Momentum-space evolution of chiral three-nucleon forces

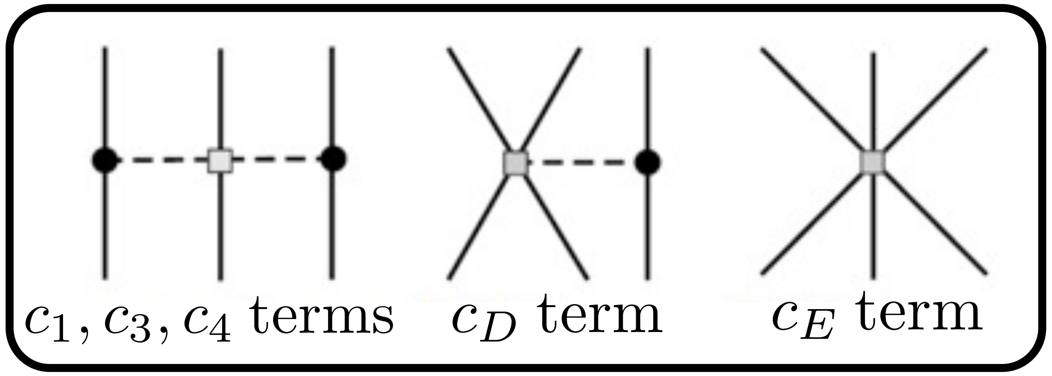
Kai Hebeler (OSU)

INT workshop:
Light nuclei from first principles

Seattle, October 8, 2012

THE OHIO STATE UNIVERSITY

RG evolution of 3N interactions



- So far (in momentum basis):

intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

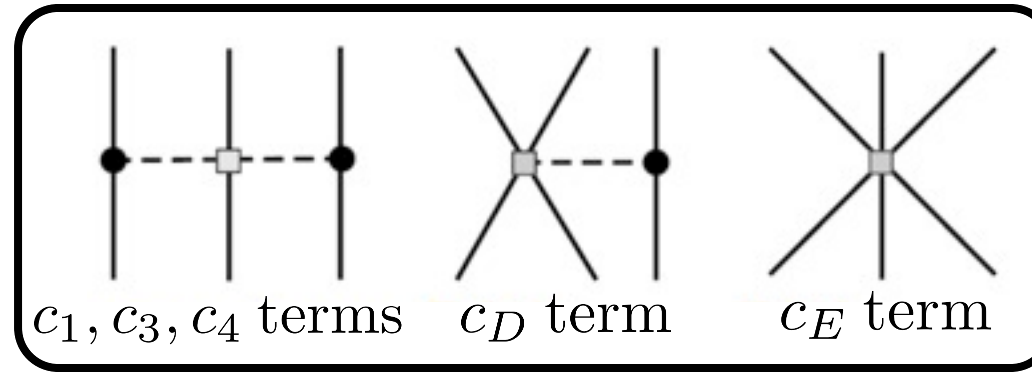
$\longrightarrow$ coupling constants of natural size

- in neutron matter contributions from c_D, c_E and c_4 terms vanish
- long-range 2π contributions assumed to be invariant under RG evolution

RG evolution of 3N interactions

- So far (in momentum basis):

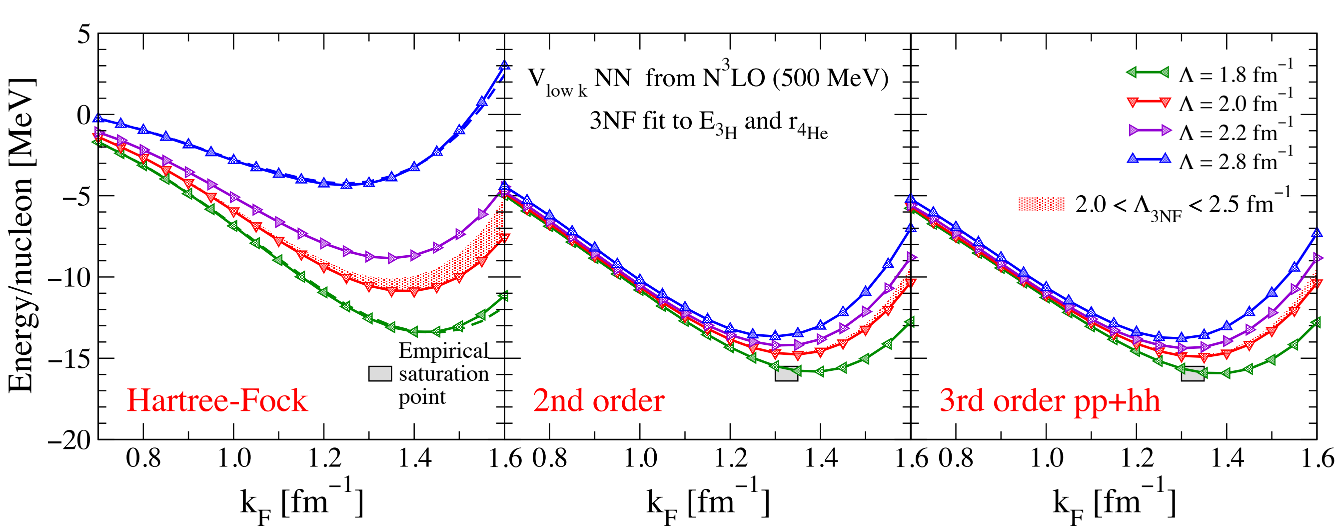
intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:



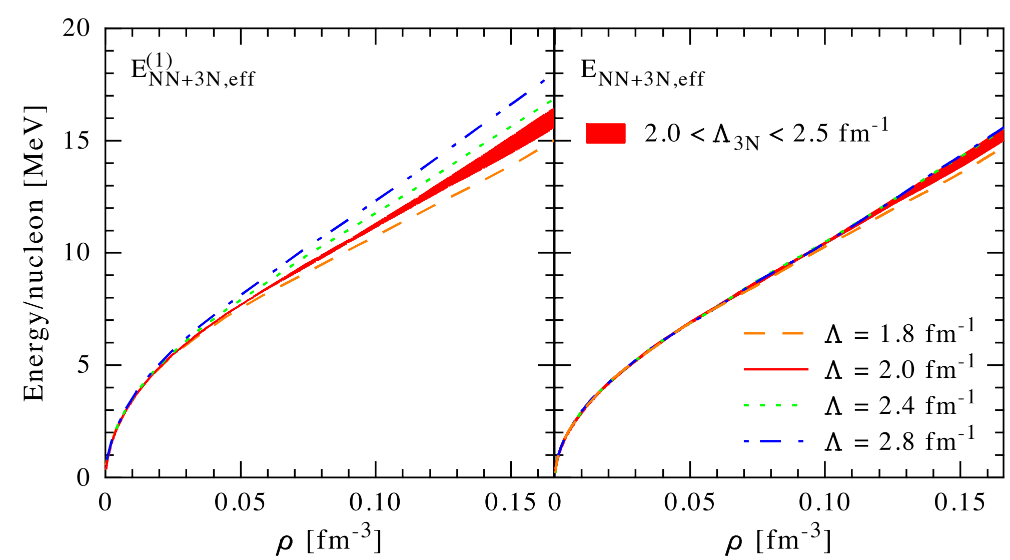
$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

$\longrightarrow$ coupling constants of natural size

- in neutron matter contributions from c_D, c_E and c_4 terms vanish
- long-range 2π contributions assumed to be invariant under RG evolution



KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

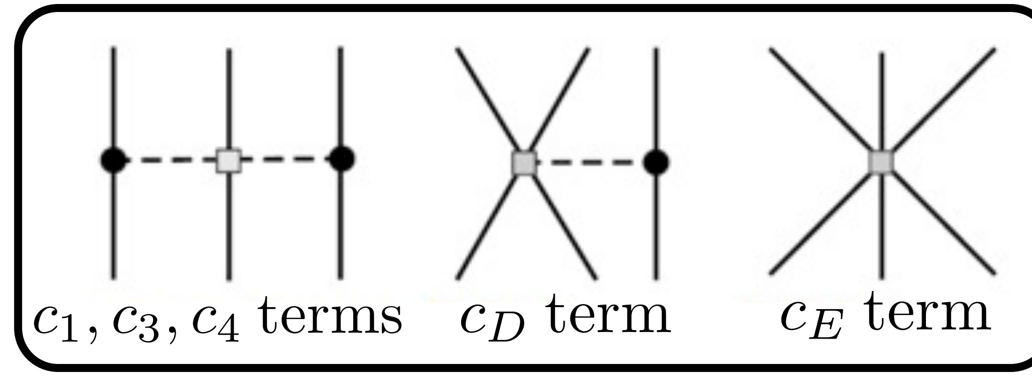


KH and Schwenk PRC 82, 014314 (2010)

RG evolution of 3N interactions

- So far (in momentum basis):

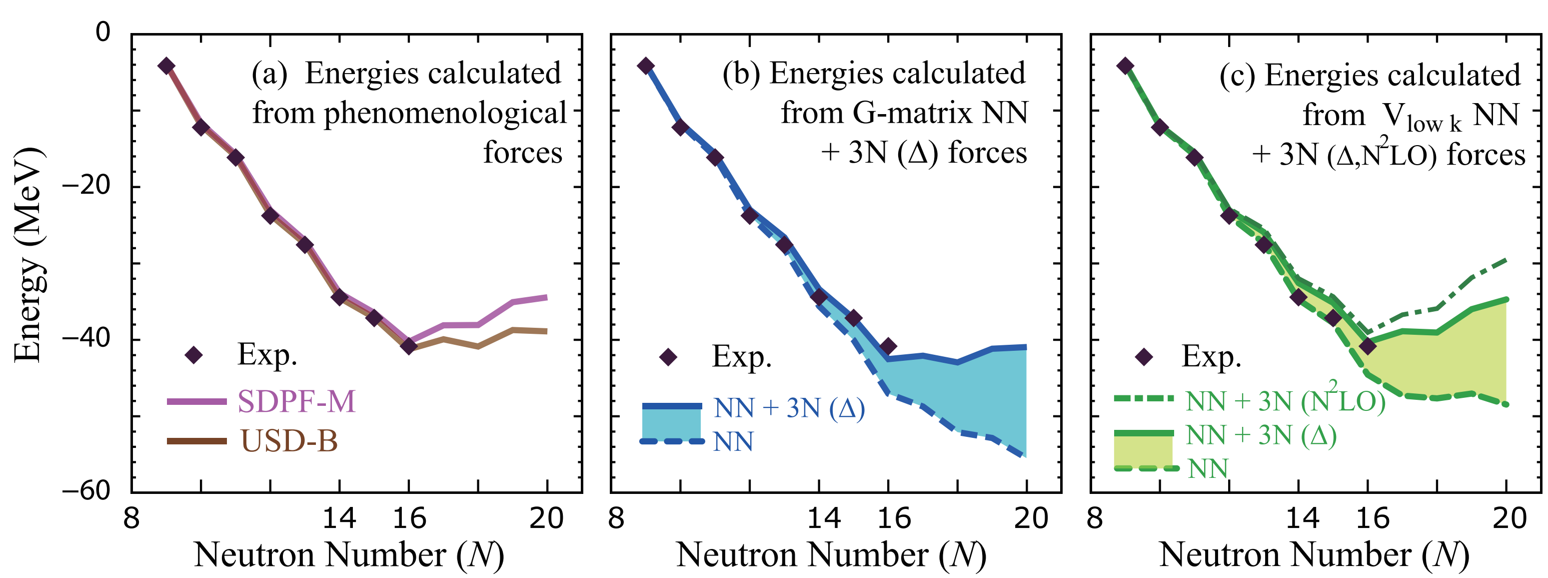
intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:



$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

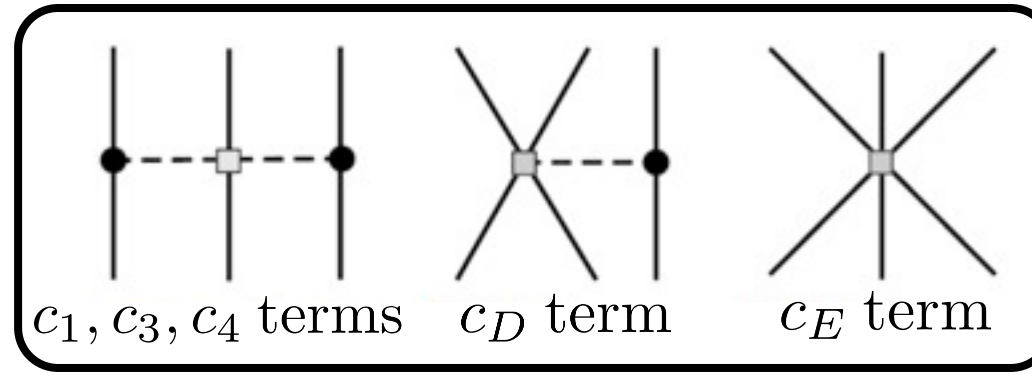
$\longrightarrow$ coupling constants of natural size

- in neutron matter contributions from c_D, c_E and c_4 terms vanish
- long-range 2π contributions assumed to be invariant under RG evolution



Otsuka et al. PRL 105, 032501 (2010)

RG evolution of 3N interactions



- So far (in momentum basis):
intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

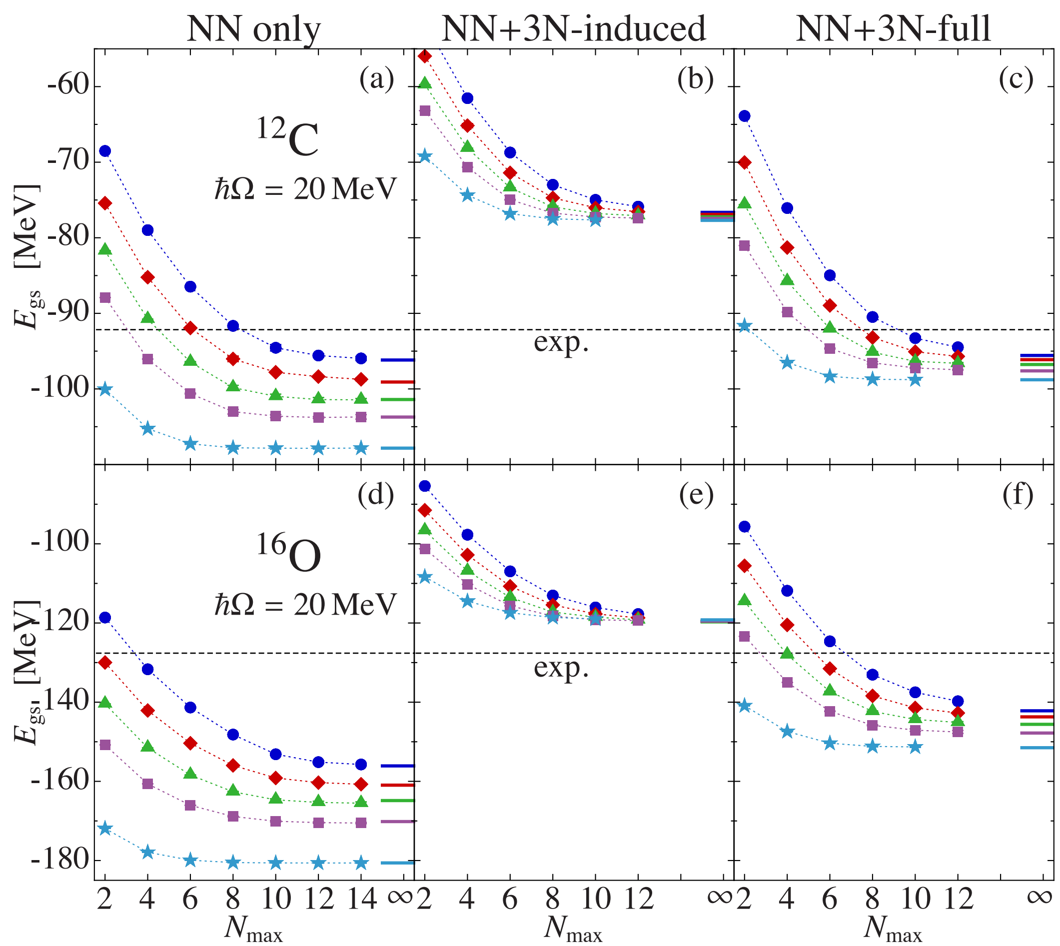
$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

$\longrightarrow$ coupling constants of natural size

  - in neutron matter contributions from c_D, c_E and c_4 terms vanish
  - long-range 2π contributions assumed to be invariant under RG evolution

- Ideal case: evolve 3NF consistently with NN within the SRG
  - has been achieved in oscillator basis (Jurgenson, Roth)
  - promising results in very light nuclei
  - puzzling effects in heavier nuclei (higher-body forces?)
  - not immediately applicable to infinite systems
  - limitations on $\hbar\Omega$

RG evolution of 3N interactions



Roth et al. PRL 107, 072501 (2011)

- Ideal case: evolve 3NF consistently with NN within the SRG
  - has been achieved in oscillator basis (Jurgenson, Roth)
  - promising results in very light nuclei
  - puzzling effects in heavier nuclei (higher-body forces?)
  - not immediately applicable to infinite systems
  - limitations on $\hbar\Omega$

Current/future directions

- application to infinite systems
  - equation of state
  - systematic study of induced many-body contributions, scaling behavior
  - include initial N3LO 3N interactions

Current/future directions

- application to infinite systems
  - equation of state (in this talk: first results for neutron matter)
  - systematic study of induced many-body contributions, scaling behavior
  - include initial N3LO 3N interactions

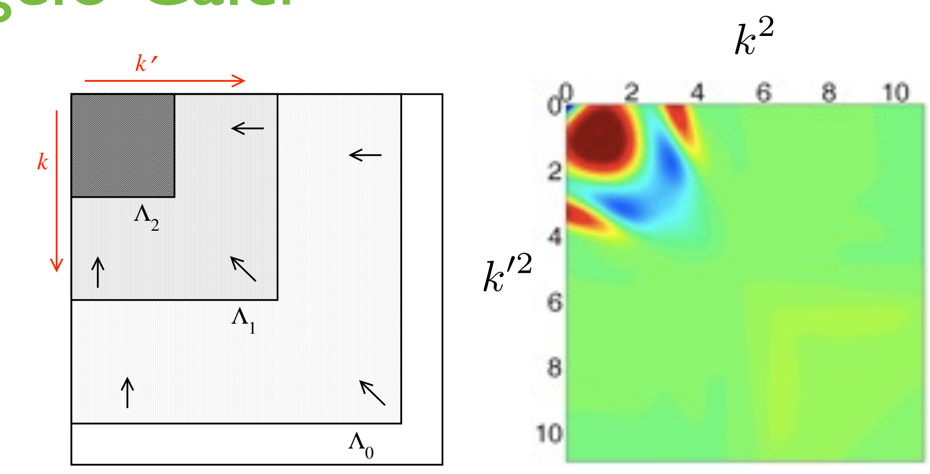
Current/future directions

- application to infinite systems
  - equation of state (in this talk: first results for neutron matter)
  - systematic study of induced many-body contributions, scaling behavior
  - include initial N3LO 3N interactions
- transformation of evolved interactions to oscillator basis
  - application to finite nuclei, complimentary to HO evolution (no core shell model, coupled cluster)

    in collaboration with Robert Roth and Angelo Calci

Current/future directions

- application to infinite systems
  - equation of state (in this talk: first results for neutron matter)
  - systematic study of induced many-body contributions, scaling behavior
  - include initial N3LO 3N interactions
- transformation of evolved interactions to oscillator basis
  - application to finite nuclei, complimentary to HO evolution (no core shell model, coupled cluster)
    in collaboration with Robert Roth and Angelo Calci
- study of various generators
  - different decoupling patterns (e.g. $V_{\text{low k}}$)
  - improved efficiency of evolution
  - suppression of many-body forces?

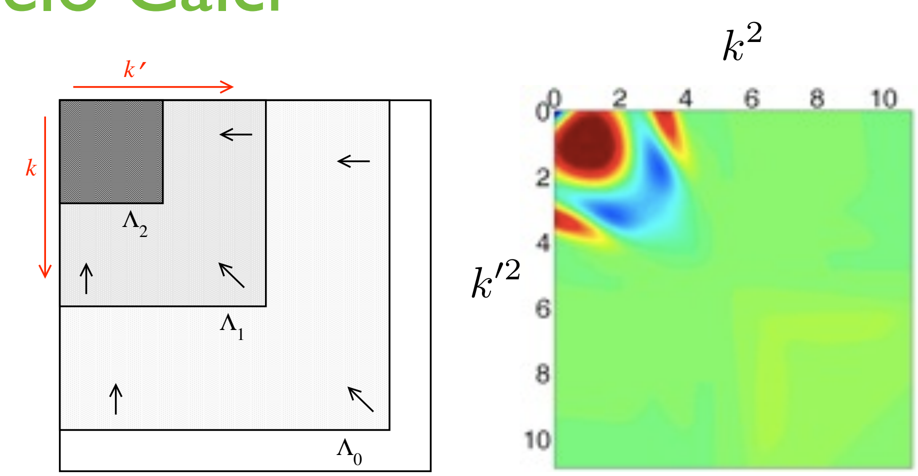


Anderson et al., PRC 77, 037001 (2008)

Current/future directions

- application to infinite systems
  - equation of state (in this talk: first results for neutron matter)
  - systematic study of induced many-body contributions, scaling behavior
  - include initial N3LO 3N interactions
- transformation of evolved interactions to oscillator basis
  - application to finite nuclei, complimentary to HO evolution (no core shell model, coupled cluster)

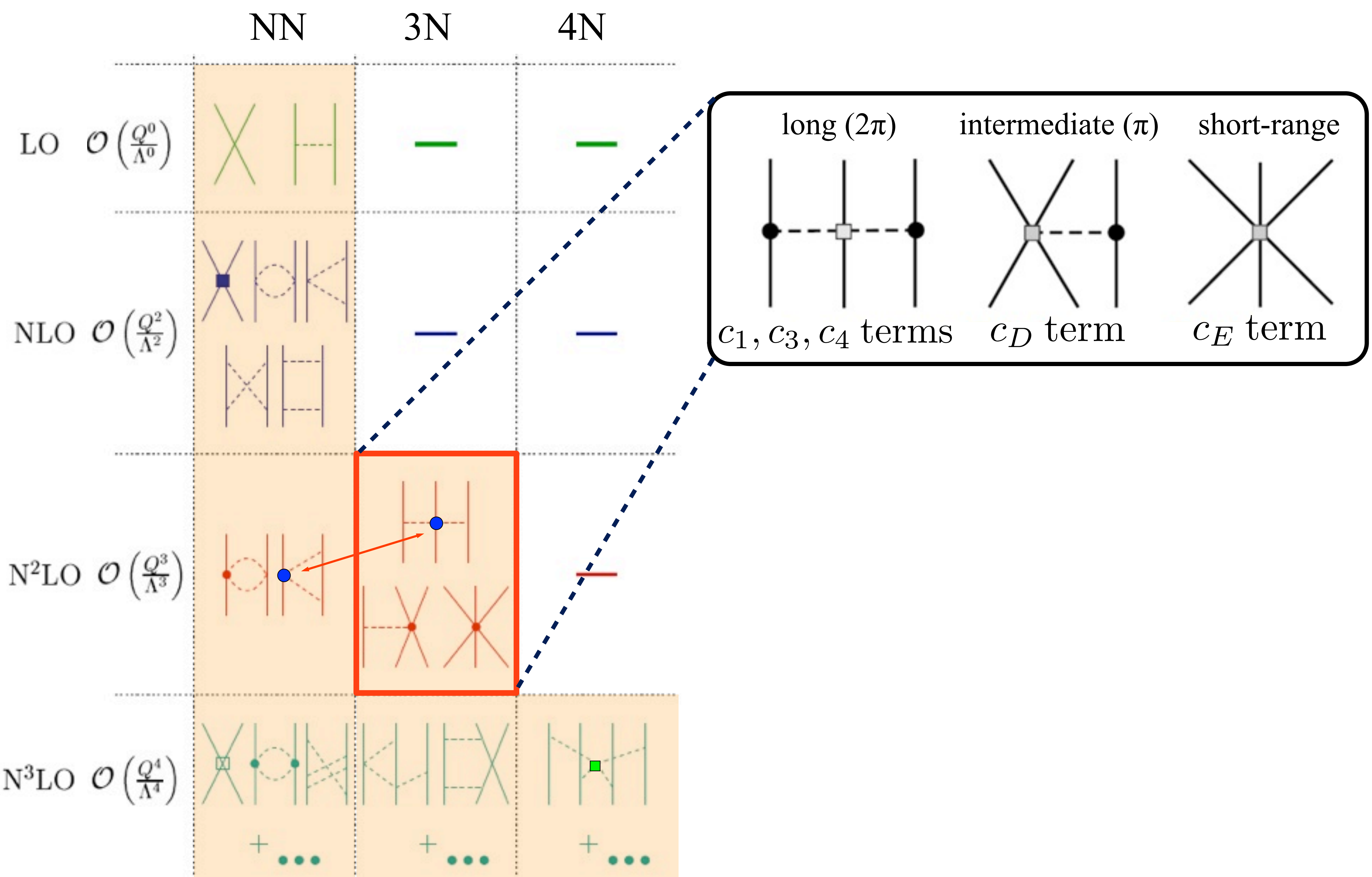
  in collaboration with Robert Roth and Angelo Calci
- study of various generators
  - different decoupling patterns (e.g. $V_{\text{low } k}$)
  - improved efficiency of evolution
  - suppression of many-body forces?



