High-momentum tails from low-momentum theories

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In collaboration with:

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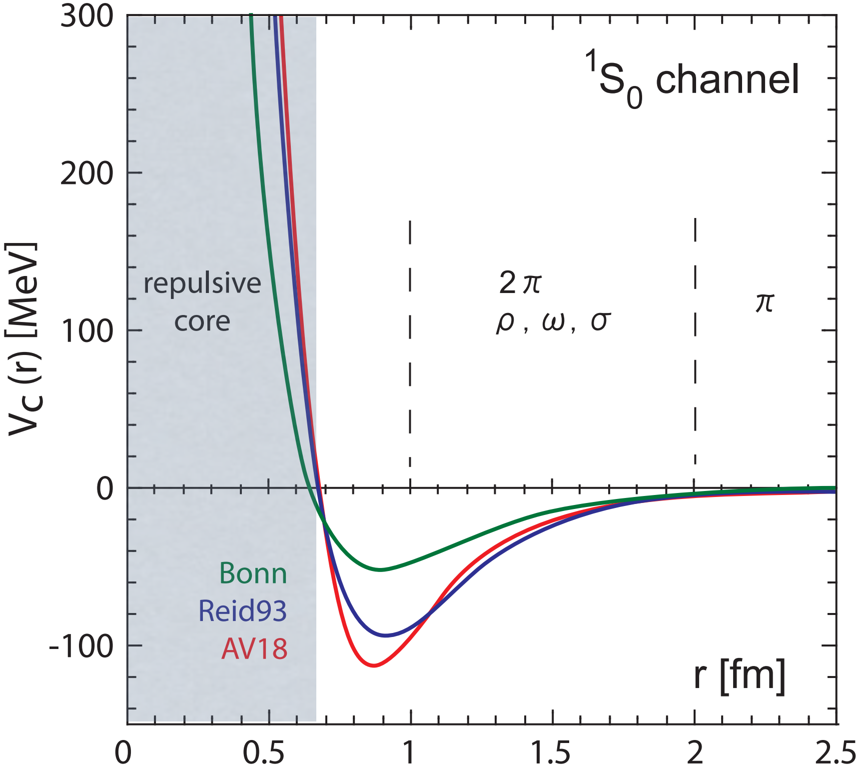
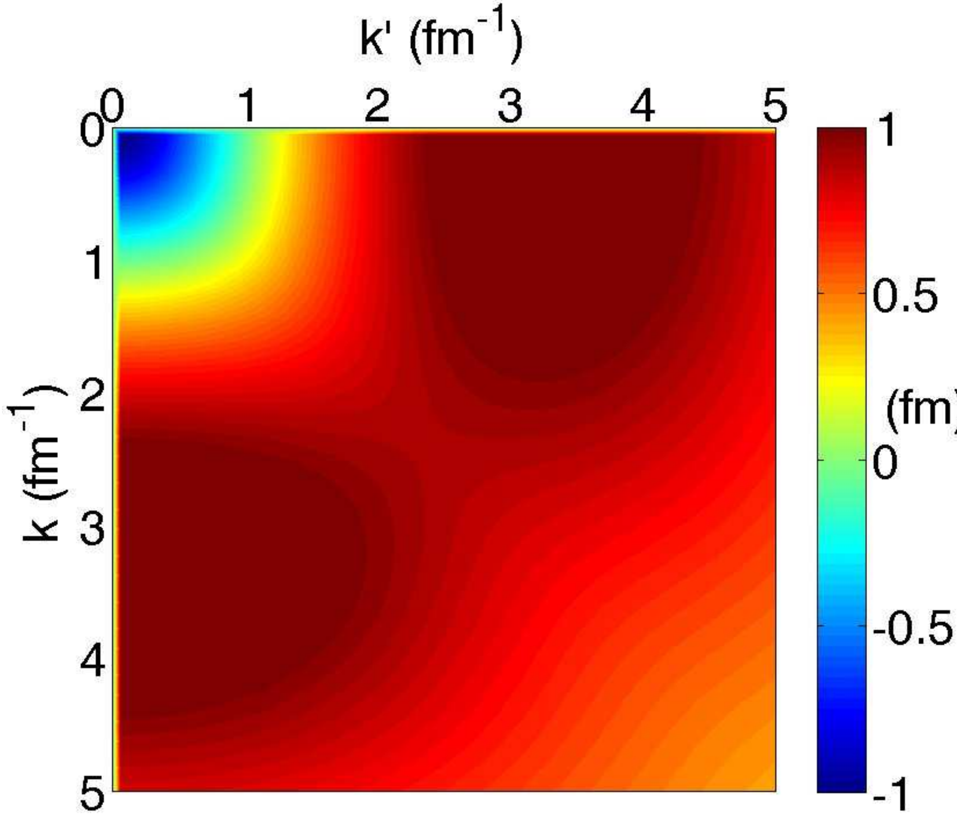
Dietrich Roscher

Anderson et al., PRC **82** 054001 (2010)
SKB and D. Roscher, arXiv:1208.1734

Why are nuclear many-body problems hard?

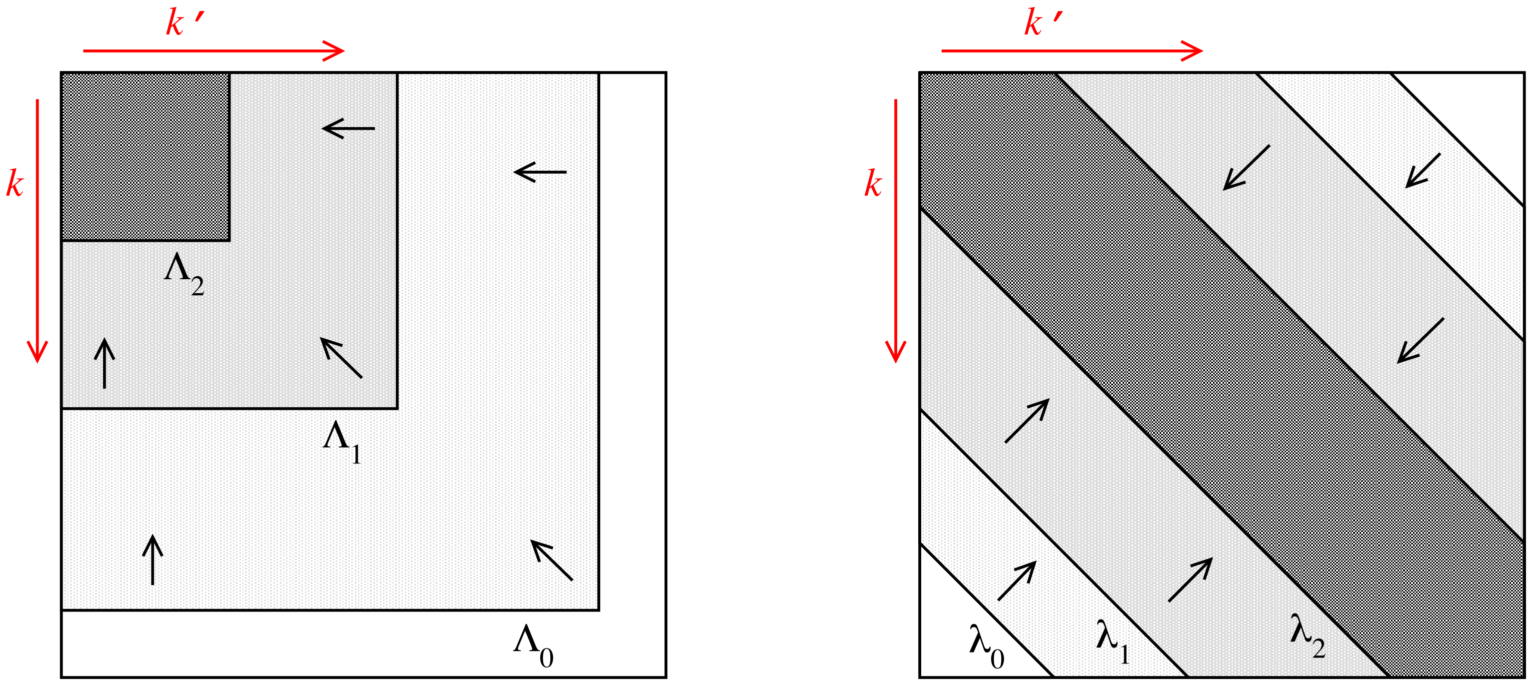




Coupling of low/high-k modes: non-perturbative, strong correlations,...

