Carbon on the lattice: From graphene to the anthropic principle

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Light Nuclei from First Principles (Program INT 12-3)
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Topics to be covered

JÜLICH FORSCHUNGSZENTRUM

- **Hybrid Monte Carlo vs. Metropolis**

  Theories with dynamical fermions

  Algorithms for global Monte Carlo updates

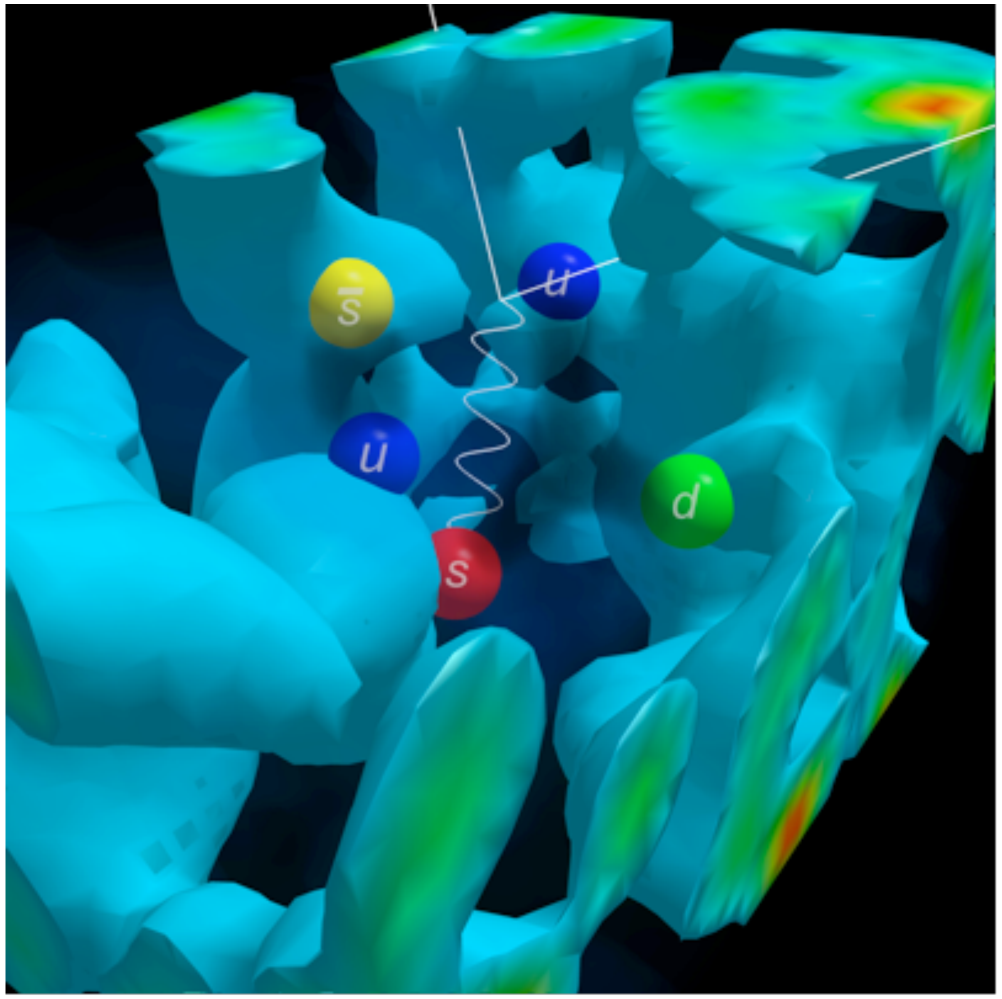
- **Two examples of applications**

  Dirac theory of graphene

  Carbon production in stars and the anthropic principle

I.

HYBRID MONTE CARLO: DYNAMICAL FERMIONS



Monte Carlo for dynamical fermions

- **Dynamical fermions:** Monte Carlo evaluation of path integrals, repeated computation of determinants ...

$$\mathcal{Z} = \int \mathcal{D}A_0 \, \underline{\exp(-S_{\text{eff}}[A_0])} \qquad S_{\text{eff}}[A_0] = -N_f \ln\det(D[A_0]) + S_E^g[A_0]$$

Positive definite probability measure
for MC calculation

$$\sigma = \frac{1}{V\mathcal{Z}} \int \mathcal{D}A_0 \, \mathrm{Tr}(D^{-1}[A_0]) \exp(-S_{\text{eff}}[A_0])$$

$$\langle \bar{\psi}_b \psi_b \rangle = \frac{1}{V} \langle \mathrm{Tr}\left[D^{-1}[A_0]\right] \rangle$$

- **Metropolis algorithm:** evolution via random lattice updates, changes accepted with probability p ...

$$p \equiv \frac{P[\theta']}{P[\theta]} = \exp(-\Delta S) \qquad \Delta S = S_{\text{eff}}[\theta'] - S_{\text{eff}}[\theta]$$

Problem: large random updates give a vanishingly small acceptance rate - only **local** updates possible!

$t_{\text{metropolis}} \sim V^3$

Global Hybrid Monte Carlo (HMC) updates

- **Step I:** Introduce random Gaussian noise:
  Physics unaffected ...

$$H = \sum_{\mathbf{n}} \frac{\pi_{\mathbf{n}}^2}{2} + S_E[\theta]$$

- **Step II:** Classical Hamiltonian dynamics:
  Global evolution of field and conjugate momentum ...

$$(\theta_{\mathbf{n}}, \pi_{\mathbf{n}}) \rightarrow (\theta'_{\mathbf{n}}, \pi'_{\mathbf{n}}) \qquad \dot{\theta}_i = \frac{\partial H[\theta,\pi]}{\partial \pi_i}, \qquad \dot{\pi}_i = -\frac{\partial H[\theta,\pi]}{\partial \theta_i}$$

- **Step III:** H should be conserved, however:
  Finite stepsize in numerical integration ...

$$p \equiv \exp(-\Delta H), \quad \Delta H \equiv H[\theta'] - H[\theta]$$

- **Step IV:** Refresh the random Gaussian noise
  and return to step II ...

Repeat until a sufficient # of decorrelated configurations are obtained

"Hybrid" algorithm: Correct for non-conservation of H with a Metropolis step

S. Duane *et al.*,
Phys. Lett. B **195**, 216 (1987)

Determinantal Hybrid Monte Carlo (DHMC)

- Application of HMC to problems with dynamical fermions:

$$\mathcal{Z} = \int \mathcal{D}A_0 \, \exp(-S_{\mathrm{eff}}[A_0]) \qquad S_{\mathrm{eff}}[A_0] = -N_f \ln\det(D[A_0]) + S_E^g[A_0]$$

- Fields and momenta are repeatedly evolved using the HMC equations of motion:

$$\dot{\theta}_i = \frac{\partial H[\theta,\pi]}{\partial \pi_i}, \qquad \dot{\pi}_i = -\frac{\partial H[\theta,\pi]}{\partial \theta_i}$$

How to deal with the "fermion force term"?

Inverse (may be) extremely costly !!

$$\frac{\partial \det(K[\lambda])}{\partial \lambda} = \det(K[\lambda]) \, \mathrm{Tr}\left(K^{-1}[\lambda] \frac{\partial K}{\partial \lambda}\right)$$

- DHMC is feasible if the size of the fermion operator is small:
  - Ultracold Fermi gasis
  - Chiral EFT for light nuclei

typically: $t_{\mathrm{DHMC}} \sim V^2$

HMC + pseudofermions (ϕ-algorithm)

- If the inverse of the fermion operator is large (for example the size of the space-time lattice), DHMC is unworkable. Introduce **pseudofermions:**

$$\det(Q) \propto \int \mathcal{D}\phi^\dagger \mathcal{D}\phi \, \exp(-S_E^p) \qquad S_E^p = \sum_{\mathbf{n},\mathbf{m}} \phi_{\mathbf{n}}^\dagger \, Q_{\mathbf{n},\mathbf{m}}^{-1}[\theta] \, \phi_{\mathbf{m}} = \sum_{\mathbf{n}} \xi_{\mathbf{n}}^\dagger \xi_{\mathbf{n}}$$

A "stochastic" evaluation of the fermion determinant:
- sample \phi from Gaussian noise \xi

- HMC + pseudofermions = **ϕ-algorithm:**
  - Lattice QCD
  - QED(2+1), Thirring, graphene

$$H = \sum_{\mathbf{n}} \frac{\pi_{\mathbf{n}}^2}{2} + S_E^g + S_E^p$$

$$\dot{\theta}_{\mathbf{n}} = \frac{\delta H}{\delta \pi_{\mathbf{n}}} = \pi_{\mathbf{n}},$$

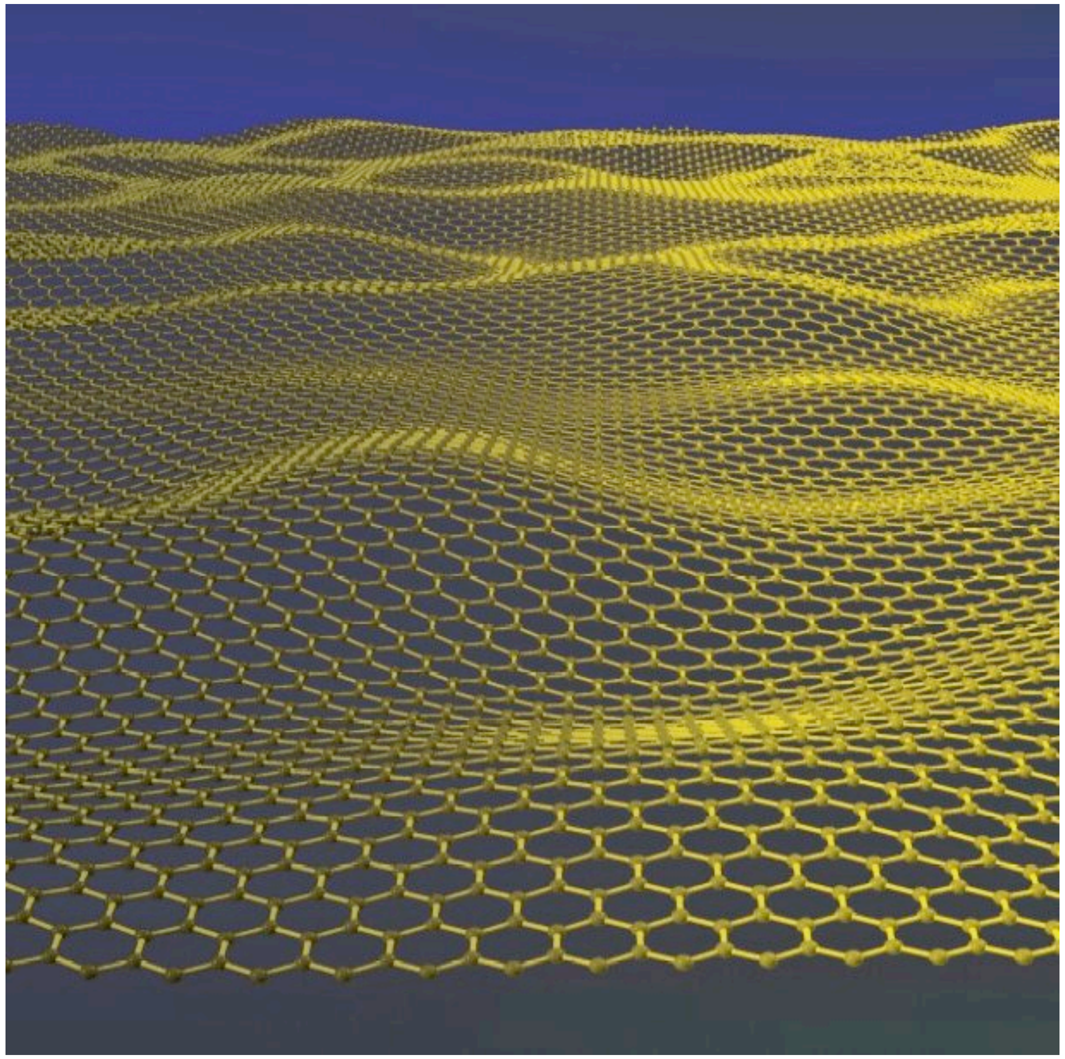
$$\dot{\pi}_{\mathbf{n}} = -\frac{\delta H}{\delta \theta_{\mathbf{n}}} \equiv F_{\mathbf{n}}^g + F_{\mathbf{n}}^p$$

