

Three- and four-particle scattering

A. Deltuva

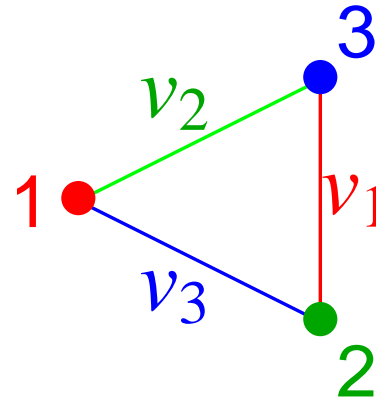
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Few-particle scattering

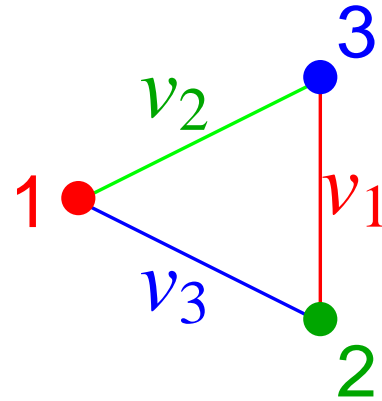
- Three-particle scattering equations
- Three-nucleon system
- Three-body direct nuclear reactions
- Four-particle scattering equations
- Four-nucleon system
- Four-boson universal physics

Three-particle system

Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$



Three-particle system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

- Faddeev equations

$$(E - H_0 - v_{\alpha}) |\Psi_{\alpha}\rangle = v_{\alpha} \sum_{\sigma} \bar{\delta}_{\alpha\sigma} |\Psi_{\sigma}\rangle$$

$$|\Psi\rangle = \sum_{\alpha} |\Psi_{\alpha}\rangle$$

Alt, Grassberger, and Sandhas equations

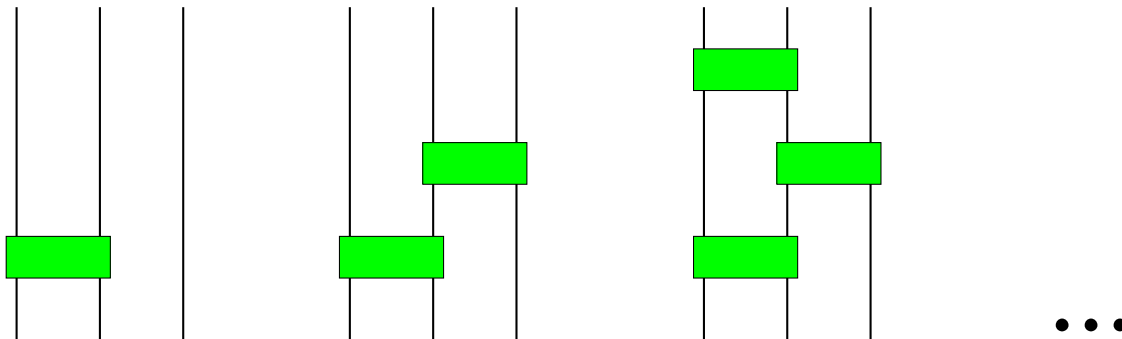
$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$T_{\sigma} = v_{\sigma} + v_{\sigma} G_0 T_{\sigma}$$

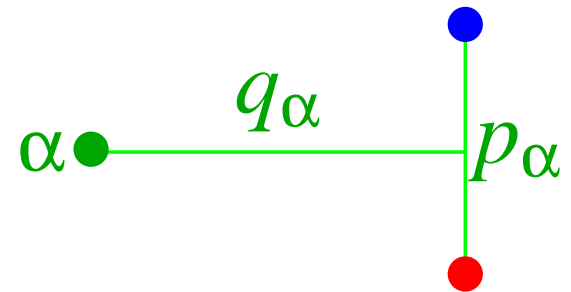
$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $(E - H_0 - v_{\alpha})|\phi_{\alpha}\rangle = 0$



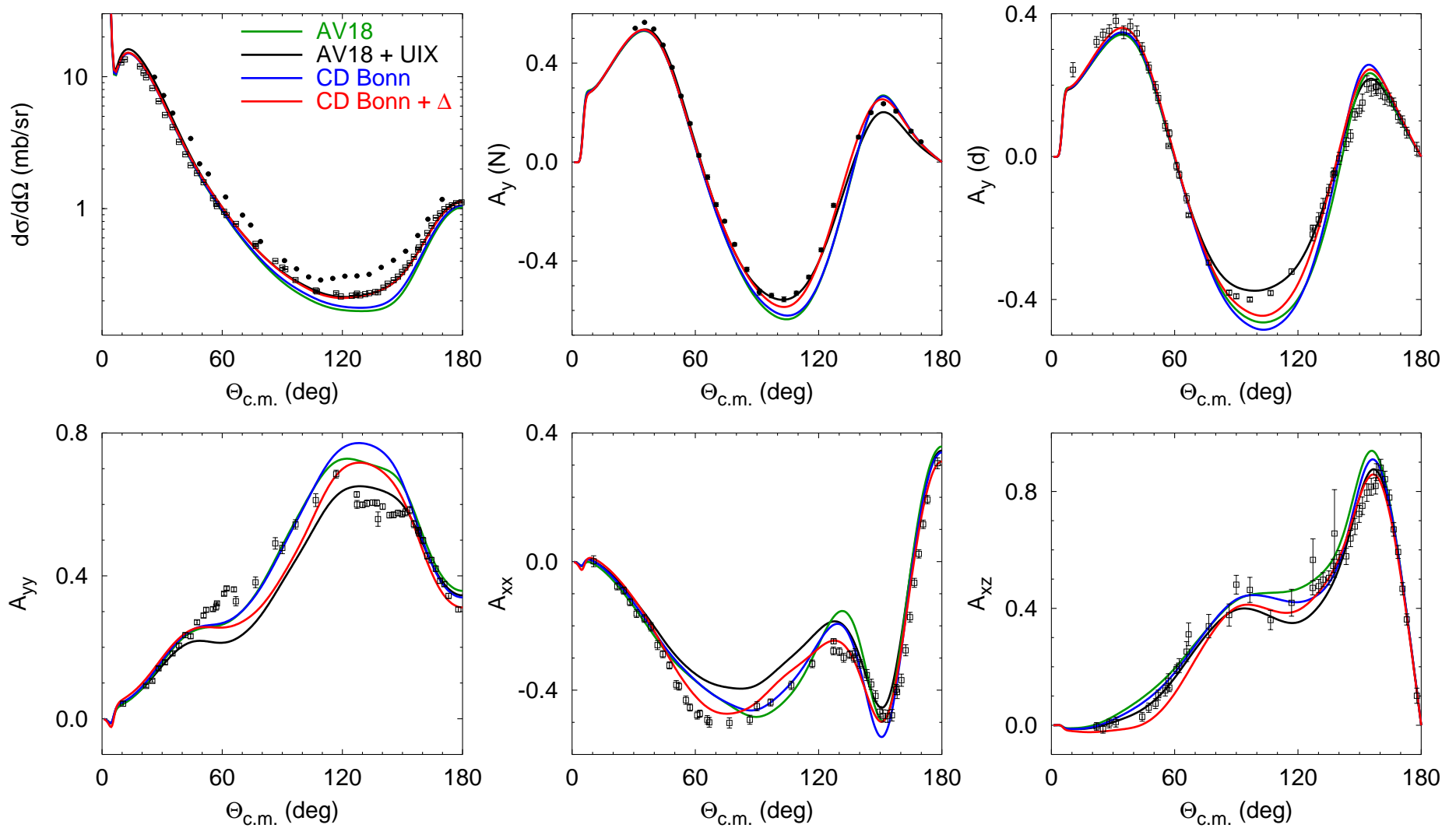
AGS equations: numerical solution

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$



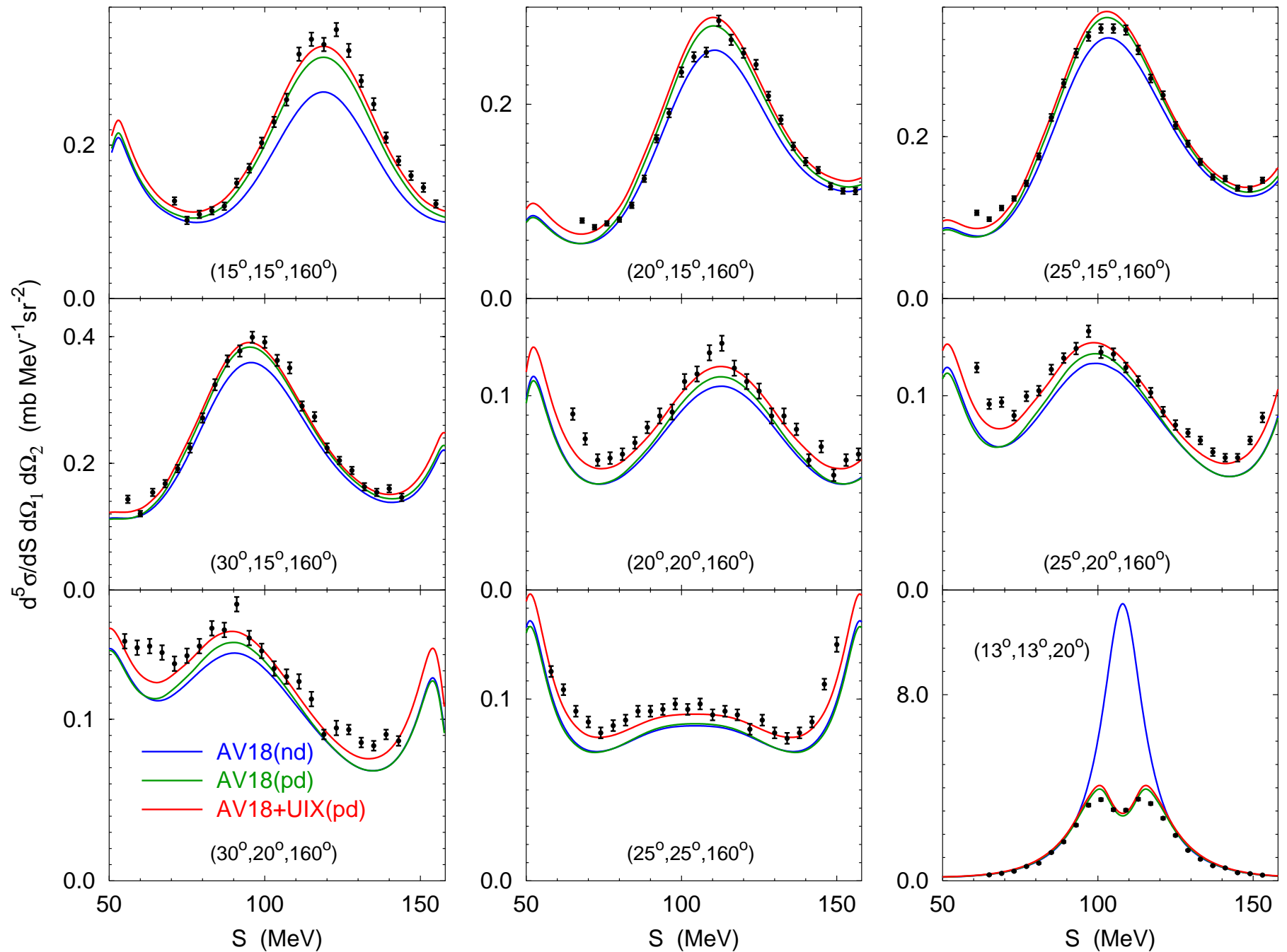
- 3 sets of Jacobi momenta
- momentum-space partial wave basis
- set of coupled 2-variable integral equations
- integrable singularities in kernel
- Coulomb interaction: screening and renormalization

Proton-deuteron elastic scattering at $E_p = 135$ MeV

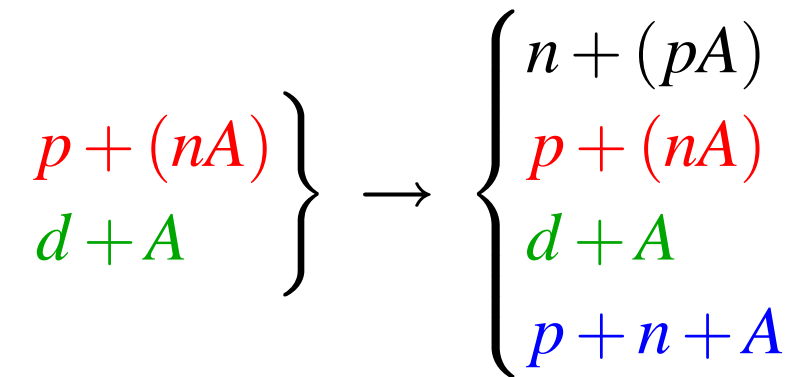


[PRC 80, 064002]

