Three-Nucleon Forces and the Structure of Exotic Nuclei

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<u>Based on</u>

- T. Otsuka, T. Suzuki, JDH, A. Schwenk, Y. Akaishi, PRL (1010)
- JDH, J. Menendez, A. Schwenk, arXiv:1108.2680
- JDH, T. Otsuka, A. Schwenk, T. Suzuki JPG (2012)
- **JDH, J. Menendez, A. Schwenk, arXiv:1207.1590**
- **JDH, J. Menendez, A. Schwenk, in prep.**
- **JDH and J. Engel, in prep.**
- D. Lincoln, JDH *et al*., submited to PRL

Supported by BMBF under 06DA7047I (NuSTAR.DA)

Outline

Goal: understand the role of 3N forces for structure of medium-mass exotic nuclei

- What are the limits of nuclear existence?

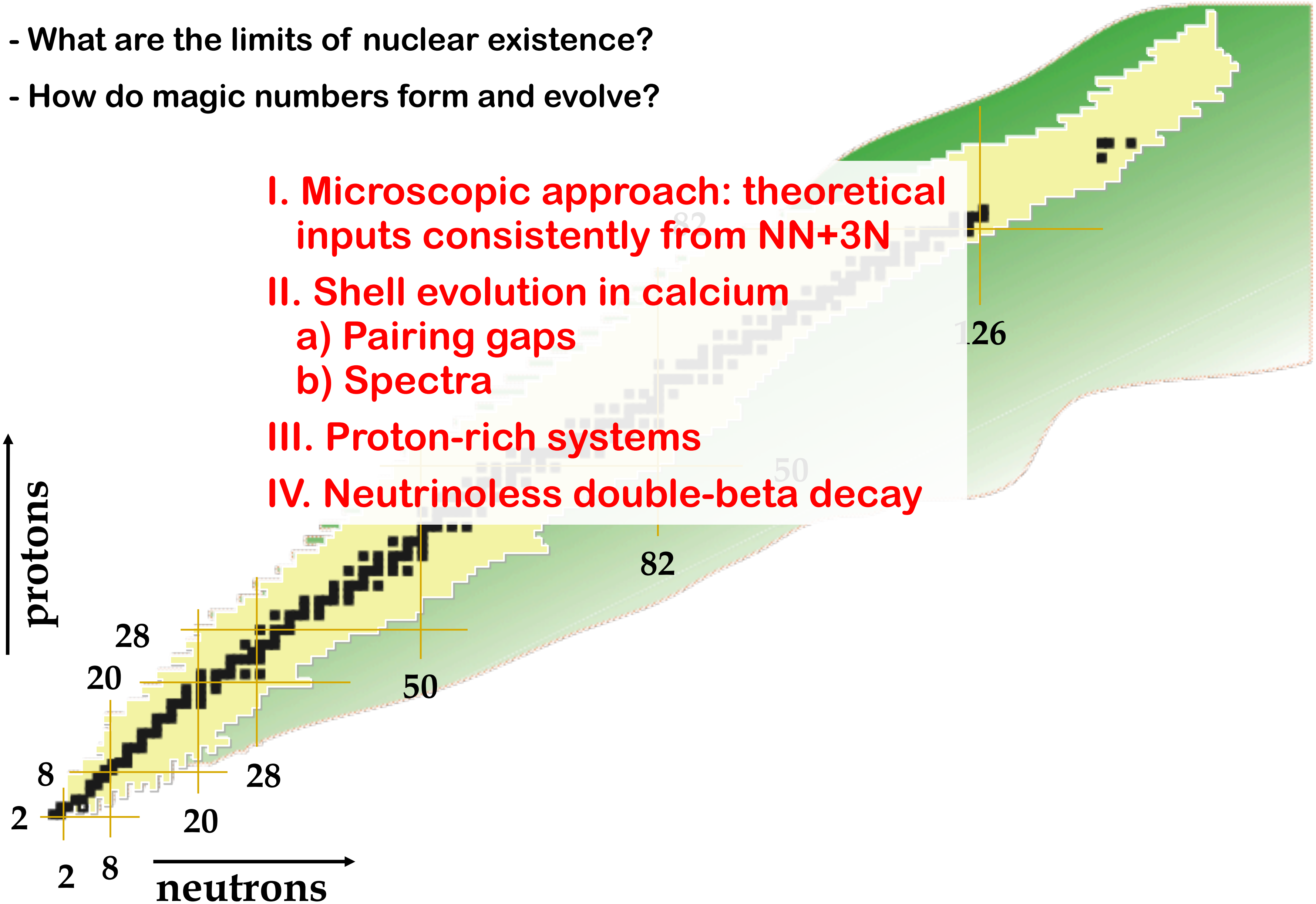
- How do magic numbers form and evolve?

I. Microscopic approach: theoretical inputs consistently from NN+3N

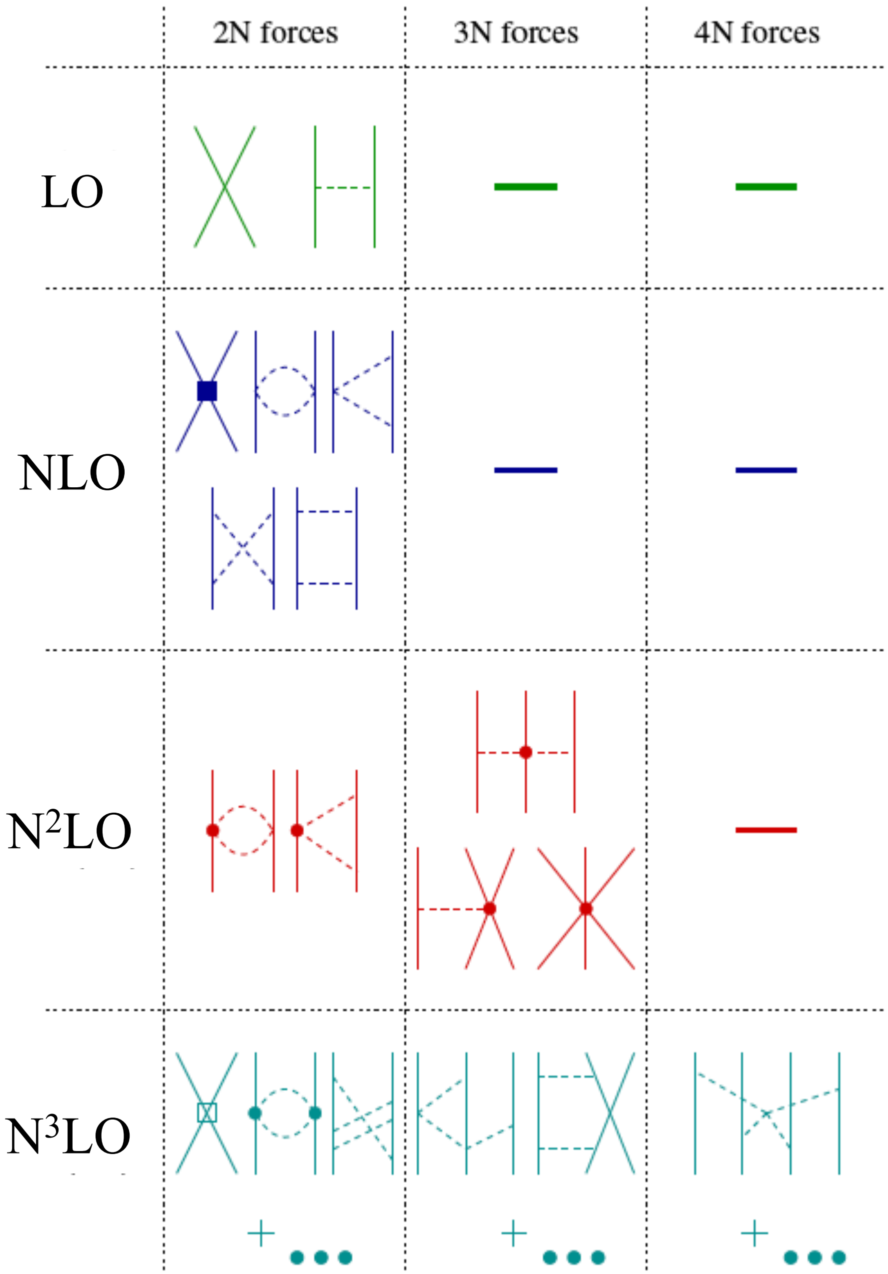
II. Shell evolution in calcium
   a) Pairing gaps
   b) Spectra

III. Proton-rich systems

IV. Neutrinoless double-beta decay



Chiral Effective Field Theory: Nuclear Forces



Nucleons interact via pion exchanges and contact interactions

Hierarchy: $V_{NN} > V_{3N} > \ldots$

Consistent treatment of NN, 3N, … electroweak operators

Couplings fit to experiment once

Evolve to **low-momentum** $V_{\mathrm{low}\,k}$
(Improved convergence behavior)

3N constants fit to properties of light nuclei at low momentum

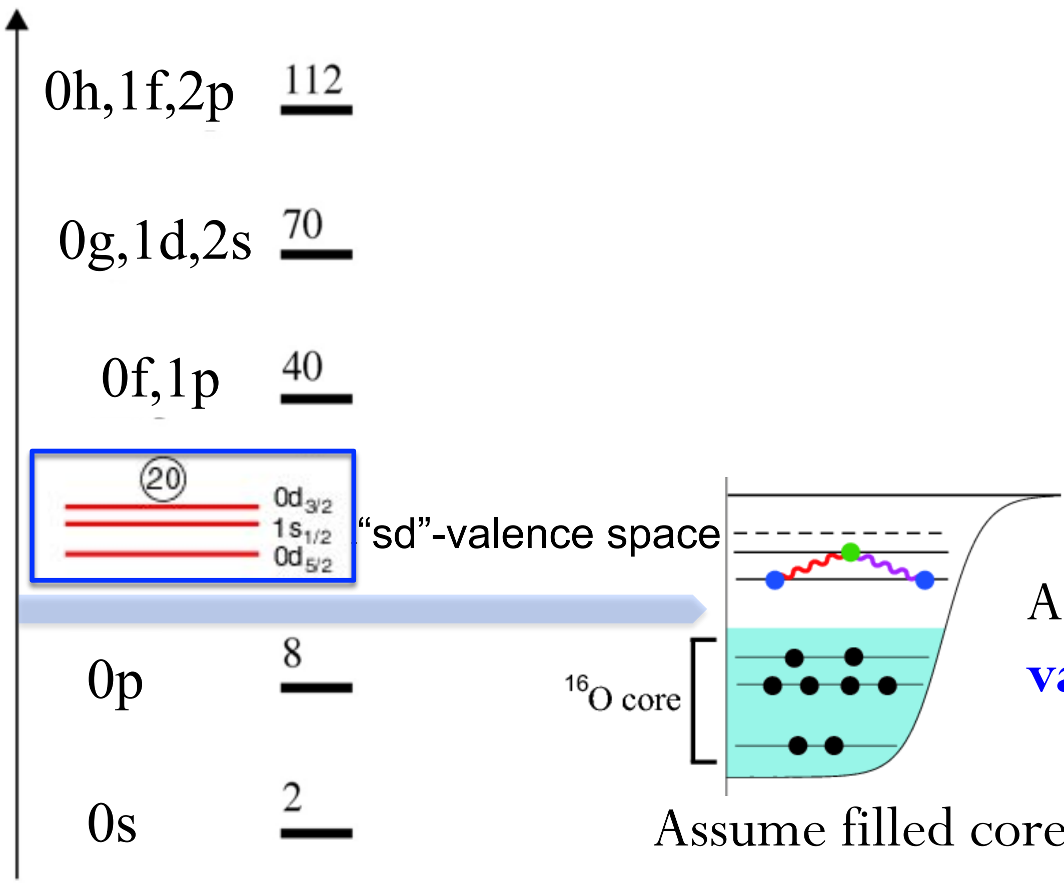
Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meissner,…

Solving the Nuclear Many-Body Problem

Nuclei understood as many-body system starting from closed shell, add nucleons

Interaction and energies of valence space orbitals from $V_{\text{low }k}$

Does not reproduce experimental data



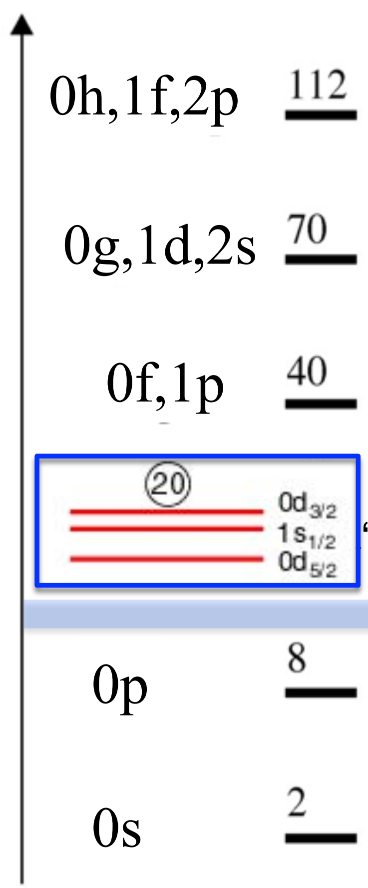
Active nucleons occupy **valence space**

Solving the Nuclear Many-Body Problem

Nuclei understood as many-body system starting from closed shell, add nucleons

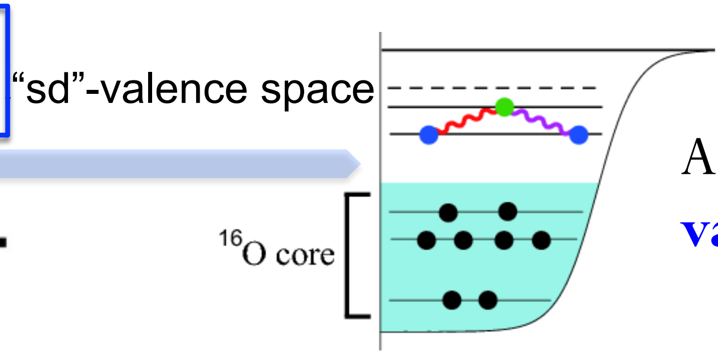
Interaction and energies of valence space orbitals from $V_{\text{low } k}$

Does not reproduce experimental data – **allow explicit breaking of core**



Strategy

1) Effective valence space interaction – 3rd order MBPT
2) HO basis of 13 major shells
3) Single-particle energies calculated self-consistently
4) Work in valence space as large as possible
5) Consistent treatment of chiral NN and 3N forces



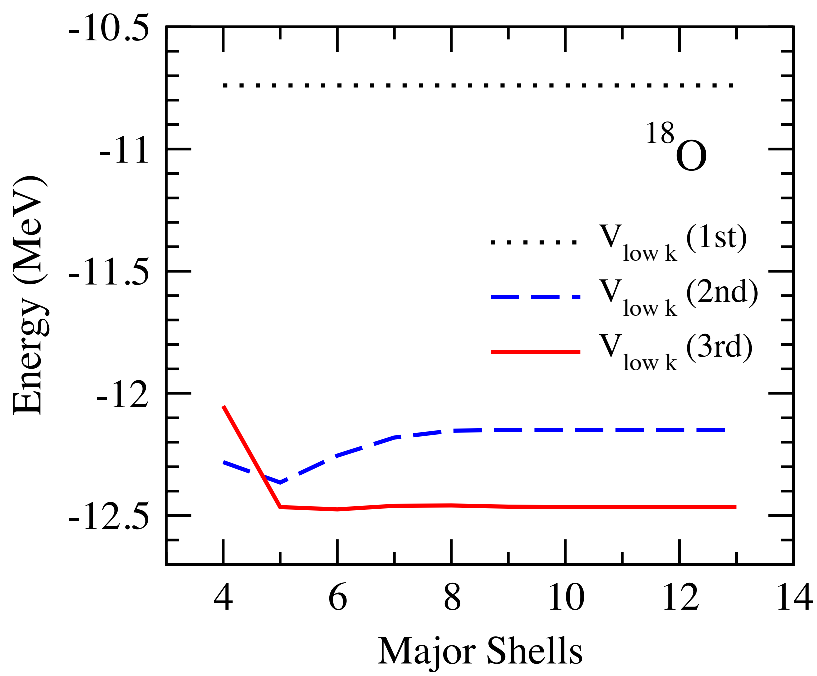
Assume filled core

Active nucleons occupy **valence space**

Convergence Properties

NN matrix elements derived from:

- Chiral N^3LO (Machleidt, 500 MeV) using smooth-regulator $V_{\text{low } k}$
- Third order in MBPT
- 13 major HO shells for intermediate state configurations



Clear convergence with HO basis size

Promising order-by-order behavior

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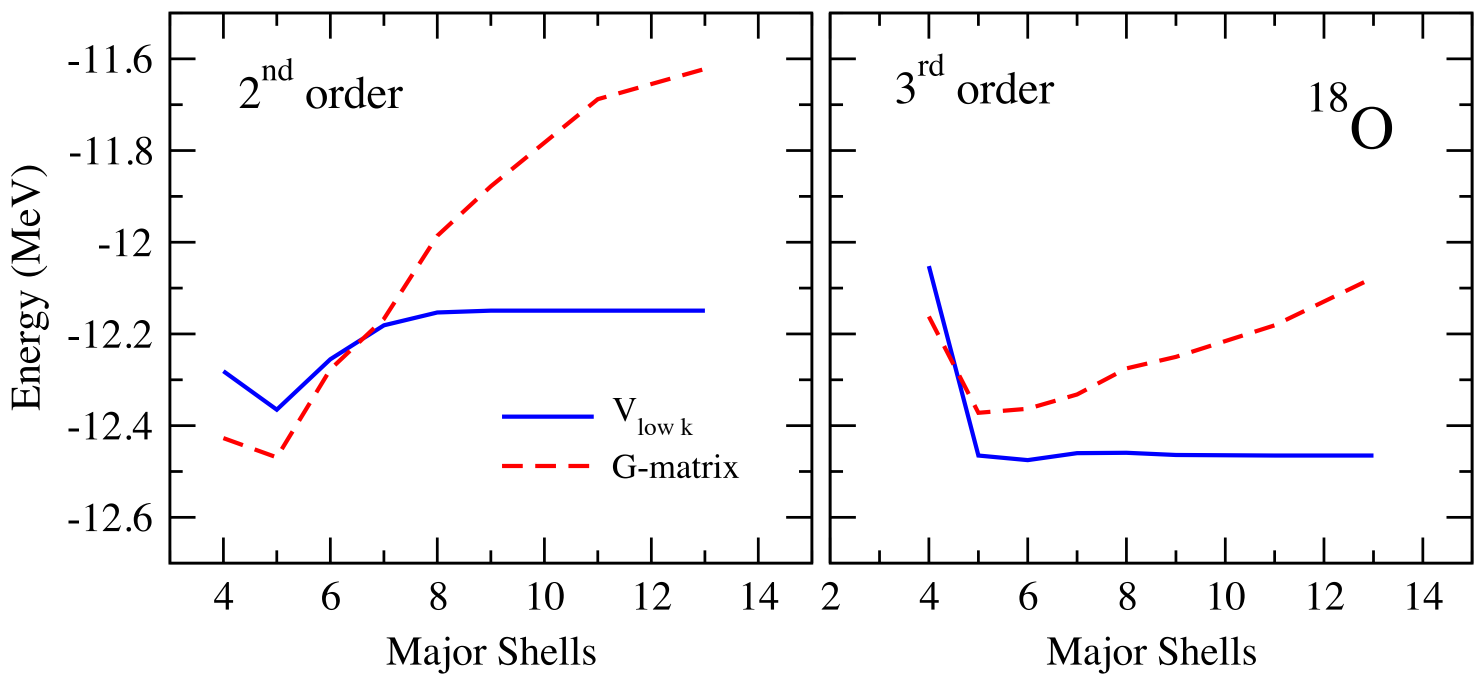


