Ground-state properties and microscopic description of clustering in ^{6}He

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erc

Outline

- Introduction - Ab initio towards the driplines
- Ground-state properties - Scales in NCSM
- A microscopic description of clustering
- Outlook - Stepping into the continuum

Ab Initio Towards The Driplines

From quarks to nuclei

Nuclear structure

Low-energy QCD

First step: Realistic interactions

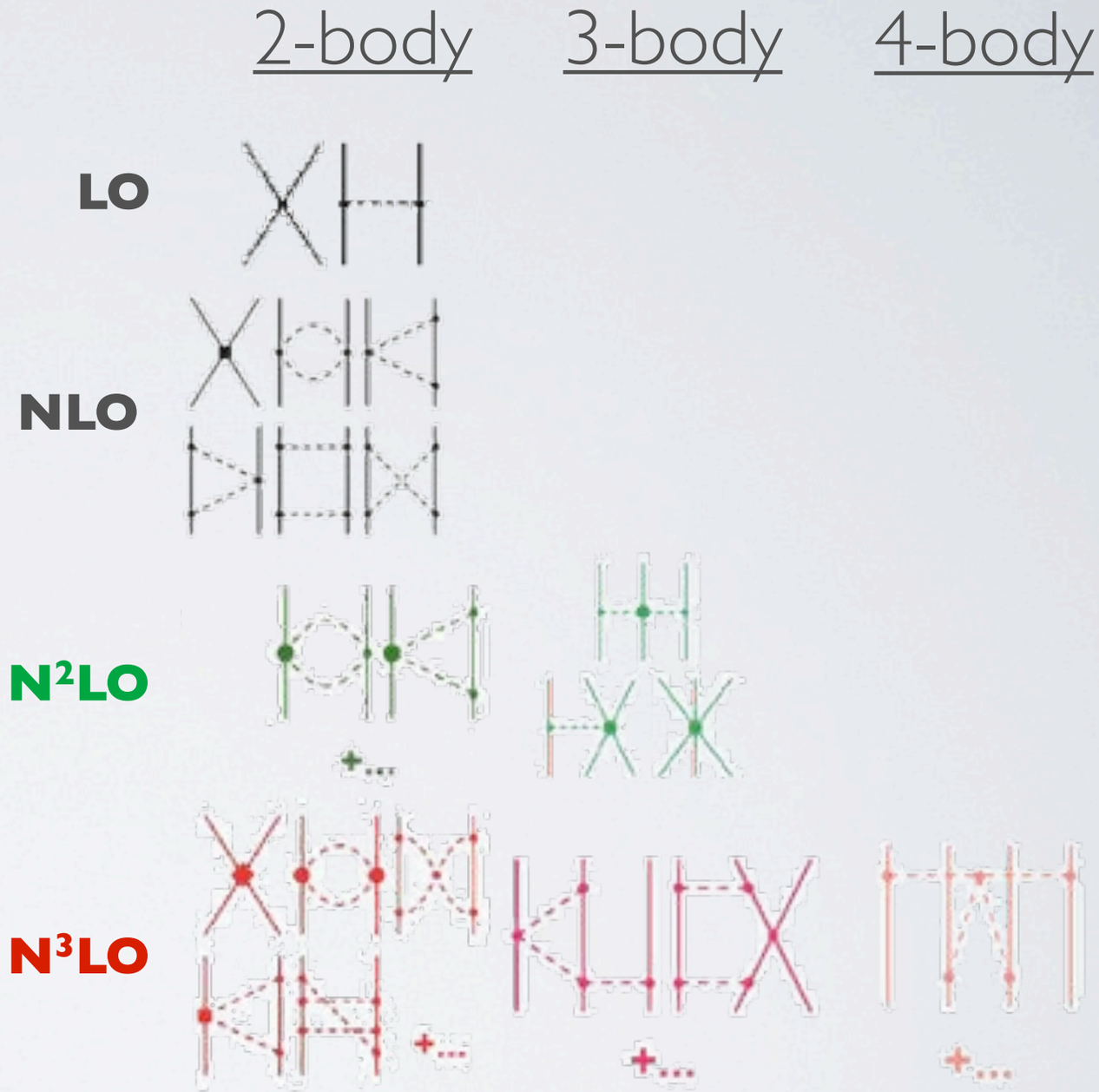
Nuclear structure

Realistic nuclear interactions

Low-energy QCD

Chiral EFT-based nuclear Hamiltonians

- EFT for relevant degrees of freedom (**π**,N) based on symmetries of QCD
- Hierarchy of consistent NN, 3N, 4N,... interactions
- See work by: Weinberg, van Kolck, Epelbaum, Meissner, Krebs, Entem, Machleidt, ...
- In the following we use:
  - NN: N^{3}LO of Entem & Machleidt with Λ = 500 MeV
  - 3NF: N^{2}LO



Chiral EFT
- E. Epelbaum, H. Hammer, U. Meissner Rev. Mod. Phys. **81** (2009) 1773
- R. Machleidt, D. Entem, Phys. Rep. **503** (2011) 1

Advanced many-body methods

Nuclear structure

Many-body methods

Realistic nuclear interactions

Low-energy QCD

A = 3,4: Several exact methods

- Faddev-Yakubovsky, SVM, GFMC, HH variational, EIHH, NCSM
- Benchmark paper: Kamada et al, Phys. Rev. C64(2001)044001
- Very important observables for testing realistic nuclear Hamiltonians

A > 4: Very few (ab initio) methods available

Intermediate Step: Unitary Transformation

Nuclear structure

Many-body methods

Unitary transformations

- adapt realistic potential to the available model space
- conserve experimentally constrained properties

Realistic nuclear interactions

Low-energy QCD

Intermediate Step: Unitary Transformation

$$V(r) \propto g_1 \frac{e^{-\kappa_1 r}}{r} - g_2 \frac{e^{-\kappa_2 r}}{r}$$

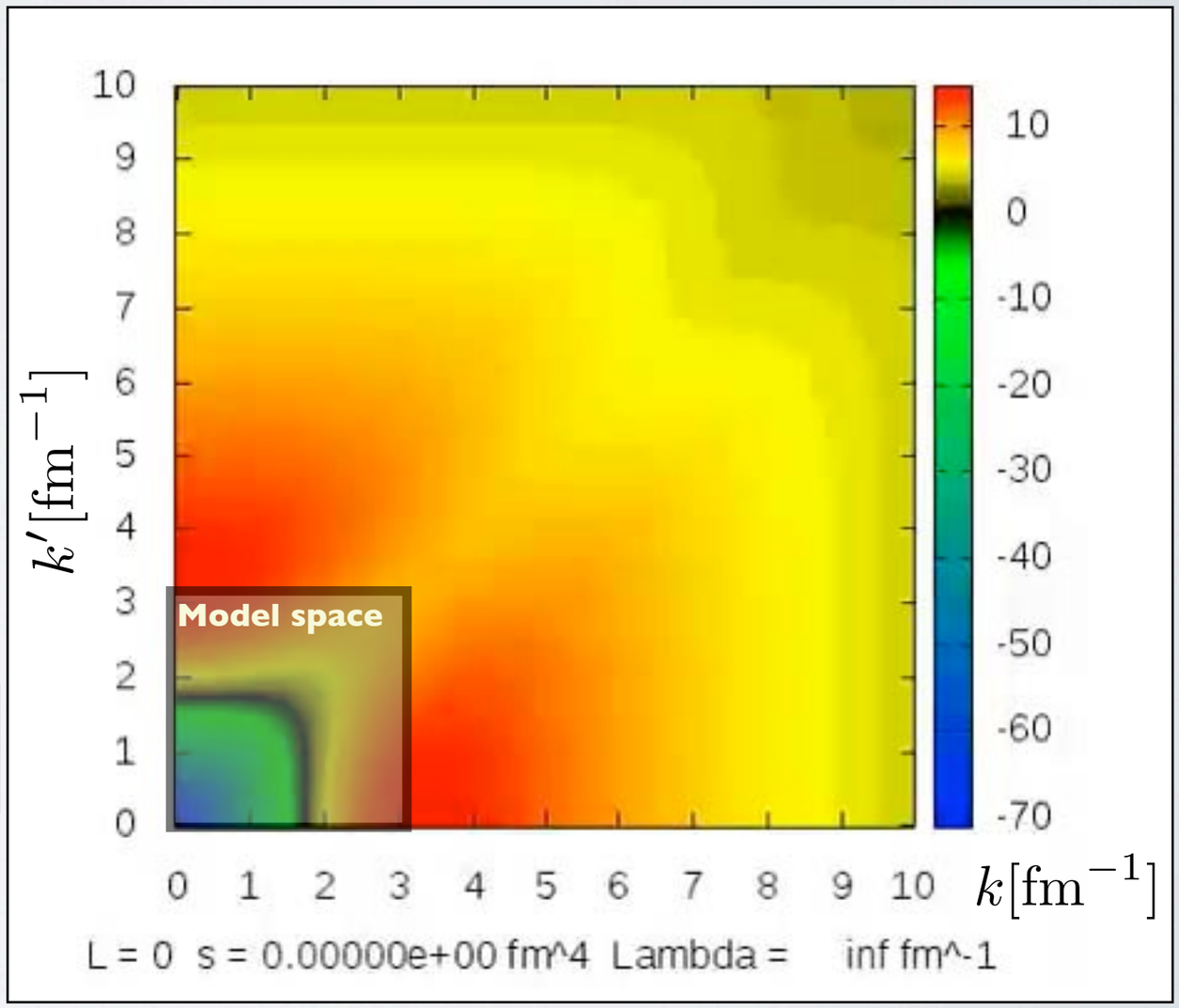
$$\mathcal{F}_r(k - k')$$

- Unitary transformation as a function of a flow parameter:

$$\tilde{H}_s = U_s^\dagger H U_s$$

- SRG evolution equation:

$$\frac{d}{ds}\tilde{H}_s = \left[\eta_s, \tilde{H}_s\right], \quad \eta_s \propto \left[T_{\text{int}}, \tilde{H}_s\right]$$



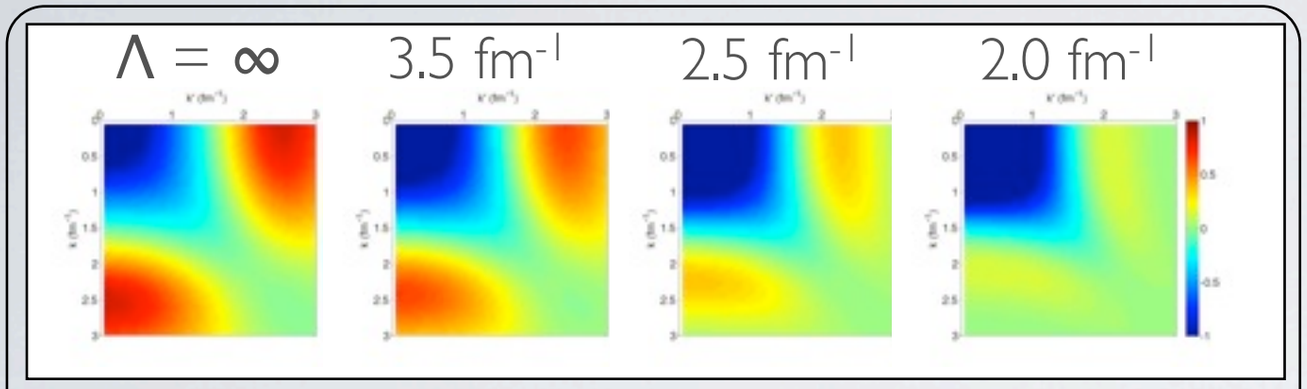
$\Lambda_{HO=20MeV;\, n=17} \sim 3$ fm^{-1}

SRG

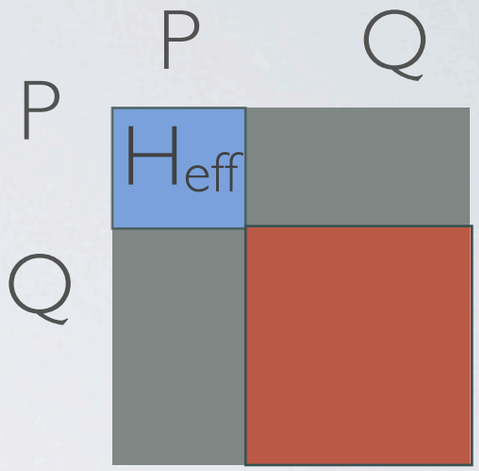
- S. Bogner, R. Furnstahl, A. Schwenk, Prog. Part. Nucl. Phys. **65** (2010) 94

Effective-interaction approaches

Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. 65 (2010) 94



- SRG, V-UCOM, V-lowk.
- H_{eff} is independent of model space.
- Variational calculation
  (but without A-body terms the converged result typically depends on the flow parameter)



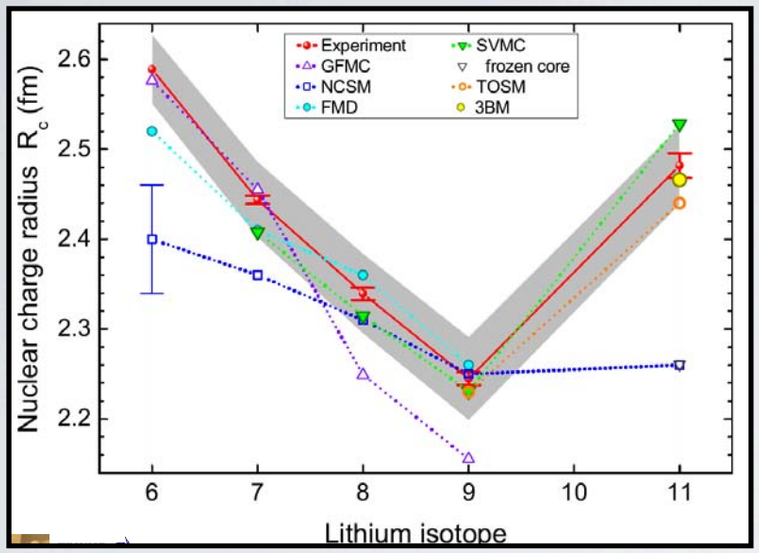
- Lee-Suzuki-Okamoto
- H_{eff} is model-space dependent.
- Not variational
  (but even with only 2-body terms it will converge to exact result)

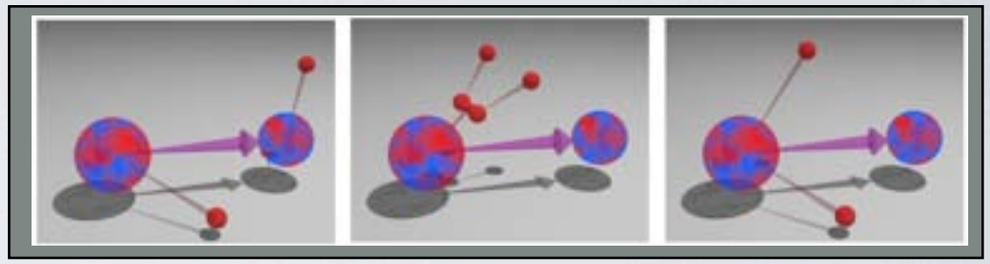
Many-body nuclear physics studied at low resolution scales comes at a price: the appearance of **many-body forces**.

C. Forssén, INT, Oct. 31, 2012

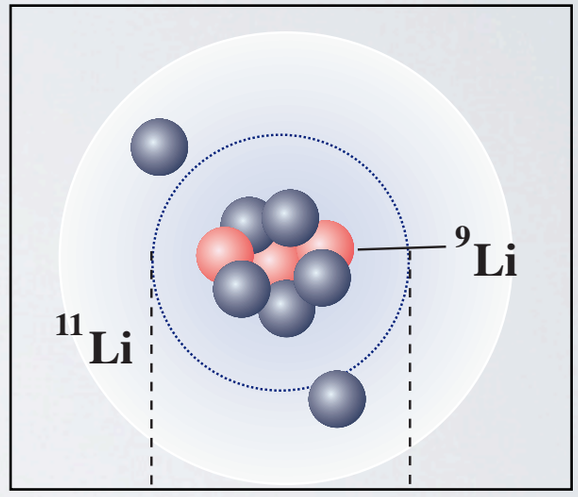
Extremes of the (light) nuclear landscape

Observables: anomalous trends





Exotic decay modes



Halo states, Borromean systems

Clusterization

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Ground-State Properties - scales in the NCSM

^{6}He Ground-State Properties

- Very accurate charge-radius measurements using laser spectroscopy.

6-He references

- P. Mueller et al., Phys. Rev. Lett. **99** (2007) 252501.
- M. Brodeur et al. Phys. Rev. Lett. **108** (2012) 052504
- S. Bacca. et al. Phys. Rev. C**86**, (2012) 034321

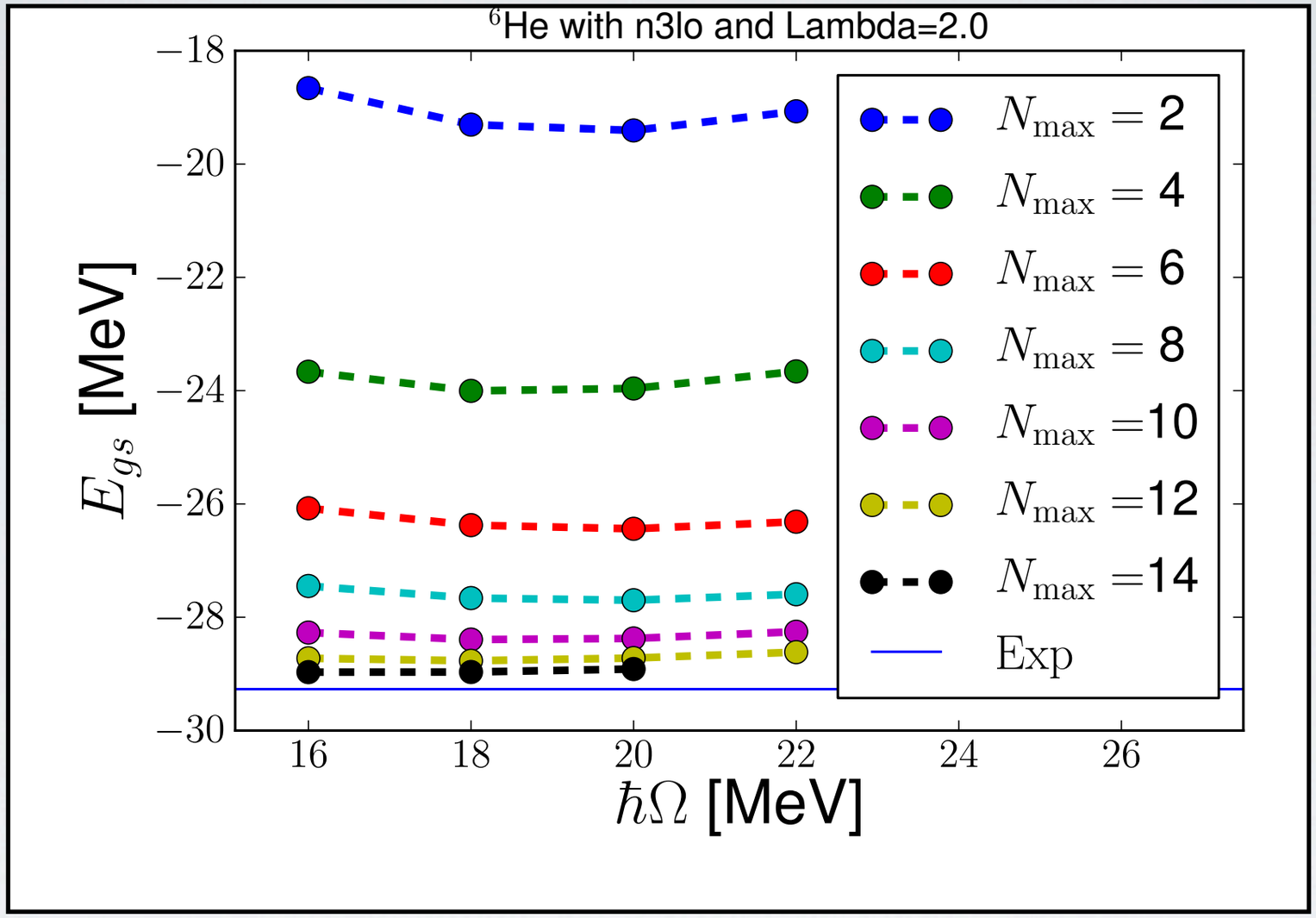
- Very accurate mass measurement with a Penning trap mass spectrometer.

