A Halo EFT description of Helium-6

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Canada's national laboratory for particle and nuclear physics
Laboratoire national canadien pour la recherche en physique nucléaire et en physique des particules

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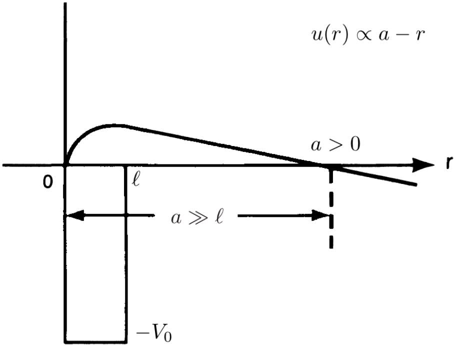
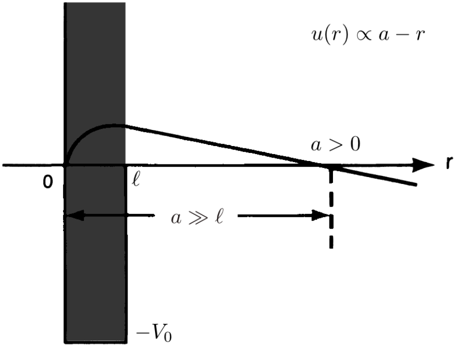
Universal features at low energies

- **Separation of scales:**
  $a \gg \ell$
- **2-body universality:**
  $B_2 = 1/Ma^2$



Universal features at low energies

- **Separation of scales:**
  $a \gg \ell$
- **2-body universality:**
  $B_2 = 1/Ma^2$
- Physics at large distance is insensitive to physics at short distance
- Large-distance physics is studied in ℓ/a expansion
- Effects from SR-dynamics can be included in perturbation theory

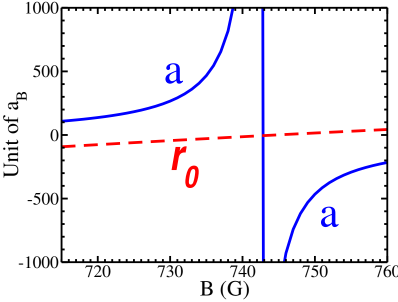


Large-scattering-length physics

Universal Physics exists in systems with $\ell \ll a$

- Atomic Physics
  - **Cold atomic gases (^{133}Cs, ^{7}Li, ^{39}K):**
    r_0 and a varies near Feshbach resonance
  - **^{4}He atoms (dimer, trimer, tetramer):**
    $\ell_{vdw} \sim 7\text{Å}$, $a \sim 100\text{Å}$
- Nuclear Physics
  - **Few-nucleon systems (^{3}H, ^{3}He, ^{4}He):**
    i.e., $\ell^t_{np} \sim 1.7$ fm, $a^t_{np} \sim 5.4$ fm
  -

^{7}Li atoms [Gross *et al.* '09]



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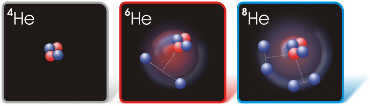
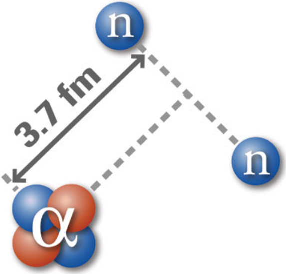


- Nuclear Physics
  - **Few-nucleon systems (^{3}H, ^{3}He, ^{4}He):**
    i.e., $\ell_{np}^{t} \sim 1.7$ fm, $a_{np}^{t} \sim 5.4$ fm
  - **Halo nuclei**
    a tightly bound core with one/few loosely bound valence nucleons
    ^{6}He: $\epsilon_{\alpha}^{*} \sim 20\text{MeV}$, $\epsilon_{^6\text{He}}^{*} \sim 1\text{MeV}$
    $r_{\alpha} \sim 1.45$ fm, $r_{\alpha-nn} \sim 3.7$ fm



[www.anl.gov]

Effective field theory

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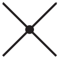
- **An approach to systems with a separation of scales**
  - **Systems with $\ell \ll a$** $\rightarrow$ an EFT with contact interactions
    - Atomic systems $\rightarrow$ zero-range EFT
    - Few-nucleon systems $\rightarrow$ pionless EFT
    - Halo nuclei $\rightarrow$ halo EFT
  - **Physical quantities are expanded in powers of ℓ/a**

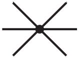
- **Contact interactions at LO**
  - **2-body contact interaction (LO)**



$$= -iC_0$$

C_0 determined by a 2-body observable

  - **3-body contact interaction (LO)**



$$= -iD_0$$

D_0 determined by a 3-body observable

Effective field theory

TRIUMF

- **An approach to systems with a separation of scales**
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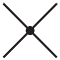
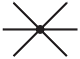
    - Few-nucleon systems $\rightarrow$ pionless EFT
    - Halo nuclei $\rightarrow$ halo EFT
  - **Physical quantities are expanded in powers of ℓ/a**

