Electromagnetic Reactions in Ultracold Atoms

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INT Workshop
Electroweak properties of light nuclei
7 November, 2012

האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

Outline

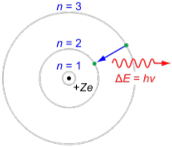
References:

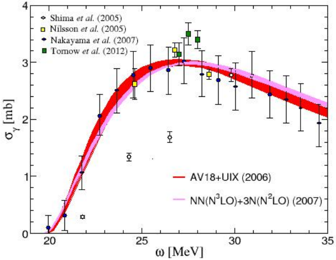
- E. Liverts, B. Bazak, and N. Barnea, Phys. Rev. Lett. **108**, 112501 (2012)
- B. Bazak, E. Liverts, and N. Barnea, Phys. Rev. A **86**, 043611 (2012)
- B. Bazak, E. Liverts, and N. Barnea, Few-Body Systems 10.1007/s00601-012-0437-8 (2012)
- B. Bazak and N. Barnea, *in preparation*

Outline

What Can We Learn From Photo Reactions?

1. Understanding of the systems at hand.
2. A test of the Hamiltonian at regimes not accessible by elastic reactions.
3. Reaction rates as input for experiments or applications (e.g. astrophysics).
4. Underlying degrees of freedom.
5. The transition from single particle to collective behavior.





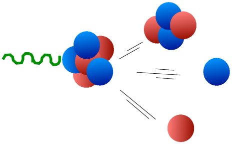
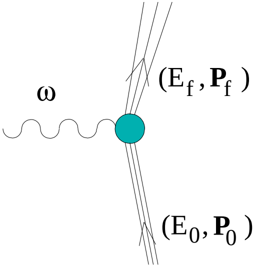


Photo Reactions

The Interaction Hamiltonian between the photon field $\boldsymbol{A}(\boldsymbol{x})$ and the atomic/nuclear system

$$H_I = -\frac{e}{c}\int d\boldsymbol{x}\boldsymbol{A}(\boldsymbol{x})\cdot\boldsymbol{J}(\boldsymbol{x})$$



The current is a sum of convection and spin currents

$$\boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{J}_c(\boldsymbol{x}) + \nabla\times\boldsymbol{\mu}(\boldsymbol{x})$$

$$H_I = -\frac{e}{c}\int d\boldsymbol{x}\left\{\boldsymbol{A}(\boldsymbol{x})\cdot\boldsymbol{J}_c(\boldsymbol{x}) + \boldsymbol{B}(\boldsymbol{x})\cdot\boldsymbol{\mu}(\boldsymbol{x})\right\}$$

- Classically, the convection current $J_c = \sum_i Z_i v_i$ is the flow of the charged particles.
- In nuclear physics, the convection current is dominant
- Ultracold atoms are neutral $J_c(x) = 0$ and the current $\mu(x)$ is dominated by the spins.

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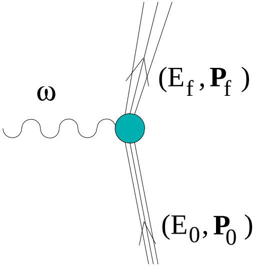

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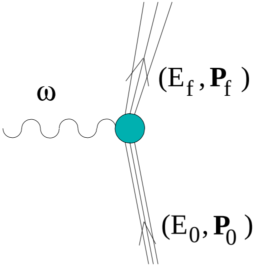

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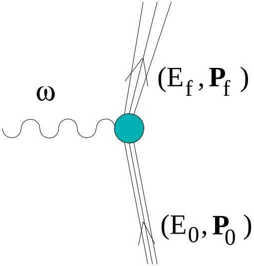

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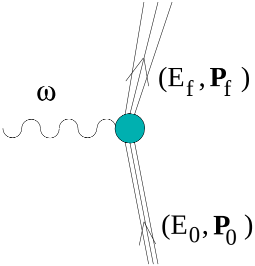

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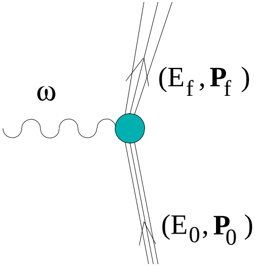

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Efimov Physics and Universality

- Borromean regime: A 3-body bound state $E_3 < 0$ exists even if the 2-body system is unbound $E_2 > 0$.
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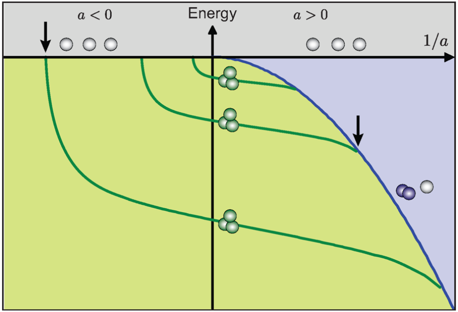