- Several ab initio calculations

- Most recently by S. Bacca et al

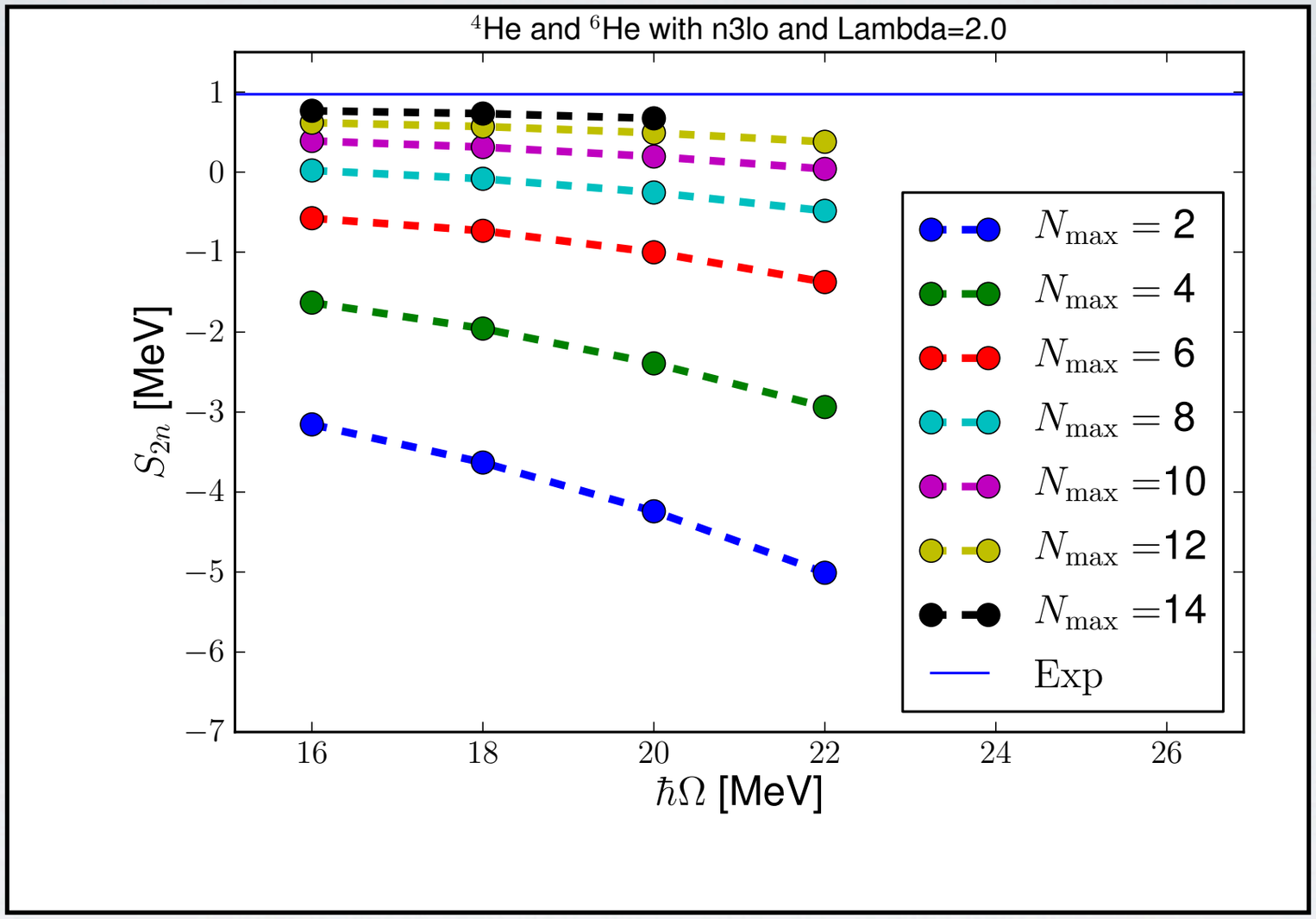
  - using EIHH and V_{lowk} NN potential based on I-N^{3}LO.
  - Study of Vlowk cutoff-dependence and observable correlations.

Ground-state energy of ^{6}He



Typical variational pattern: large $\hbar\Omega$ cuts off wf and small $\hbar\Omega$ cuts off potential.
⇒ trade-off between IR and UV regulators

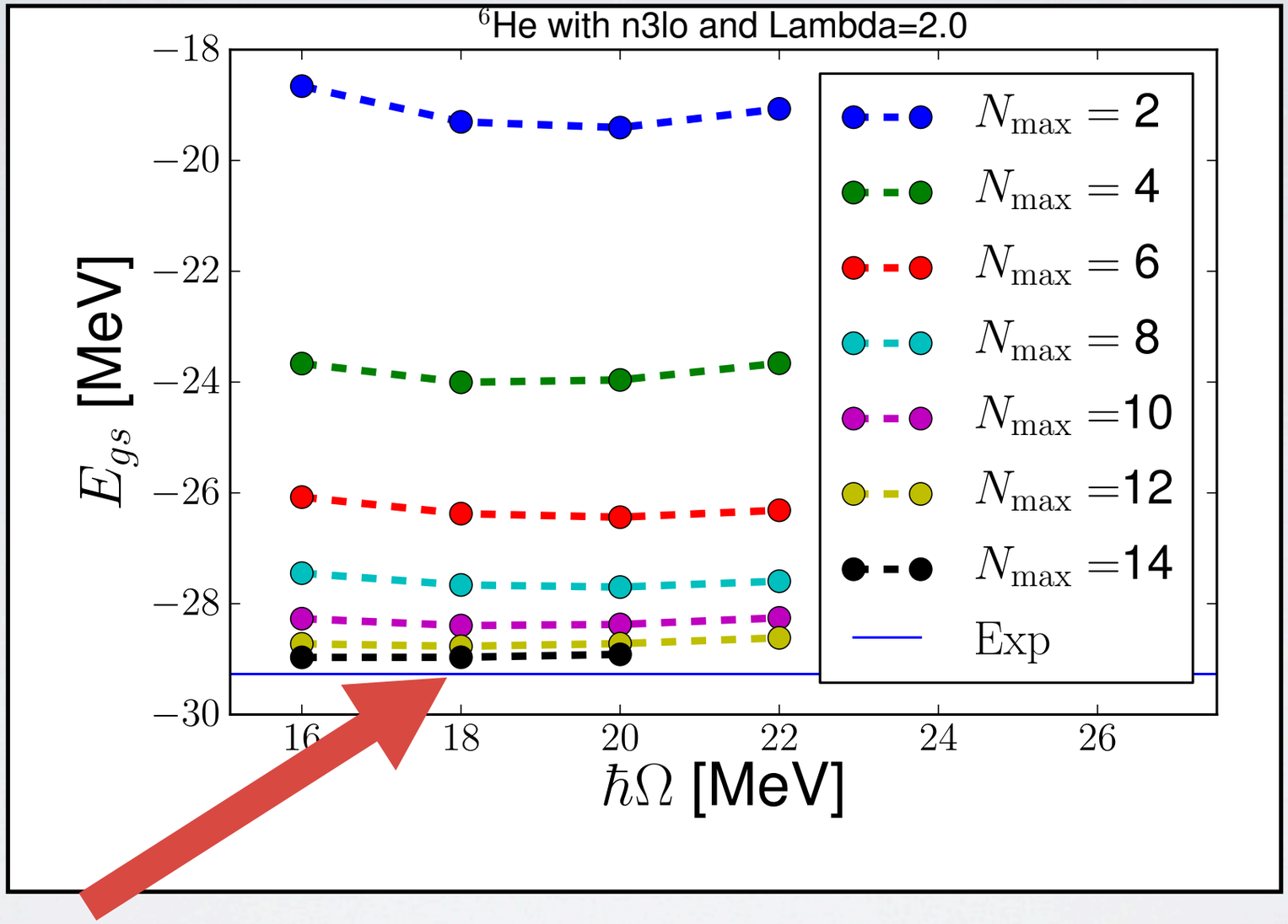
^{6}He: Two-neutron separation energy



Note that no induced three-body terms were included
⇒ dependence on the SRG flow-parameter in these calculations.

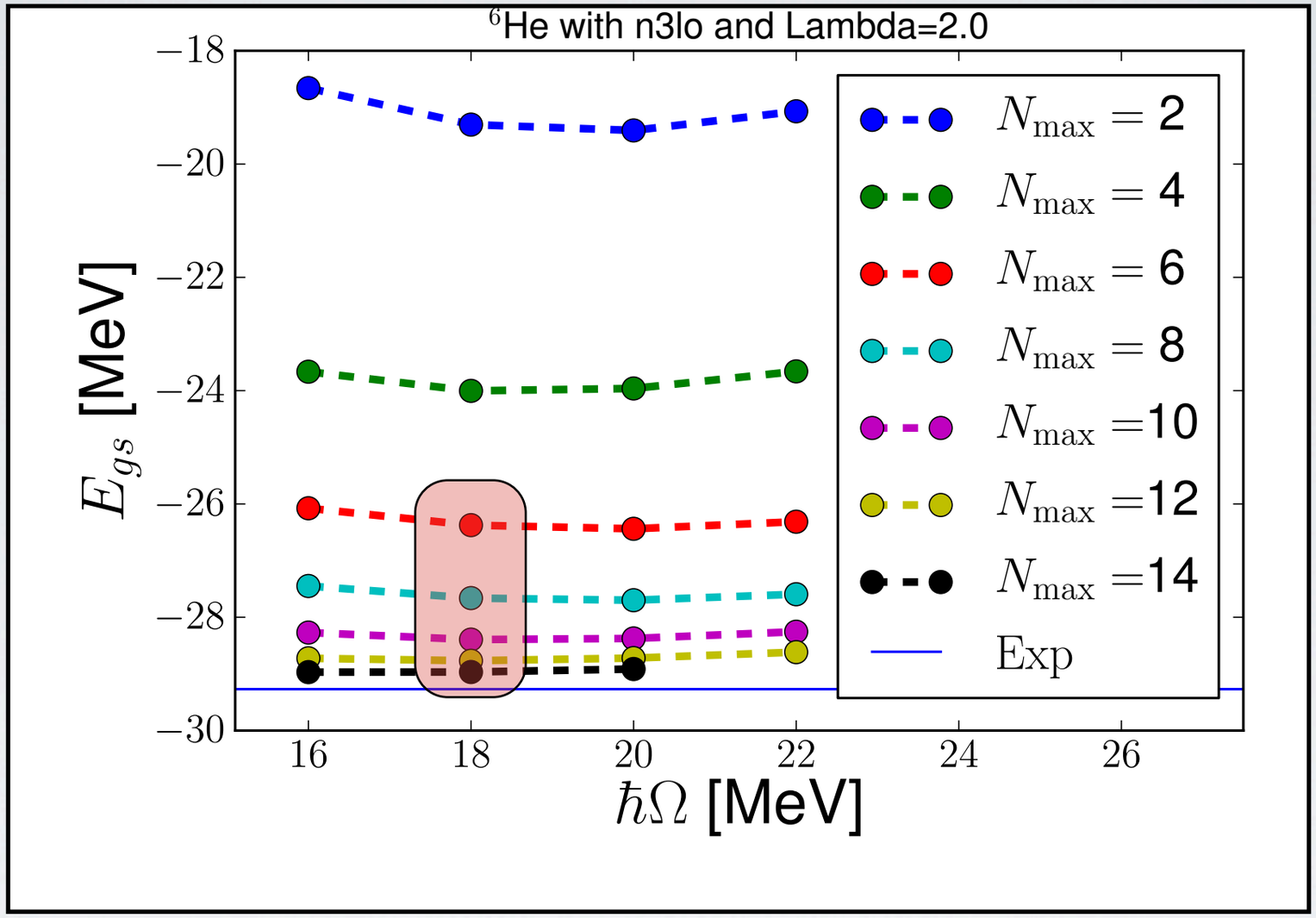
Changes S_{2n} by approximately +/-200 keV in [1.8, 2.2] fm^{-1} range

Ground-state energy of ^{6}He



E(^{6}He)	-28.967 MeV	Single (N_{max}, $\hbar\Omega$)

Ground-state energy of ^{6}He



E(^{6}He)	-28.967 MeV	Single (N_{max}, $\hbar\Omega$)
E(^{6}He)	-29.221 MeV	Exp. fit , Single $\hbar\Omega$

$$E(N_{\max}) = E_\infty + c_0 \exp\left(-c_1 N_{\max}\right)$$

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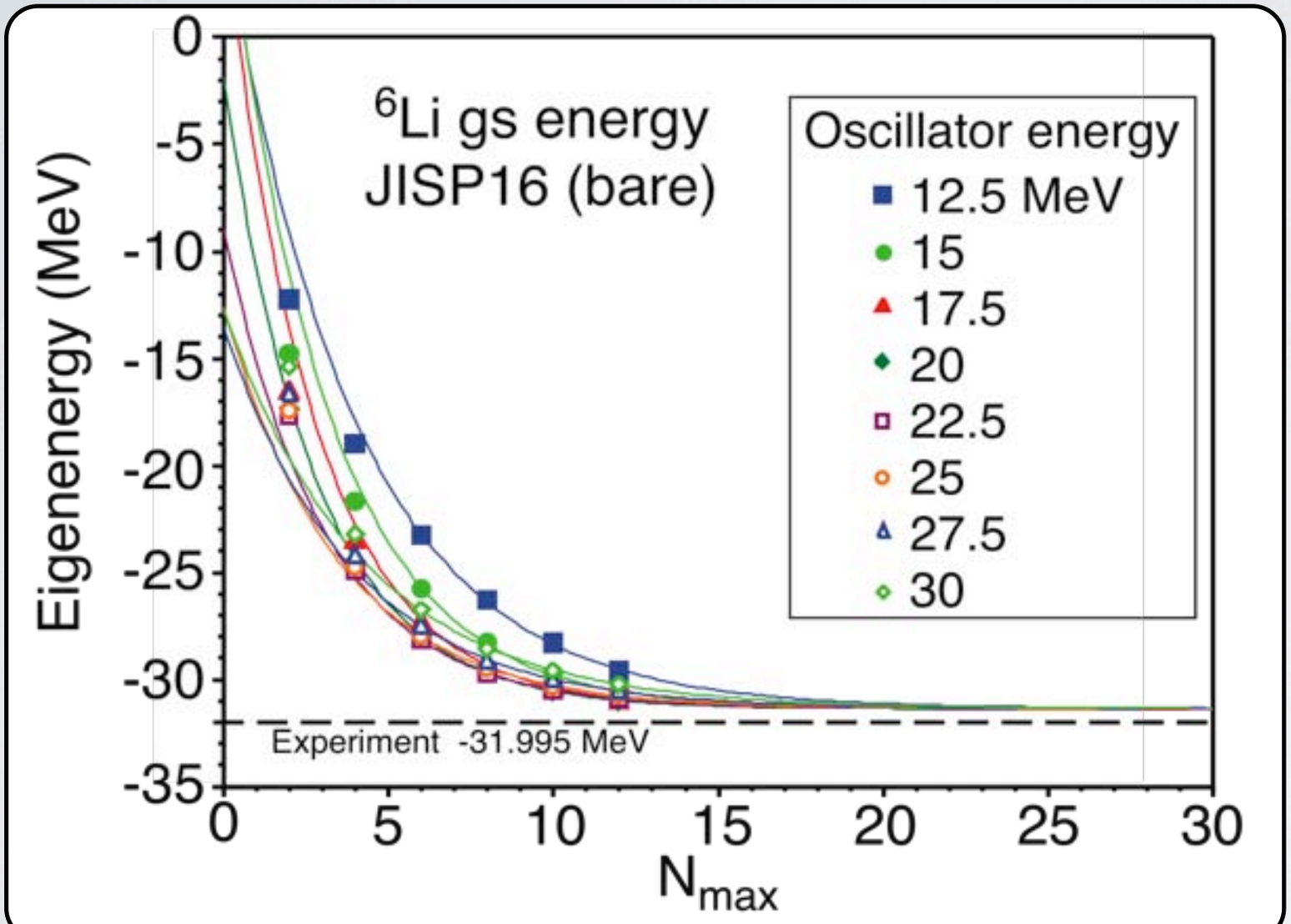
Converging sequences

Use results obtained at different HO frequencies to perform a **constrained fit**



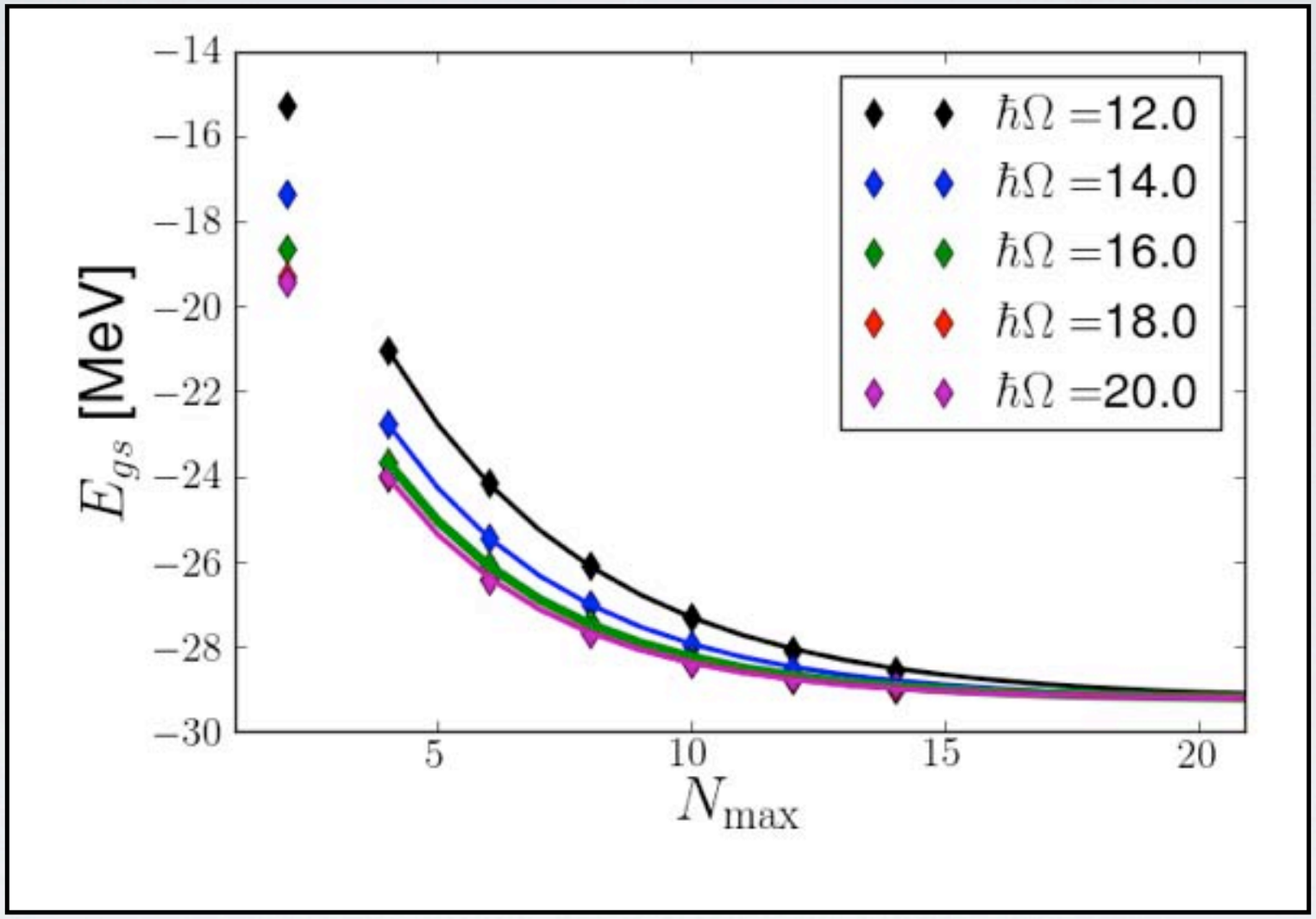
Powerful property

Any sequence that returns the initial Hamiltonian as the model space is increased will converge to the exact result.

$$E(N_{\max}, \hbar\Omega) = E_\infty + c_{0,\hbar\Omega} \exp\left(-c_{1,\hbar\Omega} N_{\max}\right)$$

CF, J. Vary, E. Caurier, P. Navrátil, Phys. Rev. C 77 (2008) 024301

Ground-state energy of ^{6}He



E(^{6}He)	-28.967 MeV	Single (N_{max}, $\hbar\Omega$)
E(^{6}He)	-29.221 MeV	Exp. fit , Single $\hbar\Omega$
E(^{6}He)	-29.240(93) MeV	Constrained fit

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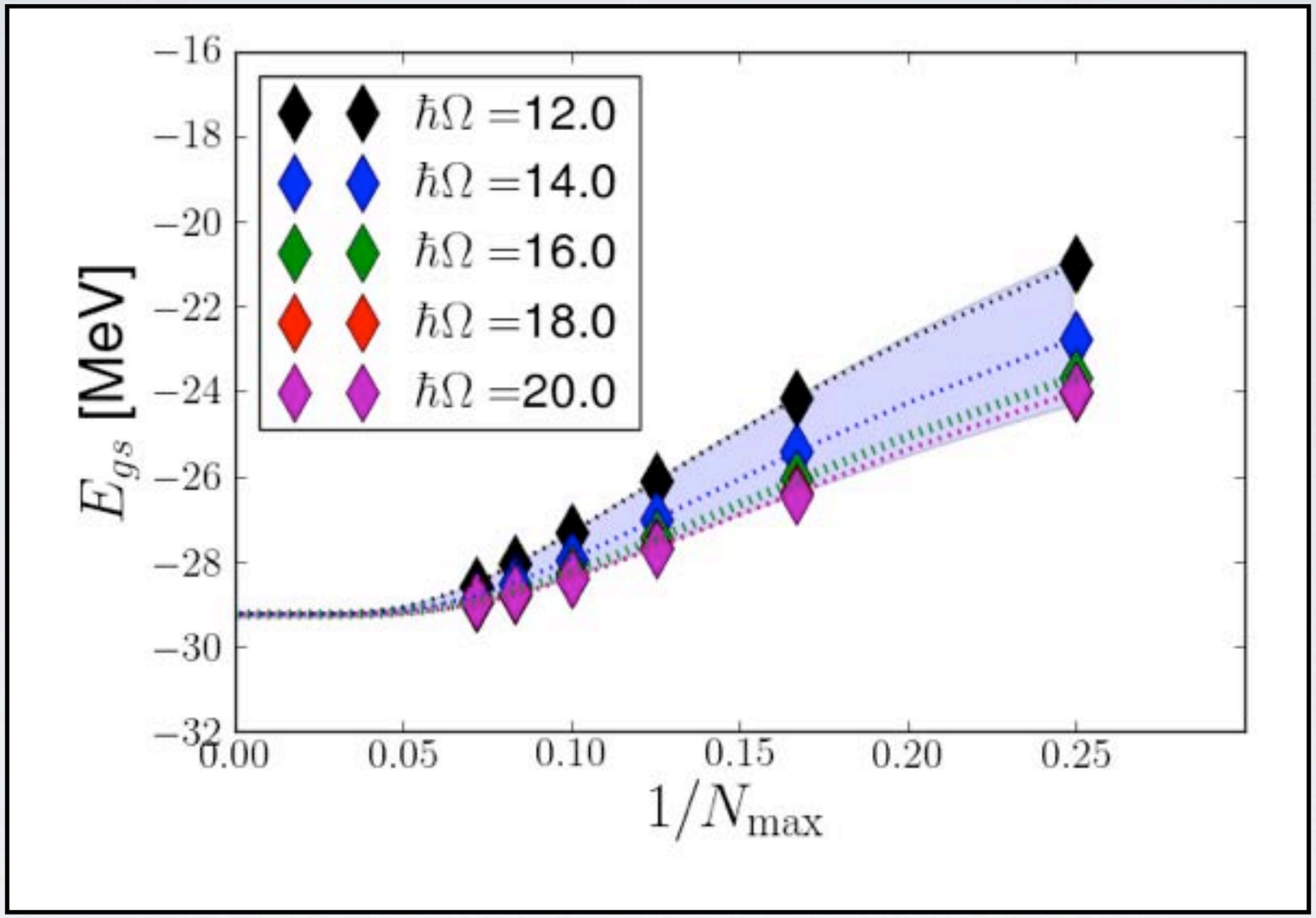
Ground-state energy of ^{6}He



