High-Accuracy Analysis of Compton Scattering off Protons and Deuterons in Chiral EFT

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How do constituents of the nucleon react to external fields?

How to reliably extract neutron and spin polarisabilities?

Comprehensive Theory Effort:
hg, J. McGovern (Manchester), D. R. Phillips (Ohio U)
arXiv:1210.4104 (proton)

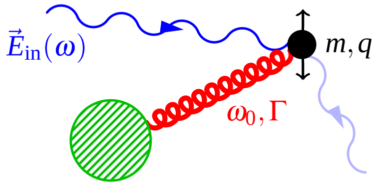
$+$ G. Feldman (GW): **Prog. Part. Nucl. Phys. 67 (2012) 841**

Precursors: Hildebrandt/Hemmert/Pasquini/hg. . . 2000-05, . . . ,Beane/Malheiro/McGovern/Phillips/van Kolck 1999-2005; Choudhury Shukla/Phillips 2005-08; Friar 1975, Arenhövel/Weyrauch 1980-83, Karakowski/Miller 1999, Levchuk/L'vov 1994-2000

(a) Energy-Dependent (Dynamical) Polarisabilities

hg/Hemmert/+Hildebrandt/Pasquini 2002/03

Example: induced electric dipole radiation from harmonically bound charge, damping Γ Lorentz/Drude 1900/1905

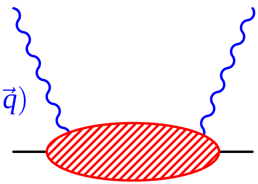


$$\vec{d}_{\rm ind}(\omega) = \underbrace{\frac{q^2}{m}\,\frac{1}{{\omega_0}^2 - \omega^2 - \mathrm{i}\Gamma\omega}}_{=:\,4\pi\,\alpha_{E1}(\omega)}\,\vec{E}_{\rm in}(\omega)$$

$$\mathcal{L}_{\rm pol} = 2\pi\Big[\underbrace{\alpha_{E1}(\omega)\vec{E}^2 + \beta_{M1}(\omega)\vec{B}^2}_{\text{electric, magnetic scalar dipole}} + \dots\Big]$$

Energy- (ω)-dependent multipole-decomposition dis-entangles scales, symmetries & mechanisms of interactions with & among constituents.

$\Longrightarrow$ Clean, perturbative probe of $\Delta(1232)$ properties, nucleon spin-constituents, χiral symmetry of pion-cloud & its breaking (proton-neutron difference).

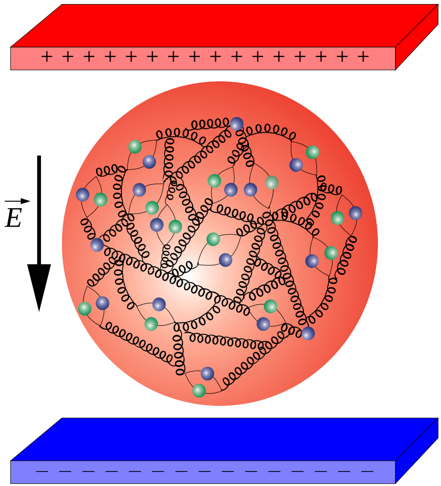


- **proton $\Longleftrightarrow$ neutron** iso-spin breaking:
  $\Longrightarrow$ elmag. p-n self-energy difference from $\beta_{M1}^{\rm p} - \beta_{M1}^{\rm n}$ Walker-Loud/. . . 2012
- 2γ contribution to Lamb shift in muonic H (β_{M1})

(b) Towards Polarisabilities from First Principles: Lattice QCD

Pioneering: Quenched, chiral fermions

Lee/Zhou/Wilcox/Christensen 2005-06



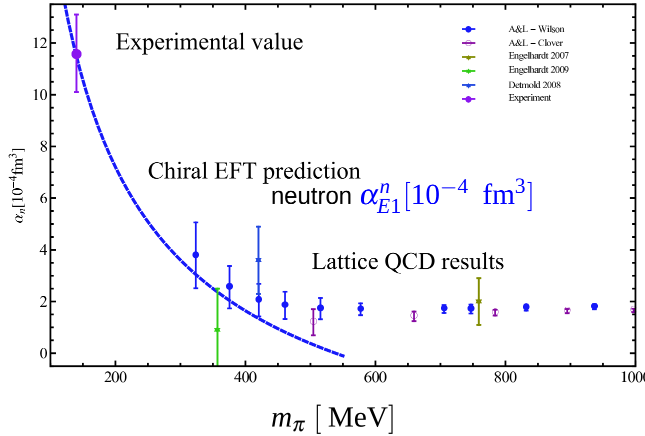
Ongoing:

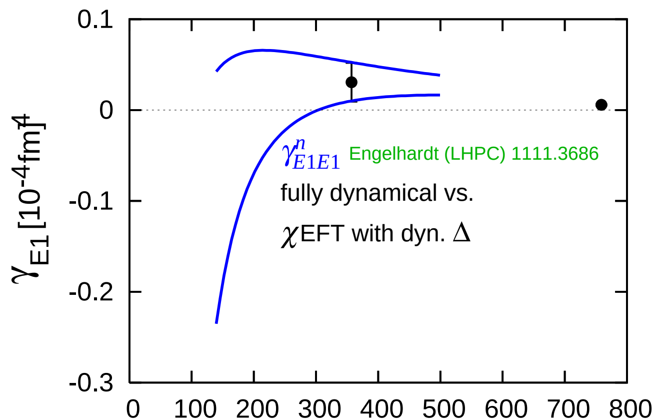
spin-polarisabilities, unquenching,

$m_\pi \searrow 200\ \text{MeV}$, larger volumes,

more statistics,. . .

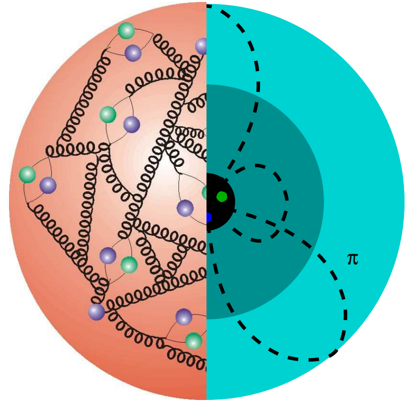
Lee/. . . 2005-, LHPC 2006-, Detmold/Tiburzi/Walker-Loud 2006-





(c) Separation of Scales: Effective Field Theory from QCD

Wilson, Weinberg, . . .



Theory of strong interactions: Quantum Chromo Dynamics QCD

$$\mathcal{L}_{\text{QCD}} = \bar{\Psi}_q [\mathrm{i}\,\not{\partial} + g\,\not{A} - m_q] \Psi_q - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

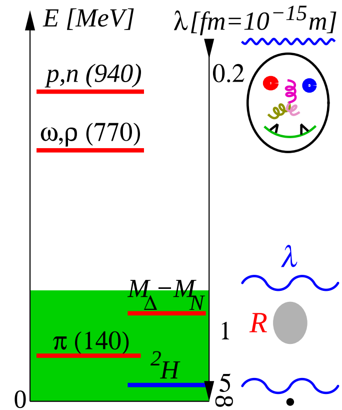
$\Longrightarrow$ Effective low-energy degrees of freedom: Nucleons, Pions, $\Delta(1232)$

Systematic ordering in $Q = \dfrac{\text{typ. momentum } \sim m_\pi}{\text{breakdown scale } \sim 1\text{ GeV}} \approx \dfrac{1}{5\ldots7}$

Controlled approximation: model-independent, error-estimate.

$$\mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2\, \pi^a \pi^a + \ldots$$

$$+ N^\dagger [\mathrm{i}\, D_0 + \frac{\vec{D}^2}{2M} + \frac{g_A}{2 f_\pi}\, \vec{\sigma} \cdot \vec{D}\pi + \ldots] N + C_0 \left(N^\dagger N\right)^2 + H_0 \left(N^\dagger N\right)^3 + \ldots$$



Correct long-range Physics + all interactions allowed by symmetries.

Short-range: encode ignorance into minimal parameter-set at given order.

– gauge, Lorentz, iso-spin, . . . symmetries

– chiral: pions light & weakly coupled: Goldstone bosons of Chiral SSB

$\Longrightarrow$ Chiral Effective Field Theory χEFT $\equiv$ low-energy QCD

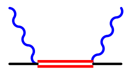
(a) Including the $\Delta(1232)$ in χEFT

$\Delta(1232)$ lowest hadronic excitation above 1-pion threshold m_π, below χEFT breakdown scale $\Lambda_\chi \approx 1000$ MeV

$\Longrightarrow$ **Expand in** $\delta = \dfrac{M_\Delta - M_N}{\Lambda_\chi} \approx \sqrt{\dfrac{m_\pi}{\Lambda_\chi}} = \dfrac{p_{\text{typ}}}{\Lambda_\chi} \ll 1$ **(numerical fact)** Pascalutsa/Phillips 2002 **and** $\dfrac{\omega}{\Lambda_\chi}$.

Low régime $\omega \lesssim m_\pi \Longrightarrow \omega \sim \delta^2$

High régime $\omega \sim M_\Delta - M_N \sim 300$ MeV $\Longrightarrow \omega \sim \delta^1$

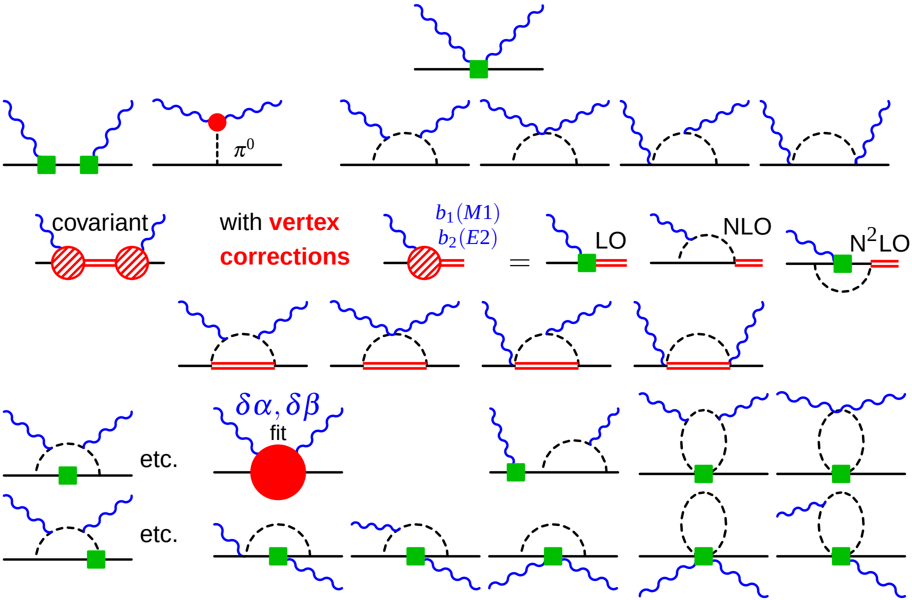
$\omega \to M_\Delta - M_N$: Δ propagator enhanced  $\propto \dfrac{1}{\omega - (M_\Delta - M_N)} \sim \dfrac{1}{\delta^3}$

$\Longrightarrow$ **Re-order & dress**  $+ \ldots = \dfrac{1}{E - (M_\Delta - M_N) - \text{[loop]}}$  $+$ relativity

Probe non-zero Δ width, $M1$ and $E2$ transition strengths.