Anderson et al., PRC 77, 037001 (2008)

- explicit calculation of unitary 3N transformation
  - RG evolution of operators
  - study of correlations in nuclear systems $\longrightarrow$ factorization

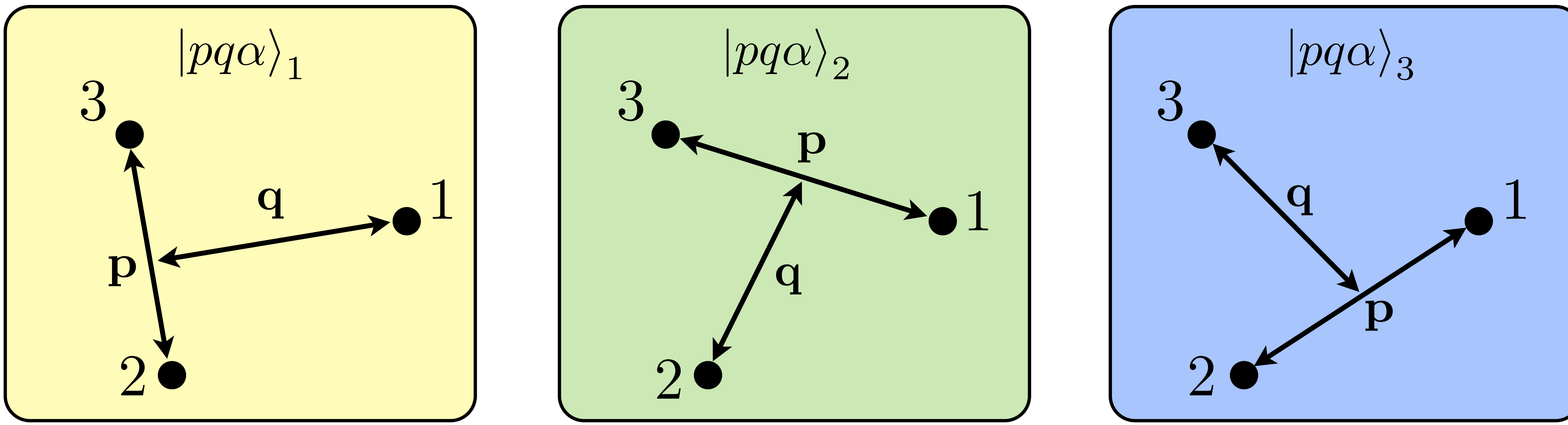
Leading-order 3N forces in chiral EFT



RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

$$|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(ls_i)j]\,\mathcal{J}\mathcal{J}_z(Tt_i)\mathcal{T}\mathcal{T}_z\rangle$$



Faddeev bound state equations:

$$|\psi_i\rangle = G_0\left[2t_iP + (1+t_iG_0)\,V^i_{3N}(1+2P)\right]|\psi_i\rangle$$

$${}_i\langle pq\alpha|P|p'q'\alpha'\rangle_i = {}_i\langle pq\alpha|p'q'\alpha'\rangle_j$$

SRG flow equations of NN and 3N forces in Faddeev basis

$$\boxed{\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \eta_s = [T_{\rm rel}, H_s]}$$

$$H = T + V_{12} + V_{13} + V_{23} + V_{123}$$

- spectators correspond to delta functions, matrix representation of H_s ill-defined
- solution: explicit separation of NN and 3N flow equations

$$\begin{aligned}
\frac{dV_{ij}}{ds} &= [[T_{ij}, V_{ij}], T_{ij} + V_{ij}], \\
\frac{dV_{123}}{ds} &= [[T_{12}, V_{12}], V_{13} + V_{23} + V_{123}] \\
&+ [[T_{13}, V_{13}], V_{12} + V_{23} + V_{123}] \\
&+ [[T_{23}, V_{23}], V_{12} + V_{13} + V_{123}] \\
&+ [[T_{\rm rel}, V_{123}], H_s]
\end{aligned}$$

- only connected terms remain in $\dfrac{dV_{123}}{ds}$, 'dangerous' delta functions cancel

see Bogner, Furnstahl, Perry PRC 75, 061001(R) (2007)

SRG evolution in momentum space

- evolve the antisymmetrized 3N interaction

$$\overline{V}_{123} = {}_i\langle pq\alpha|\,(1+P_{123}+P_{132})V_{123}^{(i)}(1+P_{123}+P_{132})\,|p'q'\alpha'\rangle_i$$

- embed NN interaction in 3N basis:

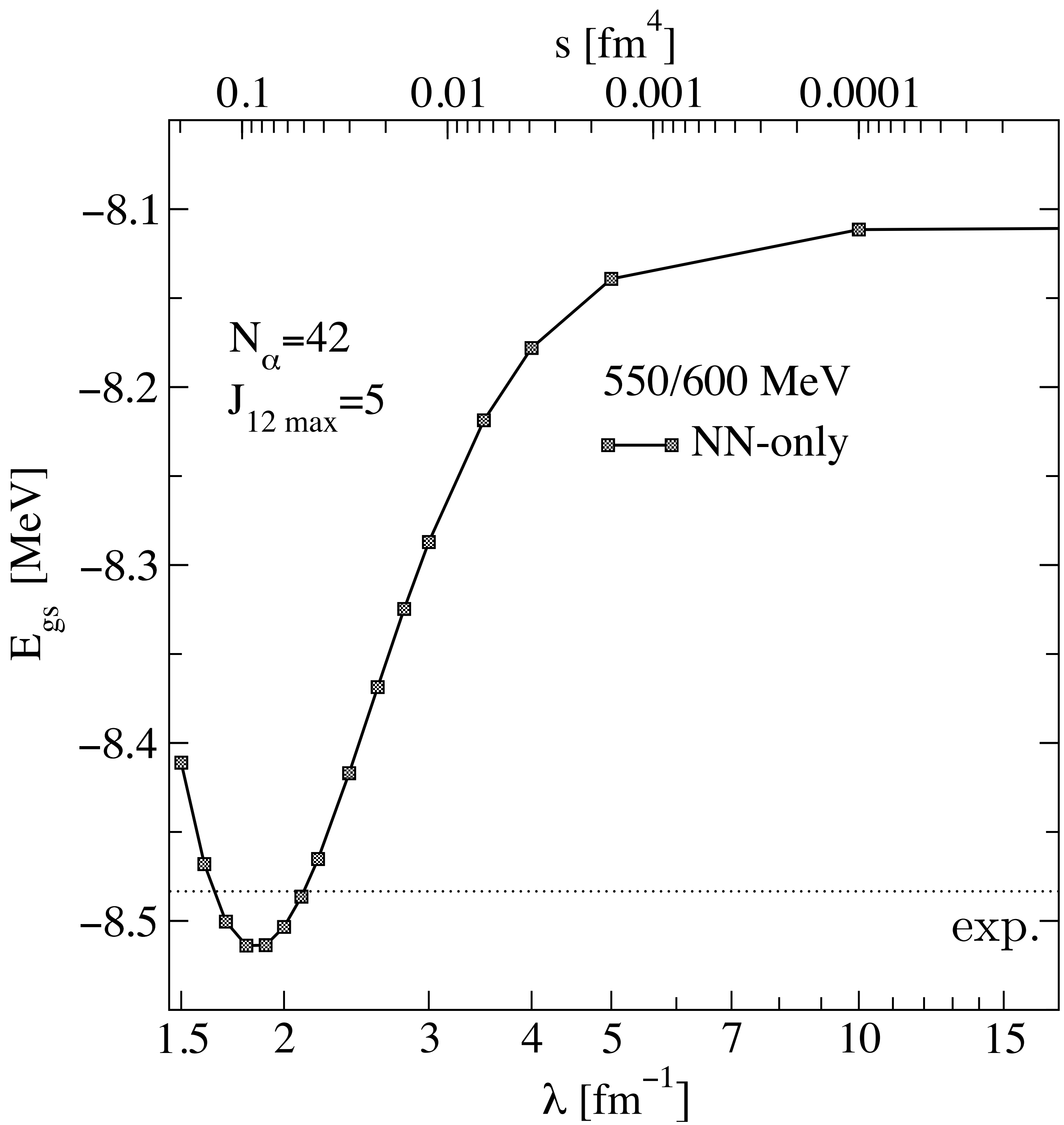
$$V_{13} = P_{123}V_{12}P_{132}, \quad V_{23} = P_{132}V_{12}P_{123}$$

with $\;{}_3\langle pq\alpha|V_{12}|p'q'\alpha'\rangle_3 = \langle p\tilde{\alpha}|V_{\rm NN}|p'\tilde{\alpha}'\rangle\,\delta(q-q')/q^2$

- use $P_{123}\overline{V}_{123} = P_{132}\overline{V}_{123} = \overline{V}_{123}$

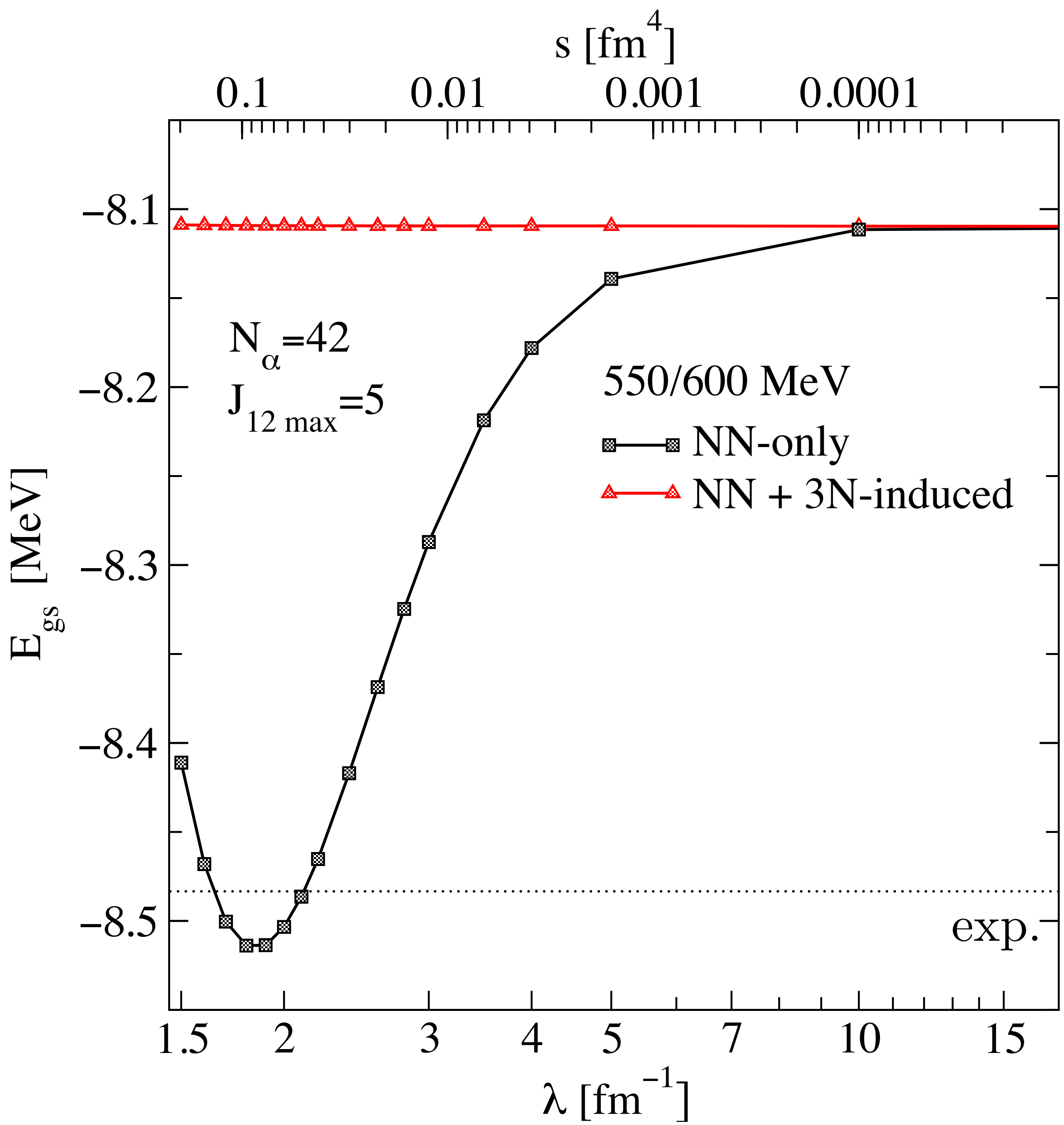
$$\Rightarrow \quad \begin{aligned} d\overline{V}_{123}/ds = {} & C_1(s,T,V_{\rm NN},P) \\ & + C_2(s,T,V_{\rm NN},\overline{V}_{123},P) \\ & + C_3(s,T,\overline{V}_{123}) \end{aligned}$$

RG evolution of 3N interactions in momentum space: Results for the Triton



see KH, PRC(R) 85, 021002 (2012)

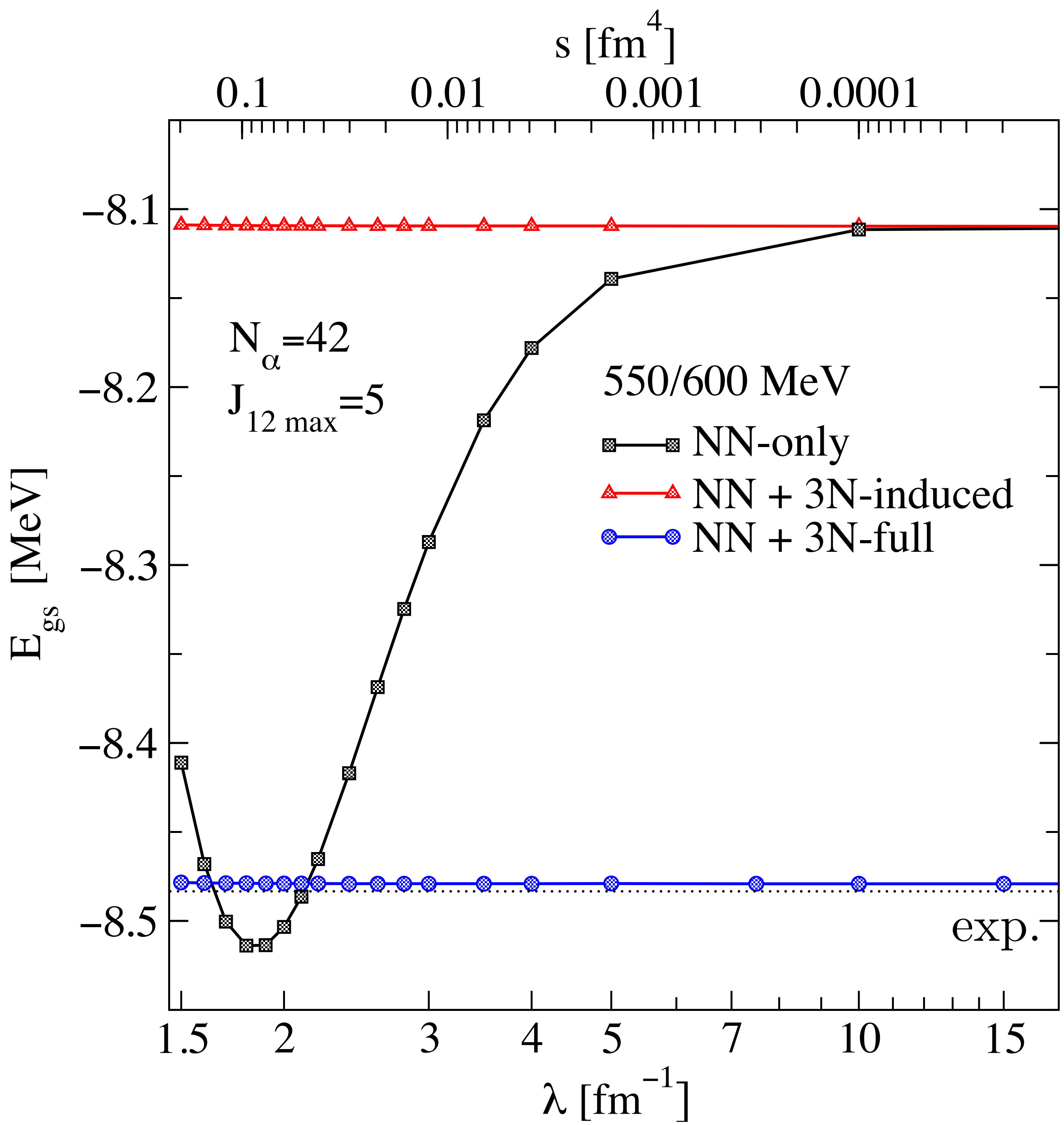
RG evolution of 3N interactions in momentum space: Results for the Triton



see KH, PRC(R) 85, 021002 (2012)

Invariance of $E_{\mathrm{gs}}^{^3H}$ within $\leq 1\,\mathrm{eV}$ for consistent chiral interactions at $\mathrm{N^2LO}$

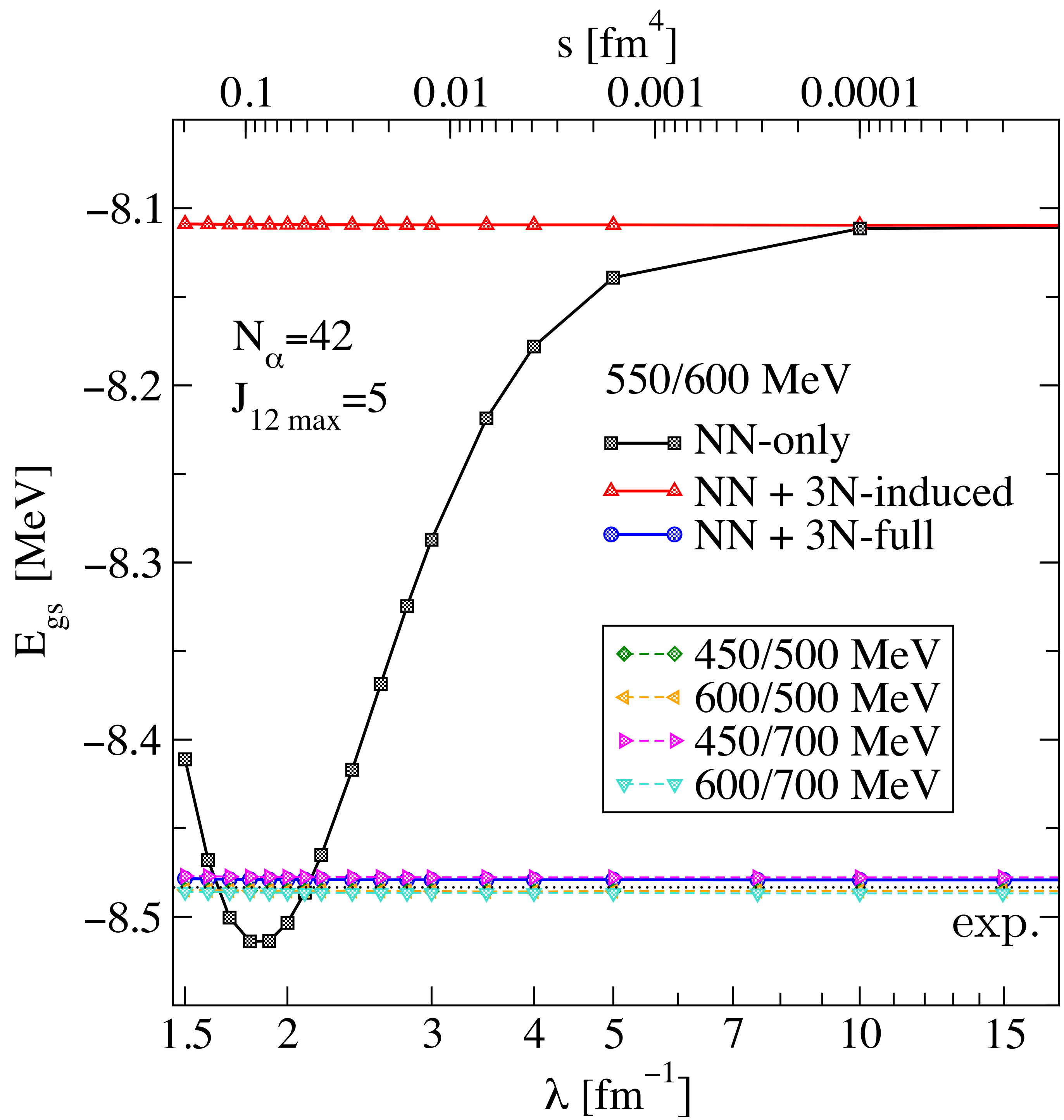
RG evolution of 3N interactions in momentum space: Results for the Triton



see KH, PRC(R) 85, 021002 (2012)

Invariance of $E_{\rm gs}^{^3H}$ within $\leq 1\,\mathrm{eV}$ for consistent chiral interactions at $\mathrm{N^2LO}$

RG evolution of 3N interactions in momentum space: Results for the Triton

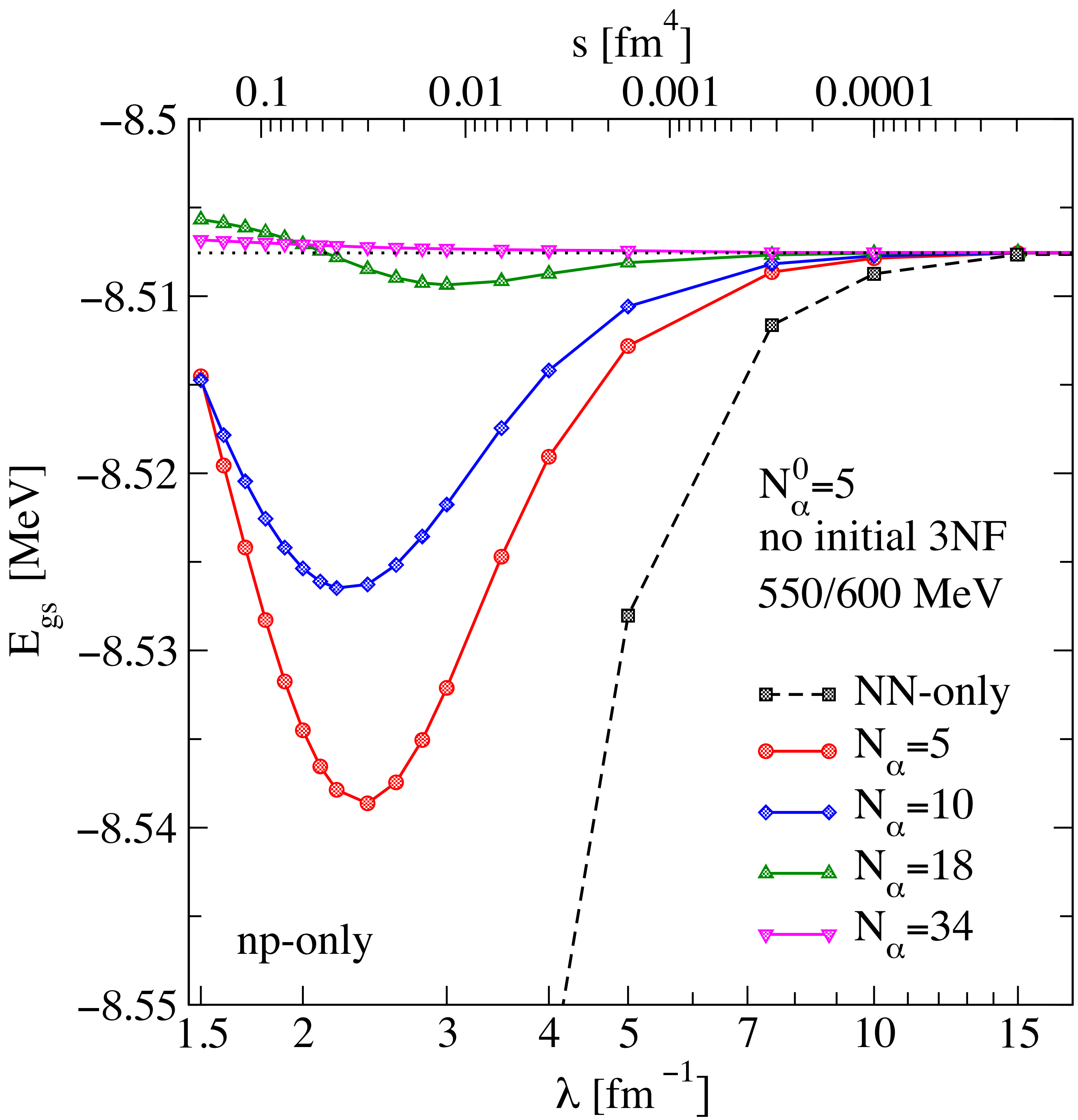


see KH, PRC(R) 85, 021002 (2012)

Invariance of $E_{\mathrm{gs}}^{^3H}$ within $\leq 1\,\mathrm{eV}$ for consistent chiral interactions at $\mathrm{N}^2\mathrm{LO}$

Unitarity of SRG evolution

- Faddeev basis not complete under permutation of particles
- embedding of NN forces in finite 3N basis not exact: $V_{12} = PV_{23}P^{-1}, \ldots$

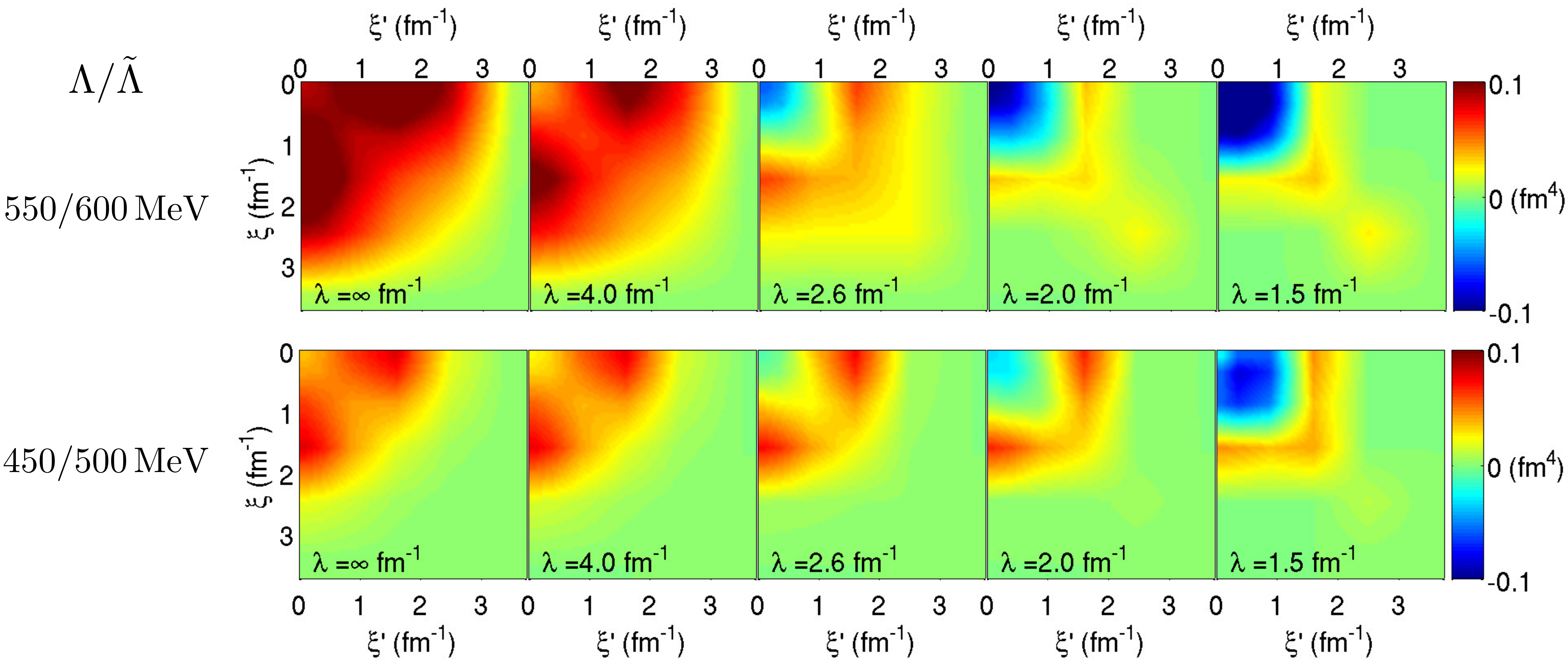


KH, PRC(R) 85, 021002 (2012)

violation of unitarity can be systematically reduced by increasing the model space

Decoupling of matrix elements

$$\theta = \frac{\pi}{12} \qquad \mathcal{T} = \mathcal{J} = \frac{1}{2}$$



KH, PRC(R) 85, 021002 (2012)

hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2$ hyperangle: $\tan\theta = \frac{2\,p}{\sqrt{3}\,q}$

