Update on the No Core Shell Model

Bruce R. Barrett
University of Arizona, Tucson

THE UNIVERSITY OF ARIZONA.
Arizona's First University.

INT Program 12-3 September 17, 2012

OUTLINE

I. Brief Overview of the No Core Shell Model (NCSM)

II. Applications of the NCSM

III. Approaches for Extending the NCSM to Heavier Mass Nuclei

IV. Summary and Outlook

MICROSCOPIC NUCLEAR-STRUCTURE THEORY

1. Start with the bare interactions among the nucleons

2. Calculate nuclear properties using nuclear many-body theory

No Core Shell Model

"*Ab Initio*" approach to microscopic nuclear structure calculations, in which all A nucleons are treated as being active.

Want to solve the A-body Schrödinger equation

$$H_A \Psi^A = E_A \Psi^A$$

R P. Navrátil, J.P. Vary, B.R.B., PRC 62, 054311 (2000)

P. Navratil, et al., J. Phys. G: Nucl. Part. Phys. 36, 083101 (2009)

From few-body to many-body

Ab initio
No Core Shell Model

Realistic NN & NNN forces

↓

Effective interactions in cluster approximation

↓

Diagonalization of many- body Hamiltonian

↓

Many-body experimental data

No-Core Shell-Model Approach

- Start with the purely intrinsic Hamiltonian

$$H_A = T_{rel} + \mathcal{V} = \frac{1}{A}\sum_{i<j=1}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j=1}^{A} V_{NN} \left(+ \sum_{i<j<k}^{A} V_{ijk}^{3b} \right)$$

Note: There are <u>no</u> phenomenological s.p. energies**!**

Can use <u>any</u> NN potentials

Coordinate space: Argonne V8’, AV18, Nijmegen I, II

Momentum space: CD Bonn, EFT Idaho

No-Core Shell-Model Approach

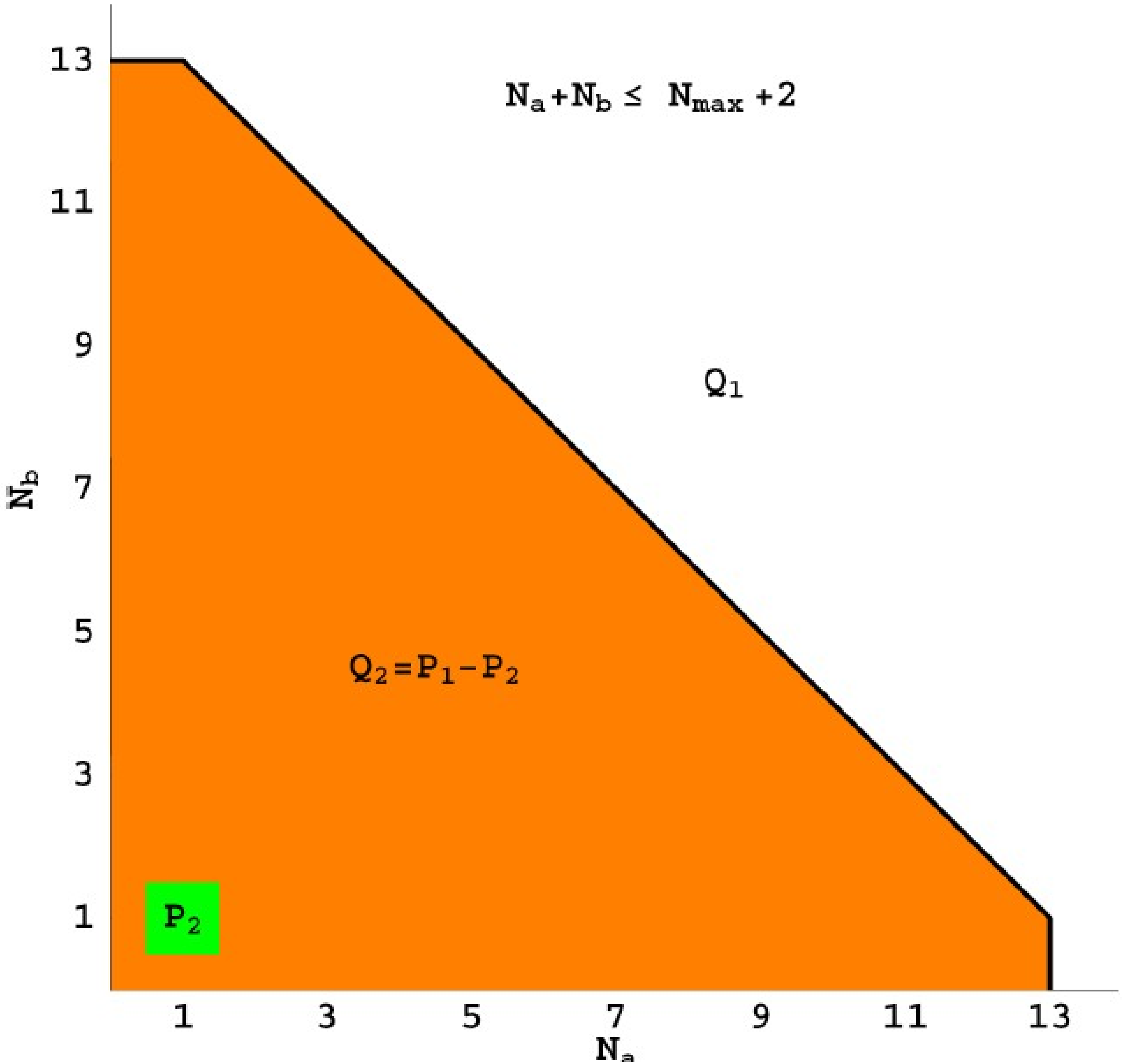
- Next, add CM harmonic-oscillator Hamiltonian

$$H_{CM}^{HO} = \frac{\vec{P}^{\,2}}{2Am} + \frac{1}{2} Am\Omega^2 \vec{R}^{\,2}; \quad \vec{R} = \frac{1}{A}\sum_{i=1}^{A} \vec{r}_i\,, \quad \vec{P} = Am\dot{\vec{R}}$$

To H_A, yielding

$$H_A^{\Omega} = \sum_{i=1}^{A}\left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2}m\Omega^2\vec{r}_i^{\,2}\right] + \underbrace{\sum_{i<j=1}^{A}\left[V_{NN}(\vec{r}_i - \vec{r}_j) - \frac{m\Omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2\right]}_{V_{ij}}$$

Defines a basis (*i.e.* HO) for evaluating V_{ij}



13
11
9
7
5
3
1
N_b
1
3
5
7
9
11
13
N_a
$N_a + N_b \leq N_{max} + 2$
Q_1
$Q_2 = P_1 - P_2$
P_2

$$H\Psi_\alpha = E_\alpha\Psi_\alpha \quad \text{where} \quad H = \sum_{i=1}^{A} t_i + \sum_{i\leq j}^{A} v_{ij}.$$

$$\mathcal{H}\Phi_\beta = E_\beta\Phi_\beta$$

$$\Phi_\beta = P\Psi_\beta$$

P is a projection operator from S into $\mathcal{S}$

$$< \tilde{\Phi}_\gamma|\Phi_\beta >= \delta_{\gamma\beta}$$

$$\mathcal{H} = \sum_{\beta\epsilon\mathcal{S}} |\Phi_\beta > E_\beta < \tilde{\Phi}_\beta|$$

Effective Hamiltonian for NCSM

Solving $H^{\Omega}_{A,\, a=2}\Psi_{a=2} = E^{\Omega}_{A,\, a=2}\Psi_{a=2}$ in "infinite space" 2n+l = 450 relative coordinates

P + Q = 1; P – model space; Q – excluded space;

$$E^{\Omega}_{A,2} = U_2 H^{\Omega}_{A,2} U_2^{\dagger}$$

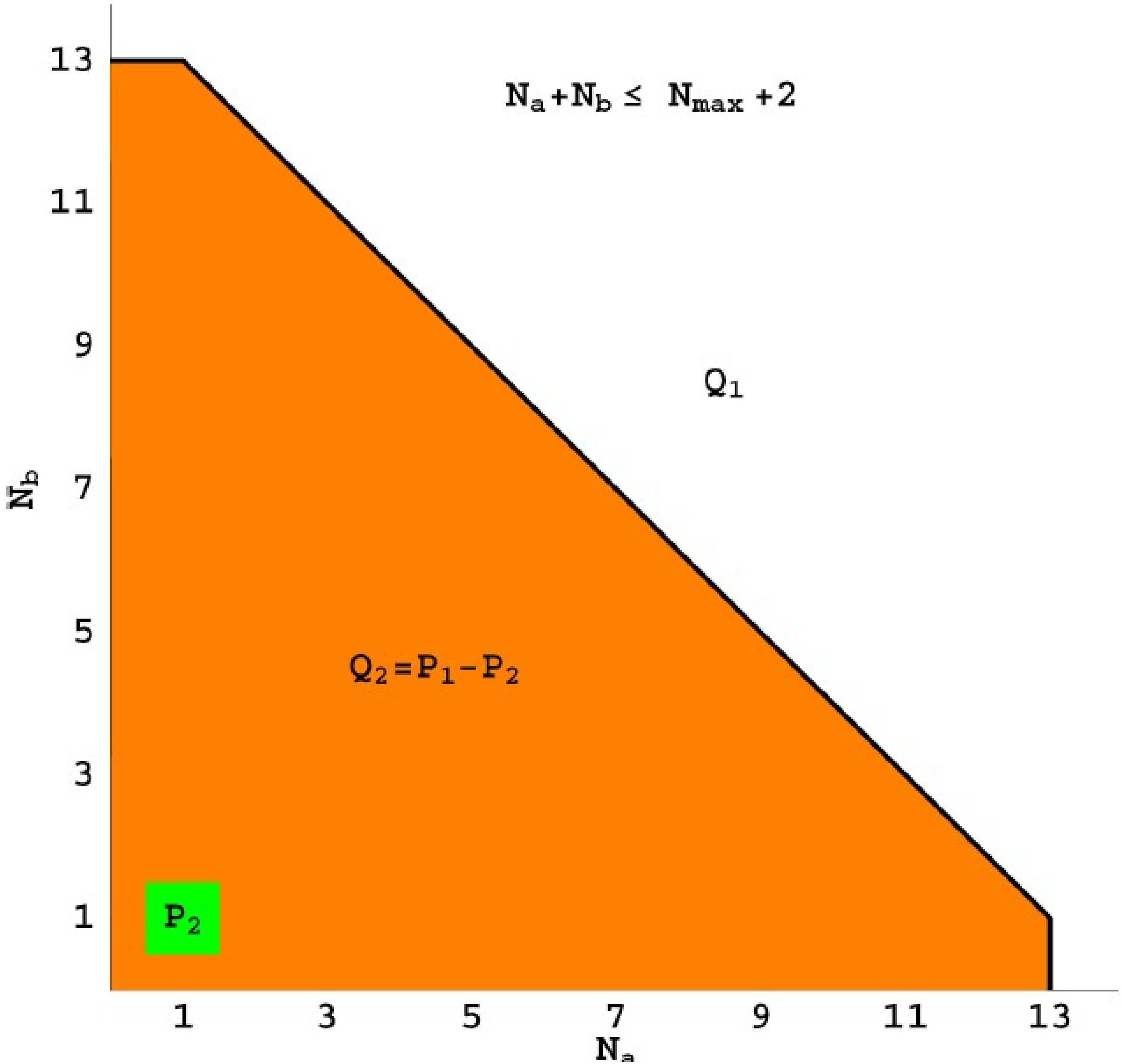
$$U_2 = \begin{pmatrix} U_{2,P} & U_{2,PQ} \\ U_{2,QP} & U_{2,Q} \end{pmatrix} \qquad E^{\Omega}_{A,2} = \begin{pmatrix} E^{\Omega}_{A,2,P} & 0 \\ 0 & E^{\Omega}_{A,2,Q} \end{pmatrix}$$

$$H^{N_{\max},\Omega,\mathrm{eff}}_{A,2} = \frac{U^{\dagger}_{2,P}}{\sqrt{U^{\dagger}_{2,P}U_{2,P}}} E^{\Omega}_{A,2,P} \frac{U_{2,P}}{\sqrt{U^{\dagger}_{2,P}U_{2,P}}}$$

Two ways of convergence:

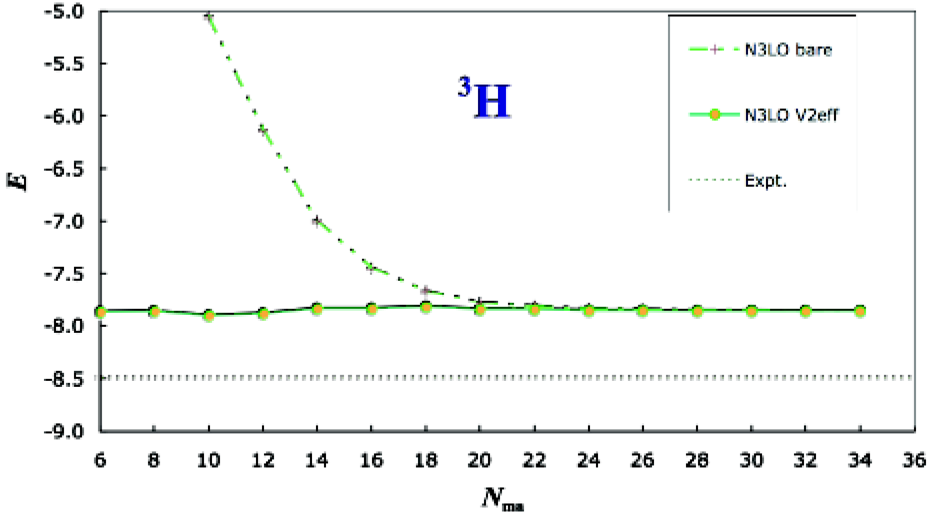
1) For $P \to 1$ and fixed a: $H^{eff}_{A,a=2} \to H_A$