(b) All Contributions

Low régime $\omega \lesssim m_\pi \sim \delta^2 \Longleftrightarrow$ **High régime** $\omega \sim M_\Delta - M_N \sim \delta^1 \approx 300$ MeV



$\omega \lesssim m_\pi$	$\sim M_\Delta - M_N$
$e^2\delta^0$ **LO**	$e^2\delta^0$ ↘NLO
$e^2\delta^2$ N^2LO	$e^2\delta^1$ N^2LO
$e^2\delta^3$ N^3LO	$e^2\delta^{-1}$ ↗**LO**
$e^2\delta^3$ N^3LO	$e^2\delta^1$ N^2LO
$e^2\delta^4$ N^4LO	$e^2\delta^2$ N^3LO

Unified Amplitude: gauge & RG invariant set of all contributions which are

in low régime $\omega \lesssim m_\pi$ **at least N^4LO** ($e^2\delta^4$)**: accuracy** $\delta^5 \lesssim 2\%$;

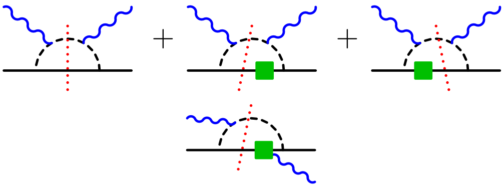
or **in high régime** $\omega \sim M_\Delta - M_N$ **at least NLO** ($e^2\delta^0$)**: accuracy** $\delta^2 \lesssim 20\%$.

Unknowns: $\delta\alpha, \delta\beta \Longleftrightarrow \alpha_{E1}, \beta_{M1}$, $\gamma N\Delta$ **strengths** $b_1(M1)$, $b_2(E2)$

Watson's Theorem: $\mathrm{Re}[\textbf{Amplitude}] = 0$ **at resonance peak by unitarity.**

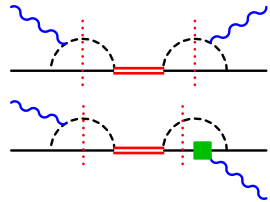
$\Longrightarrow$ Unified Amplitude must contain additional pieces:

πN loops	order for	**$\gamma\Delta N$ vertex corrections**
N^{2}LO, N^{4}LO for $\omega \sim m_\pi$	$\omega \sim M_\Delta - M_N$	N^{7}LO, N^{9}LO for $\omega \sim m_\pi$



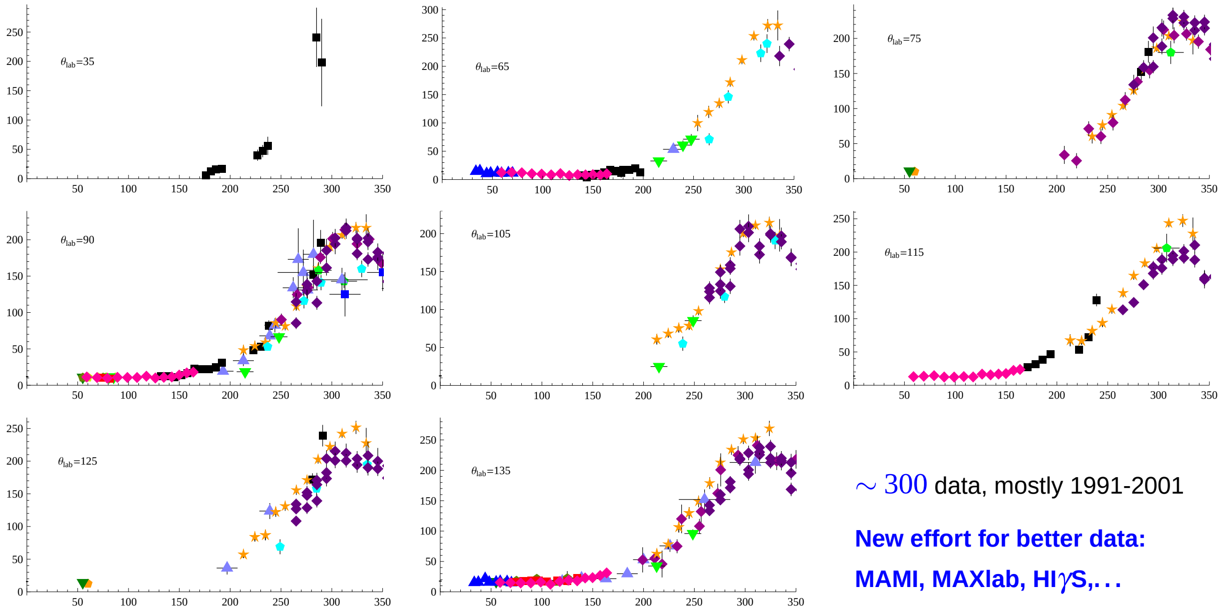
$e^2\delta^1$ N^{2}LO

$e^2\delta^2$ N^{3}LO



$\Longrightarrow$ Keep these vertex corrections, albeit higher-order everywhere; check.

(d) Creating a Consistent Proton Compton Database



~ 300 data, mostly 1991-2001

New effort for better data:

MAMI, MAXlab, HIγS,. . .

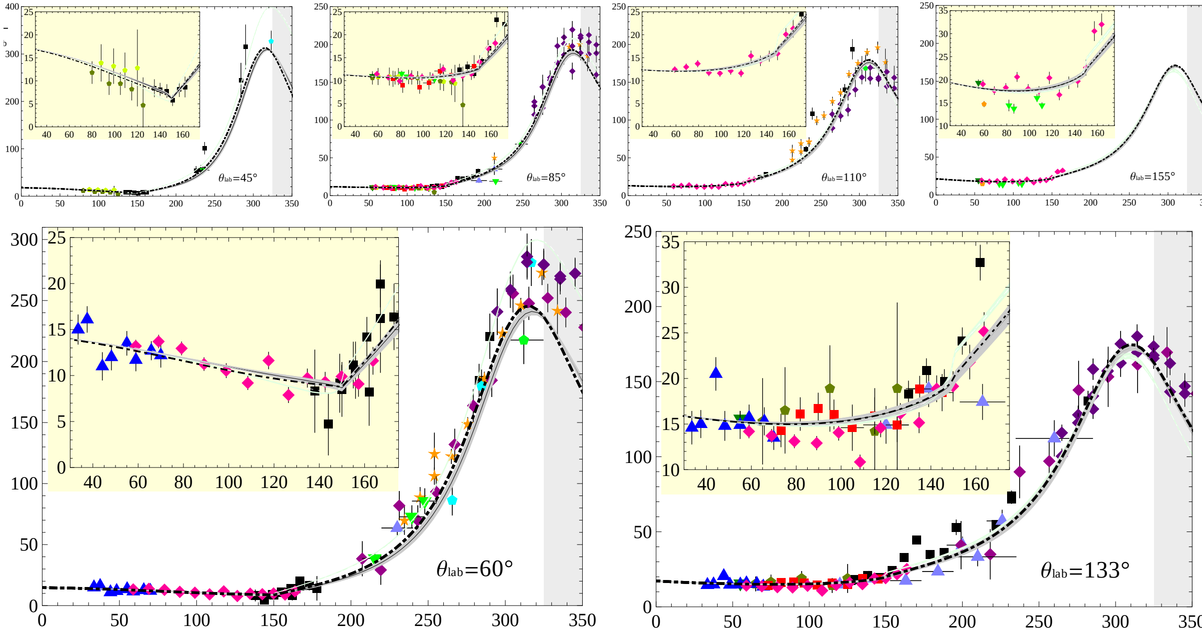
Gaps for: $\omega \in [160; 250]$ MeV; $\theta \to 0^\circ$: Baldin check; $\theta \to 180^\circ$ for $\Delta(1232)$!

Small quoted systematics $\Longrightarrow$ tensions: MAMI vs. LEGS, but also others $\Longrightarrow$ no $\frac{\chi^2}{\text{d.o.f.}} \approx 1$ without pruning.

Not *more*, but *more reliable* data needed for unpolarised proton.

(e) Polarisabilities from Consistent Database

McGovern/Phillips/hg 2012
database: +Feldman PPNP 2012



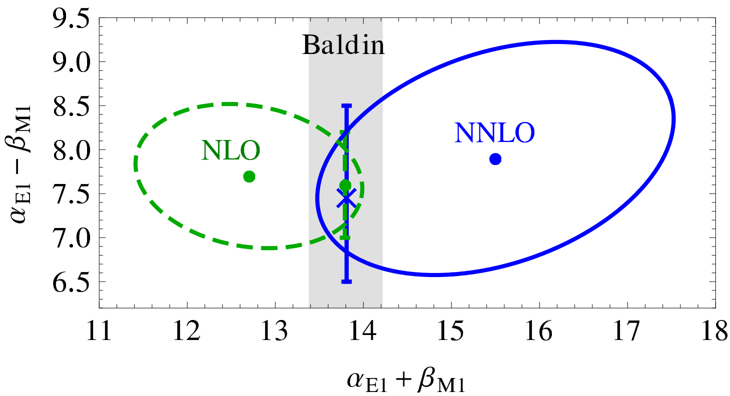
> 200 MeV: Δ **fix** $b_2/b_1 = -0.34$ Vanderhaeghen/Pascalutsa 2006, **fit** $b_1 = 3.61 \pm 0.02$; $\Longrightarrow$ < 170 MeV: **polarisabilities**

	α^p_{E1} $[10^{-4}\ \text{fm}^3]$	β^p_{M1} $[10^{-4}\ \text{fm}^3]$	χ^2/d.o.f.
Baldin constrained $\alpha^p_{E1} + \beta^p_{M1} = 13.8 \pm 0.4$	$10.7 \pm 0.4_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}}$	$3.1 \mp 0.4_{\text{stat}} \pm 0.2_{\Sigma} \mp 0.3_{\text{theory}}$	$113.2/135$

(f) Fit Discussion: Parameters and Uncertainties

Fit to α, β only: Combined πN loops *and* Δ pushes intermed. angles too high.

$\Longrightarrow$ **Must also fit** $\gamma_{M1M1} \approx 2.2 \pm 0.5_{\text{stat}}$ for good χ^2, fwd. spin-polarisability $\gamma_0^p \approx -1$ (cf. Disp. Rel.).



1σ-contours

N^{2}LO marginalised over γ_{M1M1}

Consistent with Baldin Σ Rule

$$\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_{\nu_0}^{\infty} d\nu \, \frac{\sigma(\gamma p \to X)}{\nu^2}$$

$= 13.8 \pm 0.4$ Olmos de Leon 2001

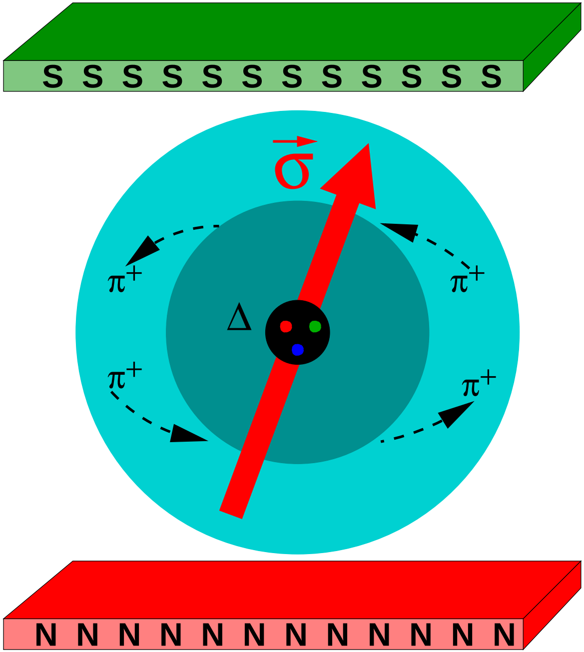
need more forward data to constrain.

