Asymptotic Wave Functions in Light Nuclei

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Workshop on "Structure of light nuclei"
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Context:

- "Corrections to nuclear energies and radii in finite oscillator spaces," rjf, G. Hagen, T. Papenbrock, arXiv:1207.6100 plus S. More at OSU
- Nuclear wave functions at large momenta from low-momentum (e.g., SRG) point of view (fate of SRCs, factorization scales, ...) [E. Anderson, S. Bogner, K. Hebeler, S. More, R. Perry, K. Wendt, ...]

Outline

Motivation: Extrapolations in finite bases

Nature and implications of infrared cutoffs

High-momentum behavior of wave functions

Combined IR and UV extrapolations

Summary and open questions

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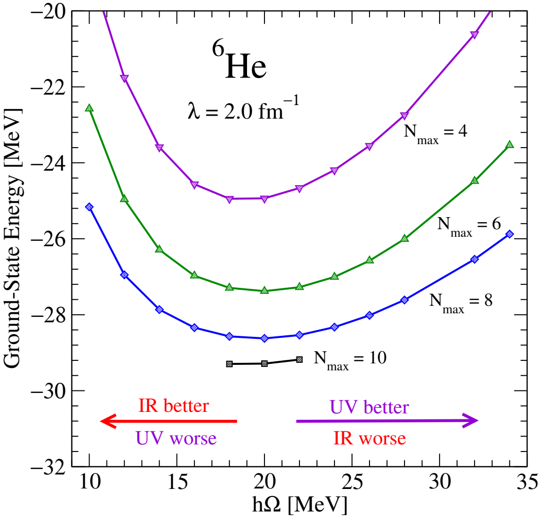
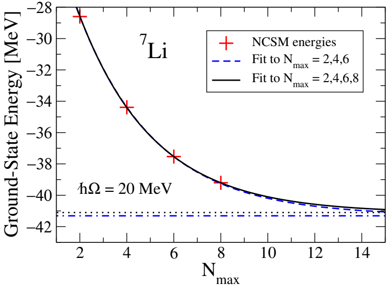
Summary and open questions

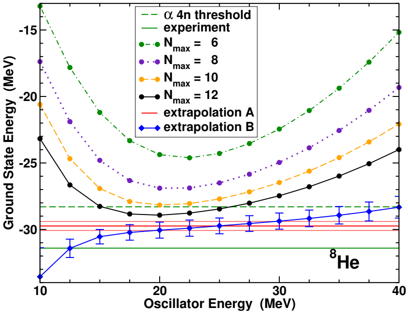
How do we correct for finite harmonic oscillator spaces?



- NCSM m-scheme results from Bogner et al. (2008) [NN-only N^{3}LO (500 MeV) np softened via SRG]
- Typical variational pattern: large $\hbar\Omega$ cuts off wf and small $\hbar\Omega$ cuts off potential $\Longrightarrow$ minimum: IR and UV corrections both needed
- Empirical extrapolation: $E(N_{\max}) = E_\infty + A_0 e^{-A_1 N_{\max}}$ (with uninterpreted A_0, A_1) at fixed $\hbar\Omega$ (near minimum)
- One suggested justification: inverse power law in no. of states

Successes of “conventional” extrapolation





- ^{7}Li extrapolation at fixed $\hbar\Omega$ from Bogner et al. (2008)

  $$E(N_{\rm max}) = E_\infty + A_0 e^{-A_1 N_{\rm max}}$$

- Consistent results from different $N_{\rm max}$ truncations

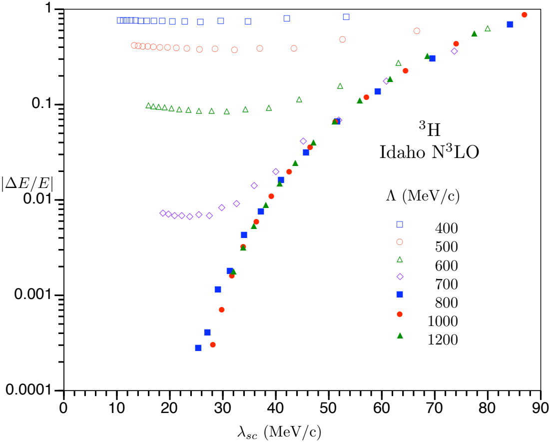
- Maris et al. developed $N_{\rm max}$ extrapolation schemes [PRC **79**, 014308 (2009)]
- ^{8}He: Difficult test nucleus using JISP16 interaction
- Successes but many open questions (e.g., A_0, A_1?)
- Other extrapolations on the market (e.g., $N_{\rm max}^{-1}$ powers)

Effective (field) theory treatments

- Harmonic Oscillator Basis Effective Theory (HOBET)
  - W. Haxton [PRC **77**,034005 (2008)], C.-L. Song, T. Luu, and . . .
  - Use Bloch-Horowitz to *factorize* Q-space UV and IR
  - Re-sum Q-space IR (so no IR extrapolation)
  - HO-based contact-gradient expansion for Q-space UV
- Effective field theory (EFT) for no-core shell model (NCSM)
  - I. Stetcu, B. Barrett, U. van Kolck [PLB **653** (2007) 358] et al.
  - Apply EFT directly within NCSM model space
  - UV from fit contact interactions within NCSM truncation
  - On-going debate about need for IR limit $\hbar\Omega \to 0$
- Convergence properties of *ab initio* calculations in a HO basis
  - Coon, Avetian, Kruse, van Kolck, Maris, Vary [arXiv:1205.3230]
  - Extrapolate in HO IR (λ,λ_{sc}) and UV (Λ_{UV}) cutoffs (cf. $N_{\rm max}$, $\hbar\Omega$)
- EFT for bound-state reflection (cf. Lüscher method for PBC's)
  - M. Pine, D. Lee [arXiv:1008.5187, 1206.6280]. More later!

Switching to IR and UV cutoffs as variables [S. Coon et al.]

- For many N and $\hbar\Omega$ combinations, calculate $|\Delta E/E|$ for the triton
- Plot as function of $\lambda_{sc} = \sqrt{m\hbar\Omega/(N+3/2)}$ for a range of $\Lambda = \sqrt{m(N+3/2)\hbar\Omega}$
- Universal dependence on λ_{sc} over wide range of $\Delta E/E$
- Fit shows exponential in $1/\lambda_{sc}$
- Plateaus to the left from UV corrections



rjf, Hagen, Papenbrock: identify nature and form of IR, UV corrections

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Truncated basis cuts off s.p. wave functions

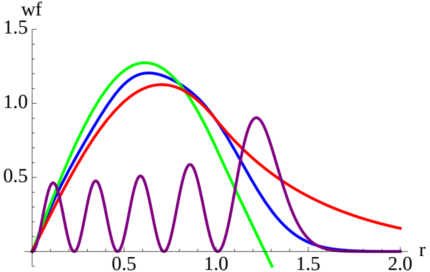
- First estimate of cutoffs: $\frac{1}{2}m\Omega^2 r_{\max}^2 = \frac{1}{2m}p_{\max}^2 = (N+3/2)\hbar\Omega$

$$\Longrightarrow \quad \Lambda_{UV} = \sqrt{2(N+3/2)}\hbar/b \quad \text{and} \quad L_0 = \sqrt{2(N+3/2)}b$$

with $b = \sqrt{\hbar/m\Omega}$ (note $\sqrt{2}$'s)

- Improved estimate for L from intercept of tangent at $r = L_0$:

$$L_{NLO} \approx L_0 + 0.54437\, b\,(L_0/b)^{-1/3}$$



- Square-well wave functions with mass $m = 1$, radius $R = 1$, and depth $V_0 = 4$
- Exact (red) is compared to HO with $\hbar\Omega = 10$ and $N = 8$ (blue) and to boundary condition at $r = L$ (green) and to $n = 4$ wf squared (purple)

$E_{\text{exact}} = -1.51$, $E_{\text{HO}} = -1.23$, $E_L = -0.72$ $[L \propto \sqrt{N}]$

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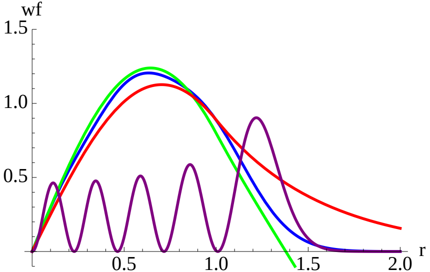

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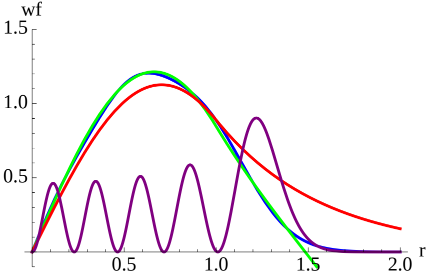

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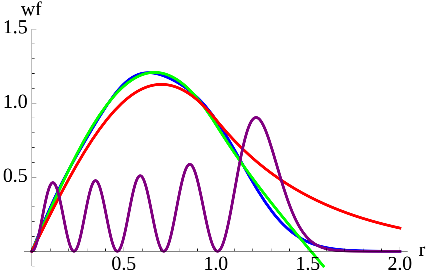

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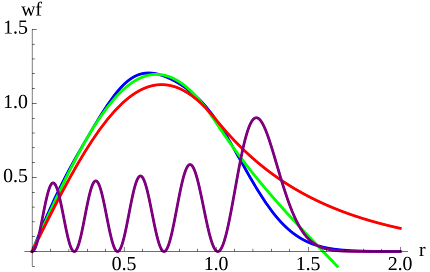

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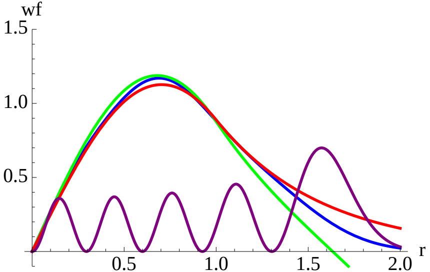

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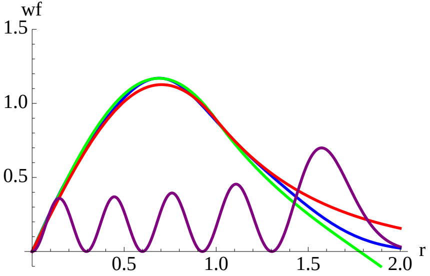

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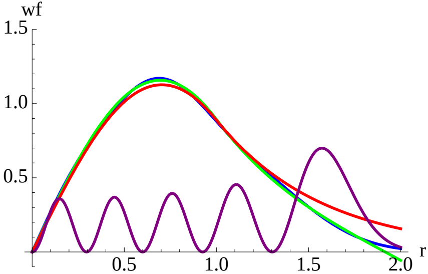

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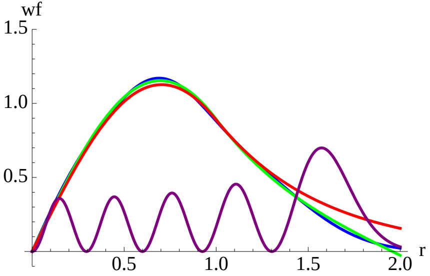

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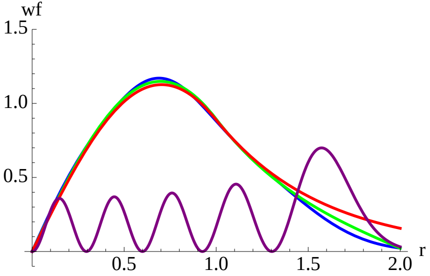

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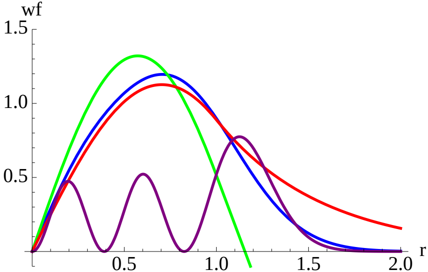