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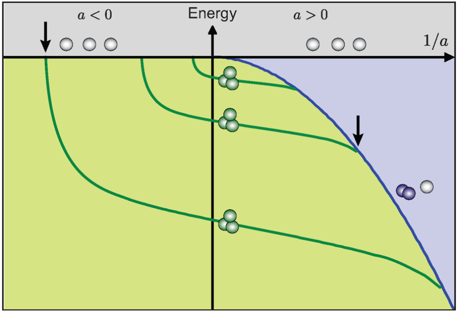

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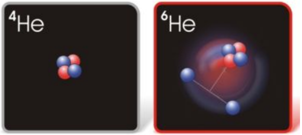

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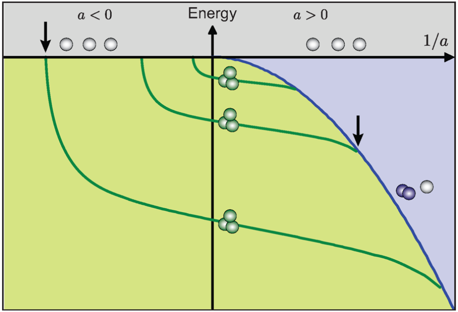

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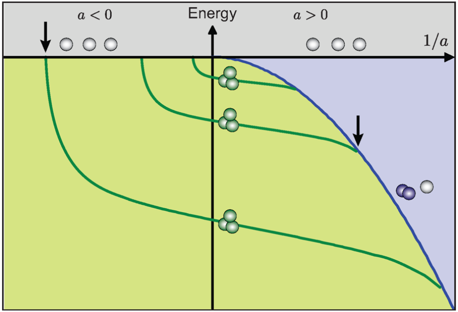

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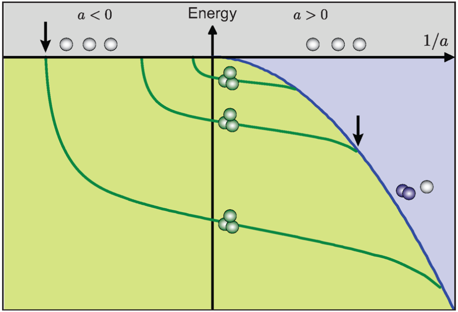

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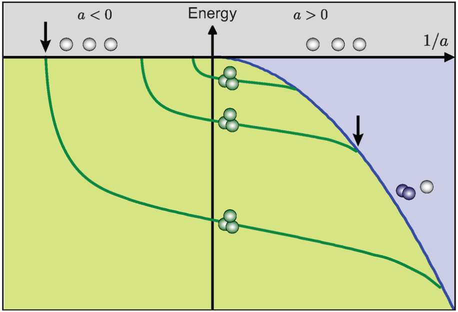

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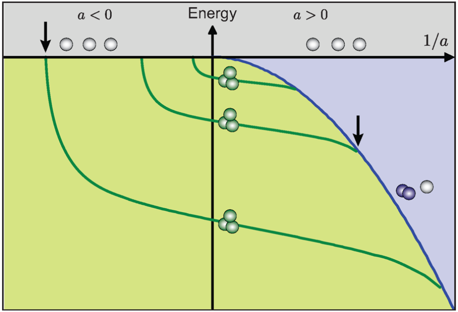

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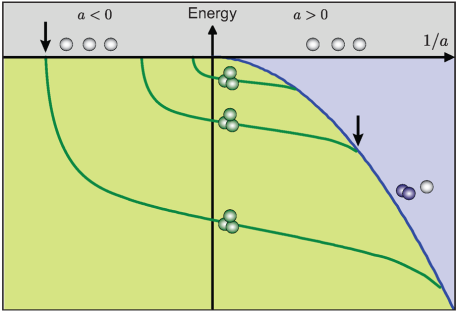

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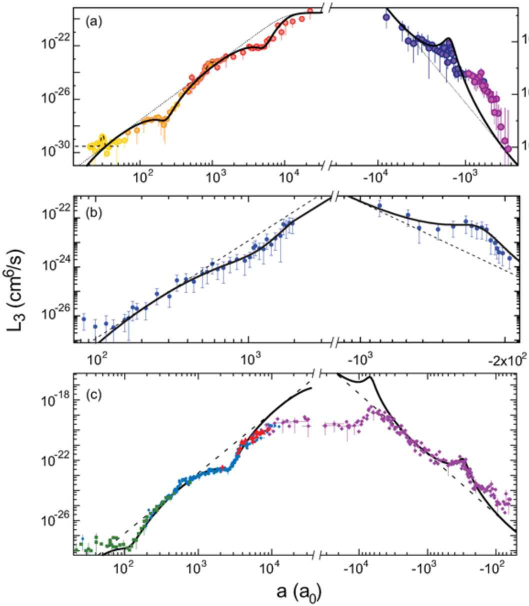