$t_\phi \sim V^{5/4}$

Exactness of HMC preserved by pseudofermions

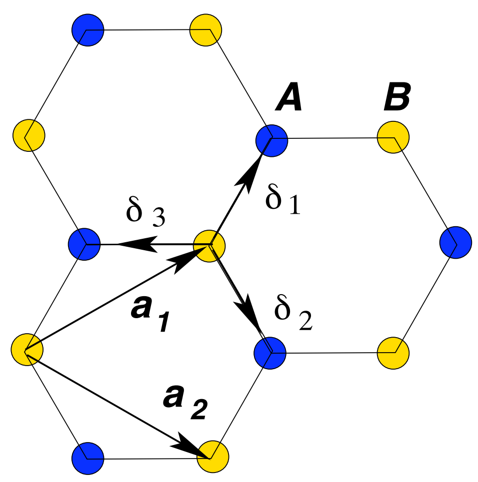
2.

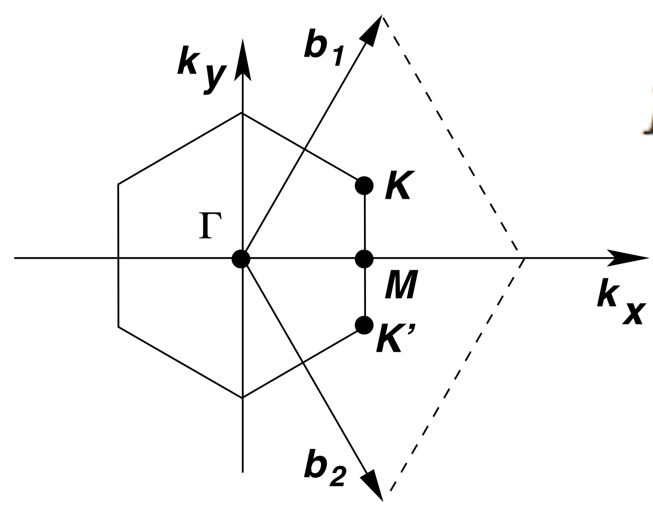
LATTICE MONTE CARLO: GRAPHENE



Electronic band structure of graphene

Hexagonal lattice of carbon atoms



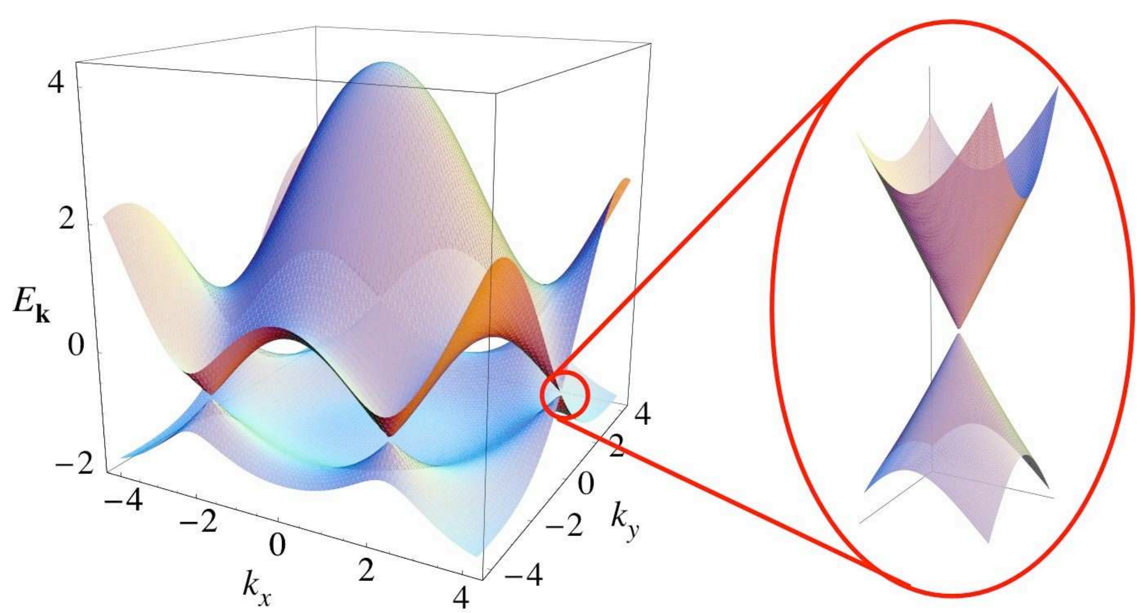


$$H = -t \sum_{\langle i,j\rangle, \sigma=\uparrow,\downarrow} \left(a^\dagger_{\sigma,i} b_{\sigma,j} + \text{H.c.} \right)$$
$$-t' \sum_{\langle\langle i,j\rangle\rangle, \sigma=\uparrow,\downarrow} \left(a^\dagger_{\sigma,i} a_{\sigma,j} + b^\dagger_{\sigma,i} b_{\sigma,j} + \text{H.c.} \right)$$

In the vicinity of a "Dirac point":
Emergent "relativistic" behavior

G. Semenoff,
Phys. Rev. Lett. **54**, 2449 (1984)

J. E. Drut, D. T. Son,
Phys. Rev. B **77**, 075115 (2008)



$$\psi_G = \begin{pmatrix} \psi_{KA} \\ \psi_{KB} \\ \psi_{K'A} \\ \psi_{K'B} \end{pmatrix}$$

$$E_k \simeq vk$$

$$\boxed{v \simeq c/300}$$

Dirac theory of interacting electrons in graphene

$$S_E = -\sum_{a=1}^{N_f} \int d^2x\, dt\, \bar{\psi}_a\, D[A_0]\, \psi_a + \frac{1}{2g^2} \int d^3x\, dt\, (\partial_i A_0)^2$$

$$D[A_0] = \gamma_0(\partial_0 + iA_0) + v\gamma_i\partial_i, \quad i = 1,2$$

Content of theory:

- Dynamical fermions (in 2+1 dimensions)
- Gauge field (single component in 3+1 dimensions)

$$\mathcal{Z} = \int \mathcal{D}A_0 \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{-S_E[\bar{\psi}_a, \psi_a, A_0]} = \int \mathcal{D}A_0\, e^{-S_E^g[A_0]} (\det[D[A_0]])^{N_f}$$

Non-perturbative region:
"graphene fine-structure constant"

$$g^2 = e^2/\epsilon_0$$

$$\alpha_g \equiv \frac{e^2}{4\pi\epsilon_0\hbar v} \simeq 300\alpha \sim 1$$

Staggered fermions à la Lattice QCD

- Gauge action:
  (\theta = lattice gauge field, \beta = bare lattice coupling)

$$S_E^g[\theta_0] = \frac{\beta}{2} \sum_n \left[\sum_{i=1}^{3} \left(\theta_{0,n} - \theta_{0,n+\hat{e}_i}\right)^2 \right]$$

Spatial lattice volume Lx^3, Lt steps in time dimension

- Staggered fermion action (with bare mass term):

$$S_E^f[\bar{\chi}, \chi, U] = -\sum_{\mathbf{n},\mathbf{m}} \bar{\chi}(\mathbf{n}) D_s[U, \mathbf{n}, \mathbf{m}] \chi(\mathbf{m})$$

$$D_s[U,\mathbf{n},\mathbf{m}] = \frac{1}{2}(\delta_{\mathbf{n}+\mathbf{e}_0,\mathbf{m}} U(\mathbf{n}) - \delta_{\mathbf{n}-\mathbf{e}_0,\mathbf{m}} U^\dagger(\mathbf{m})) + \frac{v}{2} \sum_i \eta^i(\mathbf{n})(\delta_{\mathbf{n}+\mathbf{e}_i,\mathbf{m}} - \delta_{\mathbf{n}-\mathbf{e}_i,\mathbf{m}}) + m_0 \delta_{\mathbf{n},\mathbf{m}}$$

- Gauge links, staggered phases:

$$U(\mathbf{n}) = \exp\{i\theta(\mathbf{n})\}$$

$$\begin{aligned} \eta^0(\mathbf{n}) &= 1 \\ \eta^1(\mathbf{n}) &= (-1)^{n_0} \\ \eta^2(\mathbf{n}) &= (-1)^{n_0+n_1} \end{aligned}$$

Fermion doubling problem solved for Nf = 2

Other possibilities:
overlap fermions, hexagonal lattice ...

Calculational strategy on the Lattice

Step I: The bare (input) parameters are:
- the lattice coupling \beta
- the fermion mass m0
- in principle also the number of flavors Nf

Phase diagram (chiral condensate / physical mass) as a function of (\beta, m0)

Step II: Physical predictions: where in the phase diagram is physical graphene located?
- use observed vF to fix lattice \beta

If (for example) a gap is observed, the physical lattice spacing can be determined
- scale can be set for dimensionful quantities

Step III: Compute more difficult observables, such as response functions
- conductivity and viscosity of the electrons in graphene