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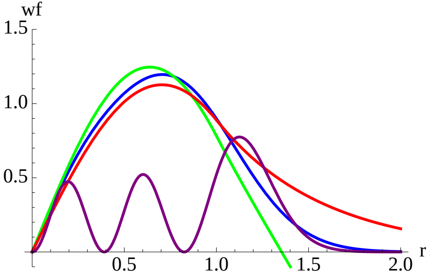

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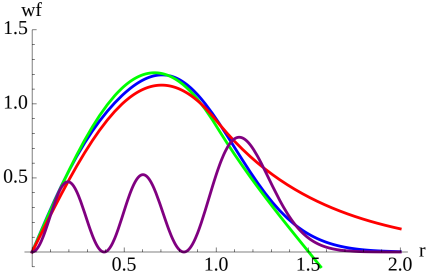

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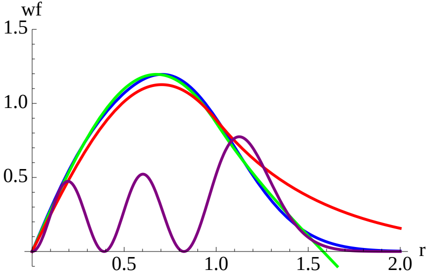

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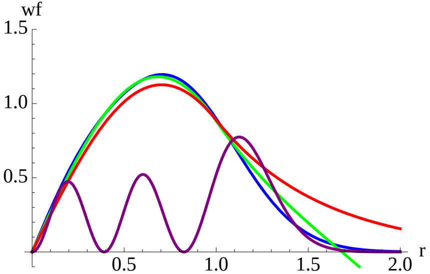

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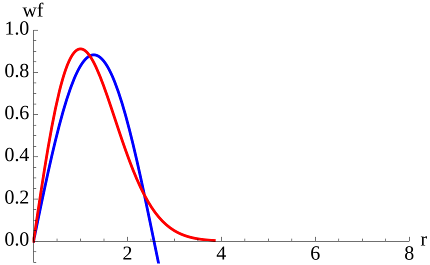
What is the range of momenta in a model space?

- Claim in NCSM EFT papers: minimal accessible non-zero momentum in oscillator basis with fixed $\hbar\Omega$ is $\lambda = \hbar/b$
  - $b = \sqrt{\hbar^2/m\hbar\Omega}$
  - Implication is that one needs to take $\hbar\Omega \to \infty$ limit
- Counterclaim: minimum momentum as in box of size L
  - $p_{\min} = \pi/L$ and r-space eigenfunction $\propto \sin(p_{\min} r)$
  - For oscillator, $L \sim \sqrt{2N}b$ (extent of phase space)
- Test by calculating eigenvalues, eigenfunctions of $\widehat{P}^2$ in HO basis



$N = 0,\ \hbar\Omega = 1 \implies L_{NLO} = 2.19$

$p_{\min} = 1.23$ (minimum eigenvalue)

$\pi/L_{NLO} = 1.44$ (box estimate)

What is the range of momenta in a model space?

- Claim in NCSM EFT papers: minimal accessible non-zero momentum in oscillator basis with fixed $\hbar\Omega$ is $\lambda = \hbar/b$
  - $b = \sqrt{\hbar^2/m\hbar\Omega}$
  - Implication is that one needs to take $\hbar\Omega \to \infty$ limit
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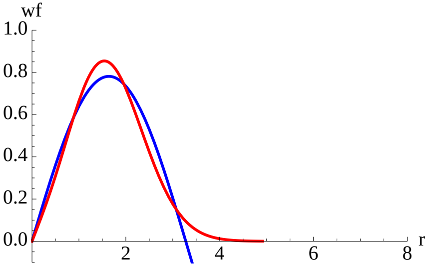

$N = 2$, $\hbar\Omega = 1 \implies L_{NLO} = 3.04$

$p_{\min} = 0.96$ (minimum eigenvalue)

$\pi/L_{NLO} = 1.03$ (box estimate)

What is the range of momenta in a model space?

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  - $b = \sqrt{\hbar^2/m\hbar\Omega}$
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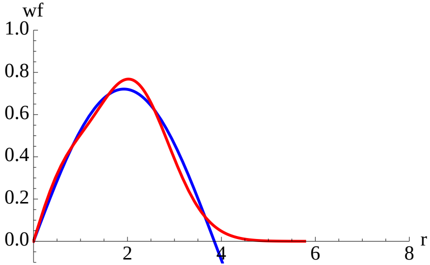

$N = 4$, $\hbar\Omega = 1 \implies L_{NLO} = 3.68$

$p_{\min} = 0.82$ (minimum eigenvalue)

$\pi/L_{NLO} = 0.85$ (box estimate)

What is the range of momenta in a model space?

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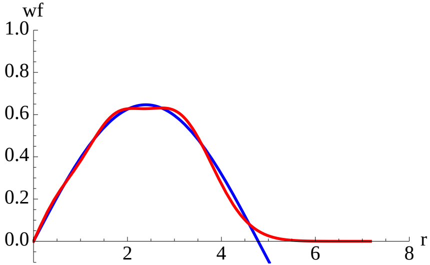
  - $b = \sqrt{\hbar^2/m\hbar\Omega}$
  - Implication is that one needs to take $\hbar\Omega \to \infty$ limit
- Counterclaim: minimum momentum as in box of size L
  - $p_{\rm min} = \pi/L$ and r-space eigenfunction $\propto \sin(p_{\rm min} r)$
  - For oscillator, $L \sim \sqrt{2N}b$ (extent of phase space)
- Test by calculating eigenvalues, eigenfunctions of $\widehat{P}^2$ in HO basis



$N = 8,\ \hbar\Omega = 1 \implies L_{NLO} = 4.69$

$p_{\rm min} = 0.66$ (minimum eigenvalue)

$\pi/L_{NLO} = 0.67$ (box estimate)

What is the range of momenta in a model space?

- Claim in NCSM EFT papers: minimal accessible non-zero momentum in oscillator basis with fixed $\hbar\Omega$ is $\lambda = \hbar/b$
  - $b = \sqrt{\hbar^2/m\hbar\Omega}$
  - Implication is that one needs to take $\hbar\Omega \to \infty$ limit
- Counterclaim: minimum momentum as in box of size L
  - $p_{\min} = \pi/L$ and r-space eigenfunction $\propto \sin(p_{\min} r)$
  - For oscillator, $L \sim \sqrt{2N}b$ (extent of phase space)
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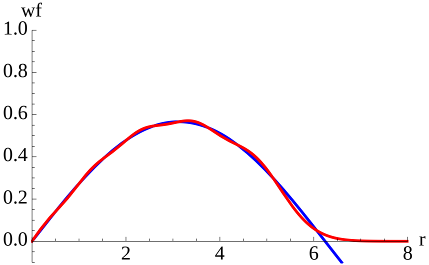


$N = 16$, $\hbar\Omega = 1 \implies L_{NLO} = 6.22$

$p_{\min} = 0.50$ (minimum eigenvalue)

$\pi/L_{NLO} = 0.50$ (box estimate)

Completeness considerations [T. Papenbrock]

- Space of N oscillator wave functions $\phi_i(x)$, $i = 0, \ldots, N-1$ in 1D
- Usual completeness relation is replaced by

$$\sum_{i=0}^{N-1} \phi_i^*(x)\phi_i(y) \equiv \rho_N(x,y) \ .$$

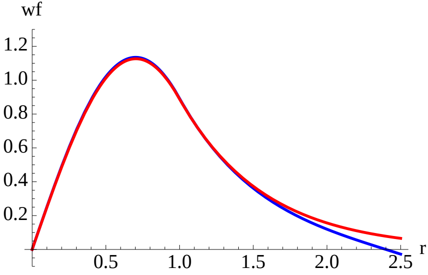
  - For $N \to \infty$ one finds $\rho_\infty(x,y) = \delta(x-y) \Longrightarrow$ completeness
  - For finite N, $\rho_N(x,y)$ equals density matrix of the ground-state wf of N spin-polarized fermions in 1D HO
- For large $N \gg 1$, the density $\rho_N(x,x) \longrightarrow$ Wigner semicircle:

$$\rho_N(x,x) \approx \frac{1}{\pi b^2}\sqrt{2Nb^2 - x^2} \ .$$

  - Valid in the semiclassical limit. We see that there is "no completeness" beyond $|x| > \sqrt{2N}b \approx L_0$
  - Note that squared wf is relevant to determine extent in x

Wave functions in a spherical box

- Forget about harmonic oscillator except to use $\hbar\Omega$ and N to determine size L of box
- Start with wave function without a box $\Longrightarrow E_\infty$
- Increase the energy $\Longrightarrow$ node moves in from $r = \infty$ to $r = L$



$E_\infty = -1.51$, $E = -1.50 \Longrightarrow L = 2.4$

- Square-well wave functions with mass $m = 1$, radius $R = 1$, and depth $V_0 = 4$
- Wave function for E_∞ (red) is compared to wf for $E > E_\infty$ (blue)

- Find $E(L)$, then the desired IR correction comes from $E(L_{\text{HO}})$

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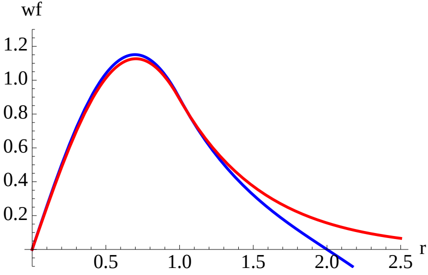

$E_\infty = -1.51$, $E = -1.46 \Longrightarrow L = 2.0$

- Square-well wave functions with mass $m = 1$, radius $R = 1$, and depth $V_0 = 4$
- Wave function for E_∞ (red) is compared to wf for $E > E_\infty$ (blue)

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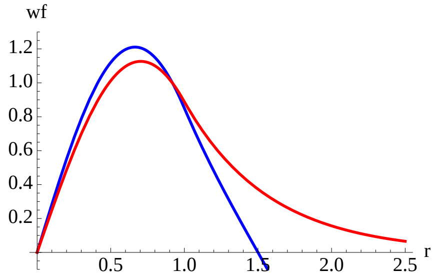


$E_\infty = -1.51$, $E = -1.21 \Longrightarrow L = 1.5$

- Square-well wave functions with mass $m = 1$, radius $R = 1$, and depth $V_0 = 4$
- Wave function for E_∞ (red) is compared to wf for $E > E_\infty$ (blue)

- Find $E(L)$, then the desired IR correction comes from $E(L_{\mathrm{HO}})$

Linear energy method to estimate corrections [Djajaputra]

- Let $u_E(r)$ be the radial solution regular at $r = 0$ with energy E, then

$$u_L(r) \equiv u_{E_L}(r) \approx u_\infty(r) + \Delta E_L \left.\frac{du_E(r)}{dE}\right|_{E_\infty} \quad \text{where} \quad E_L = E_\infty + \Delta E_L$$

$$\text{So } u_L(L) = 0 \quad \Longrightarrow \quad \Delta E_L \approx -u_\infty(L) \left(\left.\frac{du_E(L)}{dE}\right|_{E_\infty} \right)^{-1}$$

- Now $u_E(r) \xrightarrow{r \gg R} A_E(e^{-k_E r} + \alpha_E e^{+k_E r})$ with $u_\infty(r) \xrightarrow{r \gg R} A_\infty e^{-k_\infty r}$ and k_∞ from nucleon separation energy $S = \frac{\hbar^2 k_\infty^2}{2m}$
- Take the derivative and evaluate at $E = E_\infty$:

$$\left.\frac{du_E(r)}{dE}\right|_{E_\infty} = +A_\infty \left.\frac{d\alpha_E}{dE}\right|_{E_\infty} e^{+k_\infty r} + \mathcal{O}\left(e^{-k_\infty r}\right)$$

Substituting at $r = L$, we obtain our correction formula to fit:

$$\Delta E_L \approx -\left[\left.\frac{d\alpha_E}{dE}\right|_{E_\infty} \right]^{-1} e^{-2k_\infty L} + \mathcal{O}(e^{-4k_\infty L}) \quad \Longrightarrow \quad E_L = E_\infty + a_0 e^{-2k_\infty L}$$

