Properties of neutron matter, symmetry energy and neutron stars

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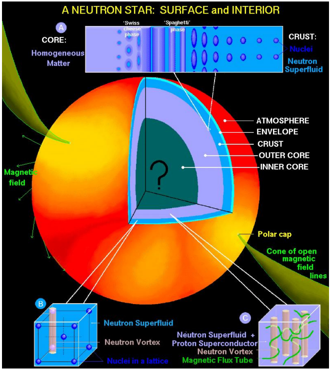
Light nuclei from first principles
INT, University of Washington, Seattle, September 25, 2012

Neutron stars

Neutron star is a wonderful natural laboratory

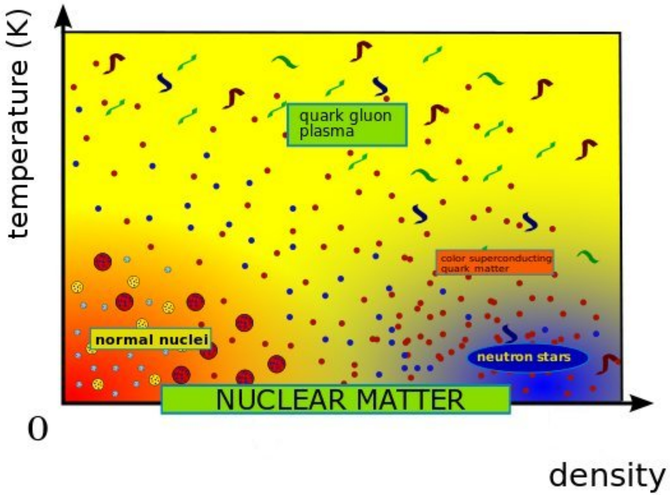


D. Page

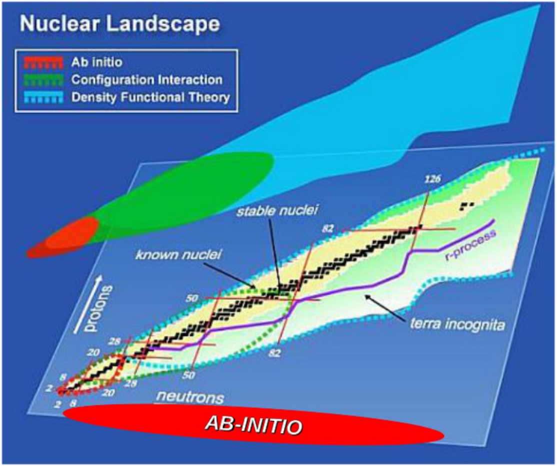
- Atmosphere: atomic and plasma physics
- Crust: physics of superfluids (neutrons, vortex), solid state physics (nuclei)
- Inner crust: deformed nuclei, pasta phase
- Outer core: nuclear matter
- Inner core: hyperons? quark matter? π or K condensates?

There is job for very different fields, from condensed matter to string theory, you can publish results related to neutron stars in PRL, PRA, PRB, PRC, PRD and PRE!!

Homogeneous neutron matter



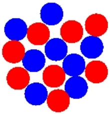
Inhomogeneous neutron matter



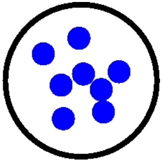
W. Nazarewicz – UNEDF

Neutron drops

Why study neutron drops?
Are they nothing more than a pure simple toy model?