- **Contact interactions at LO**
  - **2-body contact interaction (LO)** introduce a dimer field

 $= -iC_0$ $\xRightarrow{C_0=g^2/\Delta}$  $= -i\sqrt{2}g$

C_0 determined by a 2-body observable

  - **3-body contact interaction (LO)**

 $= -iD_0$ $\xRightarrow{D_0=-3hg^2/\Delta^2}$  $= ih$

D_0 determined by a 3-body observable Bedaque, Hammer, van Kolck '99

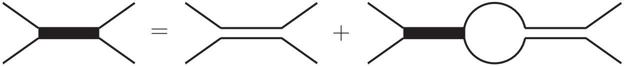
A general EFT for 3-body systems

- **EFT Lagrangian for 3 identical bosons at LO**

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - d^\dagger \left(i\partial_0 + \frac{\nabla^2}{4m} - \Delta \right) d - \frac{g}{\sqrt{2}} \left(d^\dagger \psi\psi + \text{h.c} \right) + h d^\dagger d \psi^\dagger \psi + \cdots$$

  - terms with more derivatives are at higher orders
  - nuclear physics: add spin and isospin *d.o.f.*

- **Non-perturbative features at LO**
  - particle-particle scattering (tune g)

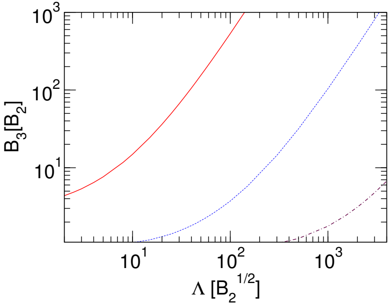


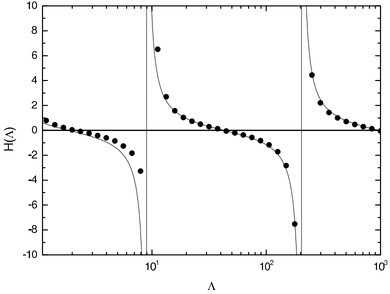
$\propto \frac{1}{1/a + ik}$

  - particle-dimer scattering (tune h)

LO renormalization

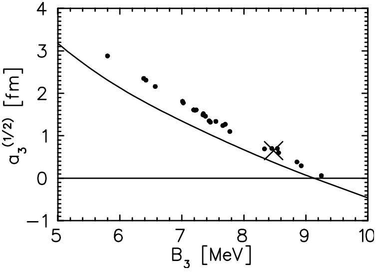
- **Without 3BF:**
  - 3-body spectrum:
    cutoff dependent ($\Lambda \sim 1/\ell$)
    Platter '09
- **LO 3BF h:**
  - tune $H(\Lambda) = \Lambda^2 h/2mg^2$:
    fix one 3-body observable
  - limit cycle:
    $H(\Lambda)$ periodic for $\Lambda \to \Lambda(22.7)^n$
    Wilson '71, Bedaque *et al.* '00



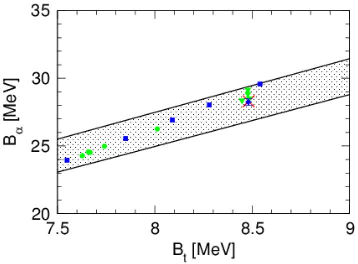


Universal correlation in nuclear systems

- **3-body correlation:**
  Phillips line (Phillips '68)
  correlation btw nd scattering length and triton binding energy
- **3- and 4-body correlation:**
  Tjon line (Tjon '75)
  correlation btw binding energies of triton and α-particle (no 4BF)



Phillips line (Bedaque *et al.* '00)



Tjon line (Platter *et al.* '04)

EFT in halo nuclei

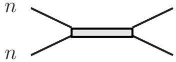
- Halo EFT is suitable for nuclei with $E^*_{core} \gg E_{sep}$
  - Core + valence nucleons
  - $M_{lo} \sim (M_N E_{sep})^{1/2}$; $M_{hi} \sim (M_N E^*_{core})^{1/2}$
  - study properties of halo nculei in M_{lo}/M_{hi} expansion

- **1-neutron halo and resonant state**
  - ^{11}Be (n-^{10}Be): E1 transition [Hammer, Phillips '11]
  - ^{5}He (n-α): p-wave resonance [Bertulani *et al.* '02, Bedaque *et al.* '02]
- **2-neutron halo:**
  - ^{11}Li, ^{12}Be, ^{20}C: n-core in s-wave resonance [Canham, Hammer '08]
  - ^{6}**He:** n-α in p-wave resonance
    - EFT + Gamow shell model [Rotureau, van Kolck '12]
    - EFT + Faddeev Equations Ji, Elster, Phillips

