Range effects on Efimov physics in cold atoms

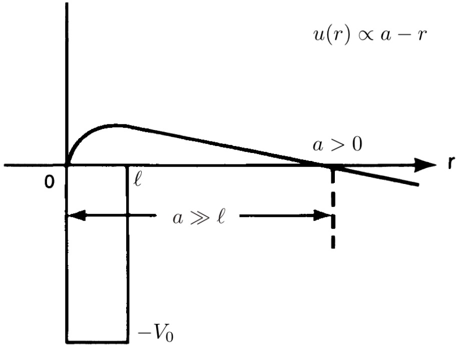
An Effective field theory approach

Chen Ji ‖ TRIUMF

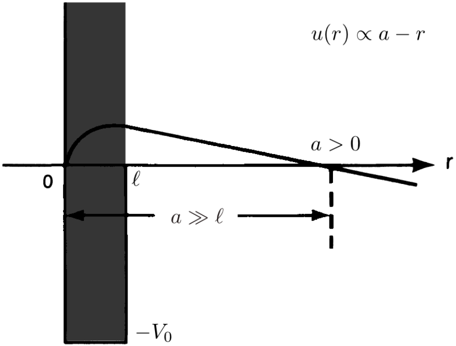
in collaboration with D. R. Phillips, L. Platter

INT workshop, November 7 2012

- **Separation of scales:**
  $a \gg \ell$
- **2-body S-wave universality:**
  $B_d = 1/Ma^2$

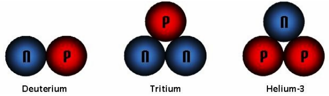


- **Separation of scales:**
  $a \gg \ell$
- **2-body S-wave universality:**
  $B_d = 1/Ma^2$
- Physics at large distance is insensitive to physics at short distance
- Large-distance physics is studied in ℓ/a expansion
- Effects from SR-dynamics can be included in perturbation theory

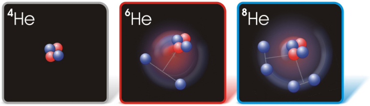


Universal Physics exists in systems with
$\ell \ll a$

- Nuclear Physics
  - **Few-nucleon systems (NN, NNN):**
    i.e., $\ell_{np}^{t} \sim 1.7$ fm, $a_{np}^{t} \sim 5.4$ fm
  - **Halo nuclei (^{6}He, ^{11}Li)**
    $E_{\text{sep}} \ll E_{\text{core}}$



[www.theurbn.com]



[www.anl.gov]

Large-scattering-length physics

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- Nuclear Physics
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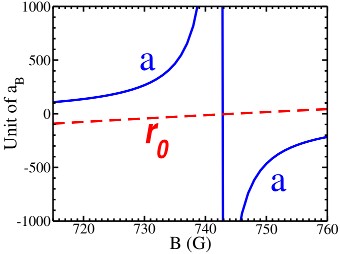
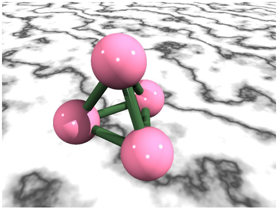
  - **Halo nuclei (^{6}He, ^{11}Li)**
    $E_{\rm sep} \ll E_{\rm core}$
- Atomic Physics
  - **Cold atomic gases (^{133}Cs, ^{7}Li, ^{39}K):**
    r_0 and a varies near Feshbach resonance
  - **^{4}He atoms (dimer, trimer, tetramer):**
    $\ell_{vdw} \sim 7$Å, $a \sim 100$Å



^{7}Li atoms [Gross *et al.* '09]



^{6}He tetramer [D. Blume]

Effective field theory

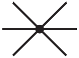
- An approach to systems with a separation of scales
  - Systems with $\ell \ll a \to$ an EFT with contact interactions
    - Few-nucleon systems $\to$ pionless EFT
    - Halo nuclei $\to$ halo EFT
    - Atomic systems $\to$ short-range EFT
  - Physical quantities are expanded in powers of r_0/a