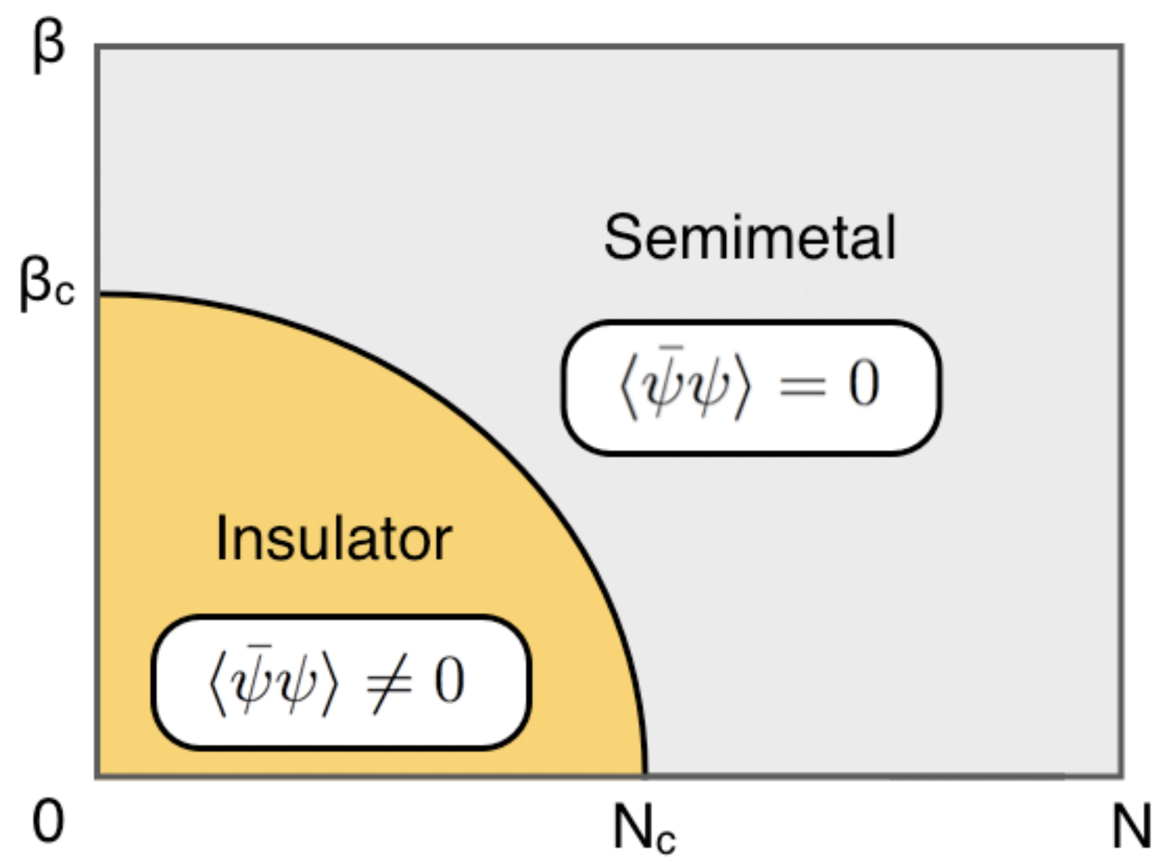
The scenario of spontaneous gap formation

- Compute the chiral condensate (and susceptibility) as a function of \beta and m0 ...

$$\langle \bar{\psi}_b \psi_b \rangle = \frac{1}{V} \langle \mathrm{Tr} \left[D^{-1}[A_0] \right] \rangle$$

$$\chi_{\bar{\psi}\psi} = \frac{1}{V} \left[\langle \mathrm{Tr}^2 \left[D^{-1} \right] \rangle - \langle \mathrm{Tr} \left[D^{-2} \right] \rangle - \langle \mathrm{Tr} \left[D^{-1} \right] \rangle^2 \right]$$

- Zero-temperature phase diagram:
  - Critical coupling \beta_c
  - Critical number of flavors N_c

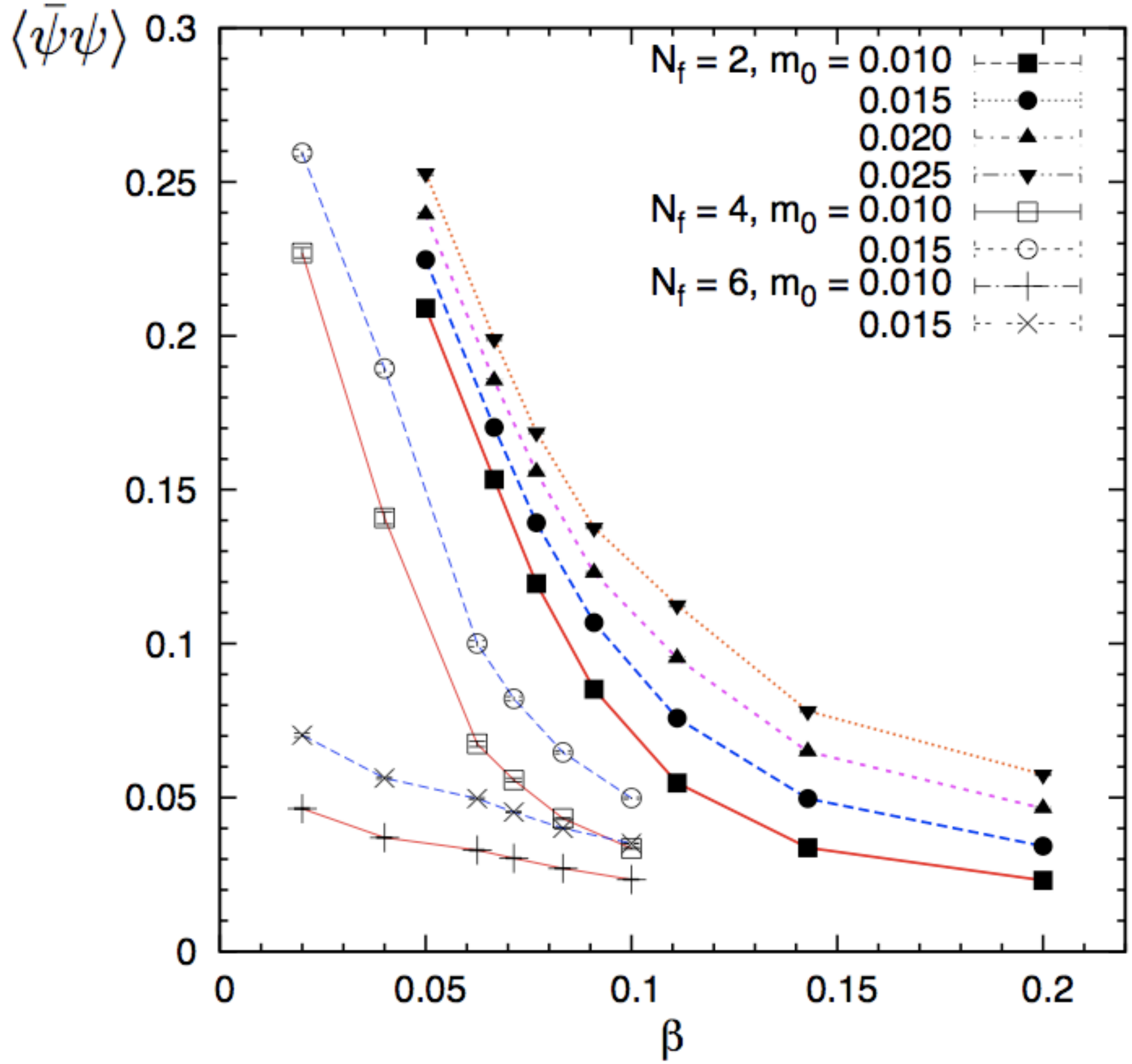


Chiral condensate (Metropolis algorithm)

First results on small lattices (L = 16 cube)

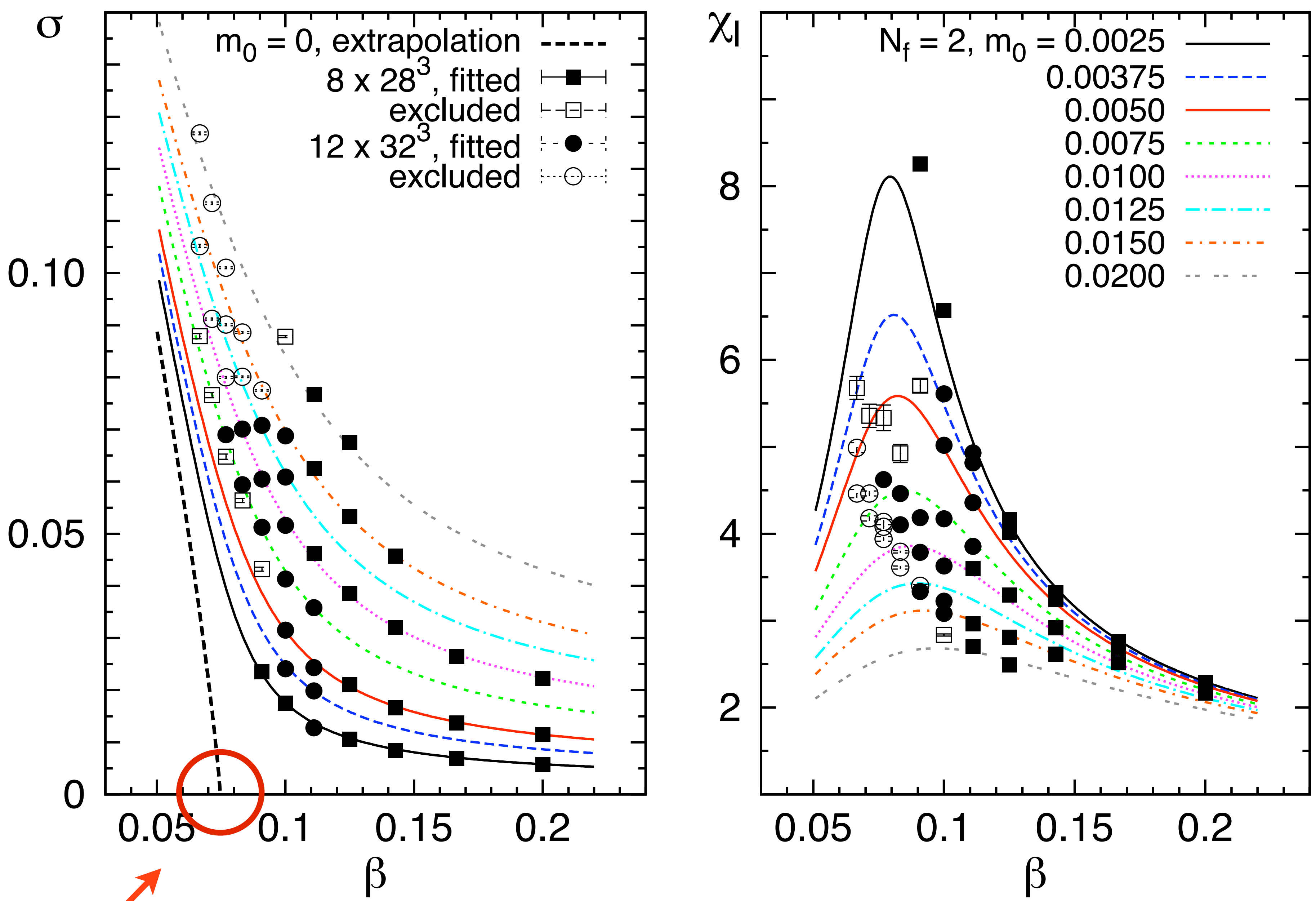
- $N_f = 2$
  Possible transition below $\beta \sim 0.10$
- $N_f = 4$
  Possible transition below $\beta \sim 0.05$
- $N_f = 6$
  No transition observed

$4 < N_{crit} < 6$



J. E. Drut, T. A. Lähde, Phys. Rev. Lett. **102**, (2009) 026802

Chiral condensate (HMC algorithm)



Analysis: "Equation of State" (EOS)

Finite size effects need to be better understood

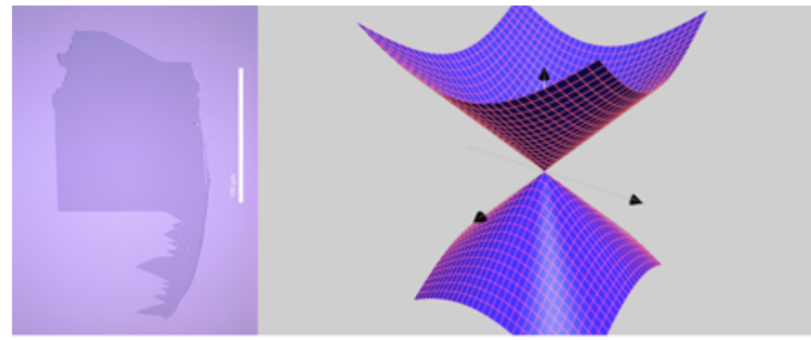
$\beta_c \sim 0.073 \pm 0.002$

Best estimate so far, appears robust, however: critical exponents difficult

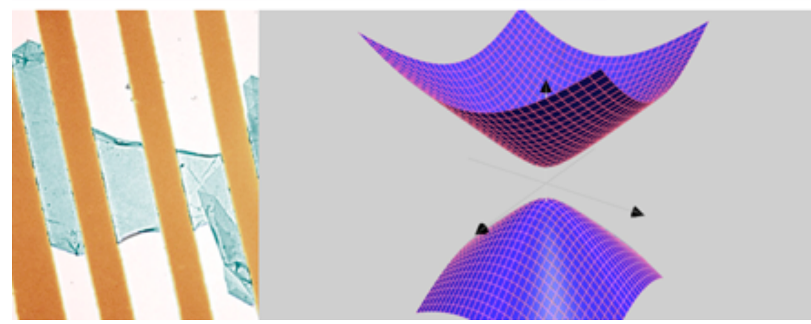
Phenomenology of electron-electron interactions

Coulomb coupling:
Likely to be larger in suspended graphene ...