Coulomb vs 3NF: $^1\text{H}(d,pp)n$ at $E_d = 130$ MeV



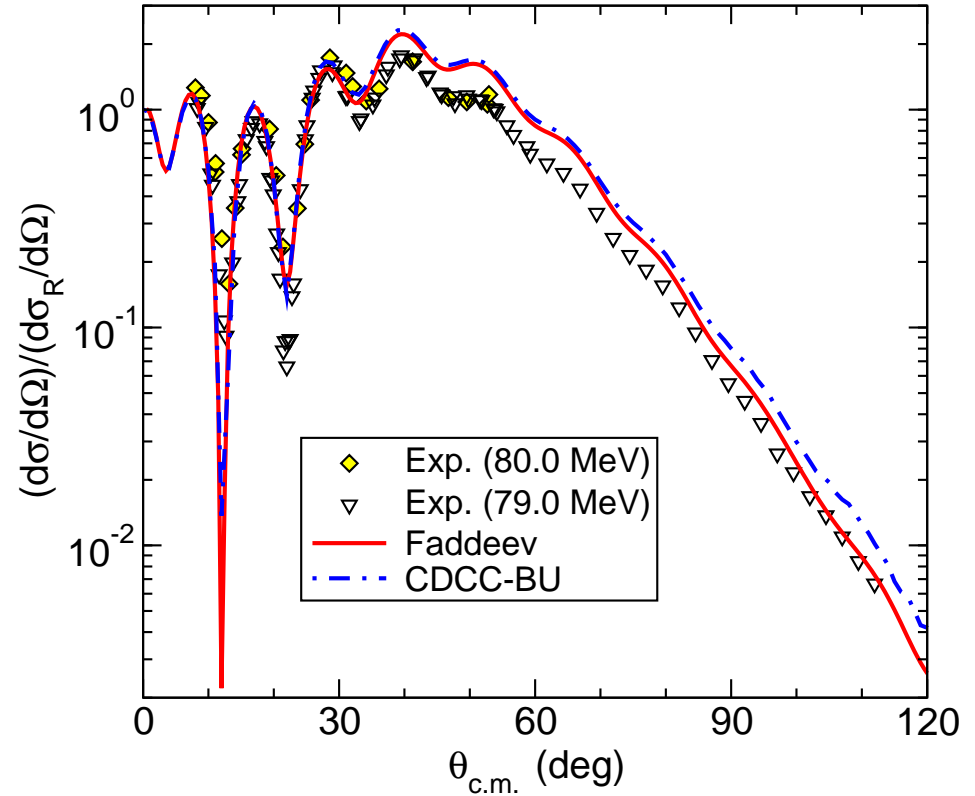
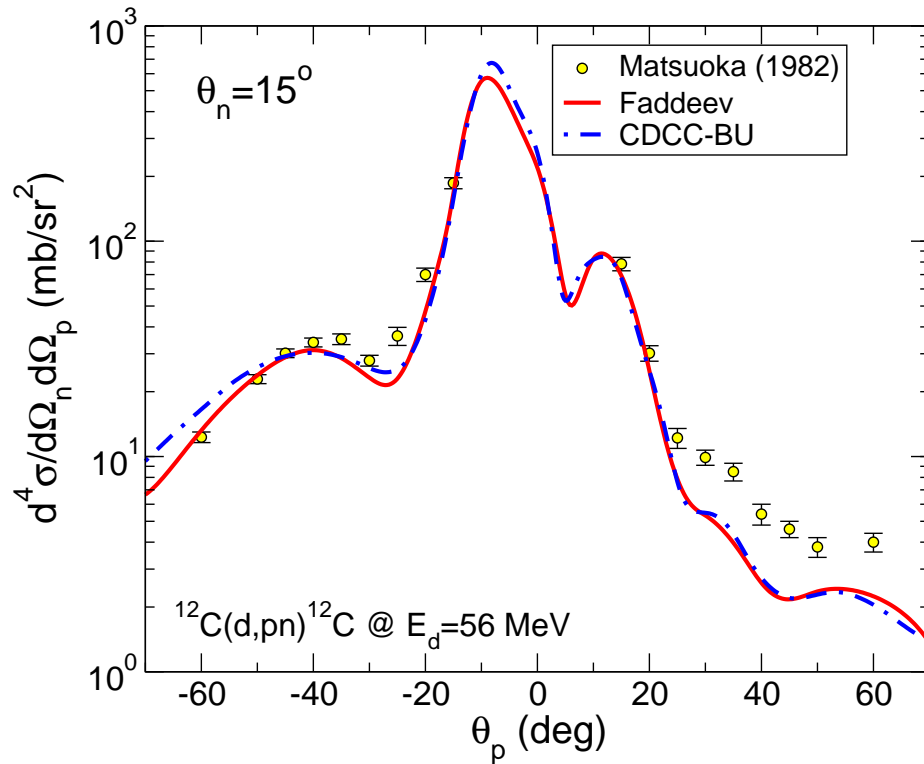
Application to 3-body nuclear reactions



with $A = {}^4\text{He}, {}^{10}\text{Be}, {}^{12}\text{C}, {}^{14}\text{C}, {}^{16}\text{O}, {}^{28}\text{Si}, {}^{40}\text{Ca}, {}^{48}\text{Ca}, {}^{58}\text{Ni}, \dots$

- Validity test of approximate nuclear reaction methods: DWBA, ADWA, CDCC, ...
- Novel dynamic input: nonlocal potentials, ...

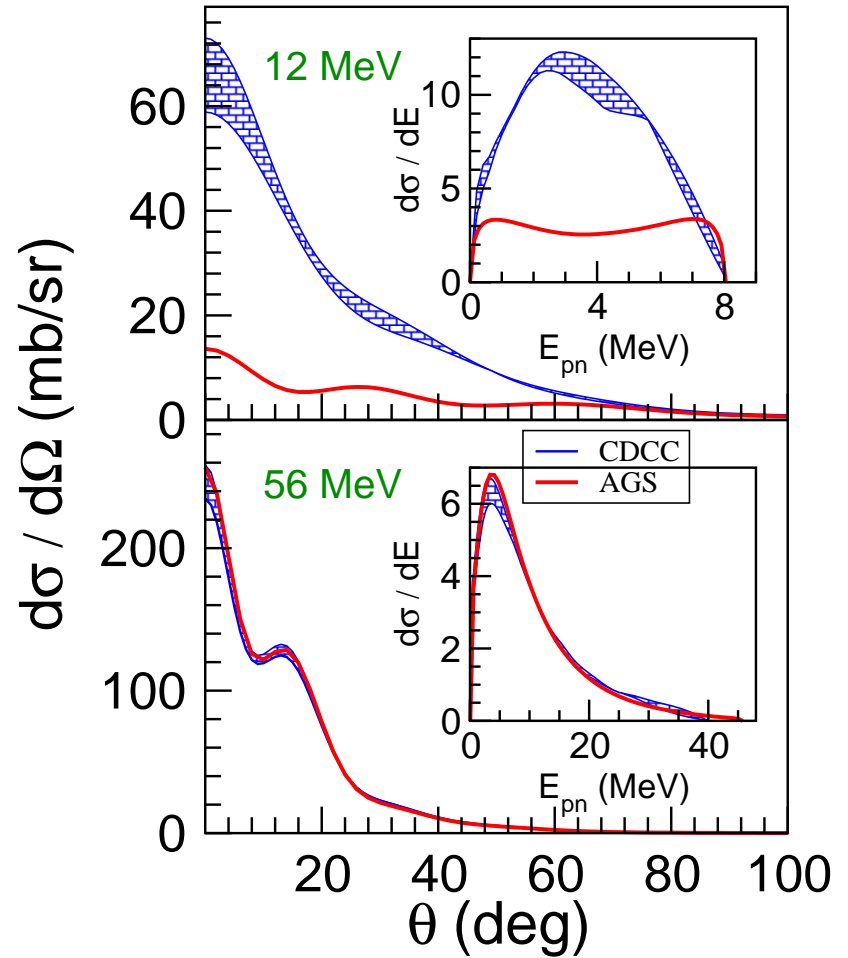
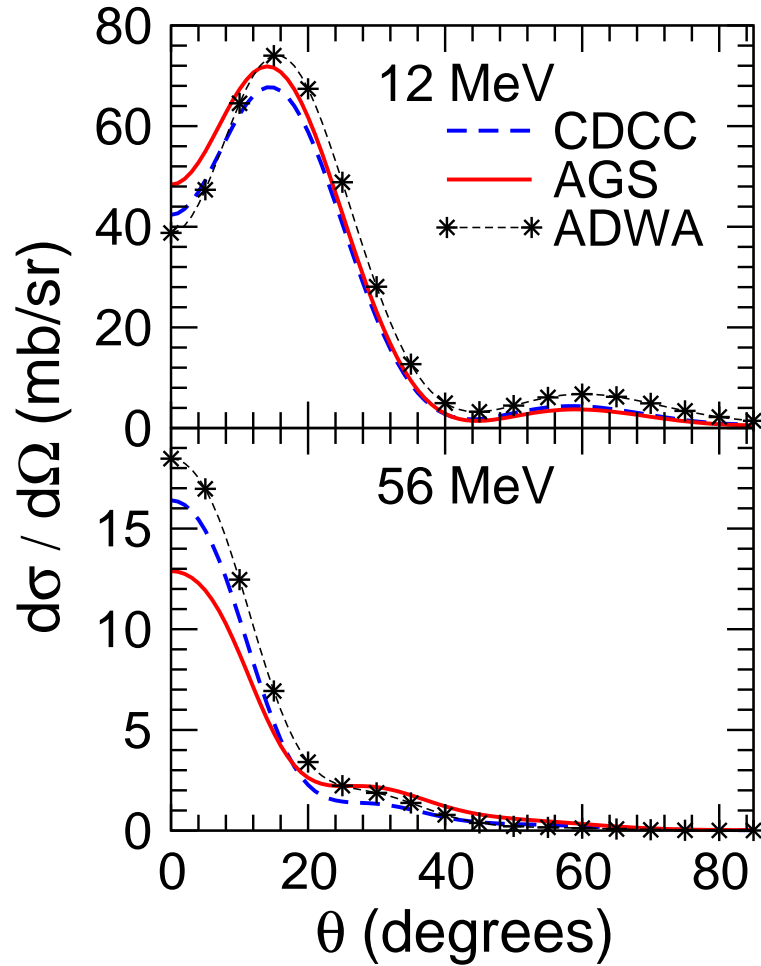
CDCC test: $^{12}\text{C}(d, pn)^{12}\text{C}$ & $^{58}\text{Ni}(d, d)^{58}\text{Ni}$



CDCC: A. M. Moro & F. M. Nunes

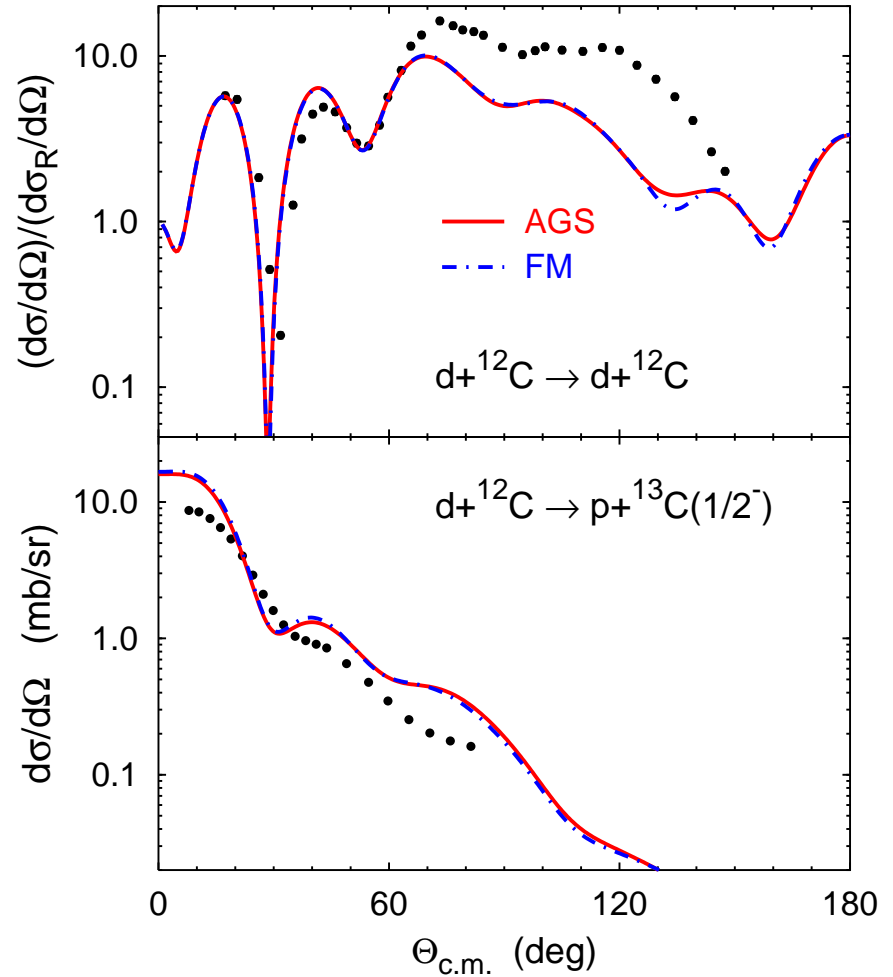
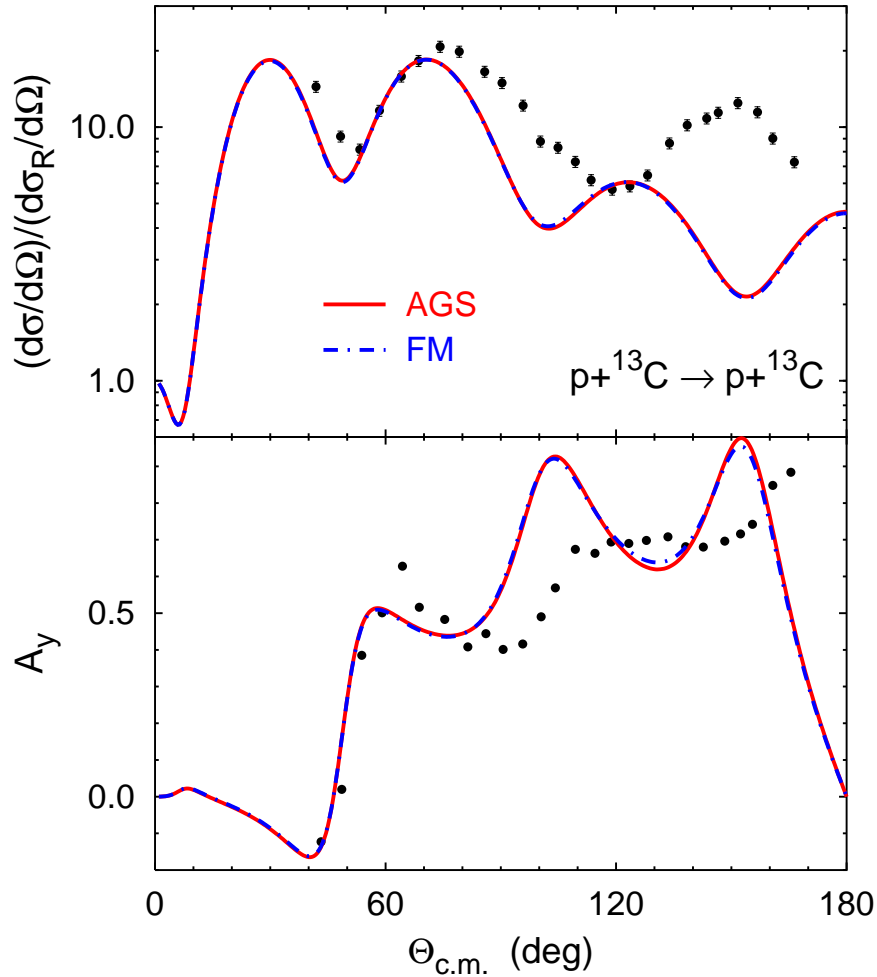
[PRC 76, 064602 (2007)]