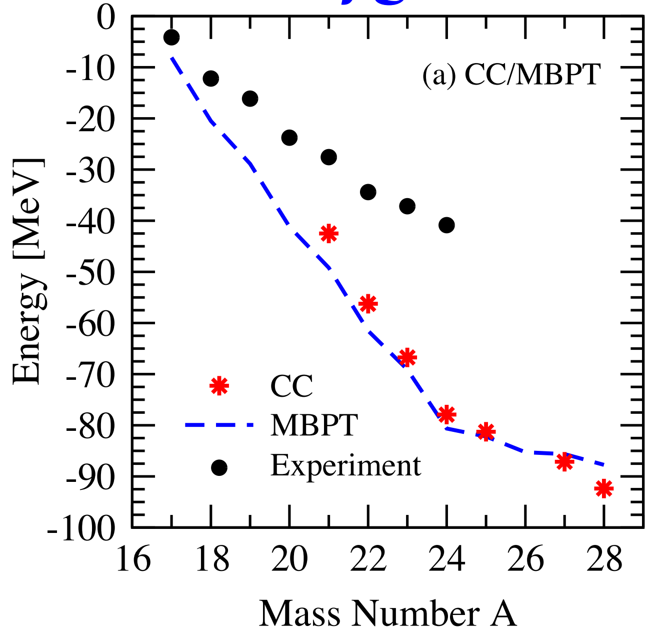
G-matrix – no sign of convergence

Comparison to Coupled Cluster

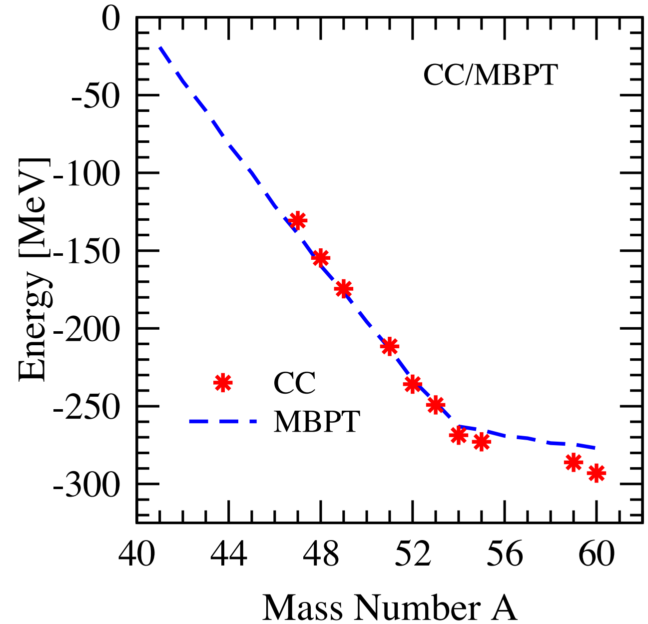
Benchmark against *ab-initio* Coupled Cluster at NN level

SPEs: one-particle attached CC energies in ^{17}O and ^{41}Ca

Oxygen



Calcium



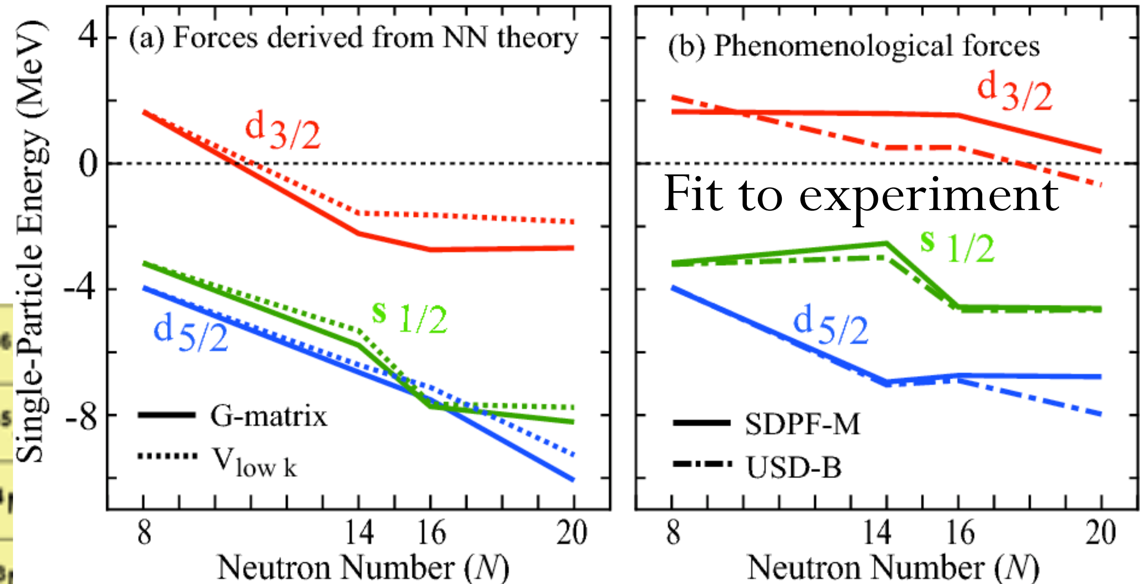
Energies relative to ^{16}O and ^{40}Ca

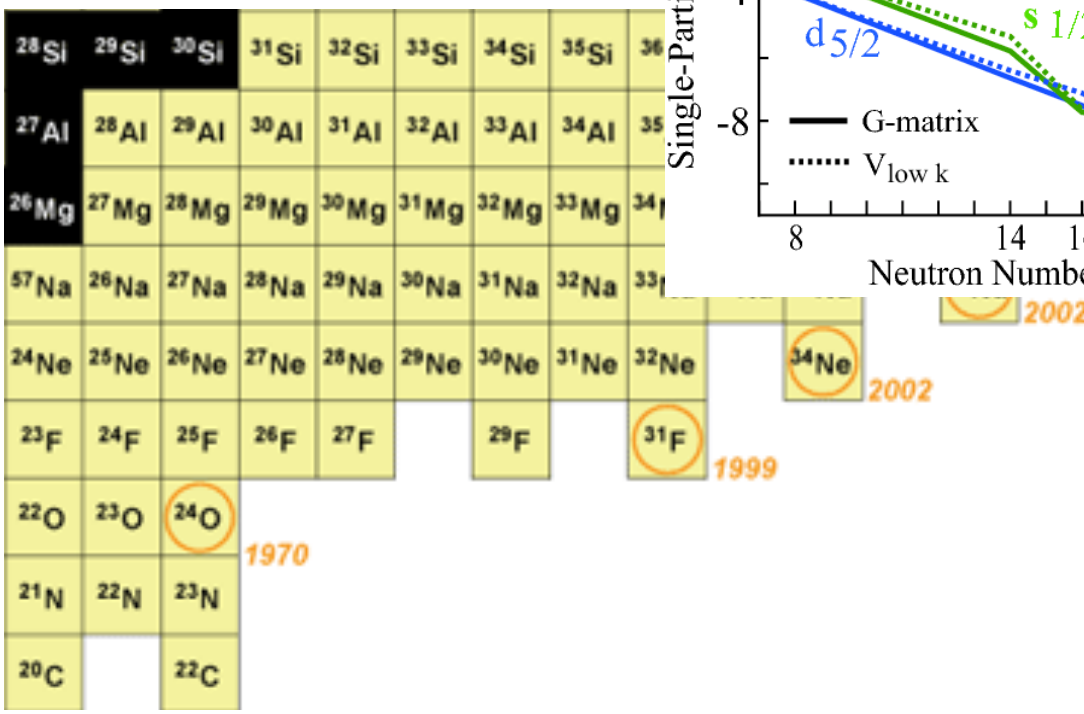
Small difference in many-body methods ~5%

Limits of Nuclear Existence: Oxygen Anomaly

Microscopic picture:

NN-forces too attractive





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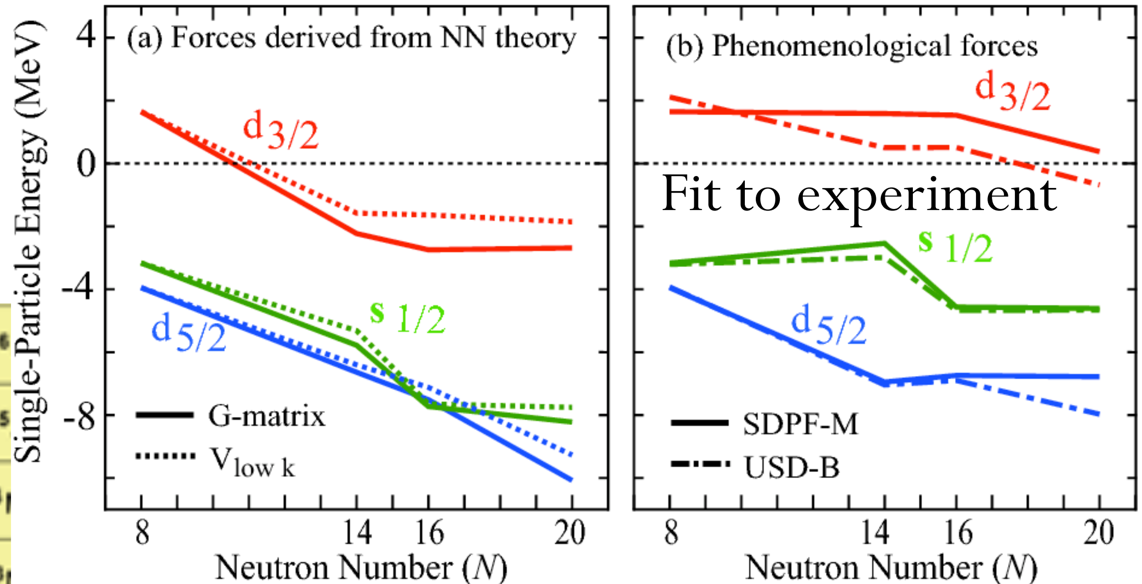
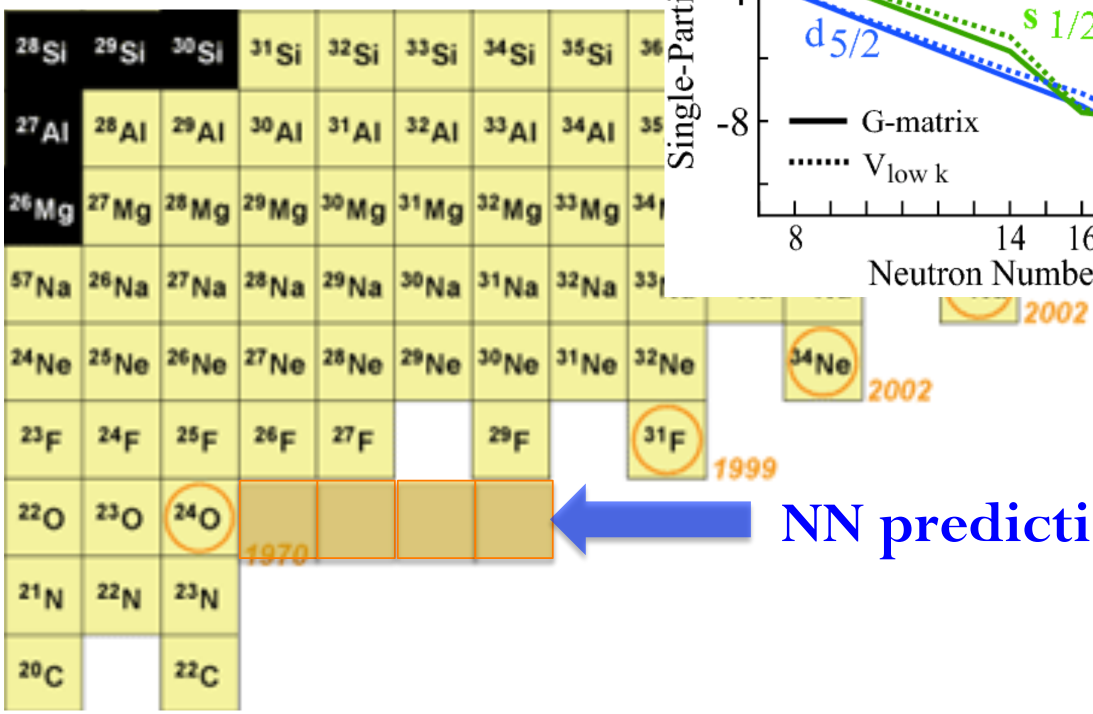
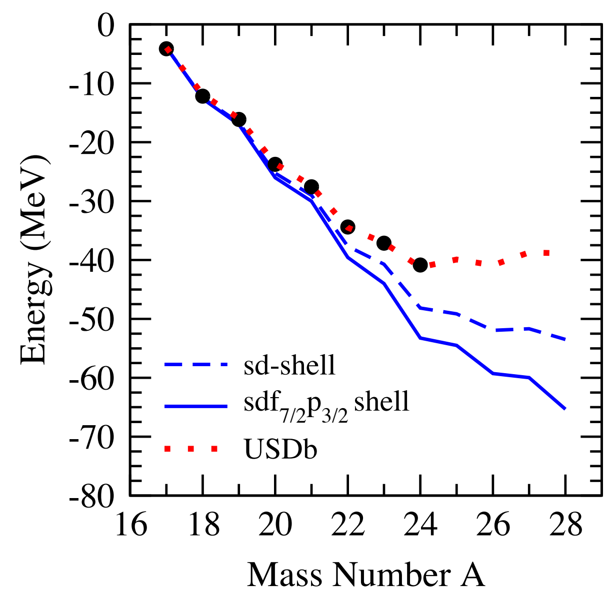




NN prediction

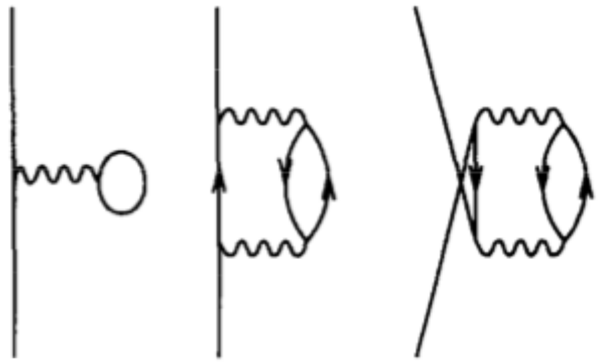


Incorrect prediction of oxygen dripline

Extended-space – more binding

Single Particle Energies

SPEs self-consistently from one-body diagrams



+ 3rd order

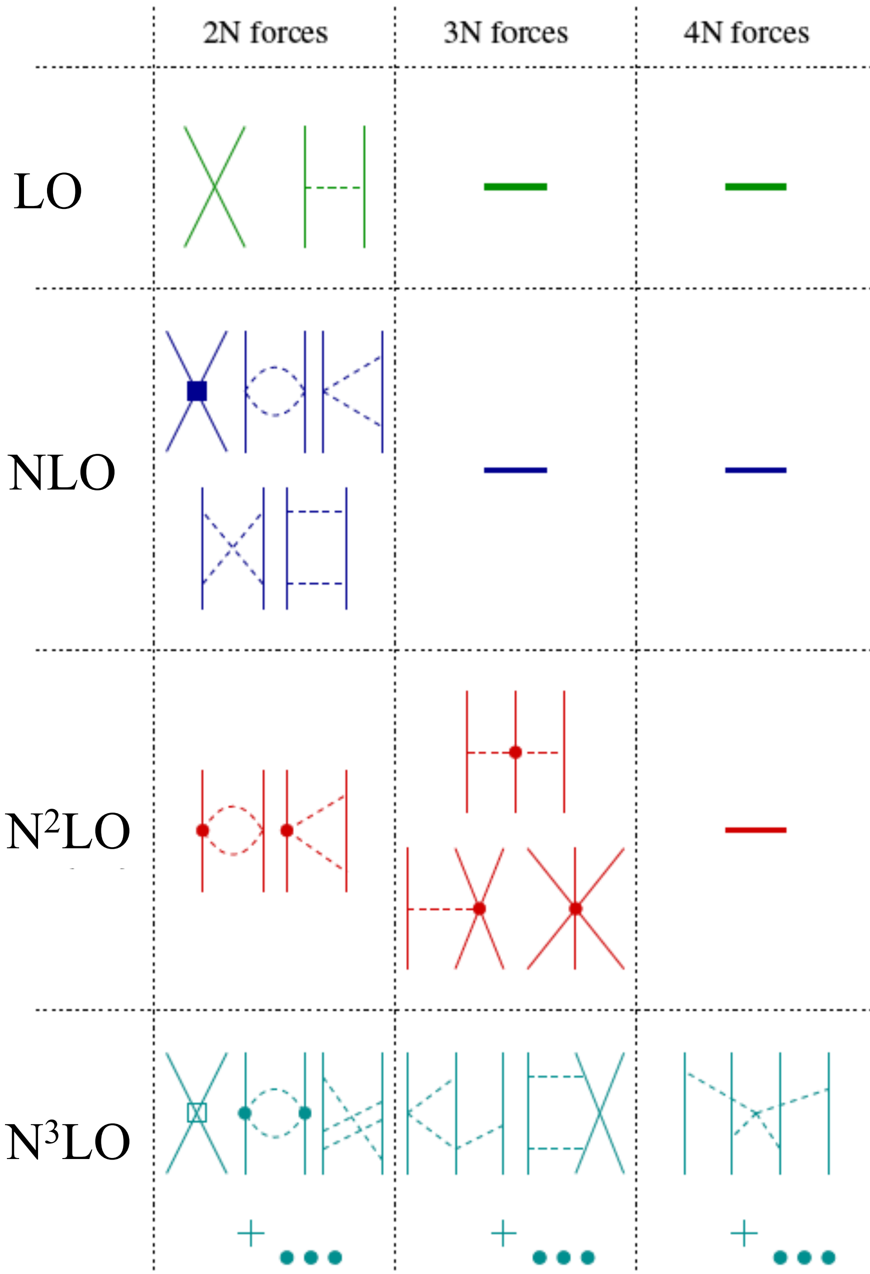
sd-shell: overbound, unreasonable spacing

Orbit	"Exp"	USDb	$T+V_{NN}$ (3rd)
$d_{5/2}$	-4.14	-3.93	**-5.43**
$s_{1/2}$	-3.27	-3.21	**-5.32**
$d_{3/2}$	0.944	2.11	**-0.97**

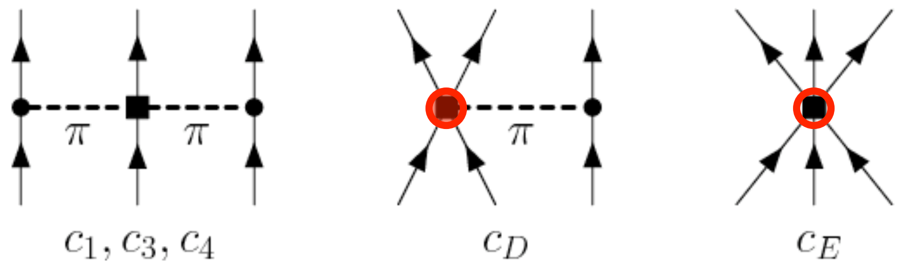
Typical approach: use empirical SPEs

3N forces eliminate need for adjusted parameters?