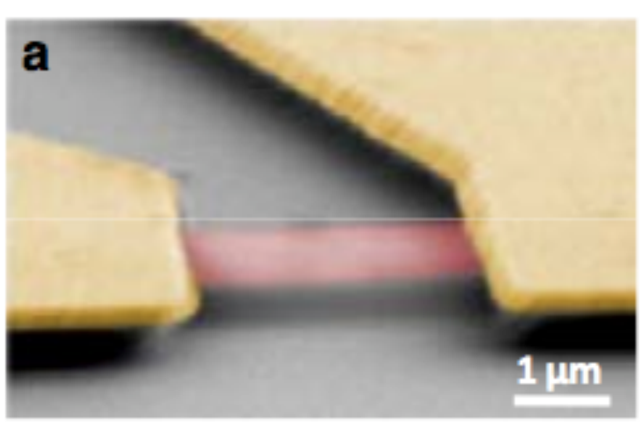
On a SiO_2 substrate
$\alpha_g \sim 0.80$



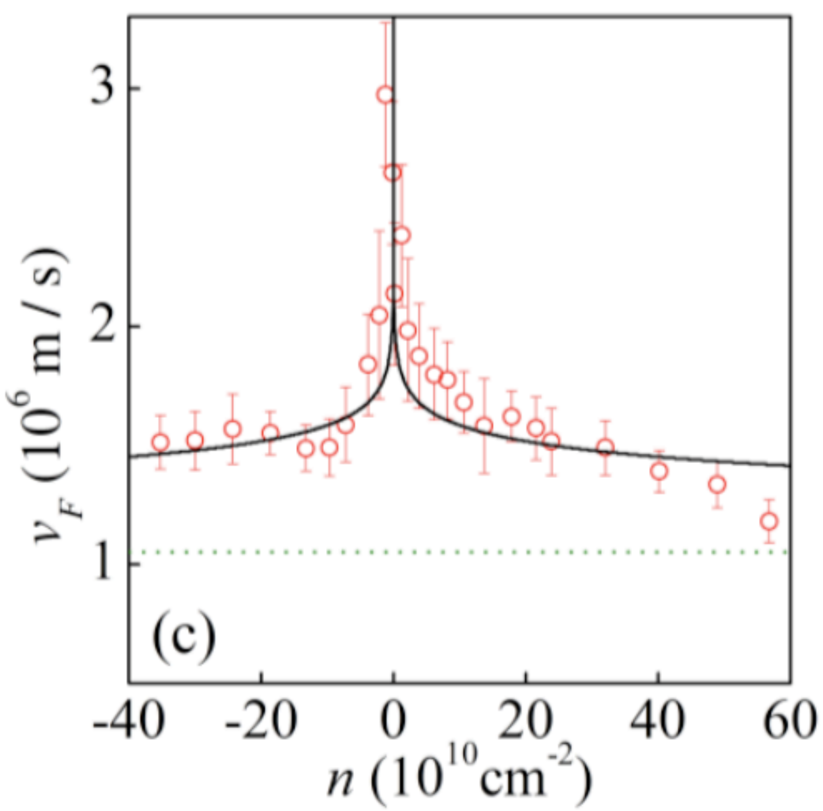
Suspended graphene
$\alpha_g \sim 2.16$



Experiment:
So far no gap observed, but strong interaction-induced velocity renormalization ...



D. C. Elias *et al.*,
Nature Phys. **7**, 701 (2011)



Fermi velocity from Lattice propagator

Staggered fermion propagator

- Fermion mass and velocity in an interacting system
- Interactions renormalize the bare parameters

$$C_f(x,y,t) \equiv \langle \chi(x,y,t)\bar{\chi}(x_0,y_0,t_0)\rangle$$

Lattice correlators

- Both "timeslice" and "spaceslice" correlators are considered
- Analysis of vF requires correlators for non-zero momenta

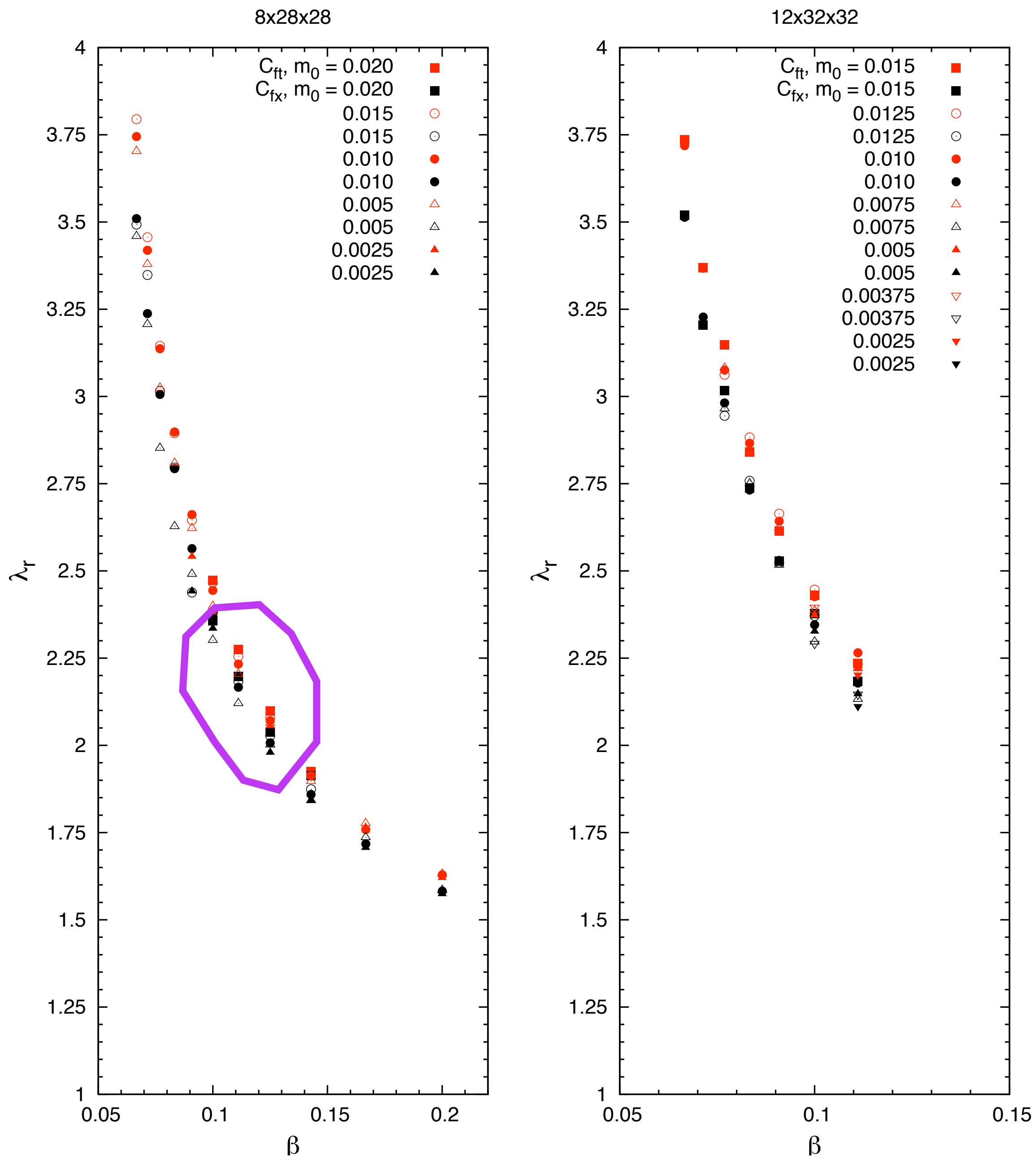
$$C_{ft}(p_1,p_2,t) \equiv \sum_{x,y} \exp(-ip\cdot x)\, C_f(x,y,t) \qquad p_0 = \frac{2\pi(n-1/2)}{N_t}, \quad n = 0,\ldots,N_t/4$$

$$C_{fx}(p_0,p_2,t) \equiv \sum_{t,y} \exp(-ip\cdot x)\, C_f(x,y,t) \qquad p_1 = \frac{2\pi n}{N_x}, \quad p_2 = \frac{2\pi n}{N_x}, \quad n = 0,\ldots,N_x/4$$

Consistent results are found by
measuring vF in both the temporal and
spatial directions

Fixing the lattice (inverse) coupling

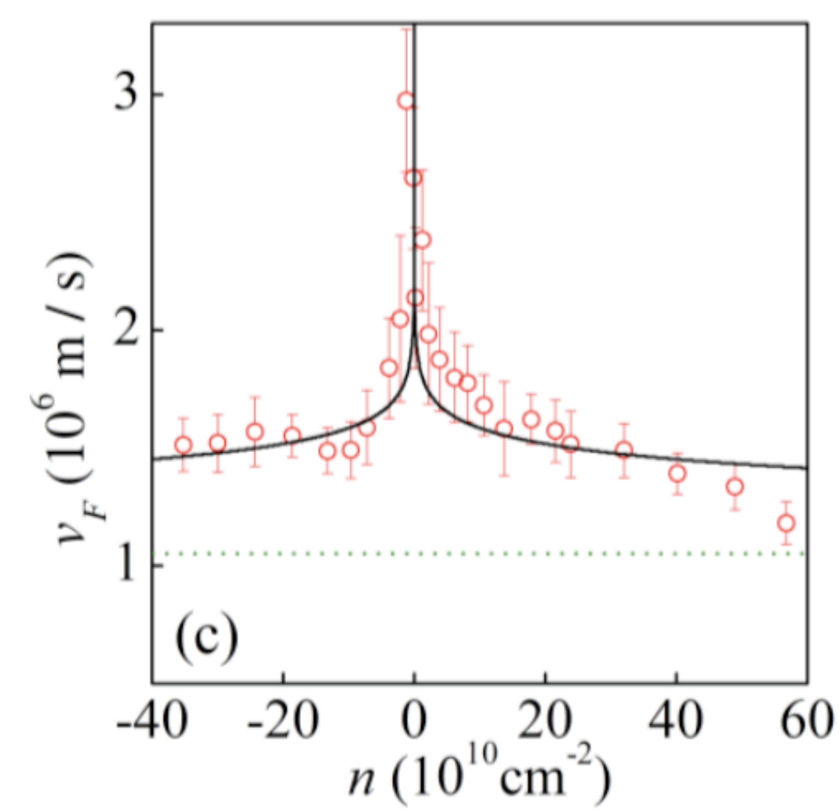
Results for vFR/vF at strong coupling (preliminary)