Remedy: Use RG to **decouple**

Renormalization Group Transformations

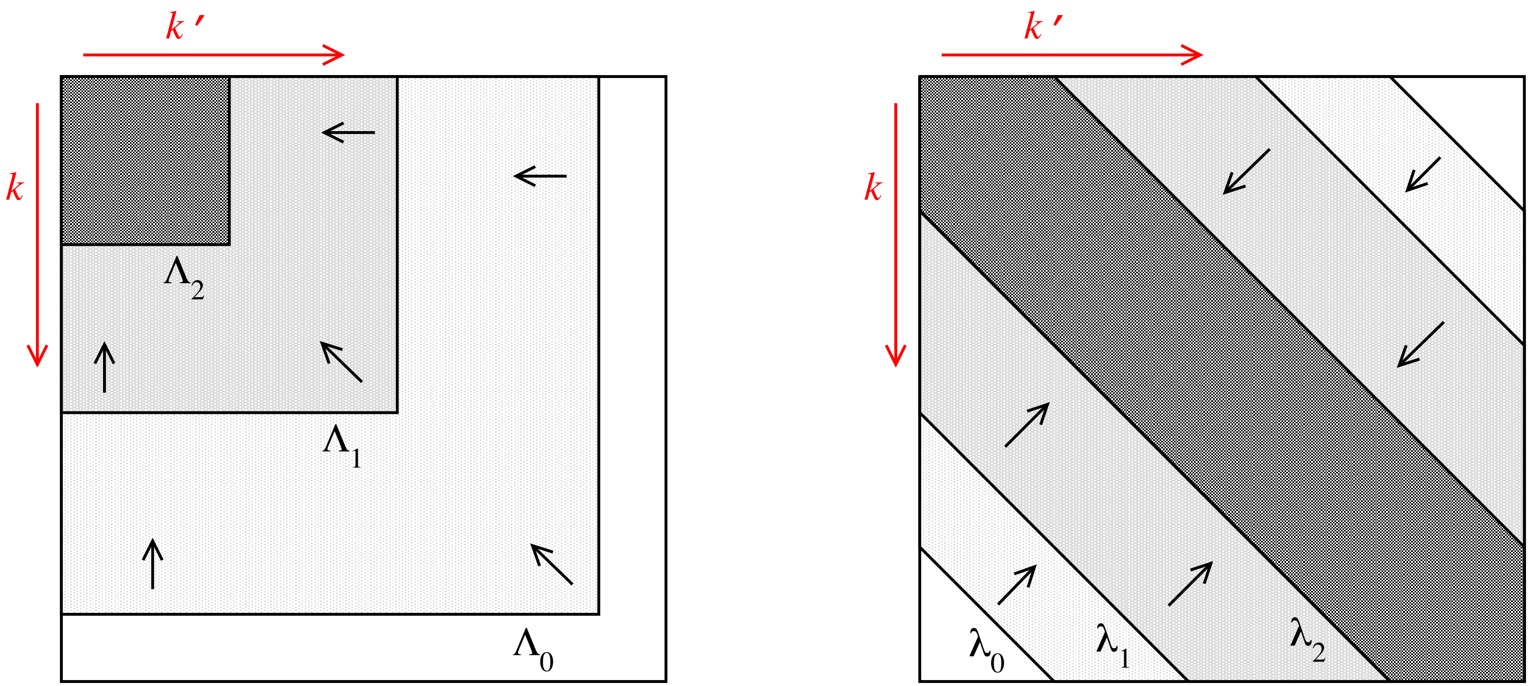


Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. **65** (2010)

Weaker correlations, faster convergence, more perturbative

(cf. talks of Holt, Schwenk, Roth, Hergert, Hebeler, Quaglioni, Furnstahl, Vary, Langhammer)

Renormalization Group Transformations



Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. **65** (2010)

What about operators other than H(Λ)? If Ψ(Λ) "simple" at lower Λ, are evolved operators *O*(Λ) more complicated?

What about high-q operators? What happens if q >> Λ?

The Similarity Renormalization Group

Unitary transformation via flow equations:

$$\frac{dH_\lambda}{d\lambda} = [\eta(\lambda), H_\lambda] \qquad \text{with} \qquad \eta(\lambda) \equiv \frac{dU(\lambda)}{d\lambda} U^\dagger(\lambda)$$

Engineer η to do different things as λ => 0

$$\lambda \equiv s^{-1/4}$$

$$\eta(\lambda) = [\mathcal{G}_\lambda, H_\lambda]$$

$\mathcal{G}_\lambda = T \;\Rightarrow\;$ H_λ driven towards diagonal in k – space

⋮

Rule of thumb: Take $\mathcal{G}_\lambda = H_\lambda^D$ where $H_\lambda = H_\lambda^D + H_\lambda^{OD}$

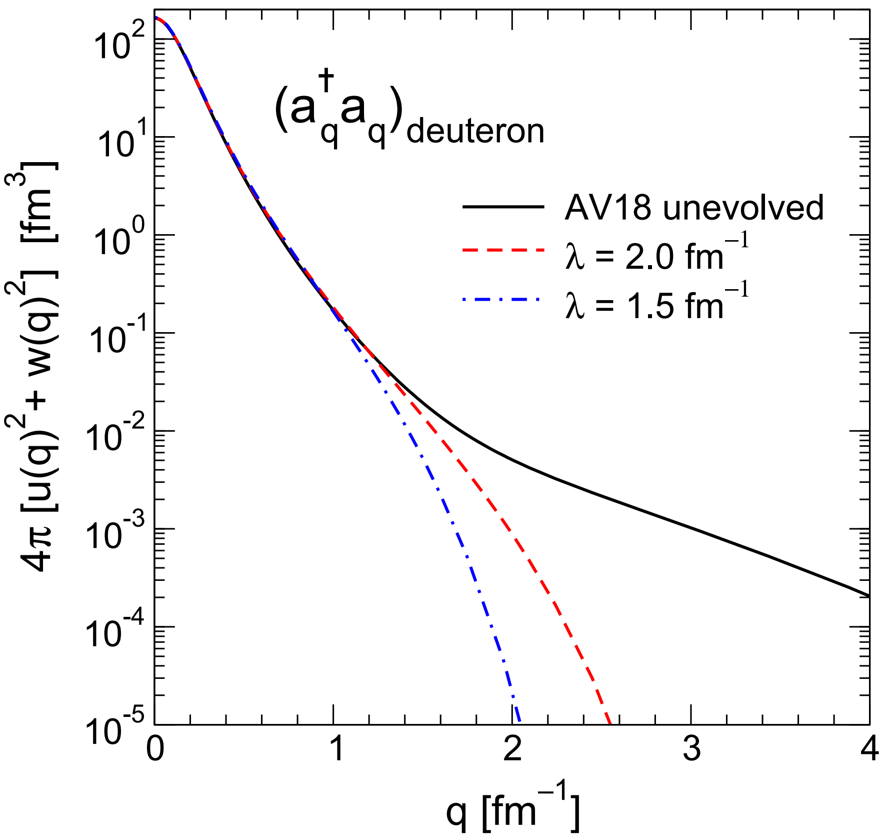
All Operators Evolve

Expectation values of bare $(a^\dagger a)_q$ "run" with λ

Expectation values of ***evolved*** operators are λ-independent

$$O_\lambda = U_\lambda O U_\lambda^\dagger$$

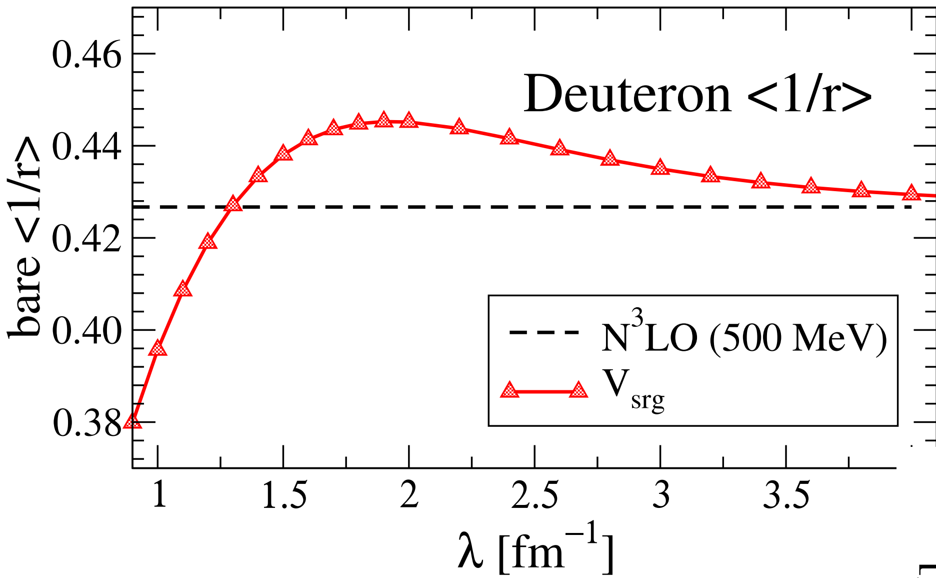
$$\therefore \frac{dO_\lambda}{d\lambda} = [\eta_\lambda, O_\lambda]$$

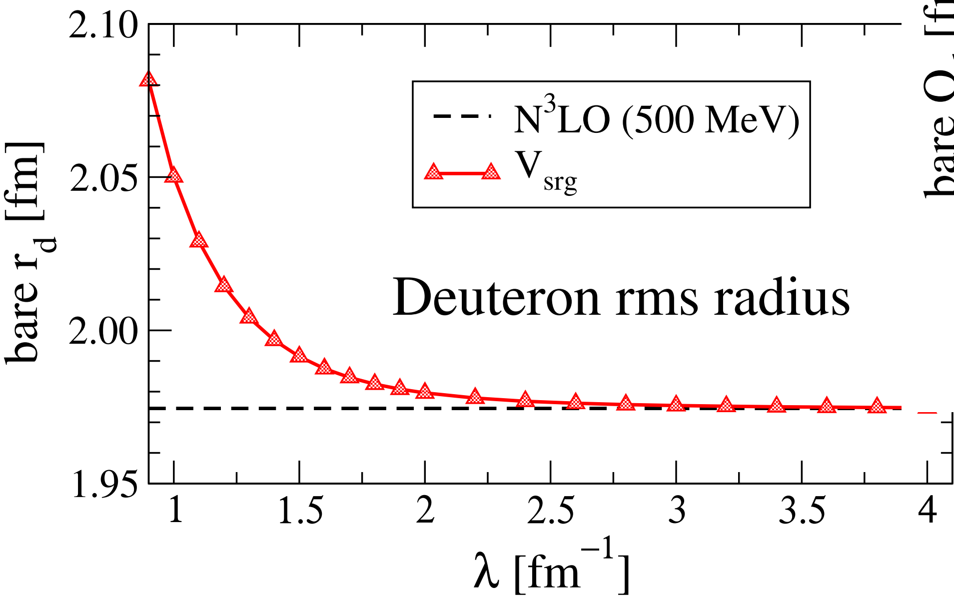


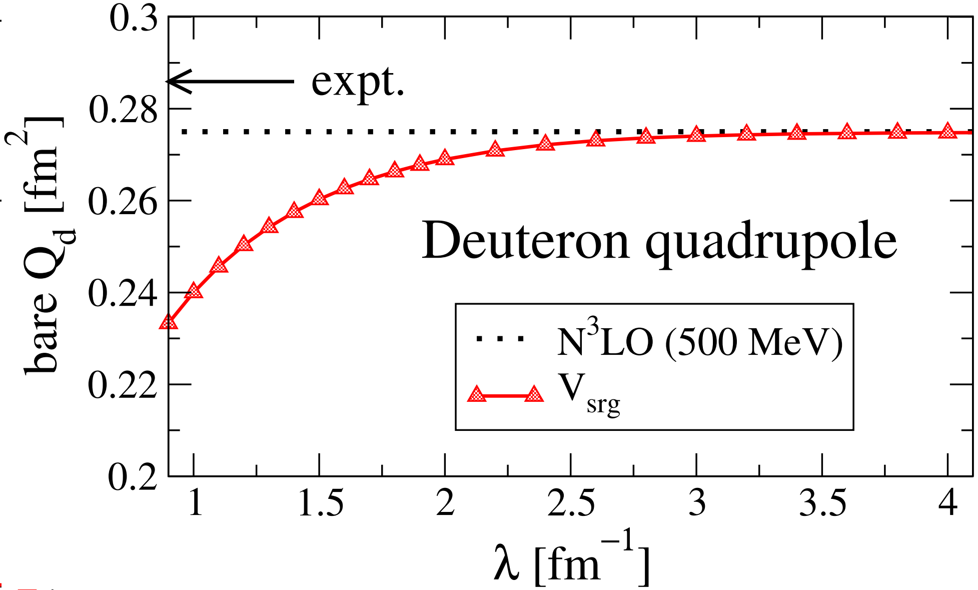
Stronger renormalization for operators sensitive to high-momentum physics

