# Chiral three-nucleon forces up to $N^4LO$

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Light Nuclei from First Principles
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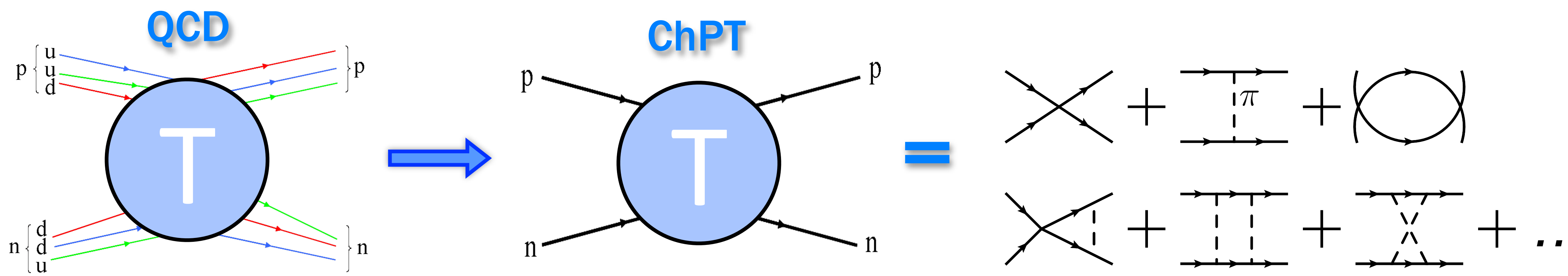
# Outline

- From QCD to nuclear physics
- Nuclear forces in chiral EFT
- Three-nucleon forces up to $N^3LO$
- Long-range part of three-nucleon forces up to $N^4LO$
- Summary & Outlook

- EFT with explicit delta
- Small scale expansion and explicit decoupling
- Convergence of NN-forces
- Pion-nucleon scattering up to $\epsilon^3$
- $N^3LO$ three-nucleon force with explicit delta
- Summary & Outlook

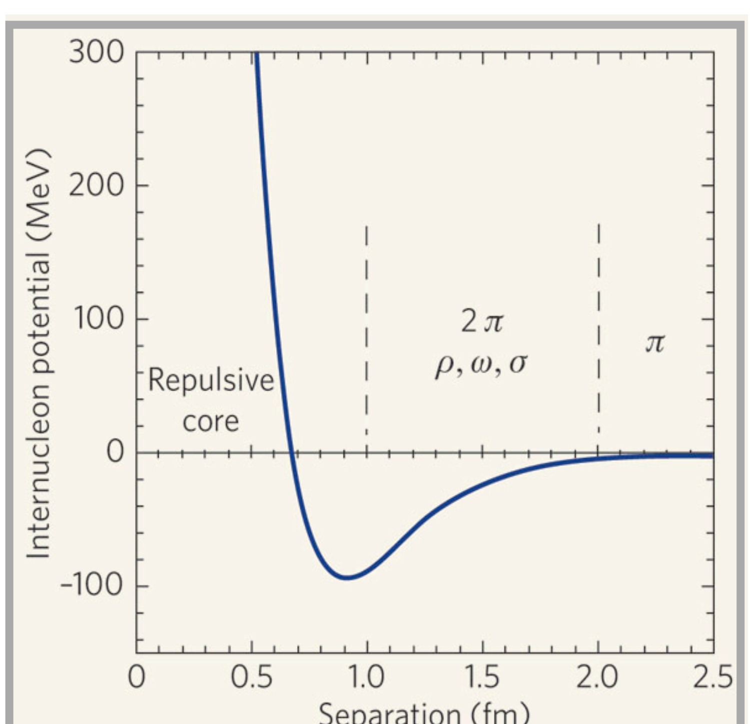
# From QCD to nuclear physics

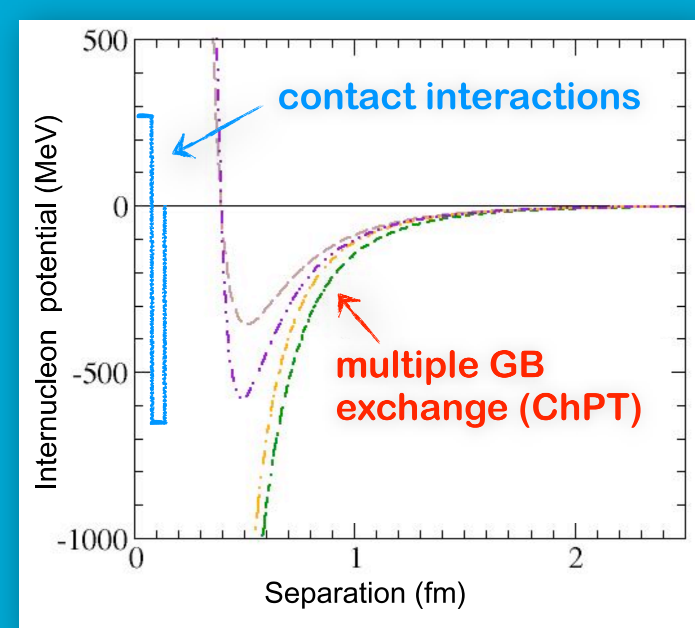



- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⟹ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

Weinberg '91




- unified description of $\pi\pi$, $\pi$N and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, $\pi$-prod., ...)
- precision physics with/from light nuclei

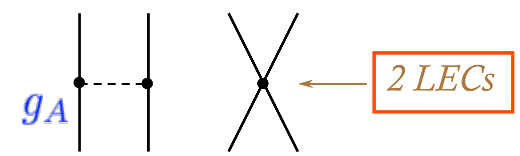
# Nucleon-nucleon force up to N³LO

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; …

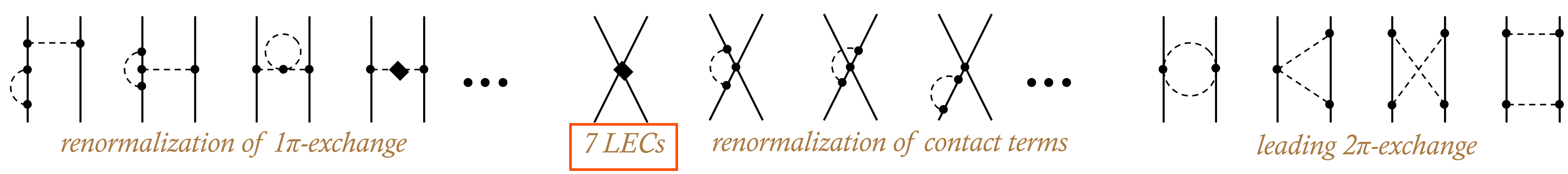
**Chiral expansion for the 2N force:**

$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \dots$$

- **LO:**

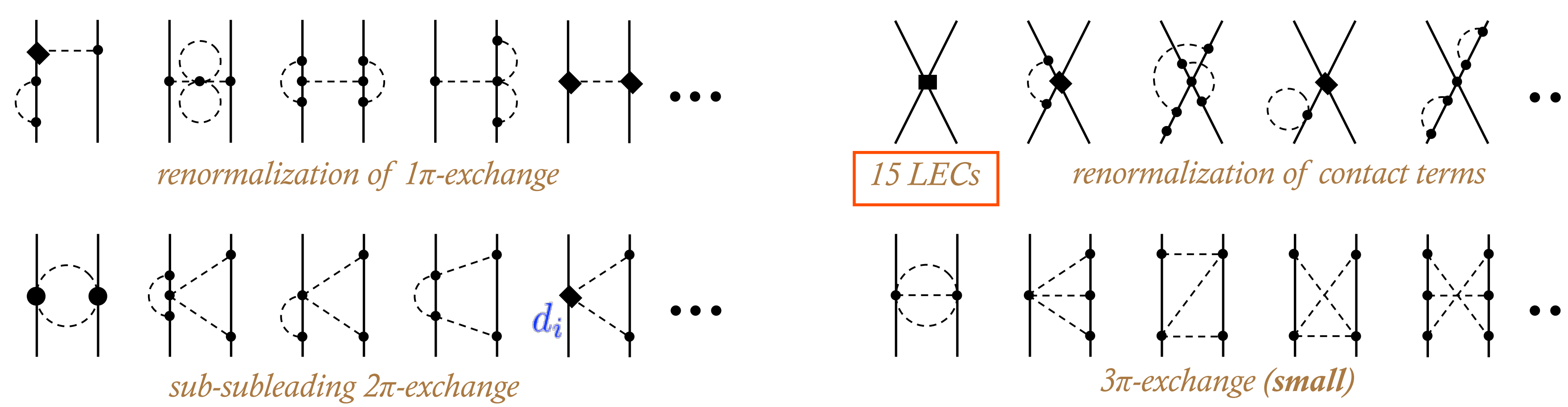



- **NLO:**



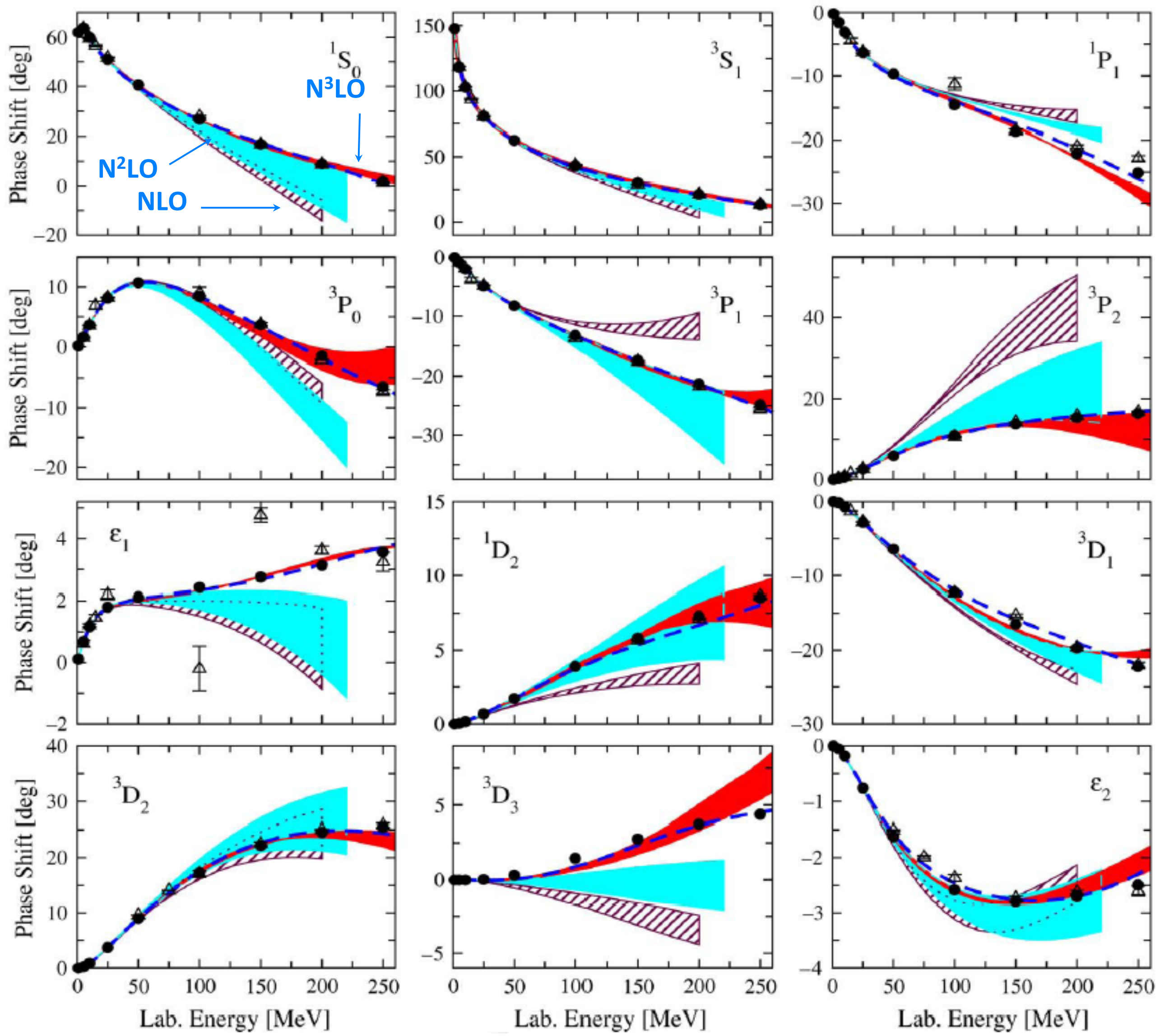

- **N²LO:**


- **N³LO:**

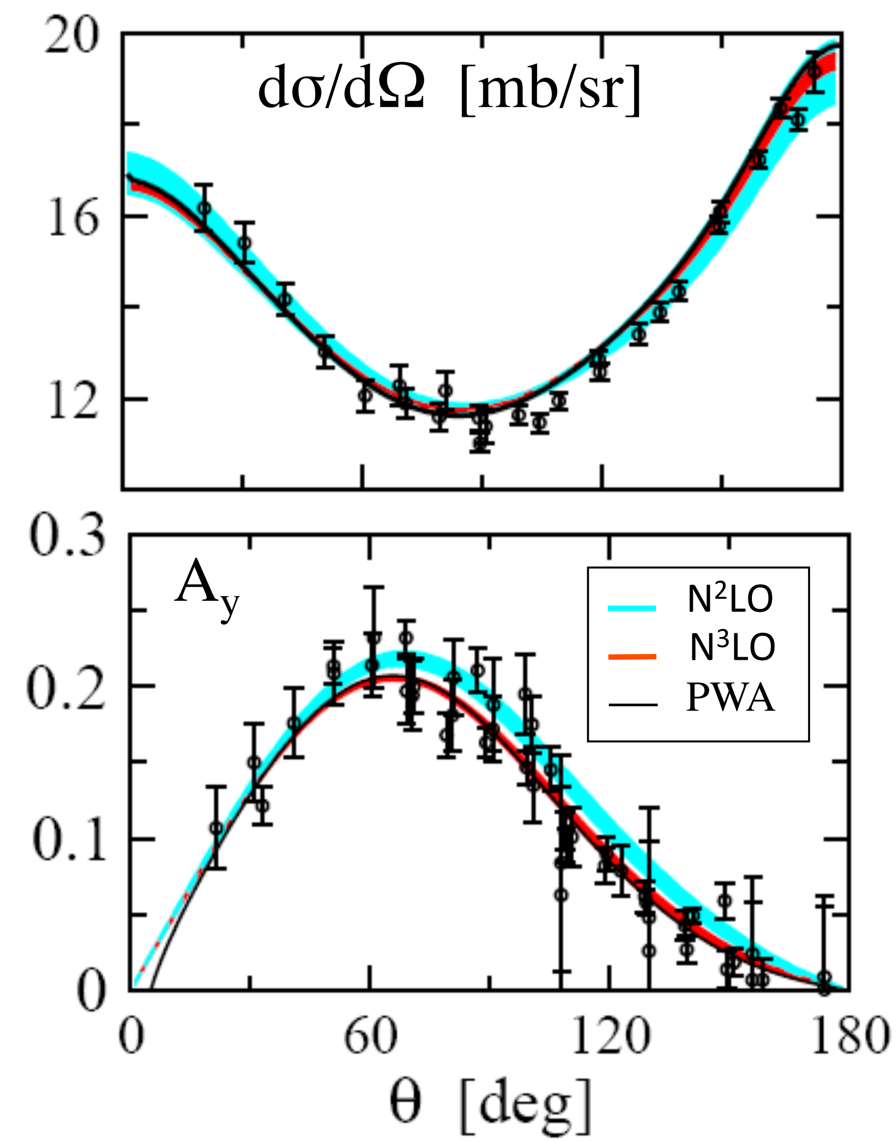



+ 1/m and isospin-breaking corrections…

## Neutron-proton phase shifts up to $N^3LO$




## np scattering at 50 MeV



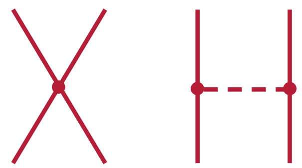
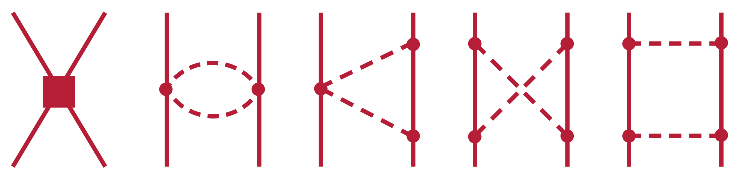
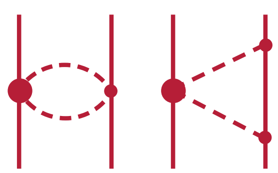
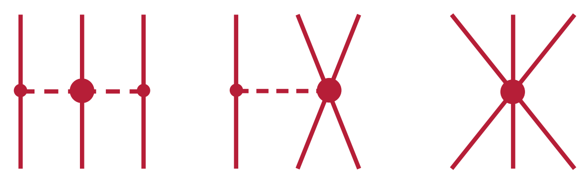
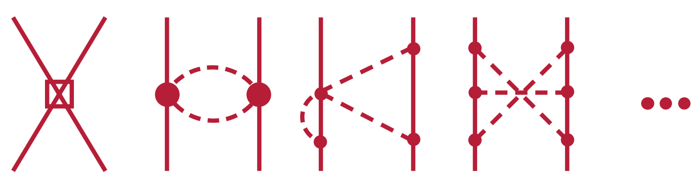
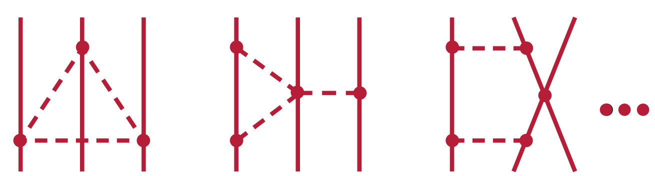
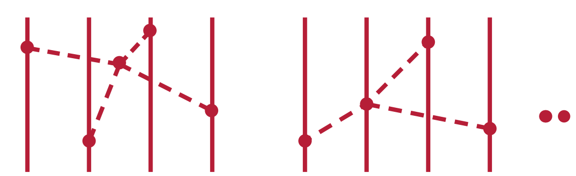

## Deuteron binding energy & asymptotic normalizations $A_s$ and $\eta_d$

| | NLO | $N^2LO$ | $N^3LO$ | Exp |
|---|---|---|---|---|
| $E_d$ [MeV] | $-2.171\ldots-2.186$ | $-2.189\ldots-2.202$ | $-2.216\ldots-2.223$ | $-2.224575(9)$ |
| $A_S$ $[\mathrm{fm}^{-1/2}]$ | $0.868\ldots0.873$ | $0.874\ldots0.879$ | $0.882\ldots0.883$ | $0.8846(9)$ |
| $\eta_d$ | $0.0256\ldots0.0257$ | $0.0255\ldots0.0256$ | $0.0254\ldots0.0255$ | $0.0256(4)$ |

*Entem & Machleidt ‘03; Epelbaum, Glöckle & Meißner ‘05*

# Nuclear forces up to $N^3LO$

## dimensional analysis counting

| | Two-nucleon force | Three-nucleon force | Four-nucleon force |
|---|---|---|---|
| LO ($Q^0$) |  | — | — |
| NLO ($Q^2$) |  | — | — |
| $N^2LO$ ($Q^3$) |  |  | — |
| $N^3LO$ ($Q^4$) |  |  |  |

**Two-nucleon force:**
- converged
- accurate description of NN at least up to $E_{lab}$ ~ 200 MeV

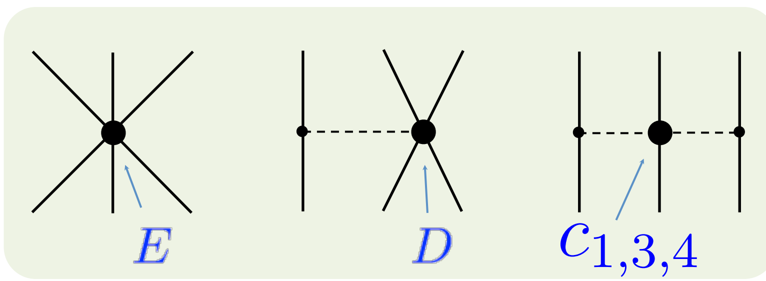
**Three-nucleon force:**
- not yet converged
- higher orders in progress
- impact on few- & many-N systems?