Chiral Effective Field Theory: 3N Forces



Two new couplings at N^2LO



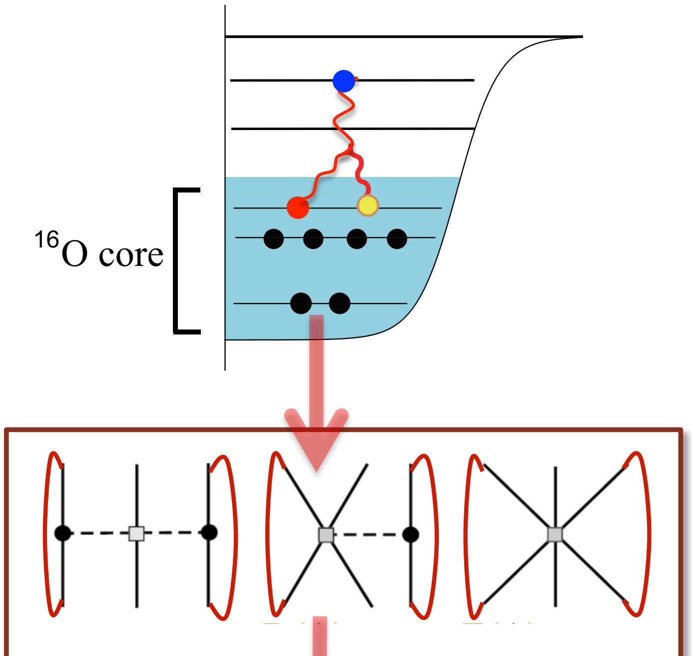
c terms given from NN fits: constrained by NN, πN data

c_D c_E fit to properties of light nuclei: Triton binding energy, ^{4}He radius

3N Forces for Valence-Shell Theories

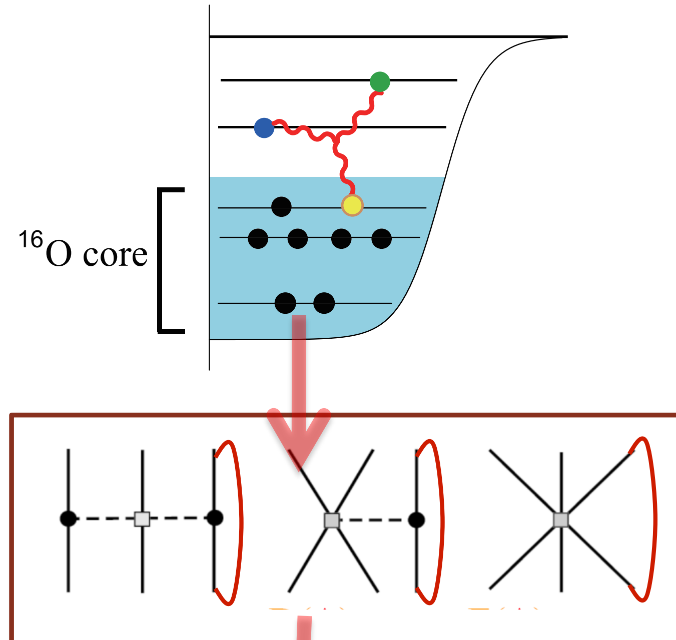
Normal-ordered 3N: contribution to valence nucleon interactions

Effective one-body



$$\langle a|\, V_{3N,\mathrm{eff}} \,|a'\rangle = \frac{1}{2} \sum_{\alpha\beta = core} \langle \alpha\beta a|\, V_{3N} \,|\alpha\beta a'\rangle$$

Effective two-body

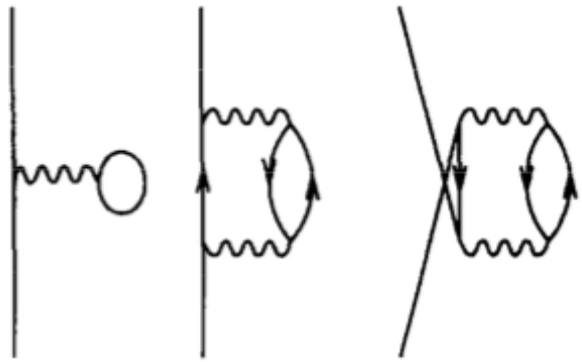


$$\langle ab|\, V_{3N,\mathrm{eff}} \,|a'b'\rangle = \sum_{\alpha = core} \langle \alpha ab|\, V_{3N} \,|\alpha a'b'\rangle$$

Combine with microscopic NN (**Third Order**): no empirical adjustments

Extended Valence Space SPEs

3N forces: additional repulsion – comparable to phenomenology



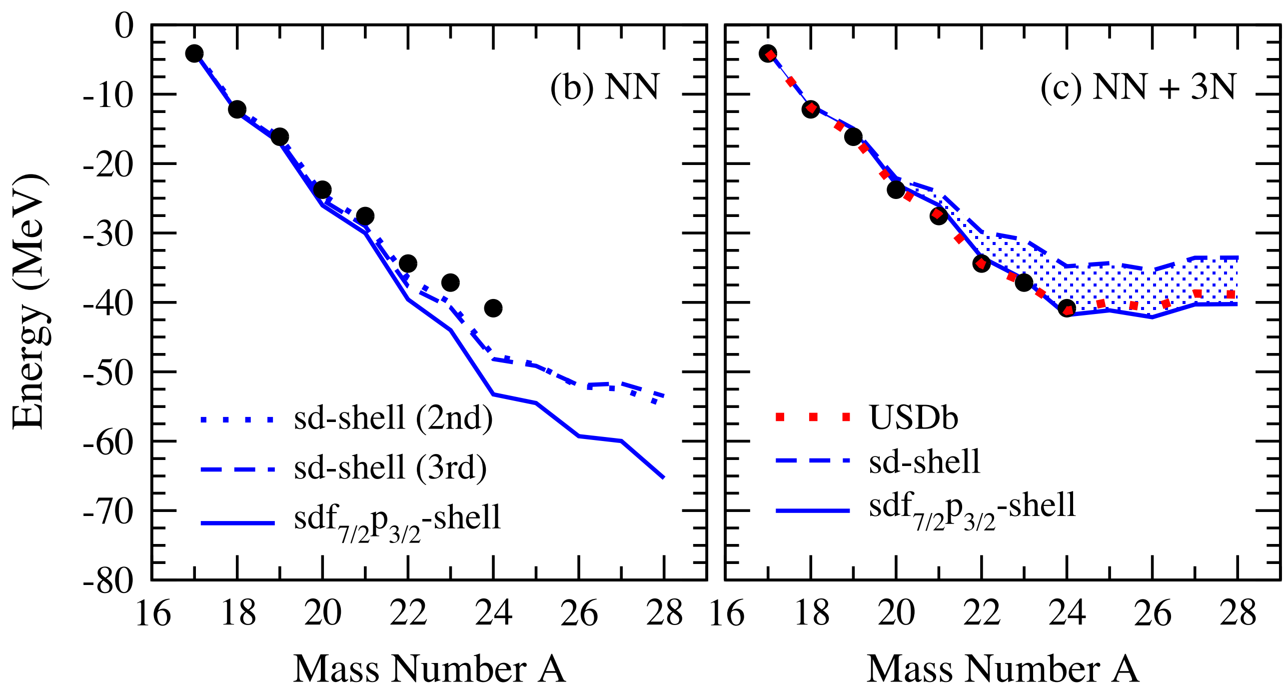
+ 3rd + 3N

Orbit	USDb	$T+V_{NN}+V_{3N}$	SDPF-M	$T+V_{NN}+V_{3N}$
$d_{5/2}$	-3.93	-3.78	-3.95	-3.46
$s_{1/2}$	-3.21	-2.42	-3.16	-2.20
$d_{3/2}$	2.11	1.45	1.65	1.92
$f_{7/2}$			3.10	3.71
$p_{3/2}$			3.10	7.72

Similar behavior in standard/extended spaces

Ground-State Energies of Oxygen Isotopes

Valence-space interaction and SPEs from NN+3N



JDH, Menendez, Schwenk, arXiv:1108.2680

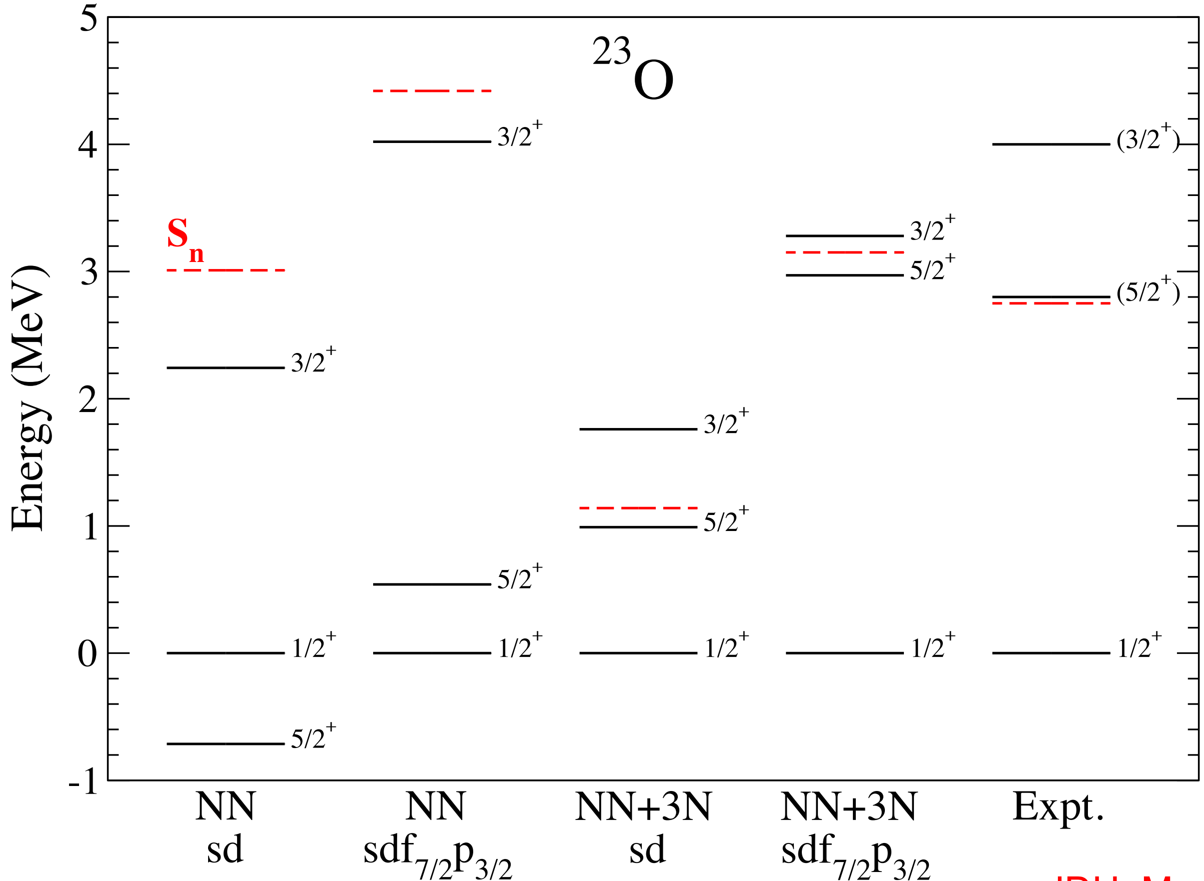
Repulsive character improves agreement with experiment

sd-shell results underbound; improved in **extended space** $sdf_{7/2}\ p_{3/2}$

Impact on Spectra: ^{23}O

Neutron-rich oxygen spectra with NN+3N

$5/2^+$, $3/2^+$ indicate position of $d_{5/2}$ and $d_{3/2}$ orbits



sd*-shell *NN-only

Wrong ground state

$5/2^+$ too low

$3/2^+$ bound

NN+3N

Improvements in extended valence space

JDH, Menendez, Schwenk, arXiv:1108.2680

Shell Formation/Evolution in Calcium Isotopes

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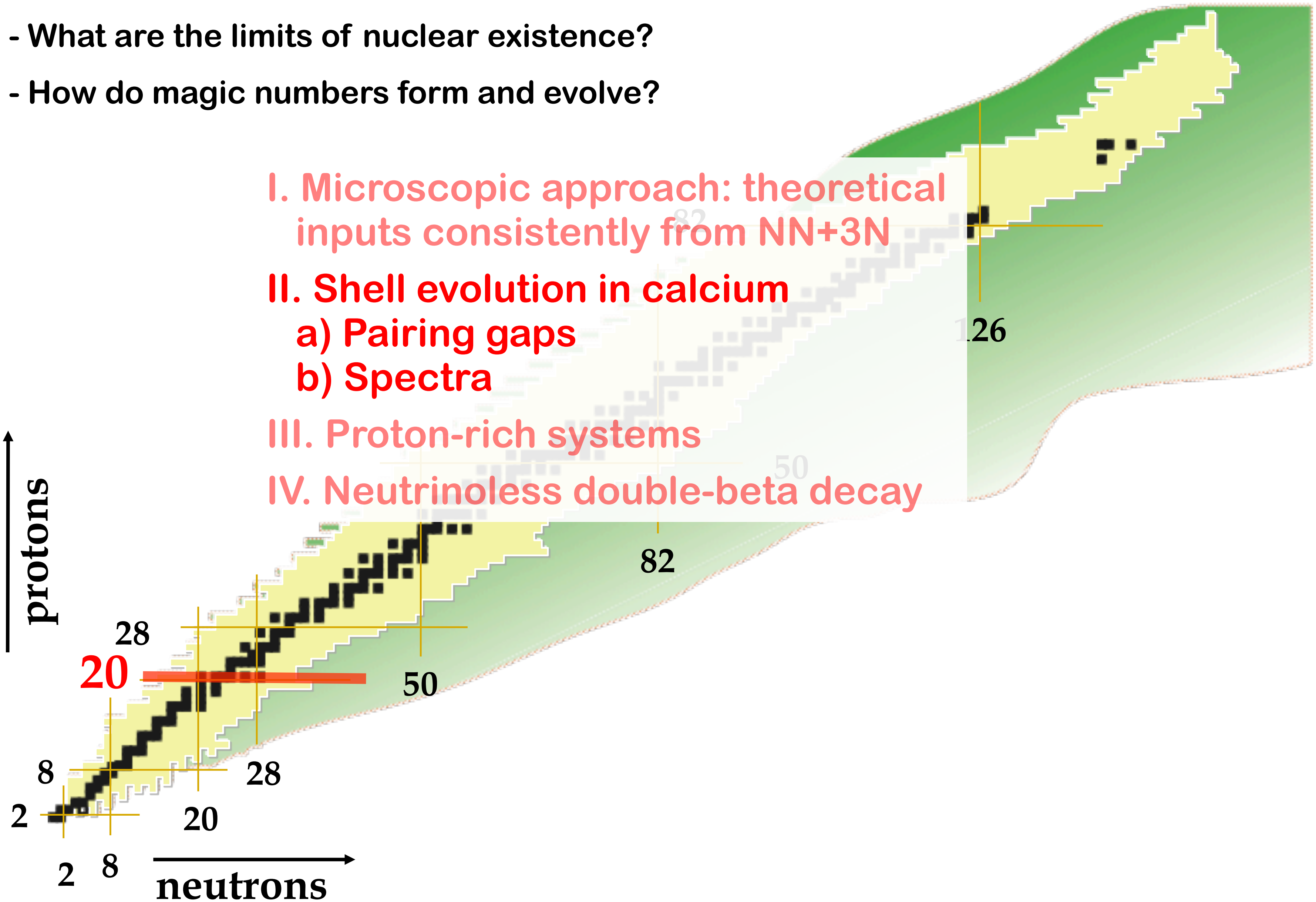
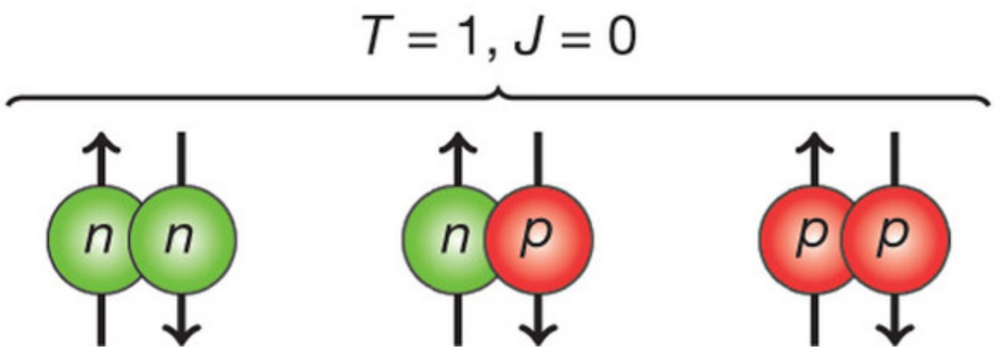


Nuclear Pairing



Pairing of even number of nucleons – even/odd staggering

Pairing gaps deduced from **3-point mass difference**:

$$\Delta_n^{(3)} = \frac{(-1)^N}{2}\left[BE(N+1,Z) + BE(N-1,Z) - 2BE(N,Z)\right]$$

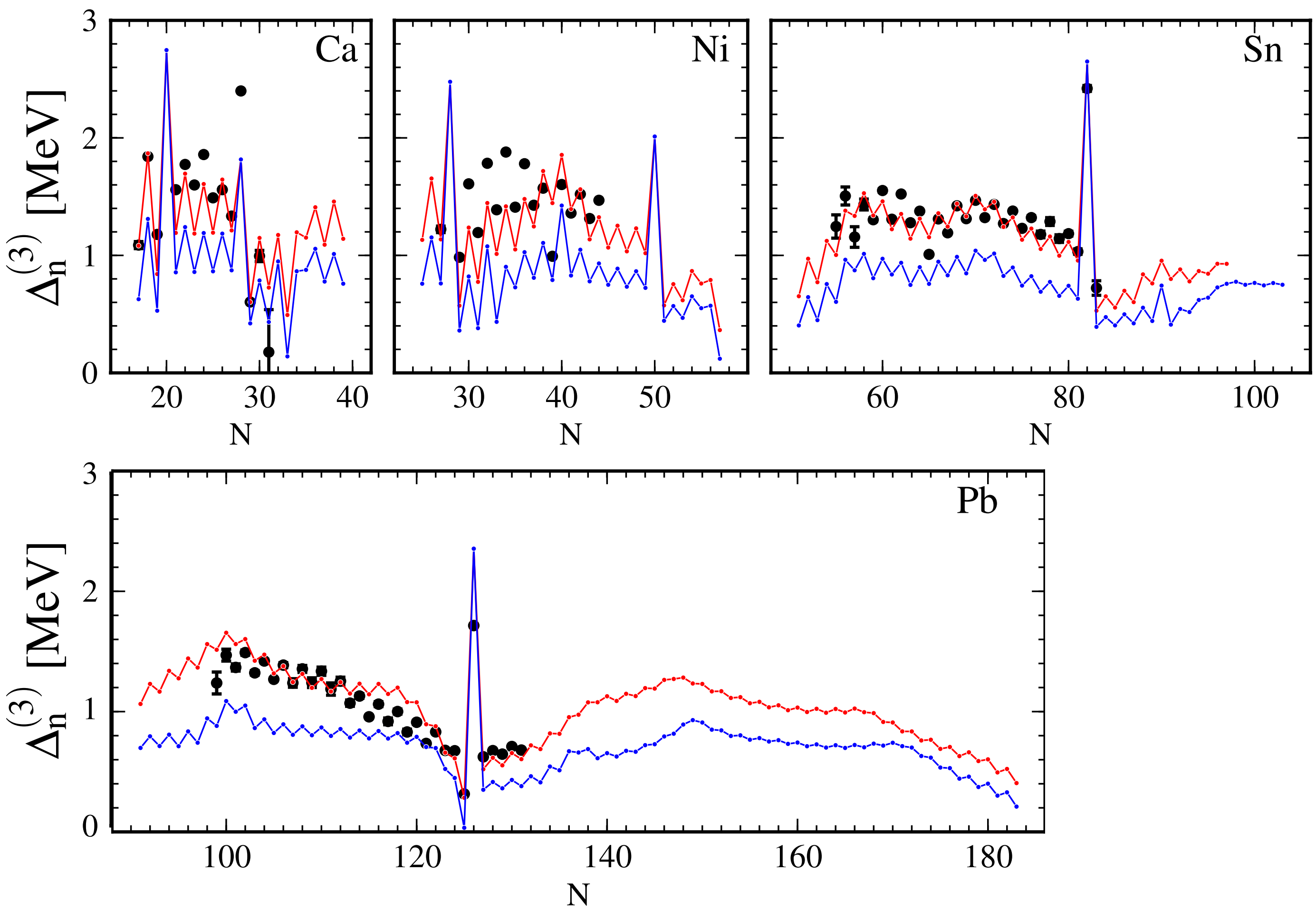
Allows comparison with experiment

Relative peak in pairing strength indicates **shell closure**

Pairing in EDF with 3N Forces

In Energy Density Functional theory: 3N forces lower gaps systematically ~30%

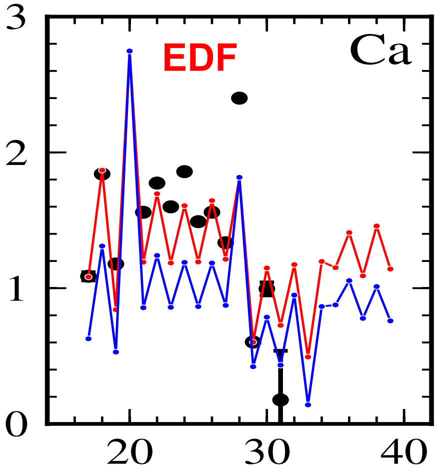
Lesinski, Hebeler, Duguet, Schwenk, JPG (2012)



