

Spin Phenomena in Elastic Scattering of ⁶He off Protons

Ch. Elster A. Orazbayev, S.P. Wepper



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RIKEN: ⁶He(p,p)⁶He @ 71 MeV

S. Sakaguchi et al. Phys.Rev. C84 (2011) 024604

Physics Challenge: Optical Potential for Halo Nucleus



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We adopted a standard Woods-Saxon optical potential with a spin-orbit term of the Thomas form:

$$U_{\text{OM}}(R) = -V_0 f_r(R) - i W_0 f_i(R) + 4i a_{id} W_d \frac{d}{dR} f_{id}(R) + V_s \frac{2}{R} \frac{d}{dR} f_s(R) L \cdot \sigma_p + V_{\text{C}}(R) \quad (1)$$

with

$$f_x(R) = \left[1 + \exp\left(\frac{R - r_{0x}A^{1/3}}{a_x}\right)\right]^{-1}$$
(2)
(x = r, i, id, or s).



Challenges for ⁶He (and similar exotic nuclei)

- ⁶He is loosely bound nucleus
 - with cluster structure:
 - Alpha core + 2 valance neutrons
- ⁶He is spin-0 nucleus
 - NOT a closed-shell nucleus

Tradítional microscopic optical potentials do NOT consider those properties



p+A Scattering as multiple scattering problem

> Spectator Expansion:

Written down by Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

2 Active Nucleons Single Scattering **3** Active 0 Nucleons **Double Scattering 4** Active 0 Nucleons **Triple Scattering**

Expansion in:

- \cdot particles active in the reaction
- ·Antisymmetrized in active particles



- o i : NN interaction
- i (A-1) core : e.g. mean field force

Phenomenological Optical Potentials parameterize single scattering term

Microscopic Optical Potentials "Folding Models" for closed shell nuclei

- Watson Multiple Scattering
 - Elster, Weppner, Chinn, Thaler, Tandy, Redish
 - Separation of p-A and n-A optical potential
 - Based on NN t-matrix as interaction input
 - Treating of interaction with (A-1)-core via mean field and as implicit three-body problem
- Kerman-McManus-Thaler (KMT)
 - Crespo, Johnson, Tostevin, Thompson
 - Based on NN t-matrix as input
 - Couple explicitly to (A-1) core
 - Introduce cluster ansatz for halo targets within coupled channels
- G-matrix folding
 - Arellano, Brieva, Love
 - Based on a g-matrix folding with local density approximation
 - Picked up by Amos, Karataglidis and extended to exotic nuclei

Scattering: Lippmann-Schwinger Equation

- LSE: $T = V + V G_0 T$
- Hamiltonian: $H = H_0 + V$
- Free Hamiltonian: $H_0 = h_0 + H_A$

 $-h_0$: kinetic energy of projectile '0'

– H_A: target hamiltonian with H_A $|\Phi\rangle$ = E_A $|\Phi\rangle$

- V: interactions between projectile '0' and target nucleons 'i' $V = \Sigma^{A}_{i=0} v_{0i}$
- Propagator is (A+1) body operator

 $- G_0(E) = (E - h_0 - H_A + i\epsilon)^{-1}$

Elastic Scattering

- In- and Out-States have the target in ground state Φ_0
- Projector on ground state $P = |\Phi_0\rangle\langle\Phi_0|$
 - With 1=P+Q and $[P,G_0]=0$
- For elastic scattering one needs
- $PTP = PUP + PUPG_{0}(E)PTP$

• Or

- $T = U + U G_0(E) P T$ $U = V + V G_0(E) Q U \iff \text{``optical potential''}$

Single Scattering: $U^{(1)} \approx \Sigma^{A}_{i=0} \tau_{0i}$ (1st order)

with $\tau_{0i} = v_{0i} + v_{0i} G_0(E) Q \tau_{0i}$

$\tau_{0i} = v_{0i} + v_{0i} G_0(E) Q \tau_{0i}$

- $G_0(E) = (E h_0 H_A + i\epsilon)^{-1} == (A+1)$ body operator
 - Standard "impulse approximation":
 - Average over $H_A \Rightarrow c$ -number
 - $\rightarrow G_0(e) ==:$ two body operator
- Handle operator Q
 - Define "two-body" operator t_{0i} free by
 - $t_{0i}^{\text{free}} = v_{0i} + v_{0i} G_0(e) t_{0i}^{\text{free}}$
 - and relate via integral equation to au_{oi}
 - $\tau_{oi} = t_{0i}^{free} t_{0i}^{free} G_0(e) \tau_{oi}$ [integral equation]
 - Important for keeping correct iso-spin character of optical potential

$$U^{(1)} = Σ^{A}_{i=1} τ_{oi} =: N τ_{n} + Z τ_{p}$$

Chinn, Elster, Thaler, PRC47, 2242 (1993)