**Four-nucleon force:**
- converged ??
- presently out of reach for few- & many-N studies

# Three-nucleon forces

- Three-nucleon forces in chiral EFT start to contribute at NNLO

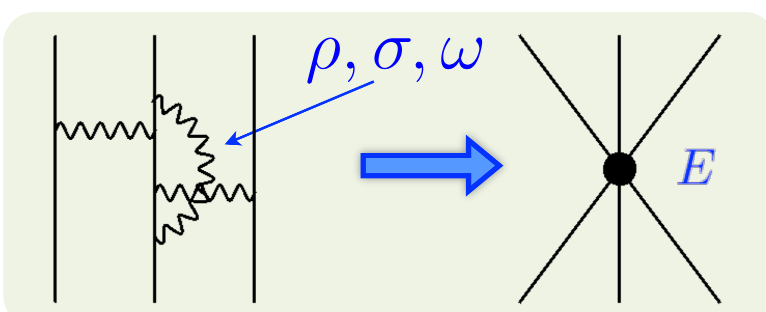
*(U. van Kolck ´94; Epelbaum et al. ´02; Nogga et al. ´05; Navratil et al. ´07)*



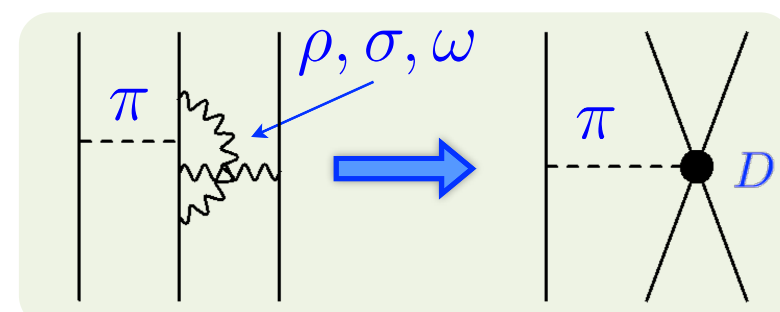

$c_{1,3,4}$ from the fit to $\pi N$-scattering data

$D, E$ from $^3\mathrm{H}, {}^4\mathrm{He}, {}^{10}\mathrm{B}$ binding energy + coherent $nd$ - scattering length

- LECs $D$ and $E$ incorporate short-range contr.




Resonance saturation interpretation of LECs




- Delta contributions encoded in LECs

*(Bernard, Kaiser & Meißner ’97)*

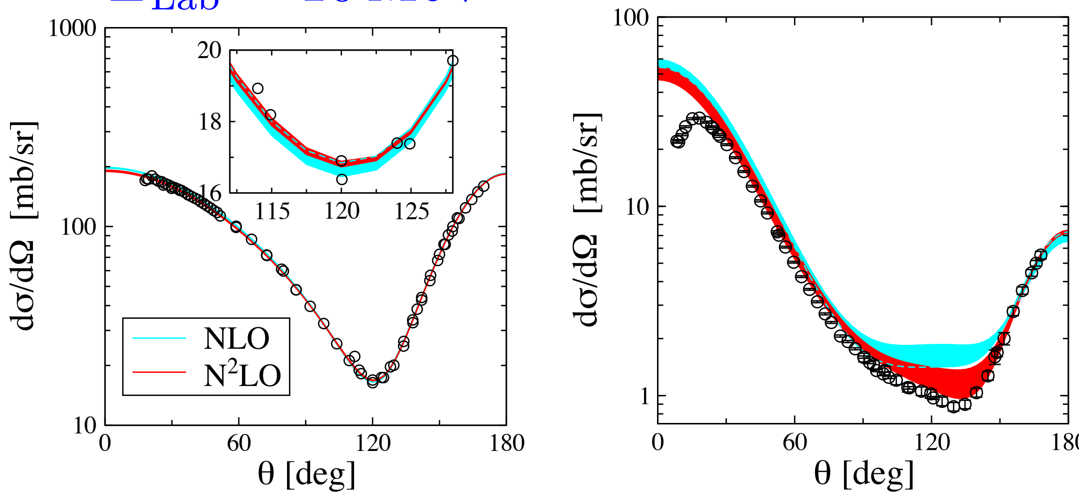

Delta-resonance saturation

$$c_3 = -2c_4 = c_3(\not\Delta) - \frac{4h_A^2}{9\Delta}$$

Enlargement due to Delta contribution

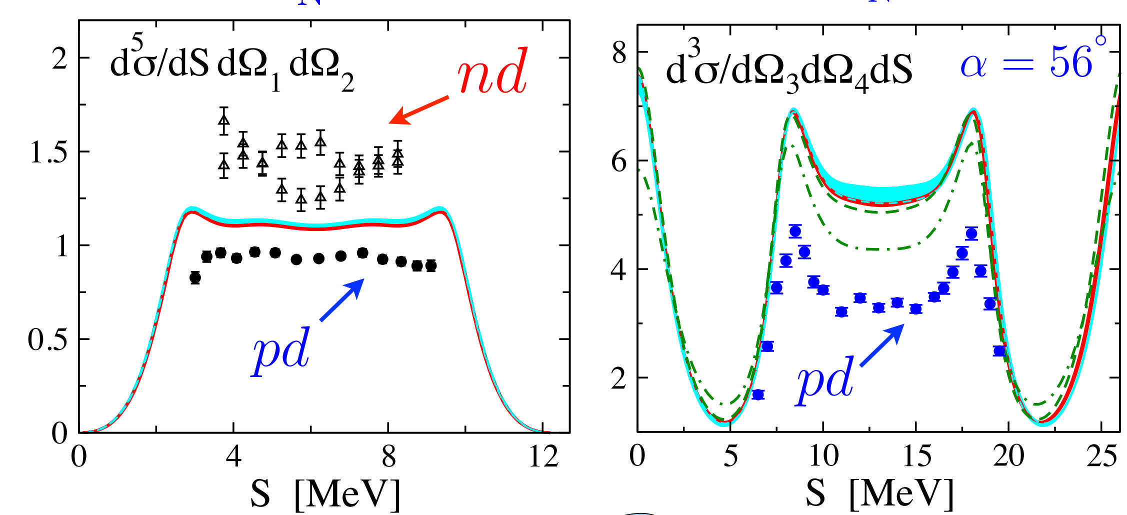
# nd elastic scattering

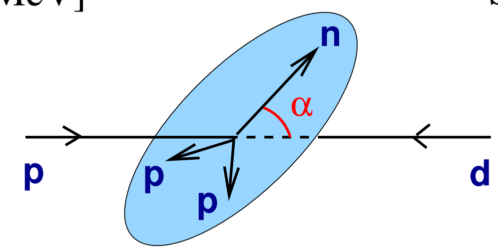
$E_{\mathrm{Lab}} = 10\,\mathrm{MeV}$ $E_{\mathrm{Lab}} = 65\,\mathrm{MeV}$




# nd break-up $[\mathrm{mb\,MeV^{-1}sr^{-2}}]$

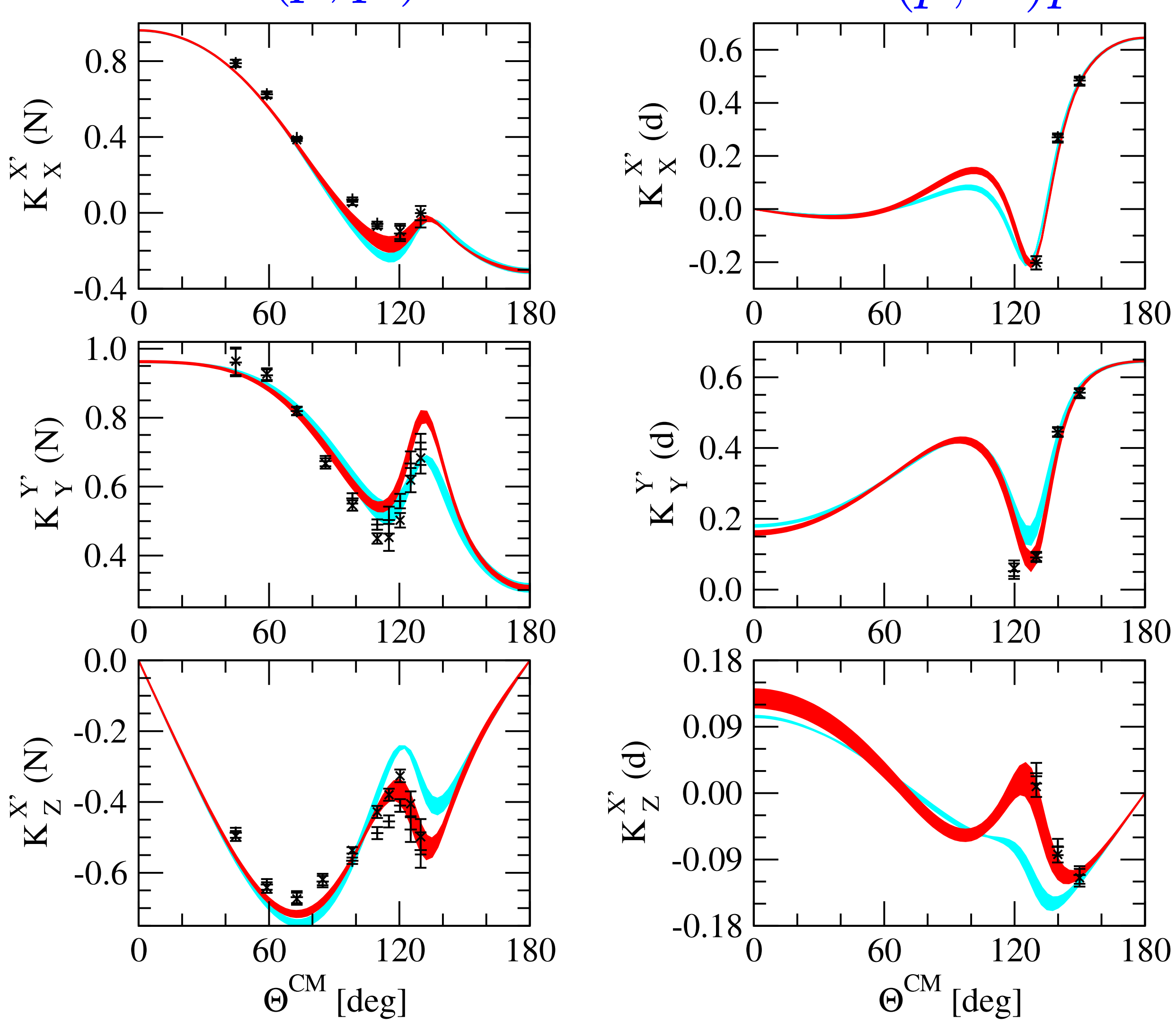
SST at $E_N = 12\,\mathrm{MeV}$ SCRE at $E_N = 19\,\mathrm{MeV}$




# polarization transfer: $E_p^{\mathrm{Lab}} = 22.7\,\mathrm{MeV}$

$d(\vec{p},\vec{p})d$ $d(\vec{p},\vec{d})p$




For references see recent reviews:

Epelbaum, Prog. Part Nucl. Phys. 57 (06) 654

Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773

Entem, Machleidt, Phys. Rept. 503 (11) 1

Epelbaum, Meißner, arXiv:1201.2136, to appear in Ann. Rev. Nucl. Part. Sci.

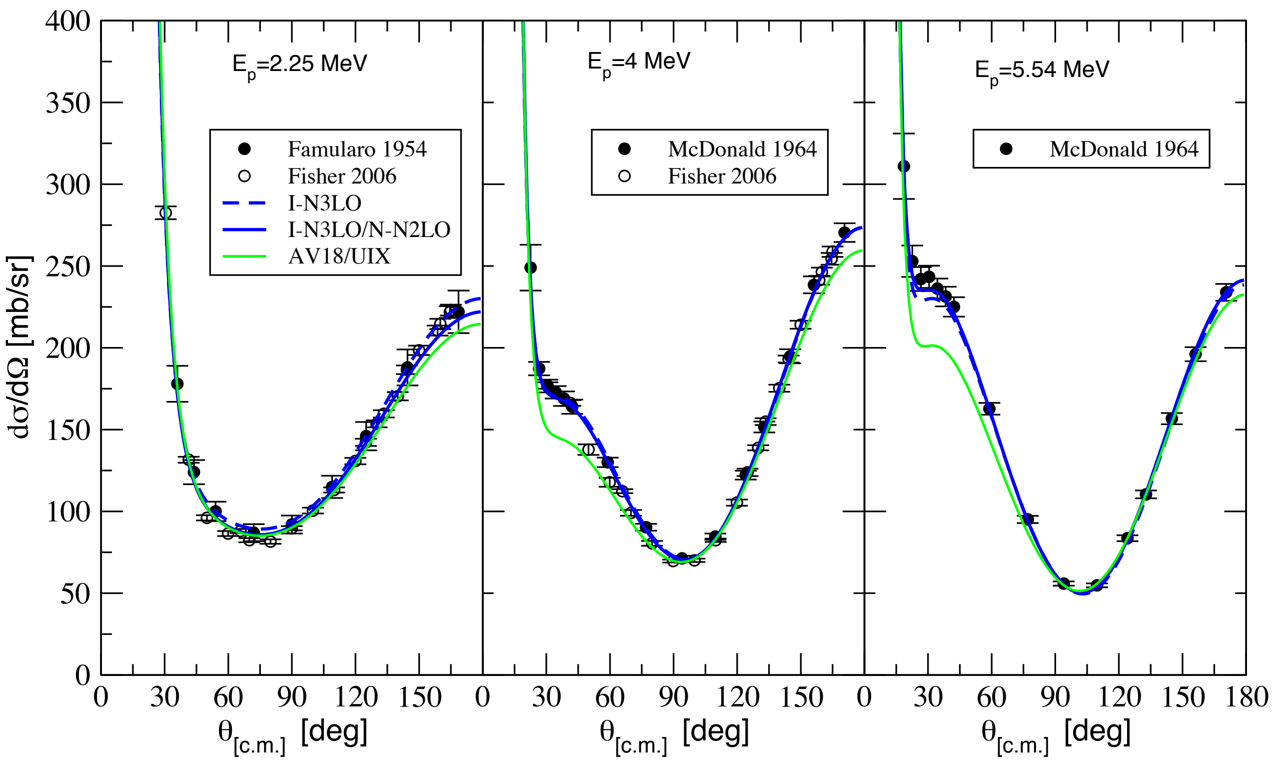
Kalantar et al. Rep. Prog. Phys. 75 (12) 016301

- Generally good description of data. But some discrepancies arise. E.g. break-up observables for SCRE/SST configuration at low energy
- Hope for improvement at N³LO

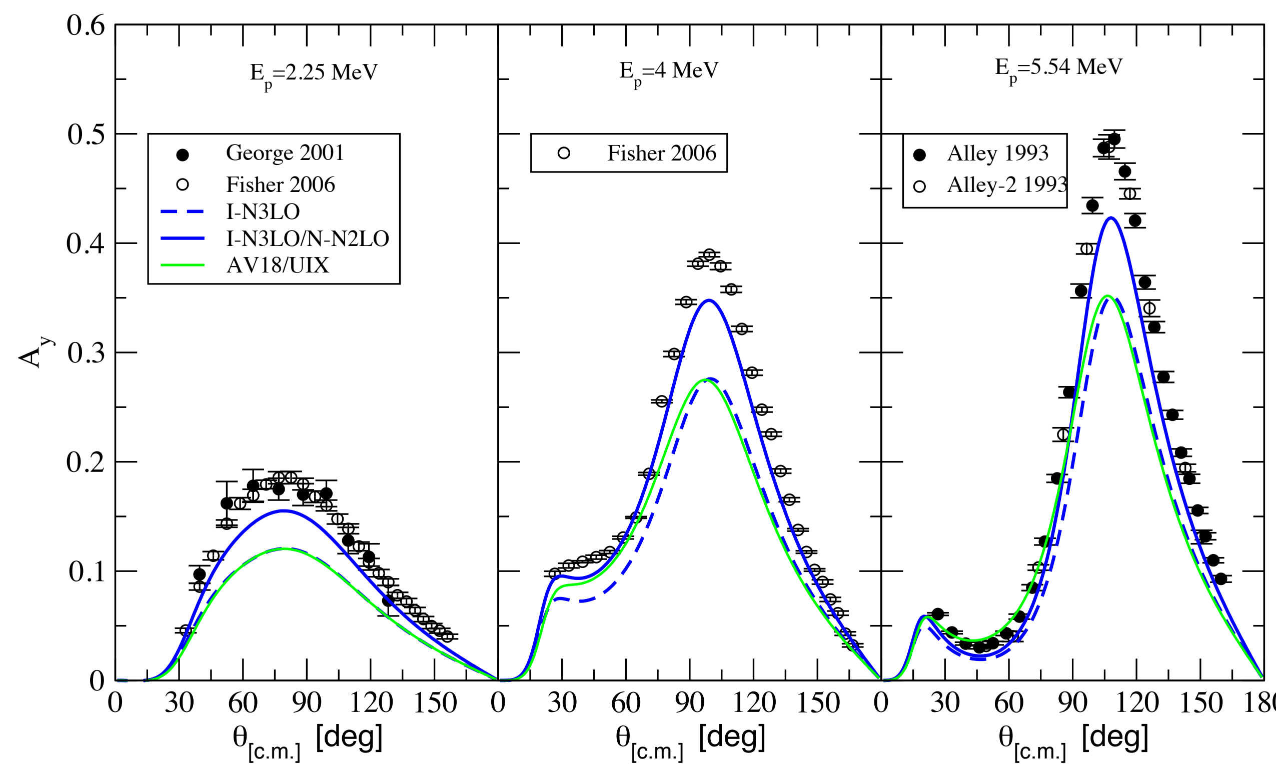
# Proton-$^3$He elastic scattering

*Viviani, Girlanda, Kievsky, Marcucci, Rosati arXiv: 1004.1306*

## p-$^3$He differential cross section at low energies




## proton vector analyzing power $A_y$-puzzle




As in n-d scattering case N$^2$LO 3NF's are not enough to resolve underprediction of $A_y$

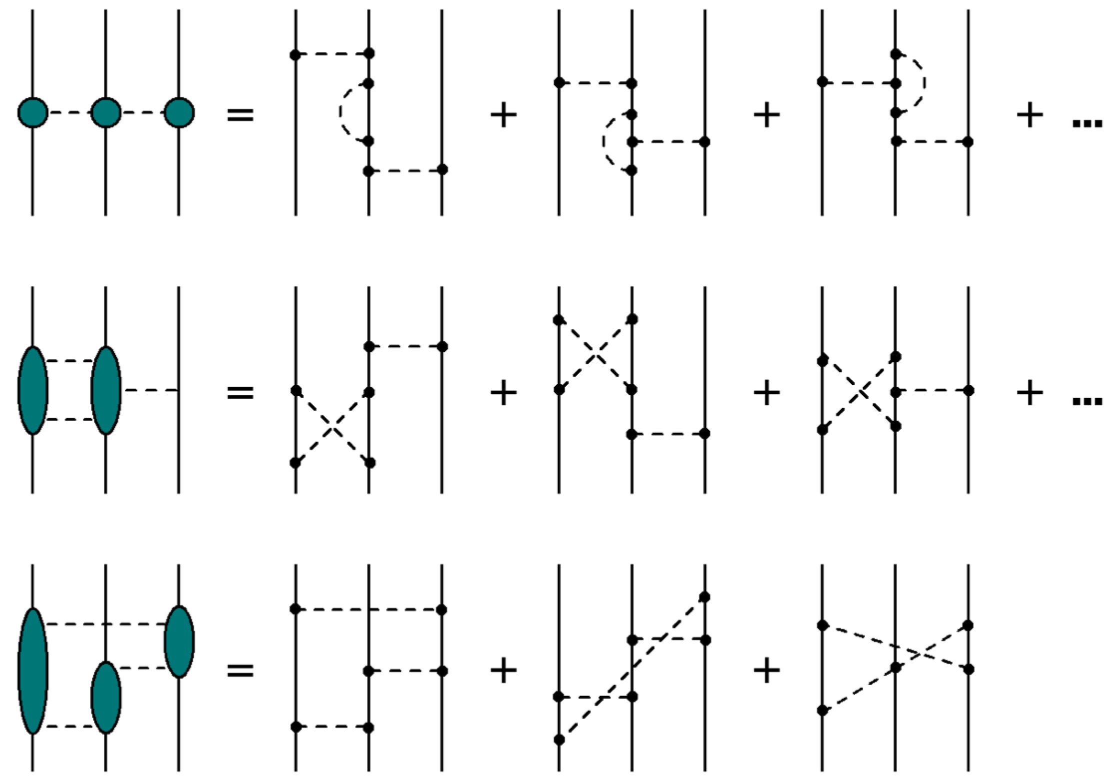
→ Hope for improvement at higher orders

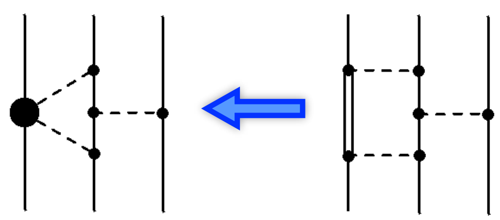
# Three-nucleon forces

- Three-nucleon forces at $N^3LO$

**Long range contributions** *Bernard, Epelbaum, H.K., Meißner ´08; Ishikawa, Robilotta ´07*

- No additional free parameters
- Expressed in terms of $g_A, F_\pi, M_\pi$
- Rich isospin-spin-orbit structure
- Δ(1232)-contr. are important