What are the contributions from neglected many-body effects? (Core polarization)

Pairing in Calcium Isotopes: Ladders

Compare with $\Delta_n^{(3)}$ calculated from microscopic NN+3N in calcium



HFB iterates ladders microscopically in pairing channel

Compare with ***pp, hh* ladders to 3rd order**

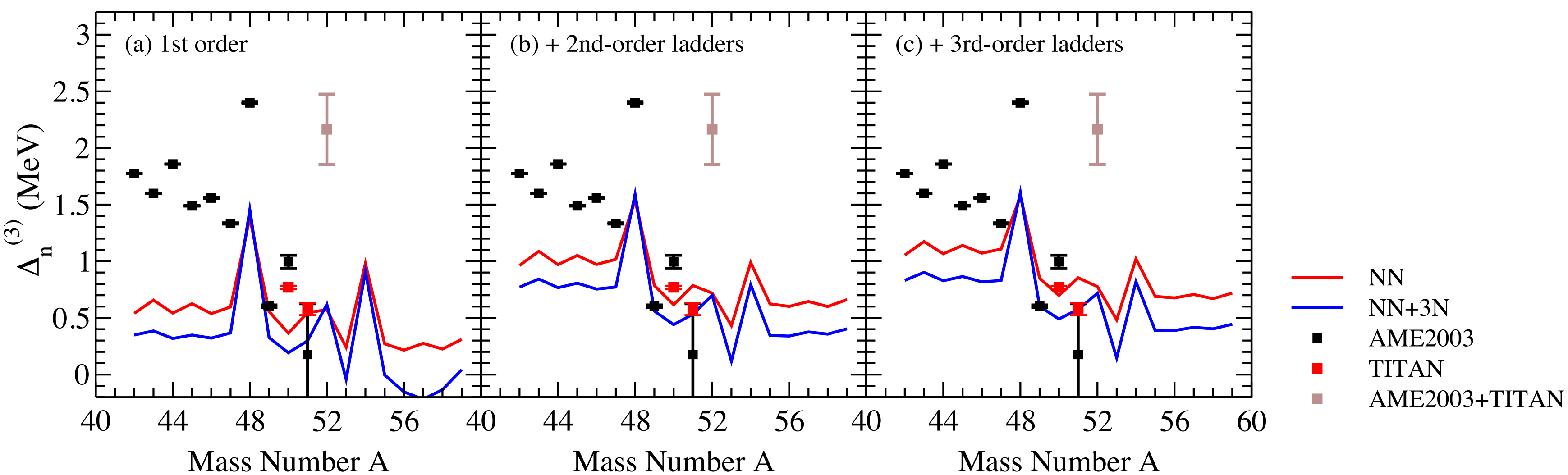
Improved agreement with experiment

Convergence in order-by-order ladders

Suppression from 3N forces as in EDF

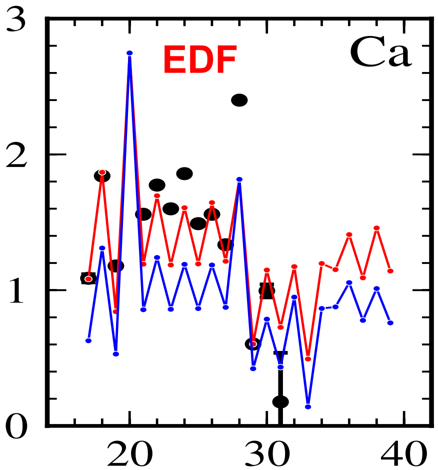
Incorrect odd/even staggering

JDH, Menendez, Schwenk, in prep.



Pairing in Calcium Isotopes: Full 3rd order

Compare with $\Delta_n^{(3)}$ calculated from microscopic NN+3N in calcium



Full 3rd-order MBPT

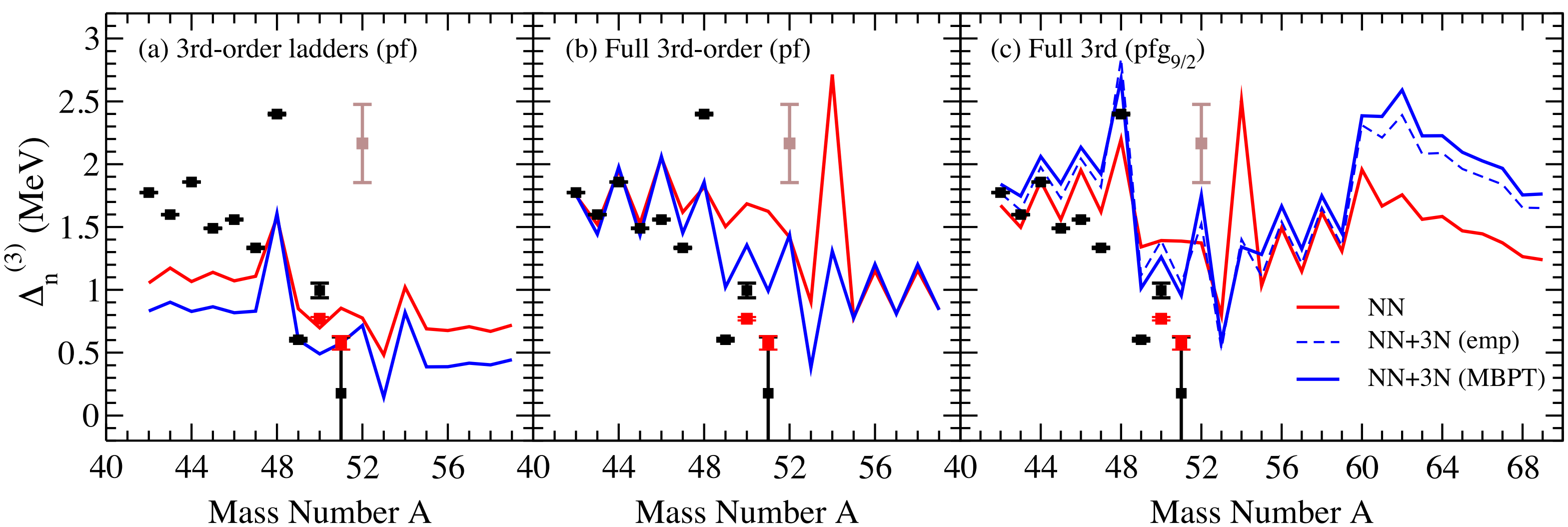
Further increases gaps

Correct odd/even staggering; more pronounced

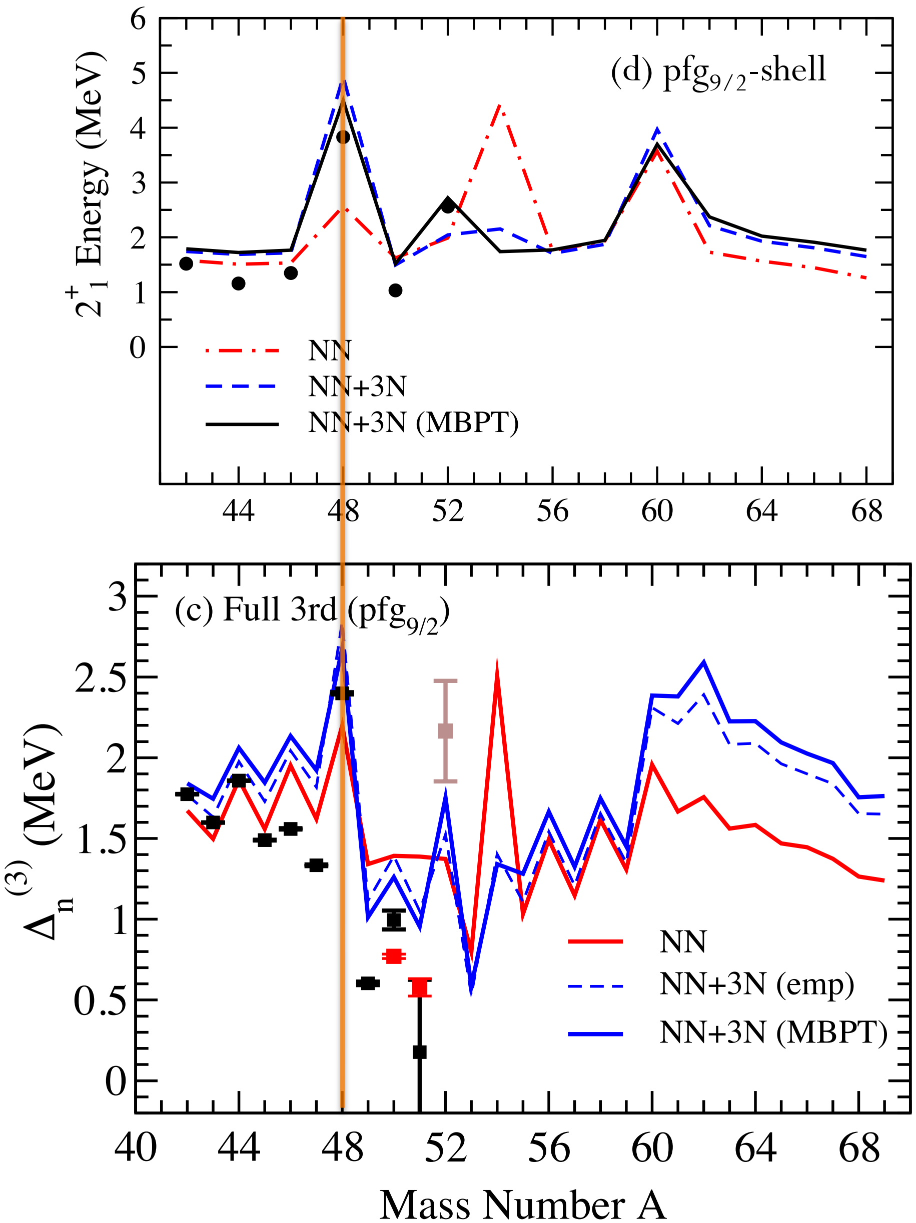
Good experimental reproduction with 3rd-order NN+3N

Can account for missing physics in EDF calculations

JDH, Menendez, Schwenk, in prep.



Pairing for Shell Evolution N=28

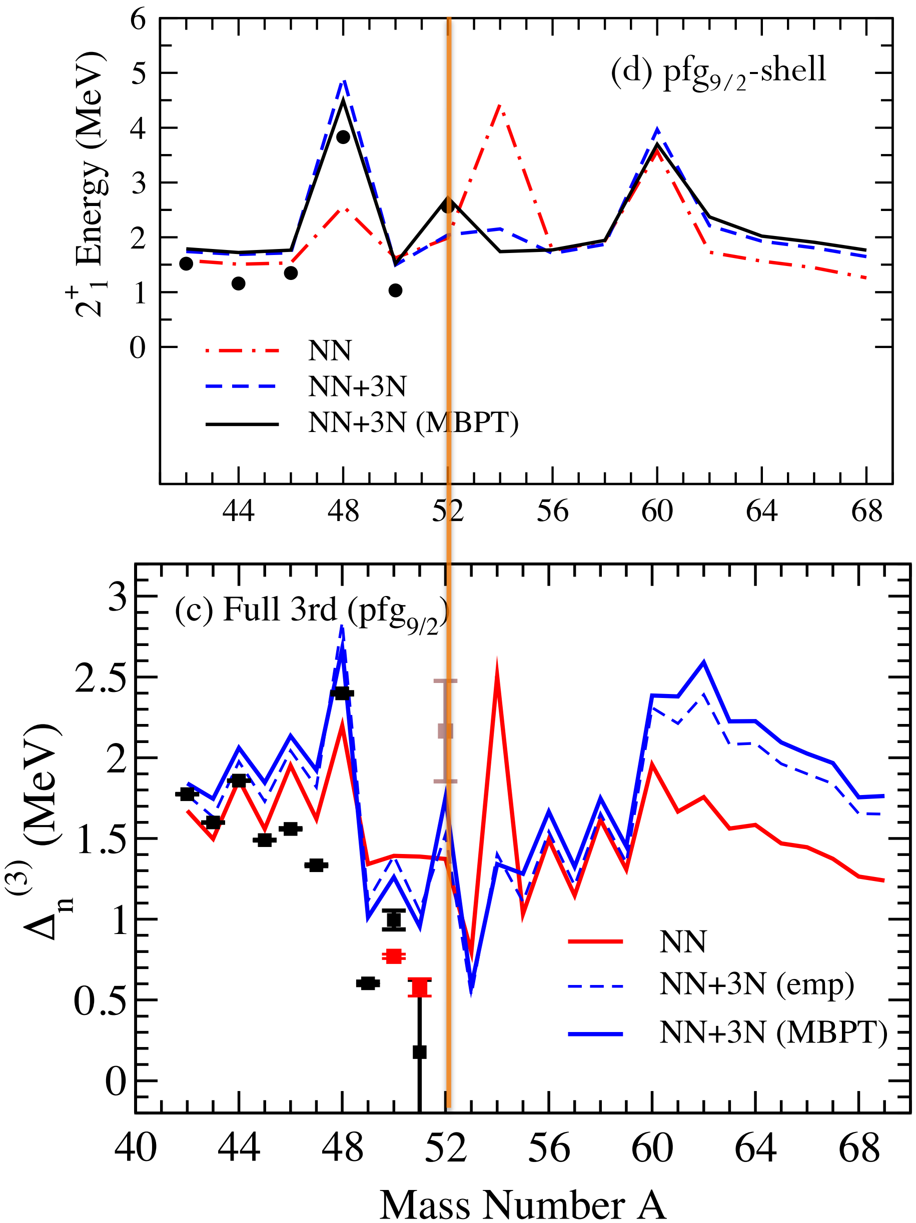


Peak in pairing gaps: complementary signature for shell closure

Compare with 2^+ energies for Ca

N=28: strong peak, overprediction in both cases

Pairing for Shell Evolution N=32



Peak in pairing gaps: complementary signature for shell closure

Compare with 2^+ energies for Ca

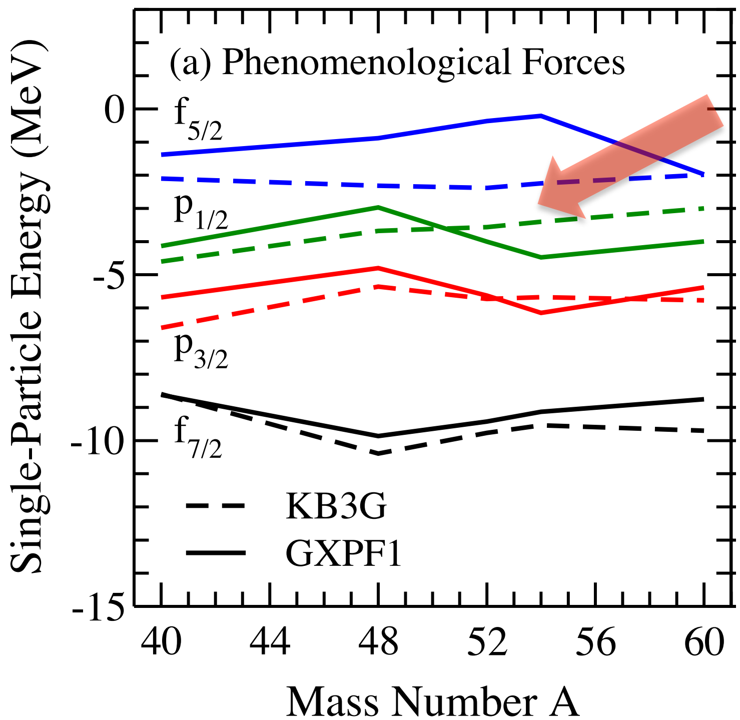
N=32: moderate peak

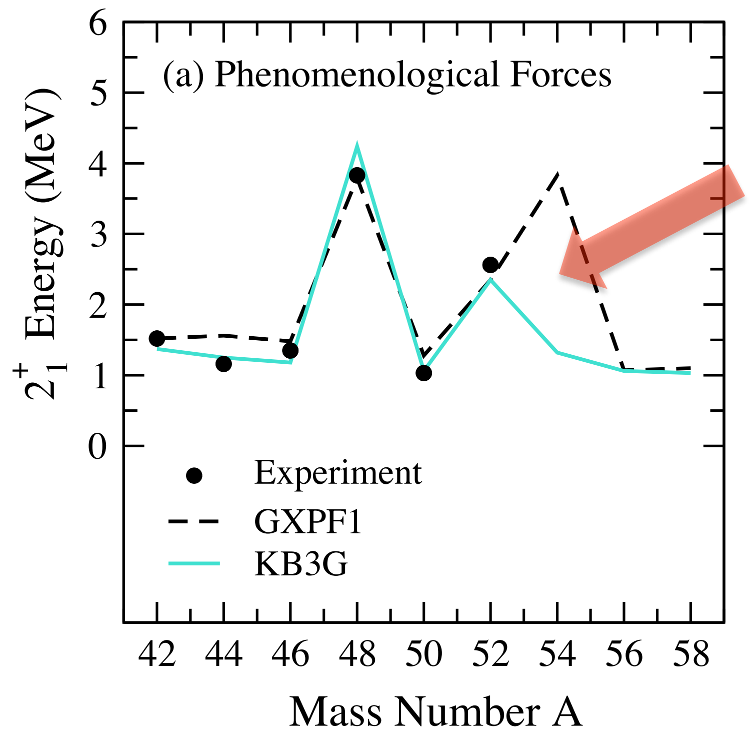
Close to experimental value with new TITAN data

Experimental measurement of ^{53}Ca mass needed to reduce uncertainty

Evolution of Magic Numbers: N=34

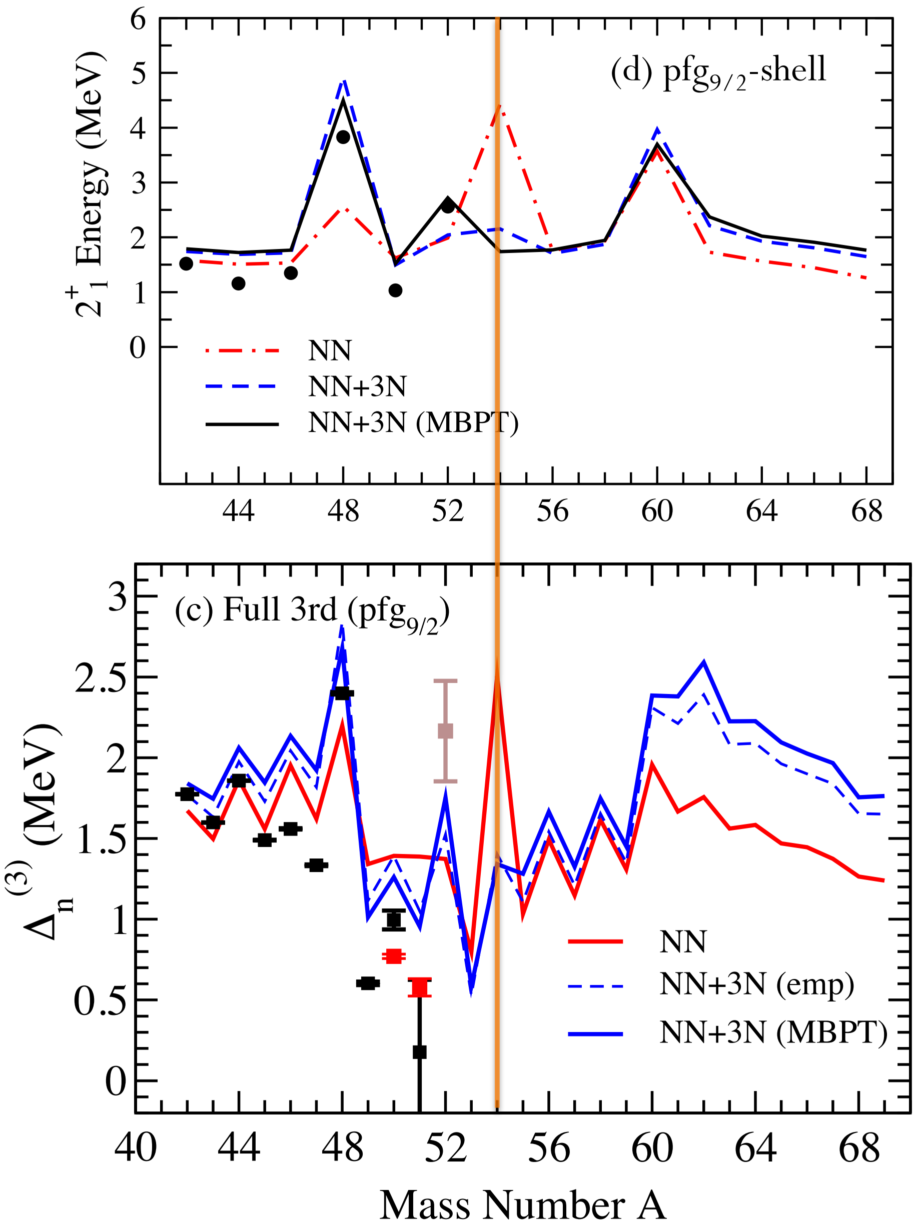
N=34 magic number in calcium?