Studies of ^{6}He

- **experiment in ^{6}He**
  - matter radius Tanihata *et al.* '92, Alkhazov *et al.* '97, Kislev *et al.* '05
  - charge radius Wang *et al.* '04, Mueller *et al.* '07
  - ^{6}He mass Brodeur *et al.* '12
- **cluster model**
  - separable potential Ghovanlou, Lehman '74
  - density-dependent nn contact interaction Esbensen *et al.* '97
- *ab intio* **calculation**
  - no-core shell model Navrátil *et al.* '01
  - hyperspherical harmonics Bacca *et al.* '12
  - Green's function Monte Carlo Pieper *et al.* '01
- **halo EFT**
  - explore **universal physics** in halo nuclei
  - understand the role of **3BF ($nn\alpha$)**
  - compare **predictions** with experiments and other theories

- nn interaction is dominated by the $^1S_0$ state

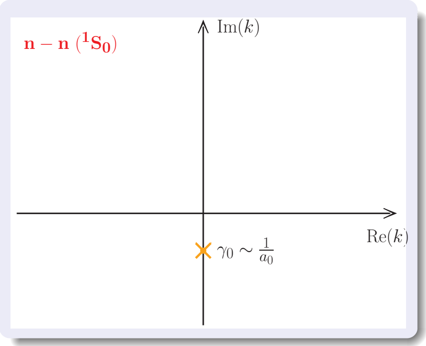


$$= \frac{1}{4\pi^2\mu_{nn}} \frac{1}{-1/a_0 + r_0 k^2/2 - ik}$$

$a_0 = -18.7$ fm, $r_0 = 2.75$ fm González Trotter *et al.* '99

- nn EFT power counting:
  - EFT: $a_0 \sim M_{lo}^{-1}$ $\quad r_0 \sim M_{hi}^{-1}$
  - $M_{lo}/M_{hi} \sim 0.15$
- $^1S_0 \rightarrow$ shallow virtual-bound state
  - $\gamma_0 \sim M_{lo}$
- LO nn t-matrix in halo EFT

$$t_{nn} = \frac{1}{4\pi^2\mu_{nn}} \cdot \frac{1}{\gamma_0 + ik}$$



$n - \alpha$ interaction

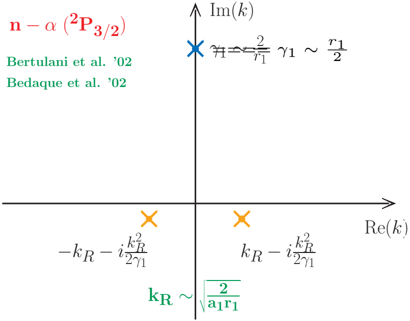
- $n\alpha$ interaction is dominated by the $^2P_{\frac{3}{2}}$ state

$$= \frac{1}{4\pi^2 \mu_{n\alpha}} \frac{\vec{p}\cdot\vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

$a_1 = -62.95$ fm^3, $r_1 = -0.8819$ fm^{-1} Ardnt et al. '73

- $n\alpha$ EFT power counting: Bedaque, Hammer, van Kolck '02
  - $a_1 \sim M_{lo}^{-2} M_{hi}^{-1}$ $r_1 \sim M_{hi}$
  - $M_{lo}/M_{hi} \sim 0.15$
- $^2P_{\frac{3}{2}}$: $\begin{cases} \text{shallow resonance :} \\ \quad k_R \sim M_{lo},\ \Gamma \sim M_{lo}^2/M_{hi} \\ \text{deep bound state :}\ \gamma_1 \sim M_{hi} \end{cases}$
- LO $n\alpha$ t-matrix in halo EFT

$$t_{n\alpha} = \frac{3}{4\pi^2 \mu_{n\alpha}} \cdot \frac{\vec{p}\cdot\vec{q}}{\gamma_1 \left(k^2 - k_R^2\right)}$$



Note: $n - \alpha$ $^1S_{\frac{1}{2}}$ and $^2P_{\frac{1}{2}}$ interactions are at higher orders above LO

^{6}He in Jacobi coordinates

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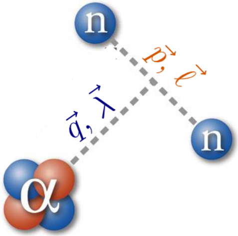
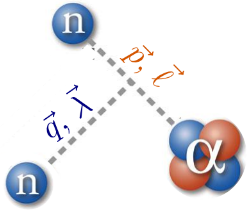
- **Jacobi-momentum**