2) For $a \to A$ and fixed P: $H^{eff}_{A,a} \to H_A$

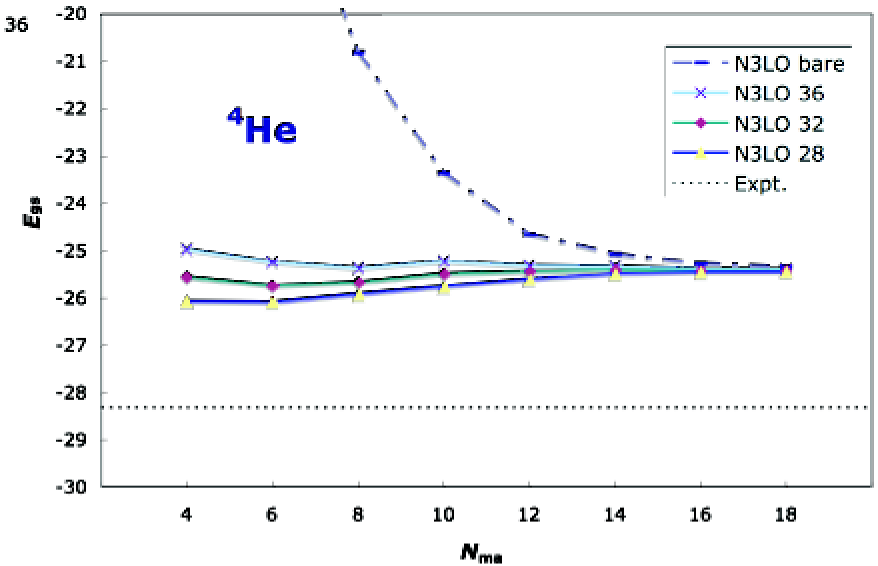


$N_a+N_b \leq N_{max}+2$
Q_1
$Q_2=P_1-P_2$
P_2
N_b
N_a
13
11
9
7
5
3
1
1
3
5
7
9
11
13

- NCSM convergence test
  - Comparison to other methods

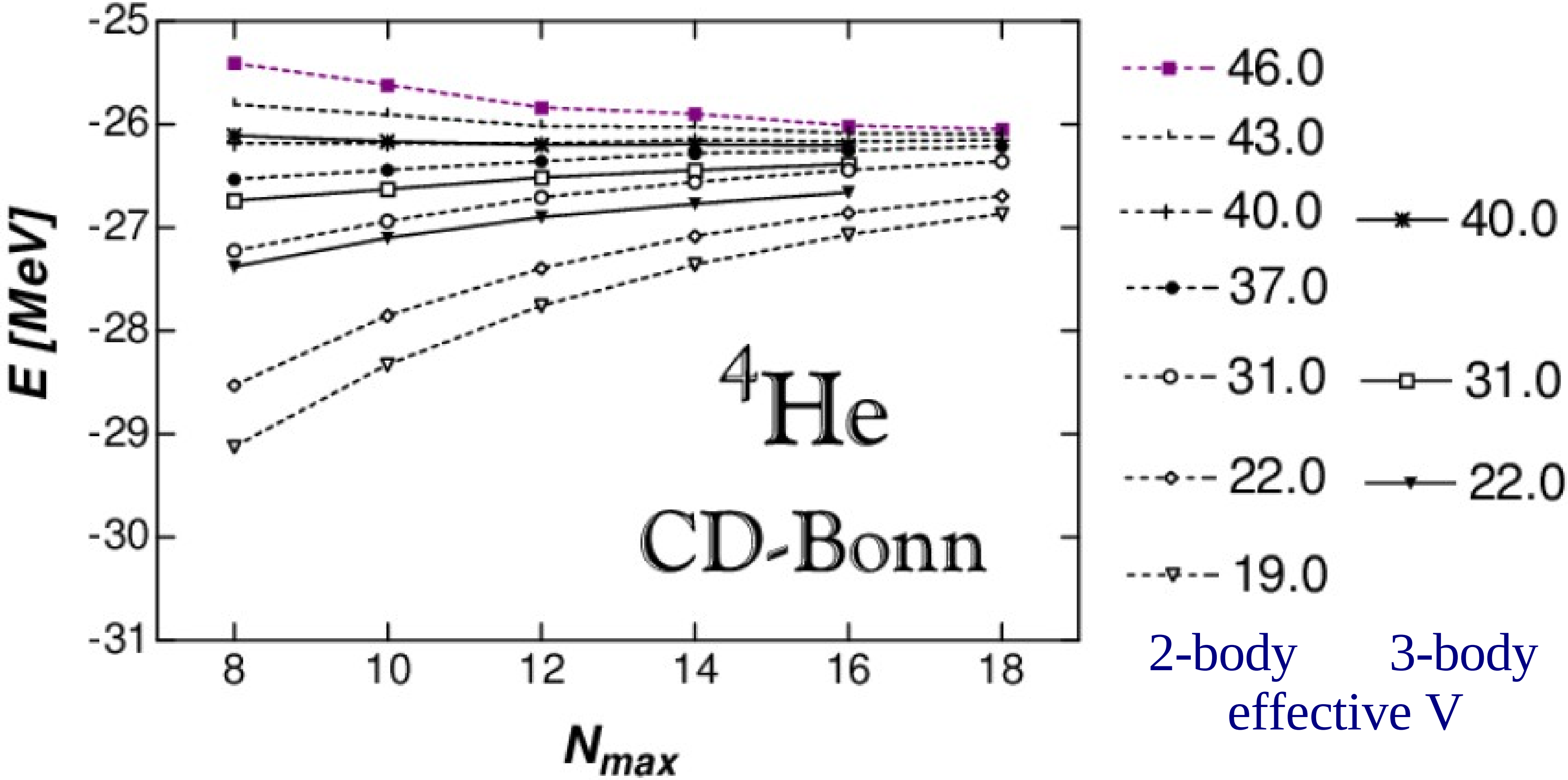


N^3LO NN	NCSM	FY	HH
^{3}H	7.852(5)	7.854	7.854
^{4}He	25.39(1)	25.37	25.38

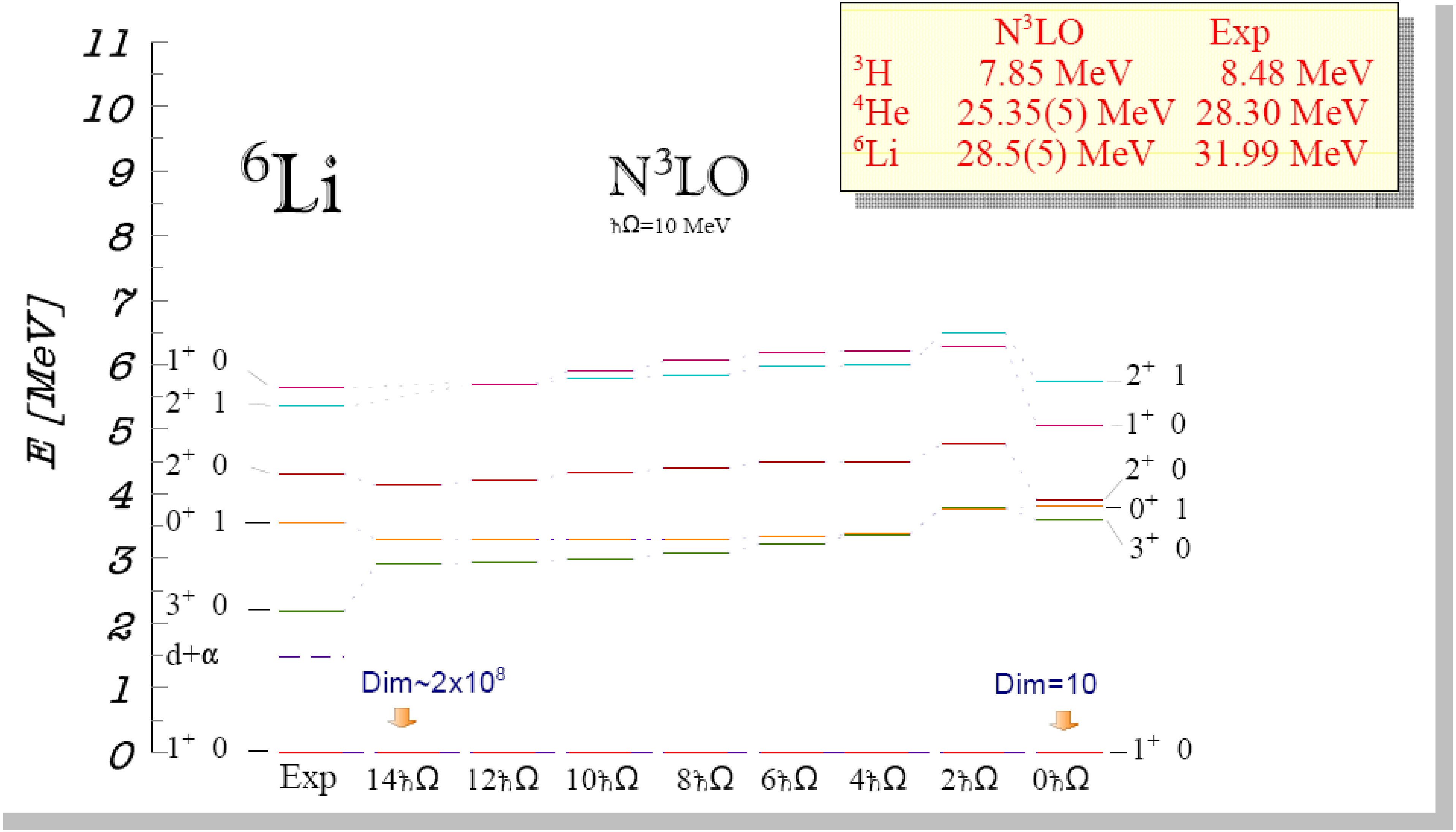


- Short-range correlations ⇒ effective interaction
- Medium-range correlations ⇒ multi-$\hbar\Omega$ model space
- Dependence on
  - size of the model space (N_{max})
  - HO frequency ($\hbar\Omega$)
- Not a variational calculation
- Convergence OK
- NN interaction insufficient to reproduce experiment

P. Navratil, INT Seminar, November 13, 2007, online



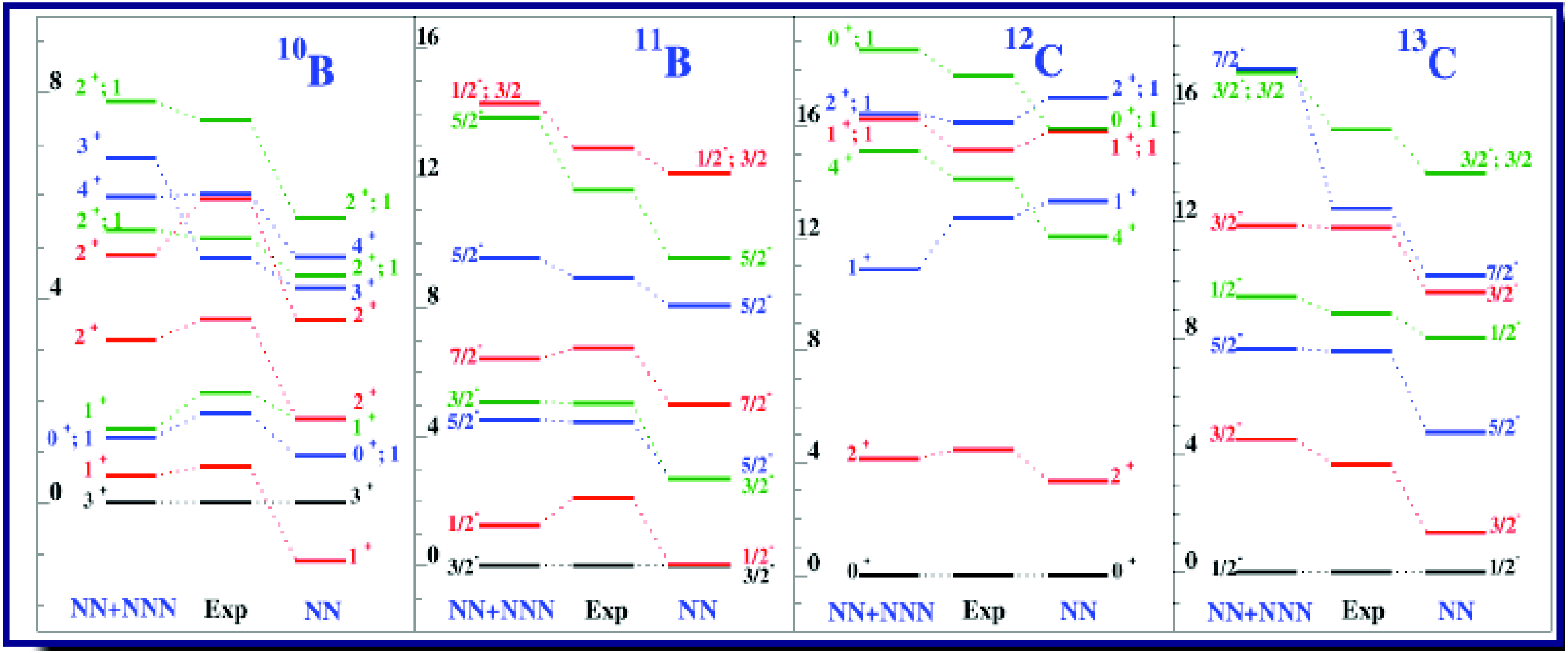
-25
-26
-27
-28
-29
-30
-31
E [MeV]
8
10
12
14
16
18
N_{max}
^{4}He
CD-Bonn
46.0
43.0
40.0
37.0
31.0
22.0
19.0
40.0
31.0
22.0
2-body
3-body
effective V



P. Navrátil and E. Caurier, Phys. Rev. C **69**, 014311 (2004)

P. Navratil, et al., Phys. Rev. Letters 99, 042501 (2007)

N3LO Interaction: D.R. Entem, et al., Phys. Rev. C 86, 041001 (2007)



II. Applications of the NCSM

H. Kamada, *et al.*, Phys. Rev. C 64, 044001 (2001)

PHYSICAL REVIEW C, VOLUME 64, 044001

Benchmark test calculation of a four-nucleon bound state

In the past, several efficient methods have been developed to solve the Schrödinger equation for four-nucleon bound states accurately. These are the Faddeev-Yakubovsky, the coupled-rearrangement-channel Gaussian-basis variational, the stochastic variational, the hyperspherical variational, the Green's function Monte Carlo, the no-core shell model, and the effective interaction hyperspherical harmonic methods. In this article we compare the energy eigenvalue results and some wave function properties using the realistic AV8′ *NN* interaction. The results of all schemes agree very well showing the high accuracy of our present ability to calculate the four-nucleon bound state.