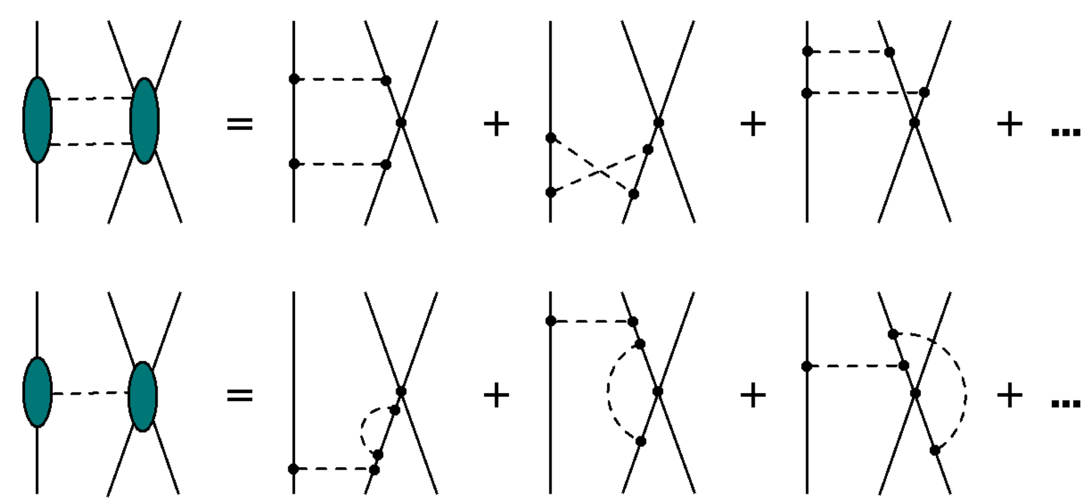


Large $c_i \sim$ 

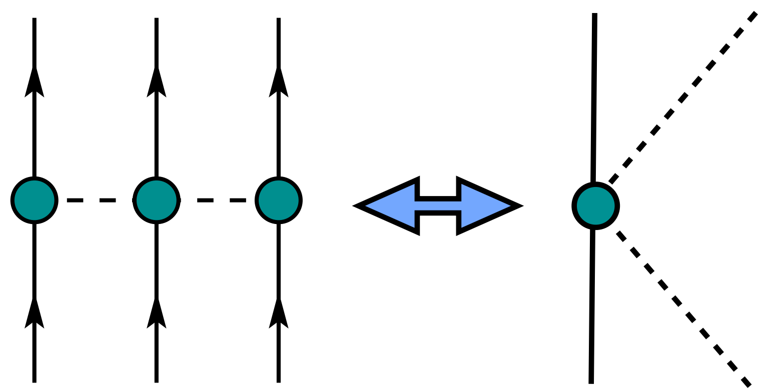
**Shorter range contributions** *Bernard, Epelbaum, H.K., Meißner ´11*

- LECs needed for shorter range contr.

  $g_A, F_\pi, M_\pi, C_T$
- Central NN contact interaction does not contribute
- Unique expressions in the static limit for a renormalizable 3NF



# Two-pion-exchange 3NF



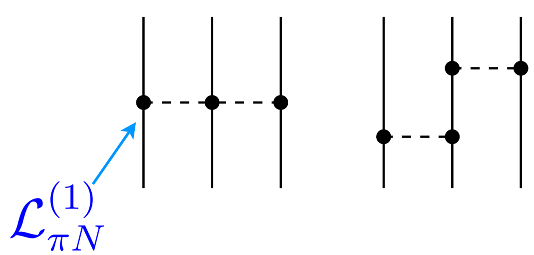
- Two-pion-exchange 3NF is connected to pion-nucleon scattering amplitude

  *Ishikawa, Robilotta ´07*

  - The same linear combinations of LECs
  - The same renormalization

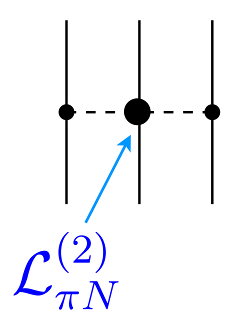
$$V_{2\pi} \;=\; \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big( \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

**NLO - contr.**

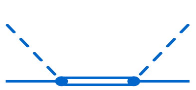



← yield vanishing 3NF contributions

**N²LO - contr.**




← first nonvanishing 3NF, encodes information about the Δ:

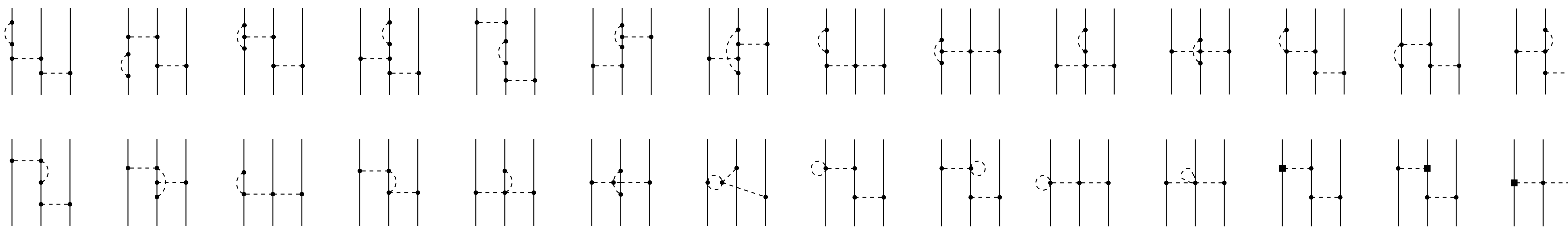

↔


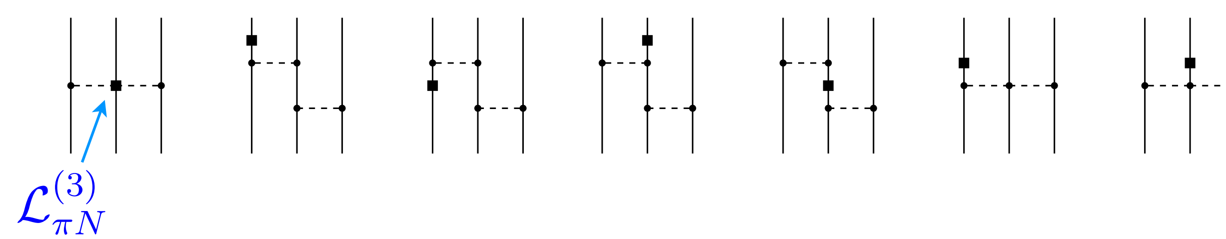
$$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4}\Big( (2c_3 - 4c_1) M_\pi^2 + c_3 q_2^2 \Big) \,, \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

*van Kolck ´94*

# Two-pion-exchange 3NF

**N$^3$LO - contr. (leading 1 loop)**






renormalization nontrivial !

$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2 + 1\right) M_\pi^3 + 2\left(g_A^2 + 1\right) M_\pi q_2^2\right],$$

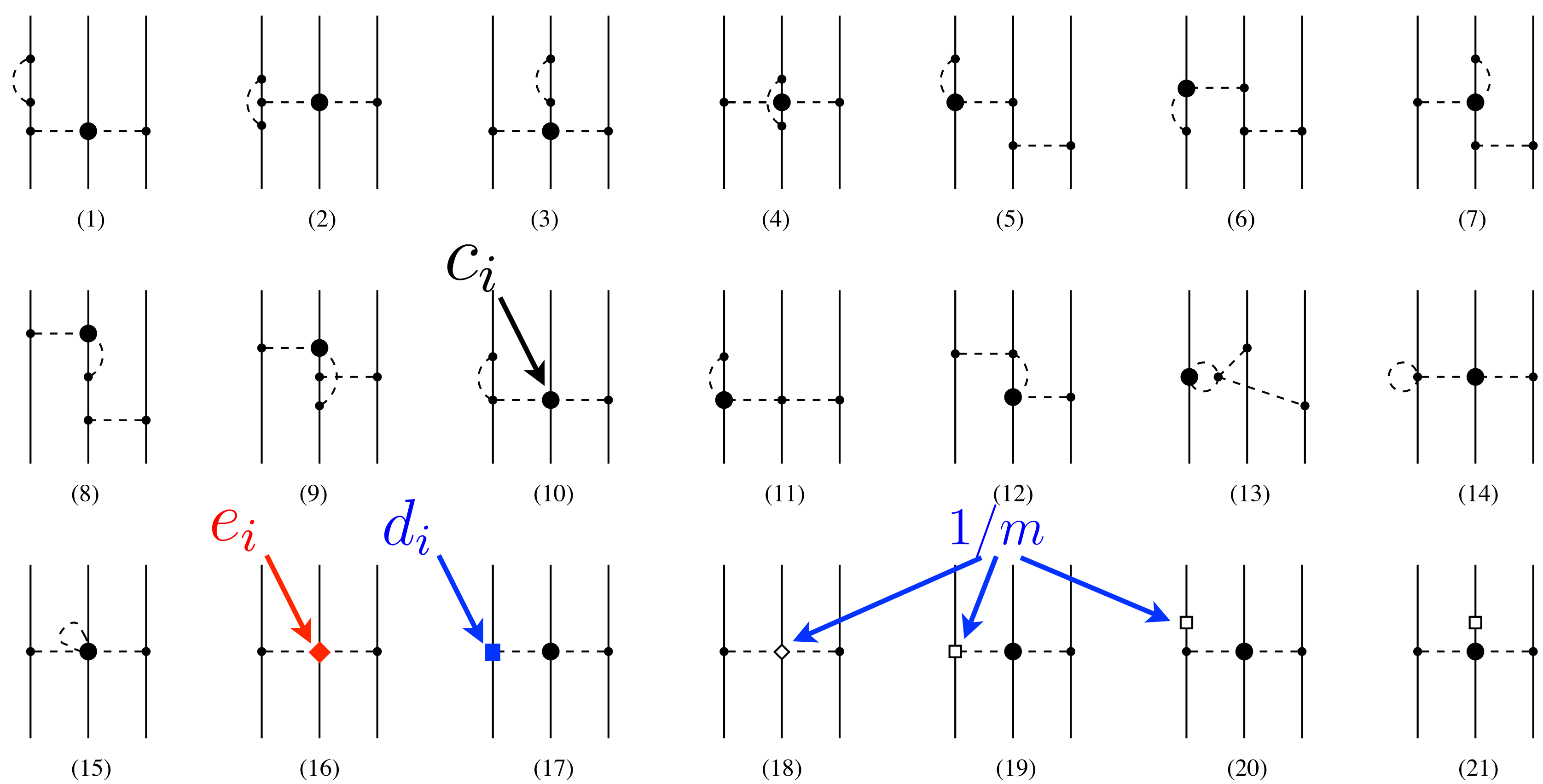
$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2 + 1)M_\pi\right]$$

*Ishikawa, Robilotta ´07,*
*Bernard, Epelbaum, HK, Meißner ´07*

- No unknown parameters at this order
- Everything is expressed in terms of loop function $A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$
- Additional unitarity transformations required for proper renormalization

# Two-pion-exchange 3NF

**$N^4LO$ - contr. (subleading 1 loop)** *Epelbaum, Gasparyan, H.K., ´12*




$c_i$´s LECs from $\mathcal{L}_{\pi N}^{(2)}$, $d_i$´s LECs from $\mathcal{L}_{\pi N}^{(3)}$, $e_i$´s LECs from $\mathcal{L}_{\pi N}^{(4)}$: fitted to $\pi N$ - scattering data

- Leading $\Delta$ - contributions are taken into account through $c_i$´s
- Vanishing $1/m$ - contributions at this order

# Two-pion-exchange 3NF at N$^4$LO

$$\mathcal{A}^{(5)}(q_2) = \frac{g_A}{4608\pi^2 F_\pi^6}\Big[M_\pi^2 q_2^2 \big(F_\pi^2 \big(2304\pi^2 g_A(4\bar{e}_{14}+2\bar{e}_{19}-\bar{e}_{22}-\bar{e}_{36}) - 2304\pi^2 \bar{d}_{18} c_3\big)$$
$$+\, g_A(144c_1 - 53c_2 - 90c_3)\big) + M_\pi^4 \big(F_\pi^2 \big(4608\pi^2 \bar{d}_{18}(2c_1 - c_3) + 4608\pi^2 g_A(2\bar{e}_{14}+2\bar{e}_{19}-\bar{e}_{36}-4\bar{e}_{38})\big)$$
$$+\, g_A\left(72\left(64\pi^2 \bar{l}_3 + 1\right) c_1 - 24c_2 - 36c_3\right)\big) + q_2^4\left(2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2g_A(5c_2 + 18c_3)\right)\Big]$$
$$-\, \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2)\left(M_\pi^2 + 2q_2^2\right)\left(4M_\pi^2(6c_1 - c_2 - 3c_3) + q_2^2(-c_2 - 6c_3)\right)$$

$$\mathcal{B}^{(5)}(q_2) = -\frac{g_A}{2304\pi^2 F_\pi^6}\Big[M_\pi^2\left(F_\pi^2\left(1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A(2\bar{e}_{17}+2\bar{e}_{21}-\bar{e}_{37})\right) + 108 g_A^3 c_4 + 24 g_A c_4\right)$$
$$+\, q_2^2\left(5g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A\right)\Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2)\left(4M_\pi^2 + q_2^2\right)$$

Some LECs can be absorbed by shifting $c_i$´s

$$c_1 \to c_1 - 2M_\pi^2\left(\bar{e}_{22} - 4\bar{e}_{38} - \frac{\bar{l}_3 c_1}{F_\pi^2}\right),$$
$$c_3 \to c_3 + 4M_\pi^2\left(2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36} + 2\frac{\bar{l}_3 c_1}{F_\pi^2}\right),$$
$$c_4 \to c_4 + 4M_\pi^2(2\bar{e}_{21} - \bar{e}_{37}),$$

$$L(q) = \frac{\sqrt{q^2 + 4M_\pi^2}}{q}\log\frac{\sqrt{q^2+4M_\pi^2}+q}{2M_\pi}$$

- No $d_i$ dependence of TPE-contr. besides $d_{18}$
- Pion-nucleon scattering does strongly depend on $d_i$´s

$$g_{\pi NN} = \frac{g_A\, m}{F_\pi}\left(1 - \frac{2M_\pi^2 \bar{d}_{18}}{g_A}\right)$$ ⟸ Violation of Goldberger-Treiman relation

# Pion-nucleon scattering

Heavy baryon calculation up to order $q^4$ *Fettes, Meißner Nucl. Phys. A676 (2000) 311*

1/m power counting used in FM work $\Longrightarrow$ $\frac{p}{m} \sim \frac{q}{\Lambda_\chi}$

- Difference in Weinberg's power counting for NN $\Longrightarrow$ $\frac{p}{m} \sim \left(\frac{q}{\Lambda_\chi}\right)^2$

Refit of $d_i$ and $e_i$ LECs is needed

$$\pi^a(q_1) + N(p_1) \to \pi^b(q_2) + N(p_2)$$

$$T^{ba}_{\pi N} = \frac{E+m}{2m}\left(\delta^{ba}\left[g^+(\omega,t) + i\,\vec\sigma\cdot\vec q_2\times\vec q_1\,h^+(\omega,t)\right] + i\,\epsilon^{bac}\tau^c\left[g^-(\omega,t) + i\vec\sigma\cdot\vec q_2\times\vec q_1\,h^-(\omega,t)\right]\right)$$

CMS kinematics: $\omega = q_1^0 = q_2^0, \quad E = E_1 = E_2 = \sqrt{\vec q^{\,2} + m^2}, \quad \vec q_1^{\,2} = \vec q_2^{\,2} = \vec q^{\,2}, \quad t = (q_1 - q_2)^2$

Partial wave amplitudes: $f^{\pm}_{l\pm}(s) = \frac{E+m}{16\pi\sqrt{s}}\int_{-1}^{1} dz\left[g^{\pm}P_l(z) + \vec q^{\,2}h^{\pm}\left(P_{l\pm1}(z) - zP_l(z)\right)\right]$

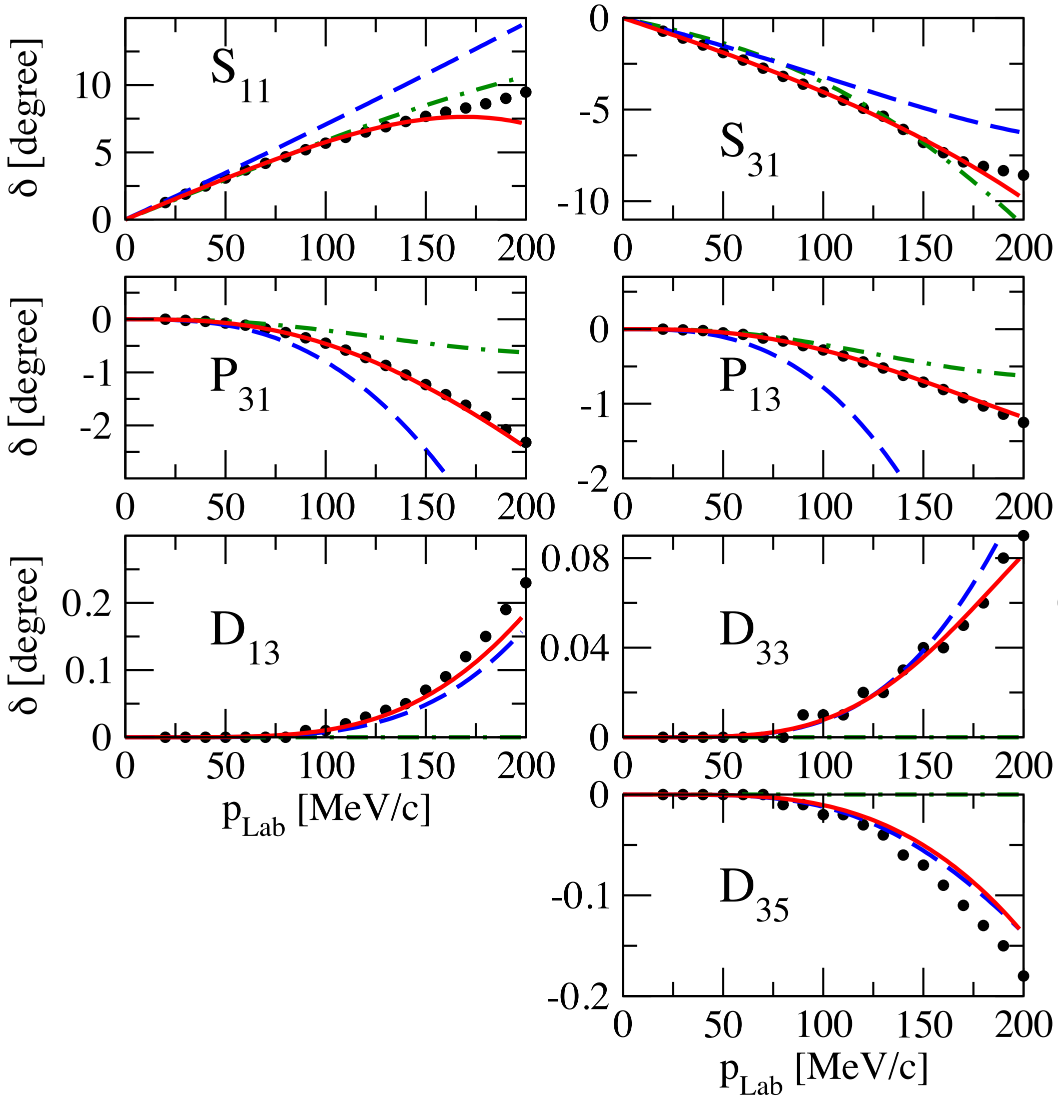
In the isospin basis: $f^{1/2}_{l\pm} = f^{+}_{l\pm} + 2f^{-}_{l\pm}, \quad f^{3/2}_{l\pm} = f^{+}_{l\pm} - f^{-}_{l\pm}$