α spectator



n spectator

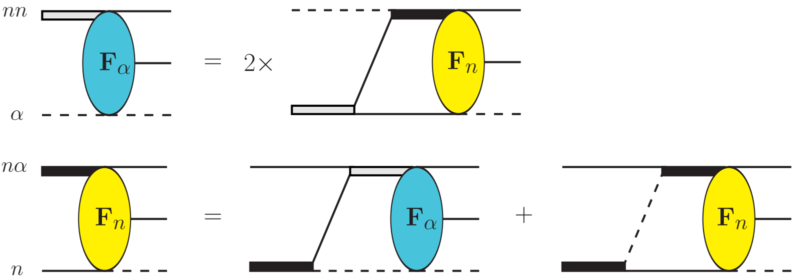


spin-orbit coupling for ^{6}He ground state ($J = 0^+$)

pair, spec	pair	spectator	total L, S	total J
$nn,\ \alpha$	$\ell = 0,\ s_1 = 0$	$\lambda = 0,\ s_2 = 0$	$L = 0,\ S = 0$	$J = 0^+$
$n\alpha,\ n$	$\ell = 1,\ s_1 = \frac{1}{2}$	$\lambda = 1,\ s_2 = \frac{1}{2}$	$L = 0,\ S = 0$	
			$L = 1,\ S = 1$	

Faddeev equations

- decompose ^{6}He ground-state wave function into Faddeev components
  - $|\Psi_{^6\mathrm{He}}\rangle = |\psi_\alpha\rangle + (1 - \mathcal{P}_{nn})\,|\psi_n\rangle$
- introduce 2-body dressed propagators $\rightarrow$ redefine Faddeev components
  - $|\psi_\alpha\rangle = G_0 t_\alpha |F_\alpha\rangle \qquad |\psi_n\rangle = G_0 t_n |F_n\rangle$
  - F_n (F_α) depends only on q_n (q_α) after partial-wave projection
- parameters in 2-body t-matrices from experiments (underlying theory)
- ^{6}He is studied in a coupled-channel integral equations



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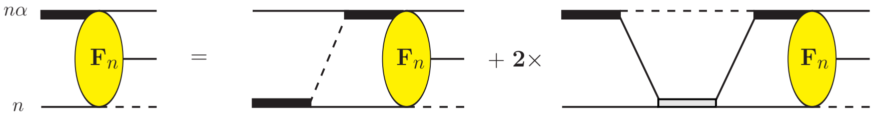
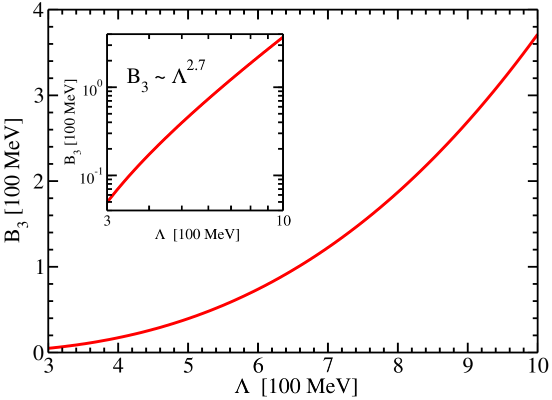


reduce to a single-channel equation

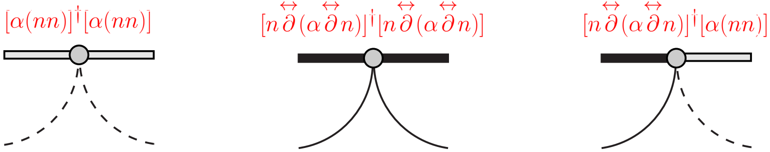
Cutoff dependence

- **Before inserting $nn\alpha$ 3-body force:**
  - Use a hard cutoff Λ
  - calculate $2n$ separtion energy S_{2n}
  - S_{2n} is strongly cutoff dependent: $S_{2n} \sim \Lambda^{2.7}$ ← need 3BF!



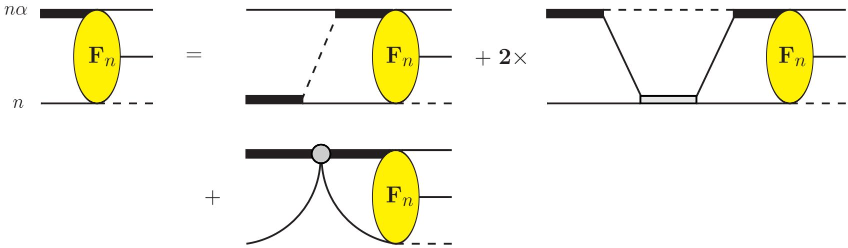
$nn\alpha$ 3-body force

- **candiates for $nn\alpha$ counterterms**

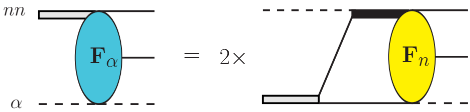


- **only the $[n\overleftrightarrow{\partial}(\alpha\overleftrightarrow{\partial}n)]^\dagger[n\overleftrightarrow{\partial}(\alpha\overleftrightarrow{\partial}n)]$ counterterm is needed**
  - Pauli principle
  - A similar three-body counterterm is discovered by Rotureau, van Kolck arXiv:1201.3351v1 (2012)