To be published

Experiment

- vFR/vF ≈ 2 - 2.5



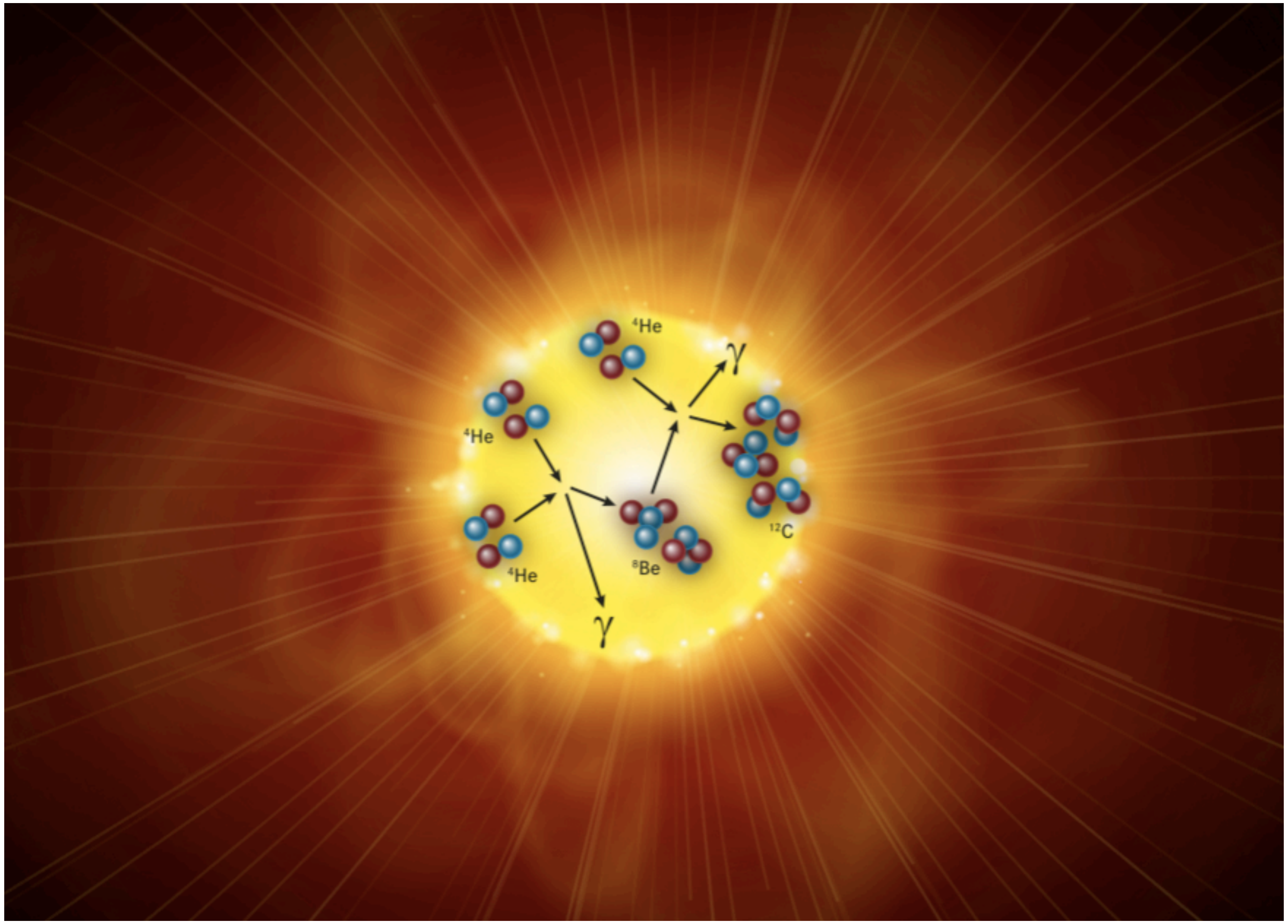
The experimental vFR/vF is reached at \beta ≈ 0.10 ...

Coulomb coupling not (yet?) strong enough to generate a gap

3.

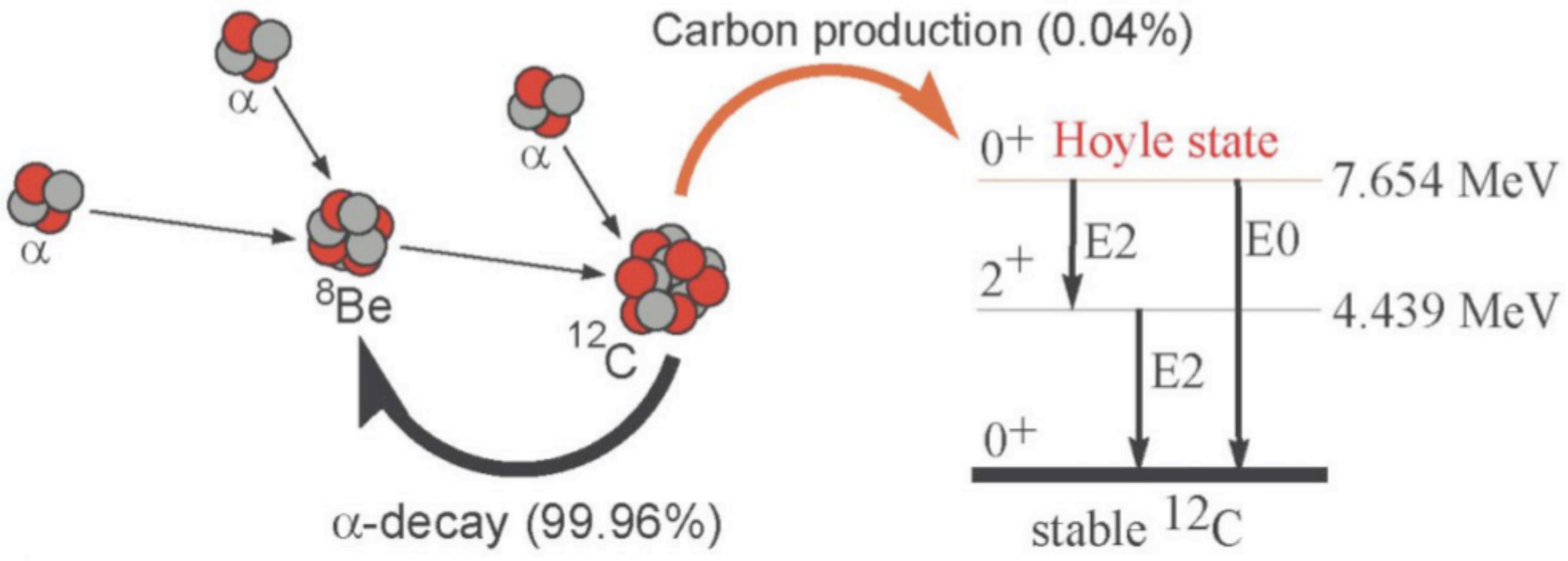
Figure produced by
Jülich Supercomputing Center (JSC)

NUCLEAR LATTICE SIMULATIONS



Carbon production in red giant stars

The triple alpha process



--> Reaction rate given by:

Experiment: 379.47 ± 0.18 keV

$$r_{3\alpha} = 3^{\frac{3}{2}} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_B T}\right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left(-\frac{\Delta E_{h+b}}{k_B T}\right)$$

$$\Delta E_{h+b} \equiv \Delta E_h + \Delta E_b = E_{12}^\star - 3E_4$$

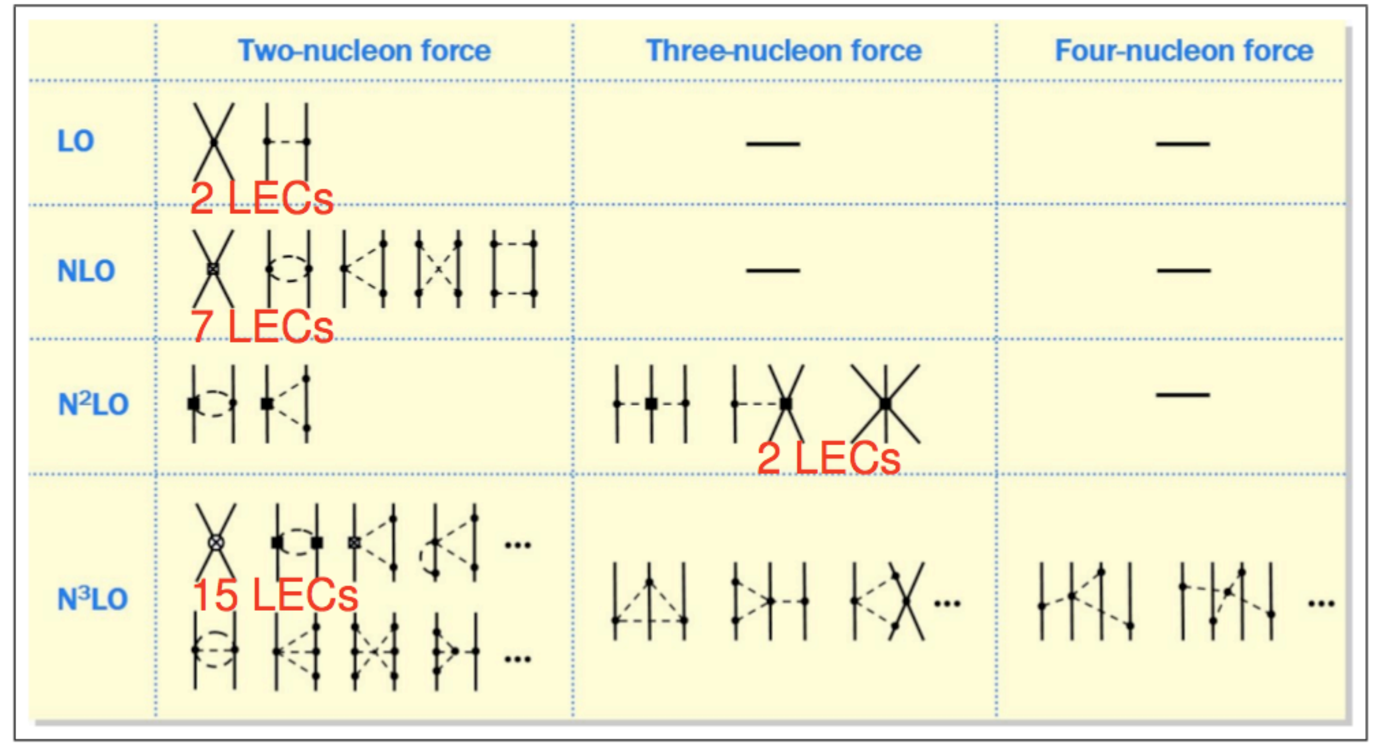
Anthropic arguments

What happens if the fundamental constants (esp. quark masses) shift slightly?