Fit Stability: floating norms within exp. sys. uncertainties; vary dataset, b_1, vertex dressing,. . .

Residual Theoretical Uncertainty from convergence pattern: $\delta^2 \approx \frac{1}{6}$ of LO$\rightarrow$NLO change $\delta(\alpha - \beta) = 3.5$

	α_{E1}^p $[10^{-4}\ \text{fm}^3]$	β_{M1}^p $[10^{-4}\ \text{fm}^3]$	χ^2/d.o.f.
LO parameter-free Bernard/Kaiser/Meißner 1994	12.5	1.25	no fit
N^{2}LO Baldin constrained $\alpha_{E1}^p + \beta_{M1}^p = 13.8 \pm 0.4$	$10.7 \pm 0.4_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}}$	$3.1 \mp 0.4_{\text{stat}} \pm 0.2_{\Sigma} \mp 0.3_{\text{theory}}$	$113.2/135$
Olmos de Leon 2001	$12.1 \pm 1.2_{\text{stat+model}} \pm 0.4_{\Sigma}$	$1.6 \mp 1.2_{\text{stat+model}} \pm 0.4_{\Sigma}$	

Optical Activity: Response of **spin-degrees of freedom**, experimentally untested.



$$\mathcal{L}_{\mathrm{pol}} = 4\pi\, N^\dagger \times$$

$$\Big\{ \frac{1}{2}\Big[\alpha_{E1}(\omega)\,\vec{E}^2 + \beta_{M1}(\omega)\,\vec{B}^2\Big]$$ **scalar dipole**

$$+\frac{1}{2}\Big[\gamma_{E1E1}(\omega)\,\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}(\omega)\,\vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}})$$

"pure" spin-dependent dipole

$$-2\,\gamma_{M1E2}(\omega)\,\sigma_i\,B_j\,E_{ij} + 2\,\gamma_{E1M2}(\omega)\,\sigma_i\,E_j\,B_{ij}\Big]$$

"mixed" spin-dependent dipole

$$+\ldots\Big\}\,N$$ quadrupole etc.

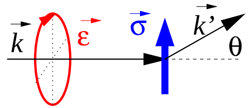
$E_{ij} := \frac{1}{2}(\partial_i E_j + \partial_j E_i)$ etc.

(h) Spin-Polarisabilities from Circular-Polarised Photon

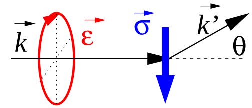
$\mathcal{O}(e^2\delta^3)$: hg/Hildebrandt/. . . 2003
$\mathcal{O}(e^2\delta^4)$: hg/McGovern in prep.
exp: MAMI 2011-

Proton Best: Incoming γ circularly polarised, sum over final states. N-spin in $(\vec{k},\vec{k}')$-plane, perpendicular to $\vec{k}$:

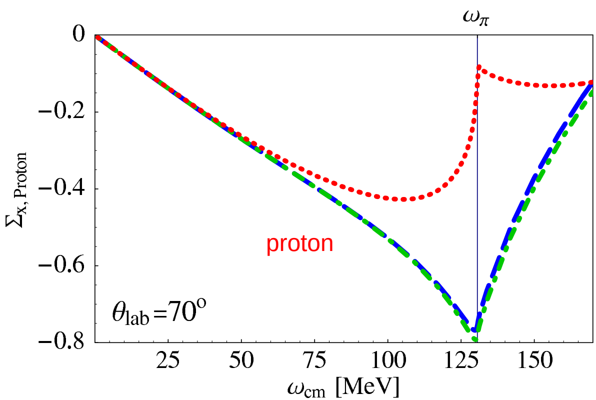
$$\Sigma_x = \frac{(\uparrow\rightarrow)-(\uparrow\leftarrow)}{(\uparrow\rightarrow)+(\uparrow\leftarrow)}$$

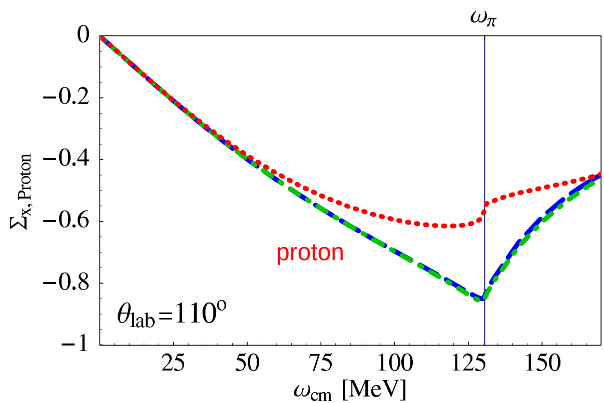


vs.



– – – full $N\pi+\Delta$ · · · · · · no γ_i's - · - · - no $l \geq 2$





– Dominated by structure – Clear γ_i-dep. – Higher pols negligible – HIγS?

Also good signal for linear polarisations.

(a) Iso-Vector Polarisabilities

Proton-neutron difference $\alpha_{E1}^{v} := \alpha_{E1}^{p} - \alpha_{E1}^{n}$ etc. probes details:

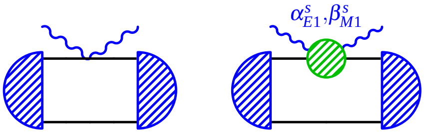
Explicit χiral-symmetry-breaking in pion-cloud,. . . , elmag. p-n self-energy difference $[0 \pm 1]\ \text{MeV} \propto \beta_{M1}^{p} - \beta_{M1}^{n}$

Appears only in NLO χEFT; compatible with $\approx \frac{1}{10}$th of iso-scalar polarisabilities in Dispersion Relations.

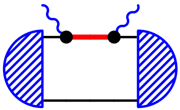
No free neutron targets $\Longrightarrow$ χEFT for model-independent subtraction of nuclear binding.

One-body: electric, magnetic moment couplings

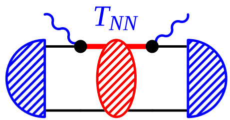
$\omega \sim \frac{Q^2}{M} \approx 20$ MeV $\omega \sim Q \approx 100$ MeV



LO, N^{3}LO   LO, $\nearrow$ NLO



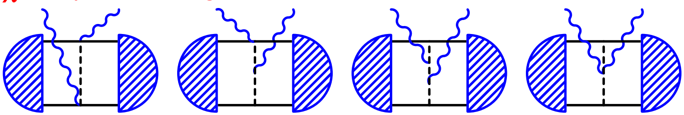
$\sum$ partial waves



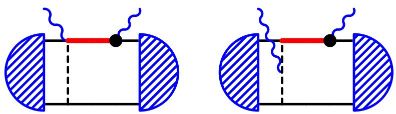
LO $\searrow$ NLO, N^{3}LO

2N correlated $\frac{i}{B_d \pm \omega - \frac{q^2}{M}}$ uncorrelated

χEFT pion-exchange currents: Beane et al. 1999-2005; hg/...2005

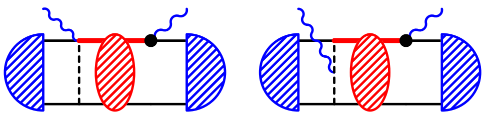


NLO $\rightarrow$ NLO



NLO $\searrow$ N^{2}LO

$\sum$ part. waves



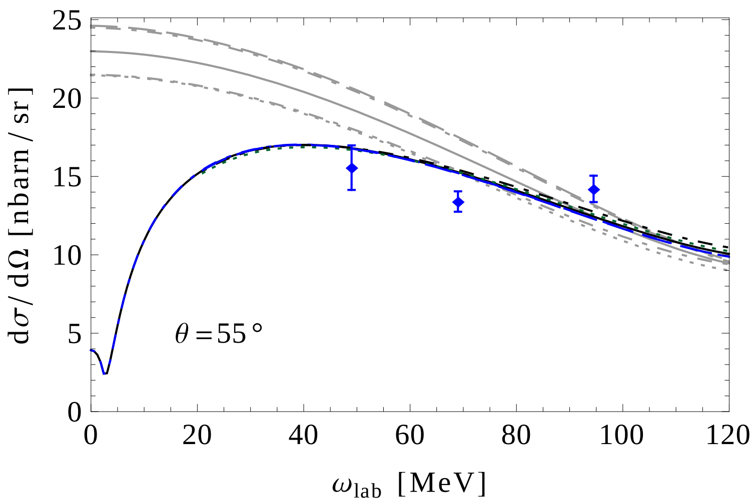
NLO $\searrow$ N^{3}LO, pert.

Full LO T_{NN} pivotal for current conservation. Arenhövel 1980

Low-Energy Theorem: Thomson limit $\mathcal{A}(\omega=0) = -\frac{e^2}{M_d}\vec{\varepsilon}\cdot\vec{\varepsilon}\,'$.

Thirring 1950, Friar 1975, Arenhövel 1980: Thomson limit $\Longleftrightarrow$ current conservation $\Longleftrightarrow$ gauge invariance.

Exact Theorem $\Longrightarrow$ At each χEFT order $\Longrightarrow$ Checks numerics.

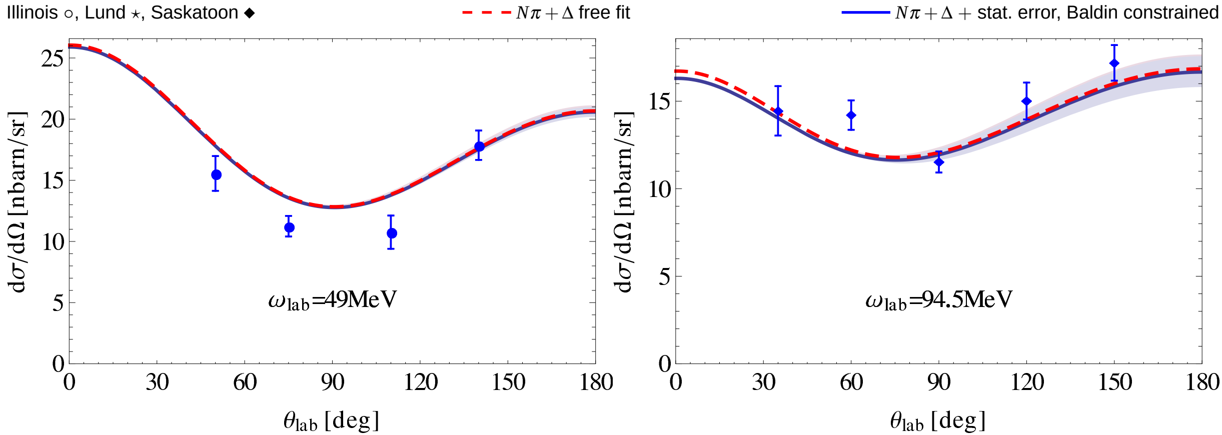


Significantly reduces cross section for $\omega \lesssim 70$ MeV. Urbana, Lund data

Numerically confirmed to $\lesssim 0.2\%$, irrespective of deuteron wave function & potential. model-independence

Wave function & potential dependence significantly reduced even as $\omega \to 150$ MeV $\Longrightarrow$ **gauge invariance**.