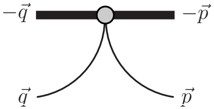
- **Add $nn\alpha$ 3BF to Faddeev equation**
- **renormalize F_n by reproducing one 3-body observable**



- **F_α is simultaneously renormalized without additional 3BFs**

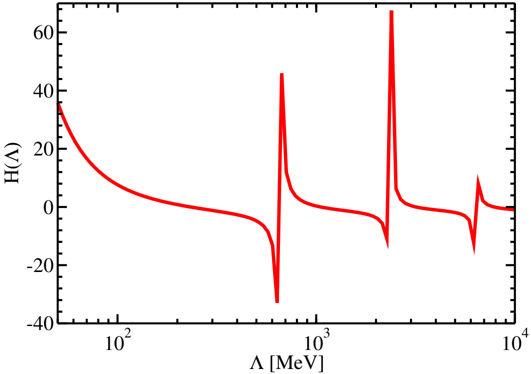


- **3BF parameter:**



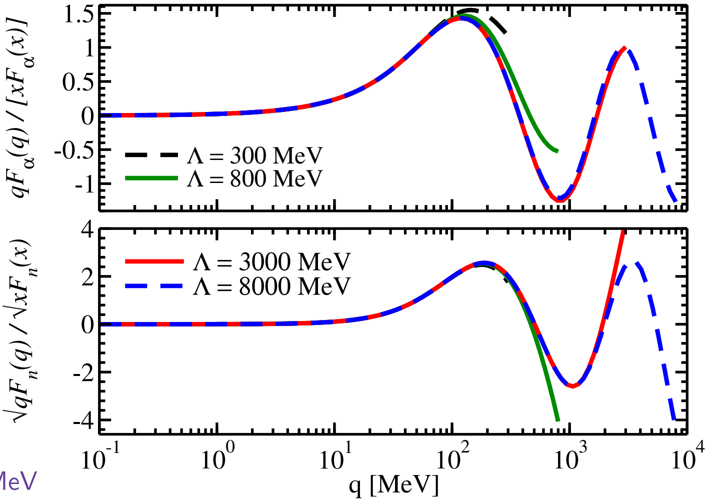
$$= M_n qp \frac{H(\Lambda)}{\Lambda^2}$$

- reproduce $S_{2n} = 0.973$MeV
- log oscillation
- No limit cycle (c.f. 3-body in S-wave)



Renormalized Faddeev Components

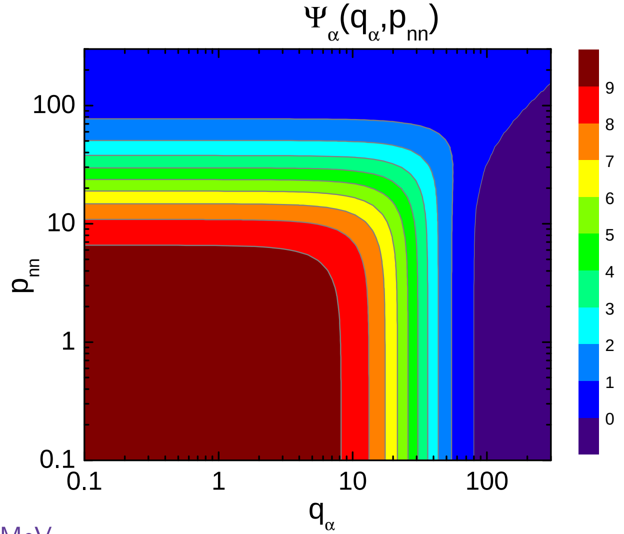
$F_\alpha(\alpha, nn)$ and $F_n(n, \alpha n)$: cutoff independent



$x = 50$ MeV

^{6}He ground-state wave function ($\alpha - nn$)

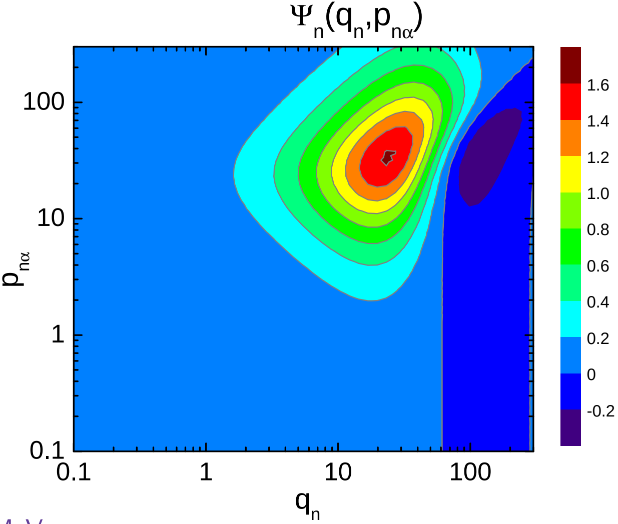
- $|\Psi\rangle$ in $\alpha - (nn)$ Jacobi representation