Comparison to Lüscher formula for bound states

- Lüscher: energy shifts to bound states from the finite size of a box with periodic boundary conditions
  - Here: size of box is spatial extent of the oscillator basis
  - We effectively have Dirichlet boundary conditions on sphere
- Usual Lüscher formula ($\kappa = \sqrt{mE_\infty}$ is binding momentum):

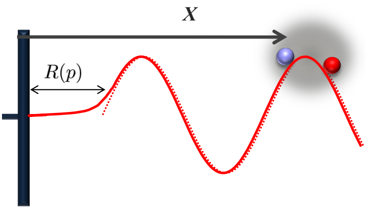
$$\Delta E_L = E_L - E_\infty = +24\pi |A|^2 \frac{e^{-\kappa L}}{mL} + \mathcal{O}(e^{-\sqrt{2}\kappa L})$$

  - Independent of form of potential V (pole properties only)
  - See S. Koenig et al. [arXiv:1109.4577] for a simple derivation
  - cf. other formulas derived more recently for lattice applications
- PBCs: S-wave energy lowered by periodic images of the potential
  - Here: energy is always *increased* by the shift of a node from $r = \infty$ to $r = L$ (cf. p-wave [H.-W. Hammer talk])
  - Consistent with variational nature of truncated expansion

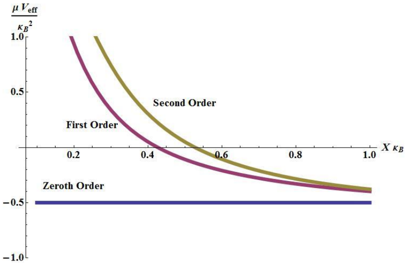
EFT for Bound-State Reflection

[M. Pine, D. Lee, arXiv:1008.5187, 1206.6280]

- See Michelle Pine's INT talk [Sept. 28]
- Motivated by lattice EFT for nuclei
  - Hard-wall cube in d-dimensions
  - Shallow bound states: $\kappa_B = 1/a_B$
- Apply adiabatic expansion in soft scattering limit
  - Use method of images for BC's
  - Systematic effective potential [1st-order $d = 1$ correction]
- Adapt to spherical hard wall
  - Effectively one dimensional
  - Depends on k_∞, A_∞ in general!



$$T^{(1)}(X) + V^{(1)}(X) = \frac{\kappa_B^2 M^2}{m_1 m_2}\left[\frac{e^{-\kappa_B x_+(X)}}{m_1} + \frac{e^{-\kappa_B |x_-(X)|}}{m_2}\right]$$



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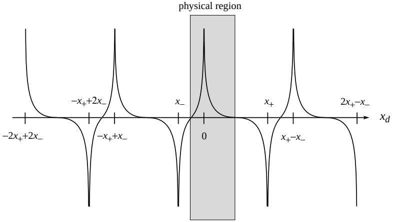
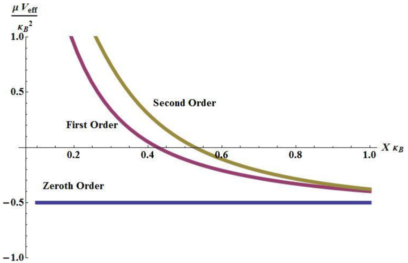


$$T^{(1)}(X) + V^{(1)}(X)$$

$$= \frac{\kappa_B^2 M^2}{m_1 m_2}\left[\frac{e^{-\kappa_B x_+(X)}}{m_1} + \frac{e^{-\kappa_B |x_-(X)|}}{m_2}\right]$$



Correction for radius (or other long-distance operators)

- Use $u_L(r) \approx u_\infty(r) + \Delta E_L \left.\frac{du_E(r)}{dE}\right|_{E_\infty}$ to evaluate

$$\Delta\langle r^2\rangle_L = \langle r^2\rangle_L - \langle r^2\rangle_\infty = \frac{\int_0^L |u_L(r)|^2\, r^2\, dr}{\int_0^L |u_L(r)|^2\, dr} - \frac{\int_0^\infty |u_\infty(r)|^2\, r^2\, dr}{\int_0^\infty |u_\infty(r)|^2\, dr}$$

- For leading L dependence, use $u_\infty(r) \longrightarrow A_\infty e^{-k_\infty r}$ and

$$\left.\frac{du_E(r)}{dE}\right|_{E_\infty} \approx -\frac{A_\infty}{\Delta E_L} e^{-2k_\infty L} e^{+k_\infty r} \quad \Longrightarrow \quad \Delta\langle r^2\rangle_L \propto \langle r^2\rangle_\infty (2k_\infty L)^3 e^{-2k_\infty L}$$

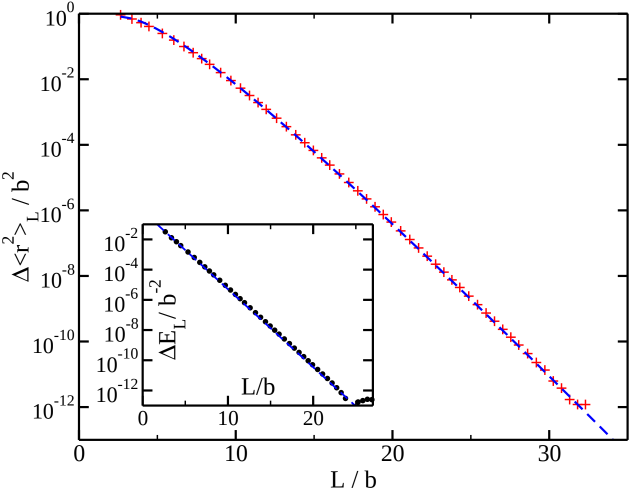
- The NLO correction scales as $(2k_\infty L)\exp(-2k_\infty L)$, so

$$\langle r^2\rangle_L \approx \langle r^2\rangle_\infty [1 - (c_0\beta^3 + c_1\beta)e^{-\beta}] \quad \text{with } \beta \equiv 2k_\infty L$$

  - $\langle r^2\rangle_\infty$, c_0, and c_1 are fit parameters while k_∞ from energy fit
  - Valid in the asymptotic regime where $\beta = 2k_\infty L \gtrsim 3$

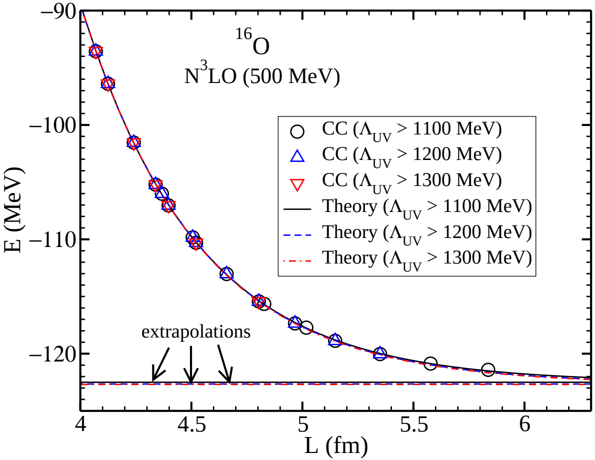
- Both E and r corrections apply to A-body system in lab coordinates

Test case: Toy model calculation: $H = p^2/2 - v_0 e^{-x^2}$ in 1D



- Theory and numerical data agree over 10 orders of magnitude
- Other model calculations also validate fit function

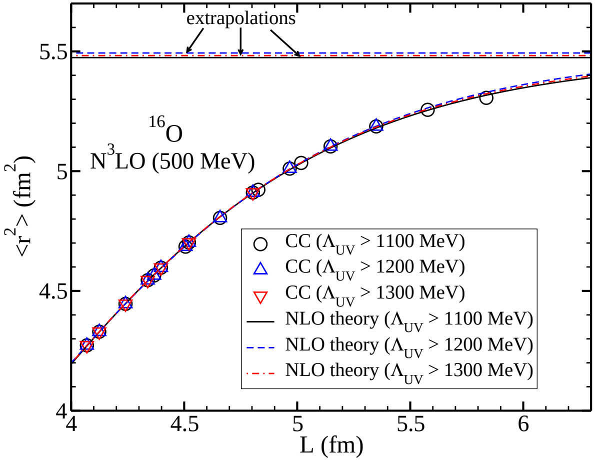
Infrared ($L \to \infty$) energy extrapolation of CCSD(T) results



- Frequencies: 42 MeV $\leq \hbar\Omega \leq$ 76 MeV
- $N = 12, 14$
- $E_L = E_\infty + a_0 e^{-2k_\infty L}$
- Use large Λ_{UV} for UV convergence
- Weak dependence on choices of UV cutoff

- Fits yield $E_\infty \approx -122.6$ MeV (± 0.2 MeV) and $k_\infty \approx 0.95$ fm^{-1}
- k_∞ agrees with decay of the $p_{1/2}$ orbital $\Longrightarrow$ the tail of the density

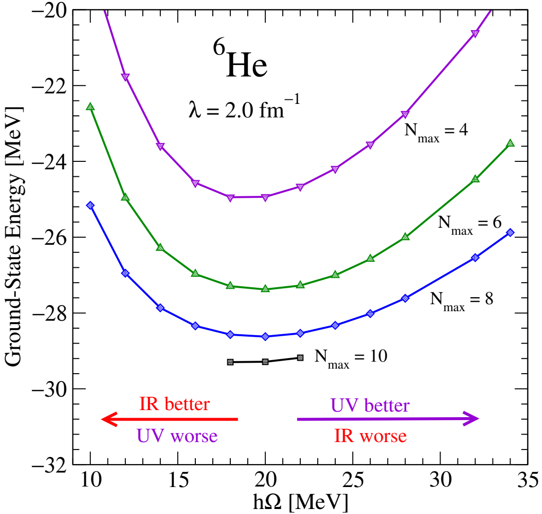
Infrared ($L \to \infty$) radius extrapolation of CCSD(T) results



- Frequencies: $42\,\text{MeV} \leq \hbar\Omega \leq 76\,\text{MeV}$
- $N = 12, 14$
- $\langle r^2 \rangle_L \approx \langle r^2 \rangle_\infty \times [1 - (c_0\beta^3 + c_1\beta)e^{-\beta}]$ with $\beta \equiv 2k_\infty L$

- Fits yield $r \approx 2.34\,\text{fm}$ using $k_\infty \approx 0.95\ \text{fm}^{-1}$ from energy fit
- Extrapolation works well with just the $\Lambda_{UV} > 1300\,\text{MeV}$ points

Infrared ($L \to \infty$) extrapolations of NCSM results for ^{6}He



- NCSM results from Bogner et al. (2008)
- Use data from right of minima to ensure small UV corrections
- $N = N_{\text{max}} + 1$ for ^{6}He

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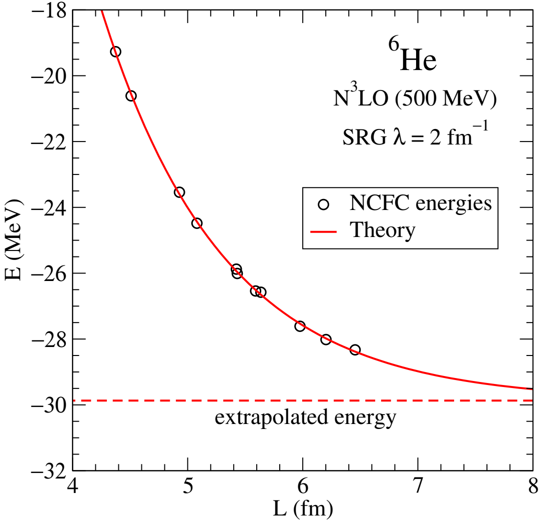

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- Fit: $E_\infty \approx -29.87$ MeV
- But fit $\hbar k_\infty \approx 93$ MeV $\Longrightarrow$ interpretation?

