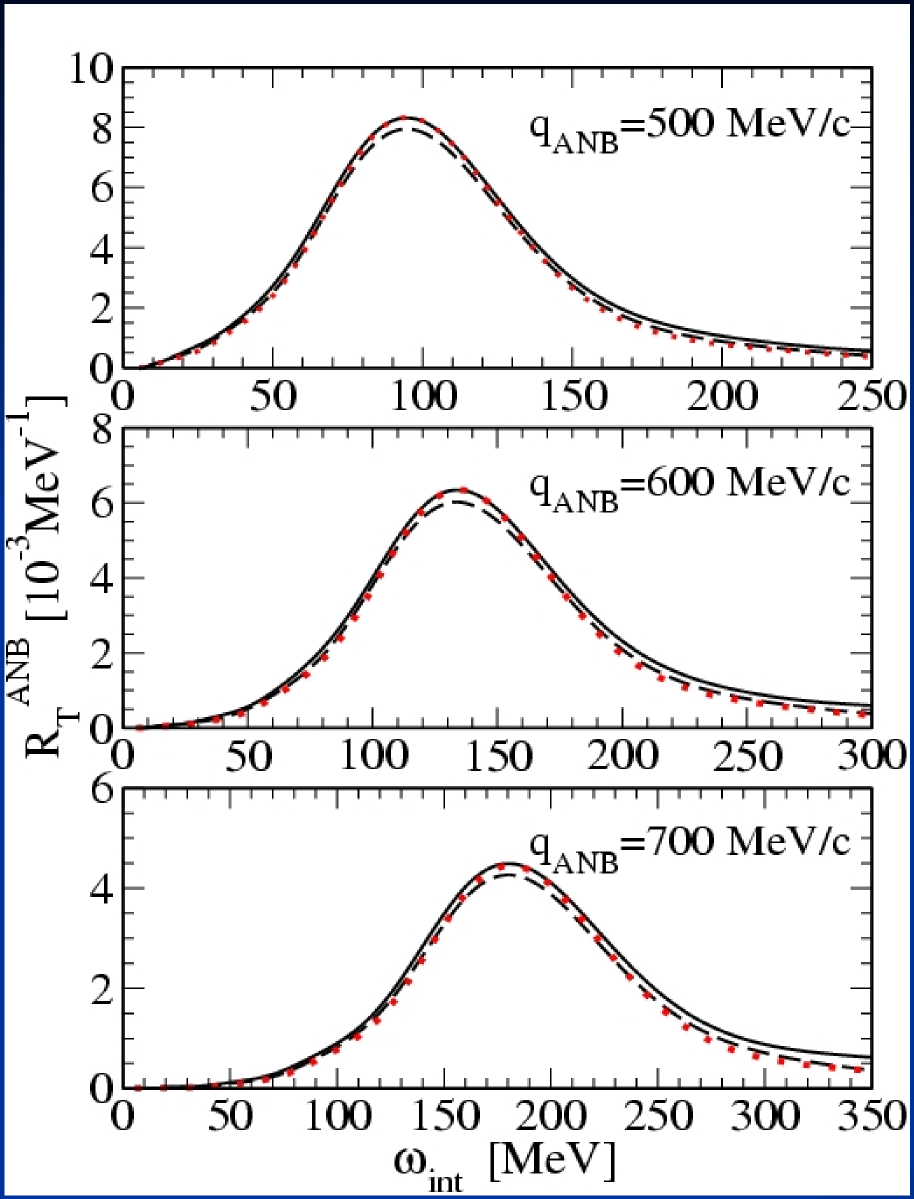
Solid: same but with two-fragment model

Deltuva et al. (PRC70, 034004,2004):

Calculation of R_T of ^{3}He with CDBonn and CDBonn+Δ:

no Δ effects in peak region!