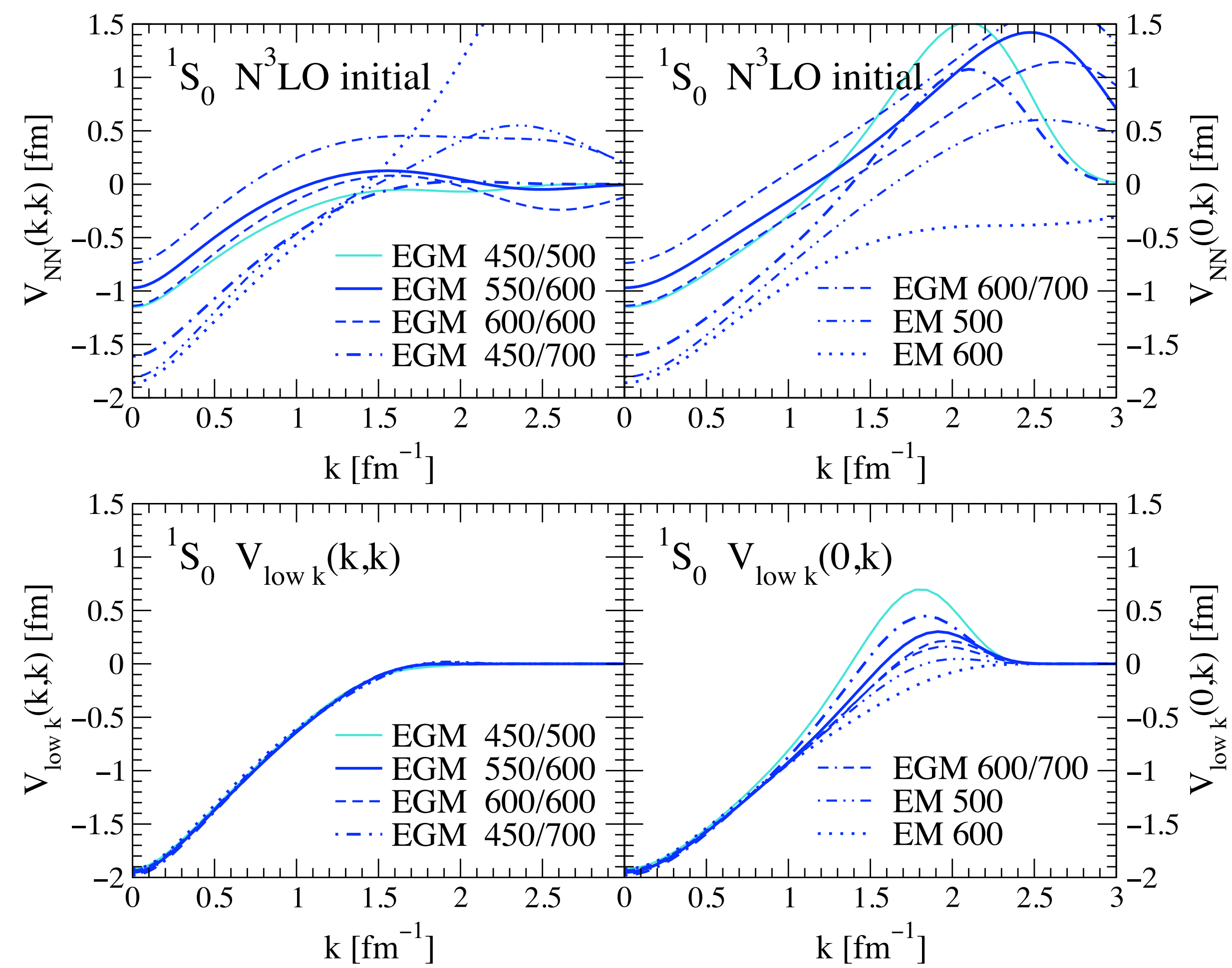
same decoupling patterns like in NN interactions

Universality in 3N interactions at low resolution

phase-shift equivalence

common long-range physics

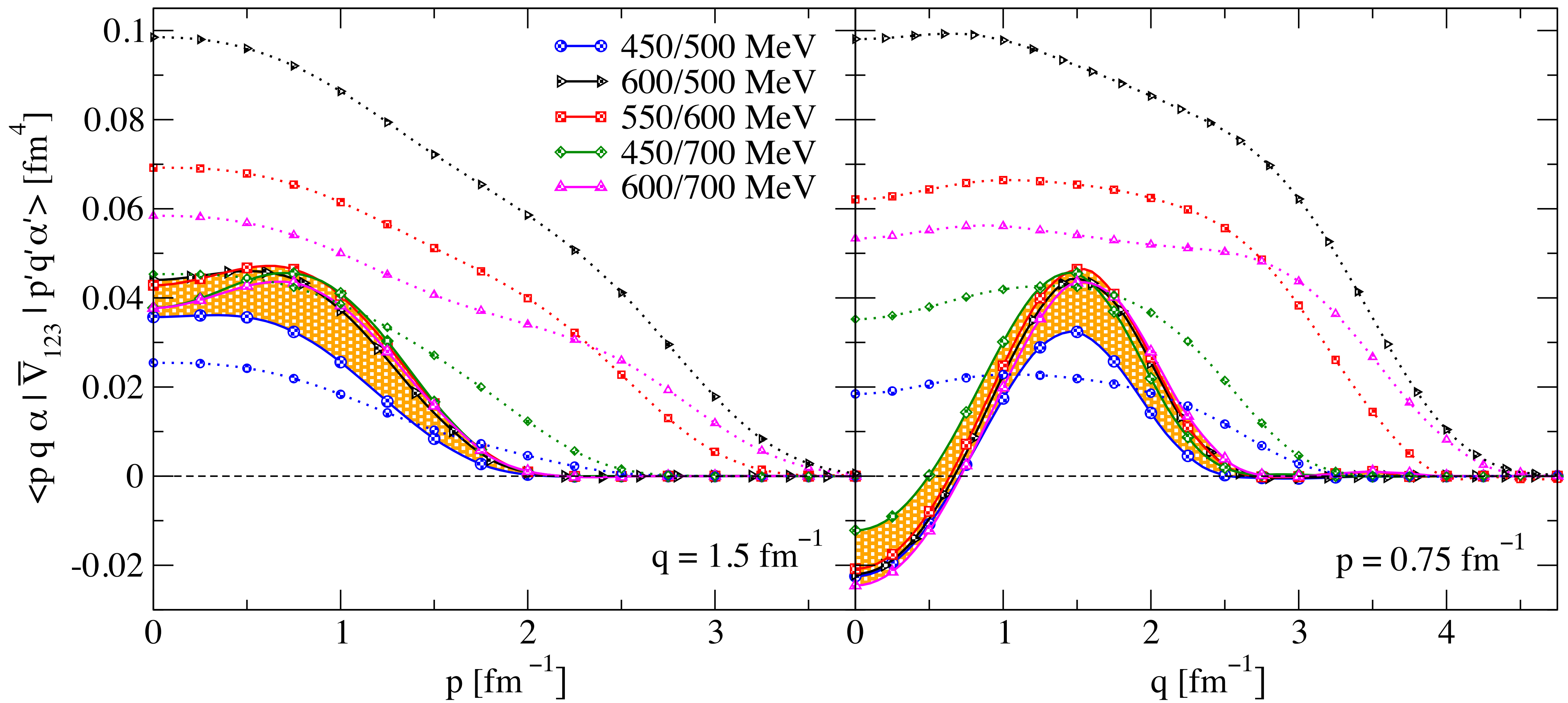
(approximate) universality of low-resolution NN interactions



To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants c_D and c_E
- 3N interactions give only subleading contributions to observables

Universality in 3N interactions at low resolution



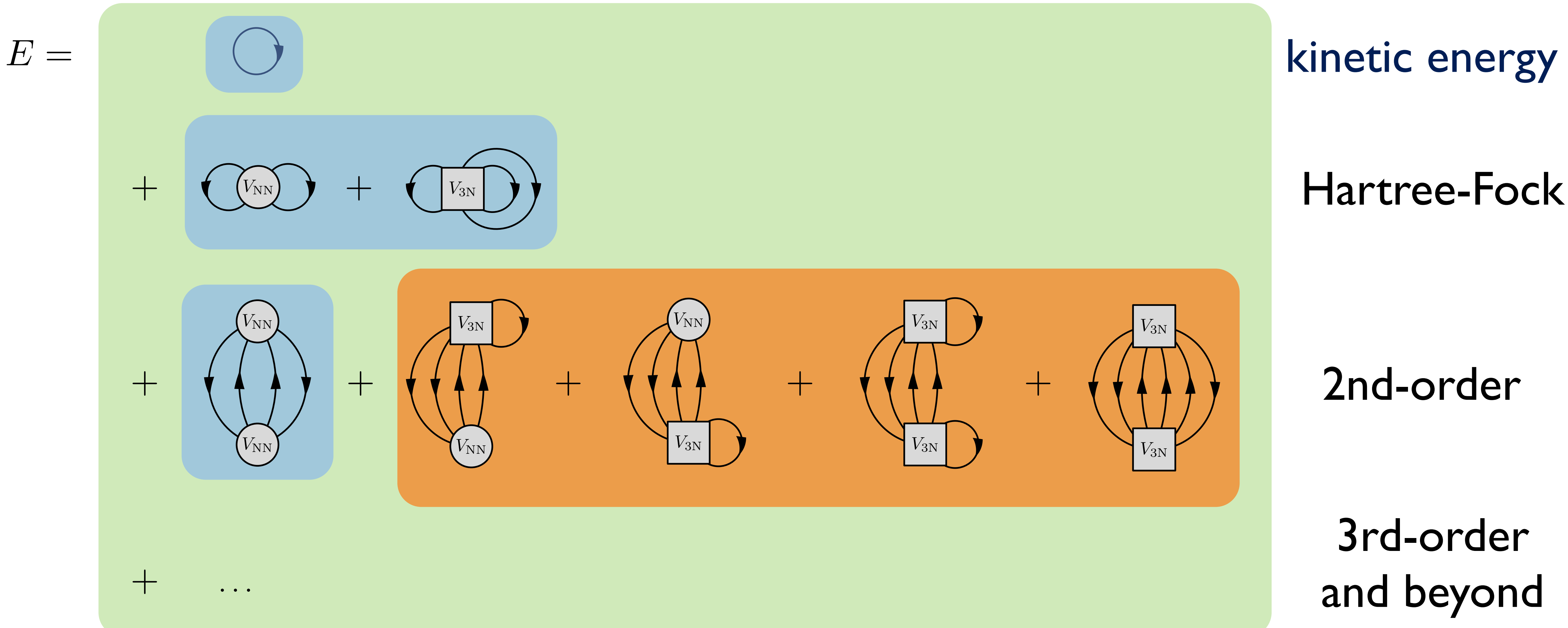
KH, PRC(R) 85, 021002 (2012)

- remarkably reduced scheme dependence for typical momenta $\sim 1\,\mathrm{fm}^{-1}$,

matrix elements with significant phase space well constrained at low resolution

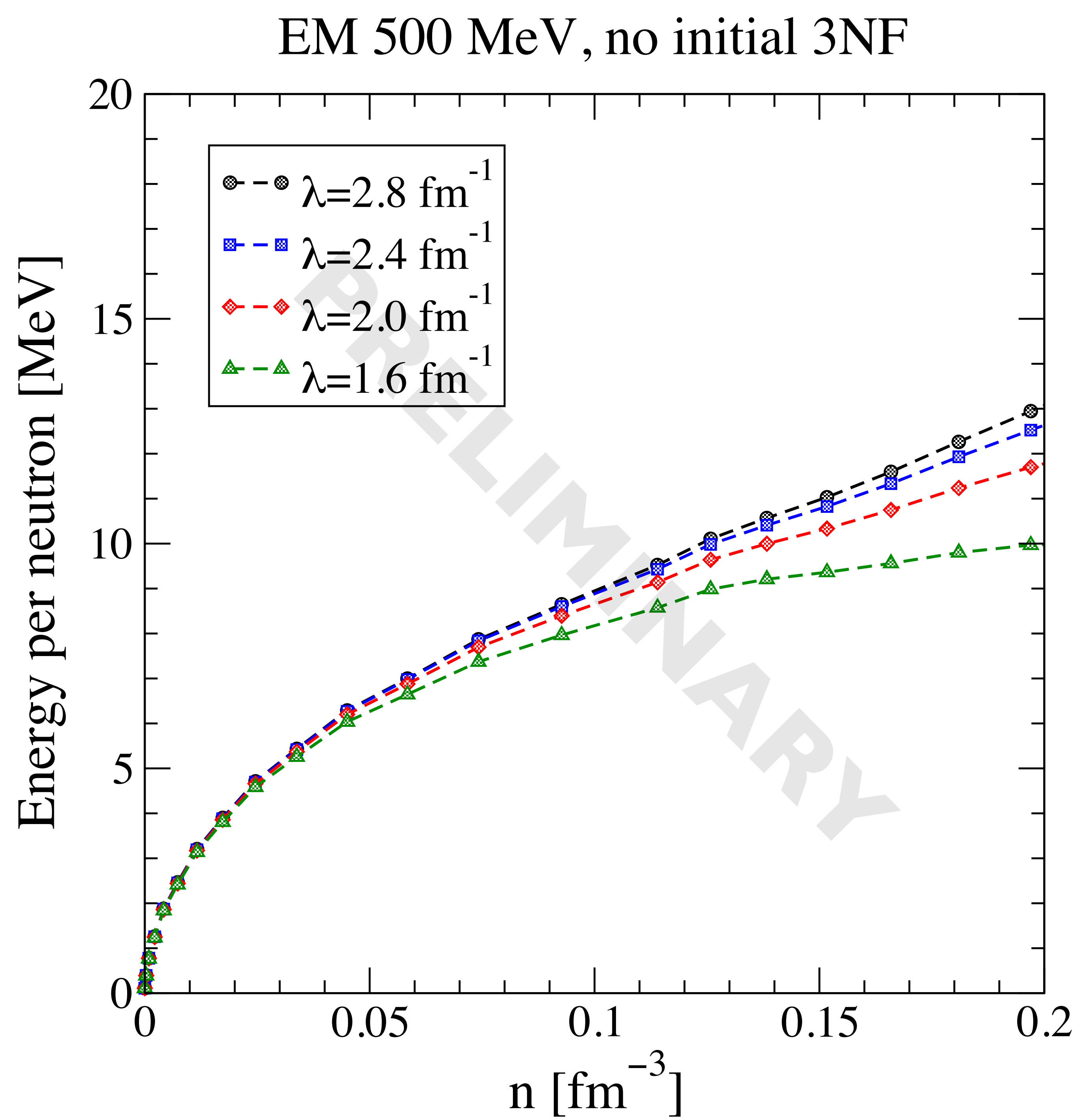
- new momentum structures induced at low resolution
- study based on $\mathrm{N^2LO}$ chiral interactions, improved universality at $\mathrm{N^3LO}$?

Application to neutron matter: Equation of state



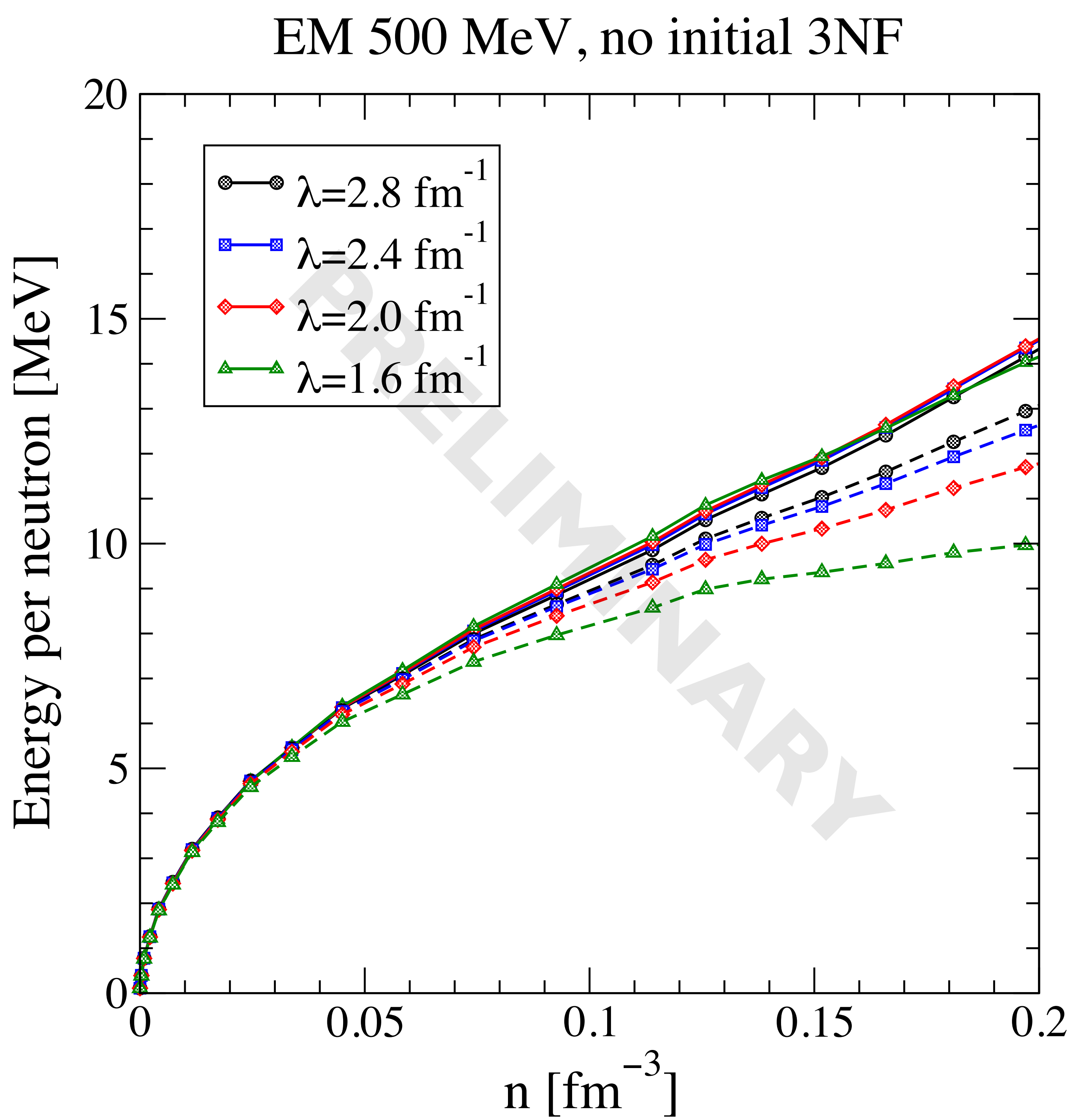
- evolve consistently NN + 3NF in the isospin $\mathcal{T} = 3/2$ channel
- calculate EOS by taking all blue-boxed contributions into account
- in this approximation NN and 3NF contributions factorize

First results for neutron matter



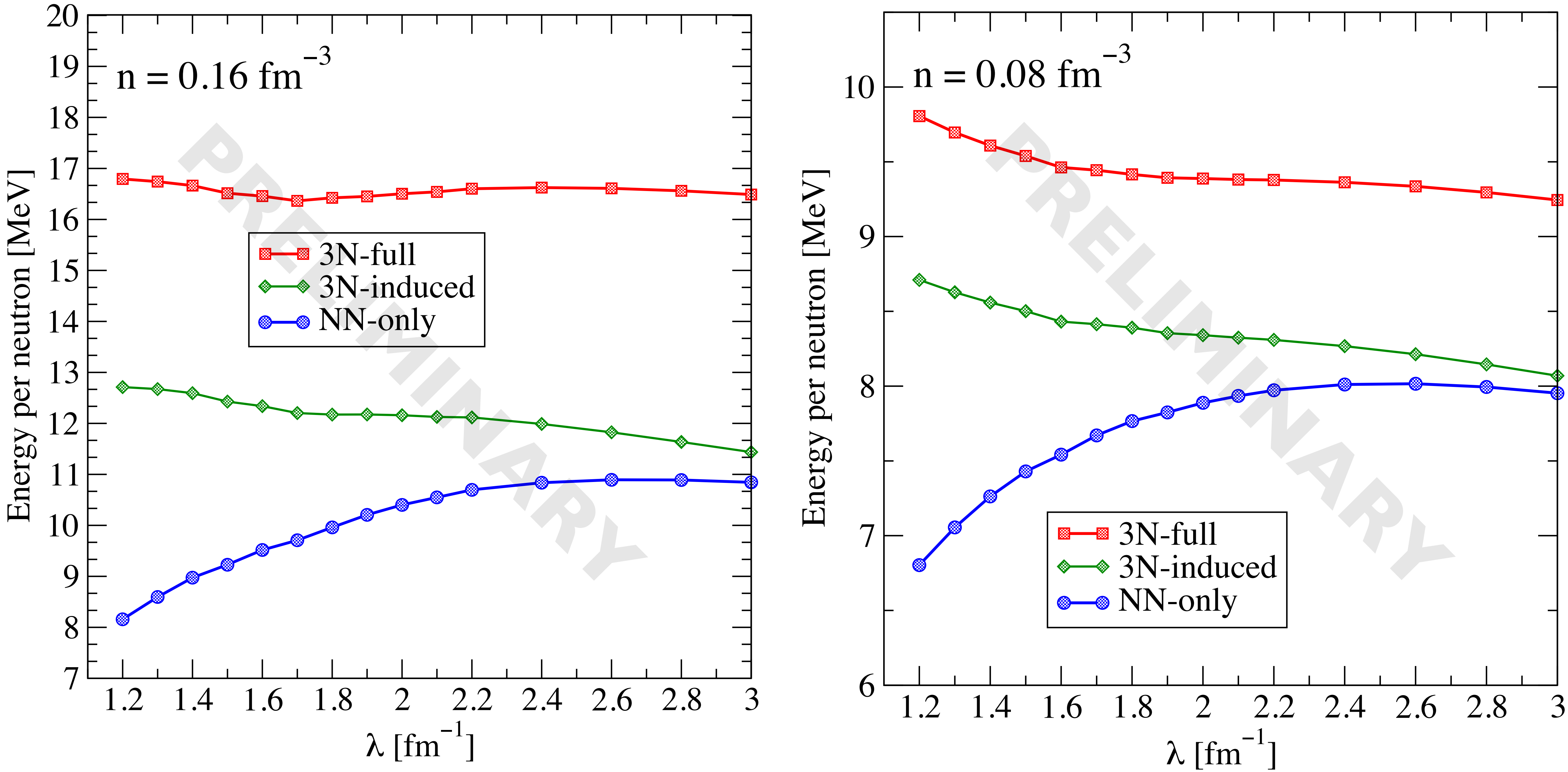
- all channels included up to $\mathcal{J} = 7/2$ in SRG evolution and EOS calculation

First results for neutron matter

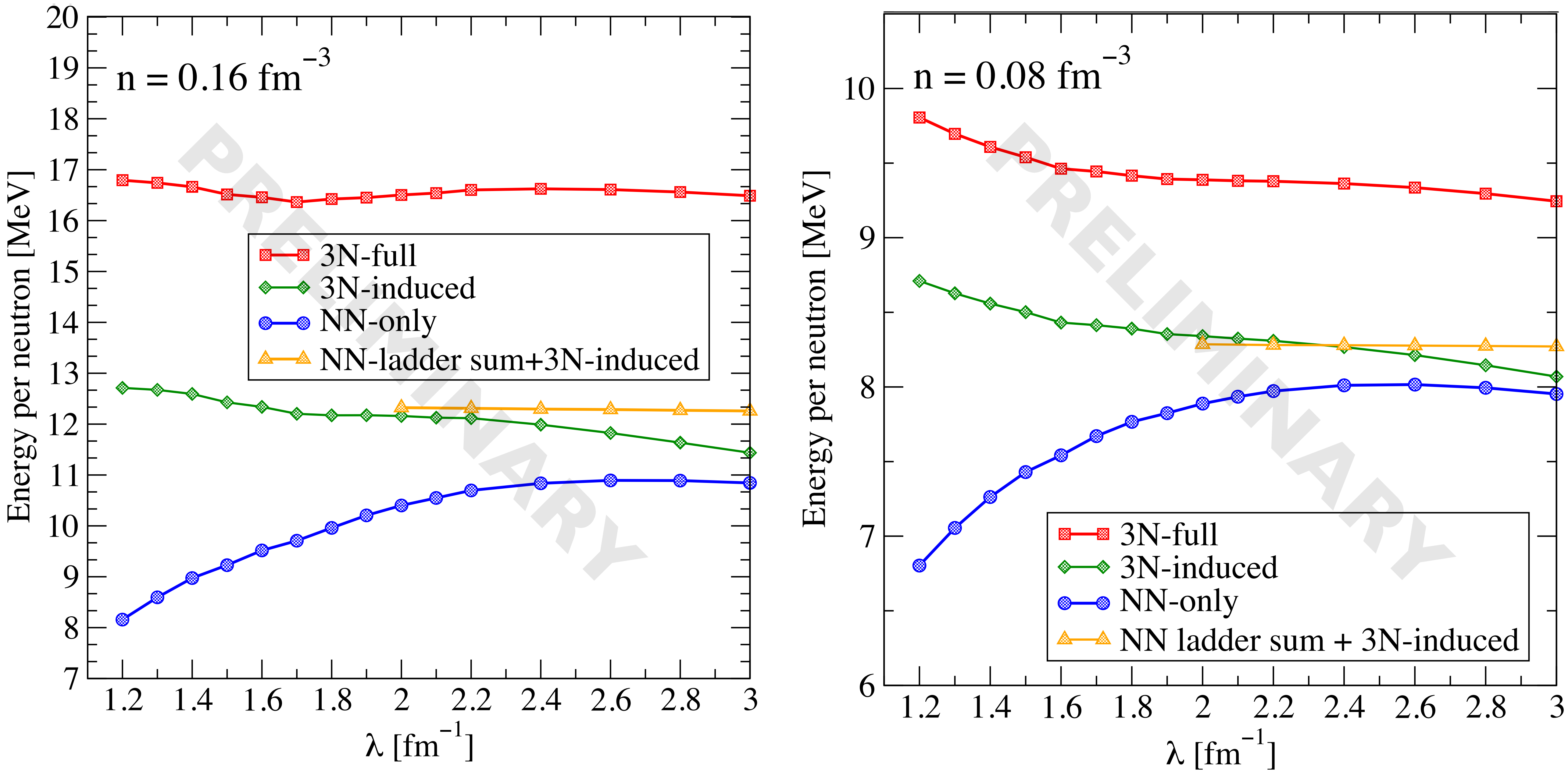


- all channels included up to $\mathcal{J} = 7/2$ in SRG evolution and EOS calculation

Resolution-scale dependence (HF + 2nd-order NN)