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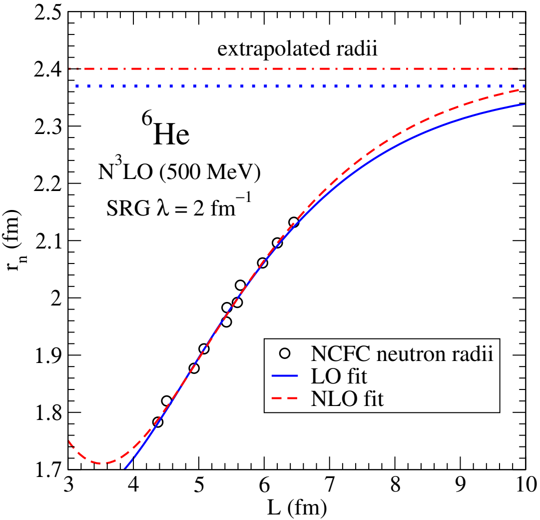

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- Radius at NLO: 2.40 fm

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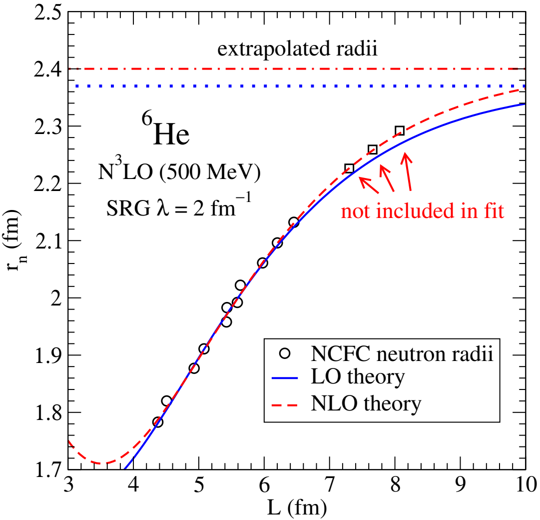

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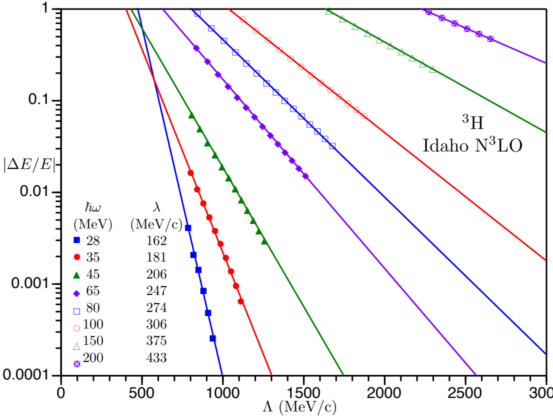
- $N = N_{\max} + 1$ for ^{6}He
- Fit: $E_\infty \approx -29.87\,\text{MeV}$
- But fit $\hbar k_\infty \approx 93\,\text{MeV}$ $\Longrightarrow$ interpretation?
- Radius at LO: 2.37 fm
- Radius at NLO: 2.40 fm
- Note the $N_{\max} = 10$ points!

Application of IR correction formula to S. Coon et al. results

- Plotted against Λ_{UV} but UV converged
- Lines at fixed $\hbar\Omega$ (or λ) $\Longrightarrow$ plotting against $\sqrt{N}$ $\Longrightarrow$ equivalent to varying L
- Replot against L



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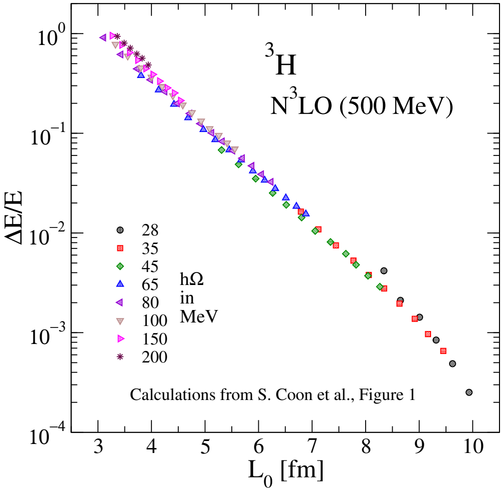
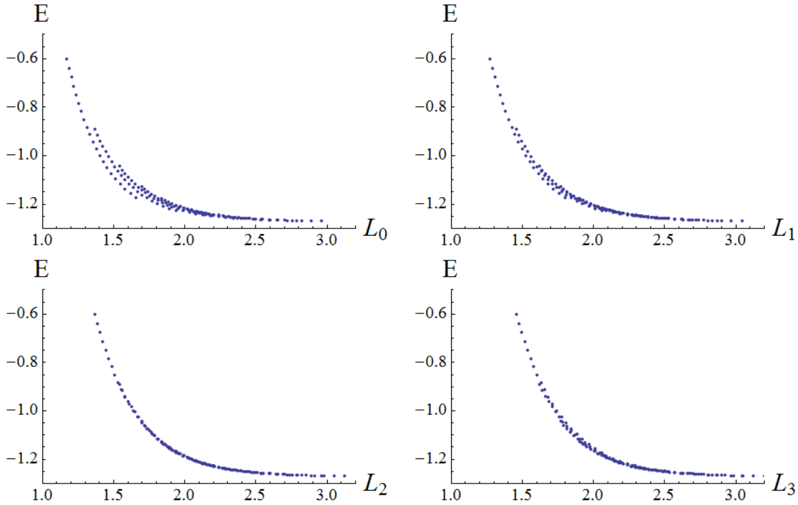
  $\Longrightarrow$ plotting against $\sqrt{N}$
  $\Longrightarrow$ equivalent to varying L
- Replot against L
  $\Longrightarrow$ exponential over wide range of $\Delta E/E$
- All other figures consistent with IR dependence $\Delta E/E \propto e^{-k_\infty L}$



Testing with models [Sushant More (OSU)]

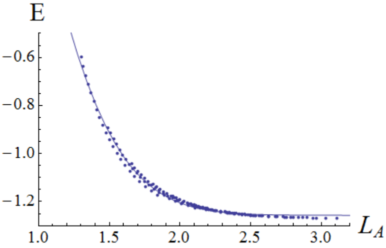
Optimal definition of L? Look at scatter vs. $L \propto \sqrt{N + \Delta N}$

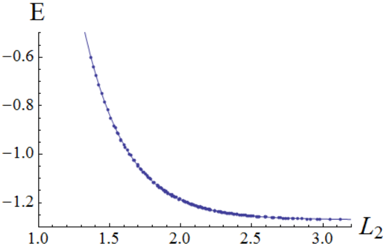


Winner: $L_2 \propto \sqrt{N + 3/2 + 2}$ (slightly larger than $L_{\rm NLO}$) $\Longrightarrow$ better results!

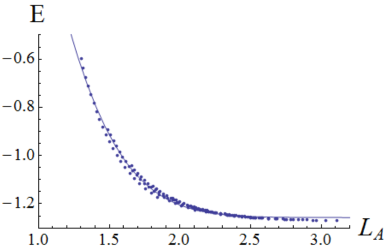
Testing with models [Sushant More (OSU)]

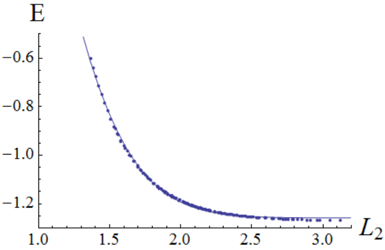
Fit to exponential with $L_{\rm NLO}$ on left and $L_2 \propto \sqrt{N+3/2+2}$ on right:





Compare to gaussian extrapolations:

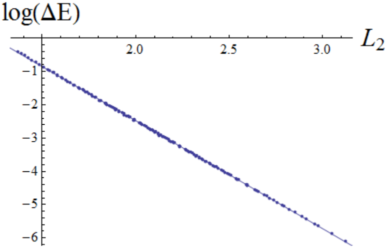


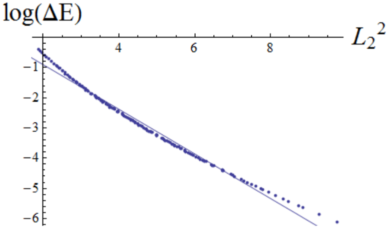


Testing with models [Sushant More (OSU)]

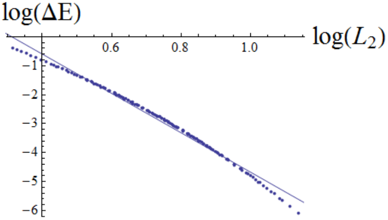
Are we sure that ΔE is an exponential in L?

- Test for exponential on right
- Test gaussian below left
- Test power law below right
- Seems to be an exponential!





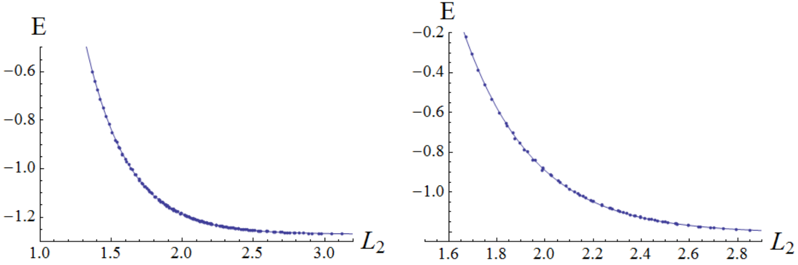
s



Testing with models [Sushant More (OSU)]

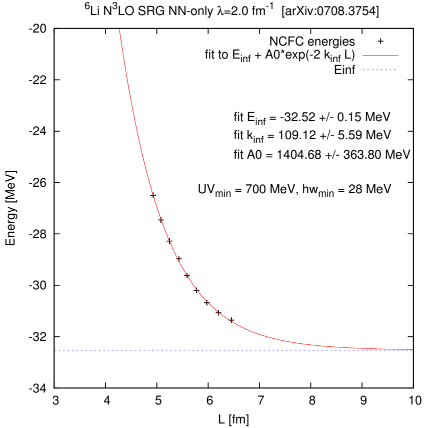
What about excited states?

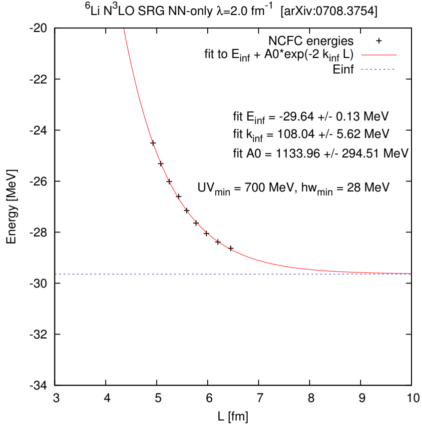
- Derivations unchanged $\Longrightarrow$ expect exponential corrections again
- Compare ground state (left) to excited state (right)



Looks like the exponential fit works for both (different k_∞, of course)!

Test on excited state [preliminary!]

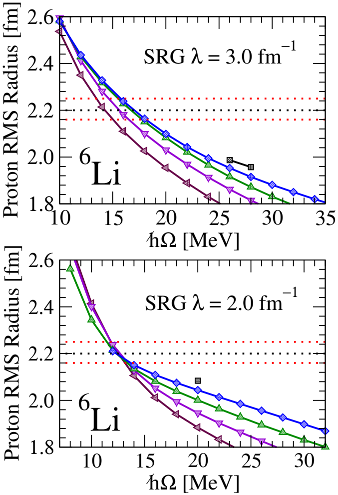




- NCSM NN-only calculations for ^{6}Li
- Ground state on left, first excited on right
- Exponential fits (seem to) work for excited states!

NCSM radii revisited [preliminary!!]

- “Pivot point” phenomena for radii vs. $\hbar\Omega$ at different $N_{\max}$:



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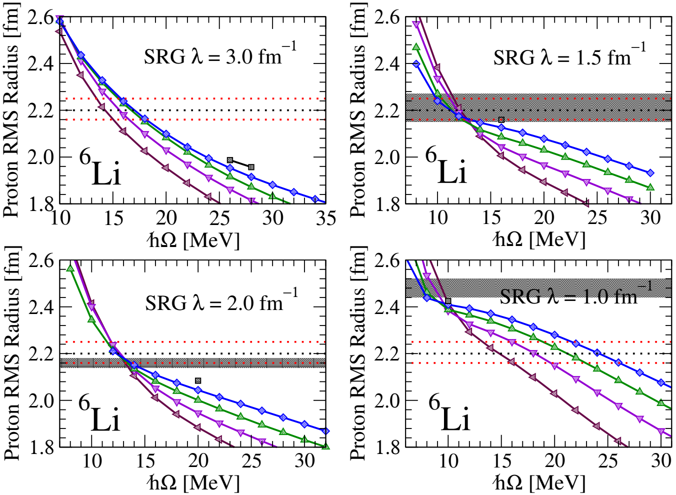


- IR extrapolations give gray bands (error from fit — reliable?)