Partial compensation of Δ-IC and 3NF

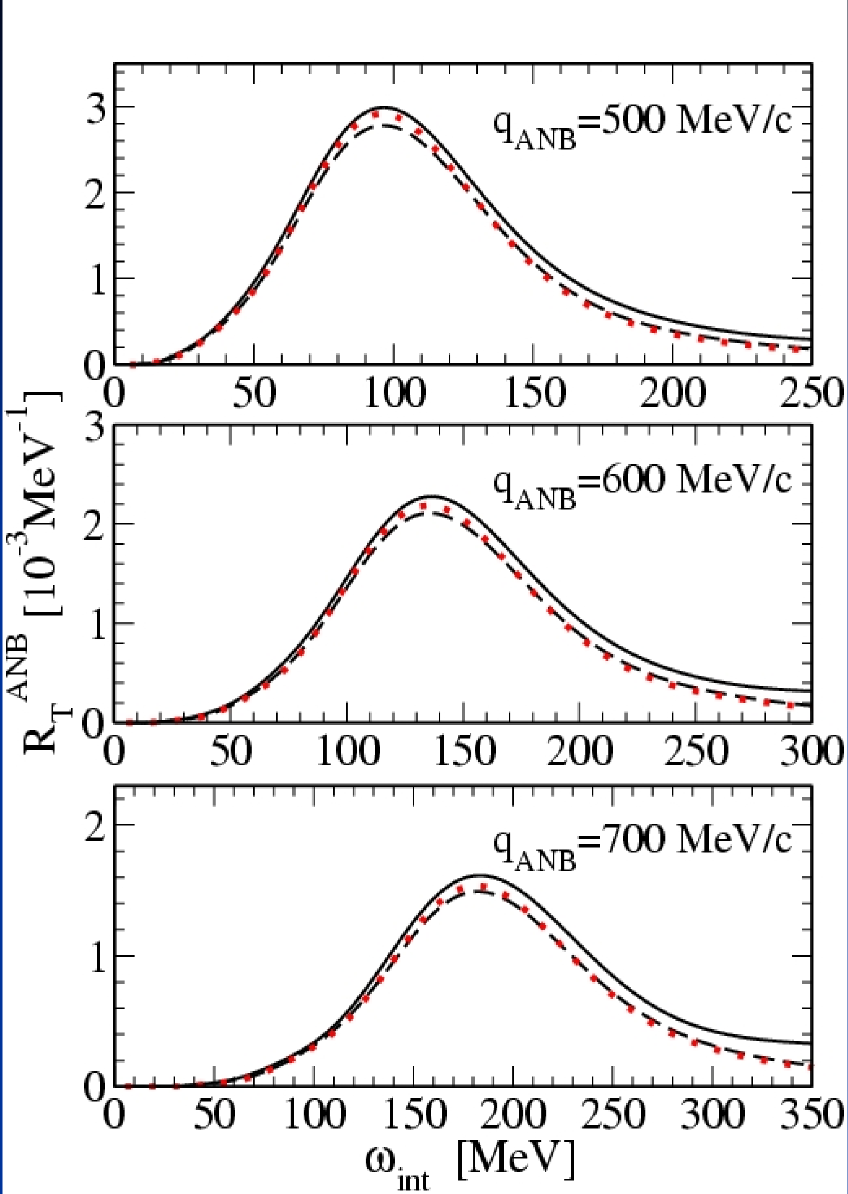


Dotted: no Δ and no 3NF

Dashed: no Δ but with 3NF

Solid: with Δ and with 3NF

No Δ effect in peak region
In a CC calculation!



Only Isospin channel T=3/2

Dotted: no Δ and no 3NF

Dashed: no Δ but with 3NF

Solid: with Δ and with 3NF

Δ-IC contribution larger than 3NF effect in peak region!

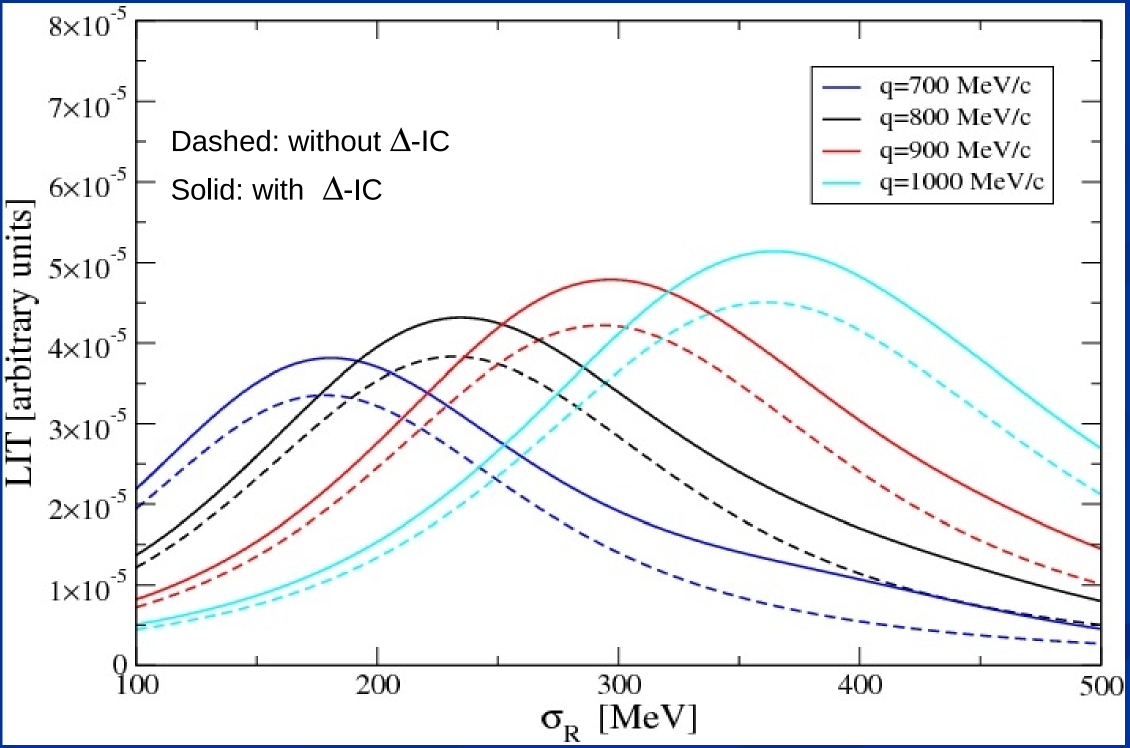
It is interesting to see what happens at even higher q

Presently we are calculating R_T in the range from 700 to 1000 MeV/c

Here only some preliminary results

Preliminary results at higher q

example: Δ-effect on LIT of sum of magnetic multipoles (T=3/2)

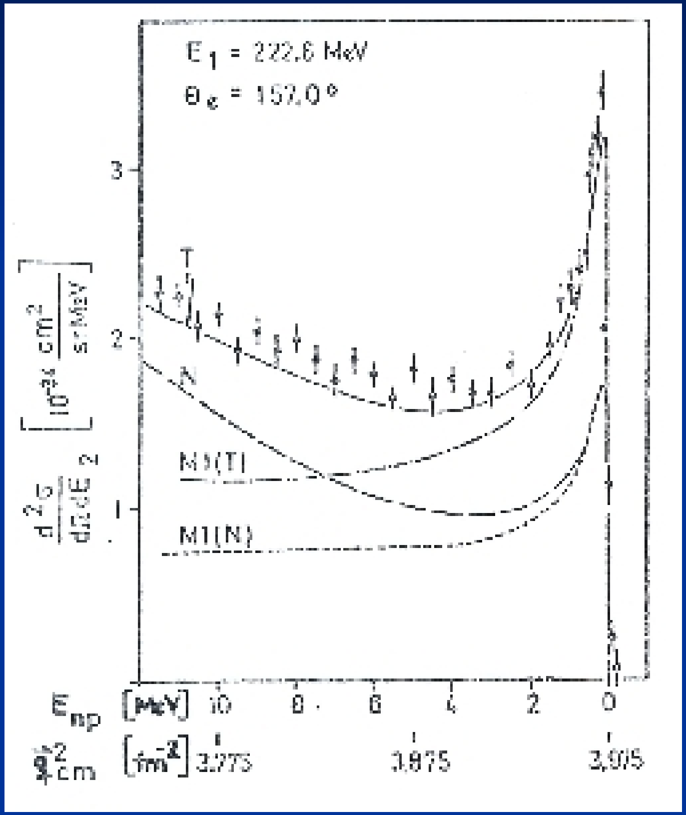


0^+ resonance in longitudinal response function R_L in $^4He(e,e')$ with LIT method