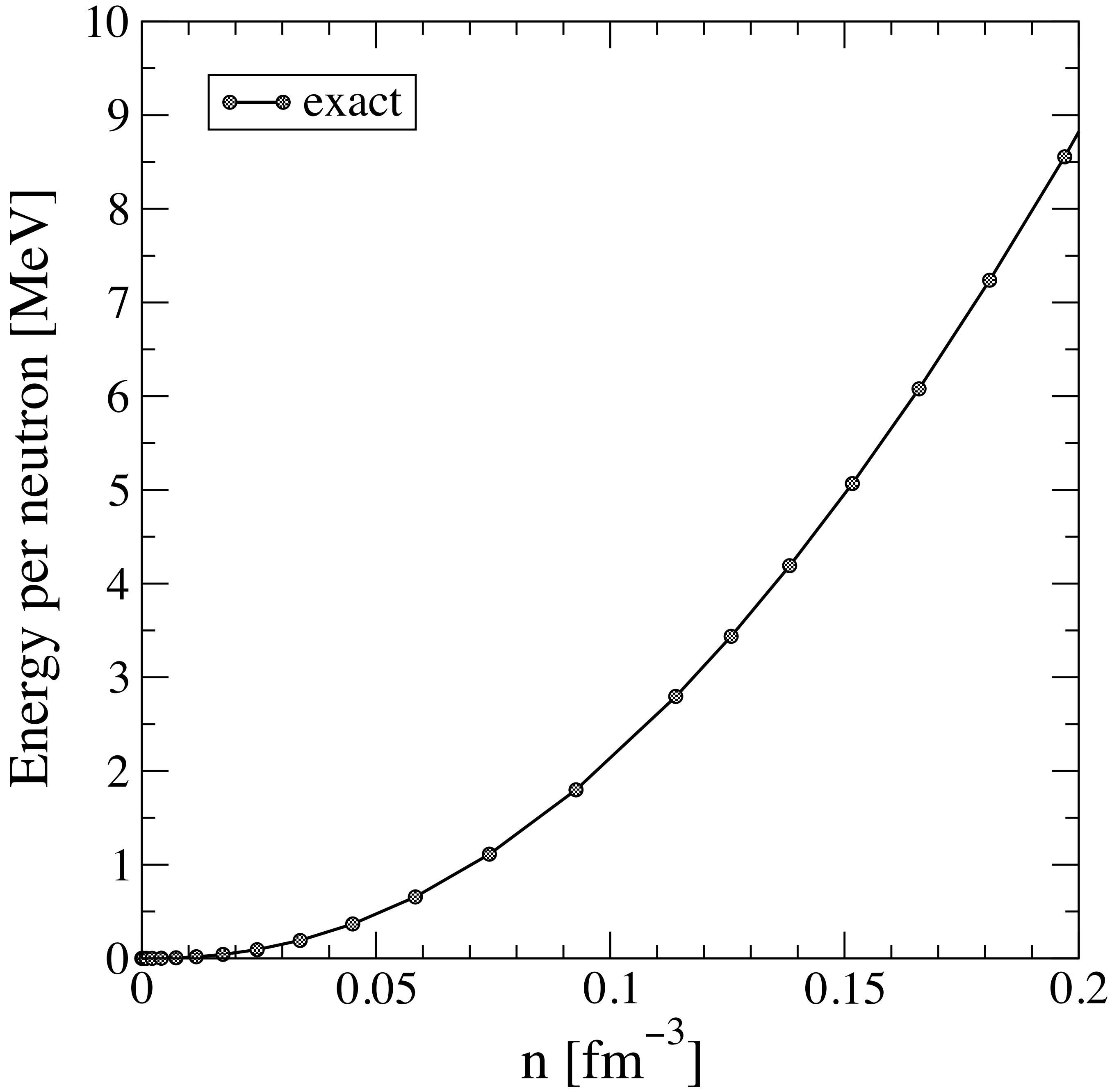


Resolution-scale dependence (HF + 2nd-order NN)

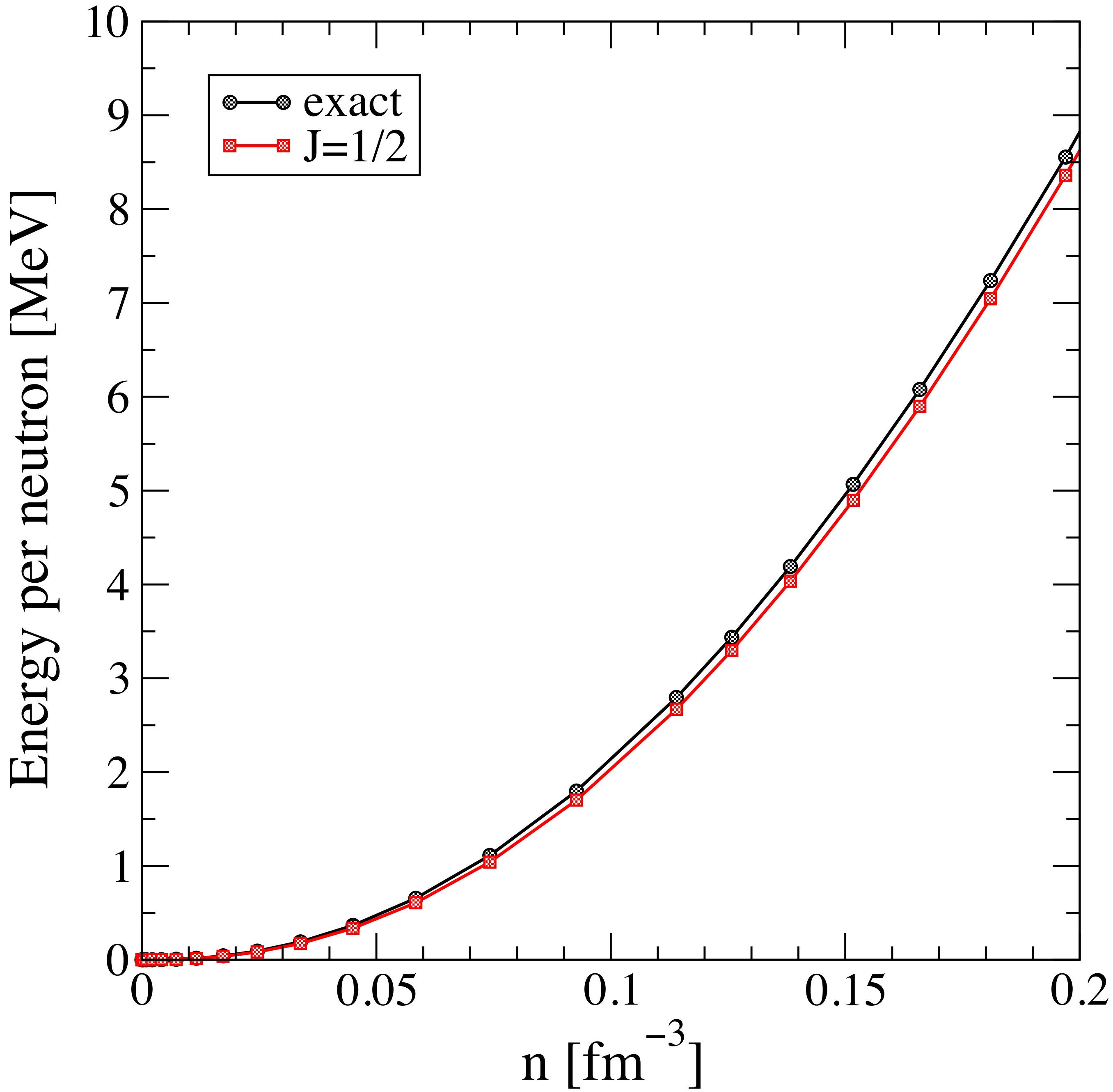


- below $\lambda \lesssim 1.8\,\mathrm{fm}^{-1}$ EOS results invariant within theoretical uncertainties
- at larger scale higher-order many-body diagrams become important

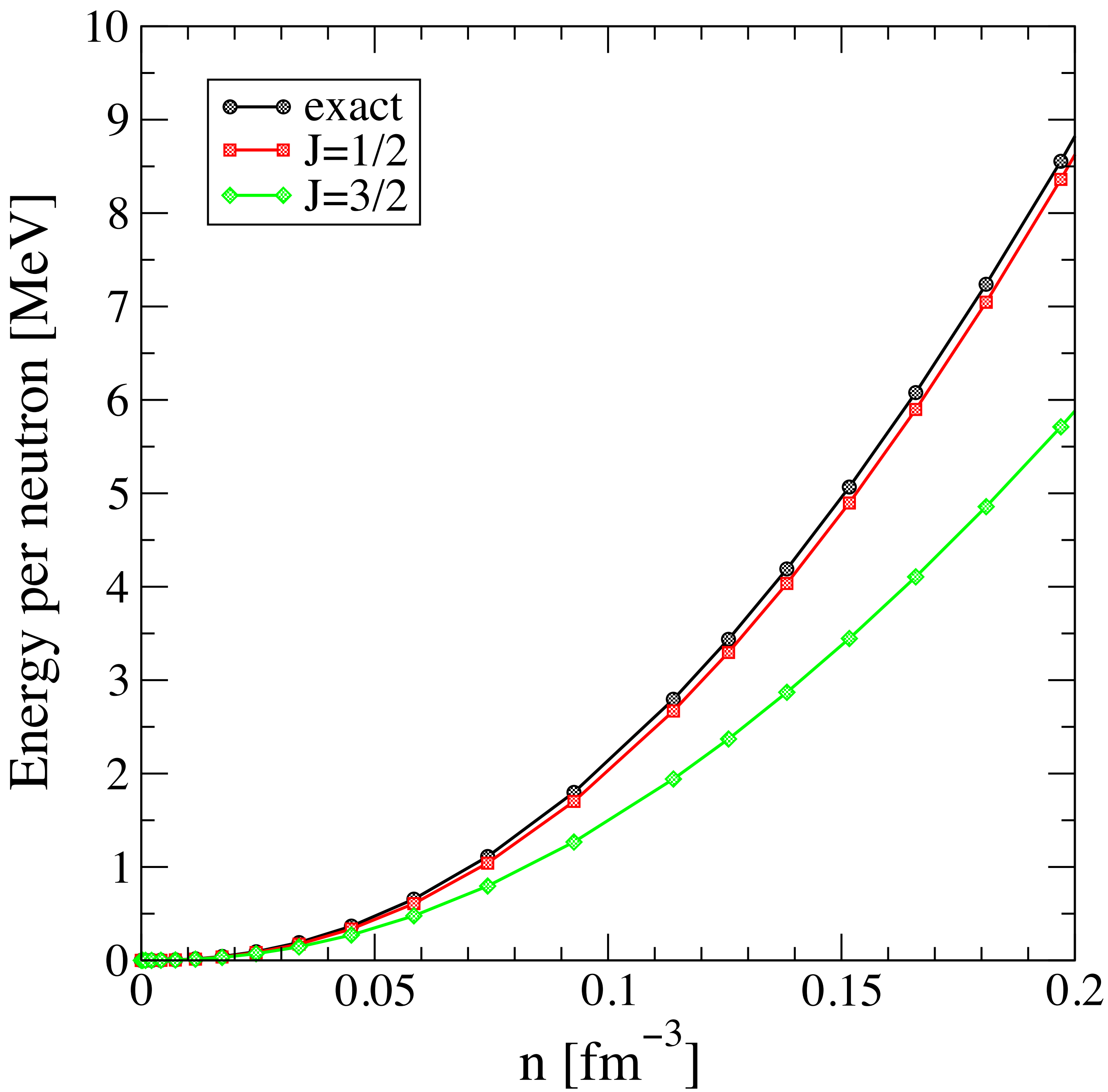
Partial-wave convergence of 3NF contributions



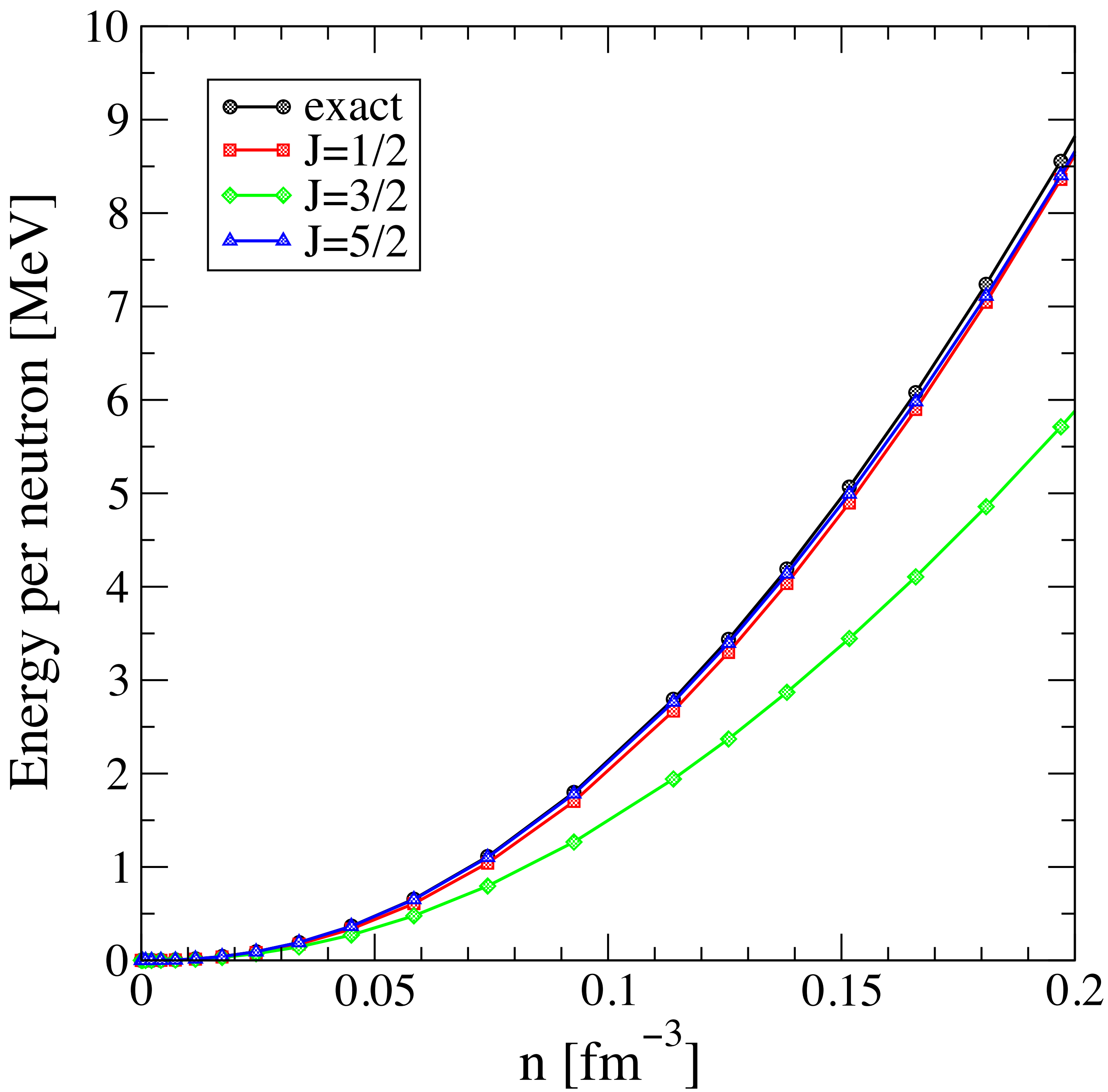
Partial-wave convergence of 3NF contributions



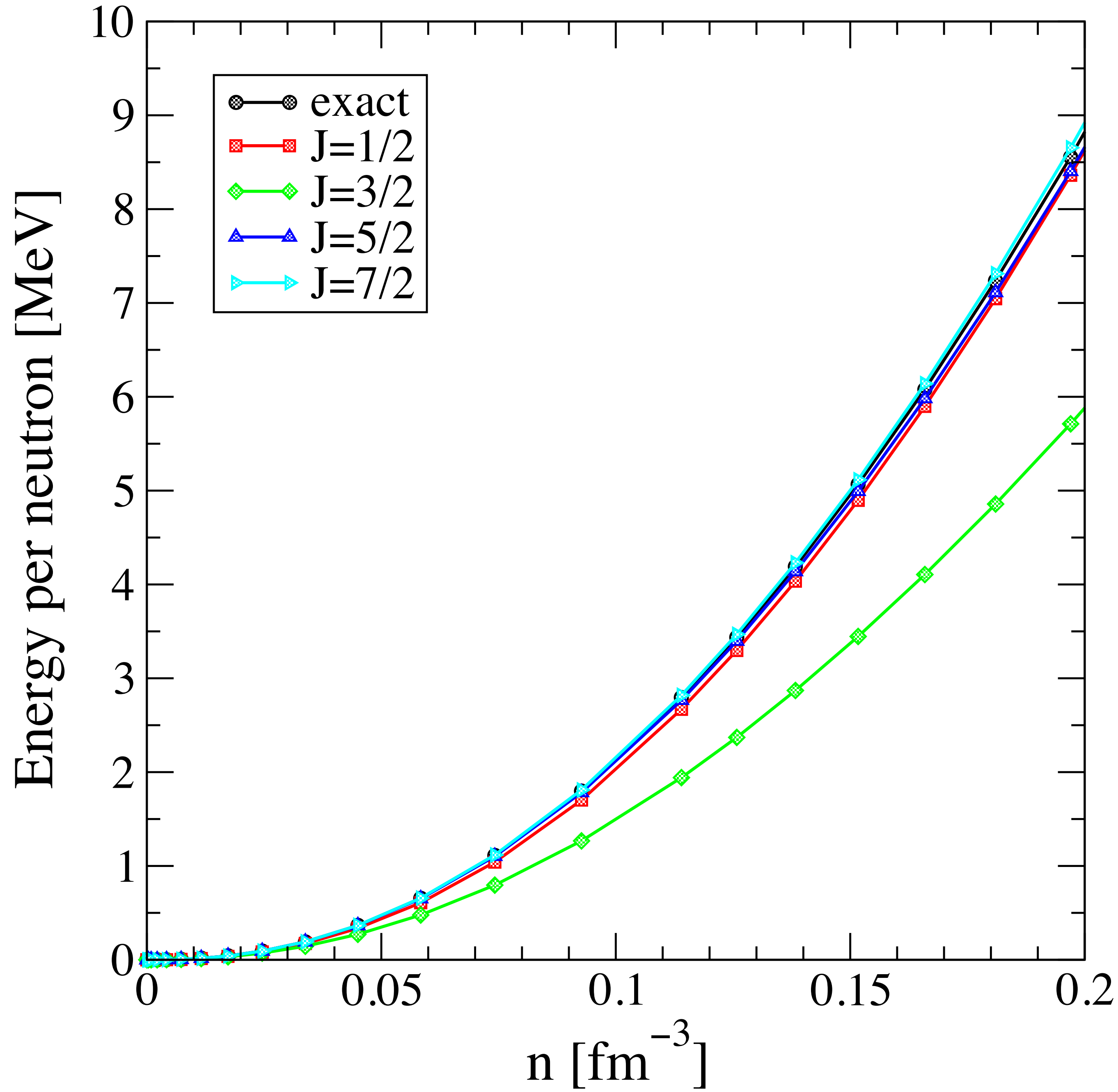
Partial-wave convergence of 3NF contributions



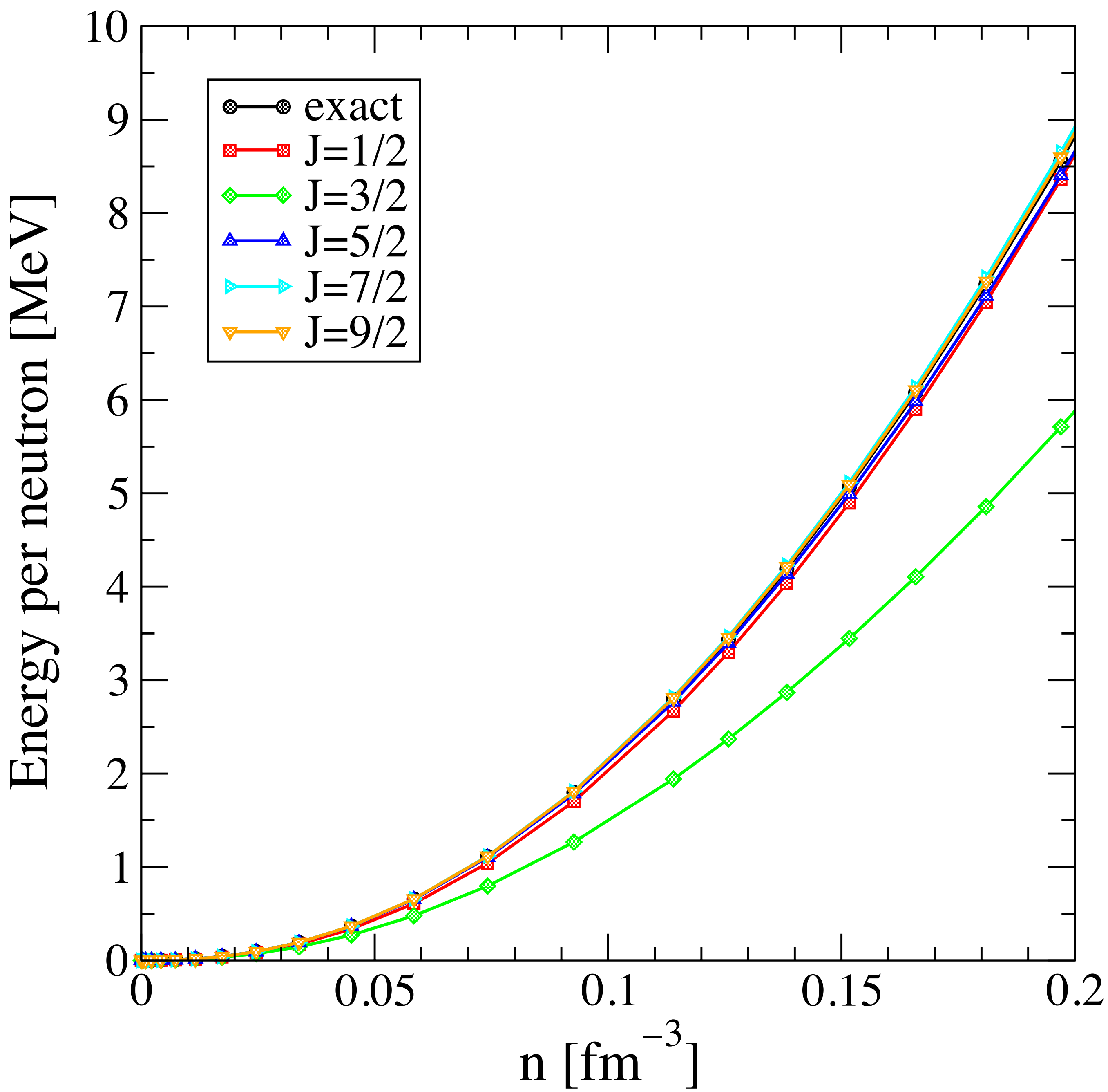
Partial-wave convergence of 3NF contributions



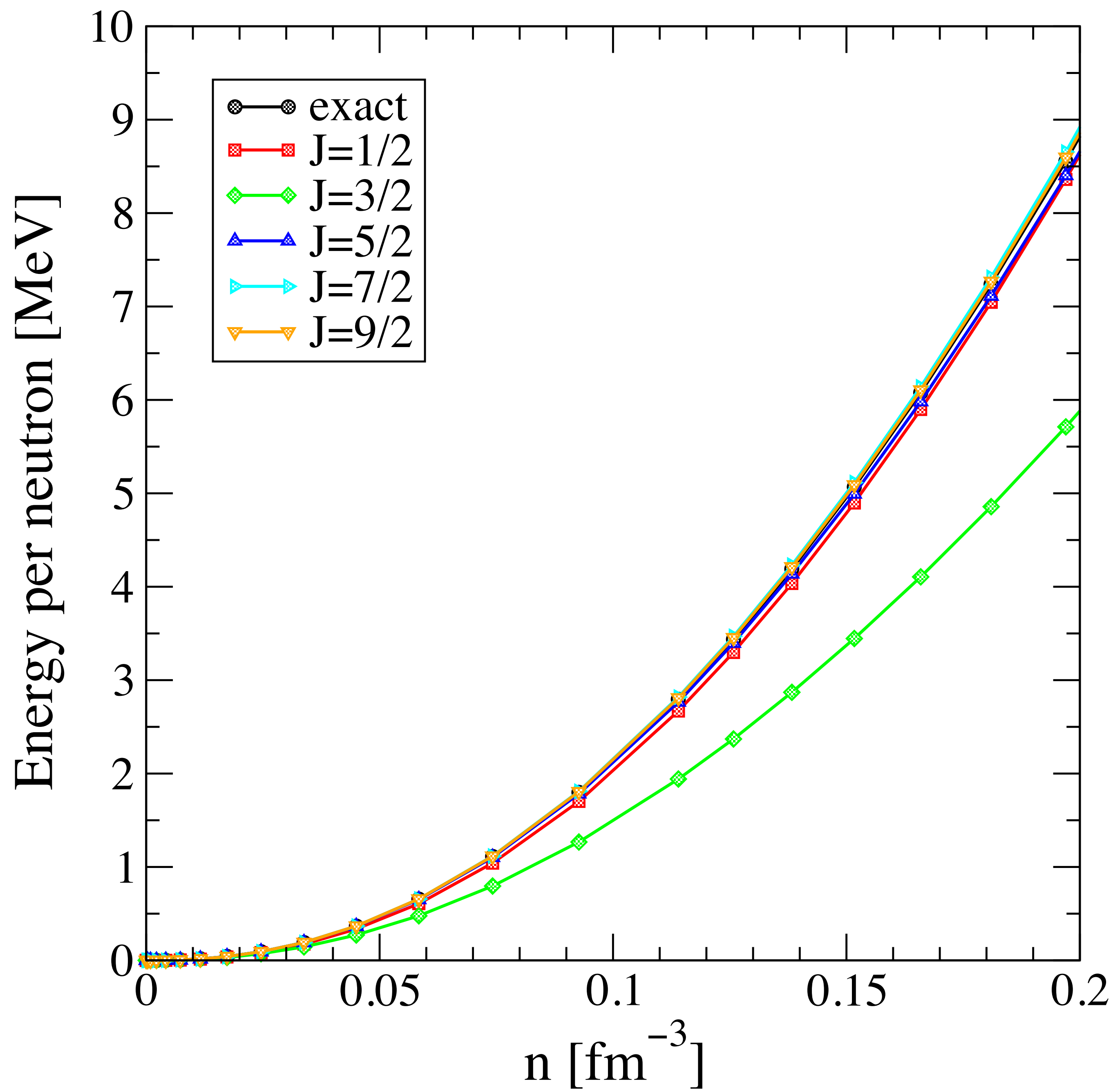
Partial-wave convergence of 3NF contributions



Partial-wave convergence of 3NF contributions



Partial-wave convergence of 3NF contributions



- energy in HF approximation agrees within $50\,\mathrm{keV}$ with exact result at nuclear saturation density for $\mathcal{J}_{\max} = 7/2$ at $s = 0$