- Contact interactions at LO
  - 2-body contact interaction (LO)

$$= -iC_0$$

C_0 determined by a 2-body observable

  - 3-body contact interaction (LO)



$$= -iD_0$$

D_0 determined by a 3-body observable

Effective field theory

- An approach to systems with a separation of scales
  - **Systems with $\ell \ll a$** $\rightarrow$ an EFT with contact interactions
    - Few-nucleon systems $\rightarrow$ pionless EFT
    - Halo nuclei $\rightarrow$ halo EFT
    - Atomic systems $\rightarrow$ short-range EFT
  - **Physical quantities are expanded in powers of r_0/a**

- Contact interactions at LO
  - **2-body contact interaction (LO)**

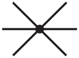
introduce a dimer field

$= -iC_0 \quad \xrightarrow{C_0 = g^2/\Delta}$

$= -i\sqrt{2}g$

C_0 determined by a 2-body observable

  - **3-body contact interaction (LO)**



$= -iD_0 \quad \xrightarrow{D_0 = -3hg^2/\Delta^2}$

$= ih$

D_0 determined by a 3-body observable

Bedaque, Hammer, van Kolck '99

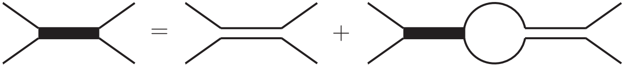
EFT for 3 identical bosons

- **LO $(r_0/a)^0$ EFT Lagrangian for 3 identical bosons**

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - d^\dagger \left(i\partial_0 + \frac{\nabla^2}{4m} - \Delta \right) d - \frac{g}{\sqrt{2}} \left(d^\dagger \psi\psi + \mathrm{h.c} \right) + h d^\dagger d \psi^\dagger \psi + \cdots$$

  - terms with more derivatives are at higher orders $(r_0/a)^n$

- **Non-perturbative features at LO**
  - atom-atom (dimer) scattering (tune g)



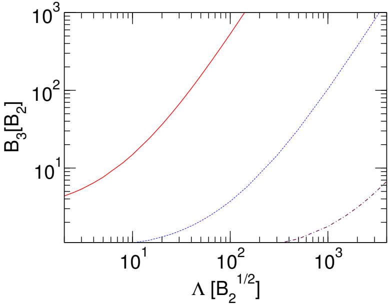
$\propto \frac{1}{1/a+ik}$

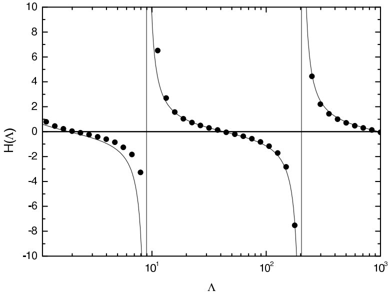
  - atom-dimer scattering (tune h)

Bedaque, Hammer, van Kolck '99

LO renormalization

- **Without 3BF:**
  - 3-body spectrum:
    cutoff dependent ($\Lambda \sim 1/\ell$)
    Platter '09
- **LO 3BF h:**
  - tune $H(\Lambda) = \Lambda^2 h/2mg^2$:
    fix one 3-body observable
  - limit cycle:
    $H(\Lambda)$ periodic for $\Lambda \to \Lambda(22.7)^n$
    Bedaque *et al.* '00
  - scaling invariance → Efimov physics
    Efimov '71