$BE_{th} \approx 25.91$ MeV $\qquad$ $BE_{exp} \approx 28.296$ MeV

H. Kamada, *et al.*, Phys. Rev. C 64, 044011 (2001)

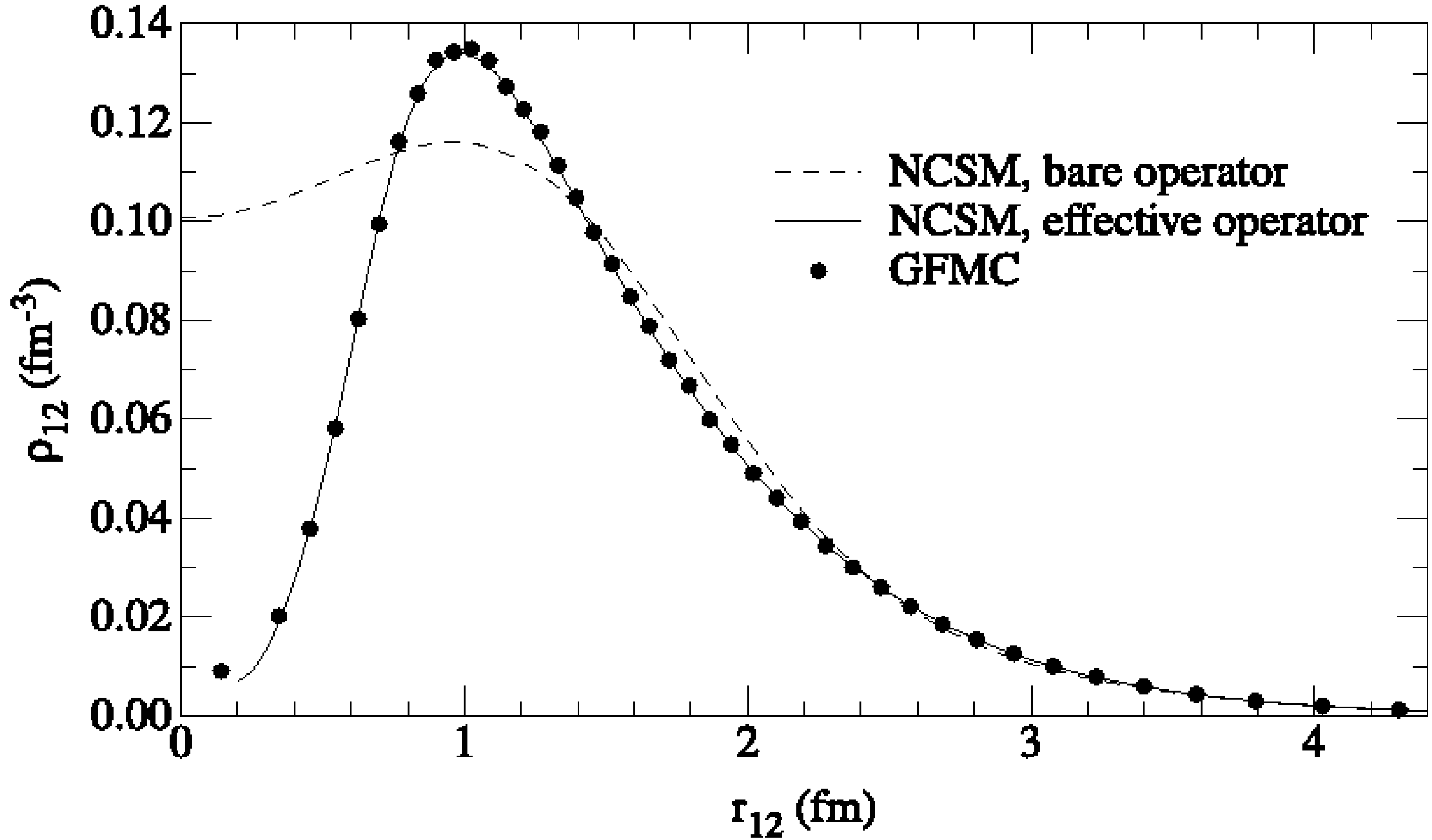


Figure 2. *NCSM and GFMC NN pair density in ^4He.*

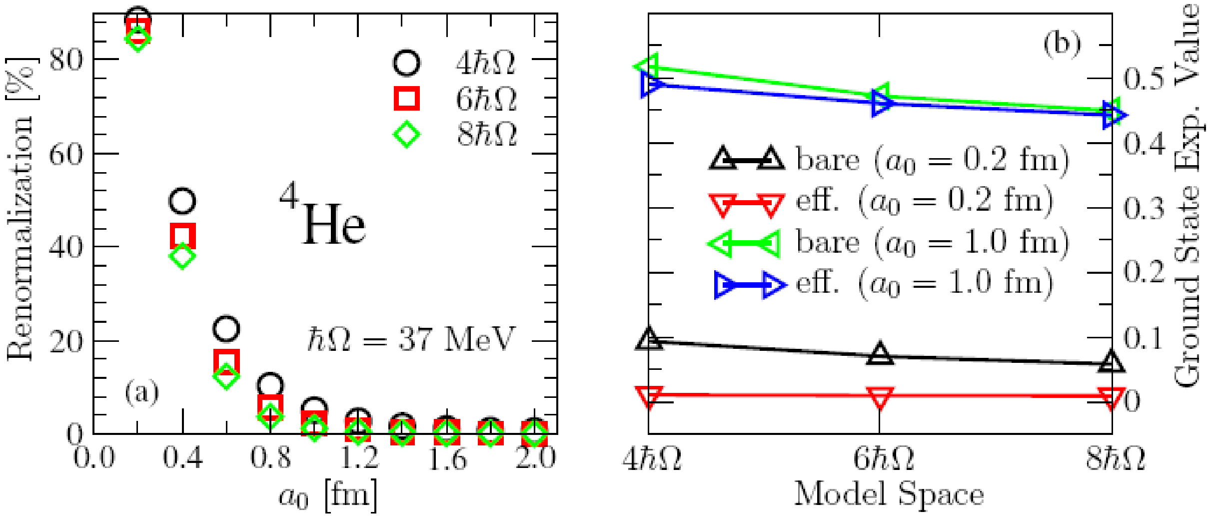
Nucleus	Observable	Model Space	Bare operator	Effective operator
^{2}H	Q_0	$4\hbar\Omega$	0.179	0.270
^{6}Li	$B(E2, 1^+0 \rightarrow 3^+0)$	$2\hbar\Omega$	2.647	2.784
^{6}Li	$B(E2, 1^+0 \rightarrow 3^+0)$	$10\hbar\Omega$	10.221	-
^{6}Li	$B(E2, 2^+0 \rightarrow 1^+0)$	$2\hbar\Omega$	2.183	2.269
^{6}Li	$B(E2, 2^+0 \rightarrow 1^+0)$	$10\hbar\Omega$	4.502	-
^{10}C	$B(E2, 2_1^+0 \rightarrow 0^+0)$	$4\hbar\Omega$	3.05	3.08
^{12}C	$B(E2, 2_1^+0 \rightarrow 0^+0)$	$4\hbar\Omega$	4.03	4.05
^{4}He	$\langle g.s.\vert T_{rel}\vert g.s.\rangle$	$8\hbar\Omega$	71.48	154.51

Stetcu, Barrett, Navratil, Vary, Phys. Rev. C 71, 044325 (2005)

- small model space: expect larger renormalization
- large variation with the model space
- three-body forces: might be important, but not the issue

- $a \rightarrow A$ for fixed model space;
- $P \rightarrow \infty$ for fixed cluster.

Range dependence



$$O \sim \exp\left[-\frac{(\vec{r}_1 - \vec{r}_2)^2}{a_0^2}\right]$$

Stetcu, Barrett, Navratil, Vary, Phys. Rev. C **71**, 044325 (2005)

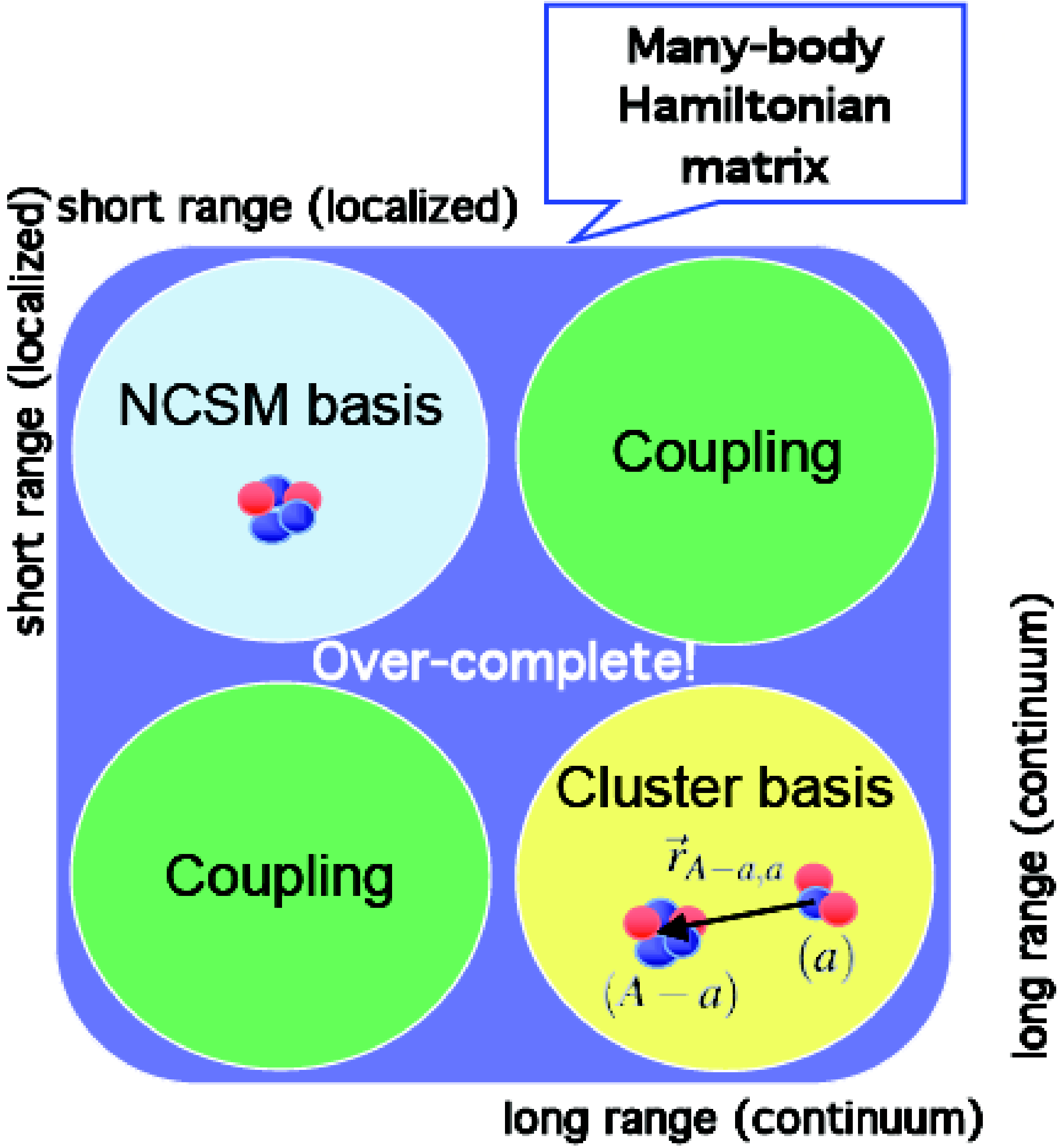
PHYSICAL REVIEW C **79**, 044606 (2009)

Ab initio many-body calculations of nucleon-nucleus scattering

Sofia Quaglioni and Petr Navrátil
Lawrence Livermore National Laboratory, P. O. Box 808, L-414, Livermore, California 94551, USA
(Received 7 January 2009; published 16 April 2009)

We develop a new *ab initio* many-body approach capable of describing simultaneously both bound and scattering states in light nuclei, by combining the resonating-group method with the use of realistic interactions, and a microscopic and consistent description of the nucleon clusters. This approach preserves translational symmetry and the Pauli principle. We outline technical details and present phase-shift results for neutron scattering on ^{3}H, ^{4}He, and ^{10}Be and proton scattering on ^{3,4}He, using realistic nucleon-nucleon (NN) potentials. Our $A = 4$ scattering results are compared to earlier *ab initio* calculations. We find that the CD-Bonn NN potential in particular provides an excellent description of nucleon-^{4}He S-wave phase shifts. In contrast, the experimental nucleon-^{4}He P-wave phase shifts are not well reproduced by any NN potential we use. We demonstrate that a proper treatment of the coupling to the n-^{10}Be continuum is successful in explaining the parity-inverted ground state in ^{11}Be.

DOI: 10.1103/PhysRevC.79.044606 PACS number(s): 21.60.De, 25.10.+s, 27.10.+h, 27.20.+n



S. Quaglioni and P. Navratil, Phys. Rev. Lett. 101, 092501 (2008)

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **81**, 021301(R) (2010)

Ab initio nuclear structure simulations: The speculative ^{14}F nucleus

P. Maris,[1] A. M. Shirokov,[1,2,*] and J. P. Vary[1]

[1]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011-3160, USA*

[2]*Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow RU-119991, Russia*

(Received 13 November 2009; published 4 February 2010)

We present results from *ab initio* no-core full configuration simulations of the exotic proton-rich nucleus ^{14}F, whose first experimental observation is expected soon. Calculations with the JISP16 NN interaction are performed up to the $N_{\max} = 8$ basis space. The binding energy is evaluated using an extrapolation technique. This technique is generalized to excitation energies, verified in calculations of ^{6}Li, and applied to ^{14}F and ^{14}B, the ^{14}F mirror, for which some data are available.