Long-distance physics =>
weak running
dominated by w.f.
renormalization

$$O_\lambda \approx Z_\lambda^2 \, O$$

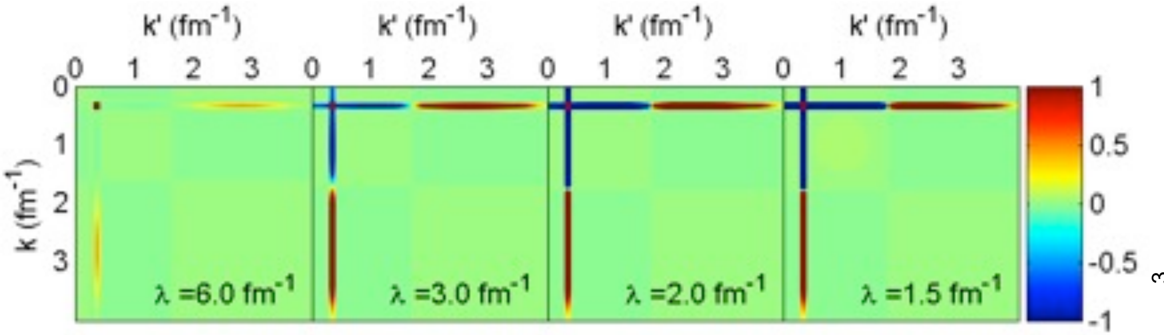




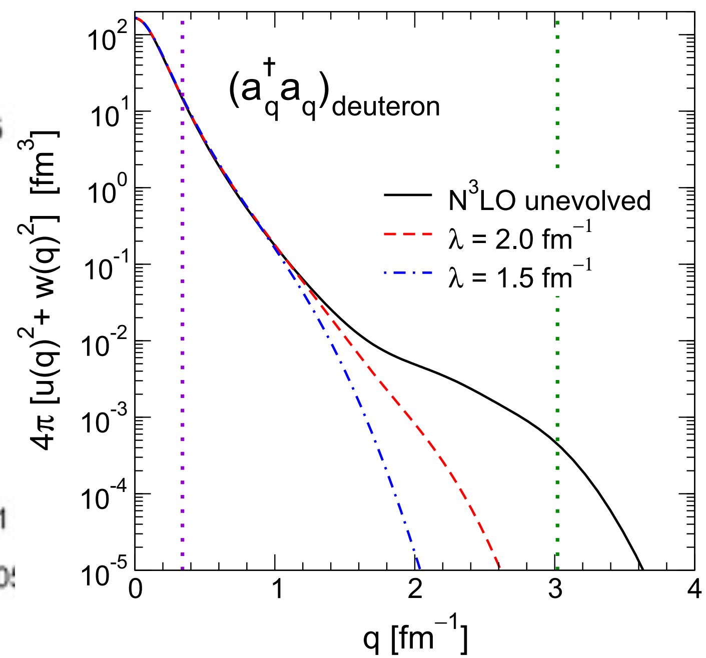


High and low momentum operators in deeton

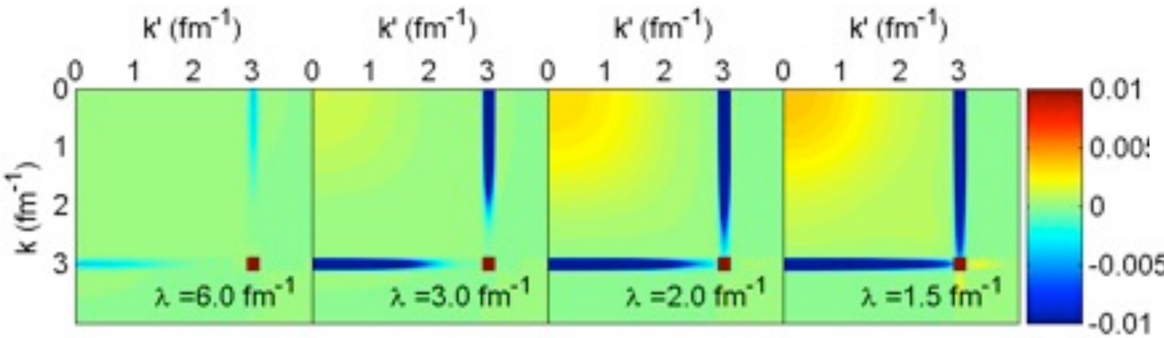
- **M.E.'s of** $\left(U_\lambda a_q^\dagger a_q U_\lambda^\dagger\right)_{kk'}$ **for** $q = 0.35\,\mathrm{fm}^{-1}$



Momentum Distribution



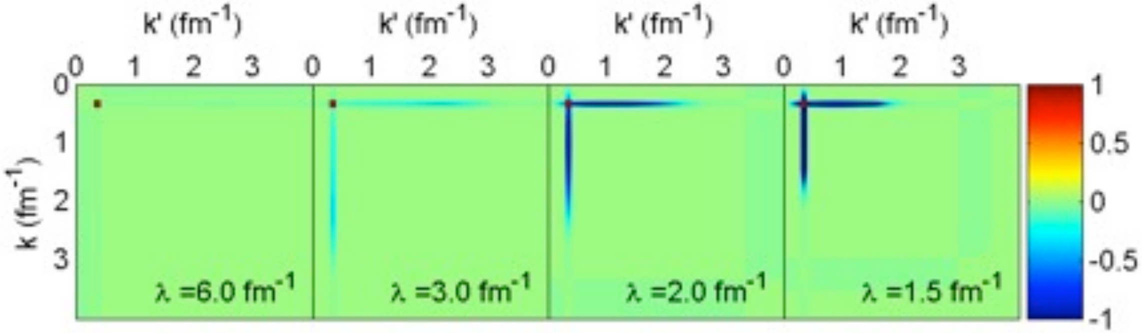
- **M.E.'s of** $\left(U_\lambda a_q^\dagger a_q U_\lambda^\dagger\right)_{kk'}$ **for** $q = 3.0\,\mathrm{fm}^{-1}$



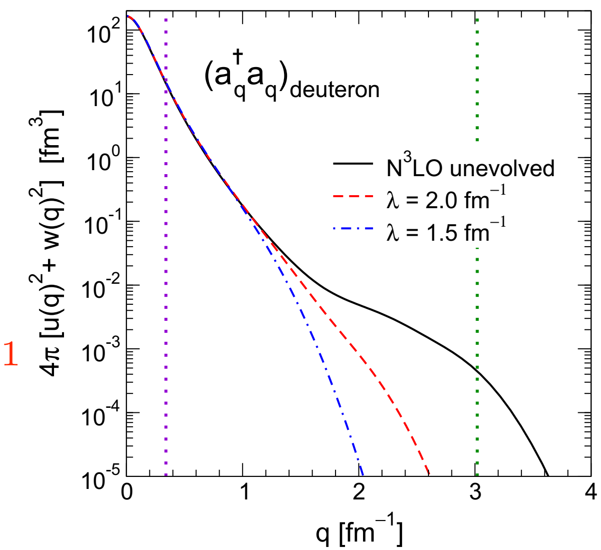
Operator structure changes (especially high q) substantially, but integrated expectation values invariant.

High and low momentum operators in deuteron

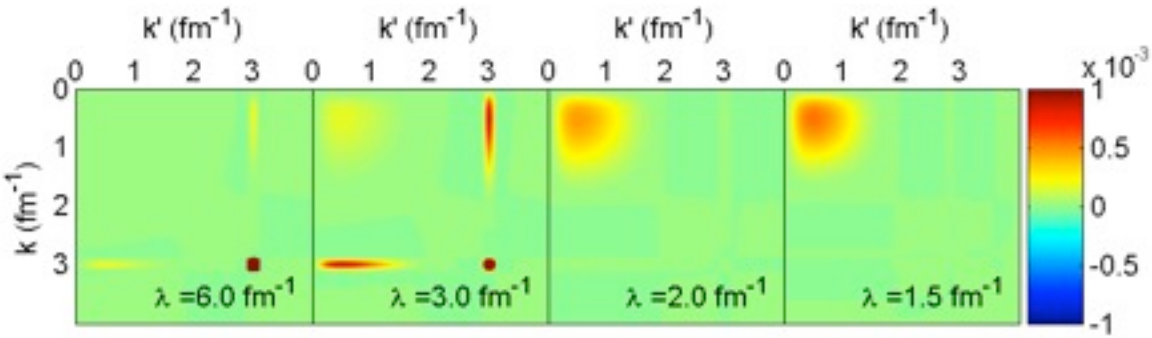
- **Integrand of** $\langle \Psi_D^\lambda | (U_\lambda a_q^\dagger a_q U_\lambda^\dagger) | \Psi_D^\lambda \rangle$ **for** $q = 0.35\,\mathrm{fm}^{-1}$



Momentum Distribution



- **Integrand of** $\langle \Psi_D^\lambda | (U_\lambda a_q^\dagger a_q U_\lambda^\dagger) | \Psi_D^\lambda \rangle$ **for** $q = 3.0\,\mathrm{fm}^{-1}$