CDCC test: $^{12}\text{C}(d, p)^{13}\text{C}$ and $^{12}\text{C}(d, pn)^{12}\text{C}$



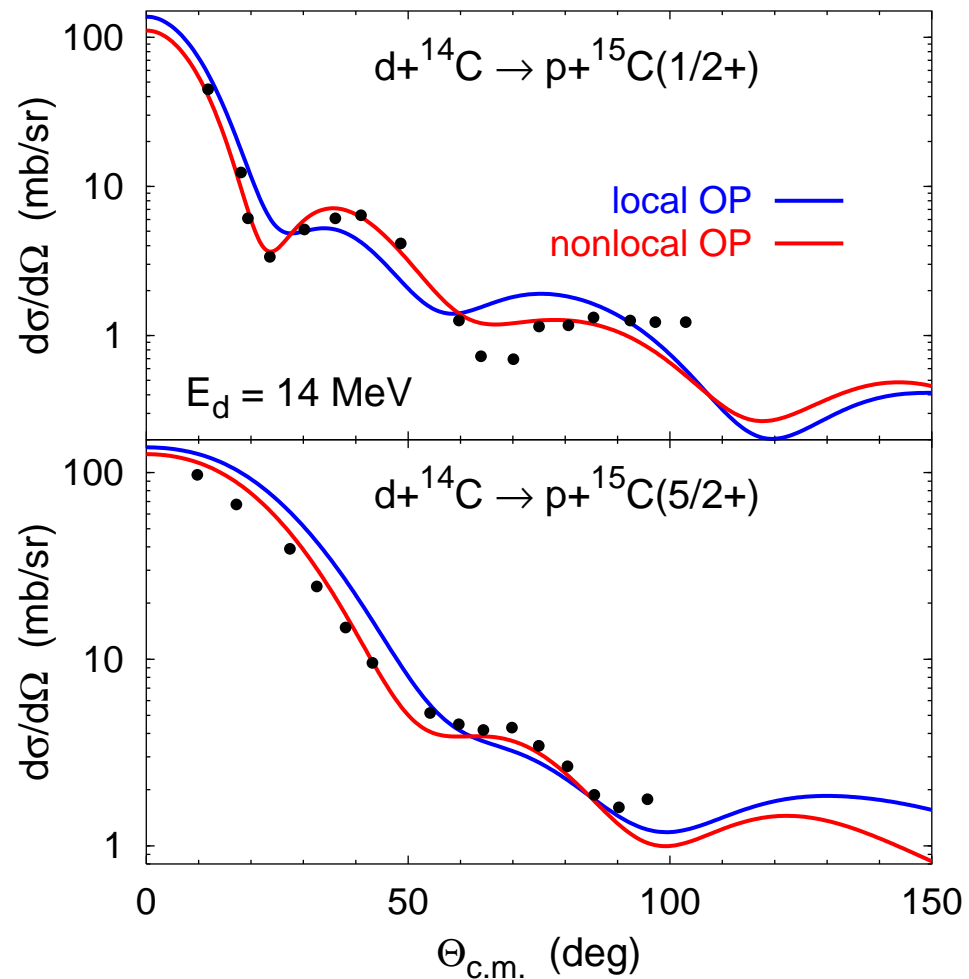
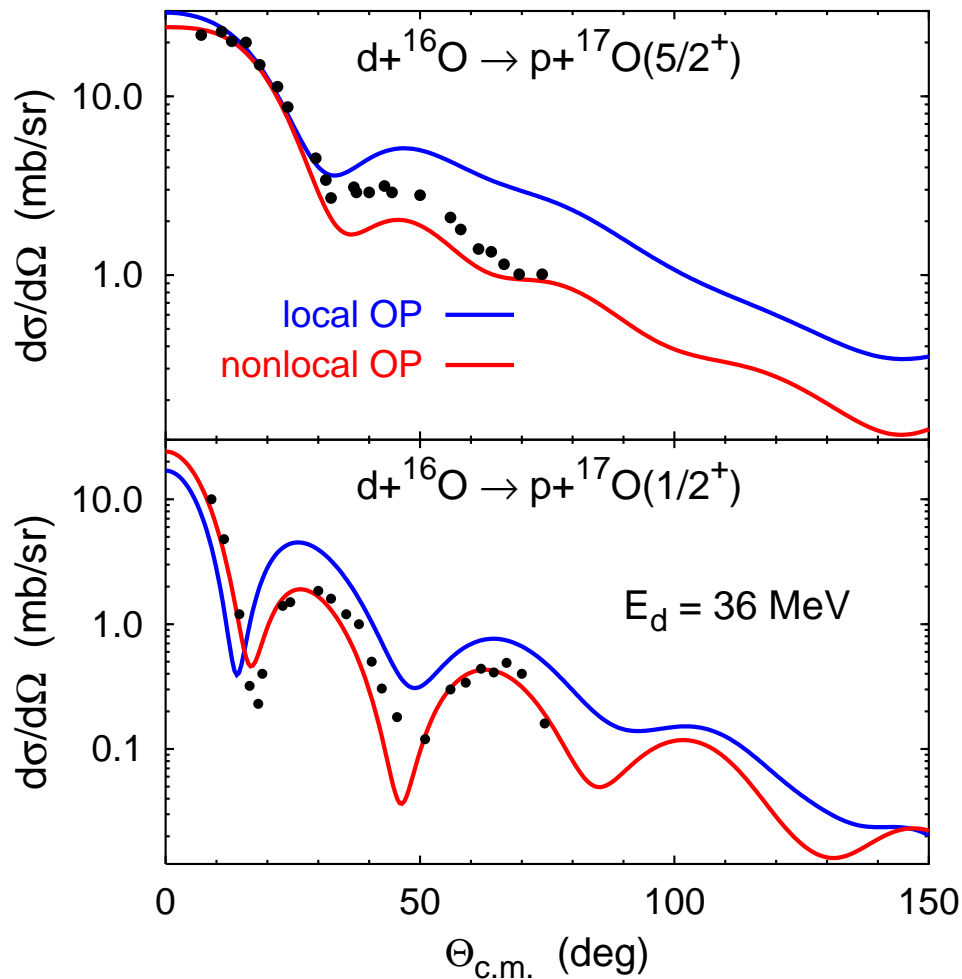
CDCC/ADWA: F. M. Nunes, N. Upadhyay [PRC 85, 054621]

Comparison with r-space FM results



FM: R. Lazauskas [arXiv:1201.4979]

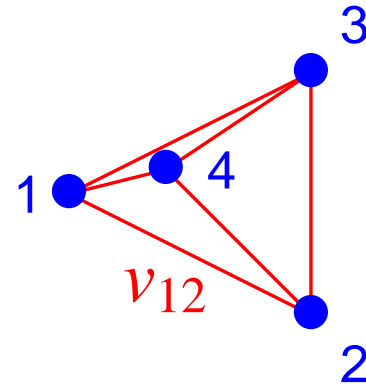
Nonlocal OP: transfer reactions



[PRC 79, 021602, PRC 79, 054603]

Four-particle scattering

Hamiltonian $H_0 + \sum_{i>j} v_{ij}$



- Wave function:
Schrödinger equation
- Wave function components:
Faddeev-Yakubovsky equations
- Transition operators:
Alt-Grassberger-Sandhas equations

Symmetrized AGS equations

$$t = v + vG_0t$$

$$G_0 = (E + i\varepsilon - H_0)^{-1}$$

$$u_j = P_j G_0^{-1} + P_j t G_0 u_j$$

$$3 + 1 : P_1 = P_{12} P_{23} + P_{13} P_{23}$$

$$2 + 2 : P_2 = P_{13} P_{24}$$

$$U_{11} = (G_0 t G_0)^{-1} \zeta P_{34} + \zeta P_{34} u_1 G_0 t G_0 U_{11} + u_2 G_0 t G_0 U_{21}$$

$$U_{21} = (G_0 t G_0)^{-1} (1 + \zeta P_{34}) + (1 + \zeta P_{34}) u_1 G_0 t G_0 U_{11}$$

$$U_{12} = (G_0 t G_0)^{-1} + \zeta P_{34} u_1 G_0 t G_0 U_{12} + u_2 G_0 t G_0 U_{22}$$

$$U_{22} = (1 + \zeta P_{34}) u_1 G_0 t G_0 U_{12}$$

$\zeta = -1$ (+1) for fermions (bosons)

basis states partially symmetrized

Scattering amplitudes: $E + i\varepsilon \rightarrow E + i0$

2-cluster reactions:

$$\begin{aligned}T_{fi} &= s_{fi} \langle \phi_f | \mathbf{U}_{fi} | \phi_i \rangle \\ |\phi_j\rangle &= G_0 t P_j |\phi_j\rangle \\ |\Phi_j\rangle &= (1 + P_j) |\phi_j\rangle\end{aligned}$$

3-cluster breakup/recombination:

$$T_{3i} = s_{3i} \langle \phi_3 | [(1 + \zeta P_{34}) \mathbf{u}_1 G_0 t G_0 \mathbf{U}_{1i} + \mathbf{u}_2 G_0 t G_0 \mathbf{U}_{2i}] | \phi_i \rangle$$

4-cluster breakup/recombination:

$$\begin{aligned}T_{4i} &= s_{4i} \{ \langle \phi_4 | [1 + (1 + P_1) \zeta P_{34}] (1 + P_1) t G_0 \mathbf{u}_1 G_0 t G_0 \mathbf{U}_{1i} | \phi_i \rangle \\ &\quad + \langle \phi_4 | (1 + P_1) (1 + P_2) t G_0 \mathbf{u}_2 G_0 t G_0 \mathbf{U}_{2i} | \phi_i \rangle \} \end{aligned}$$

Wave function

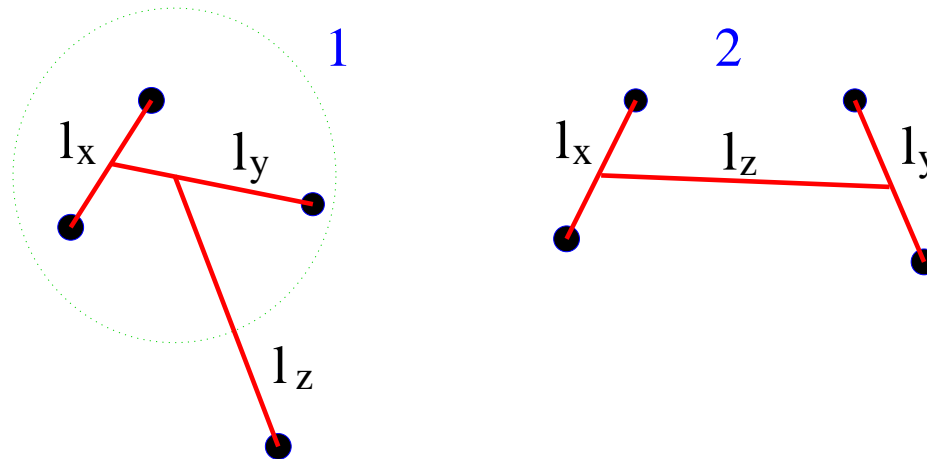
$$|\Psi_i\rangle = s_i \{ [1 + (1 + P_1)\zeta P_{34}](1 + P_1)|\Psi_{1,i}\rangle + (1 + P_1)(1 + P_2)|\Psi_{2,i}\rangle \}$$

with Faddeev-Yakubovsky components

$$|\Psi_{j,i}\rangle = \delta_{ji}|\phi_i\rangle + G_0 t G_0 u_j G_0 t G_0 U_{ji}|\phi_i\rangle$$

Solution of 4N AGS equations

$$U_{11}|\phi_1\rangle = -G_0^{-1}P_{34}P_1|\phi_1\rangle - P_{34}u_1G_0tG_0U_{11}|\phi_1\rangle + u_2G_0tG_0U_{21}|\phi_1\rangle$$



- momentum-space partial-wave basis

$$|k_x k_y k_z [l_z (\{l_y [(l_x S_x) j_x s_y] S_y \} J_y s_z) S_z] JM, [(T_x t_y) T_y t_z] T M_T \rangle_1$$

$$|k_x k_y k_z [l_z \{ (l_x S_x) j_x [l_y (s_y s_z) S_y] j_y \} S_z] JM, [T_x (t_y t_z) T_y] T M_T \rangle_2$$
- large system (up to 30000) of coupled 3-variable integral equations with integrable singularities
- Coulomb interaction: screening and renormalization
 [PRC 75, 014005, PRL 98, 162502]