(d) Determine Neutron Polarisabilities from all Deuteron Data



	α^s_{E1} $[10^{-4}\ \text{fm}^3]$	β^s_{M1} $[10^{-4}\ \text{fm}^3]$	χ^2/d.o.f.
NLO free fit	$10.5 \pm 2.0_{\text{stat}} \pm 0.8_{\text{theory}}$	$3.6 \pm 1.0_{\text{stat}} \pm 0.8_{\text{theory}}$	$24.3/24$
NLO Baldin constrained $\alpha^s_{E1} + \beta^s_{M1} = 14.5 \pm 0.3$	$10.9 \pm 0.9_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.8_{\text{theory}}$	$3.6 \mp 0.9_{\text{stat}} \pm 0.2_{\Sigma} \mp 0.8_{\text{theory}}$	$24.4/25$
N^{2}LO proton (Baldin) hg/...PPNP 2012	$10.7 \pm 0.4_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}}$	$3.1 \mp 0.4_{\text{stat}} \pm 0.2_{\Sigma} \mp 0.3_{\text{theory}}$	$113.2/135$

$\Longrightarrow$ **neutron $\approx$ proton polarisabilities**

Need better data: MAXlab taken, HIγS approved – theory: N^{2}LO, beyond pion threshold

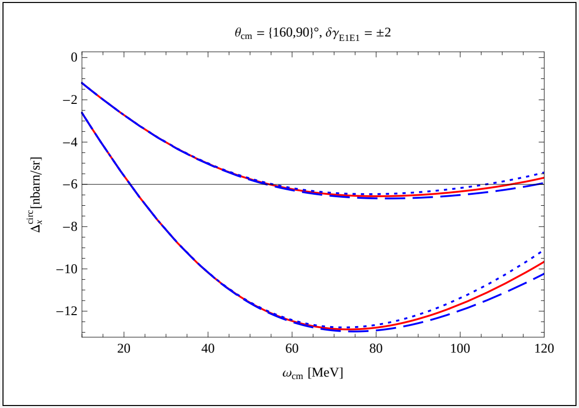
(e) Goal: Guide, Support, Analyse, Predict Experiments

hg/Shukla 2010-
hg/McGovern/Phillips 2012-

11 independent observables for proton; 23 for deuteron

$\Longrightarrow$ **Interactive** ***mathematicia 8.0*** **notebooks** from hgrie@gwu.edu

Example double-polarised on deuteron



First scatt. angle θ=160°

Second scatt. angle θ=90°

Reference frame: cm | lab

Deuteron polarisation axis: x | z

Variation by ±2 of: $\delta\gamma_{E1E1}$

χEFT order: ϵ^3

Deuteron wave function: NNLO Epelbaum 650MeV | AV18

NN potential: AV18

Range y-axis: Automatic

$\delta\gamma_{E1E1}=0$; $\delta\gamma_{E1E1}=+2$; $\delta\gamma_{E1E1}=-2$

Future: guide/support experiments at HIγS, MAMI, MAXlab

extend analysis proton to 300 MeV

full analysis deuteron (tensor-polarised), ^{3}He

Compton@Web on DAC/SAID website

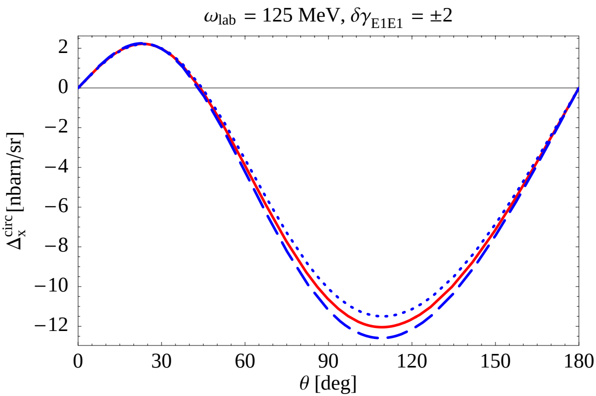
(f) Spin-Polarisabilities from Circularly Pol. Photons at 125 MeV

Deuteron Best: Incoming γ circularly polarised, sum over final states. N-spin in $(\vec{k},\vec{k}')$-plane, perpendicular to $\vec{k}$:

difference Δ_x^{circ}, asymmetry $\Sigma_x^{\text{circ}} = \frac{\Delta_x^{\text{circ}}}{\text{sum}}$

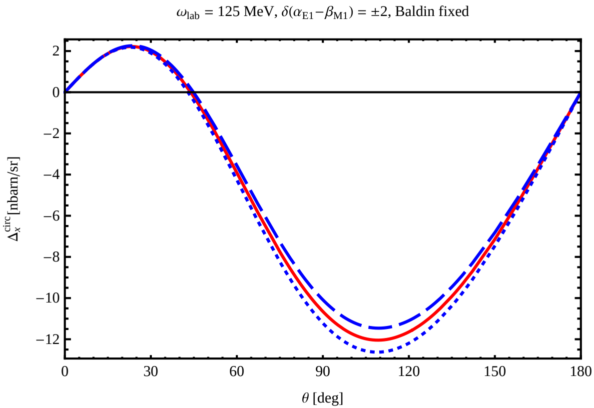
Sensitivity on neutron γ_{E1E1}

red solid line: -5.2; dashed: $-5.2+2$; dotted: $-5.2-2$



Sensitivity on neutron $\alpha_{E1} - \beta_{M1}$; Baldin-Σ fixed

red solid line: 8.2; dashed: $8.2+2$; dotted: $8.2-2$



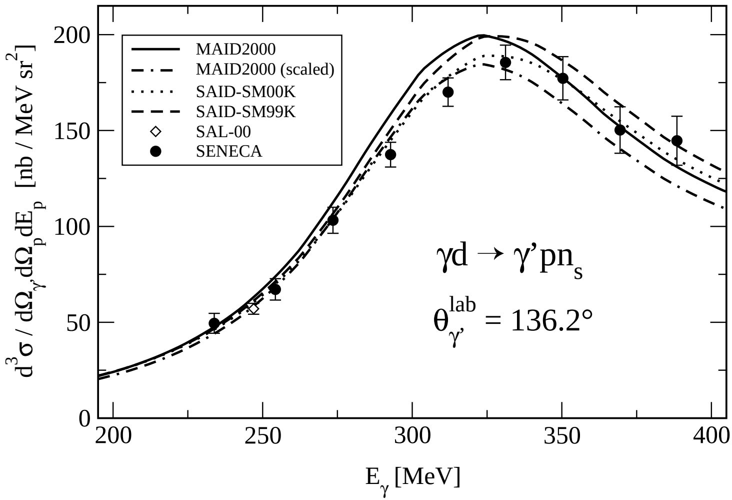
Sensitive to γ_{E1E1}, but must nail down α_{E1}, β_{M1} at lower energy.

$\Delta(1232)$ and re-scattering increase signal.

theory: Levckuk/L'vov/Petrunkin 1994-2000; Demissie/hg 2012-
exp: (Rose/... 1999); Kolb/... SAL 2000; Kossert/... MAMI 2002

Nucleon polarisabilities from centre of quasi-inelastic peak in $A(\gamma,\gamma A')N$

9 data for $d(\gamma,\gamma p)n$ for $\omega \in [230;400]$ MeV



Kossert et al. 2003 found $\alpha_{E1}^n = 12.5 \pm 1.8(\text{stat})^{+1.1}_{-0.6}(\text{syst}) \pm 1.1(\text{model})$, β_{M1} from Baldin

sys. & model-error *under-estimated?:*

π production, SAID/MAID-2000 amplitudes,

π-exchange currents not chirally consistent,

...

To Do: Theory starting up: Demissie PhD

Analyse elastic & inelastic in unified χEFT frame, test quasi-free hypothesis.

Enhancement by $\Delta(1232)$ peak $\Longrightarrow$ accurate Δ theory.

To Do: Experiment

Better data.

Lower energies.

Sensitivity of single-/double-polarised observables, breakup asymmetries.

Alternative targets: ^{3}He, ^{4}He, ^{6}Li, ..., also for proton?

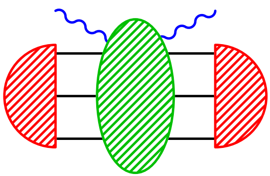
(h) Per Aspera Ad Astra: ^{3}He

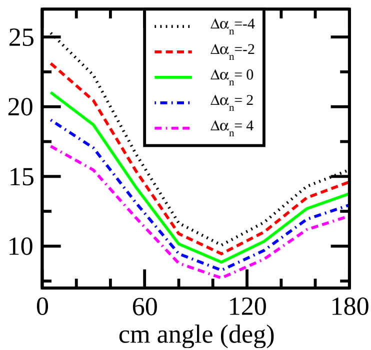
Experiment: $\frac{d\sigma}{d\Omega} \propto$ (target-charge)$^{2 \text{ to } 1}$, more & easier targets $\Longrightarrow$ *heavier nuclei*

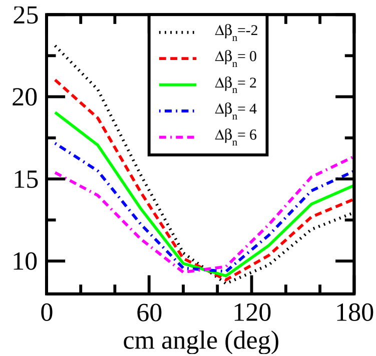
Theory: Reliable extraction needs accurate description of nuclear binding & levels $\Longrightarrow$ *lighter nuclei*

Find *sweet-spot* between competing forces: ^{3}He at HIγS, MAMI, MAXlab

Example: Sensitivity of unpolarised ^{3}He cross-section at $\omega_{\text{lab}} = 120$ MeV on α^n, β^n Shukla/Phillips/Nogga 2006-09







– First Compton cross section on ^{3}He.

– ^{3}He as effective neutron spin target.

– Extend beyond $\omega \in [80;120]$ MeV: re-scattering (Thomson, T_{NN}), explicit $\Delta(1232)$, threshold corrections

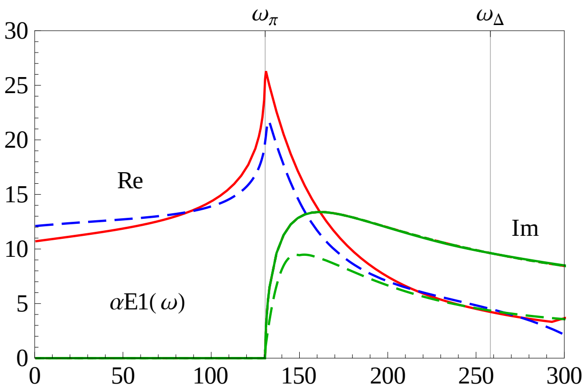
$\Longrightarrow$ Effects will become more pronounced.