Decoupling $\Rightarrow Q_\lambda | \Psi_D^\lambda \rangle \approx 0$

High-momentum strength of O_λ strongly suppressed

No fine tuning => same practical benefits as for H_λ

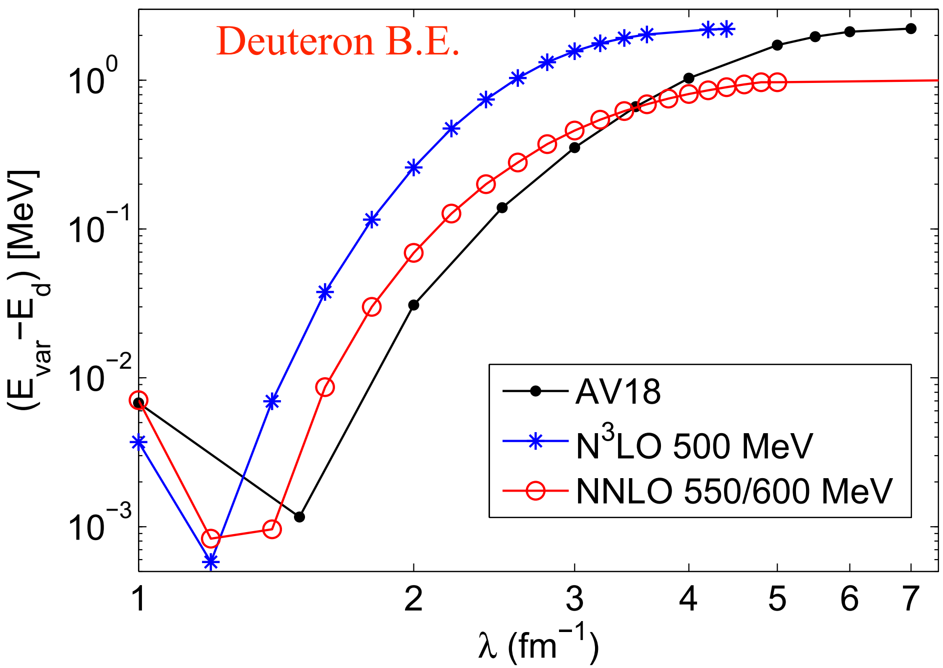
Variational Calculations in the Deuteron

Perform variational calculations to check for problematic fine-tuning in evolved operators

simple k-space trial wf's:

$$u(k) = \frac{1}{(k^2+\gamma^2)(k^2+\mu^2)} e^{-\left(\frac{k^2}{\lambda^2}\right)^2} \qquad w(k) = \frac{ak^2}{(k^2+\gamma^2)(k^2+\nu^2)^2} e^{-\left(\frac{k^2}{\lambda^2}\right)^2}$$

- Large errors in E_{var} at large λ
- Works great at small λ
- No problematic fine-tuning



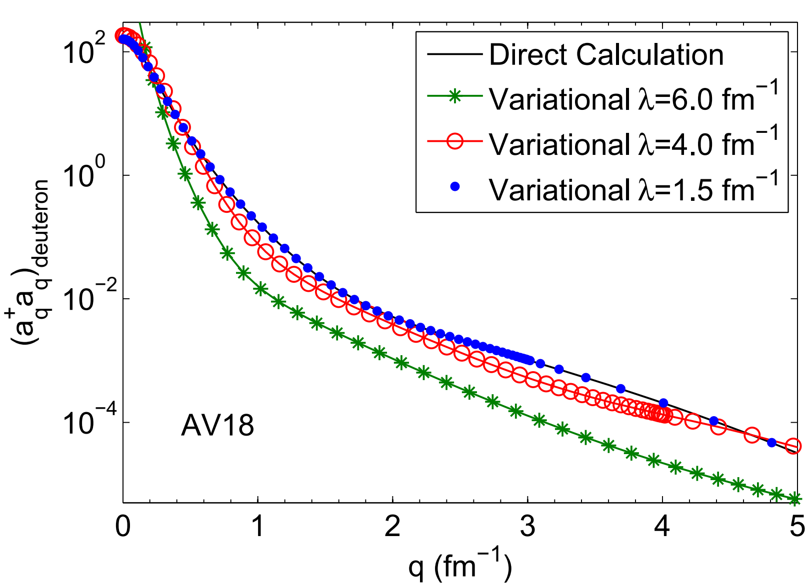
What about other operators?

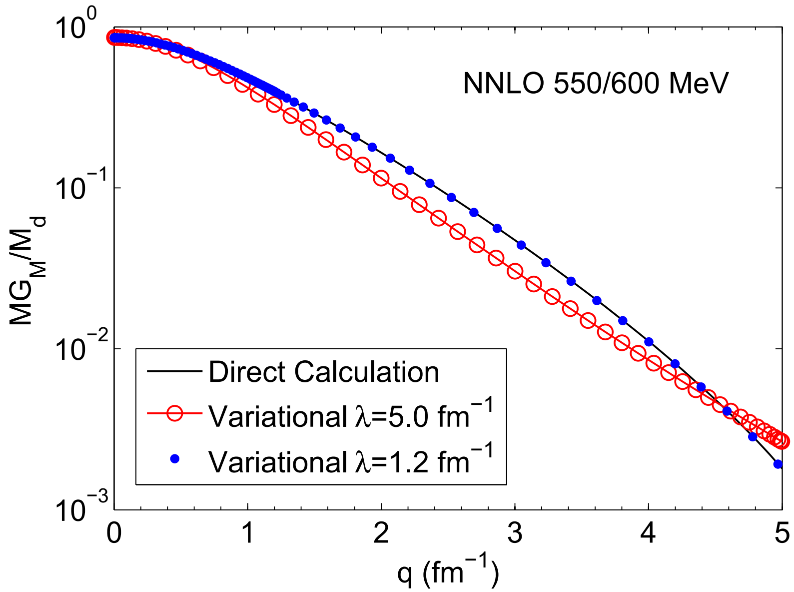
Variational Calculations in the Deuteron

Perform variational calculations to check for problematic fine-tuning in evolved operators

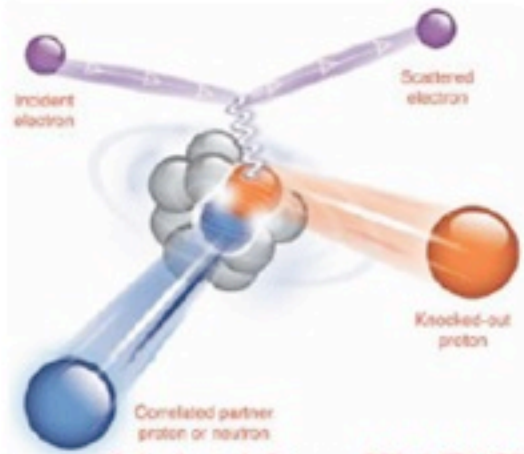
simple k-space trial wf's:

$$u(k) = \frac{1}{(k^2+\gamma^2)(k^2+\mu^2)} e^{-\left(\frac{k^2}{\lambda^2}\right)^2} \qquad w(k) = \frac{ak^2}{(k^2+\gamma^2)(k^2+\nu^2)^2} e^{-\left(\frac{k^2}{\lambda^2}\right)^2}$$

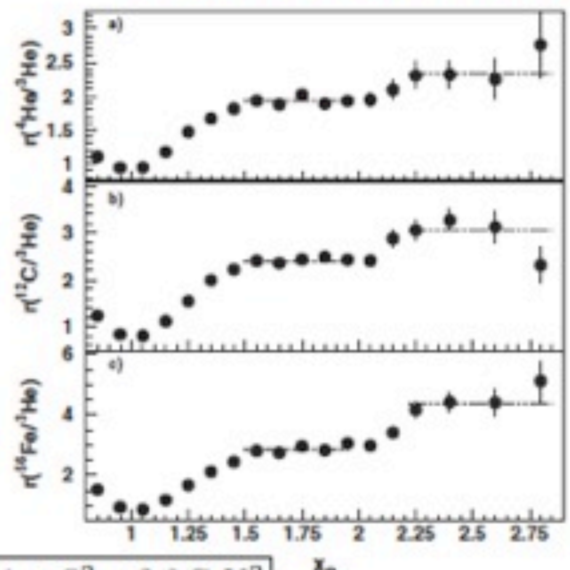




Looking for missing strength at large Q^2



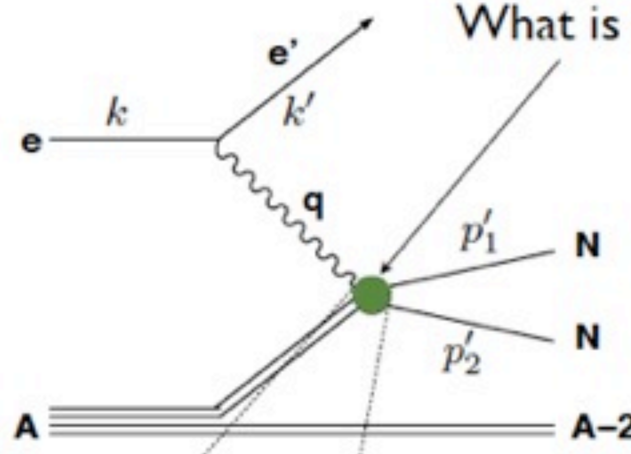
Subedi et al., Science 320, 1476 (2008)



$1.4 < Q^2 < 2.6\, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)

What is this vertex?