momenta in MeV

^{6}He ground-state wave function $(n - \alpha n)$

- $|\Psi\rangle$ in $n - (\alpha n)$ Jacobi representation



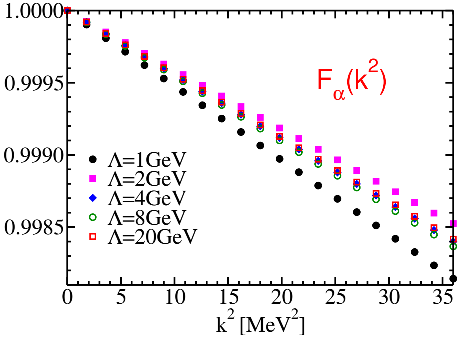
momenta in MeV

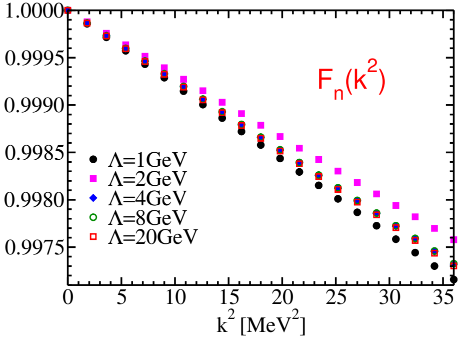
^{6}He matter density form factors

- **one-body matter form factors**

$$\mathcal{F}_i(k^2) = \frac{\int dq^3 \int dp^3 \Psi_i^\dagger(\vec{p},\vec{q})\Psi_i(\vec{p},\vec{q}-\vec{k})}{\int dq^3 \int dp^3 |\Psi_i(\vec{p},\vec{q})|^2}$$

$$\mathcal{F}_i(k^2) = 1 - \frac{1}{6}k^2\langle r_i^2\rangle + \cdots$$



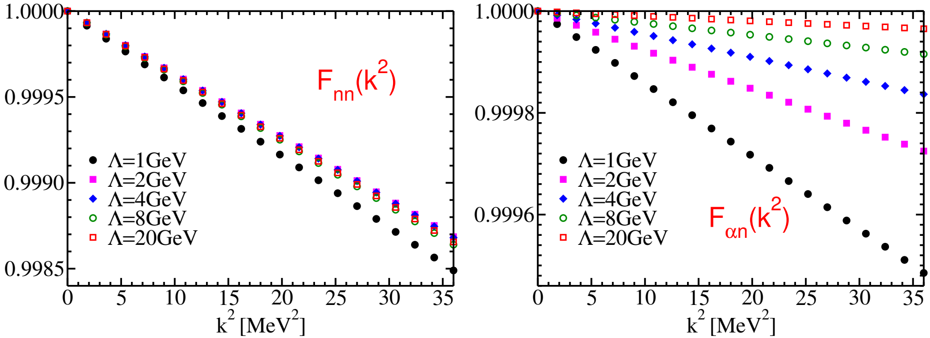


^{6}He matter density form factors

- **two-body matter form factors**

$$\mathcal{F}_{jk}(k^2) = \frac{\int dq^3 \int dp^3 \Psi_i^\dagger(\vec{p},\vec{q})\Psi_i(\vec{p}-\vec{k},\vec{q})}{\int dq^3 \int dp^3 |\Psi_i(\vec{p},\vec{q})|^2}$$

$$\mathcal{F}_{jk}(k^2) = 1 - \frac{1}{6}k^2\langle r_{jk}^2\rangle + \cdots$$



^{6}He matter density form factors

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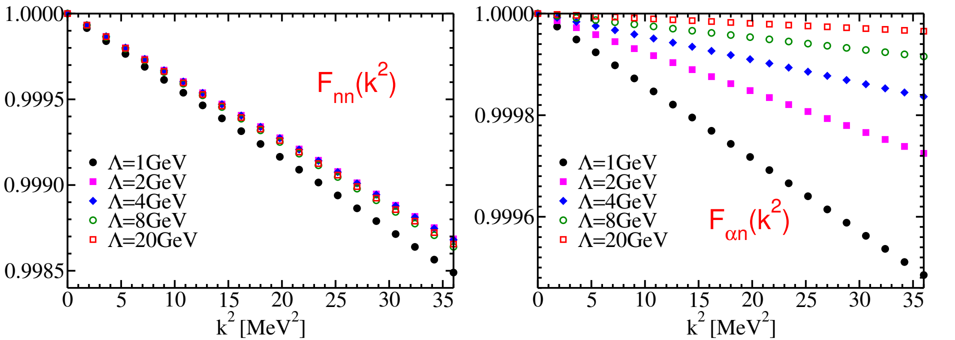
$$\mathcal{F}_{jk}(k^2) = \frac{\int dq^3 \int dp^3 \Psi_i^\dagger(\vec{p},\vec{q})\Psi_i(\vec{p}-\vec{k},\vec{q})}{\int dq^3 \int dp^3 |\Psi_i(\vec{p},\vec{q})|^2}$$

$$\mathcal{F}_{jk}(k^2) = 1 - \frac{1}{6}k^2\langle r_{jk}^2\rangle + \cdots$$

- $F_{\alpha n}$ vanishes at $\Lambda \to \infty$?
- further study $F_{\alpha n}$
- use only F_α and F_n for predictions