$$\Delta E_h \equiv E_{12}^\star - E_8 - E_4$$
$$\Delta E_b \equiv E_8 - 2E_4$$

Hamiltonian

Chiral Effective Field Theory



$$H_{\rm LO} = H_{\rm free} + V_{\rm LO}$$

$$H_{\rm free} = \frac{1}{2m} \sum_{i,j=0,1} \int d^3\vec{r}\ \vec{\nabla} a^\dagger_{i,j}(\vec{r}) \cdot \vec{\nabla} a_{i,j}(\vec{r})$$

Interaction (LO)

OPEP + 2 contact terms ...

$$V_{\rm LO} = V + V_{I^2} + V^{\rm OPEP}$$

$$\rho^{a^\dagger,a}(\vec{r}) = \sum_{i,j=0,1} a^\dagger_{i,j}(\vec{r}) a_{i,j}(\vec{r})$$

$\sim c_{11}$ $\qquad\qquad$ $\sim c_{ii} \times \tau_A \cdot \tau_B$

$$V = \frac{C}{2} \int d^3\vec{r} : \left[\rho^{a^\dagger,a}(\vec{r})\right]^2 :$$

$$V_{I^2} = \frac{C_{I^2}}{2} \sum_{I=1,2,3} \int d^3\vec{r} : \left[\rho_I^{a^\dagger,a}(\vec{r})\right]^2$$

$$V^{\rm OPEP} = \sum_{S_1,S_2,I=1,2,3} \int d^3\vec{r}_1 d^3\vec{r}_2 G_{S_1S_2}(\vec{r}_1 - \vec{r}_2) : \rho^{a^\dagger,a}_{S_1,I}(\vec{r}_1) \rho^{a^\dagger,a}_{S_2,I}(\vec{r}_2) :$$

$$G_{S_1S_2}(\vec{r}_1 - \vec{r}_2) = -\left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{q_{S_1} q_{S_2} e^{i\vec{q}\cdot(\vec{r}_1 - \vec{r}_2)}}{q^2 + m_\pi^2}$$

Shifts in the light quark masses

Equivalent to shifts in the pion masses

$$m_{\pi^\pm}^2 \sim (m_u + m_d)$$

Energies of 4He and the Hoyle state

Pion mass dependence at LO in Chiral Effective Field Theory

$$E_i = E_i(m_\pi^{\mathrm{OPE}}, m_N(m_\pi), \tilde{g}_{\pi N}(m_\pi), c_{11}(m_\pi), c_{ii}(m_\pi))$$

V^{OPEP} $\quad V \quad V_{I^2}$

$$\tilde{g}_{\pi N} \equiv g_A/(2 f_\pi)$$

Pion mass dependence

Small perturbations around the physical point …

$$\left.\frac{\partial E_i}{\partial m_\pi}\right|_{m_\pi^{\mathrm{phys}}} = \left.\frac{\partial E_i}{\partial m_\pi^{\mathrm{OPE}}}\right|_{m_\pi^{\mathrm{phys}}} + x_1 \left.\frac{\partial E_i}{\partial m_N}\right|_{m_N^{\mathrm{phys}}} + x_2 \left.\frac{\partial E_i}{\partial \tilde{g}_{\pi N}}\right|_{\tilde{g}_{\pi N}^{\mathrm{phys}}} + x_3 \left.\frac{\partial E_i}{\partial c_{11}}\right|_{c_{11}^{\mathrm{phys}}} + x_4 \left.\frac{\partial E_i}{\partial c_{ii}}\right|_{c_{ii}^{\mathrm{phys}}}$$

$$x_1 \equiv \left.\frac{\partial m_N}{\partial m_\pi}\right|_{m_\pi^{\mathrm{phys}}}, \quad x_2 \equiv \left.\frac{\partial \tilde{g}_{\pi N}}{\partial m_\pi}\right|_{m_\pi^{\mathrm{phys}}},$$

$$x_3 \equiv \left.\frac{\partial c_{11}}{\partial m_\pi}\right|_{m_\pi^{\mathrm{phys}}}, \quad x_4 \equiv \left.\frac{\partial c_{ii}}{\partial m_\pi}\right|_{m_\pi^{\mathrm{phys}}}$$

Lattice QCD and CHPT $\qquad$ **Two-nucleon problem**

Lattice formulation

Auxiliary Field Quantum Monte Carlo (AFQMC)

Borasoy, Krebs, Lee, Meißner,
Nucl. Phys. **A768** (2006) 179;
Eur. Phys. J. **A31** (2007) 105 ...

--> Discretized space-time:

$$V = L_s^3 \times L_t$$

Our Lattices:
N_x = 6
L_s = 11.8 fm
a = 1.97 fm = 100 MeV^{-1}

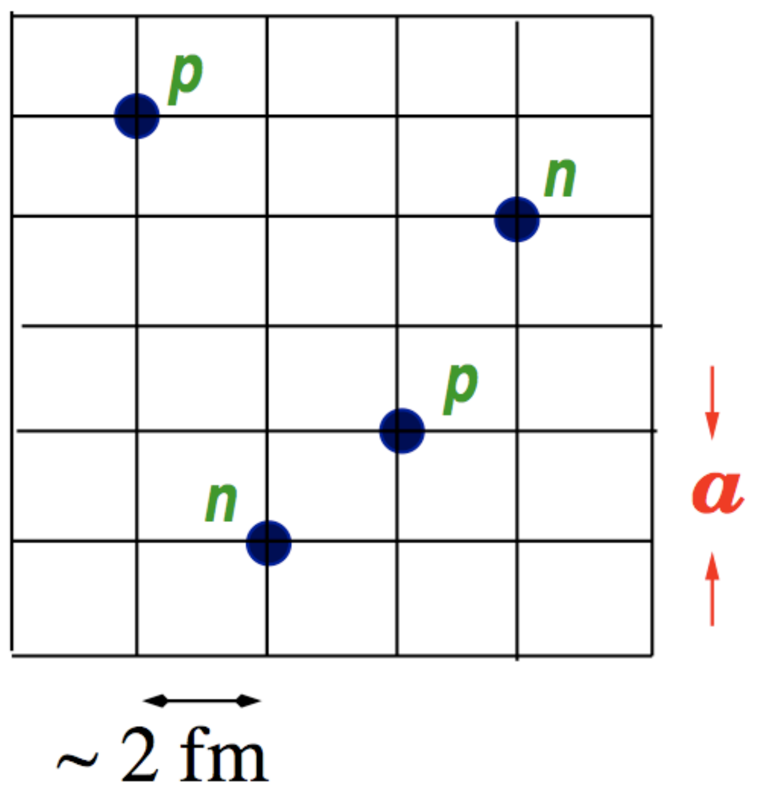
Discretized chiral potential

Pion exchange + contact interactions

--> Auxiliary fields introduced for contact interactions

Hubbard-Stratonovich transformation

$$\exp(\rho^2/2) \propto \int_{-\infty}^{\infty} ds \exp(-s^2/2 - s\rho), \quad \rho \sim a^\dagger a$$



Global Lattice updates

Hybrid Monte Carlo (pion + auxiliary fields)

Energies via Projection Monte Carlo

--> Euclidean time derivative of the correlator

For a thorough review:
D. Lee,
Prog. Part. Nucl. Phys. **63**, (2009)

$$Z_A(t) = \langle\psi_A|\exp(-tH)|\psi_A\rangle$$

Lattice Hamiltonian with Auxiliary Field

Slater determinant for A free nucleons

--> Define "transient" energy E(t):

$$E_A(t) = -\frac{d}{dt}\ln Z_A(t)$$

$$E_A^0 = \lim_{t\to\infty} E_A(t)$$

Ground state energy filtered out at large times

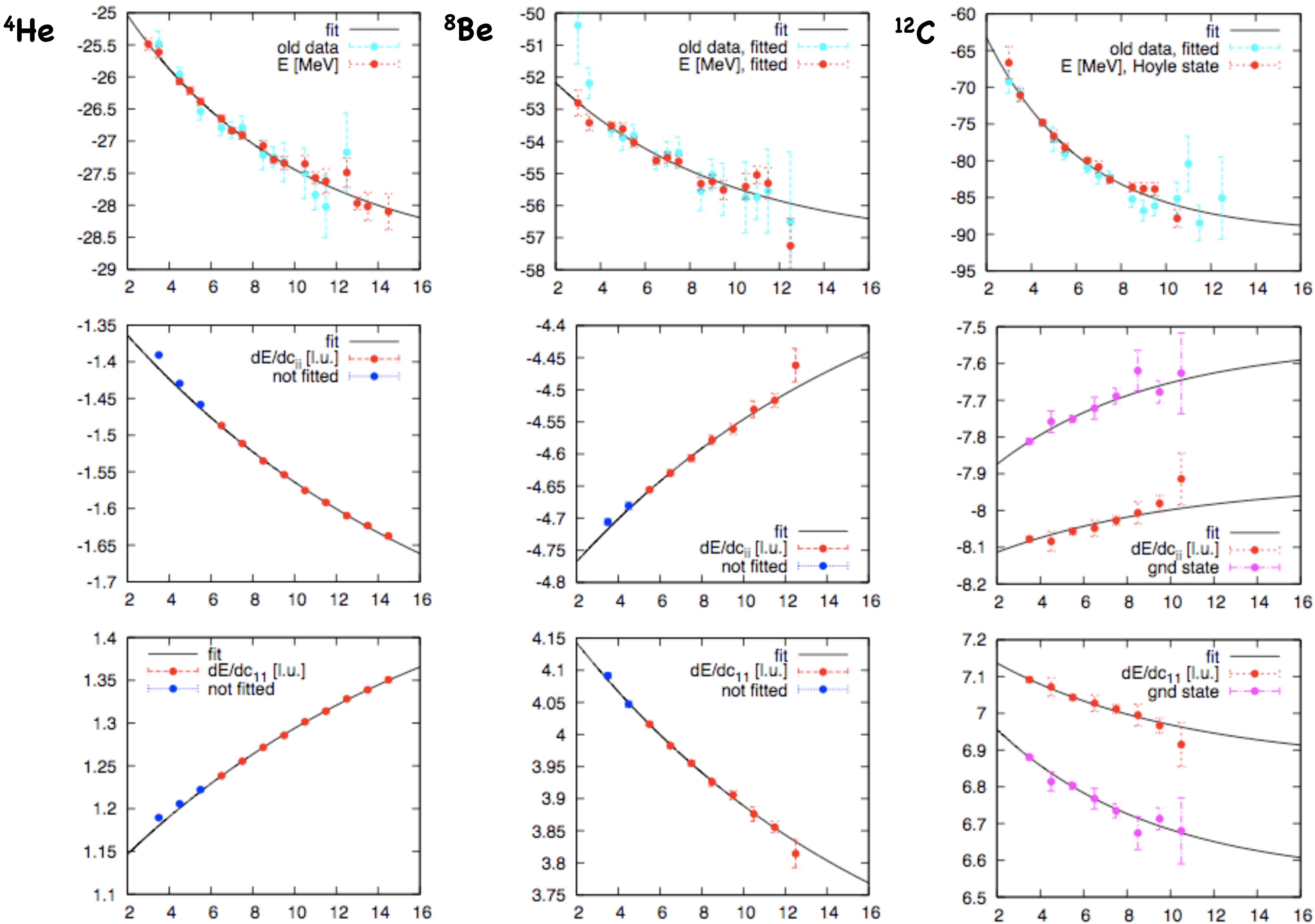
Operator expectation values

--> Projection Monte Carlo calculation of the derivatives ...

$$Z_A^{\mathcal{O}}(t) = \langle\psi_A|\exp(-tH/2)\mathcal{O}\exp(-tH/2)|\psi_A\rangle$$

$$\lim_{t\to\infty}\frac{Z_A^{\mathcal{O}}(t)}{Z_A(t)} = \langle\psi_A|\mathcal{O}|\psi_A\rangle$$

--> Extrapolation (exponential) to large Euclidean time!