New application: Resonances (from this week!)

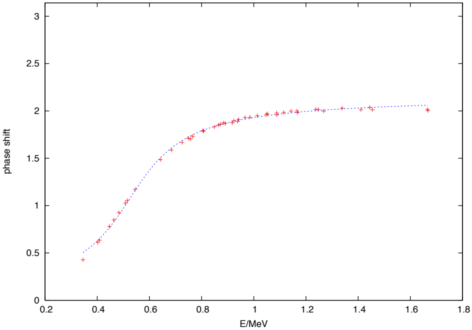
- Extract resonance parameters from a nucleus in a box
- Familiar from other contexts, e.g., J. Carlson et al., Nucl. Phys. **A424** (1984) 47 or Y. Alhassid and S.E. Koonin, Ann. Phys. **155** (1984) 108
- The asymptotic wave function $j_l(kr) - tan(\delta_l)n_l(kr)$ satisfies the Dirichlet boundary condition at $r = L$ through $\delta = kL - \pi l/2$ for angular momentum l at an energy eigenvalue
- If one knows the threshold energy S, the excitation energy E is related to k via

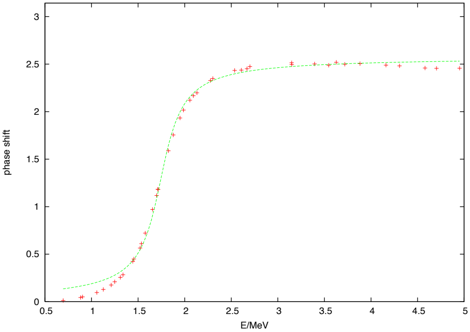
$$(E - S) = \hbar^2 k^2 / 2m$$

- Different model spaces $(N, \hbar\Omega) \Longrightarrow$ different L's $\Longrightarrow$ E's $\Longrightarrow$ δ_l's
- Plotting phase shift vs k yields the resonance at 90 degrees and the slope at 90 degrees is related to the inverse width

New application: Resonances (from this week!)

Test case: Extract phase shift for $l = 1$ (left) and $l = 2$ (right) resonances in Woods-Saxon potentials, with fits to Breit-Wigner (BW) shape.





- Compare BW fits to Gamow shell model (GSM) results $\Longrightarrow$
- Fits do well on resonance position but not so accurate on widths

	BW		GSM	
l	E_0	Γ	E_0	Γ
1	0.53	0.33	0.48	0.46
2	1.74	0.35	1.73	0.50

Unsettled questions for IR extrapolation

- What is the optimal definition of L? (Use the scatter?)
- How to weight the contributions according to L (or $N_{\rm max}, \hbar\Omega$)?
- How to make credible error estimates?
- Interpretation of k_∞? Can we extract A_∞?
- Does the interaction matter?
    - The IR corrections are independent of the potential
    - Softer interactions mean more complete UV convergence for given $\hbar\Omega$, N, so larger region with IR corrections only
    - Anything else?
- How well does extrapolation work for other operators?
- Can we systematically improve the extrapolation à la Pine/Lee?
- How can we incorporate *explicitly* the harmonic oscillator part?

Harmonic Oscillator Basis Effective Theory (HOBET)

[W. Haxton and collaborators]

- General problem: including effects of excluded space Q in model space P (with $P + Q = 1$)
- For HO basis, $Q = \sum_{\alpha_{\rm HO} > N_{\rm max}} |\alpha_{\rm HO}\rangle\langle\alpha_{\rm HO}|$, excludes both IR and UV
- Use Bloch-Horowitz framework to *factorize* IR and UV:

$$H^{\rm eff} = H + HQ\frac{1}{E - QH}QH = \underbrace{\frac{E}{E - TQ}}_{IR}\Big[T - T\frac{Q}{E}T + V + \underbrace{V\frac{1}{E - QH}QV}_{UV}\Big]\underbrace{\frac{E}{E - TQ}}_{IR}$$

  - Resummed Q-space kinetic energy puts correct tail on wf's
  - Can this justify combined UV and IR extrapolations?
- Bloch-Horowitz energy dependence of $H_{\rm eff} \Longrightarrow$ out of mainstream
  - Energy dependence claimed to be a feature, not bug; true?
  - Is technology adaptable for improved extrapolations?

Outline

Motivation: Extrapolations in finite bases

Nature and implications of infrared cutoffs

High-momentum behavior of wave functions

Combined IR and UV extrapolations

Summary and open questions

What parts of wf's can be extracted from experiment?

- Measurable: asymptotic (IR) properties like phase shifts, ANC's
- Not observables, but well-defined theoretically given a Hamiltonian: interior quantities like spectroscopic factors
  - These depend on the scale and the scheme
  - Extraction from experiment requires robust factorization of structure and reaction; only the combination is scale/scheme independent (e.g., cross sections) [What if weakly dependent?]

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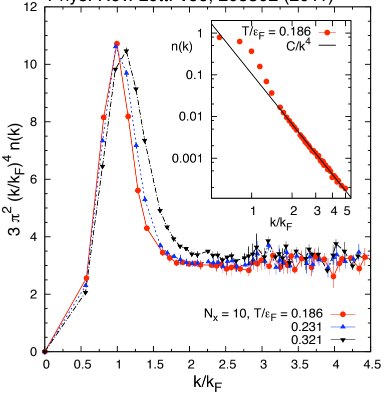
Unitary cold atoms: Is $n(k)$ observable?

- Tail of momentum distribution + contact [Tan; Braaten/Platter]

$$n(k) \xrightarrow{k\to\infty} \frac{C}{k^4}$$

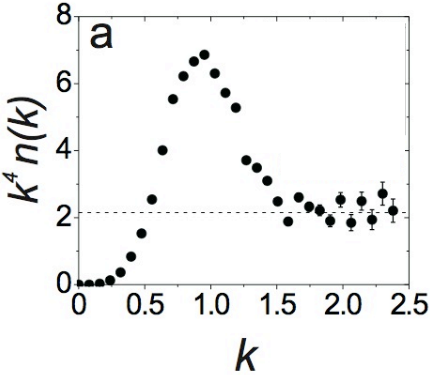
Theory (lattice)

J. E. Drut, T. A. Lähde, T. Ten
Phys. Rev. Lett. **106**, 205302 (2011)

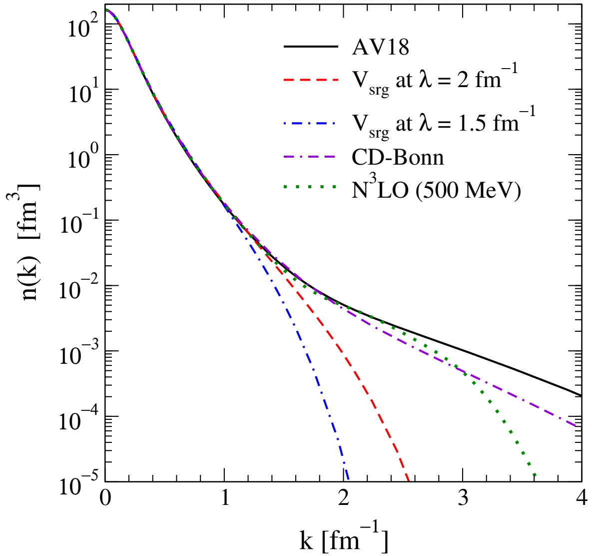


Experiment

J. T. Stewart et al
PRL **104**, 235301 (2010)



Is the tail of $n(k)$ for nuclei measurable? (cf. SRC's)



- E.g., extract from electron scattering?
- Scale- and scheme-dependent high-momentum tail!
- $n(k)$ from V_{SRG} has *no* high-momentum components!
- No region where $1/a_s \ll k \ll 1/R$ (cf. large k limit for unitary gas)

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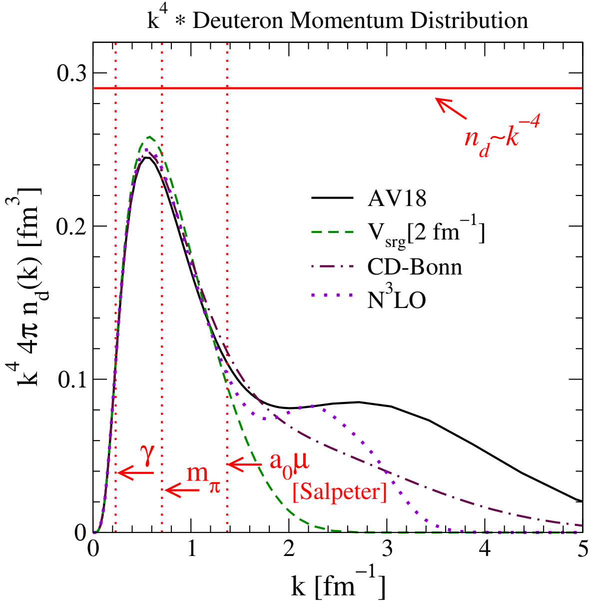

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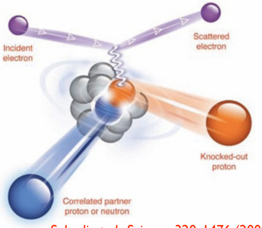
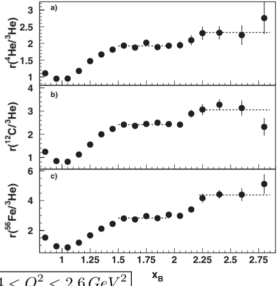
- Short-range correlations (SRCs) depend on the Hamiltonian *and* the resolution scale (cf. parton distribution functions)

Looking for missing strength at large Q^2



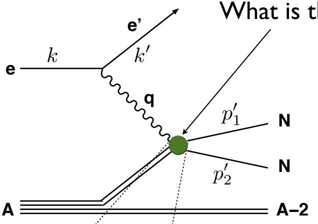
Subedi et al., Science 320, 1476 (2008)



$1.4 < Q^2 < 2.6\, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)

What is this vertex?