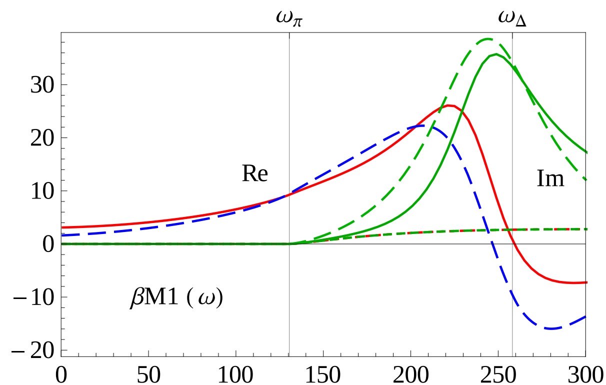
***Dynamical Polarisabilities*: Multipole decomposition of real Compton scattering at fixed energy.**

Neither more nor less information about response of constituents, but **more readily accessible.**

$\alpha_{E1}(\omega)$: Pion cusp well captured by single-$N\pi$.

$\beta_{M1}(\omega)$: para-magnetic N-to-Δ $M1$-transition.



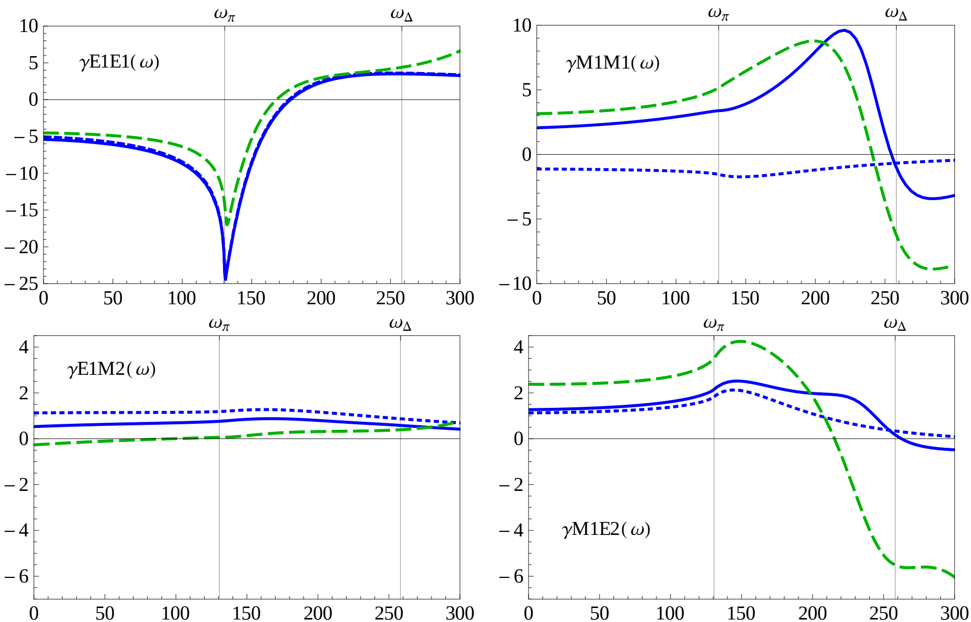


—— χEFT with $\Delta(1232)$

- - - Disp. Rel.

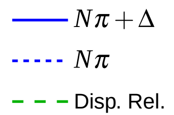
Re: **refraction**; Im: **absorption** $\Longrightarrow$ pion photo-production multipoles.

Predicted in χEFT: No N-core contributions $\Longrightarrow$ Spin-physics dominated by pion-cloud $+\ \Delta$ (γ_{M1M1}, γ_{M1E2}).



Pure polarisabilities

γ_{E1E1}, γ_{M1M1} agree.



Mixed polarisabilities

γ_{E1M2}, γ_{M1E2} small.

Uncertainties in DR?

Static values units: $[10^{-4}\ \text{fm}^4]$	χEFT iso-scalar	DR iso-scalar	MAMI proton	LEGS proton	MAMI neutron
$\bar{\gamma}_0 = -(\gamma_{E1E1}+\gamma_{M1M1}+\gamma_{E1M2}+\gamma_{M1E2})$	-0.7	-0.4	-1		??
$\bar{\gamma}_\pi - (\pi^0\text{−pole}) = -\gamma_{E1E1}+\gamma_{M1M1}-\gamma_{E1M2}+\gamma_{M1E2}$	8.6	12	~ 8	~ 18	$[12\ldots16]\pm 4_{\text{model}}$

4. Concluding Questions

Dynamical polarisabilities: Energy-dependent multipole-decomposition dis-entangles scales, symmetries & mechanisms of interactions with & among constituents: χiral symmetry of pion-cloud, iso-spin breaking, $\Delta(1232)$ properties, nucleon spin-constituents.

$\Longrightarrow$ **χEFT: unified frame-work off light nuclei: model-independent, systematic, reliable errors.**

Compton amplitude to $350\ \mathrm{MeV}$ – Scalar Dipole Polarisabilities from *all Compton data below* $200\ \mathrm{MeV}$:

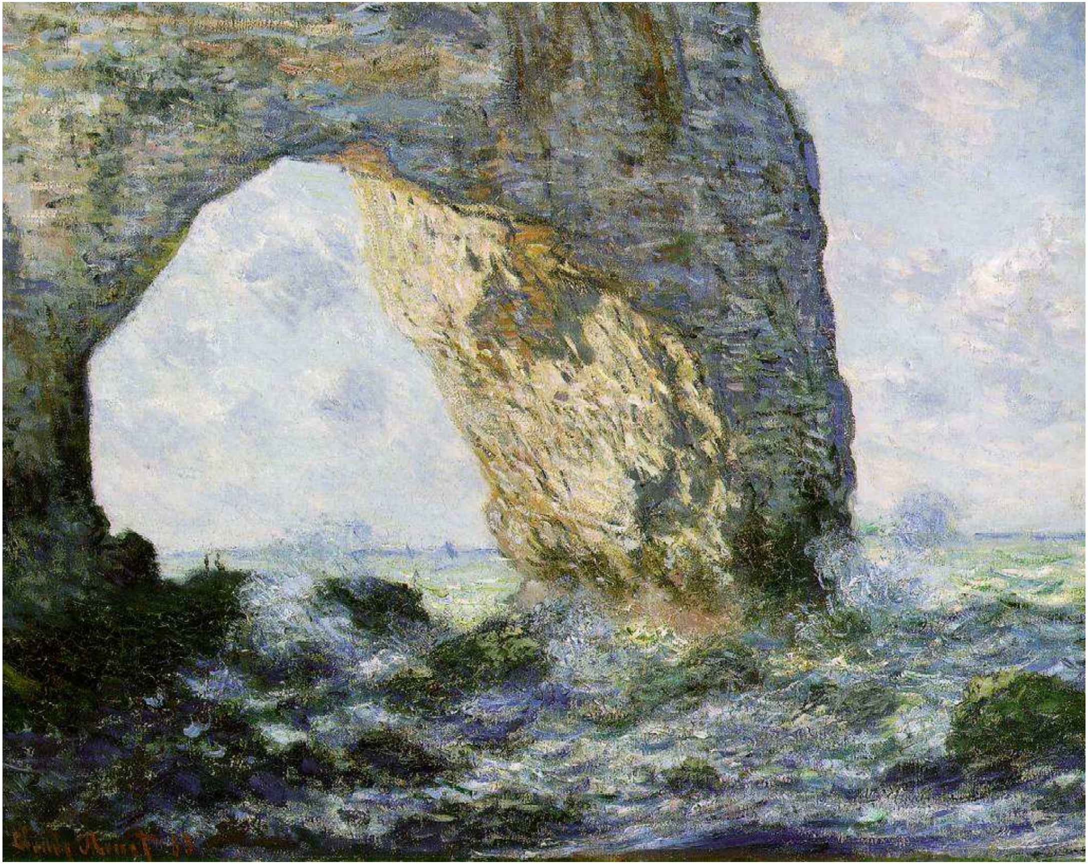
proton $\mathrm{N^2LO}$	$\alpha^p = 10.7 \pm 0.35_{\mathrm{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\mathrm{theory}}$	$\beta^p = 3.1 \mp 0.35_{\mathrm{stat}} \pm 0.2_{\Sigma} \mp 0.3_{\mathrm{theory}}$
iso-scalar NLO	$\alpha^s = 10.9 \pm 0.9_{\mathrm{stat}} \pm 0.2_{\Sigma} \pm 0.8_{\mathrm{theory}}$	$\beta^s = 3.6 \mp 0.9_{\mathrm{stat}} \pm 0.2_{\Sigma} \mp 0.8_{\mathrm{theory}}$
neutron NLO	$\alpha^n = 11.1 \pm 1.8_{\mathrm{stat}} \pm 0.4_{\Sigma} \pm 0.8_{\mathrm{theory}}$	$\beta^n = 4.1 \mp 1.8_{\mathrm{stat}} \pm 0.4_{\Sigma} \mp 0.8_{\mathrm{theory}}$

Theory To-Do List: explore host of observables: expansion $\frac{p_{\mathrm{typ}}}{\Lambda_\chi} \ll 1$ for credible error-bars. — math notebooks

- **One Nucleon:** polarisation observables, higher-order in resonance region — near-done; long-term
- **Deuteron:** embed $\mathrm{N^2LO}$; extend beyond 1π-threshold into $\Delta(1232)$ region; break-up — now focus of attention
- **^{3}He & heavier:** Thomson limit; into $\Delta(1232)$ region; break-up — starting

Data Needed: cross-sections & asymmetries – reliable systematics. $\Longrightarrow$ ***spin-polarisabilities.***

Clean probe to explore the strong force at low energies.



Claude Monet: Rock Arch West of Etretat (The Manneport), 1883

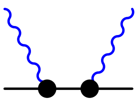
(a) Extracting Dynamical Polarisabilities

Rigorous definition from spin-independent Compton scattering amplitudes

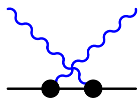
$$\begin{aligned} T(\omega,z) \;=\; & A_1(\omega,z)\,(\vec{\varepsilon}\,'^* \cdot \vec{\varepsilon}) + A_2(\omega,z)\,\left(\vec{\varepsilon}\,'^* \cdot \hat{\vec{k}}\right)\left(\vec{\varepsilon} \cdot \hat{\vec{k}}'\right) \\ & + \mathrm{i}\,A_3(\omega,z)\,\vec{\sigma}\cdot(\vec{\varepsilon}\,'^* \times \vec{\varepsilon}) + \mathrm{i}\,A_4(\omega,z)\,\vec{\sigma}\cdot(\hat{\vec{k}}' \times \hat{\vec{k}})\,(\vec{\varepsilon}\,'^* \cdot \vec{\varepsilon}) \\ & + \mathrm{i}\,A_5(\omega,z)\,\vec{\sigma}\cdot\left[\left(\vec{\varepsilon}\,'^* \times \hat{\vec{k}}\right)\left(\vec{\varepsilon}^{\,*} \cdot \hat{\vec{k}}'\right) - \left(\vec{\varepsilon} \times \hat{\vec{k}}'\right)\left(\vec{\varepsilon}\,'^* \cdot \hat{\vec{k}}\right)\right] \\ & + \mathrm{i}\,A_6(\omega,z)\,\vec{\sigma}\cdot\left[\left(\vec{\varepsilon}\,'^* \times \hat{\vec{k}}'\right)\left(\vec{\varepsilon}^{\,*} \cdot \hat{\vec{k}}'\right) - \left(\vec{\varepsilon} \times \hat{\vec{k}}'\right)\left(\vec{\varepsilon}^{\,*} \cdot \hat{\vec{k}}\right)\right] \end{aligned}$$

(1) Choose frame of reference: centre of mass; $\theta = \measuredangle(\vec{k},\vec{k}')$, $z = \cos\theta$.