see also calculations of R_L in $^4He(e,e')$ in

S. Bacca, N. Barnea, WL, G.Orlandini, PRL 102, 162501 and PRC 80, 06401

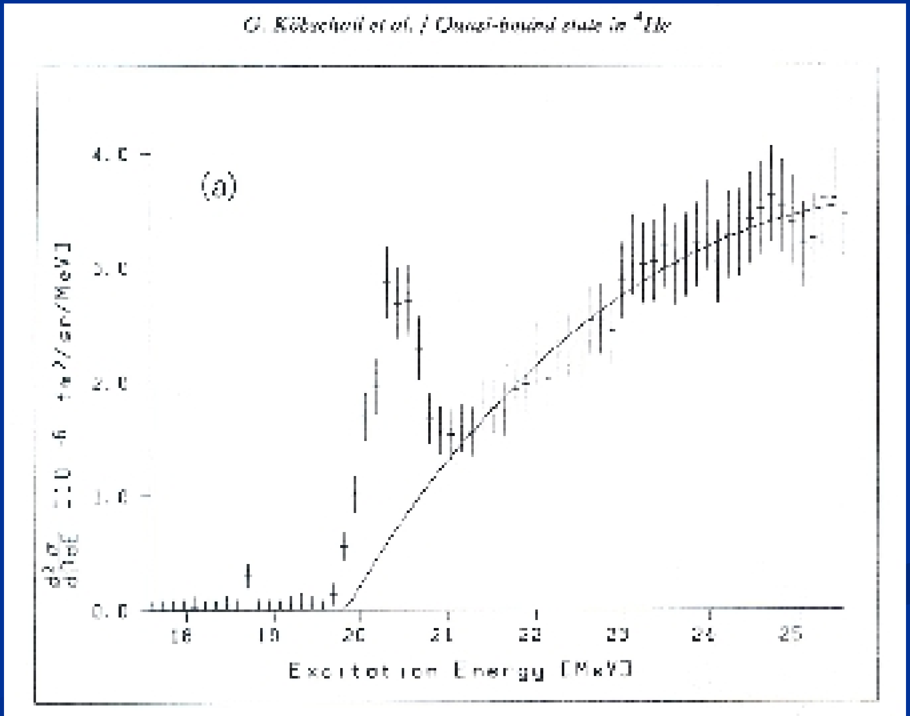
Example: ^{2}H(e,e')



G.G. Simon et al., NPA 324,277 (1979)

0^+ Resonance in the ^{4}He compound system

Resonance at E_R = -8.2 MeV, i.e. above the ^{3}H-p threshold. Strong evidence in electron scattering off ^{4}He



G. Köbschall et al., NPA 405, 648 (1983)

LIT - Inversion

Standard LIT inversion method

Take the following ansatz for the response function $R(\omega)$ (or $F_{fi}(E,E')$)

$$R(\omega') = \sum_{m=1}^{M_{max}} c_m \, \chi_m(\omega',\alpha_i)$$

with $\omega'=\omega-\omega_{th}$, given set of functions χ_m , and unknown coefficients c_m

Define: $$\tilde{\chi}_m(\sigma_R,\sigma_I,\alpha_i) = \int_0^\infty d\omega' \frac{\chi_m(\omega',\alpha_i)}{(\omega'-\sigma_R)^2+\sigma_I^2}$$

Take calculated LIT $L(\sigma_R,\sigma_I) = \langle\tilde{\psi}|\tilde{\psi}\rangle$ for many σ_R and fixed σ_I

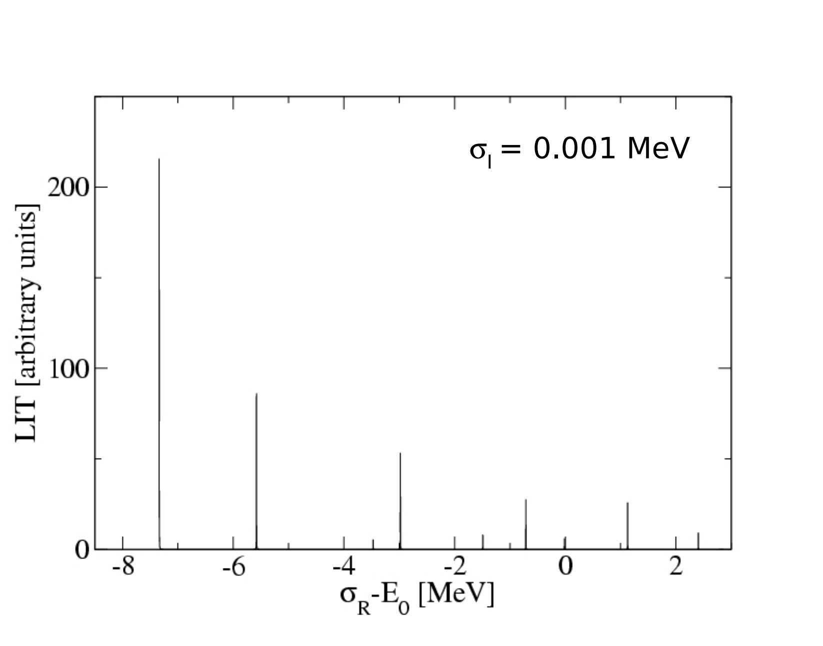
and expand in set $\tilde{\chi}_m$: $L(\sigma_R,\sigma_I) = \sum_{m=1}^{M_{max}} c_m \, \tilde{\chi}_m(\omega',\alpha_i)$