^{4}He
^{8}Be
^{12}C
fit
old data
E [MeV]
old data, fitted
E [MeV], fitted
old data, fitted
E [MeV], Hoyle state
fit
dE/dc_{jj} [l.u.]
not fitted
fit
dE/dc_{jj} [l.u.]
not fitted
fit
dE/dc_{jj} [l.u.]
gnd state
fit
dE/dc_{11} [l.u.]
not fitted
fit
dE/dc_{11} [l.u.]
not fitted
fit
dE/dc_{11} [l.u.]
gnd state

Summary of Monte Carlo results

Substantial investment of supercomputing resources (JUQUEEN, RWTH Aachen) ...

	$^3S_1$ (extr.)	$^3S_1$ (exact)	^{4}He (extr.)	^{8}Be (extr.)	^{12}C* (extr.)	^{12}C (extr.)
E	−9.070(12)	−9.078	−28.89(11)	−57.2(5)	−89.8(13)	−95.6(6)
$\Delta E(\Delta m_\pi^{\mathrm{OPE}})$	−0.003548(12)	−0.003569	−0.2290(17)	−0.477(5)	−0.802(2)	−0.778(4)
$\Delta E(\Delta m_\pi^{\mathrm{IB}})$	−0.002372(8)	−0.002379	−0.1528(11)	−0.318(3)	−0.5343(16)	−0.519(3)
$\Delta E(c_{pp})$	–	–	0.433(3)	1.02(3)	2.032(10)	1.95(2)
$\Delta E(\alpha_{em})$	–	–	0.613(2)	2.35(2)	5.54(2)	5.67(2)
$\partial E/\partial m_\pi^{\mathrm{OPE}}$	−0.000785(8)	−0.0007732	−0.0504(4)	−0.1064(12)	−0.1769(17)	−0.1697(19)
$\partial E/\partial m_N$	−0.00382(2)	−0.003809	−0.0750(7)	−0.187(6)	−0.403(5)	−0.395(5)
$\partial E/\partial \tilde{g}_{\pi N}$	0.01024(11)	0.01017	0.337(3)	0.746(12)	1.343(13)	1.285(16)
$\partial E/\partial c_{11}$	0.13897(15)	0.138867	1.527(12)	3.52(8)	6.86(3)	6.55(7)
$\partial E/\partial c_{ii}$	−0.4171(4)	−0.41660	−1.881(17)	−4.22(7)	−7.92(3)	−7.54(6)

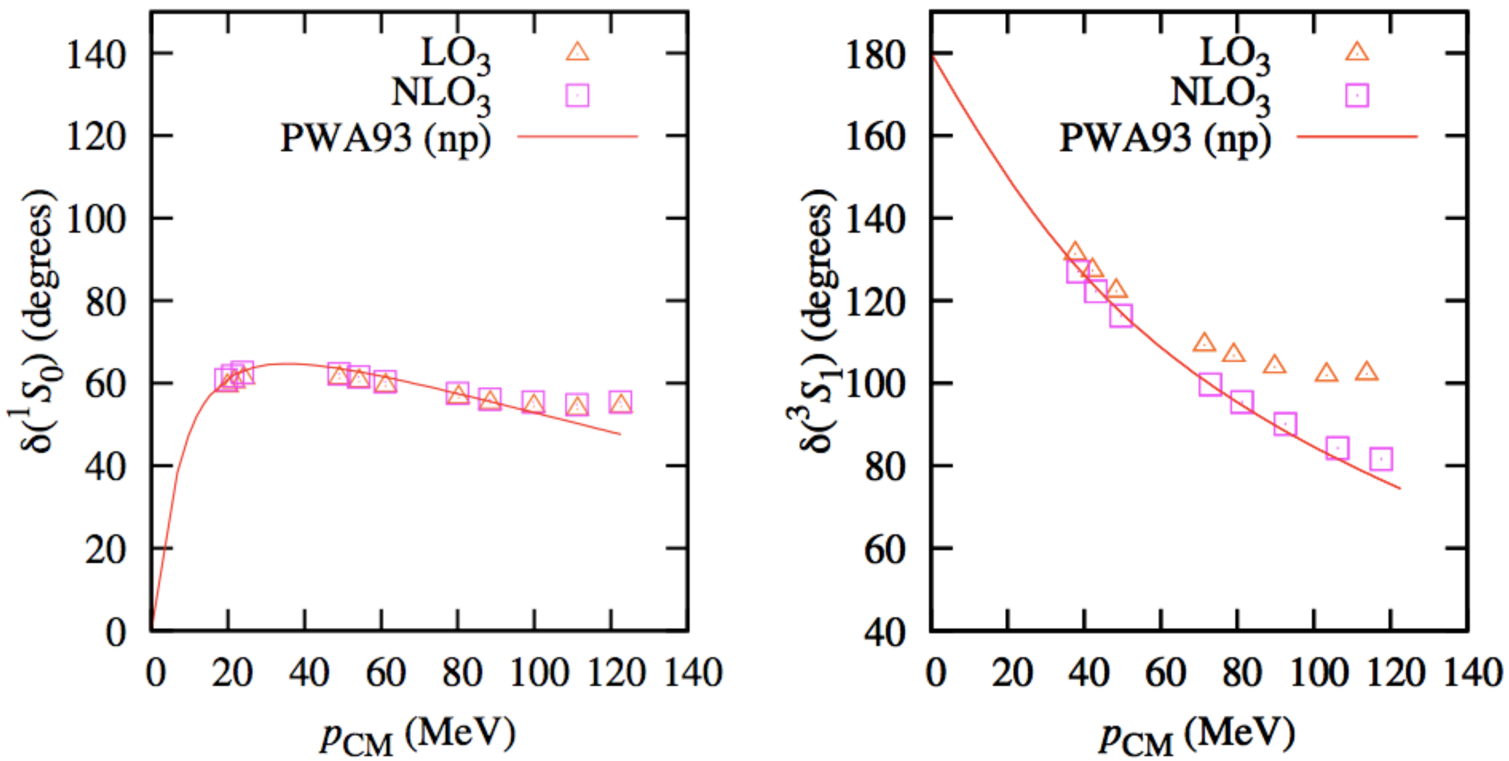
Extrapolation validated against the deuteron
(exact solution of Schrödinger equation in a periodic box)

Determination of LECs

--> Two-nucleon scattering analysis

B. Borasoy *et al.*,
Eur. Phys. J. A **35**, (2008) 343

Predictions can be made for heavier nuclei



Derivatives of LECs w.r.t. the pion mass

--> Lüscher's finite volume formula ...

$$p\cot\delta = \frac{1}{\pi L}S(\eta) \approx -\frac{1}{a}, \qquad \eta \equiv m_N E\left(\frac{L}{2\pi}\right)^2$$

Two-nucleon energy levels in a periodic cube related to S-wave phase shifts

Replace derivatives w.r.t. LECs
--> derivatives w.r.t. a^{-1} ...

$$x_3 \equiv \left.\frac{\partial c_{11}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} \qquad \bar{A}_s \equiv \left.\frac{\partial a_s^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}}$$

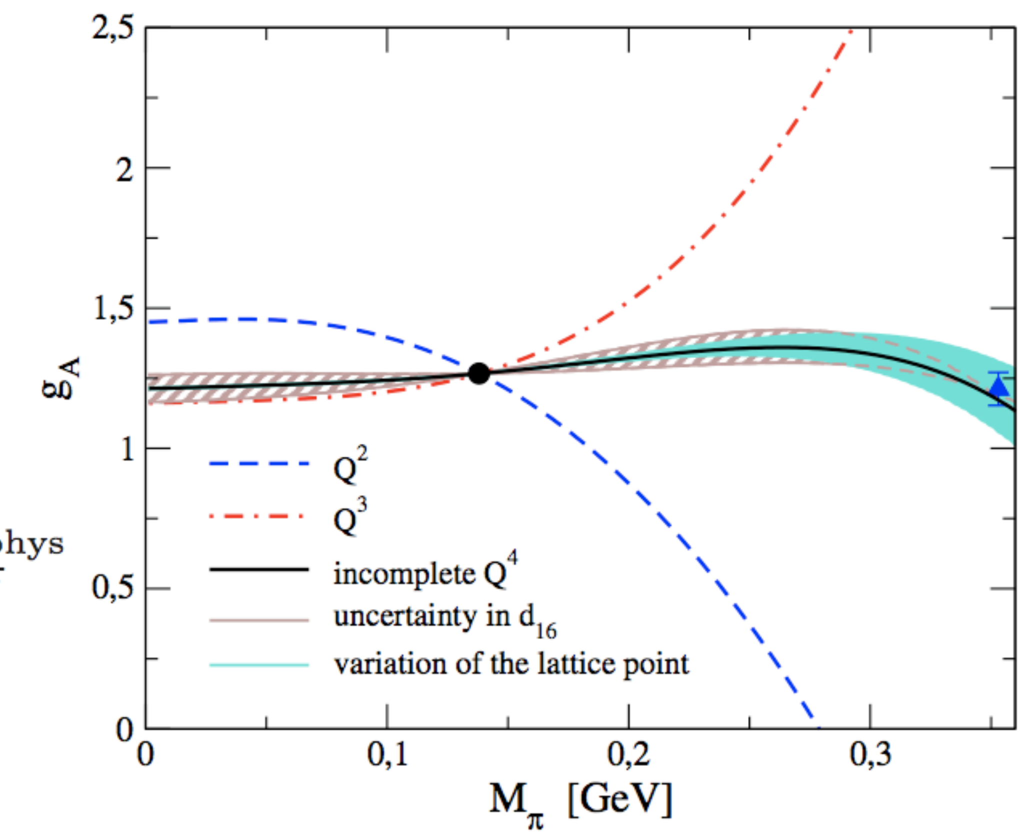
$$x_4 \equiv \left.\frac{\partial c_{ii}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} \longleftrightarrow \bar{A}_t \equiv \left.\frac{\partial a_t^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}}$$

Shifts of the ^{4}He and Hoyle states

--> Input needed from CHPT and LQCD ...

$$x_1 \equiv \left.\frac{\partial m_N}{\partial m_\pi}\right|_{m_\pi^{\rm phys}}$$

$$x_2 \equiv \left.\frac{\partial \tilde{g}_{\pi N}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} = \frac{1}{2f_\pi}\left.\frac{\partial g_A}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} - \frac{g_A}{2f_\pi^2}\left.\frac{\partial f_\pi}{\partial m_\pi}\right|_{m_\pi^{\rm phys}}$$