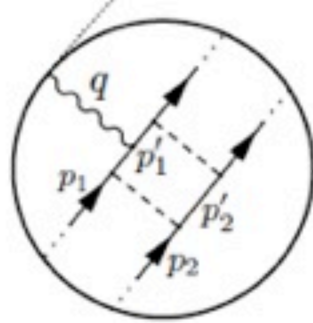
$q = k - k'$

$\nu = E_k - E_{k'}$

$Q^2 = -q^2$

$x_B = \frac{Q^2}{2m_N \nu}$

Higinbotham, arXiv:1010.4433



SRC interpretation:

NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p_1', p_2' \gg k_F$ which are knocked out by the photon

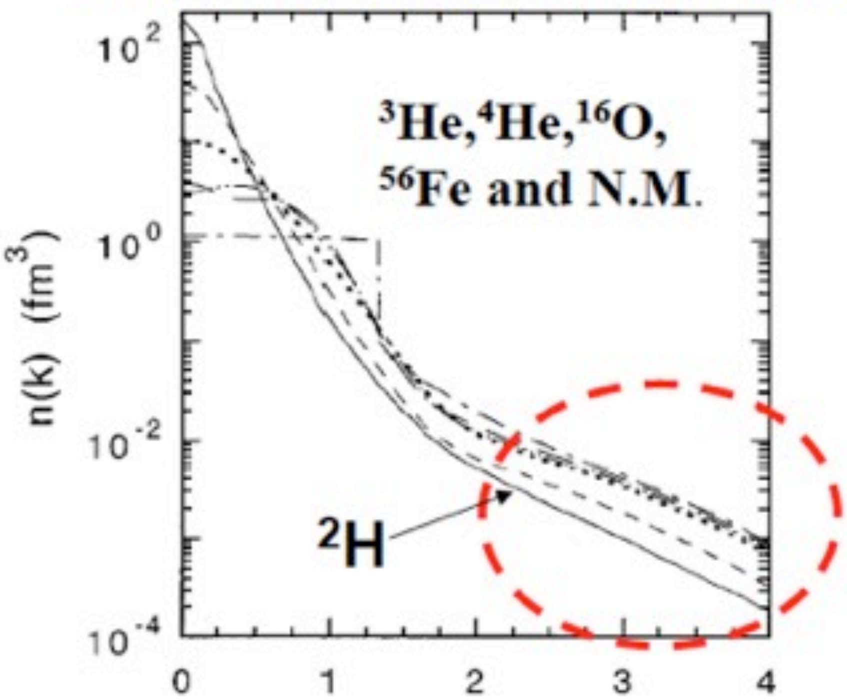
How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

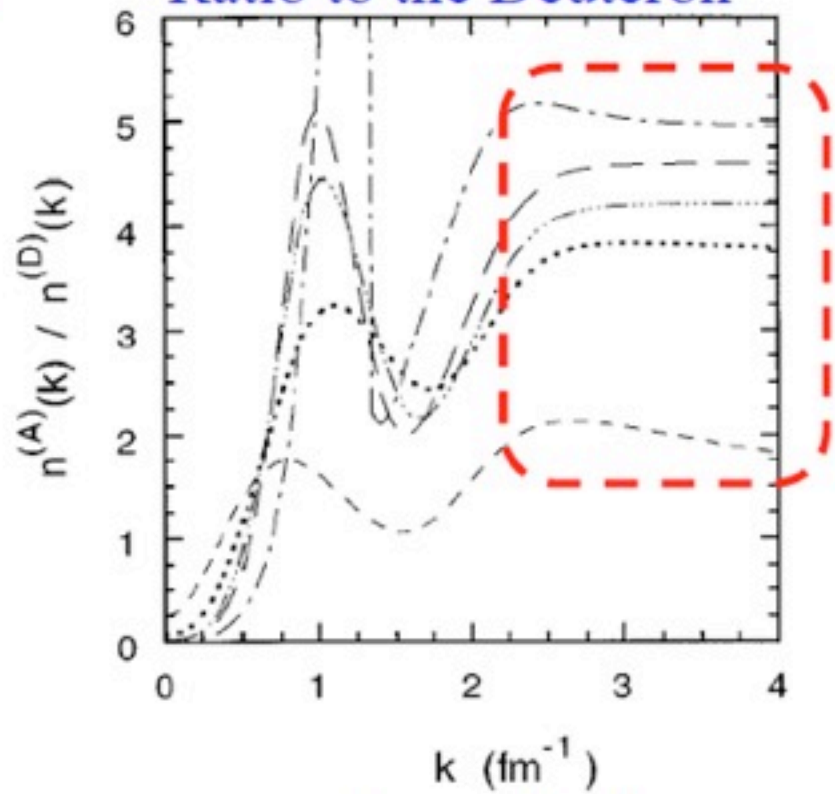
Deuteron-like scaling at high momenta

C. Ciofi and S. Simula, *Phys.Rev* **C53**, 1689(1996)

Momentum Distributions n(k)



Ratio to the Deuteron



n(k) at high Momentum regions are similar to it of the Deuteron

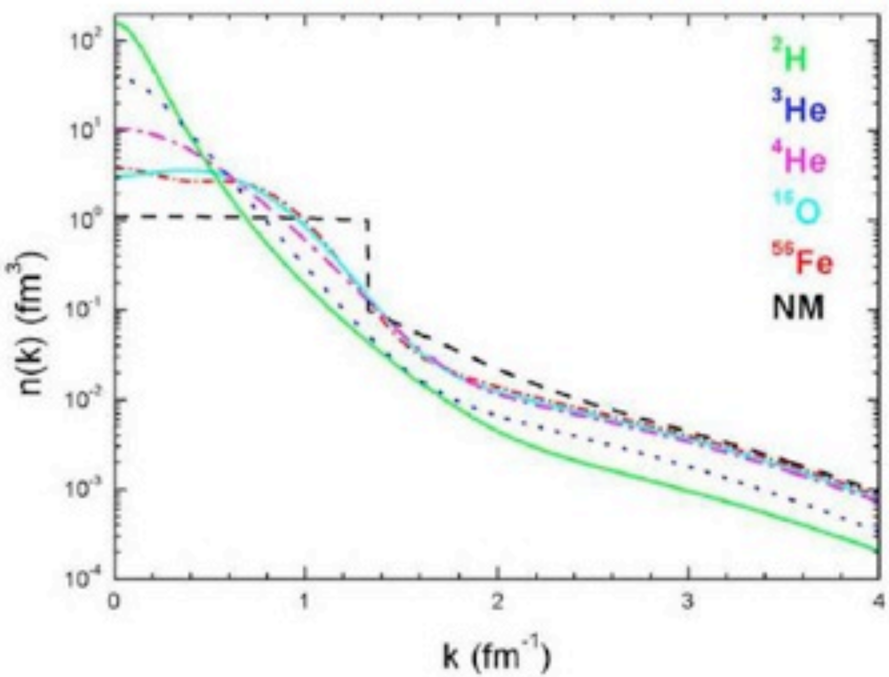
Almost Flat!

High resolution: Dominance of V_{NN} and SRCs (Frankfurt et al.)

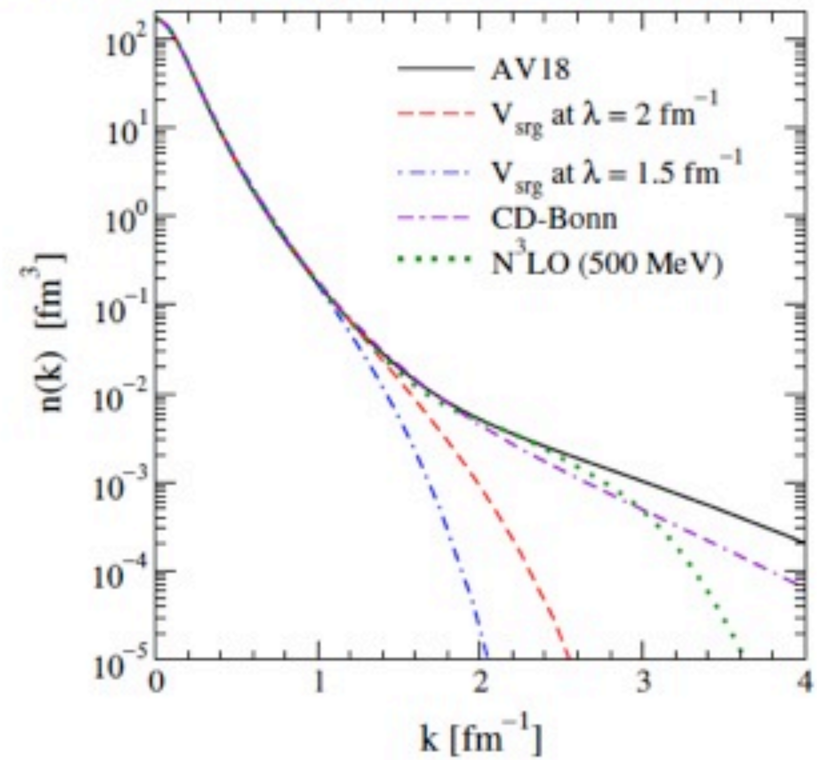
How do we understand this scaling with low-resolution interactions?

Changing the separation scale with RG evolution

- Conventional analysis has (implied) high momentum scale
  - Based on potentials like AV18 and one-body current operator



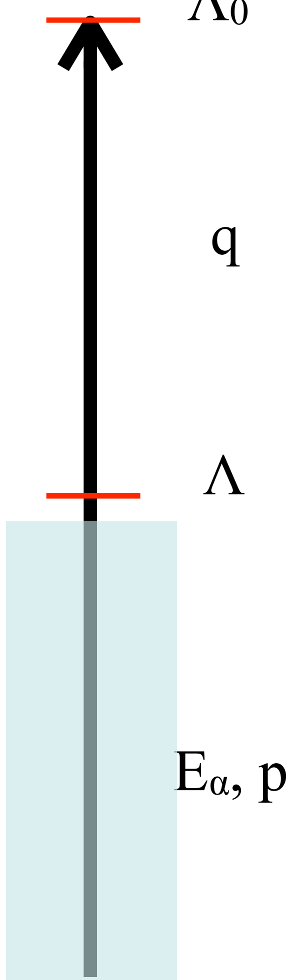
[From C. Ciofi degli Atti and S. Simula]



High q tails from low-k theories? Evolve operators!

$$\langle \Psi_n^{\Lambda_0} | \hat{O}_{\mathbf{q}}^{\Lambda_0} | \Psi_n^{\Lambda_0} \rangle \quad = \quad \langle \Psi_n^{\Lambda} | \hat{O}_{\mathbf{q}}^{\Lambda} | \Psi_n^{\Lambda} \rangle$$