NP self–bound



N confined

Neutron drops are interesting because:

- Provide a strong benchmark for microscopic calculations
- Model neutron-rich nuclei
- Calibrate Skyrme models for neutron-rich systems (useful to check $\nabla\rho$ terms in different geometries)

Outline

- The model and the method
- **Homogeneous neutron matter**
  - Three-neutron force and the equation of state of neutron matter
  - Symmetry energy
  - Neutron star structure
- **Inhomogeneous neutron matter: Skyrme vs ab-initio.**
  - Energy
  - Density and radii
- Conclusions

Nuclear Hamiltonian

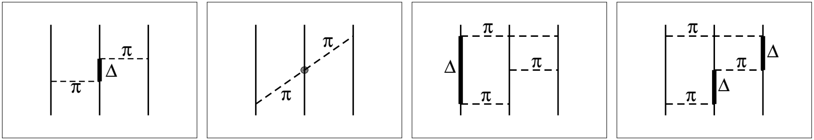
Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

v_{ij} NN (Argonne AV8') fitted on scattering data. Sum of operators:

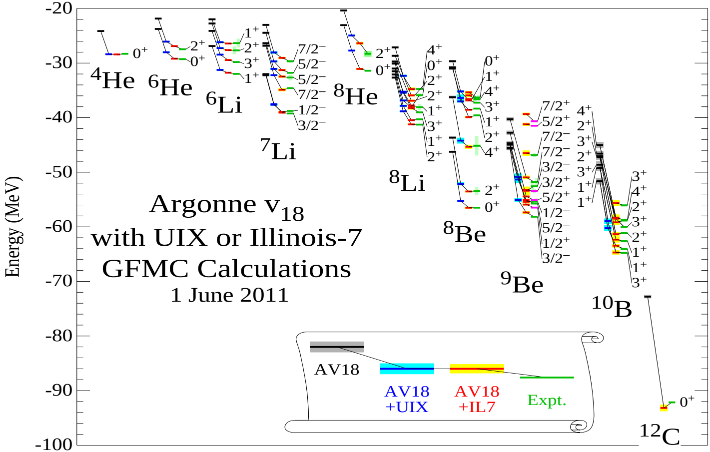
$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij}) \,, \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

Urbana–Illinois V_{ijk} models processes like



$+$ short-range correlations (spin/isospin independent).

Light nuclei spectrum computed with GFMC



Carlson, Pieper, Wiringa, many papers

Quantum Monte Carlo

Evolution of Schrodinger equation in imaginary time **t**:

$$\psi(R,t) = e^{-(H-E_T)t}\psi(R,0)$$

In the limit of $t \to \infty$ it approaches to the lowest energy eigenstate (not orthogonal to $\psi(R,0)$).

Propagation performed by

$$\psi(R,t) = \langle R|\psi(t)\rangle = \int dR' G(R,R',t)\psi(R',0)$$

$G(R,R',t)$ is an approximate propagator (small-time limit). We iterate the above integral equation many times in the small time-step limit.
$\rightarrow$ parallel codes and supercomputers.

For a given microscopic Hamiltonian, this method solves the ground–state within a systematic uncertainty of **1–2%** in a **non-perturbative way.**

Quantum Monte Carlo

Recall: propagation in imaginary-time

$$e^{-(T+V)\Delta\tau}\psi \approx e^{-T\Delta\tau}e^{-V\Delta\tau}\psi$$

Kinetic energy is sampled as a diffusion of particles:

$$e^{-\nabla^2\Delta\tau}\psi(R) = e^{-(R-R')^2/2\Delta\tau}\psi(R) = \psi(R')$$

The (scalar, local) potential gives the weight of the configuration:

$$e^{-V(R)\Delta\tau}\psi(R) = w\psi(R)$$

Algorithm for each time-step:

- do the diffusion: $R' = R + \xi$
- compute the weight w
- compute observables using the configuration R' weighted using w over a trial wave function ψ_T.

For spin-dependent potentials things are much worse!

GFMC and AFDMC

Because the Hamiltonian is state dependent, all spin/isospin states of nucleons must be included in the wave-function.