Singularities of 4N AGS equations

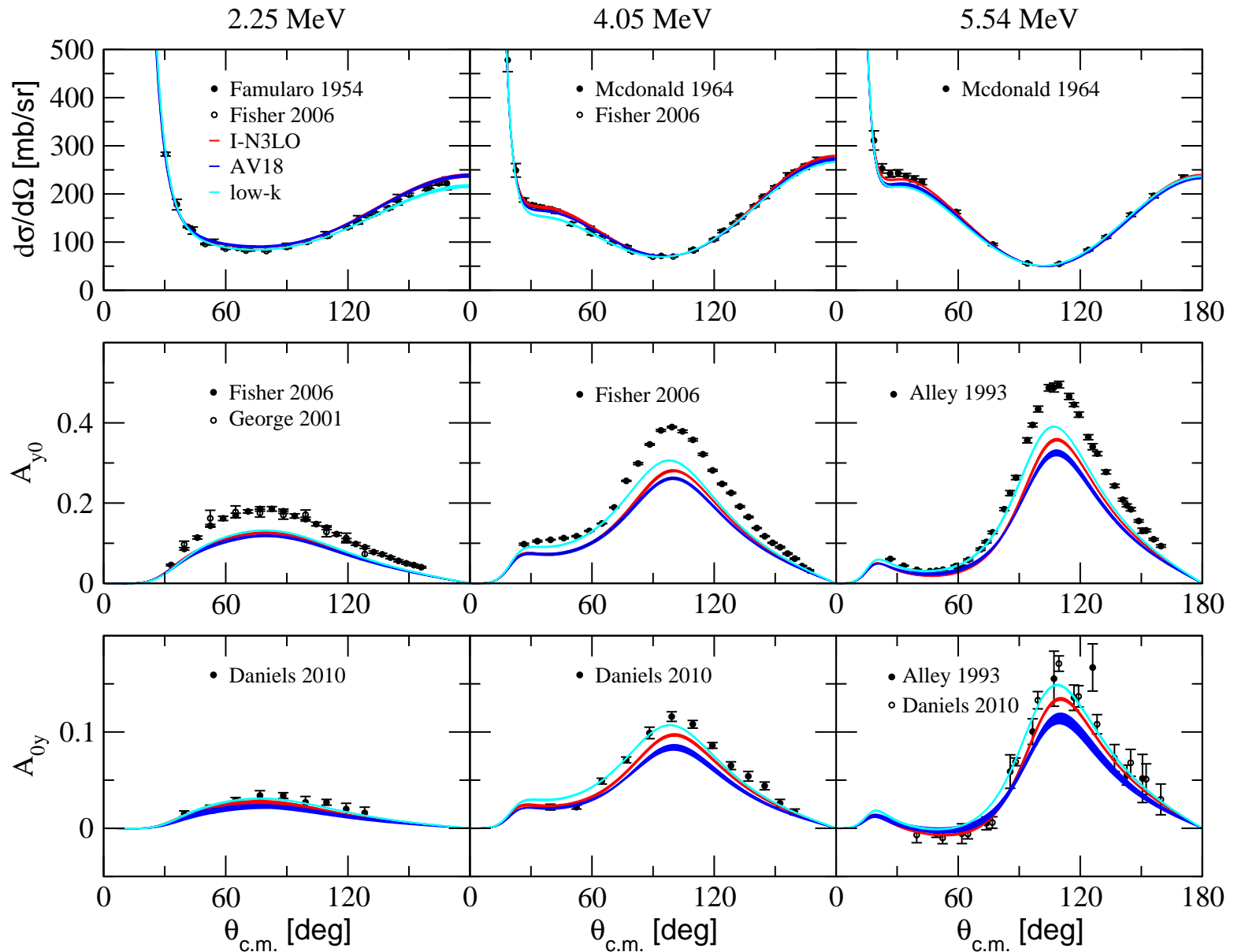
${}^3\text{H}$, ${}^3\text{He}$, or d+d bound state poles

$$G_0 u_j G_0 \rightarrow \frac{P_j |\phi_j\rangle s_{jj} \langle \phi_j| P_j}{E + i\varepsilon - E_j^b - k_z^2 / 2\mu_j}$$

treated by subtraction below 3-cluster threshold

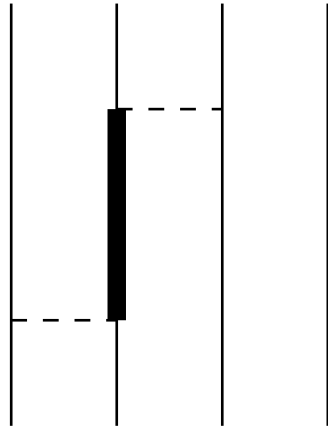
$$\begin{aligned} & \int_p^q k_z^2 dk_z \frac{F(k_z)}{k_0^2 - k_z^2 + i0} \\ &= \mathcal{P} \int_p^q k_z^2 dk_z \frac{F(k_z)}{k_0^2 - k_z^2} - \frac{1}{2} i\pi k_0 F(k_0) \\ &= \int_p^q dk_z \frac{k_z^2 F(k_z) - k_0^2 F(k_0)}{k_0^2 - k_z^2} \\ & \quad - \frac{1}{2} k_0 F(k_0) \left[i\pi + \ln \frac{(k_0 + p)(q - k_0)}{(k_0 - p)(k_0 + q)} \right] \end{aligned}$$

p - ^3He scattering

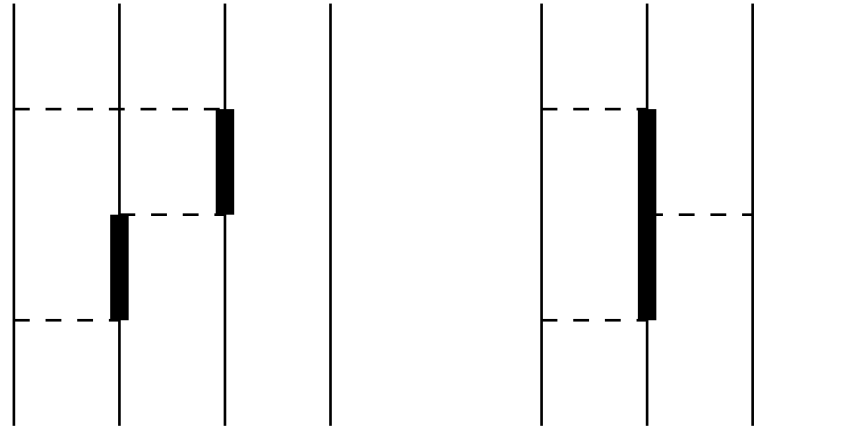


Δ -isobar excitation: effective 3N and 4N forces

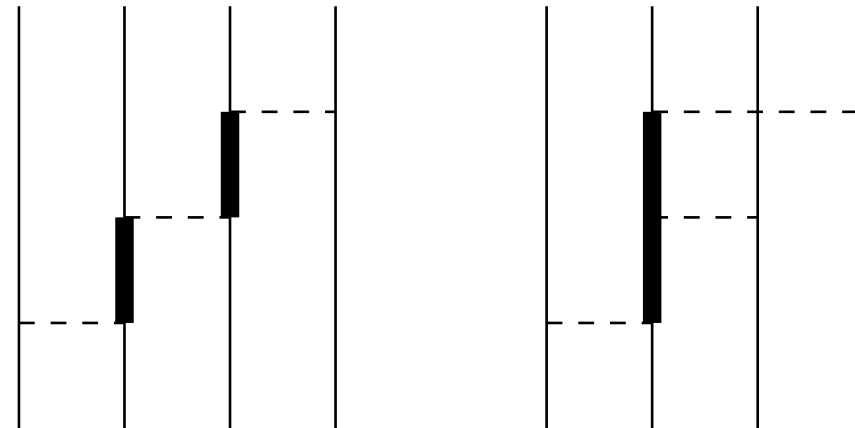
Fujita-Miyazawa



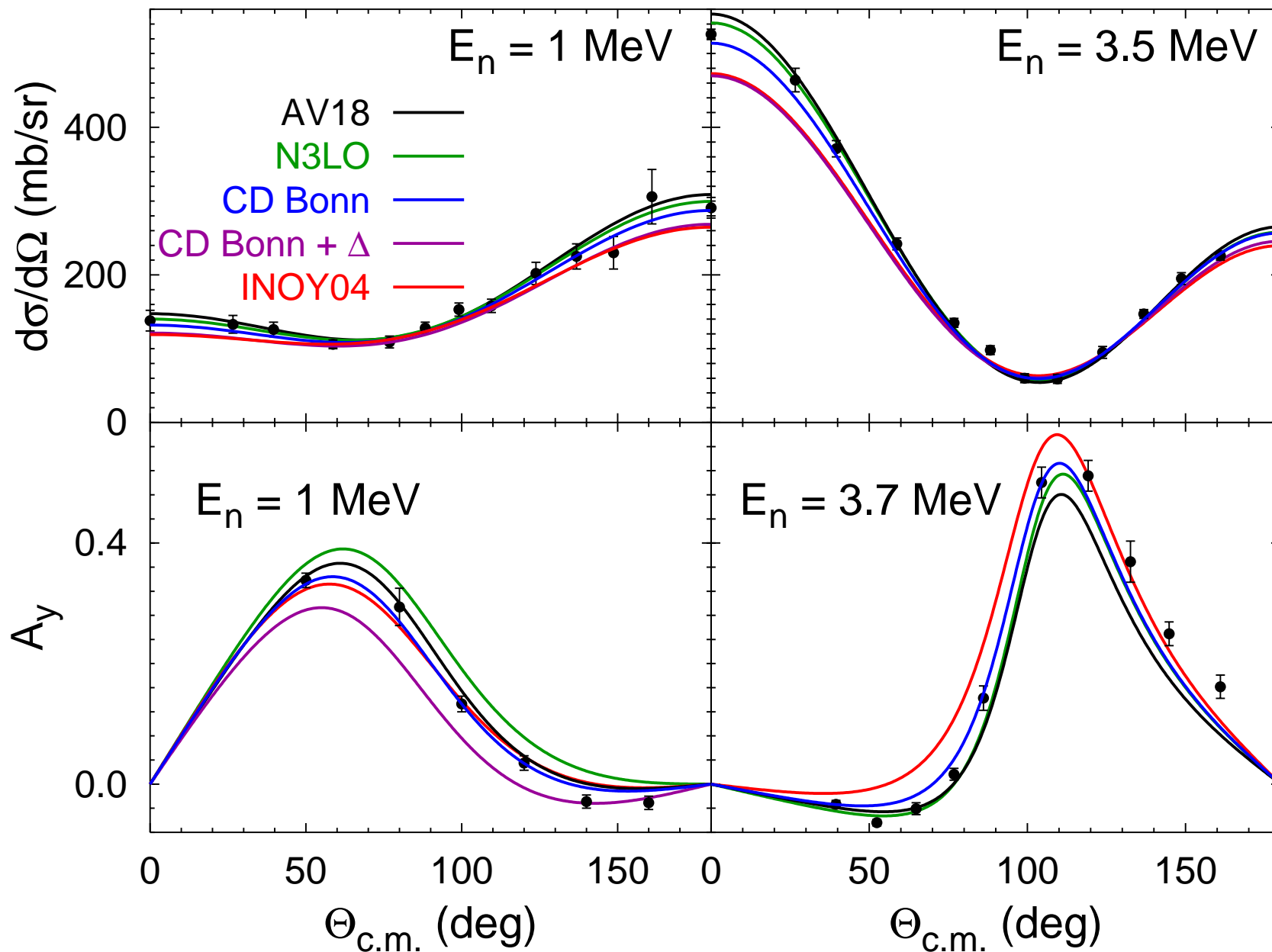
higher order 3N force



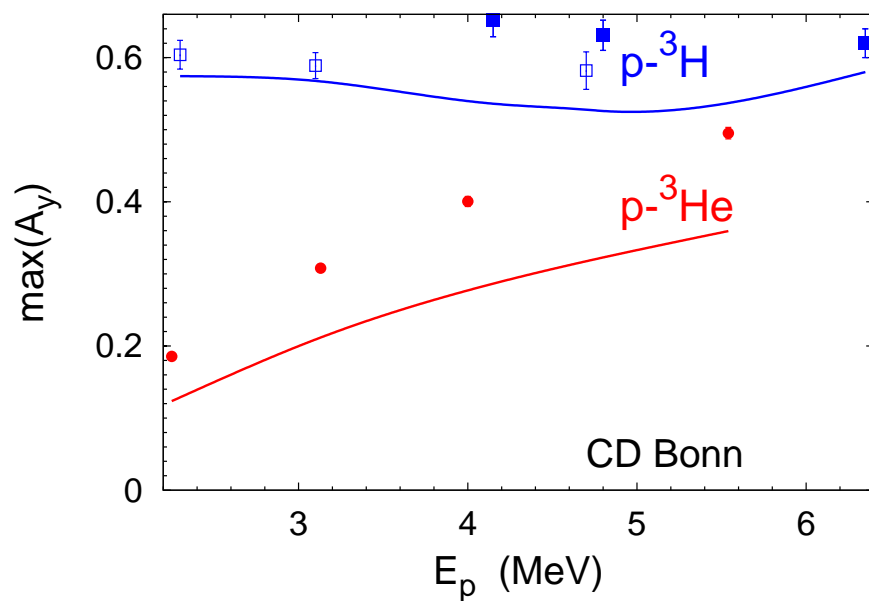
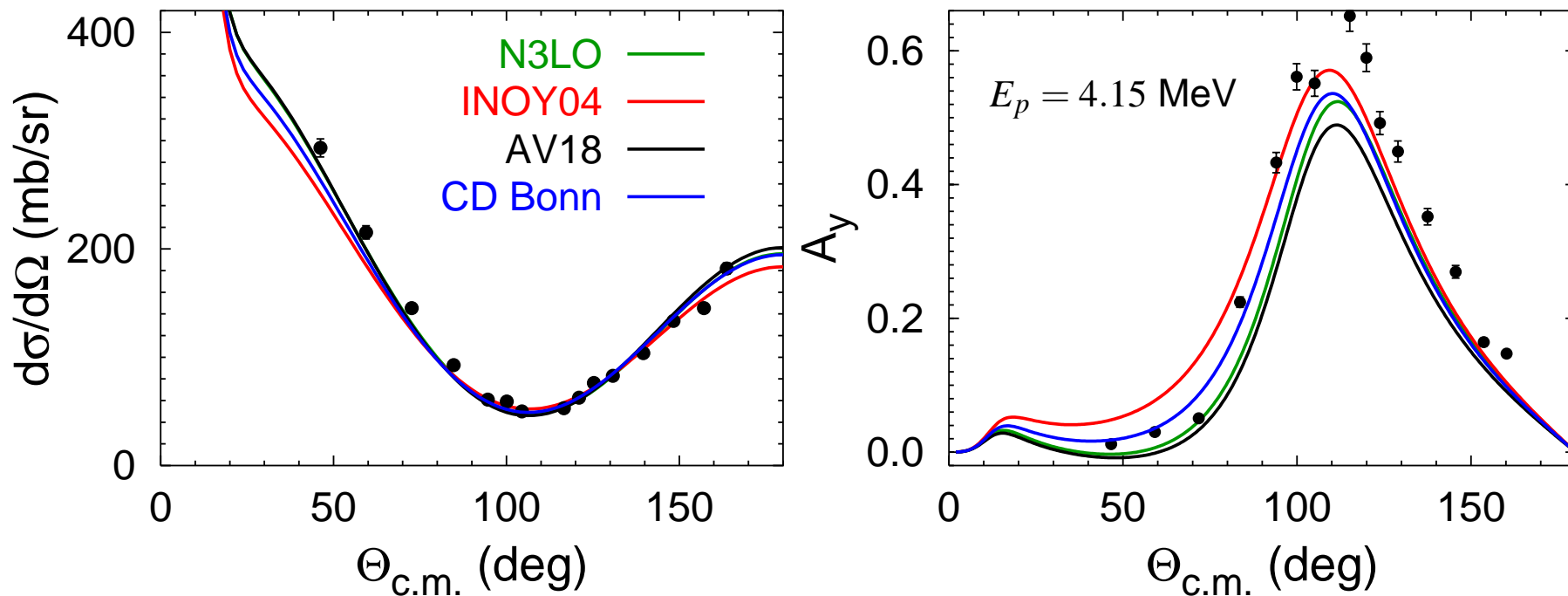
4N force