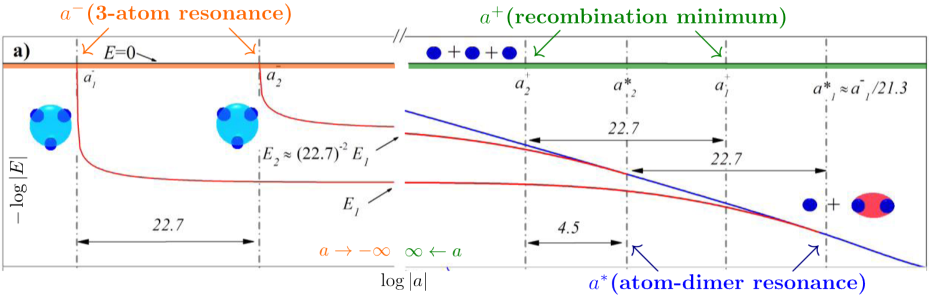




Universal physics at LO

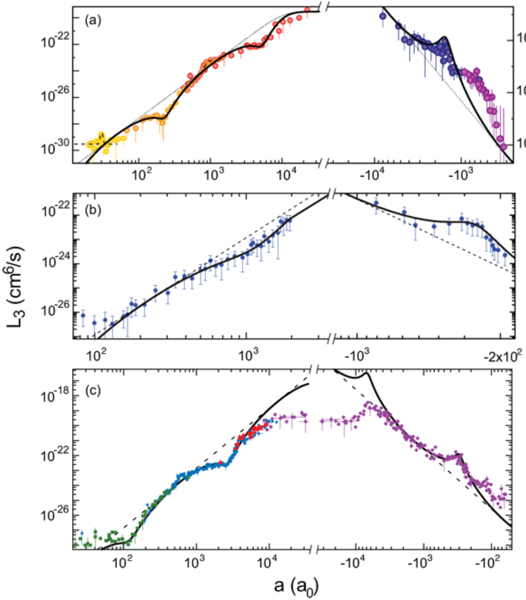
TRIUMF

- Universal features in three-body systems **(Efimov effects)**
  - 3-body spectrum: a function of scattering length a
- geometric spectrum
  - $E_n = (22.7)^{-2} E_{n-1}$ in the limit $a \to \infty$
- universal relation of recombination features
  - $a^* = a^+/4.5 = -a^-/21.3$
  - *i.e.* $a^-_{(n)} = 22.7 a^-_{(n-1)}$



Zaccanti *et al.* '09

Recombination features of cold atoms



loss rates of free atoms in ultracold atomic gases

- ^{39}K atoms
  Zaccanti *et al.* '09
- ^{7}Li atoms $|F = 1, m_F = 0\rangle$
  Gross *et al.* '09
- ^{7}Li atoms $|F = 1, m_F = 1\rangle$
  Pollack *et al.* '09

Pic. credit: Ferlaino, Grimm '10

Beyond universality: range effects

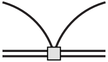
- range effects on universal physics
  - 2-body observable: $k\cot\delta_0 = -\frac{1}{a} + \frac{r_0}{2}k^2 + \cdots$
  - 3-body observables: $\rightarrow$ in r_0/a expansion
- Previous EFT calculations of r_0/a corrections (fixed a):
  - Hammer, Mehen '01 (NLO)
  - Bedaque, Rupak, Griesshammer, Hammer '03 (N^2LO, partial iteration)
  - Platter, Phillips '06 (N^2LO, partial iteration)
- We perform a rigorous perturbative caluclation
  - NLO (systems with variable a)
    - ultracold atomic gases (r_0/a varies near Feshbach resonance)
    - 3-body recombination
  - N^2LO (systems with a fixed a)
    - ^{4}He atoms ($r_0/a \sim 0.07$)
    - ^{4}He trimer (bound state), atom-dimer phase shift (scattering state)

NLO (r_0/a) range effects

- Calculate r_0/a correction to atom-dimer amplitude
  - NLO correction to atom-atom propagator **(dimer)**:

$$= ir_0 \frac{2\pi}{mg^2} \frac{1/a+ik}{-1/a+ik}$$

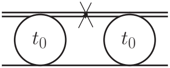
  - NLO correction to atom-dimer contact term **(3BF)**:



$$= i\frac{2mg^2}{\Lambda^2} H_1(a,\Lambda)$$

  - Contribution in 1st order perturbation theory:

NLO dimer:



NLO 3BF:

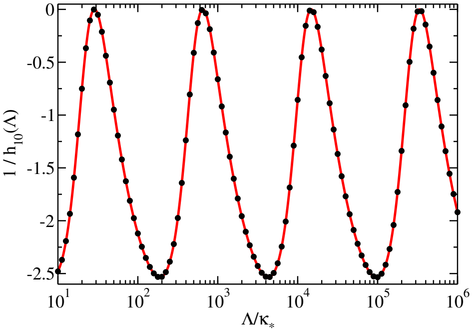
- NLO 3-body force:

$$H_1(\Lambda) = r_0\Lambda\ h_{10}(\Lambda) + r_0/a\ h_{11}(\Lambda)$$

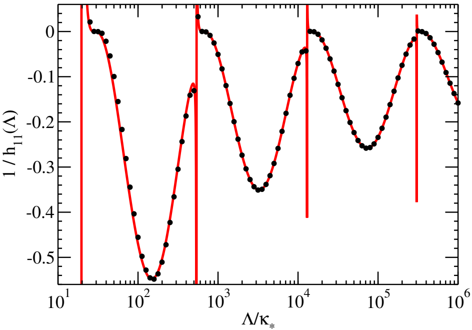
  - a **fixed:** h_{11} is absorbed (no additional 3-body input is needed)
  - a **varies:** one additional 3-body input is needed

NLO three-body 3BF

- $H_1(\Lambda) = r_0\Lambda\ h_{10}(\Lambda) + r_0/a\ h_{11}(\Lambda)$



$1/h_{10}(\Lambda)$

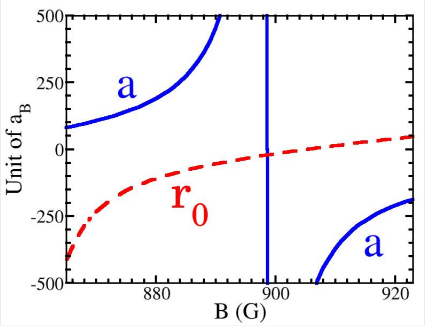


$1/h_{11}(\Lambda)$

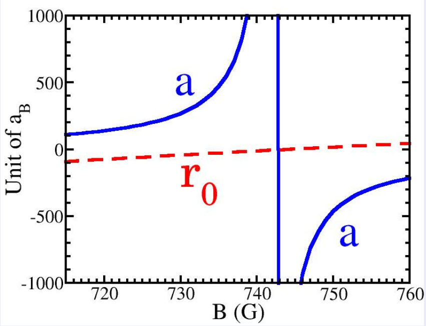
In ultracold atomic gases

a and r_0 are functions of the magnetic field

$^7Li \quad |F=1, m_F=0>$



$^7Li \quad |F=1, m_F=1>$



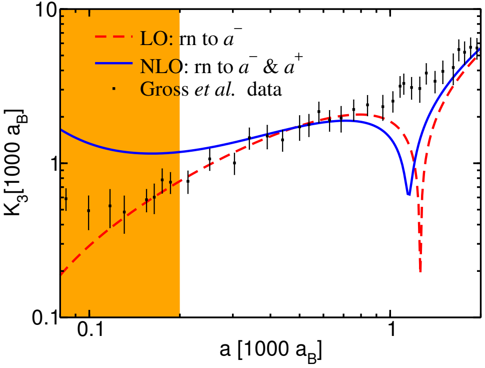
Gross *et al.* '09

Recombination of ^{7}Li Atoms

TRIUMF

$|F = 1, m_F = 0\rangle$

- **3-body recombination rate of ^{7}Li atoms**
- **NLO (r_0/a) range effects to recombination**
- **positions are not shifted by deep-dimer at LO** [Braaten *et al.* '08]



	Experiment†‡
3A res $a^-_{(-)}$ $[a_B]$	-264†
rec min a^+ $[a_B]$	1160†
Ad res a^* $[a_B]$	180‡

† Gross *et al.* '09 ‡ Machtey *et al.* '12 ⋆ Ji, Phillips, Platter '10

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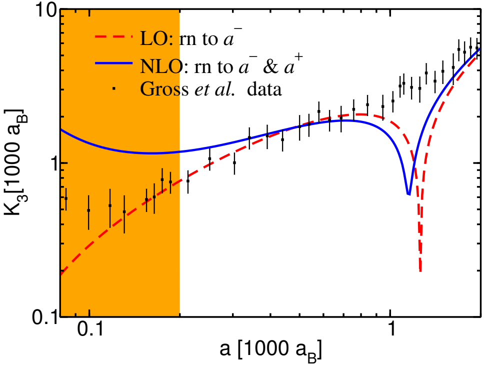

	Experiment†‡	LO
3A res $a^-_{(-)}$ $[a_B]$	-264†	-264
rec min a^+ $[a_B]$	1160†	1254
Ad res a^* $[a_B]$	180‡	281