Example: spin for 3 neutrons (radial parts also needed in real life):

GFMC wave-function:

$$\psi = \begin{pmatrix} a_{\uparrow\uparrow\uparrow} \\ a_{\uparrow\uparrow\downarrow} \\ a_{\uparrow\downarrow\uparrow} \\ a_{\uparrow\downarrow\downarrow} \\ a_{\downarrow\uparrow\uparrow} \\ a_{\downarrow\uparrow\downarrow} \\ a_{\downarrow\downarrow\uparrow} \\ a_{\downarrow\downarrow\downarrow} \end{pmatrix}$$

A correlation like

$$1 + f(r)\sigma_1 \cdot \sigma_2$$

can be used, and the variational wave function can be very good. Any operator accurately computed.

AFDMC wave-function:

$$\psi = \mathcal{A}\left[\xi_{s_1}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}\xi_{s_2}\begin{pmatrix} a_2 \\ b_2 \end{pmatrix}\xi_{s_3}\begin{pmatrix} a_3 \\ b_3 \end{pmatrix}\right]$$

We must change the propagator by using the Hubbard-Stratonovich transformation:

$$e^{\frac{1}{2}\Delta t O^2} = \frac{1}{\sqrt{2\pi}}\int dx e^{-\frac{x^2}{2}+x\sqrt{\Delta t}O}$$

Auxiliary fields x must also be sampled.

The wave-function is pretty bad, but we can deal to large systems (up to $A \approx 100$).

Operators (except the energy) are very hard to be computed, but in some case there is some trick!

Neutron matter equation of state

Motivations:

- EOS of neutron matter main ingredient to study neutron stars.
- EOS of neutron matter useful to study the symmetry energy and its slope at saturation.

Assumptions/observations:

- The two-nucleon interaction reproduces well (elastic) *pp*, *np* and *nn* scattering data up to high energies ($E_{lab} \sim 600$MeV).
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part (but zero in neutron matter). No direct $T = 3/2$ experiments available!
- In neutron matter the short-range repulsive part of three-body force is the dominant term.

Systematic uncertainties of 3-neutron forces must be understood!

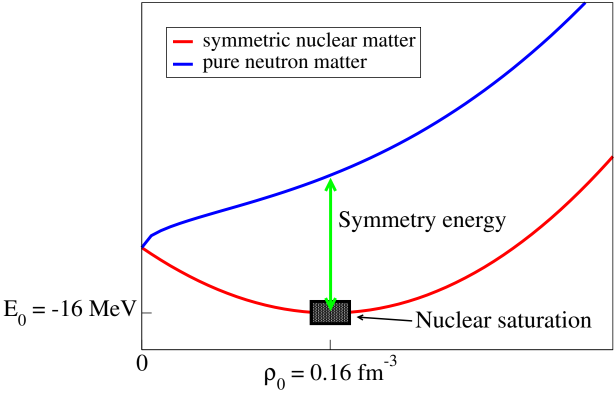
Symmetry energy

Nuclear matter EOS:

$$E(\rho, x) = E_{SNM}(\rho) + E_{sym}^{(2)}(\rho)(1-2x)^2 + \cdots$$

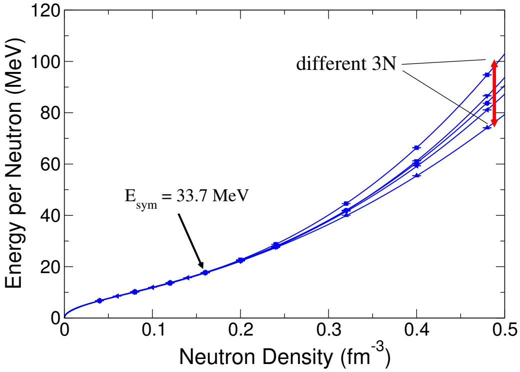
where

$$\rho = \rho_n + \rho_p\,, \quad x = \frac{\rho_p}{\rho}$$



Neutron matter

We consider different forms of three-neutron interaction by only requiring a particular value of E_{sym} at saturation.