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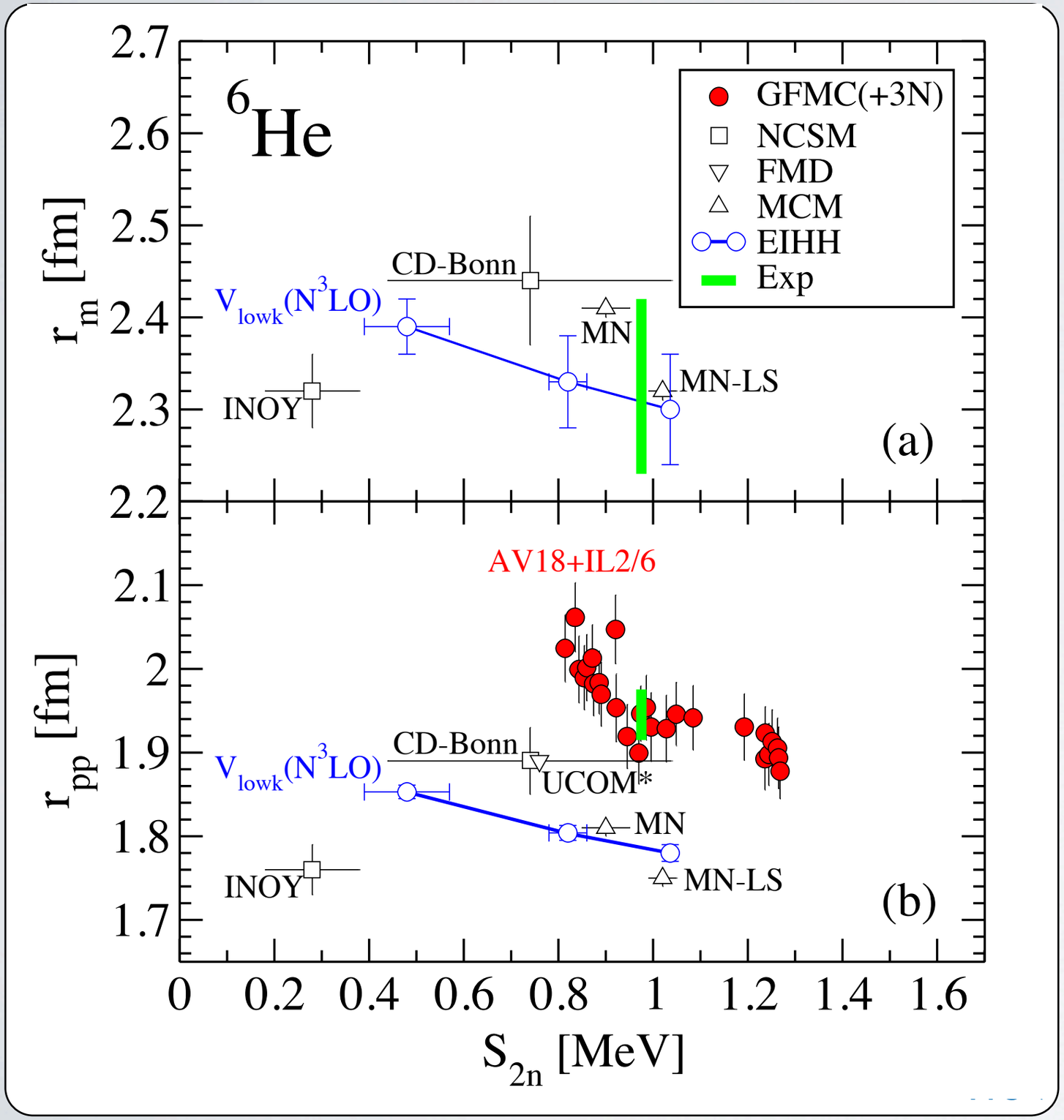
^{6}He from EIHH



- Use hypersperical coordinates, expand in HH and solve hyperradial equations

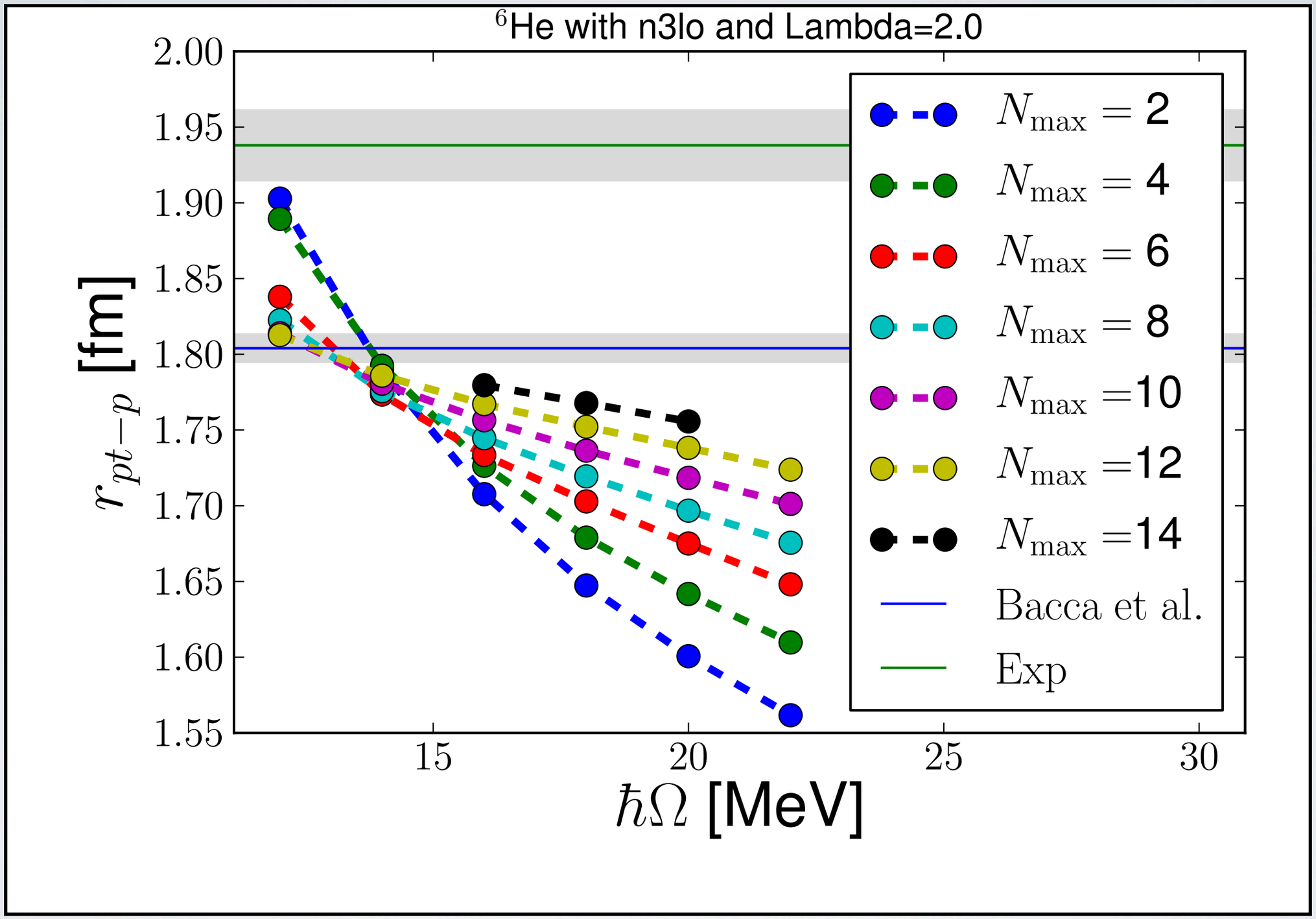
$$|\Psi\rangle = \sum_{[K]}^{K_{\max}} \sum_{\nu}^{\nu_{\max}} c_{[K],\nu} \mathcal{Y}_{[K]}(\Omega) e^{-\rho/2b} L_\nu(\rho)$$

- $V_{low\text{-}k}$ from I-N3LO with various cutoffs Λ
- Lee-Suzuki effective interaction
- Study correlations of observables

6-He measurement and EIHH calculations

- M. Brodeur et al. Phys. Rev. Lett. **108** (2012) 052504
- S. Bacca. et al. Phys. Rev. C**86**, (2012) 034321

^{6}He: Point-Proton Radius



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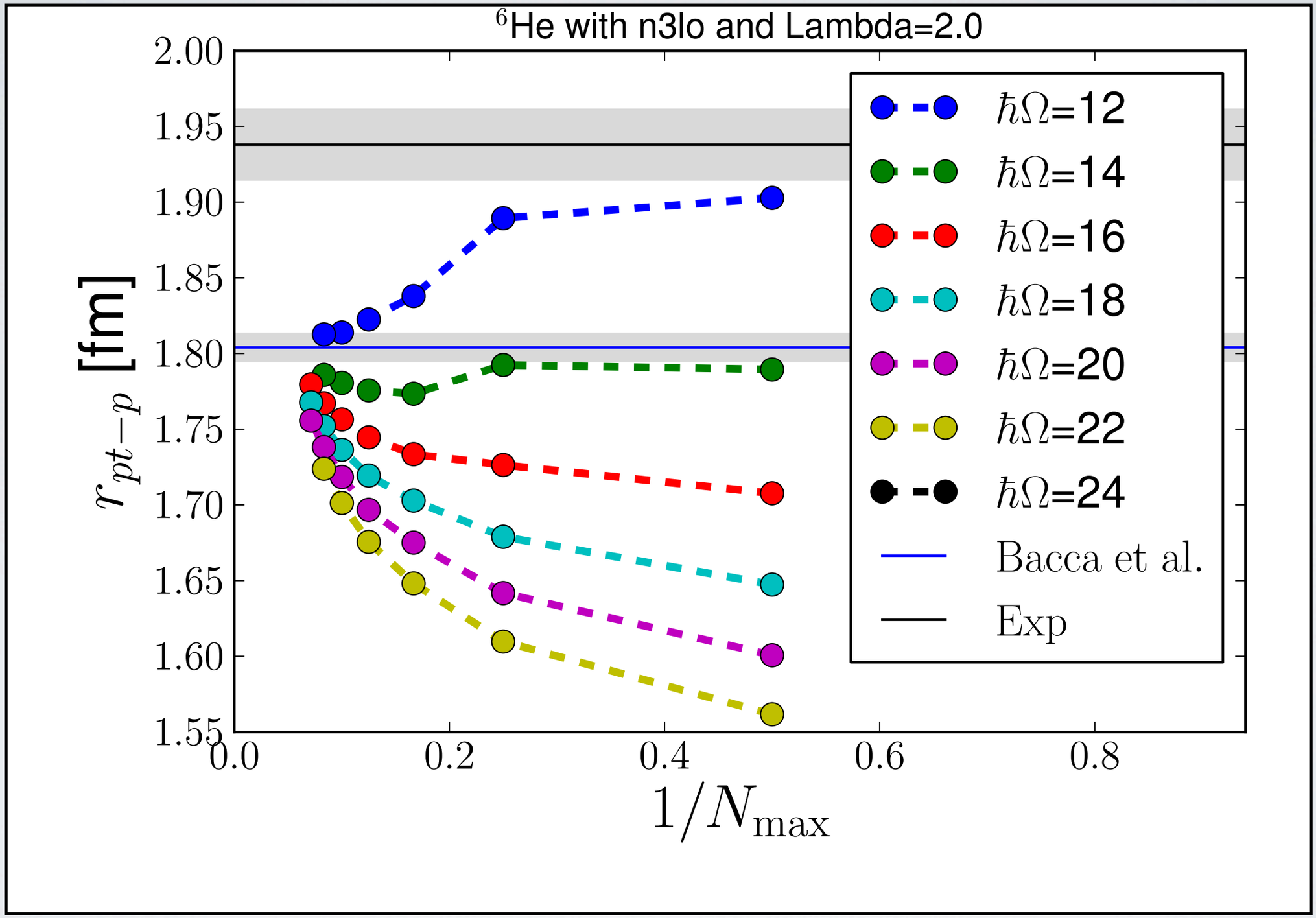
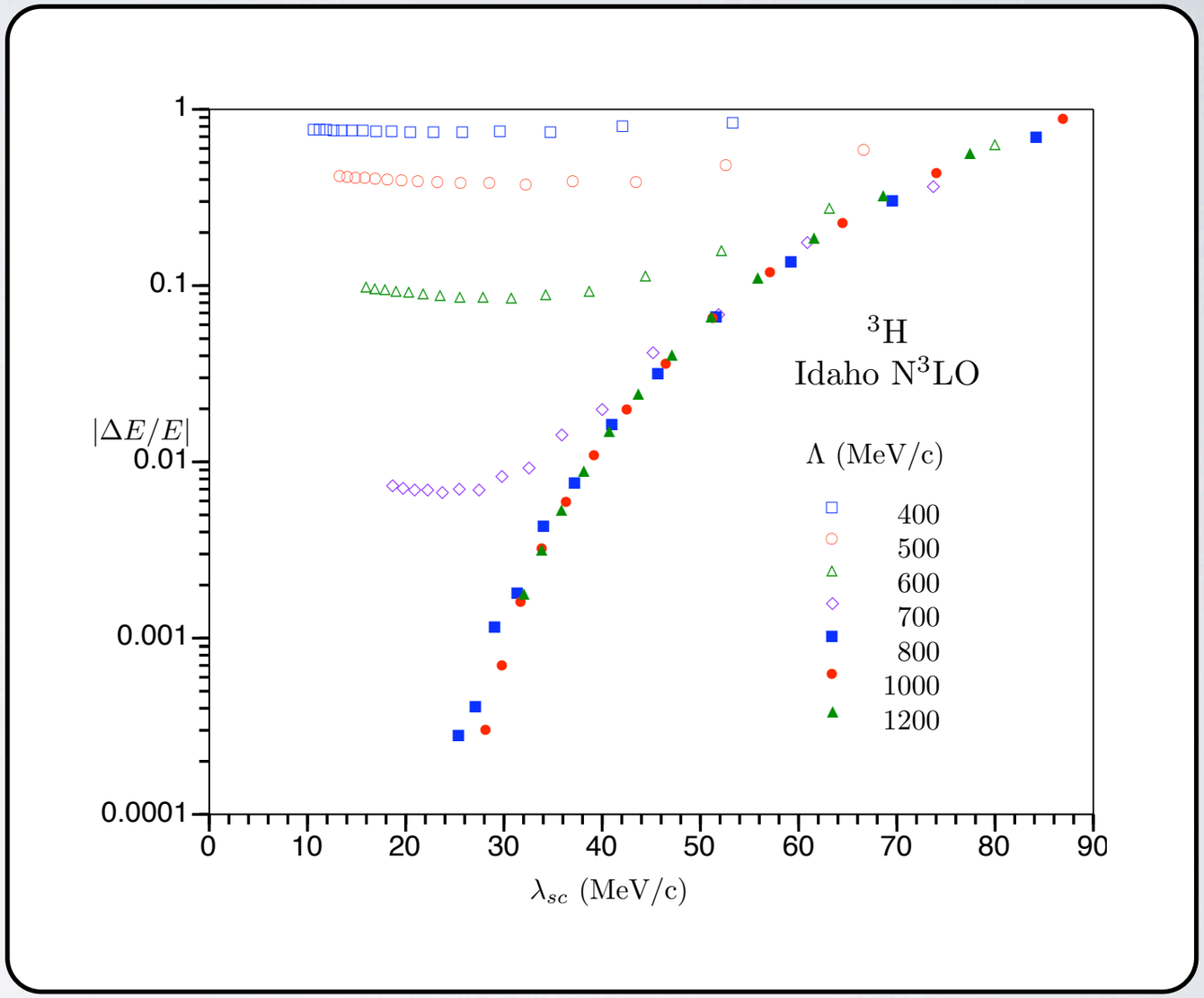


Switching to IR and UV cutoffs as variables

- Plot $\Delta E/E_{conv}$ as a function of $\lambda_{sc} \equiv \sqrt{m\hbar\Omega/(N+3/2)}$ and $\Lambda \equiv \sqrt{m\hbar\Omega(N+3/2)}$
- Universal dependence on λ_{sc} over wide range of $\Delta E/E_{conv}$.
- Fit shows exponential in $1 / \lambda_{sc}$
- Plateaus to the left from UV corrections



S. Coon et al., arXiv:1205.3230.

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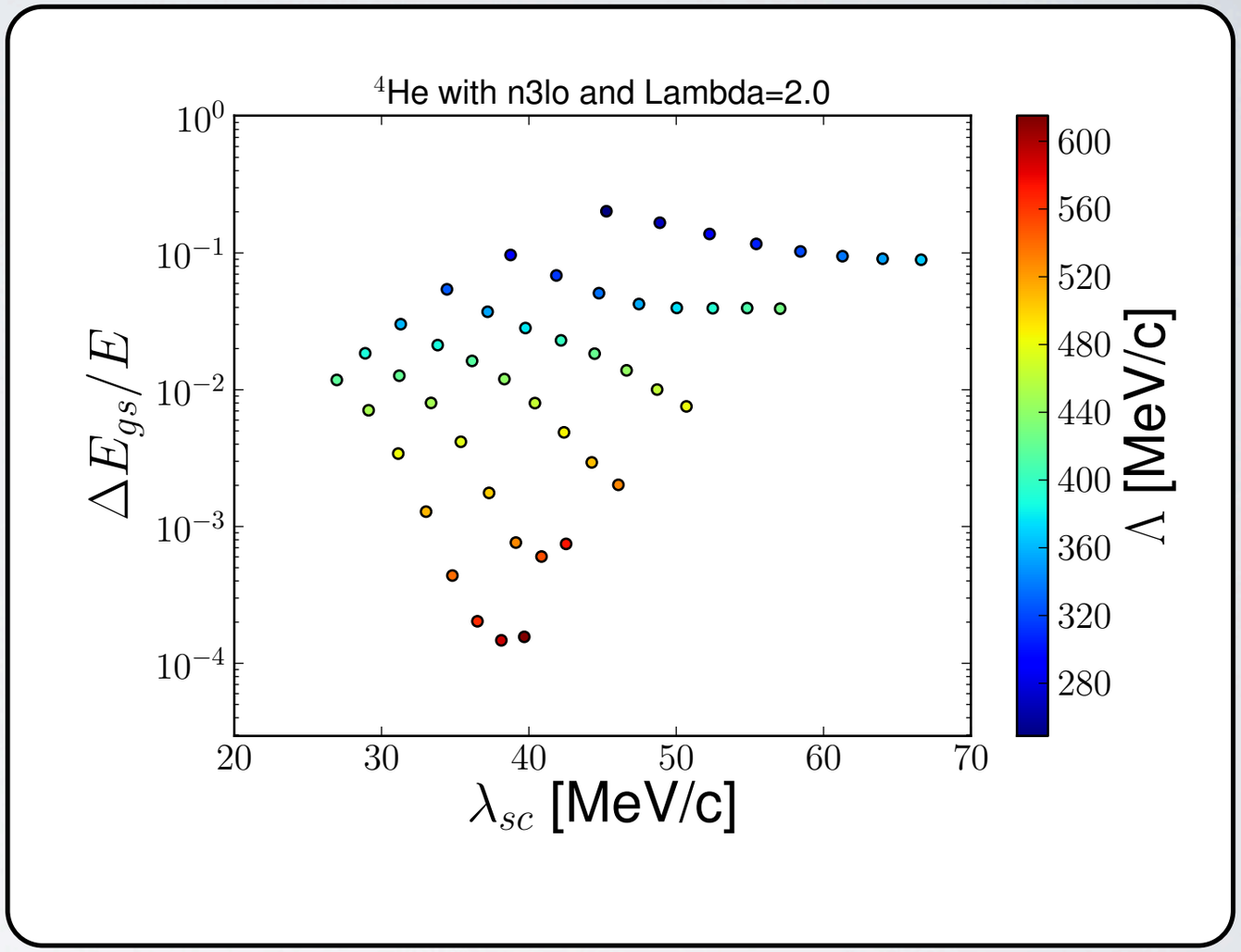


Our calculations
- not fixed Λ but color mapped

Nature and form of IR and UV corrections

Truncated basis cuts off s.p. wave functions

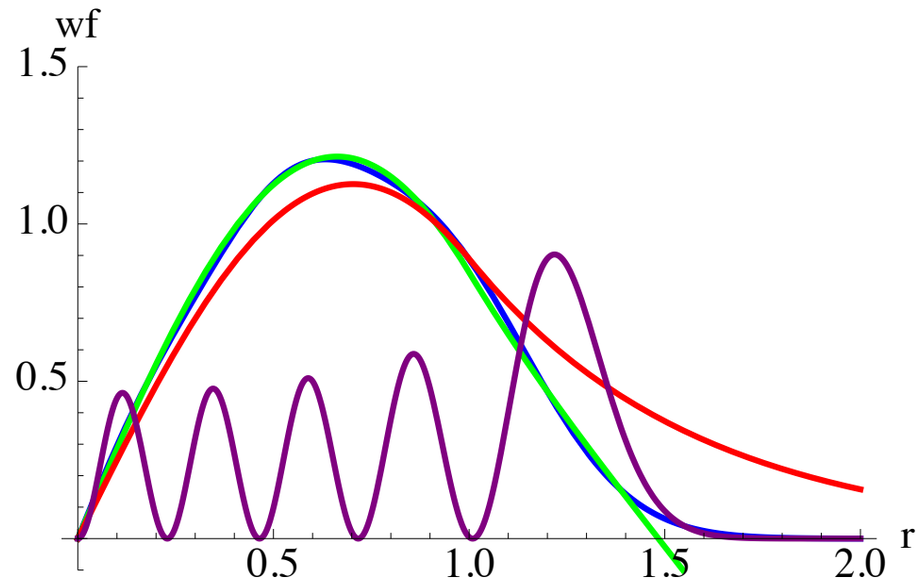
- First estimate of cutoffs: $\frac{1}{2}m\Omega^2 r_{\max}^2 = \frac{1}{2m}p_{\max}^2 = (N+3/2)\hbar\Omega$

  $$\Longrightarrow \quad \Lambda_{UV} = \sqrt{2(N+3/2)}\hbar/b \quad \text{and} \quad L_0 = \sqrt{2(N+3/2)}b$$

  with $b = \sqrt{\hbar/m\Omega}$ (note $\sqrt{2}$'s)

- Improved estimate for L from intercept of tangent at $r = L_0$:

  $$L_{NLO} \approx L_0 + 0.54437\, b\, (L_0/b)^{-1/3}$$



- Square-well wave functions with mass $m = 1$, radius $R = 1$, and depth $V_0 = 4$
- Exact (red) is compared to HO with $\hbar\Omega = 10$ and $N = 8$ (blue) and to boundary condition at $r = L$ (green) and to $n = 4$ wf squared (purple)

$E_{\text{exact}} = -1.51$, $E_{\text{HO}} = -1.23$, $E_L = -1.14$ [L improved]

R.J. Furnstahl, talk at this program, 2012-10-10
R.J. Furnstahl et al., Phys. Rev. C 86(2012)031301R.