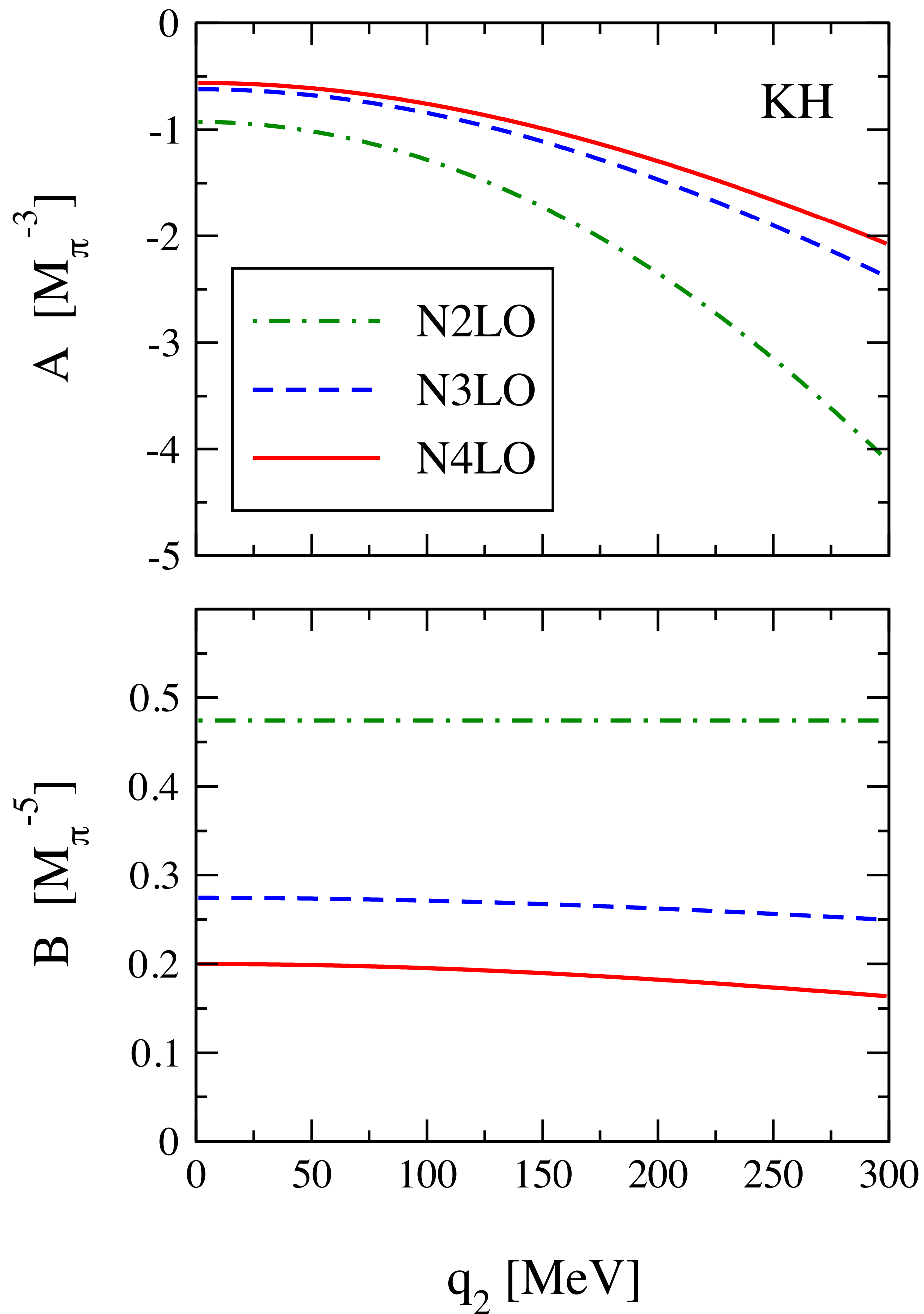
Absence of inelasticity below the two-pion production threshold

$$\delta^I_{l\pm}(s) = \arctan\left(|\vec q|\,\mathcal{R}e\, f^I_{l\pm}(s)\right)$$

# Two-pion-exchange at $N^4LO$

Data fitted for $p_{\mathrm{Lab}} < 150\,\mathrm{MeV}$



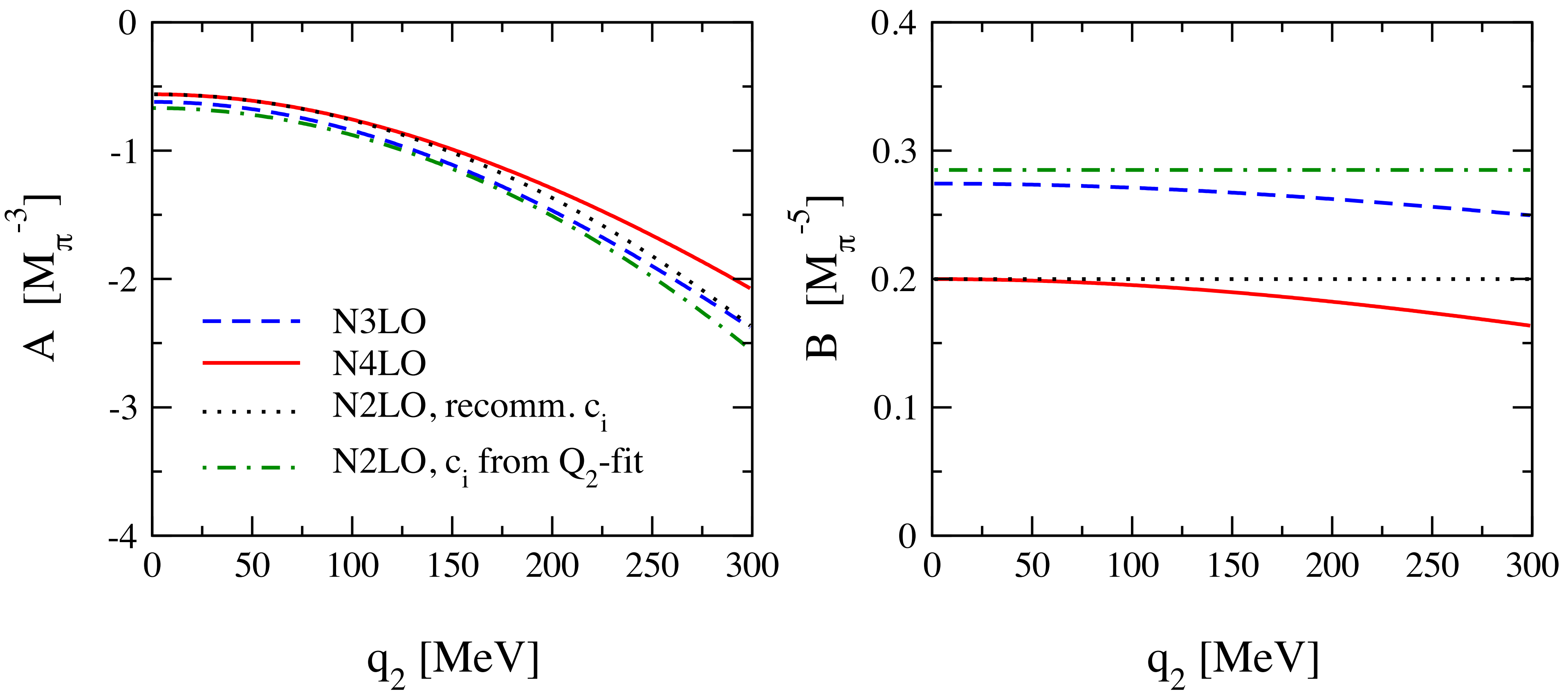

*Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707*

*Similar fit to George-Washington (GW) PWA: Arndt et al. Phys. Rev. C 74 (2006) 045205*

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $\bar{d}_1 + \bar{d}_2$ | $\bar{d}_3$ | $\bar{d}_5$ | $\bar{d}_{14} - \bar{d}_{15}$ | $\bar{e}_{14}$ | $\bar{e}_{15}$ | $\bar{e}_{16}$ | $\bar{e}_{17}$ | $\bar{e}_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GW-fit | $-1.13$ | $3.69$ | $-5.51$ | $3.71$ | $5.57$ | $-5.35$ | $0.02$ | $-10.26$ | $1.75$ | $-5.80$ | $1.76$ | $-0.58$ | $0.96$ |
| KH-fit | $-0.75$ | $3.49$ | $-4.77$ | $3.34$ | $6.21$ | $-6.83$ | $0.78$ | $-12.02$ | $1.52$ | $-10.41$ | $6.08$ | $-0.37$ | $3.26$ |

- No dependence on $d_i$´s
- $e_i$´s are of natural size
- Good convergence of TPE 3NF

# Working with N$^2$LO 3NF




Recommended $c_i$´s by working with N$^2$LO 3NF

$$c_1^{\rm KH} = -0.37\,{\rm GeV}^{-1},\quad c_3^{\rm KH} = -2.71\,{\rm GeV}^{-1},\quad c_4^{\rm KH} = 1.41\,{\rm GeV}^{-1},$$
$$c_1^{\rm GW} = -0.73\,{\rm GeV}^{-1},\quad c_3^{\rm GW} = -3.38\,{\rm GeV}^{-1},\quad c_4^{\rm GW} = 1.69\,{\rm GeV}^{-1}.$$

- With these parameters we get at $q_2 = 0$ the value and slope of N$^4$LO result
- $c_i$´s fitted to pion-nucleon $Q^2$(KH-fit) lead to slightly different results for B-function

$$c_1 = -0.25\,{\rm GeV}^{-1},\quad c_2 = 2.02\,{\rm GeV}^{-1},\quad c_3 = -2.80\,{\rm GeV}^{-1},\quad c_4 = 2.01\,{\rm GeV}^{-1}.$$

# Most general structure of a local 3NF

Epelbaum, Gasparyan, H.K., in preparation

Up to N$^4$LO, the computed contributions are local $\longrightarrow$ it is natural to switch to r-space.

A meaningful comparison requires a complete set of independent operators

| Generators $\mathcal{G}$ of 89 independent operators | $S$ | $A$ | $G_1$ | $G_2$ | $G_1(12)$ | $G_2(12)$ |
|---|---|---|---|---|---|---|
| $1$ | $\mathcal{O}_1$ | - | - | - | - | - |
| $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ | $\mathcal{O}_2$ | - | $\mathcal{O}_3$ | $\mathcal{O}_4$ | - | - |
| $\vec{\sigma}_1 \cdot \vec{\sigma}_3$ | $\mathcal{O}_5$ | - | $\mathcal{O}_6$ | $\mathcal{O}_7$ | - | - |
| $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$ | $\mathcal{O}_8$ | - | $\mathcal{O}_9$ | $\mathcal{O}_{10}$ | - | - |
| $\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$ | $\mathcal{O}_{11}$ | $\mathcal{O}_{12}$ | $\mathcal{O}_{13}$ | $\mathcal{O}_{14}$ | $\mathcal{O}_{15}$ | $\mathcal{O}_{16}$ |
| $\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$ | $\mathcal{O}_{17}$ | - | - | - | - | - |
| $\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{r}_{12} \times \vec{r}_{23})$ | $\mathcal{O}_{18}$ | - | $\mathcal{O}_{19}$ | $\mathcal{O}_{20}$ | - | - |
| $\vec{r}_{23} \cdot \vec{\sigma}_1 \vec{r}_{23} \cdot \vec{\sigma}_3$ | $\mathcal{O}_{21}$ | $\mathcal{O}_{22}$ | $\mathcal{O}_{23}$ | $\mathcal{O}_{24}$ | $\mathcal{O}_{25}$ | $\mathcal{O}_{26}$ |
| $\vec{r}_{23} \cdot \vec{\sigma}_3 \vec{r}_{12} \cdot \vec{\sigma}_1$ | $\mathcal{O}_{27}$ | - | $\mathcal{O}_{28}$ | $\mathcal{O}_{29}$ | - | - |
| $\vec{r}_{23} \cdot \vec{\sigma}_1 \vec{r}_{12} \cdot \vec{\sigma}_3$ | $\mathcal{O}_{30}$ | - | $\mathcal{O}_{31}$ | $\mathcal{O}_{32}$ | - | - |
| $\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{r}_{23} \cdot \vec{\sigma}_1 \vec{r}_{23} \cdot \vec{\sigma}_2$ | $\mathcal{O}_{33}$ | $\mathcal{O}_{34}$ | $\mathcal{O}_{35}$ | $\mathcal{O}_{36}$ | $\mathcal{O}_{37}$ | $\mathcal{O}_{38}$ |
| $\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{r}_{23} \cdot \vec{\sigma}_1 \vec{r}_{12} \cdot \vec{\sigma}_2$ | $\mathcal{O}_{39}$ | $\mathcal{O}_{40}$ | $\mathcal{O}_{41}$ | $\mathcal{O}_{42}$ | $\mathcal{O}_{43}$ | $\mathcal{O}_{44}$ |
| $\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{r}_{12} \cdot \vec{\sigma}_1 \vec{r}_{23} \cdot \vec{\sigma}_2$ | $\mathcal{O}_{45}$ | $\mathcal{O}_{46}$ | $\mathcal{O}_{47}$ | $\mathcal{O}_{48}$ | $\mathcal{O}_{49}$ | $\mathcal{O}_{50}$ |
| $\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{r}_{12} \cdot \vec{\sigma}_1 \vec{r}_{12} \cdot \vec{\sigma}_2$ | $\mathcal{O}_{51}$ | $\mathcal{O}_{52}$ | $\mathcal{O}_{53}$ | $\mathcal{O}_{54}$ | $\mathcal{O}_{55}$ | $\mathcal{O}_{56}$ |
| $\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{r}_{23} \cdot \vec{\sigma}_2 \vec{r}_{23} \cdot \vec{\sigma}_3$ | $\mathcal{O}_{57}$ | - | $\mathcal{O}_{58}$ | $\mathcal{O}_{59}$ | - | - |
| $\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{r}_{12} \cdot \vec{\sigma}_2 \vec{r}_{12} \cdot \vec{\sigma}_3$ | $\mathcal{O}_{60}$ | $\mathcal{O}_{61}$ | $\mathcal{O}_{62}$ | $\mathcal{O}_{63}$ | $\mathcal{O}_{64}$ | $\mathcal{O}_{65}$ |
| $\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{r}_{23} \cdot \vec{\sigma}_2 \vec{r}_{12} \cdot \vec{\sigma}_3$ | $\mathcal{O}_{66}$ | - | $\mathcal{O}_{67}$ | $\mathcal{O}_{68}$ | - | - |
| $\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\sigma}_3 \cdot (\vec{r}_{12} \times \vec{r}_{23})$ | $\mathcal{O}_{69}$ | - | $\mathcal{O}_{70}$ | $\mathcal{O}_{71}$ | - | - |
| $\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{r}_{23} \vec{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$ | $\mathcal{O}_{72}$ | $\mathcal{O}_{73}$ | $\mathcal{O}_{74}$ | $\mathcal{O}_{75}$ | $\mathcal{O}_{76}$ | $\mathcal{O}_{77}$ |
| $\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{r}_{23} \vec{\sigma}_2 \cdot \vec{r}_{23} \vec{\sigma}_3 \cdot (\vec{r}_{12} \times \vec{r}_{23})$ | $\mathcal{O}_{78}$ | $\mathcal{O}_{79}$ | $\mathcal{O}_{80}$ | $\mathcal{O}_{81}$ | $\mathcal{O}_{82}$ | $\mathcal{O}_{83}$ |
| $\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{r}_{12} \vec{\sigma}_2 \cdot \vec{r}_{12} \vec{\sigma}_3 \cdot (\vec{r}_{12} \times \vec{r}_{23})$ | $\mathcal{O}_{84}$ | - | $\mathcal{O}_{85}$ | $\mathcal{O}_{86}$ | - | - |
| $\boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{r}_{23} \vec{\sigma}_2 \cdot \vec{r}_{12} \vec{\sigma}_3 \cdot (\vec{r}_{12} \times \vec{r}_{23})$ | $\mathcal{O}_{87}$ | - | $\mathcal{O}_{88}$ | $\mathcal{O}_{89}$ | - | - |

Most general, local 3NF involves 89 operators, can be generated (by permutations) from 22 structures:

$$V_{3\rm N}^{\rm loc} = \sum_{i=1}^{22} \mathcal{G}_i F_i(r_{12}, r_{23}, r_{31}) + 5\,\text{perm.}$$

The structures $\mathcal{O}_i$ are defined as:

$$S(\mathcal{G}) := \frac{1}{6} \sum_{P \in S_3} P\mathcal{G}$$

$$A(\mathcal{G}) := \frac{1}{6} \sum_{P \in S_3} (-1)^P P\mathcal{G}$$

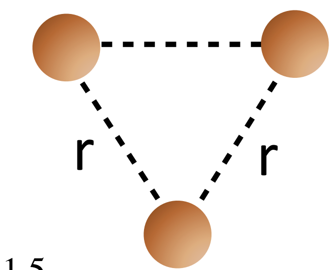
$$G_1(\mathcal{G}) := \left[ S_{13} - \frac{1}{2}(S_{23}S_{13} + S_{12}S_{13}) \right] (\mathcal{G})$$

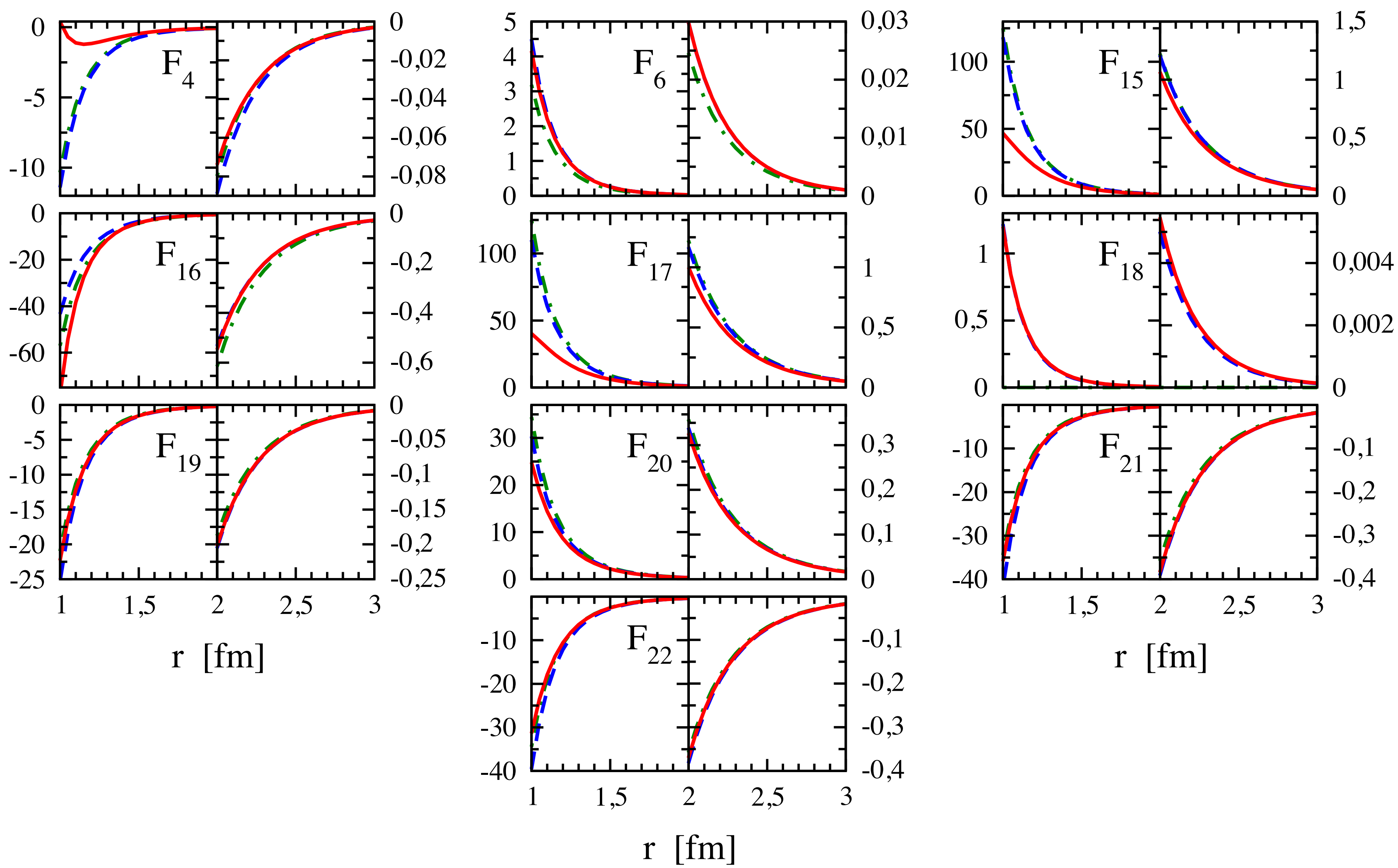
$$G_2(\mathcal{G}) := \frac{\sqrt{3}}{2} \left[ S_{23}S_{13} - S_{12}S_{13} \right] (\mathcal{G})$$