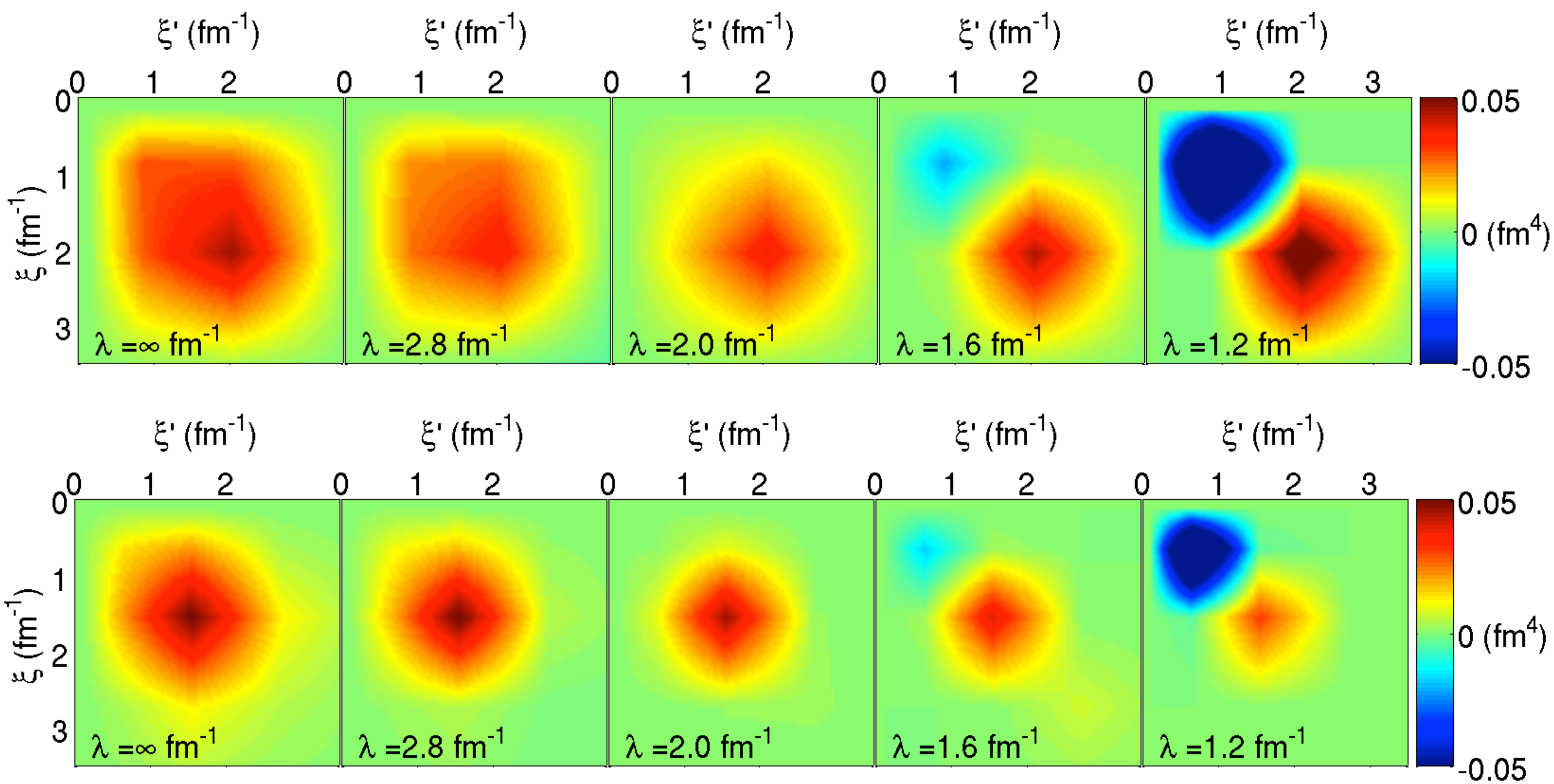
Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4} q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \frac{\pi}{20}$

$\theta = \frac{\pi}{15}$

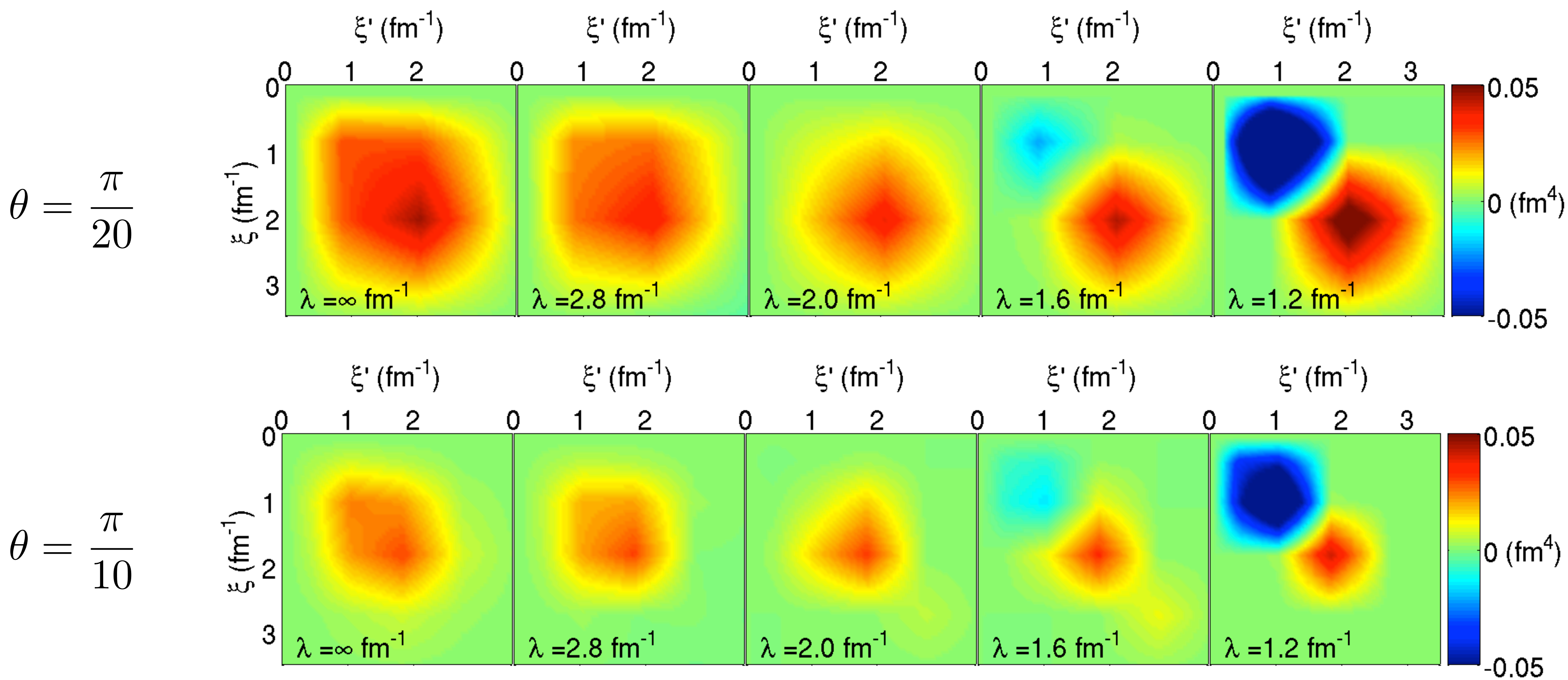


- strong renormalization effects of long-range two-pion exchange!

Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:



- strong renormalization effects of long-range two-pion exchange!

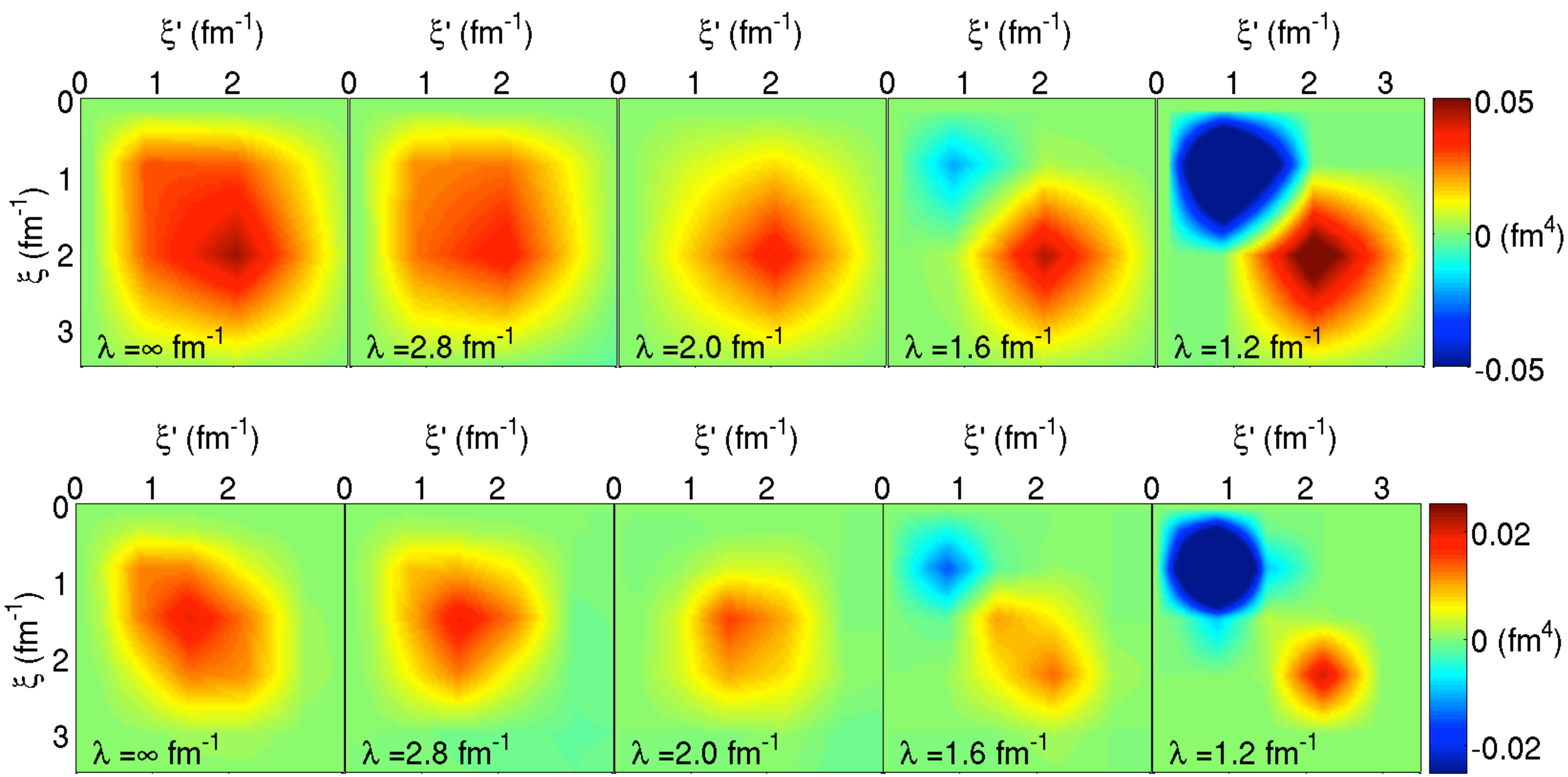
Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \dfrac{\pi}{20}$

$\theta = \dfrac{\pi}{8}$



- strong renormalization effects of long-range two-pion exchange!

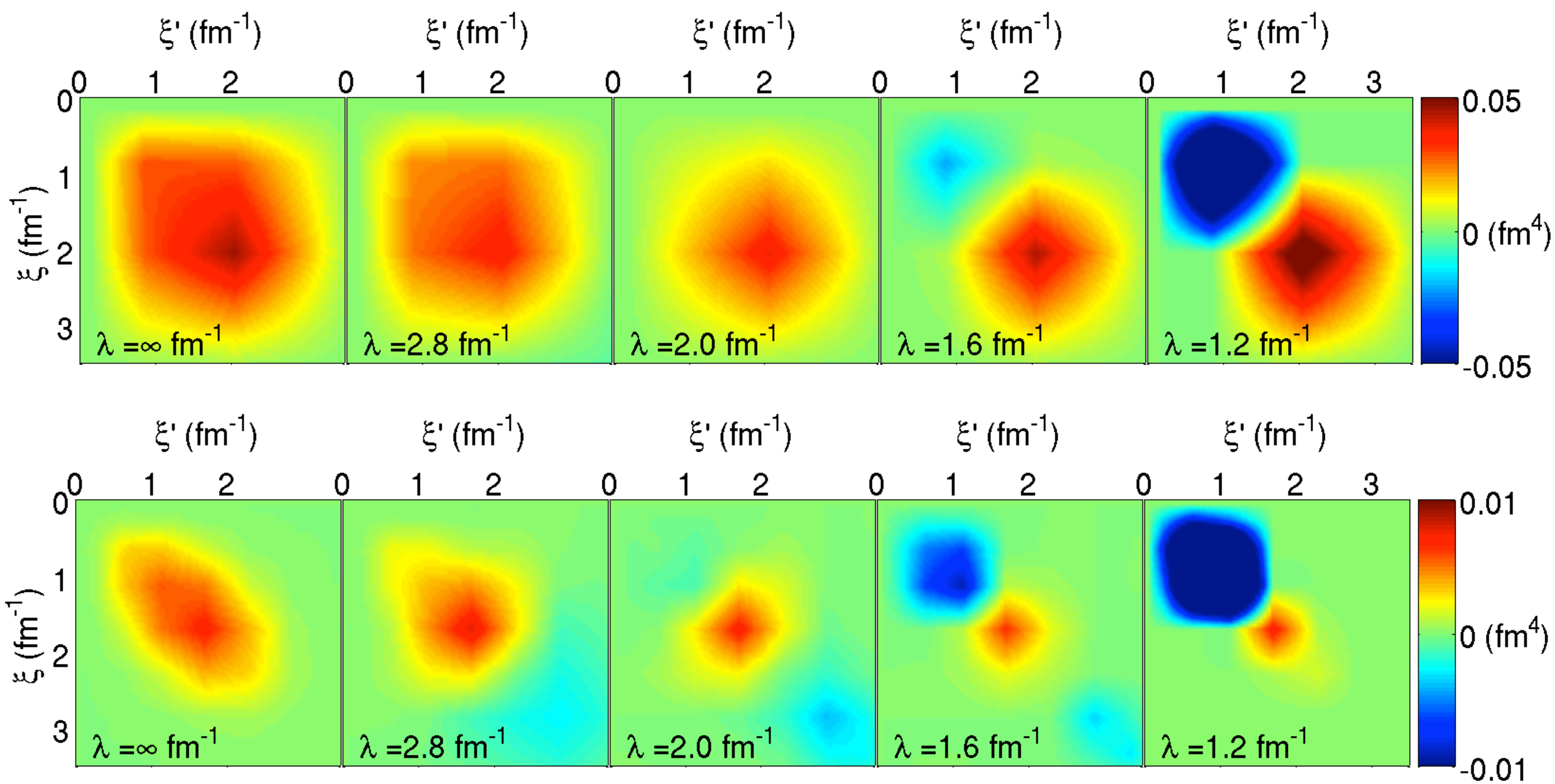
Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4} q^2 \qquad\qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \frac{\pi}{20}$

$\theta = \frac{\pi}{6}$



- strong renormalization effects of long-range two-pion exchange

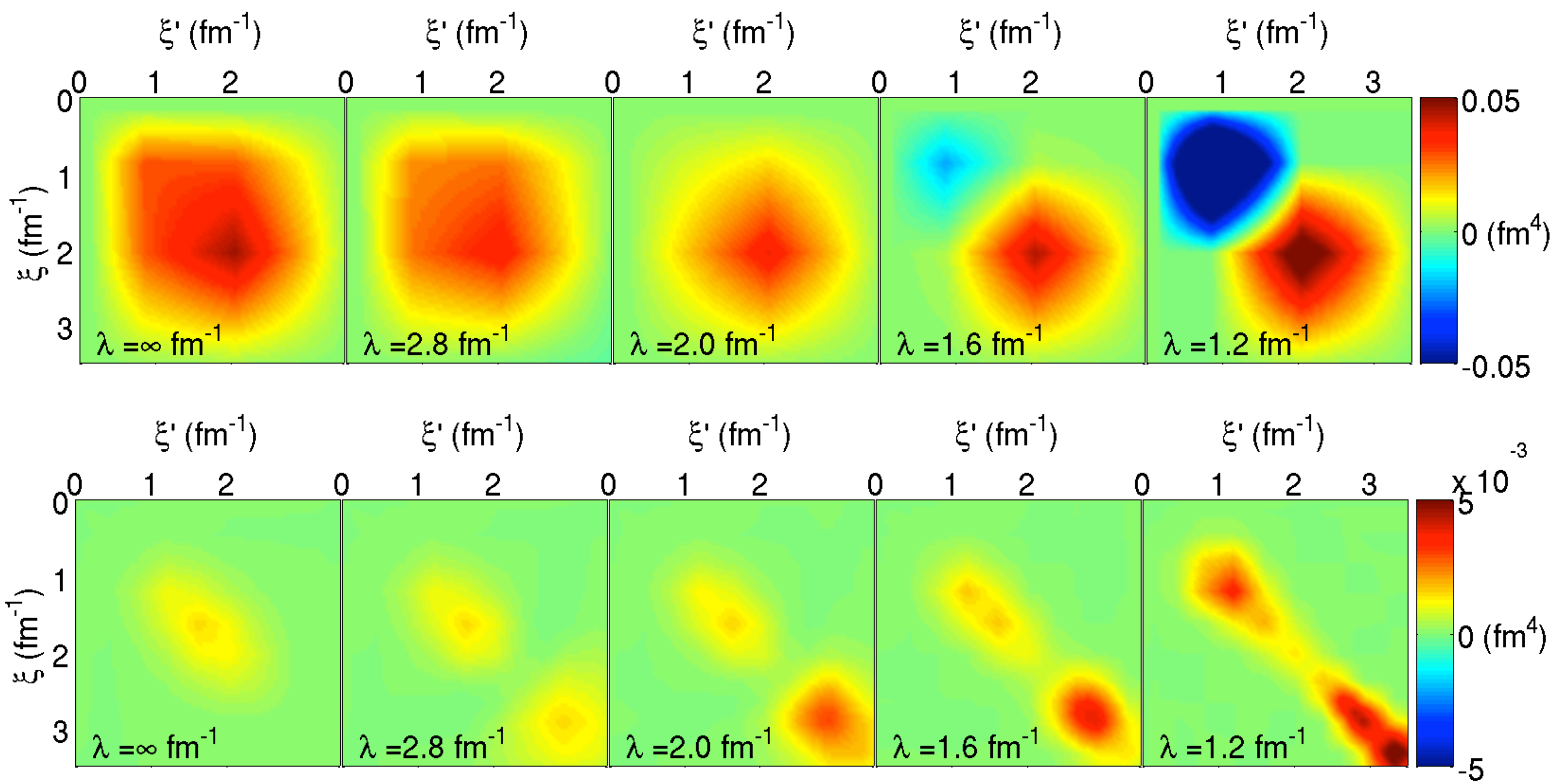
Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad\qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$$\theta = \frac{\pi}{20}$$

$$\theta = \frac{\pi}{4}$$



- strong renormalization effects of long-range two-pion exchange

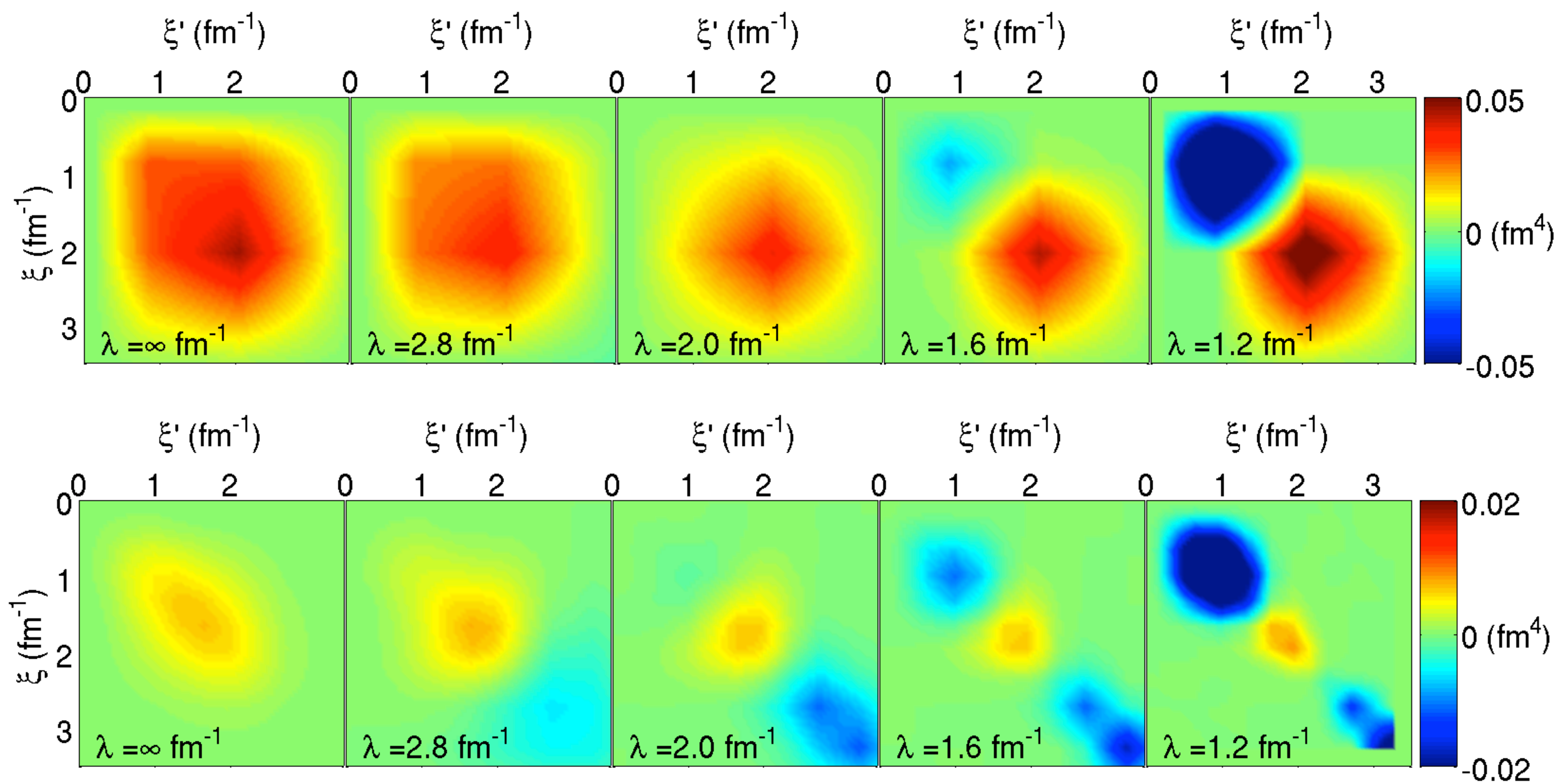
Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad\qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$$\theta = \frac{\pi}{20}$$

$$\theta = \frac{\pi}{3}$$



- strong renormalization effects of long-range two-pion exchange

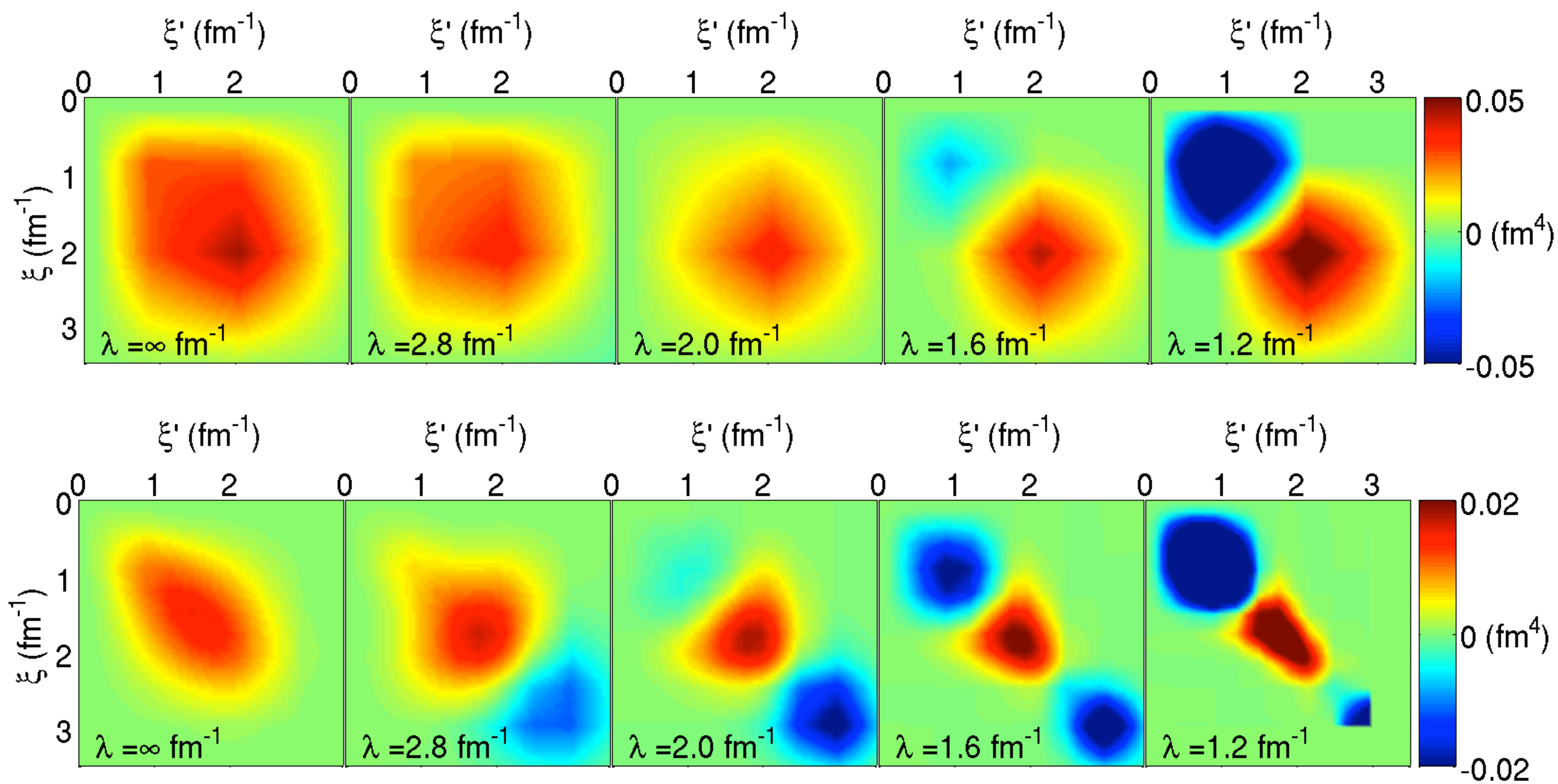
Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