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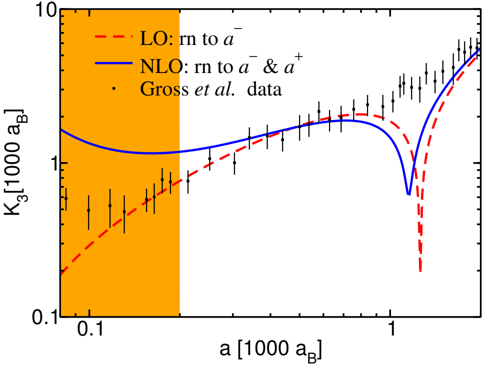

	Experiment[†‡]	LO	LO
3A res $a^-_{(\text{-})}$ $[a_B]$	-264[†]	-264	-244
rec min a^+ $[a_B]$	1160[†]	1254	1160
Ad res a^* $[a_B]$	180[‡]	281	259

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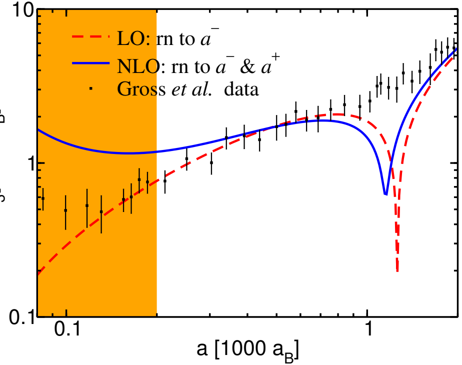

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Ad res a^* $[a_B]$	180‡	281	259	210(44)

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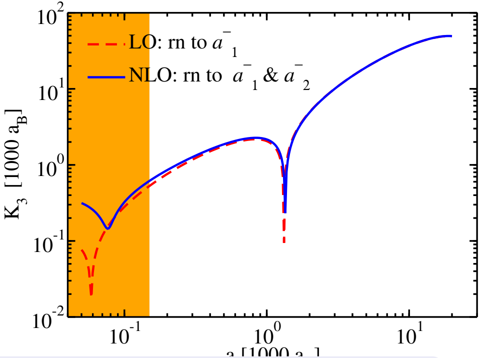
Recombination of ^{7}Li Atoms

$|F = 1, m_F = 1\rangle$

- ^{7}**Li**$(m_F = 1)$ **recombination** [Pollack *et al.* '09]
- **data disagree with universality by a factor of 2**
- **EFT analysis at NLO agrees with universality**



	Experiment [Pollack *et al.* '09]
3A res $a^-_{(-)}$ $[a_B]$	-298
3A res a^- $[a_B]$	-6301
rec min a^+ $[a_B]$	2676
Ad res a^* $[a_B]$	608

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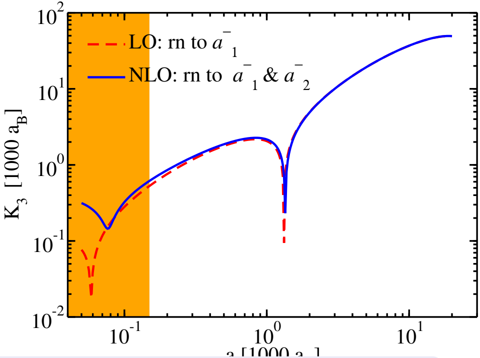

	Experiment [Pollack *et al.* '09]	**LO**
3A res $a^-_{(-)}$ $[a_B]$	-298	-298
3A res a^- $[a_B]$	-6301	-6763
rec min a^+ $[a_B]$	2676	1415
Ad res a^* $[a_B]$	608	317

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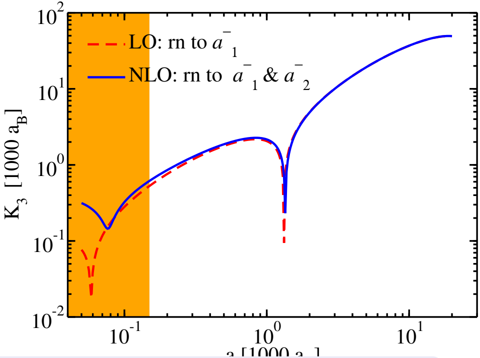