Correction for energy

Linear energy method to estimate corrections [Djajaputra]

- Let $u_E(r)$ be the radial solution regular at $r = 0$ with energy E, then

$$u_L(r) \equiv u_{E_L}(r) \approx u_\infty(r) + \Delta E_L \left.\frac{du_E(r)}{dE}\right|_{E_\infty} \quad \text{where} \quad E_L = E_\infty + \Delta E_L$$

$$\text{So } u_L(L) = 0 \implies \Delta E_L \approx -u_\infty(L) \left(\left.\frac{du_E(L)}{dE}\right|_{E_\infty} \right)^{-1}$$

- Now $u_E(r) \overset{r \gg R}{\longrightarrow} A_E(e^{-k_E r} + \alpha_E e^{+k_E r})$ with $u_\infty(r) \overset{r \gg R}{\longrightarrow} A_\infty e^{-k_\infty r}$ and k_∞ from nucleon separation energy $S = \frac{\hbar^2 k_\infty^2}{2m}$
- Take the derivative and evaluate at $E = E_\infty$:

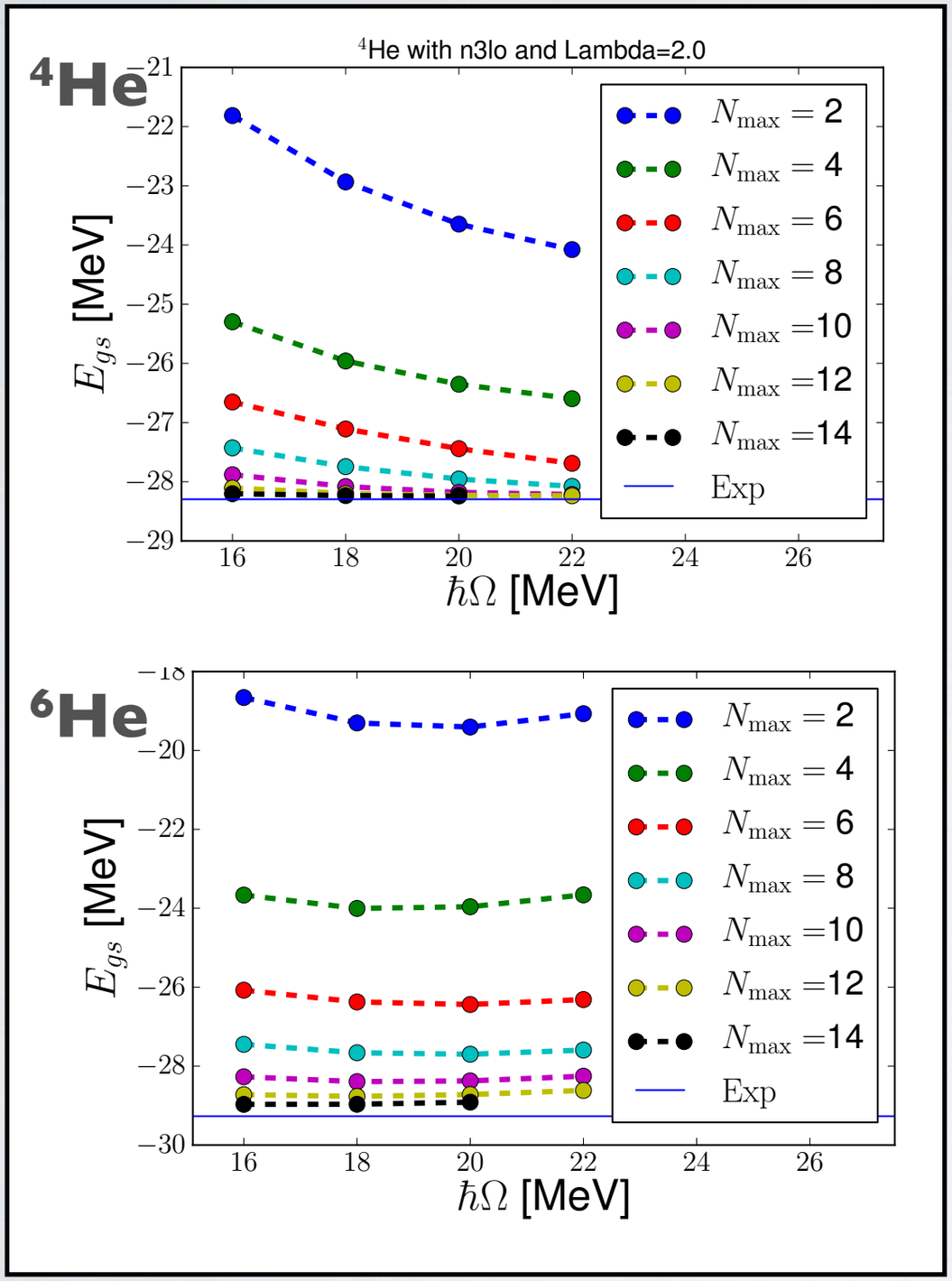
$$\left.\frac{du_E(r)}{dE}\right|_{E_\infty} = +A_\infty \left.\frac{d\alpha_E}{dE}\right|_{E_\infty} e^{+k_\infty r} + \mathcal{O}\left(e^{-k_\infty r}\right)$$

Substituting at $r = L$, we obtain our correction formula to fit:

$$\Delta E_L \approx -\left[\left.\frac{d\alpha_E}{dE}\right|_{E_\infty}\right]^{-1} e^{-2k_\infty L} + \mathcal{O}(e^{-4k_\infty L}) \implies E_L = E_\infty + a_0 e^{-2k_\infty L}$$

R.J. Furnstahl, talk at this program, 2012-10-10
R.J. Furnstahl et al., Phys. Rev. C 86(2012)031301R.

NCSM example: Energy convergence



HO basis cutoff scales

$$\Lambda_{\rm UV} = \sqrt{2(N+3/2)}\hbar/b$$

$$L_{\rm IR} = \sqrt{2(N+3/2)}b$$

$$L \approx L_{\rm IR} + 0.544b(L_{\rm IR}/b)^{-1/3}$$

Extrapolations from finite HO basis

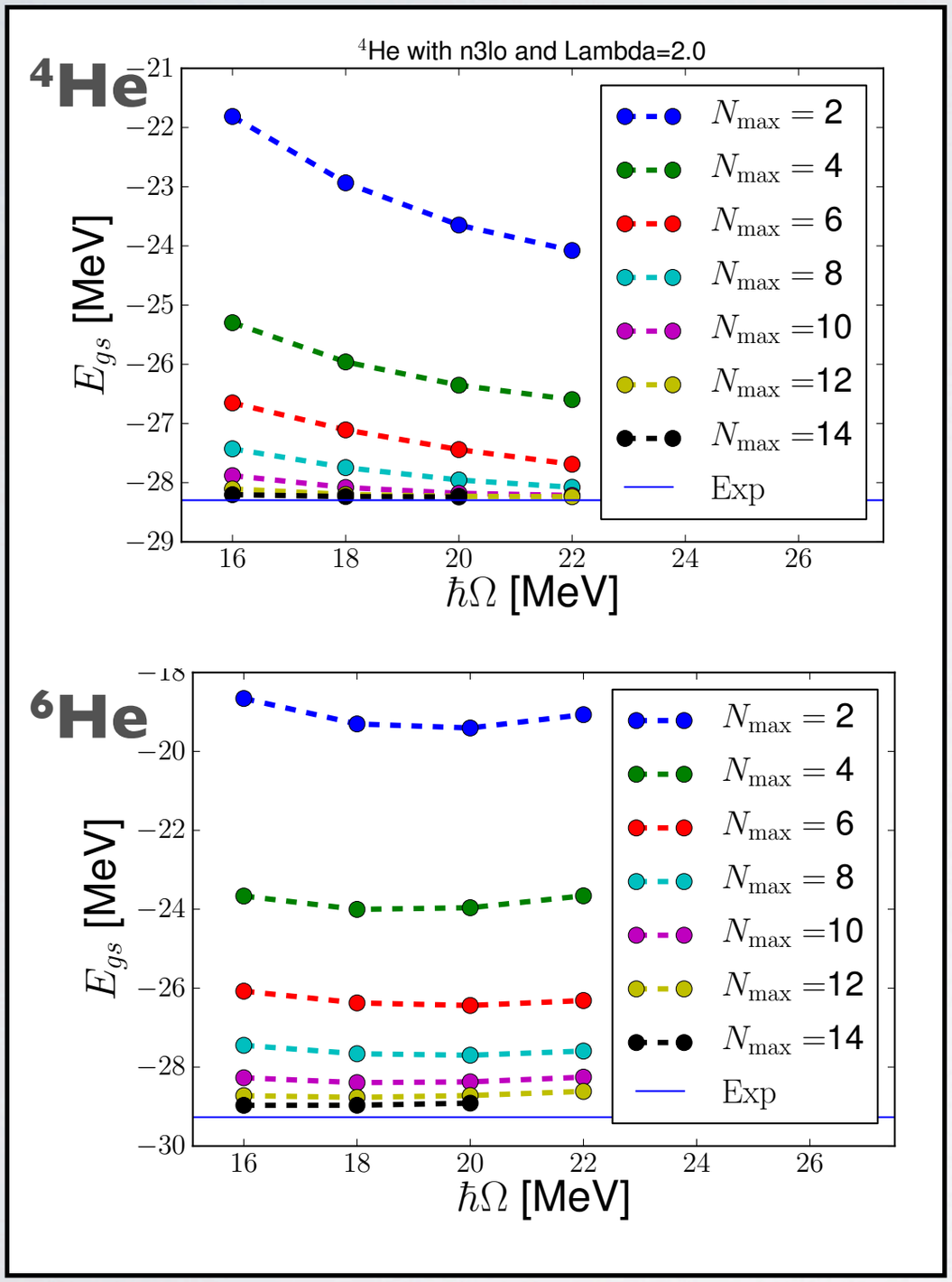
- R.J. Furnstahl et al., Phys. Rev. C 86(2012)031301R

Correction to the energy due to finite HO space

$$E_L = E_\infty + \Delta E_L$$

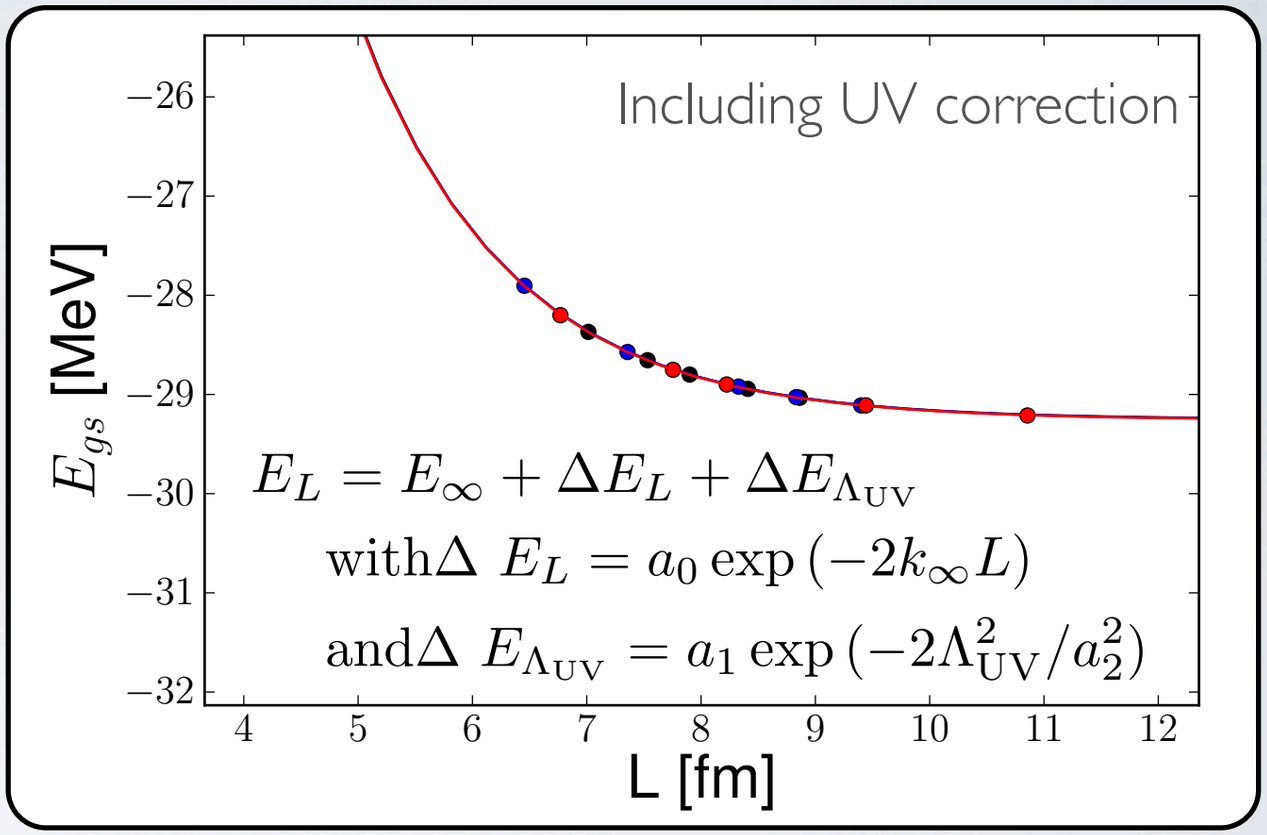
$$\text{with}\Delta\ E_L = a_0 \exp(-2k_\infty L)$$

NCSM example: Energy convergence



Binding energies

N^{3}LO, SRG (NN only, Λ = 2.0 fm^{-1})

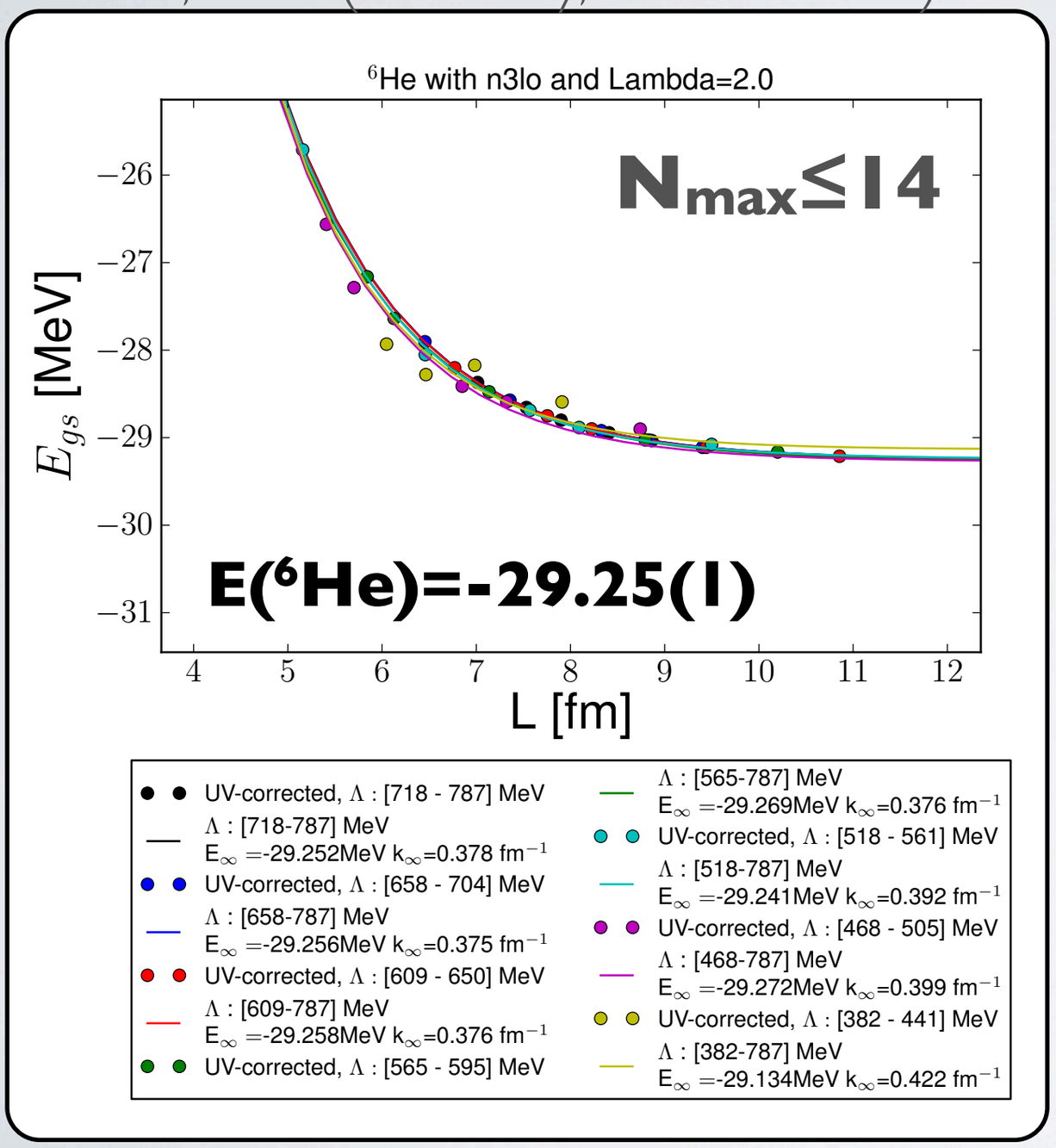


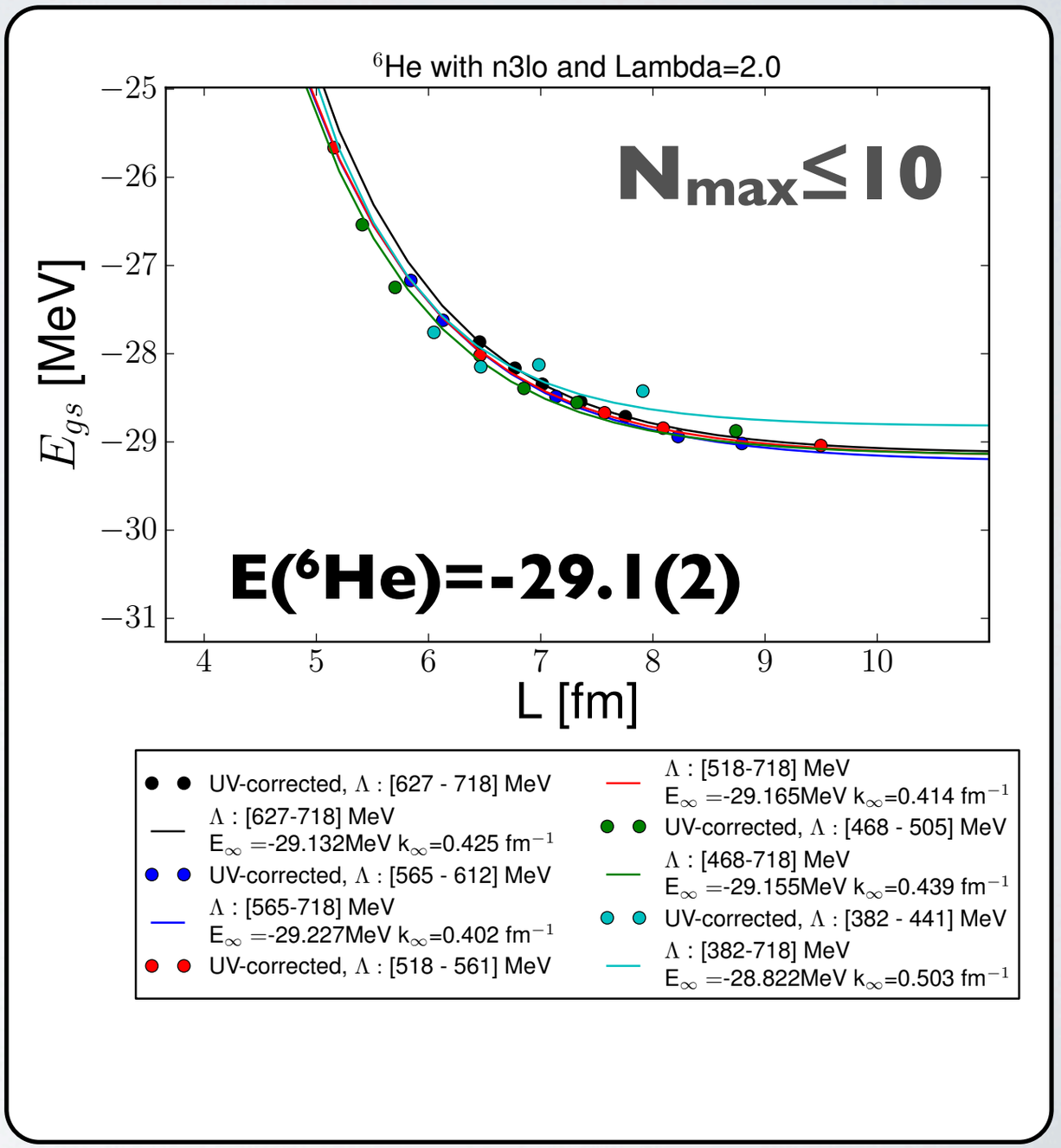
E(^{6}He)	-29.25(1) MeV
S_{2n}	1.01(1) MeV

C. Forssén, INT, Oct. 31, 2012

NCSM example: Energy convergence

N^{3}LO, SRG (NN only, Λ = 2.0 fm^{-1})