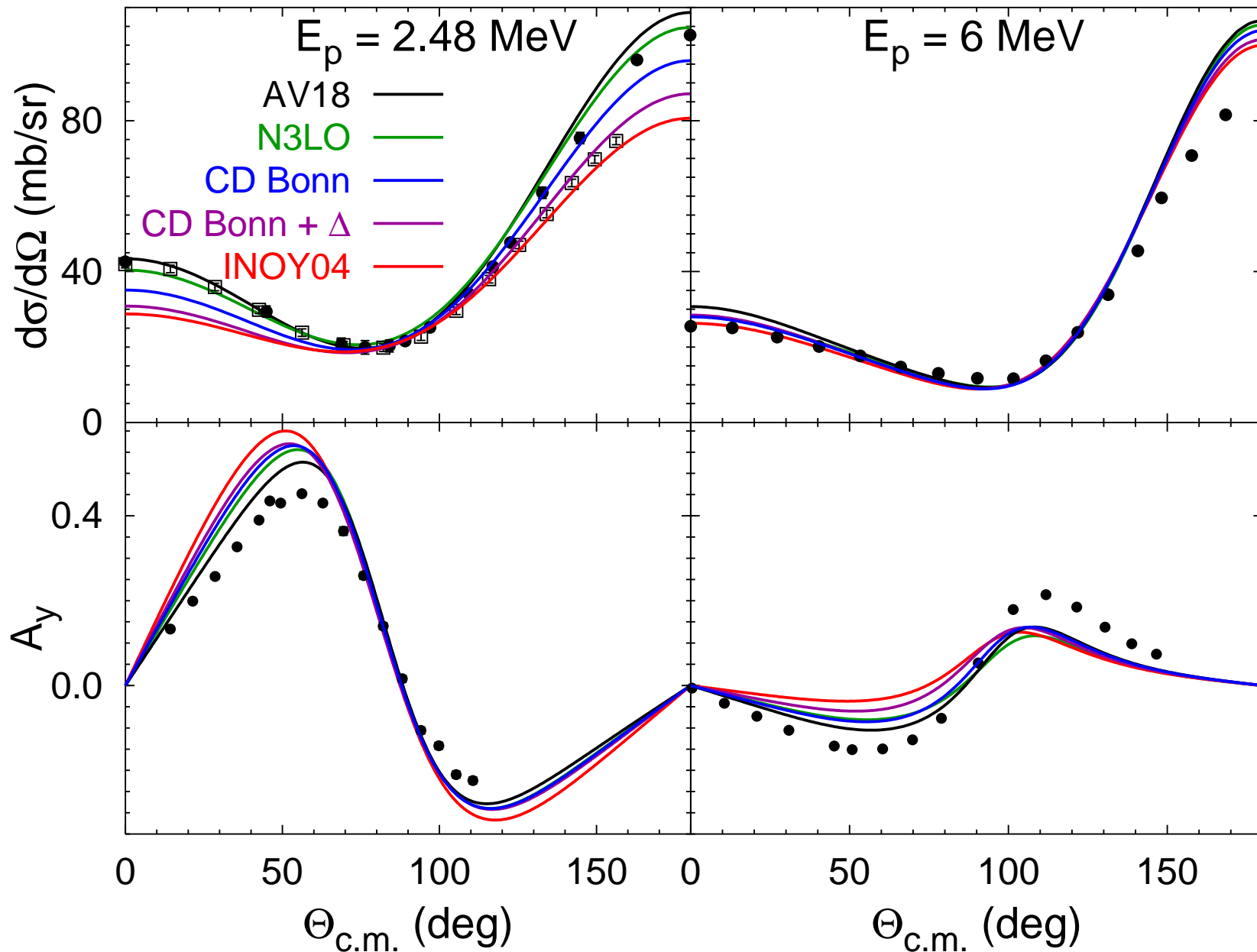
n - ^3He elastic scattering



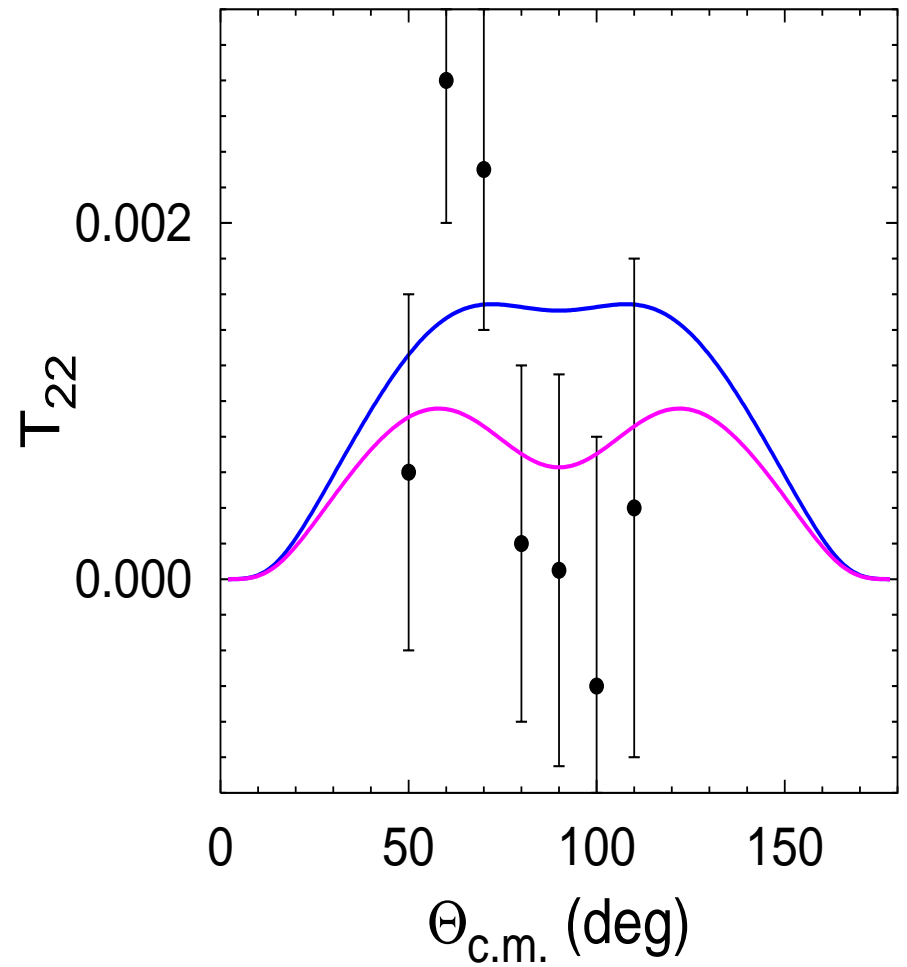
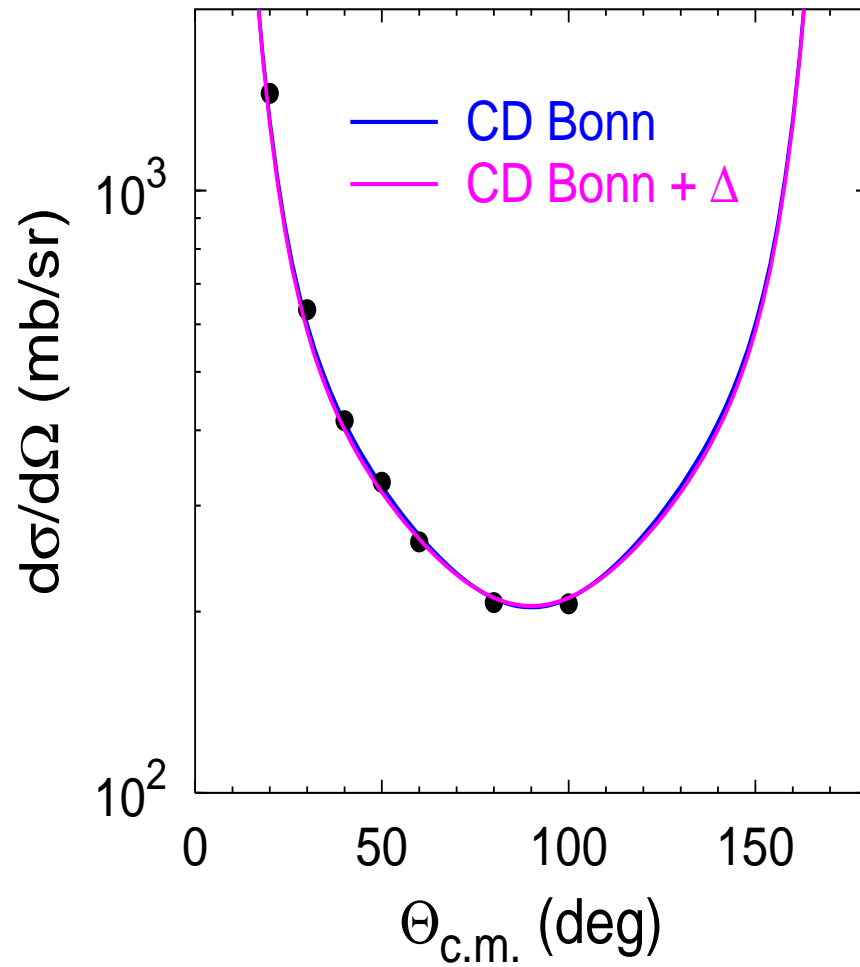
p - ^3H elastic scattering



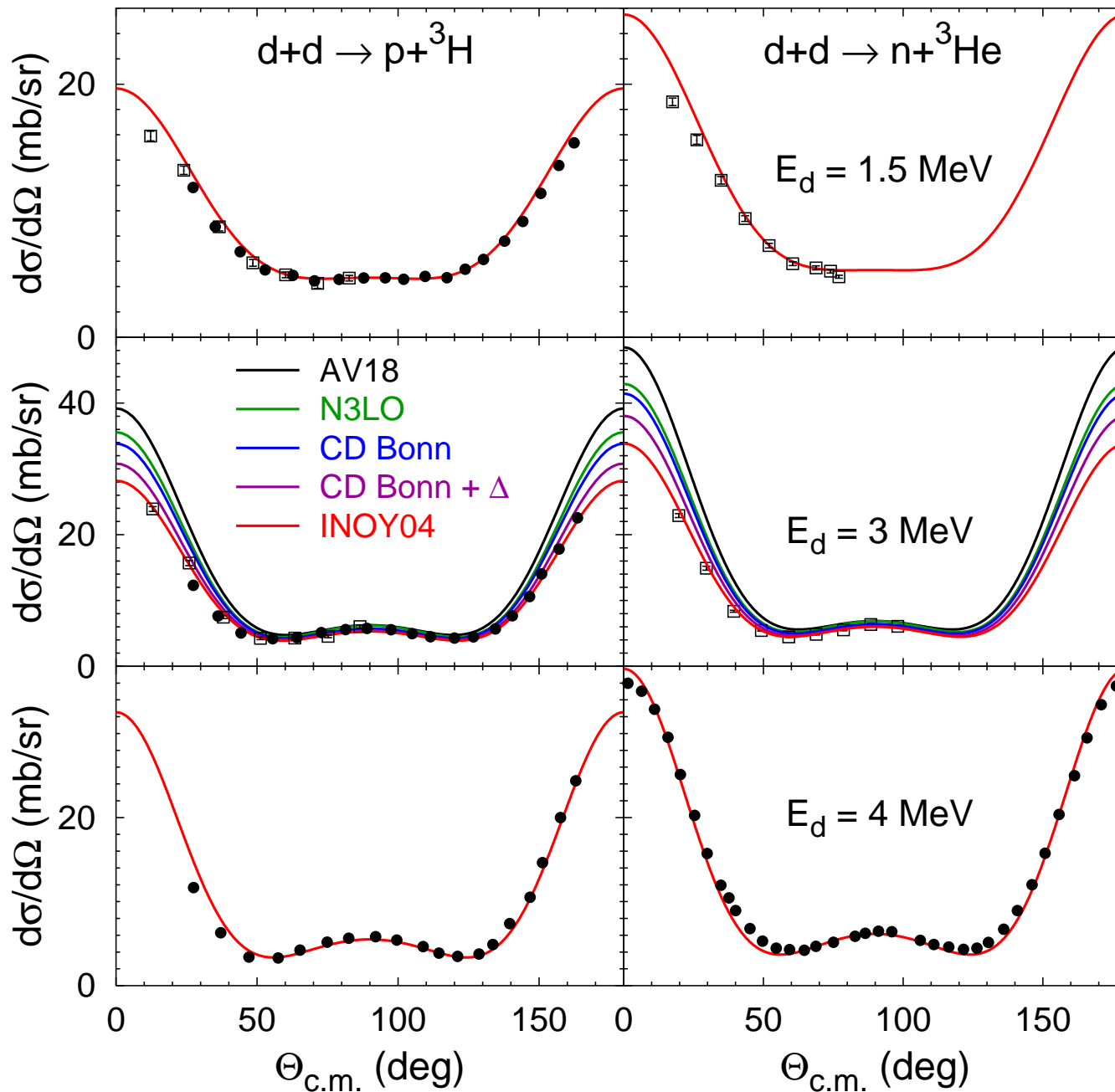
Charge exchange reaction ${}^3\text{H}(p, n){}^3\text{He}$