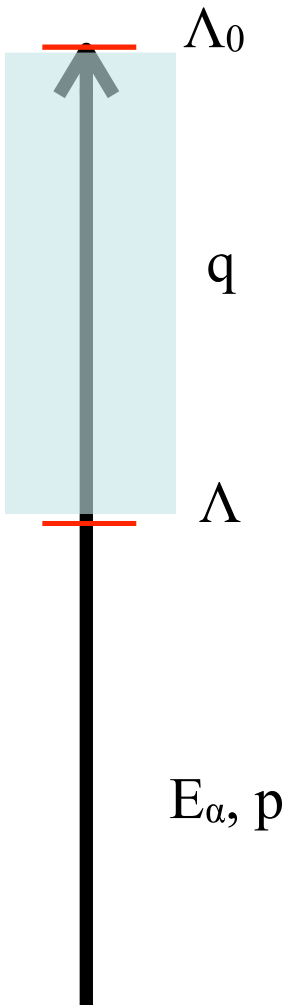
Relationship between bare and effective theory wf's



RG evolution doesn't modify long-distance structure

$$\psi_\alpha^{\Lambda_0}(\mathbf{p}) \approx Z_\Lambda \psi_\alpha^{\Lambda}(\mathbf{p})$$

Relationship between bare and effective theory wf's



Decoupling => **Factorization** of high/low-k physics

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^\Lambda d^3p\, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) + \eta(\mathbf{q};\Lambda) \int_0^\Lambda d^3p\, \mathbf{p}^2 Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) \cdots$$

state-independent ("universal") q-dependence

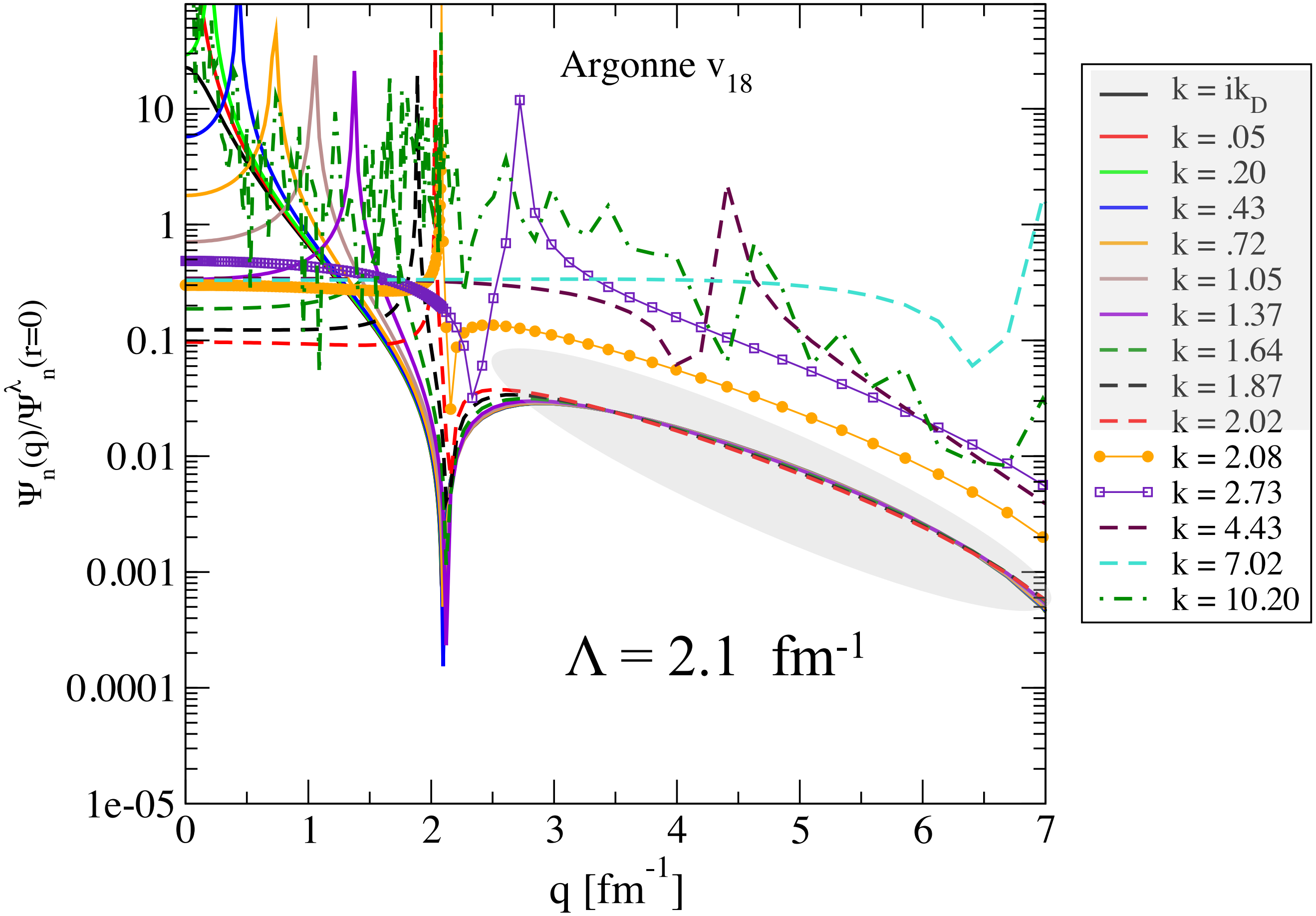
$$\gamma(\mathbf{q};\Lambda) = -\int_\Lambda^{\Lambda_0} d\mathbf{q}' \,\langle \mathbf{q}| \frac{1}{QH^{\Lambda_0}Q} |\mathbf{q}'\rangle V^{\Lambda_0}(\mathbf{q}',0)$$

$$\beta(\mathbf{q};\Lambda) = -\int_\Lambda^{\Lambda_0} d\mathbf{q}' \,\langle \mathbf{q}| \frac{1}{QH^{\Lambda_0}Q} |\mathbf{q}'\rangle \frac{\partial^2}{\partial p^2} V^{\Lambda_0}(\mathbf{q}',\mathbf{p})\Big|_{\mathbf{p}=0}$$

Anderson et al., PRC **82** 054001 (2010)
SKB and Roscher, arXiv:1208.1734

Example: leading order factorization

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^\Lambda d^3p\, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p})$$



state-independent ratio (shaded area) for well separated scales

$$\frac{\psi_\alpha^{\Lambda_0}(\mathbf{q})}{\psi_\alpha^\Lambda(\mathbf{r}=0)} \sim \gamma(\mathbf{q};\Lambda)$$

$$|E_\alpha| \lesssim \Lambda^2 \quad |\mathbf{q}| \gtrsim \Lambda$$

Implication of w.f. factorization for effective operators

$$
\begin{aligned}
\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle &= \int_0^\Lambda dp \int_0^\Lambda dp' \, \psi_\alpha^{\Lambda_0 *}(p) O(p,p') \psi_\alpha^{\Lambda_0}(p') + \int_0^\Lambda dp \int_\Lambda^{\Lambda_0} dq \, \psi_\alpha^{\Lambda_0 *}(p) O(p,q) \psi_\alpha^{\Lambda_0}(q) \\
&+ \int_\Lambda^{\Lambda_0} dq \int_0^\Lambda dp \, \psi_\alpha^{\Lambda_0 *}(q) O(q,p) \psi_\alpha^{\Lambda_0}(p) + \int_\Lambda^{\Lambda_0} dq \int_\Lambda^{\Lambda_0} dq' \, \psi_\alpha^{\Lambda_0 *}(q) O(q,q') \psi_\alpha^{\Lambda_0}(q')
\end{aligned}
$$

Implication of w.f. factorization for effective operators

$$\langle\psi_\alpha^{\Lambda_0}|\widehat{O}_{\Lambda_0}|\psi_\alpha^{\Lambda_0}\rangle = \int_0^\Lambda dp\int_0^\Lambda dp'\,\psi_\alpha^{\Lambda_0 *}(p)O(p,p')\psi_\alpha^{\Lambda_0}(p') + \int_0^\Lambda dp\int_\Lambda^{\Lambda_0} dq\,\psi_\alpha^{\Lambda_0 *}(p)O(p,q)\psi_\alpha^{\Lambda_0}(q)$$

$$+ \int_\Lambda^{\Lambda_0} dq\int_0^\Lambda dp\,\psi_\alpha^{\Lambda_0 *}(q)O(q,p)\psi_\alpha^{\Lambda_0}(p) + \int_\Lambda^{\Lambda_0} dq\int_\Lambda^{\Lambda_0} dq'\,\psi_\alpha^{\Lambda_0 *}(q)O(q,q')\psi_\alpha^{\Lambda_0}(q')$$

Now use:

1) wf factorization:

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda)\int_0^\Lambda d^3p\, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) + \cdots$$

$$\psi_\alpha^{\Lambda_0}(\mathbf{p}) \approx Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p})$$

2) scale separation:

$$O(q,p) \approx O(q,0) + \cdots$$

Implication of w.f. factorization for effective operators

$$\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle \approx Z_\Lambda^2 \langle \psi_\alpha^{\Lambda} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda} \rangle + g^{(0)}(\Lambda) \, \langle \psi_\alpha^{\Lambda} | \delta^{(3)}(\mathbf{r}) | \psi_\alpha^{\Lambda} \rangle + \cdots$$

state-independent coupling
encodes high-q physics

soft m.e. (low-k physics)
same for **all** high q probes

Implication of w.f. factorization for effective operators

$$\langle\psi_\alpha^{\Lambda_0}|\widehat{O}_{\Lambda_0}|\psi_\alpha^{\Lambda_0}\rangle \approx Z_\Lambda^2\langle\psi_\alpha^{\Lambda}|\widehat{O}_{\Lambda_0}|\psi_\alpha^{\Lambda}\rangle + g^{(0)}(\Lambda)\,\langle\psi_\alpha^{\Lambda}|\delta^{(3)}(\mathbf{r})|\psi_\alpha^{\Lambda}\rangle + \cdots$$

state-independent coupling
encodes high-q physics

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E.g.,

$$g^{(0)}(\Lambda) \equiv 2Z_\Lambda^2\int_\Lambda^{\Lambda_0} d\tilde{q}\,O(0,q)\gamma(q;\Lambda) + Z_\Lambda^2\int_\Lambda^{\Lambda_0} d\tilde{q}\int_\Lambda^{\Lambda_0} d\tilde{q}'\,\gamma^*(q;\Lambda)O(q,q')\gamma(q';\Lambda)$$