NCSM example: Energy convergence

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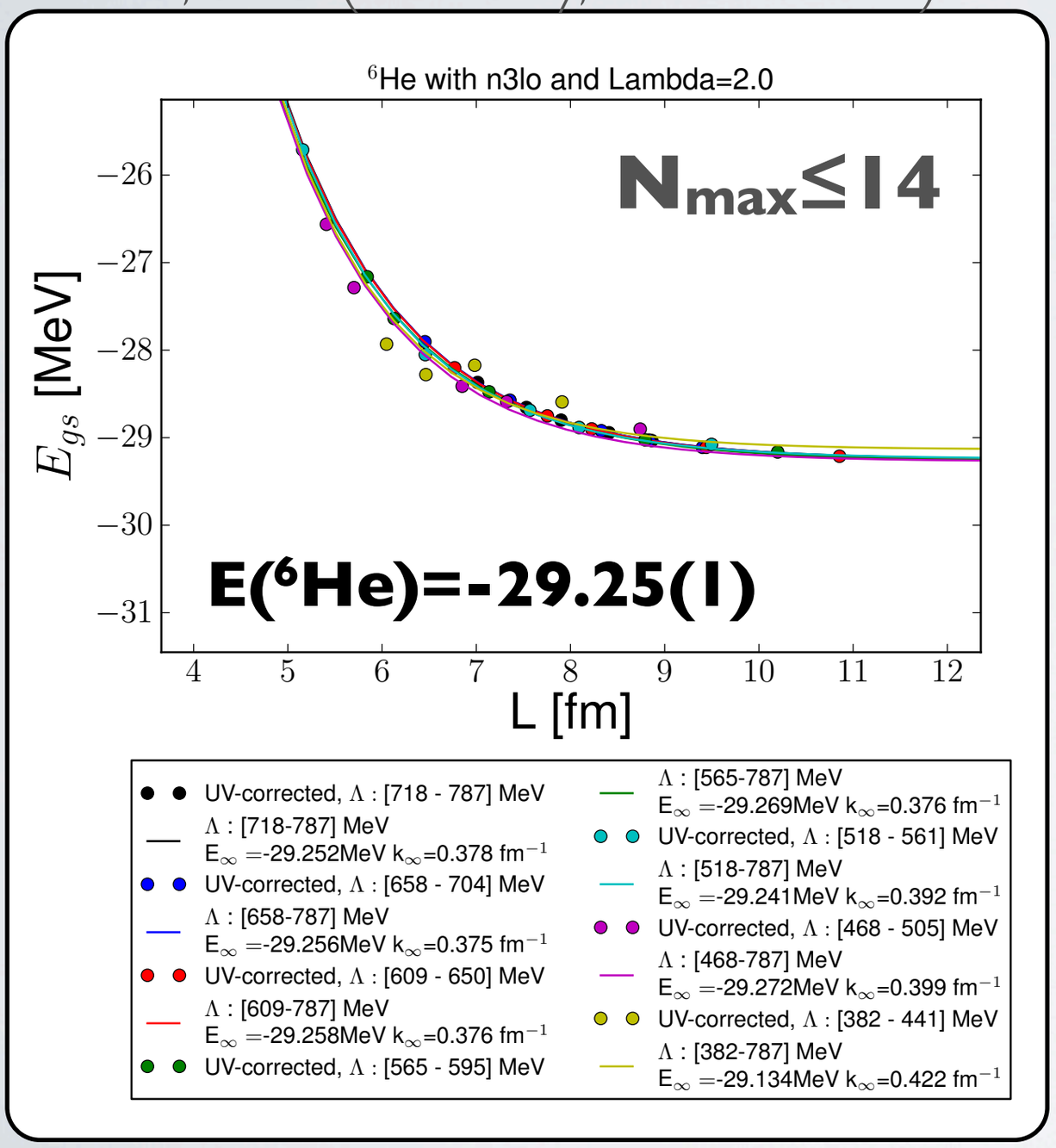
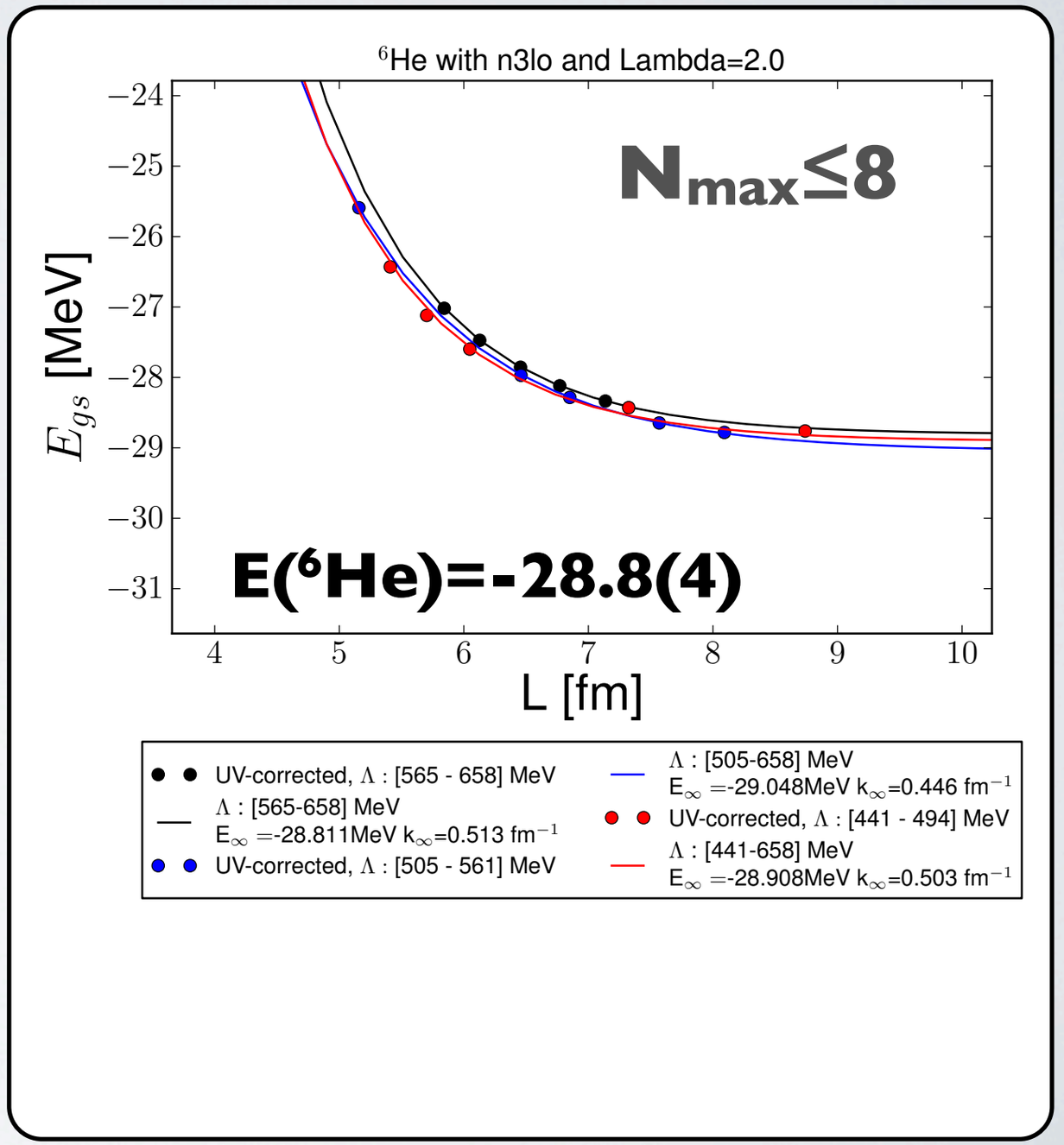




Correction for radius

Correction for radius (or other long-distance operators)

- Use $u_L(r) \approx u_\infty(r) + \Delta E_L \left.\frac{du_E(r)}{dE}\right|_{E_\infty}$ to evaluate

$$\Delta\langle r^2\rangle_L = \langle r^2\rangle_L - \langle r^2\rangle_\infty = \frac{\int_0^L |u_L(r)|^2\, r^2\, dr}{\int_0^L |u_L(r)|^2\, dr} - \frac{\int_0^\infty |u_\infty(r)|^2\, r^2\, dr}{\int_0^\infty |u_\infty(r)|^2\, dr}$$

- For leading L dependence, use $u_\infty(r) \longrightarrow A_\infty e^{-k_\infty r}$ and

$$\left.\frac{du_E(r)}{dE}\right|_{E_\infty} \approx -\frac{A_\infty}{\Delta E_L} e^{-2k_\infty L} e^{+k_\infty r} \quad \Longrightarrow \quad \Delta\langle r^2\rangle_L \propto \langle r^2\rangle_\infty (2k_\infty L)^3 e^{-2k_\infty L}$$

- The NLO correction scales as $(2k_\infty L)\exp(-2k_\infty L)$, so

$$\langle r^2\rangle_L \approx \langle r^2\rangle_\infty [1 - (c_0\beta^3 + c_1\beta)e^{-\beta}] \quad \text{with } \beta \equiv 2k_\infty L$$

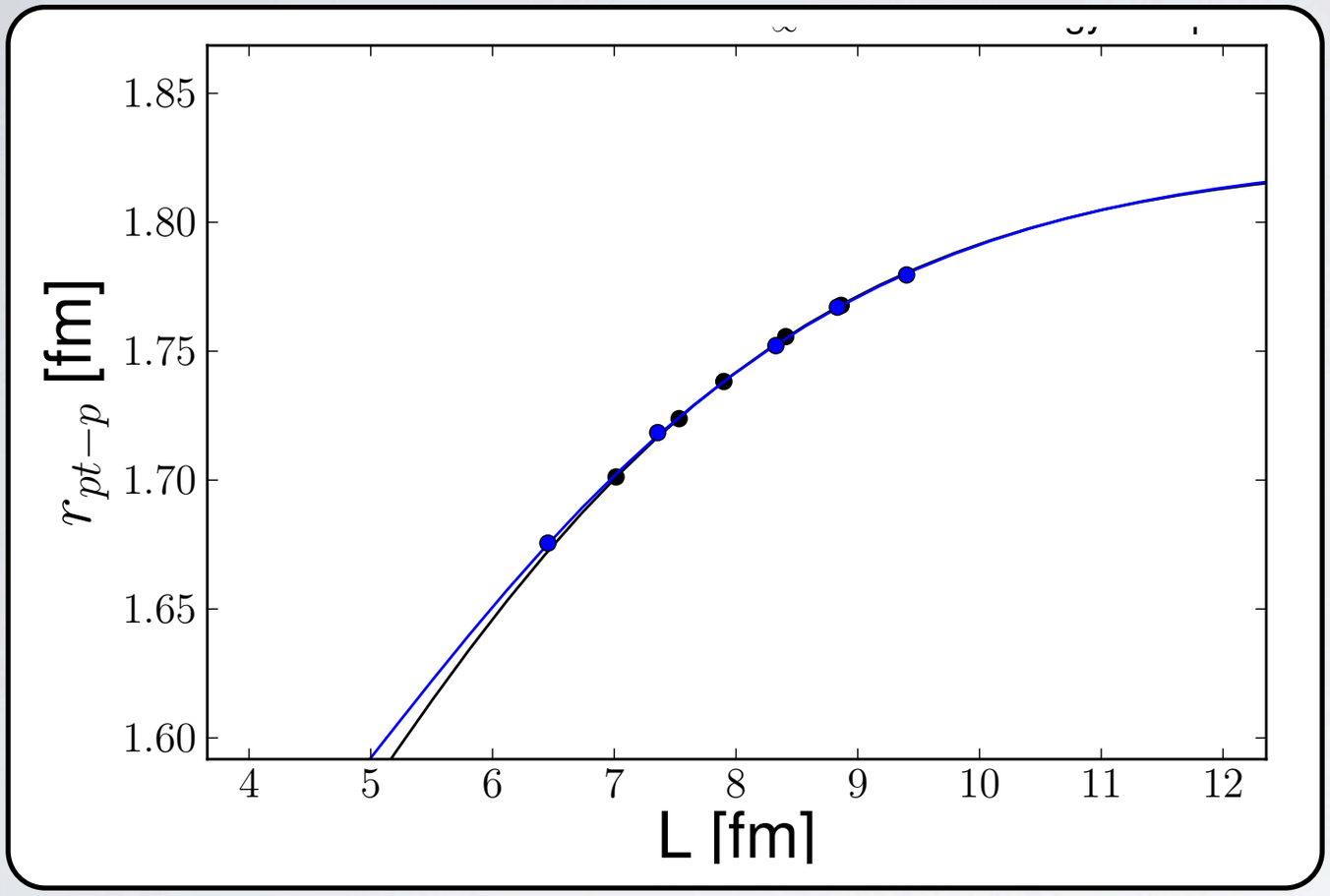
  - $\langle r^2\rangle_\infty$, c_0, and c_1 are fit parameters while k_∞ from energy fit
  - Valid in the asymptotic regime where $\beta = 2k_\infty L \gtrsim 3$

- Both *E* and *r* corrections apply to *A*-body system in lab coordinates

R.J. Furnstahl, talk at this program, 2012-10-10
R.J. Furnstahl et al., Phys. Rev. C 86(2012)031301R.

^{6}He: Point-Proton Radius

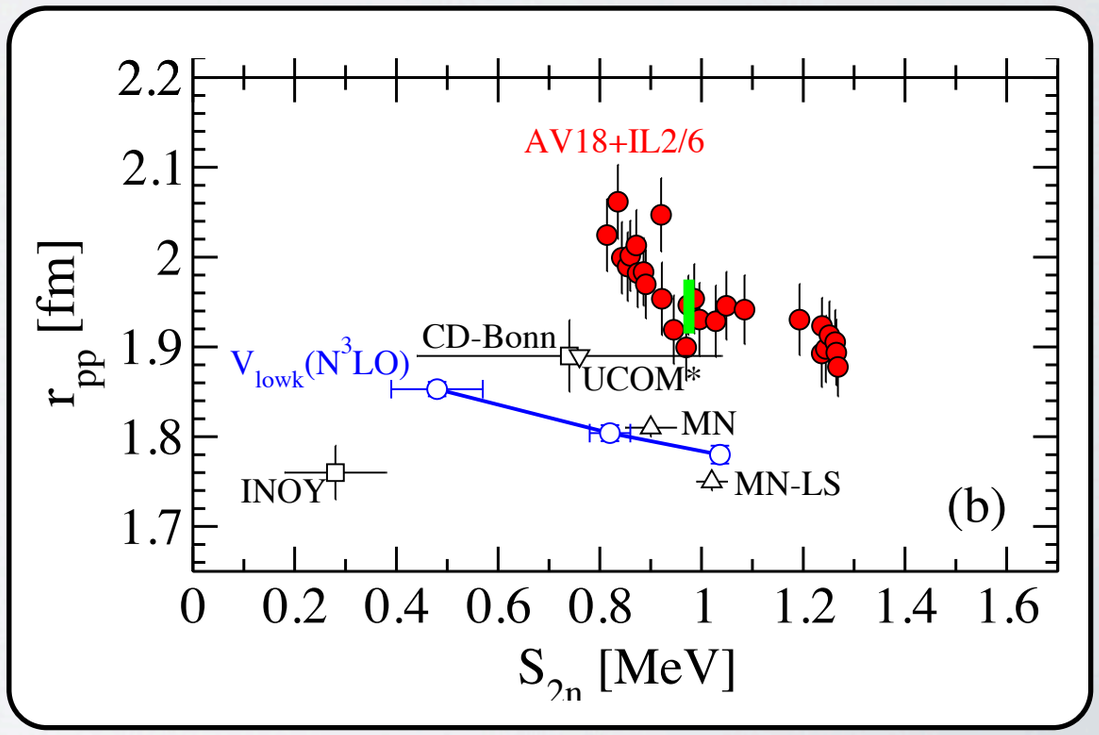
^{6}He point-proton radius



Ref.	$r_{pt\text{-}p}(^6\text{He})$
Exp	1.938(23)
NCSM: I-N3LO (Λ = 2 fm^{-1})	1.83(1)
HH (S. Bacca et al): I-N3LO (LS+V_{lowk}, 2 fm^{-1})	1.804(9)

$$\langle r^2 \rangle_L \approx \langle r^2 \rangle_\infty \left[1 - (c_0\beta^3 + c_1\beta)e^{-\beta}\right]$$
$$\text{with } \beta \equiv 2k_\infty L$$

From: R.J. Furnstahl et al., Phys. Rev. C 86(2012)031301R.



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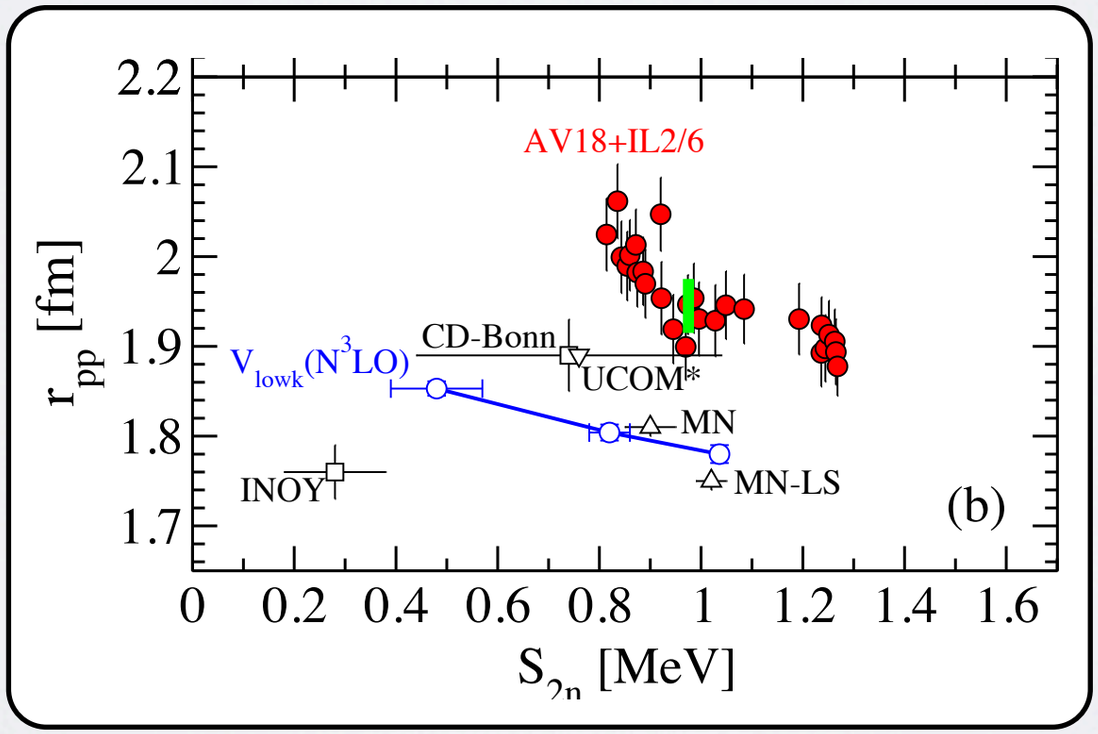
^{6}He: Point-Proton Radius

Ref.	$r_{pt\text{-}p}(^6\text{He})$ [fm]	$S_{2n}(^6\text{He})$ [fm]
Exp	1.938(23)	0.97
NCSM: I-N3LO (Λ = 1.8 fm^{-1})	1.81(1)	1.17(4)
NCSM: I-N3LO (Λ = 2.0 fm^{-1})	1.83(1)	1.01(1)
NCSM: I-N3LO (Λ = 2.2 fm^{-1})	1.85(3)	0.82(2)



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Unsettled questions for the IR extrapolation

Selected from list by R.J. Furnstahl, talk at this program, 2012-10-10

- What is the optimal definition of L?
- How to make credible error estimates?
- Does the interaction matter?
  - The IR corrections are independent of the potential
  - Softer interactions mean more complete UV convergence, so larger region with IR corrections only.
- How well does extrapolation work for other operators?