	Experiment [Pollack *et al.* '09]	LO	LO
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rec min a^+ $[a_B]$	2676	1415	1319
Ad res a^* $[a_B]$	608	317	295

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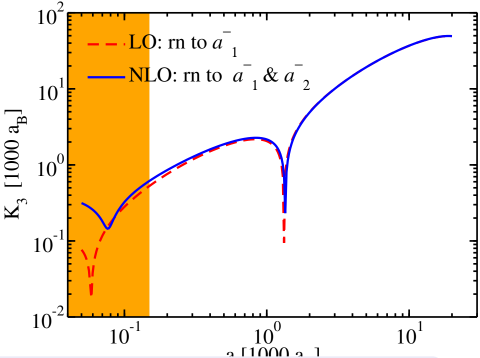

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3A res a^- $[a_B]$	-6301	-6763	-6301	-6301
rec min a^+ $[a_B]$	2676	1415	1319	1348(151)
Ad res a^* $[a_B]$	608	317	295	356(55)

Universal relations at NLO

- $\frac{1}{a} = \gamma - \frac{1}{2} r_0 \gamma^2$

Ji, Phillips, Platter '12

Universal relations at NLO

- $\frac{1}{a} = \gamma - \frac{1}{2} r_0 \gamma^2$

- **NLO corrections**

$$\gamma_0 = -0.210\gamma_-^{(-)} + r_0(a_0) \cdot \mathcal{I}_0^{(-)} {\gamma_-^{(-)}}^2$$

$$\gamma_* = -0.939\gamma_-^{(-)} + r_0(a_*) \cdot \mathcal{I}_*^{(-)} {\gamma_-^{(-)}}^2$$

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- **Linear correlation at NLO**

$$\mathcal{I}_*^{(-)} = -0.309 + 7.17\ \mathcal{I}_0^{(-)}$$

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- **Universal relation at NLO**

$$\gamma_* = \; -0.939\gamma_-^{(-)} \; -0.309\, r_0(a_*) {\gamma_-^{(-)}}^2 \; + 7.17\, \frac{r_0(a_*)}{r_0(a_0)} \left(\gamma_0 \; + 0.210\gamma_-^{(-)}\right)$$

Ji, Phillips, Platter '12

Universal relations at NLO

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- **NLO corrections**

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  $\gamma_* = \; -0.939\gamma_-^{(-)} \; -0.309 \, r_0(a_*) {\gamma_-^{(-)}}^2 \; +7.17 \, \frac{r_0(a_*)}{r_0(a_0)} \left(\gamma_0 \; +0.210\gamma_-^{(-)}\right)$

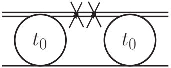
  $\gamma_* = \; 4.47\gamma_0 \; -7.02 \, r_0(a_*)\gamma_0^2 \; +0.566 \frac{r_0(a_*)}{r_0(a_-^{(-)})} \left(\gamma_-^{(-)} \; +4.76\gamma_0\right)$

Ji, Phillips, Platter '12

N^{2}LO $(r_0/a)^2$ range effects

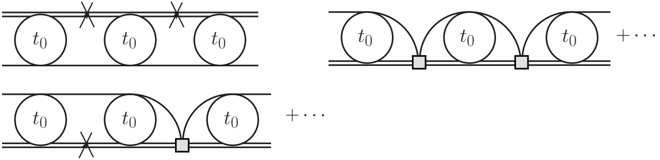
- N^{2}LO corrections to atom-dimer scattering amplitude:
  - in 2$^{\text{nd}}$ order perturbation theory ($\sim r_0^2/a^2$):

N^{2}LO dimer:



N^{2}LO 3BF:

two NLO terms:

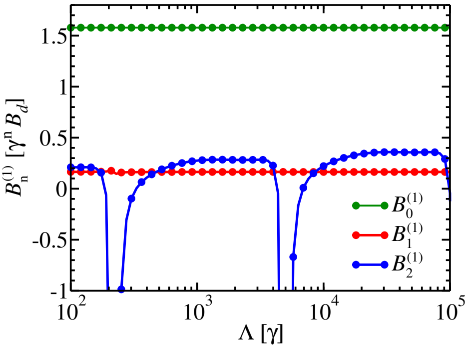


- N^{2}LO 3-body force:
  $H_2(E,\Lambda) = r_0^2\Lambda^2\; h_{20}(\Lambda) + r_0^2 m E_3\; h_{22}(\Lambda)$
  $\rightarrow$ one additional 3-body input is needed (even when a is fixed)

c.f. Bedaque *et al.* '03 & Platter, Phillips '06

N^2LO renormalization

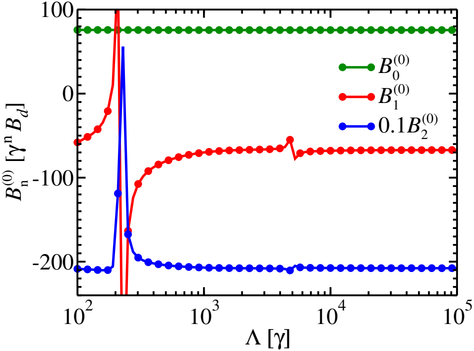
- $H_2 = r_0^2 \Lambda^2 \; h_{20}$



$a_{ad} \to$ LO/NLO/N^2LO

$B_t^{(1)} = B_0^{(1)} + r_0 B_1^{(1)} + r_0^2 B_2^{(1)}$

- $H_2 = r_0^2 \Lambda^2 \; h_{20} + r_0^2 m E_t \, h_{22}$



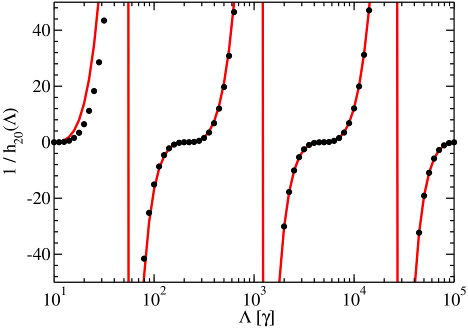
$a_{ad} \to$ LO/NLO/N^2LO

$B_t^{(1)} \to N^2$LO

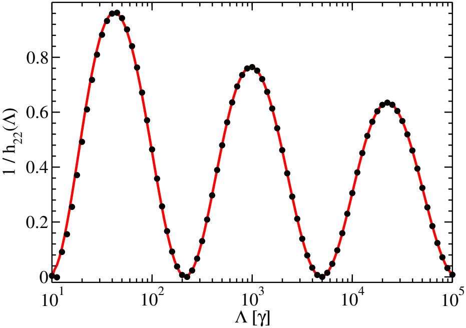
$B_t^{(0)} = B_0^{(0)} + r_0 B_1^{(0)} + r_0^2 B_2^{(0)}$

NLO three-body 3BF

- $H_2(\Lambda) = r_0^2\Lambda^2\ h_{20}(\Lambda) + r_0^2 m E_t\ h_{22}(\Lambda)$



$1/h_{20}(\Lambda)$



$1/h_{22}(\Lambda)$ [partial version]

^{4}He trimer

Input	$B_t^{(1)}$ $[B_d]$	$B_t^{(0)}$ $[B_d]$	a_{ad} $[\gamma^{-1}]$	r_{ad} $[\gamma^{-1}]$
TTY potential	1.738	96.33	1.205	

Input		$B_t^{(1)}$ $[B_d]$	$B_t^{(0)}$ $[B_d]$	a_{ad} $[\gamma^{-1}]$	r_{ad} $[\gamma^{-1}]$
TTY potential		1.738	96.33	1.205	
a_{ad}	LO	1.723	97.12	1.205	0.8352
a_{ad}	NLO	1.736	89.72	1.205	0.9049
a_{ad}, $B_t^{(1)}$	N^{2}LO	1.738	116.9	1.205	0.9132

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$B_t^{(1)}$	LO	1.738	99.37	1.178	0.8752
$B_t^{(1)}$	NLO	1.738	89.77	1.201	0.9130
$B_t^{(1)}$, a_{ad}	N^{2}LO	1.738	115.9	1.205	0.9135