Determine c_m via best fit

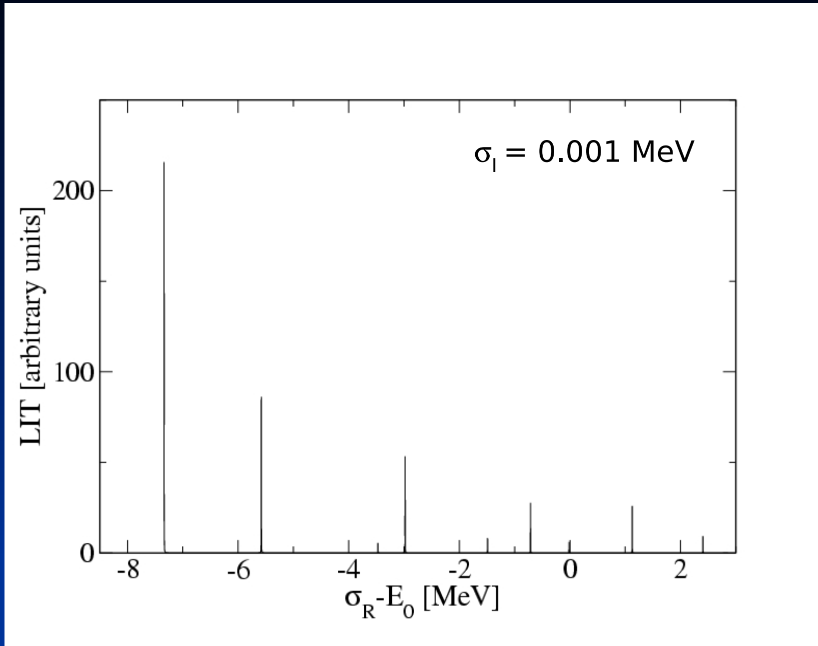
Increase M_{max} up to the point that stable result is obtained for $R(\omega)$. Even further increase of M_{max} might lead to oscillations in $R(\omega)$

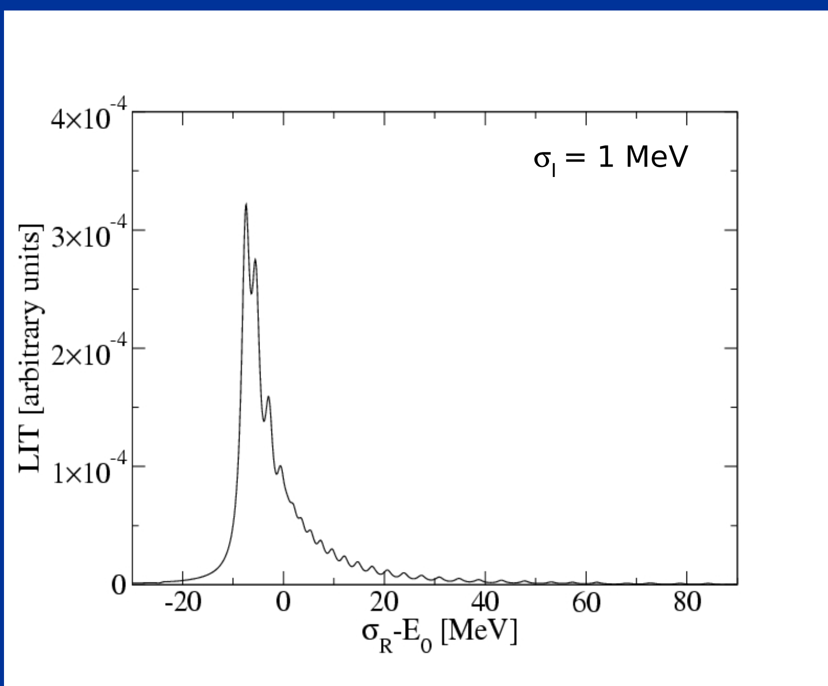
As basis set χ_m we normally use

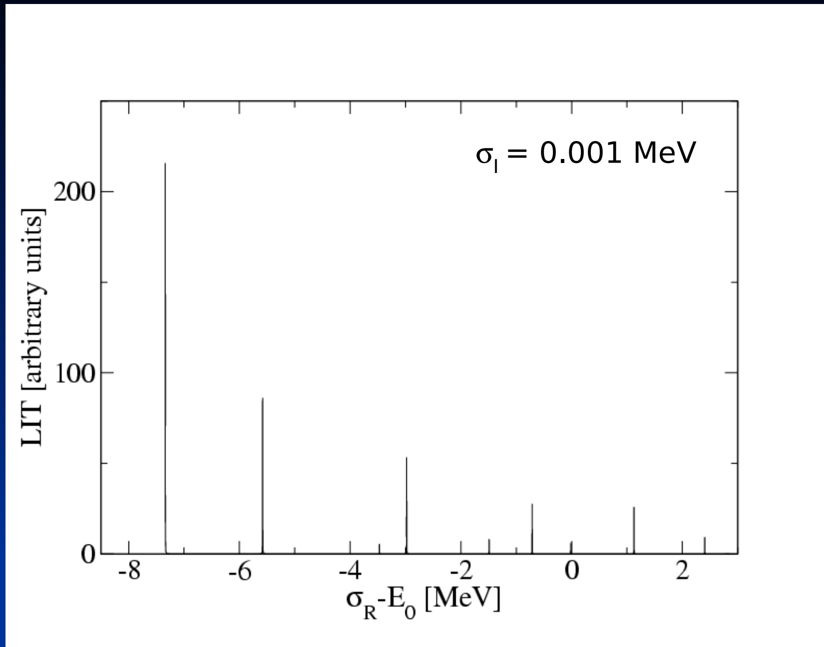
$$\chi_m(\omega',\alpha_i) = (\omega')^{\alpha_1} \exp(-\alpha_2 \omega'/m)$$