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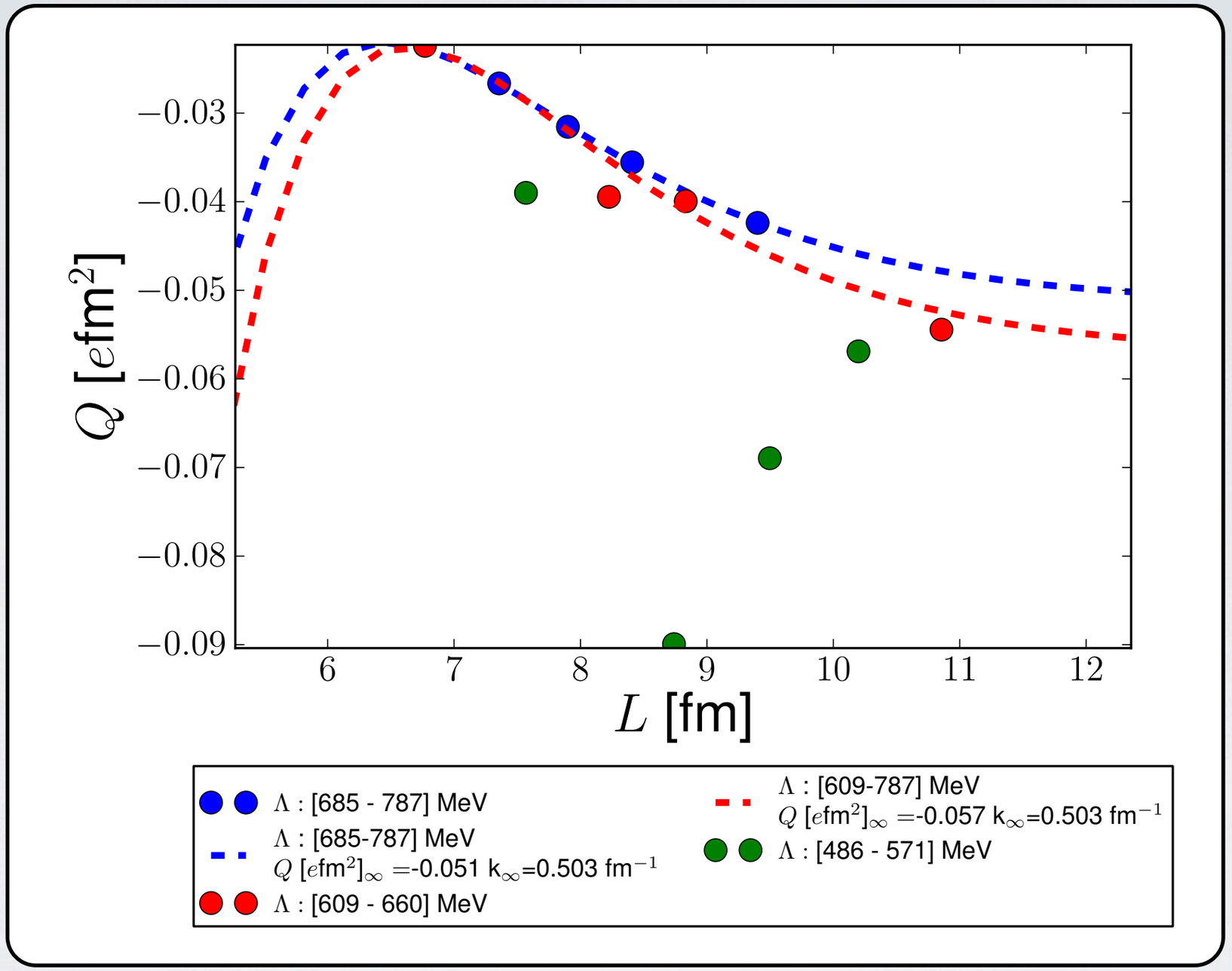
$Q(^6\mathrm{Li}, 1^+)$



$$\langle O \rangle_L \approx \langle O \rangle_\infty \left[1 - (c_0 \beta^3 + c_1 \beta) e^{-\beta} \right]$$

$$\text{with } \beta \equiv 2 k_\infty L$$

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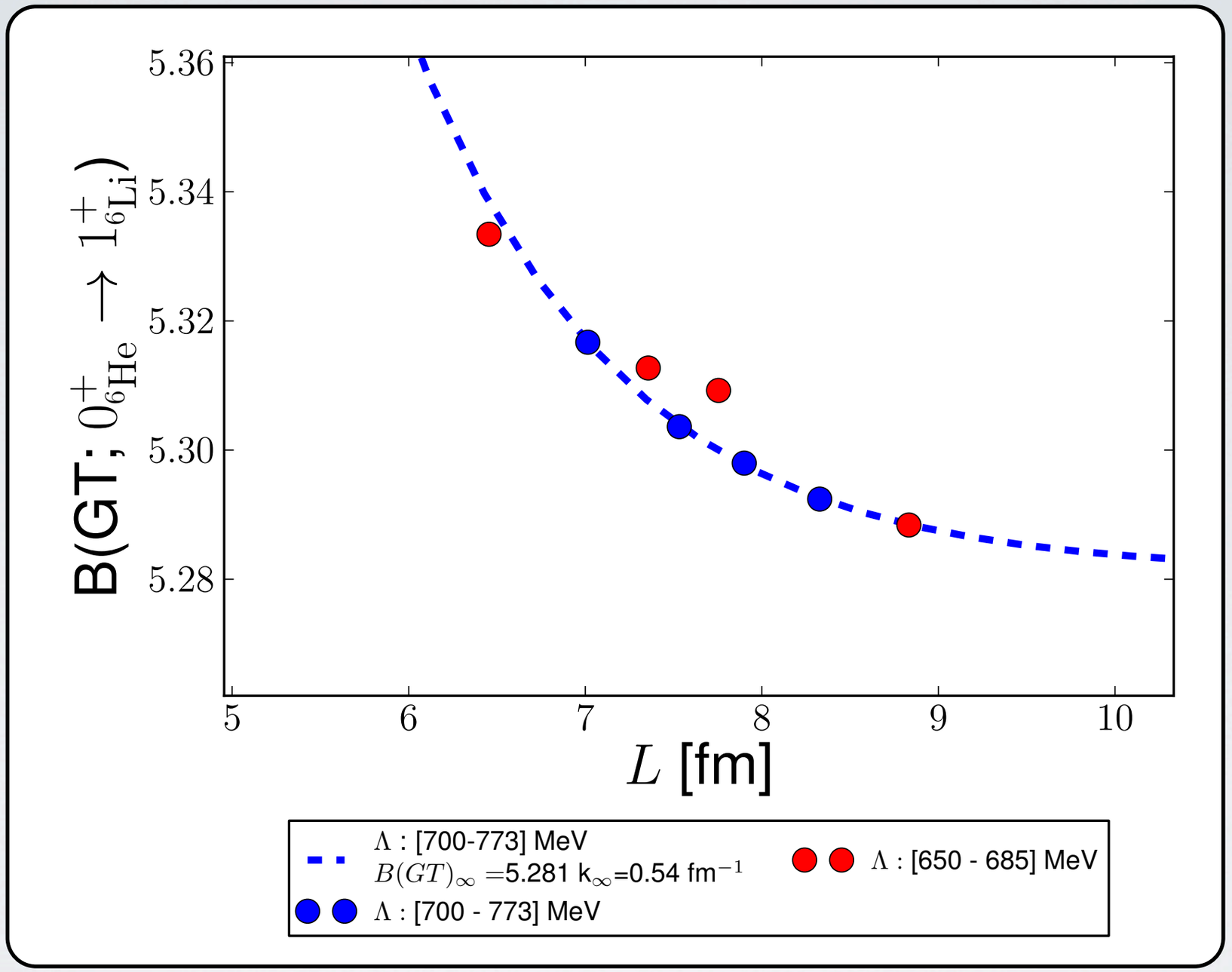
B(GT): ^{6}He → ^{6}Li



$$\langle O \rangle_L \approx \langle O \rangle_\infty \left[1 - (c_0\beta^3 + c_1\beta)e^{-\beta}\right]$$

$$\text{with } \beta \equiv 2k_\infty L$$

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Conclusions - Cutoff scales

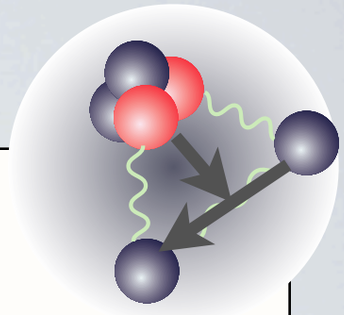
- Introduction of UV and IR scales - Combination of results from different N_{max}, $\hbar\Omega$
- Optimization of run sequence - BUT still need several large N_{max} computations
- Prefer UV converged results and perform IR extrapolation
- Transformation of operators

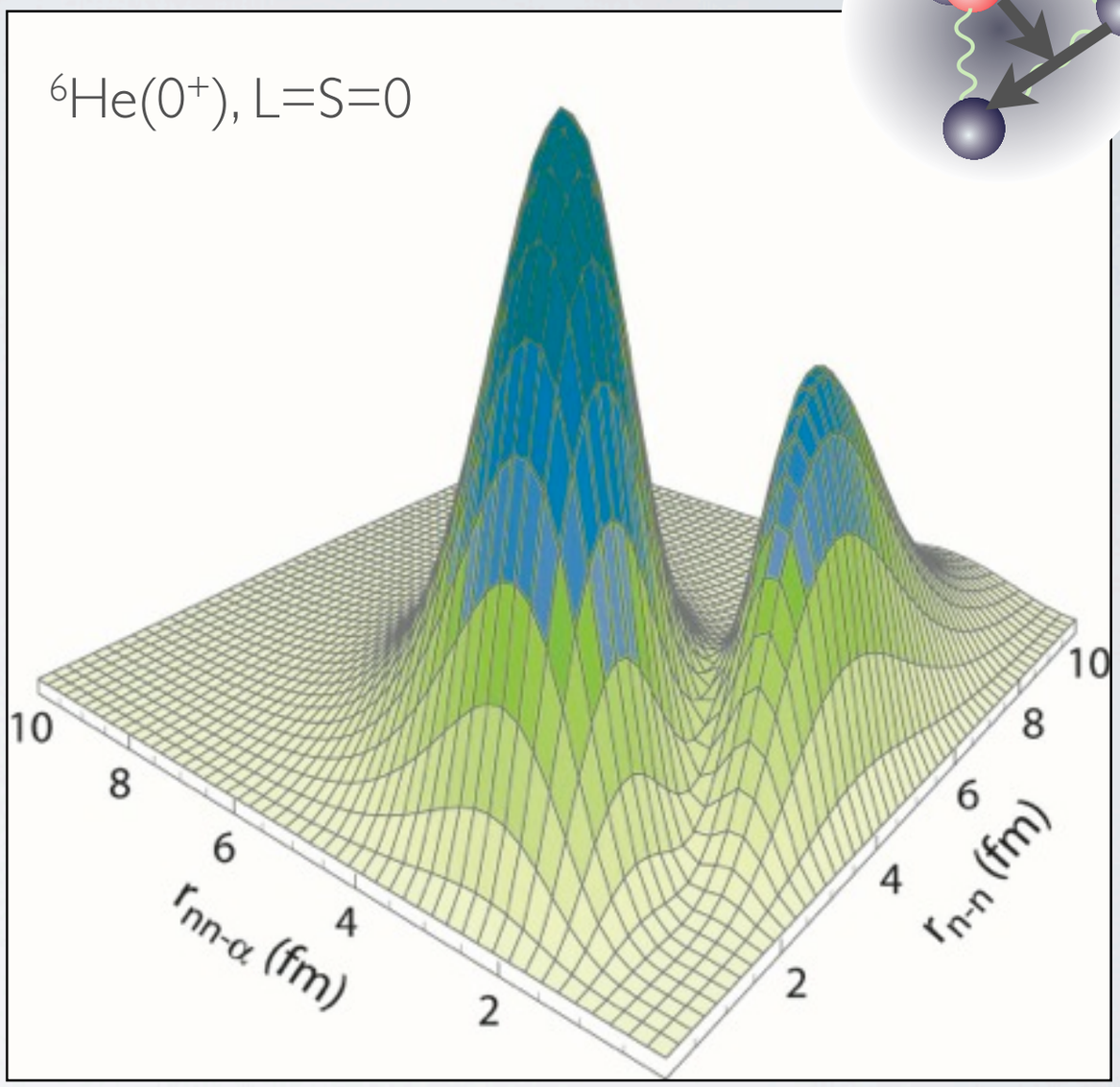
A Microscopic Description Of Clustering

^{6}He as a three-body system

- Borromean nucleus
- HH and CSF three-body models with inert cluster.
  V_{nn} and $V_{n\alpha}$
  - Core polarization needed
    $r_{n\alpha}=1.03r_{n\alpha(free)}$
    (cf. three-body force)
  - Repulsive s-wave potential
    ("Pauli core")
  - HH expansion
    K=2 (90%) with L=S=0 (80%)
    and L=S=1(10%)
- Pauli focusing.

^{6}He = ^{4}He+n+n

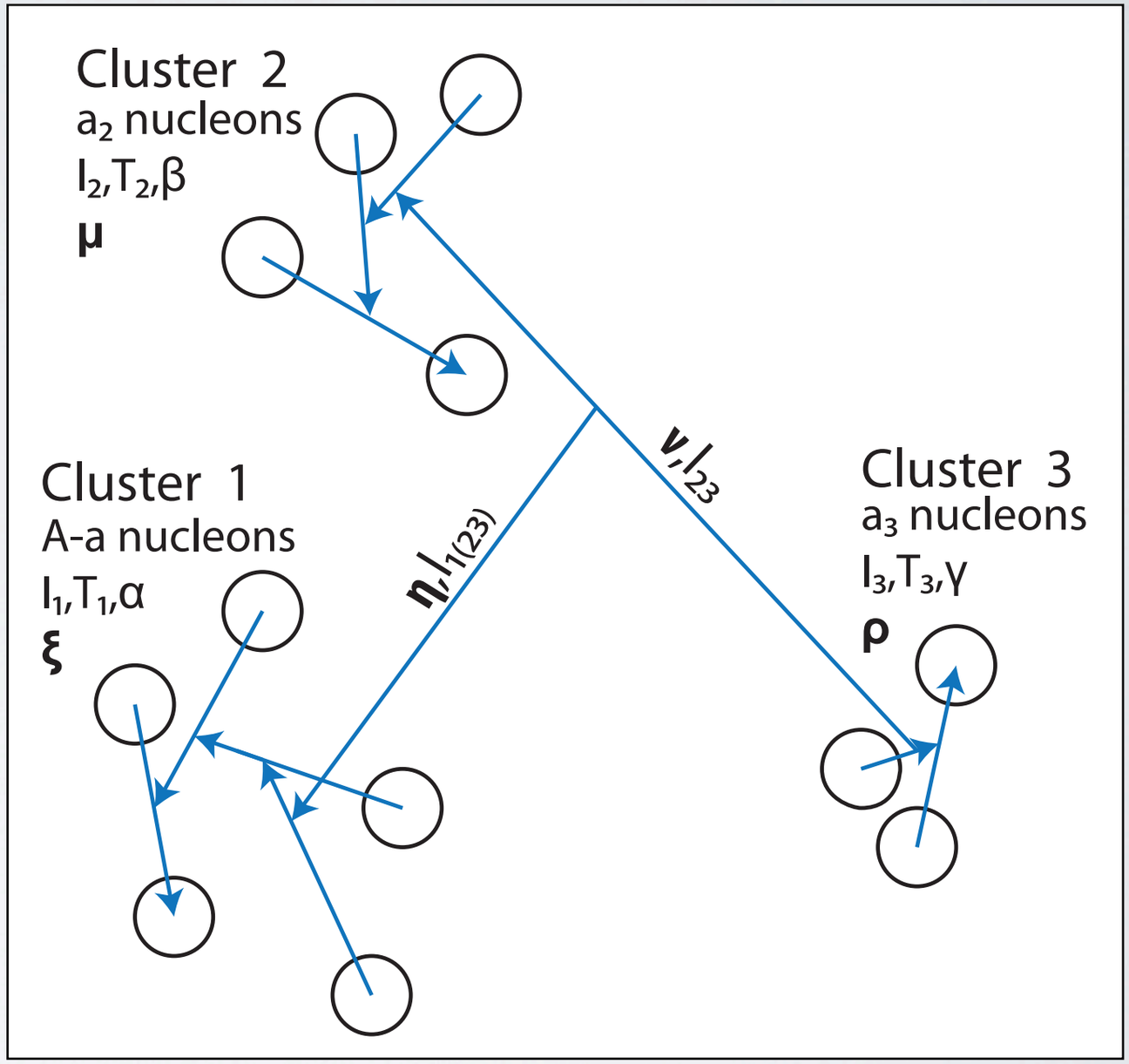




M.V. Zhukov et al.— Phys. Rep. **231**, 151 (1993),

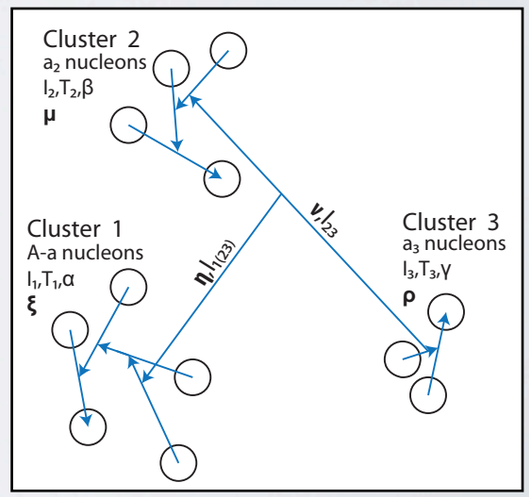
Cluster structures in light nuclei

- Investigate clustering in NCSM wave functions
- Preserve translational invariance
- Harmonic oscillator SD many-body basis
- Transformation between single-particle and Jacobi coordinates



Three-body cluster overlap functions

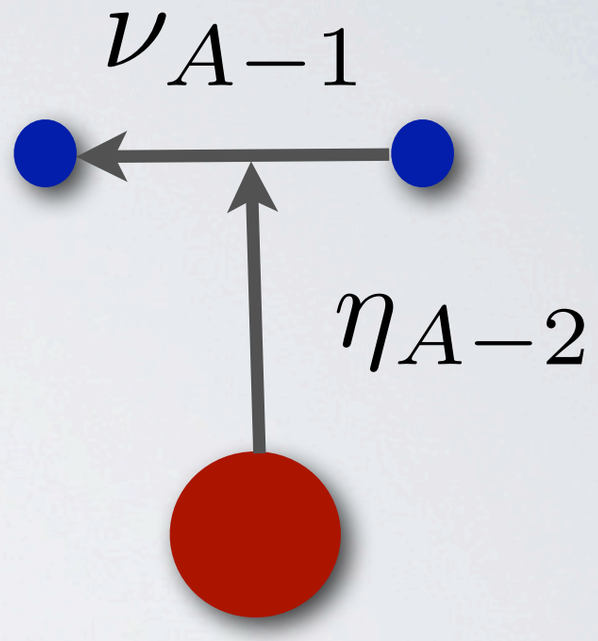
$$
\begin{aligned}
& u^{A\lambda JT}_{A-a\alpha I_1 T_1, a_2\beta I_2 T_2, a_3\gamma I_3 T_3; LS}(\eta_{A-a}, \nu_{A-a+1}) \\
&= \left\langle A\lambda JT \middle| \mathcal{A}_{A-a,a_2,a_3} \Phi^{(A-a,a_2,a_3)JT}_{\alpha I_1 T_1, \beta I_2 T_2, \gamma I_3 T_3; LS} : \delta_{\eta_{A-a}}, \delta_{\nu_{A-a+1}} \right\rangle \\
&= \sum_{n_{1(23)}, n_{23}} R_{n_{1(23)} l_{1(23}}(\eta_{A-a}) R_{n_{23} l_{23}}(\nu_{A-a+1}) \\
&\times \sqrt{\frac{A!}{(A-a)! a_2! a_3!}} \left\langle A\lambda JT \middle| \Phi^{(A-a,a_2,a_3)JT}_{\alpha I_1 T_1, \beta I_2 T_2, \gamma I_3 T_3; LS} : n_{1(23)} l_{1(23)}, n_{23} l_{23} \right\rangle
\end{aligned}
$$



Overlap function for core+N+N

Start with the **core+N+N** case:

- Do a couple of coordinate transformations (between relative and s.p.)
- Do a number of spin re-couplings
- Integrate over coordinates



Overlap function for core+N+N

$$u^{A\lambda JT}_{A-2\alpha I_1 T_1,\frac{1}{2}\frac{1}{2},\frac{1}{2}\frac{1}{2};LS}(\eta_{A-2},\nu_{A-1})$$

$$= \sum \frac{R_{n_{1(23)}l_{1(23)}}(\eta_{A-2})R_{n_{23}l_{23}}(\nu_{A-1})}{\left\langle n_{1(23)}l_{1(23)}00l_{1(23)}\right|\left. 00n_{1(23)}l_{1(23)}l_{1(23)}\right\rangle_{\frac{2}{A-2}}}$$

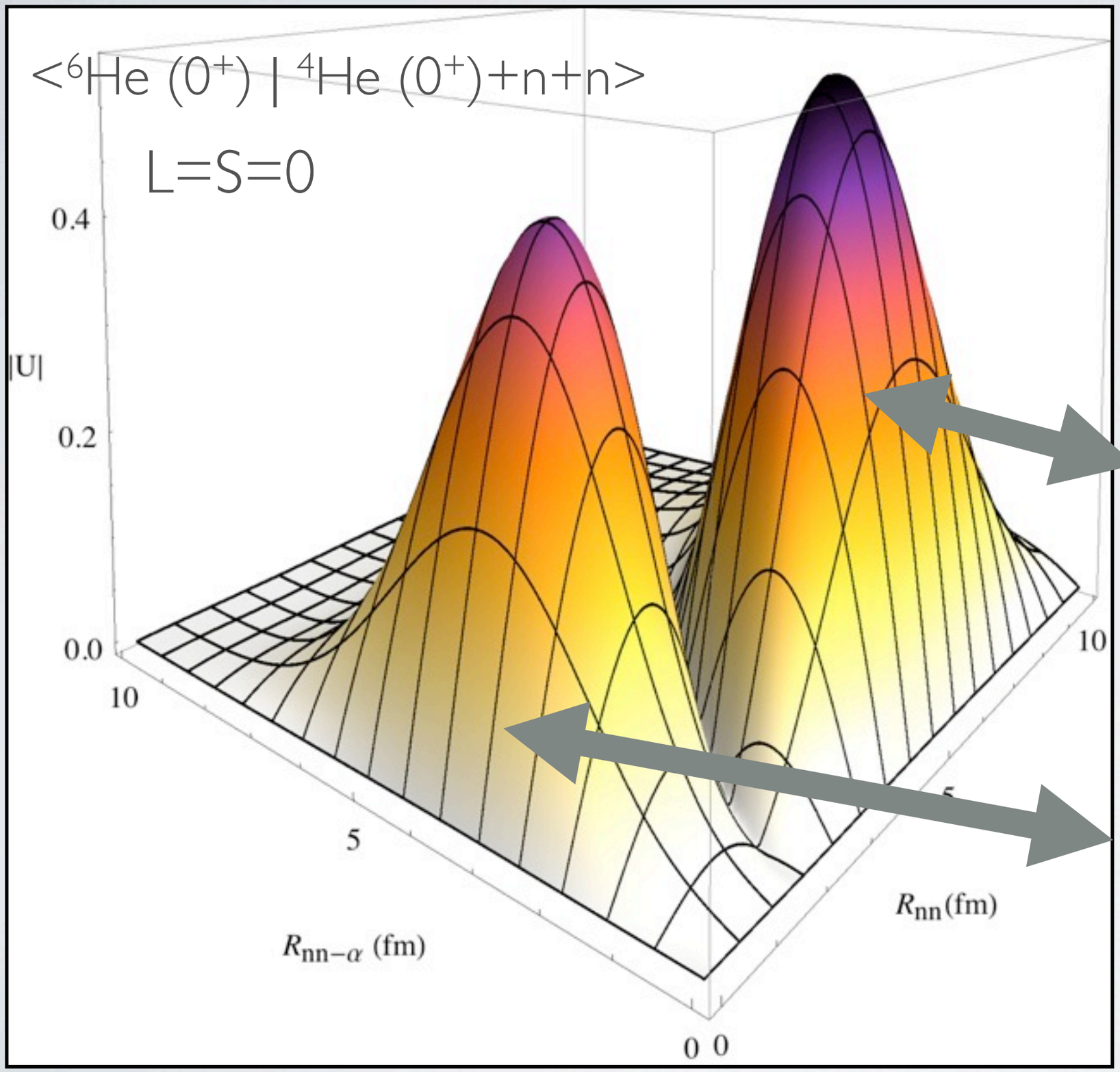
$$\times \left\langle n_a l_a n_b l_b L\right|\left. n_{1(23)}l_{1(23)}n_{23}l_{23}L\right\rangle_1 (-1)^{3I_1+I_{23}+I_{ab}-T_{23}-S+L}$$

$$\times \frac{\hat{L}\hat{S}\hat{I}^2_{ab}\hat{j}_a\hat{j}_b}{\hat{J}\hat{T}} \begin{Bmatrix} L & I_{23} & I_{ab} \\ I_1 & J & S \end{Bmatrix} \begin{Bmatrix} l_a & l_b & L \\ \frac{1}{2} & \frac{1}{2} & I_{23} \\ j_a & j_b & I_{ab} \end{Bmatrix}$$

$$\times {}_{SD}\!<A\lambda JT|||\left[a^\dagger_{n_a l_a j_a t_a} a^\dagger_{n_b l_b j_b t_b}\right]^{I_{ab}T_2}|||(A-2)\alpha I_1 T_1>_{SD}.$$