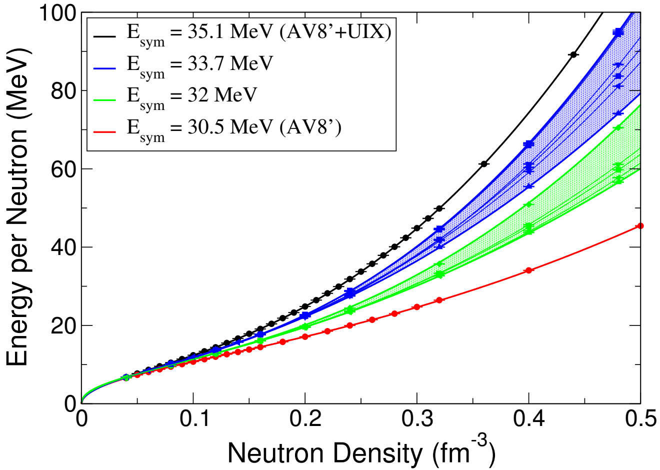


different 3N:

- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V_R^{\mu}$ (several μ)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$
- ...

Neutron matter and symmetry energy

We then try to change the neutron matter energy at saturation:

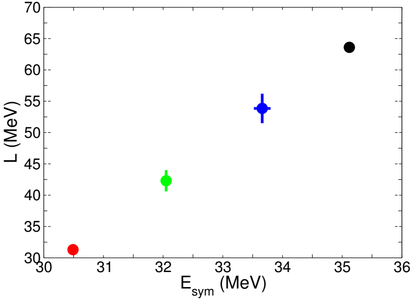


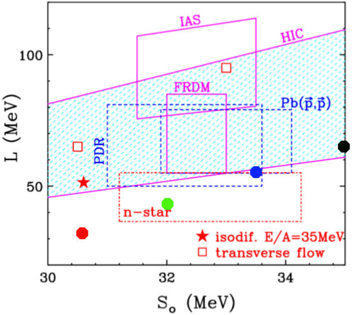
Gandolfi, Carlson, Reddy, PRC (2012).

Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around ρ_0 using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$





Tsang *et al.*, PRC (2012)

Very weak dependence to the model of 3N force for a given E_{sym}.

Neutron star structure

Given an EOS, we can study the neutron star structure by integrating the Tolman-Oppenheimer-Volkoff (TOV) equations:

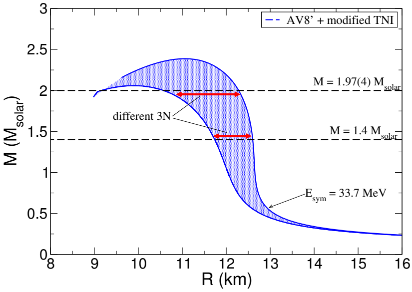
$$\frac{dP}{dr} = -\frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]}, \quad \frac{dm(r)}{dr} = 4\pi\epsilon r^2 ,$$

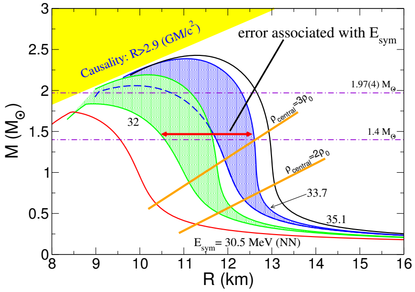
where $P = \rho^2(\partial E/\partial\rho)$ and $\epsilon = \rho(E + m_N)$.

What we get is the maximum mass M of a neutron star as a function of its radius R.

Neutron star structure

EOS used to solve the TOV equations.