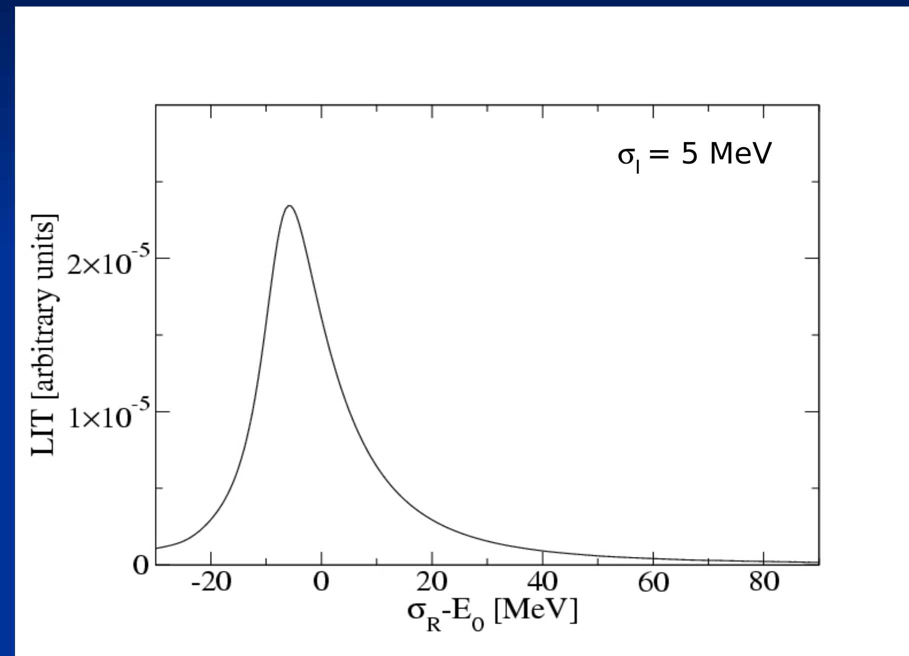


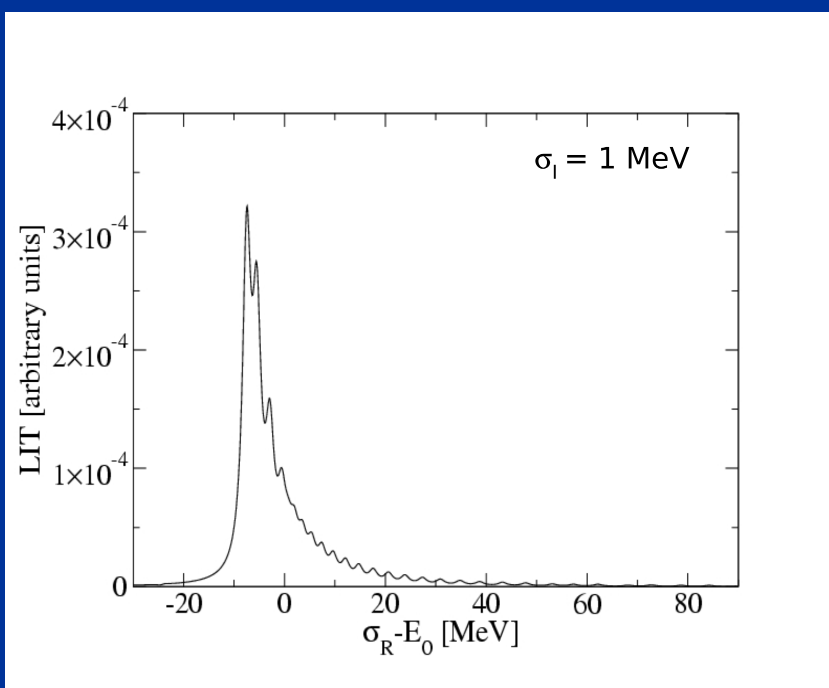
σ_I = 0.001 MeV
LIT [arbitrary units]
0
100
200
-8
-6
-4
-2
0
2
σ_R-E_0 [MeV]


σI = 0.001 MeV
LIT [arbitrary units]
200
100
0
-8
-6
-4
-2
0
2
σR-E0 [MeV]


σI = 1 MeV
LIT [arbitrary units]
4×10-4
3×10-4
2×10-4
1×10-4
0
-20
0
20
40
60
80
σR-E0 [MeV]


σI = 0.001 MeV
LIT [arbitrary units]
0
100
200
-8
-6
-4
-2
0
2
σR-E0 [MeV]


σI = 5 MeV
LIT [arbitrary units]
0
1×10-5
2×10-5
-20
0
20
40
60
80
σR-E0 [MeV]


σI = 1 MeV
LIT [arbitrary units]
0
1×10-4
2×10-4
3×10-4
4×10-4
-20
0
20
40
60
80
σR-E0 [MeV]

The present precision of the calculation does not allow to resolve the shape of the resonance, therefore the width cannot be determined.

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However, the strength of the resonance can be determined!

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Of course not by taking the strength to the discretized state, but by rearranging the inversion in a suitable way:

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However, the strength of the resonance can be determined!