# Two-pion-exchange up to N$^4$LO

*Epelbaum, Gasparyan, H.K., in preparation*

Chiral expansion of TPE „structure functions“ $F_i$ (in MeV) in the equilateral-triangle configuration

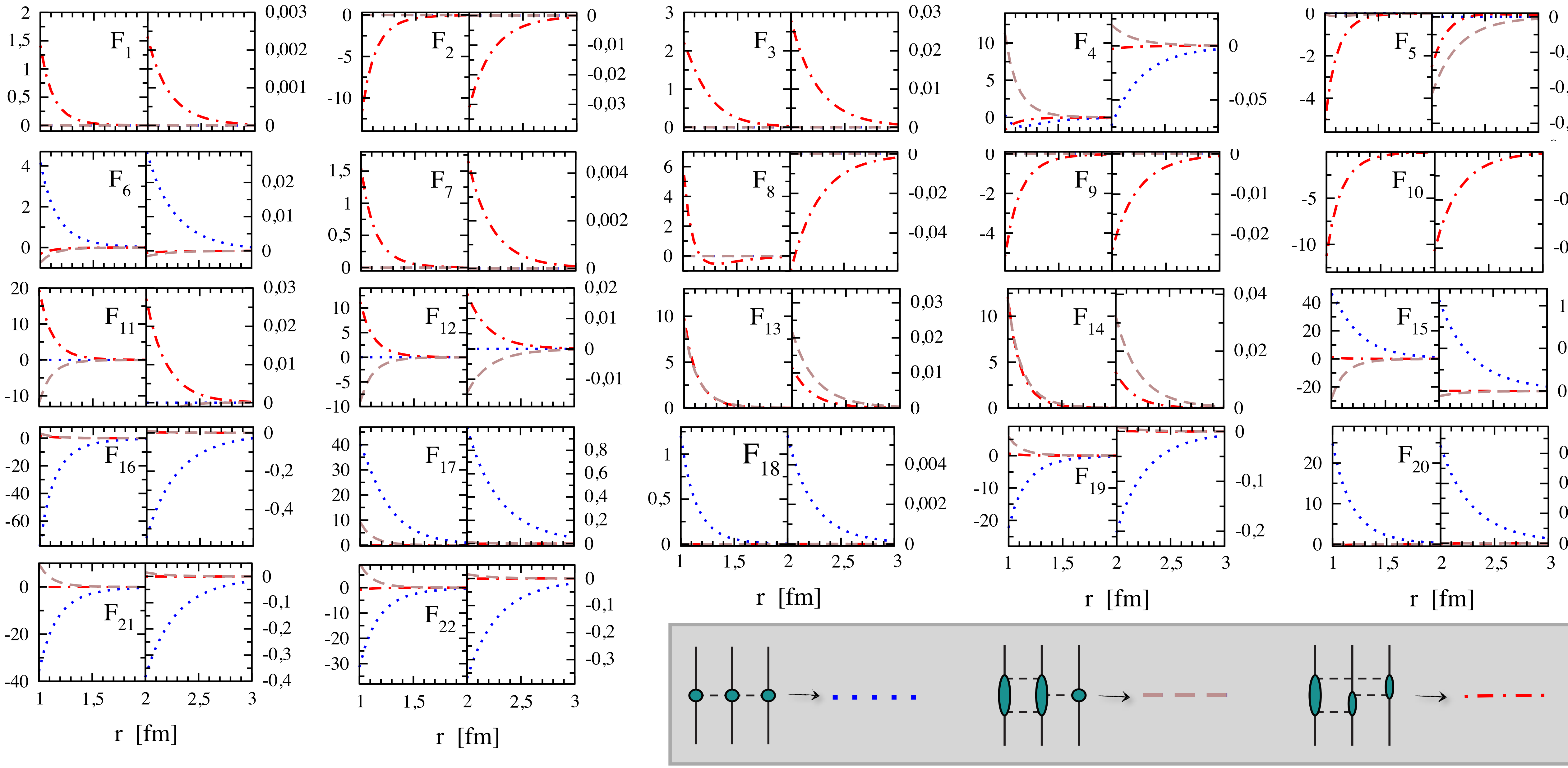



Excellent convergence of TPE-force at distance r $\geq 2\,\mathrm{fm}$

# Complete long-range 3NF up to N$^4$LO

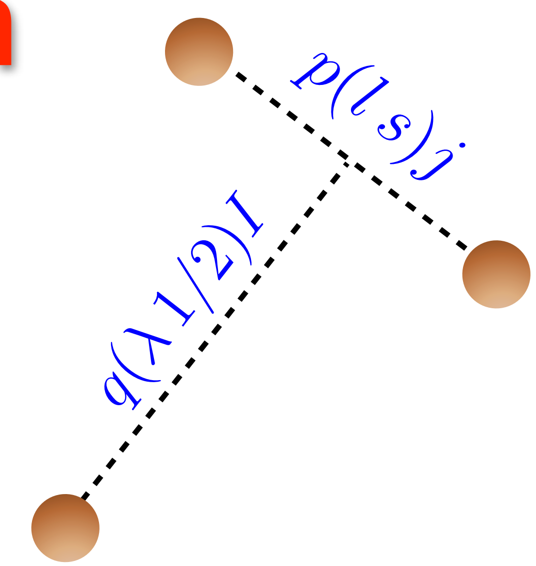
*Epelbaum, Gasparyan, H.K., in preparation*




- Predictions based entirely on chiral symmetry + input from $\pi N$, benchmarks for lattice-QCD
- Implications for Nd, light nuclei & nuclear matter? *(work in progress ...)*
- $2\pi - 1\pi$ and ring-topology: already converged? ChPT with explicit $\Delta$´s *(work in progress ...)*

# Partial wave decomposition

*Golak et al. Eur. Phys. J. A 43 (2010) 241*




- Faddeev equation is solved in the partial wave basis

$$|p, q, \alpha\rangle \equiv |pq(ls)j(\lambda\frac{1}{2})I(jI)JM_J\rangle\, |(t\frac{1}{2})TM_T\rangle$$

- Too many terms for doing PWD by hand ⟹ Automatization

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{\text{matrix} \sim 10^5 \times 10^5} = \int \underbrace{d\hat{p}'\, d\hat{q}'\, d\hat{p}\, d\hat{q}}_{\text{can be reduced to 5 dim. integral}} \sum_{m_l,\ldots} \Big(\text{CG coeffs.}\Big)\Big(Y_{l,m_l}(\hat{p})\, Y_{l',m_l'}(\hat{p}')\ \ldots\Big)\underbrace{\langle m'_{s_1} m'_{s_2} m'_{s_3}|V|m_{s_1} m_{s_2} m_{s_3}\rangle}_{\text{depends on spin \& isospin}}$$

- Ring-diagram-contr. expensive to calculate on the fly

We prestore ring-contr. to 3nf's on a fine momentum grid ⟹ Numerical interpolation of ring terms

Matrix-elements are so far calculated up to $j_{max}$=2 and J=5/2

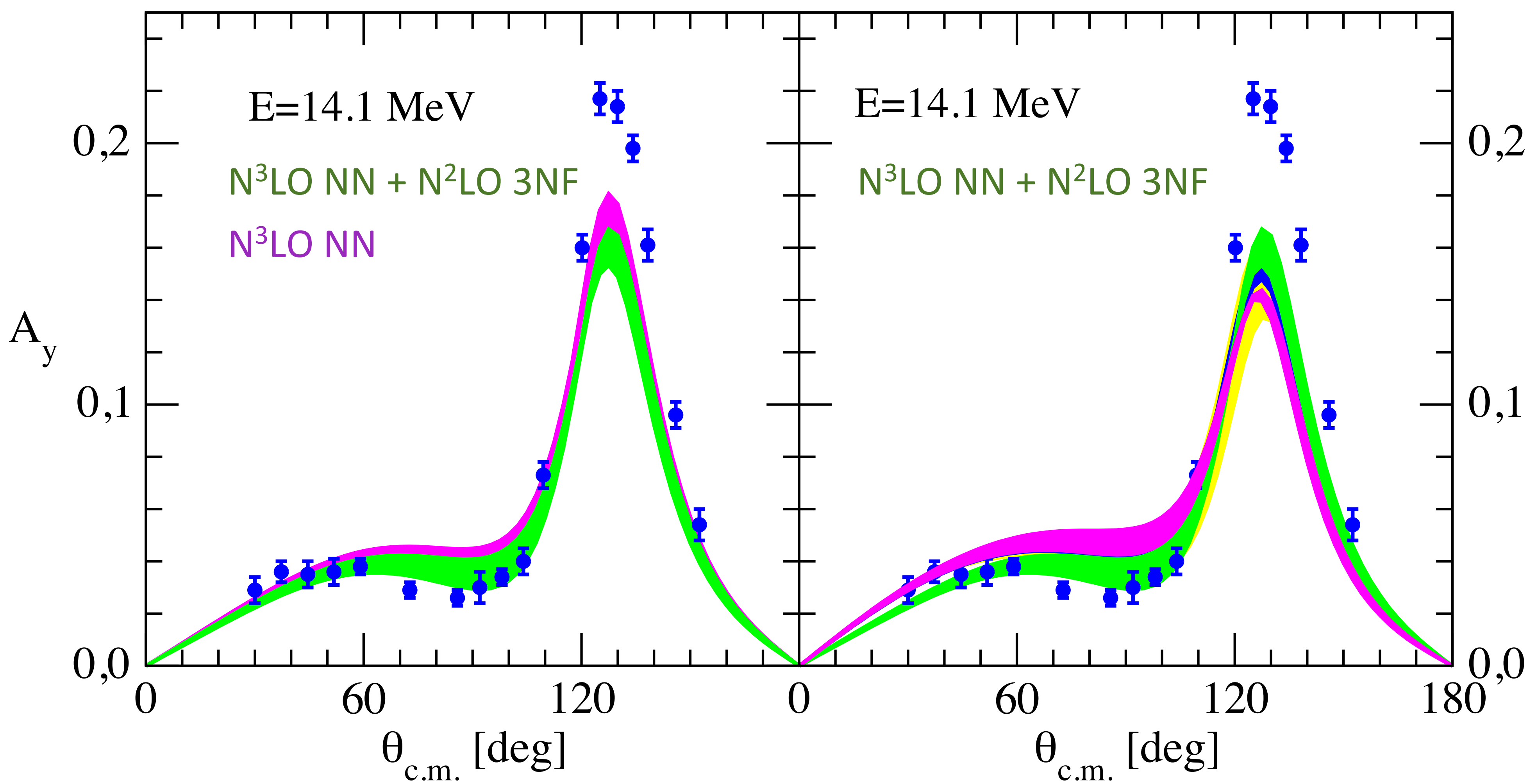
Supercomputers used: JUGENE in FZ-Jülich and OSC in Ohio State University

- PWD matrix-elements can be used to produce matrix-elements in harmonic oscillator basis
  *see talk by Robert Roth & Kai Hebeler*

Straightforward implementation of high order 3nf's in many-body calc. within No-Core Shell Model

# $A_y$-puzzle in elastic nd scattering

*Witala et al. Proceedings of Few Body 20*




Right panel: X = $N^3LO$ NN + $N^2LO$ 3NF + $N^3LO$ 3NF (1π-cont.) + $N^3LO$ 3NF (cont.)

- (yellow) = X + $N^3LO$ 3NF (2π-exch.)
- (blue) = X + $N^3LO$ 3NF (2π-exch. & 2π-1π-exch.)
- (magenta) = X + $N^3LO$ 3NF (2π-exch. & 2π-1π-exch. & ring)

Incomplete results: $N^3LO$ 3NF (2π-cont. & 1/m-corr.) are missing

# Summary

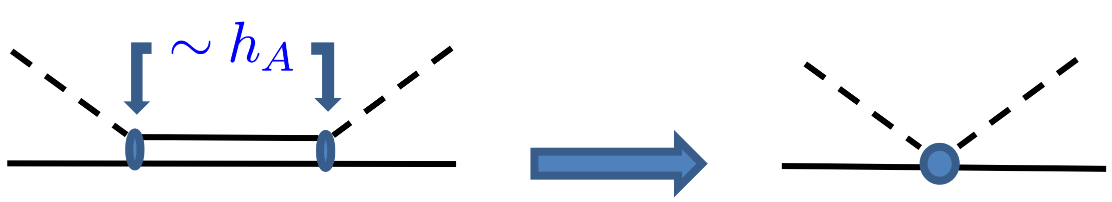
- Chiral nuclear forces are analyzed up to $N^3LO$
- Long-range part of chiral three-nucleon forces is analyzed up to $N^4LO$
- In general there are 89 spin-isospin structures in local 3NF's built out of 22 + perm.
- Two-pion-exchange part dominates 3NF but does not fill all 22 structures
- With two-pion-one-pion-exchange and ring diagrams all 22 structures are filled
- First (incomplete) results for $A_y$ in nd elastic scattering with $N^3LO$ 3NF's

# Outlook

- Partial wave decomposition of $N^3LO$ three-nucleon forces
- Complete study of 3NF and 4NF up to $N^4LO$ with explicit delta-isobar
- Implementations in Nd, light nuclei & nuclear matter

# EFT with explicit delta

- Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293\ \mathrm{MeV}$
- Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$ *(Hemmert, Holstein & Kambor '98)*
- Delta contributions encoded in LECs
  *(Bernard, Kaiser & Meißner ´97)*

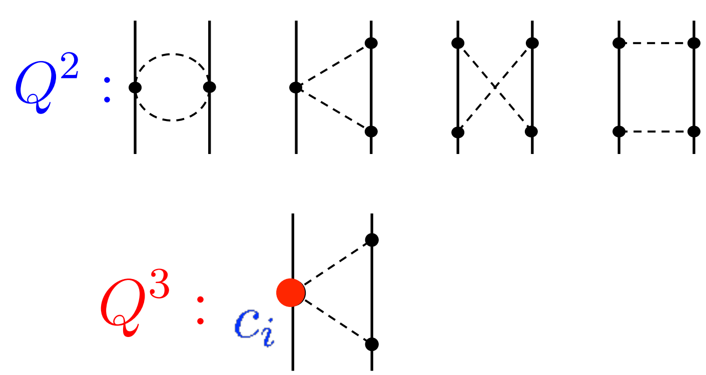


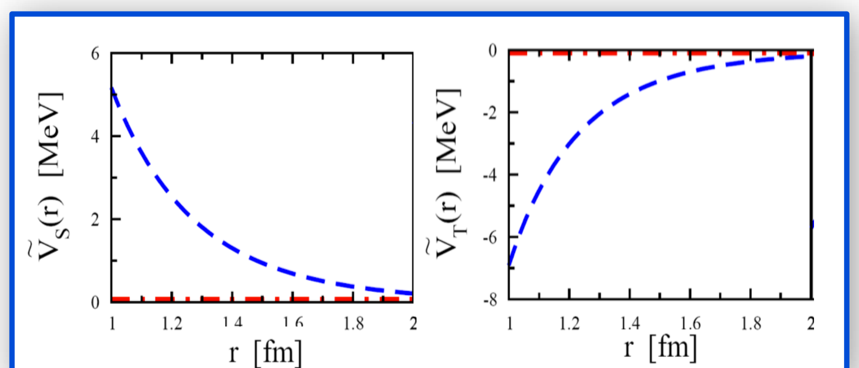

Delta-resonance saturation

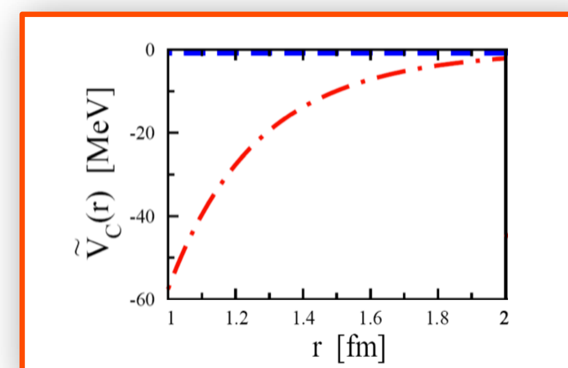
$$c_3 = -2c_4 = c_3(\not\Delta) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

- Convergence of EFT potential




The subleading contributions are larger than the leading one!

Expectation from inclusion of Δ explicitely

- more natural size of LECs
- better convergence
- applicability at higher energies

# Explicit decoupling

Do the positive powers of Δ not spoil the convergence?

Small scale expansion parameter $\Delta/\Lambda_\chi \sim \frac{1}{3}$ is not that small!