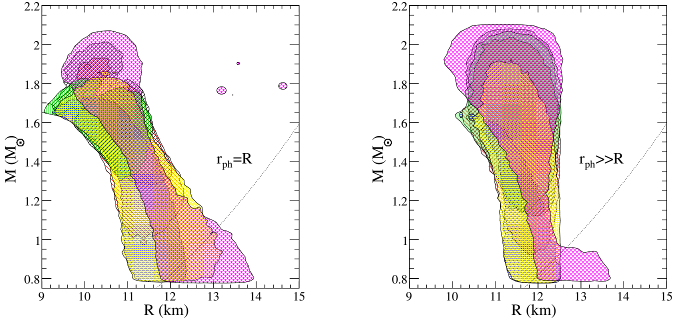


Accurate measurement of E_{sym} would put a constraint to the radius of neutron stars, **OR** observation of M and R would constrain E_{sym}!

$M = 1.97\ M_{solar}$ recently observed – Nature (2010).

Neutron stars

Observations of the mass-radius relation of neutron stars are becoming available:



Steiner, Lattimer, Brown, ApJ (2010)

→ We can **fit** an EOS to observations.

Neutron star matter

We model neutron star matter as

$$E_{NSM} = a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta}, \qquad \rho < \rho_t$$

(form suggested by QMC simulations),

and a high density model for $\rho > \rho_t$

i) two polytropes

ii) polytrope+quark matter model, Alford et al., ApJ (2005).

By changing ρ_t and the high density model we can understand systematic errors in E_{NSM} parametrization.

From observations we can extract all the parameters a, α, ...

Observations

What can we learn by fitting our model to observations?

- Symmetry energy and its slope:

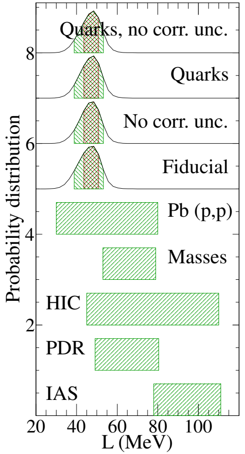
$$E_{sym} = a + b + 16\,, \quad L = 3(a\alpha + b\beta)$$

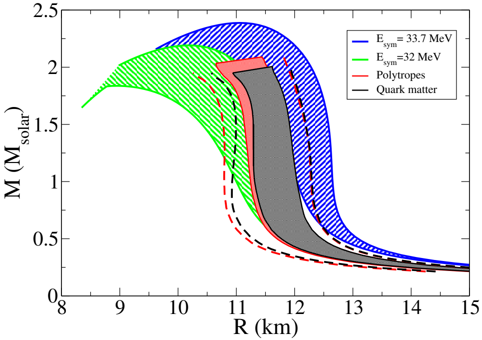
- Strength of 3N:

3N force	$E_{\rm sym}$ (MeV)	L (MeV)	a (MeV)	α	b (MeV)	β
none	30.5	31.3	12.7	0.49	1.78	2.26
$V_{2\pi} + V^R_{\mu=300}$	32.0	40.6	12.8	0.488	3.19	2.20
$V_{2\pi} + V^R_{\mu=600}$	32.0	41.3	12.8	0.488	3.19	2.20
$V_{2\pi} + V_R$	32.1	41.3	12.7	0.476	3.34	2.22
$V_{3\pi} + V_R$	32.0	44.0	13.0	0.49	3.21	2.47
$V_{2\pi} + V_R$	33.7	52.9	13.3	0.512	4.38	2.39
$V_{3\pi} + V_R$	33.8	56.2	13.0	0.50	4.71	2.49
UIX	35.1	63.6	13.4	0.514	5.62	2.436

Note: a and α don't depend too much to the model of 3N!

Neutron star observations





$32 < E_{sym} < 34\,MeV,\ 43 < L < 52\,MeV$

Steiner, Gandolfi, PRL (2012).

Can we use neutron star observations to constrain the model of 3N?