"First order Watson optical potential" $U^{(1)} = \sum_{i=1}^{A} \tau_{oi} =: \sum_{i=1}^{N} \tau_{n} + \sum_{i=1}^{P} \tau_{p}$

- Important for treating N≠Z nuclei
- Sensitive to proton vs. neutron scattering
- In general

 $- \quad \textbf{t}_{pp} \neq \textbf{t}_{np} \quad \text{and} \ \ \rho_p \neq \rho_n$

• These differences enter in a non-linear fashion into first order Watson optical potential

 $\tau_{\alpha} = t_{\alpha} - t_{\alpha} G_0^{\alpha}(e) \tau_{\alpha}, \quad \alpha = n, p$

 This formulation allows a more complicated structure of the optical potential, e.g. a cluster ansatz

More explicit:

P:= projector on ground state

- Elastic scattering : $T_{el} = PUP + PUPG_0(E)PT_{el}$.
- First order Watson O.P.:

$$\langle \mathbf{k}' | \langle \phi_A | PUP | \phi_A \rangle \mathbf{k} \rangle \equiv U_{el}(\mathbf{k}', \mathbf{k}) = \sum_{i=n,p} \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{0i}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$$



 $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

Proton scattering: $U_{el}(\mathbf{k}', \mathbf{k}) = Z \langle \hat{\tau}_{01}^{pp} \rangle + N \langle \hat{\tau}_{01}^{np} \rangle$

Calculate: $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

 $\langle \mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3 \mathbf{k}_4 \dots \mathbf{k}_A | \phi_A \rangle = \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4 \dots + \mathbf{k}_A - \mathbf{p}_0) \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 \dots \zeta_{\mathbf{A}-1} | \phi_A \rangle.$

$$\langle \hat{\tau}_{01} \rangle = \int \prod_{j=1}^{A} d\mathbf{k}_{j}' \int \prod_{l=1}^{A} d\mathbf{k}_{l} \, \langle \phi_{A} | \zeta_{1}' \zeta_{2}' \zeta_{3}' \zeta_{4}' \dots \zeta_{A-1}' \rangle \delta(\mathbf{p}' - \mathbf{p}_{0}') \, \langle \mathbf{k}' \mathbf{k}_{1}' | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} \mathbf{k}_{1} \rangle$$

$$\prod_{j=2}^{A} \delta(\mathbf{k}_{j}' - \mathbf{k}_{j}) \delta(\mathbf{p} - \mathbf{p}_{0}) \, \langle \zeta_{1} \zeta_{2} \zeta_{3} \zeta_{4} \dots \zeta_{A-1} | \phi_{A} \rangle,$$

$$(2.48)$$

With single particle density matrix :

$$\rho(\zeta_{1}',\zeta_{1}) \equiv \int \prod_{l=2}^{A-1} d\zeta_{1}' \int \prod_{j=2}^{A-1} d\zeta_{j} \langle \phi_{A} | \zeta_{1}' \zeta_{2}' \zeta_{3}' \zeta_{4}' \dots \zeta_{A-1}' \rangle \langle \zeta_{1} \zeta_{2} \zeta_{3} \zeta_{4} \dots \zeta_{A-1} | \phi_{A} \rangle.$$

$$\langle \hat{\tau}_{01} \rangle = \int d\zeta_{1}' \int d\zeta_{1} \langle \mathbf{k}' \zeta_{1}' + \frac{\mathbf{p}_{0}'}{A} | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} \zeta_{1} + \frac{\mathbf{p}_{0}}{A} \rangle \rho(\zeta_{1}',\zeta_{1})$$

$$\delta(\frac{A-1}{A}\mathbf{p}_{0}' - \zeta_{1}' - \frac{A-1}{A}\mathbf{p}_{0} + \zeta_{1}).$$

Elster, Weppner, PRC 57. 189 (1998)