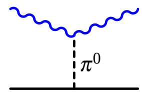
(2) Subtract "nucleon pole" terms in



s-channel,



u-channel,



t-channel,

: two-photon interaction with point-like spin-$\frac{1}{2}$ nucleon of magnetic moment κ (Powell) $+$ pion-pole term.

$\Longrightarrow$ **Structure-dependent part** of Compton amplitude:

$$\bar{A}_i(\omega,\cos\theta) = A_i(\omega,\cos\theta) - A_i^{\mathrm{pole}}(\omega,\cos\theta)$$

(3) Multipole decomposition into photon transitions $Tl \to T'l'$ $(T = E, M)$ at fixed energy ω:

static: $\alpha_{E1}(\omega = 0) = \bar{\alpha}$ etc. $W = \omega + \sqrt{M^2 + \omega^2}$: cm energy

$$\bar{A}_1(\omega,z) = \frac{4\pi W}{M}\left[\Big(\alpha_{E1}(\omega) + \cos\theta\,\beta_{M1}(\omega)\Big)\omega^2 + \frac{1}{12}\Big(\cos\theta\,\alpha_{E2}(\omega) + (2\cos^2\theta - 1)\,\beta_{M2}(\omega)\Big)\omega^4 + \dots\right]$$

$$\bar{A}_2(\omega,z) = -\frac{4\pi W}{M}\,\beta_{M1}(\omega)\,\omega^2 + \dots,$$

$$\bar{A}_3(\omega,z) = -\frac{4\pi W}{M}\left[\gamma_{E1E1}(\omega) + \cos\theta\,\gamma_{M1M1}(\omega) + \gamma_{E1M2}(\omega) + \cos\theta\,\gamma_{M1E2}(\omega)\right]\omega^3 + \dots$$

$$\bar{A}_4(\omega,z) = \frac{4\pi W}{M}\left[-\gamma_{M1M1}(\omega) + \gamma_{M1E2}(\omega)\right]\omega^3 + \dots$$

$$\bar{A}_5(\omega,z) = \frac{4\pi W}{M}\,\gamma_{M1M1}(\omega)\,\omega^3 + \dots \qquad \bar{A}_6(\omega,z) = \frac{4\pi W}{M}\,\gamma_{E1M2}(\omega)\,\omega^3 + \dots$$

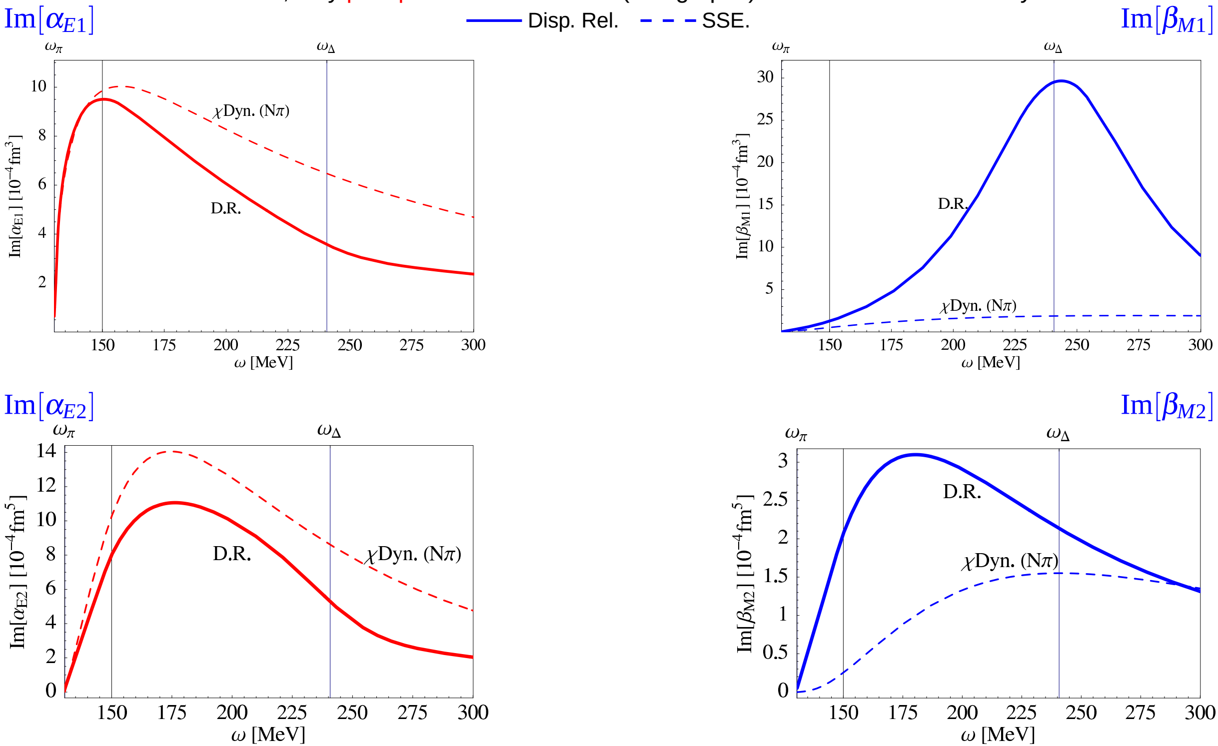
Dynamical polarisabilities:

Response of internal degrees of freedom to external, real photon field of definite multipolarity & non-zero energy.

Neither more nor less information about temporal response/dispersive effects of nucleon constituents, but information more readily accessible.

(b) Imaginary Parts of Iso-Scalar Polarisabilities

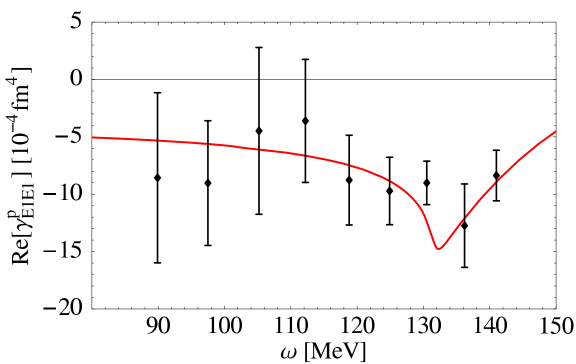
LO MSSE: Δ without width; only pion production threshold ($N\pi$ graphs). Non-zero Δ width clearly seen in DR.

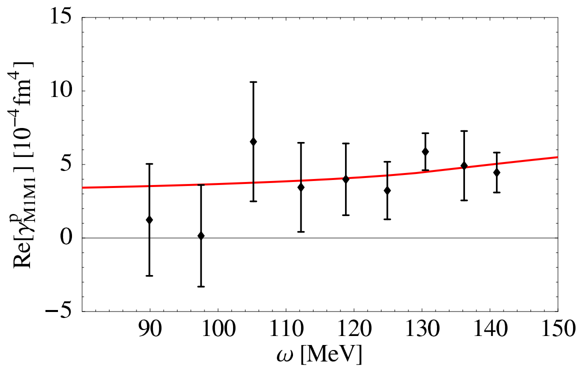


$$4\pi\, N^\dagger \left\{ \frac{1}{2} \left[\alpha_{E1}(\omega)\, \vec{E}^2 + \beta_{M1}(\omega)\, \vec{B}^2 \right] \right.$$ spin-indep dipole

$$+ \frac{1}{2} \left[\gamma_{E1E1}(\omega)\, \vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}(\omega)\, \vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}}) \right.$$ "pure" spin-dep dipole

$$\left. \left. -2\,\gamma_{M1E2}(\omega)\, \sigma_i\, B_j\, E_{ij} + 2\,\gamma_{E1M2}(\omega)\, \sigma_i\, E_j\, B_{ij} \right] + \dots \right\} N$$ "mixed" spin-dep dipole





Assumptions: $\alpha_{E1}(\omega)$, $\beta_{M1}(\omega)$ well captured, only $\gamma_{E1E1}(\omega)$, $\gamma_{M1M1}(\omega)$ large $\Longrightarrow$ superficial fit to data.

Spin-physics dominated by pion-cloud $+\Delta$. No N-core contributions.

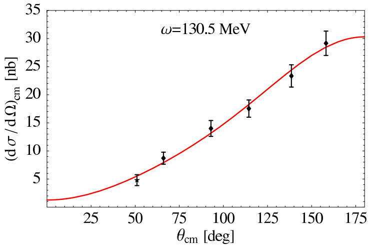
$$
\begin{aligned}
4\pi\, N^\dagger \Big\{ & \tfrac{1}{2}\left[\alpha_{E1}(\omega)\,\vec{E}^2 + \beta_{M1}(\omega)\,\vec{B}^2\right] && \text{spin-indep dipole} \\
& + \tfrac{1}{2}\Big[\gamma_{E1E1}(\omega)\,\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}(\omega)\,\vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}}) && \text{“pure” spin-dep dipole} \\
& -2\,\gamma_{M1E2}(\omega)\,\sigma_i\,B_j\,E_{ij} + 2\,\gamma_{E1M2}(\omega)\,\sigma_i\,E_j\,B_{ij}\Big] + \dots \Big\}\, N && \text{“mixed” spin-dep dipole}
\end{aligned}
$$

Spin-physics dominated by pion-cloud $+\Delta$. No N-core contributions.

$$4\pi\, N^\dagger \Big\{ \tfrac{1}{2} \Big[\alpha_{E1}(\omega)\, \vec{E}^2 + \beta_{M1}(\omega)\, \vec{B}^2\Big]$$ spin-indep dipole

$$+ \tfrac{1}{2} \Big[\gamma_{E1E1}(\omega)\, \vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}(\omega)\, \vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}})$$ "pure" spin-dep dipole

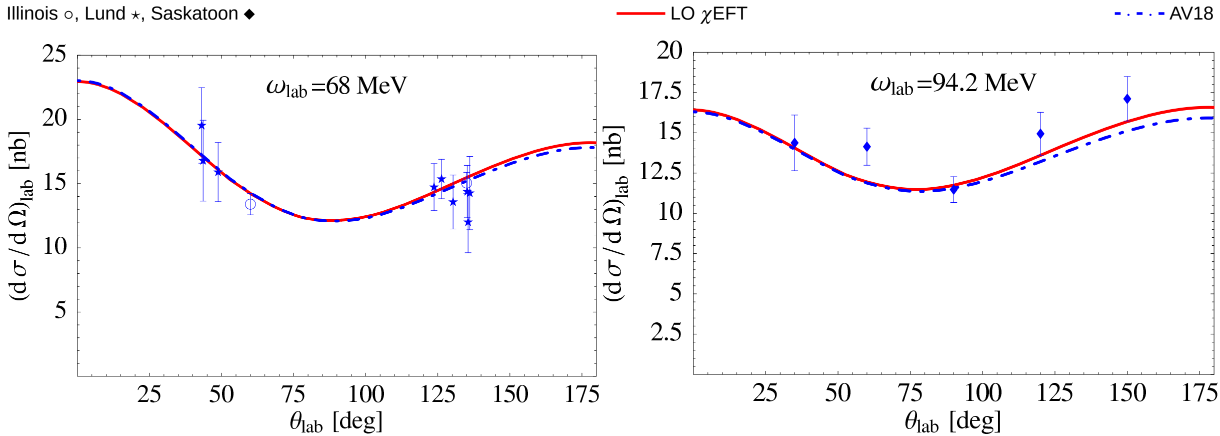
$$-2\,\gamma_{M1E2}(\omega)\, \sigma_i\, B_j\, E_{ij} + 2\,\gamma_{E1M2}(\omega)\, \sigma_i\, E_j\, B_{ij}\Big] + \dots \Big\}\, N$$ "mixed" spin-dep dipole

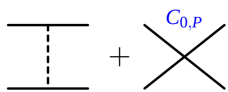


Large error-bars because γ_i-effects $\propto \cos^2\theta$, while data nearly linear.