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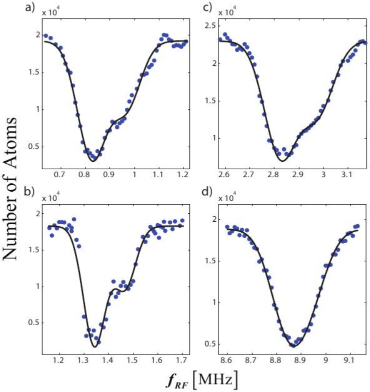
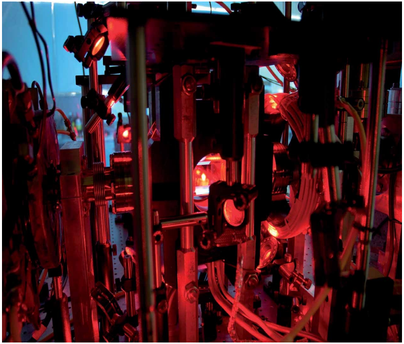
Photoassociation of Atomic Molecules

RF-induce atom loss resonances for different values of bias magnetic fields.





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Outline

The Static Response - Inelastic Reactions

- The response of an A-particle system is closely related to the static moments of the charge density

$$\rho(\boldsymbol{x}) = \sum_i^A Z_i \delta(\boldsymbol{x} - \boldsymbol{r}_i)$$

- The Fourier Transform

$$\rho(\boldsymbol{k}) = \int d\boldsymbol{x} \rho(\boldsymbol{x}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} = \sum_i^A Z_i e^{i\boldsymbol{k}\cdot\boldsymbol{r}_i}$$

- In the long wavelength limit $k \longrightarrow 0$

- For a system of **identical** particles

- **Conclusion A:** In general the Dipole is the leading term.
- **Conclusion B:** For identical particles the leading terms are $\hat{R}^2$ and $\hat{Q}$.

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- **Conclusion A:** In general the Dipole is the leading term.
- **Conclusion B:** For identical particles the leading terms are $\hat{R}^2$ and $\hat{Q}$.

Photo Reactions with Ultracold Atoms

- For RF photons in the few MHz region the wave length is meters so $kR \ll 1$.
- The atoms reside in a strong magnetic field, thus spins are "frozen"

$$|\Psi_0\rangle = \Phi_0(\boldsymbol{r}_i)|m_F^1 m_F^2 \ldots m_F^A\rangle$$

- In the final state the photon can either change one of the spins or leave them untouched.
- Spin-flip reaction

$$\boxed{|m_F^1 m_F^2 \ldots m_F^A\rangle \longrightarrow |m_F^1 m_F^2 \pm 1 \ldots m_F^A\rangle}$$

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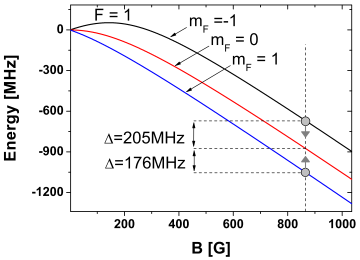

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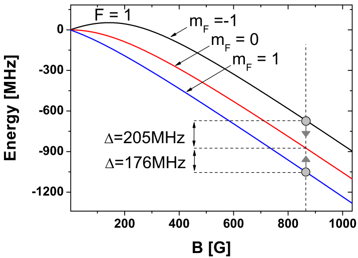

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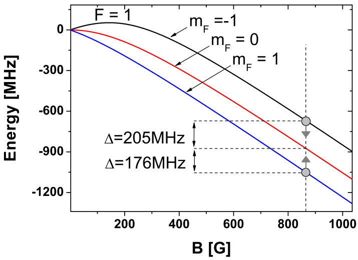

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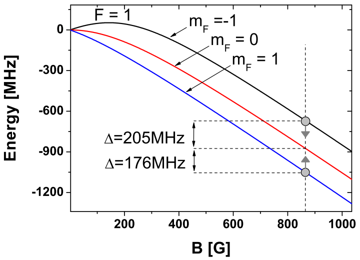

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Photo Reactions with Ultracold Atoms

- For Spin-flip reactions the Franck-Condon factor dominates the transition

$$R(\omega) = Ck \sum_{f,\lambda} \left| \langle \Phi_f | \Phi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$

- For Frozen-Spin reactions we get a sum of the monopole operator $\hat{M} = R^2 = \sum r_i^2$ and the Quadrupole operator $\hat{Q} = \sum r_i^2 Y_2(\hat{r}_i)$

$$\boldsymbol{O} = \alpha \hat{M} + \beta \hat{Q}$$

- The response is given by

$$R(\omega) = k^5 \sum_{f,\lambda} \left| \langle \Phi_f | \boldsymbol{O} | \Phi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$

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Outline

Photoassociation of the Atomic Dimer

- For the dimer case the response function can be written as

$$R(\omega) = C\omega^5 \left[\frac{1}{6^2} |\langle \psi_0 \| \hat{M} \| \varphi_0(q) \rangle|^2 + \frac{1}{5 \cdot 15^2} |\langle \psi_0 \| \hat{Q} \| \varphi_2(q) \rangle|^2 \right]$$