Analogous to multipole expansion (cf. Lepage). “Universal” form

$$\widehat{O}_\Lambda = Z_\Lambda^2\,\widehat{O}_{\Lambda_0} + g^{(0)}(\Lambda)\,\delta(\mathbf{r}) + g^{(2)}(\Lambda)\,\nabla^2\delta(\mathbf{r}) + \cdots$$

Factorization of high-q operators

$$\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle \approx Z_\Lambda^2 \langle \psi_\alpha^{\Lambda} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda} \rangle + g^{(0)}(\Lambda) \, \langle \psi_\alpha^{\Lambda} | \delta^{(3)}(\mathbf{r}) | \psi_\alpha^{\Lambda} \rangle + \cdots$$

$$= \quad 0$$

$$\text{since} \qquad Q_\Lambda | \psi_\alpha^{\Lambda} \rangle \approx 0$$

Ex: momentum distribution (large q, low-E state)

$$\langle \psi_\alpha^{\Lambda_0} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | \psi_\alpha^{\Lambda_0} \rangle \approx \gamma^2(\mathbf{q}; \Lambda) \, Z_\Lambda^2 | \langle \psi_\alpha^{\Lambda} | \delta(\mathbf{r}) | \psi_\alpha^{\Lambda} \rangle |^2$$

All low-E A=2 states have the same large-q tails

How to generalize beyond A=2 system?

High q tails of nuclear momentum distributions

$$a_{\mathbf{q}}^{(\Lambda)\dagger} = a_{\mathbf{q}}^{\dagger} + \sum_{\mathbf{k_1},\mathbf{k_2}} C_{\mathbf{q}}^{\Lambda}(\mathbf{k_1},\mathbf{k_2}) a_{\mathbf{k_1}}^{\dagger} a_{\mathbf{k_2}}^{\dagger} a_{\mathbf{k_1}+\mathbf{k_2}-\mathbf{q}} + \cdots \equiv a_{\mathbf{q}}^{\dagger} + \delta a_{\mathbf{q}}^{(\Lambda)\dagger}$$

Fixed from A=2

Claim: 1) $C_{\mathbf{q}}^{\Lambda}(\mathbf{p},-\mathbf{p}) \approx Z_{\Lambda}\, \gamma(\mathbf{q};\Lambda)$ $\quad (\mathbf{p} \ll \Lambda \ll \mathbf{q})$

2) $C_{\mathbf{p}'}^{\Lambda}(\mathbf{p},-\mathbf{p}) \approx (Z_{\Lambda} - 1)\, \delta_{\mathbf{p},\mathbf{p}'}$ $\quad (\mathbf{p},\mathbf{p}' \ll \Lambda)$

$$\psi_{\alpha}^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^{\Lambda} d^3p\, Z_{\Lambda} \psi_{\alpha}^{\Lambda}(\mathbf{p}) + \eta(\mathbf{q};\Lambda) \int_0^{\Lambda} d^3p\, \mathbf{p}^2 Z_{\Lambda} \psi_{\alpha}^{\Lambda}(\mathbf{p}) \cdots$$

$$\psi_{\alpha}^{\Lambda_0}(\mathbf{p}) \approx Z_{\Lambda} \psi_{\alpha}^{\Lambda}(\mathbf{p})$$

High q tails of nuclear momentum distributions

$$\begin{aligned}\langle\psi^{\Lambda_0}_{\alpha,A}|a^\dagger_{\mathbf{q}}a_{\mathbf{q}}|\psi^{\Lambda_0}_{\alpha,A}\rangle &= \langle\psi^{\Lambda}_{\alpha,A}|a^\dagger_{\mathbf{q}}a_{\mathbf{q}} + \delta a^\dagger_{\mathbf{q}}a_{\mathbf{q}} + a^\dagger_{\mathbf{q}}\delta a_{\mathbf{q}} + \delta a^\dagger_{\mathbf{q}}\delta a_{\mathbf{q}}|\psi^{\Lambda}_{\alpha,A}\rangle \\ &\approx \langle\psi^{\Lambda}_{\alpha,A}|\delta a^\dagger_{\mathbf{q}}\delta a_{\mathbf{q}}|\psi^{\Lambda}_{\alpha,A}\rangle\end{aligned}$$

$$\Lambda \ll \mathbf{q} \ll \Lambda_0$$

High q tails of nuclear momentum distributions

$$\langle\psi^{\Lambda_0}_{\alpha,A}|a^\dagger_{\mathbf{q}}a_{\mathbf{q}}|\psi^{\Lambda_0}_{\alpha,A}\rangle = \langle\psi^{\Lambda}_{\alpha,A}|a^\dagger_{\mathbf{q}}a_{\mathbf{q}} + \delta a^\dagger_{\mathbf{q}}a_{\mathbf{q}} + a^\dagger_{\mathbf{q}}\delta a_{\mathbf{q}} + \delta a^\dagger_{\mathbf{q}}\delta a_{\mathbf{q}}|\psi^{\Lambda}_{\alpha,A}\rangle$$

$$\approx \langle\psi^{\Lambda}_{\alpha,A}|\delta a^\dagger_{\mathbf{q}}\delta a_{\mathbf{q}}|\psi^{\Lambda}_{\alpha,A}\rangle$$

$$\Lambda \ll \mathbf{q} \ll \Lambda_0$$

$$\approx \gamma^2(\mathbf{q};\Lambda) \times \sum_{\mathbf{k},\mathbf{k}',\mathbf{K}}^{\Lambda} Z^2_\Lambda \langle\psi^{\Lambda}_{\alpha,A}|a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}|\psi^{\Lambda}_{\alpha,A}\rangle$$

- short-distance
- Universal (state-indep)
- fixed from A=2

- long-distance structure
- same for all high-q probes
- A-dependent scale factor

$$\left[a^\dagger_{\mathbf{q}}a_{\mathbf{q}}\right]^{\Lambda} \approx \gamma^2(\mathbf{q};\Lambda)Z^2_\Lambda \sum_{\mathbf{K},\mathbf{k}',\mathbf{k}} \left[a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}\right]^{\Lambda_0}$$

Connection to OPE? (cf. Braaten and Platter). Links few- and many-body.

High q tails of nuclear momentum distributions

$$\langle\psi^{\Lambda_0}_{\alpha,A}|a^\dagger_{\mathbf{q}}a_{\mathbf{q}}|\psi^{\Lambda_0}_{\alpha,A}\rangle = \langle\psi^{\Lambda}_{\alpha,A}|a^\dagger_{\mathbf{q}}a_{\mathbf{q}} + \delta a^\dagger_{\mathbf{q}}a_{\mathbf{q}} + a^\dagger_{\mathbf{q}}\delta a_{\mathbf{q}} + \delta a^\dagger_{\mathbf{q}}\delta a_{\mathbf{q}}|\psi^{\Lambda}_{\alpha,A}\rangle$$

$$\approx \langle\psi^{\Lambda}_{\alpha,A}|\delta a^\dagger_{\mathbf{q}}\delta a_{\mathbf{q}}|\psi^{\Lambda}_{\alpha,A}\rangle$$

$$\Lambda \ll \mathbf{q} \ll \Lambda_0$$

$$\approx \gamma^2(\mathbf{q};\Lambda) \times \sum_{\mathbf{k},\mathbf{k}',\mathbf{K}}^{\Lambda} Z^2_\Lambda \langle\psi^{\Lambda}_{\alpha,A}|a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}|\psi^{\Lambda}_{\alpha,A}\rangle$$

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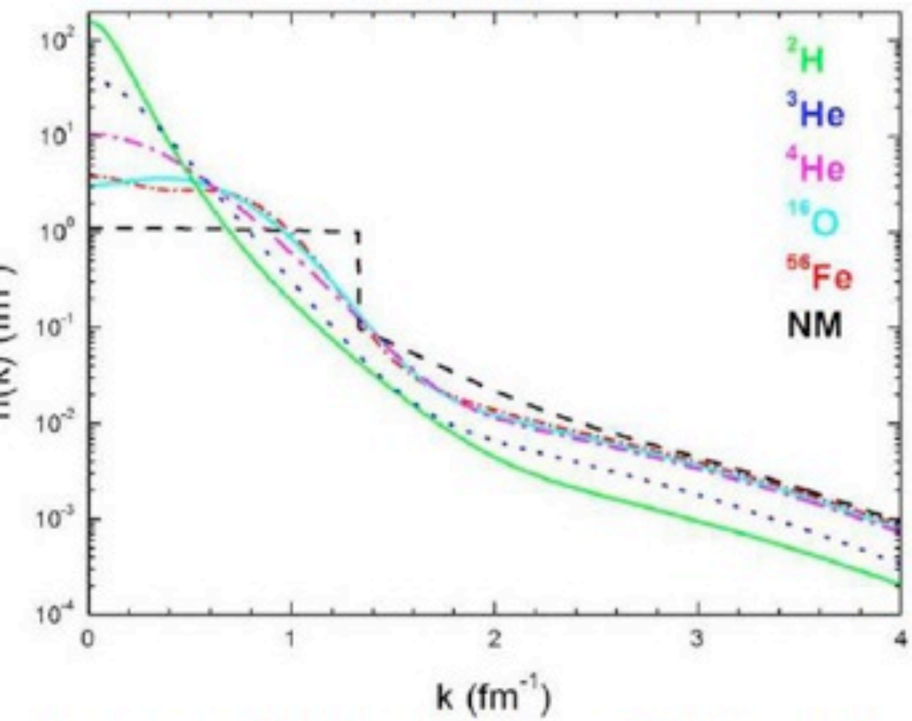
- same for all high-q probes
- A-dependent scale factor