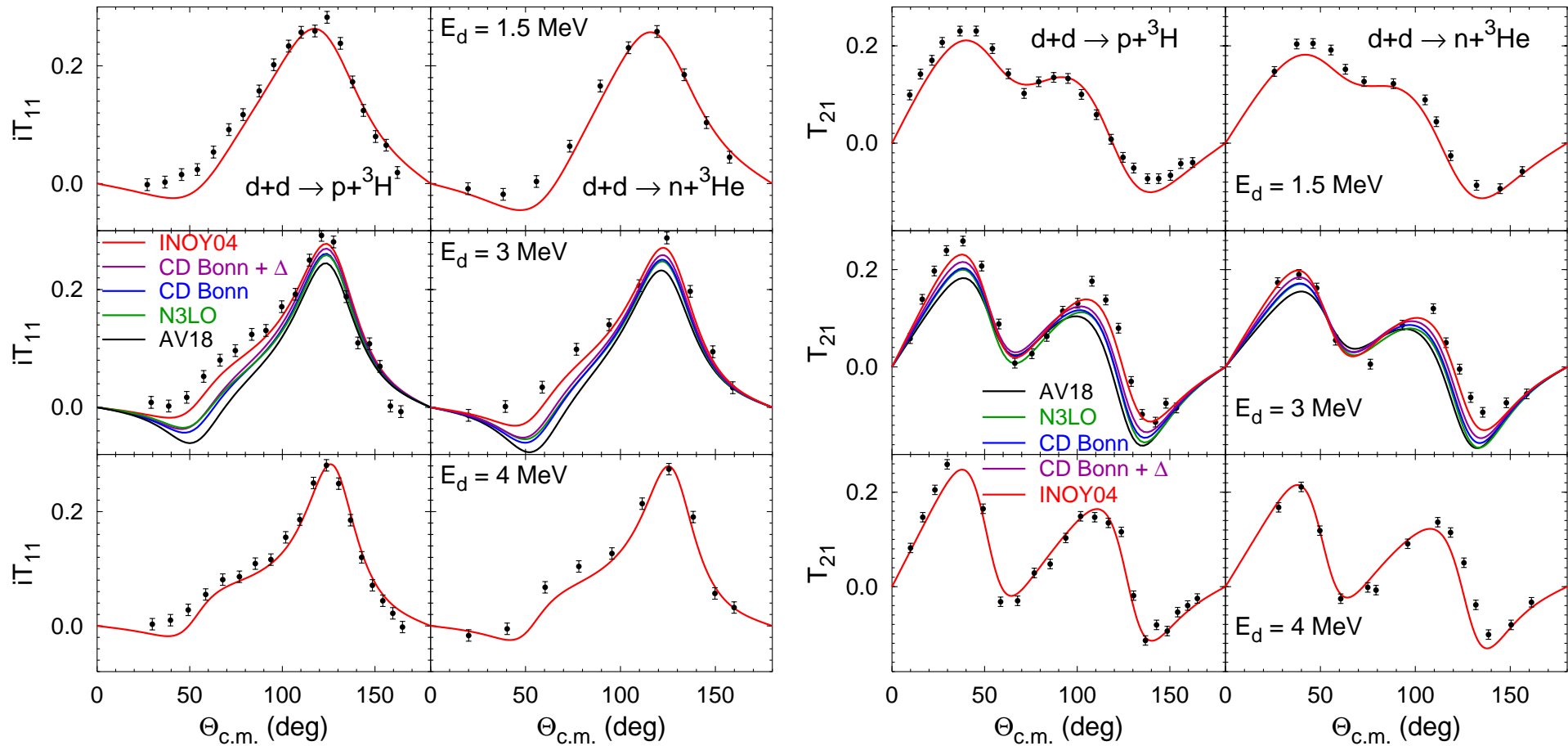
d - d elastic scattering at $E_d = 3$ MeV



${}^2\text{H}(d,p){}^3\text{H}$ and ${}^2\text{H}(d,n){}^3\text{He}$



${}^2\text{H}(d,p){}^3\text{H}$ and ${}^2\text{H}(d,n){}^3\text{He}$



Above breakup: additional singularities in AGS equations

deuteron bound state poles

$$t \rightarrow \frac{v|\phi_d\rangle\langle\phi_d|v}{E + i\varepsilon - e_d - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

free resolvent

$$G_0 \rightarrow \frac{1}{E + i\varepsilon - k_x^2/2\mu_j^x - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

Above breakup: additional singularities in AGS equations

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free resolvent

$$G_0 \rightarrow \frac{1}{E + i\varepsilon - k_x^2/2\mu_j^x - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

treated by complex-energy method:

1. solve for $U_{fi}(E + i\varepsilon)$ with finite $\varepsilon = \varepsilon_1, \dots, \varepsilon_n$
2. extrapolate to $\varepsilon \rightarrow 0$ for physical amplitudes $U_{fi}(E + i0)$

[H. Kamada *et al*, Prog. Theor. Phys. 109, 869L (2003)]

Integration with special weights

accuracy & efficiency of the complex-energy method is greatly improved by a special integration

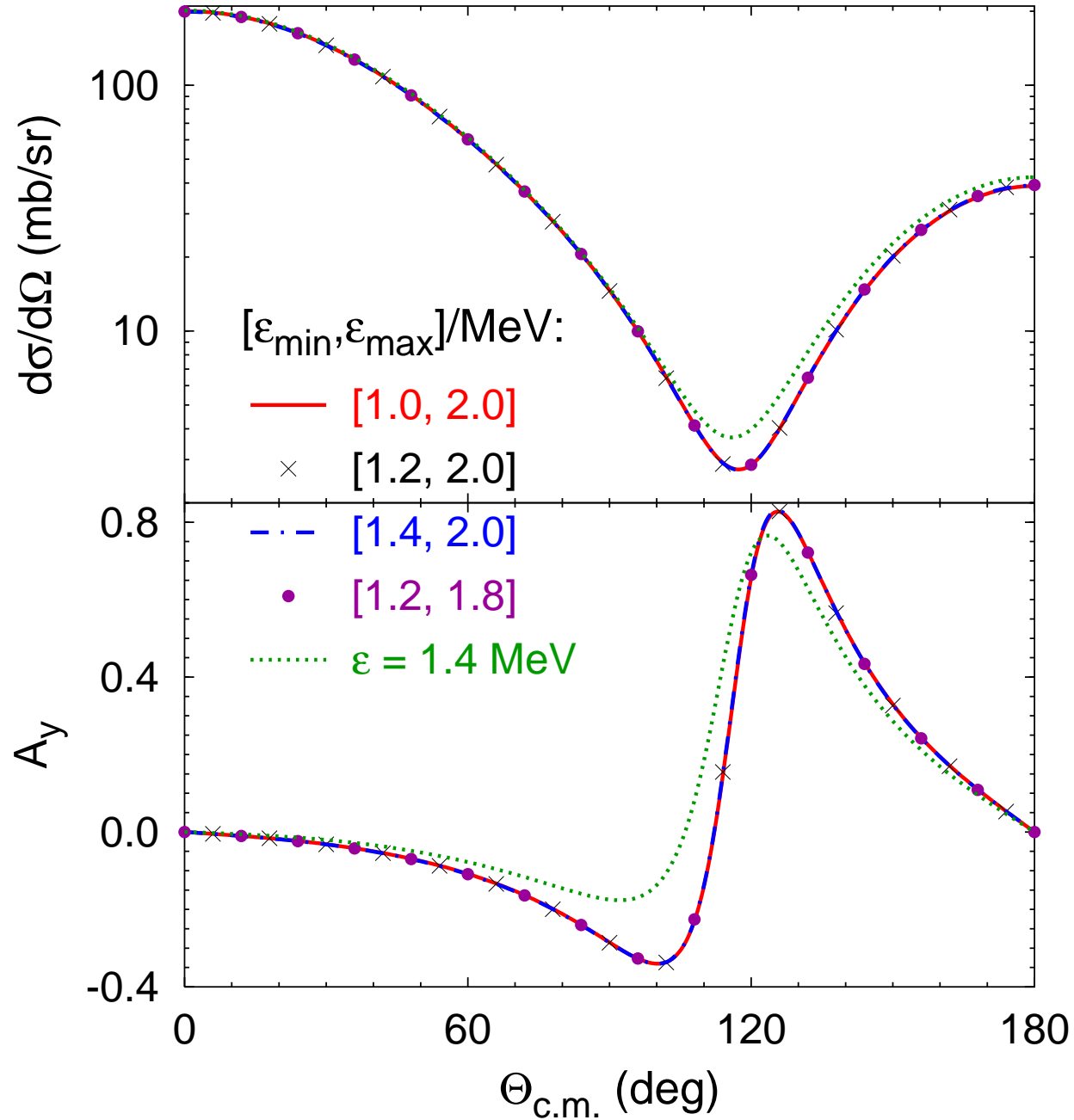
$$\int_a^b \frac{f(x)}{x_0^n + iy_0 - x^n} dx \approx \sum_{j=1}^N f(x_j) w_j(n, x_0, y_0, a, b)$$

where the quasi-singular factor is absorbed into special weights

$$w_j(n, x_0, y_0, a, b) = \int_a^b \frac{S_j(x)}{x_0^n + iy_0 - x^n} dx$$

that may be calculated using spline functions $\{S_j(x)\}$ for standard Gaussian grid $\{x_j\}$ [PRC 86, 011001]

Extrapolation $\varepsilon \rightarrow 0$: $n+{}^3\text{H}$ at 22.1 MeV

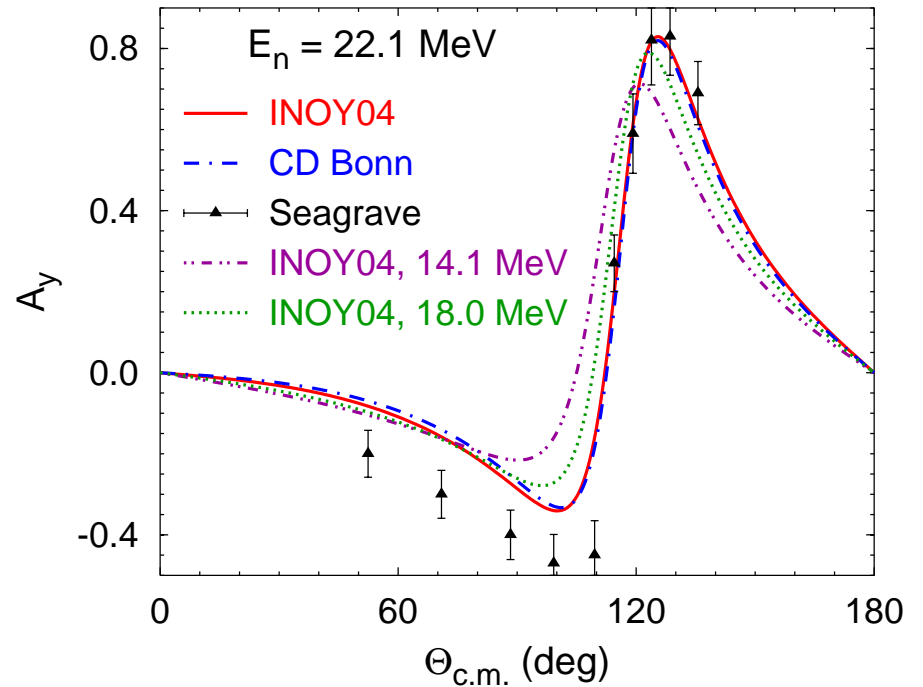
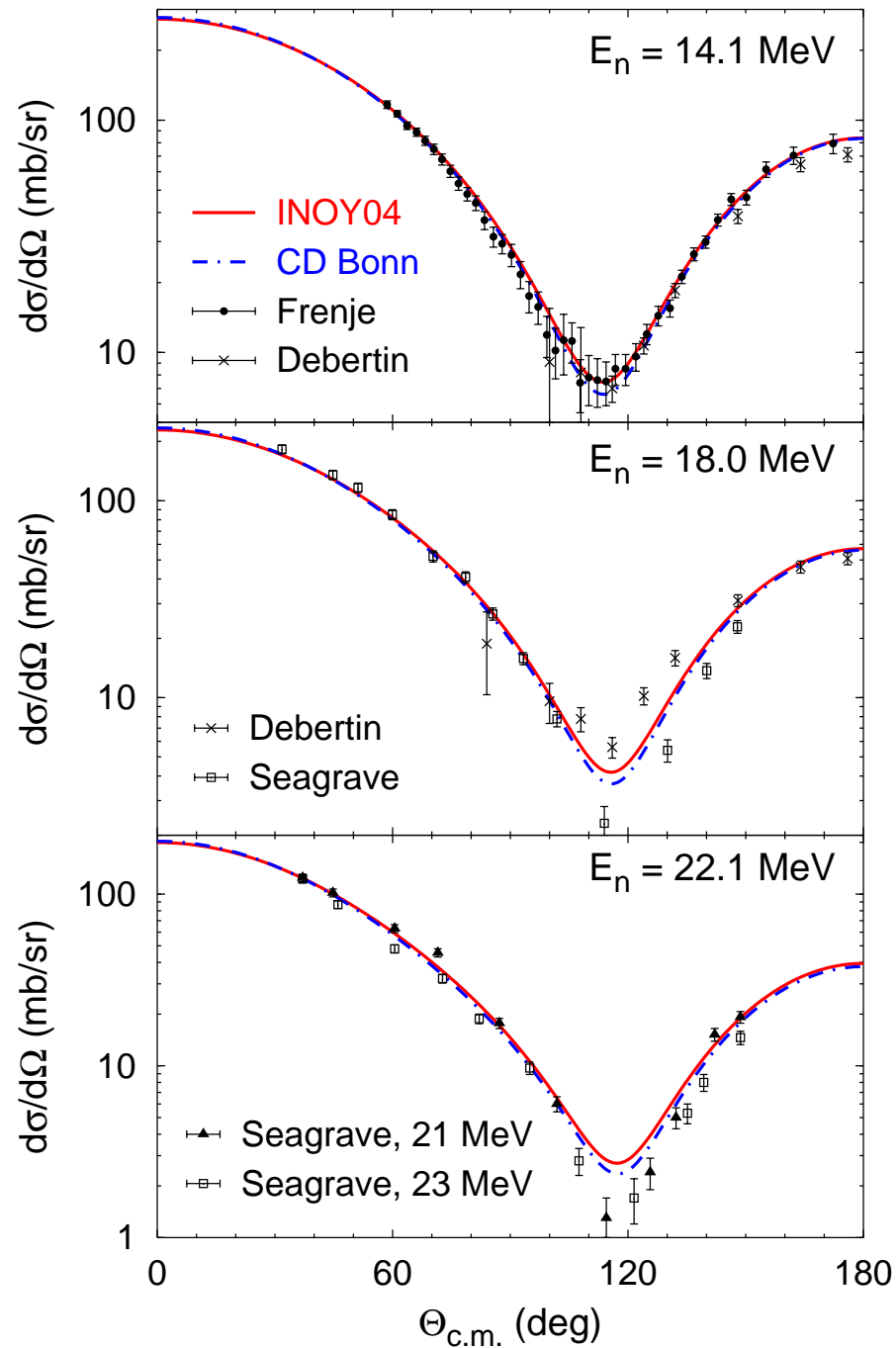