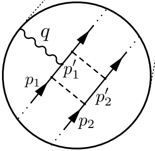
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2m_N \nu}$$

Higinbotham, arXiv:1010.4433

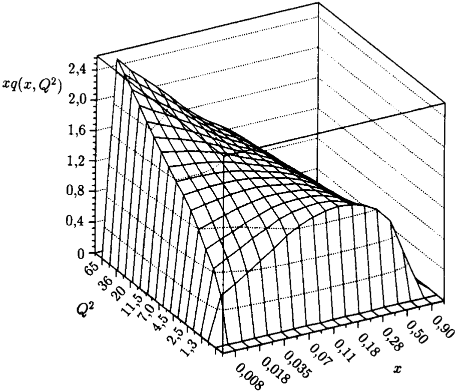


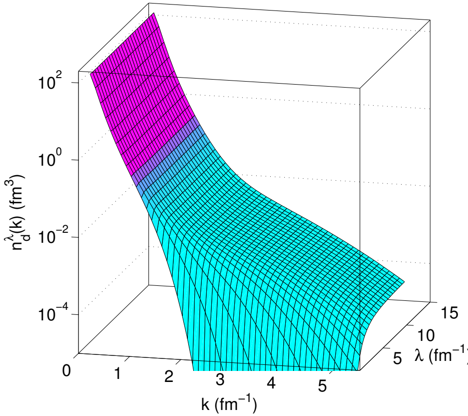
SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p'_1, p'_2 \gg k_F$ which are knocked out by the photon

How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

Parton vs. nuclear momentum distributions





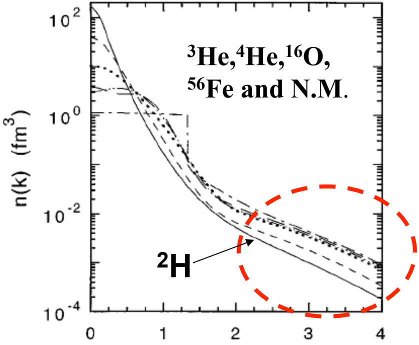
- The quark distribution $q(x, Q^2)$ is scheme *and* scale dependent
- $x\ q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval

- Deuteron momentum distribution is scheme *and* scale dependent
- Initial AV18 potential evolved with SRG from $\lambda = \infty$ to $\lambda = 1.5\,\text{fm}^{-1}$

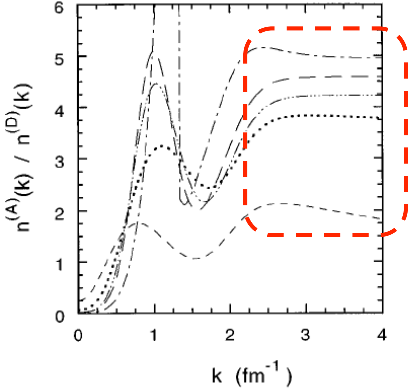
Deuteron-like scaling at high momenta

C. Ciofi and S. Simula, *Phys.Rev* **C53**, 1689(1996)

Momentum Distributions n(k)



Ratio to the Deuteron



n(k) at high Momentum regions are similar to it of the Deuteron

Almost Flat!

High resolution: Dominance of V_{NN} and SRCs (Frankfurt et al.)

Lower resolution $\Longrightarrow$ lower separation scale $\Longrightarrow$ fall-off depends on V_λ

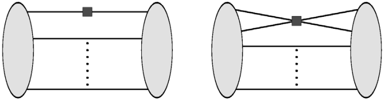
A dependence of the EMC effect is long-distance physics!

- EFT treatment by Chen and Detmold [Phys. Lett. B 625, 165 (2005)]

$$F_2^A(x) = \sum_i Q_i^2 \, x q_i^A(x) \qquad \Longrightarrow \qquad R_A(x) = F_2^A(x)/AF_2^N(x)$$

"The x dependence of $R_A(x)$ is governed by short-distance physics, while the overall magnitude (the A dependence) of the EMC effect is governed by long distance matrix elements calculable using traditional nuclear physics."

- Match matrix elements: leading-order nucleon operators to isoscalar twist-two quark operators



$$\Longrightarrow \langle x^2 \rangle_q v^{\mu_0} \cdots v^{\mu_n} N^\dagger N [1 + \alpha_n N^\dagger N] + \cdots$$

$$R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)\mathcal{G}(A) \quad \text{where} \quad \mathcal{G}(A) = \langle A|(N^\dagger N)^2|A\rangle / A\Lambda_0$$

$\Longrightarrow$ the slope $\frac{dR_A}{dx}$ scales with $\mathcal{G}(A)$ [Why is this not cited more?]

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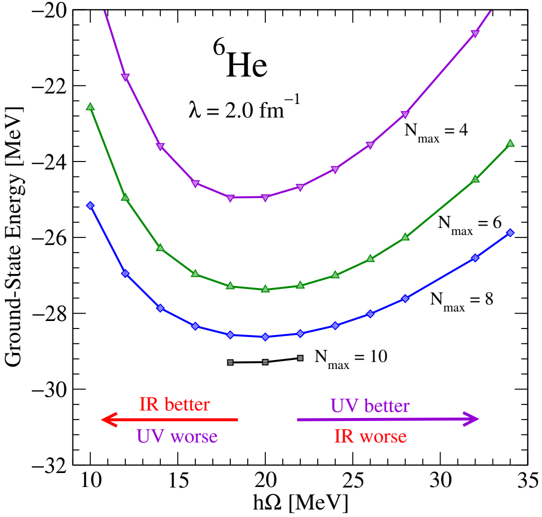
  - Consider cold atoms in the unitary regime
  - Compare to nuclear case
- Short-range correlations (SRCs) depend on the Hamiltonian *and* the resolution scale (cf. parton distribution functions)
- So might expect Hamiltonian- and resolution-dependent but *A*-independent high-momentum tails of wave functions [T. Neff]
  - Universal extrapolation for different *A*, but $\lambda_{\rm SRG}$ dependent

Ultraviolet ($\Lambda_{UV} \to \infty$) extrapolations of NCSM results for ^{6}He



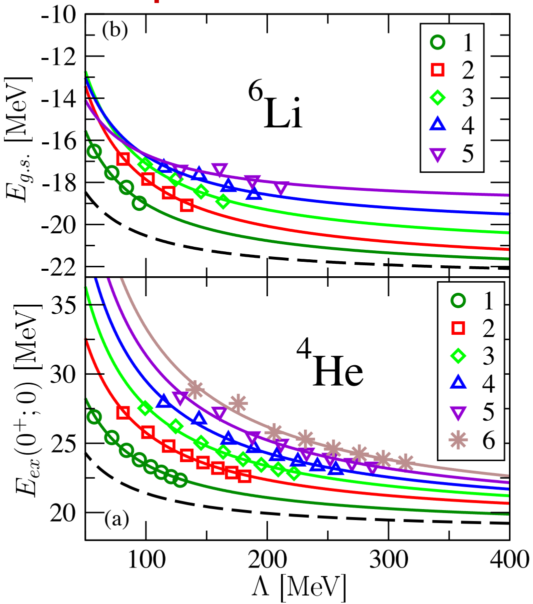
- NCSM results from Bogner et al. (2008)
- Use data from *left* of minima to ensure small IR corrections
- $N = N_{\text{max}} + 1$ for ^{6}He

What form do we expect for UV extrapolations?

- Consider NCSM as an EFT [I. Stetcu et al. PLB **653**, 358 (2007); I. Stetcu and J. Rotureau, arXiv:1206.0234]
- Choice of extrapolation guided by LO running of bound-state energy in the continuum:

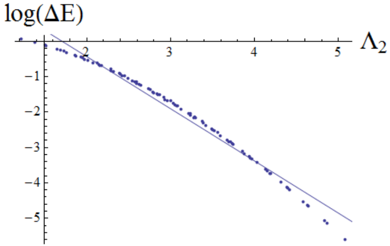
$$E = E_0(\hbar\Omega) + A(\hbar\Omega)/\Lambda_{UV}$$

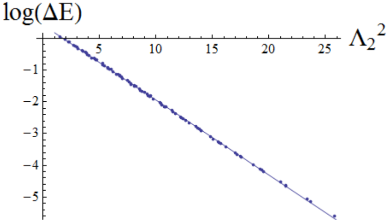
- Extrapolate $E_0(\hbar\Omega)$ to $E_0(0)$
- Study of SRG decoupling by Jurgenson et al. (2008) found power-law dependence on imposed UV cutoff of potential

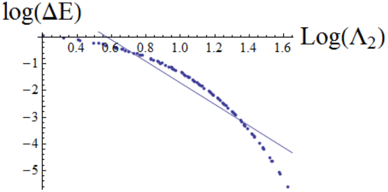


What do we expect for UV from models? [S. More]

- Points lie on curve vs. Λ_{UV}
- Test for exponential on right
- Test gaussian below left
- Test power law below right
- $\Delta E \propto e^{-a\Lambda_{UV}^2}$ favored!

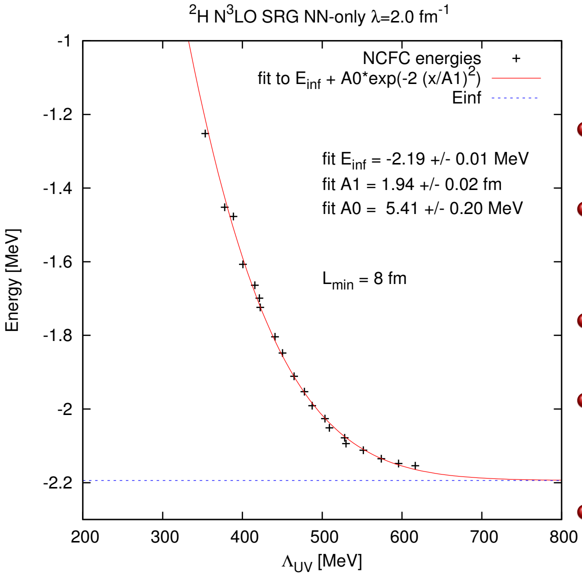






- Gaussian in $\Lambda_{UV}^2 \propto N_{\max}$ also found empirically by Haxton and Song (HOBET study) and by Coon et al., but not explained

Ultraviolet (Λ_{UV}) extrapolations of NCSM results [Empirical!]



- NCSM results from Bogner et al. (2008)
- Points lie on curve for $\Delta E_{\Lambda_{UV}} \propto e^{-2\Lambda_{UV}^2/A_1^2}$, so Λ_{UV} is correct variable
- $\Lambda_{UV} = \sqrt{2(N+3/2)}\hbar/b$ is used here
- Fits *do not* work with other curves: power laws, $e^{-c\Lambda_{UV}^n}$ for $n \neq 2$
- Fitted $A_1 \approx \lambda_{\rm SRG}$!

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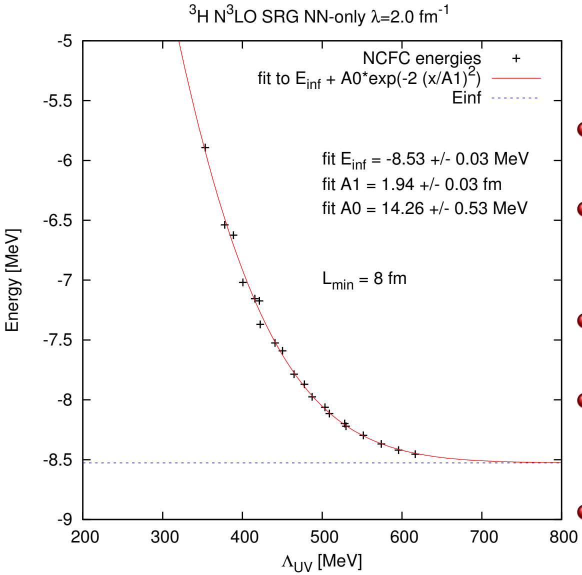

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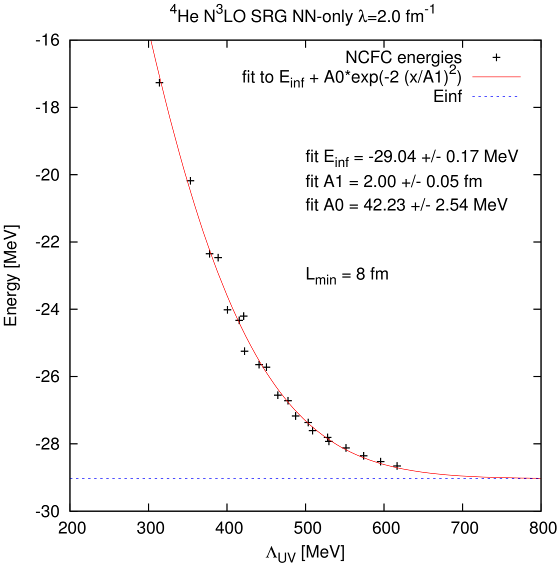

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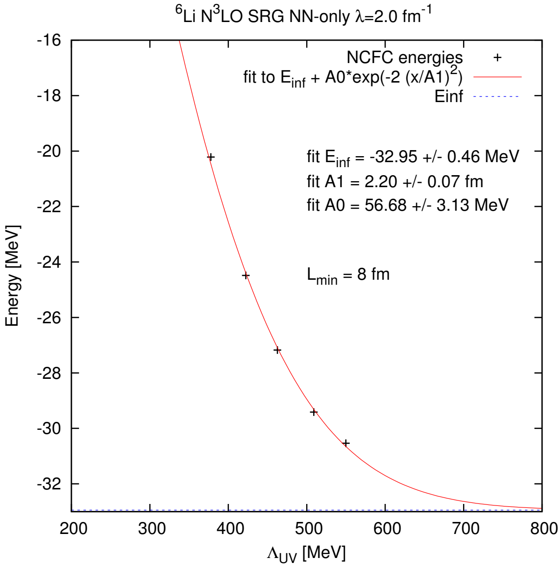