- Where the bound state wave function is given by

$$\psi_0 = Y_0 \sqrt{2\kappa} e^{-\kappa r}/r \quad ; \quad \kappa \approx 1/a_s$$

- The continuum state is given by $\varphi_\ell(q) = Y_\ell(\hat{r})\chi_\ell(r)/r$

$$\chi_\ell(r) = 2qr[\cos\delta_\ell j_\ell(qr) - \sin\delta_\ell n_\ell(qr)]$$

- The $\ell = 0$ matrix element

$$|\langle \psi_0 \| \hat{M} \| \varphi_0(q) \rangle|^2 = \frac{1}{4\pi} \left(\frac{4q\sqrt{2\kappa}}{(q^2+\kappa^2)^3} \right)^2 \left[\cos\delta_0 (3\kappa^2 - q^2) - \sin\delta_0 \frac{\kappa}{q} (3q^2 - \kappa^2) \right]^2$$

- The $\ell = 2$ matrix element, assuming $\delta_2 = 0$

$$|\langle \psi_0 \| \hat{Q} \| \varphi_2(q) \rangle|^2 = \frac{5}{4\pi} \left[\frac{16q^3\sqrt{2\kappa}}{(q^2+\kappa^2)^3} \right]^2$$

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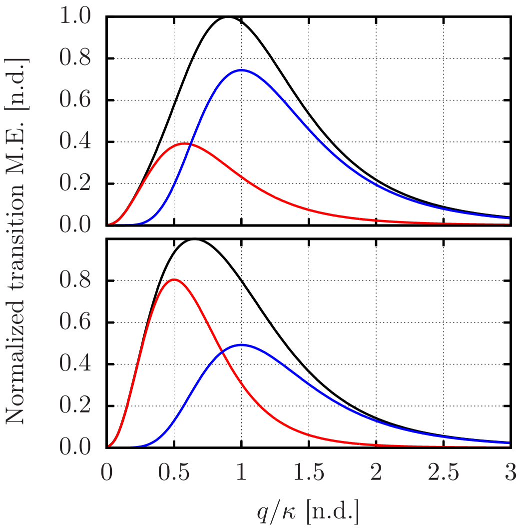
The s-wave and d-wave components in the response function

- upper panel $a/r_{eff} = 2$
- lower panel $a/r_{eff} = 200$
- red - r^2 monopole
- blue - quadrupole
- black - their sum



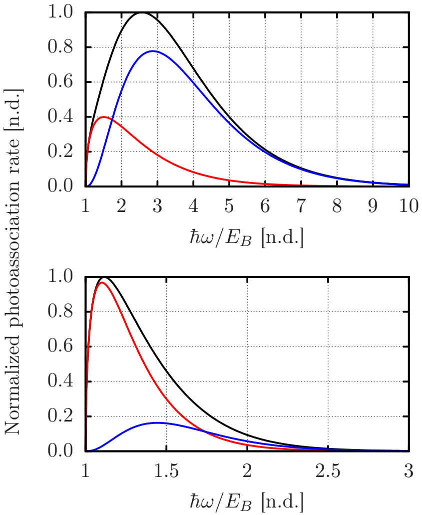
Dimer Photoassociation Rates

Photoassociation of ^{7}Li dimers

$a_s = 1000a_0$
$T = 25\mu K$ (upper panel)
$T = 5\mu K$ (lower panel)

red - r^2 monopole, blue - quadrupole, black - sum

The relative contribution to the peak



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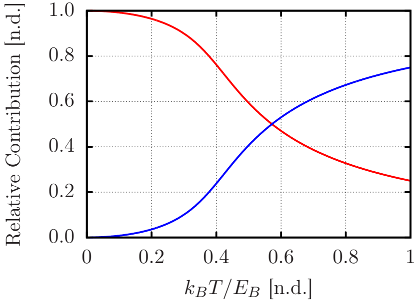
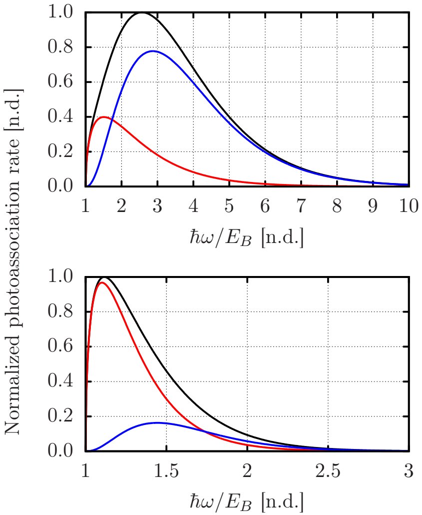
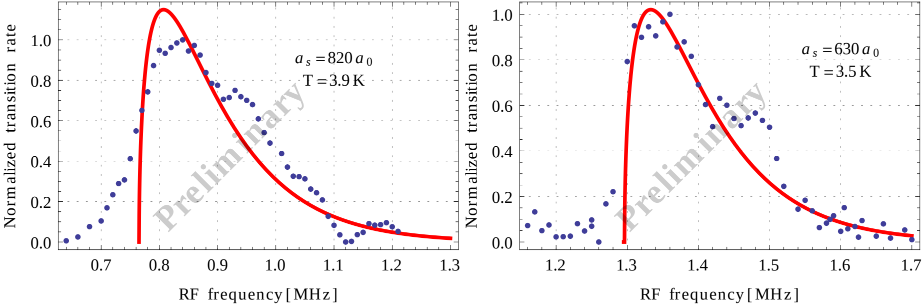