Neutron drops

Now let's study **inhomogeneous neutron matter.**

We confine neutrons by adding an external potential:

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i V_{ext}(r_i)$$

V_{ext} is a Wood-Saxon or Harmonic well:

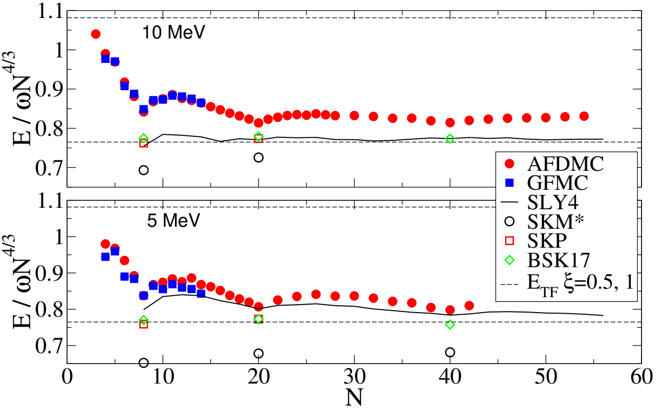
$$V_{WS} = -\frac{V_0}{1 + exp[(r-R)/a]}$$

$$V_{HO} = \frac{1}{2} m\omega^2 r^2$$

$\Longrightarrow$ different geometries and densities.

Neutron drops, harmonic oscillator well

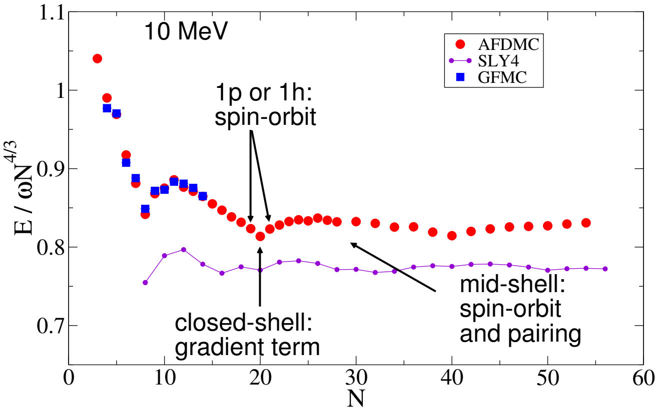
External well: harmonic oscillator with $\hbar\omega$=5, 10 MeV.



Skyrme systematically overbind neutron drops.

Neutron drops, harmonic oscillator well

Fixing Skyrme force:

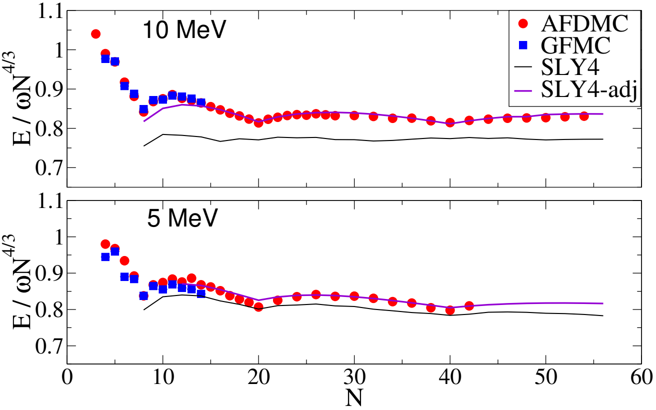


The correction is very similar in all the Skyrme forces we considered.

Neutron drops, adjusted Skyrme force

Note: bulk term of Skyrme fit neutron matter.

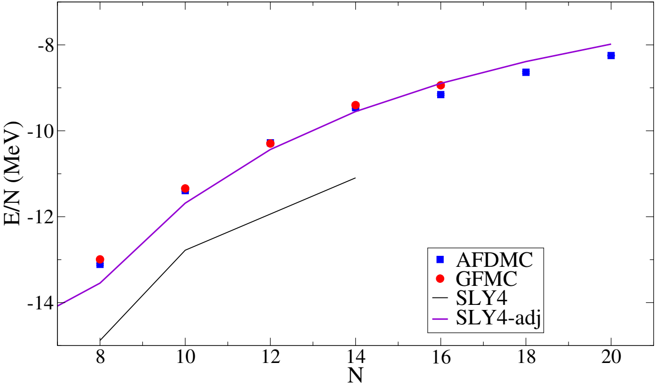
We add the **missing repulsion** by adjusting the gradient term $G_d[\nabla\rho_n]^2$, the pairing and spin-orbit terms.