Of course not by taking the strength to the discretized state, but by rearranging the inversion in a suitable way:

Reduce strength to the state up to the point that the inversion does not show any resonant structure at the resonance energy $E_{\mathbf{R}}$:

$$\mathrm{LIT}(\sigma_R, \sigma_I) \rightarrow \mathrm{LIT}(\sigma_R, \sigma_I) - f_R \,/\, [(E_{\mathbf{R}} - \sigma_R)^2 + \sigma_I^2] \equiv \mathrm{LIT}(\sigma_R, \sigma_I, f_R)$$

with resonance strength f_R

Determination of resonance strength f_R

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Include in the inversion a basis function with resonant structure

$$\chi_1(E') = 1 \,/\, [(E_R - E')^2 + \Gamma^2 / 4]$$

and check inversion result.

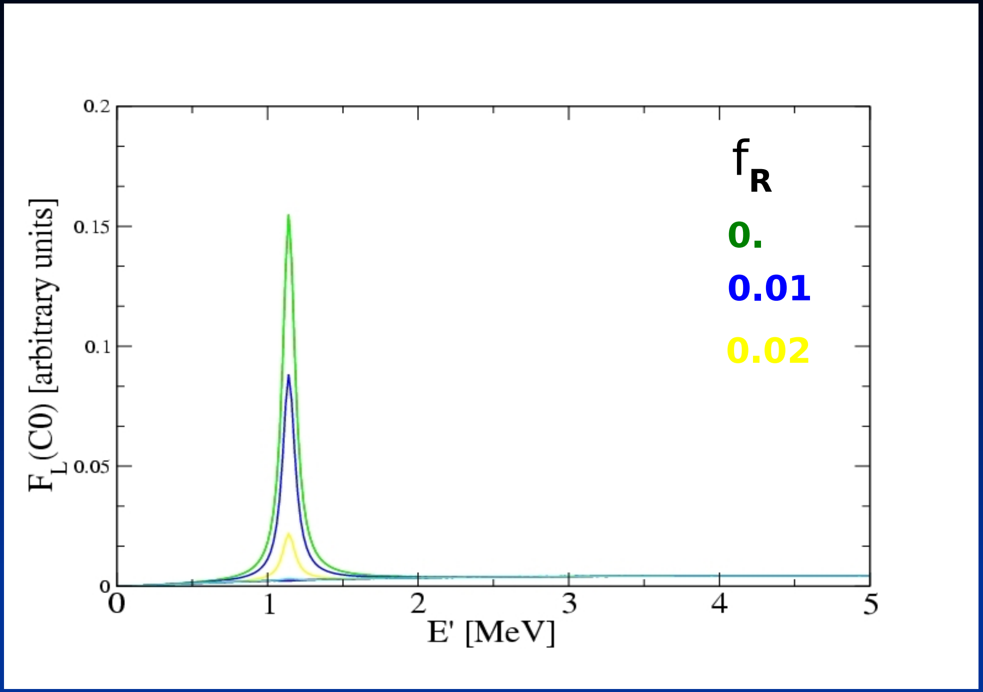
Determination of resonance strength f_R

Include in the inversion a basis function with resonant structure

$$\chi_1(E') = 1 \, / \, [(E_R - E')^2 + \Gamma^2 / 4]$$

and check inversion result.

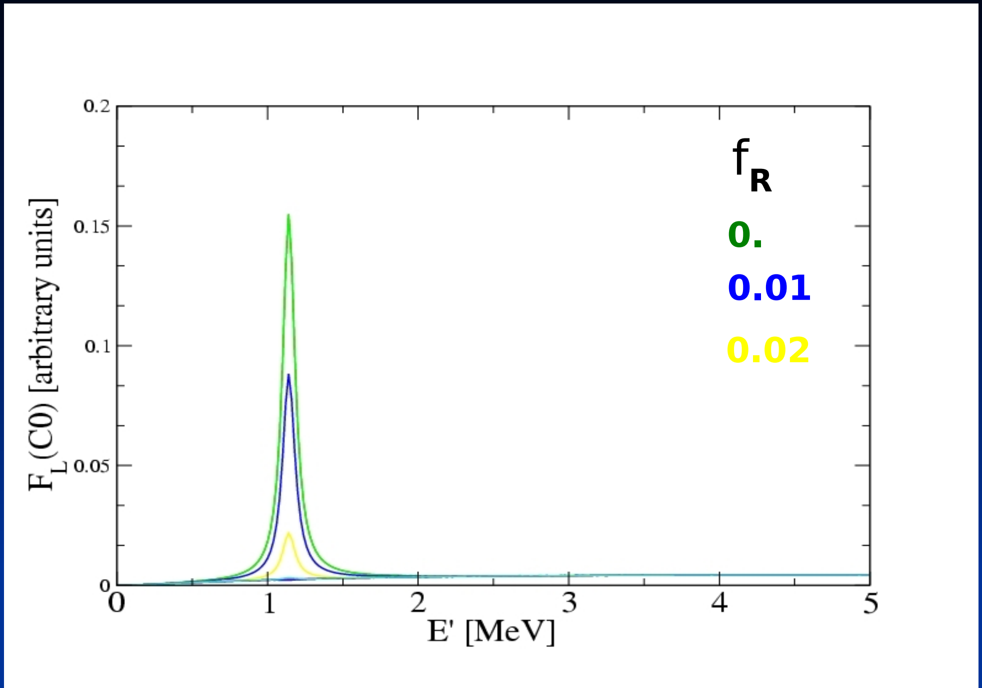
Vary LIT$(\sigma_R, \sigma_I, f_R)$ by changing f_R up to the point that no resonant structure is present. Then f_R corresponds to the resonance strength.