DOI: 10.1103/PhysRevC.81.021301 PACS number(s): 21.60.De, 21.10.Dr, 27.20.+n

Origin of the Anomalous Long Lifetime of ^{14}C

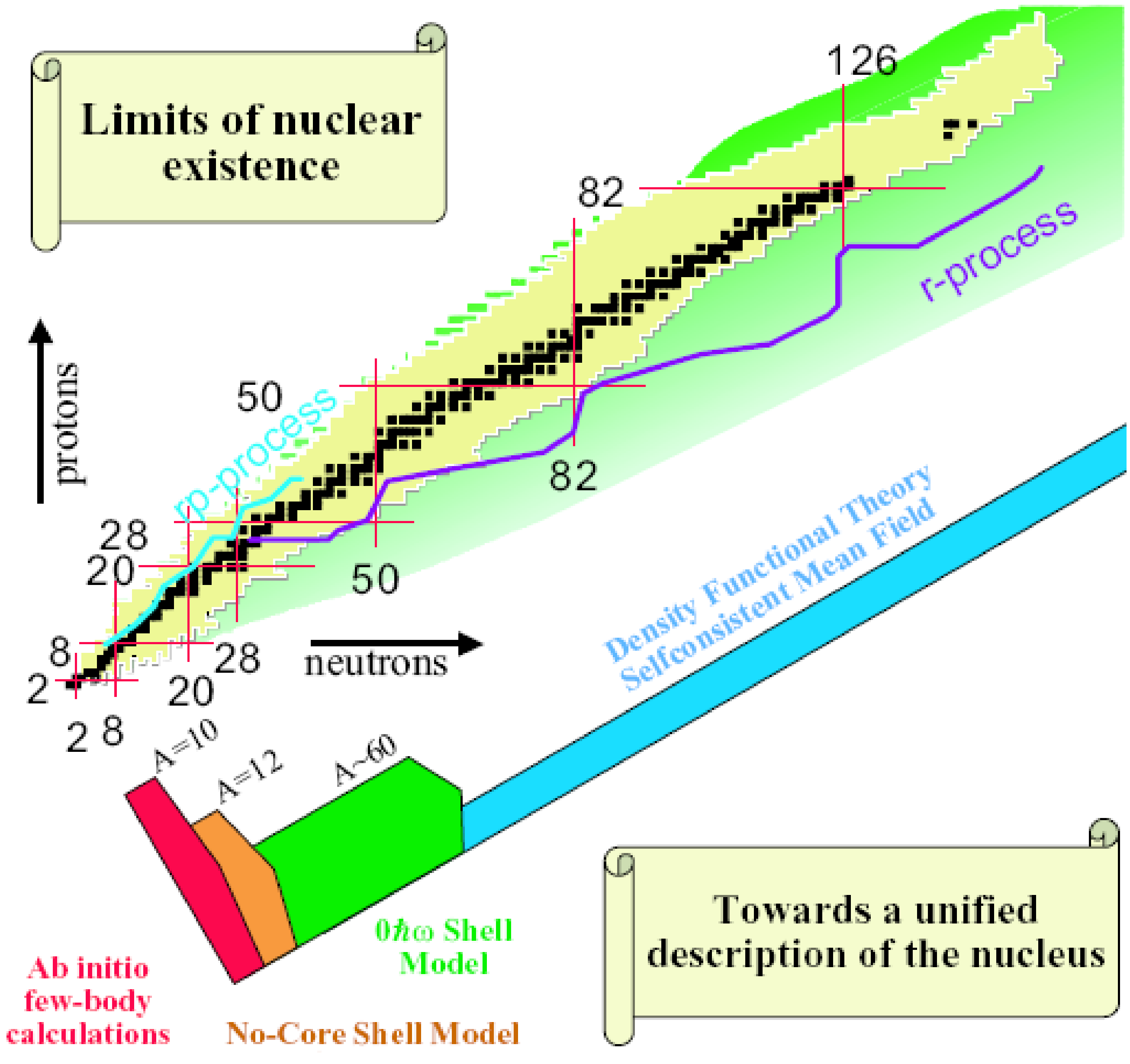
P. Maris,[1] J. P. Vary,[1] P. Navrátil,[2,3] W. E. Ormand,[3,4] H. Nam,[5] and D. J. Dean[5]

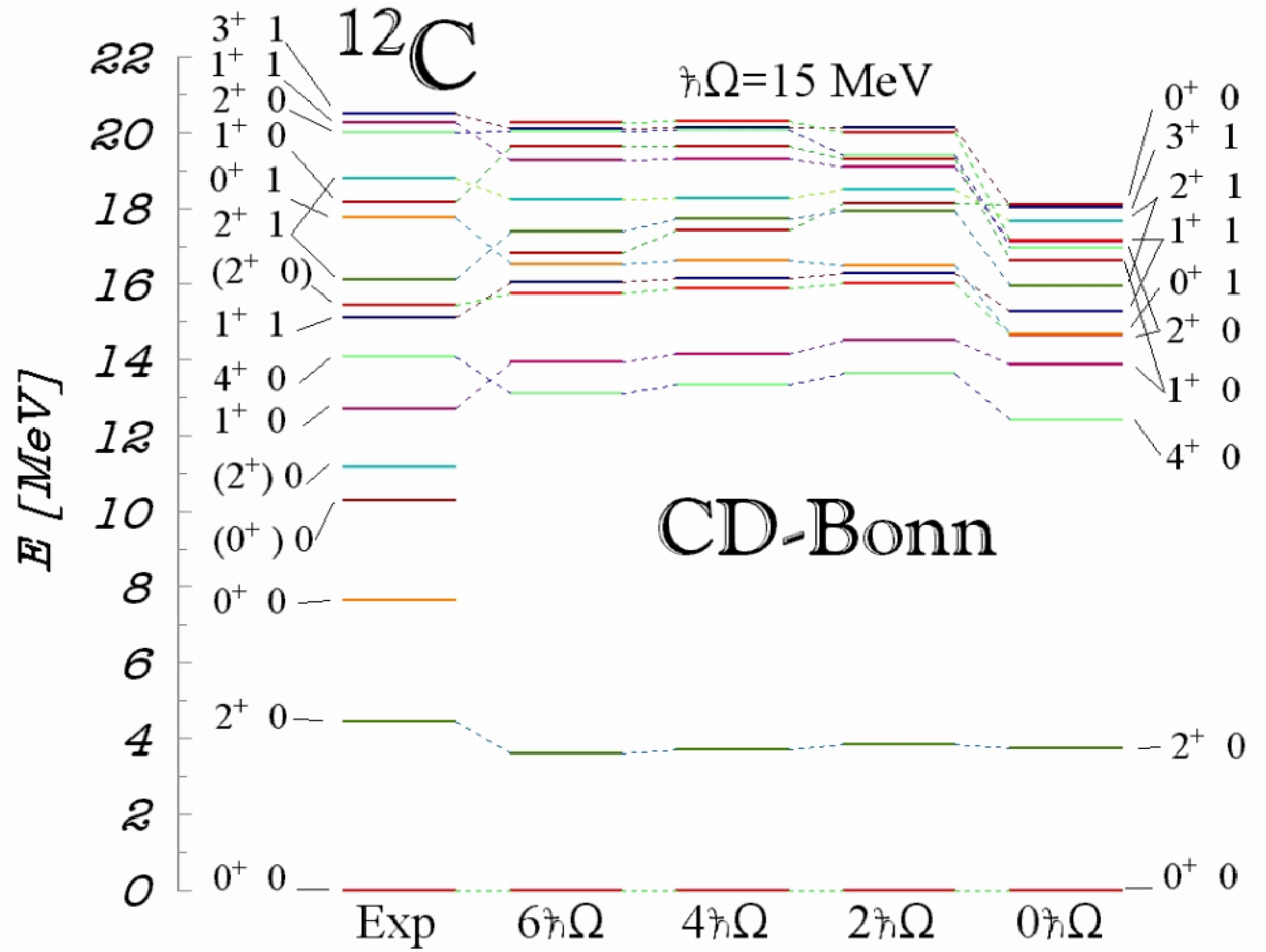
[1]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011-3160, USA*
[2]*TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, V6T 2A3, Canada*
[3]*Lawrence Livermore National Laboratory, L-414, P.O. Box 808, Livermore, California 94551, USA*
[4]*Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*
[5]*Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, Tennessee 37831, USA*
(Received 27 January 2011; published 20 May 2011)

We report the microscopic origins of the anomalously suppressed beta decay of ^{14}C to ^{14}N using the *ab initio* no-core shell model with the Hamiltonian from the chiral effective field theory including three-nucleon force terms. The three-nucleon force induces unexpectedly large cancellations within the *p* shell between contributions to beta decay, which reduce the traditionally large contributions from the nucleon-nucleon interactions by an order of magnitude, leading to the long lifetime of ^{14}C.

DOI: 10.1103/PhysRevLett.106.202502 PACS numbers: 21.10.Tg, 21.60.De, 23.40.−s, 27.20.+n

III. Extending the NCSM to Heavier Mass Nuclei


Limits of nuclear existence
protons
neutrons
126
82
50
28
20
8
2
rp-process
r-process
Density Functional Theory
Selfconsistent Mean Field
A=10
A=12
A~60
Ab initio few-body calculations
No-Core Shell Model
0ħω Shell Model
Towards a unified description of the nucleus



P. Navrátil, J. P. Vary and B. R. B., Phys. Rev. C 62, 054311 (2000)

Beyond the No Core Shell Model

1. The ab initio Shell Model with a Core

2. Importance Truncation

3. The NCSM in an Effective Field Theory (EFT) Framework

4. MC-NCSM (U of Tokyo/Iowa State U)

5. In-Medium Similarity Renormalization Group

6. Other approaches

1. The *ab initio* Shell Model with a Core

PHYSICAL REVIEW C **78**, 044302 (2008)

Ab-initio shell model with a core

A. F. Lisetskiy,[1,*] B. R. Barrett,[1] M. K. G. Kruse,[1] P. Navratil,[2] I. Stetcu,[3] and J. P. Vary[4]

[1]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*

[2]*Lawrence Livermore National Laboratory, Livermore, California 94551, USA*

[3]*Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

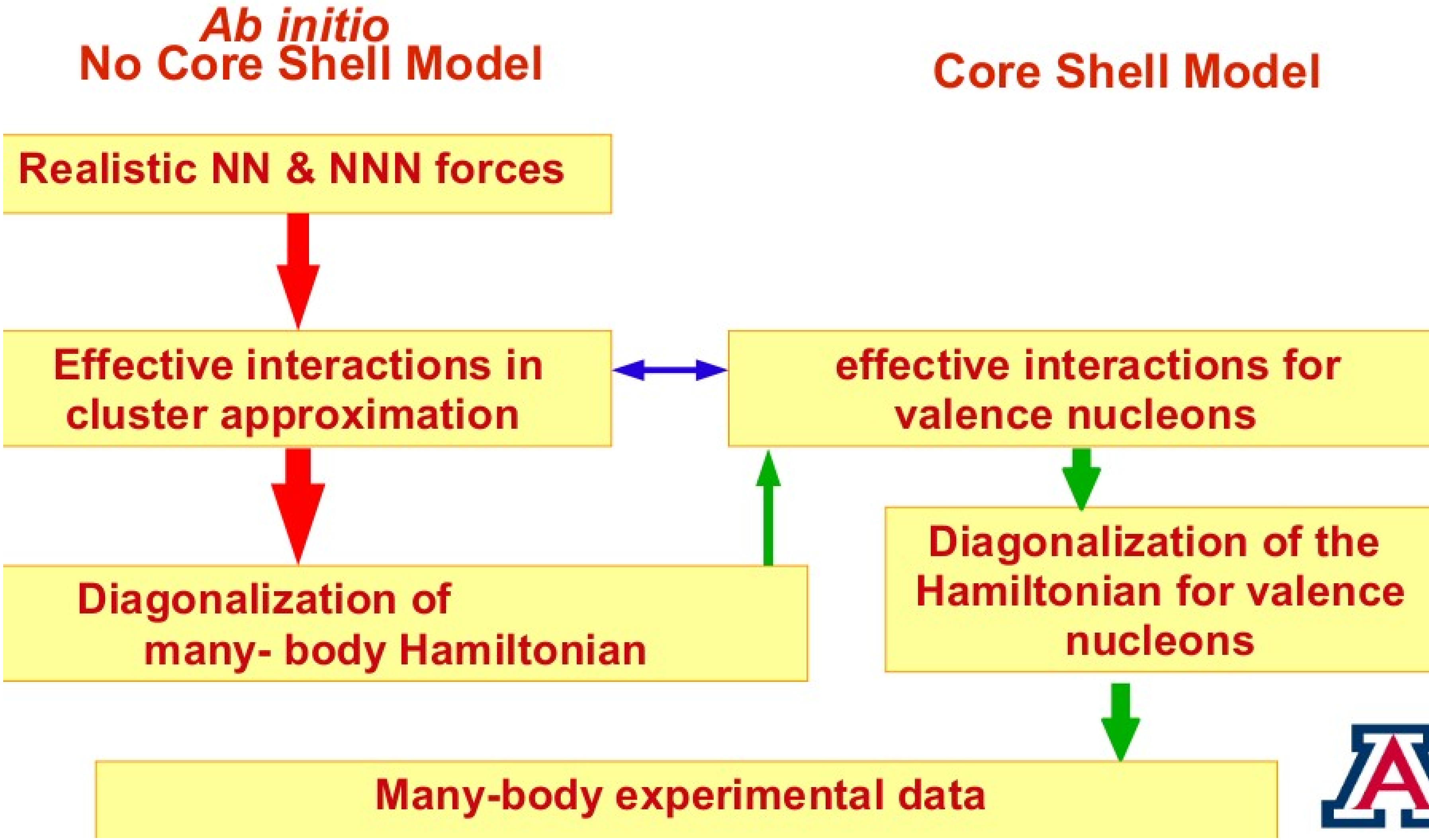
[4]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*

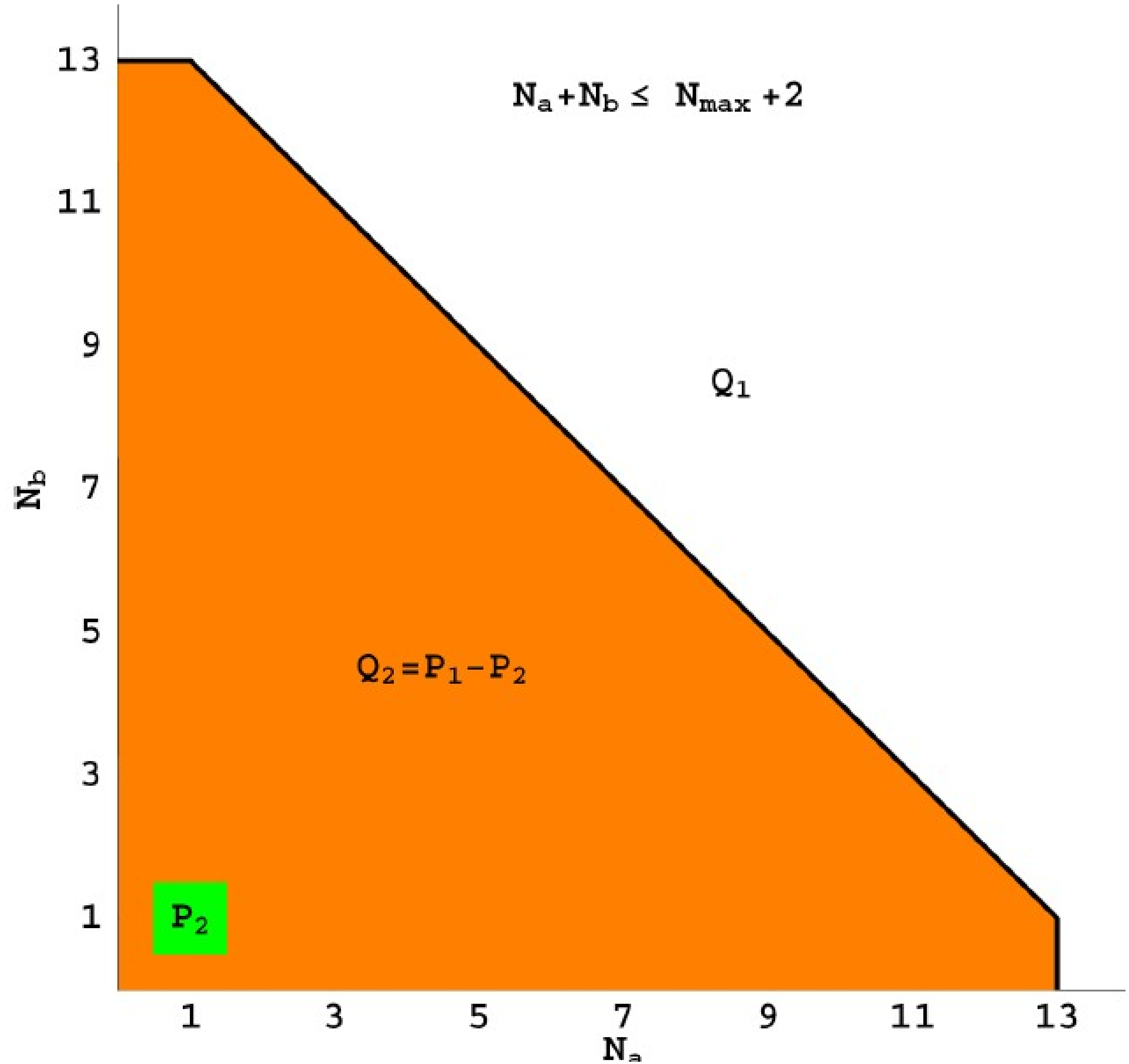
(Received 20 June 2008; published 10 October 2008)

We construct effective two- and three-body Hamiltonians for the p-shell by performing $12\hbar\Omega$ *ab initio* no-core shell model (NCSM) calculations for $A = 6$ and 7 nuclei and explicitly projecting the many-body Hamiltonians onto the $0\hbar\Omega$ space. We then separate these effective Hamiltonians into inert core, one- and two-body contributions (also three-body for $A = 7$) and analyze the systematic behavior of these different parts as a function of the mass number A and size of the NCSM basis space. The role of effective three- and higher-body interactions for $A > 6$ is investigated and discussed.