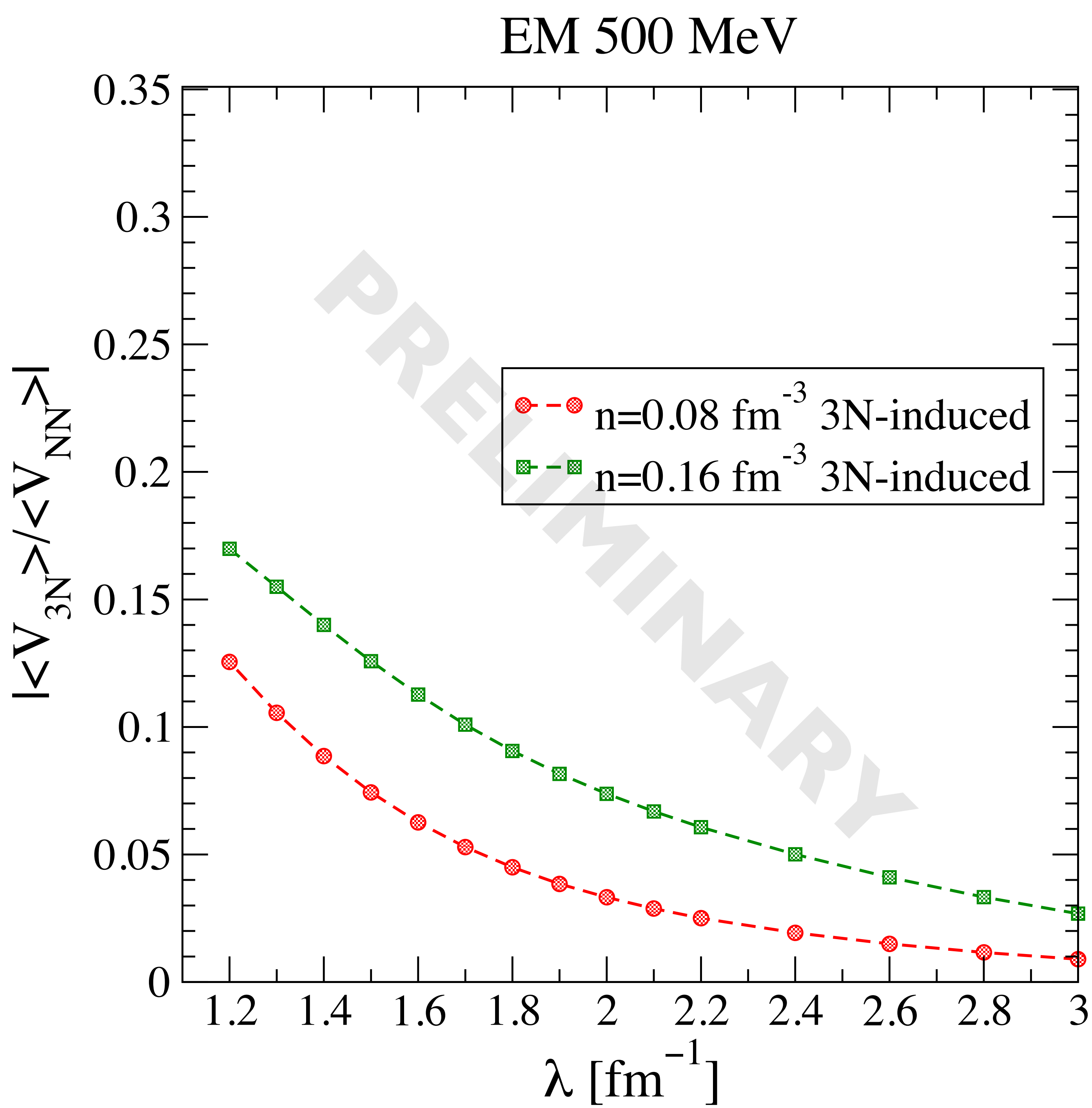
$$\theta = \frac{\pi}{20}$$

$$\theta = \frac{\pi}{2.5}$$

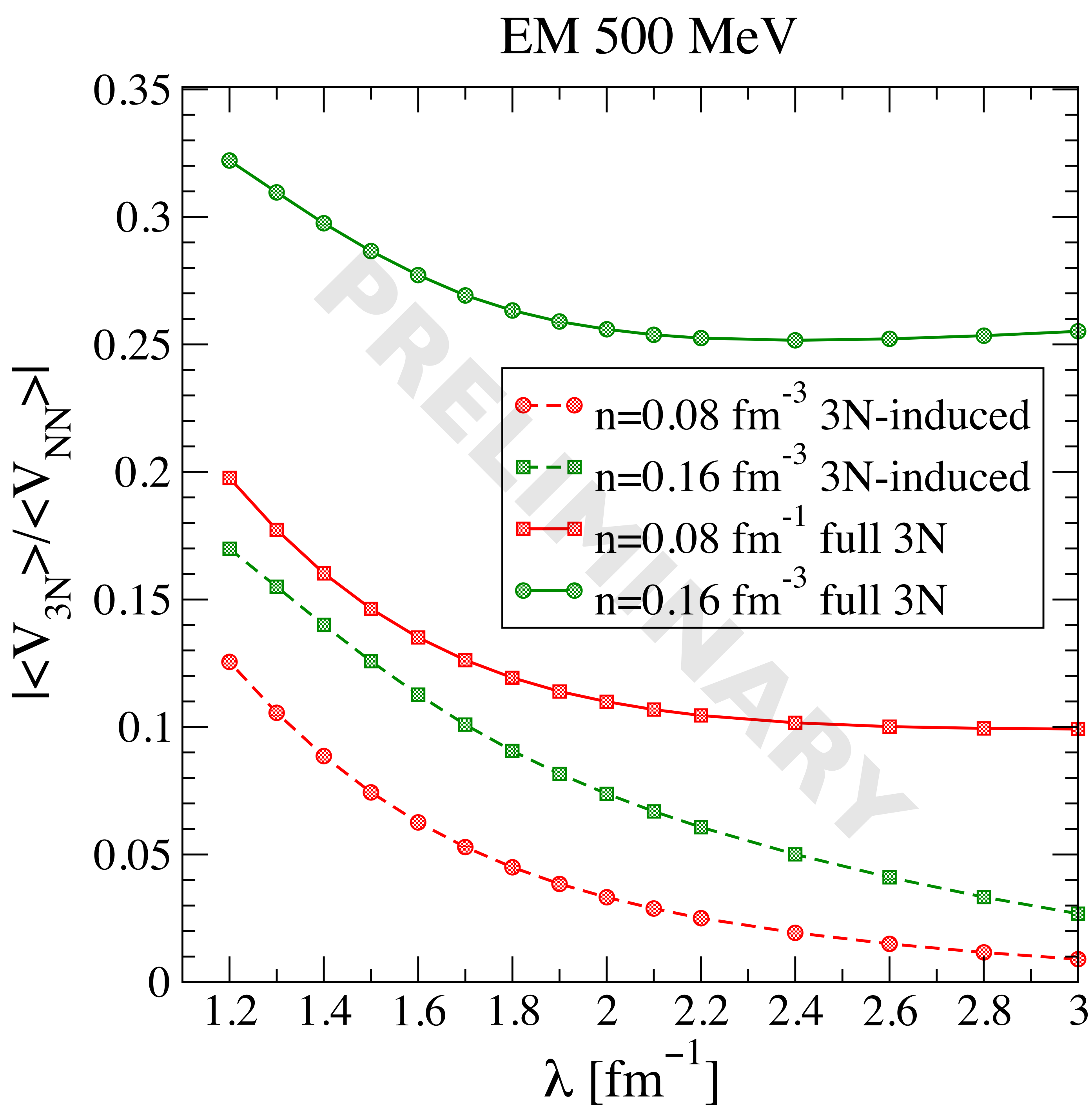


- strong renormalization effects of long-range two-pion exchange

Scaling of three-body contributions



Scaling of three-body contributions



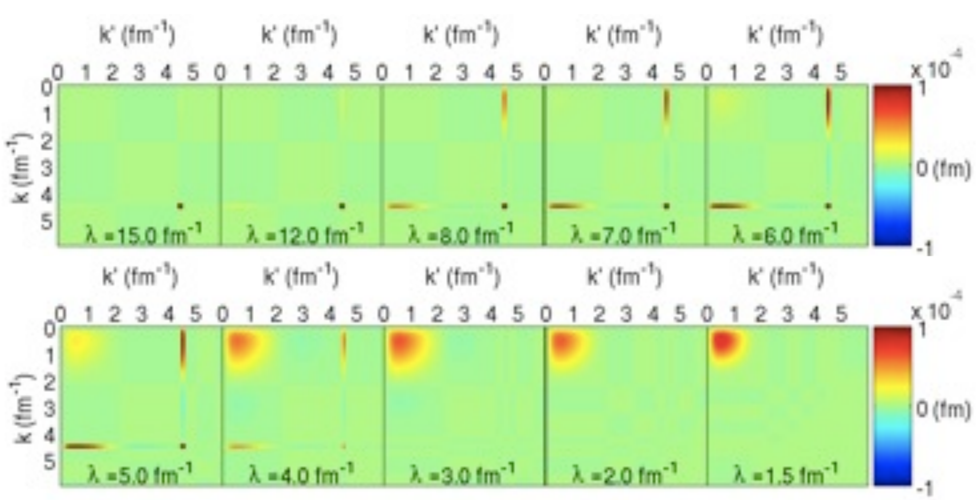
- at larger resolution scales ratio depends on details of the interaction
- at low resolution, 3N contributions seem to grow systematically, however apparently no simple power law (still under investigation)

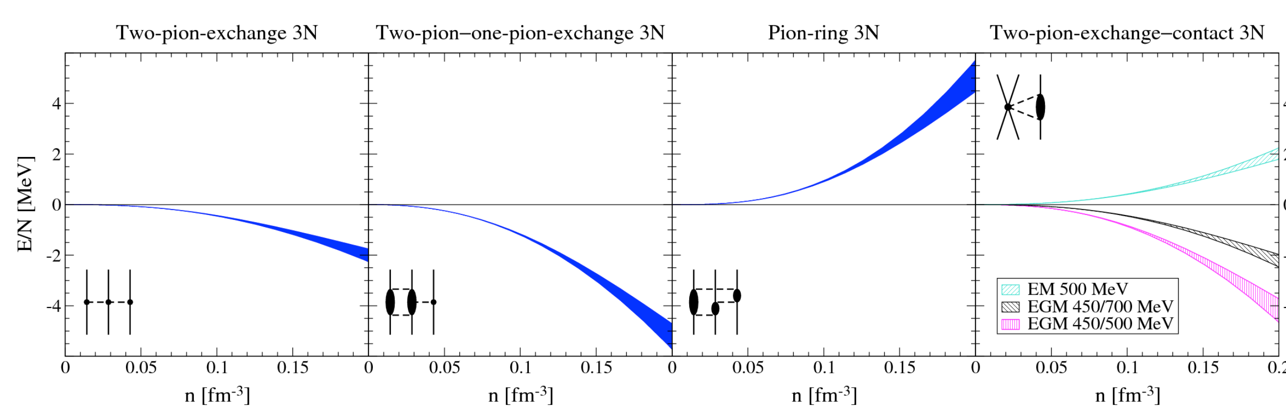
Summary

- demonstrated the feasibility of SRG evolution of NN+3NF in momentum space
- first results of neutron matter based on consistently evolved NN+3NF interactions
- strong renormalization effects of chiral two-pion exchange interaction in neutron matter
- 4N force contributions seem to be small in neutron matter down to $\lambda = 1.2\,\mathrm{fm}^{-1}$

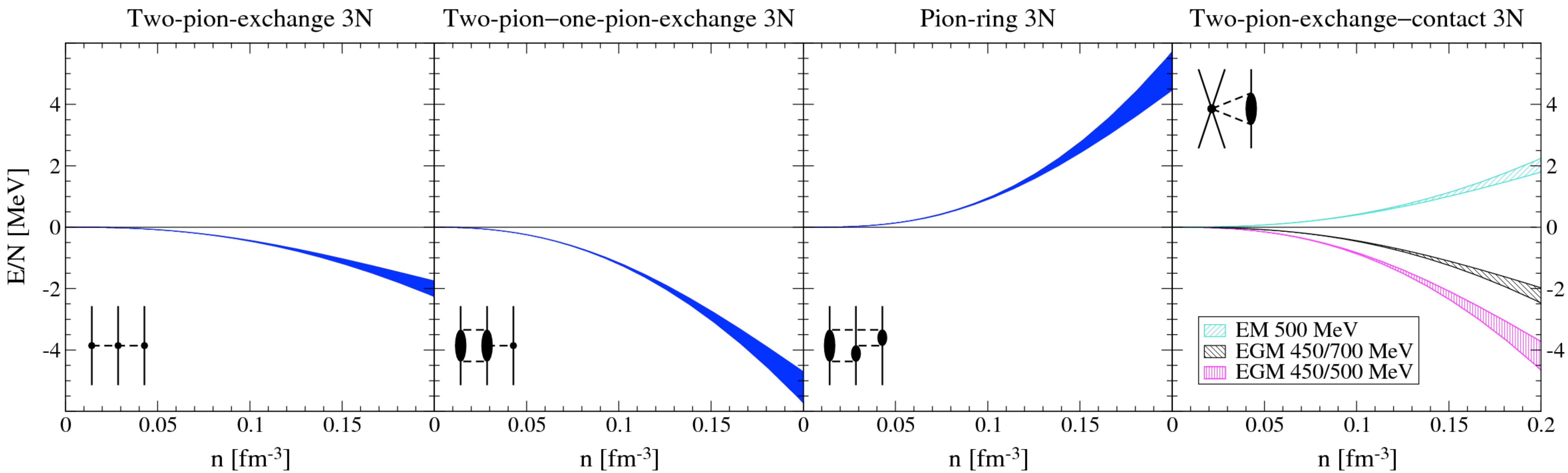
Outlook

- inclusion of N3LO contributions
- extend RG evolution to $\mathcal{T} = 1/2$ channels, application to nuclear matter and finite nuclei
- RG evolution of operators: nuclear scaling and correlations in nuclear systems





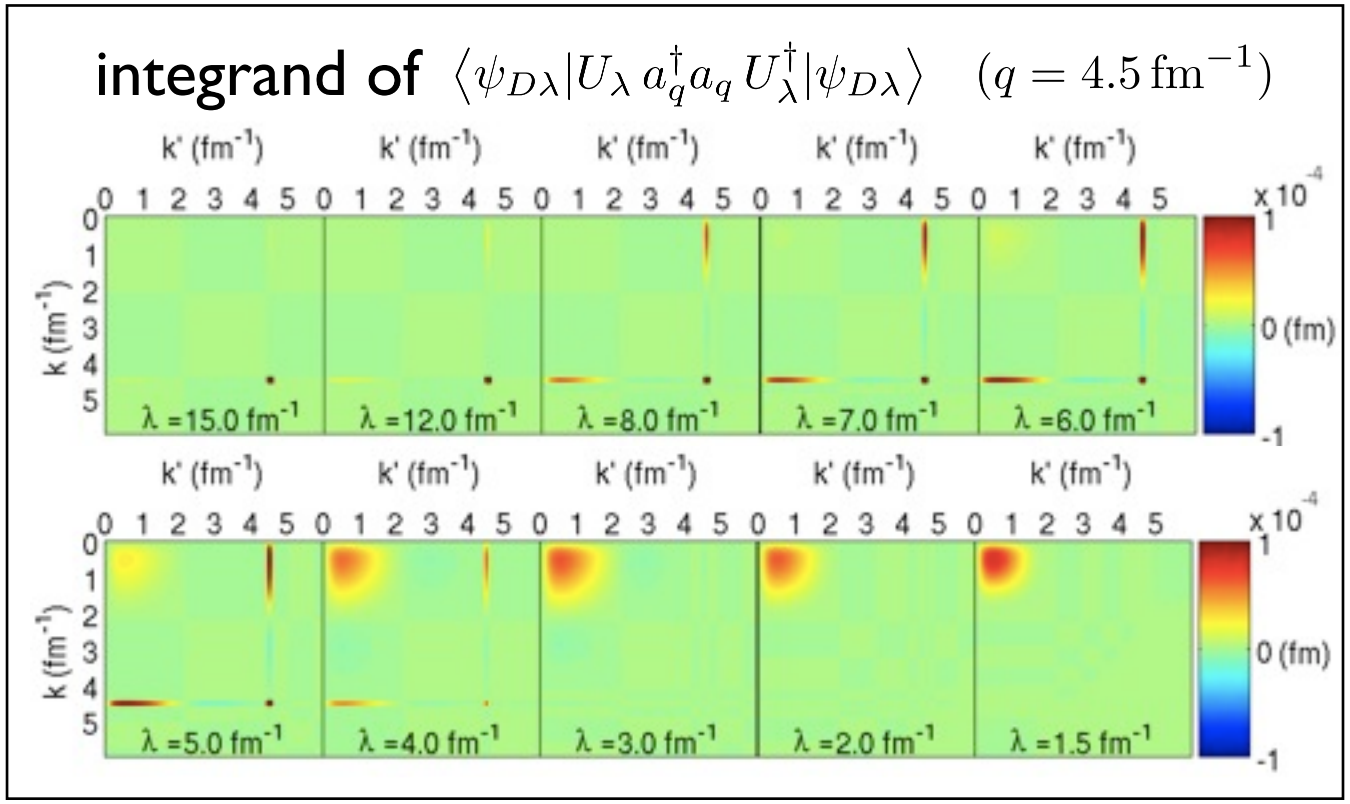
Effects of N3LO 3NF contributions: Neutron matter

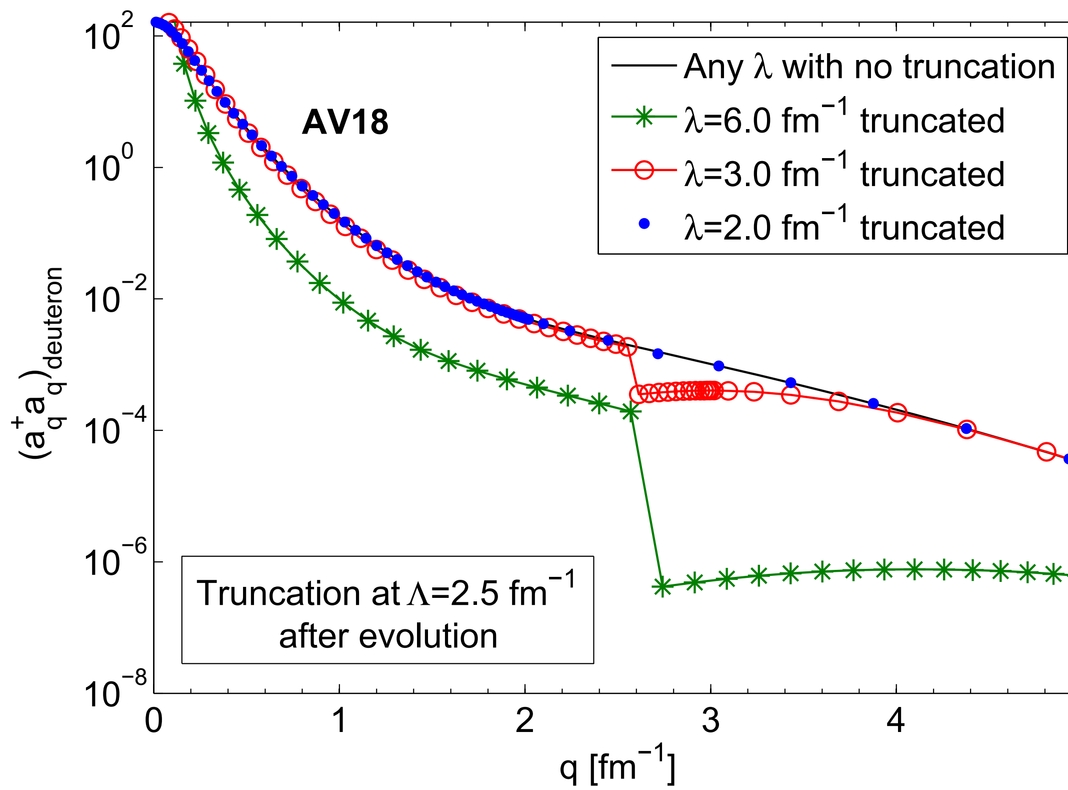


Thews, Krueger, KH and Schwenk, arXiv:1206.0025

Reminder:
SRG evolution of operators (see Scott's and Dick's talk)

Instructive test case: density operator in the deuteron



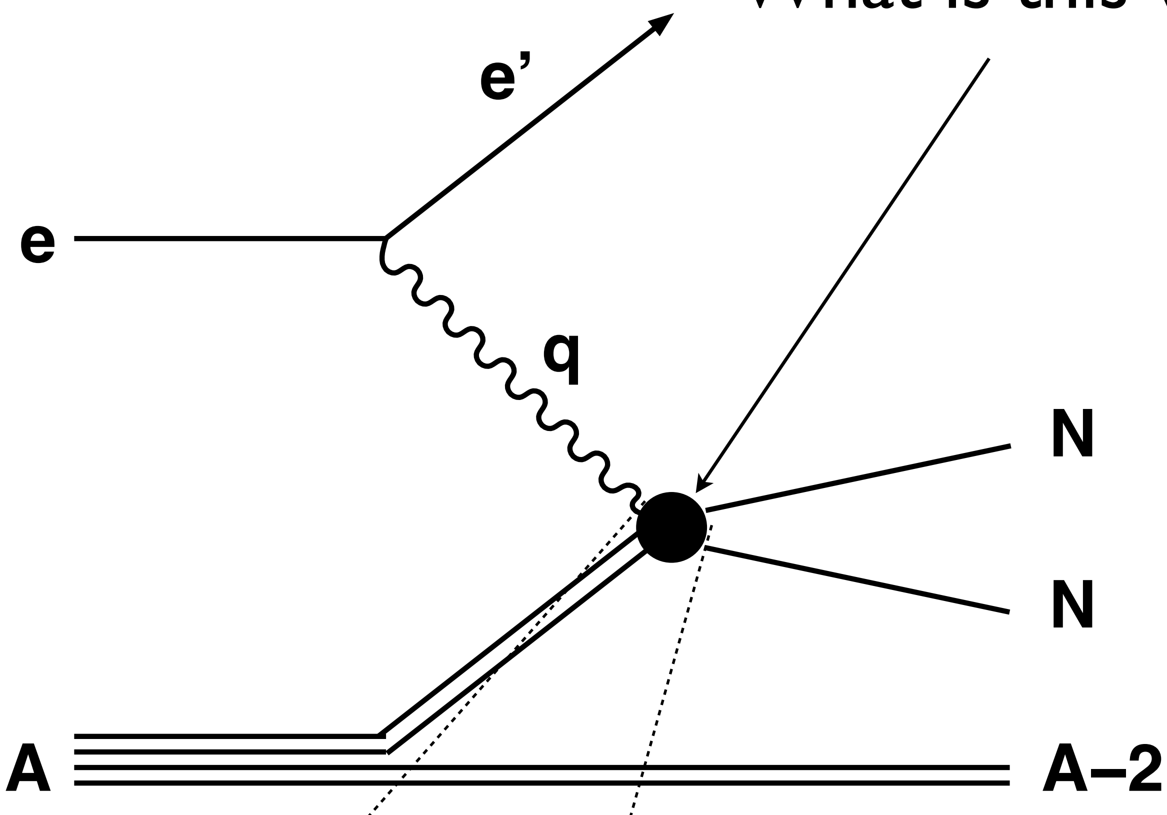


Anderson, Bogner, Furnstahl, Perry, PRC 82, 054001 (2010)

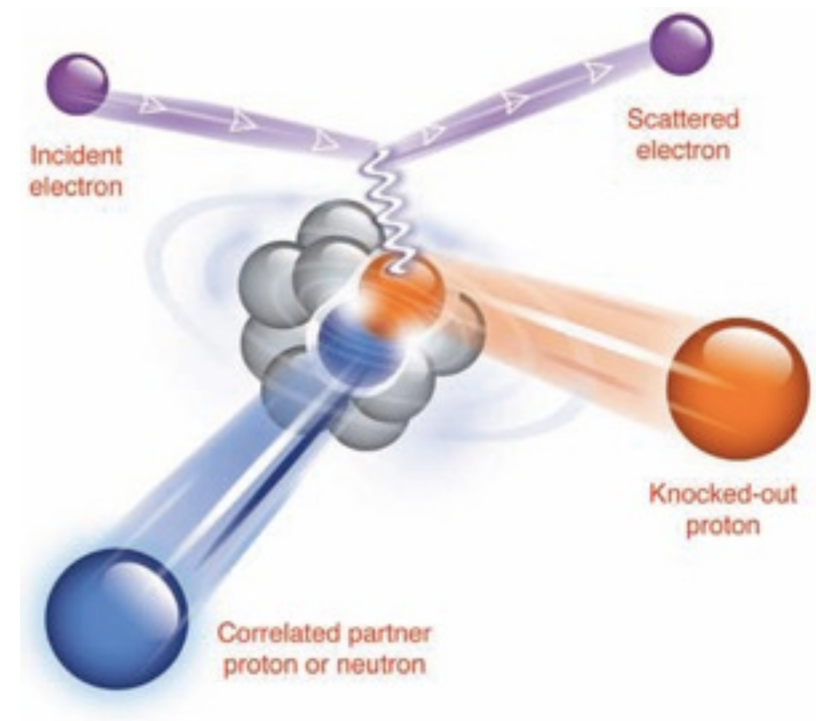
- perfect invariance of momentum distribution function with evolved density operator
- strong/slight evolution of short/long-distance operators
- $U_\lambda(k, q)$ factorizes for $k < \lambda$ and $q \gg \lambda$: $U_\lambda(k, q) \approx K_\lambda(k) Q_\lambda(q)$

Correlations in nuclear systems (→talk by Dick Furnstahl next week)

What is this vertex?



Higinbotham, arXiv:1010.4433

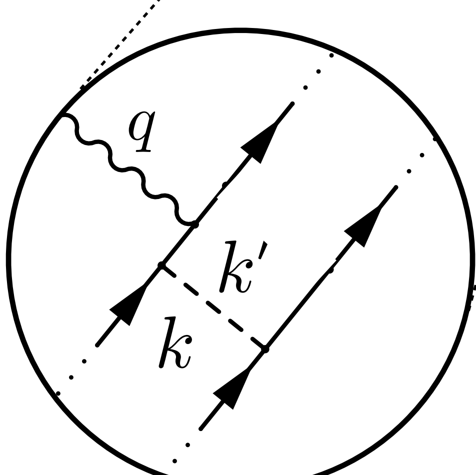


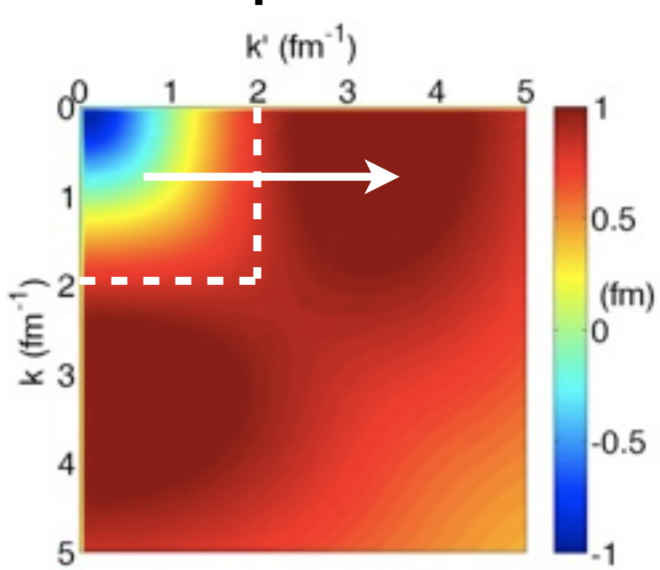
- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

Subedi et al., Science 320, 1476 (2008)

Explanation in terms of low-momentum interactions?