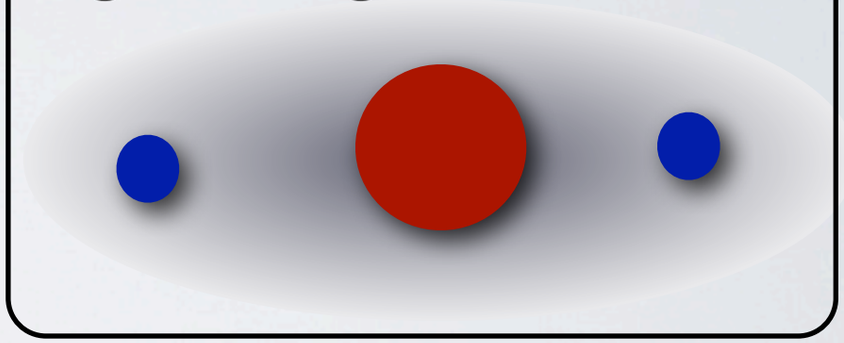
D. Sääf and CF, - in preparation

Ab initio < ^{6}He | ^{4}He+n+n > overlap

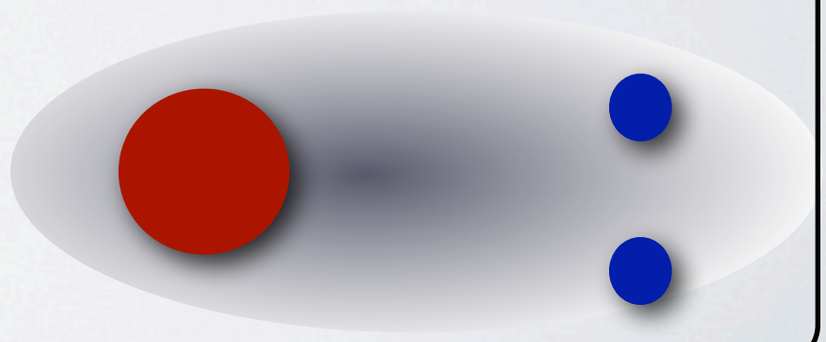


N^{3}LO, SRG
(NN only, Λ = 2.0 fm^{-1})

Cigar configuration



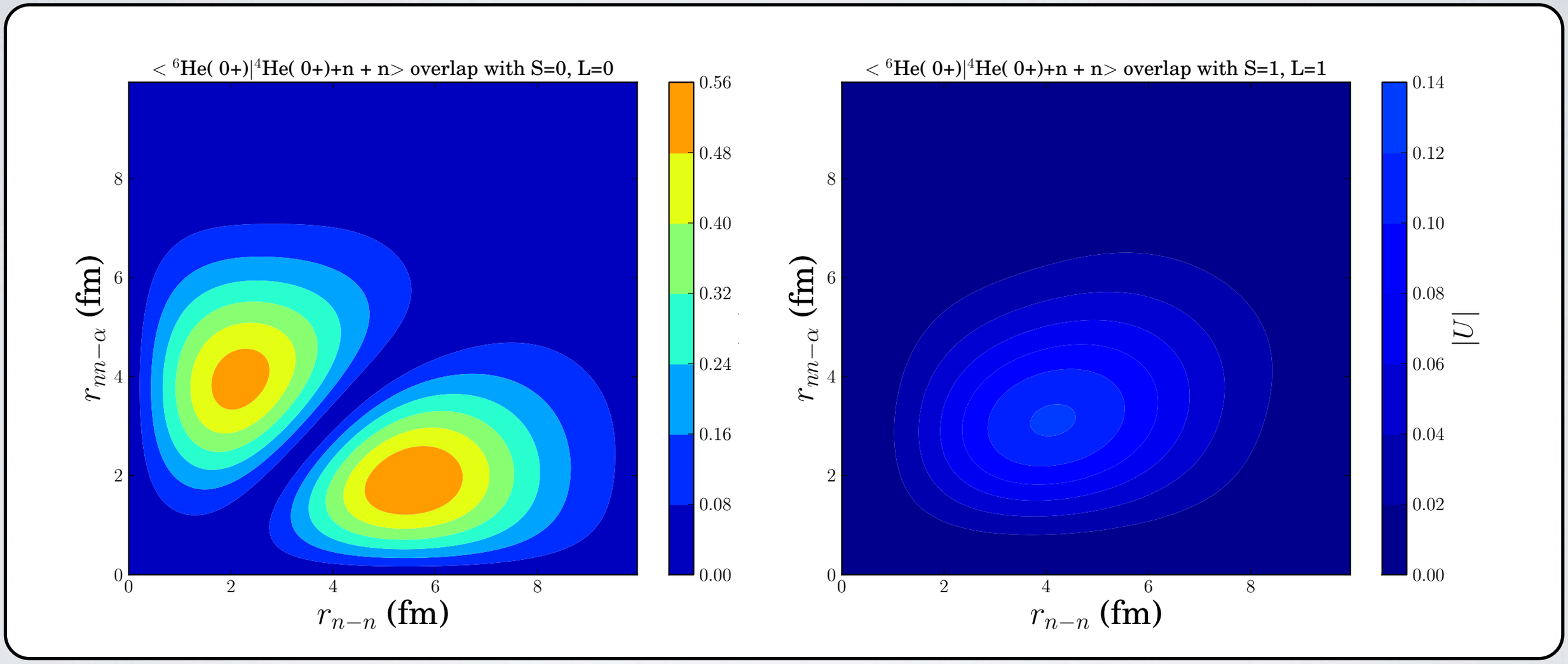
Di-neutron configuration



Ab initio < ^{6}He | ^{4}He+n+n > overlap

^{3}LO, SRG
NN only, Λ = 2.0 fm^{-1},
N_{max}=14, HO=20 MeV

<^{6}He (0^+) | ^{4}He (0^+)+n+n>



L=S=0

L=S=1

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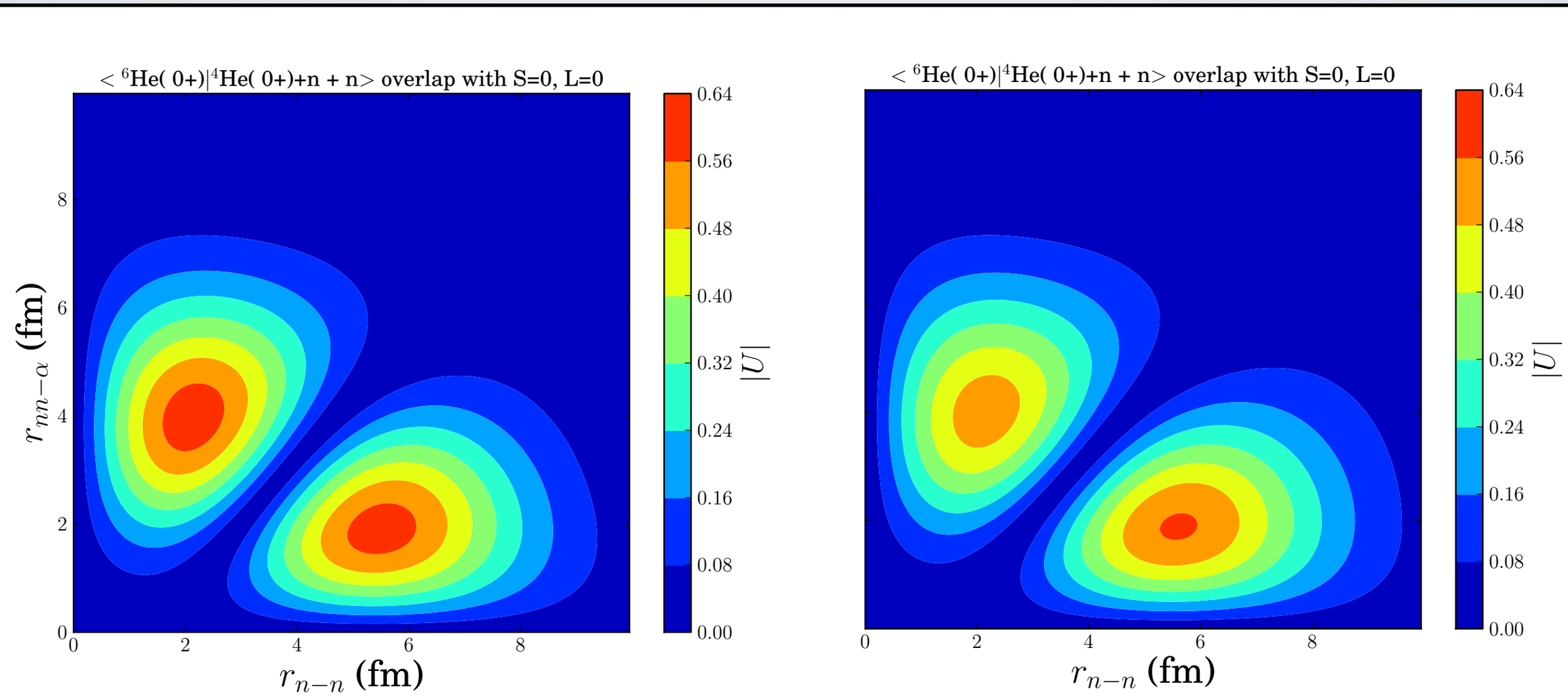
Ab initio < ^{6}He | ^{4}He+n+n > overlap

N^3LO, NN only
SRG, Λ = 2.0 fm^{-1},
N_{max}=8, HO=16 MeV

N^3LO, NN+3NF
SRG, Λ = 2.0 fm^{-1},
N_{max}=8, HO=16 MeV



with 3NF

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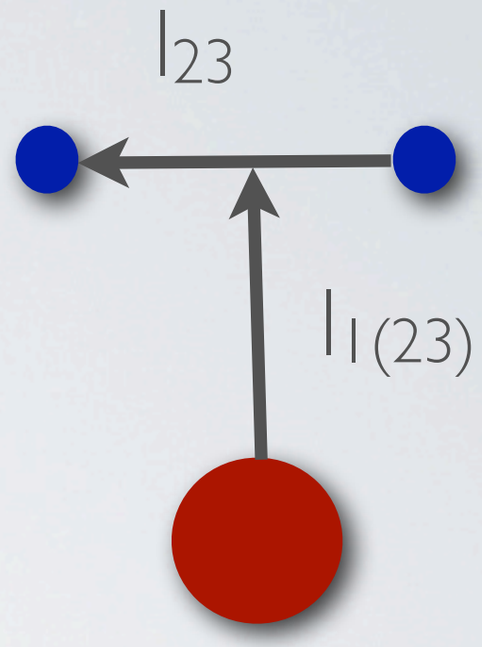
Pauli focusing

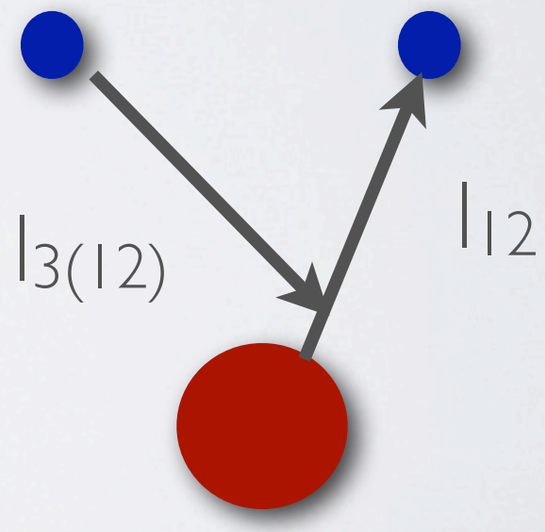
$<^6\mathrm{He}\ (0^+) \mid {}^4\mathrm{He}\ (0^+)+n+n>$

- Dominance of the $l_{1(23)} = l_{23} = 0$ component.



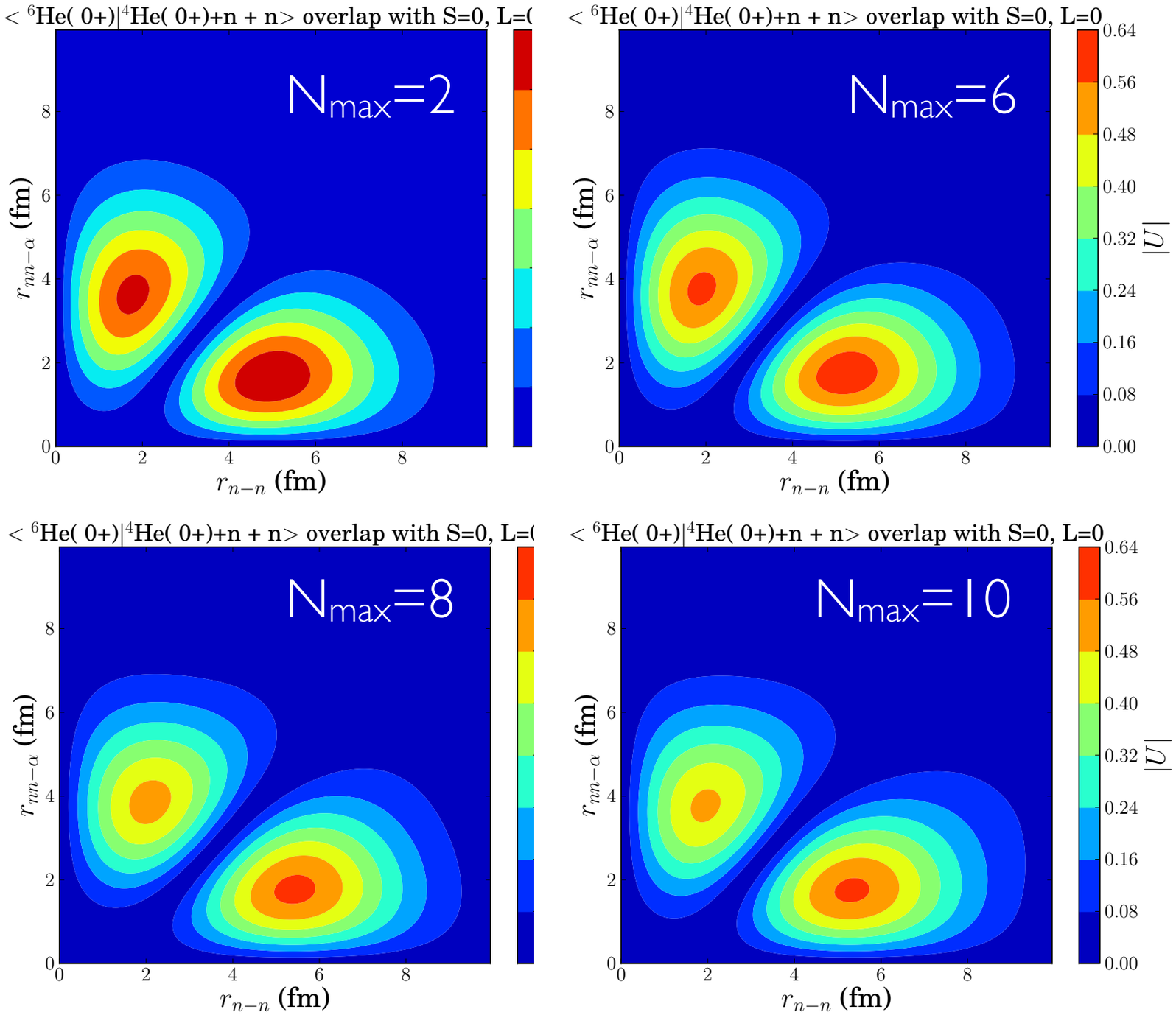
- RR coefficients determine HH under coordinate-system transformation.

- E.g. with $l_{1(23)} = l_{23} = 0$ we get:
  - $l_{3(12)} = l_{12} = 0$ for K=0 (almost Pauli forbidden)
  - Dominating $l_{3(12)} = l_{12} = 1$ for K=2
  - Hyperangular function for this component gives the two-peak correlation density.



$< {}^6\mathrm{He} \mid {}^4\mathrm{He}+n+n >$ overlap: N_{max} dependence

$<{}^6\mathrm{He}\ (0^+) \mid {}^4\mathrm{He}\ (0^+)+n+n>$

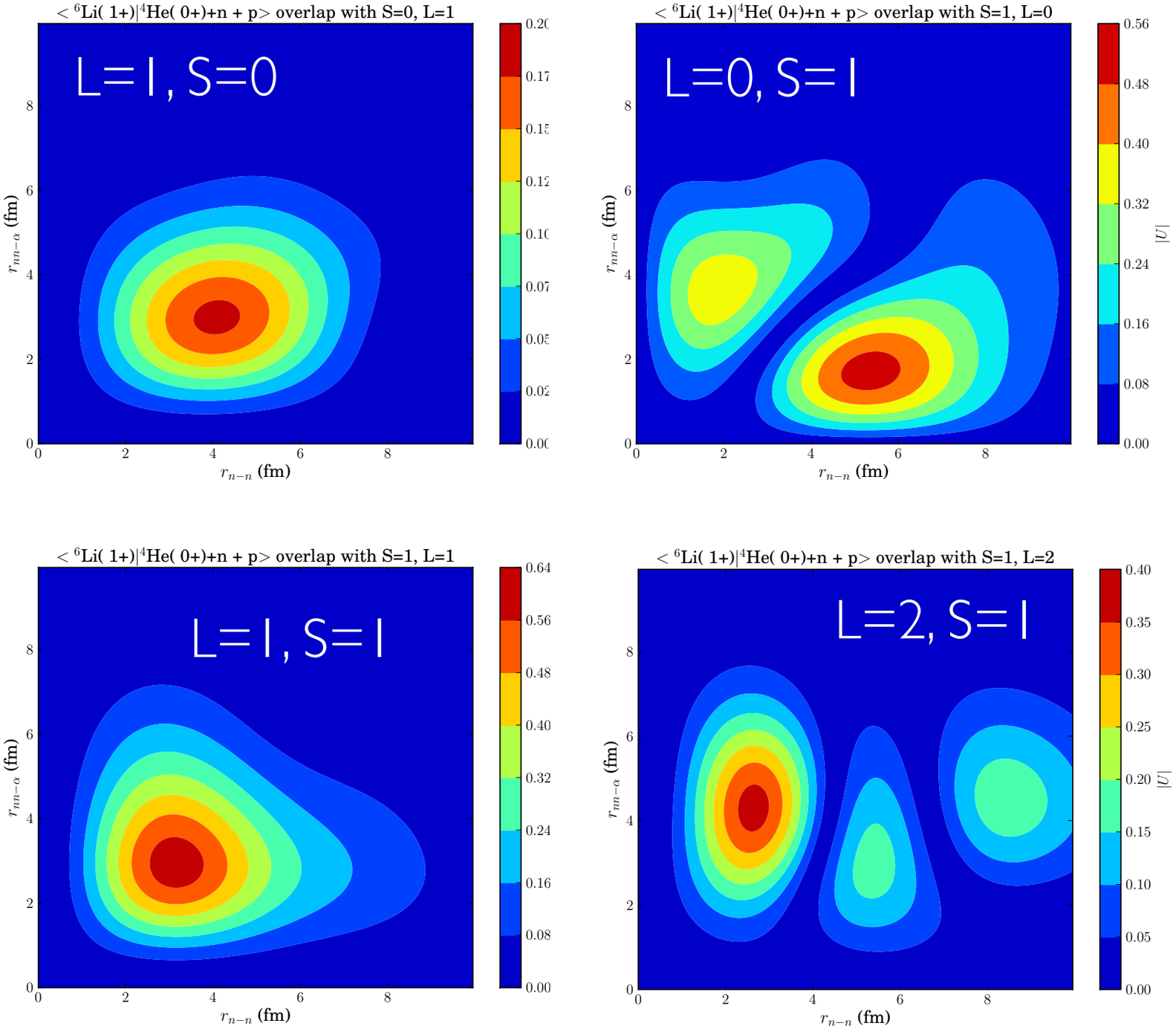


N^{3}LO, SRG
NN only, Λ = 2.0 fm^{-1},
N_{max}=14, HO=20 MeV

Conclusion:
This is a Pauli focusing effect

Ab initio < ^{6}Li | ^{4}He+n+p > overlap

<^{6}Li (1^{+}) | ^{4}He (0^{+})+n+n>



N^{3}LO, SRG
NN only, Λ = 2.0 fm^{-1},
N_{max}=14, HO=20 MeV

Note: different scales

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Conclusion and Outlook

- Introduction of UV and IR scales - Combination of results from different N_{max}, $\hbar\Omega$
- Optimization of run sequence -
  BUT still need several large N_{max} computations
- Prefer UV converged results and perform IR extrapolation
- Study of tw-body operators, and the transformation of operators
- Microscopic description of clustering
  - Calculate core swelling: r_{pp}
  - Study projection on HH basis
- Continuum effects in ab initio calculations

Acknowledgments

Many thanks to my collaborators

- Lucas Platter (ANL), Jimmy Rotureau, Mikhail Zhukov, Emil Ryberg, **Daniel Sääf**, Boris Carlsson (Chalmers) http://fy.chalmers.se/subatom/nt
- Petr Navrátil (TRIUMF), Robert Roth (Darmstadt), James Vary and Pieter Maris (Iowa), and the NCSM collaboration.
- Witek Nazarewicz and Nicolas Michel (UT/ORNL), Marek Ploszajczak (GANIL), George Papadimitriou and Bruce Barrett (UA)