DOI: 10.1103/PhysRevC.78.044302 PACS number(s): 21.10.Hw, 21.60.Cs, 23.20.Lv, 27.20.+n

From few-body to many-body





13
11
9
7
5
3
1
N_b
$N_a + N_b \leq N_{max} + 2$
Q_1
$Q_2 = P_1 - P_2$
P_2
1
3
5
7
9
11
13
N_a

2-body Valence Cluster approximation for A=6

$$\mathcal{H}_{A,a_1=6}^{0,N_{\max}} = V_0^{A,4} + V_1^{A,5} + V_2^{A,6}$$

Need NCSM results in N_{max} space for

^{4}He

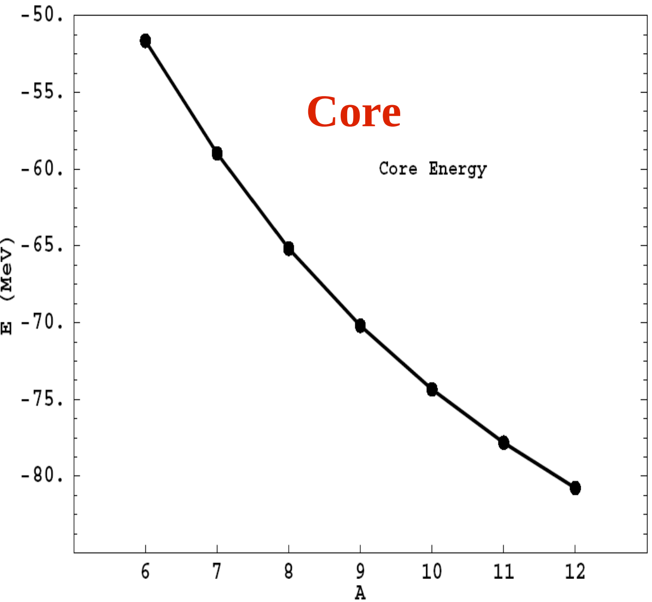
^{5}He ^{5}Li

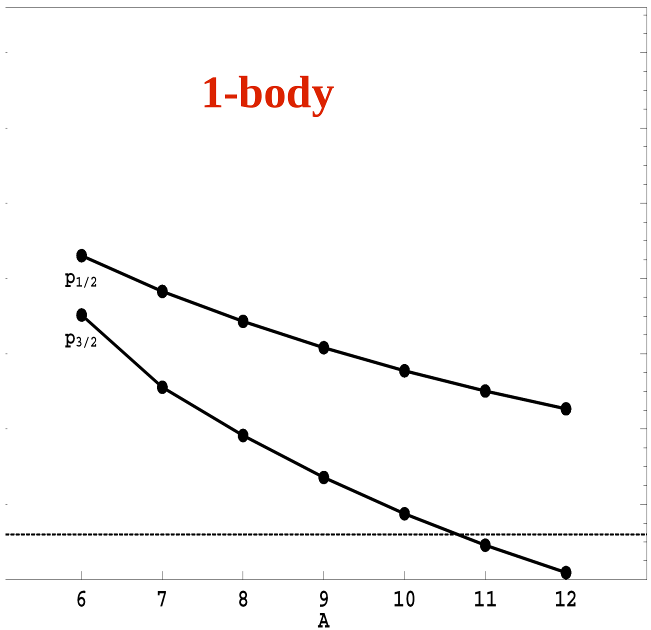
^{6}He ^{6}Li ^{6}Be

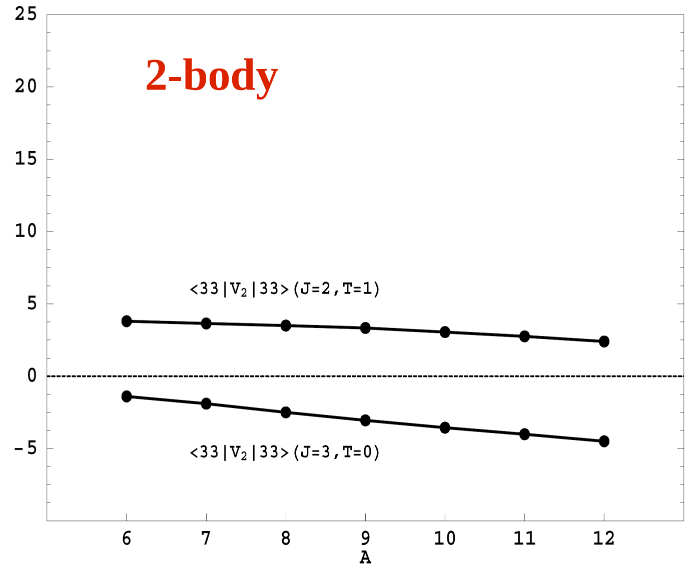
$N_{max} = 6$

With effective interaction for A !!!

$$H_{A,2}^{N_{\max},\Omega,\mathrm{eff}}$$







2-body Valence Cluster approximation for A=7

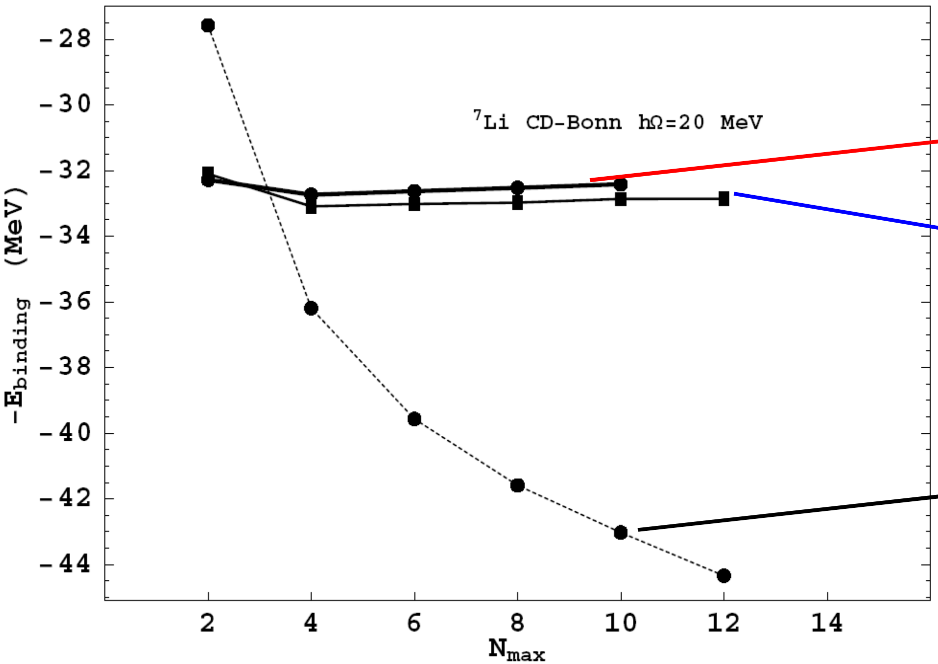
$$\mathcal{H}_{A,a_1=6}^{0,N_{max}} = V_0^{A,4} + V_1^{A,5} + V_2^{A,6}$$

Need NCSM results in N_{max} space for

^{4}He **^{5}He ^{5}Li** **^{6}He ^{6}Li ^{6}Be**

With effective interaction for A=7 !!!

$$H_{A,2}^{N_{max},\Omega,\mathrm{eff}}$$



Exact NCSM

SSM with A-dependent core

SSM with inert core

3-body Valence Cluster approximation for A>6

$$\mathcal{H}^{0,N_{\max}}_{A,a_1=7} = V_0^{A,4} + V_1^{A,5} + V_2^{A,6} + V_3^{A,7}$$

Need NCSM results in N_{max} space for

^{4}He | **^{5}He ^{5}Li** | **^{6}He ^{6}Li ^{6}Be** | **^{7}He ^{7}Li ^{7}B ^{7}Be**

With effective interaction for A !!!

$$H_{A\ \ ,2}^{N_{\max},\Omega,\text{eff}}$$

Construct 3-body interaction in terms of 3-body matrix elements: Yes

$$V_3^{A,7} = \mathcal{H}^{0,N_{\max}}_{A,7} - \mathcal{H}^{0,N_{\max}}_{A,6}$$

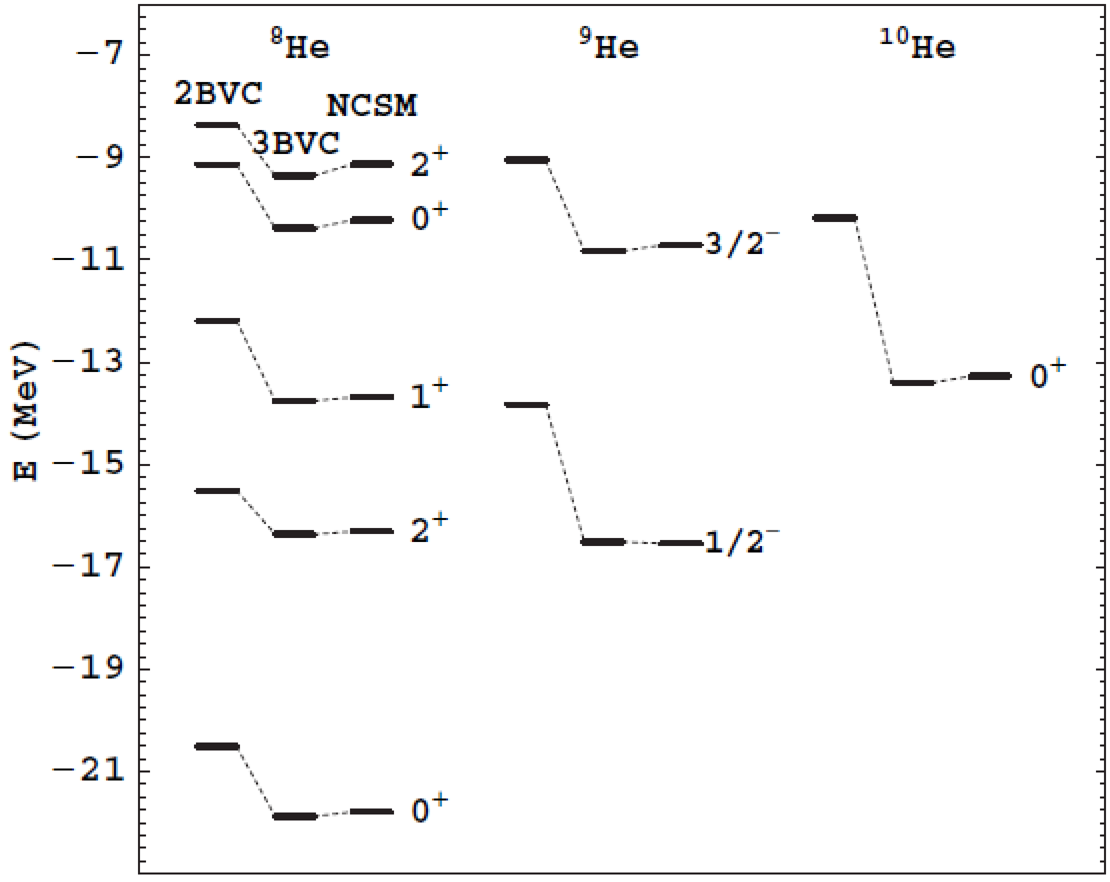


FIG. 9. Comparison of spectra for ^{8}He, ^{9}He, and ^{10}He from SSM calculations using the effective 2BVC and 3BVC Hamiltonians and from exact NCSM calculation for $N_{\max} = 6$ and $\hbar\Omega = 20$ MeV using the CD-Bonn interaction.