^{4}He trimer

TRIUMF

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- **Difference in 2 renormalization schemes (LO→NLO→N^{2}LO):**
  - atom-dimer effective range r_{ad}: 5% → 0.9% → 0.02%
  - ground-state trimer $B_t^{(0)}$: 2% → 0.07% → 0.9%

Input		$B_t^{(1)}$ $[B_d]$	$B_t^{(0)}$ $[B_d]$	a_{ad} $[\gamma^{-1}]$	r_{ad} $[\gamma^{-1}]$
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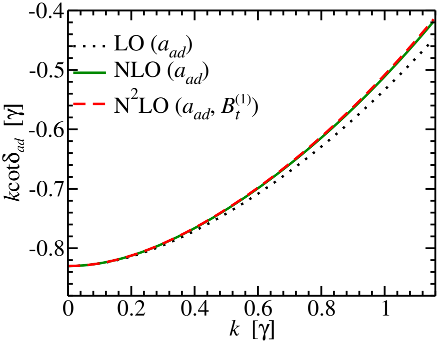
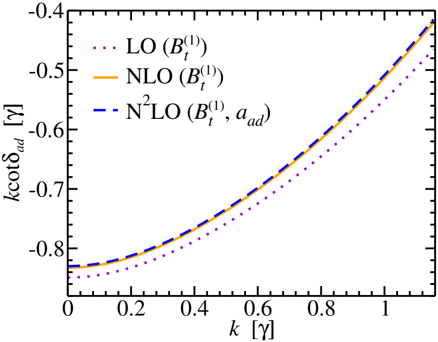
  - atom-dimer effective range r_{ad}: 5% → 0.9% → 0.02%
  - ground-state trimer $B_t^{(0)}$: 2% → 0.07% → 0.9%
- **Compare with TTY:**
  - $\Delta B_t^{(0)}$ ~20%
    - $r_0^2 B_t^{(0)} \sim 0.5$
    - EFT expansion needs to be corrected for deep trimer with U_{vdw} [Gao '98]

He-4 trimer phase shift at N^{2}LO

- a_{ad} → LO/NLO/N^{2}LO
- $B_t^{(1)}$ → N^{2}LO

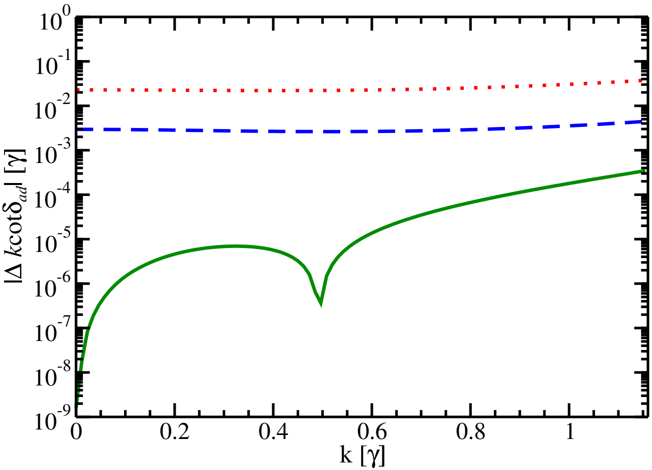


- $B_t^{(1)}$ → LO/NLO/N^{2}LO
- a_{ad} → N^{2}LO



Difference btw 2 renormalization

- LO $\rightarrow$ NLO$\rightarrow$ N^{2}LO



Conclusion

TRIUMF

- We studied effective-range corrections on Efimov physics in perturbation theory
- NLO for varying a:
  - recombination in cold atoms
  - $H_1(\Lambda) = r_0\Lambda\ h_{10}(\Lambda) + r_0/a\ h_{11}(\Lambda)$
  - one additional 3-body input is needed for NLO renormalization
- N^{2}LO for fixed a:
  - He-4 trimer
  - $H_2(E,\Lambda) = r_0^2\Lambda^2\ h_{20}(\Lambda) + r_0^2 mE_3\ h_{22}(\Lambda)$
  - one additional 3-body data is needed for N^{2}LO renormalization

- Range corrections are also important in nuclear physics