(c) Spin-Dependent Dynamical Polarisabilities from Multipole Analysis

Dependence of T_{NN} on NN-potential $\cong$ short-distance, for $\omega \to 0$ clear from Thomson.



LO χEFT-potential: 

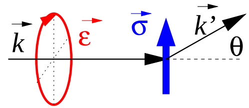
$\sim Q^{-1}$

Consistent for Compton at NLO: $\mathcal{O}(Q^0)$-correction of NN-potential presumed zero.

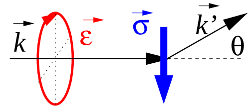
AV18 provides $< 3\%$ corrections $\implies$ suggests higher-order indeed $Q^1 \approx \left(\frac{1}{7}\right)^2$.

Deuteron Best: Incoming γ circularly polarised, sum over final states. N-spin in $(\vec{k},\vec{k}')$-plane, perpendicular to $\vec{k}$:

difference Δ_x^{circ}, asymmetry $\Sigma_x^{\text{circ}} = \dfrac{\Delta_x^{\text{circ}}}{\text{sum}}$

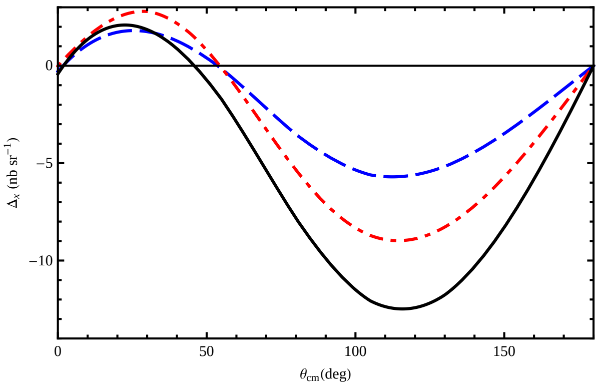


vs.



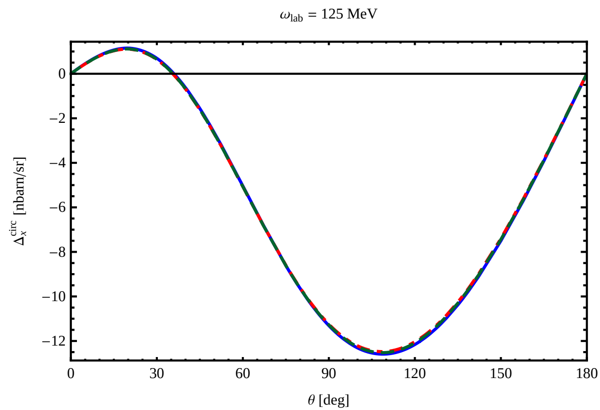
Sensitivity on Δ & NN-rescattering:

- - - - $N\pi$, no NN; - · - · $N\pi+\Delta$, no NN; —— $N\pi+\Delta+NN$



Sensitivity on wave-function:

NNLO Epelbaum 650 MeV, AV18, Nijmegen 93



– More pronounced by explicit $\Delta(1232)$

– Thomson (NN rescatt.) important even at high $\omega = 125$ MeV

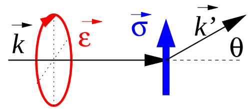
– No residual deuteron wave-function dependence

– Higher pols negligible

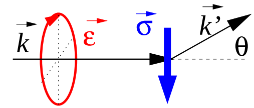
(d) Spin-Polarisabilities from Circularly Pol. Photons at 125 MeV

Deuteron Best: Incoming γ circularly polarised, sum over final states. N-spin in $(\vec{k},\vec{k}')$-plane, perpendicular to $\vec{k}$:

difference Δ_x^{circ}, asymmetry $\Sigma_x^{\text{circ}} = \dfrac{\Delta_x^{\text{circ}}}{\text{sum}}$

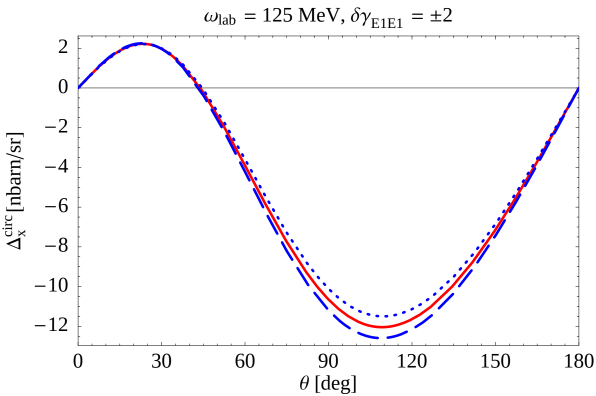


vs.



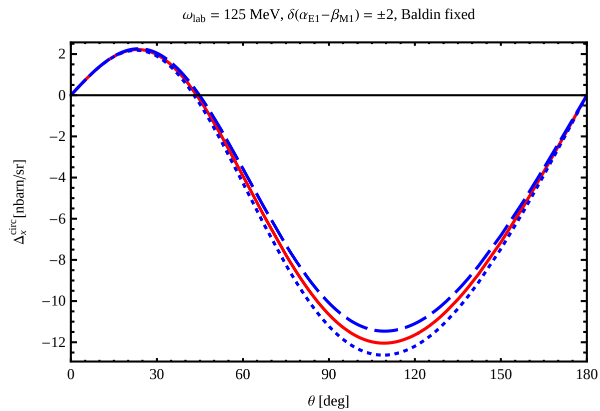
Sensitivity on neutron γ_{E1E1}

—— -5.2; - - - - - $-5.2+2$; ······ $-5.2-2$



Sensitivity on neutron $\alpha_{E1}-\beta_{M1}$; Baldin-Σ fixed

—— 8.2; - - - - - $8.2+2$; ······ $8.2-2$



Sensitive to γ_{E1E1}, but must nail down α_{E1}, β_{M1} at lower energy.

Similarly good signal for linear polarisation Δ_x^{lin}.

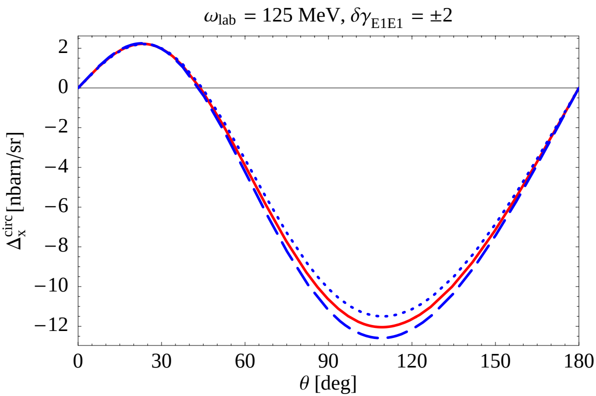
(d) Spin-Polarisabilities from Circularly Pol. Photons at $125\ \mathrm{MeV}$

Deuteron Best: Incoming γ circularly polarised, sum over final states. N-spin in $(\vec{k},\vec{k}')$-plane, perpendicular to $\vec{k}$:

difference Δ_x^{circ}, asymmetry $\Sigma_x^{\text{circ}} = \frac{\Delta_x^{\text{circ}}}{\text{sum}}$

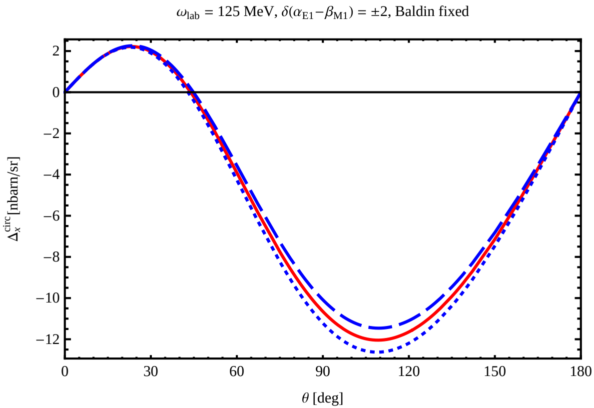
Sensitivity on neutron γ_{E1E1}

—— -5.2; - - - - - $-5.2+2$; ······ $-5.2-2$



Sensitivity on neutron $\alpha_{E1}-\beta_{M1}$; Baldin-Σ fixed

—— 8.2; - - - - - $8.2+2$; ······ $8.2-2$

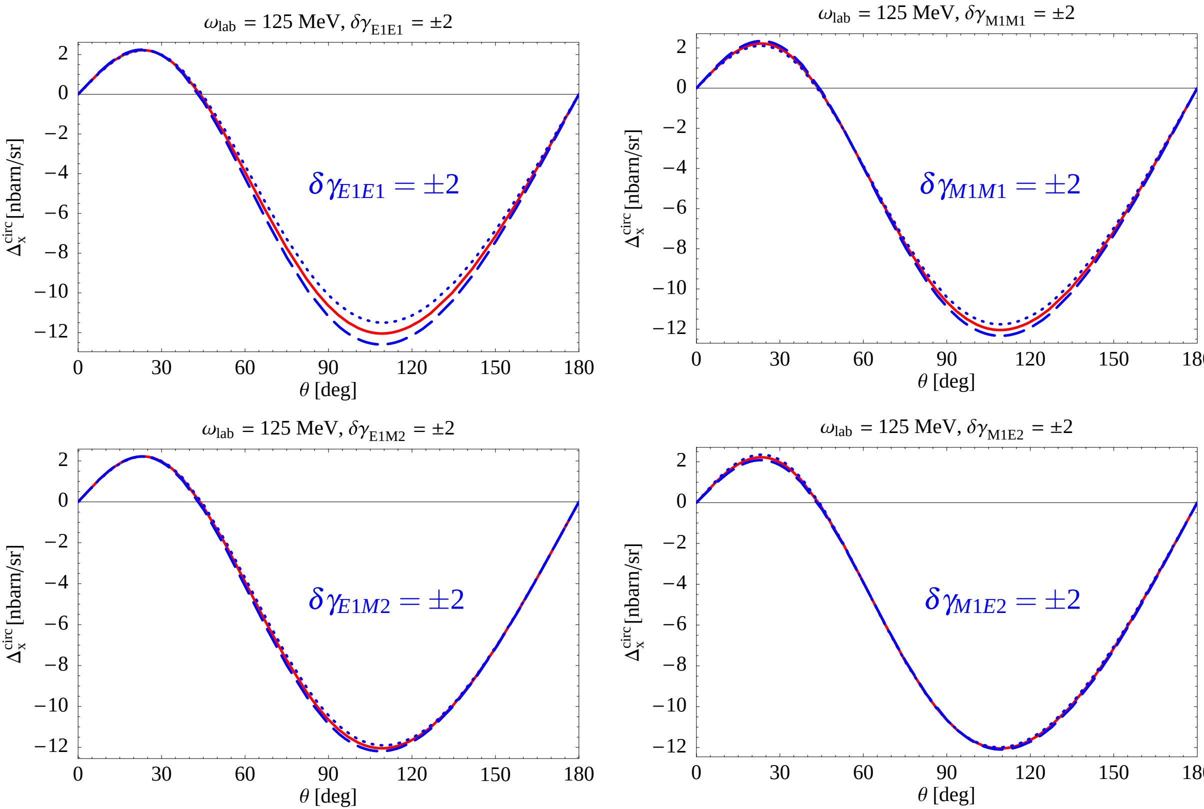


Sensitive to γ_{E1E1}, but must nail down α_{E1}, β_{M1} at lower energy.