Comparison to the Khaykovich group data:



- The fitted values of a_s and T are in reasonable agreement with the estimates of the experimental group.
- High amplitude RF causes power broadening
- Finite time effect
- Disagreement are due to 3-body (4-body?) association.
- Effects of $\delta_2 \neq 0$ are negligible.

Outline

Road-map for Efimov Physics

To get analytical results for the 3-body problem,

- Assume short-range interaction and large scattering length
- Remove center of mass and adopt the hyper-spherical coordinates

$$(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \rightarrow (\mathbf{R}_{CM}, \mathbf{x}_i, \mathbf{y}_i) \rightarrow (\mathbf{R}_{CM}, \rho, \alpha_i, \hat{x}_i, \hat{y}_i)$$

- Use the adiabatic expansion (Born-Oppenheimer like), where ρ is the slow coordinate

$$\Psi(\rho, \Omega) = \sum_n \rho^{-5/2} f_n(\rho) \Phi_n(\rho, \Omega)$$

- Decompose into Faddeev amplitudes to impose symmetry and boundary condition

$$\Phi_n(\rho, \Omega) = \sum_i \phi_{n,i}(\rho, \Omega_i)$$

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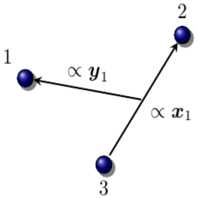
The Hyper-Spherical Coordinates

- To eliminate center of mass, we use the Jacobi coordinates, $(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) \to (\boldsymbol{R}_{CM}, \boldsymbol{x}_i, \boldsymbol{y}_i)$:

$$\boldsymbol{x}_i = \frac{\boldsymbol{r}_j - \boldsymbol{r}_k}{\sqrt{2}}, \quad \boldsymbol{y}_i = \sqrt{\frac{2}{3}} \left(-\boldsymbol{r}_i + \frac{\boldsymbol{r}_j + \boldsymbol{r}_k}{2} \right)$$



- Now using the hyper-spherical coordinates, $(x_i, y_i) \to (\rho, \alpha_i, \hat{x}_i, \hat{y}_i)$:

$$\rho^2 = x_i^2 + y_i^2, \quad \tan \alpha_i = x_i / y_i,$$

- The Hamiltonian $\mathcal{H} = (T + \sum_{i<j} V(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$ reads,

$$T = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial \rho^2} + \frac{5}{\rho} \frac{\partial}{\partial \rho} - \frac{\hat{K}^2}{\rho^2} \right)$$

where

$$\hat{K}^2 = -\frac{1}{\sin 2\alpha} \frac{\partial^2}{\partial \alpha^2} \sin 2\alpha + \frac{\hat{l}_x^2}{\sin^2 \alpha} + \frac{\hat{l}_y^2}{\cos^2 \alpha} - 4$$

and

$$\sum_{i<j} V(|\boldsymbol{r}_i - \boldsymbol{r}_j|) = \sum_i V(\sqrt{2} \rho \sin \alpha_i)$$

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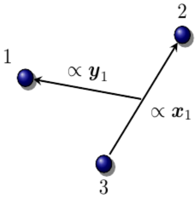

- Now using the hyper-spherical coordinates, $(\boldsymbol{x}_i, \boldsymbol{y}_i) \to (\rho, \alpha_i, \hat{x}_i, \hat{y}_i)$:

$$\rho^2 = x_i^2 + y_i^2, \quad \tan\alpha_i = x_i/y_i,$$

- The Hamiltonian $\mathcal{H} = (T + \sum_{i<j} V(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$ reads,

$$T = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} - \frac{\hat{K}^2}{\rho^2}\right)$$

where

$$\hat{K}^2 = -\frac{1}{\sin 2\alpha}\frac{\partial^2}{\partial\alpha^2}\sin 2\alpha + \frac{\hat{l}_x^2}{\sin^2\alpha} + \frac{\hat{l}_y^2}{\cos^2\alpha} - 4$$

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$$\sum_{i<j} V(|\boldsymbol{r}_i - \boldsymbol{r}_j|) = \sum_i V(\sqrt{2}\rho\sin\alpha_i)$$

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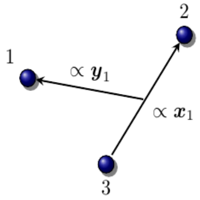