Extrapolation $\varepsilon \rightarrow 0$: $n+{}^3\text{H}$ at 22.1 MeV

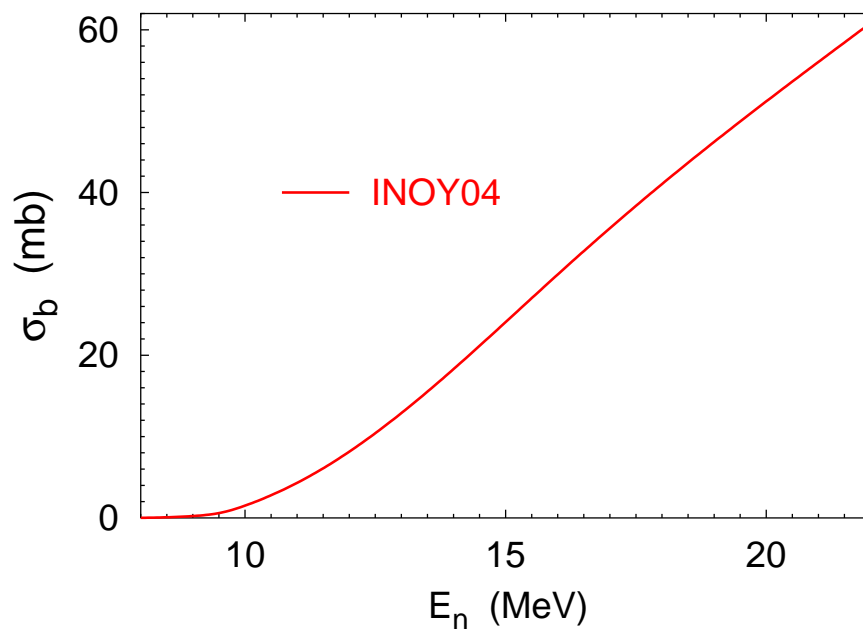
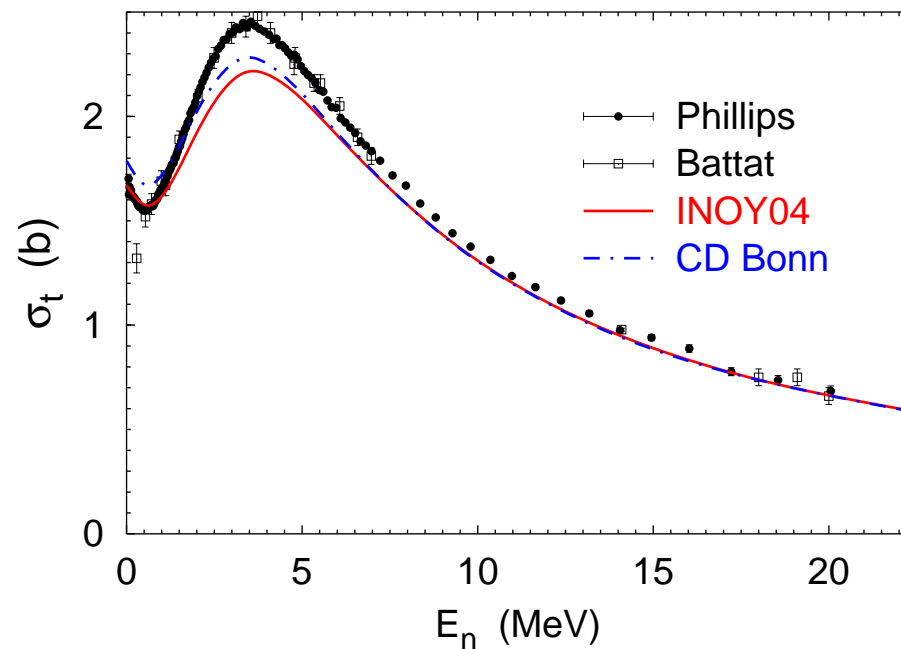
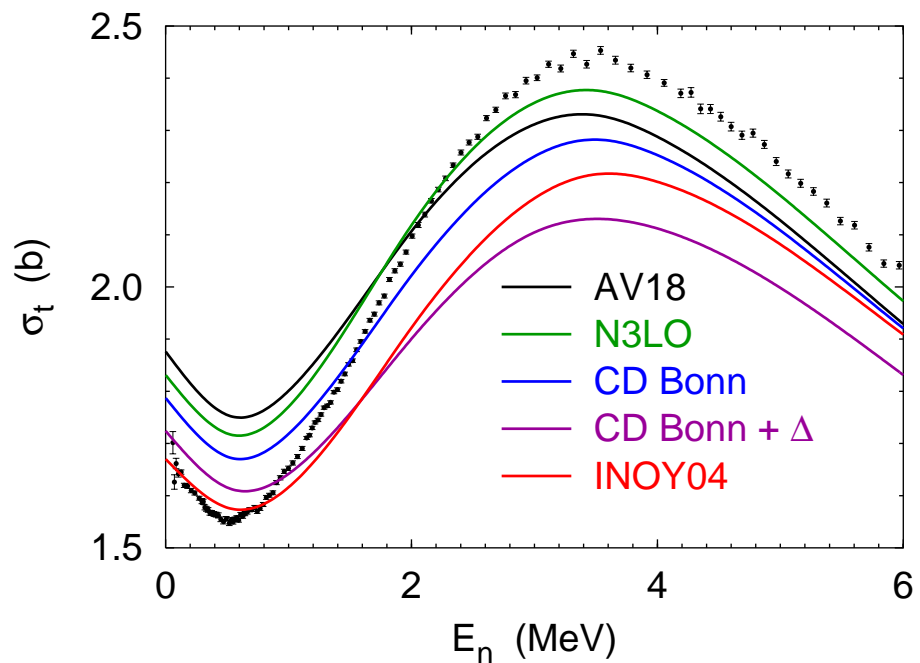
$[\varepsilon_{\min}, \varepsilon_{\max}]$	$\delta({}^1S_0)$	$\eta({}^1S_0)$	$\delta({}^3P_0)$	$\eta({}^3P_0)$	$\delta({}^3P_2)$	$\eta({}^3P_2)$
[1.0, 2.0]	62.63	0.990	43.03	0.959	65.27	0.950
[1.2, 2.0]	62.60	0.991	43.04	0.959	65.29	0.951
[1.4, 2.0]	62.67	0.991	43.03	0.958	65.27	0.950
[1.2, 1.8]	62.65	0.992	43.03	0.959	65.28	0.950
1.4	73.37	0.916	44.77	0.840	67.38	0.933

[PRC 86, 011001]

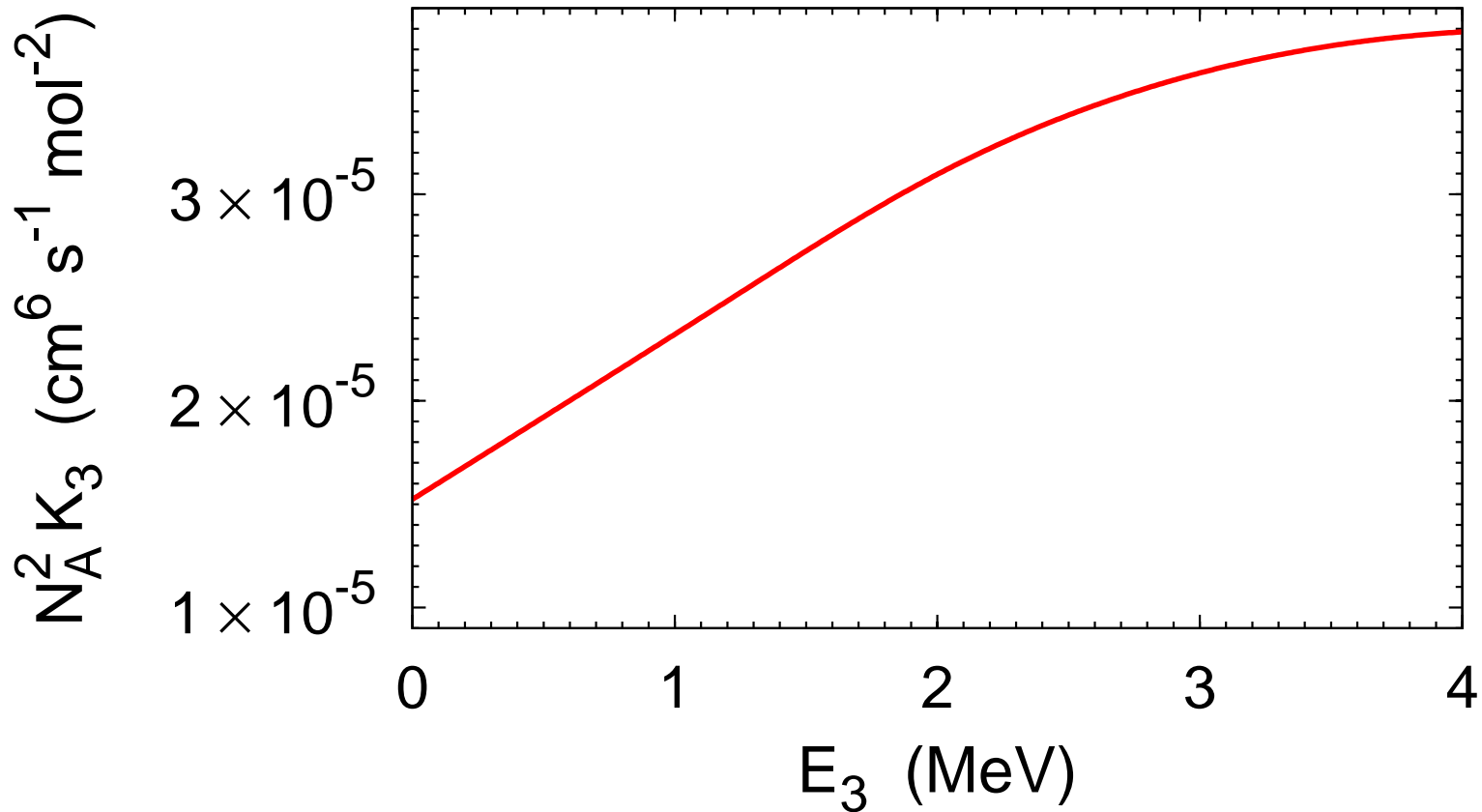
$n+{}^3\text{H}$ elastic scattering



$n+{}^3\text{H}$ total and breakup cross sections

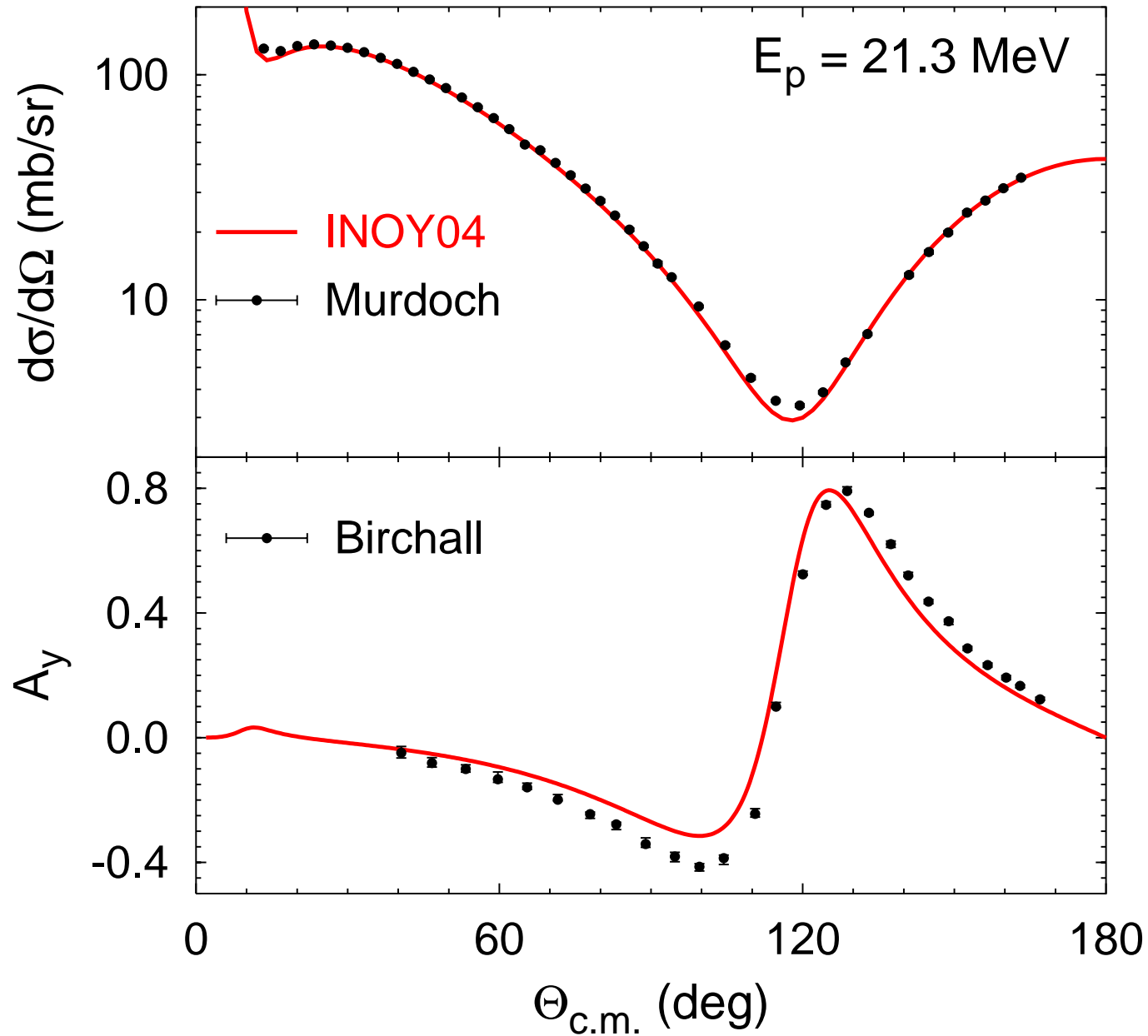


Recombination reaction ${}^2\text{H} + \text{n} + \text{n} \rightarrow \text{n} + {}^3\text{H}$