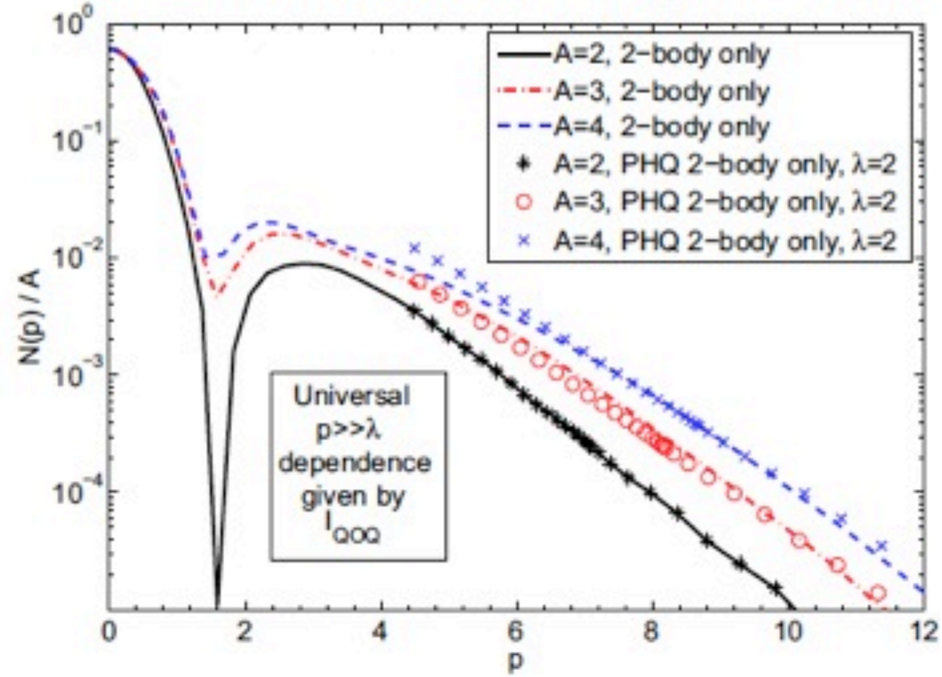
$$C(A,2) \equiv \frac{n_A(\mathbf{q})}{n_D(\mathbf{q})} \sim \frac{\sum_{\mathbf{k},\mathbf{k}',\mathbf{K}}\langle\psi^{\Lambda}_{\alpha,A}|a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}|\psi^{\Lambda}_{\alpha,A}\rangle}{\sum_{\mathbf{k},\mathbf{k}',\mathbf{K}}\langle\psi^{\Lambda}_{\alpha,D}|a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}|\psi^{\Lambda}_{\alpha,D}\rangle}$$

$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



[From C. Ciofi degli Atti and S. Simula]

Test case: A bosons in toy 1D model



[Anderson et al., arXiv:1008.1569]

$$C(A,2) \equiv \frac{n_A(\mathbf{q})}{n_D(\mathbf{q})} \sim \frac{\sum\limits_{\mathbf{k},\mathbf{k}',\mathbf{K}} \langle \psi^{\Lambda}_{\alpha,A} | a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi^{\Lambda}_{\alpha,A} \rangle}{\sum\limits_{\mathbf{k},\mathbf{k}',\mathbf{K}} \langle \psi^{\Lambda}_{\alpha,D} | a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi^{\Lambda}_{\alpha,D} \rangle}$$

High q tails of static structure factors

$$\langle\psi^{\Lambda_0}_{\alpha,A}|\widehat{S}(\mathbf{q})|\psi^{\Lambda_0}_{\alpha,A}\rangle \approx \left\{2\gamma(\mathbf{q};\Lambda) + \sum_{\mathbf{P}}\gamma(\mathbf{P}+\mathbf{q};\Lambda)\gamma(\mathbf{P};\Lambda)\right\}$$

$$\times \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\Lambda} Z^2_{\Lambda}\,\langle\psi^{\Lambda}_{\alpha,A}|a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}|\psi^{\Lambda}_{\alpha,A}\rangle$$

Reproduce known results for unitary Fermi gas, electron gas, 1d bosons w/delta function V(r) ...

SKB and Roscher, arXiv:1208.1734

Factorization of general high-q probes (schematic)

$$\langle \Psi_n^{\Lambda_0} | \hat{O}_{\mathbf{q}}^{\Lambda_0} | \Psi_n^{\Lambda_0} \rangle \quad = \quad \langle \Psi_n^{\Lambda} | \hat{O}_{\mathbf{q}}^{\Lambda} | \Psi_n^{\Lambda} \rangle \qquad \Lambda \ll \mathbf{q} \ll \Lambda_0$$

examples:

$$\hat{O}_{\mathbf{q}}^{\Lambda_0} = a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} \,, \sum_{\mathbf{p},\mathbf{p}'} a_{\mathbf{p}+\mathbf{q}}^{\dagger} a_{\mathbf{p}} a_{\mathbf{p}'}^{\dagger} a_{\mathbf{p}'+\mathbf{q}} \cdots$$

Factorization of high-q probes (schematic)

$$\langle \Psi_n^{\Lambda_0} | \hat{O}_{\mathbf{q}}^{\Lambda_0} | \Psi_n^{\Lambda_0} \rangle \quad = \quad \langle \Psi_n^{\Lambda} | \hat{O}_{\mathbf{q}}^{\Lambda} | \Psi_n^{\Lambda} \rangle \qquad \Lambda \ll \mathbf{q} \ll \Lambda_0$$

Expand evolved operator as polynomial in creation/annihilation operators at $\mathbf{\Lambda}_0$

$$\hat{O}_{\mathbf{q}}^{\Lambda} = \sum_{\alpha} g_{\mathbf{q}}^{\alpha}(\Lambda)\, \hat{A}_{\alpha}^{\Lambda_0}$$

c-number coeff

string of creation/annihilation operators

$\alpha = \mathbf{p}_1 \ldots \mathbf{p}_\beta$

Factorization of high-q probes (schematic)

$$\langle \Psi_n^{\Lambda_0} | \hat{O}_{\mathbf{q}}^{\Lambda_0} | \Psi_n^{\Lambda_0} \rangle \quad = \quad \langle \Psi_n^{\Lambda} | \hat{O}_{\mathbf{q}}^{\Lambda} | \Psi_n^{\Lambda} \rangle \qquad \Lambda \ll \mathbf{q} \ll \Lambda_0$$

$$= \sum_{\alpha} g_{\mathbf{q}}^{\alpha}(\Lambda) \, \langle \Psi_n^{\Lambda} | \hat{A}_{\alpha}^{\Lambda_0} | \Psi_n^{\Lambda} \rangle$$

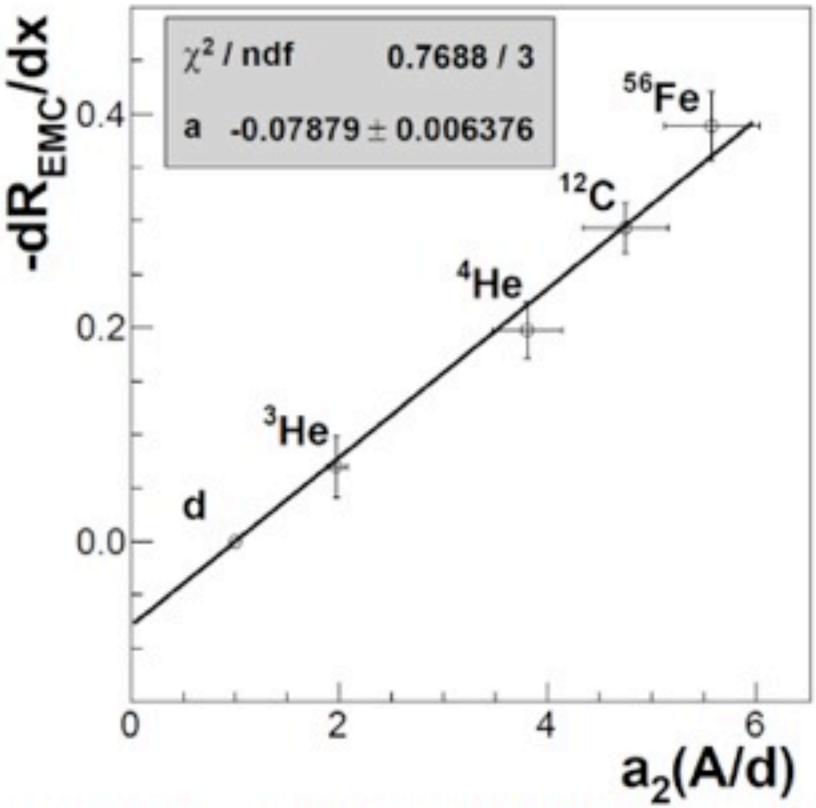
1) Decoupling => only modes $p < \mathbf{\Lambda}$ in α contribute

2) Taylor expand c-# coefficients about $p = 0$

=> **q**-dependence **factorizes**
=> state-dependence from soft matrix elements A_α

Short Range Correlations and the EMC Effect

- Deep inelastic scattering ratio at $Q^2 \geq 2\,\text{GeV}^2$ and $0.35 \leq x_B \leq 0.7$ and inelastic scattering at $Q^2 \geq 1.4\,\text{GeV}^2$ and $1.5 \leq x_B \leq 2.0$
- Strong linear correlation between slope of ratio of DIS cross sections (nucleus A vs. deuterium) and nuclear scaling ratio
- SRG Factorization at leading order:
  $\rightarrow$ Dependence on high-q is *independent* of A
  $\rightarrow$ A-dependence from low momentum matrix element *independent* of operator
  $\Rightarrow$ Ratios are linearly correlated



L.B. Weinstein, et al., Phys. Rev. Lett. 106, 052301 (2011)

Can nuclear scaling and EMC effect be quantitatively explained via factorization of operators and low momentum structure of the nuclei?

- In progress: Calculation of a_2 in MBPT
- Same dependence on nuclear structure for high momentum operators $\Rightarrow$ EMC effect?

Summary

- RG evolution of high-q operators => factorization
  - separation of long- and short-distance physics
  - effective operators w/universal q-dependence (few-body); predictions in many-body systems
  - connection to operator product expansion?
  - tool to connect resolution-dependent qty's (SF, occupation #'s, etc.) at different resolutions?
  - electron scattering at medium/high energies?