-1



Elster, Weppner, Chinn, PRC 56, 2080 (1997)

$$U_{el}(\mathbf{q}, \mathbf{K}) = \sum_{i=n,p} \int d\mathbf{P} \hat{\tau}_{0i}(\mathbf{q}, \frac{1}{2}(\frac{A+1}{A}\mathbf{K} - \mathbf{P}), \hat{\mathcal{E}}) \ \rho_i(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q})$$

Depends on |q|, |K|, $\cos(\theta)qK$

$$\vec{q} = \vec{k}' - \vec{k}$$

$$\vec{K} = \frac{1}{2} \left(\vec{k}' + \vec{k} \right)$$

$$\vec{n} = \vec{k} \times \vec{k}'$$

$$\hat{\mathcal{E}} = E_{NA} - \frac{(\mathbf{k} + \mathbf{k}_1)^2}{4m} = E_{NA} - \left(\frac{(\frac{A-1}{A}\mathbf{K} + \mathbf{P})^2}{4m}\right)$$



Weppner, Elster, PRC 85, 044617 (2012)

Cluster Folding Optical Potential $(n+n+\alpha)$

$$\mathbf{p}_{j_i} = \frac{1}{A} (A_{s_i} \mathbf{p}_i - A_i \mathbf{p}_{s_i})$$

Correlation Density

Jacobi momenta

$$\rho_{corr}(\mathbf{p}_{j_1}, \mathbf{p}_{j_1}') \equiv \int \prod_{l=2}^{N_c} d\mathbf{p}_{j_l}' \int \prod_{m=2}^{N_c} d\mathbf{p}_{j_m} \langle \phi_A | \mathbf{p}_{j_1}' \mathbf{p}_{j_2}' ... \mathbf{p}_{j_{N_c}}' \rangle \langle \mathbf{p}_{\mathbf{j}_1} \mathbf{p}_{\mathbf{j}_2} ... \mathbf{p}_{\mathbf{j}_{N_c}} | \phi_A \rangle$$

$$p_{3/2} \mathcal{HO} \text{ state}$$

Cluster optical potential

$$\begin{aligned} U_{el}(\mathbf{q}, \mathbf{K}) &= \sum_{c=1, N_c} \sum_{i=n_c, p_c} \int d\mathbf{P} \ d\mathcal{P}_{j_c} \ \rho_{corr}(\mathcal{P}_{j_c}) \\ &\hat{\tau}_{0i}\left(\mathbf{q}, \frac{1}{2}\left(\frac{A+1}{A}\mathbf{K} - \mathbf{P}\right), \mathcal{E}\right) \ \rho_{ci}\left(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}\right) \end{aligned}$$

Cluster folding potential for ⁶He+p

$${}^{^{6}\text{He}}U_{el}(\mathbf{q},\mathbf{K}) = U_{\alpha} + 2U_{n} =$$

$$\sum_{i=n,p} \int d\mathbf{P} \, d\mathcal{P}_{j_{\alpha}} \, \rho_{corr}(\mathcal{P}_{j_{\alpha}}) \, \hat{\tau}_{0i}\left(\mathbf{q},\frac{1}{2}\left(\frac{A+1}{A}\mathbf{K}-\mathbf{P}\right),\mathcal{E}\right) \, \rho_{\alpha i}\left(\mathbf{P}-\frac{A-1}{2A}\mathbf{q},\mathbf{P}+\frac{A-1}{2A}\mathbf{q}\right)$$

$$+ 2\int d\mathbf{P} \, d\mathcal{P}_{j_{n}} \, \rho_{corr}(\mathcal{P}_{j_{n}}) \, \hat{\tau}_{0n}\left(\mathbf{q},\frac{1}{2}(\frac{A+1}{A}\mathbf{K}-\mathbf{P}),\mathcal{E}\right) \, \rho_{n}\left(\mathbf{P}-\frac{A-1}{2A}\mathbf{q},\mathbf{P}+\frac{A-1}{2A}\mathbf{q}\right).$$

For calculation: NN t-matrix: Nijmegen II potential Densities: COSMA density == s & p- shell harmonic oscillator wave functions Fitted to give rms radius of ⁶He (older value)

and for ⁴He: Gogny density with coupling to medium

⁶He (p,p) ⁶He @ 71 MeV



⁶He (p,p) ⁶He @ 71 MeV



⁶He (p,p) ⁶He



⁶He (p,p) ⁶He



Black: COSMA single particle





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More explicit:

P:= Projector on ground state

- $T_{el} = PUP + PUPG_0(E)PT_{el}.$ Elastic scattering :
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$$\langle \mathbf{k}' | \langle \phi_A | P U P | \phi_A \rangle \mathbf{k} \rangle \equiv U_{el}(\mathbf{k}', \mathbf{k}) = \sum_{i=n,p} \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{0i}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$$



 $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

Proton scattering: $U_{el}(\mathbf{k}', \mathbf{k}) = Z \langle \hat{\tau}_{01}^{pp} \rangle + N \langle \hat{\tau}_{01}^{np} \rangle$

Ansatz for ⁶He Density Matrix

HO ansatz for s and p shell

$$\psi_s^m(p) = (2\pi)^{3/2} \left(\frac{4}{\sqrt{\pi\nu_s^{3/2}}}\right)^{1/2} \frac{1}{4\pi} e^{-p^2/2\nu_s} \mathcal{Y}_{0m}^{\frac{1}{2}}(\hat{p})$$
$$\psi_p^m(p) = (2\pi)^{3/2} \left(\frac{4}{\sqrt{\pi\nu_p^{3/2}}}\right)^{1/2} \sqrt{\frac{2}{3}} \frac{p}{\sqrt{\nu_p}} e^{-p^2/2\nu_p} \mathcal{Y}_{1m}^{\frac{3}{2}}(\hat{p})$$

With valence neutrons in $p_{3/2}$ shell

$$\psi_{p_{3/2}}(p) := f_{p_{3/2}}(p) \frac{1}{\sqrt{4}} \left(\mathcal{Y}_{1\frac{3}{2}}^{\frac{3}{2}}(\hat{p}) - \mathcal{Y}_{1\frac{1}{2}}^{\frac{3}{2}}(\hat{p}) + \mathcal{Y}_{1-\frac{1}{2}}^{\frac{3}{2}}(\hat{p}) - \mathcal{Y}_{1-\frac{3}{2}}^{\frac{3}{2}}(\hat{p}) \right)$$

Ansatz for ⁶He Density Matrix

Parameters for 1st calculation:

Zhukov, M.V. et al. Phys.Rept. 231 (1993) 151-199 Korsheninnikov, A.A. et al. Phyle.Lett. B316 (1993) 38-44

Table 1: Charge Radii and Oscillator Parameters of ⁴He, ⁶He and ⁸He.

Isotope	Charge	Matter	Oscillator	Oscillator
He	radius	radius	parameter	parameter
	$_{\mathrm{fm}}$	$_{\mathrm{fm}}$	$\nu_s \ {\rm fm}^{-2}$	$\nu_p \ {\rm fm}^{-2}$
$^{4}\mathrm{He}$	1.676[37]	1.676[37]	0.534	None
⁶ He	2.054 [34]	2.320 [35]	0.355	0.322
⁸ He	1.929 [38]	2.490 [39]	0.403	0.224

NN t-matrix [Wolfenstein Representation]

 $t_{NN}(\mathbf{q}, \mathbf{K}, \varepsilon) = A(\mathbf{q}, \mathbf{K}, \varepsilon) \mathbf{1} + i \left(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)} \right) \cdot \hat{\mathbf{n}} C(\mathbf{q}, \mathbf{K}, \varepsilon)$

Closed Shell Nucleí:

$$\left\langle \Phi_{A} \left| \vec{\sigma} \cdot \vec{K} \right| \Phi_{A} \right\rangle = 0$$

see e.g. Elster, Cheon, Redish, Tandy PRC 41, 814 (1990)

Expectation Values for p-shell

$$\Psi_p(\hat{\mathbf{p}}')\boldsymbol{\sigma}^{(2)} \cdot \hat{\mathbf{n}} \Psi_p(\hat{\mathbf{p}}) = \frac{2}{9\pi^{3/2}\nu_p^{5/2}} p p' \exp\left(-\frac{p^2}{2\nu_p} - \frac{p'^2}{2\nu_p}\right) \sin\alpha_{pp'} \cos\beta$$
$$\Psi_p(\hat{\mathbf{p}}')\boldsymbol{\sigma}^{(2)} \cdot \hat{\mathbf{q}} \Psi_p(\hat{\mathbf{p}}) = 0$$
$$\Psi_p(\hat{\mathbf{p}}')\boldsymbol{\sigma}^{(2)} \cdot \hat{\mathbf{K}} \Psi_p(\hat{\mathbf{p}}) = \frac{2}{9\pi^{3/2}\nu_p^{5/2}} p p' \exp\left(-\frac{p^2}{2\nu_p} - \frac{p'^2}{2\nu_p}\right) \sin\alpha_{pp'} \cos\delta$$