Strong phenomenological disagreement for neutron-rich calcium

Pairing for Shell Evolution N=34



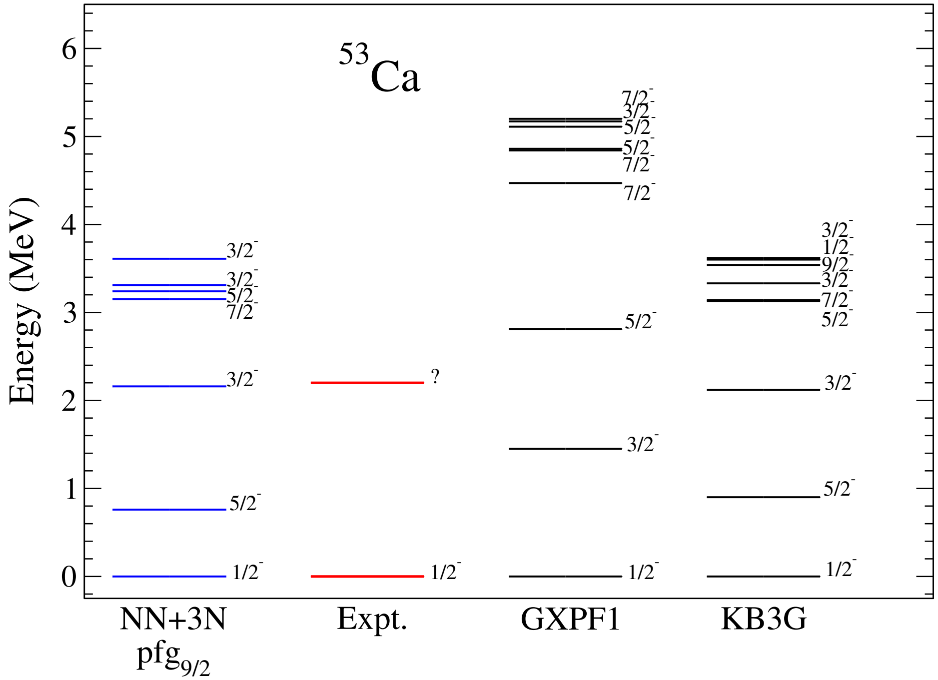
Peak in pairing gaps: complementary signature for shell closure

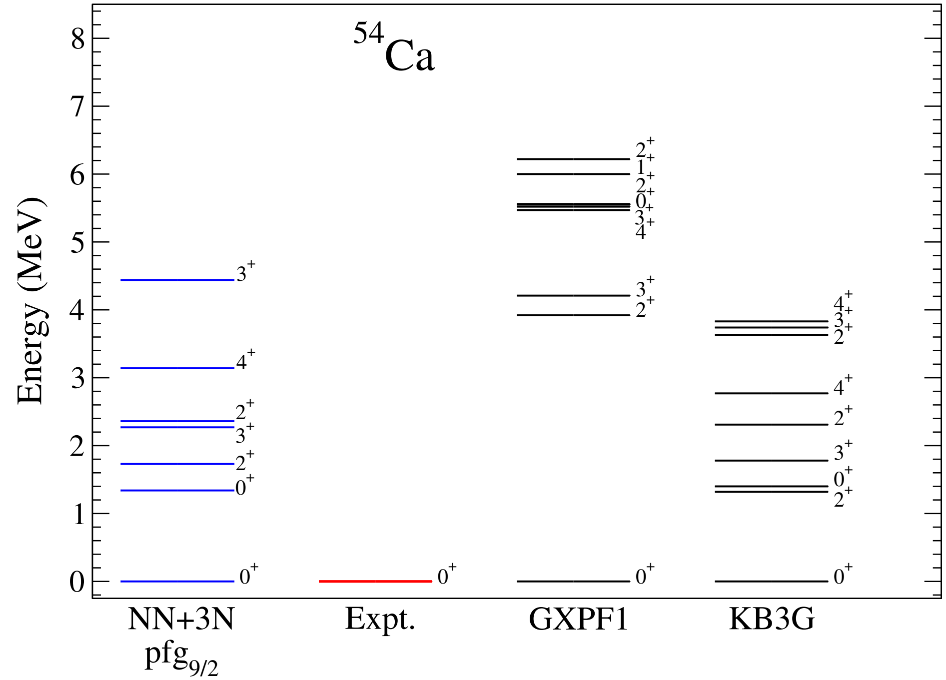
Compare with 2^+ energies for Ca

N=34: suppressed with 3N forces

Neutron-Rich Ca Spectra Near N=34

Neutron-rich calcium spectra with NN+3N





JDH, Menendez, Schwenk, in prep.

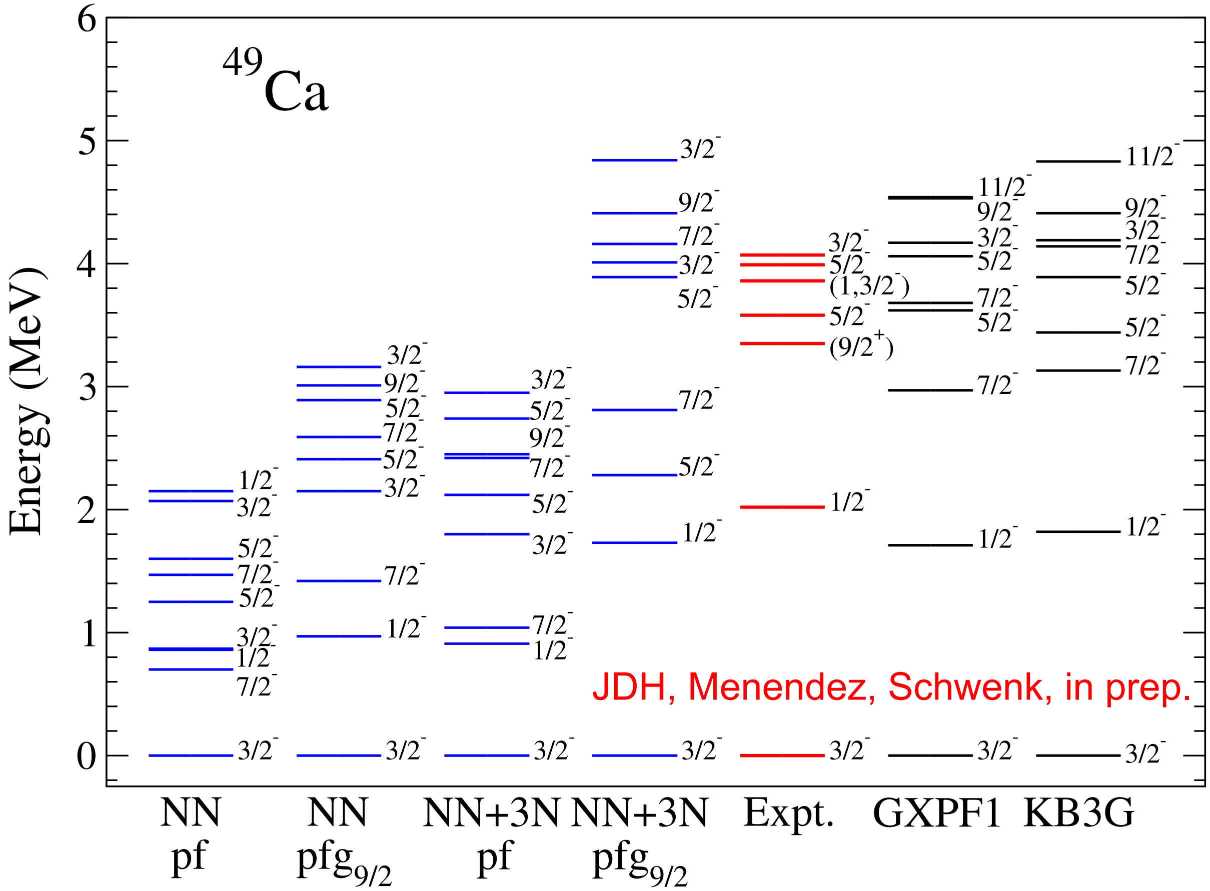
Dramatic difference in phenomenology

NN+3N similar to KB3G – no indication of *N=34* magic number

Consistent with predictions from Coupled-Cluster theory

Impact on Spectra: ^{49}Ca

Neutron-rich calcium spectra with NN+3N



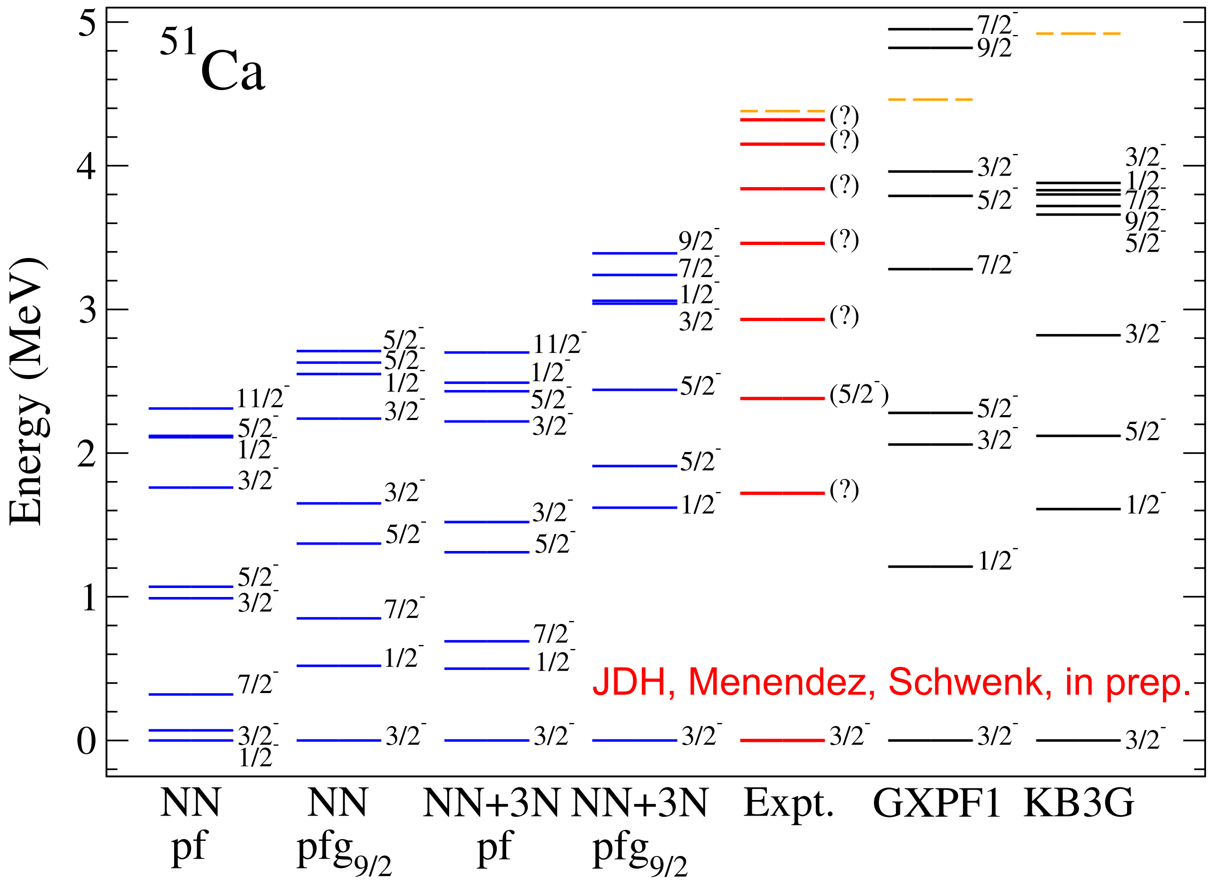
Spectrum typically too compressed

NN+3N in $pfg_{9/2}$ correct $1/2^-$

Comparable to phenomenology (as for all lighter Ca isotopes)