Matter radius and point-proton radius

		EFT [fm]	Exp [fm]
He-4	$r_{pp}[\alpha]$	—	1.455(1)
	$r_m[\alpha]$	—	1.455(1)
He-6			

Matter radius and point-proton radius

		EFT [fm]	Exp [fm]
He-4	$r_{pp}[\alpha]$	—	1.455(1)
	$r_m[\alpha]$	—	1.455(1)
He-6	$r_{\alpha(nn)}$	3.235	—
	$r_{n(\alpha n)}$	4.096	—

Matter radius and point-proton radius

		EFT [fm]	Exp [fm]
He-4	$r_{pp}[\alpha]$	—	1.455(1)
	$r_m[\alpha]$	—	1.455(1)
He-6	$r_{\alpha(nn)}$	3.235	—
	$r_{n(\alpha n)}$	4.096	—
	$r_\alpha = \frac{1}{3} r_{\alpha(nn)}$	1.078	—
	$r_n = \frac{5}{6} r_{n(\alpha n)}$	3.413	—

Matter radius and point-proton radius

		EFT [fm]	Exp [fm]
He-4	$r_{pp}[\alpha]$	—	1.455(1)
	$r_m[\alpha]$	—	1.455(1)
He-6	$r_{\alpha(nn)}$	3.235	—
	$r_{n(\alpha n)}$	4.096	—
	$r_\alpha = \frac{1}{3} r_{\alpha(nn)}$	1.078	—
	$r_n = \frac{5}{6} r_{n(\alpha n)}$	3.413	—
	$r_{pp}(^6\text{He}) = \sqrt{r_\alpha^2 + r_{pp}^2[\alpha]}$	1.811(1)	1.938(23), 1.953(22)
	$r_m(^6\text{He}) = \sqrt{\frac{1}{6}\left(4r_\alpha^2 + 2r_n^2 + 4r_m^2[\alpha]\right)}$	2.464(1)	2.33(4), 2.30(7), 2.37(5)

Matter radius and point-proton radius

		EFT [fm]	Exp [fm]
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	$r_\alpha = \frac{1}{3} r_{\alpha(nn)}$	1.078	—
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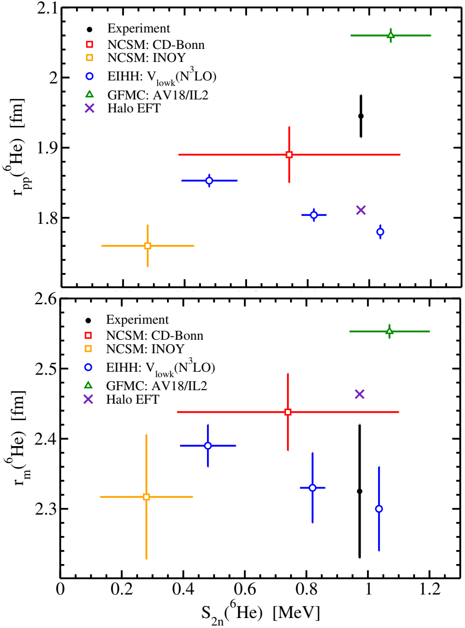
	$r_{pp}(^6\text{He}) = \sqrt{r_\alpha^2 + r_{pp}^2[\alpha]}$	1.811(1)	1.938(23), 1.953(22)
	$r_m(^6\text{He}) = \sqrt{\frac{1}{6}\left(4r_\alpha^2 + 2r_n^2 + 4r_m^2[\alpha]\right)}$	2.464(1)	2.33(4), 2.30(7), 2.37(5)

EFT discrepancy from experiment: (consistent with EFT expansion)