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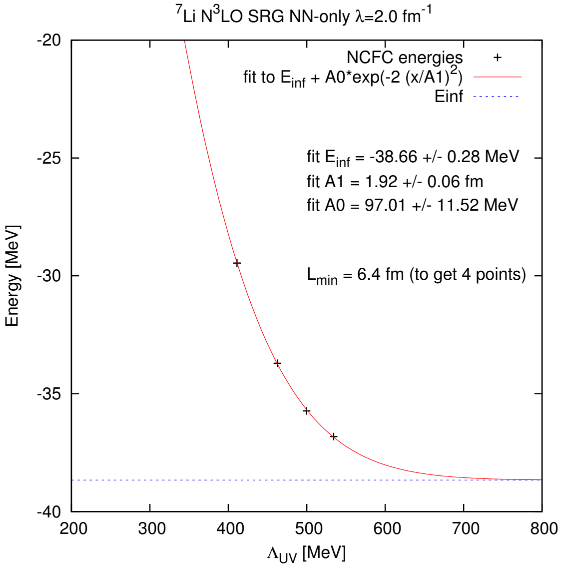

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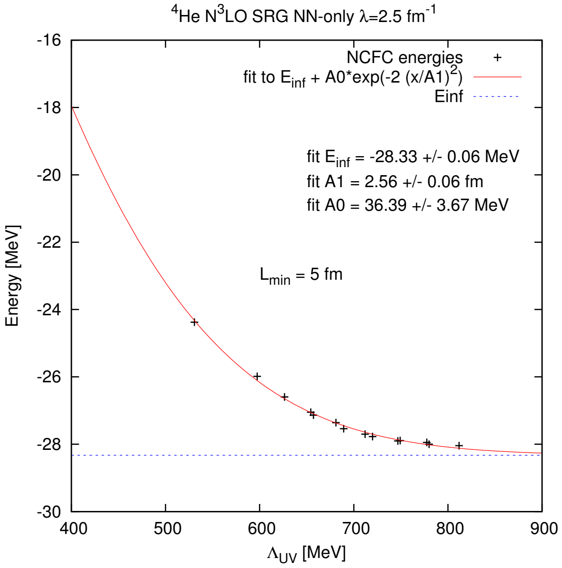

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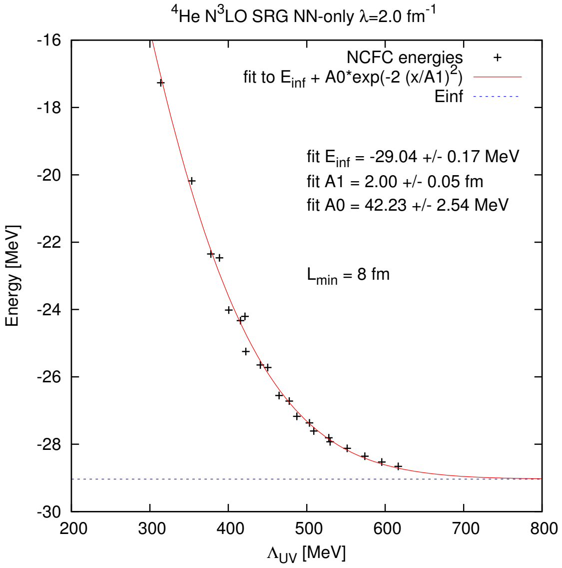

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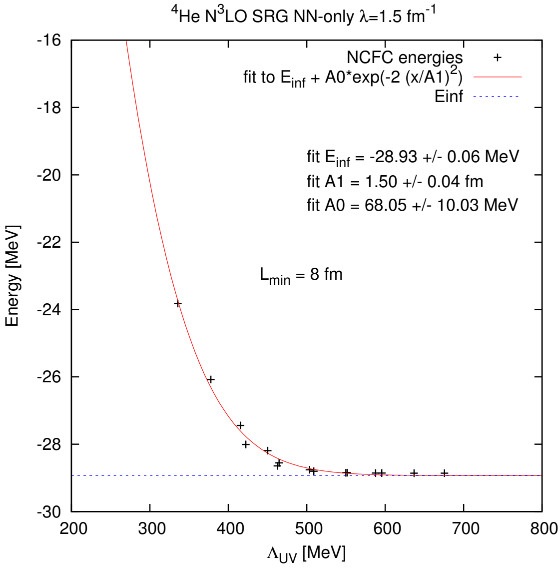

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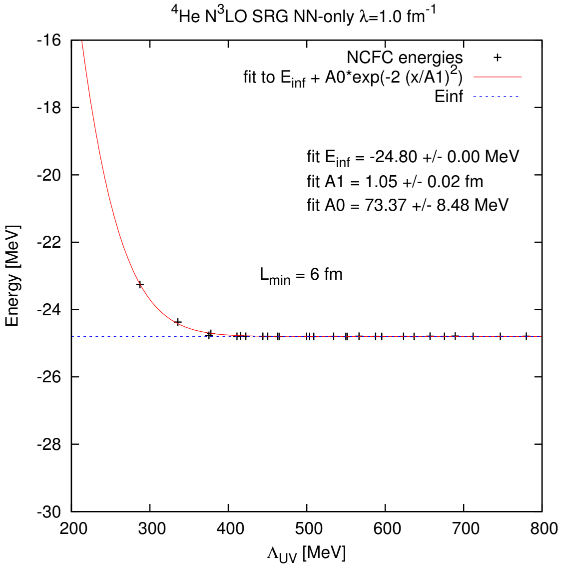

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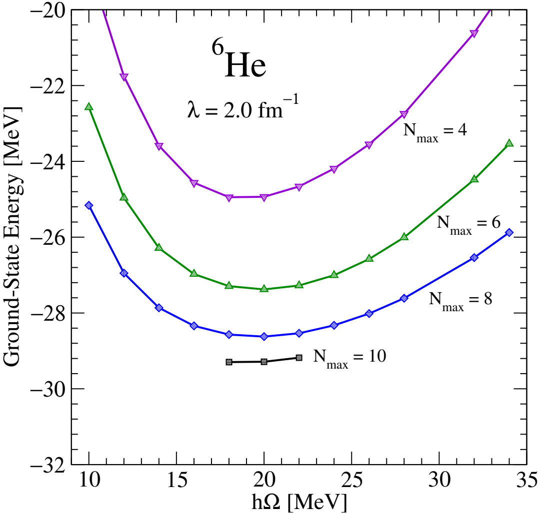
Summary and open questions

Combined IR and UV energy fit of NCSM results



- Assume that IR and UV corrections near minimum are additive (factorized):

$$E(L, \Lambda_{UV}) = E_\infty + \Delta E_L + \Delta E_{\text{UV}}$$

- $\Delta E_L(L) = A_1 e^{-2k_\infty L}$ with fit k_∞ to optimize
- $\Delta E_{\text{UV}}(\Lambda_{UV}) = A_0 e^{-2\left(\frac{\Lambda_{UV}}{\lambda}\right)^2}$ with fit λ to optimize

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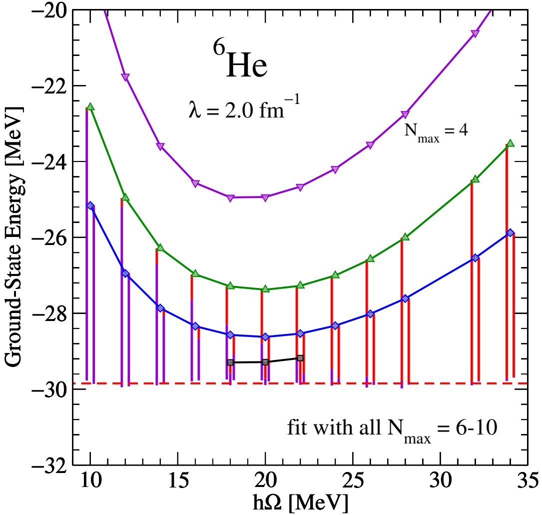

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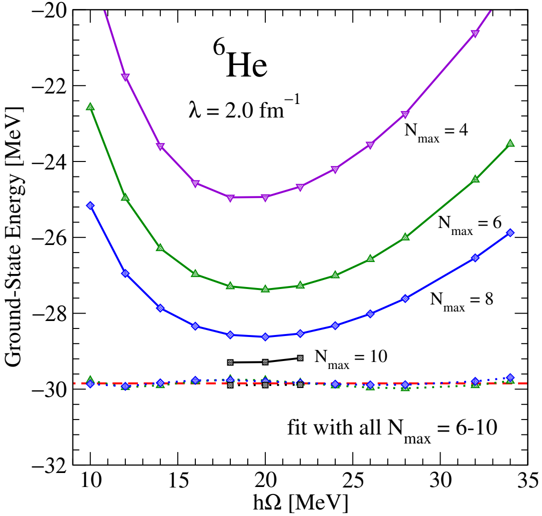

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- Fit all $N_{\rm max} = 6$–10 points
- $E_\infty \approx -29.84\,\text{MeV}$
- $\lambda_{\rm fit} \approx 2.2\ \text{fm}^{-1}$

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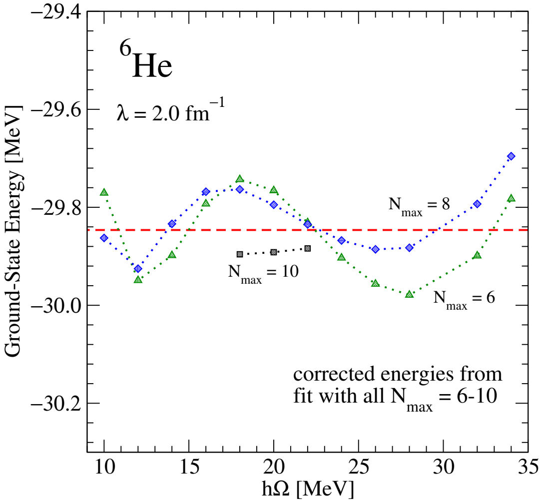

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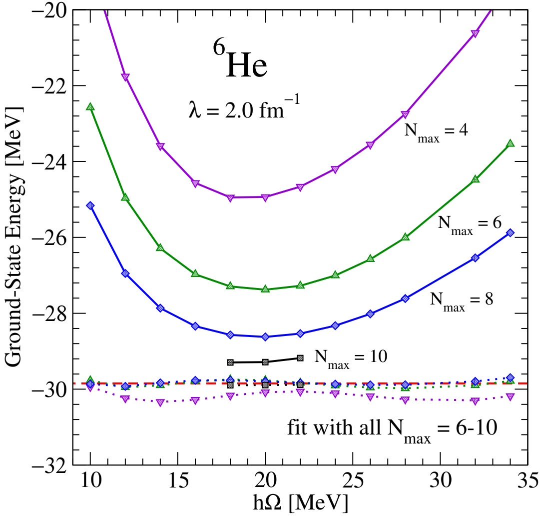

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- $E_\infty \approx -29.84\,$MeV
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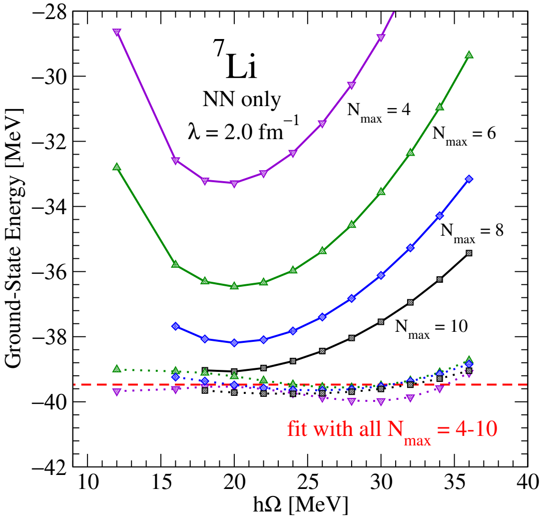
- Corrected $N_{\rm max} = 4$ energies (not fit) slightly overbound

Combined IR and UV energy fit of NCSM results

[Preliminary from Jurgenson, Maris et al.]