Gandolfi, Carlson, Pieper, PRL (2011).

Neutron drops, adjusted Skyrme force

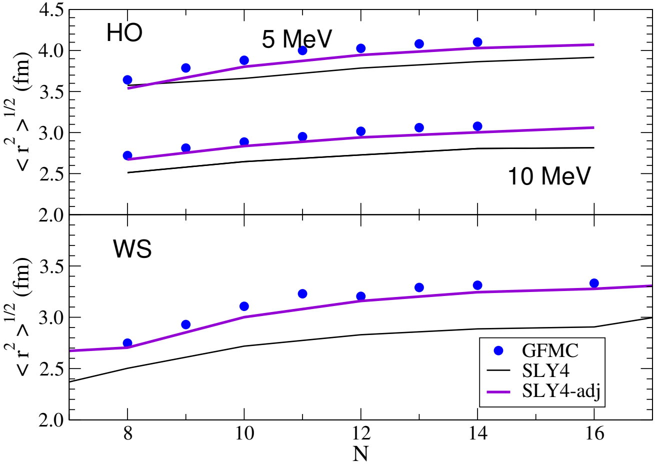
Neutrons in the Wood-Saxon well are also better reproduced by the adjusted SLY4.



Gandolfi, Carlson, Pieper, PRL (2011).

Neutron drops: radii

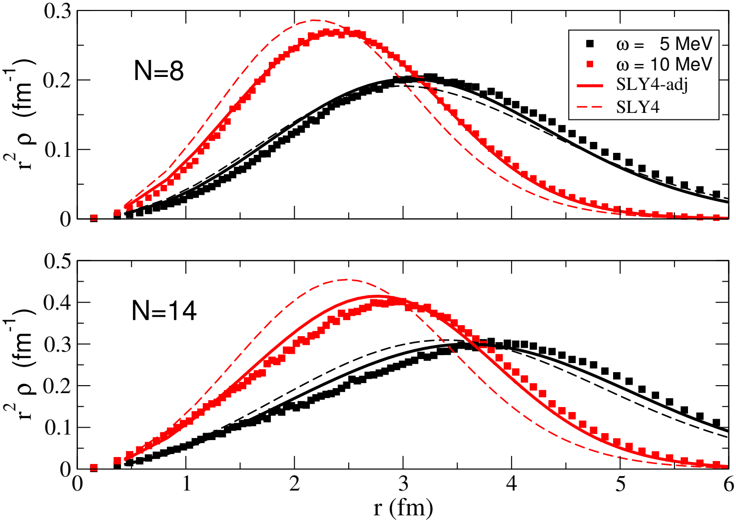
Correction to radii using the adjusted-SLY4.



Gandolfi, Carlson, Pieper, PRL (2011).

Neutron drops: radial density

Neutron radial density:



Gandolfi, Carlson, Pieper, PRL (2011).

Gradient term

Where is the gradient term important?

Just few examples:

- Medium large neutron-rich nuclei
- Phases in the crust of neutron stars
- Isospin-asymmetry energy of nuclear matter

Conclusions

- Effect of three–neutron forces to high-density neutron matter; the systematic uncertainty due to 3N is relatively small.
- E_{sym} strongly constrain L. Weak dependence to the model of 3N.
- Uncertainty of the radius of neutron stars mainly due E_{sym} rather than 3N.
- Neutron star observations becoming competitive with terrestrial experiments.
- Skyrme can be better constrained by ab-initio calculations.

Thanks for the attention