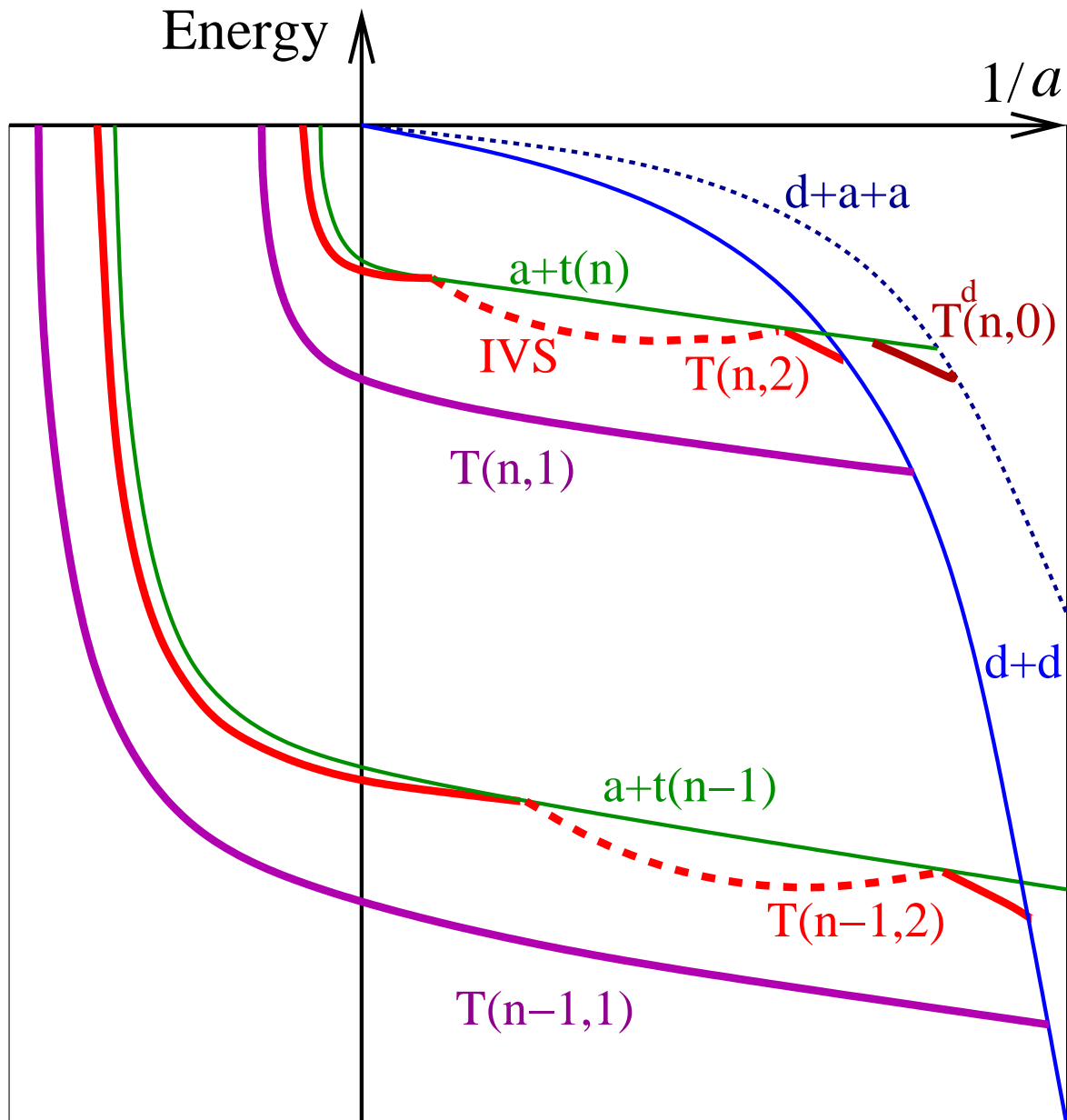


$$\frac{d\rho_t}{dt} = K_2^\gamma \rho_d \rho_n + K_3 \rho_d \rho_n^2 + \dots$$

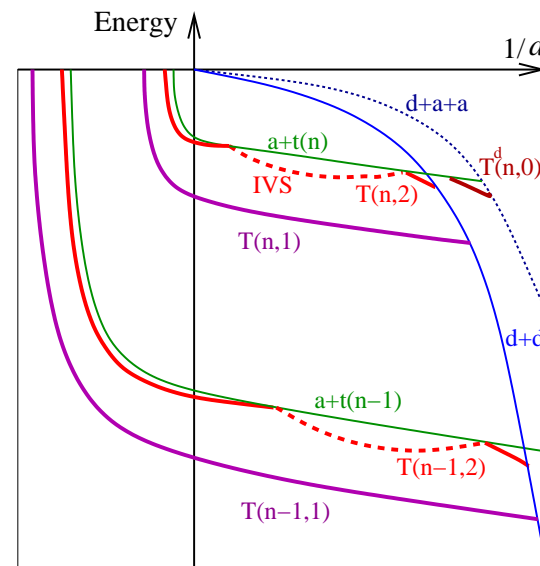
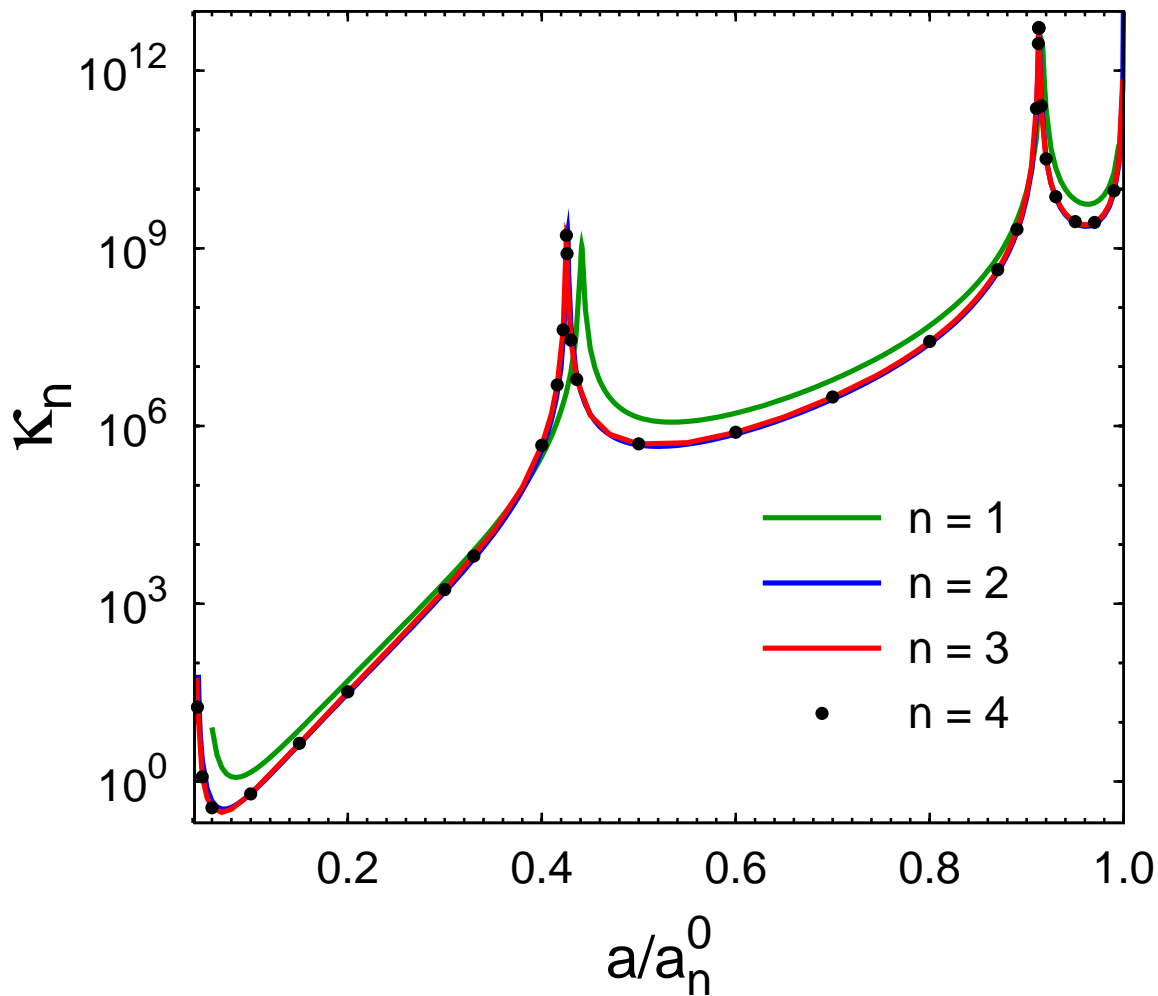
$p+{}^3\text{He}$ elastic scattering



4-boson system with resonant interactions



Four-atom recombination at threshold



$$a_n^0 : b_n = 0$$

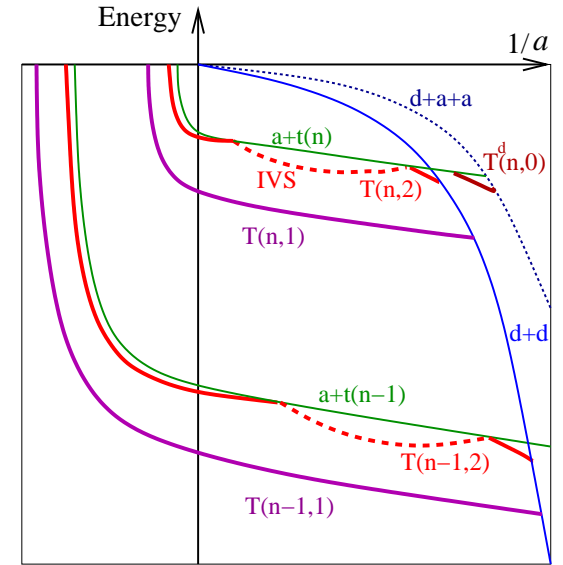
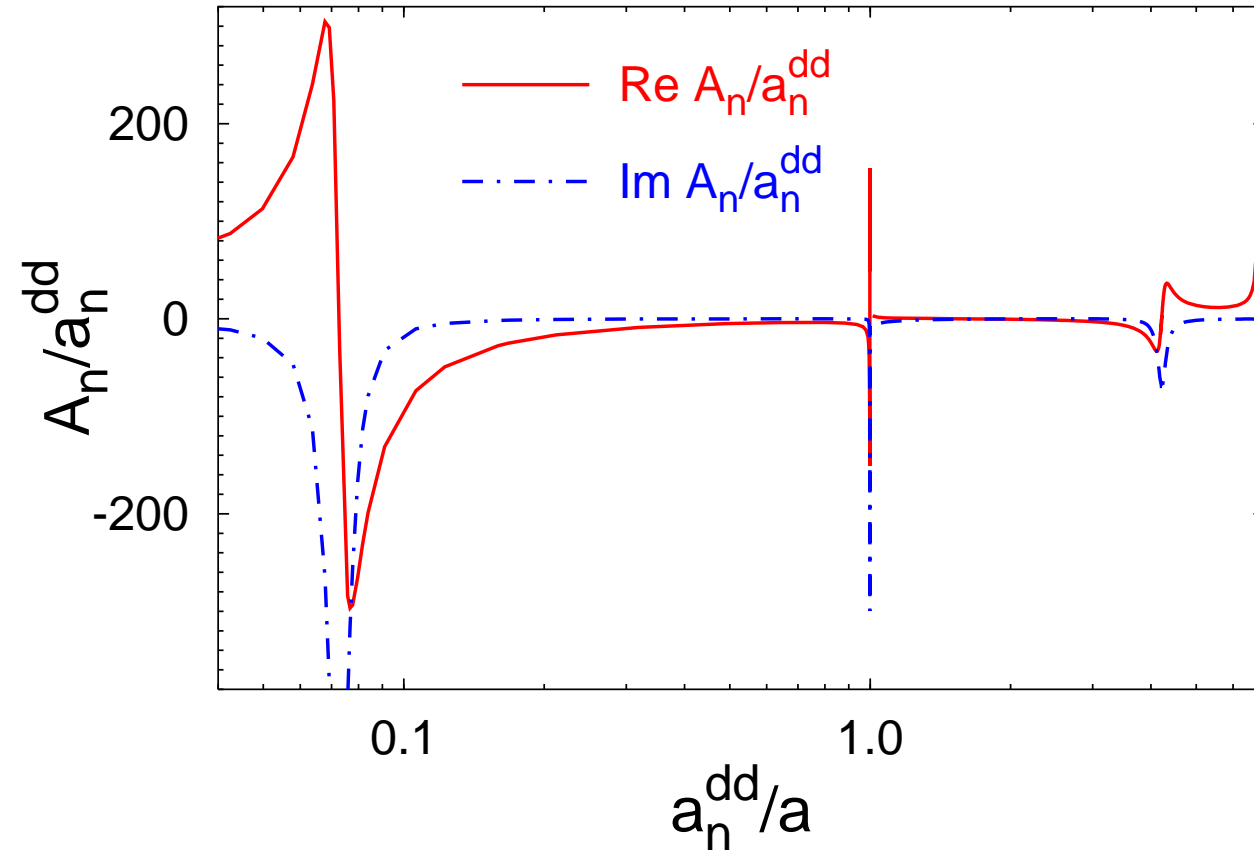
$$a_{n,k}^0 : B_{n,k} = 0$$

$$a_{n,1}^0 / a_n^0 = 0.4254$$

$$a_{n,2}^0 / a_n^0 = 0.9125$$

[PRA 85, 012708]

Atom-trimer scattering length



$$a_n^{dd} : b_n = 2b_d$$

[EPL 95, 43002, PRA 85, 042705]

Summary

- 3/4-body Faddeev/AGS equations in momentum space
- complex-energy method
with special integration weights

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- 3/4-body Faddeev/AGS equations in momentum space
- complex-energy method
with special integration weights
- 3N scattering
- 3-body nuclear reactions
- 4N scattering
- universal 4-boson physics