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The Adiabatic Expansion

- Next we apply the adiabatic expansion,

$$\Psi(\rho,\Omega)=\sum_n \rho^{-5/2} f_n(\rho)\Phi_n(\rho,\Omega),$$

- $\Phi_n(\rho,\Omega)$ is the solution of the hyper angular equation corresponding to the eigenvalue ν_n^2,

$$\left(\hat{K}^2+\frac{2m}{\hbar^2}\rho^2\sum_i V(\sqrt{2}\rho\sin\alpha_i)+4\right)\Phi_n(\rho,\Omega)=\nu_n^2\Phi_n(\rho,\Omega).$$

- $f_n(\rho)$ is the solution of the hyper-radial equation,

$$\left(-\frac{\partial^2}{\partial\rho^2}+\frac{2m}{\hbar^2}(V_{\text{eff}}(\rho)-E)\right)f_n(\rho)=\sum_{n\neq n'}(2P_{nn'}\frac{\partial}{\partial\rho}+Q_{nn'})f_{n'}(\rho)$$

- where the effective potential is

$$V_{\text{eff}}(\rho)=\frac{\hbar^2}{2m}\frac{\nu_n^2(\rho)-1/4}{\rho^2}-Q_{nn}$$

and the non-adiabatic couplings are

$$P_{nn'}(\rho)=\left\langle \Phi_n(\rho,\Omega)\left|\frac{\partial}{\partial\rho}\right|\Phi_{n'}(\rho,\Omega)\right\rangle_\Omega$$

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$$\Phi_n(\rho,\Omega) = \sum_i \phi_{n,i}(\rho,\Omega_i)$$

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- Now the solution is,

$$\phi_{n,i}(\rho,\Omega_i) = \frac{g_{\nu,L}(\alpha_i)}{\sin(2\alpha_i)} Y^{L,M}_{l_x,l_y}(\hat{x}_i,\hat{y}_i)$$

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$$Y^{L,M}_{l_x,l_y}(\hat{x},\hat{y}) = \sum_{m_x,m_y} \langle l_x m_x l_y m_y | LM \rangle Y^{m_x}_{l_x}(\hat{x}) Y^{m_y}_{l_y}(\hat{y})$$

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$$\left[\frac{1}{2\alpha_i\Phi}\frac{\partial}{\partial\alpha_i}2\alpha_i\Phi\right]_{\alpha_i=0} = -\sqrt{2}\rho\frac{1}{a_s}$$

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Applying Boundary Condition

- Plugging the angular wave functions, the equation for $L = 0$ reads,

$$\nu \cos(\nu\pi/2) - \frac{8}{\sqrt{3}} \sin(\nu\pi/6) = \frac{\sqrt{2}\rho}{a} \sin(\nu\pi/2)$$

- For $L = 2$ the equation reads,

$$\nu(4-\nu^2)\cos(\nu\pi/2) + 24\nu\cos(\nu\pi/6) + \frac{8}{\sqrt{3}}(\nu^2-10)\sin(\nu\pi/6) = -\frac{\rho}{a}(\nu^2-1)\sin(\nu\pi/2)$$

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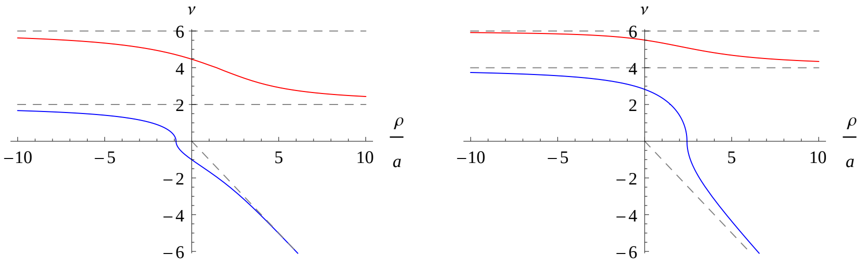

$L = 0$ (negative ν = imaginary values) $L = 2$

The Unitary Limit

- In the unitary limit, $|a| \to \infty$, ν is not depend on ρ, and therefore $P_{n,n'} = 0 = Q_{n,n'}$!
- The hyper-radial equation is similar to the Bessel equation,

$$-\frac{d^2 f(\rho)}{d\rho^2} + \frac{\nu_L^2(\rho) - 1/4}{\rho^2} f(\rho) = \epsilon f(\rho)$$

with $\nu_0 \approx 1.00624i$, and $\nu_2 \approx 2.82334$.