Impact on Spectra: ^{51}Ca

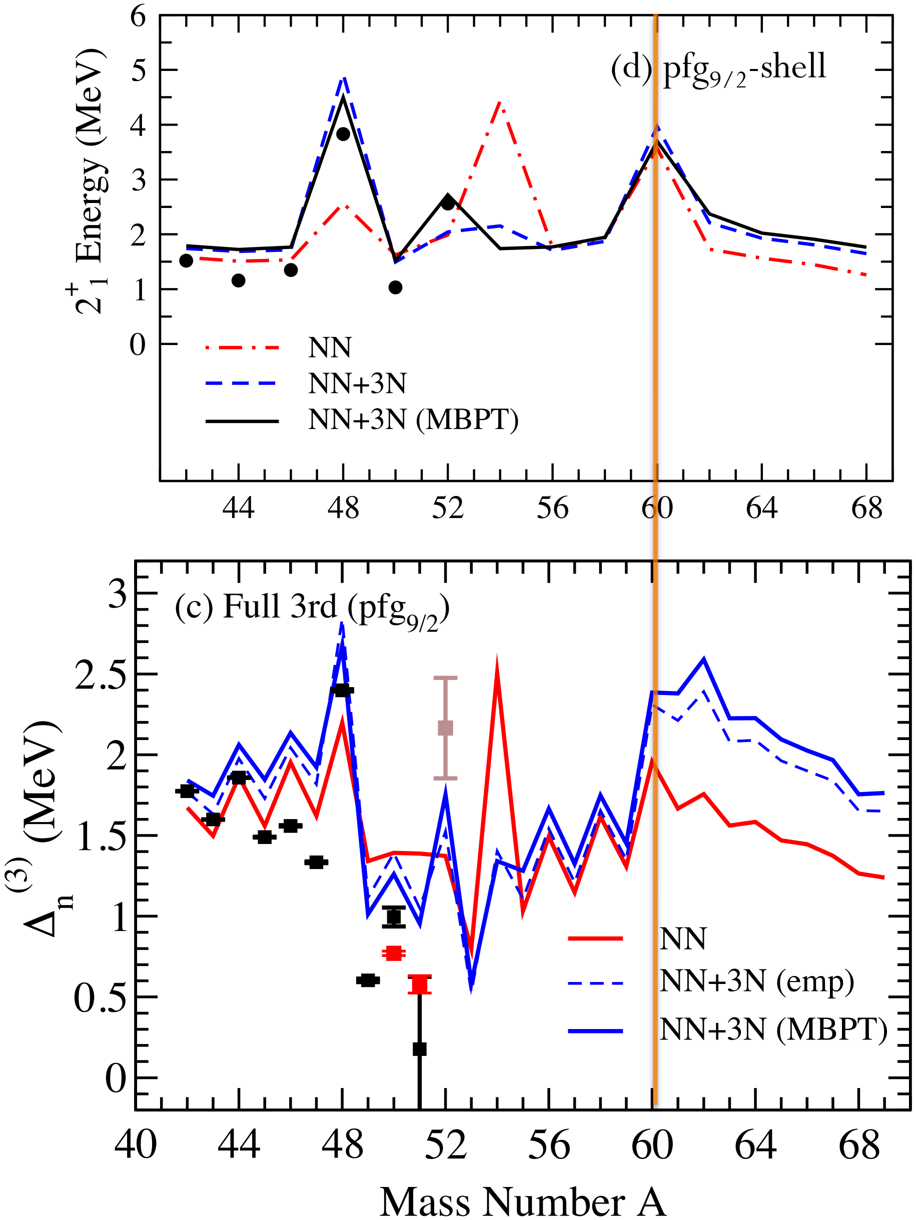
Neutron-rich calcium spectra with NN+3N



Possibility to assign spin/parity where unknown

Gamma-ray spectroscopy needed

Pairing for Shell Evolution N=40



Peak in pairing gaps: complementary signature for shell closure

Compare with 2^+ energies for Ca

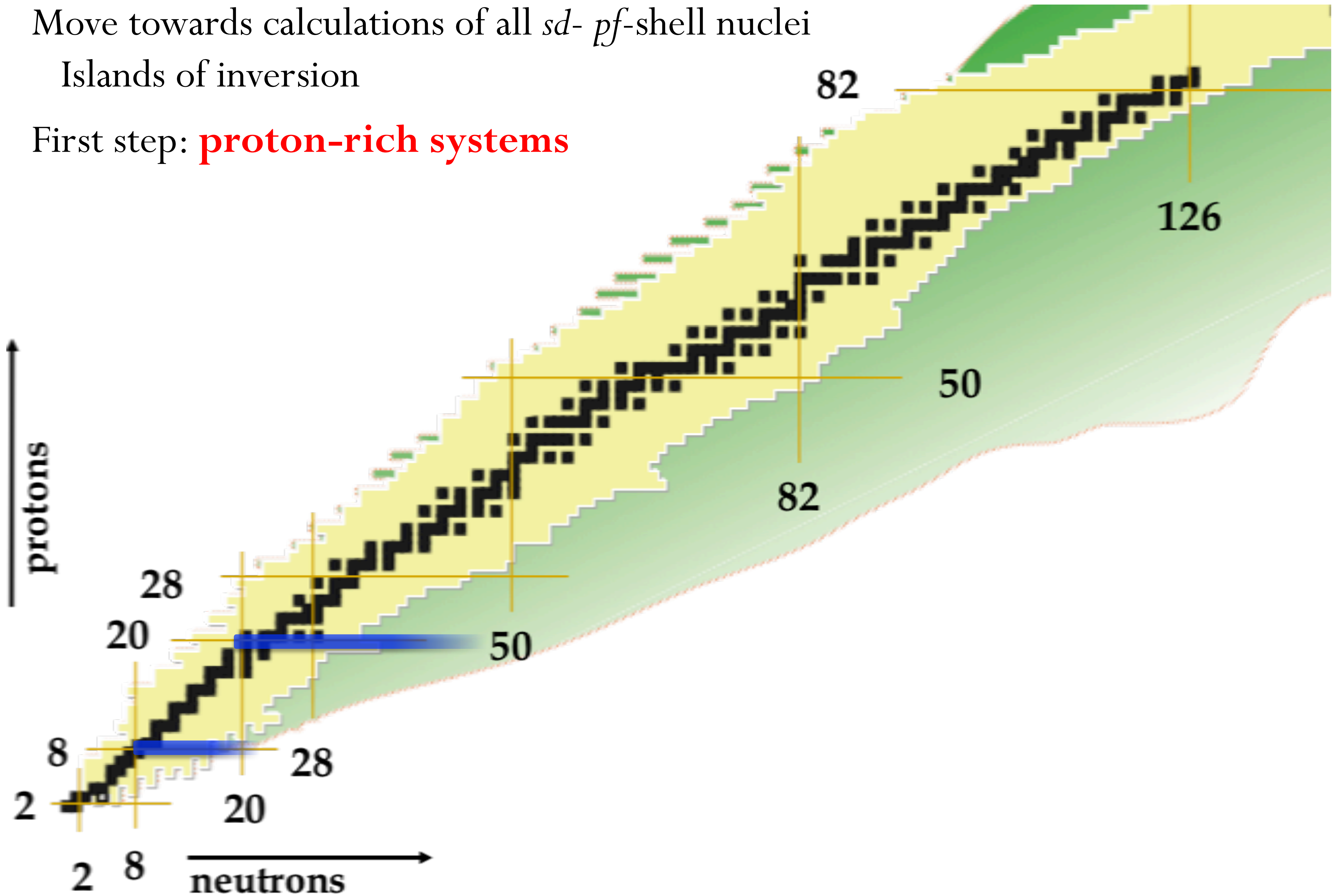
N=40: robust signature of shell closure

Proton-Rich Systems

Move towards calculations of all *sd*- *pf*-shell nuclei

Islands of inversion

First step: **proton-rich systems**



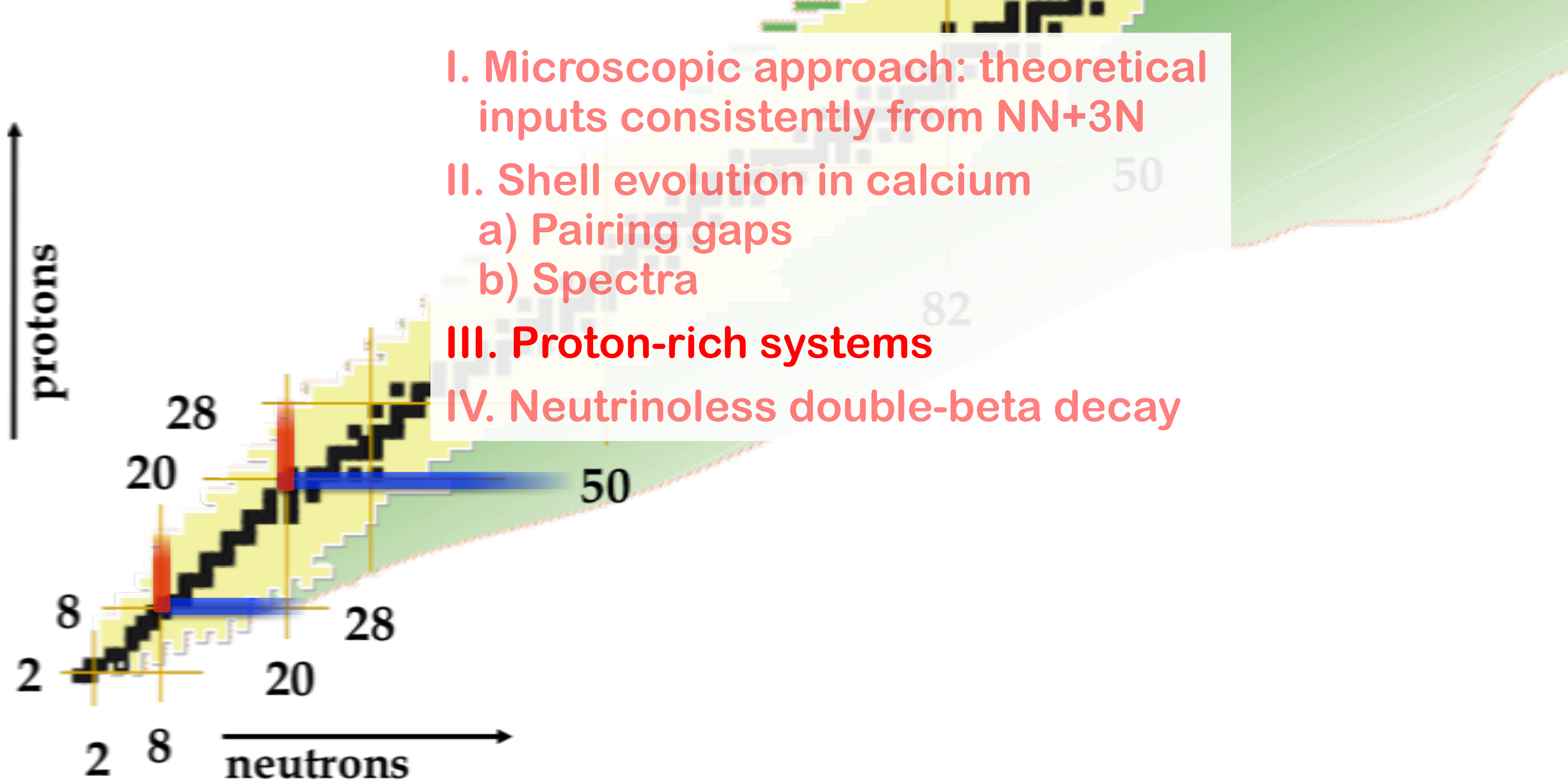
Proton-Rich Systems

Move towards calculations of all *sd*- *pf*-shell nuclei

Islands of inversion

First step: proton-rich systems

Study N=8, 20 isotone chains



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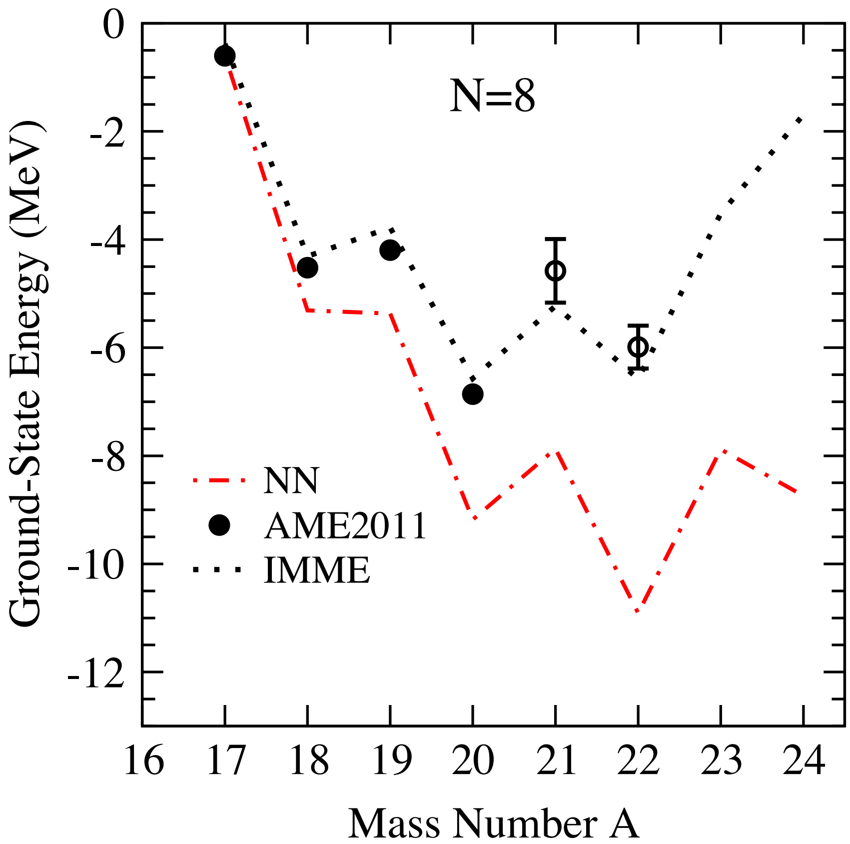
Ground-State Energies of N=8 Isotones



Data limited – use phenomenological isobaric multiplet mass equation (IMME)

$$E(A,T,T_z) = E(A,T,-T_z) + 2b(A,T)T_z$$

$$b = 0.7068A^{2/3} - 0.9133$$

NN-only: overbound

JDH, Menendez, Schwenk, arXiv:1207.1590

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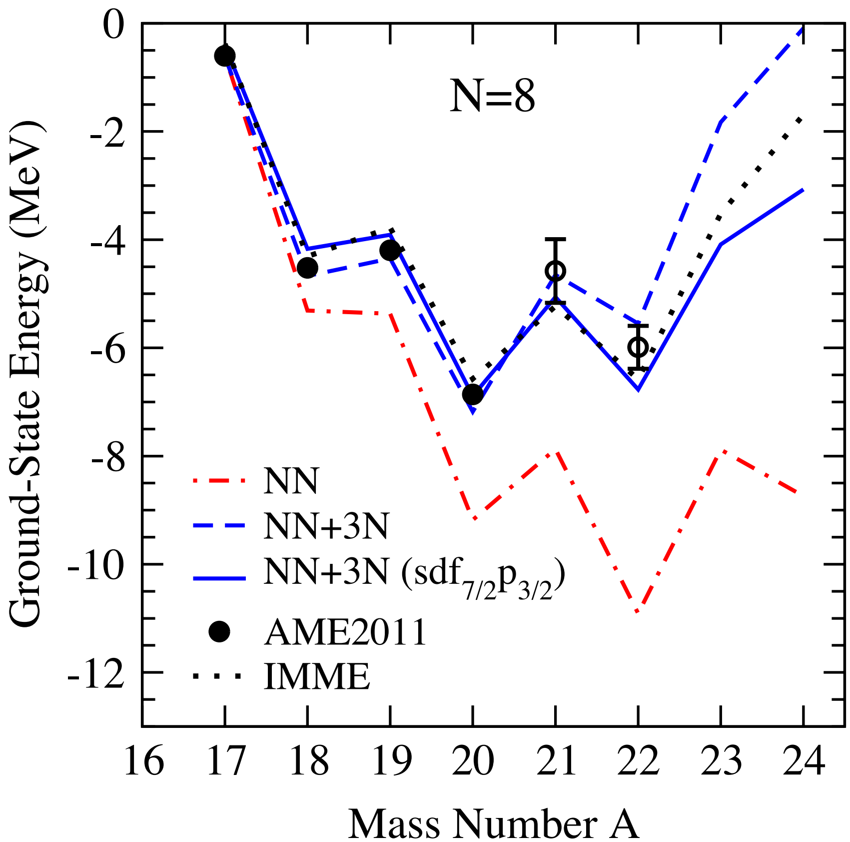

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NN+3N: improved agreement with experiment/IMME

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Dripline uncertain: Unbound in AME2011, NN+3N; bound in IMME

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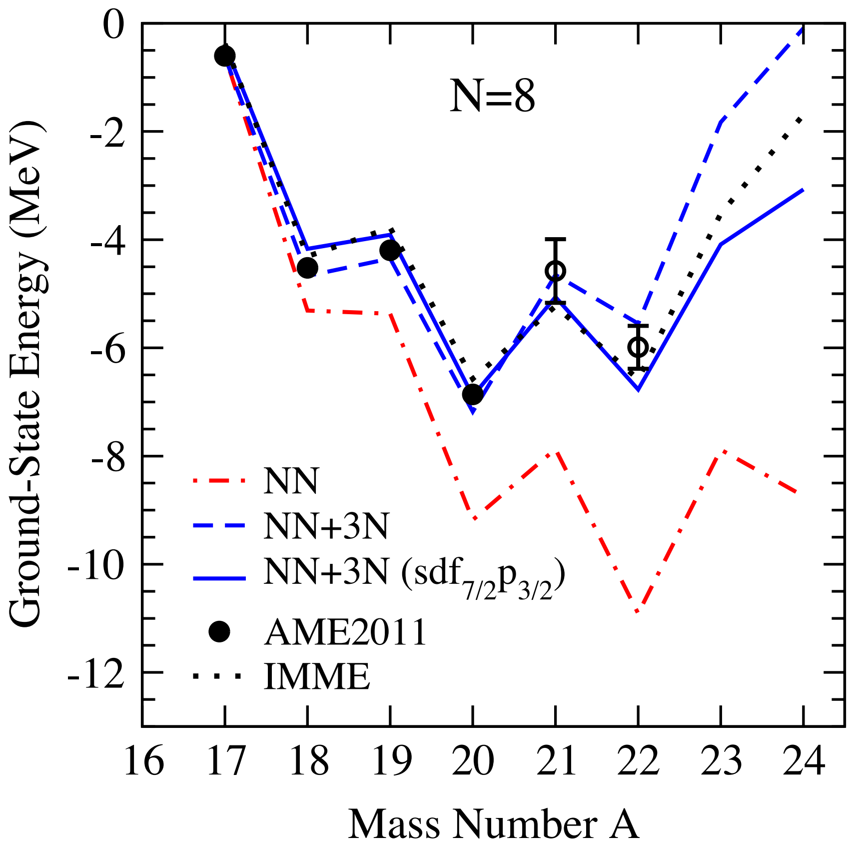

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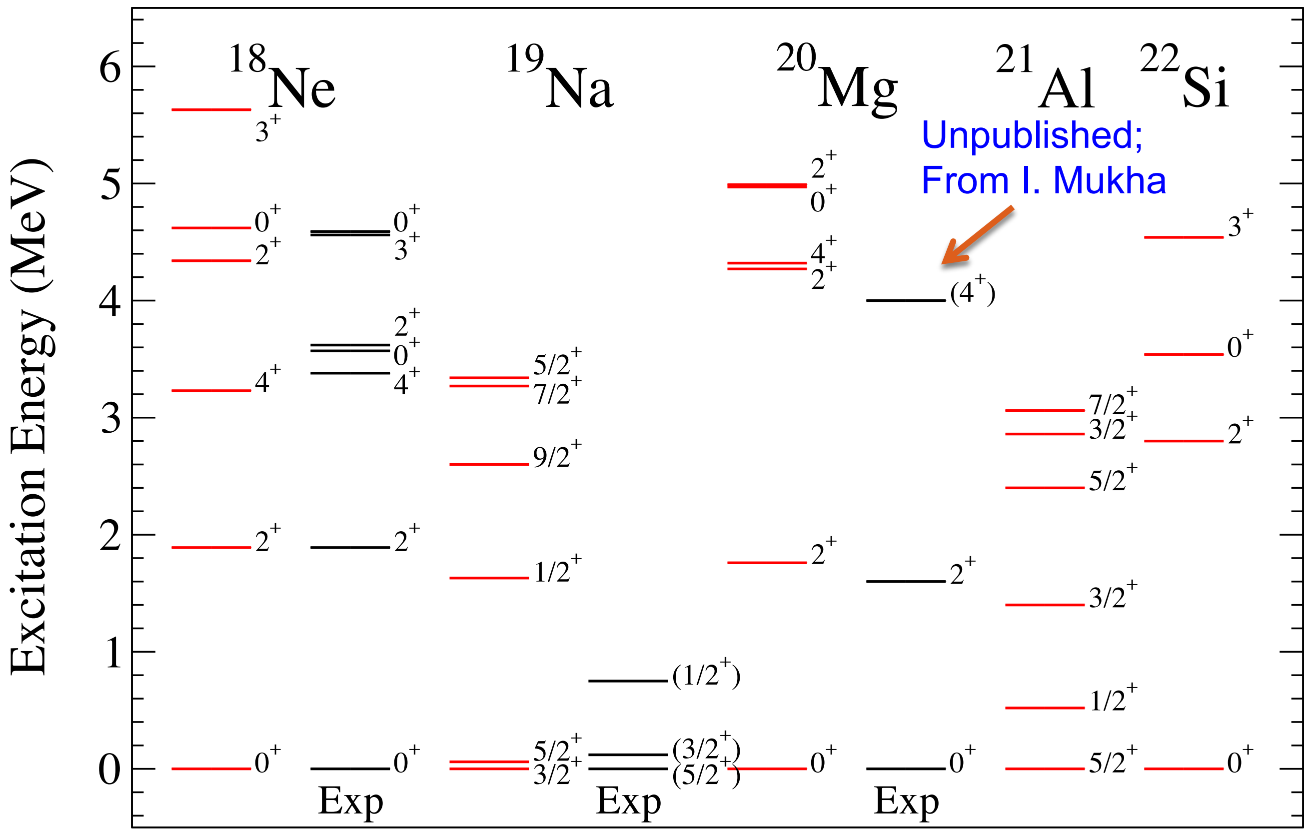
Dripline uncertain: Unbound in AME2011, NN+3N; bound in IMME

^{22}Si possible two-proton emitter

S_{2p}	IMME	NN+3N (*sd*)	NN+3N ($sdf_{7/2}p_{3/2}$)
	0.01 MeV	-1.63 MeV	-0.12 MeV

Spectra of N=8 Isotones



JDH, Menendez, Schwenk, arXiv:1207.1590

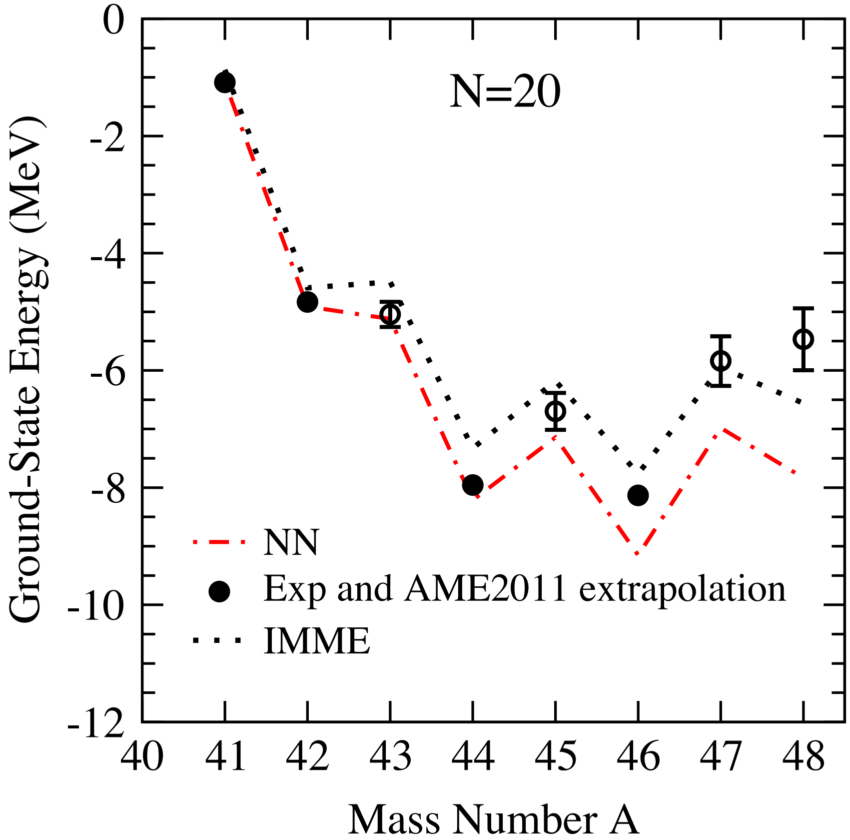
NN+3N: reasonable agreement with experiment