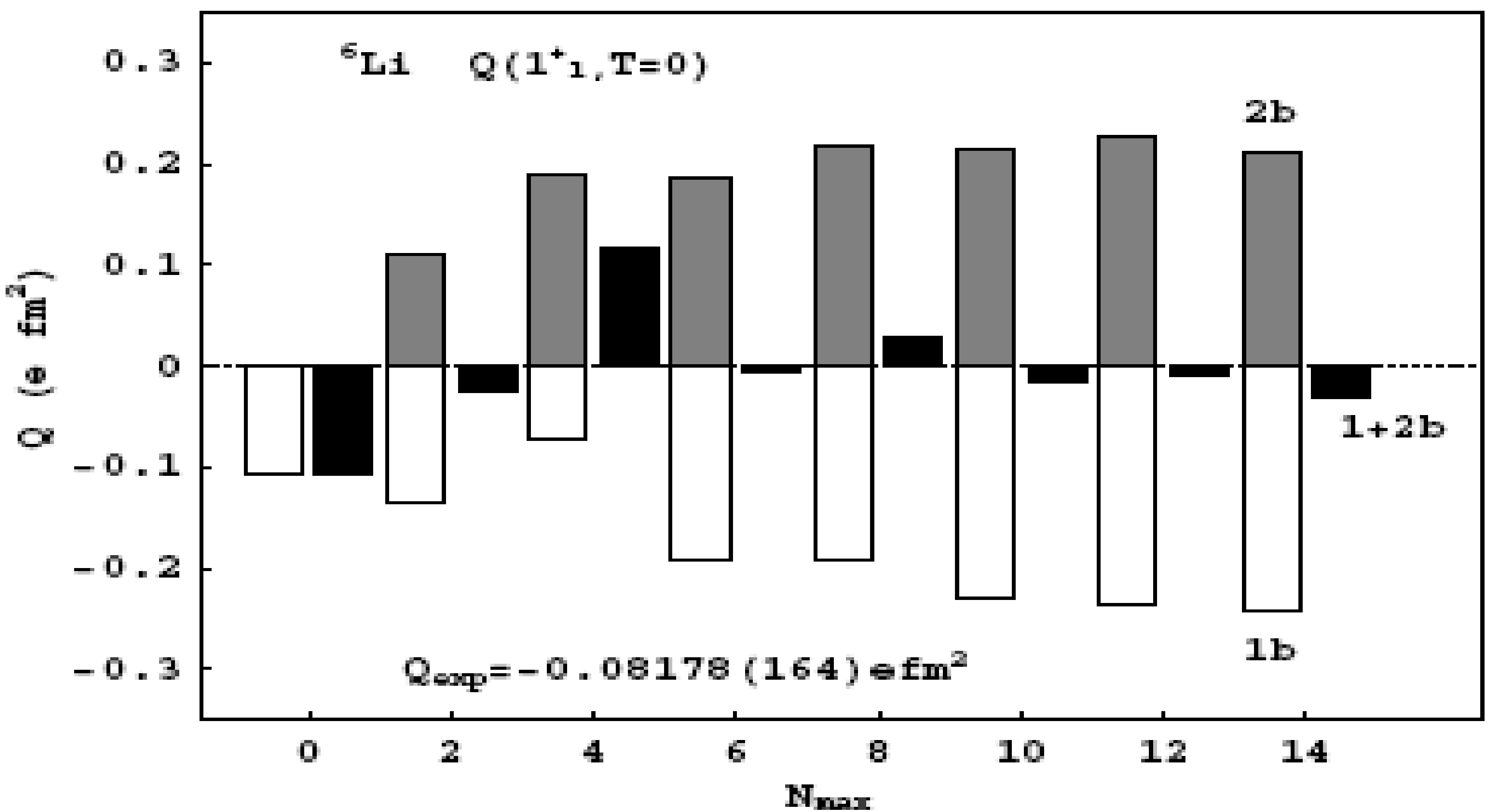
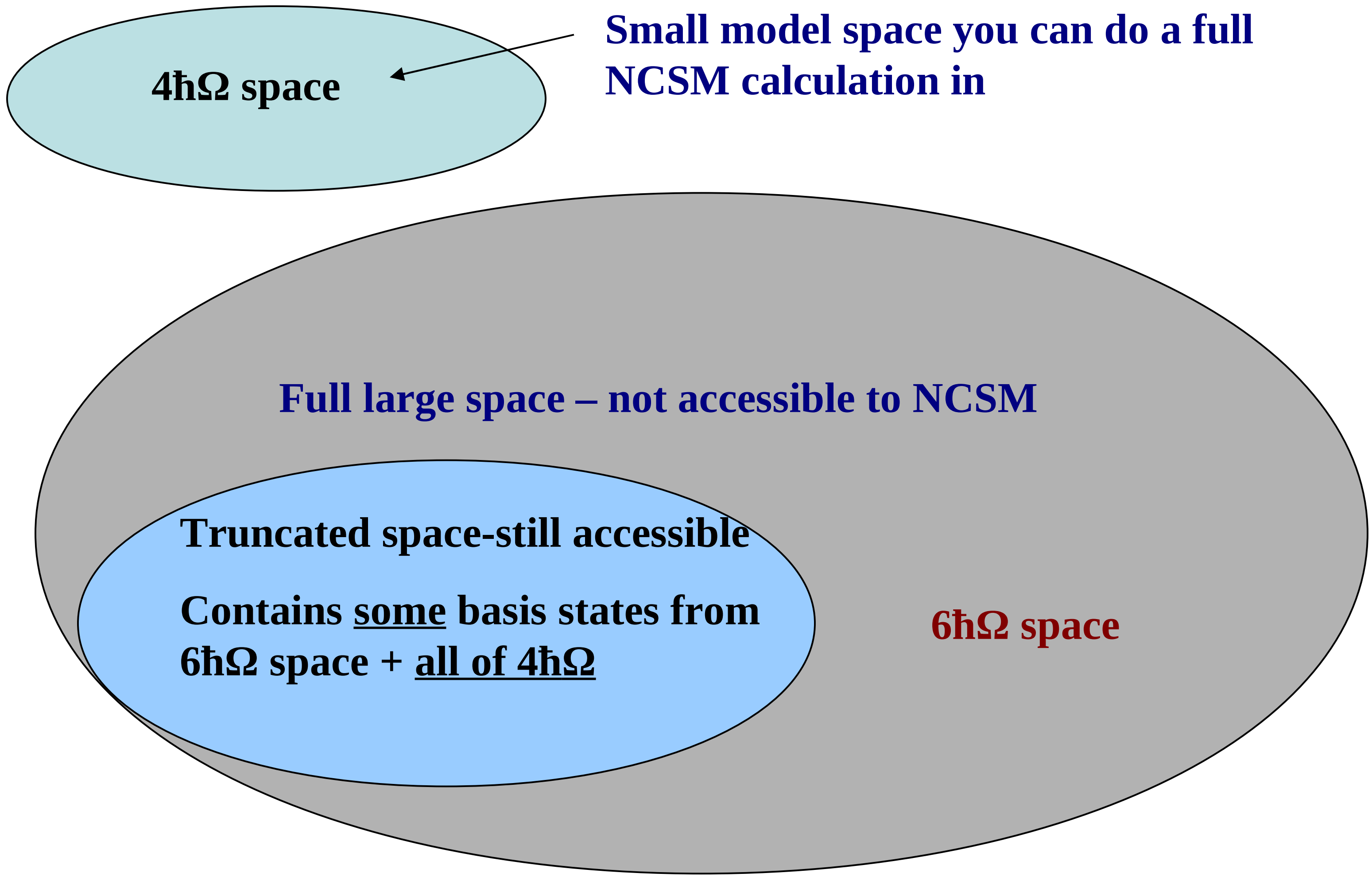


FIG. 6: The quadrupole moment of the ground state for ^{6}Li ($1^+(T = 0)$) is shown in terms of one- and two-body contributions as a function of increasing model space size.

2. Importance Truncation

The idea of Importance Truncation



Formalism of Importance truncation.

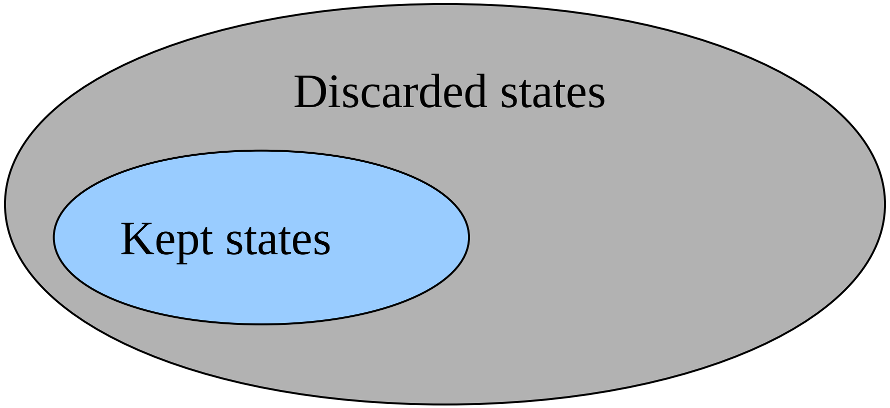
- First order multi-configurational perturbation theory gives...

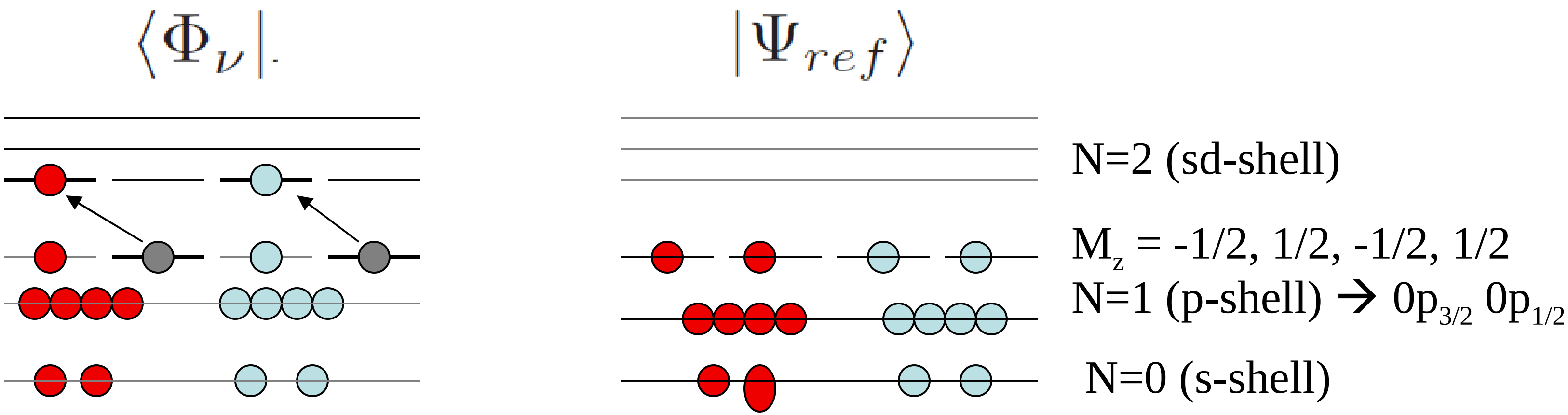
$$|\Psi^{(1)}\rangle = -\sum_{\nu \notin \mathcal{M}_{\text{ref}}} \frac{\langle \Phi_\nu | W | \Psi_{\text{ref}} \rangle}{\epsilon_\nu - \epsilon_{\text{ref}}} |\Phi_\nu\rangle$$
$$= -\sum_{\nu \notin \mathcal{M}_{\text{ref}}} \frac{\langle \Phi_\nu | H | \Psi_{\text{ref}} \rangle}{\epsilon_\nu - \epsilon_{\text{ref}}} |\Phi_\nu\rangle.$$

$$W = H - H_0$$

Importance truncation schematically

$$\kappa_\nu = \frac{|\langle \Phi_\nu | H | \Psi_{ref} \rangle|}{\epsilon_\nu - \epsilon_{ref}}$$





O16 – one possible configuration

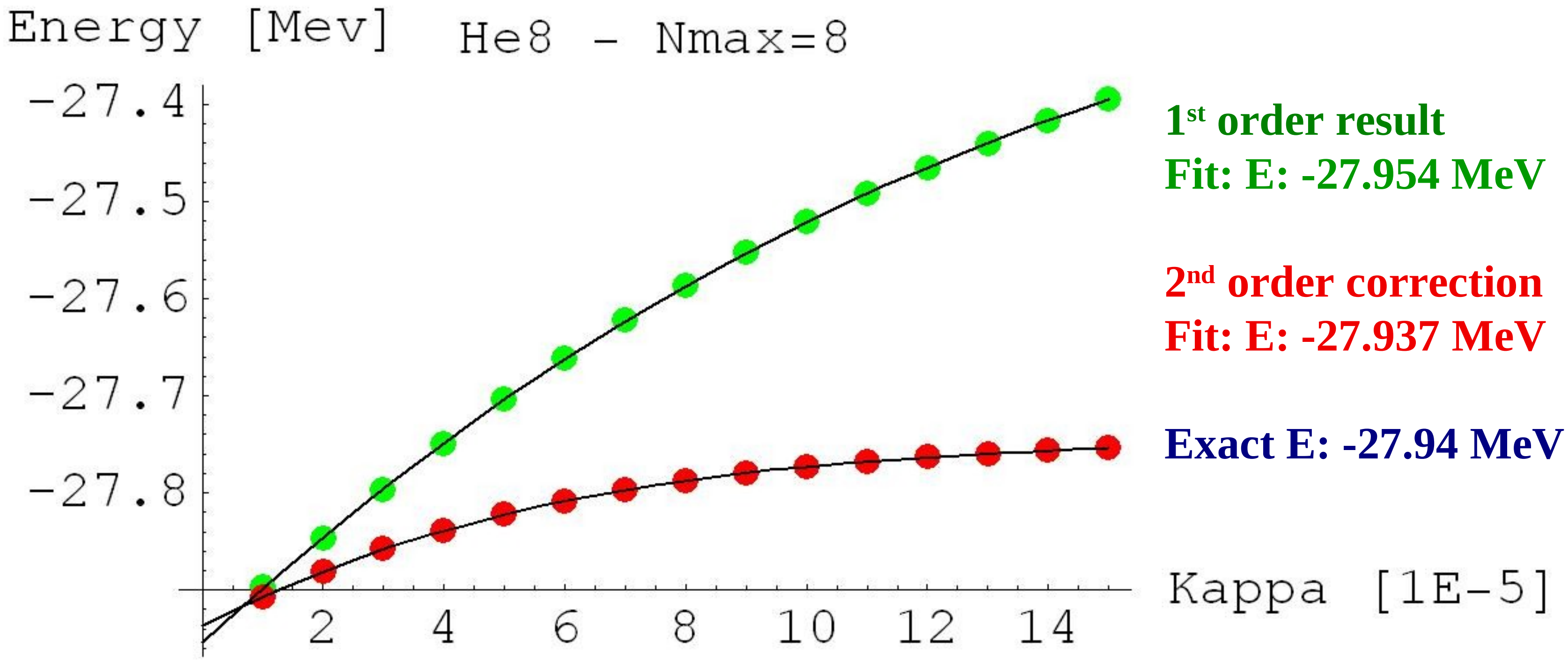
O16 - 0ħΩ configuration

Corrections to the energy

- 2nd order perturbation theory gives you an estimate of the correction to the energy from the discarded state. The first order result is equal to zero.

$$\Delta_{\text{excl}}(\kappa_{\min}) = - \sum_{\nu \notin \mathcal{M}(\kappa_{\min})} \frac{|\langle \Phi_\nu | H | \Psi_{\text{ref}} \rangle|^2}{\epsilon_\nu - \epsilon_{\text{ref}}}$$

^{8}He: IT started at N_{max} = 6, final space N_{max} = 8



1st order result
Fit: E: -27.954 MeV

2nd order correction
Fit: E: -27.937 MeV

Exact E: -27.94 MeV

Interaction: ^{8}He SRG N3LO

3. The NCSM in an Effective Field Theory (EFT) Framework

Available online at www.sciencedirect.com

ScienceDirect

Physics Letters B 653 (2007) 358–362

PHYSICS LETTERS B

www.elsevier.com/locate/physletb

No-core shell model in an effective-field-theory framework

I. Stetcu[a,b,*], B.R. Barrett[a], U. van Kolck[a]

[a] *Department of Physics, University of Arizona, Tucson, AZ 85721, USA*

[b] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

Received 5 March 2007; received in revised form 8 July 2007; accepted 31 July 2007

Available online 9 August 2007

Editor: W. Haxton

Abstract

We present a new approach to the construction of effective interactions suitable for many-body calculations by means of the no-core shell model (NCSM). We consider an effective field theory (EFT) with only nucleon fields directly in the NCSM model spaces. In leading order, we obtain the strengths of the three contact interactions from the condition that in each model space the experimental ground-state energies of ^{2}H, ^{3}H and ^{4}He be exactly reproduced. The first $(0^+;0)$ excited state of ^{4}He and the ground state of ^{6}Li are then obtained by means of NCSM calculations in several spaces and frequencies. After we remove the harmonic-oscillator frequency dependence, we predict for ^{4}He an energy level for the first $(0^+;0)$ excited state in remarkable agreement with the experimental value. The corresponding ^{6}Li binding energy is about 70% of the experimental value, consistent with the expansion parameter of the EFT.