--> Use conservative error estimates for x_1 and x_2 (effects shown in red):

$$x_1 = 0.73\,(0.57\ldots 0.97) \qquad x_2 = -0.024\,(-0.058\ldots 0.008)\ \text{l.u.}$$

$$\left.\frac{\partial E_4}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} = -0.339(5)\left.\frac{\partial a_s^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} - 0.697(4)\left.\frac{\partial a_t^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} + 0.0380(14)^{+0.008}_{-0.006}$$

$$\left.\frac{\partial E_{12}^\star}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} = -1.588(11)\left.\frac{\partial a_s^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} - 3.025(8)\left.\frac{\partial a_t^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} + 0.178(4)^{+0.026}_{-0.021}$$

Small changes in the fundamental parameters

--> Light quark masses + EM fine structure constant ...

$$\delta(\Delta E_{h+b}) \approx \left.\frac{\partial \Delta E_{h+b}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} \times \delta m_\pi + \left.\frac{\partial \Delta E_{h+b}}{\partial \alpha_{em}}\right|_{\alpha_{em}^{\rm phys}} \times \delta \alpha_{em}$$

$$= \left.\frac{\partial \Delta E_{h+b}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} \times K_\pi^q \, m_\pi \left(\frac{\delta m_q}{m_q}\right) + Q(\Delta E_{h+b}) \left(\frac{\delta \alpha_{em}}{\alpha_{em}}\right)$$

$$\boxed{\left.\frac{\partial \Delta E_{h+b}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} = -0.572(19) \left.\frac{\partial a_s^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} - 0.933(15) \left.\frac{\partial a_t^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} + 0.064(6)^{+0.010}_{-0.009}}$$

Models of stellar evolution

--> Produce ^{12}C, do not convert it all to ^{16}O ...

$$|\delta(\Delta E_{h+b})| < 100 \text{ keV}$$

H. Oberhummer, A. Csótó, H. Schlattl, Science **289**, 88 (2000) ...

--> Feasibility of carbon-based life:

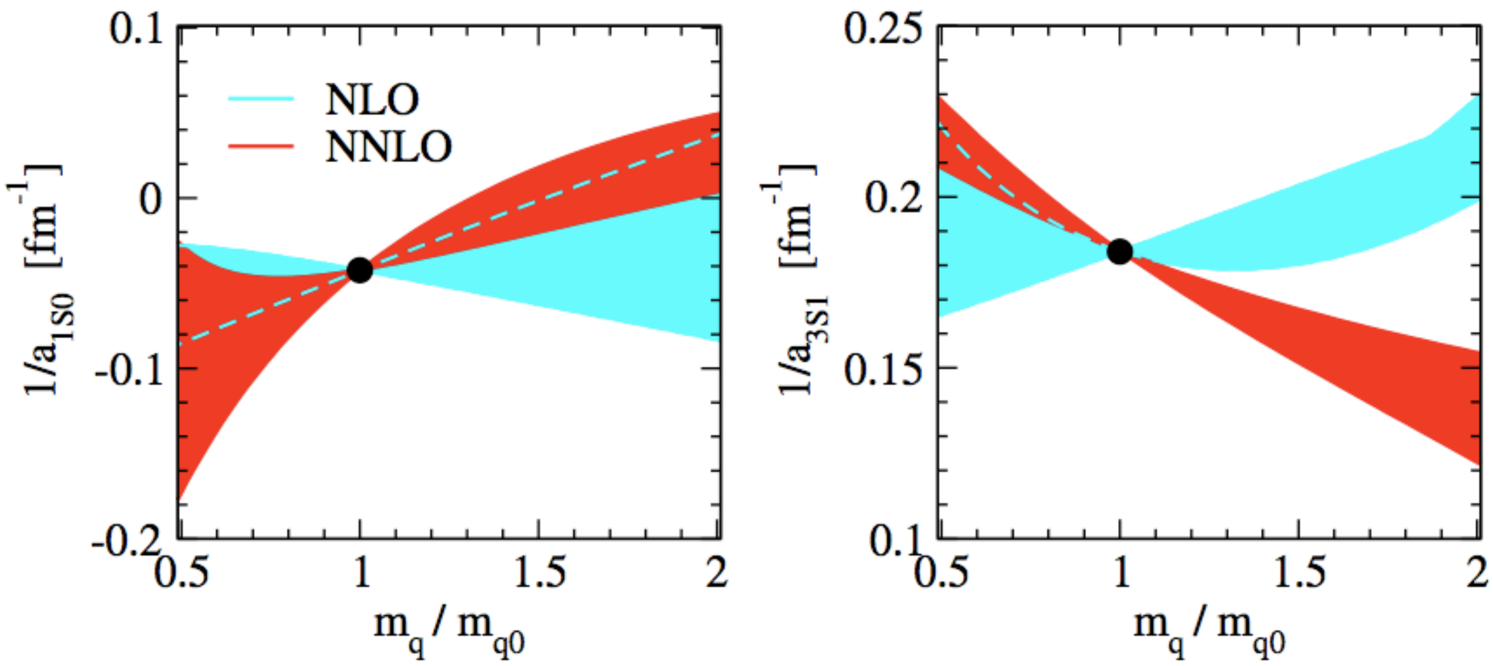
$$\boxed{\left| \left[0.572(19) \bar{A}_s + 0.933(15) \bar{A}_t - 0.064(6) \right] \times \left(\frac{\delta m_q}{m_q}\right) \right| < 0.15\%}$$

How well are the remaining parameters known?

--> Information from Chiral EFT and Lattice QCD ...

Quark mass variation in Chiral EFT:
E. Epelbaum *et al.*, to be published ...

$$-\frac{\partial a_s^{-1}}{\partial m_\pi} \equiv \frac{A_s}{a_s m_\pi}, \quad A_s \equiv \frac{K^q_{a_s}}{K^q_\pi}, \qquad -\frac{\partial a_t^{-1}}{\partial m_\pi} \equiv \frac{A_t}{a_t m_\pi}, \quad A_t \equiv \frac{K^q_{a_t}}{K^q_\pi}$$



$$K^q_{a_s} = 2.3^{+1.9}_{-1.8}, \qquad K^q_{a_t} = 0.32^{+0.17}_{-0.18}$$

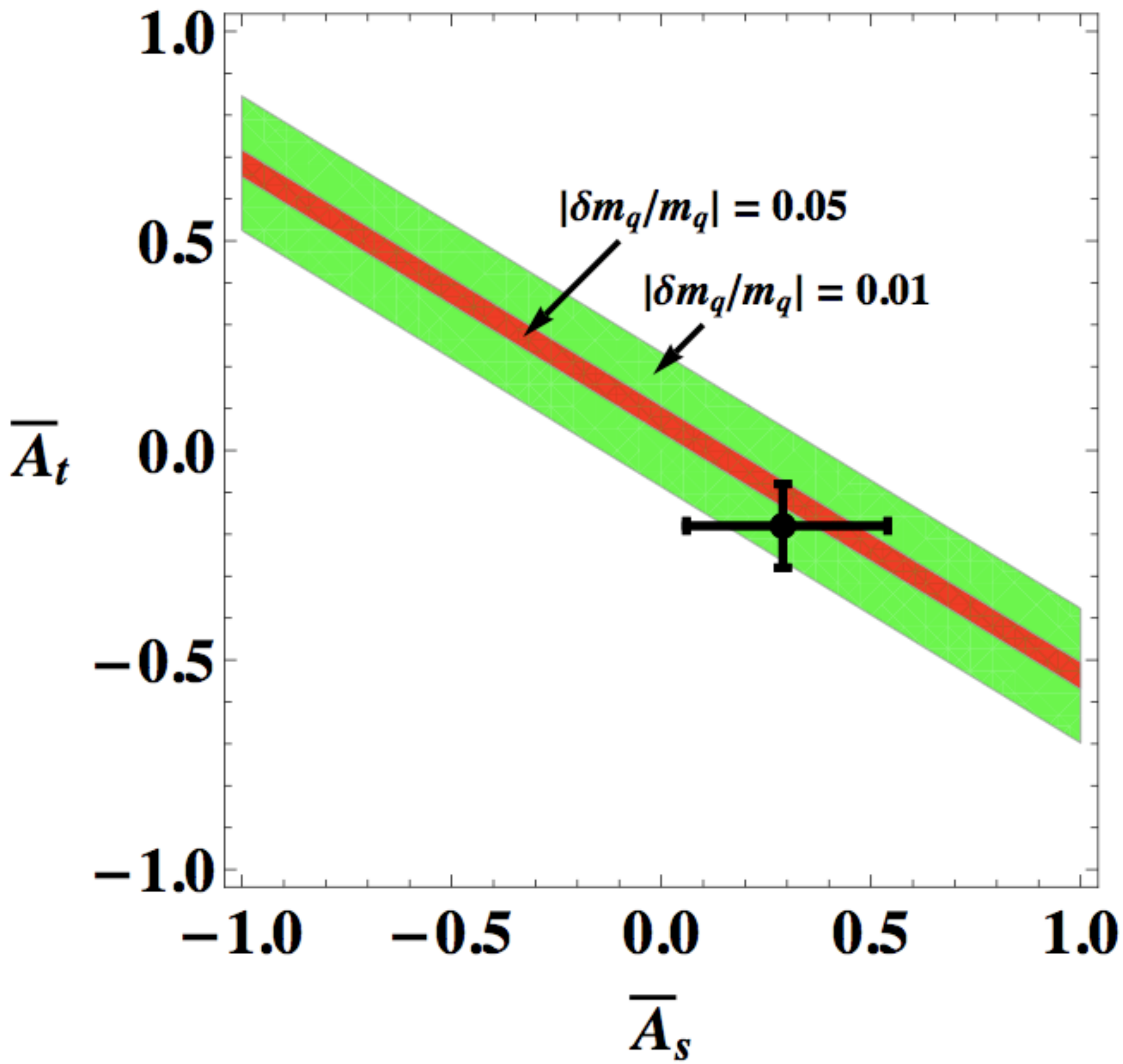
Relatively large uncertainty
Will be improved dramatically
by Lattice QCD
(note: preliminary!)

$$\bar{A}_t \equiv \left.\frac{\partial a_t^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} = \frac{-197.33 \text{ MeV fm}}{5.4 \text{ fm} \times 134.98 \text{ MeV}} \times \frac{0.32^{+0.17}_{-0.18}}{0.494^{+0.009}_{-0.013}} \simeq -0.18^{+0.10}_{-0.10}$$

$$\bar{A}_s \equiv \left.\frac{\partial a_s^{-1}}{\partial m_\pi}\right|_{m_\pi^{\rm phys}} = \frac{-197.33 \text{ MeV fm}}{-23.8 \text{ fm} \times 134.98 \text{ MeV}} \times \frac{2.3^{+1.9}_{-1.8}}{0.494^{+0.009}_{-0.013}} \simeq 0.29^{+0.25}_{-0.23}$$

How does our Universe compare with the predictions?

--> The END OF THE WORLD plot :)



The black dot = our Universe: Non-anthropic scenario falls within current bounds

How about EM effects (variation of fine structure constant)?

--> Carbon-based possible within 2% variation (preliminary AFQMC results)