Manifest decoupling through the choice of renormalization conditions (no positive powers of Δ)

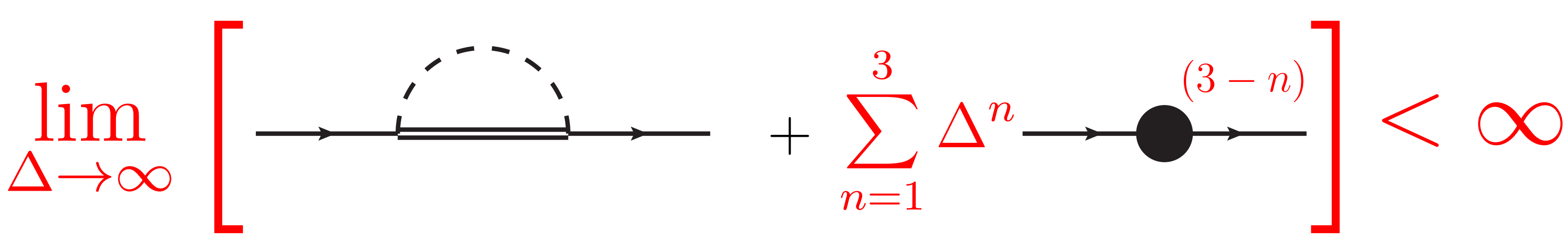
*Decoupling theorem due to Appelquist & Carrazone Phys. Rev. 11 (1974) 2856*

$$\mathcal{L}^{\rm SSE}_{piN} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \Delta\mathcal{L}^{(1)}_{\pi N} + \Delta\mathcal{L}^{(2)}_{\pi N} + \Delta^2\mathcal{L}^{(1)}_{\pi N} + \mathcal{O}(\epsilon^4)$$

Choose finite part of these LECs such that

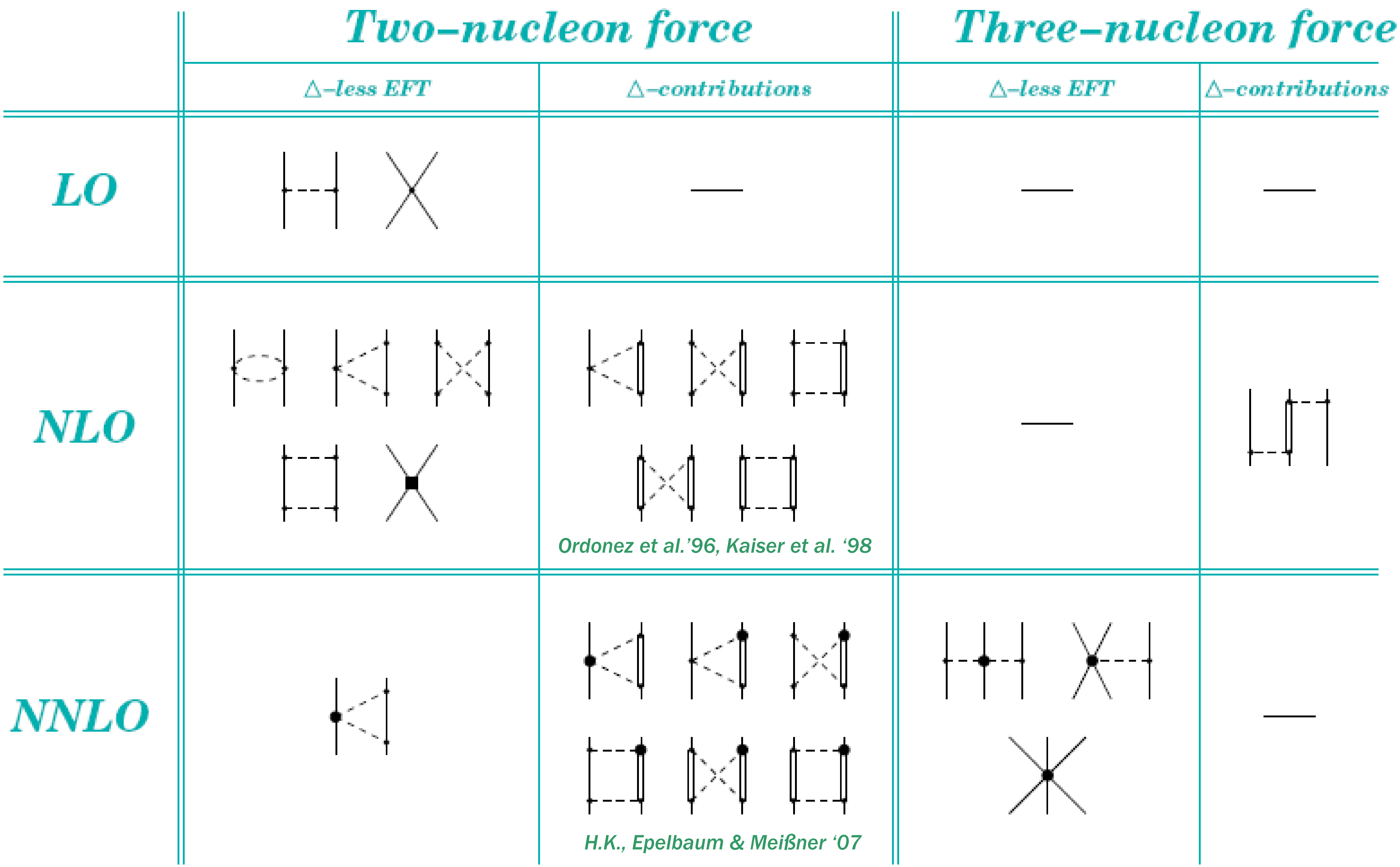
$$\lim_{\Delta\to\infty} \text{Green Function} < \infty$$

*Bernard, Fearing, Hemmert, Meißner NPA635 (1998) 121*



# Few-nucleon forces with the Delta

Isospin-symmetric contributions

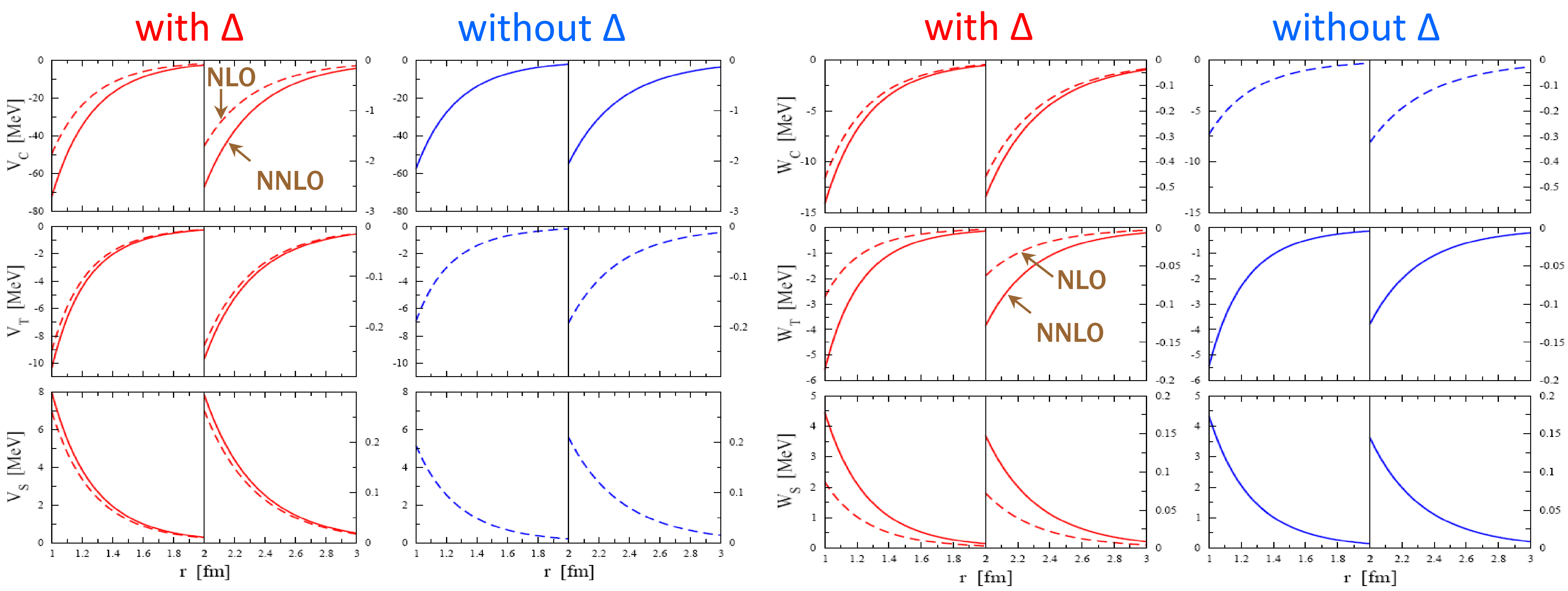


# NN potential with explicit Δ

*Epelbaum, H.K., Meißner, Eur. Phys. J. A32 (2007) 127*

$$V_{\rm eff} = V_C + W_C \vec{\tau}_1\cdot\vec{\tau}_2 + [V_S + W_S \vec{\tau}_1\cdot\vec{\tau}_2]\vec{\sigma}_1\cdot\vec{\sigma}_2 + [V_T + W_T \vec{\tau}_1\cdot\vec{\tau}_2](3\,\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2)$$

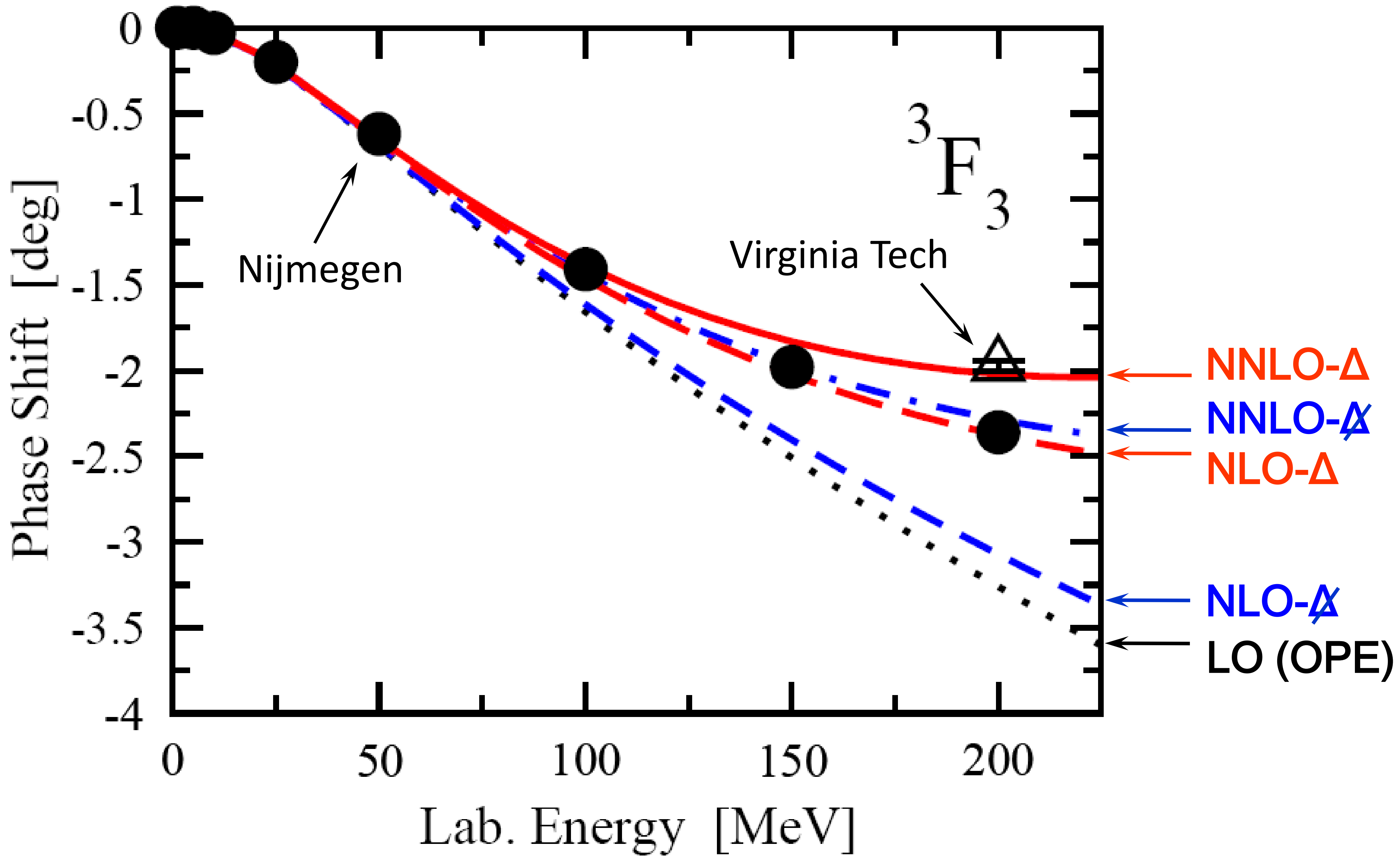
**Chiral 2π- exchange potential up to NNLO**




**Advantages when Δ is included explicitly**

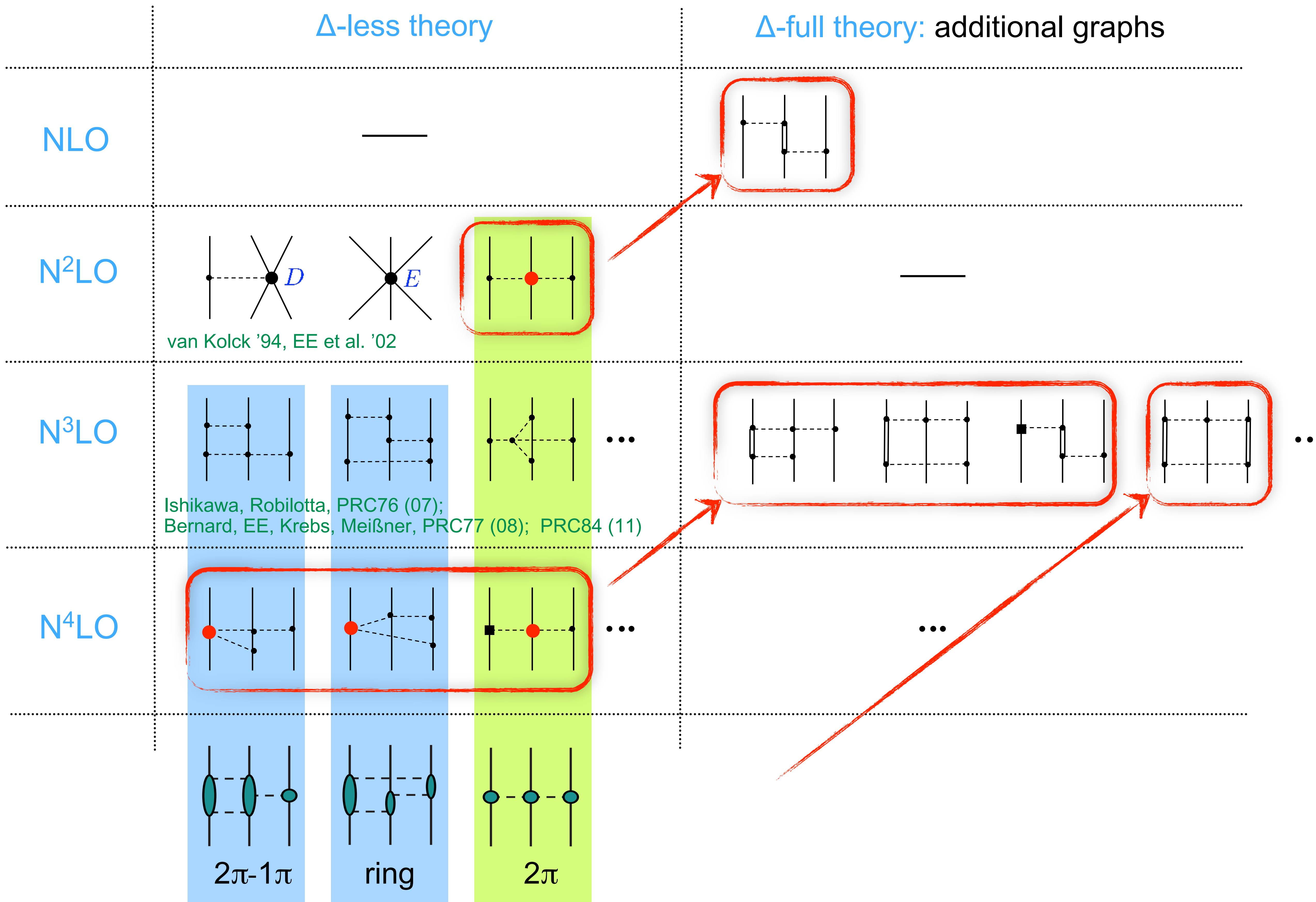
- Dominant contributions already at NLO
- Much better convergence in all potentials

# $^3F_3$ partial waves up to NNLO with and without Δ

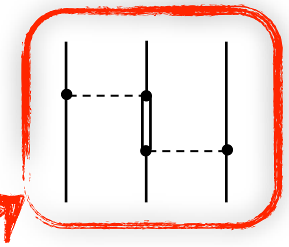
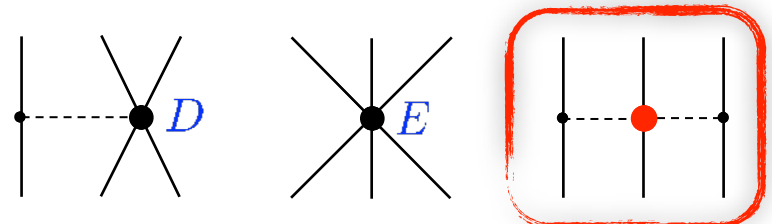
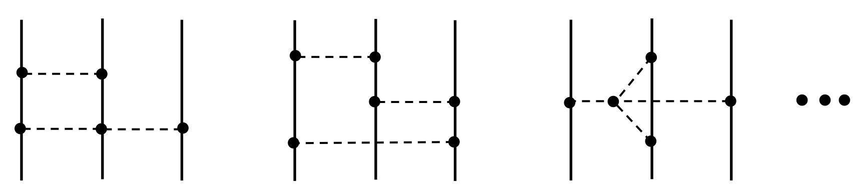
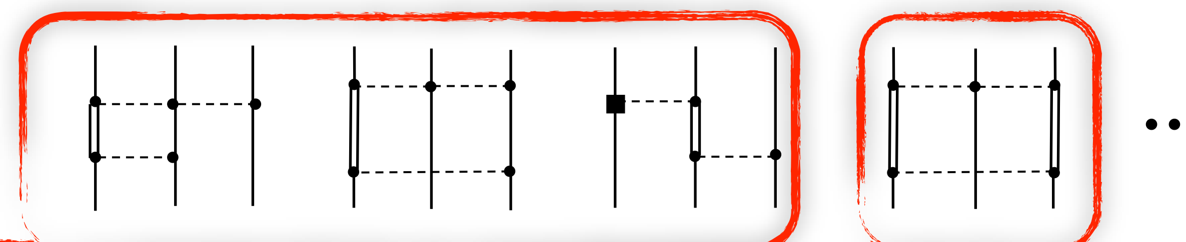
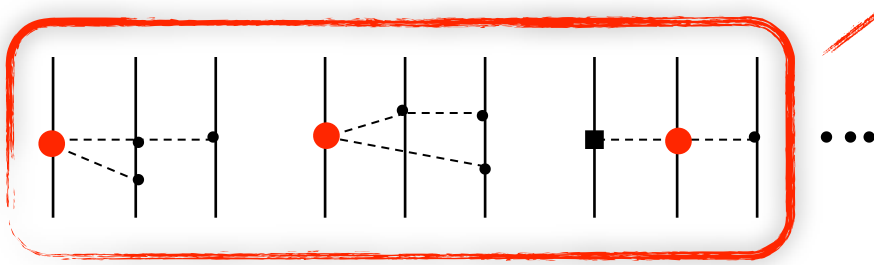



(calculated in the first Born approximation)

# Small scale expansion of 3NF



# Small scale expansion of 3NF

| | Δ-less theory | Δ-full theory: additional graphs |
|---|---|---|
| NLO | — |  |
| $N^2LO$ |  van Kolck ’94, EE et al. ’02 | — |
| $N^3LO$ |  Ishikawa, Robilotta, PRC76 (07); Bernard, EE, Krebs, Meißner, PRC77 (08); PRC84 (11) |  |
| $N^4LO$ |  | |

- no effect up to $N^2LO$ (modulo reshuffling)
- expect large contributions to the ring & $2\pi$-$1\pi$-topologies saturating some of the $N^{4,5,6}LO$ graphs in the Δ-less theory
- What is more efficient: Δ-less $N^4LO$ (and beyond?) vs Δ-full $N^3LO$ ??

# Computational strategy

- d-dim one loop tensor integrals by Passarino-Veltman reduction

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_{\mu_1} \dots l_{\mu_n}}{[l^2 - M^2][(l+p)^2 - M^2]} = T^{(1)}_{\mu_1 \dots \mu_n}(p) f_1(p^2) \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - M^2][(l+p)^2 - M^2]} + T^{(2)}_{\mu_1 \dots \mu_n}(p) f_2(p^2) \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - M^2]}$$

Tensors in $p$

$f_1(p^2)$ and $f_2(p^2)$ include in general non-physical singularities which cancel in final result

- Dimensional-shift reduction Davydychev '91

Combinatorial factors

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_{\mu_1} \dots l_{\mu_n}}{[l^2 - M^2][(l+p)^2 - M^2]} = \sum_{ij} T^{(i)}_{\mu_1 \dots \mu_n}(p) \int \frac{d^{d+2i} l}{(2\pi)^{d+2i}} \frac{c_{ij}}{[l^2 - M^2]^{n_{ij}}[(l+p)^2 - M^2]^{m_{ij}}}$$

- Partial integration techniques provide recursion relations

$$\int \frac{d^d l}{(2\pi)^d} \frac{\partial}{\partial l_\mu} f(l) = 0$$ ⟸ Connection betw. Dimensional-shift and Passarino-Veltman red.