Boost from NN frame to N-Nucleus frame



Optical Potential for Valence Neutrons of ⁶**He**

± indicate spin-flip amplitudes

$$\begin{aligned} \mathbf{U}_{\text{val}} & (\mathbf{q}, \ \mathcal{K}) = \\ \mathbf{N} \int \mathbf{t}_{2 \ \mathbf{B}} \left(\mathbf{q}, \ \mathcal{K}, \mathbf{p}, \mathbf{p}' \right) \ \mathcal{P}_{\mathbf{j} = \frac{3}{2}, \mathbf{l} = 1}^{\text{neutron}} \left(\mathbf{q}, \ \mathcal{K}, \mathbf{p}, \mathbf{p}' \right) \ d^{3}\mathbf{p} \ d^{3}\mathbf{p}' = \\ \mathbf{N} \int \left(\mathbf{f}_{\mathbf{j} = \frac{3}{2}, \mathbf{l} = 1} \left(\mathbf{p} \right) \ \mathbf{f}_{\mathbf{j} = \frac{3}{2}, \mathbf{l} = 1} \left(\mathbf{p}' \right) \right)_{\text{neutron}} \left(\left(\mathcal{F} \pm \mathbf{C} \right) \left(\frac{\pi}{2} \ \mathbf{P}_{\mathbf{l} = 1} \left(\cos[\gamma] \right) \right) \\ & + \left(\frac{\mathbf{i} \ \pi \ \sin[\gamma]}{6} \right) \left(\mathbf{C} \ \cos[\beta] \pm \mathcal{M} \ \cos[\beta] + \\ & \mathbf{G} + \mathcal{H} \right) \left(\frac{1}{2 \ |\mathcal{K}_{\mathrm{m}}|} \left(|\mathbf{k}_{\mathrm{m}}| + |\mathbf{k}'_{\mathrm{m}}| \ \mathbf{e}^{\left(\mp \mathbf{i} \gamma_{\mathrm{m}} \right)} \right) \ \cos[\alpha] \right) + \\ & \mathcal{D} \left(\frac{1}{|\mathbf{q}_{\mathrm{m}}|} \left(-|\mathbf{k}_{\mathrm{m}}| + |\mathbf{k}'_{\mathrm{m}}| \ \mathbf{e}^{\left(\mp \mathbf{i} \gamma_{\mathrm{m}} \right)} \right) \ \cos[\alpha] \right) \right) d^{3}\mathbf{p} \ d^{3}\mathbf{p}', \end{aligned}$$

Give zero contribution integrated with p-3/2 states!

Optical Potential for ⁶He with all terms from the valence neutrons

$$U_{^{6}He}(\mathbf{q},\mathbf{K}) = \sum_{i=N,P} U_{core}(\mathbf{q},\mathbf{K}) + U_{val}(\mathbf{q},\mathbf{K})$$

$$U_{val_{central}} = U_A(\mathbf{q}, \mathbf{K}) + U_A^C(\mathbf{q}, \mathbf{K})$$
$$U_{val_{spin-orbit}} = U_C(\mathbf{q}, \mathbf{K}) + U^M(\mathbf{q}, \mathbf{K})$$





⁶He(p,p) at 71MeV/nucleon



Explore sensitivity to matter radius



⁶He(p,p) at 200 MeV/nucleon



Explore sensitivity to matter radius

Experiment with ⁸He available soon (October)



arXiv:1207.0545

Challenges for ⁶He

• ⁶He is a loosely bound nucleus ⇒ Cluster ansatz

- Correlation visible in $d\sigma/d\Omega$ at forward angles
- Only microscopic calculation with a negative A_v at 71 MeV
- Good description of ⁴He important
- Cluster ansatz can be implemented for ⁸He in similar fashion
- ⁶He is not a closed shell nucleus (as is ⁸He)
 - In microscopic OP in principle all six Wolfenstein amplitudes contribute
 - Projection on ground state p-3/2: G, H, D do not contribute
 - Additional terms to central and spin-orbit potential
- Finally the additional p-shell terms will also have to be combined with the cluster description
- Polarization \Rightarrow Interference can amplify small contributions