Inversion results with different f_R values

AV18+UIX, q=300 MeV/c

(Γ = 0.1 MeV)



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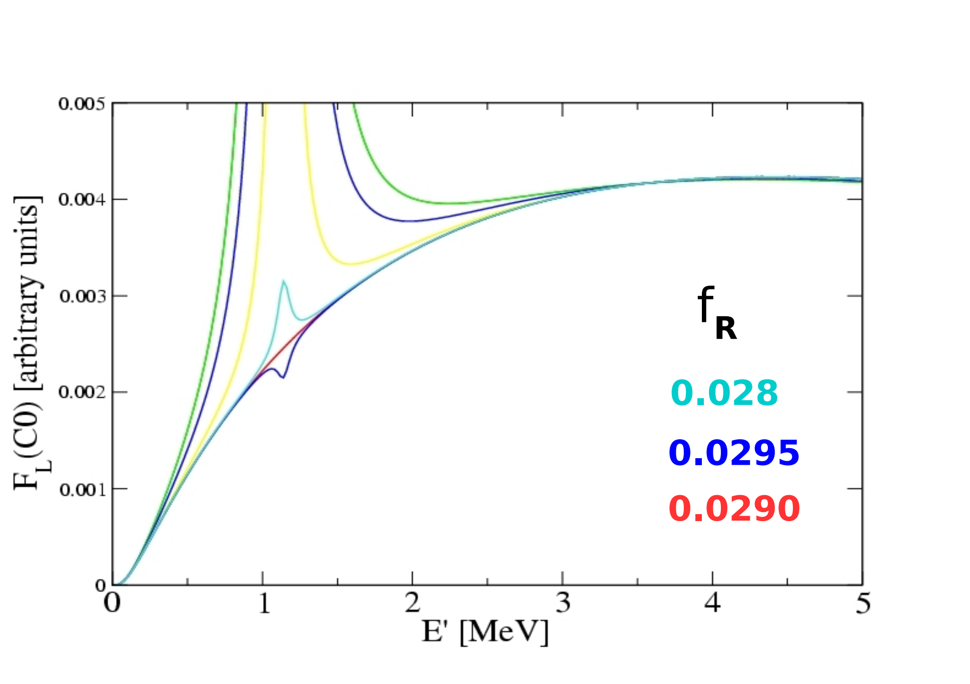

Results for the resonance strength and comparison to experimental data

In Giuseppina's talk on Friday

Density excitation response in bulk atomic ^{4}He at T = 0

with the Sumudu transform

(A.Roggero, F. Pederiva, G.Orlandini)

MONTE CARLO METHODS ARE APPLIED TO CALCULATE

$$\Phi(t) = \int <|\Theta^{\dagger}(t, \mathbf{x})\, \Theta(0, 0)|> d^3\mathbf{x} \;\longrightarrow\; \int e^{-itE}\, S(E)\, dE$$

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$$\downarrow$$

$$e^{-\tau E}$$

Imaginary time $\tau = it$

Laplace kernel

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Imaginary time $\tau = it$

$$\downarrow$$

$$e^{-\tau E}$$

Laplace kernel

In Condensed Matter Physics:

Θ = Density Operator

$S(E)$ = Dynamical Structure Function

In Nuclear Physics:

Θ= Charge or current density operator

$S(E) = R(E)$ "Response" Function

In QCD

Θ = quark operators

$S(E)$ = Hadronic Spectral Function

A good kernel for Monte Carlo methods:

(A.Roggero, F. Pederiva, G.Orlandini 2012)

combination of Sumudu kernels:

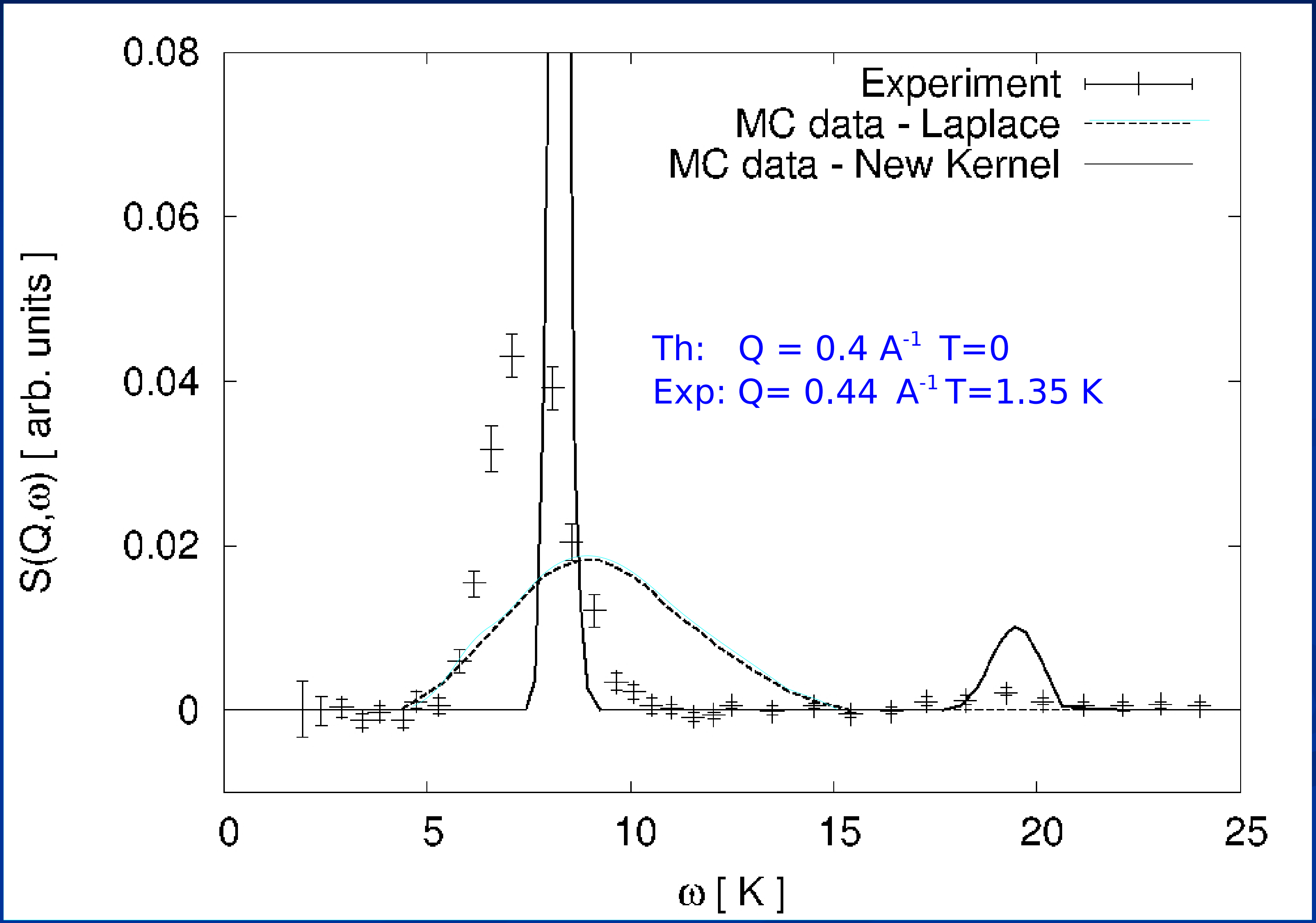
$$K_P(\omega , \sigma) = N\, (e^{-\mu\, \omega/\sigma} - e^{-\nu\, \omega/\sigma})^P$$