- $r_{pp} \sim 7\% \to r_\alpha \sim 20\%$
- $r_m \sim 6\% \to r_n \sim 25\%$



cf. Bacca, Barnea, Schwenk '12

- **He-6 point-proton radius**

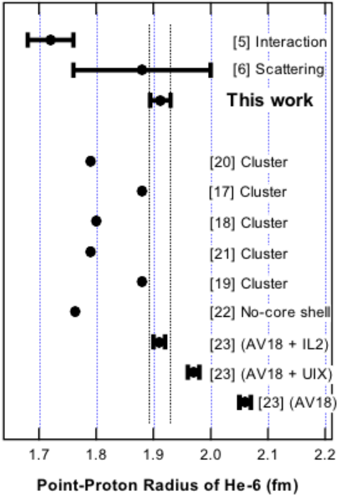
- **He-6 matter radius**

Conclusion and Outlook

- **^{6}He is a $2n$ halo with $n\alpha$ interacting in $^{2}P_{\frac{3}{2}}$ resonance**
- **LO halo EFT analysis:**
  - A p-wave $n(n\alpha)$ 3BF is needed for proper renormalization
  - Tune 3BF to reproduce $S_{2n}(^{6}He) = 0.973$ MeV
  - Halo EFT predictions of r_m and r_{pp} is consistent with EFT expansion
- **two-body form factors need to be checked**
- **application to ^{11}Li**

BACK UP

point-proton charge radius of ^{6}He



Wang *et al.* '04

Cluster-model calculations

2-body Yamaguchi potential *[Ghovanlou, Lehman '74]*

$$V_{nn}(\vec{p},\vec{q}) = \frac{\lambda_0}{4\pi} g_0(p) g_0(q) \qquad g_0(p) = 1/\left(p^2+\beta_0^2\right)$$

$$V_{n\alpha}(\vec{p},\vec{q}) = \frac{3\lambda_1}{4\pi} g_1(p)\ \vec{p}\cdot\vec{q}\ g_1(q) \qquad g_1(p) = 1/\left(p^2+\beta_1^2\right)^2$$

nn	β_0 [fm^{-1}]	λ_0 [fm^{-2}]	a_0 [fm]	r_0 [fm]
	1.13	-0.3484	-16.4	2.84
$n\alpha$	β_1 [fm^{-1}]	λ_1 [fm^{-4}]	a_1 [fm^3]	r_1 [fm^{-1}]
	1.45	-7.969	-71.5	-0.851

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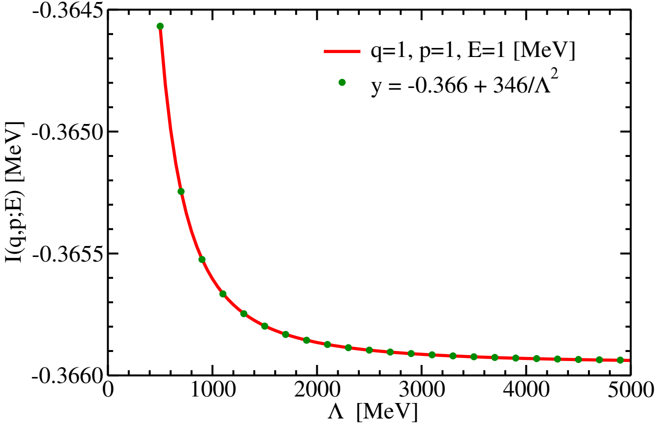
- **^{6}He ground-state binding energy:**
  - **Prediction:** $\mathbf{B(^6He) = 0.78}$ **MeV**
  - **Experiment:** $\mathbf{B(^6He) = 0.97}$ **MeV**

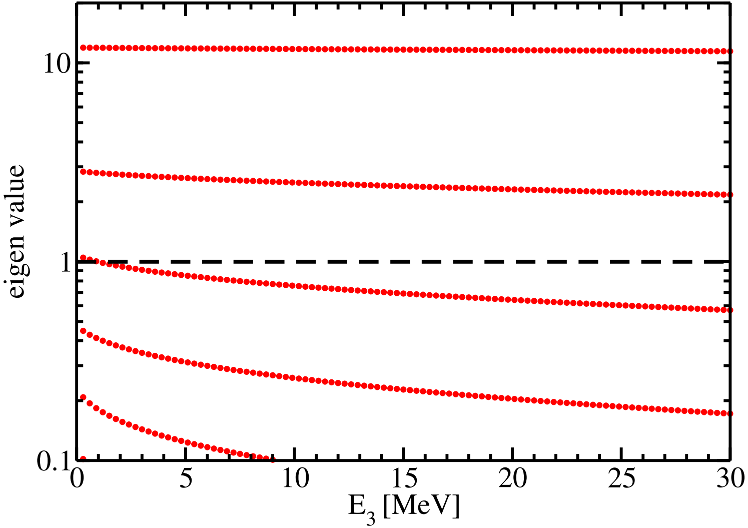
  underbind? $\rightarrow \mathrm{nn}\alpha$ **counterterm**

nn	β_0 [fm^{-1}]	λ_0 [fm^{-2}]	a_0 [fm]	r_0 [fm]
	1.13	-0.3484	-16.4	2.84
$n\alpha$	β_1 [fm^{-1}]	λ_1 [fm^{-4}]	a_1 [fm^3]	r_1 [fm^{-1}]
	1.45	-7.969	-71.5	-0.851

loop integral

TRIUMF





10
1
0.1
0
5
10
15
20
25
30
eigen value
E_3 [MeV]