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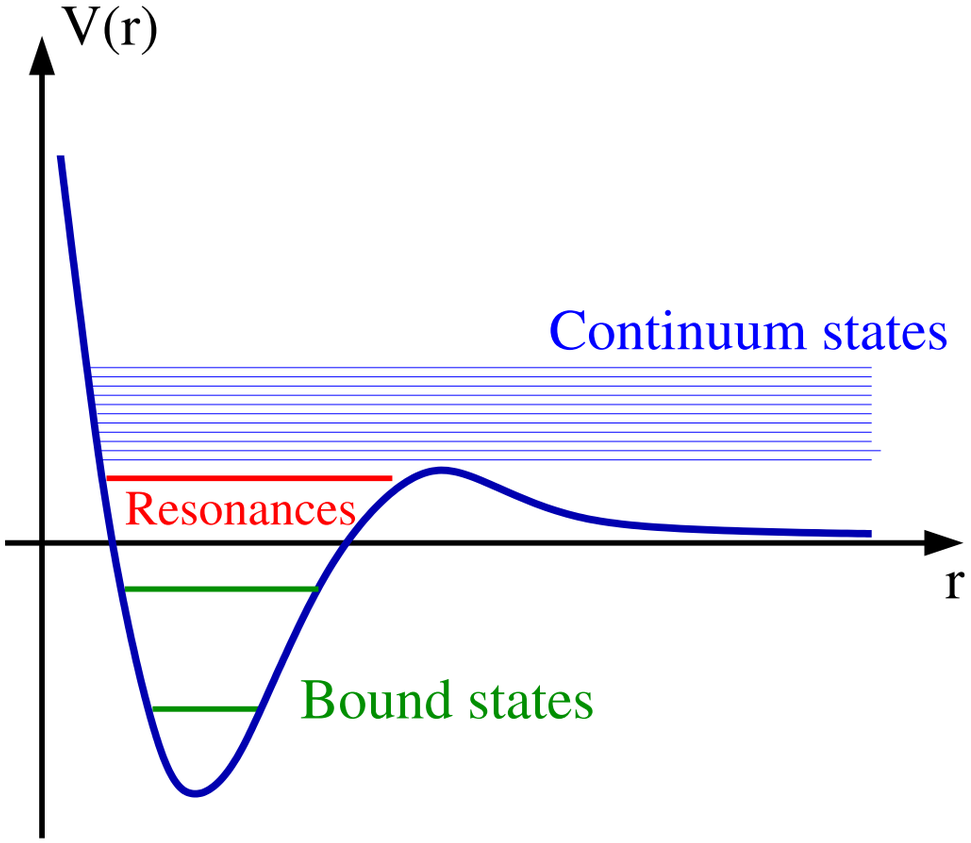
- The Swedish Research Council
- European Research Council

Stepping Into The Continuum

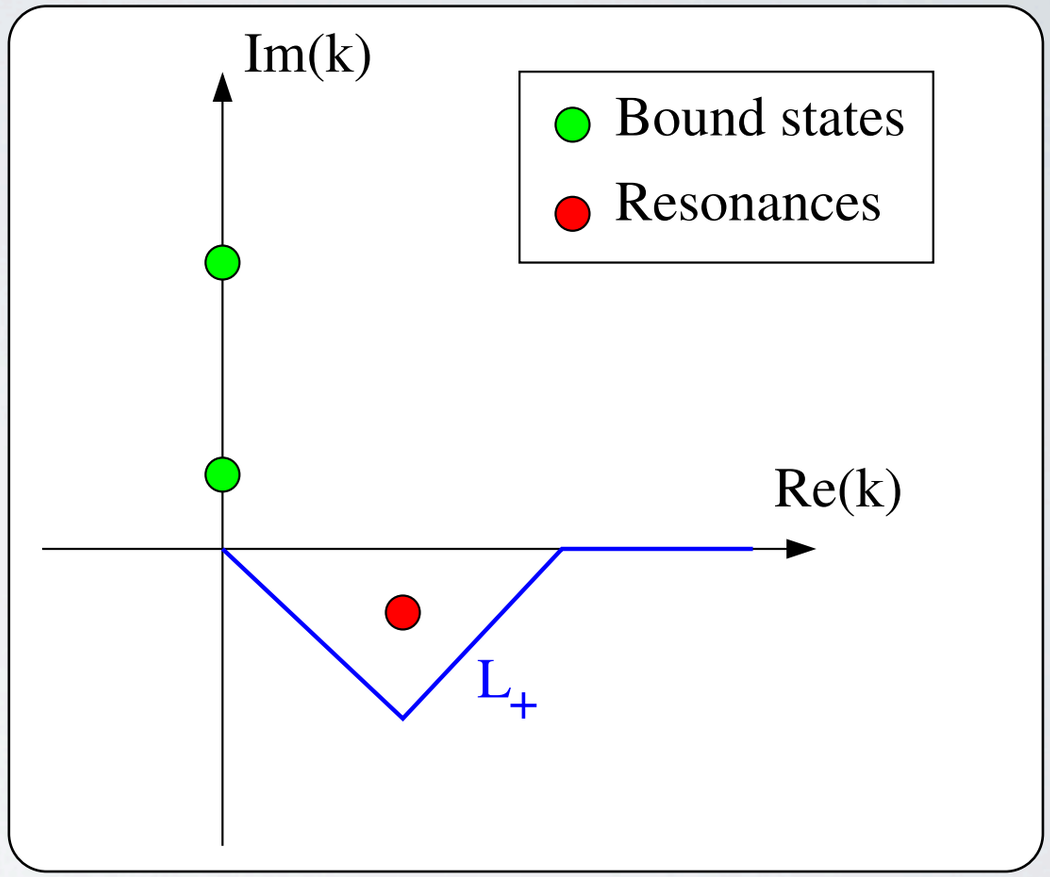
Gamow states and the Berggren basis

Gamow Shell Model



Gamow Shell Model
- N. Michel et al Phys. Rev. Lett. **89**(2002) 042502;
- J. Rotureau et al. (2006)
- G. Hagen et al. (2010)

Berggren basis



$$\sum_{n=b,r} |u_n\rangle\langle\bar{u}_n| + \frac{1}{\pi}\int_{L_+} |u(k)\rangle\langle u^*(k)|dk = 1$$

Gamow states and completeness
- T. Berggren, Nucl. Phys A **109**(1968)265; NPA**389**(1982)261
- T. Lind, Phys. Rev. C **47**(1993)1903

Gamow shell Model

(i) discretization of continuum contour

$$\sum_{n=b,r} |u_n\rangle\langle u_n| + \sum_i |u_{k,i}\rangle\langle u_{k,i}| \approx 1$$

(ii) construction of many-body basis

$$|\mathrm{SD}_i\rangle = |u_{i1}, \ldots, u_{iA}\rangle$$

(iii) construction of Hamiltonian matrix (complex symmetric matrix)

$$\langle \mathrm{SD}_i | H | \mathrm{SD}_j \rangle$$

(iv) many-body spectrum contains: bound, resonant and "spurious" continuum states

Gamow Shell Model

- N. Michel et al, PRL 89 (2002) 042502; PRC67 (2003) 054311; PRC70 (2004) 064313; JPG (2009) 013101
- G. Hagen et al, PRC71 (2005) 044314
- J. Rotureau et al, PRL 97 (2006) 110603
- G.Papadimitriou et al, PRC(R) 84 (2011) 051304

- Pole approximation is 0th order approximation:

$$H^{\mathrm{p.a.}} |\Psi^{\mathrm{p.a.}}\rangle = E^{\mathrm{p.a.}} |\Psi^{\mathrm{p.a.}}\rangle$$

- Many-body resonance (bound) state have large overlap:

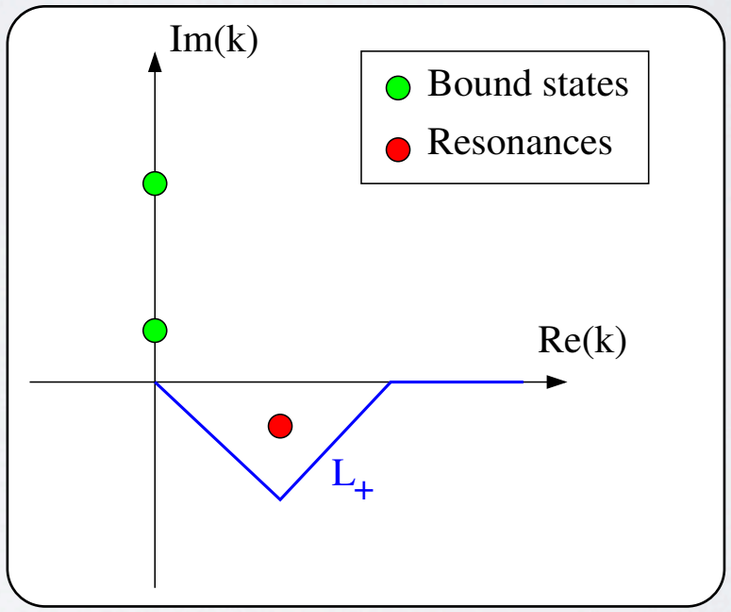
$$|\langle \Psi^{\mathrm{p.a.}} | \Psi \rangle|$$

ab initio Calculations in the Berggren Basis

- NN potential
  - Realistic 2b interactions V_{ij} : Argonne V18 or chiral I-N^{3}LO
  - softened by $V_{low\text{-}k}$

- R. Wiringa et al., Phys. Rev. C, **51** (1995)38
- D. Entem et al., Phys. Rev. C, **68**(2003)041001(R)
- S. Bogner et al., Phys. Rep., **386**(2003)1
- J. Rotureau et al., Phys. Rev. Lett., **97**(2006)110603

- Single-particle states
  - s- and p-shells from HF potential
  - $l > 1$ shells from HO potential



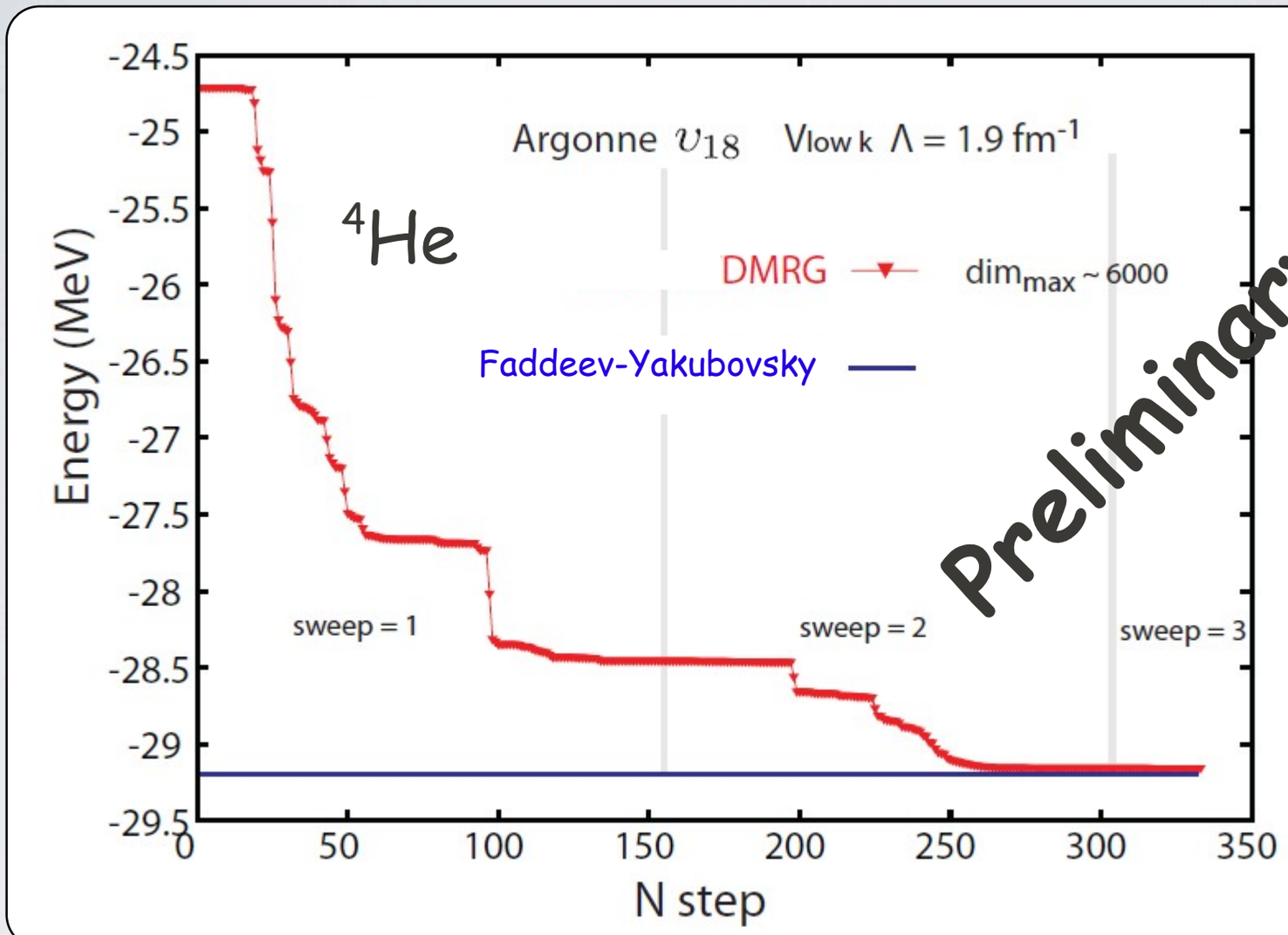
- Diagonalization
  - many-body Schrödinger equation

$$H_A = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{NN}\left(\vec{r}_i - \vec{r}_j\right) + \ldots$$

  - using J-coupled DMRG (basis truncation)

Ab initio Benchmark calculations in a Gamow Basis

^{4}He



Preliminary

Shells

$0s_{1/2}$(p) :E=-23.304 MeV

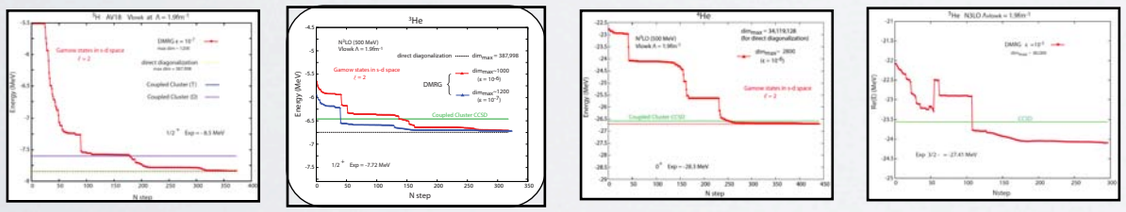
$0s_{1/2}$(n) :E=-24.334 MeV

$s_{1/2}$,$p_{3/2}$,$p_{1/2}$,$s_{1/2}$ real energy continua

d,f,g H.O states

156 s.p. in total

^{3}H ^{3}He ^{4}He ^{5}He

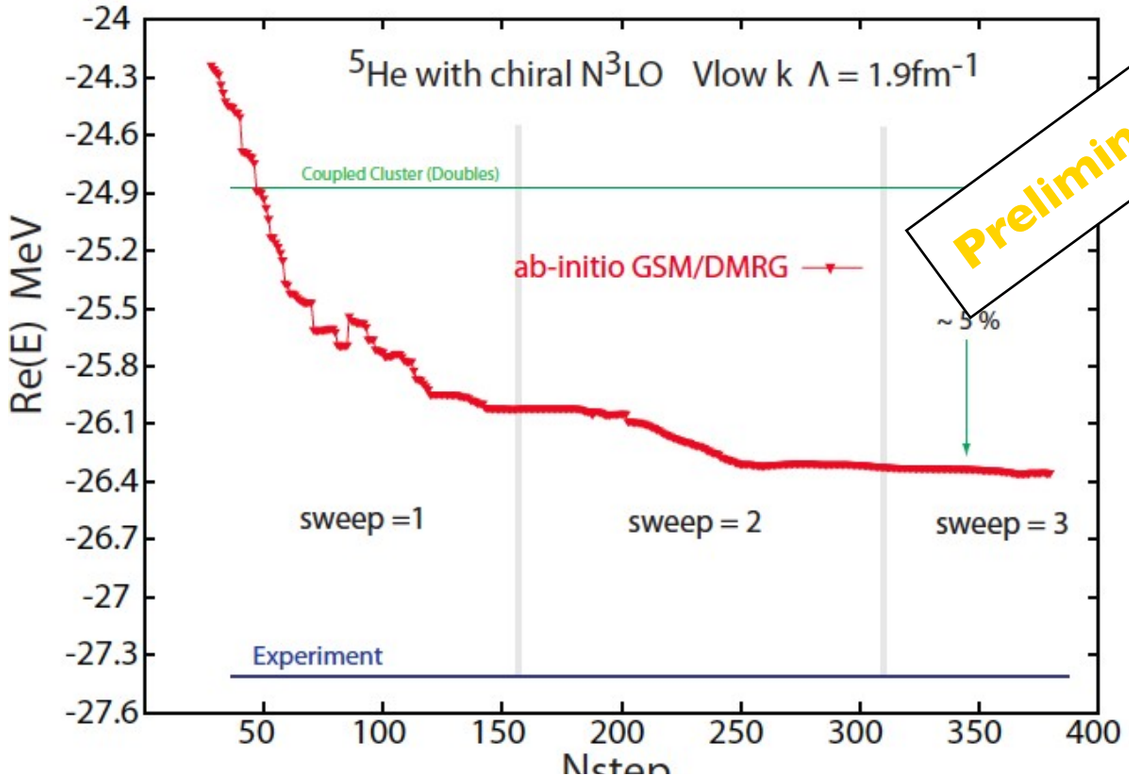


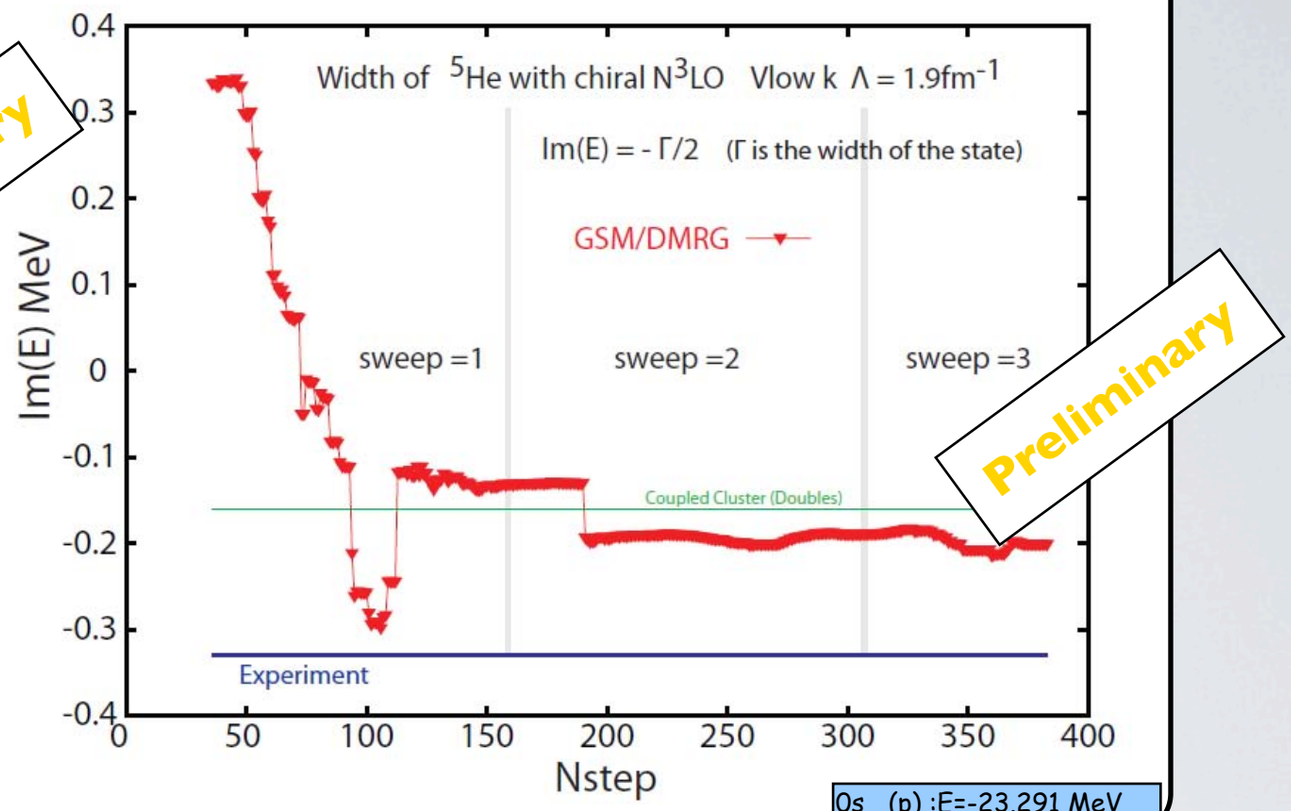
Ab initio NCGSM:

- **G. Papadimitrio, J. Rotureau**,, B. Barrett, M. Ploszajczak, W. Nazarewicz, N. Michel
- See FUSTIPEN workshop, March 2012

Ab initio Benchmark calculations in a Gamow Basis

^{5}He





$0s_{1/2}$(p) :E=-23.291 MeV
$0s_{1/2}$(n) :E=-23.999 MeV

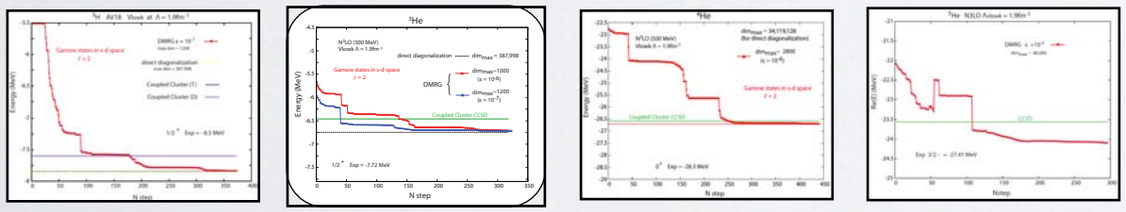
$0p_{3/2}$(n): E (1.194,-0.633)
$p_{3/2}$(n) complex contour (discretized)

$s_{1/2}$,$p_{3/2}$,$p_{1/2}$,$s_{1/2}$ real energy continua

d,f,g H.O states

157 s.p. in total

^{3}H ^{3}He ^{4}He ^{5}He



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