1. Bound state, $E_n = -\hbar^2 \kappa_n^2 / 2m < 0$:

$$f_B^{(n)}(\rho) \propto \kappa_n \sqrt{\rho} K_{\nu_0}(\kappa_n \rho)$$

where to ignore the Thomas collapse, a 3-body repulsive force is to be introduced, for example $U(\rho \le \rho_0) = \infty$ for some finite ρ_0, resulting in the famous Efimov spectrum,

$$\frac{E_n}{E_0} = e^{-2\pi n/|\nu_0|} \approx 515^{-n}.$$

2. Scattering state, $E = \hbar^2 q^2 / 2m > 0$:

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Matrix Elements Calculation

- The r^2 operator reads $\sum_i r_i^2 = \rho^2 + 3R_{CM}^2$.
- For the $\hat{Q}$ operator, $\boldsymbol{r}_i = \boldsymbol{R} - \sqrt{\frac{2}{3}}\boldsymbol{y}_i$,

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$$\left|\langle f|\hat{H}_I|i\rangle\right|^2 \propto \left[\frac{1}{6^2}\left|\left\langle \psi_B \| \sum_i r_i^2 Y_0 \| \psi_s \right\rangle\right|^2 + \frac{1}{15^2}\left|\left\langle \psi_B \| \sum_i r_i^2 Y_2(\hat{r}_i) \| \psi_d \right\rangle\right|^2\right]$$

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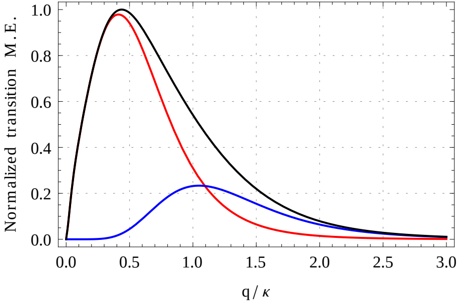
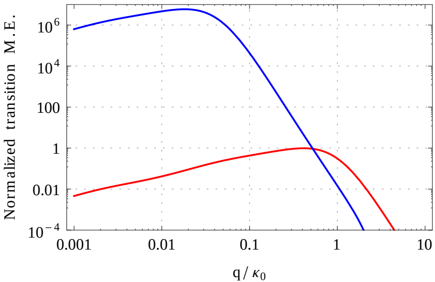
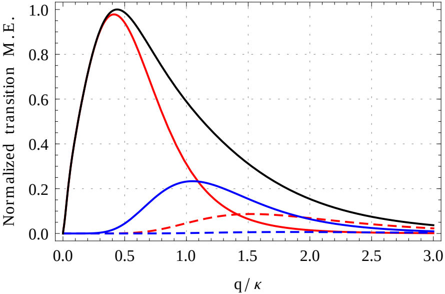


Matrix Elements Calculation

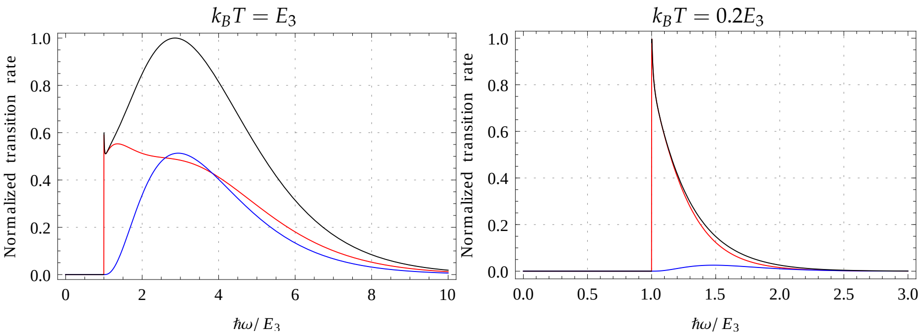
- Note that the transition rate should include all appropriate final and initial states, conserving energy and angular momentum.
- $\sum_{n,\nu} \left|\langle f_n|\hat{H}_I|i_\nu\rangle\right|^2 \delta(E_f - E_0 - \hbar\omega)$ is to be calculated.





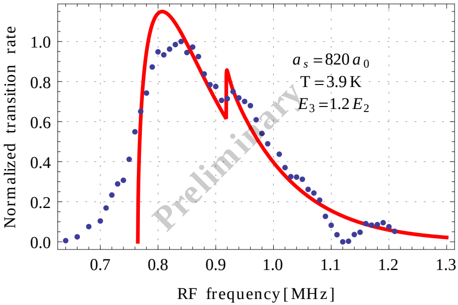
- Higher Efimov states scales as κ_n^{-5} but contribute at lower energy.
- Only $\nu_{0,1} \approx 4.465$ and $\nu_{2,1} \approx 5.508$ (dashed) contribute.

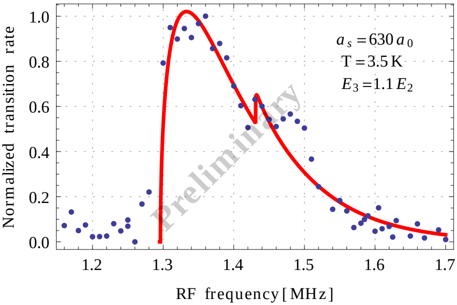
Trimer Photoasociation: Results



red - r^2 monopole, blue - quadrupole, and black - their sum

Comparison to Experiment

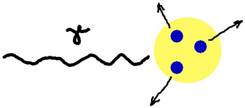




Outline

Quadrupole Response

The quadrupole response of the trimer photo-disintegration -



$$R(\omega) = C \sum_{f,\lambda} \left| \langle \Phi_f | \hat{Q} | \Phi_0 \rangle \right|^2 \delta(E_f - E_0 - \hbar\omega)$$