PACS: 21.30.-x; 21.60.Cs; 24.10.Cn; 45.50.Jf

IOP Publishing Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. **37** (2010) 064033 (11pp) doi:10.1088/0954-3899/37/6/064033

Effective interactions for light nuclei: an effective (field theory) approach

I Stetcu[1], J Rotureau[2], B R Barrett[2,3] and U van Kolck[2]

[1] Department of Physics, University of Washington, Seattle, WA 98195, USA
[2] Department of Physics, University of Arizona, Tucson, AZ 85721, USA

E-mail: bbarrett@physics.arizona.edu

Received 12 December 2009
Published 26 April 2010
Online at stacks.iop.org/JPhysG/37/064033

Abstract

One of the central open problems in nuclear physics is the construction of effective interactions suitable for many-body calculations. We discuss a recently developed approach to this problem, where one starts with an effective field theory containing only fermion fields and formulated directly in a no-core shell-model space. We present applications to light nuclei and to systems of a few atoms in a harmonic-oscillator trap. Future applications and extensions, as well as challenges, are also considered.

PHYSICAL REVIEW C **85**, 034003 (2012)

Two and three nucleons in a trap, and the continuum limit

J. Rotureau,[1,2] I. Stetcu,[3,4] B. R. Barrett,[2] and U. van Kolck[2]

[1]*Fundamental Physics, Chalmers University of Technology, 412 96 Göteborg, Sweden*

[2]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*

[3]*Department of Physics, University of Washington, Box 351560, Seattle, Washington 98195-1560, USA*

[4]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

(Received 1 December 2011; published 14 March 2012; publisher error corrected 20 March 2012)

We describe systems of two and three nucleons trapped in a harmonic-oscillator potential with interactions from the pionless effective field theory up to next-to-leading order (NLO). We construct the two-nucleon interaction using two-nucleon scattering information. We calculate the trapped levels in the three-nucleon system with isospin $T = 1/2$ and determine the three-nucleon force needed for stability of the triton. We extract neutron-deuteron phase shifts, and show that the quartet scattering length is in good agreement with experimental data.

DOI: 10.1103/PhysRevC.85.034003 PACS number(s): 21.30.Fe, 21.45.−v

Effective Field Theory (1/3)

i) Separation of scale :

M_{QCD} ~ 1 GeV (mass of nucleon)

M_{nucl} ~ 100 MeV (typical momentum in a nucleus)

M_{struct} ~ 10 MeV (binding energy of a nucleon in a nucleus)

-> details of physics at short distance (high energy) are irrelevant for low energy physics.

-> in EFT low energy degrees of freedom are explicitly included (high momenta are integrated out).

ii) The Lagrangian / potential consistent with symmetries is expanded as a Taylor Series:

$$V(\vec{p}\,', \vec{p}) = \sum_{i,j} C_{i,j} (\vec{p})^i (\vec{p}\,')^j$$

Effective Field Theory (2/3)

iii) Regularization and renormalization :

-> cut-off Λ (separation between low and high energy physics)

$$V(\vec{p}\,',\vec{p}) => \sum_{i,j} C_{i,j}(\Lambda)(\vec{p})^i(\vec{p}\,')^j$$

-> no dependence on cut-off for observables (for a high enough cut-off), dependence absorbed by coupling constants (fitted with observables).

Effective Field Theory (3/3)

iv) Find the power counting ("truncation of the Taylor series"):

-> hierarchy between the different contributions

-> results improvable order by order (Leading Order, Next-to-Leading-Order, Next-to-Next-to-Leading-Order......)

Why EFT + NCSM?

EFT:

1. Captures the relevant degrees of freedom/symmetries
2. Builds in the correct long-range behavior
3. Has a systematic way for including the short-range behavior/order by order
4. Many-body and two-body interactions treated in the same framework
5. Explains naturally the hierarchy of the (many-body) forces

NCSM:

1. Flexible many-body method/easy to implement
2. Equivalent SD and Jacobi formulations
3. Can handle both NN and NNN interactions
4. In principle applies to any nucleus/extensions to heavier nuclei

CUTOFFS

1. Ultraviolet Cutoff: Want convergence as Lamda increases.

$$\Lambda = \sqrt{m_N (N_{\max} + 3/2)\hbar\omega}$$

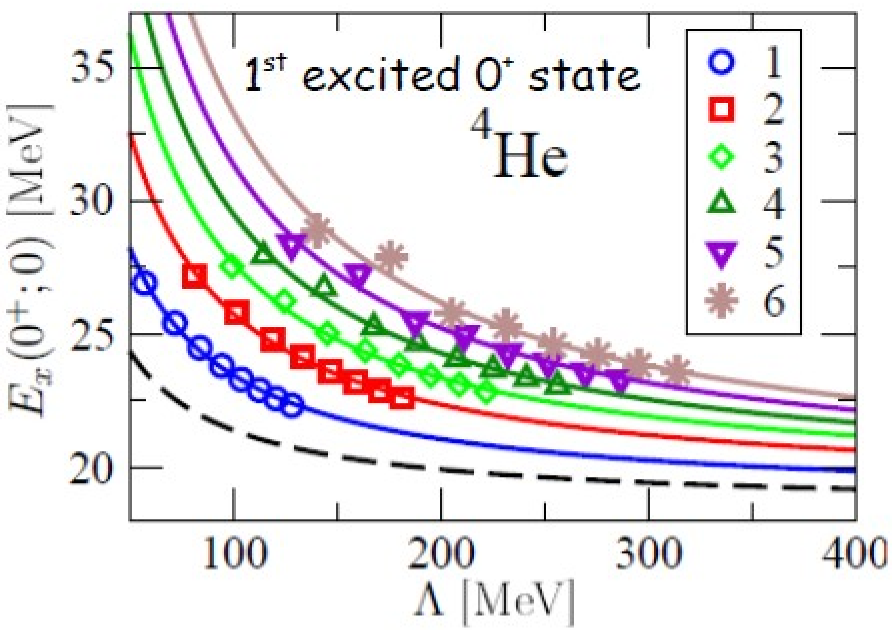
2. Infrared Cutoff: Want convergence as omega decreases.

$$\lambda = \sqrt{m_N \hbar\omega}$$

Pionless EFT for nuclei within the NCSM:
Without pions--> Breakdown momentum roughly 100 MeV/c

$$H = \frac{1}{2m_N A}\sum_{[i<j]}(\vec{p}_i - \vec{p}_j)^2 + C_0^1 \sum_{[i<j]^1}\delta(\vec{r}_i - \vec{r}_j) + C_0^0 \sum_{[i<j]^0}\delta(\vec{r}_i - \vec{r}_j) + D_0 \sum_{[i<j<k]}\delta(\vec{r}_i - \vec{r}_j)\delta(\vec{r}_j - \vec{r}_k),$$

Stetcu et. al., 2007



-> calculation at **Leading order** : two N-N contact interactions in the $^3S_1$, $^1S_0$ channel and a three-body contact interaction in the 3-nucleon $S_{1/2}$ channel

-> coupling constants fitted to the binding energy of the deuteron, triton and ^{4}He.

Difficulties:

fixing the couplings to few-body states is cumbersome

HO: bound states only

no immediate connection to the scattering observables

Question : How to construct an EFT within a bound many-body model space beyond **Leading-Order** ?

Answer : by trapping nuclei in a harmonic potential

T. Busch, et al., Found. Phys. 28, 549 (1998)

$$\frac{\Gamma\left(\frac{3}{4}-\frac{E}{2\hbar\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E}{2\hbar\omega}\right)} = -\frac{bk}{2}\cot\delta$$

energy in the trap (bound state physics)

phase shift (scattering physics)

$$k\,\cot\delta = -\frac{1}{a_2} + \frac{1}{2} r_2 k^2 + \ldots,$$

Effective Range Expansion

LO Renormalization:

$$\Psi(\vec{r}) = \sum_{n=0}^{N_{max}/2} A_n \varphi_n(\vec{r})$$

$$\left[b^2 p^2 + \frac{r^2}{b^2} + 2\mu C_0(N_{max}) b^2 \delta^{(3)}(\vec{r}) \right] \Psi(\vec{r}) = 2\frac{E}{\omega} \Psi(\vec{r})$$

$$\frac{1}{C_0(N_{max})} = -\sum_{n=0}^{N_{max}/2} \frac{|\varphi_n(0)|^2}{2n + 3/2 - \varepsilon}$$

Fix from Busch's formula

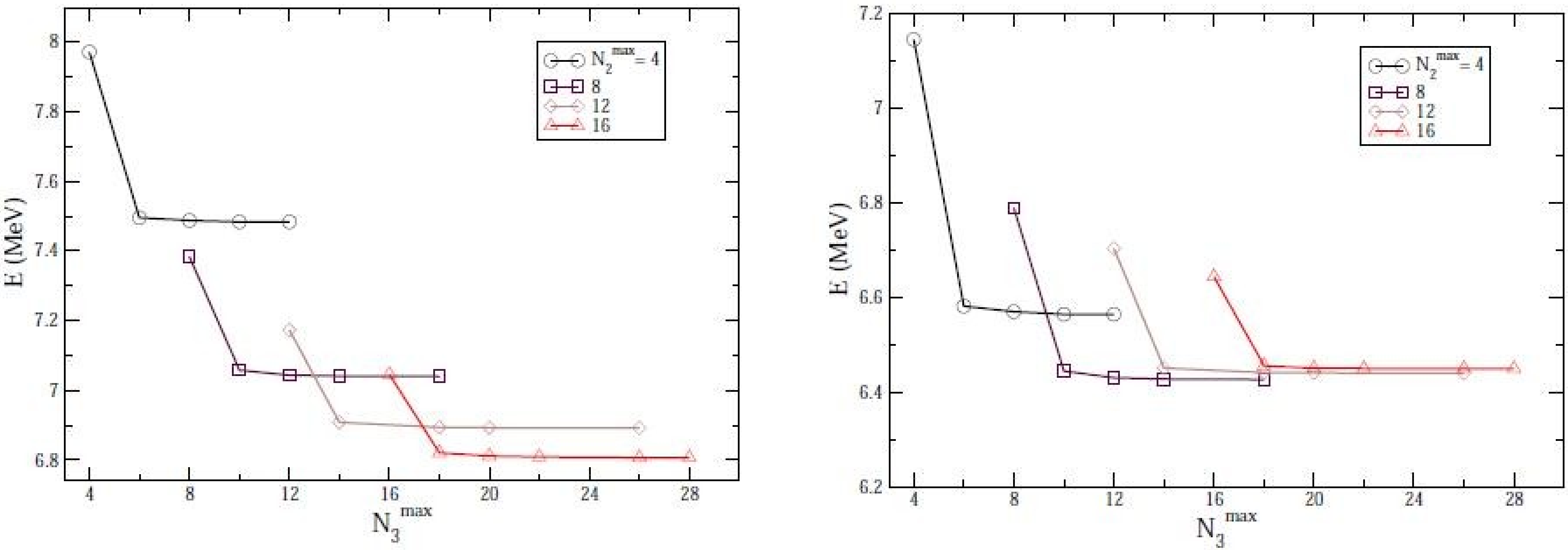


FIG. 6: Ground-state energy of the trapped three-nucleon system coupled to $T = 1/2$, $J^\pi = 3/2^+$ as function of the three-body model-space size N_3^{max}, for $\omega = 3$ MeV: LO (left panel) and NLO (right panel). Results are shown for different values of the two-body model-space size N_2^{max}.

SUMMARY

The NCSM is an *ab initio* method for calculating nuclear structure.

It has been applied to nuclei throughout the 0p-shell, where it has been able:

a.) to predict new results, e.g., the spectrum of 14-F,

b.) to explain previously non-understood observations, e.g., the lifetime of 14-C by including three nucleon forces,

c.) to describe the binding energies, low-lying spectra and other observables for 0p-shell nuclei,

d.) to serve as input for *ab initio* nuclear reaction calculations, *etc.*

But there are challenges and much else still to do.