Short-range-correlation interpretation:



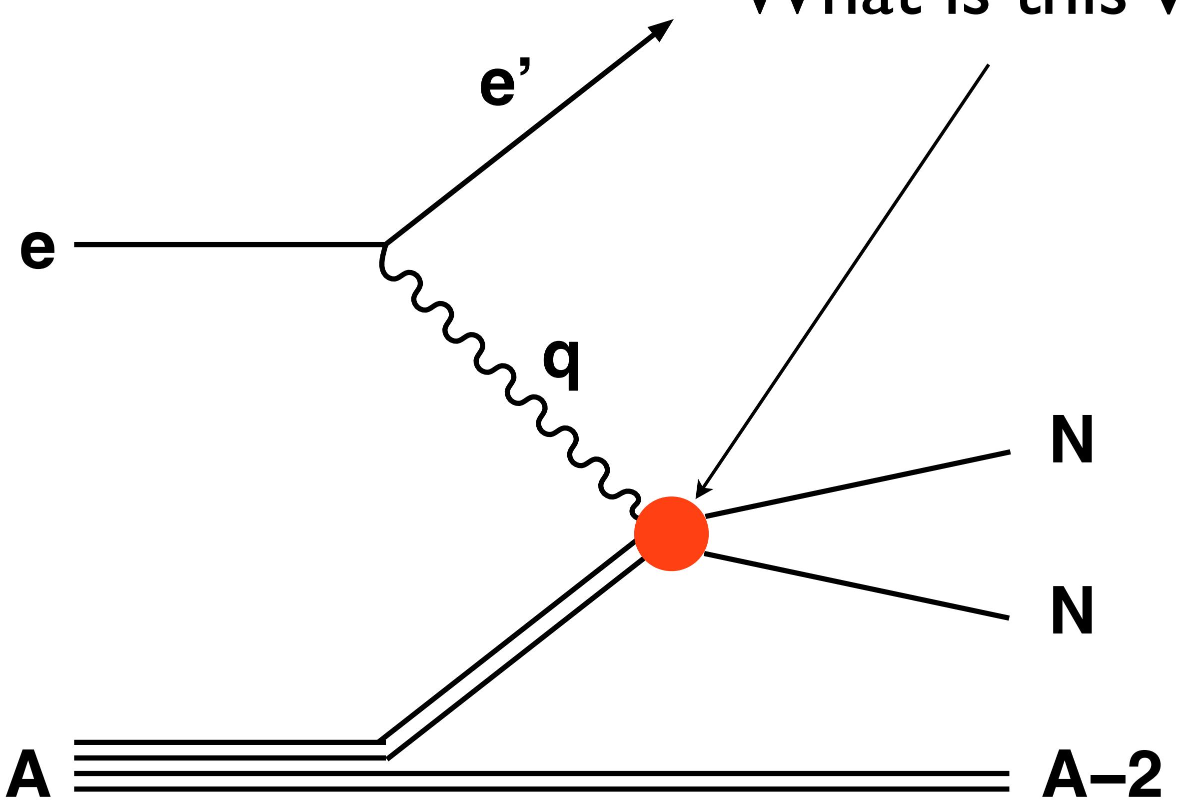


k : low rel. momentum

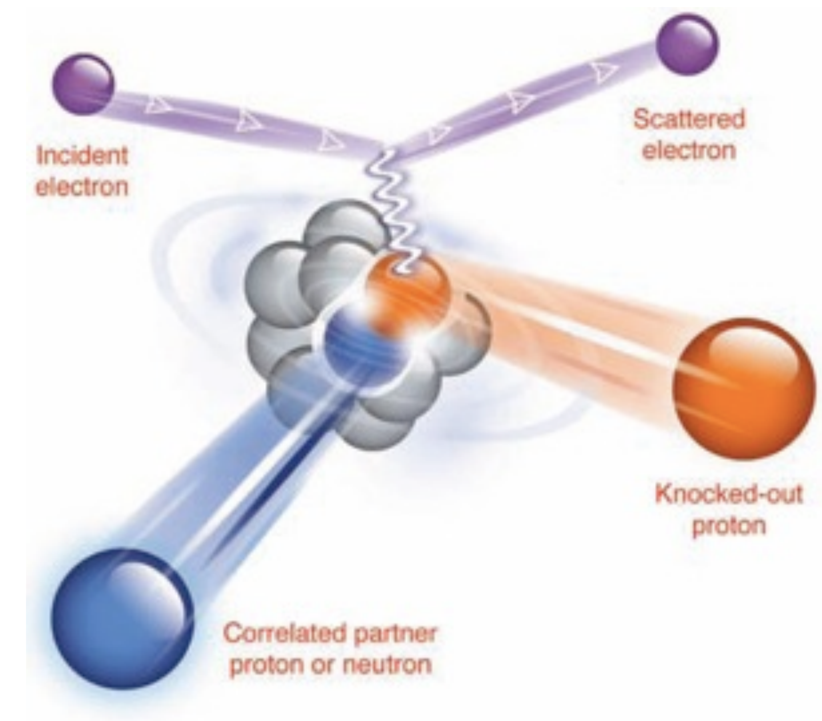
k' : high rel. momentum

Correlations in nuclear systems (⟶talk by Dick Furnstahl next week)

What is this vertex?



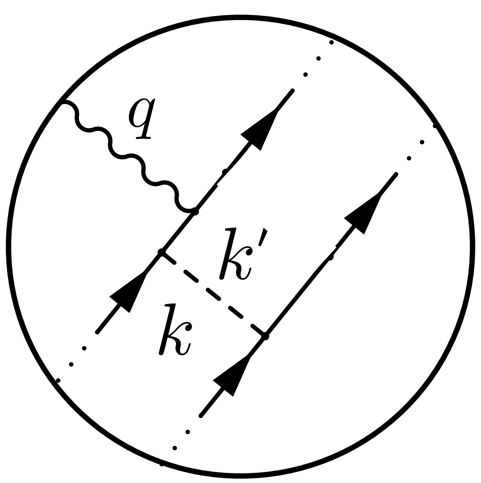
Higinbotham, arXiv:1010.4433



- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

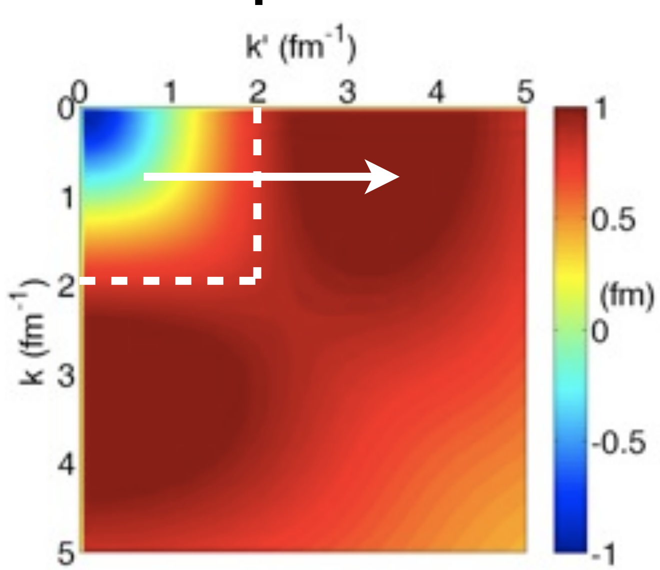
Subedi et al., Science 320, 1476 (2008)

Short-range-correlation interpretation:



k : low rel. momentum

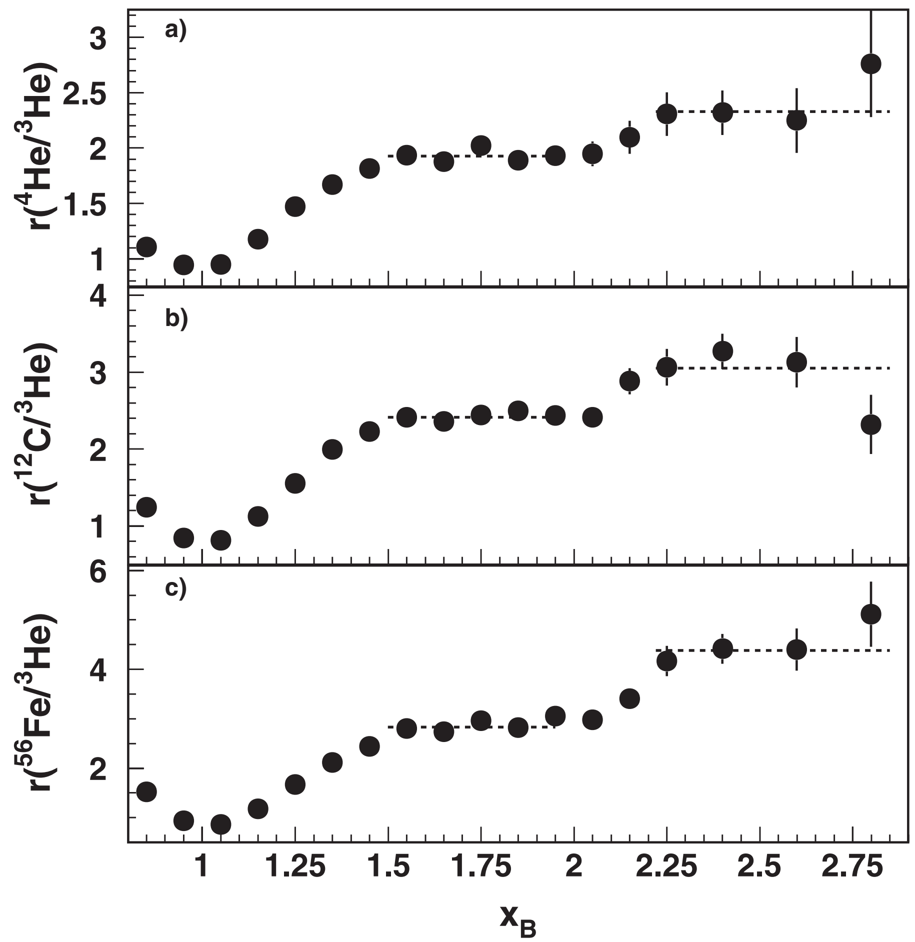
k' : high rel. momentum



Explanation in terms of low-momentum interactions?

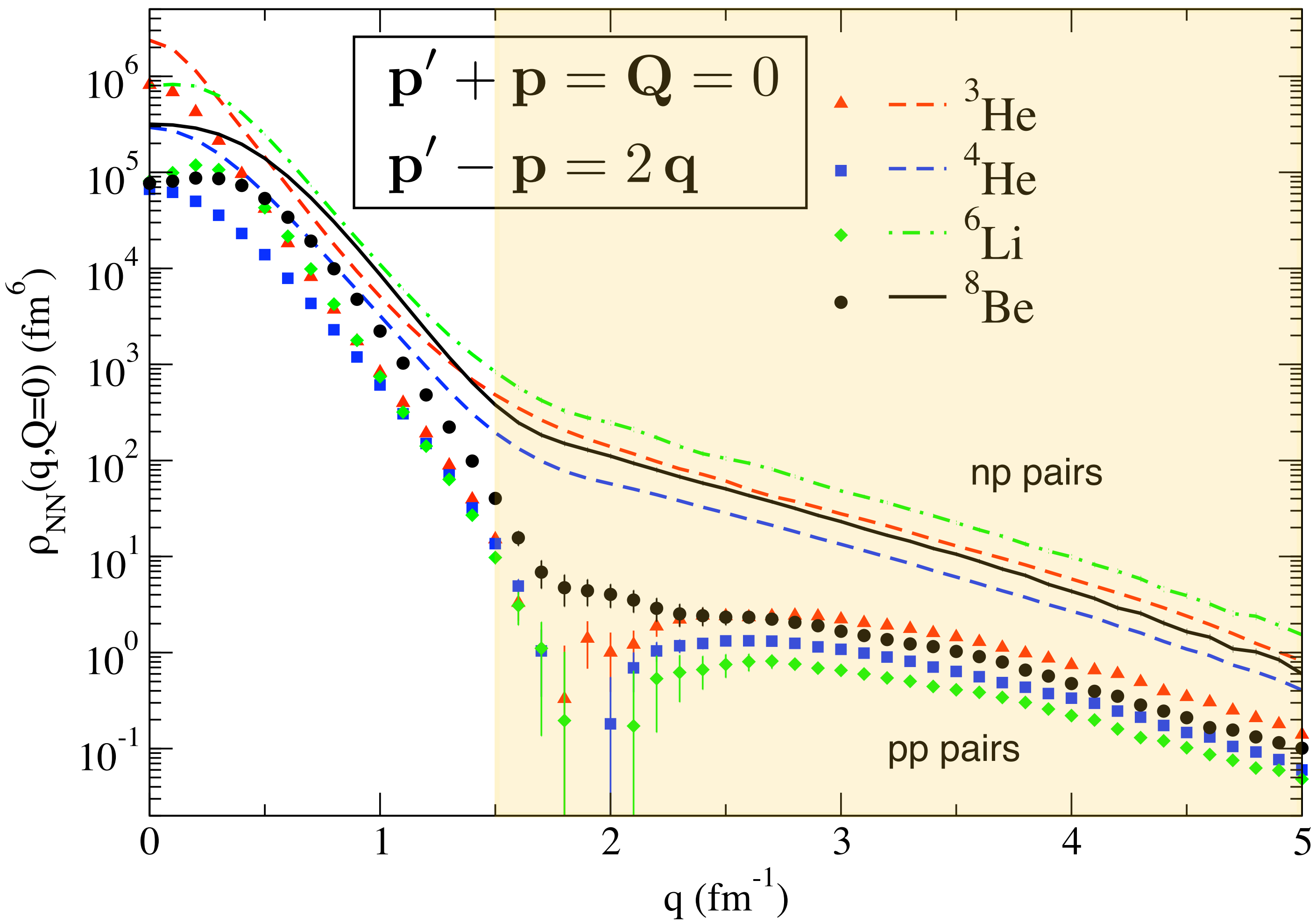
Vertex depends on the resolution!
RG provides systematic way to calculate such processes at low resolution.

Scaling in nuclear systems



$1.4 < Q^2 < 2.6\,GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)



Schiavilla et al., PRL 98, 132501 (2007)

- scaling behavior of momentum distribution function:

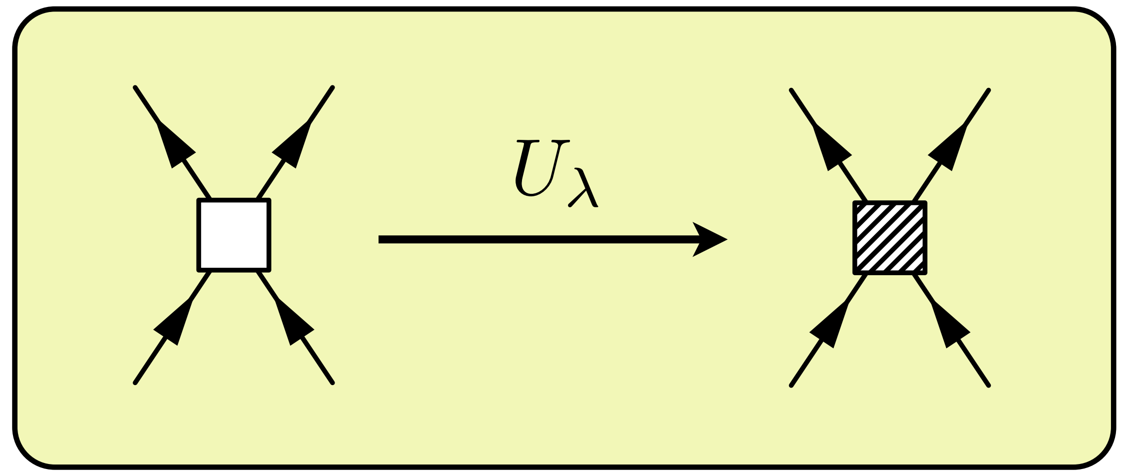
$$\rho_{\mathrm{NN}}(q, Q=0) \approx C_A \times \rho_{\mathrm{NN,Deuteron}}(q, Q=0)$$ at large q

- dominance of np pairs over pp pairs
- “hard” (high resolution) interaction used, calculations hard!
- dominance explained by short-range tensor forces

Nuclear scaling at low resolution

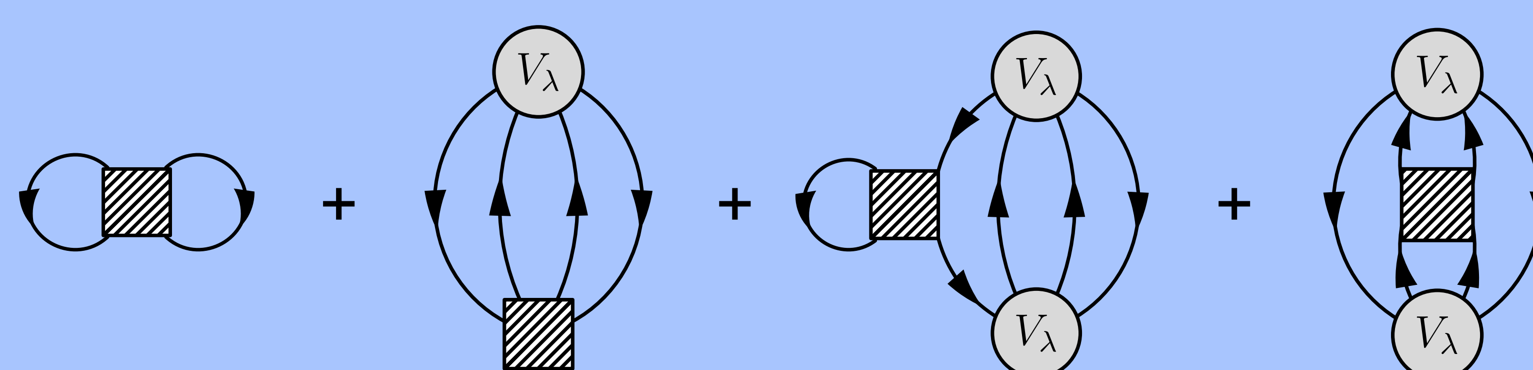
$\langle \psi_\lambda | O_\lambda | \psi_\lambda \rangle$ **factorizes** into a low-momentum structure and a **universal** high momentum part if the initial operator only weakly couples low and high momenta $\longrightarrow$ explains scaling!

RG transformation of pair density operator (induced many-body terms neglected):

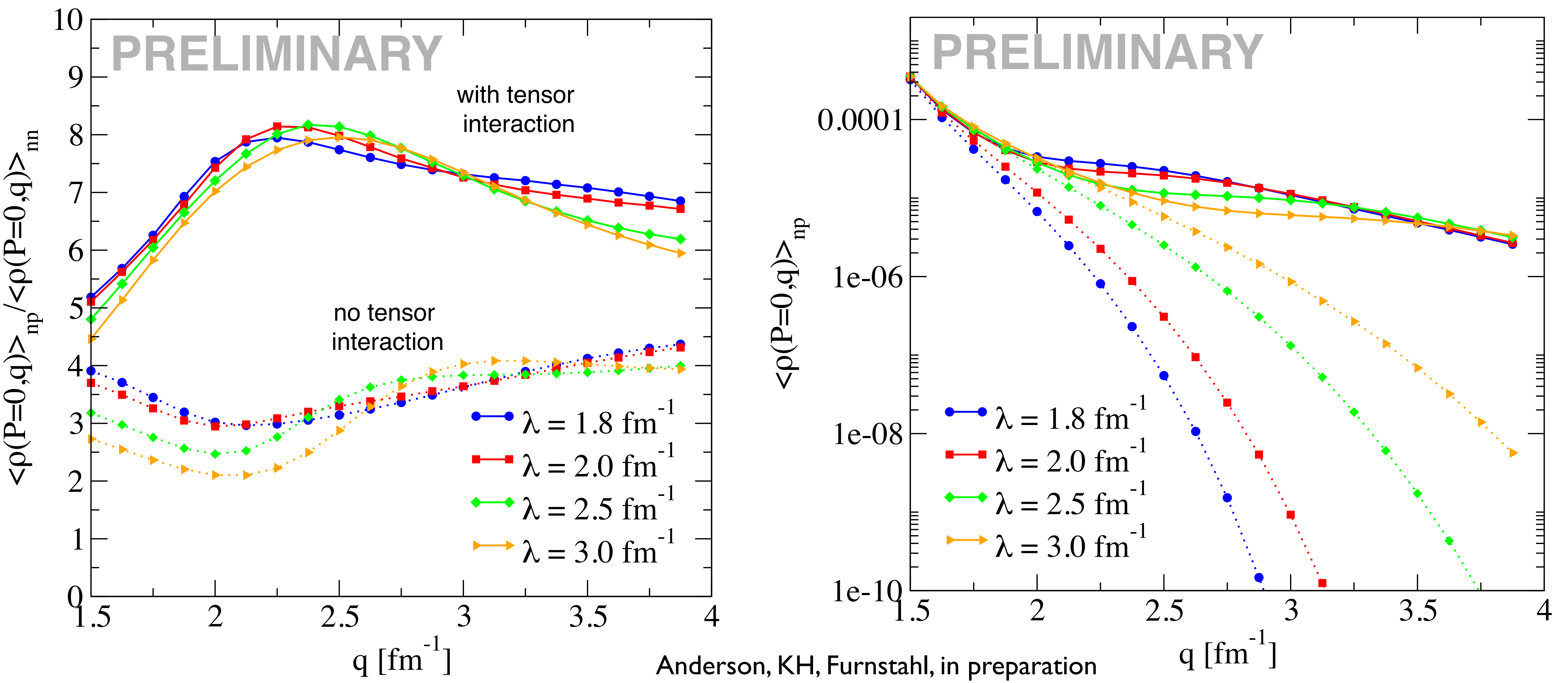


"simple" calculation of pair density at low resolution in nuclear matter:

$\langle \rho(\mathbf{P}, \mathbf{q}) \rangle =$



Nuclear scaling at low resolution



Anderson, KH, Furnstahl, in preparation

- pair-densities approximately resolution independent
- significant enhancement of np pairs over nn pairs due to tensor force
- reproduction of previous results using a “simple” calculation at low resolution

Nuclear scaling at low resolution

$\langle\psi_\lambda|O_\lambda|\psi_\lambda\rangle$ **factorizes** into a low-momentum structure and a **universal** high momentum part if the initial operator only weakly couples low and high momenta $\longrightarrow$ explains scaling!

key: $U_\lambda(k,q) \approx K(k)Q(q)$ for $k < \lambda$ and $q \gg \lambda$

factorization!

That leads to:

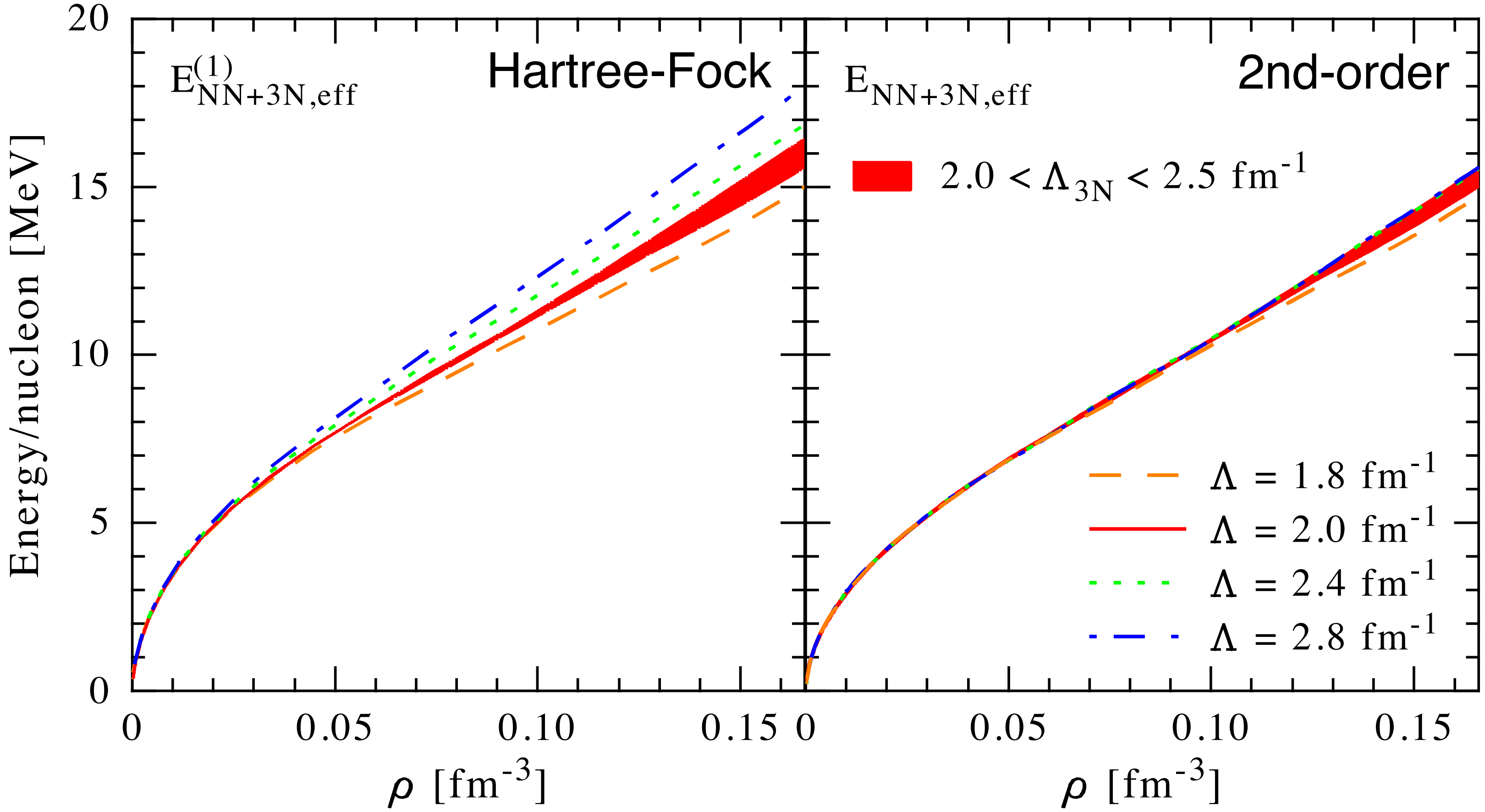
$$\begin{aligned}
\langle\psi_\lambda|\,O_\lambda\,|\psi_\lambda\rangle &= \int_0^\lambda dk\,dk' \int_0^\infty dq\,dq' \psi^\dagger(k) U_\lambda(k,q) O(q,q') U_\lambda(q',k') \psi_\lambda(k') \\
&\approx \int_0^\lambda dk\,dk'\,\psi_\lambda^\dagger(k) \left[\int_0^\lambda dq\,dq' K(k)K(q)O(q,q')K(q')K(k') + I_{QOQ} K(k)K(k')\right] \psi_\lambda(k')
\end{aligned}$$