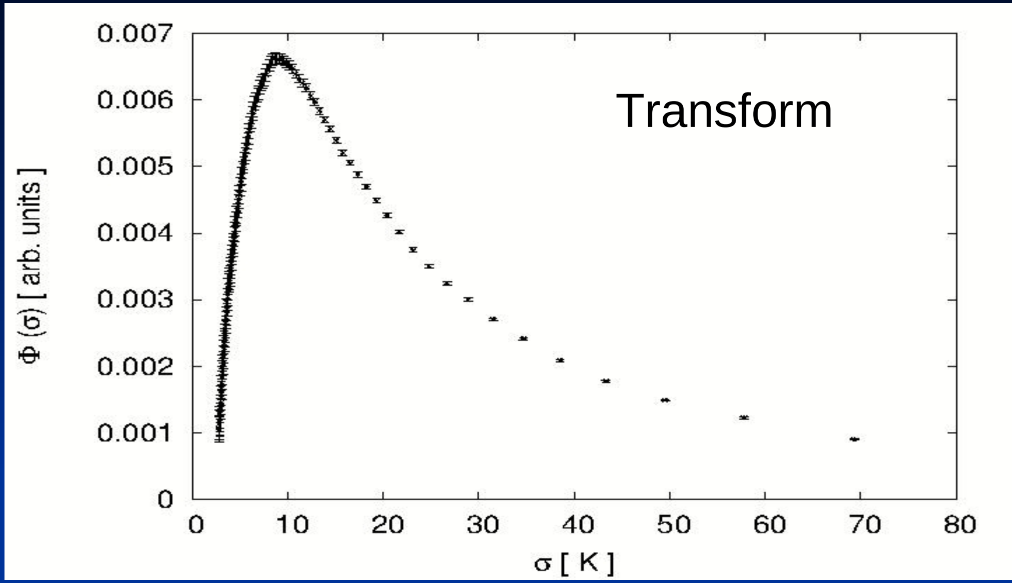
$\nu/\mu = b/a$ $\nu - \mu = (\mathit{ln}\,[b] - \mathit{ln}\,[a])/(b-a)$

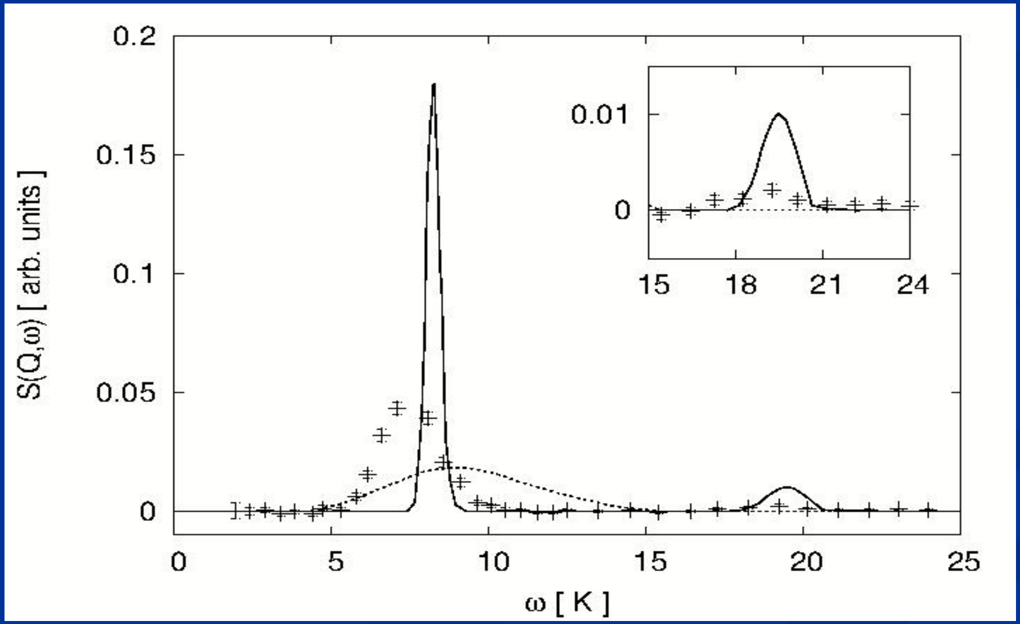
b > a >0 integer

$$K_P(\omega , \sigma) \xrightarrow[P \to \infty]{} \delta\,(\omega - \sigma)$$

Density excitation response in bulk atomic ^{4}He at T = 0




Transform
Φ (σ) [arb. units]
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0 10 20 30 40 50 60 70 80
σ [K]


S(Q,ω) [arb. units]
0.2
0.15
0.1
0.05
0
0 5 10 15 20 25
ω [K]
0.01
0
15 18 21 24