Using the hyper-spherical harmonics (HH) expansion up to $K_{max} = 70$, we calculate the Lorentz integral transform (LIT)

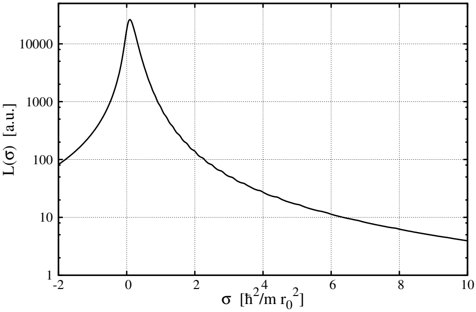
$$L(\sigma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \langle \tilde{\Psi}(\sigma) | \tilde{\Psi}(\sigma) \rangle$$

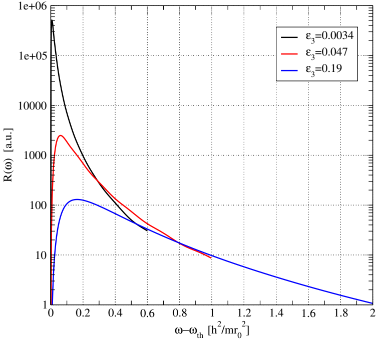
where

$$(\hat{H} - E_0 - \sigma - i\Gamma) | \tilde{\Psi}(\sigma) \rangle = \hat{Q} | \Psi_0 \rangle$$

and invert the transform to get the response $R(\omega)$

Quadrupole Response





Gaussian potential with $g = 2.3$
Lorentzian width is $\Gamma = 0.1\hbar^2/mr_0^2$.

Photodisintegration Sum Rules

$$S_n \equiv \int_{\omega_{th}}^{\infty} d\omega\, \omega^n\, R(\omega)$$

The sum rule S_n

- Exists if $R(\omega) \longrightarrow 0$ faster than ω^{-n-1}.
- Can be expressed as GS observable utilizing the closure of the eigenstates of H.

$$\begin{aligned} S_1 &= \langle 0| \left[\mathbf{O}, [H, \mathbf{O}]\right] |0\rangle = \langle 0|\mathbf{O}\,(H - E_0)\,\mathbf{O}|0\rangle \\ S_0 &= \langle 0|\mathbf{O}\mathbf{O}|0\rangle \\ S_{-1} &= \langle 0|\mathbf{O}\,\frac{1}{H - E_0}\,\mathbf{O}|0\rangle \end{aligned}$$

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Naive Scaling

- We use $a_s < 0$, therefore the only energy scale is the trimer energy
- Using simple dimensional arguments we expect that

$$r \sim 1/\sqrt{E}$$

- The Quadrupole operator behaves as r^2 so

$$R(\omega) \sim r^4/E \sim 1/E^3$$

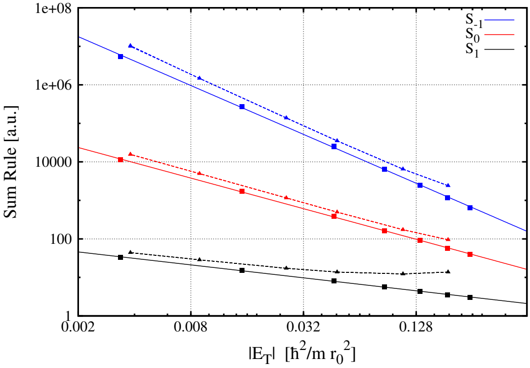
- It follows that the sum rules should have the relations

$$S_n \sim 1/E^{2-n}$$

- or

$$S_0 \sim 1/E^2$$
$$S_{-1} \sim 1/E^3$$
$$S_0/S_{-1} \sim E$$

Calculated Sum Rules



- Gauss potential (squares, solid)
- Yukawa potential (triangles, dashed)

Fitted lines

$$S_{-1} = A_{-1}E^{-2.13}$$

$$S_0 = A_0E^{-1.34}$$

$$S_1 = A_1E^{-0.55}$$

Naive Scaling Does Not Work !!!

- For S_1 we got a power of 0.55 instead of 1.
- For S_0 we got a power of 1.33 instead of 2.
- For S_{-1} we got a power of 2.13 instead of 3.
- The ration $S_n/S_{n-1} \sim E^{0.8}$ instead of $S_n/S_{n-1} \sim E$.
- The results seems to be independent of the short range specifications of the potential.

Outline

Summary and Conclusions

1. The new RF experiments in ultracold-atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
2. For spin-flip reaction, the Franck-Condon factor is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
3. For frozen-spin reactions the monopole R^2 and the Quadrupole are the leading terms, and $R(\omega) \propto \omega^5$.
4. We have studied the dimer formation and found that the reaction mechanism changes from monopole to quadrupole with increasing gas temperature.
5. The trimer formation was studied, with similar dependence on temperature.
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