**Implement Heavy-Baryon extension of these techniques in Mathematica/FORM**

# Pion-nucleon scattering

Heavy baryon SSE calculation up to $\varepsilon^3$ : *Fettes & Meißner NPA679 (2001) 629*

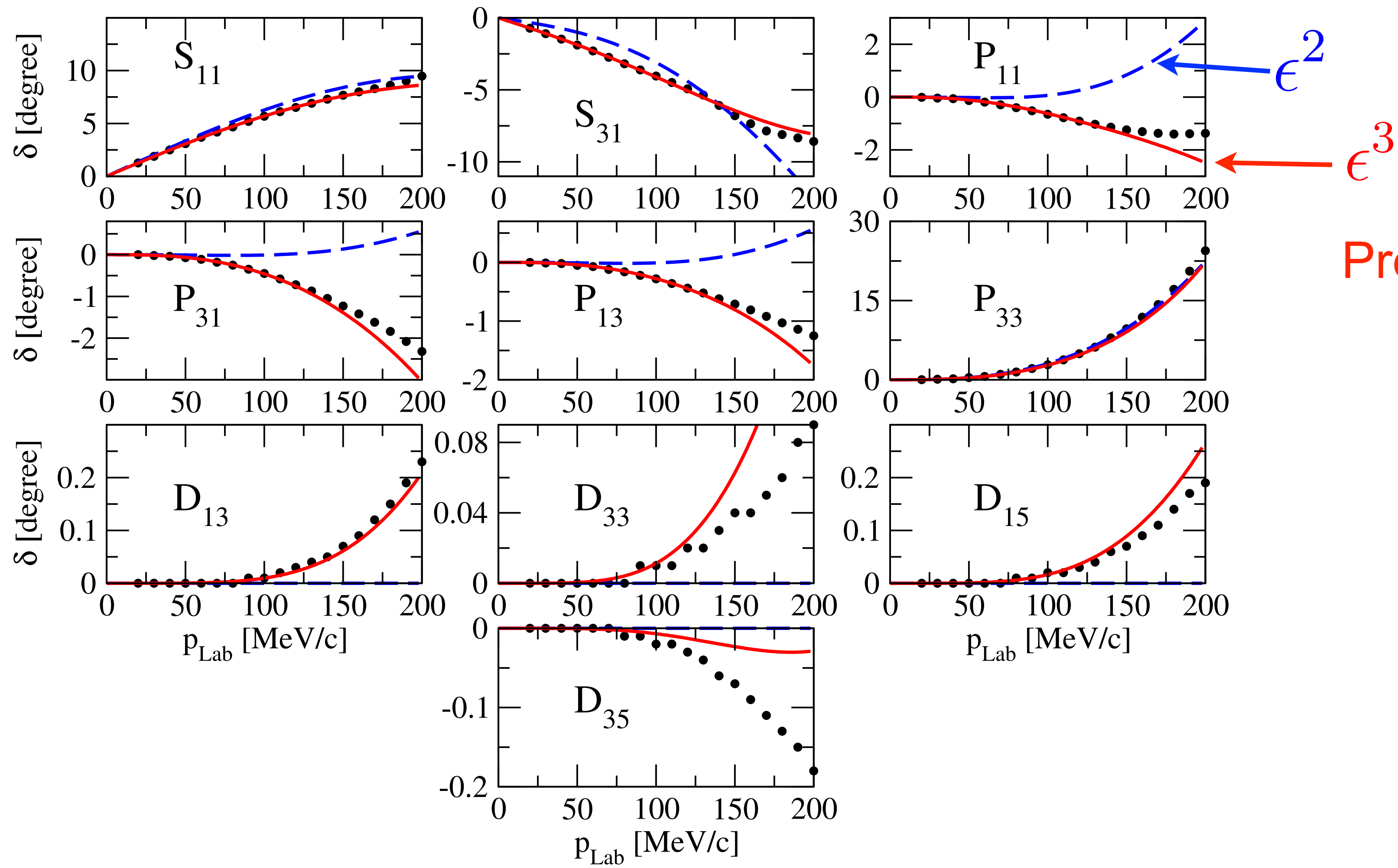
Recalculation needed due to different power-counting in 1/m

After renormalization of πNΔ-constant $h_A$ and appropriate shift of $c_i$´s and $d_i$´s we do not find any dependence on new LECs from $\mathcal{L}^{(2)}_{\pi N\Delta}$ & $\mathcal{L}^{(3)}_{\pi N\Delta}$

⇩

Additional LECs at $\varepsilon^3$ : πNΔ-constant $h_A$ & πΔΔ-constant $g_1$

*Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707*



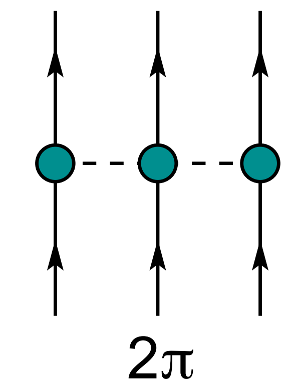

Preliminary

# Two-pion-exchange 3NF

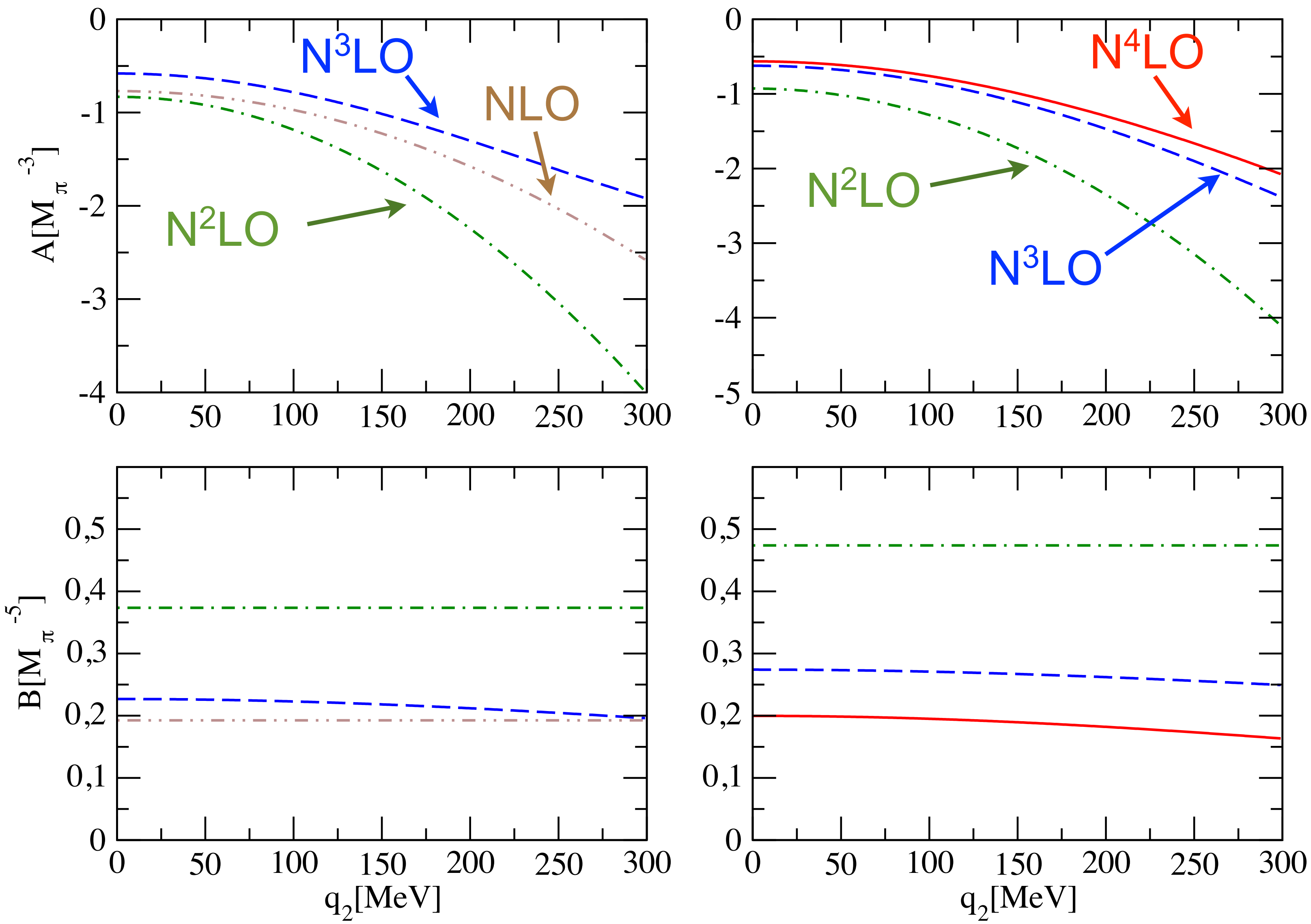
Preliminary

*Epelbaum, Gasparyan, HK. forthcoming*

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big( \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$




Explicit-Δ calc. Δ-less calc.




- Similar results in both cases for $N^2LO$ and $N^3LO$

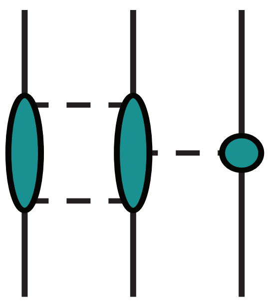
Difference btw. $N^2LO$ and $N^3LO$ is given by loops

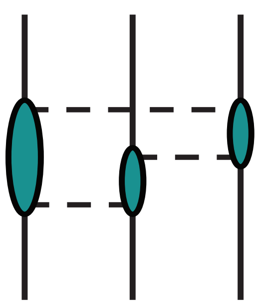
Loop contr. of Δ-dofs to two-pion-exchange 3NF are small

- Explicit-Δ $N^3LO$ TPE-3NF is between Δ-less $N^3LO$ and $N^4LO$ results
- We expect small explicit-Δ $N^4LO$ contributions to two-pion-exchange 3NF

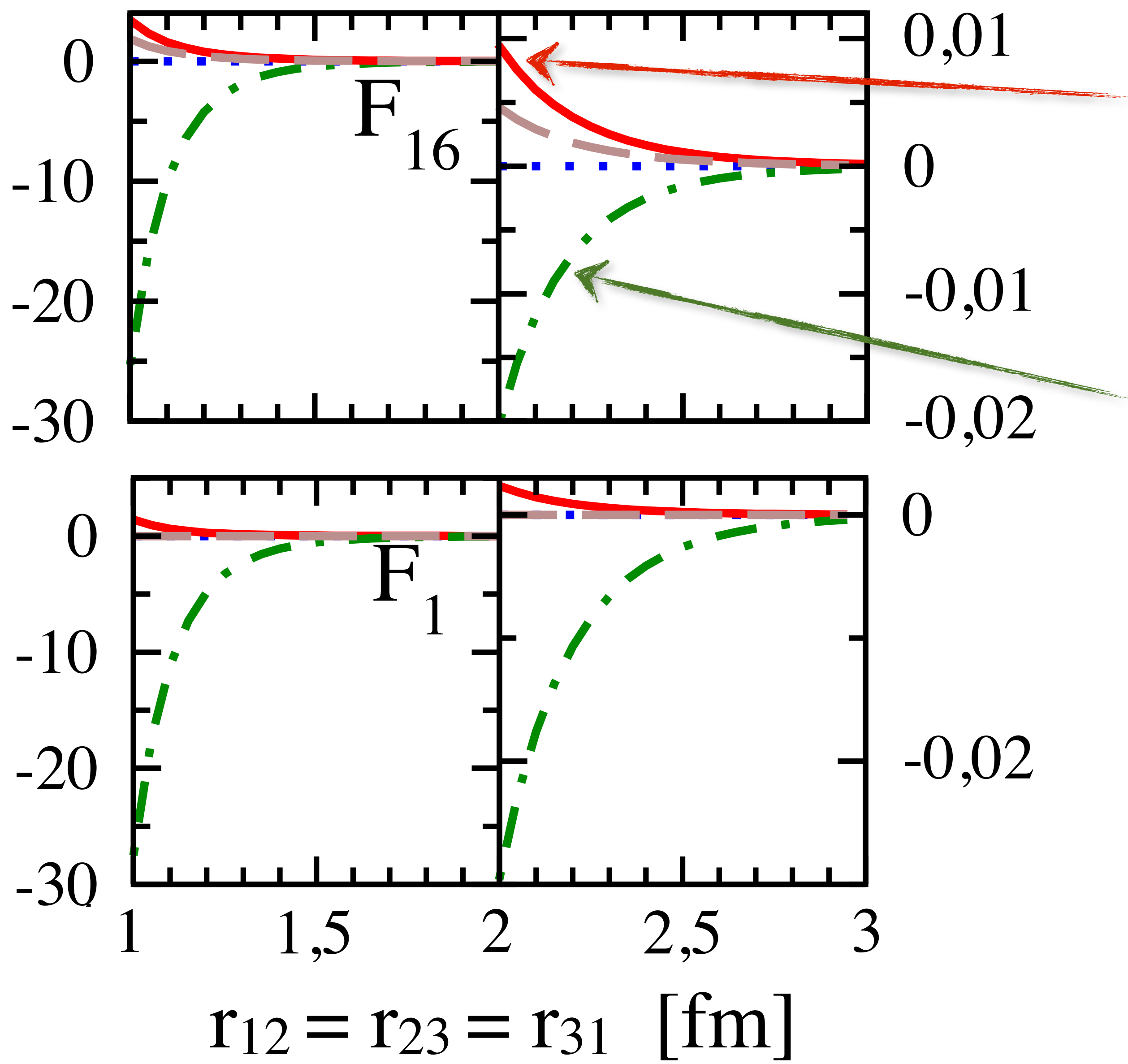
# Other topologies (exemplified)

*Epelbaum, Gasparyan, HK. forthcoming*





Preliminary




includes terms $\sim \frac{1}{m_\Delta - m_N}$

but **NOT** $\sim \frac{1}{(m_\Delta - m_N)^2}, \ldots$

**resumes terms**

$\sim \frac{1}{m_\Delta - m_N}, \; \frac{1}{(m_\Delta - m_N)^2}, \; \ldots$

- ······ $N^3LO$ Δ-less
- – · – $N^3LO$ Δ-full
- —— $N^4LO$ Δ-less
- - - - - N4LO Δ-less with $c_i$´s from Δ-res. saturation

- Difference btw. res. sat. and $N^4LO$ indicate contr. of $c_i$´s coming not from Δ-dofs
- Difference btw. res. sat. and explicit-Δ $N^3LO$ indicates contr. ~$O(1/\Delta^2)$
- For $F_1$ and $F_{16}$ EFT with explicit Δ(1232) seems to be more efficient

# Summary

- Long-range part of 3NFs is analyzed up to $N^3LO$ with explicit Δ-dof
- Small loop-contr. with Δ-dofs to two-pion-exchange part of 3NF
- Two-pion-exchange part seems to be converged
- Most of sizable $F_i$ structures are similar in explicit-Δ $N^3LO$ and Δ-less $N^4LO$ calc.
- Some missing sizable Δ-contr. in $N^4LO$ results like central attractive force ~$O(1/\Delta^2)$

# Outlook

- Partial wave decomposition of $N^3LO$ three-nucleon forces
- Explicit-Δ $N^3LO$ calc. of shorter range part of 3NF
- $N^4LO$ with explicit-Δ of long range part of 3NF

# LECs in two-pion-exchange 3NF

## Δ-less $N^4LO$

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $\bar{d}_1+\bar{d}_2$ | $\bar{d}_3$ | $\bar{d}_5$ | $\bar{d}_{14}-\bar{d}_{15}$ | $\bar{e}_{14}$ | $\bar{e}_{15}$ | $\bar{e}_{16}$ | $\bar{e}_{17}$ | $\bar{e}_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GW-fit | $-1.13$ | 3.69 | $-5.51$ | 3.71 | 5.57 | $-5.35$ | 0.02 | $-10.26$ | 1.75 | $-5.80$ | 1.76 | $-0.58$ | 0.96 |
| KH-fit | $-0.75$ | 3.49 | $-4.77$ | 3.34 | 6.21 | $-6.83$ | 0.78 | $-12.02$ | 1.52 | $-10.41$ | 6.08 | $-0.37$ | 3.26 |

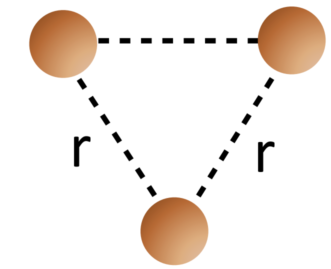
## explicit-Δ $N^3LO$

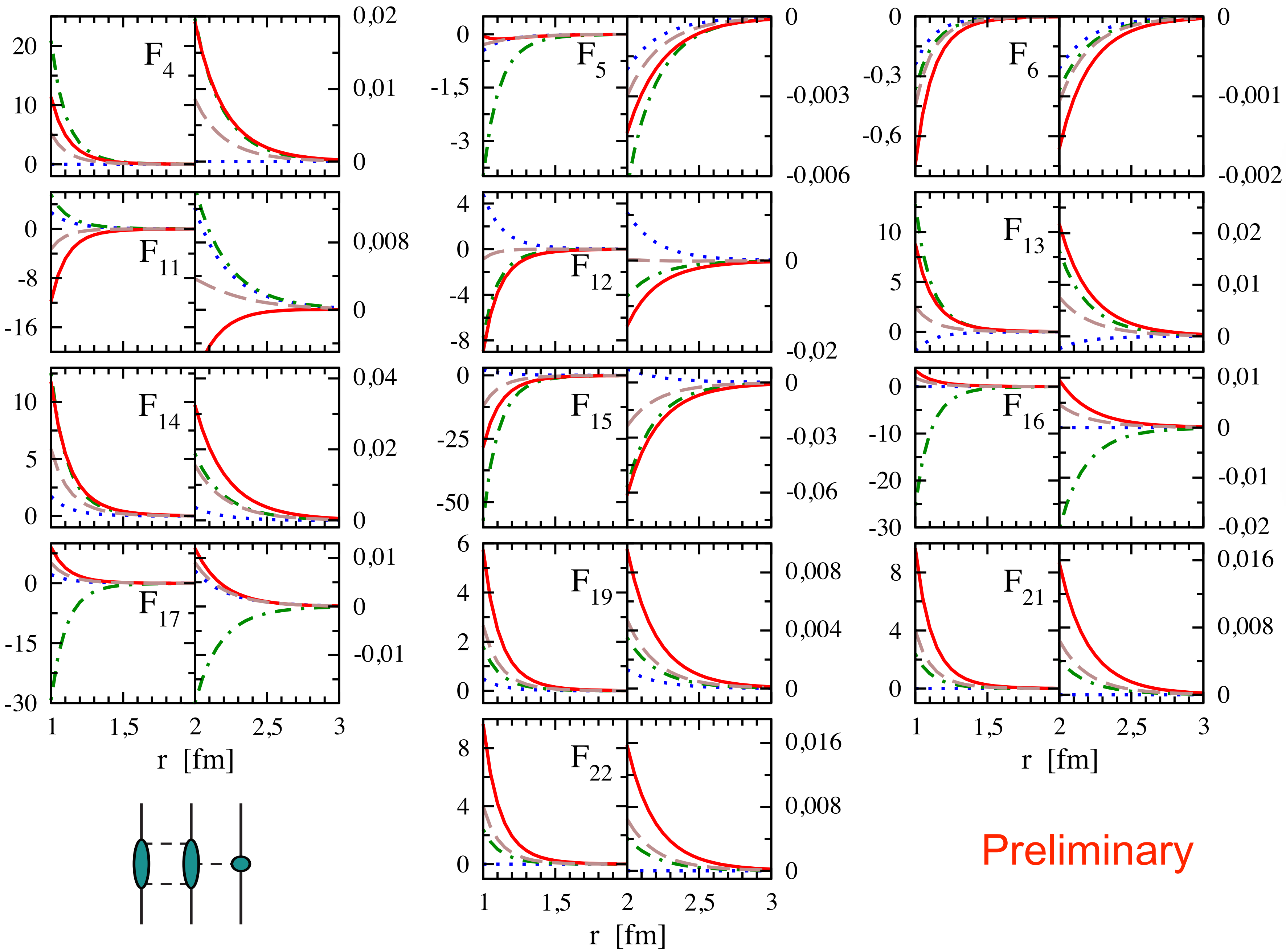
| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $\bar{d}_1+\bar{d}_2$ | $\bar{d}_3$ | $\bar{d}_5$ | $\bar{d}_{14}-\bar{d}_{15}$ |
|---|---|---|---|---|---|---|---|---|
| GW-fit | $-1.70$ | 1.19 | $-3.52$ | 1.85 | 0.10 | $-1.26$ | 0.71 | $-1.17$ |
| KH-fit | $-1.41$ | 1.40 | $-3.43$ | 1.80 | 0.45 | $-2.36$ | 1.43 | $-2.18$ |

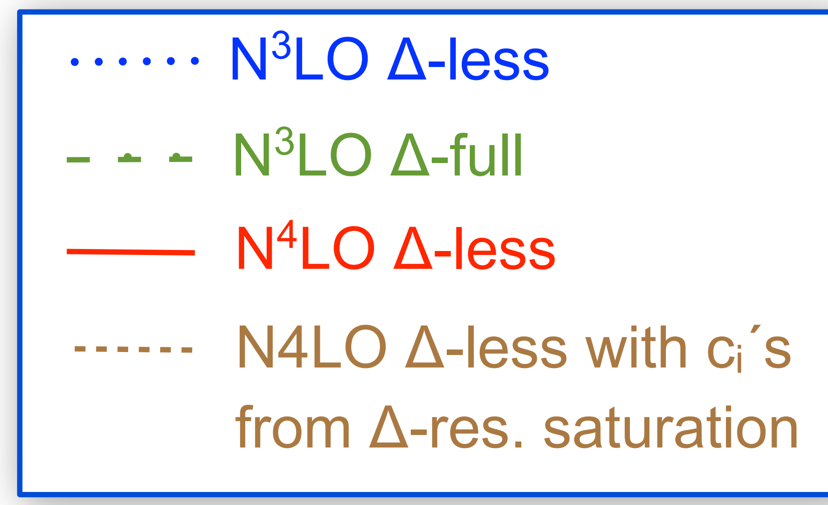
LECs $c_i$ & $d_i$ become smaller once Δ-dofs are taken explicitly

# Two-pion-one-pion-exchange 3NF

Coordinate space to discuss the long-range part at equilateral-triangle conf.