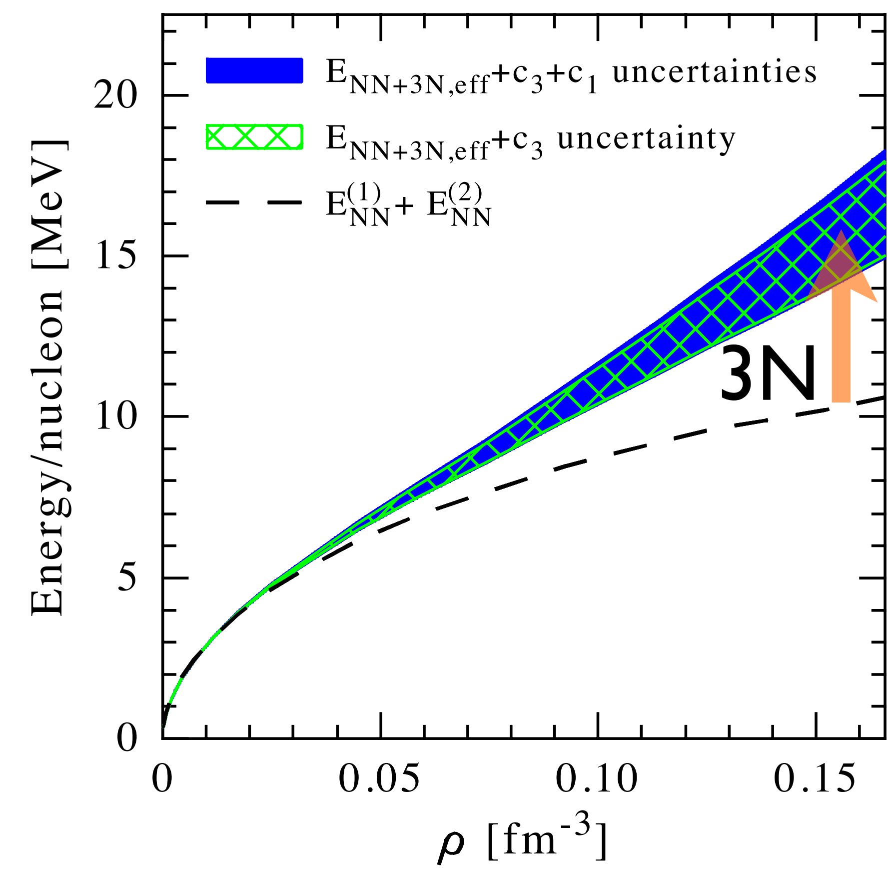
with the **universal** quantity:

$$I_{QOQ} = \int_\lambda^\infty dq\,dq' Q(q)O(q,q')Q(q')$$

valid if initial operator weakly couples low and high momenta

Equation of state of pure neutron matter

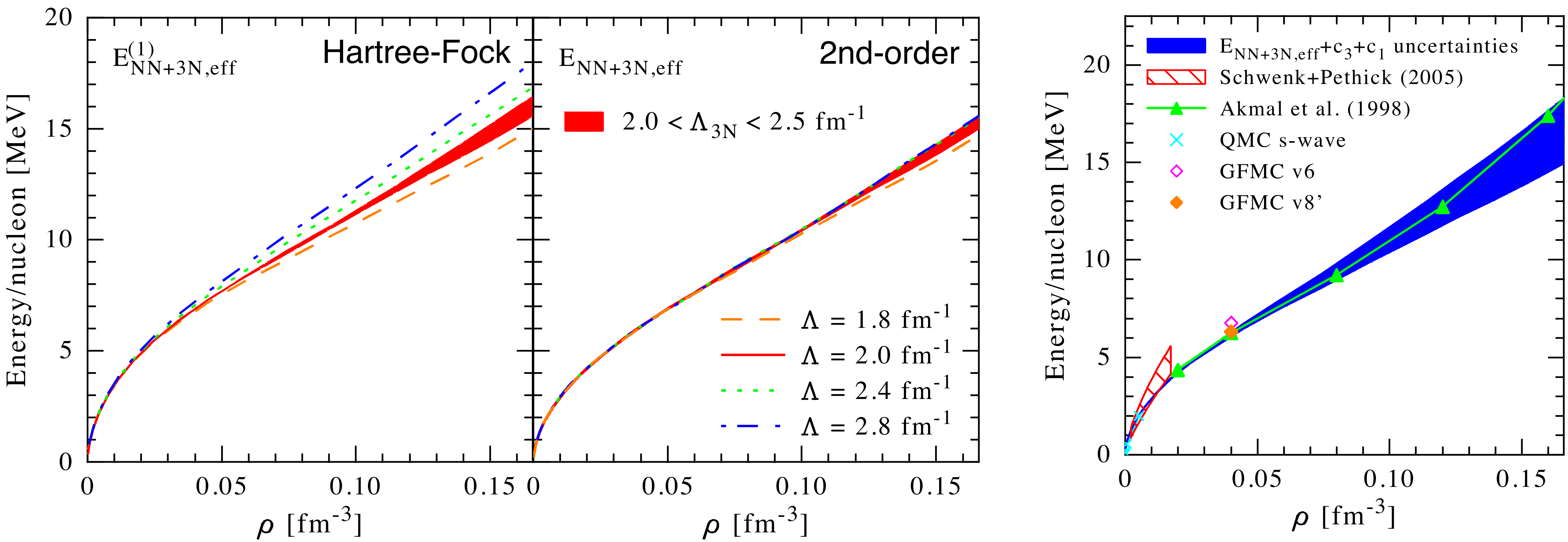




KH and Schwenk PRC 82, 014314 (2010)

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence

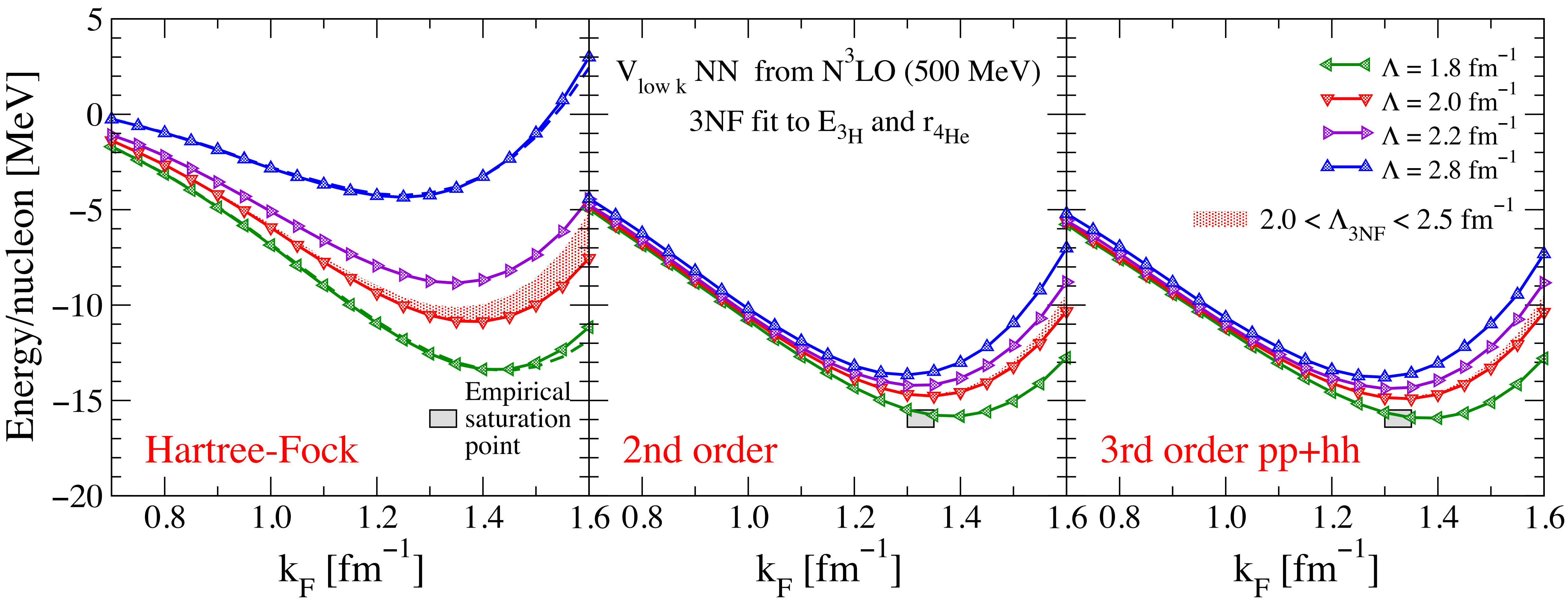
Equation of state of pure neutron matter



KH and Schwenk PRC 82, 014314 (2010)

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence
- good agreement with other approaches (different NN interactions)

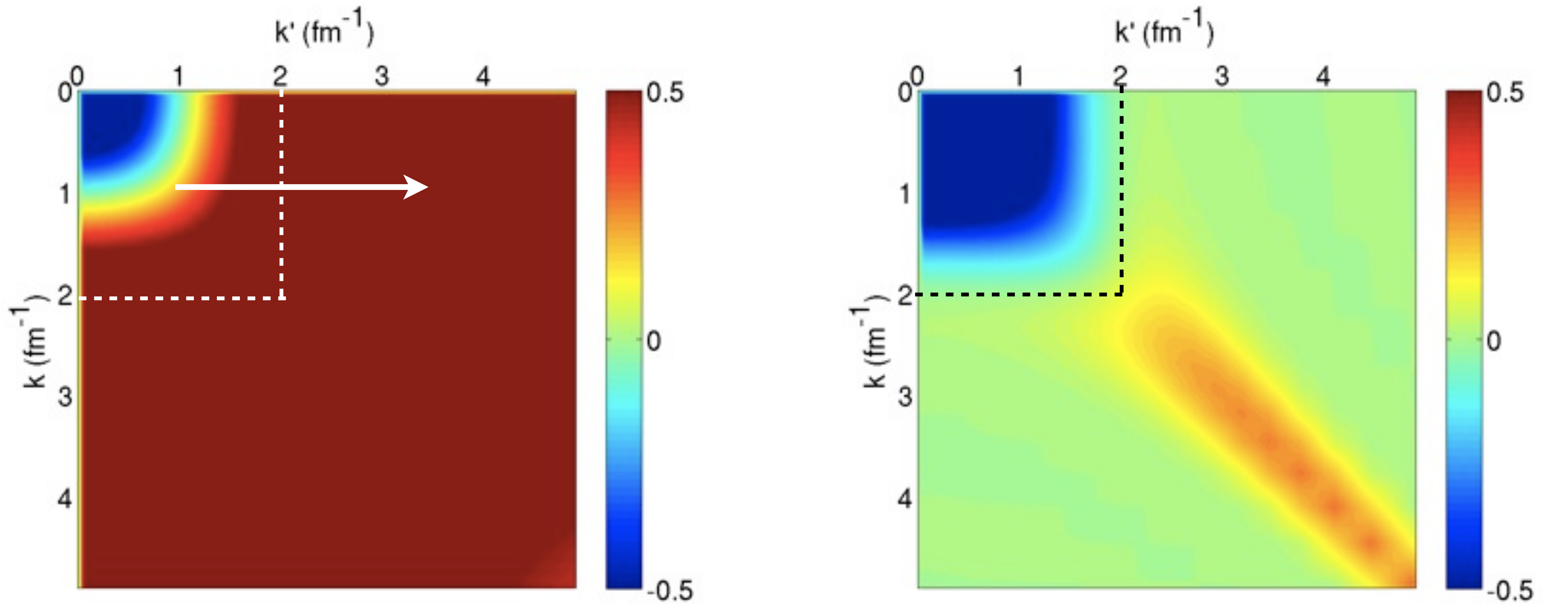
Equation of state of symmetric nuclear matter, Nuclear saturation



KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

- saturation point consistent with experiment, without free parameters
- cutoff dependence at 2nd order significantly reduced
- 3rd order contributions small
- cutoff dependence consistent with expected size of 4N force contributions

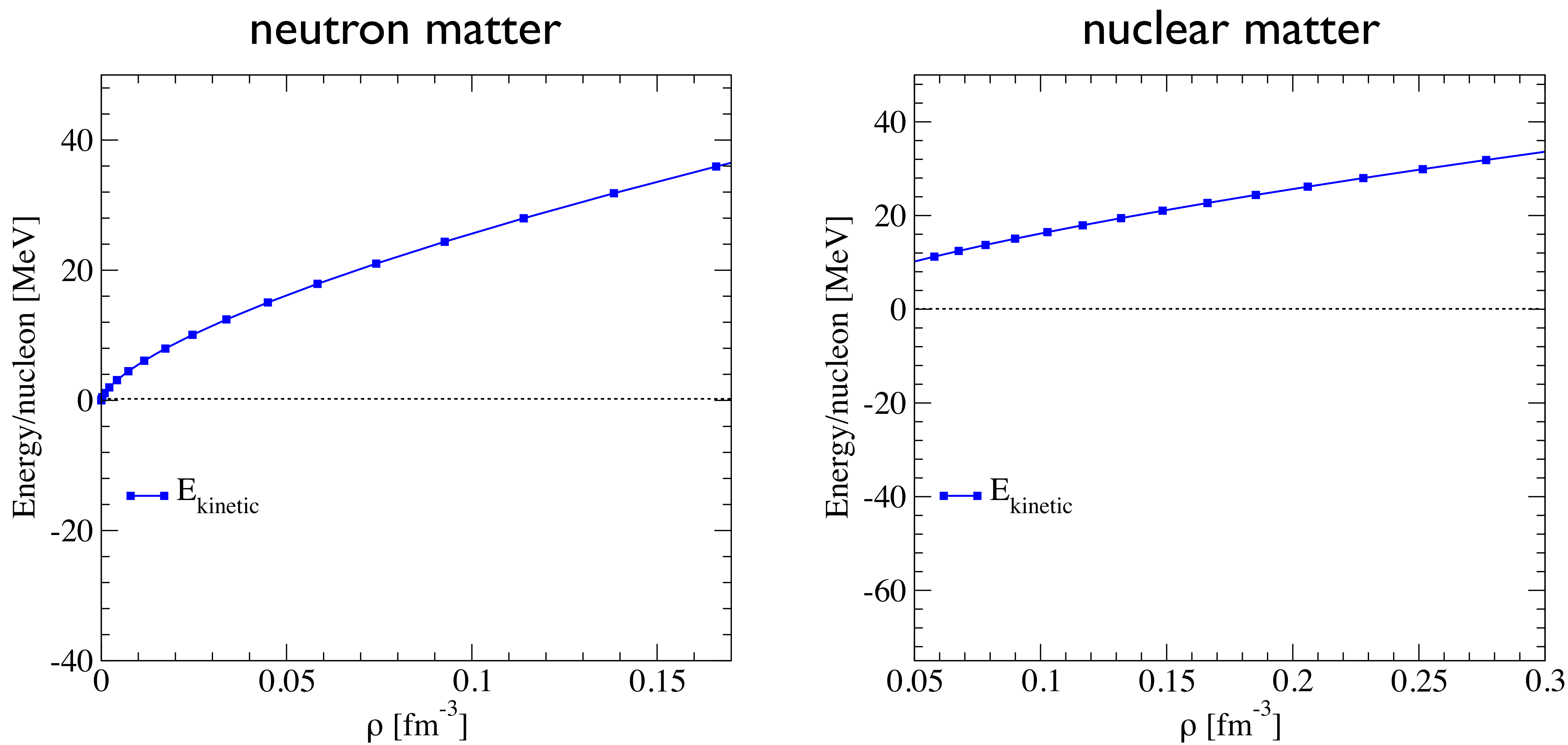
Changing the resolution:
The (Similarity) Renormalization Group



- elimination of coupling between low- and high momentum components, calculations much easier
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions.

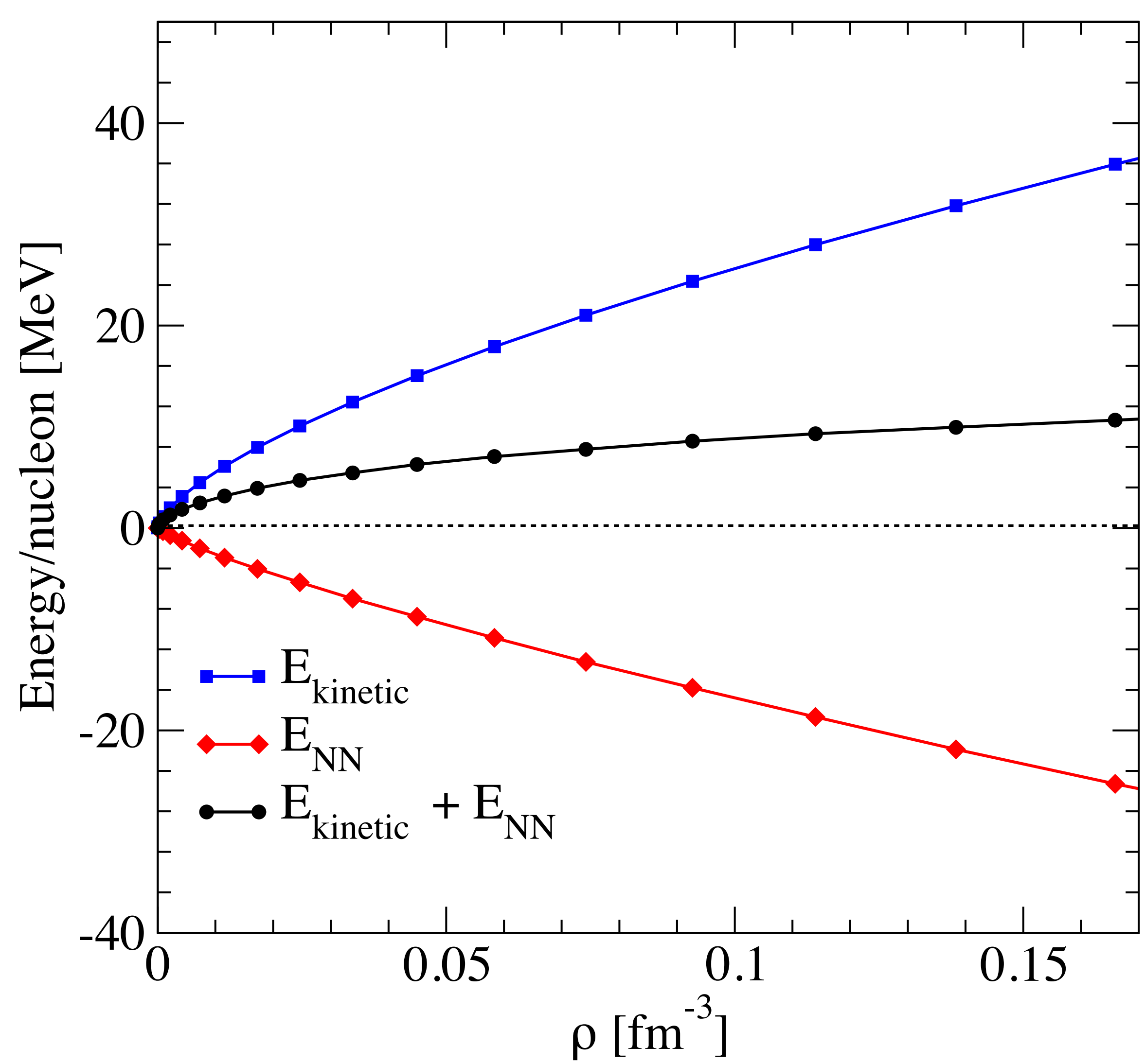
Hierarchy of many-body contributions



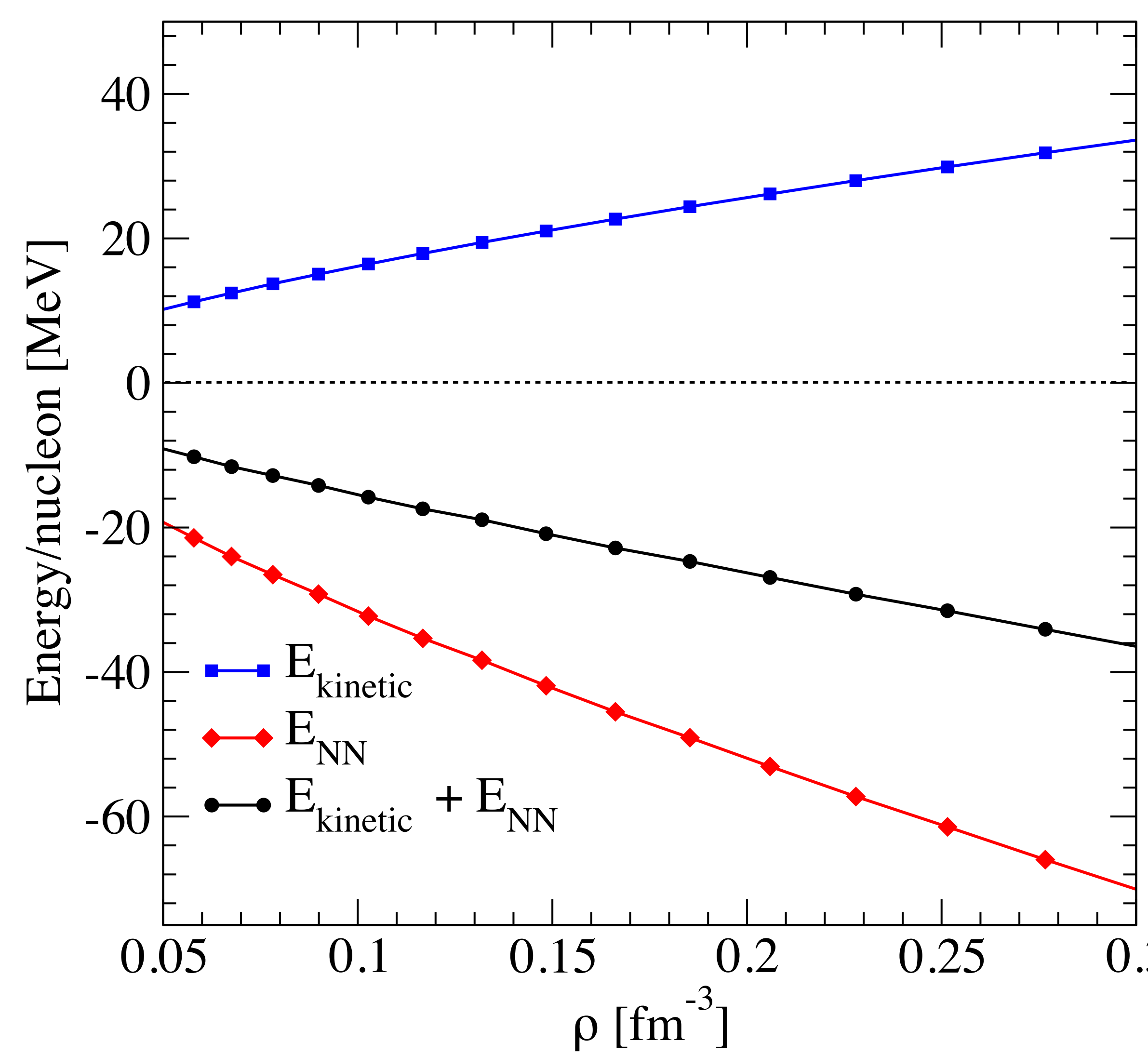
- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

Hierarchy of many-body contributions

neutron matter

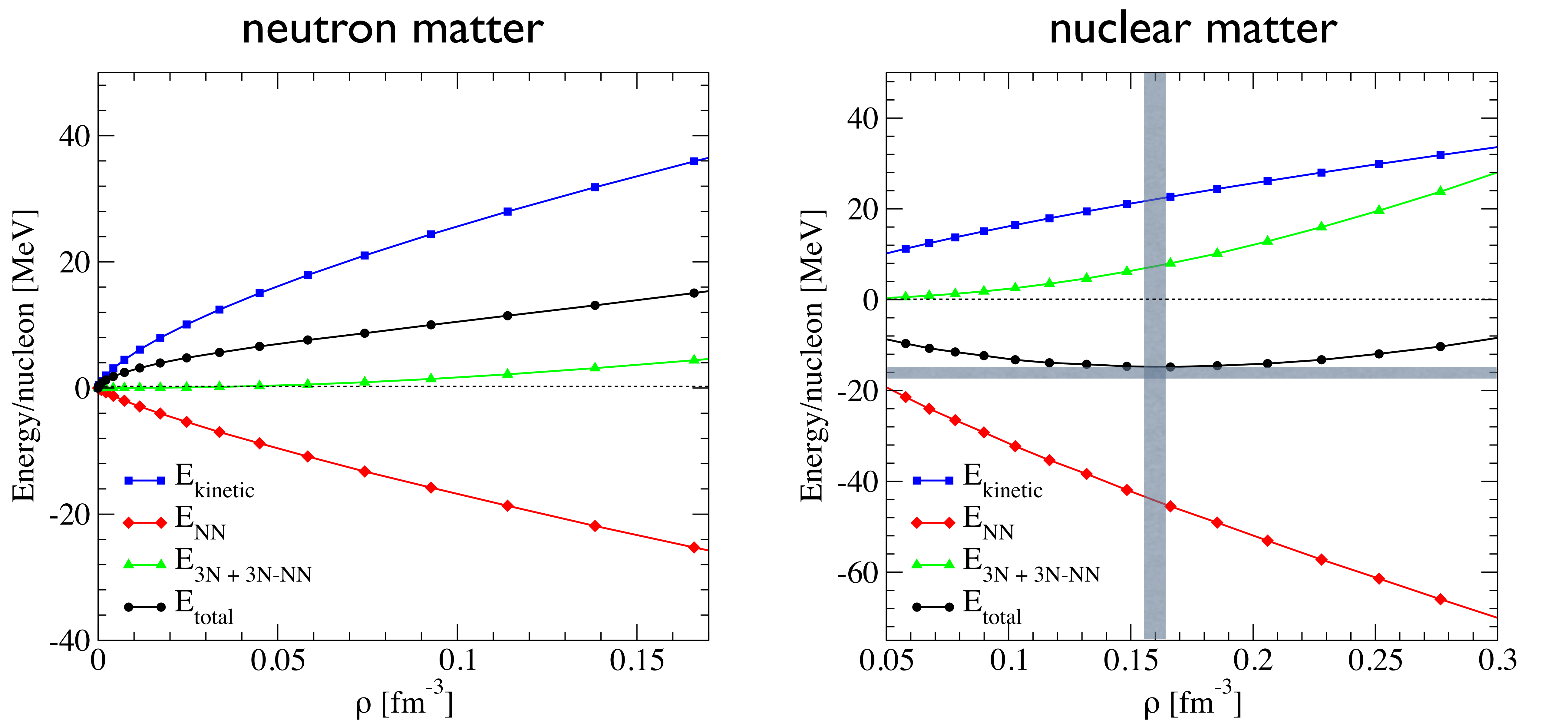


nuclear matter



- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

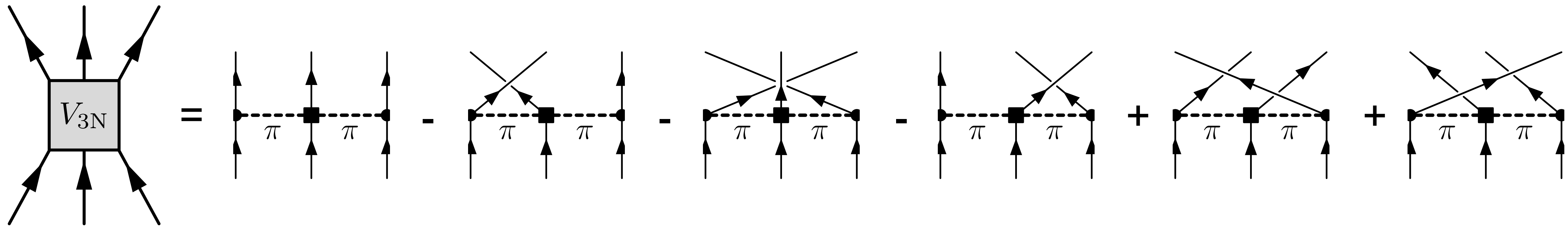
Hierarchy of many-body contributions



- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

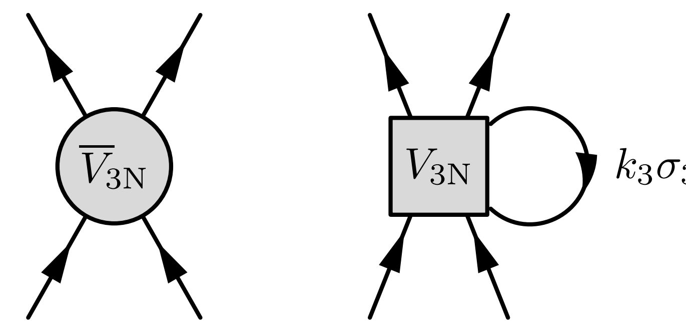
Chiral 3N interaction as density-dependent two-body interaction

(1) calculate antisymmetrized 3N interaction



(2) construct effective density-dependent NN interaction

Basic idea:
Sum one particle over occupied states in the Fermi sea



(3) combine with free-space NN interaction

combinatorial factor c depends on type of diagram