- ^{7}Li NN only
- Combine UV and IR corrections and fit all $N_{\text{max}} = 4$–10 points
- $E_\infty \approx -39.47$ MeV
- $\lambda_{\text{fit}} \approx 2.1\ \text{fm}^{-1}$
- $k_\infty \approx 102$ MeV

Combined IR and UV energy fit of NCSM results

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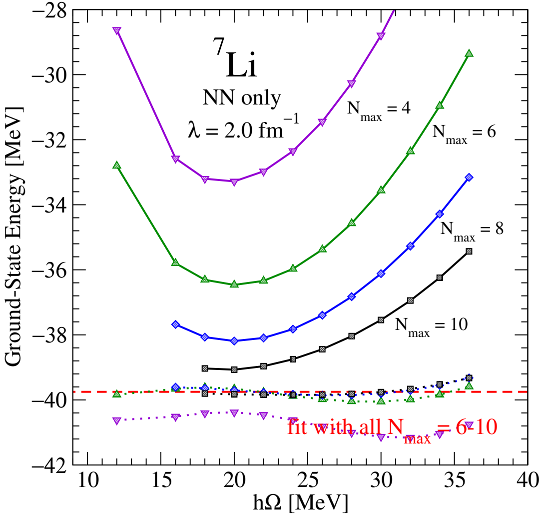

- ^{7}Li NN only
- Combine UV and IR corrections and fit all $N_{\rm max} = 6$–10 points
- $E_\infty \approx -39.75\,\text{MeV}$
- $\lambda_{\rm fit} \approx 2.2\ \text{fm}^{-1}$
- $k_\infty \approx 103\,\text{MeV}$

Combined IR and UV energy fit of NCSM results

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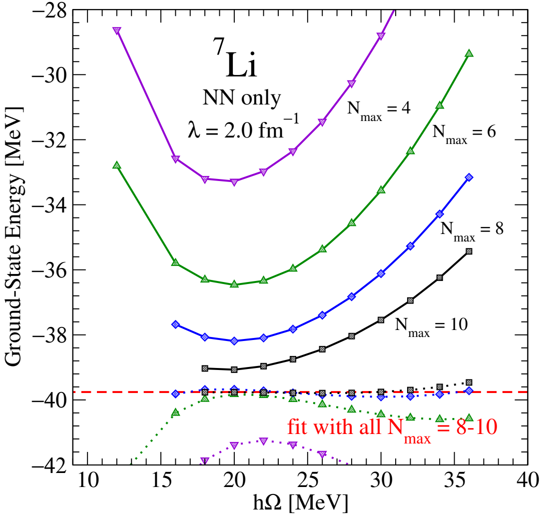
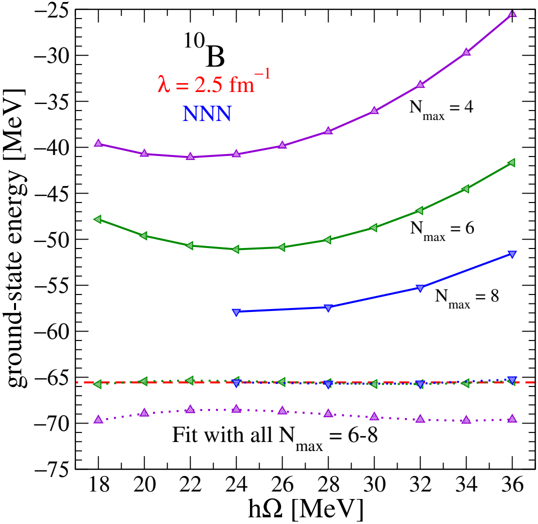


- ^{7}Li NN only
- Combine UV and IR corrections and fit all $N_{\text{max}} = 8$–10 points
- $E_\infty \approx -39.76\,\text{MeV}$
- $\lambda_{\text{fit}} \approx 2.1\ \text{fm}^{-1}$
- $k_\infty \approx 109\,\text{MeV}$

Combined IR and UV energy fit of NCSM results

[Preliminary from Jurgenson, Maris et al.]



- ^{10}B NN+NNN
- Combine UV and IR corrections and fit all $N_{\max} = 6$–8 points
- $E_\infty \approx -65.6\,\text{MeV}$
- $\lambda_{\text{fit}} \approx 2.8 \pm 0.1\ \text{fm}^{-1}$
- $k_\infty \approx 108 \pm 5\,\text{MeV}$

Combined IR and UV energy fit of NCSM results

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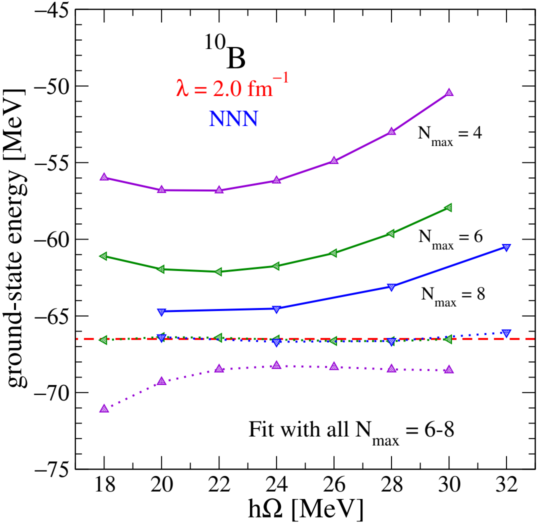
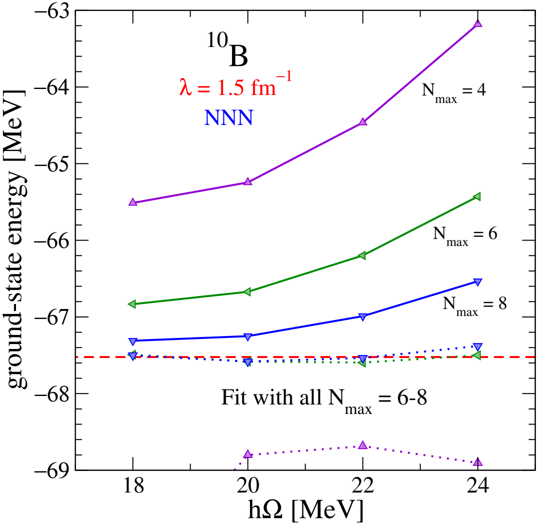


- ^{10}B NN+NNN
- Combine UV and IR corrections and fit all $N_{\text{max}} = 6$–8 points
- $E_\infty \approx -66.5\,\text{MeV}$
- $\lambda_{\text{fit}} \approx 1.9 \pm 0.1\ \text{fm}^{-1}$
- $k_\infty \approx 119 \pm 14\,\text{MeV}$

Combined IR and UV energy fit of NCSM results

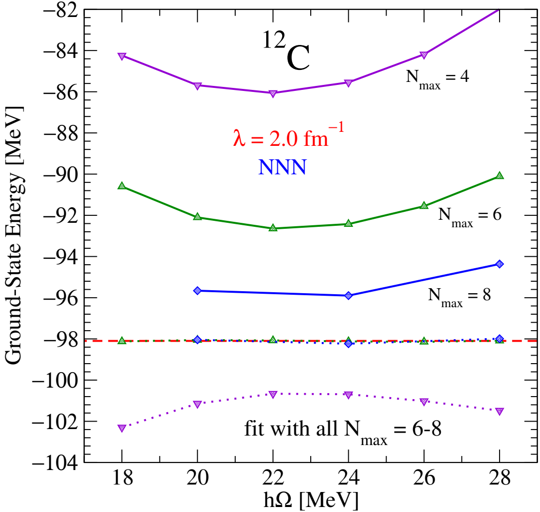
[Preliminary from Jurgenson, Maris et al.]



- ^{10}B NN+NNN
- Combine UV and IR corrections and fit all $N_{\text{max}} = 6$–8 points
- $E_\infty \approx -65.6\,\text{MeV}$
- $\lambda_{\text{fit}} \approx 1.1 \pm 1.4\ \text{fm}^{-1}$
- $k_\infty \approx 150 \pm 33\,\text{MeV}$

Combined IR and UV energy fit of NCSM results

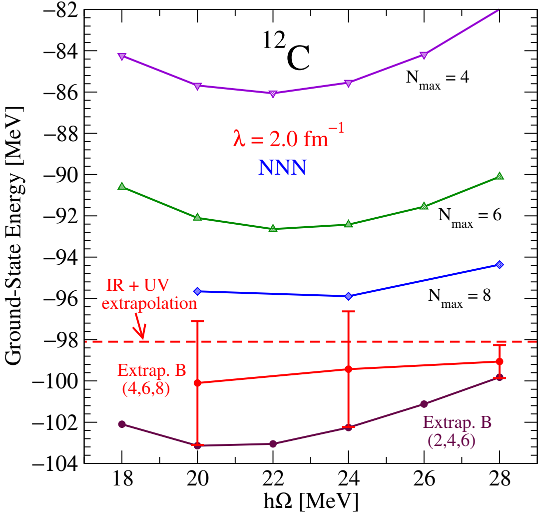
[Preliminary from Jurgenson, Maris et al.]



- ^{12}C NN+NNN
- Combine UV and IR corrections and fit all $N_{\max} = 6$–8 points
- $E_\infty \approx -98.1\,\text{MeV}$
- $\lambda_{\text{fit}} \approx 2.1 \pm 0.1\ \text{fm}^{-1}$
- $k_\infty \approx 136 \pm 8\,\text{MeV}$

Comparison to standard extrapolation [P. Maris "B"]

[Preliminary from Jurgenson, Maris et al.]



- ^{12}C NN+NNN
- Combine UV and IR corrections and fit all $N_{\max} = 6$–8 points $\Longrightarrow E_\infty \approx -98.1$ MeV
- Extrapolation B $\Longrightarrow E_\infty = -99.1 \pm 1$ MeV
- Similar systematic difference with JISP16 comparisons (preliminary!)

Outline

Motivation: Extrapolations in finite bases

Nature and implications of infrared cutoffs

High-momentum behavior of wave functions

Combined IR and UV extrapolations

Summary and open questions

Summary: Exploiting finite oscillator spaces

- A truncated oscillator basis essentially puts the nucleus in a box in both space and momentum $\Longrightarrow$ Turn a bug into a feature!
- Only IR corrections for sufficiently large $\Lambda_{UV} \sim \sqrt{2(N+3/2)\hbar\Omega}$
  - To the right of the E vs. $\hbar\Omega$ minimum
  - "Sufficiently large" depends on the interaction (soft is better)
  - Treat as nucleons in box $\Longrightarrow$ energy and radius corrections, phase shifts, ...
  - Fit parameters independent of interaction (k_∞, A_∞)
- Only UV corrections for sufficiently large $L \sim \sqrt{2(N+3/2)/\hbar\Omega}$
  - To the left of the E vs. $\hbar\Omega$ minimum
  - Fit parameters pick up scale(s) from interactions
  - Form of fit function not yet derived ...
- Combined UV and IR corrections seem to work (so far!)
  - Consistent extrapolated energies compared to UV or IR alone
- Many more things to try, test, and refine!

Open questions

- What range of $\hbar\Omega$ should you use?
- What are the optimal definitions of L and Λ_{UV}? (Use the scatter?)
- How to weight the contributions according to L (or $N_{\rm max}, \hbar\Omega$)?
- How to make credible error estimates?
- How to explain the form of UV scaling?
- Is a combined IR/UV extrapolation justified (e.g., by HOBET)?
- Interpretation of k_∞? Can we extract A_∞?
- How well does extrapolation work for other operators?
- Does it work with other basis expansions (e.g., hyperspherical harmonics)?
- Can we systematically improve the IR and UV extrapolations?
- How can we incorporate *explicitly* the harmonic oscillator part?
- ...