$\Delta(1232)$ and re-scattering increase signal.

(d) Spin-Polarisabilities from Circularly Pol. Photons at 125 MeV

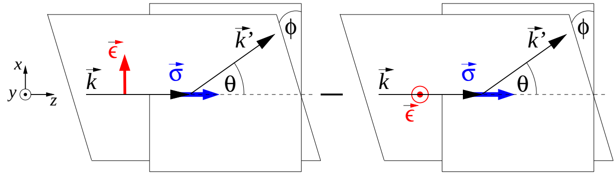
Deuteron Best: Incoming γ circularly polarised, sum over final states. N-spin in $(\vec{k}, \vec{k}')$-plane, perpendicular to $\vec{k}$:



(e) Spin-Polarisabilities from Linearly Pol. Photons at $125\,\mathrm{MeV}$

Deuteron Best: Incoming γ linearly polarised, sum over final states. N-spin in $(\vec{k},\vec{k}')$-plane, parallel to $\vec{k}$:

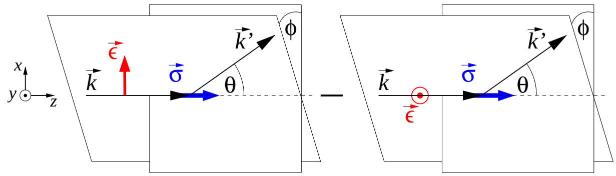
difference Δ_z^{lin}, asymmetry $\Sigma_z^{\text{lin}} = \dfrac{\Delta_z^{\text{lin}}}{\text{sum}}$



(e) Spin-Polarisabilities from Linearly Pol. Photons at 125 MeV

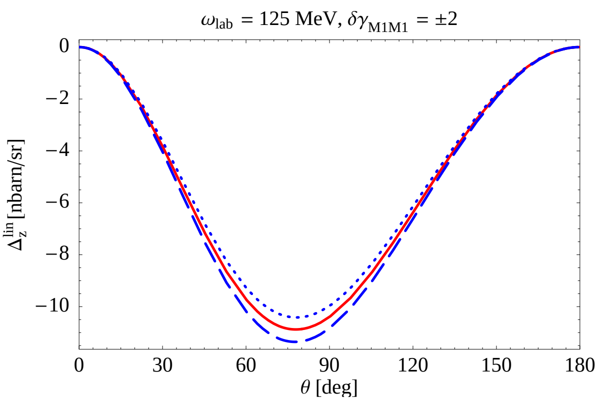
Deuteron Best: Incoming γ linearly polarised, sum over final states. N-spin in $(\vec{k},\vec{k}')$-plane, parallel to $\vec{k}$:

difference Δ_z^{lin}, asymmetry $\Sigma_z^{\text{lin}} = \frac{\Delta_z^{\text{lin}}}{\text{sum}}$



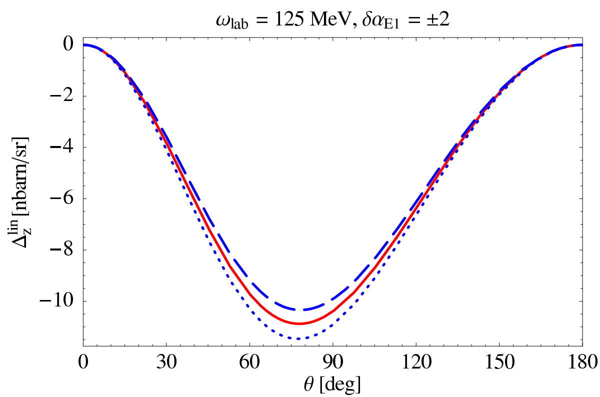
Sensitivity on neutron γ_{M1M1}

—— 3.2; - - - - 3.2 + 2; ······ 3.2 − 2



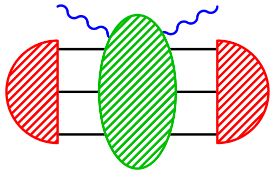
Sensitivity on neutron α_{E1}

—— 11.3; - - - - 11.3 + 2; ······ 11.3 − 2

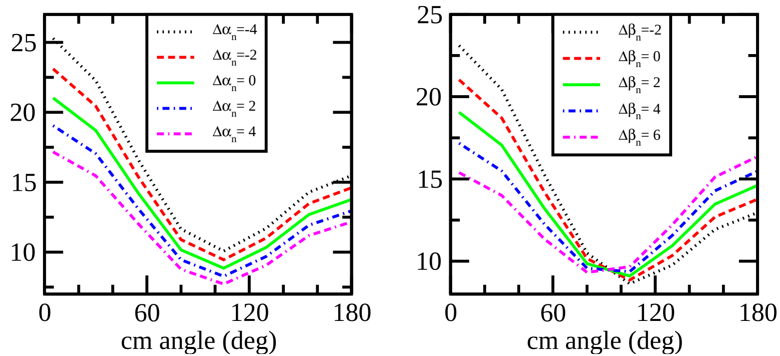


Sensitive to γ_{M1M1}, but must nail down α_{E1}, β_{M1} at lower energy.

(f) Per Aspera Ad Astra: ^{3}He



Example: Sensitivity of unpolarised cross-section at $\omega_{\text{lab}} = 120$ MeV on α^n, β^n Shukla/Phillips/Nogga 2006-09



– First Compton cross section on ^{3}He.

– ^{3}He as effective neutron spin target.

– Extend beyond $\omega \in [80;120]$ MeV: re-scattering (Thomson, T_{NN}), explicit $\Delta(1232)$, threshold corrections

$\Longrightarrow$ Effects will become more pronounced.

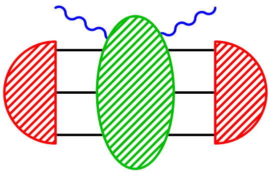
(f) Per Aspera Ad Astra: ^{3}He

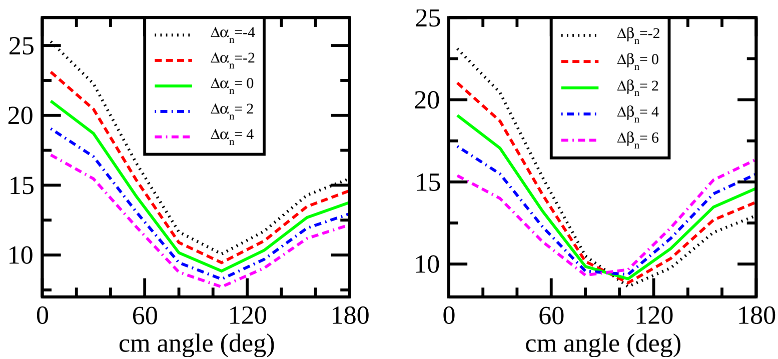
Experiment: $\frac{d\sigma}{d\Omega} \propto$ (target-charge)$^{2\text{ to }1}$, more & easier targets $\Longrightarrow$ *heavier nuclei*

Theory: Reliable extraction needs accurate description of nuclear binding & levels $\Longrightarrow$ *lighter nuclei*

Find *sweet-spot* between competing forces: ^{3}He at HIγS, MAMI, MAXlab

Example: Sensitivity of unpolarised ^{3}He cross-section at $\omega_{\text{lab}} = 120$ MeV on α^n, β^n Shukla/Phillips/Nogga 2006-09





– First Compton cross section on ^{3}He.

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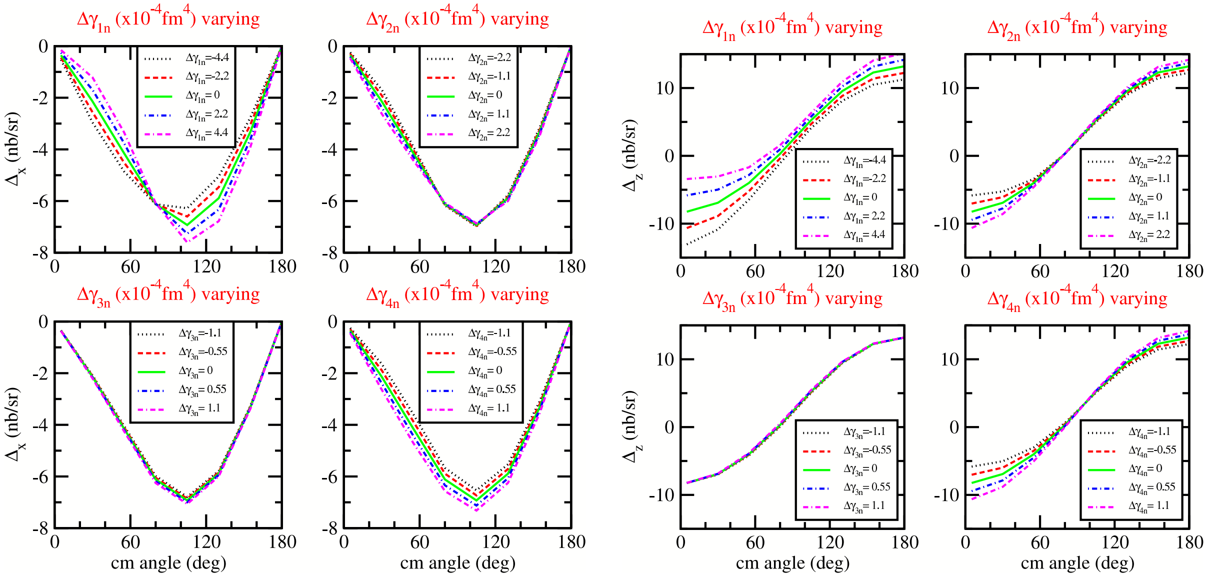
– Extend beyond $\omega \in [80;120]$ MeV: re-scattering (Thomson, T_{NN}), explicit $\Delta(1232)$, threshold corrections

$\Longrightarrow$ Effects will become more pronounced.

(g) Starting with ^{3}He: Doubly-Polarised

Choudhury Shukla/Phillips/Nogga 2006-09

Example: Sensitivity of polarised cross-section at $\omega_{\text{lab}} = 120$ MeV on γ_i^n's



– First Compton cross section on ^{3}He.

– ^{3}He as effective neutron spin target.

– Extend beyond $\omega \in [80; 120]$ MeV: re-scattering (Thomson, T_{NN}), explicit $\Delta(1232)$, threshold corrections

$\Longrightarrow$ Effects will become more pronounced.