SOME REMAINING CHALLENGES

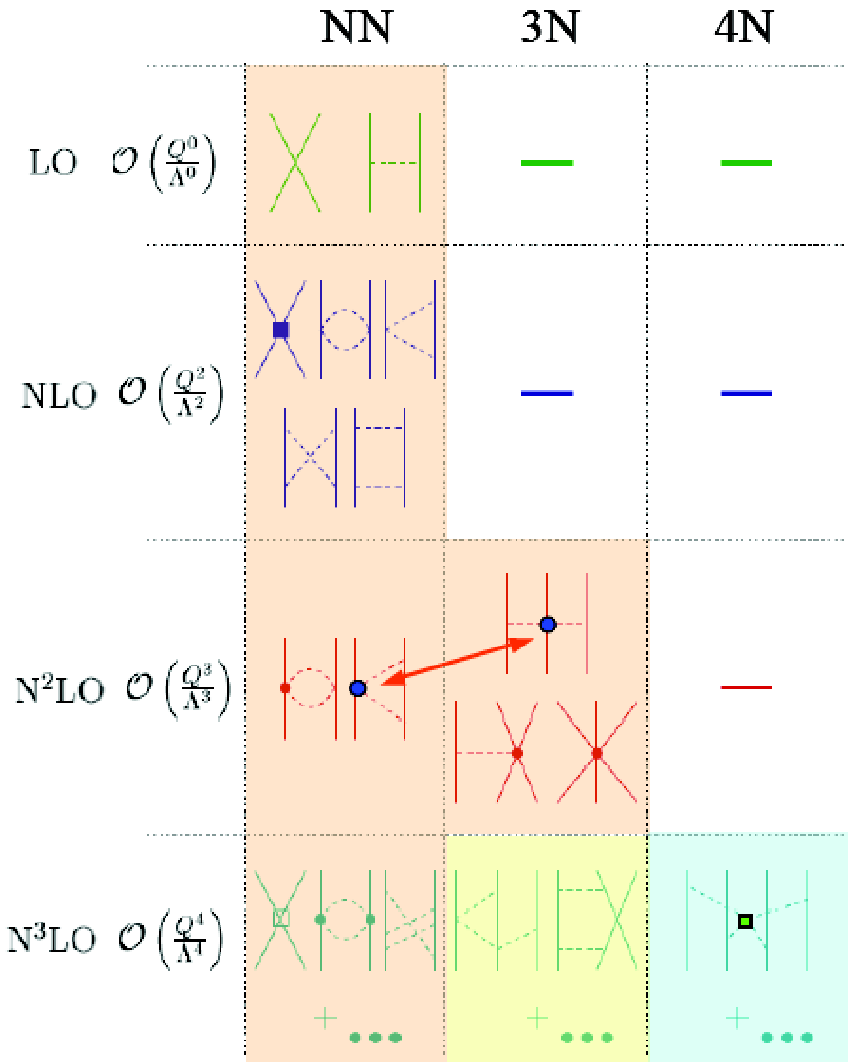
1. Understanding the fundamental interactions among the nucleons in terms of QCD, e.g., NN, NNN, ...
2. Determination of the mean field (the monopole effect).
3. Microscopic calculations of medium- to heavy-mass nuclei:
   a.) How to use the advances for light nuclei to develop techniques for heavier nuclei.
   b.) Building in more correlations among the nucleons in small model spaces, e.g., effective interactions for heavier nuclei.
4. Further extensions of these microscopic advances for nuclear structure to nuclear reactions.

COLLABORATORS

M. C. Birse, University of Manchester, UK
Sybil de Clark, University of Arizona
Michael Kruse, Lawrence Livermore National Laboratory
Alexander Lisetskiy, Mintec Inc., Tucson
Petr Navratil, TRIUMF, Vancouver, B.C. Canada
Erich Ormand, Lawrence Livermore National Laboratory
Sofia Quaglioni, Lawrence Livermore National Laboratory
Jimmy Rotureau, Chalmers U. of Tech., Goteborg, Sweden
Ionel Stetcu, Los Alamos National Laboratory
Ubirajara van Kolck, University of Arizona/Orsay, France
James P. Vary, Iowa State University

Chiral effective field theory (EFT) for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale Λ_b

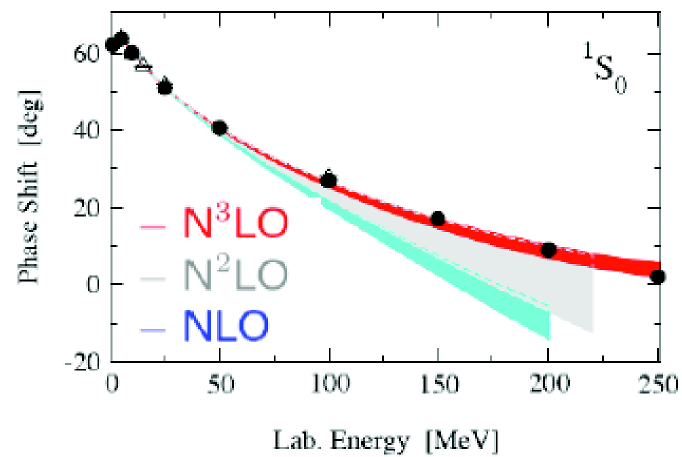


explains pheno hierarchy:
NN > 3N > 4N > …

NN-3N, πN, ππ, electro-weak,…
consistency

3N,4N: 2 new couplings to N³LO!

theoretical error estimates



A. Schwenk

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

I. Forces among nucleons

1. QCD --> EFT --> CPT --> self-consistent nucleon interactions

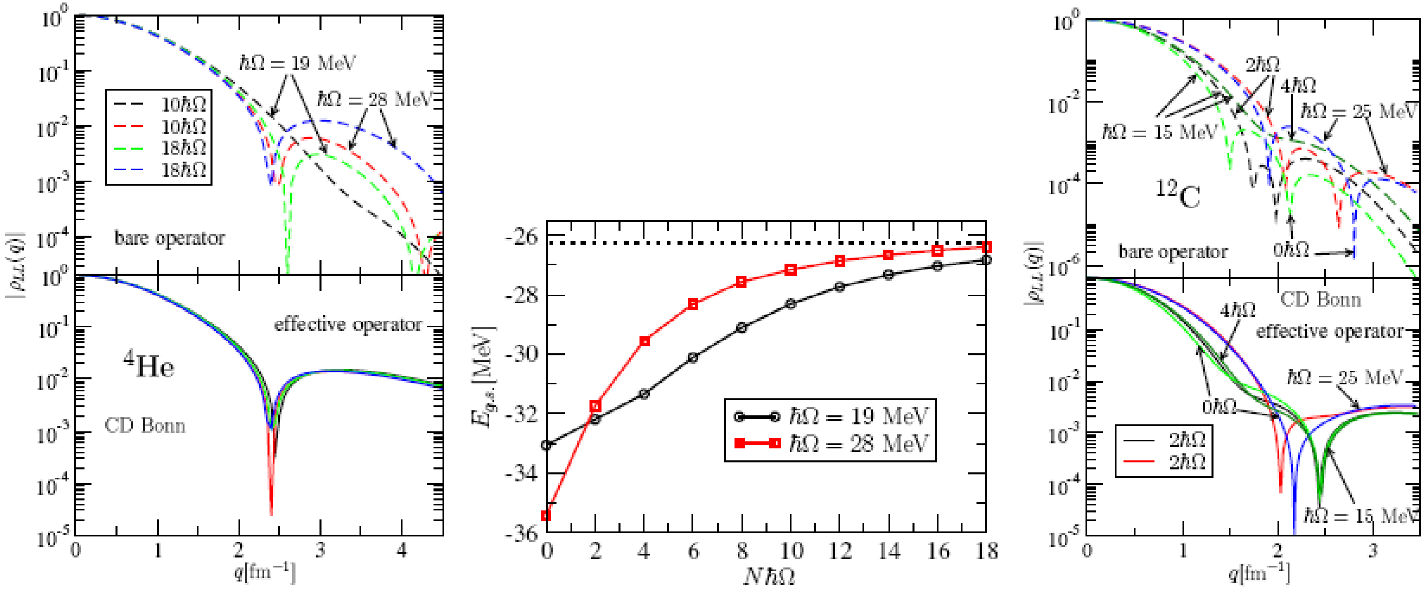
2. Need NN and NNN and perhaps also NNNN interactions

	N^3LO	Exp
^3H	7.85 MeV	8.48 MeV
^4He	25.35(5) MeV	28.30 MeV
^6Li	28.5(5) MeV	31.99 MeV

P. Navratil and E. Caurier, Phys. Rev. C 69, 014311 (2004)

Longitudinal-longitudinal distribution function

$$\rho_{LL}(q) = \frac{1}{4Z}\sum_{j\neq i}(1+\tau_z(i))(1+\tau_z(j))\langle g.s.|j_0(q|\vec{r_i}-\vec{r_j}|)|g.s.\rangle$$



Stetcu, Barrett, Navratil, Vary, nucl-th/0601076

Model space independence at high momentum transfer: good renormalization at the two-body cluster level

$$H_{int} = \frac{1}{A}\sum_{i>j=1}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i>j=1}^{A} V_{ij} + \sum_{i>j>k=1}^{A} V_{ijk} + \ldots$$

$$H = H_{int} + \frac{\vec{P}_{CM}^2}{2mA} + \frac{1}{2} m A \omega^2 \vec{R}_{CM}^2$$

$$= \sum_{i=1}^{A}\left(\frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 r_i^2\right) + \sum_{i<j=1}^{A}\left(V_{ij} - \frac{m\omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2\right) + \sum_{i<j<k=1}^{A} V_{ijk} + \ldots$$

$$h_{12} = \frac{p_1^2}{2m} + \frac{1}{2} m\omega r_1^2 + \frac{p_2^2}{2m} + \frac{1}{2} m\omega r_2^2 + V_{12} - \frac{m\omega^2}{2A}(\vec{r}_1 - \vec{r}_2)^2$$

$$h_{12} = h_{rel} + h_{CM}$$

NCSM: unitary transformation h_{rel}

Renormalization for trap $\Omega = \omega\sqrt{\frac{A-2}{A}}$

EFT FOR TWO PARTICLES IN A TRAP

Original motivation: to understand gross features of nuclear systems from a QCD perspective

At the heart of an effective theory: a truncation of the Hilbert space / all interactions allowed by symmetries are generated / power counting

$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)}=\frac{b}{2a_2}$$

$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)}=-\frac{b}{2}\left(-\frac{1}{a_2}+\frac{r_2}{b^2}\varepsilon+\ldots\right)$$

In finite model spaces:

$$V_{LO}(\vec{p},\vec{p}')=C_0$$

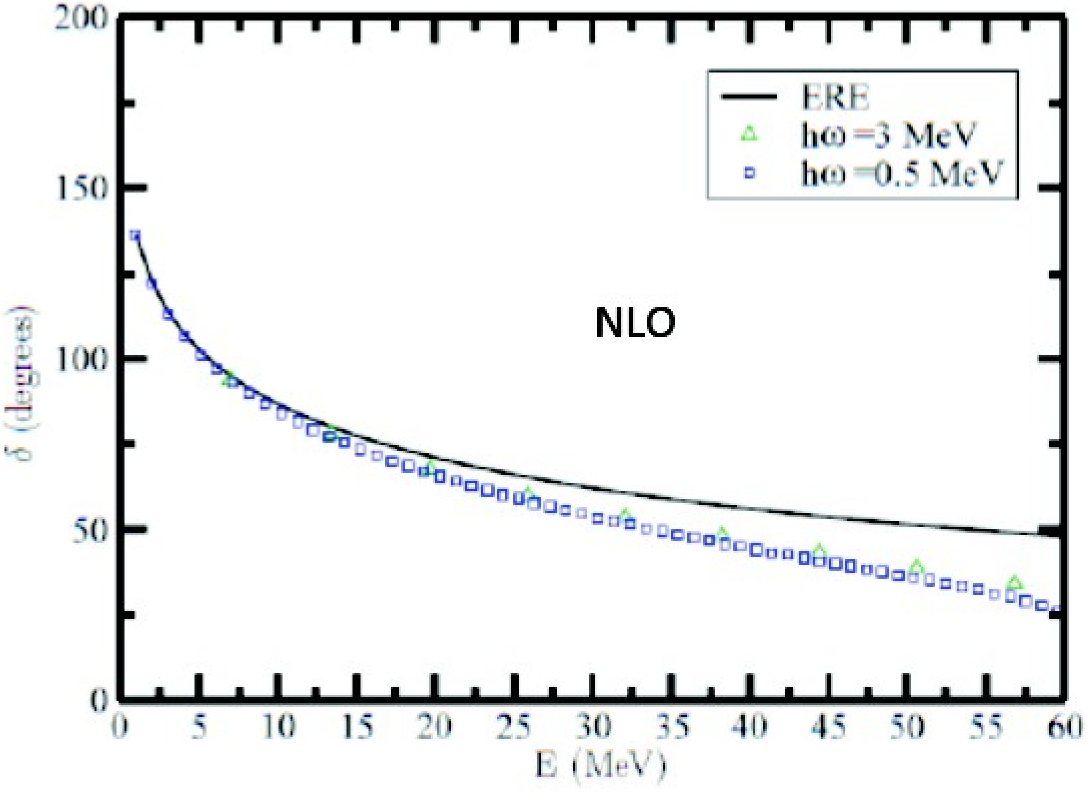
$$V_{NLO}(\vec{p},\vec{p}')=C_2(p^2+p'^2)$$

$$V_{N^2LO}(\vec{p},\vec{p}')=C_4(p^2+p'^2)^2$$

$C_0, C_2, C_4, \ldots$ Constants to be determined in each model space so that select observables are preserved

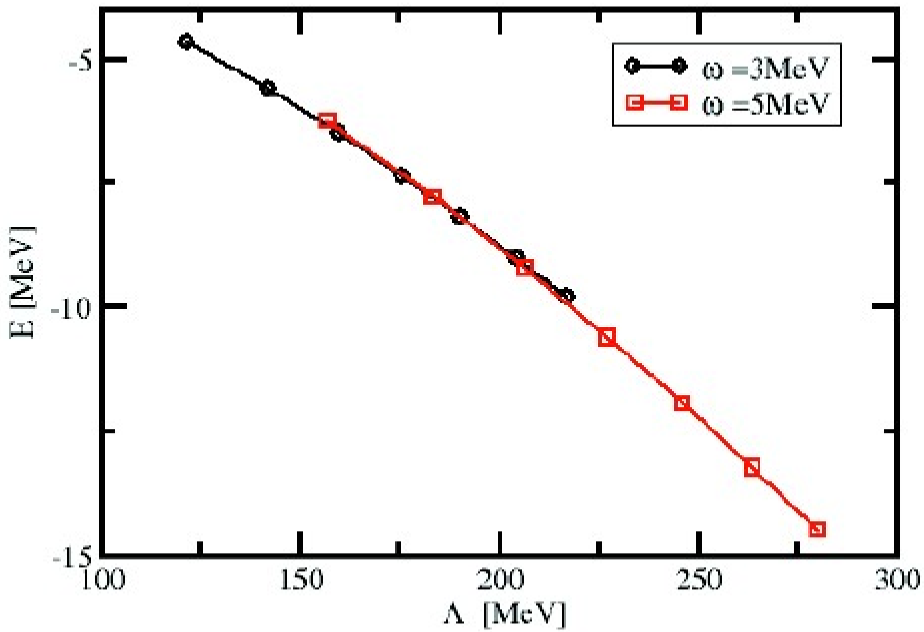
TRAPPED NUCLEONS

Triplet S NN phase shift



Rotureau et. al., in preparation

TRITON IN LO W/O A THREE-BODY FORCE



Rotureau et. al., in preparation

III. New Methods/Transformative Ideas (???)

1. “soft” NN interactions plus weak NNN interactions

2. Coupled Cluster calculations with NNN interactions

3. Universal Nuclear Energy Density Functional

4. Building more correlations into smaller model space:
   - a) Fermionic Molecular Dynamics Approach (T. Neff, et al.)
   - b) Extensions of the NCSM:
     - i) Projected NCSM/SSM
     - ii) Symplectic (3,R) NCSM (J. Draayer, et al.)
     - iii) Importance Truncated NCSM (Navratil and Roth)
     - iv) NCSM + Resonating Group Method (Navratil & Quaglioni)