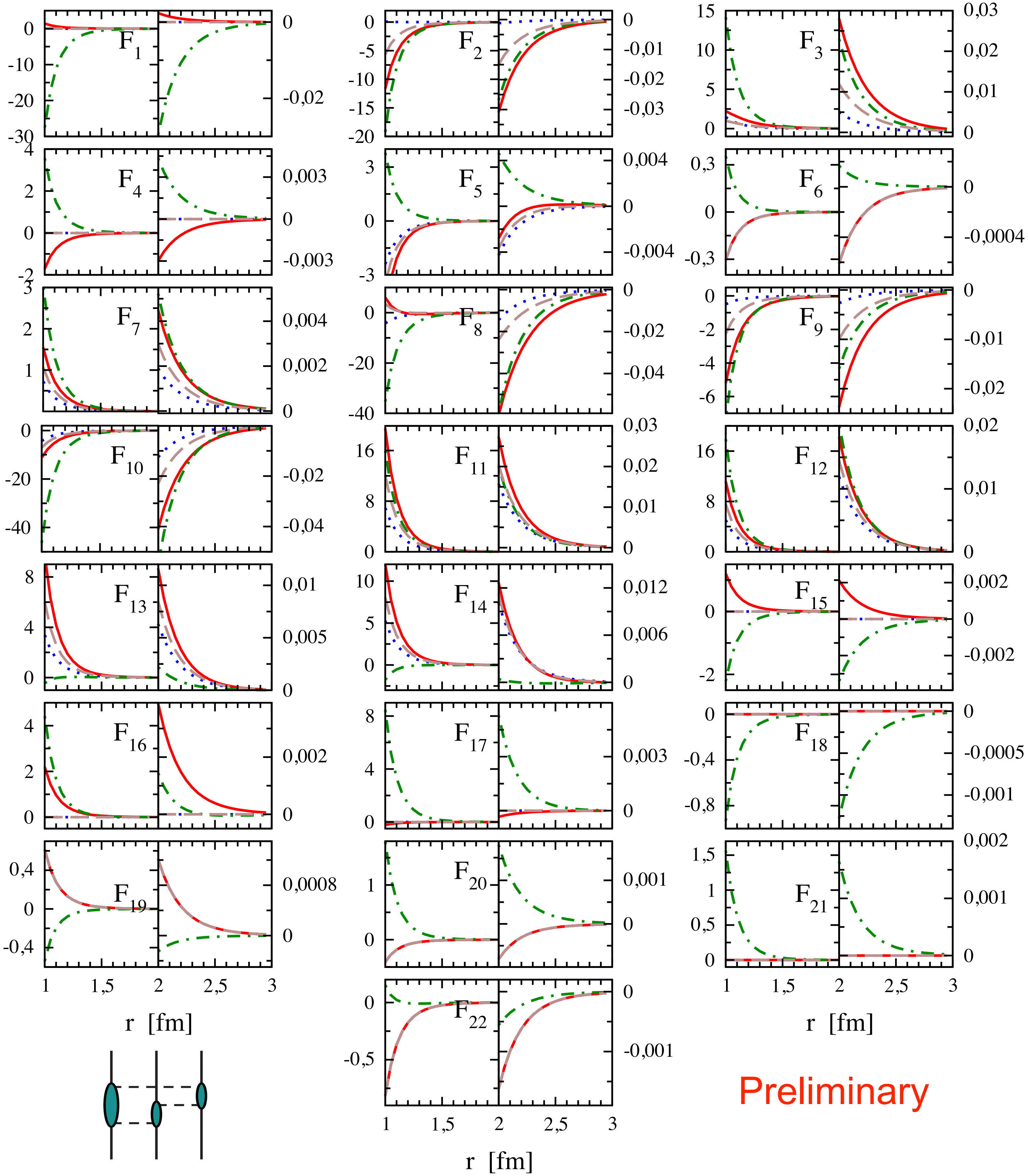




Preliminary

- Difference btw. res. sat. and $N^4LO$ indicate contr. of $c_i$´s coming not from Δ-dofs
- Difference btw. res. sat. and explicit-Δ $N^3LO$ indicates contr. ~$O(1/\Delta^2)$

- Structures with larger values like $F_4$ & $F_{15}$ look similar for explicit-Δ $N^3LO$ and Δ-less $N^4LO$
- There are sizable structures like $F_{16}$ & $F_{17}$ which in Δ-less $N^4LO$ miss important Δ-cont.
- No statement about convergence of smaller structures like $F_{11}$ can be made at this order

# Ring-contributions to 3NF

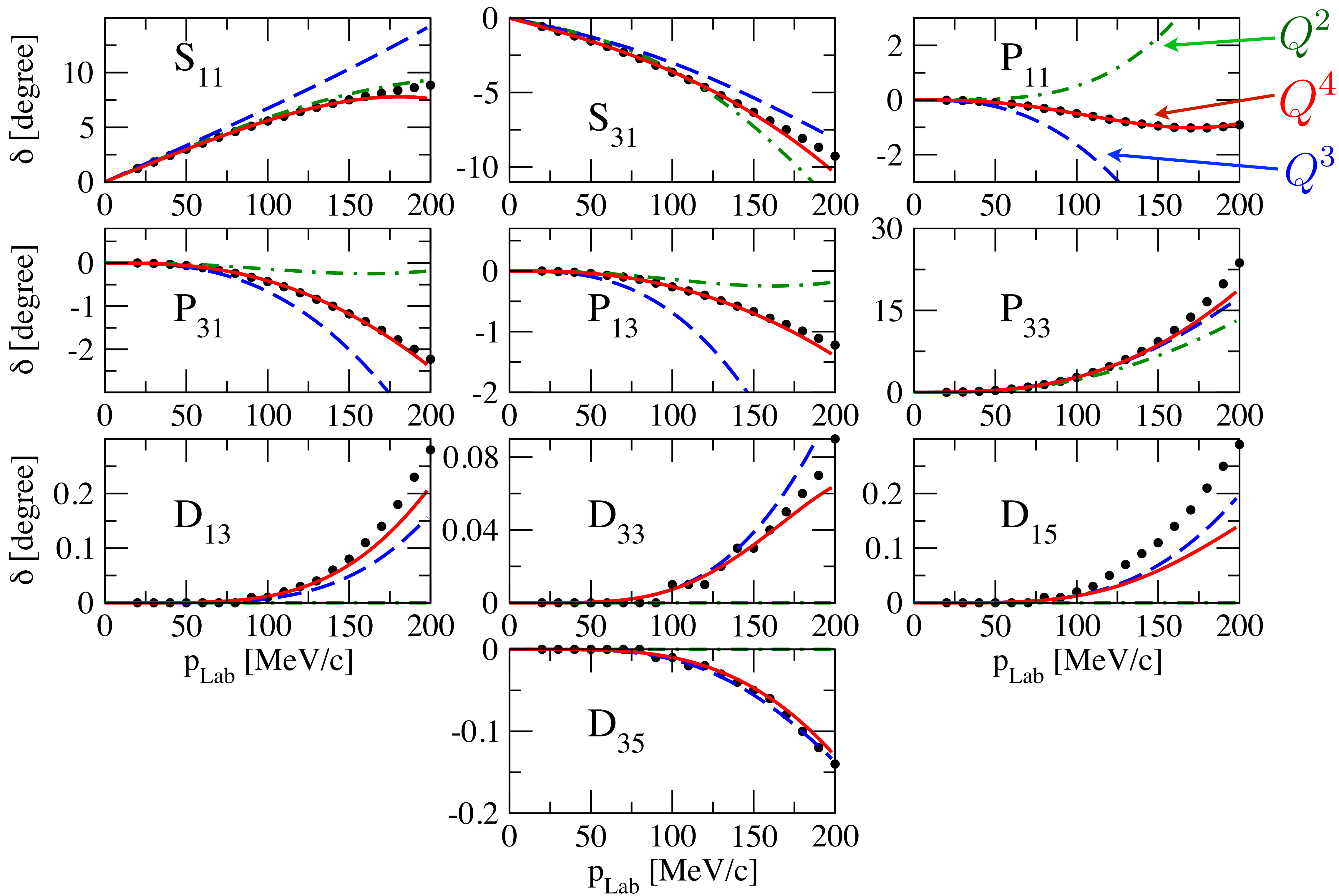




- Strong attractive central force coming from ~O($1/\Delta^2$) contr.
- Similar results btw. Δ-less $N^4LO$ and explicit-Δ $N^3LO$ results for structures with larger value like $F_2$, $F_8$, $F_{10}$, $F_{11}$ & $F_{12}$
- No statement about convergence possible for smaller structures like $F_4$, $F_5$, $F_6$, $F_{14}$ - $F_{22}$ at this order
- Explicit-Δ $N^4LO$ would be helpful to draw final conclusions about convergence of two-pion-exchange and ring contr. to 3NF

# GW-Fit to pion-nucleon scattering

GW-PWA: Arndt et al. Phys. Rev. C 74 (2006) 045205

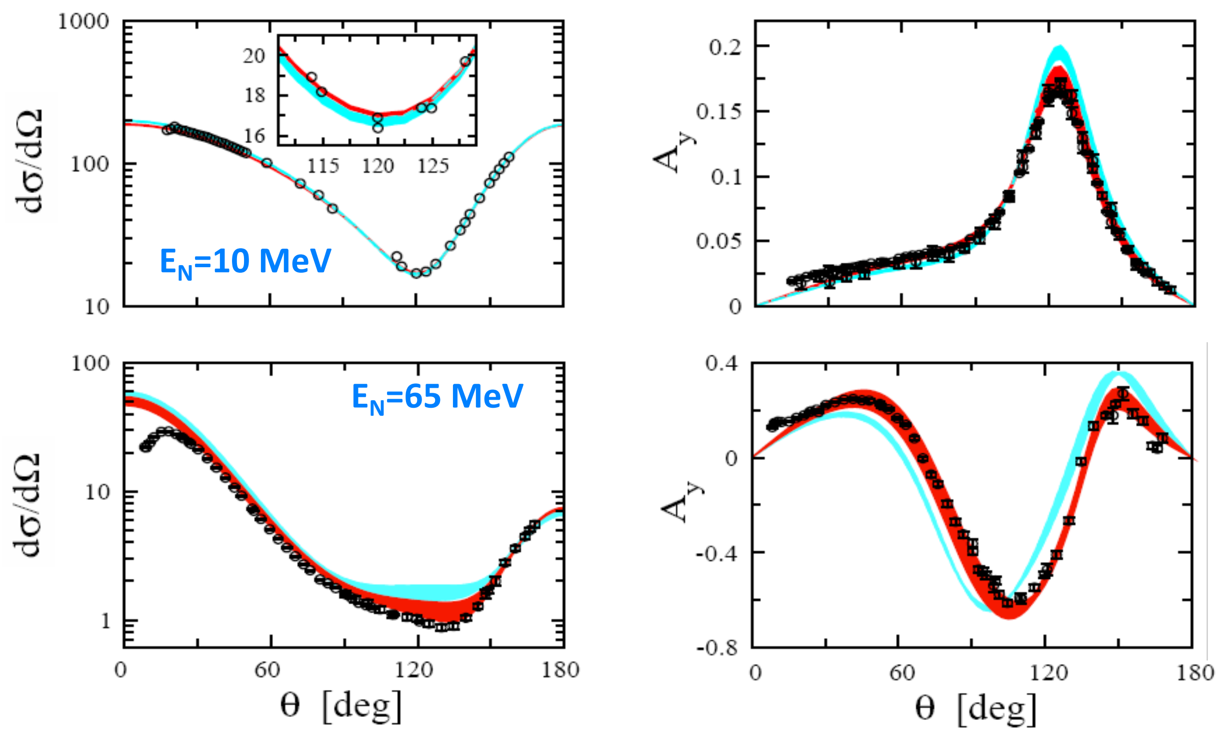



Data fitted for $\mathrm{p_{Lab}} < 150\,\mathrm{MeV}$

# Nd elastic scattering

## Cross section & vector analyzing power

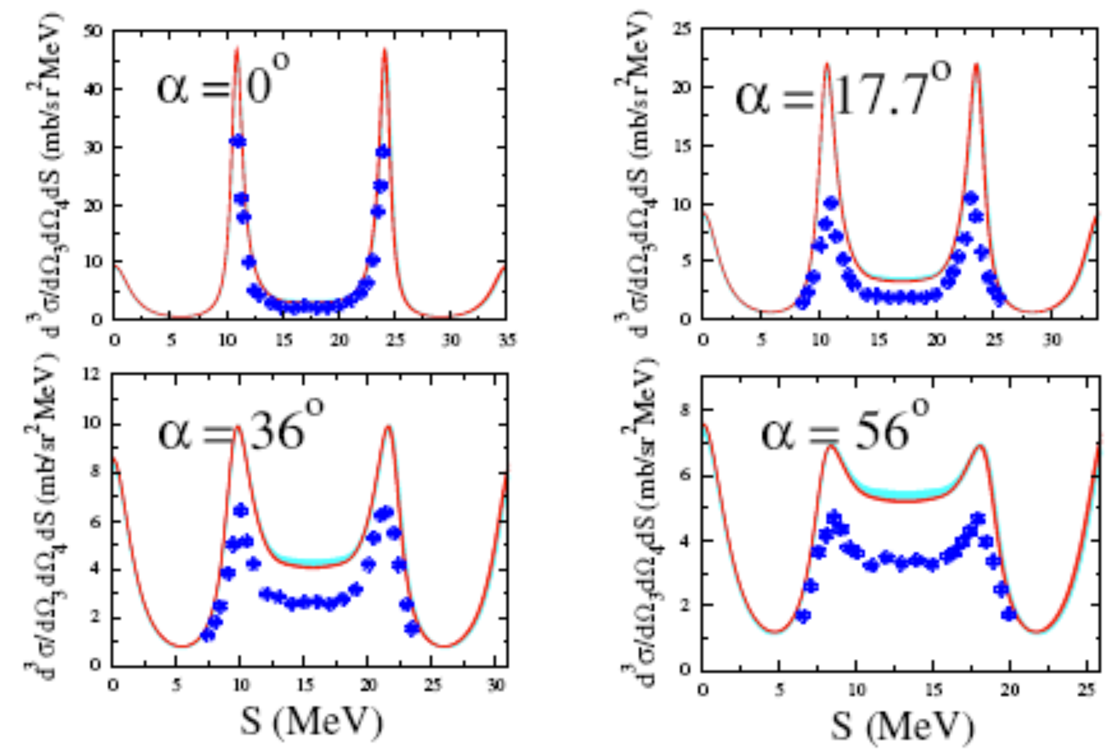
*E.pelbaum, PPNP 57 (2006) 654*




# Deuteron break-up

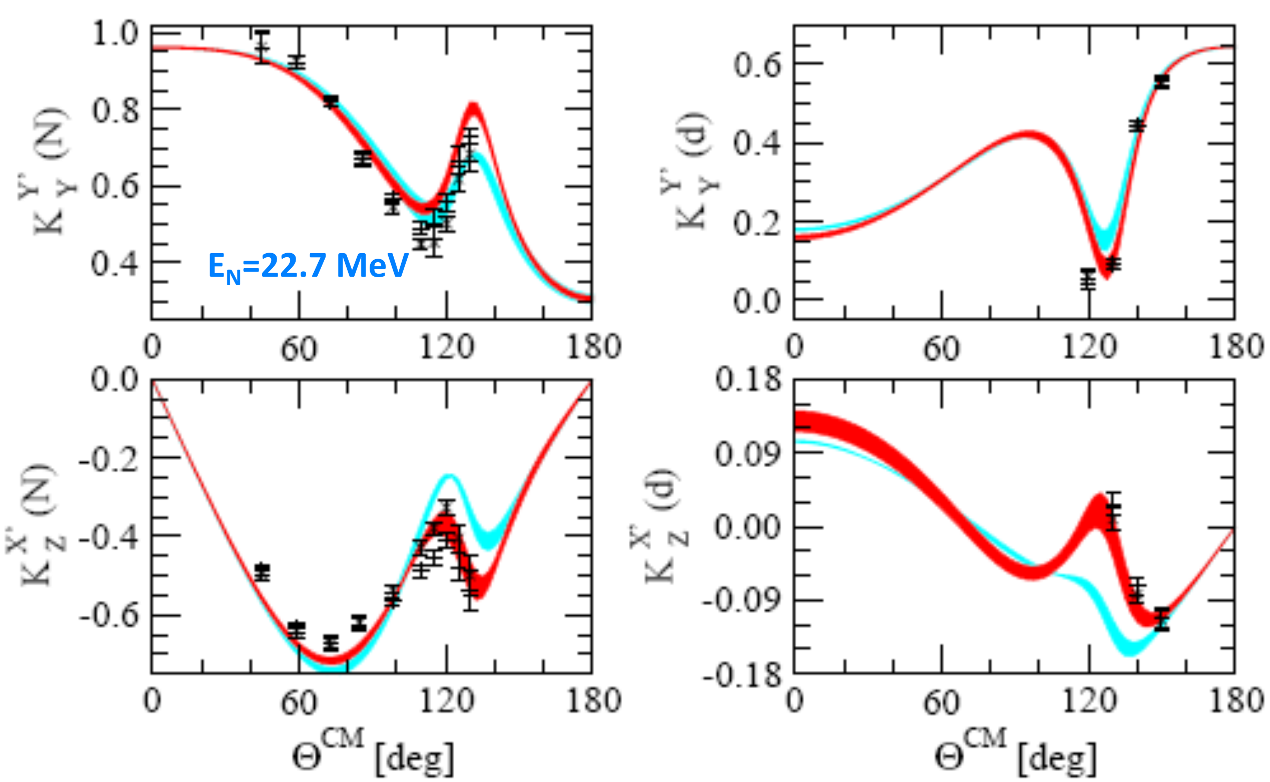
## SCRE configuration at $E_d$=19 MeV

*Ley et al., PRC 73 (2006) 064001*




## Polarization transfer coefficients

*Witała et al., PRC 73 (2006) 044004*




- Promising NNLO results for Nd elastic scattering
- Generally good description of break-up observables except for SCRE/SST break-up configuration at low energy
- Hope for improvement at $N^3LO$