New measurement: excited state in ^{20}Mg close to predicted 4^+-2^+ doublet

Predictions for proton-rich ^{21}Al, ^{22}Si spectra

Closed sub-shell signature in ^{22}Si

Ground-State Energies of N=20 Isotones



NN-only: overbound beyond ^{45}Mn

JDH, Menendez, Schwenk, arXiv:1207.1590

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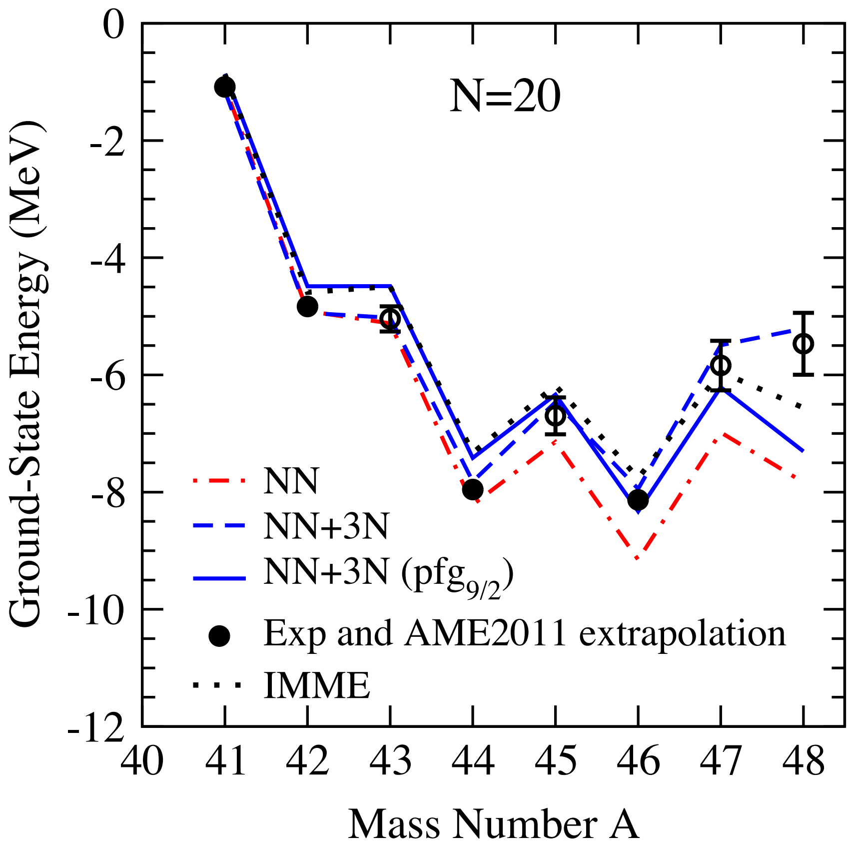


NN-only: overbound beyond ^{45}Mn

NN+3N: close to experiment/IMME

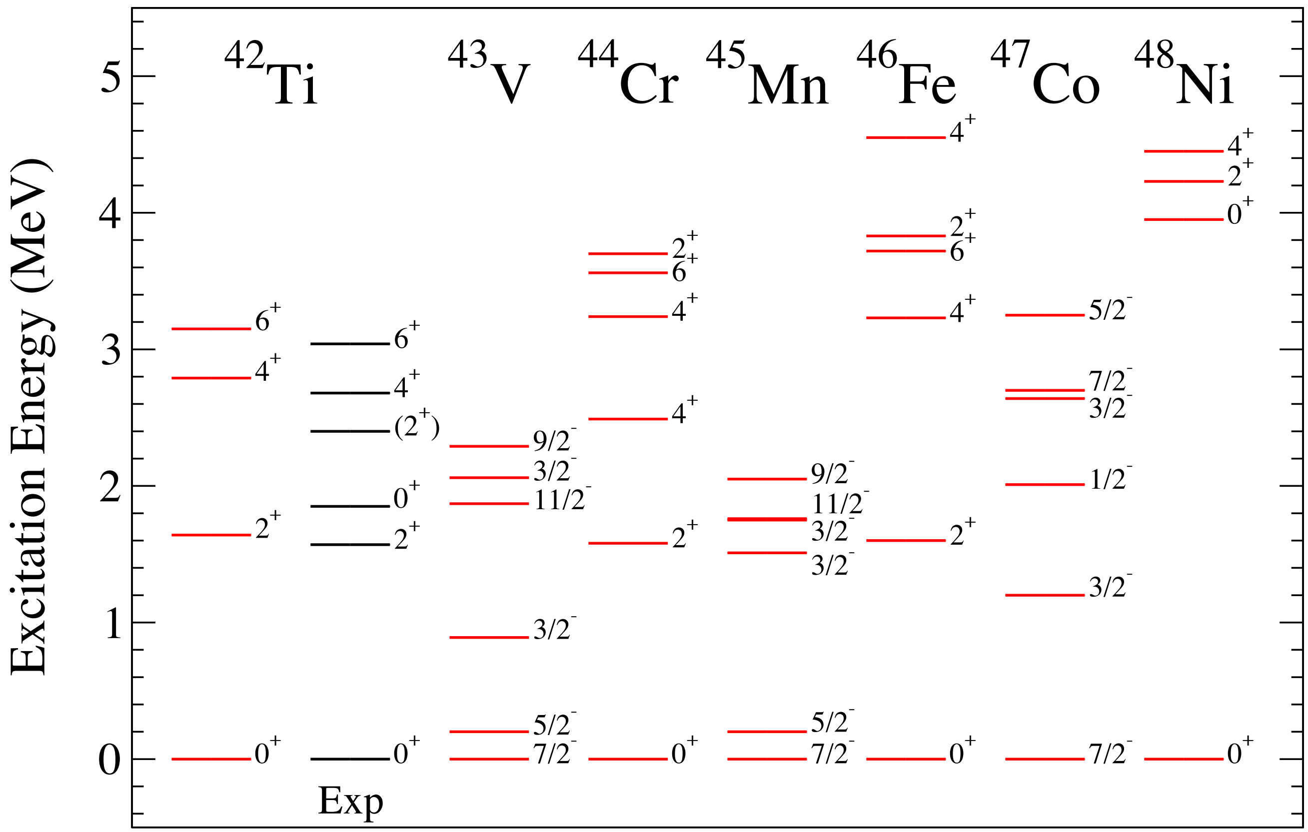
JDH, Menendez, Schwenk, arXiv:1207.1590

Dripline: Predicted to be ^{46}Fe in all calculations

	Expt.	NN+3N (pf)	NN+3N ($pfg_{9/2}$)
S_{2p}	-1.28(6) MeV	-2.73 MeV	-1.02 MeV

Prediction for ^{48}Ni within 300keV of experiment

Spectra of N=20 Isotones



JDH, Menendez, Schwenk, arXiv:1207.1590

NN+3N: reasonable agreement with measured ^{42}Ti

Predictions for proton-rich spectra

Mirror energy differences with Ca isotopes ~400keV

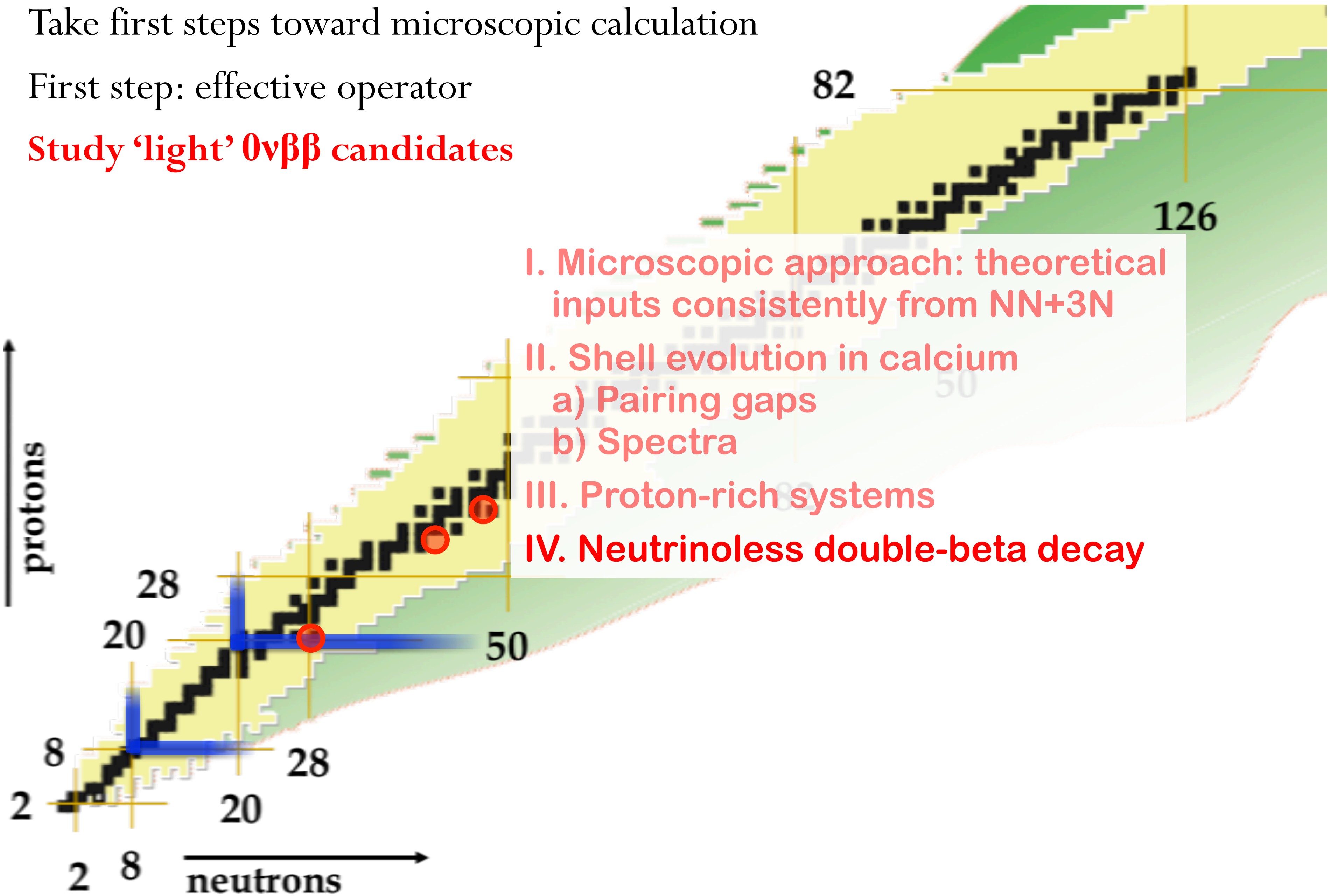
Closed-shell signature in ^{48}Ni

Neutrinoless Double-Beta Decay

Take first steps toward microscopic calculation

First step: effective operator

Study 'light' 0νββ candidates



I. Microscopic approach: theoretical inputs consistently from NN+3N

II. Shell evolution in calcium

a) Pairing gaps

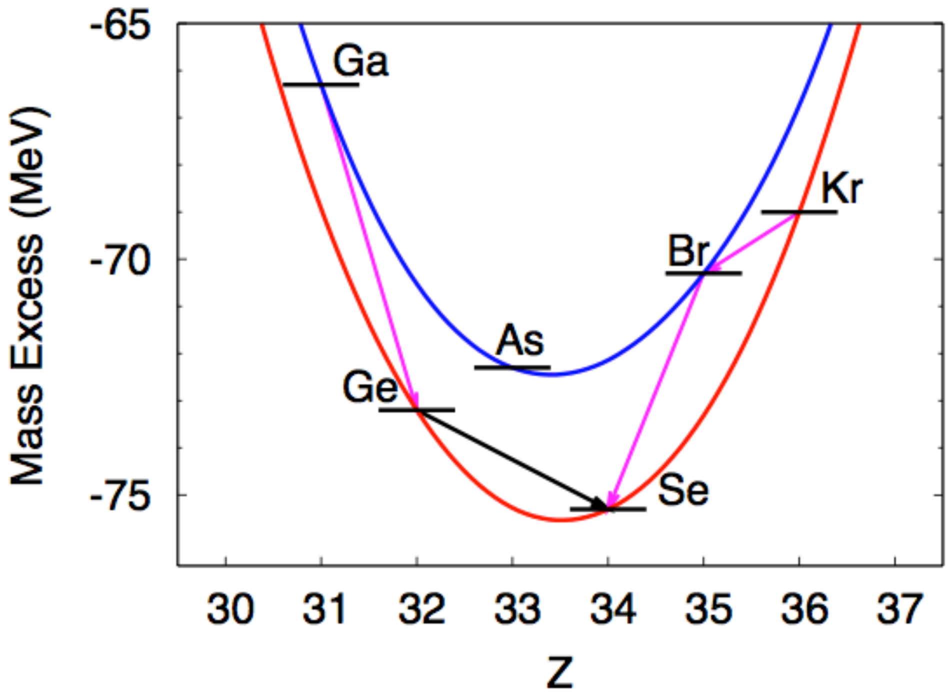
b) Spectra

III. Proton-rich systems

IV. Neutrinoless double-beta decay

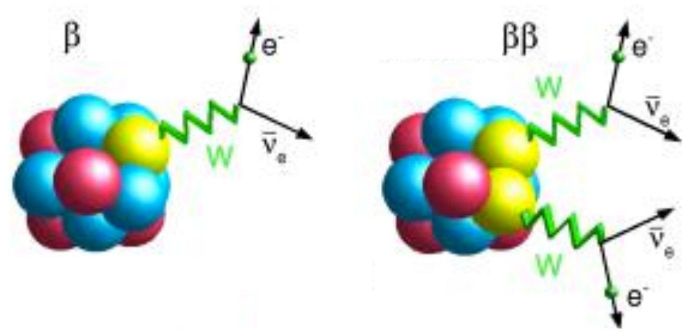
Nuclear Weak Processes: ββ-Decay

Nuclear weak processes: fundamental importance for particle physics



Rare cases when single β-decay is energetically forbidden

Can undergo ββ-decay

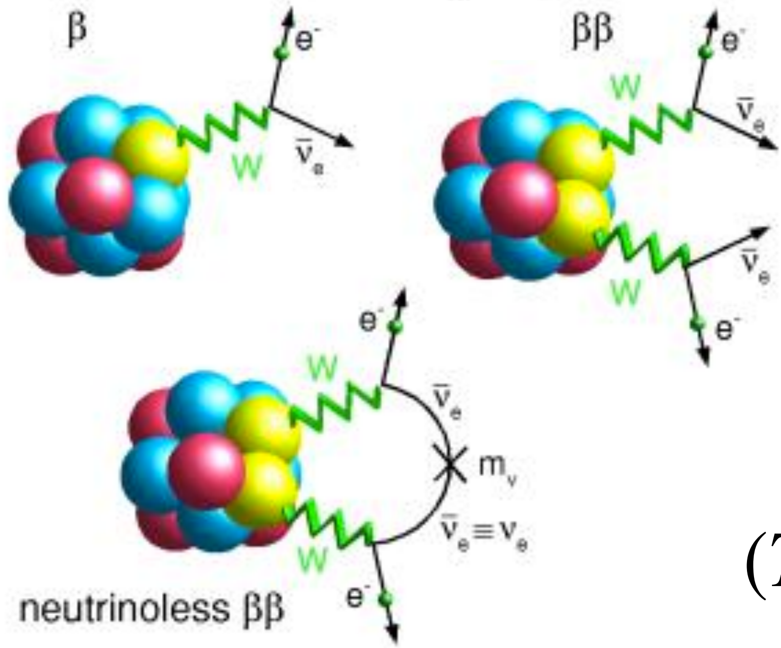


Observed in nature with:

$T_{1/2}^{2\nu\beta\beta} \gtrsim 10^{19}$ y

Nuclear Weak Processes: 0νββ-Decay

Nuclear weak processes: fundamental importance for particle physics



$$(T_{1/2}^{0\nu\beta\beta})^{-1} = G_{0\nu}(Q_{\beta\beta},Z)\left|M_{0\nu}\right|^2\left|m_{\beta\beta}\right|^2$$

Determines character of neutrino (Majorana/Dirac)

Lepton number violation

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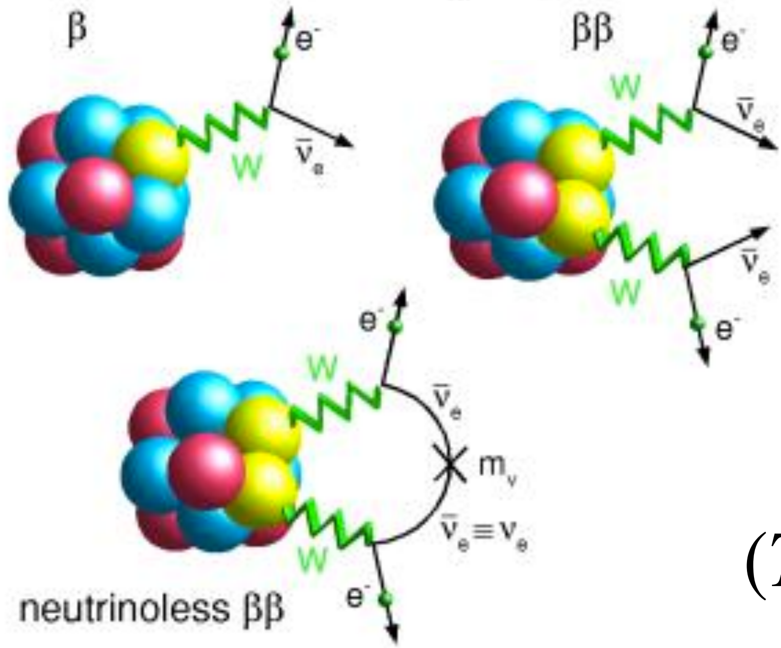

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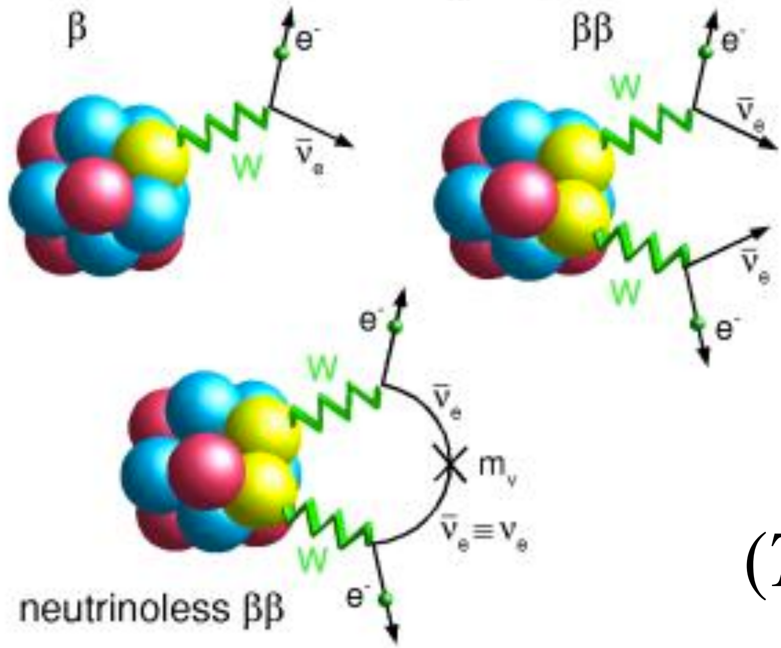

Two essential ingredients:

Q-value (experiment)

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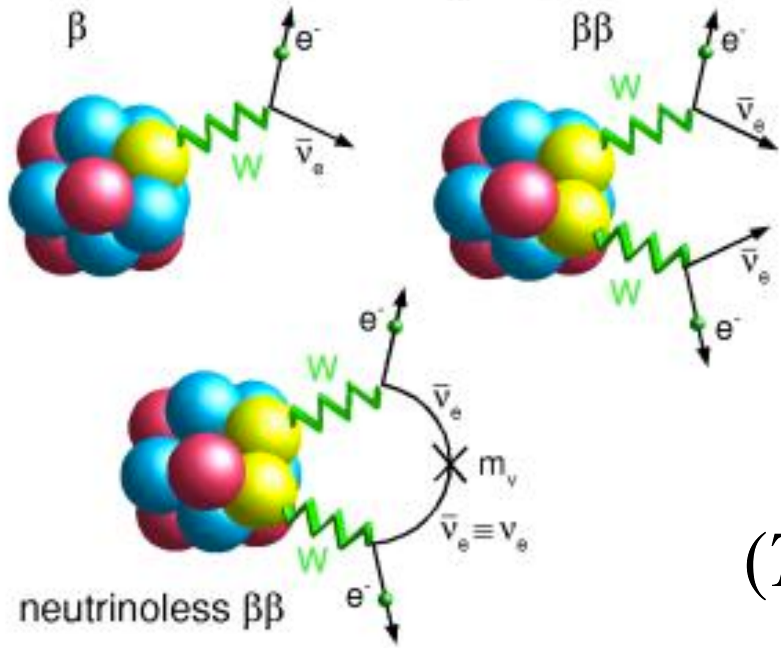
New measurement of ^{82}Se at NSCL

D. Lincoln, JDH *et al.*, submitted to PRL

Nuclear Weak Processes: 0νββ-Decay

Nuclear weak processes: fundamental importance for particle physics



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Q-value (experiment)

Nuclear matrix element

$$(T_{1/2}^{0\nu\beta\beta})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)\left|M_{0\nu}\right|^2\left|m_{\beta\beta}\right|^2$$

Determines character of neutrino (Majorana/Dirac)

Lepton number violation

Neutrino mass scale

Nuclear structure required for NME

Need microscopic framework capable of accurate prediction

Nuclear Matrix Element

$$M_{0\nu} = M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^{F} + \cdots$$

Corrections ~ 30%

$$M_{0\nu}^{GT} = \langle f | \sum_{ab} H(r_{ab}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle \quad M_{0\nu}^{F} = \langle f | \sum_{ab} H(r_{ab}) \tau_a^+ \tau_b^+ | i \rangle$$

Shell model: arbitrary correlations in small single-particle space

QRPA: simple correlations in large single-particle space

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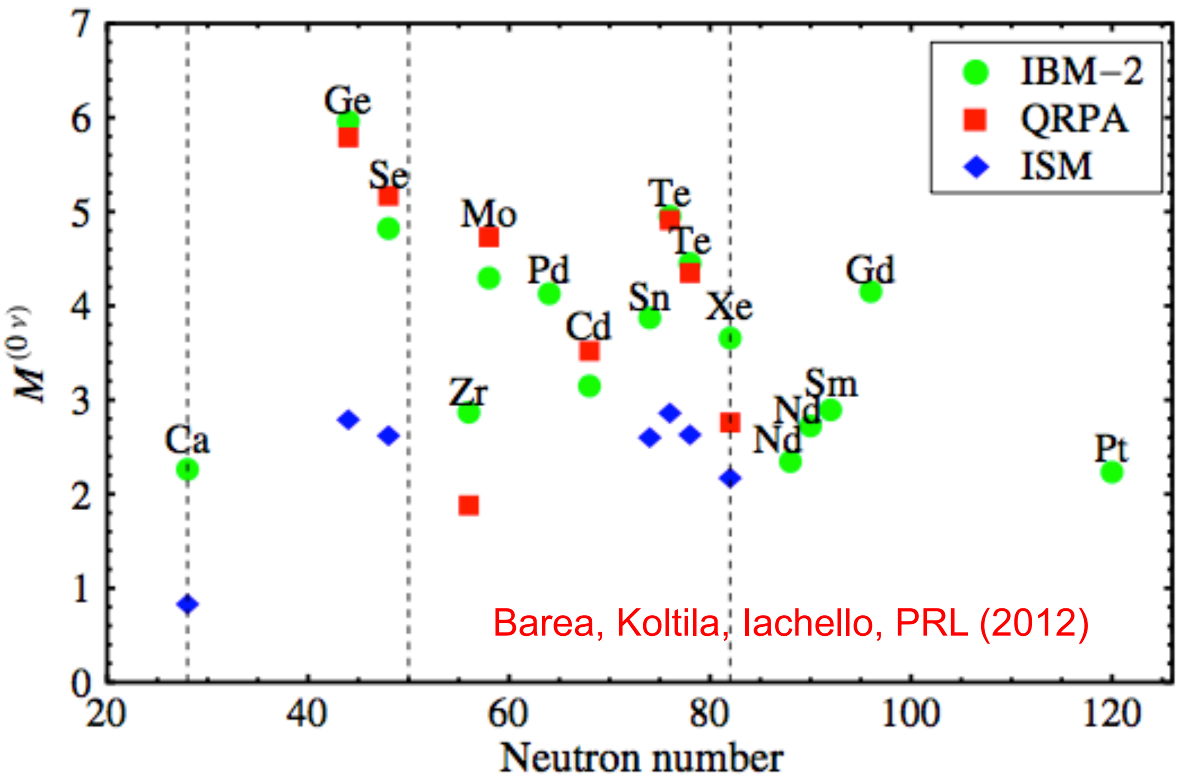


Pronounced differences for lighter candidates

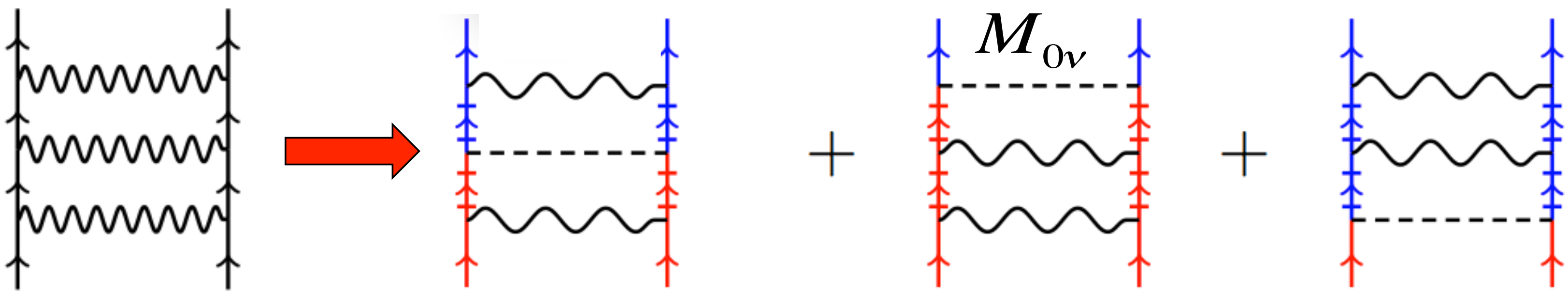
Explore shell model improvements

Effective 0νββ-Decay Operator

Standard SM approach: phenomenological wavefunctions + **bare** operator

Calculate ***effective*** 0νββ operator using formalism of effective interaction theory

Diagrammatically similar: replace one interaction vertex with $M_{0\nu}$ operator



Previous: G-matrix calculation in 2nd-order MBPT non-convergent

Engel and Hagen, PRC (2009)

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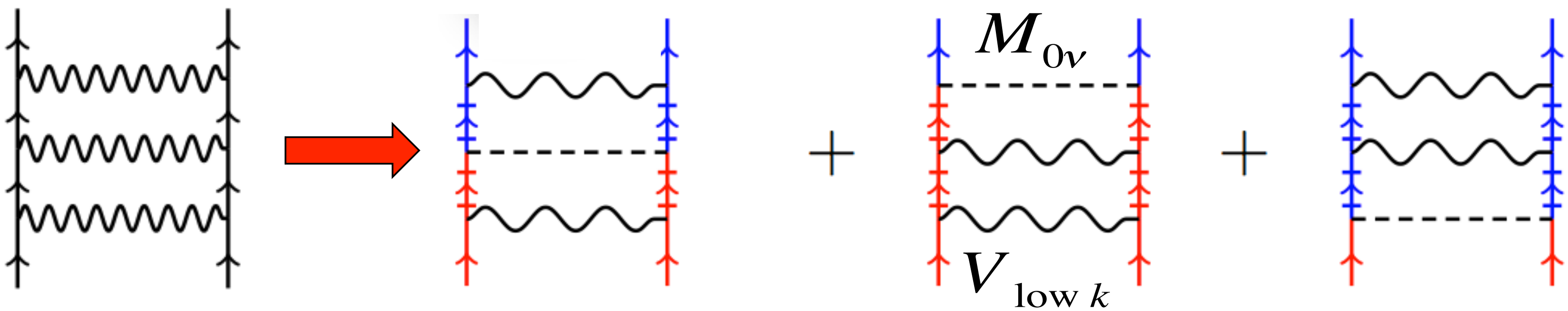


Previous: G-matrix calculation in 2[nd]-order MBPT non-convergent

Engel and Hagen, PRC (2009)

Low-momentum interactions: Improve convergence behavior?

- Chiral N[3]LO (Machleidt, 500 MeV) using smooth-regulator $V_{\text{low } k}$
- 13 major HO shells for intermediate state configuration

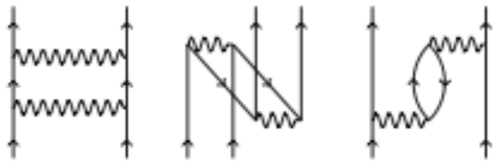
Effective 0νββ-Decay Operator

Calculate in MBPT:

2^{nd} order

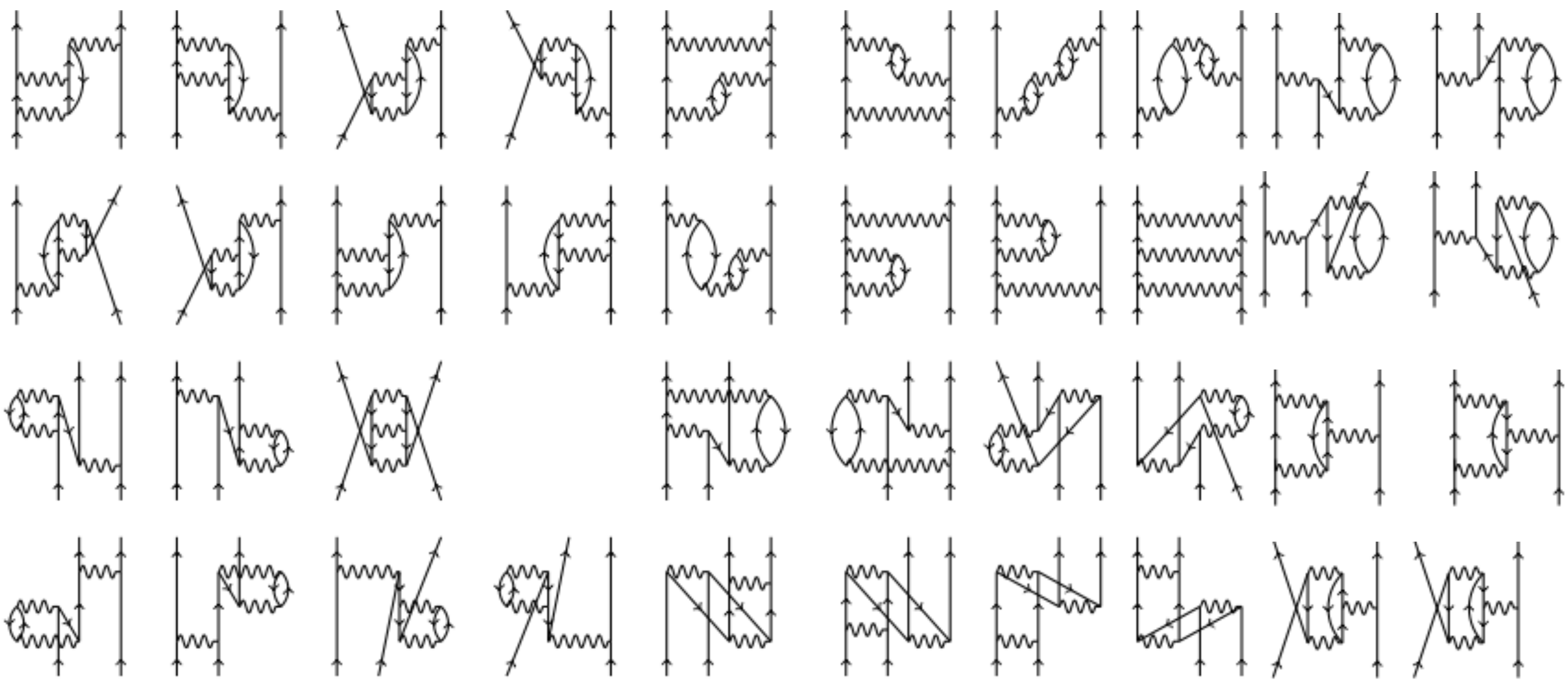
Effective 0νββ-Decay Operator

Calculate in MBPT:



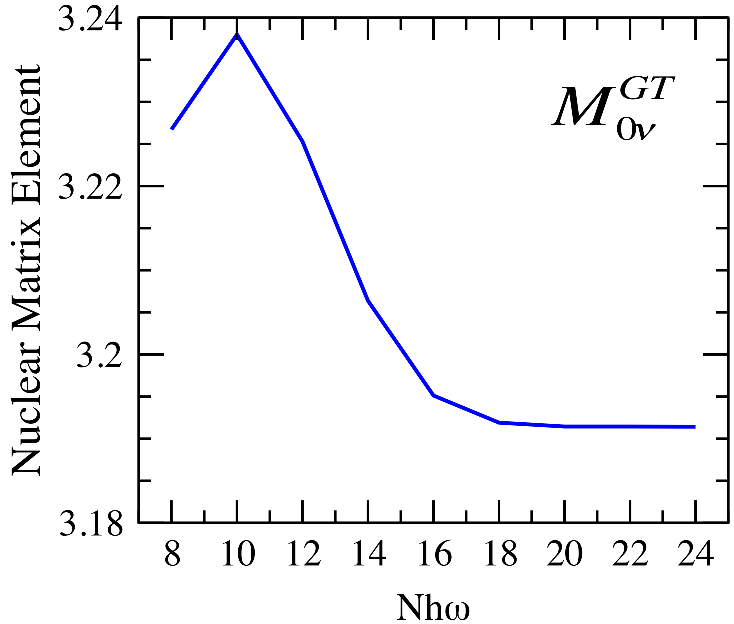
2^{nd} order

3^{rd} order



Intermediate-State Convergence

First results in ^{82}Se (with phenomenological wavefunctions from A. Poves)



JDH and Engel, in prep.

Results **well converged** in terms of intermediate state excitations

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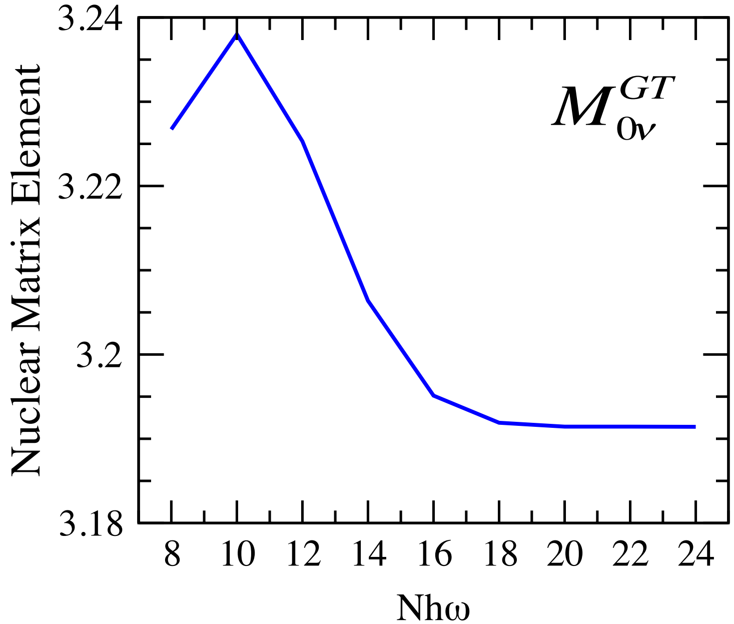

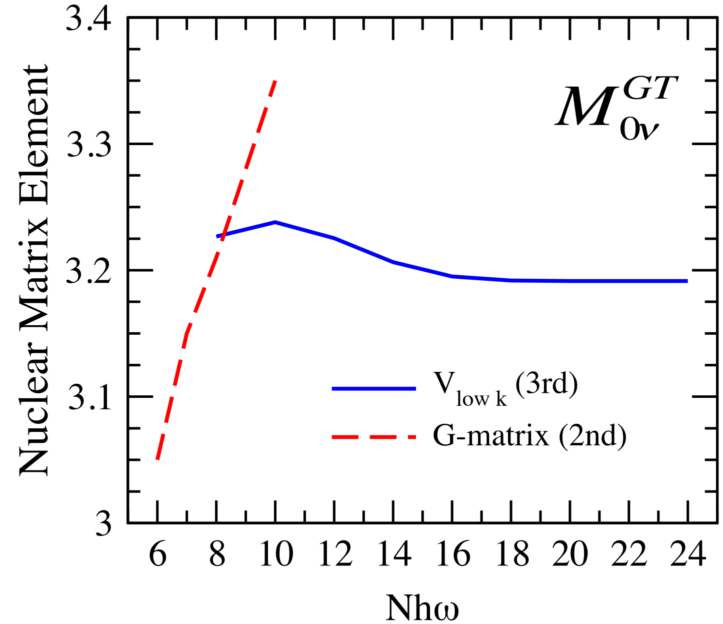

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Results **well converged** in terms of intermediate state excitations

Comparison with G-matrix – no sign of convergence

Order-by-order convergence analysis in progress…

Conclusion

- Nuclear structure theory of medium-mass nuclei with 3N forces, extended spaces
- Robust repulsive 3N mechanism for T=1 neutron/proton-rich nuclei

- **Oxygen isotopes**
  - Cures NN-only failings: dripline, shell evolution, spectra
- **Calcium isotopes** in *pf*- and $pfg_{9/2}$-shells:
  - Prediction of $N=28$ magic number in ^{48}Ca
  - Shell evolution towards the dripline: modest $N=34$ closure, quenching of $N=40$
  - Pairing gaps reflect shell structure – higher-order many-body processes essential

- **Proton-rich N=8,20 isotones**: similar improvements in g.s. energies/spectra
- First effective $0\nu\beta\beta$ operator with chiral NN interactions

- Clearly improvable upgrade path

Acknowledgments

Collaborators

EMMI

TECHNISCHE UNIVERSITÄT DARMSTADT — J. Menendez, A. Schwenk

東京大学 THE UNIVERSITY OF TOKYO — T. Otsuka

THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL — J. Engel

日本大学 — T. Suzuki (Nihon U.)

THE UNIVERSITY of TENNESSEE UT KNOXVILLE / OAK RIDGE NATIONAL LABORATORY — G. Hagen, T. Papenbrock

UNEDF SciDAC Collaboration
Universal Nuclear Energy Density Functional

NICS

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National Institute for Computational Sciences a UT ORNL Partnership

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Office of Science U.S. Department of Energy

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